NASA TECHNICAL NOTE

NASA TN D-8278 c.1

NASA TN D-8278



# RADIATIVE BEHAVIOR OF A GAS LAYER SEEDED WITH SOOT

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JULY 1976

1. Report No.
NASA TN D-8278

2. Government Accession No.

3. Recipient's Catalog No.

4. Title and Subtitle
RADIATIVE BEHAVIOR OF A GAS LAYER SEEDED WITH SOOT

5. Report Date
July 1976

6. Performing Organization Code

7. Author(s)
Robert Siegel

8. Performing Organization Report No.
E-8595

10. Work Unit No.
506-24

9. Performing Organization Name and Address
Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio 44135

11. Contract or Grant No.

13. Type of Report and Period Covered
Technical Note

12. Sponsoring Agency Name and Address
National Aeronautics and Space Administration
Washington, D. C. 20546

14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

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17. Key Words (Suggested by Author(s))
Radiation; Emittance; Transmittance; Particles; Seeded layer radiation; Particle suspension radiation; Transmittance of seeded layer; Emittance of particle suspension

18. Distribution Statement
Unclassified - unlimited
STAR Category 34

| 19. Security Classif. (of this report) | 20. Security Classif. (of this page) | 21. No. of Pages | 22. Price* |
|---|---|---|---|
| Unclassified | Unclassified | 35 | $4.00 |

* For sale by the National Technical Information Service, Springfield, Virginia 22161

# RADIATIVE BEHAVIOR OF A GAS LAYER SEEDED WITH SOOT

by Robert Siegel

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## SUMMARY

The heat transfer was examined for a gaseous layer seeded with radiation-absorbing carbon particles and flowing along a surface. The layer is subjected to an external high-temperature source of blackbody radiation. The particle absorption coefficient was taken to be inversely proportional to wavelength, which is approximately true for soot at wavelengths below about 5 micrometers. The radiative behavior was found to depend on a parameter containing particle concentration, layer thickness, and source temperature. Only a very small particle volume concentration, in the range of $10^{-5}$, was required to obtain high absorption in a 1-centimeter-thick layer for typical conditions. The results provide the distance along the surface for which the heat transfer to the wall remains within an acceptable limit and the particles remain below a temperature at which they will melt or vaporize. The wall protection by the layer lasts only until the particles vaporize or the layer becomes so hot that it reradiates substantial energy to the wall. Depending on the layer mass velocity the protection may be effective for a distance along the wall of only a few layer thicknesses. Hence, to protect greater wall lengths, it will be necessary to introduce the suspension through multiple slots or holes along the wall.

## INTRODUCTION

In devices involving high temperatures, significant amounts of energy can be transferred by both thermal radiation and convection, thereby causing high heat fluxes at the bounding walls. In the case of convection, gaseous film or transpiration cooling may be used to reduce the heat flux reaching the wall. Such a protective film, however, is usually not effective for reducing radiative heat transfer as most gases are transparent in the temperature range for which solid walls can exist. For example, hydrogen, which could be used for both propellant and coolant in a nuclear or laser rocket, is transparent at temperatures below about 5000 K. Common exceptions are carbon dioxide, water vapor, carbon monoxide, and sulfur dioxide, which absorb and radiate in distinct wavelength bands. However, in a thin layer, as would be typical for film cooling, even these gases would be quite transparent.

A technique that has been proposed for reducing the external radiative heat load reaching a surface is to flow a gas layer along the wall and seed the gas with small particles that absorb radiation. This provides a heat shield between the heat source and the wall. Possible seeding materials that have been suggested are ground carbon particles or soot, hafnium carbide, tungsten, and aluminum oxide (ref. 1). In the seeded layer a small amount of the radiant energy that is absorbed by the particles raises the particle temperature. Another portion of the absorbed energy is transferred from the particles to the gas by conduction and convection, and in the present analysis it is assumed that this process is effective enough to maintain the gas and the particles at essentially the same temperature. The remaining absorbed energy is transferred away from the particles by radiation. As the particle temperature increases, this reradiation becomes a significant portion of the absorbed energy. The wall will no longer be protected when significant absorbed energy is reradiated to the wall or when the particles become so hot that they begin to sublime or vaporize.

If there are too few particles in the gas, radiation will be readily transmitted through the layer and the layer will not be an effective heat shield. A sufficient number of particles leads to good attenuation of the incident flux, but will also provide good reradiation to the wall. A difficulty with effective radiative protection by a seeded layer is that the heat capacity per unit volume of the particulate suspension is generally small because of the low heat capacity of the gas. The particle volume concentration required to achieve significant opacity for the incident flux is usually very small, so that particles do not add significantly to the heat capacity of the suspension. This means that the gas temperature will rise quite rapidly in the flow direction unless the mass flow rates are high. Within a short distance the particles will either reach their vaporization temperature or there will be large reradiation to the wall.

In the present report a simplified analysis is made to determine the particle layer heating and wall heat flux dependency on factors such as particle concentration, suspension mass flow rate, and spectral distribution of the incident radiation, which, for the conditions considered herein, is determined by the temperature of the radiative source. The analysis assumes the use of carbon powders or soot; in this instance scattering by the particles can be neglected in comparison with absorption (refs. 2 and 3). The soot spectral absorption coefficient is obtained from the Mie equations in conjunction with measurements of the optical constants of soot.

The configuration being analyzed is shown in figure 1. A seeded gas layer of thickness $L$ is flowing with an average velocity $\bar{u}$ along a wall at temperature $T_w$. A radiative source at temperature $T_s$ supplies a uniform diffuse energy flux $q_s$ to the entire upper boundary of the layer. There is also convection from the layer to the wall. As a result a heat removal $q_w(x)$ is required at the wall to maintain the specified wall temperature. It is assumed that the heat transfer between the particles and gas is sufficiently good so that the temperature difference between the particles and the gas can be neglected. As a result of the convective and radiative processes, the suspension has a temperature variation $T_m(x)$ in the flow direction.

A nonflowing layer of suspended particles is considered first in order to develop the terms in the radiative heat balance and to obtain the magnitudes of the radiative parameters that can provide effective radiative absorption of incident energy. One of the terms obtained is the net radiative energy absorbed by the suspension. This term is then combined with the convective terms to yield the energy equation for the local mean temperature of the layer flowing along the wall. This equation is solved by numerical forward integration, and the resulting temperature variation is used in another energy balance to obtain the heat flow into the wall.

## ANALYSIS FOR NONFLOWING SUSPENSION

Figure 2 shows a stationary layer subjected to a radiative heat source at temperature $T_s$. Obtaining the relation between the wall temperature and the heat removal at the wall will give the heat removal required to keep the wall temperature at an acceptable limiting value. The results will be expressed in terms of a parameter containing the particle concentration in the suspension. The term $q_m$ is the energy per unit area of the wall that is being absorbed by the suspension and then removed by some means other than radiation to maintain the suspension at a specified $T_m$. This heat removal would ordinarily be principally by the convective transport of the suspension. In the absence of flow or any auxiliary means of internally cooling the suspension, the suspension will come to an equilibrium temperature sufficiently high so that all energy absorbed is radiated away. In this section, $q_m$ is determined in terms of a specified $T_m$. This relation is used later for the flowing layer to give the energy absorbed in the suspension in terms of its local temperature.

### Net Radiation Method

The interaction of the radiative terms can be obtained by using the net radiation method (ref. 4), which relates the incoming and outgoing fluxes at each layer boundary. Since the radiative properties of the particulate suspension depend on wavelength, a spectral analysis is required. The relations obtained will relate the wall heat flux and the energy absorbed in the suspension to the incident flux, the wall temperature, and the suspension temperature. A heat balance at the wall gives, for a small wavelength interval centered about wavelength $\lambda$ (where $q_\lambda$ is energy flux per unit wavelength interval),

$$q_{\lambda,w} = q_{\lambda i,w} - q_{\lambda o,w} \tag{1}$$

The outgoing radiation consists of emitted and reflected portions

$$q_{\lambda o,w} = \epsilon_{\lambda,w} e_{\lambda b,w} + \rho_{\lambda,w} q_{\lambda i,w}$$

$$= \epsilon_{\lambda,w} e_{\lambda b,w} + (1 - \epsilon_{\lambda,w}) q_{\lambda i,w} \tag{2}$$

where Kirchhoff's law has been used to substitute $1 - \epsilon_{\lambda,w}$ for $\rho_{\lambda,w}$. (All symbols are defined in appendix A.)

The radiative quantities at the two boundaries of the layer are related by the suspension spectral transmittance $\overline{\tau}_\lambda$. The suspension spectral absorptance is equal to $1 - \overline{\tau}_\lambda$, and from Kirchhoff's law the suspension spectral emittance is equal to the spectral absorptance. The heat flux leaving the seeded gas layer consists of transmitted and emitted portions

$$q_{\lambda o,l} = q_{\lambda o,w} \overline{\tau}_\lambda + e_{\lambda b,m}(1 - \overline{\tau}_\lambda) \tag{3}$$

Similarly, the incident energy at the wall is

$$q_{\lambda i,w} = q_{\lambda i,l} \overline{\tau}_\lambda + e_{\lambda b,m}(1 - \overline{\tau}_\lambda) \tag{4}$$

where $q_{\lambda i,l}$ is the incident spectral energy flux arriving at boundary $l$ (fig. 2) from the source external to the layer.

Another relation is a balance on the seeded layer between the incoming and outgoing spectral quantities

$$q_{\lambda,m} = q_{\lambda o,w} + q_{\lambda i,l} - q_{\lambda i,w} - q_{\lambda o,l} \tag{5}$$

where $q_{\lambda,m}$ is the net spectral energy absorbed by the suspension.

