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THE STRESS DISTRIBUTION IN TEMPERED GLASS DUE TO A CRACK

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RESEARCH

BY

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THE STRESS DISTRIBUTION IN TEMPERED GLASS DUE TO A CRACK

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ABSTRACT

A model describing the failure in tempered glass is proposed and a method of solution is presented. An infinite elastic strip is assumed to represent the glass and the loads vanish everywhere on the boundary as well as at infinity. The problem will be solved using the integral equations technique where the input will be the residual stresses in the glass.

INTRODUCTION

A sudden failure of a tempered glass is an important as well as a practical problem which requires a closer look as far as the fracture initiation is concerned. A practical model will be chosen as follows:

Consider two semi-infinite strips which have curved surfaces at one of their ends defined by a function $y = f(x)$ (See fig. 1). According to this model, the actual situation in a tempered glass will be obtained if these two curved surfaces are brought together and bonded along a line $-a < x < a$ (fig. 1). At both ends along the x axis a frictionless contact is assumed to take place ($b<|x|<h$). Hence, $f(x)$ will be the input function taking the place of residual stresses. The problem can be formulated and solved under these conditions and considering the appropriate singularities at $x = \pm a$ and $x = \pm b$.

FORMULATION OF THE PROBLEM

Let the elastic constants of the strip be E and ν. The thickness is assumed to be 2h. Then the boundary conditions will be

$$\sigma_x(\pm h,y) = \tau_{xy}(\pm h,y) = 0, \quad 0\leq y<\infty \tag{1}$$

$$\tau_{xy}(x,0) = 0 \quad , -h\leq x\leq h \tag{2}$$

$$v(x,0) = -f(x)+v_o \quad ,|x|<a \text{ and } b<|x|<h \tag{3}$$

$$\sigma_y(x,0) = 0, \quad a<|x|<b \tag{4}$$

$(\sigma_x, \sigma_y, \tau_{xy}) \to 0$ at infinity

(See Fig. 1 for v_o)

Where $\sigma_x(x,y)$, $\sigma_y(x,y)$ and $\tau_{xy}(x,y)$ are the stresses and $u(x,y)$, $v(x,y)$ will represent the displacements. A set of solution of the Navier's equations are

$$u(x,y) = -\frac{2}{\pi}\int_0^\infty \frac{A(t)}{t}\left(\frac{\kappa-1}{2} - ty\right) e^{-ty} \sin(tx)\, dt$$

$$-\frac{2}{\pi}\int_0^\infty \left\{\frac{1}{t}\left[B(t) - \frac{\kappa-1}{2} C(t)\right] \sinh(tx)\right.$$

$$\left. + x\, C(t) \cosh(tx)\right\} \cos(ty)\, dt$$

$$v(x,y) = \frac{2}{\pi}\int_0^\infty \frac{A(t)}{t}\left(\frac{\kappa+1}{2} + ty\right) e^{-ty} \cos(tx)\, dt$$

$$+\frac{2}{\pi}\int_0^\infty \left\{\frac{1}{t}\left[B(t) + \frac{\kappa+1}{2} C(t)\right] \cosh(tx)\right.$$

$$\left. + x\, C(t) \sinh(tx)\right\} \sin(ty)\, dt \tag{5}$$

where A(t), B(t), C(t) are the new unknowns

and

$$\kappa = \begin{cases} 3-4\nu & \text{, for plane strain} \\ (3-\nu)/(1+\nu), & \text{for generalized plane stress} \end{cases} \tag{6}$$

The stresses are then determined as

$$\frac{\sigma_x}{2\mu} = -\frac{2}{\pi} \int_0^\infty A(t)\,(1-ty)\,e^{-ty} \cos(tx)\,dt$$

$$-\frac{2}{\pi} \int_0^\infty [B(t)\cosh(tx) + tx\,C(t)\sinh(tx)]\cos(ty)\,dt$$

$$\frac{\sigma_y}{2\mu} = -\frac{2}{\pi} \int_0^\infty A(t)\,(1+ty)\,e^{-ty} \cos(tx)\,dt +$$

$$\frac{2}{\pi} \int_0^\infty \{[B(t) + 2\,C(t)]\cosh(tx)$$

$$+ tx\,C(t)\sinh(tx)\}\cos(ty)\,dt$$

$$\frac{\tau_{xy}}{2\mu} = -\frac{2}{\pi} \int_0^\infty yt\,A(t)\,e^{-ty} \sin(tx)\,dt$$

$$+\frac{2}{\pi} \int_0^\infty \{[B(t) + C(t)]\sinh(tx)$$

$$+ tx\,C(t)\cosh(tx)\}\sin(ty)\,dt \tag{7}$$

$$\mu = E/2(1+\nu)$$

Hence the symmetry conditions are automatically satisfied. Using the stress expressions (7) and an inversion, the first set of boundary conditions (1) can be expressed as

$$B(t) + ht\,C(t)\tanh(ht) = \Phi_1(t)$$

$$= -\frac{4t^2}{\pi\cosh(ht)} \int_0^\infty \frac{\xi\,A(\xi)}{(\xi^2 + t^2)^2} \cos(h\xi)\,d\xi$$

$$B(t)\tanh(ht) + [\tanh(ht) + ht]\,C(t) = \Phi_2(t)$$

$$= \frac{4t}{\pi \cosh (ht)} \int_0^\infty \frac{\xi^2 A(\xi)}{(\xi^2 + t^2)^2} \sin(h\xi)\, d\xi \qquad (8)$$

