## General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.
- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.
- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.
- This document is paginated as submitted by the original source.
- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

Produced by the NASA Center for Aerospace Information (CASI)

MCDONNELL DOUGLAS TECHNICAL SERVICES CO. houston astronautics division

## SPACE SHUTTLE ENGINEERING AND OPERATIONS SUPPORT

DESIGN NOTE NO. 1.4-7-33
determination of orbiter aid carrier aerodynamic COEFFICIENTS FROM LOAD CELL MEASUREMENTS

MISSION PLANNING, MISSION ANALYSIS AND SOFTWARE FORMULATION

22 March 1976

This Design Note is Submitted to NASA under Task Order No. D0408, Task Assignment A, Contract NAS 9-13970.

Separation and Recontact
Task
488-5660, Ext. 270

APPROVED BY: $\frac{y, 7 f(1)}{T} \int_{2 i}$ APPROVED BY:
T. H. Wenglinski Work Package Manager Guidance and Dynamics Branch
488-5660, Ext. 228
$\frac{\operatorname{Si} \operatorname{Cocct}}{i}$
C. L. Colwell

Task Manager
Separation and
Recontact
488-5660 Ext. 271

## TABLE OF CONTENTS

SECTION TITLE PAGE
1.0 SUMMARY ..... 1
2.0 INTRODUCTIOM ..... 2
3.0 DISCUSSION ..... 3
3.1 Assumptions ..... 4
3.2 Geometry of the Orbiter/Carrier ..... 5
3.3 Derivation of Equations ..... 6
4.0 RESULTS ..... 13
5.0 CONCLUSIONS AND RECOMMENDATIONS ..... 15
6.0 REFERENCES ..... 16

## LIST OF FIGURES

FIGURE title ..... PAGE
1 geometry of the orbiter/Carrier ..... 17
2 relative position vectors ..... 18
LIST OF TABLES
TABLE TITLE PAGE
1 DATA REQUIRED FOR ANALYSIS ..... 19
2 RESULTS OF DERIVED EQUATIONS (ORBITER) ..... 20
3 RESULTS OF DERIVED EQUATIONS (CARRIER) ..... 21

## gLOSSARY OF SYMBOLS

| ${ }^{\text {car }}$ | AERODYNAMIC REFERENCE AREA OF THE CARRIER ( $=5500 \mathrm{ft}{ }^{2}$ ) |
| :---: | :---: |
| $A_{\text {ORB }}$ | aERODYnamic reference area of the orbiter ( $=2690 \mathrm{ft}^{\mathbf{2}}$ ) |
| ALT | APPROACH AND LANDING TEST |
| ${ }^{a_{s}} \text { CAR }$ | ACCELERATION VECTOR SENSED ON THE CARRIER EXPRESSED IN THE CARRIER VRCS |
| ${ }^{\alpha_{C A R}}$ | ANGLE ATtack Cf the carrier |
| ${ }^{\text {a }}$ ( ${ }^{\text {a }}$ | ANGLE ATtack of the orbiter |
| [ $\mathrm{B}_{21}$ ] | ORBITER TO CARRIER VRCS TRANSFORMATION MATRIX |
| ${ }^{5}$ CAR | Lateral aerodynamic reference lengti of the carrier ( $=195.7 \mathrm{ft}$ ) |
| $5_{\text {ORB }}$ | LATERAL AERODYNAMIC REFERENCE LENGTH OF THE ORBITER $(=78.0567 \mathrm{ft})$ |
| $\bar{c}_{\text {CAR }}$ | Longitudinal aerodynayic reference lengit of the CARRIER ( $=27.32 \mathrm{ft}$ ) |
| $\bar{c}_{\text {ORB }}$ | LONGITUDINAL AERODYNAMIC REFEREHCE LENGTH OF THE ORBITER $(=39.5667 \mathrm{ft})$ |
| ${ }^{c_{A_{C A R}}}$ | aerodynamic axial force coefficient of the carrier |
| ${ }^{c_{A_{O R B}}}$ | aerodynamic axial force coefficient of the orbiter |
| ${ }^{C_{D_{C A R}}}$ | aerodynamic drag coefficient of the carrier |

## gLossary of sMmbols (CONT.)

| ${ }^{C_{D_{O R B}}}$ | AERODYNAMIC DRAG COEFFICIENT OF THE ORBITER |
| :---: | :---: |
| $\overrightarrow{\mathrm{C}}_{\mathrm{CAR}}$ | POSITION OF THE CARRIER cg WITH RESPECT TO THE CARRIER VRCS |
| ${\stackrel{\rightharpoonup}{\mathrm{C}} \mathrm{G}_{\mathrm{ORB}}}$ | POSITION OF THE ORBITER cg WITH RESPECT TO THE ORBITER VRCS |
| $\overrightarrow{C G}_{R E F}^{C A R}$ | POSITION OF THE CARRIER MRC WITH RESPECT TO THE CARRIER VRCS (XYZ COMPONENTS $=-1339.9,0,9.25 \mathrm{in}$ ) |
| $\stackrel{\rightharpoonup G}{R E F}_{C A R}$ | POSITION OF THE ORBITER MRC WITH RESPECT TO THE ORBITER VRCS (XYZ COMPONENTS $=-1108.75,0,25 \mathrm{in}$ ) |
| cg | CENTER OF GRAVITY |
| ${ }^{c_{L_{C A R}}}$ | AERODYMAMIC LIFT COEFFICIENT OF THE CARRIER |
| $c_{L_{\text {ORB }}}$ | AERODYMAMIC LIFT COEFFICIENT OF THE ORbIter |
| ${ }^{c_{\ell_{C A R}}}$ | AERODYNAMIC ROLLING NOMEMT COEFFICIENT OF THE CARRIER ABOUT ITS MRC |
| ${ }^{c_{\ell_{O R B}}}$ | aerodymail rolling monent coefficient of the orbiter ABOUT ITS MRC |
| ${ }^{\mathrm{C}_{\ell} \mathrm{cg}_{\mathrm{CAR}}}$ | AERODYNAMIC ROLLING MOMENT COEFFICIENT OF THE CARRIER ABOUT ITS cg |
| ${ }^{c_{\ell}}{ }_{c g_{O R B}}$ | AERODYNAHIC ROLLING MOHEMT COEFFICIENT OF THE ORBITER ABOUT ITS cg |
| $c_{m_{C A R}}$ | AERODYNAMIC PITCHING MOMENT COEFFICIENT OF THE CARRIER ABOUT ITS MRC |
| $c_{m_{O R B}}$ | AERODYNAMIC PITCHING MOMENT COEFFICIENT OF THE ORBITER ABOUT ITS MRC |

