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ABSTRACT

Several intact or broken stringers which are continuously attached to a cracked orthotropic sheet through an adhesive are considered. The effect of orthotropy on the stress intensity factors is investigated. The stringers are assumed to be partially debonded due to high stress concentrations. The shear stress distribution between the stringers and the plate and the stress intensity factors are obtained from an integral equation which represents the continuity of displacements along the bond lines.

INTRODUCTION

Several problems have been considered recently regarding the metal sheets stiffened by a number of stringers [1-5]. The stringers are assumed to help decrease the probability of fracture initiation and/or propagation of an existing crack. As a result of these studies for an isotropic sheet, it has been found that the stiffening effect of stringers depends on several parameters, i.e., the distances between the mid point of the crack and the stringers, the debond lengths, etc [5]. Also, the adverse effect of stringer breakage has been shown [5].

In this paper, the effect of orthotropy will be investigated. The loading will be on the crack surfaces (intact stringers) as well as at infinity (broken stringers) [5]. There will be no restriction as far as the number and the locations of the stringers are concerned. The method used here consists of obtaining the integral equation of the problem by the use of Green's functions. Results for orthotropic plate are compared with the case of isotropic plate to illustrate the difference.

FORMULATION OF THE PROBLEM

In formulating the problem, the following assumptions will be made: The adhesive will be treated as a shear spring and the plate will be in a state of generalized plane stress. The shear stresses will be considered as body forces in the plate analysis. The input q will represent either the uniform pressure on the crack surface or the uniform tension at infinity.

Using the notation described below*, the continuity of displacements can be expressed as [5],

$$v_p(z) - v_s(z) = \frac{h_a}{d_s \mu_a} P(z), \; z \text{ on } L \qquad (1)$$

Here, L denotes the union of straight lines L_j defined by $x = c_j$, $b_j \leq y < \infty$; $j = 1,\ldots n_s$ where n_s is the number of stringers, b_j is the half debond length of the jth stringer and c_j is the distance between the jth stringer and the midpoint of the crack.

As in [5], the displacements can be expressed as

$$v_p(z) = qk_o(z) + \int_L k_p(z,z_o)\, P(z_o) dy_o$$

$$v_s(z) = \int_L k_s(z,z_o)\, P(z_o) dy_o + C, \; z \text{ on } L \qquad (2)$$

$$y_o = \text{Im}(z_o)$$

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*$(E_x, E_y, \nu_{yx}, G_{xy})$	:	Elastic constants of the plate
(E_s, A_s)	:	Elastic modulus and cross-sectional area of the stringer
μ_a	:	Shear modulus of the adhesive
(h_p, h_a)	:	Thickness of the plate and the adhesive
a	:	Half crack length
d_s	:	Stringer width
$v_p(z)$, $v_s(z)$	:	Displacements of the plate and the stringer at z location
P(z)	:	Shear stress in the adhesive at z location

where C represents the rigid body displacements assuming a different value on each stringer. Hence,

$$C = C_j, \ j = 1,\dots n_s \ \text{for } z \text{ on } L_j \tag{3}$$

On the other hand, from [6], $k_0(z)$ can be expressed as in [5],

$$k_0(z) = \begin{cases} k^*(z), \ \text{for crack surface loading} \\ \\ k^*(z) + \frac{1}{E_y} y, \ \text{for loading at infinity} \end{cases} \tag{4}$$

$$y = \text{Im}(z)$$

and

$$k^*(z) = \text{Re}\ \{\frac{1}{\mu_1 - \mu_2} [q_1 \mu_2 (z_1 - \sqrt{z_1^2 - a^2})$$

$$- q_2 \mu_1 (z_2 - \sqrt{z_2^2 - a^2})]\} \tag{5}$$

where

$$z_m = c_j + \mu_m y, \ m = 1,2; \ z \text{ on } L_j \tag{6}$$

and for orthotropic materials and the generalized plane stress,

$$q_m = \frac{1}{E_y} (\frac{1}{\mu_m} - \nu_{yx} \mu_m), \ m = 1,2 \tag{7}$$

and μ_1, μ_2 are the roots of (see [6])

$$\mu^4 + (\frac{E_x}{G_{xy}} - 2\nu_{xy})\,\mu^2 + \frac{E_x}{E_y} = 0 \tag{8}$$

$$(E_x\nu_{yx} = E_y\nu_{xy})$$

$k_p(z,z_o)$ can be determined as in the isotropic case, by superposition and using the formulation set forth in [6]. Hence, $k_p(z,z_o)$ can be expressed as [7],

$$k_p(z,z_o) = \frac{1}{\pi h}\,[-\theta(z,z_o) + \theta(z,\bar{z}_o)] \tag{9}$$

where

$$\theta(z,z_o) = \text{Re}\,\{iq_1\,[\frac{1}{2(\mu_2-\mu_1)}\Big((\mu_2-\mu_1)c_{12}J(z_1,z_{10})$$

$$- (\mu_2-\bar{\mu}_1)\bar{c}_{12}J(z_1,\bar{z}_{10}) - (\mu_2-\bar{\mu}_2)\bar{c}_{22}J(z_1,\bar{z}_{20})\Big)$$

$$- c_{12}\log(z_1-z_{10})] - iq_2\,[\frac{1}{2(\mu_2-\mu_1)}\Big((\mu_1-\mu_2)c_{22}$$

$$J(z_2,z_{20}) - (\mu_1-\bar{\mu}_2)\bar{c}_{22}J(z_2,\bar{z}_{20}) - (\mu_1-\bar{\mu}_1)\bar{c}_{12}$$