Second boundary conditions (2) is automatically satisfied by (7). The mixed conditions (3) and(4) give

$$v(x,0) = \frac{\kappa+1}{\pi} \int_0^\infty \frac{A(t)}{t} \cos(tx)\, dt = -f(x) + v_o$$
$$|x|<a, b<|x|<h \qquad (9)$$

$$\frac{\sigma_y(x,0)}{2\mu} = -\frac{2}{\pi} \lim_{y\to 0} \{\int_0^\infty A(t)\,(1+ty)\, e^{-ty} \cos(tx)\, dt$$

$$+\frac{2}{\pi} \int_0^\infty \{[B(t)+2\, C(t)] \cosh\, (tx)$$

$$+tx\, C(t) \sinh\, (tx)\, \} \cos(ty)\, dt\} \qquad (10)$$

Defining

$\phi(x) = \frac{\partial v(x,0)}{\partial x}$ such that

$$\phi(x) = -f'(x) \text{ for } |x|<a \quad ,b<|x|<h \qquad (11)$$

and differentiating and inverting (9), A(t) can be expressed in terms of $\phi(x)$. Moreover, solving (8) for B(t) and C(t) and hence expressing these also in terms of $\phi(x)$ and substituting all these into (10) and separating singularities,one arrives at the following singular integral equation:

$$\frac{\pi(1+\kappa)}{4\mu}\sigma_y(x,0) = \int_a^b \left[\frac{1}{t+x} + \frac{1}{t-x} + k(x,t)\right]\phi(t)\, dt - g(x) = 0$$
$$a<x<b \qquad (12)$$

where

$$g(x) = \left(\int_0^a + \int_b^h\right)\left[\frac{1}{t+x} + \frac{1}{t-x} + k(x,t)\right] f'(t)\, dt \qquad (13)$$

and

$$k(x,t) = \int_0^\infty [K(x,t,\eta)\,\bar{e}^{(h-t)\eta} - K(x,-t,\eta)\,\bar{e}^{(h+t)\eta}]d\eta \qquad (14)$$

and

$$K(x,t,\eta) = \Big[\,(x\eta)\sinh(x\eta)\,[-1+2\eta(h-t)-\bar{e}^{2h\eta}\,]$$
$$+ \cosh(x\eta)\,\{\eta h-2-(2\eta h-3)(h-t)\eta + [(h-t)\eta-(h\eta+2)]$$
$$\bar{e}^{2\eta h}\}\,\Big]/\{\,[2h\eta + \sinh(2h\eta)]\bar{e}^{h\eta}\} \qquad (15)$$

where the singularity is to be taken care of appropriately and the single-valuedness of the displacements will be expressed as

$$\int_a^b \phi(t)\,dt = f(a)-f(b) \qquad (16)$$

To solve (12) and (16) non-dimensional variables will be used, i.e.:

$$x = \frac{b+a}{2} + \frac{b-a}{2}\,\chi$$
$$t = \frac{b+a}{2} + \frac{b-a}{2}\,\tau \qquad (17)$$

Then(12) and (16) become

$$\frac{1}{\pi}\int_{-1}^{1} \frac{\phi_o(\tau)}{\tau-\chi}\,d\tau + \int_{-1}^{1} k_o(\chi,\tau)\,\phi_o(\tau)\,d\tau$$
$$= g_o(\chi),\ |\chi|<1$$
$$\int_{-1}^{1} \phi_o(\tau)d\tau = \frac{2}{b-a}\,[f(a)-f(b)] \qquad (18)$$

where

$$\phi_o(\tau) = \phi(t)\ ,g_o(\chi) = g(x)/\pi$$

$$k_o(\chi,\tau) = \frac{(b-a)}{2\pi}\,[\frac{1}{t+x} + k(x,t)] \qquad (19)$$

Choosing the unknown as

$$\phi_o(\tau) = F(\tau)\sqrt{\frac{1-\tau}{1+\tau}} = F(\tau)\sqrt{\frac{b-t}{t-a}} = \phi(t)$$

$$, |\tau| < 1 \qquad (20)$$

(18) can be solved numerically [1],

$$\sum_{k=1}^{n} \frac{2(1-\tau_k)}{2n+1} \quad F(\tau_k)\left[\frac{1}{\tau_k - x_r} + \pi k_o(x_r, \tau_k)\right] = g_o(x_r)$$

$$r = 1, \text{---} n \qquad (21)$$

$$\sum_{k=1}^{n} \frac{2(1-\tau_k)}{2n+1} \quad F(\tau_k) = \frac{2}{\pi(b-a)} \left[f(a) - f(b)\right] \qquad (22)$$

for $F(\tau_k)$, k=1,---n and "b". Here,

$$\tau_k = \cos\left(\frac{2k\pi}{2n+1}\right), \qquad k=1, \text{---} n$$

$$x_r = \cos\left(\frac{2r-1}{2n+1}\pi\right) \qquad r=1, \text{---} n \qquad (23)$$

Specifying a and choosing b, equation (21) is solved for $F(\tau_k)$ and equation (22) is checked. An iteration is needed to determine b such that equ. (22) is also satisfied. The stresses on the x axis can be found from (12) and (20).