## gLOSSARY OF SYMBOLS (CONT.)

| $c_{m_{c g_{C A R}}}$ | AERODYNAMIC PITCHING MOMENT COEFFICIENT OF THE CARRIER ABOUT ITS cg |
| :---: | :---: |
| $c_{m_{c g_{O R B}}}$ | aerodynamic pitching moment coefficient of the orbiter ABOUT ITS cg |
| $\mathrm{C}_{\mathrm{N}_{\mathrm{CAR}}}$ | AERODYNAMIC NORMAL FORCE COEFFICIENT OF THE CARRIER |
| $c_{N_{0 R B}}$ | AERODYNAHIC NORMAL FORCE COEFFICIENT OF THE ORBITER |
| $c_{n_{C A R}}$ | aerodynamic yawing moment coefficient of the carrier ABOUT ITS MRC |
| $c_{n_{0 R B}}$ | aerodynamic yaning moment coefficient of the orbiter ABOUT ITS liRC |
| ${ }^{c_{n^{c g}}}$ | aerodynamic yahing moment coefficient of the carrier ABOUT ITS cg |
| ${ }^{c_{n_{c g_{O R B}}}}$ | aerodynamic yahing mohent coefficient of the orbiter ABOUT ITS cg |
| ${ }^{{ }^{y_{Y_{C A R}}}}$ | aerodynamic side force coefficient of the carrier |
| $c_{Y_{O R B}}$ | AERODYNAMIC SIDE FORCE COEFFICIENT OF THE ORBITER |
| $\Delta \theta$ | ANGLE OF INCIDENCE |
| $\mathrm{F}_{\mathrm{A}_{\mathrm{CAR}}}$ | AERODYNMIIC FORCE VECTOR ACTING ON THE CARRIER cg EXPRESSED IN THE CARRIER VRCS |
| ${ }^{F^{\text {ARBB }}}$ | AERODYHAMIC FORCE VECTOR ACTING ON THE ORBITER cg EXPRESSED IN THE ORBITER VRCS |


| $\vec{F}_{\mathbf{c}}$ | FORCE VECTOR ACTING ON THE CARRIER cg BY THE ORBITER COUPLED THRU THE ATTACH STRUCTURES EXPRESSED IN THE CARRIER VRCS |
| :---: | :---: |
| $F_{C A R}$ | TOTAL EXTERNAL FORCE VECTOR ACTING ON THE CARRIER cg EXPRESSED IN THE CARRIER VRCS |
| $\stackrel{\rightharpoonup}{F}_{\text {F }}$ | FORCE VECTOR IN THE FRONT ATTACH STRUT EXPRESSED IN THE CARRIER VRCS |
| $\stackrel{\rightharpoonup}{F}_{L A}$ | force vector in the left aft attach strut expressed IN THE CARRIER VRCS |
| $\mathrm{F}_{\text {MAT }}$ | TOTAL EXTERNAL FORCE VECTOR ACTING ON THE HATED cg EXPRESSED IN THE CARRIER VRCS |
| $F_{\text {ORB }}$ | TOTAL EXTERNAL FORCE VECTOR ACTING ON THE ORBITER cg EXPRESSED IN THE ORBITER VRCS |
| $\vec{F}_{R A}$ | FORCE VECTOR IN THE RIGHT AFT ATTACH STRUT EXPRESS:D IN THE CARRIER VRCS |
| $\overrightarrow{\mathrm{F}}_{\mathrm{T}}$ | CARRIER ENGINE THRUST EXPRESSED IN THE CARRIER VRCS |
| $\overrightarrow{\mathbf{G}}_{A_{C A R}}$ | AERODYNAMIC MOMENT VECTOR ACTING ABOUT THE CARRIER cg EXPRESSED IN THE CARRIER VRCS |
| $G_{A_{O R B}}$ | AERODYNANIC MOMENT VECTOR ACTING AEOUT THE ORBITER cg EXPRESSED IN THE ORBITER VRCS |
| $\vec{G}_{\mathbf{C}}$ | MOMENT VECTOR ACTING ABOUT THE CARRIER cg BY THE ORBITER COUPLED THRU THE ATTACH STRUCTURES EXPRESSED IN THE CARRIER VRCS |
| ${ }^{G} \text { CAR }$ | TOTAL EXTERNAL MOMENT VECTOR ACTING ABOUT THE CARRIER EXPRESSED IN THE CARRIER VRCS |