Since it is desired to obtain relations between the heat flow quantities and the suspension temperature, wall temperature, and incident energy from the external source, equations (1) to (5) are solved to obtain $q_{\lambda,w}$, $q_{\lambda,m}$, and $q_{\lambda o,l}$ in terms of $e_{\lambda b,m}$, $e_{\lambda b,w}$, and $q_{\lambda i,l}$. The algebraic manipulation is straightforward and yields the results

$$q_{\lambda,w} = \epsilon_{\lambda,w}(e_{\lambda b,m} - e_{\lambda b,w}) + \epsilon_{\lambda,w} \overline{\tau}_\lambda (q_{\lambda i,l} - e_{\lambda b,m}) \tag{6}$$

$$q_{\lambda,m} = q_{\lambda i,l} + \epsilon_{\lambda,w} e_{\lambda b,w} - e_{\lambda b,m}(1 + \epsilon_{\lambda,w}) - \bar{\tau}_\lambda \epsilon_{\lambda,w}(e_{\lambda b,w} + q_{\lambda i,l} - 2e_{\lambda b,m}) - \bar{\tau}_\lambda^2(1 - \epsilon_{\lambda,w})(q_{\lambda i,l} - e_{\lambda b,m}) \tag{7}$$

$$q_{\lambda o,l} = e_{\lambda b,m} + \bar{\tau}_\lambda \epsilon_{\lambda,w}(e_{\lambda b,w} - e_{\lambda b,m}) + (1 - \epsilon_{\lambda,w})\bar{\tau}_\lambda^2(q_{\lambda i,l} - e_{\lambda b,m}) \tag{8}$$

Equations (6) to (8) all involve spectral energy quantities; that is, they are quantities per unit of the small wavelength interval $d\lambda$ centered about a wavelength $\lambda$. To obtain energy fluxes, an integration must be made over all wavelengths. The integration of equation (6) will now be carried out; then the integration of equations (7) and (8) will follow in a similar manner. Integrating equation (6) over all $\lambda$ gives

$$\int_{\lambda=0}^{\infty} q_{\lambda,w}\, d\lambda = q_w = \int_{\lambda=0}^{\infty} \epsilon_{\lambda,w}(e_{\lambda b,m} - e_{\lambda b,w})d\lambda + \int_{\lambda=0}^{\infty} \epsilon_{\lambda,w}\bar{\tau}_\lambda(q_{\lambda i,l} - e_{\lambda b,m})d\lambda \tag{9}$$

To carry out the integrals, the spectral dependence of the wall emissivity $\epsilon_{\lambda,w}(\lambda)$ and the spectral dependence of the particulate suspension transmittance $\bar{\tau}_\lambda(\lambda)$ must be known. Also the nature of the heat source must be known so that $q_{\lambda i,l}(\lambda)$ can be specified. The terms $e_{\lambda b,w}(\lambda)$ and $e_{\lambda b,m}(\lambda)$ can be obtained from Planck's law by inserting either the wall or the suspension temperature. Some specific assumptions will be made here that have the advantage of leading to a closed-form analytical solution. This will reveal the general trends of how the heat flows are related to such quantities as wall emissivity and particle concentration in the gas.

The wall is assumed to be gray; that is, $\epsilon_{\lambda,w}$ does not depend on wavelength, and the source providing the incident energy on the layer is assumed to radiate like a blackbody at temperature $T_s$ so that $q_{\lambda i,l} = e_{\lambda b,s}$. Equation (9) then becomes

$$q_w = \epsilon_w \int_0^{\infty} (e_{\lambda b,m} - e_{\lambda b,w})d\lambda + \epsilon_w \int_0^{\infty} \bar{\tau}_\lambda(e_{\lambda b,s} - e_{\lambda b,m})d\lambda \tag{10}$$

Equation (10) is normalized with respect to the source emission to yield

$$\frac{q_w}{\int_0^{\infty} e_{\lambda b,s}\, d\lambda} = \epsilon_w\left[\frac{\int_0^{\infty} (e_{\lambda b,m} - e_{\lambda b,w})d\lambda}{\int_0^{\infty} e_{\lambda b,s}\, d\lambda} + \frac{\int_0^{\infty} \bar{\tau}_\lambda(e_{\lambda b,s} - e_{\lambda b,m})d\lambda}{\int_0^{\infty} e_{\lambda b,s}\, d\lambda}\right] \tag{11}$$

The integral of Planck's law over all wavelengths yields the total blackbody radiation $\int_0^\infty e_{\lambda b}\, d\lambda = \sigma T^4$ (the Stefan-Boltzmann law). This is applied to equation (11), except in the final term. In this term an approximation will be made so that an analytical expression can be obtained for the numerator. The same approximation will be used in both numerator and denominator so that good accuracy will be obtained for the ratio in this term. Equation (11) then becomes

$$\frac{q_w}{\epsilon_w \sigma T_s^4} = \left(\frac{T_m}{T_s}\right)^4 - \left(\frac{T_w}{T_s}\right)^4 + F \tag{12}$$

where

$$F = \frac{\int_0^\infty \bar{\tau}_\lambda (e_{\lambda b,s} - e_{\lambda b,m})\, d\lambda}{\int_0^\infty e_{\lambda b,s}\, d\lambda}$$

The term $\bar{\tau}_\lambda$ is an average transmittance that accounts for the fact that radiation is passing through the particulate suspension in all angular directions. The shortest path length through the layer is L, which corresponds to a path normal to the wall. The path length approaches infinity for directions at grazing incidence to the wall. In traveling through an absorbing layer for a distance S, spectral radiation attenuates by a factor $e^{-a_\lambda S}$ where $a_\lambda$ is the spectral absorption coefficient. Hence the transmittance along the path is $\tau_\lambda = e^{-a_\lambda S}$. When the integration is made for all possible path lengths through the layer of thickness L, the result yields the average transmittance for passage through a plane layer of diffuse radiation incident on one boundary of the layer (ref. 4, p. 563)

$$\bar{\tau}_\lambda = 2E_3(a_\lambda L) \tag{13}$$

For all values of $a_\lambda L$, the exponential integral function can be approximated quite well by the mean beam length approximation, giving

$$\bar{\tau}_\lambda = 2E_3(a_\lambda L) \approx e^{-a_\lambda C_L L} \tag{14}$$

where $C_L L$ is the mean beam length. For a plane layer the coefficient $C_L = 1.8$ is usually used (ref. 4, p. 570).

If we consider now the last term of equation (12), Planck's law is given by

$$e_{\lambda b} = \frac{2\pi C_1}{\lambda^5 \left(e^{C_2/\lambda T} - 1\right)} \tag{15}$$

As shown for example in reference 4 (pp. 22 and 23), in the wavelength regions containing practically all of the energy, equation (15) can be approximated quite well by Wien's formula

$$e_{\lambda b} \approx \frac{2\pi C_1}{\lambda^5 e^{C_2/\lambda T}} \tag{16}$$

Substituting equations (14) and (16) into the last term of equation (12) yields

$$F = \frac{\int_0^\infty \frac{1}{\lambda^5} \left\{ e^{-[a_\lambda C_L L + (C_2/\lambda T_s)]} - e^{-[a_\lambda C_L L + (C_2/\lambda T_m)]} \right\} d\lambda}{\int_0^\infty \frac{1}{\lambda^5} e^{-C_2/\lambda T_s} \, d\lambda} \tag{17}$$

## Soot Absorption Coefficient

Before equation (17) can be integrated, it is necessary to know how the absorption coefficient $a_\lambda$ varies with wavelength. This is difficult to specify in general because of the various types of particles that can be used, such as carbon, aluminum oxide, and tungsten, and the geometric forms of the particles, which can affect their optical behavior. Scattering of radiation may also be significant for some types of particles, such as reflecting metallic particles. This would require modifying the present analysis, which only accounts for absorption by the particles. One approach for obtaining $a_\lambda$ is to assume that the suspension is made of small absorbing spheres. (Soot particles have been found to be generally spherical, ref. 3.) Then the Mie equations (ref. 4) yield the absorption coefficient in terms of the volume concentration of particles and the optical constants as

$$\frac{a_\lambda}{f_v} = \frac{36\pi}{\lambda} \frac{n\kappa}{(n^2 - \kappa^2 + 2)^2 + 4n^2\kappa^2} \tag{18}$$

The optical constants are the ordinary index of refraction $n$ and the extinction coefficient $\kappa$, which is a measure of radiative attenuation within the particle material.

The calculations are carried out for soot particles as there is a reasonable amount of information available for soot carbon. For soot suspensions, scattering is very small compared with absorption (refs. 2 and 3). Also soot absorption behavior is insensitive to the temperature of the soot particles (ref. 3) and, hence, room-temperature information can be applied at elevated temperatures. In table I some measured optical constants are given for two types of soot carbon (ref. 2). To make specimens for these optical measurements, the particles were compressed on a flat plate. Table I also gives the $a_\lambda/f_v$ computed from equation (18), and these $a_\lambda/f_v$ values are plotted in figure 3 as a function of wavelength.

The peak in the blackbody emission curve occurs at a wavelength (call it $\lambda_{max}$) given by $\lambda_{max} = C_3/T$, where $C_3 = 2898$ micrometer K. Thus, at 1000 K the peak of the blackbody curve is at 2.898 micrometers and at 10 000 K it shifts to 0.2898 micrometer. Hence, for a high-temperature system the radiant energy is at the shorter end of the wavelength range shown in figure 3. Shown on the figure are two lines with a slope of -1; and it is evident that, in the shorter wavelength region of interest, the lines approximate the data reasonably well. Thus, the approximate expression

$$\frac{a_\lambda}{f_v} = \frac{k}{\lambda} \tag{19}$$

can be used. This is also well substantiated by the data in reference 5 and will be shown later to yield good values for the total emittance of a soot-seeded layer. The coefficient $k$ is found as the value of $a_\lambda/f_v$ when $\lambda = 1$ micrometer; and figure 3 yields $k_{propane} \approx 4.9$ and $k_{acetylene} \approx 4.0$. For the soot from coal flames, $k$ has been found to range from 3.7 to 7.5; for the soot from oil flames, a value of 6.3 was found (ref. 6). From equation (19) the $a_\lambda$ depends on the product $f_v k$; the exact value of $k$ is thus significant only with regard to specifying the required particle concentration. The form of the results obtained in the analysis does depend on the $a_\lambda$ dependence on $\lambda$ as given by equation (19). For some types of particles the dependence in equation (19) is only a rough approximation, as indicated by the correlations discussed in reference 4 (p. 715).

