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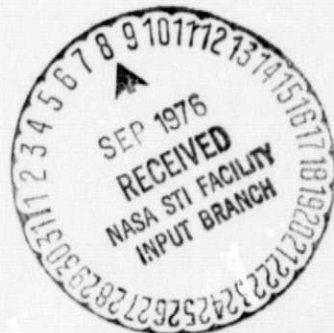
TRANSIENT ANALYSIS OF UNBALANCED SHORT CIRCUITS
OF THE ERDA-NASA 100 kW WIND TURBINE ALTERNATOR

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ABSTRACT

Four different types of unbalanced short-circuit faults on the alternator of the ERDA/NASA Mod-0 100-kW experimental wind turbine are studied. For each case, complete solutions for armature, field, and damper-circuit currents, short-circuit torque, and open-phase voltage are derived directly by a mathematical analysis. Formulated results are tabulated in appendixes II and III. For the Mod-0 wind turbine alternator, numerical calculations are given, and results are presented by graphs. Comparisons for significant points among the more important cases are summarized. For these cases the transients are found to be potentially severe. The effect of the alternator neutral-to-ground impedance is evaluated.

INTRODUCTION

In 1974-75 NASA-Lewis Research Center designed and erected an experimental 100 kW wind turbine as part of the ERDA wind energy program. This turbine, designated the Mod-0, is located at the NASA Plum Brook site near Sandusky, Ohio. The machine, which became operational in September 1975, is a horizontal-axis, propeller-type wind turbine driving a 100 kW synchronous alternator through a step-up gear box.

The NASA Mod-0 alternator is a 125 kVA, three-phase, 60-Hz, 1800 rpm, 480-volt, Y-connected synchronous generator. The complete wind turbine system including the alternator has been described in previous publications (refs. 1 and 2). Figure 1 depicts the Mod-0 wind turbine. Figure 2 shows schematic details of the drive train assembly. Figure 3 is a schematic diagram of the alternator.

Short-circuit analyses provide currents, voltage, and torque on a power system during electrical fault conditions. This information is required for analyzing the transient stability of a system under faults. The information is essential to designing an adequate protective relaying system and to determining interrupting requirements for circuit breakers used in the wind turbine generator system. Relaying systems must recog-

nize the existence of a fault and initiate circuit breaker operation to clear the fault in time to avoid any serious consequences.

Statistically most of the faults that occur on power systems are unsymmetrical faults. This report presents a mathematical analysis of four possible cases of unsymmetrical faults; namely, simultaneous unbalanced, line-to-line, line-to-ground, and double line-to-ground faults. The most significant results are presented and provide a complete solution to the important problem of unsymmetrical short circuits of the Mod-0 wind turbine alternator. Appendix A presents the nomenclature used in this report. The analytical formulas given in appendixes B and C can be used for analyzing the proposed Mod-1 wind turbine alternator as well as other wind turbine alternators. The resulting information can contribute to designing an effective relaying, switching, and control system for the proposed Mod-1 wind turbine generators. A digital computer program for numerical computations of the analytical formulas has been developed.

The purpose of the work reported here is to estimate the effects of these electrical faults on the wind turbine generator, to evaluate the system design, and to recommend, if warranted, any system improvements to reduce the transients caused by the faults.

Two basic assumptions are made in deriving the relationships presented in this report. These are assumptions recognizing conditions conventionally accepted for similar fault analyses:
  (1) Alternator speed remains constant.
  (2) Field regulation is not active.
The first assumption recognizes that the alternator is synchronized to a large power system; the second assumption recognizes that the fault transient is much faster than the field circuit reaction.

MATHEMATICAL ANALYSIS

Performance Equations of Alternators

In this report, modified Clarke's components, α, β, γ components (ref. 3), are used to analyze the unsymmetrical short-circuit conditions. The transformation from phase quantities to α, β, γ components is given by

$$\begin{bmatrix} f_\alpha \\ f_\beta \\ f_\gamma \end{bmatrix} = \begin{bmatrix} \sqrt{2/3} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} \qquad (1)$$

or conversely,

$$
\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = \begin{bmatrix} \sqrt{2/3} & 0 & \dfrac{1}{\sqrt{3}} \\ -\dfrac{1}{\sqrt{6}} & -\dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{3}} \\ -\dfrac{1}{\sqrt{6}} & \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} f_\alpha \\ f_\beta \\ f_\gamma \end{bmatrix} \tag{2}
$$

where f may represent i, e, or ψ, the current, voltage, or flux linkage, respectively. (All basic system parameters are completely defined in appendix A. Other symbols are defined in the text.)

In terms of α, β, γ components, the following performance equations can be verified when necessary (ref. 4):

$$
\begin{bmatrix} -e_f \\ e_\alpha \\ e_\beta \\ e_\gamma \\ 0 \\ 0 \end{bmatrix} = p \begin{bmatrix} -L_{ff} & -M_{af}\cos\theta & M_{af}\sin\theta & 0 & -M_{fld} & 0 \\ -Ma_f\cos\theta & -A-B\cos2\theta & B\sin2\theta & 0 & -M_{ald}\cos\theta & -M_{ald}\sin\theta \\ Ma_f\sin\theta & B\sin2\theta & -A+B\cos2\theta & 0 & M_{ald}\sin\theta & -M_{alq}\cos\theta \\ 0 & 0 & 0 & -L_0 & 0 & 0 \\ -M_{fld}' & -M_{ald}\cos\theta & M_{ald}\sin\theta & 0 & -L_{11d} & 0 \\ 0 & -M_{alq}\sin\theta & -M_{alq}\cos\theta & 0 & 0 & -L_{11q} \end{bmatrix}
$$

$$
\begin{bmatrix} i_f \\ i_\alpha \\ i_\beta \\ i_\gamma \\ i_{11d} \\ i_{11q} \end{bmatrix} - \begin{bmatrix} R_f i_f \\ R_a i_\alpha \\ R_a i_\beta \\ R_a i_\gamma \\ R_{1d} i_{11d} \\ R_{1q} i_{11q} \end{bmatrix} \tag{3}
$$

where

$$p = \frac{d}{dt}, \qquad A = \frac{L_\alpha + L_q}{2}, \qquad B = \frac{L_d - L_q}{2}$$