| $\mathrm{G}_{\text {MAT }}$ | total exterial moment vector acting about the mated eg EXPRESSED III THE CARRIER VRCS |
| :---: | :---: |
| ${ }^{\text {G ORB }}$ | total extermal moment vector acting aeout the orbiter cg EXPRESSED IM THE ORBITER VRCS |
| $\left[^{\text {CAR }}\right.$ ] | InERTIA TENSOR MATRIX OF THE CARRIER |
| [ $\mathrm{I}_{\text {MAT }}$ ] | inertia tensor matrix of the mated vehicle |
| $\left[\mathrm{I}_{\text {ORB }}\right.$ ] | Imertia tensor matrix of the orbiter |
| MDTSCO | MCDONNELL DOUGLAS TECHNICAL SERVICES COMPANY |
| MRC | MOMENT REFERENCE CENTER |
| $M_{\text {car }}$ | MASS OF THE CARRIER |
| $M_{\text {MAT }}$. | MASS Of The mated vehicle |
| MORB | MASS OF THE ORBITER |
| $\overrightarrow{\mathrm{P}}_{\mathrm{F}}$ | POSITION OF THE FRONT ATTACH STRUT (PINNED TO THE CARPIER) WITH RESPECT TO THE CARRIER VRCS (XYZ COMPOHENTS $=-680$, $0,-172$ in) |
| $\vec{P}_{\text {LA }}$ | position of the left aft attach point with respect to the CARRIER VRCS (XYZ COMPONENTS $=-1607,-96.5,-200 \mathrm{in}$ ) |
| $\vec{P}_{\mathrm{RA}}$ | POSITION OF THE RIGHT AFT ATTACH POINT WITH RESPECT TO THE CARRIER VRCS (XYZ COMPONENTS $=-1607,96.5,-200 \mathrm{in}$ ) |
| $\overrightarrow{\mathrm{P}}_{\mathrm{RA}}^{\mathrm{ORB}},$ | POSITION OF THE RIGHT AFT ATTACH POINT HITH RESPECT TO THE ORBITER VRCS (XYZ COMPONENTS $=-1317,96.5,132.4 \mathrm{in}$ ) |


| ${ }^{P}$ | position of the thrust application point with respect TO THE CARRIER VRCS (XYZ COMPONENTS $=-1331.946,0$, 77.652 in) |
| :---: | :---: |
| $\overline{9}$ | DYNAMIC PRESSURE |
| RI | ROCKWELL INTERNATIONAL |
| $\stackrel{\rightharpoonup}{R}$ | RELATIVE POSITION OF THE ORBITER cg WITH RESPLCT TO THE CARRIER gg EXPRESSED IN THE CARRIER VRCS |
| $\stackrel{\rightharpoonup}{R_{F}}$ | relative position of the front attach strut hith respect TO THE CARRIER cg EXPRESSED IN THE CARRIER VRCS |
| $\vec{R}_{\text {LA }}$ | relative positicn of the left aft attach point with respect TO THE CARRIER cg EXPRESSED IN THE CARRIER VRCS |
| $\stackrel{\rightharpoonup}{R}_{\text {RA }}$ | relative position of the right aft attach point with respect TO THE CARRIER cg EXPRESSED II THE CARRIER VRCS |
| $\vec{R}_{R A_{0 R B}}$ | relative position of the right aft attach point hith respect TO THE ORBITER cg EXPRESSED II THE ORBITER VRCS |
| $\stackrel{+}{R_{T}}$ | relative position of the thrist application point with respect to the carrier cg expressed in the carrier vrcs |
| $\vec{R}_{1}$ | relative position of the carrier ag with respect to the MATED cg EXPRESSED If The CARRIER VRCS |
| $\vec{R}_{2}$ | relative position of the orbiter cg with respect to the MATED cg EXPRESSED IN THE CARRIER VRCS |
| SVOS | SPACE VEHICLE DYNAMICS SIMULATIOH |
| VRCS | Y/.hicle referenced coordinate system |
| $\stackrel{+}{\omega}$ | angular rate vector of the carrier expressed in the carrier vrcs |
| $\dot{\mathbf{\omega}}$ | angular acceleration vector of the carrier expressec in the CARRIER VRCS |

### 1.0 SLMMARY

A method of determining orbiter and carrier total aerodynamic coefficients from load cell measurements is required to support the inert and the captive active flights of the ALT program. This report documents the derivation of a set of equations expressing the orbiter and carrier total aerodynamic coefficients in terms of the load cell measurements, the sensed dynamics of the Boeing 747 (carrier) aircraft, and the relative geometry of the orbiter/ carrier.

The requirement for a method of determining orbiter and carrier total aerodynamic coefficients is stated in Section 2.0. The assumptions, the geometry, and the derivation of the equations are presented in Section 3.0. Numerical results of the derived equa.* tions are evaluated in Section 4.0. The conclusions and recommendations are summarized in Section 5.0. Supporting reference sources are listed in Section 6.0.

### 2.0 INTRODUCTION

A method of determining orbiter and carrier total aerodynamic coefficients from load cell measurements is required to support the inert and the captive active flights of the ALT program. During the inert and captive active flights, the mated vehicle will perform the pre-separation maneuver and attain equilibrium glide at the target separation conditions. In addition, during the captive active flights, the orbiter pilot will move the elevons to several positions about the trim position. During these procedures, load cell readings in the front and aft attach struts will be recorded. This load cell data in combination with other data recorded on the flight recurders (i.e. sensed acceleration of the carrier, body angular rates and accelerations of the earrier, thrust of the carrier engines, angle of attack of the carrier , and dynamic pressure) will be used to cumpute the total aerodynamic coefficients of the orbiter and the carrier. These computed values will then be compared th the wind tunnel predicted values. If there are significant differences between the two sets of values the target separation conditions will be adjusted accordingly.