## Integration to Obtain Final Relations

Substituting equation (19) into (17) yields

$$F = \frac{\displaystyle\int_0^\infty \frac{1}{\lambda^5}\left\{e^{-[(f_v k C_L L/\lambda)+(C_2/\lambda T_s)]} - e^{-[(f_v k C_L L/\lambda)+(C_2/\lambda T_m)]}\right\} d\lambda}{\displaystyle\int_0^\infty \frac{1}{\lambda^5} e^{-C_2/\lambda T_s}\, d\lambda} \tag{20}$$

As shown in appendix B, this can be simplified to the form

$$F = \frac{1}{\left(\frac{f_v C_L kL}{C_2} T_s + 1\right)^4} - \left(\frac{T_m}{T_s}\right)^4 \frac{1}{\left(\frac{f_v C_L kL}{C_2} T_m + 1\right)^4} \tag{21}$$

Substituting this expression into equation (12) yields the heat flux transferred into the wall:

$$\frac{q_w}{\epsilon_w \sigma T_s^4} = \left(\frac{T_m}{T_s}\right)^4 - \left(\frac{T_w}{T_s}\right)^4 + \frac{1}{\left(\frac{f_v C_L kL}{C_2} T_s + 1\right)^4} - \left(\frac{T_m}{T_s}\right)^4 \frac{1}{\left(\frac{f_v C_L kL}{C_2} T_m + 1\right)^4} \tag{22}$$

Returning now to equation (7) and making the assumptions of a gray wall and blackbody external source give, after integrating over all wavelengths,

$$q_m = \int_0^\infty e_{\lambda b, s}\, d\lambda + \epsilon_w \int_0^\infty e_{\lambda b, w}\, d\lambda - (1 + \epsilon_w) \int_0^\infty e_{\lambda b, m}\, d\lambda$$

$$- \epsilon_w \int_0^\infty \bar{\tau}_\lambda (e_{\lambda b, w} + e_{\lambda b, s} - 2e_{\lambda b, m}) d\lambda - (1 - \epsilon_w) \int_0^\infty \bar{\tau}^2 (e_{\lambda b, s} - e_{\lambda b, m}) d\lambda$$

Nondimensionalize by dividing through by $\int_0^\infty e_{\lambda b, s}\, d\lambda$ to yield

$$\frac{q_m}{\sigma T_s^4} = 1 + \epsilon_w\left(\frac{T_w}{T_s}\right)^4 - (1 + \epsilon_w)\left(\frac{T_m}{T_s}\right)^4$$

$$- \epsilon_w \frac{\int_0^\infty \overline{\tau}_\lambda(e_{\lambda b, w} + e_{\lambda b, s} - 2e_{\lambda b, m})d\lambda}{\int_0^\infty e_{\lambda b, s}\, d\lambda} - (1 - \epsilon_w) \frac{\int_0^\infty \overline{\tau}_\lambda^2(e_{\lambda b, s} - e_{\lambda b, m})d\lambda}{\int_0^\infty e_{\lambda b, s}\, d\lambda}$$

Insert Wien's law for the $e_{\lambda b}$ and equation (14) for the $\overline{\tau}_\lambda$ with the $a_\lambda$ from equation (19) to obtain

$$\frac{q_m}{\sigma T_s^4} = 1 + \epsilon_w\left(\frac{T_w}{T_s}\right)^4 - (1 + \epsilon_w)\left(\frac{T_m}{T_s}\right)^4$$

$$- \epsilon_w \frac{\int_0^\infty \frac{1}{\lambda^5}\left\{e^{-(1/\lambda)[f_v k C_L L + (C_2/T_w)]} + e^{-(1/\lambda)[f_v k C_L L + (C_2/T_s)]} - 2e^{-(1/\lambda)[f_v k C_L L + (C_2/T_m)]}\right\} d\lambda}{\int_0^\infty \frac{1}{\lambda^5} e^{-C_2/\lambda T_s}\, d\lambda}$$

$$- (1 - \epsilon_w) \frac{\int_0^\infty \frac{1}{\lambda^5}\left\{e^{-(1/\lambda)[2f_v k C_L L + (C_2/T_s)]} - e^{-(1/\lambda)[2f_v k C_L L + (C_2/T_m)]}\right\} d\lambda}{\int_0^\infty \frac{1}{\lambda^5} e^{-C_2/\lambda T_s}\, d\lambda}$$

Applying the reduction in appendix B then provides the final expression

$$\frac{q_m}{\sigma T_s^4} = 1 + \epsilon_w\left(\frac{T_w}{T_s}\right)^4 - (1 + \epsilon_w)\left(\frac{T_m}{T_s}\right)^4$$

$$- \epsilon_w\left[\left(\frac{T_w}{T_s}\right)^4 \frac{1}{\left(\frac{f_v k C_L L T_w}{C_2} + 1\right)^4} + \frac{1}{\left(\frac{f_v k C_L L T_s}{C_2} + 1\right)^4} - \left(\frac{T_m}{T_s}\right)^4 \frac{2}{\left(\frac{f_v k C_L L T_m}{C_2} + 1\right)^4}\right]$$

$$- (1 - \epsilon_w)\left[\frac{1}{\left(2\frac{f_v k C_L L T_s}{C_2} + 1\right)^4} - \left(\frac{T_m}{T_s}\right)^4 \frac{1}{\left(2\frac{f_v k C_L L T_m}{C_2} + 1\right)^4}\right] \qquad (23)$$

In a similar fashion, equation (8) for the energy coming out of the particulate layer becomes

$$\frac{q_{o,l}}{\sigma T_s^4} = \left(\frac{T_m}{T_s}\right)^4 + \epsilon_w\left[\left(\frac{T_w}{T_s}\right)^4 \frac{1}{\left(\frac{f_v C_L k L T_w}{C_2} + 1\right)^4} - \left(\frac{T_m}{T_s}\right)^4 \frac{1}{\left(\frac{f_v C_L k L T_m}{C_2} + 1\right)^4}\right]$$

$$+ (1 - \epsilon_w)\left[\frac{1}{\left(2\frac{f_v C_L k L T_s}{C_2} + 1\right)^4} - \left(\frac{T_m}{T_s}\right)^4 \frac{1}{\left(2\frac{f_v C_L k L T_m}{C_2} + 1\right)^4}\right] \qquad (24)$$

## Soot Total Emittance

To further examine whether the assumptions used in the absorption coefficient are reasonable, a comparison can be made with the results for the emittance of soot suspensions in reference 2. To do this, start with the $q_w$ as obtained from equation (22),

$$q_w = \epsilon_w \sigma \left[ T_m^4 - T_w^4 + \frac{T_s^4}{\left(\frac{f_v C_L k L}{C_2} T_s + 1\right)^4} - \frac{T_m^4}{\left(\frac{f_v C_L k L}{C_2} T_m + 1\right)^4} \right] \tag{25}$$

For a black wall $\epsilon_w = 1$; and for low source and wall temperatures, this reduces to

$$q_w = \left[ 1 - \frac{1}{\left(\frac{f_v C_L k L}{C_2} T_m + 1\right)^4} \right] \sigma T_m^4 \tag{26}$$

Equation (26) represents a condition where the only radiation is from the suspension. Since the flux to the black wall can be written in terms of a suspension emittance as $q_w = \epsilon_m \sigma T_m^4$, it is evident that

$$\epsilon_m = 1 - \frac{1}{\left(\frac{f_v C_L k L}{C_2} T_m + 1\right)^4} \tag{27}$$

where the $C_L L$ is the mean path length (mean beam length) through the gas. Figure 6 of reference 2 gives the gas emittance for propane soot as calculated numerically without using the foregoing assumptions of Wien's law and a dependence of the absorption coefficient on $1/\lambda$. The results for $\epsilon_m$ are given as a function of $T_m$ and of the particle volume fraction multiplied by the path length (i.e., $f_v C_L L$ in the present notation). With k taken from figure 3 as 4.9 for propane soot and $C_2$ as 1.4388 centimeter K (the second radiation constant), equation (27) also gives $\epsilon_m$ in terms of the suspension temperature and the product of concentration and path length. Some comparisons are given in table II. The approximate emittances are in good agreement; hence, the approximations made appear to be satisfactory to provide reasonable results for the heat flow quantities in the present situation.

As can be shown from equation (27) the particulate suspension is somewhat wavelength selective in its behavior, meaning that its emittance for radiation at its equilibrium temperature is different from its absorptance for radiation from the incident source at a higher temperature. This arises from the fact that the absorption coefficient $a_\lambda$ varies appreciably with wavelength and the source radiation is predominatly in a shorter wavelength region than the layer reradiation. The parameter M is defined as $M = f_v C_L k L T_s / C_2$ and hence the emittance in equation (27) is $\epsilon_m = 1 - 1/[M(T_m/T_s) + 1]^4$. If the suspension were at temperature $T_s$, the emitted energy spectrum would then be the same as the incident spectrum. The emissivity would then become equal to the absorptance as both of these quantities would have the same weighting of their spectral properties over the incident or emitted spectrum. Thus when $T_m = T_s$ the absorptance of the layer becomes equal to its emittance and hence $\alpha_m = 1 - 1/(M + 1)^4$. Figure 4 shows how the suspension emittance increases as its temperature rises. The $\epsilon_m$ reaches the value of $\alpha_m$ when $T_m/T_s = 1$. Thus initially when the suspension is introduced at low temperature, it will absorb well but will reemit poorly toward the wall. This selective behavior will aid in protecting the wall from the incident flux.

## Behavior When Wall and Seeded Gas Temperatures are Much Less Than Source Temperature

To obtain some insight into the heat transfer behavior of the particle-seeded layer, a simplified case will be considered. If the source temperature is high, the material limitations of the wall and the particles will result in the ratios $(T_m/T_s)^4$ and $(T_w/T_s)^4$ being much less than unity. In this instance, equations (22) to (24) reduce to

$$\frac{q_w}{\sigma T_s^4} = \frac{\epsilon_w}{\left(\dfrac{f_v C_L k L}{C_2} T_s + 1\right)^4} \tag{28}$$

$$\frac{q_m}{\sigma T_s^4} = 1 - \frac{\epsilon_w}{\left(\dfrac{f_v C_L k L}{C_2} T_s + 1\right)^4} - \frac{1 - \epsilon_w}{\left(2\dfrac{f_v C_L k L}{C_2} T_s + 1\right)^4} \tag{29}$$

$$\frac{q_{o,l}}{\sigma T_s^4} = \frac{1 - \epsilon_w}{\left(2 \frac{f_v C_L kL}{C_2} T_s + 1\right)^4} \tag{30}$$

For a black wall ($\epsilon_w = 1$), there is not any means for energy to be directed back toward the source. The black wall is nonreflecting, and scattering by the particulate suspension has been neglected; therefore, $q_{o,l} = 0$. The dimensionless heat flows are a function of two parameters, $\epsilon_w$ and $(f_v C_L kL/C_2)T_s$; for convenience the latter parameter has been called M. Equations (28) to (30) are plotted in figures 5(a) to (c), and from energy conservation the results in these three figures total unity at each M and $\epsilon_w$.