The α, β, γ components of flux linkages in the armature circuit are given by

$$
\begin{bmatrix} \psi_\alpha \\ \psi_\beta \\ \psi_\gamma \end{bmatrix}
=
\begin{bmatrix}
M_{af} \cos\theta & (A + B\cos 2\theta) & -B\sin 2\theta & 0 & M_{ald}\cos\theta & M_{ald}\sin\theta \\
-M_{af}\sin\theta & -B\sin 2\theta & (A - B\cos 2\theta) & 0 & -M_{ald}\sin\theta & M_{alq}\cos\theta \\
0 & 0 & 0 & L_0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} i_f \\ i_\alpha \\ i_\beta \\ i_\gamma \\ i_{11d} \\ i_{11q} \end{bmatrix}
\tag{4}
$$

The torque equation is

$$T = K\frac{P}{2}(\psi_\beta i_\alpha - \psi_\alpha i_\beta) \tag{5}$$

Under any unsymmetrical short-circuit conditions, it is possible to solve the matrix equations (3) simultaneously for the six unknown currents. Then, by using equations (2) the phase currents can be determined. The torque can be found by applying equation (5). Finally, the open-phase voltage can be evaluated by simply differentiating the corresponding flux linkage expression. A detailed derivation of the solution is omitted in this report; only the results are presented. Similar detailed derivation is presented in previous work (ref. 5).

Specification of Fault Conditions

Summary of unbalanced short-circuit cases. -

Case 1: Simultaneous unbalanced short-circuits

As shown in figure 4, a short-circuit is applied between phase a and the neutral through an impedance Zg, and another short-circuit is applied simultaneously between phases b and c. For the present case, with the machine unloaded, the terminal conditions are:

$$e_a = e_{bc} = 0, \qquad i_b = -i_c \tag{6}$$

In terms of the α, β, γ components, these terminal conditions become

$$e_\alpha = -\frac{1}{\sqrt{2}} e_\gamma, \qquad e_\beta = 0, \qquad i_\alpha = \sqrt{2}\, i_\gamma \tag{7}$$

Case 2: Line-to-line fault on phases b-c, with line A open in figure 4

The corresponding terminal conditions are:

$$i_a = 0, \qquad i_b = -i_c, \qquad e_{bc} = 0 \tag{8}$$

or

$$i_\alpha = 0, \qquad i_\gamma = 0, \qquad e_\beta = 0 \tag{9}$$

Case 3: Line-to-ground fault on phase a, with open-circuit lines B and C.

The terminal conditions are:

$$e_a = 0, \qquad i_b = i_c = 0 \tag{10}$$

or

$$e_\alpha = -\frac{1}{\sqrt{2}} e_\gamma, \qquad i_\alpha = \sqrt{2/3}\, i_a,$$

$$\tag{11}$$

$$i_\beta = 0, \qquad i_\gamma = \frac{1}{\sqrt{2}} i_\alpha$$

Case 4: Double line-to-ground fault on phases b and c with line A open.

The terminal conditions are given by:

$$e_b = e_c = 0, \qquad i_a = 0 \tag{12}$$

or

$$e_\alpha = \sqrt{2}\, e_\gamma, \qquad e_\beta = 0, \qquad i_\gamma = -\sqrt{2}\, i_\alpha \tag{13}$$

The analytical formulas given in appendixes B and C can be used to calculate the currents, torque, and open-phase voltage caused by the fault in each listed case. If the alternator is initially loaded, fault condition values can be determined simply by adding the initial values existing before the fault to the results obtained from the given formulas with the fault condition.

Numerical Calculations

System parameters. -

The 100 kW Mod-0 wind turbine alternator has the following per unit parameters:

$$R_a = 0.0180 \text{ p.u.} \qquad L_q'' = 0.1930 \text{ p.u.}$$
$$R_f = 0.00405 \text{ p.u.} \qquad L_{ff} = 2.9650 \text{ p.u.}$$
$$R_{1d} = 0.0466 \text{ p.u.} \qquad L_{11d} = 2.1600 \text{ p.u.}$$
$$R_{1q} = 0.0493 \text{ p.u.} \qquad L_{11q} = 1.1600 \text{ p.u.}$$
$$R_0 = 0.0180 \text{ p.u.} \qquad M_{f1d} = 2.1600 \text{ p.u.}$$
$$L_d = 2.2100 \text{ p.u.} \qquad M_{af} = 2.1600 \text{ p.u.}$$
$$L_q = 1.0640 \text{ p.u.} \qquad M_{ald} = 2.1600 \text{ p.u.}$$
$$L_0 = 0.0058 \text{ p.u.} \qquad M_{alq} = 1.0050 \text{ p.u.}$$
$$L_d' = 0.1650 \text{ p.u.} \qquad F_f = 1.0000 \text{ p.u.}$$
$$L_d'' = 0.1280 \text{ p.u.} \qquad I_{f0} = \sqrt{3/2} \text{ p.u.}$$
$$\omega = 377 \text{ rad/sec}$$
$$= 1.0000 \text{ p.u.}$$

The impedance between alternator neutral and ground, Z_G, has been set to zero.