The objective of this MDTSCO "Determination of Orbiter and Carrier Aerodynamic Cocfficients from Load Cel: Measurements" is to present a straight forward derivation of the equations necessary to determine the orbiter and carrier total aerody: itic coefficients from data reconded during the ALT flichts.

### 3.0 DISCUSSION

This section presents the assumptions, the geametry, and the derivation of the equations necessary to determine the orbiter and carrier total aerodynamic coefficients. Maximum utilization of previous analyses is made and source data is referenced accordingly in the subsequent text.

The derivation of the equations of the orbiter and carrier total aerodynamic coefficients is performed in five steps. The first step is to write the total external forces and moments acting on the mated vehicle at its cg in terms of the total external forces and moments acting on the orbiter and carrier at their respective cg's and also in terms of parameters recorded during the ALT filght. In the sacond step, the total external forces and monents acting on the carrier at its cg are written in terns of parameters recorded during the ALT flight. The third step is to solve the equations from steps 1 and 2 simultaneously to obtain equations expressing the total external forces and moments acting on the orbiter at ite cg. In the fourth step, the forces and moments due to engine thrust are subtracted from the forces and moments from steps 2 and 3 to obtain the total aerodynamic forces and moments acting on the orbiter and carrier at their respective cg's. In step 5, the forces and moments from step 4 are used to determine the aerodynamic axial, side, and normal force coefficients and the rolling, pitching, and yawing moment coefficients of both the orbiter and carrier at their respective cg's. Also, the aerodynamic forces of the orbiter and
carrier from step 4 are transformed to their respective stability axes and transferred to their respective $H$ RC's to determine the aerodynamic drag and lift coefficients and the rolling, pitching, and yawing moment coefficients of both the orbiter and carrier at their respective MRC's.

### 3.1 Assumptions

There are two categories of assumptions used in this report. The first category is assumptions pertaining to the form of the recorded data while the assumptions of the second category serve to simplify the problem.

It is assumed that the following parameters will te expressed (where applicable) in the vehicle referenced coordinate system (VRCS) of the carrier and will be given in the units indicated.

1) The force vectors in the front, left aft, and right aft aitach struts, $\vec{F}_{F}, \vec{F}_{L A}$, and $\vec{F}_{R A}$ respectively in 1 b .
2) The force vector due to carrier engine thrust, $\vec{F}_{T}$ in 1 b .
3) The sensed acceleration vector of the carrier, ${ }^{a_{s}}{ }_{C A R}$ in $\mathrm{ft} / \mathrm{sec}^{2}$
4) The angular rate vector of the carrier, $\vec{\omega}$ in $\mathrm{rad} / \mathrm{sec}$.
5) The angular acceleration vecter of the carrier, $\stackrel{\stackrel{\rightharpoonup}{\dot{w}}}{ }$ in $\mathrm{rad} / \mathrm{sec}^{2}$.
6) The dynamic pressure sensed on the carrier, $\bar{q}$ in $1 b / \mathrm{ft}^{2}$.
7) The angle of attack of the carrier $\alpha_{C A R}$ in rad.

To simplify the problem, the following assumptions are made.

1) The mated vehicle is considered to be two rigid bodies (the orbiter and the carrier) constrained to move as one rigid body.
2) The dynamic pressure of the orbiter is equal to that of the carrier. This assumption is necessary since during the time that the orbiter's air data probe is in proximity to the carrier, it will make incorrect measurements.

### 3.2 Geometry of the Orbiter/Carrier

The orbiter is oriented in the vehicle's plane of symmetry with respect to the carrier at an angle of incidence. The incidence angle and location and orientation of the VRCS of both vehicles are illustrated in Figure 1. The transformation of the orbiter VRCS to the carrier VRCS may $b$ : expressed as

$$
\left[B_{21}\right]^{=}\left[\begin{array}{ccc}
\cos \Delta \theta & 0.0 & \sin \Delta \theta  \tag{3.2.1}\\
0.0 & 1.0 & 0.0 \\
-\sin \Delta \theta & 0.0 & \cos \Delta \theta
\end{array}\right]
$$

Figure 2 illustrates the relative positions of the attach points, the orbiter cg, the carrier cg, and the mated cg.

The mass properties of the mated vehicle, in terms of the mass properties of the orbiter and carrier and the geometry, may be written as

$$
\begin{gather*}
M_{\text {MAT }}=M_{C A R}+M_{O R B} \text { and }  \tag{3.2.2}\\
{\left[I_{M A T}\right]=\left[I_{C A R}\right]+\left[B_{21}\right]\left[I_{O R B}\right]\left[B_{21}\right]^{\top}+\frac{M_{C A R} M_{O R B}}{M_{M A T}}\left[R_{R-R R}^{+}\right]} \tag{3.2.3}
\end{gather*}
$$

From the VRCS positions of the attach points and the thrust application point (Reference 1) the relative position vectors shown in Figure 2 may be written as

$$
\begin{align*}
\vec{R}_{F} & =\vec{P}_{F}-\overrightarrow{C G}_{C A R},  \tag{3.2.4}\\
\vec{R}_{L A} & =\vec{P}_{L A}-\overrightarrow{C G}_{C A R},  \tag{3.2.5}\\
\vec{R}_{R A} & =\vec{P}_{R A}-\overrightarrow{C G}_{C A R},  \tag{3.2.6}\\
\vec{R}_{R A_{O R B}} & =\vec{P}_{R A_{O R B}}-\overrightarrow{C G}_{O R B},  \tag{3.2.7}\\
\vec{R}_{T} & =\vec{P}_{T}-\overrightarrow{C G}_{C A R},  \tag{3.2.8}\\
\vec{R} & =\vec{R}_{R A}-\left[B_{21}\right] \vec{R}_{R A_{O R B}},  \tag{3.2.9}\\
\vec{R}_{1} & =-\frac{M_{O R B}}{M_{M A T}}, \text { and }  \tag{3.2.10}\\
\vec{R}_{2} & =-\frac{M_{C A R}}{M_{M A T}} \tag{3.2.11}
\end{align*}
$$