## RESULTS AND DISCUSSION FOR NONFLOWING SUSPENSION

Since the particulate suspension and wall are both at low temperature, figure 5 represents the effect of radiation from the source only. Figure 5(a), which is a plot of equation (28), represents the fraction of energy transmitted through the particulate layer and absorbed in the wall. The absorption by the wall depends directly on the wall absorptivity, which is equal to the wall emissivity for the case of a gray wall. The abscissa has in it the thickness of the layer, the particle volumetric concentration, and the coefficient k that is in the absorption coefficient. The figure shows how the energy penetrating through the layer decreases as any of these quantities is increased. Increasing the source temperature also decreases the fraction of energy that penetrates through the particulate layer. This results from the absorption coefficient increasing as the wavelength of the radiation decreases, as indicated by equation (19). According to Wien's displacement law the radiation from the blackbody source shifts to shorter wavelengths as the source temperature increases; thus with increasing $T_s$ the radiation shifts into a region of increased absorption by the particulate layer.

The energy coming back out of the layer (eq. (30) and fig. 5(c)) is that which has reached the wall, been reflected, and then transmitted back through the layer. This radiation has undergone two passes through the layer, which accounts for the factor of 2 multiplying the quantity containing the layer thickness in equation (30). Figure 5(b) shows how the absorption by the suspension increases with the parameter M. The absorption also increases as $\epsilon_w$ decreases as this corresponds to increased reflection from the wall and thus an increase in the amount of energy that is undergoing two passes through the particulate suspension.

In figure 5(a) for low M (e.g., $M < 0.01$) the suspension is becoming essentially transparent. Hence the curves giving the fraction of incident energy absorbed $q_w/\sigma T_s^4$

each approach their value of $\epsilon_w$ as M becomes very small. (Note that $\epsilon_w$ is equal to the wall absorptivity because the wall is gray.) When $M \approx 0.2$, the energy absorbed by the wall is reduced to about $\epsilon_w/2$; when $M \approx 0.8$, the energy absorbed is reduced to about $\epsilon_w/10$.

## ANALYSIS FOR FLOWING SUSPENSION

The behavior of a particulate suspension flowing along a wall will now be considered (fig. 6). Convective energy transport is assumed to dominate along the flow direction so that radiative transport can be neglected in that direction. The previous radiative analysis can then be applied locally along the layer. Actually, as found later, the temperature can change quite rapidly in the x-direction because of the low heat capacity of the layer, so that there can be significant radiation in the flow direction. Nevertheless the solution given here should indicate the length along the wall over which the particulate layer is effective in shielding the wall from the external radiation. The convective flux $q_{w,c}$ from the layer to the wall is expressed in terms of a convective heat transfer coefficient h(x), which is assumed to be available from known correlations. If, for any particular numerical case, the suspension temperature goes above the temperature at which the particles can exist, the results will no longer be valid as the analysis assumes that the particles are present. The results can then be regarded as providing the location at which the suspension will cease to protect the wall.

From equation (5), $q_m$ is the net amount of radiation that is absorbed in the suspension, and an expression for $q_m$ is given by equation (23). The heat balance using the terms in figure 6 gives the net radiation absorbed as being equal to the convective terms

$$q_m = \frac{d}{dx}\left\{L\bar{u}\left[(1 - f_v)\rho_g c_{p,g} + f_v \rho_p c_{p,p}\right]T_m\right\} + h(x)(T_m - T_w) \tag{31}$$

Although the mass flow of the constituents remains constant, there are thermal expansion effects as the layer becomes heated. These effects are extremely complicated because the upper boundary of the layer is a free boundary. No attempt was made to account for these expansion effects.

Substituting equation (23) into equation (31) and letting all quantities be constant except for $T_m$ yield the equation

$$
\sigma T_s^4 \left\{ 1 + \epsilon_w \left(\frac{T_w}{T_s}\right)^4 - (1 + \epsilon_w)\left(\frac{T_m}{T_s}\right)^4 \right.
$$

$$
- \epsilon_w \left[ \left(\frac{T_w}{T_s}\right)^4 \frac{1}{\left(M \frac{T_w}{T_s} + 1\right)^4} + \frac{1}{(M + 1)^4} - 2\left(\frac{T_m}{T_s}\right)^4 \frac{1}{\left(M \frac{T_m}{T_s} + 1\right)^4} \right]
$$

$$
\left. - (1 - \epsilon_w) \left[ \frac{1}{(2M + 1)^4} - \left(\frac{T_m}{T_s}\right)^4 \frac{1}{\left(2M \frac{T_m}{T_s} + 1\right)^4} \right] \right\} = L\bar{u}\, \overline{\rho c}_p \frac{dT_m}{dx} + h(T_m - T_w) \qquad (32)
$$

After $T_m(x)$ is obtained from this relation, the heat flow to the wall $q_w$ is obtained by using the radiative portion given by equation (25) in combination with the convection to yield

$$
q_w = \epsilon_w \sigma \left[ T_m^4 - T_w^4 + \frac{T_s^4}{(M + 1)^4} - \frac{T_m^4}{\left(M \frac{T_m}{T_s} + 1\right)^4} \right] + h(T_m - T_w) \qquad (33)
$$

Equations (32) and (33) are placed in dimensionless form to yield

$$\frac{dt_m}{dX} = 1 + \epsilon_w t_w^4 - (1 + \epsilon_w)t_m^4 - \epsilon_w\left[\frac{1}{(M + 1)^4} + \frac{t_w^4}{(Mt_w + 1)^4} - \frac{2t_m^4}{(Mt_m + 1)^4}\right]$$

$$- (1 - \epsilon_w)\left[\frac{1}{(2M + 1)^4} - \frac{t_m^4}{(2Mt_m + 1)^4}\right] - H(t_m - t_w) \qquad (34)$$

$$\frac{q_w}{\sigma T_s^4} = \epsilon_w\left[t_m^4 - t_w^4 + \frac{1}{(M + 1)^4} - \frac{t_m^4}{(Mt_m + 1)^4}\right] + H(t_m - t_w) \qquad (35)$$

Equation (34) is a nonlinear first-order differential equation and can be readily solved by numerical forward integration by the Runge-Kutta method starting from an initial value of $t_m = t_{m,o}$. The $t_m(x)$ is then substituted into equation (35) to yield $Q_{w,t}(x)$; this is the energy that must be removed from the wall in order to keep the wall cooled to the specified value of $t_w$.

## RESULTS AND DISCUSSION FOR FLOWING SUSPENSION

### Ranges of Parameters

Before carrying out some typical examples it is necessary to examine what the typical ranges are of the parameters $M$, $\epsilon$, $t_{m,o}$, $t_w$, and $H$. From figure 5(a) it is found that an almost transparent layer would correspond to $M \approx 0.02$, one that is about half transparent to $M \approx 0.2$, and one that is almost opaque to $M \approx 1.0$. Typical values of $\epsilon_w$ might be 0.1 to 1.0 although it may be difficult to obtain the low values for the materials required in a high-temperature system. The $t_{m,o}$ is the ratio of the particulate suspension entrance temperature to the radiative source temperature. If the source were at 5000 K, a $t_{m,o}$ of 0.1 would mean the gas enters at 500 K. For a high source temperature, the $t_{m,o}$ would typically be lower, say 0.05 to 0.01. For a wall temperature of 1000 K and source temperatures of 5000 K or above, the $t_w$ might typically be 0.2 to 0.05. The convective heat transfer coefficient from the flowing gas-particle suspension would generally be low because the heat transfer fluid is primarily a gas. The h might typically be 5 to 25 $W/(m^2)(K)$ for pressures near atmospheric.

For a $T_s$ = 10 000 K this yields H of about $10^{-4}$ to $5\times10^{-4}$. The H values can be appreciably higher in high-pressure systems. Some typical solutions were evaluated for these ranges by numerical integration and are discussed in the next section.

## Typical Results for Flowing Suspension

Some of the most important quantities that can be regulated, such as particle concentration and layer thickness, are contained in the parameter $M = (f_v C_L kL/C_2)T_s$. Figure 7 shows the effect of various M values on the suspension temperature and heat flow to the surface as functions of dimensionless length along the surface. Part (a) is for a black wall, and part (b) is for a gray wall with $\epsilon_w = 0.5$. Recall from figure 5(a) that for M = 0.02 the layer is quite transparent, for M = 0.2 the layer is about one-half transmitting, and for M = 1 and M = 2 the layer is quite strongly absorbing. For M = 0.02 the suspension heats slowly in the X-direction because of its low absorption. One set of results for this M value is shown in figure 7(b), where a log scale is used for the abscissa because of the relatively large X involved. For this low M the heat transfer to the wall is essentially all by absorption of radiation directly transmitted through the quite transparent layer. The suspension radiates relatively little even when it is hot; hence, the heat flux to the wall rises only slightly as the gas temperature becomes high in the region of large X.

For a high M, such as 1 or 2, the incident flux is strongly absorbed by the suspension, so the heat load directly transmitted to the wall is very small. At small X there is also little radiation by the suspension because it is still cool, and thus little radiant energy reaches the wall in this region. As a result of the strong absorption, however, the suspension heats up quickly in the flow direction and then radiates to the wall, which rapidly diminishes the wall's protection from excessive heating. If the suspension temperature becomes too high, the protection is lost by vaporization of the particles. Thus, it is possible to shield the wall only to a limited X value by use of a high-M layer. As shown by figure 7 the suspension temperature variation with X is not significantly altered by increasing M beyond about 1.0 since for this large an M essentially all the energy is already being absorbed by the particles. Increasing M to 2 causes the particulate layer to be almost opaque, as evidenced by the energy transmission to the wall being very small at small X. At larger X the energy is reaching the wall by suspension reradiation.