$$J(z_2,\bar{z}_{10})\Big) + \bar{c}_{22}\log(z_2-z_{20})]\} \tag{10}$$

$$z_{mo} = c_j + \mu_m y_o,\ m = 1,2;\ z \text{ on } L_j \tag{11}$$

and

$$J(z,z_0) = \log[\sqrt{z^2-a^2}\ \sqrt{z_0^2-a^2} + zz_0 - a^2]$$

$$- \log[z + \sqrt{z^2-a^2}] \tag{12}$$

and

$$c_{11} = \frac{c_1}{c_3},\ c_{12} = \frac{c_2}{c_3}$$

$$c_{21} = \frac{c_4}{c_6},\ c_{22} = \frac{c_5}{c_6} \tag{13}$$

and

$$c_1 = \mu_1[\mu_2 + \bar{\mu}_2 + \bar{\mu}_1\ (1 + \mu_2\bar{\mu}_2\nu_{yx})]$$

$$c_2 = \mu_1[\mu_2\bar{\mu}_2 + \nu_{xy} + \bar{\mu}_1(\mu_2+\bar{\mu}_2)]$$

$$c_3 = (\mu_1-\bar{\mu}_2)(\mu_1-\bar{\mu}_1)(\mu_1-\mu_2)$$

$$c_4 = \mu_2\ [\mu_1 + \bar{\mu}_1 + \bar{\mu}_2(1 + \mu_1\bar{\mu}_1\nu_{yx})] \tag{14}$$

$$c_5 = \mu_2\ [\mu_1\bar{\mu}_1 + \nu_{xy} + \bar{\mu}_2(\mu_1+\bar{\mu}_1)]$$

$$c_6 = (\mu_2-\bar{\mu}_1)(\mu_2-\bar{\mu}_2)(\mu_2-\mu_1)$$

Finally, $k_s(z,z_0)$ is given in [5] as:

For intact stringers

$$k_s(z,z_0) = \begin{cases} \left.\begin{array}{ll} \dfrac{y}{A_s E_s}, & y<y_0 \\ \dfrac{y_0}{A_s E_s}, & y>y_0 \end{array}\right] & \text{if z and } z_0 \text{ are on the same stringer} \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

and $C \equiv 0$, $y = \mathrm{Im}(z)$, $y_0 = \mathrm{Im}(z_0)$.

For broken stringers

$$k_s(z,z_0) = \begin{cases} \left.\begin{array}{ll} 0 & , y<y_0 \\ \dfrac{y_0 - y}{A_S E_S}, & y>y_0 \end{array}\right] & \text{if z and } z_0 \text{ are on the same stringer} \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

and C_j, $j = 1,\ldots n_s$ are unknown constants.

Hence, in the case of broken stringers, there will be n_s additional equilibrium equations to determine C_j, i.e.,

$$\int_{L_j} P(z_0)dy_0 = \begin{cases} -\dfrac{E_S A_S q}{E_y} & \text{if the load at infinity is also transferred to the stringer} \\ 0 & \text{if the end of the stringer at infinity is stress free} \end{cases}$$

$$j = 1,\ldots n_s \tag{17}$$

From (1) and (2), the integral equation of the problem can be obtained as

$$P(z) + \int_L k(z,z_0)P(z_0)dy_0 + \frac{d_s \mu_a}{h_a} C$$
$$= \frac{d_s \mu_a}{h_a} qk_0(z), \; z \text{ on } L \tag{18}$$

which will be solved together with (17) to obtain the shear stress distribution. Here,

$$k(z,z_0) = \frac{d_s \mu_a}{h_a} [k_s(z,z_0) - k_p(z,z_0)] \tag{19}$$

STRESS INTENSITY FACTOR

The stress intensity factor will be defined as

$$K_1 = \lim_{x \to a} [\sqrt{2(x-a)}]\sigma_y(x,o)$$

$$K_2 = \lim_{x \to a} [\sqrt{2(x-a)}]\tau_{xy}(x,o) \tag{20}$$

and given as [5], [7],

$$\frac{K_1}{\sqrt{a}} + \frac{K_2/\sqrt{a}}{\mu_2} = q + \int_L [\theta(\bar{z}_0) - \theta(z_0)]P(z_0)dy_0 \tag{21}$$

$$y_0 = Im(z_0)$$

where

$$\theta(z_0) = \frac{1}{2\pi i h a^* \mu_2} [(\mu_2-\mu_1)C_{12}J_0(z_{10})$$

$$- (\mu_2-\bar{\mu}_1)\bar{C}_{12}J_0(\bar{z}_{10}) - (\mu_2-\bar{\mu}_2)C_{22}J_0(\bar{z}_{20})] \tag{22}$$

and

$$J_0(z_0) = (a^* - z_0 + \sqrt{z_0^2-a^{*2}})/(a^* - z_0) \tag{23}$$

$$a^* = \begin{cases} a \text{ for right tip} \\ -a \text{ for left tip} \end{cases} \tag{24}$$

K_1 and K_2 can be found from (21) by comparing the real and the imaginary parts of the equation. For symmetric problem (w.r.t x axis), $K_2 \equiv 0$.

NUMERICAL RESULTS AND CONCLUSIONS

Numerical results will be obtained for uniformly spaced stringers, i.e.,

$$c_j = d_o + (j-1)d_1, \; j = 1,\ldots n_s \tag{25}$$

where d_1 is the stringer spacing and d_o is the distance of the first stringer to the mid-point of the crack. The following data will be used:

<u>Boron-Epoxy</u>

Plate:
$E_x = 3.5 \times 10^6$ psi
$E_y = 3.24 \times 10^7$ psi
$\nu_{yx} = 0.23$
$G_{xy} = 1.23 \times 10^6$ psi
$h_p = 0.09$ in

Stringer:
$A_S = 0.165$ in^2
$E_S = 1.24 \times 10^7$ psi

Adhesive:
$\mu_a = 1.65 \times 10^5$ psi
$h_a = 0.004$ in

q = constant

Since there are two types of orthotropic materials [8], the numerical results can be obtained for both types. However, for simplicity, only the boron-epoxy considered above (a Type I material) will be used. Aluminum plate ($\nu = 0.30$, $E = 10^7$ psi) will represent the isotropic case whenever a comparison is made.