### 3.3 Derivation of Equations

The total external forces acting on the mated vehicle may be expressed, in terms of the total external forces acting on the orbiter and the carrier, as

$$
\begin{equation*}
\vec{F}_{\mathrm{MAT}}=\vec{F}_{\mathrm{CAR}}+\left[B_{21}\right] \vec{F}_{\mathrm{ORB}} . \tag{3.3.1}
\end{equation*}
$$

Similarily, the total external moments acting on the mated vehicle may be expressed as

$$
\begin{equation*}
\vec{G}_{\text {MAT }}=\vec{G}_{C A R}+\left[B_{21}\right] \vec{G}_{O R B}+\vec{R}_{1} \times \vec{F}_{C A R}+\vec{R}_{2} \times\left[B_{21}\right] \vec{F}_{O R B} . \tag{3.3.2}
\end{equation*}
$$

By substituting equations [3.2.10] and [3.2.11] for $\vec{R}_{1}$ and $\vec{R}_{2}$, equation [3.3.2] may also be written as

$$
\begin{equation*}
\vec{G}_{\text {MAT }}=\vec{G}_{C A R}+\left[B_{21}\right] \vec{G}_{O R B}-\frac{M_{O R B}}{M_{\text {MAT }}} \overrightarrow{R x F}_{C A R}+\frac{M_{\text {CAR }}}{\text { M}_{\text {MAT }}}{\left.\overrightarrow{R x\left[B_{21}\right.}\right] \vec{F}_{\text {ORB }}}^{+} \tag{3.3.3}
\end{equation*}
$$

Also, the total external forces and moments on the mated vehicle may be expressed, in terms of data recorded during the flight, as

$$
\begin{align*}
& \vec{F}_{\text {MAT }}=M_{\text {MAT }}\left\{\vec{a}_{\text {SCAR }}-\stackrel{+}{\left.\omega \times R_{1}-\overrightarrow{\omega \times}\left(\overrightarrow{\omega \times R_{2}}\right)\right\}}\right.  \tag{3.3.4}\\
& G_{\text {MAT }}=\left[1_{\text {MAT }}\right] \overrightarrow{\dot{\omega}}+\vec{\omega} \times\left[I_{\text {MAT }}\right] \vec{\omega} . \tag{3.3.5}
\end{align*}
$$

By substituting equations [3.2.3.] and [3.2.10] for $\left[\mathrm{I}_{\text {MAT }}\right.$ ] and $R_{1}$, equations [3.3.4] and [3.3.5] may be written as

$$
\begin{align*}
& \vec{f}_{\text {MAT }}=M_{\text {MAT }} \vec{a}_{\text {S }_{\text {CAR }}}+M_{O R B} \vec{\omega} \times \vec{R}+M_{O R B}\{\vec{\omega} \times(\vec{\omega} \times \vec{R})\}  \tag{3.3.6}\\
& G_{\text {MAT }}=\left[I_{\text {CAR }}\right] \overrightarrow{\dot{\omega}}+\vec{\omega} \times\left[I_{\text {CAR }}\right] \vec{\omega}+  \tag{3.3.7}\\
& {\left[E_{21}\right]\left[I_{O R B}\right]\left[B_{21}\right]^{\top} \stackrel{+}{\omega}+\vec{\omega} \times\left[B_{22}\right]\left[I_{O R B}\right]\left[B_{21}\right]^{\top} \vec{\omega}+}
\end{align*}
$$

$$
\begin{align*}
& \vec{F}_{C}=M_{C A R} \vec{a}_{S_{C A R}}-\vec{F}_{C A R}  \tag{3.3.8}\\
& \vec{F}_{C}=-\left(\vec{F}_{F}+\vec{F}_{L A}+\vec{F}_{R A}\right)  \tag{3.3.9}\\
& \left.\vec{G}_{C}=\left[I_{C A R}\right] \vec{\omega}+\vec{\omega}_{\omega} \times I_{C A R}\right] \vec{\omega}-\vec{G}_{C A R}  \tag{3.3.10}\\
& \vec{G}_{C}=-\left(\vec{R}_{F} \times \vec{F}_{F}+\vec{R}_{L A} \times \vec{F}_{L A}+\vec{R}_{R A} \times \vec{F}_{R A}\right)
\end{align*}
$$

From equations [3.3.8] thru [3.3.11], the total external forces and moments acting on the carrier at its cg may be solved for in terms of the recorded data, mass properties, and geometry.

$$
\begin{align*}
& \vec{F}_{C A R}=M_{C A R} \vec{a}_{S C A R}+\vec{F}_{F}+\vec{F}_{L A}+\vec{F}_{R A} .  \tag{3.3.12}\\
& \vec{G}_{C A R}=\left[I_{C A R}\right] \stackrel{+}{\omega}+\vec{\omega} \times\left[I_{C A R}\right] \vec{\omega}+\vec{R}_{F} \times \vec{F}_{F}+\vec{R}_{L A} \times \vec{F}_{L A}+\vec{R}_{R A} \times \vec{F}_{R A} \tag{3.3.13}
\end{align*}
$$

By solving equations [3.3.1], [3.3.3], [3.3.6], [3.3.7], [3.3.12], and [3.3.13] simultaneously, the total external forces and moments acting on the orbiter at its cg may be written in terms of the recorded data, the mass properties, and the relative geometry of the orbiter/carrier.