Experience has shown that 70 to 80 percent of transmission line faults are single-phase faults and that they are the most severe (ref. 6). For that reason, numerical calculations are given for three cases of short circuit faults; namely,

(1) simultaneous unbalanced
(2) line-to-line
(3) line-to-ground

Field, damper, and armature currents, as well as short circuit torque and open phase voltages, were calculated using a digital computer program. The results are shown in figures 5 to 10. Time is measured in radians with ω = 377 rad/sec = 1.0 p.u. A representative value of the phase angle, $\theta_0 = \pi/3$, is assumed in all calculations.

RESULTS AND DISCUSSIONS

Short-Circuit Torques

Figure 5 shows that case 1 seems to be the least severe; it not only contains the smallest double frequency component, but also has a field decrement factor, $F(t)$ smaller than either of the other two cases. The case of line-to-line short circuit is most severe, as the absolute peak value of the transient torque is equal to about 21 per unit. This transient torque occurs at a time equal to 2.9 radians or about 7.7 milliseconds. As one per unit torque referred to the rotor shaft of the Mod-0 wind turbine is equal to 22 023 foot-pounds, the transient torque amounts to about 460 000 foot-pounds. Furthermore, this transient torque may be even much higher if the extreme value of θ_0 is chosen as $\pi/2$ instead of $\pi/3$. This transient torque reduces gradually, but at the end of the first three cycles its absolute peak value still remains nearly 6 per unit or about 130 000 foot-pounds on the rotor shaft.

The transient torque for the line-to-ground case is not much less than that of the line-to-line case. At steady-state, case 1 has a very small torque nearly equal to zero, and cases 2 and 3 have a peak torque less than 0.17 per unit (given by computer results) or about 3700 foot-pounds on the rotor shaft. For all cases, the output of the alternator finally reduces to a very small value nearly equal to zero.

Armature Currents

Figure 6 shows that each phase current in case 1 is nearly equal to the corresponding phase current of cases 2 or 3. The currents in phase a for cases 1 and 3 reach a maximum value nearly equal to 16 per unit, but the maximum currents in phase b for cases 1 and 2 are equal to only about 9 per unit. In each case, the transient armature currents contains a d.c. component. In addition to the odd harmonic series, each of cases 2 and 3 has only the even harmonic cosine series, while both the even harmonic cosine and sine series appear in each phase current of case 1. However, all the even harmonic series will decay to zero according to the armature time constants. The steady-state armature currents for all cases

are nearly sinusoidal. The above statements can be verified either by the results given in figure 6 or by expanding the analytical expressions for armature currents into Fourier series.

Open-Phase Voltages

Figure 7 shows that all the open-phase voltages contain an appreciable amount of harmonics. The case of line-to-ground seems to be most severe, as its peak voltage is almost five times its rated value (1 per unit). Such a high voltage would be dangerous to both life and the insulation of the armature winding.

Damper Circuit Currents

Figure 8 shows that the d-axis damper circuit currents for all cases rise to a peak value nearly equal to 13 per unit during the first cycle after the faults occur. The currents in the q-axis damper circuit (fig. 9) rise to an absolute maximum value nearly equal to 6 per unit for cases 2 and 3. Case 1 has relatively small current in the same circuit. The transient currents in both damper circuits for each case contain both even and odd harmonics. However, all the odd harmonics will decay to zero according to armature time constants, and only the even harmonics will persist in the steady-state currents. As the steady-state armature currents for all cases are nearly sinusoidal, the corresponding damper circuit currents contain very small even harmonic terms in addition to the initial d.c. currents. The statements regarding the damper circuit currents can be easily verified from the results given in figures 8 and 9, or by expanding the corresponding analytical expressions given in appendix C into Fourier series (ref. 3).

Field Currents

The Mod-0 alternator, similar to most practical machines, has a K_f factor equal to zero. Therefore, the direct-axis damper circuit is effective in preventing initially any harmonics in the transient field current caused by armature circuit changes. This condition is verified in figure 10. The current induced in the field, for each case, initially rises exponentially, then decays to a value slightly greater than 1.0 per unit. Figure 10 also shows that case 1 (simultaneous faults) has a maximum value of field current greater than that of either case 2 or 3. However, the peak value is only slightly above 4.0 per unit. This value will not cause serious damage to the insulation. The rated field current is equal to about 3.0 per unit.

Effect of Neutral-to-Ground Impedance

The calculations presented in this report have been made with zero impedance between the alternator neutral and ground in order to determine the maximum possible values of the transients.

Standard design practice is to include as a minimum an impedance equal to three times the subtransient reactance between the alternator neutral and ground in order to limit transients involving circuitry through the neutral (ref. 7). This minimum value of impedance for the Mod-0 wind turbine (0.38 p.u. or 0.7 ohm) reduces the transients 80 percent. The maximum current is reduced from 15.6 p.u. to 3.1 p.u., and the maximum torque from 17.8 p.u. to 3.8 p.u.

The line-to-ground transients for current and torque can be limited to approximately their per unit values (i_a = 0.94 p.u., torque = 1.2 p.u.) with the insertion of a 1.5 p.u. (2.8 ohm) impedance. Considering the acceptable magnitude of the transients and the reasonable size of the reactance (7.5 mh at 150 amp), the use of a 1.5 p.u. impedance is the choice for use in the Mod-0 wind turbine design.