Intact stringers

Figure 4 illustrates the $K/q\sqrt{a}$ variation vs. b_1/a for both orthotropic and isotropic plates. It is seen that the trend for both materials are the same. Hence, the conclusions for the isotropic case [5] remain the same for the orthotropic case. Since the contribution from a third stringer is rather insignificant, only one or two stringers are considered. In any case, an increasing debond length, especially beyond $b_1/a > 2\sim3$, increasesthe stress intensity factor appreciably. Figure 5 shows that the stiffening effect of the stringers diminish rather rapidly for $d_0/a > 2$ for both isotropic as well as orthotropic materials. Hence, the following conclusion more or less holds true for both isotropic [5] and orthotropic materials: To have a low stress intensity factor, there should be as many perfectly bonded stringers as possible between the two crack tips.

Broken stringers

Figures 6 and 7 show the dependence of $K/q\sqrt{a}$ values on the debond length b_1 and d_0 in the case of crack surface loading. This is given to compare with the intact stringer problem. An obvious observation for both isotropic as well as orthotropic materials is the sudden increase in the stress intensity factors when the breakage occurs which is quite significant.

However, the actual problem is the one where the loads are applied at infinity. This case is shown in Figures 8 and 9 where it is observed that the stress intensity factors increase beyond unity. This indicates that breakage is very important since it makes the structure more susceptible to fracture. Also note that the loads at infinity are applied to both the plate and the stringers simultaneously.

ACKNOWLEDGEMENT

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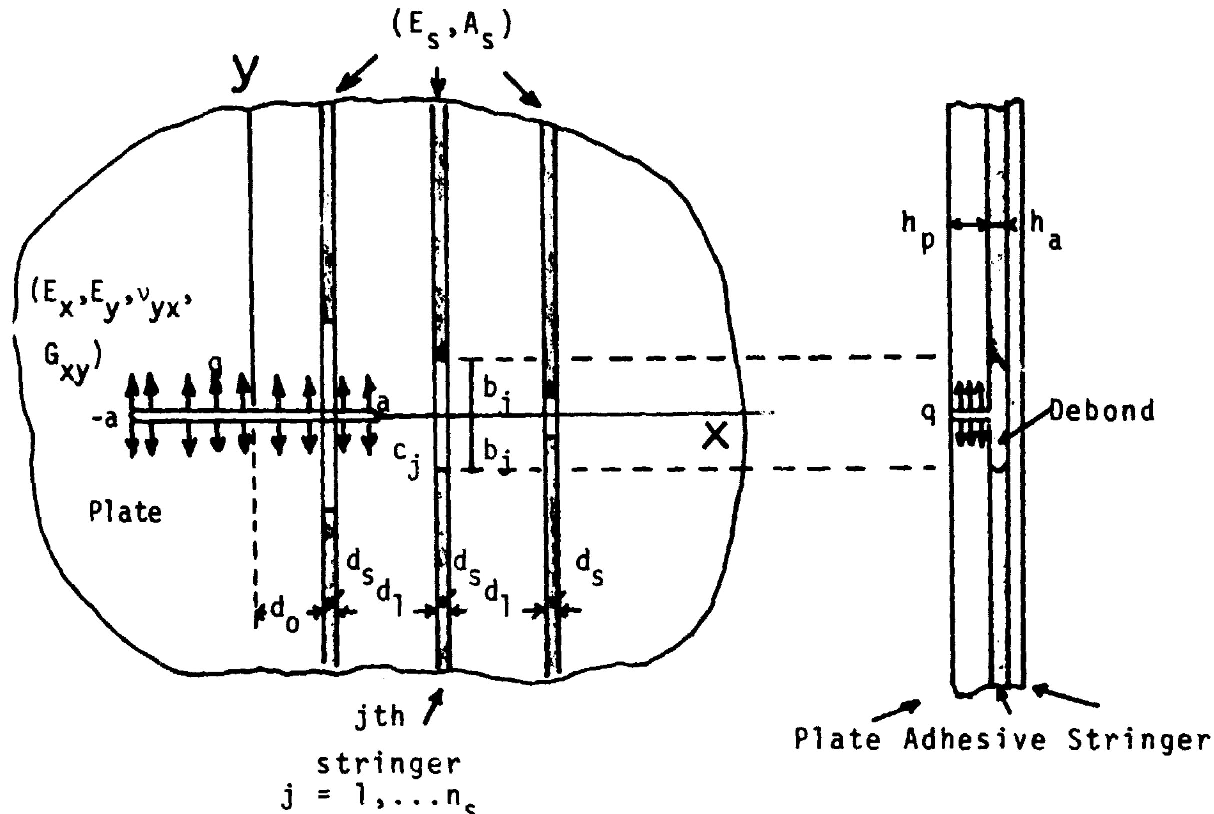


Figure 1 - Geometry of the problem (intact stringers)