It is worthwhile examining the physical significance of the dimensionless coordinate

$$X = \frac{x}{L}\,\frac{\sigma T_s^3}{\bar{u}\left[(1 - f_v)\rho_g c_{p,g} + f_v \rho_p c_{p,p}\right]} = \frac{x\sigma T_s^4}{L\bar{u}\,\overline{\rho c_p}\,T_s}$$

The numerator is the radiation incident on the layer from the inlet to x. The denominator is the maximum energy that the layer can absorb; that is, it is the energy required to heat the layer from zero temperature to $T_s$. Thus, if all the energy incident on the layer were absorbed and retained in the layer, a layer starting at zero temperature would reach the source temperature at $X = 1$. This is shown by the long- and short-dashed curve in figure 7(a). Thus, X can be approximately regarded as the number of such ideal heating lengths required in the actual heat transfer process.

To better understand the physical significance of the values of the dimensionless quantities, it is well to consider a numerical example. When $M = 1$, the layer is a good absorber. To obtain the magnitude of a typical particle concentration, let $T_s = 5000$ K and the layer thickness be $L = 1$ centimeter. Then from the definition of M, the particle volume concentration is

$$f_v = \frac{MC_2}{C_L kLT_s} = \frac{1\times 1.44\times 10^{-2}\ \mathrm{m\ K}}{1.8\times 4.9\times 10^{-2}\ \mathrm{m} \times 5000\ \mathrm{K}} = 3.27\times 10^{-5}$$

Thus a very small particle volume concentration is required to obtain good absorption of radiation.

To compute the thermal behavior of the flowing layer, the $\overline{\rho c}_p$ is needed. Assume that the suspension is supplied at room temperature and atmospheric pressure and is composed of carbon particles in nitrogen. Then

$$\overline{\rho c}_p = (1 - f_v)\rho_g c_{p,g} + f_v \rho_p c_{p,p} = (1.14\ \mathrm{kg/m^3})(1.04\times 10^3\ \mathrm{W\ sec/kg\ K})$$

$$+ (3.27\times 10^{-5})(1300\ \mathrm{kg/m^3})(1.26\times 10^3\ \mathrm{W\ sec/kg\ K})$$

$$\overline{\rho c}_p = 1.186\times 10^3 + 0.054\times 10^3 = 1.240\times 10^3\ \mathrm{W\ sec/m^3\ K}$$

The particles contribute only about 5 percent to the value of $\overline{\rho c}_p$. If the carrier gas were hydrogen, the result would be about the same. (Hydrogen has about 1/14 the density but 14 times the specific heat of nitrogen, so the product $\rho_g c_{p,g}$ is about the same.)

From graphs such as figure 7, most of the layer heating is seen to occur when X is near unity. If the layer velocity is taken to be introduced at a velocity of 10 meters per second, the length coordinate x for which the layer has become quite heated and thus loses its ability to protect the wall is

$$x = \frac{XL\bar{u}\,\overline{\rho c}_p}{\sigma T_s^3} = \frac{(1)(10^{-2}\ \mathrm{m})(10\ \mathrm{m/sec})(1.24\times 10^3\ \mathrm{W\ sec/m^3\ K})}{(5.73\times 10^{-8}\ \mathrm{W/m^2\ K^4})(5000^3\ \mathrm{K^3})} = 0.0173\ \mathrm{m}$$

$$= 1.73\ \mathrm{cm}$$

By going to higher pressures the mass flow can be increased, thereby increasing the effective length of the film protection. This will also increase the convective heat transfer to the wall. It appears that for many practical conditions it will be required to inject the layer along the wall at intervals of the order of the thickness of the seeded layer. If the injection is through holes, the wall would become somewhat like a perforated plate.

In figure 8 a layer of intermediate absorbing ability (M = 0.2) is used to illustrate the effect of various wall emissivities. The dimensionless suspension temperature and wall heat flux are given as functions of X for three different $\epsilon_w$. When the suspension and wall are both cool, figure 5(a) shows that for M = 0.2 about one-half of the energy from the source is transmitted to the wall. For $\epsilon_w = 1.0$ all of this energy is absorbed at the wall; hence for a small X in figure 8 the curve of $q_w/\sigma T_s^4$ for $\epsilon_w = 1$ starts at about 0.5, and the curves for smaller $\epsilon_w$ are proportionately lower as a result of reflection from the wall. As the particle suspension becomes heated it begins to reradiate the energy that was absorbed in it. With increasing X an equilibrium is reached wherein all the absorbed energy is being reradiated and the suspension temperature no longer increases. For M = 0.2 about one-half of the incident energy is directly absorbed in the suspension; and if the wall is black, it absorbs the remaining half of the energy. When the layer is sufficiently hot to reradiate the absorbed energy, one-half of this energy is reradiated to the wall and the other half back toward the source. Hence, for M = 0.2 and $\epsilon_w = 1$, about three-fourths of the incident energy is absorbed in the wall as the suspension temperature becomes high.

The parameter H contains the heat transfer coefficient from the flowing particulate layer to the wall. The H values are small for the conditions of interest herein, where the heat transfer coefficient from the gaseous layer to the wall is not very high and the radiation originates from a high-temperature source. A typical set of cases were calculated for H from $10^{-5}$ to $10^{-2}$, and no significant effect was found for this range (fig. 9). The results are dominated by radiative effects and by the energy transport being carried along by the flow. Most of the results given in the other figures of this report were calculated for $H = 10^{-4}$ but should apply for any H less than about $10^{-2}$.

Figure 10 illustrates the effect of the dimensionless wall temperature on the suspension temperature and on the heat flux to the wall. This parameter is generally of minor importance for two reasons. Compared with radiative effects, a relatively small amount of energy is being transferred by convection between the wall and the particle suspension (as indicated by the results in fig. 9). With regard to radiation from the wall, since this depends approximately on the fourth power of temperature, $T_w$ has to be an appreciable fraction of $T_s$ before the wall radiation becomes significant relative to that from the source. The curves show a small decrease in heat flux to the wall as $T_w$ is increased, as a result of decreased convection to the wall and increased radiation from the wall.

The final parameter to be discussed is the entrance temperature of the particulate suspension. Results for $T_{m,o}/T_s$ of 0.1 and 0.2 are given in figure 11 for a moderately absorbing suspension, $M = 0.2$, and a strongly absorbing suspension, $M = 1$. As would be expected the higher entrance temperature increases the wall heating and the suspension temperature at each location along the surface.

## CONCLUSIONS

The interaction between a seeded gas layer flowing along a surface and radiation from a blackbody source has been analyzed. The analysis yields the energy being absorbed by the surface and the temperature rise of the flowing seeded layer. The radiative behavior of the layer was found to be governed by the parameter $M = (f_v C_L kL/C_2)T_s$ and the ratio of suspension temperature to source temperature $T_m/T_s$. The parameter M contains the volume fraction of particles $f_v$, the layer thickness L, and the source temperature $T_s$, as well as the ratio of mean beam length to layer thickness $C_L$, the constant k in the absorption coefficient, and the constant $C_2$ in Planck's energy distribution. When $M \approx 2$ the layer absorbs practically all the radiation incident on it, and when $M \approx 0.2$ about one-half of the incident radiation is absorbed. The spectral absorption coefficient of the particulate suspension is approximately proportional to the reciprocal of the wavelength of the incident radiation. This is what causes the M to increase with increasing $T_s$ as the blackbody spectrum of the incident radiation is thereby shifted to smaller wavelengths.

The heat transferred into the wall can be reduced by increasing the value of M so that direct transmission of radiation through the layer is made small. This is a workable approach only if the layer temperature stays low since with an increased value of M the layer will tend to emit well as its temperature increases. The heat flux to the wall can also be reduced by making the wall a good reflector for incident radiation. Although it was not considered here, a spectrally selective wall (e.g., a wall with a white ceramic coating) would be beneficial. This wall would tend to reflect away the incident energy from the high-temperature source and would emit well the longer wavelength radiation characteristic of the wall temperature. The heat transferred into the wall would thereby be reduced.

To obtain good absorption in the suspension, the M value must be at least near unity. The value of M depends on the product of particle volume concentration, layer thickness, and source temperature. For a typical layer thickness of 1 centimeter the required particle volume concentrations $f_v$ were found to be quite small, being in the range $10^{-5}$ to $5\times10^{-5}$ for source temperatures of 5000 to 20 000 K.

The results provide the distance along the surface for which the wall heat transfer remains within an acceptable limit and the particles remain below a temperature at which they will melt or vaporize. If the layer becomes hot enough to vaporize the particles or radiate appreciably to the wall, the layer is no longer effective in providing surface protection. The effective distances are usually short, being of the order of a few layer thicknesses as indicated by the numerical example in the discussion of the flowing layer. This is because the heat capacity per unit volume is small for the combination of particles and gas. This follows from the fact that the particle volume concentration required to obtain good absorption is quite small and hence the particles do not appreciably increase the volume heat capacity of the flowing layer. The heat capacity remains low as is typical for a gas, and the layer will heat rapidly as radiative energy is absorbed. The layer heat capacity per unit volume can be raised by increasing the suspension pressure, thereby increasing the length over which the layer is effective.

In a practical design it would probably be necessary to introduce the suspension at frequent intervals along the surface, the spacing between introduction locations depending on the mass flow of the layer. The wall might be made in the form of a perforated plate with the coolant being introduced through many holes; this would be somewhat like a transpiration-cooled wall but with a seeded gas to absorb and carry the incident radiation away from the wall.

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506-24.