For any value of impedance used to limit ground transients it is necessary to fuse the alternator at its terminals. This protection is necessary since the impedance between alternator neutral and ground does not limit the transients for the other two cases (line-to-line and simultaneous faults), and these faults, although not so frequent as line-to-ground faults, produce electrical transients large enough to be destructive to equipment. Fusing and circuit breakers must be used as protective devices for these faults.

SUMMARY OF RESULTS

In this report numerical results are presented for three severe short-circuit conditions that occur most frequently for an alternator:

(1) Simultaneous unbalanced
(2) Line-to-line, and
(3) line-to-ground

This analytical study of potential short-circuit characteristics of the Mod-0 100-kilowatt wind turbine has indicated the following results:

1. The severe electrical transients possible from short-circuit faults can be effectively reduced by the alternator neutral-to-ground impedances.

2. Protection of wind turbine system from transients resulting from line-to-line and simultaneous faults must be provided by circuit breakers and fusing at the alternator terminals.

3. For future wind turbine generator systems, it is recommended that these cases of short-circuits be analyzed and the effects of faults on the over-all system be taken into consideration in designing the control and protective subsystems.

Analytical expressions given in appendixes B and C can be readily modified to suit cases of faults applied at points on transmission lines at substantial distances from the wind turbine alternator by adding impedances external to the alternator in the volt-ampere equations.

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APPENDIX A

NOMENCLATURE

L_d, L_q d- and q-axis synchronous inductances

L_d', L_d'' d- axis transient and subtransient inductances

L_{ff} self-inductance of field winding

L_q'' q-axis subtransient inductance

L_0, L_2 zero-sequence and negative-sequence inductances

L_{11d}, L_{11q} self-inductances of d- and q-axis damper circuits

M_{af} reciprocal mutual inductance between d-axis armature circuit and field circuit

M_{ald}, M_{alq} mutual inductance between d-axis (or q-axis) armature circuit and d-axis (or q-axis) damper circuit

M_{fld} mutual inductance between field and d-axis damper circuit

R_a, R_f armature resistance per phase and field resistance

R_g, L_g resistance and inductance from the neutral to ground

R_{1d}, R_{1q} resistances of d- and q-axis damper circuits

$T_{a\alpha}, T_{a\beta}$ armature time constants of α- and β-axis circuits

T_d', T_d'' short-circuit transient and subtransient time constants in d-axis

T_{d0}', T_{d0}'' open-circuit transient and subtransient time constants in d-axis

APPENDIX B

EXPRESSIONS FOR PARAMETERS AND TIME CONSTANTS

Parameter	Simultaneous	Line-to-line	Line-to-ground	Double line-to-ground
A''	$x_d''\left(x_q'' + \dfrac{x_0'}{2}\right)$	x_d''	$x_d'' + \dfrac{x_0'}{2}$	$x_d''(x_q'' + 2x_0')$
B''	$x_q''\left(x_d'' + \dfrac{x_0'}{2}\right)$	x_q''	$x_q'' + \dfrac{x_0'}{2}$	$x_q''(x_d'' + 2x_0')$
x_2		$\sqrt{x_d''x_q''}$	$\sqrt{A''B''} - \dfrac{x_0'}{2}$	$\dfrac{x_d''x_q'' + \sqrt{A''B''}}{x_d'' + x_q'' + 2x_0'}$
x_e	$\dfrac{x_d''(B'' - x_d''x_q'')}{x_d''x_q'' + \sqrt{A''B''}}$		$x_2 + x_0'$	$\dfrac{x_2 x_0'}{(x_2 + x_0')}$
T_{d0}'	$\dfrac{L_{ff}}{R_f}$	$\dfrac{L_{ff}}{R_f}$	$\dfrac{L_{ff}}{R_f}$	$\dfrac{L_{ff}}{R_f}$
T_{d0}''	$\dfrac{L_{ff}L_{11d} - M_{f1d}^2}{R_{1d}L_{ff}}$	$\dfrac{L_{ff}L_{11d} - M_{f1d}^2}{R_{1d}L_{ff}}$	$\dfrac{L_{ff}L_{11d} - M_{f1d}^2}{R_{1d}L_{ff}}$	$\dfrac{L_{ff}L_{11d} - M_{f1d}^2}{R_{1d}L_{ff}}$
T_d'	$\dfrac{x_d' + x_e}{x_d + x_e}T_{d0}'$	$\dfrac{x_d' + x_2}{x_d + x_2}T_{d0}'$	$\dfrac{x_d' + x_e}{x_d + x_e}T_{d0}'$	$\dfrac{x_d' + x_e}{x_d + x_e}T_{d0}'$
T_d''	$\dfrac{x_d'' + x_e}{x_d' + x_e}T_{d0}'$	$\dfrac{x_d'' + x_2}{x_d' + x_2}T_{d0}''$	$\dfrac{x_d'' + x_e}{x_d' + x_e}T_{d0}''$	$\dfrac{x_d'' + x_e}{x_d' + x_e}T_{d0}''$
$T_{a\alpha}$	$\dfrac{0.5\,x_2'\sqrt{A''B''}}{\omega r(\sqrt{A'B''} - x_a''x_q'')}$	$\dfrac{x_2}{\omega R_a}$	$\dfrac{x_2 + 0.5\,x_0'}{\omega r}$	$\dfrac{x_2 + 2x_0'}{\omega r}$
$T_{a\beta}$	$\dfrac{0.5\,x_0'\sqrt{A''B''}}{\omega R_a[(x_d' + 0.5\,x_0')(x_q'' + 0.5\,x_0') - \sqrt{A''B''}]}$			$\dfrac{x_2}{\omega R_a}$
	$\omega(L_0 + 3L_g)$		$\omega(L_0 + 3L_g)$	$\omega(L_0 + 3L_g)$
R_0	$R_a + 3R_g$		$R_a + 3R_g$	$R_a + 3R_g$
r	$R_a + \dfrac{R_0}{2}$		$R_a + \dfrac{R_0}{2}$	$R_a + 2R_0$
K_f	$\dfrac{M_{af}L_{11d} - M_{f1d}M_{a1d}}{L_{ff}L_{11d} - M_{f1d}^2}\dfrac{L_{ff}}{M_{af}}$	Same	Same	Same