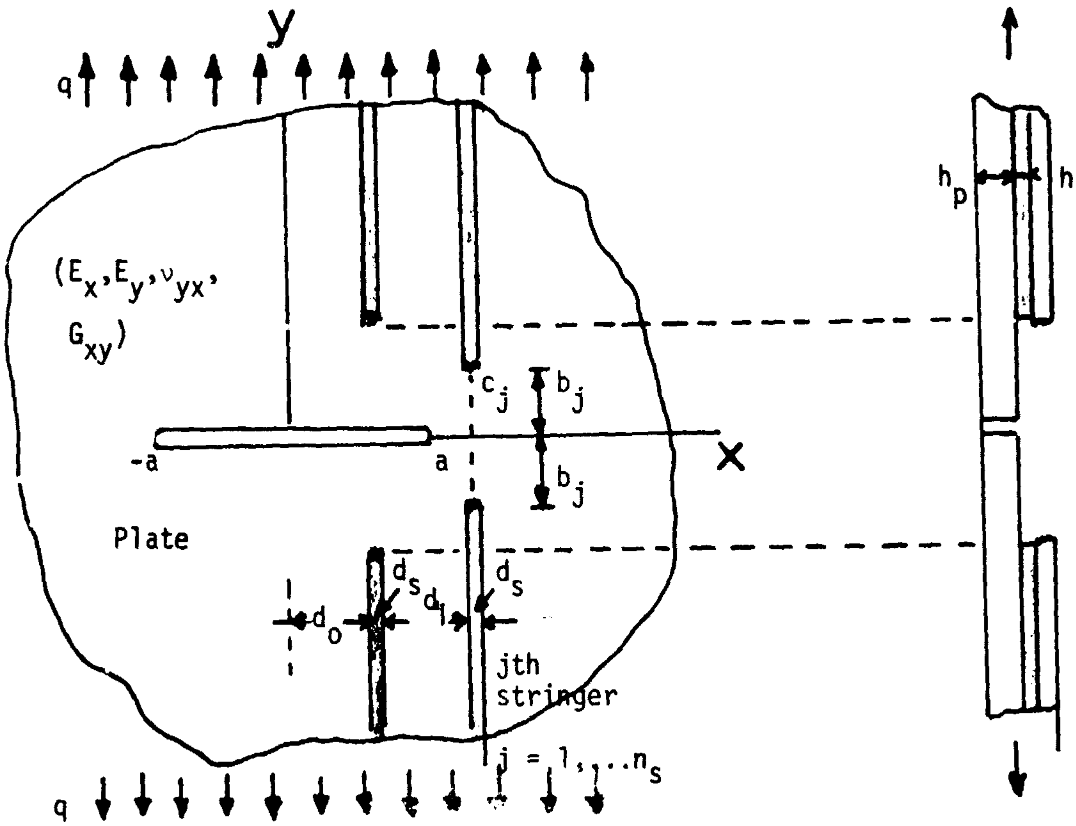
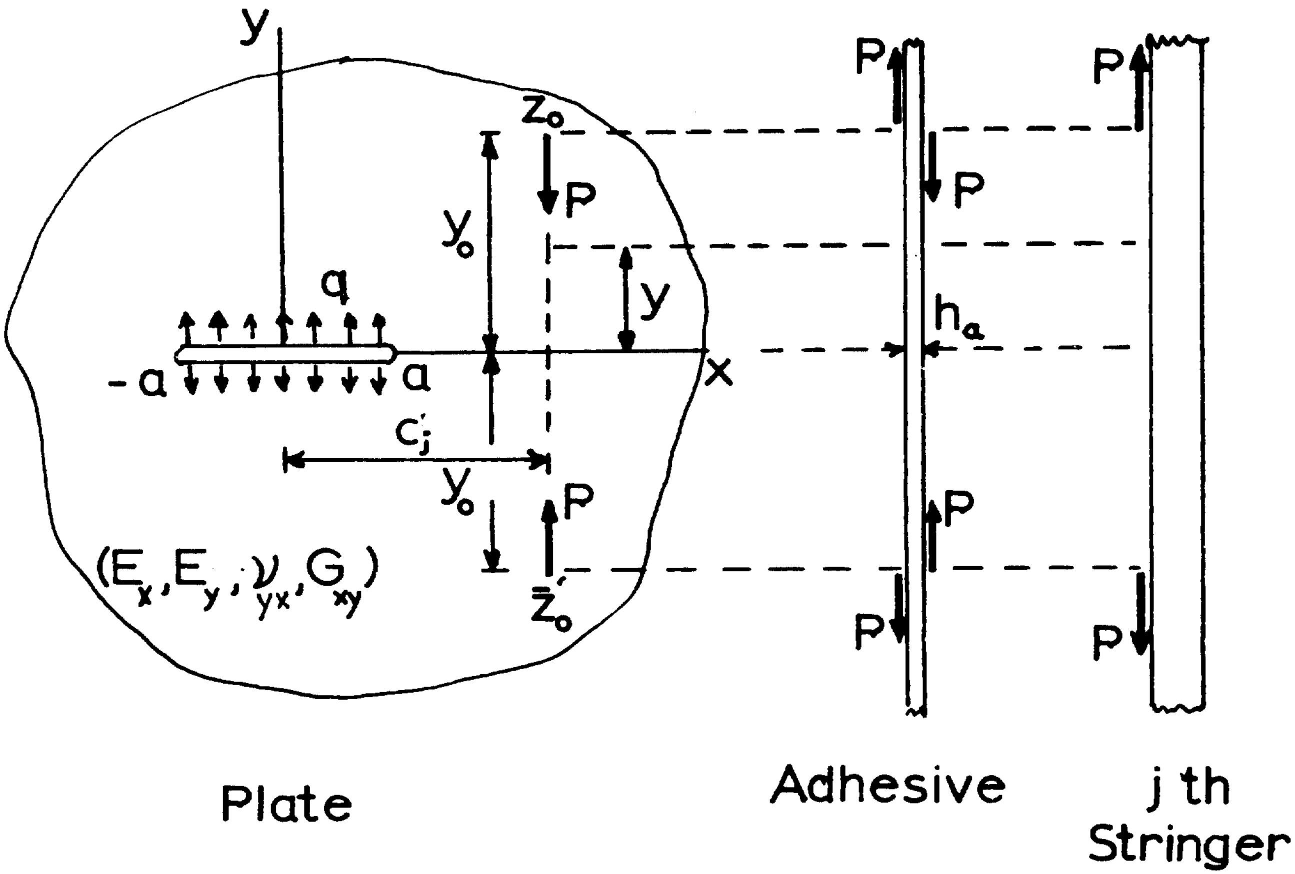