## APPENDIX A

## SYMBOLS

| | |
|---|---|
| $a$ | absorption coefficient |
| $C_L$ | ratio of mean beam length to layer thickness |
| $C_1, C_2$ | constants in Planck's spectral energy distribution |
| $C_3$ | constant in Wien's displacement law |
| $c_p$ | specific heat |
| $E_3$ | third exponential integral function |
| $e$ | radiative emissive power |
| $F$ | quantity defined in eq. (17) |
| $f_v$ | volume of particles per unit volume of particulate suspension |
| $H$ | dimensionless quantity defined as $h/\sigma T_s^3$ |
| $h$ | convective heat transfer coefficient |
| $k$ | coefficient in absorption coefficient |
| $L$ | thickness of seeded layer |
| $M$ | parameter defined as $(f_v C_L kL/C_2)T_s$ |
| $n$ | index of refraction |
| $q$ | energy flux; energy per unit time and area |
| $T$ | absolute temperature |
| $t_w, t_m$ | dimensionless quantities $T_w/T_s$, $T_m/T_s$ |
| $\bar{u}$ | average velocity in x-direction |
| $X$ | dimensionless quantity defined as $(x/L)\left(\sigma T_s^3 / \bar{u}\, \overline{\rho c}_p\right)$ |
| $x$ | length coordinate along plate from origin of seeded layer |
| $\alpha$ | absorptivity |
| $\epsilon_m$ | suspension emittance |
| $\epsilon_w$ | wall emissivity |
| $\kappa$ | extinction coefficient |
| $\lambda$ | wavelength |
| $\rho_\lambda$ | spectral reflectivity |

| | |
|---|---|
| $\rho_g$ | density of gas |
| $\rho_p$ | density of particles |
| $\overline{\rho c}_p$ | quantity equal to $(1 - f_v)\rho_g c_{p,g} + f_v \rho_p c_{p,p}$ |
| $\sigma$ | Stefan-Boltzmann constant |
| $\overline{\tau}$ | transmittance |

Subscripts:

| | |
|---|---|
| b | blackbody |
| c | convective |
| g | gas |
| i | incoming energy |
| $l$ | outer boundary of seeded layer |
| m | gas-particle suspension (mixture) |
| max | at maximum of blackbody curve |
| o | outgoing energy |
| p | particle |
| s | source of radiation |
| w | wall |
| $\lambda$ | at one wavelength; spectral quantity |

## APPENDIX B

## EVALUATION OF INTEGRALS OVER ALL WAVELENGTHS

The integrals in equation (20) are all of the form

$$I = \int_{\lambda=0}^{\infty} \frac{1}{\lambda^5} e^{-K/\lambda} \, d\lambda \tag{B1}$$

where K is independent of $\lambda$. Let $\eta = K/\lambda$. Then $d\eta = -\lambda^{-2}K \, d\lambda = -(\eta^2/K)d\lambda$. Substitute into equation (B1) to obtain

$$I = -\int_{\eta=0}^{\infty} \frac{\eta^5}{K^5} e^{-\eta} \frac{K}{\eta^2} \, d\eta = \frac{1}{K^4} \int_0^{\infty} \eta^3 e^{-\eta} \, d\eta \tag{B2}$$

Applying this to equation (20) yields

$$F = \frac{\dfrac{1}{\left(f_v k C_L L + \dfrac{C_2}{T_s}\right)^4} \displaystyle\int_0^{\infty} \eta^3 e^{-\eta} \, d\eta - \dfrac{1}{\left(f_v k C_L L + \dfrac{C_2}{T_m}\right)^4} \displaystyle\int_0^{\infty} \eta^3 e^{-\eta} \, d\eta}{\dfrac{1}{\left(C_2/T_s\right)^4} \displaystyle\int_0^{\infty} \eta^3 e^{-\eta} \, d\eta}$$

$$F = \frac{1}{\left(\dfrac{f_v k C_L L T_s}{C_2} + 1\right)^4} - \frac{1}{\left(\dfrac{f_v k C_L L T_m}{C_2} + 1\right)^4 \left(\dfrac{T_s}{T_m}\right)^4} \tag{B3}$$

This is the result given in equation (21).

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TABLE I. - OPTICAL CONSTANTS OF ACETYLENE AND PROPANE SOOTS (REF. 2)

| Wavelength, $\lambda$, $\mu m$ | Acetylene soot | | | Propane soot | | |
|---|---|---|---|---|---|---|
| | Index of refraction, n | Extinction coefficient, $\kappa$ | Absorption coefficient per particle volume fraction, $a_\lambda/f_v$, $\mu m^{-1}$ | Index of refraction, n | Extinction coefficient, $\kappa$ | Absorption coefficient per particle volume fraction, $a_\lambda/f_v$, $\mu m^{-1}$ |
| 0.4358 | 1.56 | 0.46 | 9.37 | 1.57 | 0.46 | 9.29 |
| .4500 | 1.56 | .48 | 9.45 | 1.56 | .50 | 9.83 |
| .5500 | 1.56 | .46 | 7.42 | 1.57 | .53 | 8.44 |
| .6500 | 1.57 | .44 | 5.96 | 1.56 | .52 | 7.07 |
| .8065 | 1.57 | .46 | 5.02 | 1.57 | .49 | 5.34 |
| 2.5 | 2.31 | 1.26 | 1.97 | 2.04 | 1.15 | 2.34 |
| 3.0 | 2.62 | 1.62 | 1.44 | 2.21 | 1.23 | 1.75 |
| 4.0 | 2.74 | 1.64 | .998 | 2.38 | 1.44 | 1.24 |
| 5.0 | 2.88 | 1.82 | .747 | 2.07 | 1.72 | 1.30 |
| 6.0 | 3.22 | 1.84 | .505 | 2.62 | 1.67 | .727 |
| 7.0 | 3.49 | 2.17 | .270 | 3.05 | 1.91 | .484 |
| 8.5 | 4.22 | 3.46 | .213 | 3.26 | 2.10 | .357 |
| 10.0 | 4.80 | 3.82 | .143 | 3.48 | 2.46 | .271 |

TABLE II. - COMPARISON OF APPROXIMATE SUSPENSION EMITTANCES WITH VALUES FROM REFERENCE 2

| Concentration path length product, $f_vC_LL$, cm | Suspension temperature, $T_m$, K | Suspension emittance, $\epsilon_m$, | |
|---|---|---|---|
| | | Equation (27) | Reference 2 |
| $0.01\times10^{-4}$ | 1000 | 0.0135 | 0.013 |
| .10 | 1000 | .125 | .125 |
| 1.00 | 1000 | .690 | .64 |
| .01 | 2000 | .0268 | .030 |
| .10 | 2000 | .232 | .250 |
| 1.00 | 2000 | .875 | .90 |

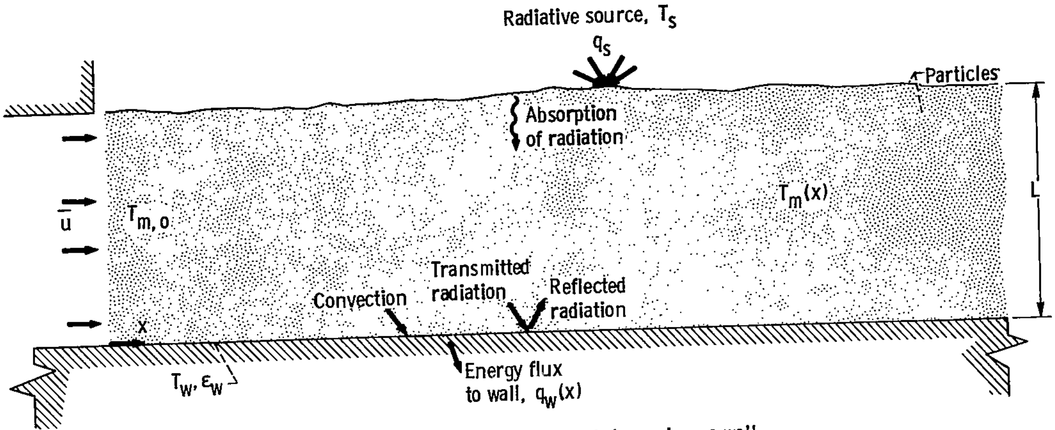



Figure 1. - Injection of particle-seeded gas along a wall.

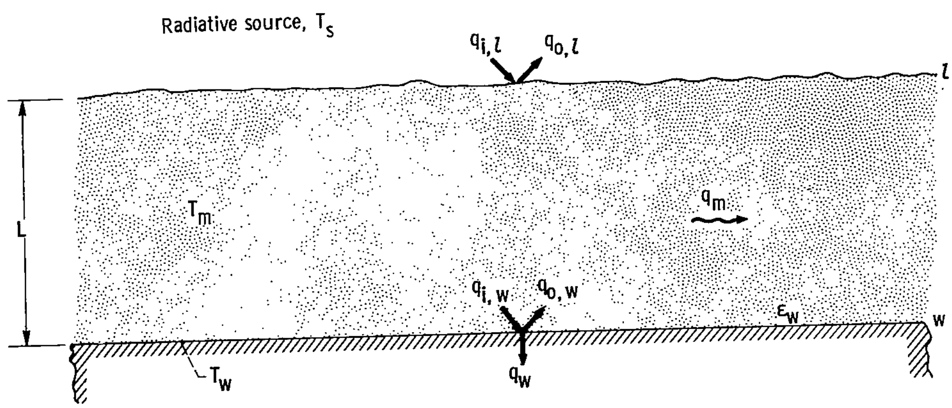



Figure 2. - Radiative quantities for stationary layer of particle-seeded gas.

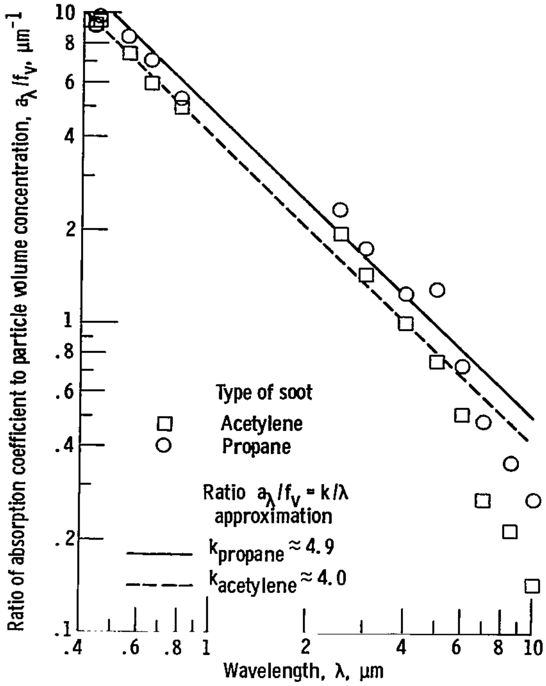



Figure 3. - Ratio of absorption coefficient to particle volume concentration as a function of wavelength for soot particles.