APPENDIX C

EXPRESSION FOR ANALYTIC FUNCTIONS

Variable	Simultaneous	Line-to-line	Line-to-ground	Double line-to-ground
D	$A'' + B'' - (A'' - B'')\cos 2\theta$	$x_d'' + x_q'' - (x_d'' - x_q'')\cos 2\theta$	$A'' + B'' + (A'' + B'')\cos 2\theta$	$A'' + B'' - (A'' - B'')\cos 2\theta$
$F(t)$	$\dfrac{x_d'' + x_e}{x_d + x_e} + \left(\dfrac{x_d'' + x_e}{x_d' + x_e} - \dfrac{x_d'' + x_e}{x_d + x_e}\right)e^{-t/T_d'}$ $\qquad + \left(1 - \dfrac{x_d'' + x_e}{x_d' + x_e}\right)e^{-t/T_d''}$	$\dfrac{x_d'' + x_2}{x_d + x_2} + \left(\dfrac{x_d'' + x_2}{x_d' + x_2} - \dfrac{x_d'' + x_2}{x_d + x_2}\right)e^{-t/T_d'}$ $\qquad + \left(1 - \dfrac{x_d'' + x_2}{x_d' + x_2}\right)e^{-t/T_d''}$	$\dfrac{x_d'' + x_e}{x_d + x_e} + \left(\dfrac{x_d'' + x_e}{x_d' + x_e} - \dfrac{x_d'' + x_e}{x_d + x_e}\right)e^{-t/T_d'}$ $\qquad + \left(1 - \dfrac{x_d'' + x_e}{x_d' + x_e}\right)e^{-t/T_d''}$	$\dfrac{x_d'' + x_e}{x_d + x_e} + \dfrac{x_d'' + x_e}{x_d' + x_e} - \dfrac{x_d'' + x_e}{x_d + x_e}\, e^{-t/T_d'}$ $\qquad + \left(1 - \dfrac{x_d'' + x_e}{x_d' + x_e}\right)e^{-t/T_d''}$
$G_1(t)$	$e^{-t/T_{a\alpha}}$	$e^{-t/T_{a\alpha}}$	$e^{-t/T_{a\alpha}}$	$e^{-t/T_{a\alpha}}$
$G_2(t)$	$e^{-t/T_{a\beta}}$	$e^{-t/T_{a\beta}}$	$e^{-t/T_{a\beta}}$	$e^{-t/T_{a\beta}}$
i_a	$-\dfrac{3}{2}\dfrac{E_f}{D}\,[2x_q'' F(t)\cos\theta - (x_d'' + x_q'')$ $\quad\cdot\, G_1(t)\cos\theta_0 + (x_d' - x_q'')G_1(t)\cos\theta_0$ $\quad\cdot\cos 2\theta + (x_d'' - x_q'')G_2(t)\sin\theta_0\sin 2\theta]$	0	$\dfrac{3E_f}{D}\,[F(t)\cos\theta - G_1(t)\cos\theta_0]$	0
i_b	$-\dfrac{\sqrt{3}}{2}\dfrac{E_f}{D}\,[2(x_q'' + x_0')F(t)\sin\theta - (x_d'' + x_q'' + x_0')$ $\quad\cdot\, G_2(t)\sin\theta_0 - (x_d' - x_q'')G_2(t)\sin\theta_0$ $\quad\cdot\cos 2\theta + (x_d'' - x_q'')G_1(t)\cos\theta_0\sin 2\theta]$	$-\sqrt{3}\,E_f\,\dfrac{F(t)\sin\theta - G_1(t)\sin\theta_0}{x_d'' + x_q'' - (x_d'' - x_q'')\cos 2\theta}$	0	$\dfrac{E_f}{D}\Big\{[3x_q''\cos\theta - \sqrt{3}(x_q'' + 2x_0')\cdot\sin\theta]$ $\quad\cdot\, F(t) - \dfrac{3}{2}\,[(x_d'' + x_q'') - (x_d'' - x_q'')$ $\quad\cdot\,(\cos 2\theta - \tfrac{1}{\sqrt{3}}\sin 2\theta)]\cos\theta_0\, G_1(t)$ $\quad\cdot\,\dfrac{\sqrt{3}}{2}\,[(x_d'' + x_q'' + 4x_0') + (x_d'' - x_q'')$ $\quad(\cos 2\theta + \sqrt{3}\sin 2\theta)\sin\theta_0\, G_2(t)\Big\}$
i_c	$-i_b$	$-i_b$	0	$\dfrac{E_f}{D}\Big\{[3x_q''\cos\theta + \sqrt{3}(x_q'' + 2x_0')\sin\theta]$ $\quad\cdot\, F(t) - \dfrac{3}{2}\,[(x_d'' + x_q'') - (x_d'' - x_q'')$ $\quad\cdot\,(\cos 2\theta + \tfrac{1}{\sqrt{3}}\sin 2\theta)\cos\theta_0\, G_1(t)$ $\quad-\dfrac{\sqrt{3}}{2}\,[(x_d'' + x_q'' + 4x_0') + (x_d'' - x_q'')$ $\quad\cdot\,(\cos 2\theta - \sqrt{3}\sin 2\theta)]\sin\theta_0\, G_2(t)\Big\}$
i_d	$\sqrt{\tfrac{2}{3}}\, i_a\cos\theta + \sqrt{2}\, i_b\sin\theta$	$\sqrt{2}\, i_b\sin\theta$	$\sqrt{\tfrac{2}{3}}\, i_a\cos\theta$	$-\dfrac{1}{\sqrt{6}}(i_b + i_c)\cos\theta + \dfrac{1}{\sqrt{2}}(i_b - i_c)\sin\theta$