Figure 2 - Geometry of the problem (broken stringers)



$$z_o = c_j + iy_o$$

For uniformly spaced stringers: $c_j = d_o + (j-1)d_1$, $j = 1, \ldots n_s$

Figure 3 - Free-body diagrams

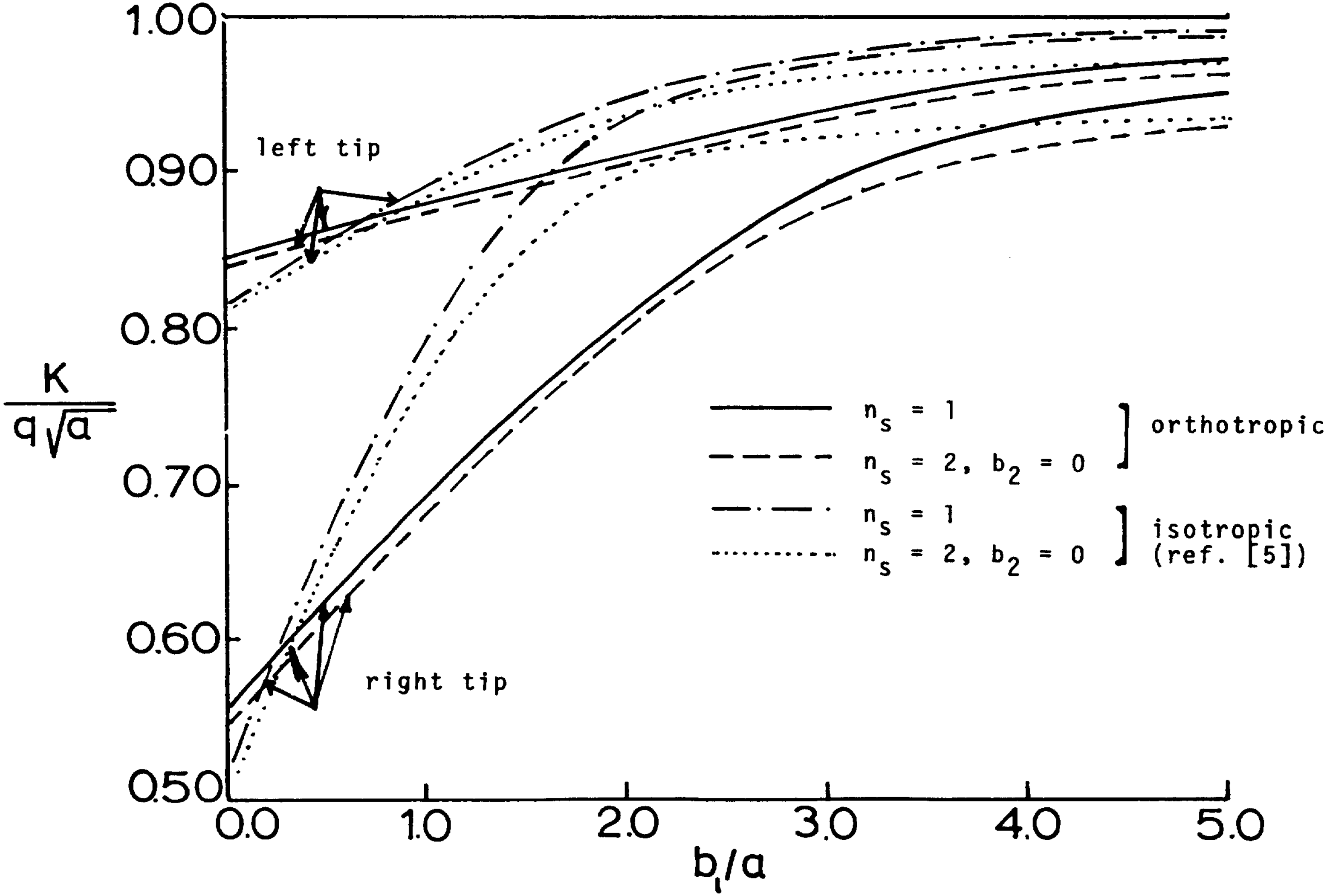


Figure 4 - $K/q\sqrt{a}$ vs. b_1/a (intact stringer, $d_0/a = 0.5$, $d_1/a = 1.0$, $d_s/a = 0.2$)