$$
\begin{equation*}
\vec{F}_{\text {ORB }}=\left[B_{21}\right]^{\top}\left\{M_{\text {ORB }}\left[\vec{a}_{S_{C A R}}+\overrightarrow{\omega \times R}+\overrightarrow{\omega \times}(\overrightarrow{ } \overrightarrow{ } \times \vec{R})\right]-\left(\vec{F}_{F}+\vec{F}_{L A}+\vec{F}_{R A}\right)\right\} \tag{3.3.14}
\end{equation*}
$$

$$
\begin{aligned}
& {\left[B_{22}\right]^{\top}\left\{\left(\vec{R}-\vec{R}_{F}\right) \times \vec{F}_{F}+\left(\vec{R}-\vec{R}_{L A}\right) \times \vec{F}_{L A}+\left(\vec{R}^{-R_{R A}}\right) \times \vec{F}_{R A}\right\}}
\end{aligned}
$$

To obtain the total aerodynamic forces and moments, the forces and moments due to the engine thrust must be subtracted from the total external forces and moments. Therefore the aerodynamic forces and moments acting on the orbiter and carrier at their respective cos may be written as

$$
\begin{align*}
& \vec{F}_{A_{O R B}}=\vec{F}_{O R B},  \tag{3.3.16}\\
& \vec{G}_{A_{O R B}}=\vec{G}_{O R B},  \tag{3.3.17}\\
& \vec{F}_{A_{C A R}}=\vec{F}_{C A R}-\vec{F}_{T}, \text { and }  \tag{3.3.18}\\
& \vec{G}_{A_{C A R}}=\vec{G}_{C A R}-\vec{R}_{T} \times \vec{F}_{T} . \tag{3.3.19}
\end{align*}
$$

From the forces and moments given by equations [3.3.16] and [3.3.17], the orbiter aerodynamic coefficients $C_{A_{O R B}}, C_{Y_{O R B}}, C_{N_{O R B}},{ }_{\ell_{\ell_{C g}}}$, ${ }^{\mathrm{C}_{\mathrm{Cg}_{0 R B}}}$, and $\mathrm{C}_{\mathrm{n}_{\mathrm{Cg}}^{\mathrm{ORB}}}$ may be written as

$$
\begin{equation*}
c_{A_{O R B}}=-F_{X_{A_{O R B}}} / \bar{q} \cdot A_{O R B} \text {, } \tag{3.3.20}
\end{equation*}
$$

$$
\begin{align*}
& C_{Y_{O R B}}=F_{Y_{A_{O R B}}} / \bar{q} \cdot A_{O R B},  \tag{3.3.21}\\
& c_{N_{O R B}}=-F_{Z_{A_{O R B}}} / \bar{a} \cdot A_{O R B},  \tag{3.3.22}\\
& c_{\ell_{c g_{O R B}}}={ }^{G_{X_{A_{O R B}}}} / \bar{q} \cdot A_{O R B} \cdot \bar{b}_{O R B},  \tag{3.3.23}\\
& c_{m_{c g_{O R B}}}=G_{Y_{A_{O R B}}} / \bar{q} \cdot A_{O R B} \cdot \bar{c}_{O R B} \text {, and }  \tag{3.3.24}\\
& c_{n_{c g_{O R B}}}=G_{Z_{A_{O R B}}} / \bar{q} \cdot A_{\text {ORB }} \cdot \bar{b}_{O R B} . \tag{3.3.25}
\end{align*}
$$

By transforming the forces given by equation [3.3.16] to the stability axis system of the orbiter and transferring them to the orbiter's MRC, the aerodynamic coefficients $C_{D_{O R B}}, C_{L_{O R B}}, C_{\ell_{O R B}}$, $C_{\text {morB }}$, and $C_{n_{\text {ORB }}}$ may be written as

$$
\begin{align*}
& c_{D_{O R B}}=-\left[F_{X_{A_{O R B}}} \cdot \cos \left(\alpha_{C A R}+\Delta \theta\right)+F_{Z_{A_{O R B}}} \cdot \sin \left(\alpha_{C A R}+\Delta \theta\right)\right] / \bar{a} \cdot A_{O R B},[3.3 .26] \\
& c_{L_{O R B}}=-\left[-F_{X_{A_{O R B}}} \cdot \sin \left(\alpha_{C A R}+\Delta \theta\right)+F_{Z_{A_{O R B}}} \cdot \sin \left(\alpha_{C A R}+\Delta \theta\right)\right] / \bar{q} \cdot A_{O R E}, \quad[3.3 .27] \tag{3.3.27}
\end{align*}
$$

$$
\begin{align*}
& c_{m_{O R B}}=\left[G_{Y_{A_{O R B}}} / c_{O R B}+\left(C G_{Z_{R E F}}-{ }^{-C G_{Z_{O R B}}}\right) \cdot F_{X_{A_{O R B}}}-\left(C G_{X_{R E F}}{ }_{O R B}-C G_{X_{R E F}}\right) \cdot F_{Z_{A_{O R B}}}\right] / \bar{q} \cdot A_{O R B}, \tag{3.3.29}
\end{align*}
$$