i_q	$\sqrt{\tfrac{2}{3}}\, i_a\sin\theta - \sqrt{2}\, i_b\cos\theta$	$-\sqrt{2}\, i_b\cos\theta$	$\sqrt{\tfrac{2}{3}}\, i_a\sin\theta$	$-\dfrac{1}{\sqrt{6}}(i_b + i_c)\sin\theta - \dfrac{1}{\sqrt{2}}(i_b - i_c)\cos\theta$
T	$-K\dfrac{P}{2}\,[M_{af}i_{fo}F(t) + (L_d'' - L_q'')i_d]i_q$	$-K\dfrac{P}{2}\,[M_{af}i_{fo}F(t) + (L_d'' - L_q'')i_d]i_q$	$-K\dfrac{P}{2}\,[M_{af}i_{fo}F(t) + (L_d'' - L_q'')i_d]i_q$	$-K\dfrac{P}{2}\,[M_{af}i_{fo}F(t) + (L_d'' - L_q'')i_d]i_q$
$I_{11d}(t)$	$-\sqrt{\tfrac{3}{2}}\, E_f\dfrac{M_{af}}{M_{fld}}\dfrac{1 - K_f}{x_d + x_e}\left[1 + \dfrac{x_d - x_d'}{x_d' + x_e}e^{-t/T_d'}\right.$ $\quad\left. - \dfrac{x_d' + x_e}{x_d'' + x_e}e^{-t/T_d''}\right]$	$-\sqrt{\tfrac{3}{2}}\, E_f\dfrac{M_{af}}{M_{fld}}\dfrac{1 - K_f}{x_d + x_2}\left[1 + \dfrac{x_d - x_d'}{x_d' + x_2}\right.$ $\quad\left.\cdot\, e^{-t/T_d'} - \dfrac{x_d' + x_2}{x_d'' + x_2}e^{-t/T_d''}\right]$	$-\sqrt{\tfrac{3}{2}}\, E_f\dfrac{M_{af}}{M_{fld}}\dfrac{1 - K_f}{x_d + x_e}\left[1 + \dfrac{x_d - x_d'}{x_d' + x_e}\right.$ $\quad\left.\cdot\, e^{-t/T_d'} - \dfrac{x_d' + x_e}{x_d'' + x_e}e^{-t/T_d''}\right]$	$-\sqrt{\tfrac{3}{2}}\, E_f\dfrac{M_{af}}{M_{fld}}\dfrac{1 - K_f}{x_d + x_e}\left[1 + \dfrac{x_d - x_d'}{x_d' + x_e}e^{-t/T_d'}\right.$ $\quad\left. - \dfrac{x_d' + x_e}{x_d'' + x_e}e^{-t/T_d''}\right]$
i_{11d}	$I_{11d}(t) - (1 - K_f)\dfrac{M_{af}}{M_{fld}}i_d$	$I_{11d}(t) - (1 - K_f)\dfrac{M_{af}}{M_{fld}}i_d$	$I_{11d}(t) - (1 - K_f)\dfrac{M_{af}}{M_{fld}}i_d$	$I_{11d}(t) - (1 - K_f)\dfrac{M_{af}}{M_{fld}}i_d$
i_{11q}	$-\dfrac{M_{alq}}{L_{11q}}i_q$	$-\dfrac{M_{alq}}{L_{11q}}i_q$	$-\dfrac{M_{alq}}{L_{11q}}i_q$	$-\dfrac{M_{alq}}{L_{11q}}i_q$
$i_f(t)$	$-\sqrt{\tfrac{3}{2}}\, E_f\dfrac{M_{af}}{L_{ff}}\left[\dfrac{K_f}{x_d + x_e}(1 - e^{-t/T_d'})\right.$ $\quad\left. + \dfrac{1 - K_f}{x_d' + x_e}(e^{-t/T_d''} - e^{-t/T_d'})\right]$	$-\sqrt{\tfrac{3}{2}}\, E_f\dfrac{M_{af}}{L_{ff}}\left[\dfrac{K_f}{x_d + x_2}(1 - e^{-t/T_d'})\right.$ $\quad\left. + \dfrac{1 - K_f}{x_d' + x_2}(e^{-t/T_d''} - e^{-t/T_d'})\right]$	$-\sqrt{\tfrac{3}{2}}\, E_f\dfrac{M_{af}}{L_{ff}}\left[\dfrac{K_f}{x_d + x_e}(1 - e^{-t/T_d'})\right.$ $\quad\left. + \dfrac{1 - K_f}{x_d' + x_2}(e^{-t/T''} - e^{-t/T'})\right]$	$-\sqrt{\tfrac{3}{2}}\, E_f\dfrac{M_{af}}{L_{ff}}\left[\dfrac{K_f}{x_d + x_e}(1 - e^{-t/T_d'})\right.$ $\quad\left. + \dfrac{1 - K_f}{x_d' + x_2}(e^{-t/T_d''} - e^{-t/T_d'})\right]$
i_f	$i_{fo} + I_f(t) - K_f\dfrac{M_{af}}{L_{ff}}i_d$	$i_{fo} + I_f(t) - K_f\dfrac{M_{af}}{L_{ff}}i_d$	$i_{fo} + I_f(t) - K_f\dfrac{M_{af}}{L_{ff}}i_d$	$i_{fo} + I_f(t) - K_f\dfrac{M_{af}}{L_{ff}}i_d$
e_a	0	$-p\Psi_a = -p\Big\{\sqrt{\tfrac{2}{3}}\,[M_{af}i_{fo}F(t)$ $\quad + (L_c'' - L_q'')i_d]\cos\theta\Big\}$	0	$-p\Psi_a = -p\Big\{\sqrt{\tfrac{2}{3}}\,[M_{af}i_{fo}F(t) + (L_d'' - L_q'' - L_0')i_d]$ $\quad\cdot\cos\theta - L_0'' i_q\sin\theta\Big\}$
e_{b-c}	0	0	$\sqrt{2}\, p\Psi_\delta = -p(\sqrt{2}\,[M_{af}i_{fo}F(t)$ $\quad + (L_d'' - L_q'')i_d]\sin\theta)$	0