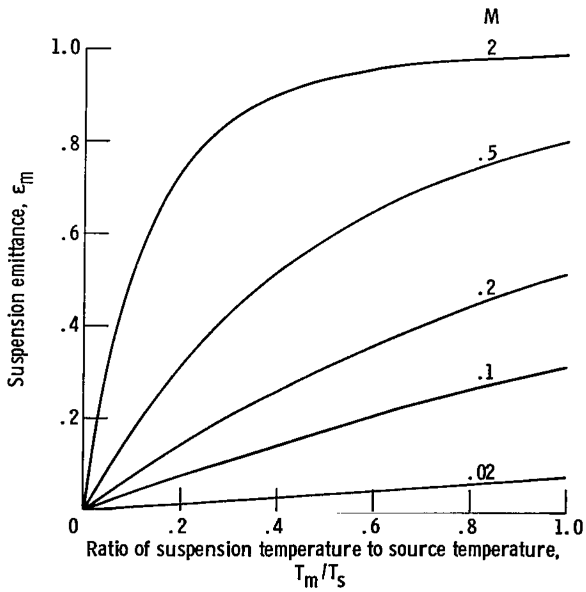
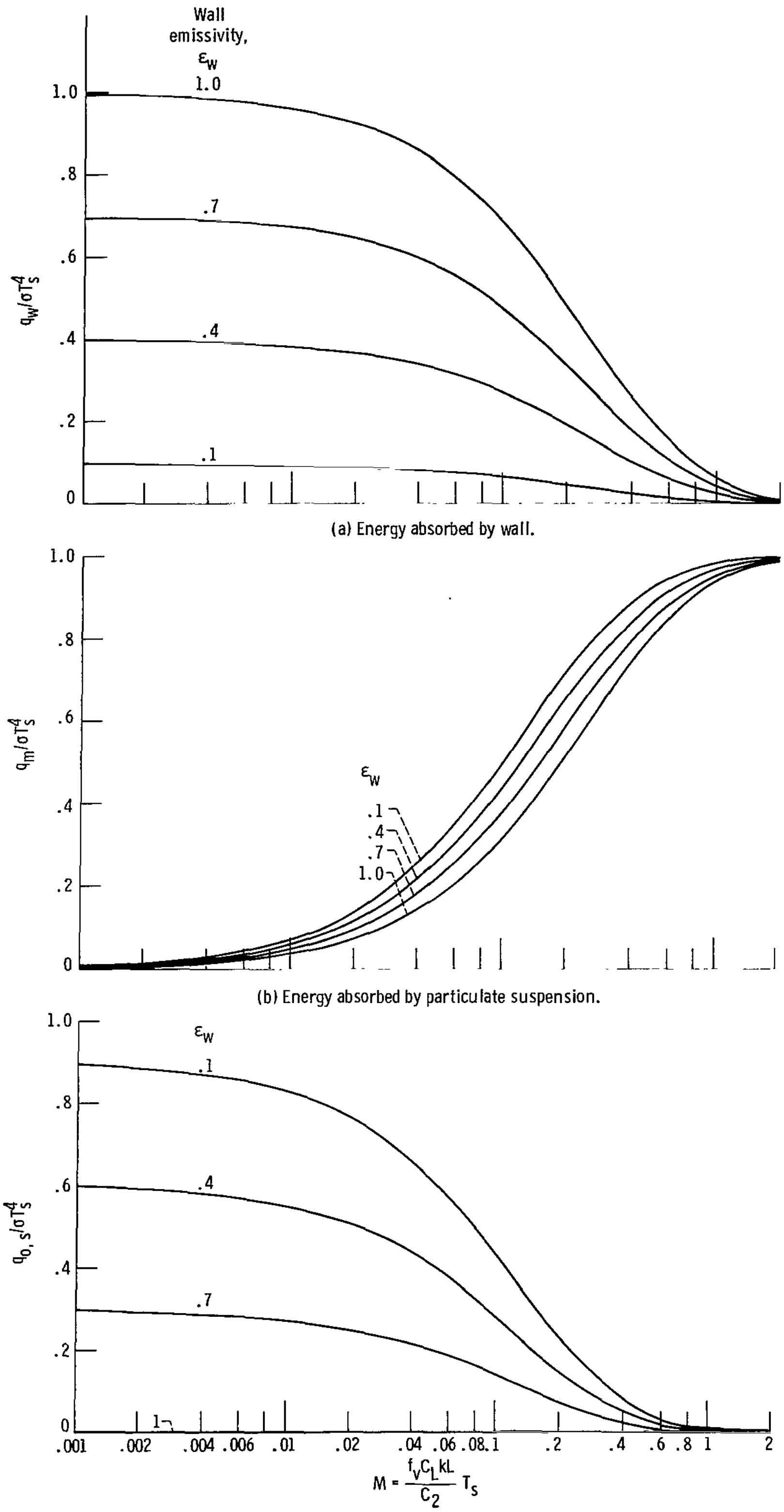



Figure 4. - Emittance of suspension as a function of its temperature.




(a) Energy absorbed by wall.



(b) Energy absorbed by particulate suspension.



(c) Energy going back out of particulate layer.

Figure 5. - Characteristics of nonflowing layer for suspension and wall at low temperature.

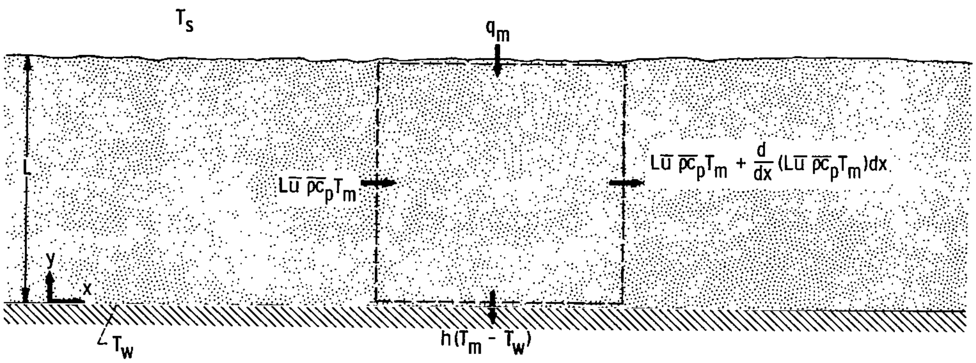
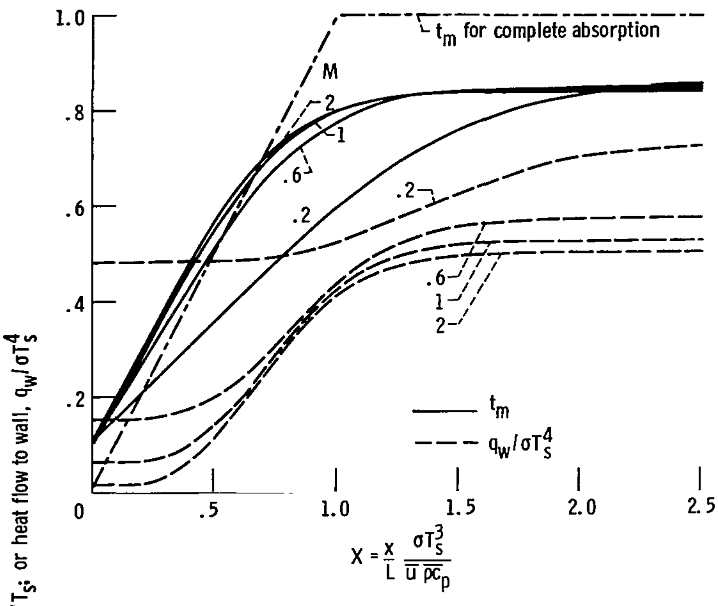
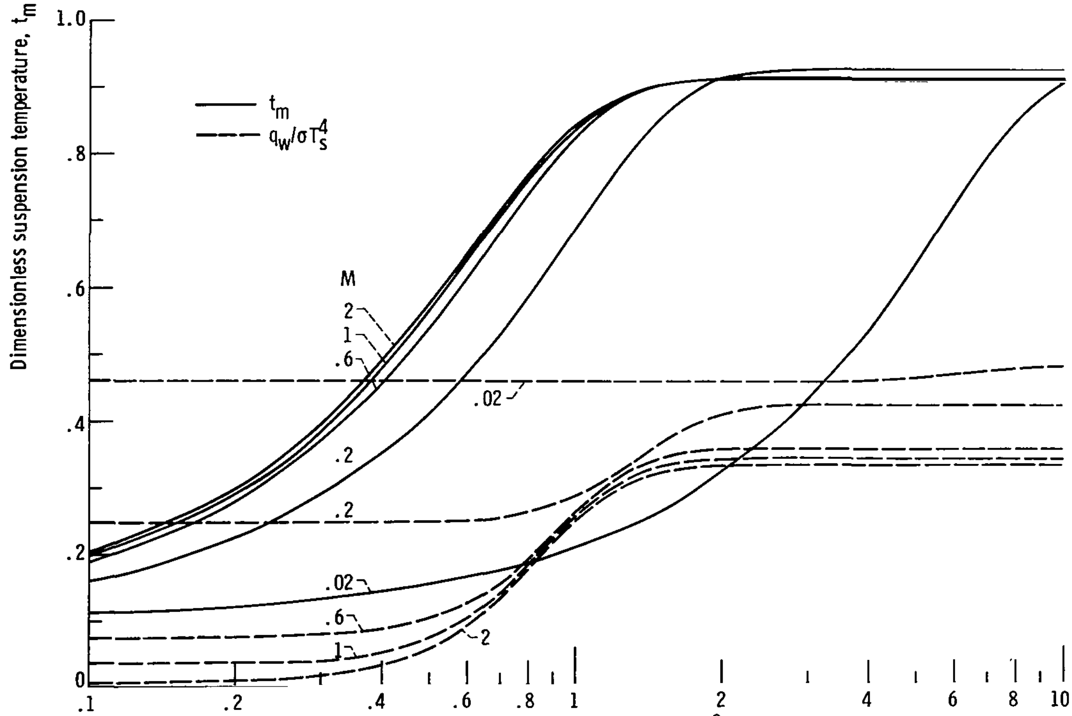



Figure 6. - Radiative and convective heat flow quantities for flowing particulate suspension.