Similarily from equations [3.3.18] and [3.3.19], the carrier aerodynamic coefficients $C_{A_{C A R}},{ }_{Y_{Y_{C A R}}}, C_{N_{C A R}}, C_{D_{C A R}}, C_{L_{C A R}}, C_{{ }_{l} C_{C A R}}$, ${ }^{c_{m_{c g_{C A R}}}},{ }^{c_{n_{c g_{C A R}}}},{ }^{c_{l_{C A R}}},{ }^{c_{m_{C A R}}}$, and $c_{n_{C A R}}$ may be written as

$$
\begin{equation*}
c_{A_{C A R}}=-F_{X_{A_{C A R}}} / \bar{q} \cdot A_{C A R} \text {, } \tag{3.3.31}
\end{equation*}
$$

$$
\begin{equation*}
C_{Y_{C A R}}=F_{Y_{A_{C A R}}} / \bar{q} \cdot A_{C A R}, \tag{3.3.32}
\end{equation*}
$$

$$
\begin{equation*}
c_{N_{C A R}}=-F_{Z_{A}} / \bar{q} \cdot A_{C A R} \tag{3.3.33}
\end{equation*}
$$

$$
\begin{aligned}
& C_{D_{C A R}}=-\left[F_{X_{A}} \cdot{ }^{C O S} \alpha_{C A R}+F_{Z_{A_{C A R}}} \cdot{ }^{\text {SINa }}{ }_{C A R}\right] / \bar{q} \cdot A_{C A R} \text {, } \\
& r_{L_{C A R}}=-\left[-F_{X_{A_{C A R}}} \cdot \operatorname{SIN\alpha _{CAR}}+F_{Z_{A_{C A R}}} \cdot \cos \alpha_{C A R}\right] / \bar{q} \cdot A_{\text {CAR }} \text {, } \\
& c_{\ell_{C g_{C A R}}}=G_{X_{A}} / \bar{q} \cdot A_{C A R} \cdot \sigma_{C A R}, \\
& c_{m_{C g}}=G_{Y_{A R}} / \bar{q} \cdot A_{C A R} \cdot \bar{C}_{C A R} \text {, } \\
& c^{n^{c g_{C A R}}}={ }^{G_{Z_{A}}}{ }_{C A R} / \bar{q} \cdot A_{C A R} \cdot \sigma_{C A R},
\end{aligned}
$$

$$
\begin{aligned}
& \text { [3.3.40] }
\end{aligned}
$$

$$
\begin{aligned}
& \text { [3.3.41] }
\end{aligned}
$$

## RESULTS

To evaluate numerical results of the derived equations [3.3.20] thru [3.3.41], the Space Vehicle Dynamics Simulation (SVDS) Program is used to generate the parameters required that will be recorded during an ALT flight. The mass properties used in this example are for flight number 1 with the tailcone on as defined in Reference 3 , the incidence angle is set at 6.5 deg , and the time point is at the instant separation would occur. Tabulated in Table 1 are the parameters required, their SVDS mnemonic names, their values, and their units as output by the SVDS. The results of the derived equations ([3.3.20] thru [3.3.41]) for the orbiter and carrier aerodynamic coefficients are tabulated in Tables 2 and 3 respectively. For the ease of comparison, the actual values of the coefficients computed by the SVDS and the percent error in the hand computed coefficients are also tabulated in Tables 2 and 3. In this analysis, all computations were carried out to 8 significant digits.

Table 2 shows that the range of error in the hand computed values of the orbiter coefficients compared to the values output by the SVDS is $0.087 \%$ to $0.0936 \%$. This is much smaller than the $5 \%$ accuracy of the dynamic pressure (see Reference 1).

Table 3 shows no difference in the carrier coefficients out to 5 significant digits. The largest error encountered is less than $0.00032 \%$ and the smallest is $0 \%$. These errors are accounted for by the truncation inherent in digit computers.

### 5.0 CONCLUSIONS AND RECOMMENDATIONS

From the above plus the results discussed in Section 4.0, it is concluded that the equations derived in Section 3.0 will yield acceptable results within the operating region of the ALT flights. It is therefore recommended that equations [3.3.20] thru [3.3.41] be used in the determination of the orbiter and carrier aerodynamic coefficients from load cell measurements.

### 6.0 REFERENCES

1. RI document SD73-SH-0180E, "Space Shuttle Separation System Data Book", dated November 1975.
2. MDTSCO TM No. 1.4-7-0-18, "Orbiter/747 Loads Model in SVDS", dated 16 May 1975.
3. MDTSCO TM No. 1.4-7-159, "Data Base Update for ALT Orbiter/ Carrier Separation Analysis", dated 23 December 1975.

FIGURE 1 GEOMETRY OF THE ORBITER/CARRIER

figure 2 relative position yectors


TABLE 1 DATA REQUIRED FOR ANALYSIS

| PARAMETER | SVOS MNEMONIC | VALUE | UNITS |
| :---: | :---: | :---: | :---: |
| $\vec{F}_{F}$ | $\begin{aligned} & \operatorname{FFV}(1) \\ & \operatorname{FFV}(2) \\ & \operatorname{FFV}(3) \end{aligned}$ | $\begin{array}{r} 1.5504 \times 10^{3} \\ -1.6446 \times 10^{-4} \\ 2.6494 \times 10^{4} \end{array}$ | LB |
| $\vec{F}_{L A}$ | $\begin{aligned} & \text { FAL }(1) \\ & \text { FAL (2) } \\ & \text { FAL (3) } \end{aligned}$ | $\begin{aligned} & 1.3177 \times 10^{3} \\ & 0.0 \\ & 2.6758 \times 10^{4} \end{aligned}$ | LB |
| $\vec{F}_{R A}$ | $\begin{aligned} & \operatorname{FAR}(1) \\ & \operatorname{FAR}(2) \\ & \operatorname{FAR}(3) \end{aligned}$ | $\begin{array}{r} 1.2637 \times 10^{3} \\ -1.8641 \times 10^{4} \\ 2.6363 \times 1 \end{array}$ | LB |
| $\stackrel{3}{a}_{\text {SAR }}$ | SAC47X <br> SACA7Y <br> Sf:472 | $\begin{aligned} & -4.3488 \\ & -1.3629 \times 10^{-2} \\ & -3.1595 \times 10^{1} \end{aligned}$ | $\mathrm{ft} / \mathrm{sec}^{2}$ |
| $\omega$ | OHEGX OMEGY CMEGZ | $\begin{array}{r} 7.3742 \times 10^{-3} \\ -1.5785 \times 10^{-2} \\ 2.3054 \times 10^{-2} \end{array}$ | deg/sec |
| $\stackrel{+}{\omega}$ | OMEGXD OHEGYD ONEGZD | $\begin{array}{r} -2.0051 \times 10^{-4} \\ 5.6490 \times 10^{-3} \\ 8.9711 \times 10^{-5} \end{array}$ | deg/sec ${ }^{2}$ |
| $\vec{F}_{T}$ | FBTMX FBTMY FBTMZ | $\begin{aligned} & 9.8046 \times 10^{3} \\ & 0.0 \\ & -4.2834 \times 10^{2} \end{aligned}$ | Lb |
| $\bar{\square}$ | QBAR | $2.3482 \times 10^{2}$ | $L B / f t^{2}$ |
| ${ }^{\alpha}$ CAR | ALPHA | 2.6111 | deg |