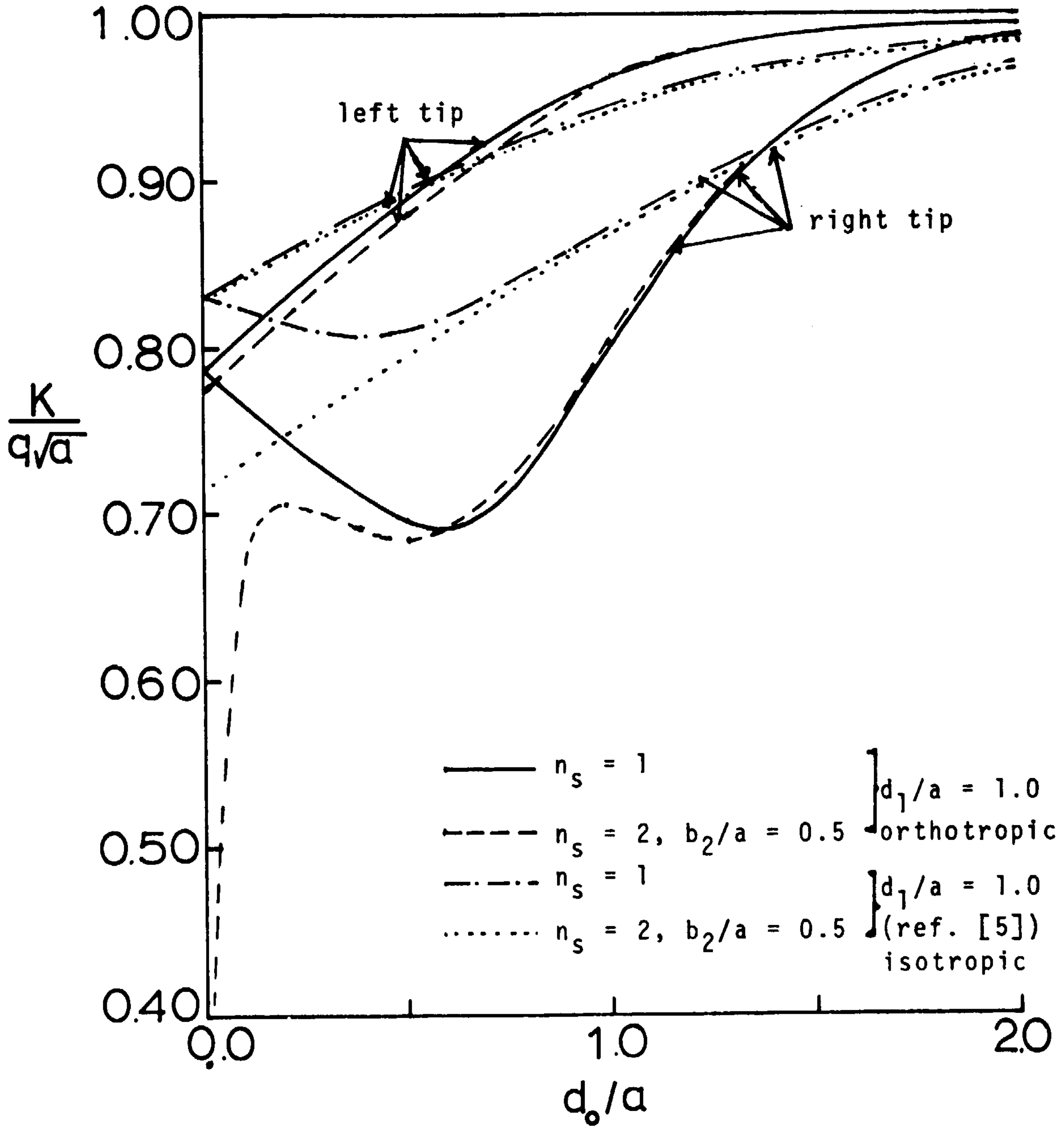


Figure 5 - $K/q\sqrt{a}$ vs. d_0/a
(intact stringer, $b_1/a = 1.0$, $d_s/a = 0.2$)

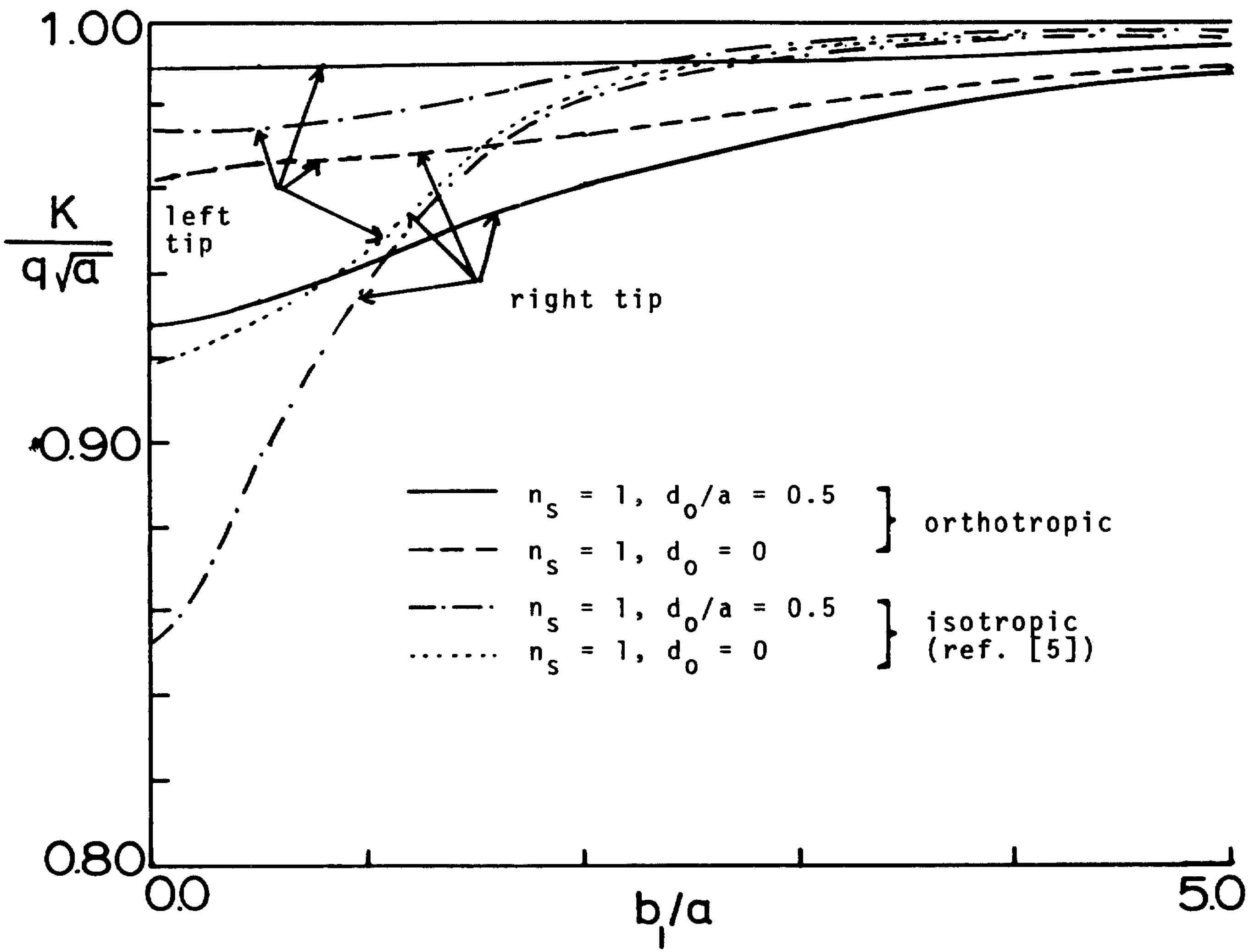


Figure 6 - $K/q\sqrt{a}$ vs. b_1/a (broken stringer, loading on the crack surfaces, $d_1/a = 0.5$, $d_s/a = 0.2$)