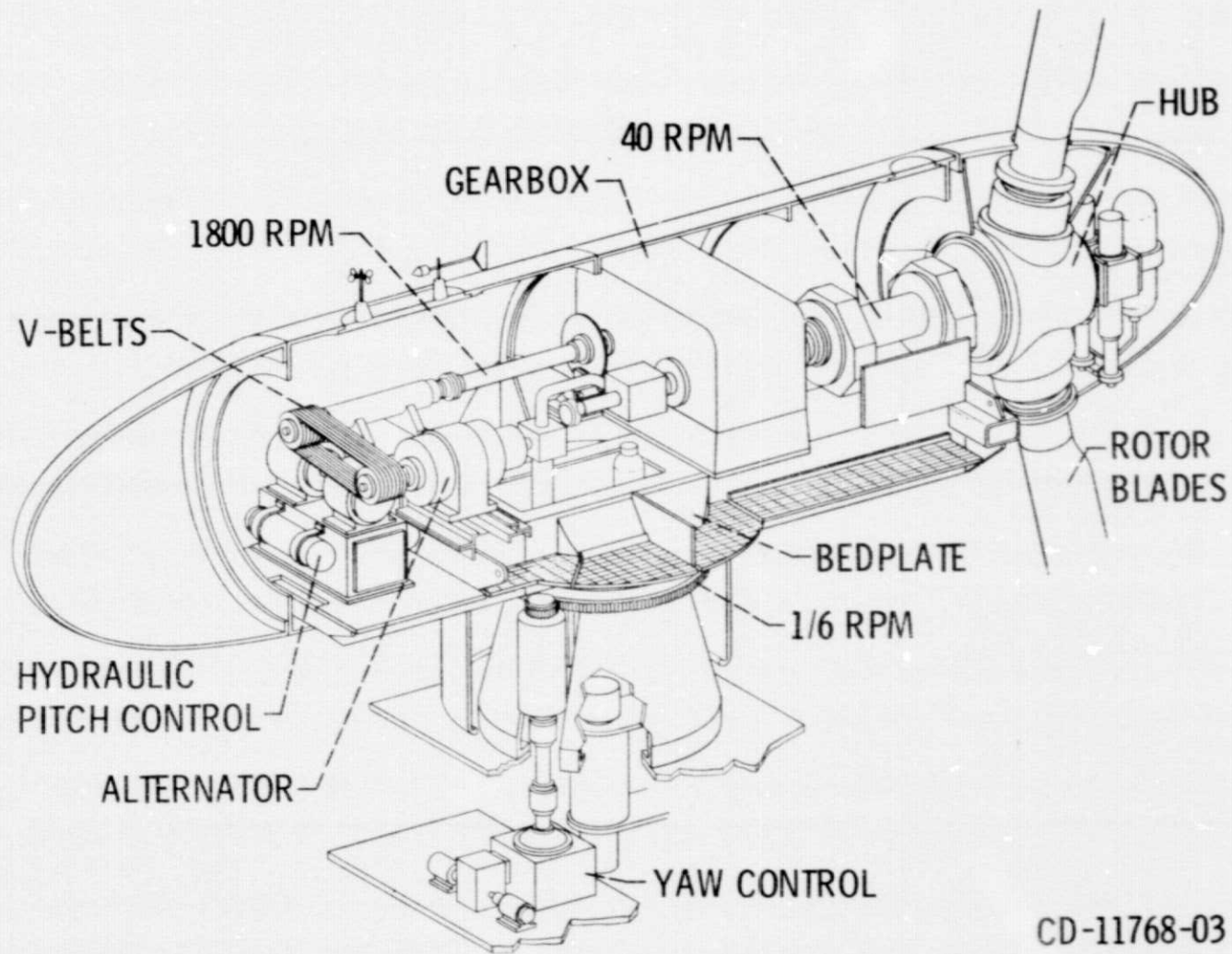Figure 1. - ERDA/NASA 100 KW experimental wind turbine.