THE STRESS INTENSITY FACTOR

The stress intensity factors can be found from (12) and [2] by investigating the behavior of the Cauchy integrals around the end points. Hence defining

$$K(a) = \lim_{x \to a} \sqrt{2(a-x)} \quad \sigma_y(x,0) \qquad (24)$$

one obtains

$$K(a) = \frac{4\mu}{1+\kappa} \sqrt{2(b-a)} \quad F(-1) \qquad (25)$$

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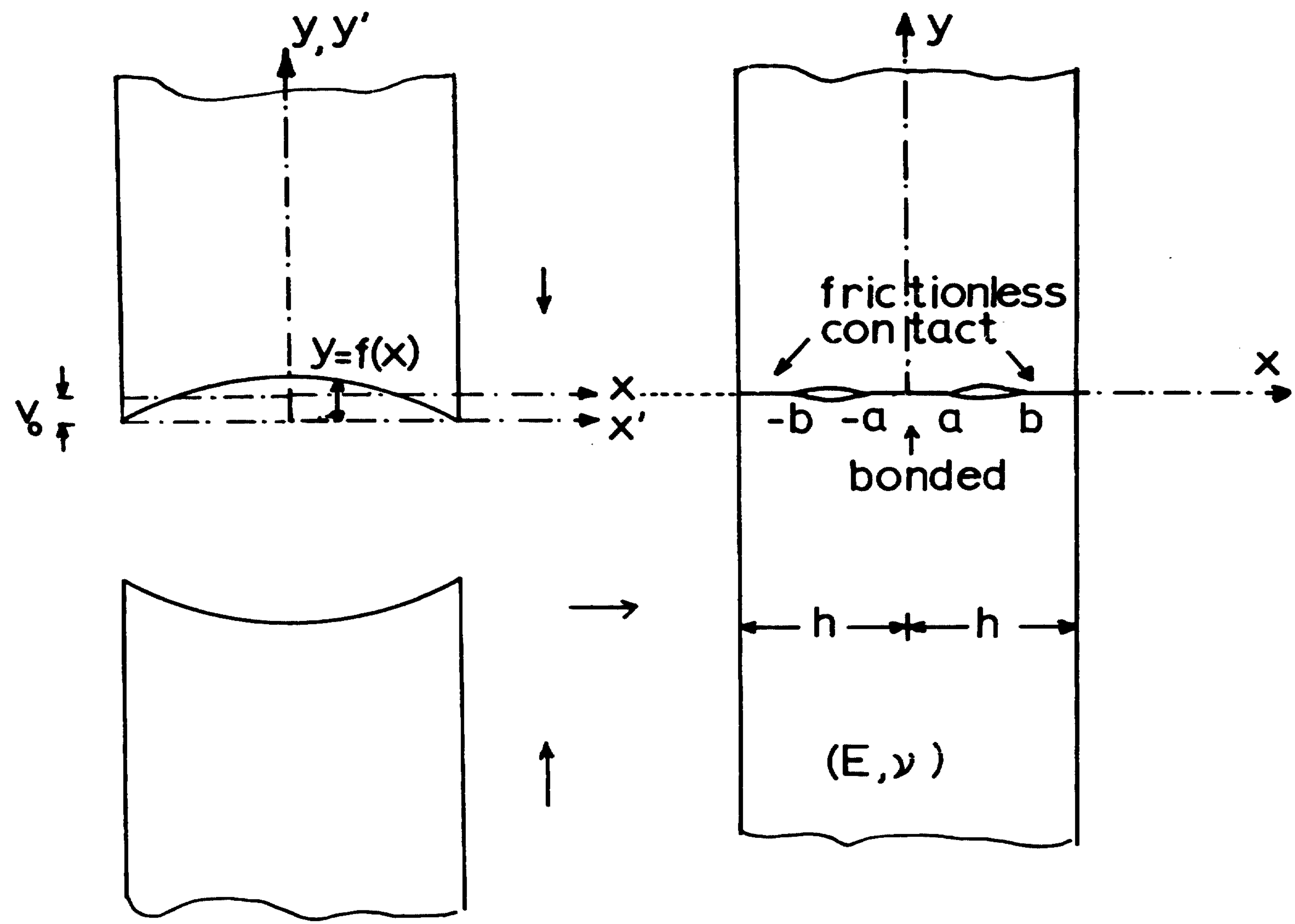

Fig. 1

The Geometry of the Problem