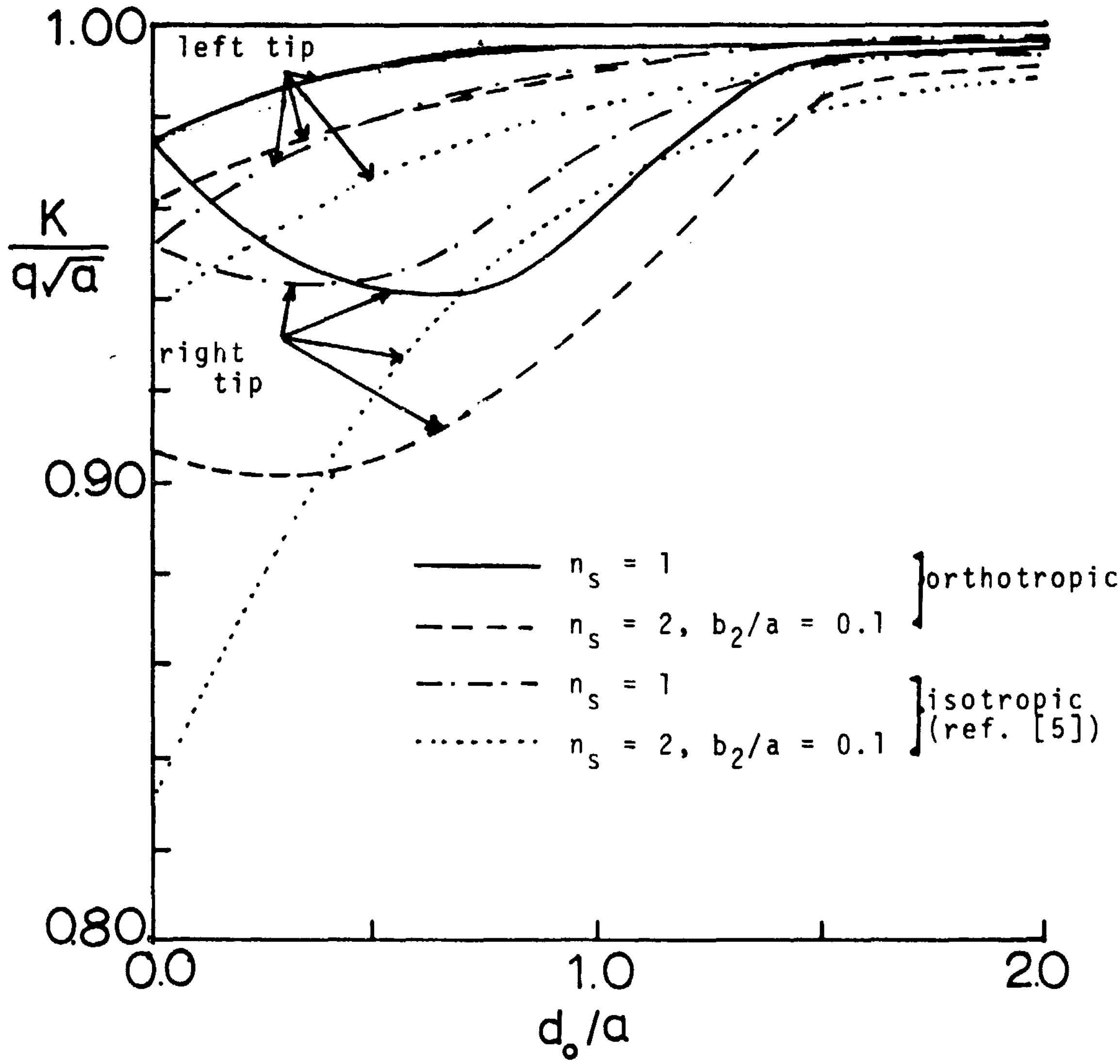


Figure 7 - $K/q\sqrt{a}$ vs. d_0/a
(broken stringer, loading on the crack surfaces, $b_1/a = 1.0$, $d_1/a = 0.5$, $d_s/a = 0.2$)