? 2 E 2 RESULTS OF DERIVED EQUATIONS (ORBITER.)

| COEFFICIENT | SVDS MNEMONIC | HAND COMPUTED | OUTPUT BY SVDS | PERCENT ERROR |
| :---: | :---: | :---: | :---: | :---: |
| ${ }^{C_{A}}{ }_{\text {ORB }}$ |  | $-2.2565 \times 10^{-3}$ | $-2.2586 \times 10^{-3}$ | 0.0932 |
| ${ }^{C_{Y}}{ }_{\text {ORB }}$ | CY | $-9.8334 \times 10^{-5}$ | $-9.8424 \times 10^{-5}$ | 0.0907 |
| ${ }^{C^{\text {N }} \text { ORB }}$ |  | $3.6130 \times 10^{-1}$ | $3.6164 \times 10^{-1}$ | 0.0935 |
| ${ }^{C_{D_{\text {ORE }}}}$ | CD | $5.4983 \times 10^{-2}$ | $5.5034 \times 10^{-2}$ | 0.0930 |
| $C_{L_{\text {OP, }}}$ | CL | $3.5710 \times 10^{-1}$ | $3.5743 \times 10^{-1}$ | 0.0936 |
| ${ }^{c_{\ell}} \mathrm{cg}_{0 R B}$ | CLL | $1.1839 \times 10^{-4}$ | $1.1850 \times 10^{-4}$ | 0.0926 |
| $\mathrm{C}_{\mathrm{m}_{\mathrm{CG}}^{0 R B}}$ | CM | $1.4610 \times 10^{-2}$ | $1.4624 \times 10^{-2}$ | 0.0936 |
| ${ }^{c_{n}}{ }_{C g_{0 R B}}$ | CLII | $1.1642 \times 10^{-6}$ | $1.1652 \times 10^{-6}$ | 0.0870 |
| ${ }^{C_{\ell}}{ }_{\text {ORB }}$ |  | $1.2156 \times 10^{-2}$ | $1.2167 \times 10^{-2}$ | 0.0935 |
| $c_{m_{0 R B}}$ |  | -1.5346 | -1.5360 | 0.0935 |
| $c_{n_{O R B}}$ |  | $4.9799 \times 10^{-4}$ | $4.9844 \times 10^{-4}$ | 0.0910 |


| COEFFICIENT | SVDS MINEMOMIC | $\begin{gathered} \text { HAND } \\ \text { COMPUTED } \end{gathered}$ | $\begin{aligned} & \text { OUTPUT BY } \\ & \text { SVOS } \end{aligned}$ | PERCENT ERROR |
| :---: | :---: | :---: | :---: | :---: |
| ${ }^{c^{\text {CAR }}}$ |  | $4.0403 \times 10^{-2}$ | $4.0403 \times 10^{-2}$ | less than .00032 |
| ${ }^{c_{Y}}{ }_{\text {CAR }}$ | CY | $-1.1430 \times 10^{-4}$ | $-1.1430 \times 10^{-4}$ |  |
| ${ }^{\mathrm{N}_{\text {CAR }}}$ |  | $1.9965 \times 10^{-1}$ | $1.9965 \times 10^{-1}$ |  |
| ${ }^{\mathrm{D}_{\text {CAR }}}$ | CD | -4.9456 $\times 10^{-2}$ | $4.9456 \times 10^{-2}$ |  |
| ${ }^{c_{L_{\text {CAR }}}}$ | CL | $1.9760 \times 10^{-1}$ | $1.9760 \times 10^{-1}$ |  |
| ${ }^{c_{\ell}{ }_{c g_{C A R}}}$ | CLL | -1.2846 $\times 10^{-5}$ | $-1.2846 \times 10^{-5}$ |  |
| $c_{m_{c g_{C A R}}}$ | CN | -1.3553 $\times 10^{-2}$ | -1.3553 $\times 10^{-2}$ |  |
| $c_{n_{c g_{C A R}}}$ | CLN | $2.0738 \times 10^{-6}$ | $2.0738 \times 10^{-6}$ |  |
| ${ }^{c_{\ell_{C A R}}}$ |  | $2.9100 \times 10^{-4}$ | $2.9100 \times 10^{-4}$ |  |
| ${ }^{\text {cmar }}$ |  | $7.3718 \times 10^{-2}$ | $7.3718 \times 10^{-2}$ |  |
| ${ }^{\text {n }}$ cAR |  | $-1.0270 \times 10^{-4}$ | $-1.0270 \times 10^{-4}$ | $V$ |