(a) Wall emissivity, $\varepsilon_w$, 1.0.




(b) Wall emissivity, $\varepsilon_w$, 0.5.

Figure 7. - Effect of parameter $M = f_v C_L k L T_s / C_2$ on suspension temperature and heat flow to wall. Dimensionless heat transfer coefficient, H, $10^{-4}$; dimensionless wall temperature, $t_w$, 0.1; dimensionless entrance suspension temperature, $t_{m,o}$, 0.1.

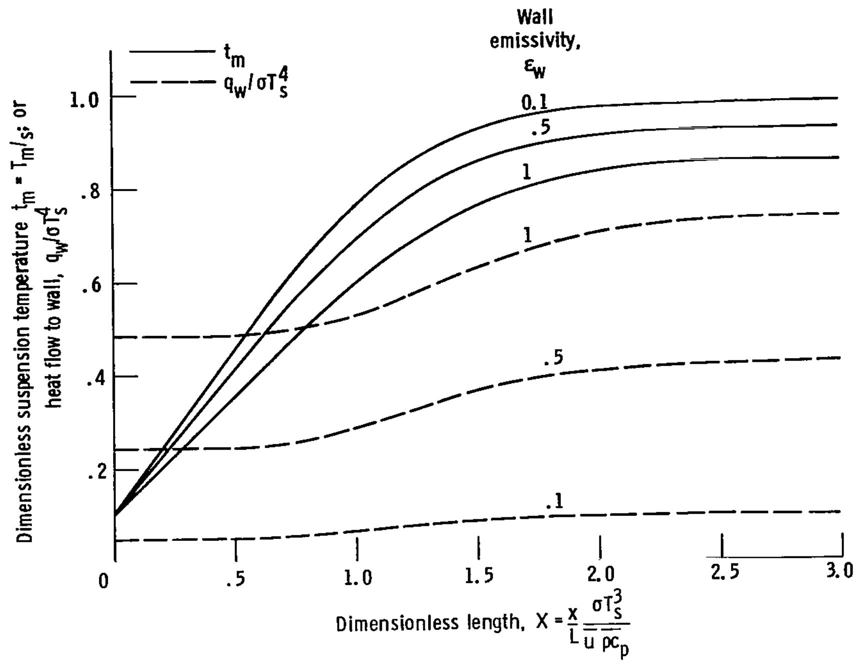



Figure 8. - Effect of wall emissivity on suspension temperature and heat flow to wall. Dimensionless parameter, M, 0.2; dimensionless heat transfer coefficient, H, $10^{-4}$; dimensionless wall temperature, $t_w$, 0.1; dimensionless entrance suspension temperature, $t_{m,o}$, 0.1.

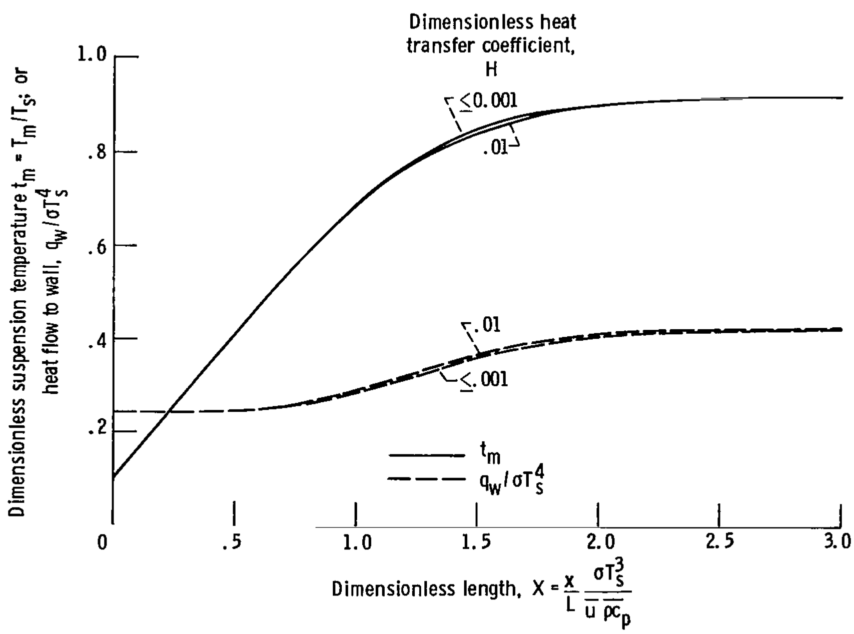



Figure 9. - Effect of dimensionless heat transfer coefficient on suspension temperature and heat flow to wall. Dimensionless parameter, M, 0.2; wall emissivity, $\varepsilon_w$, 0.5; dimensionless wall temperature, $t_w$, 0.1; dimensionless entrance suspension temperature, $t_{m,o}$, 0.1.

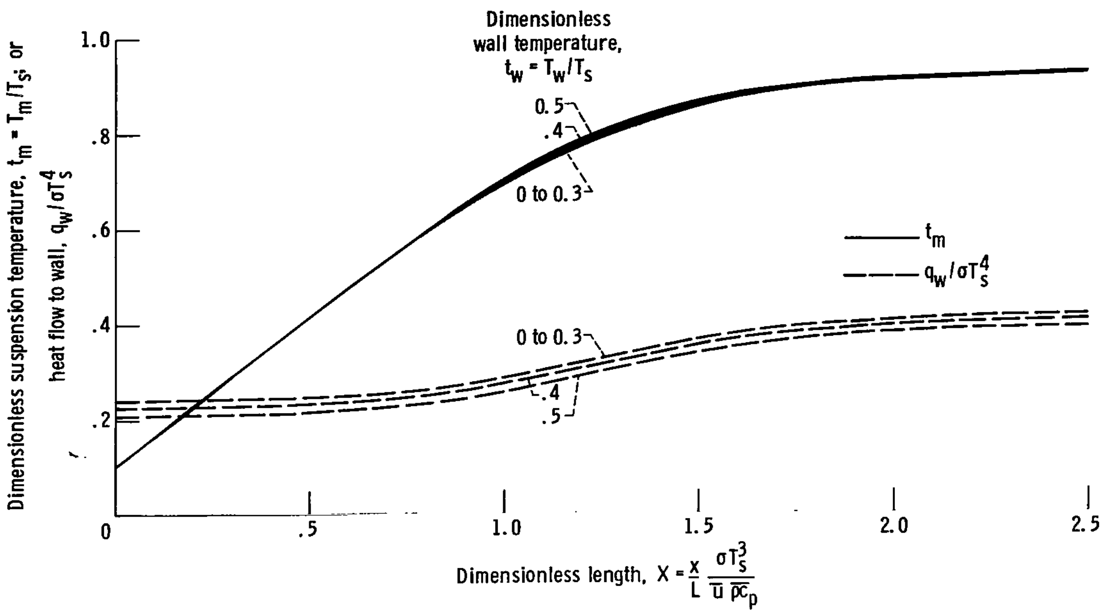



Figure 10. - Effect of dimensionless wall temperature on suspension temperature and heat flow to wall. Dimensionless parameter, M, 0.2; wall emissivity, $\epsilon_w$, 0.5; dimensionless heat transfer coefficient, H, $10^{-4}$; dimensionless entrance suspension temperature, $t_{m,o}$, 0.1.

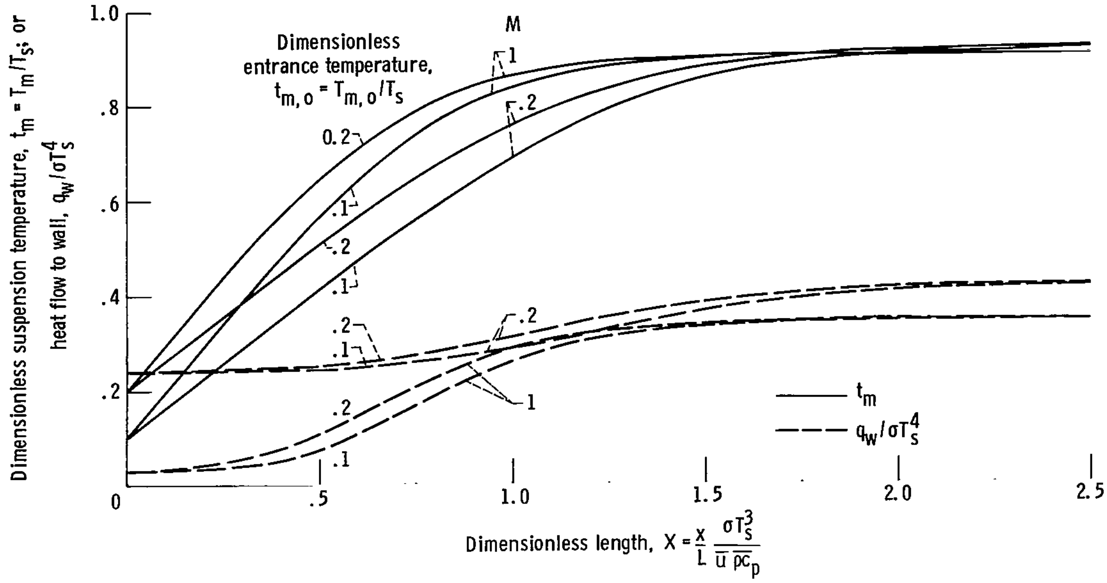



Figure 11. - Effect of entrance suspension temperature on heating of seeded layer and heat flow to wall. Wall emissivity, $\epsilon_w$, 0.5; dimensionless heat transfer coefficient, H, $10^{-4}$; dimensionless wall temperature, $t_w$, 0.2.



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