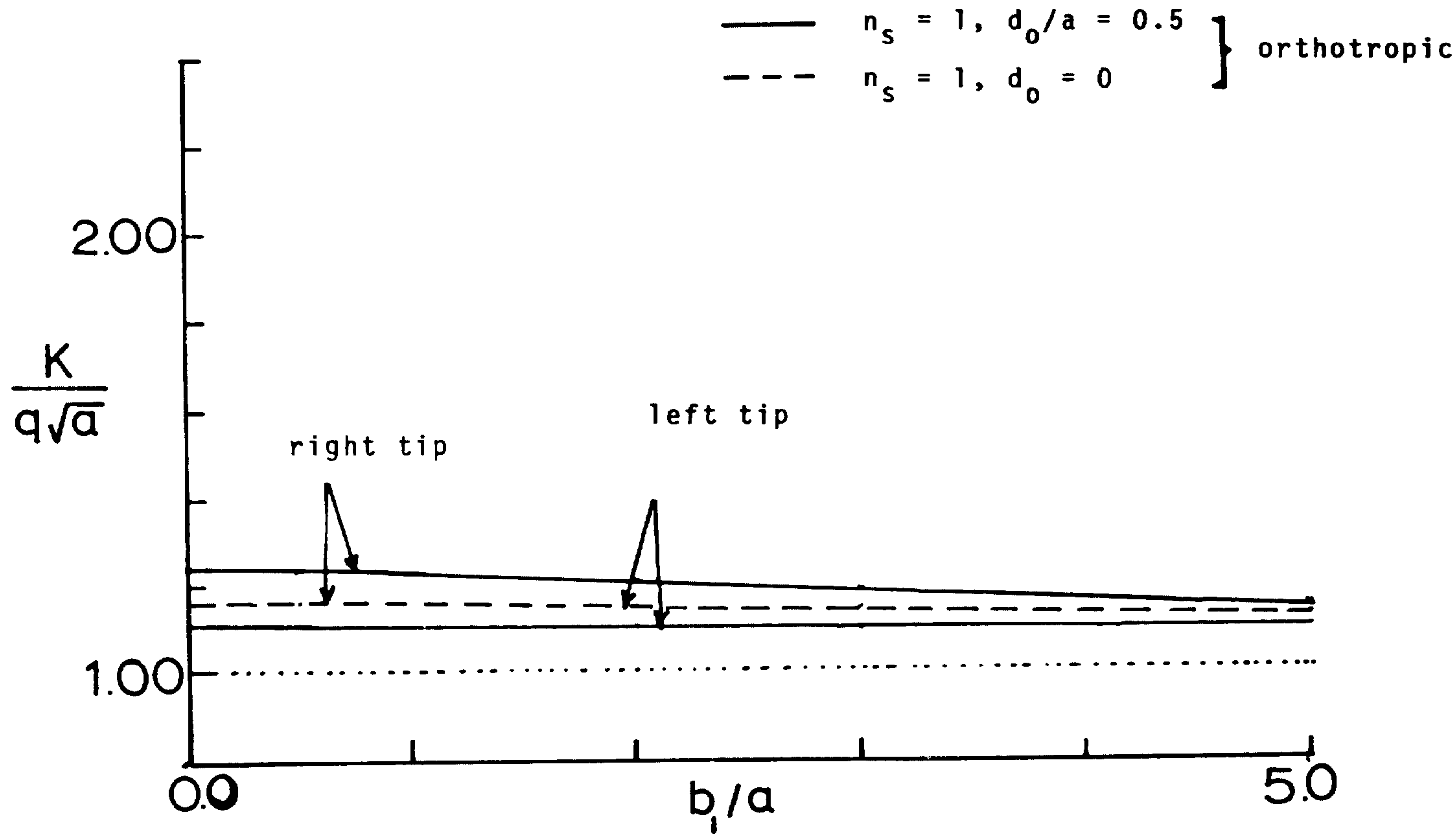


Figure 8 - $K/q\sqrt{a}$ vs. b_1/a (broken stringer, loading at infinity, $d_1/a = 0.5$, $d_s/a = 0.2$)

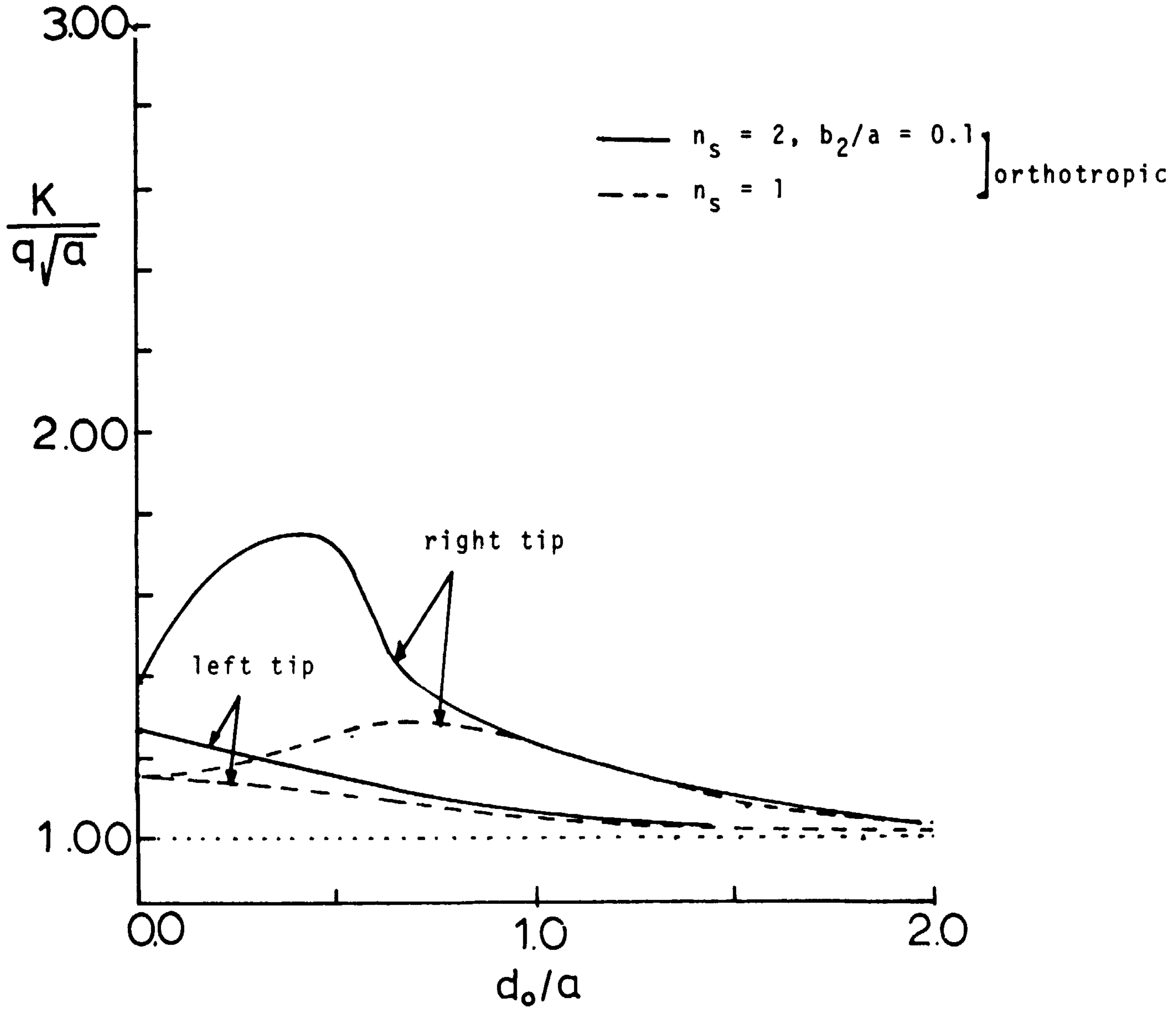


Figure 9 - $K/q\sqrt{a}$ vs. d_o/a (broken stringer, loading at infinity, $d_1/a = 0.5$, $b_1/a = 1.0$, $d_s/a = 0.2$)