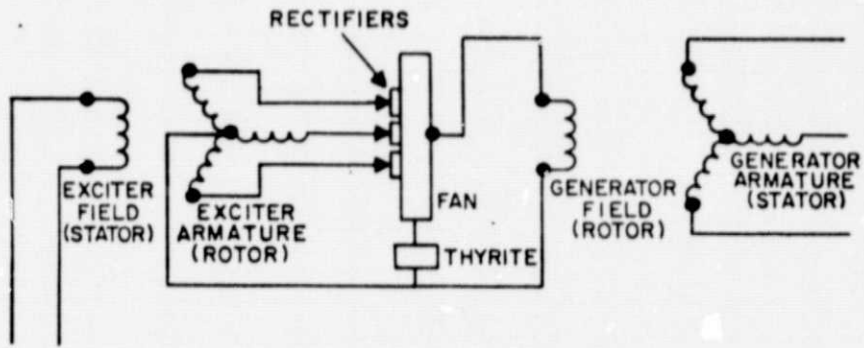Figure 2. - 100-kilowatt wind turbine drive train assembly and yaw system.

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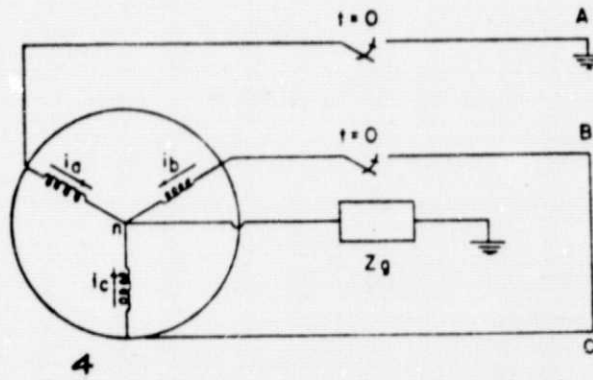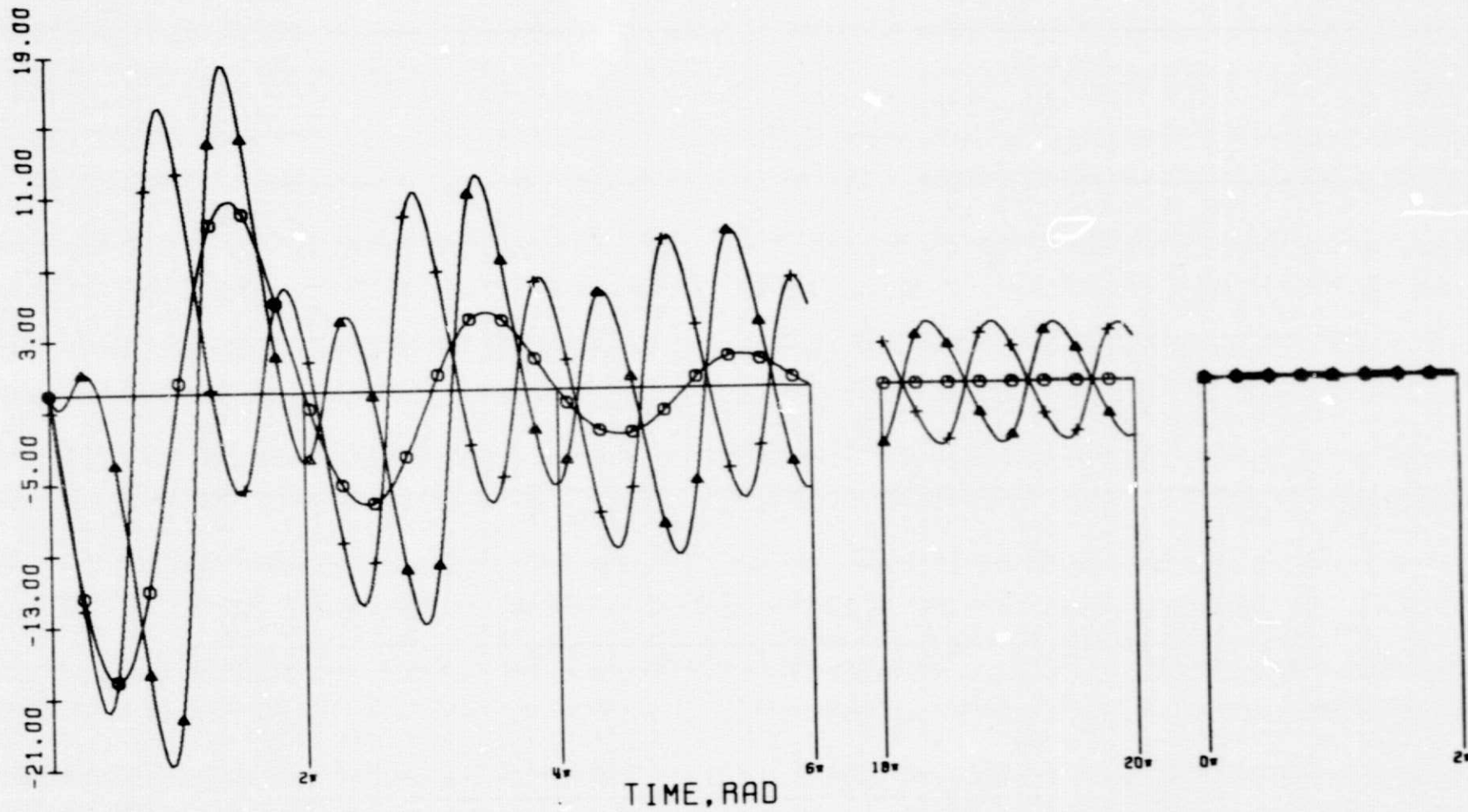Fig. 3. - ERDA-NASA MOD-O wind turbine alternator
schematic.

Fig. 4   Simultaneous unbalanced short circuits
of a synchronous machine.

SHORT CIRCUIT TORQUE

Fig. 5
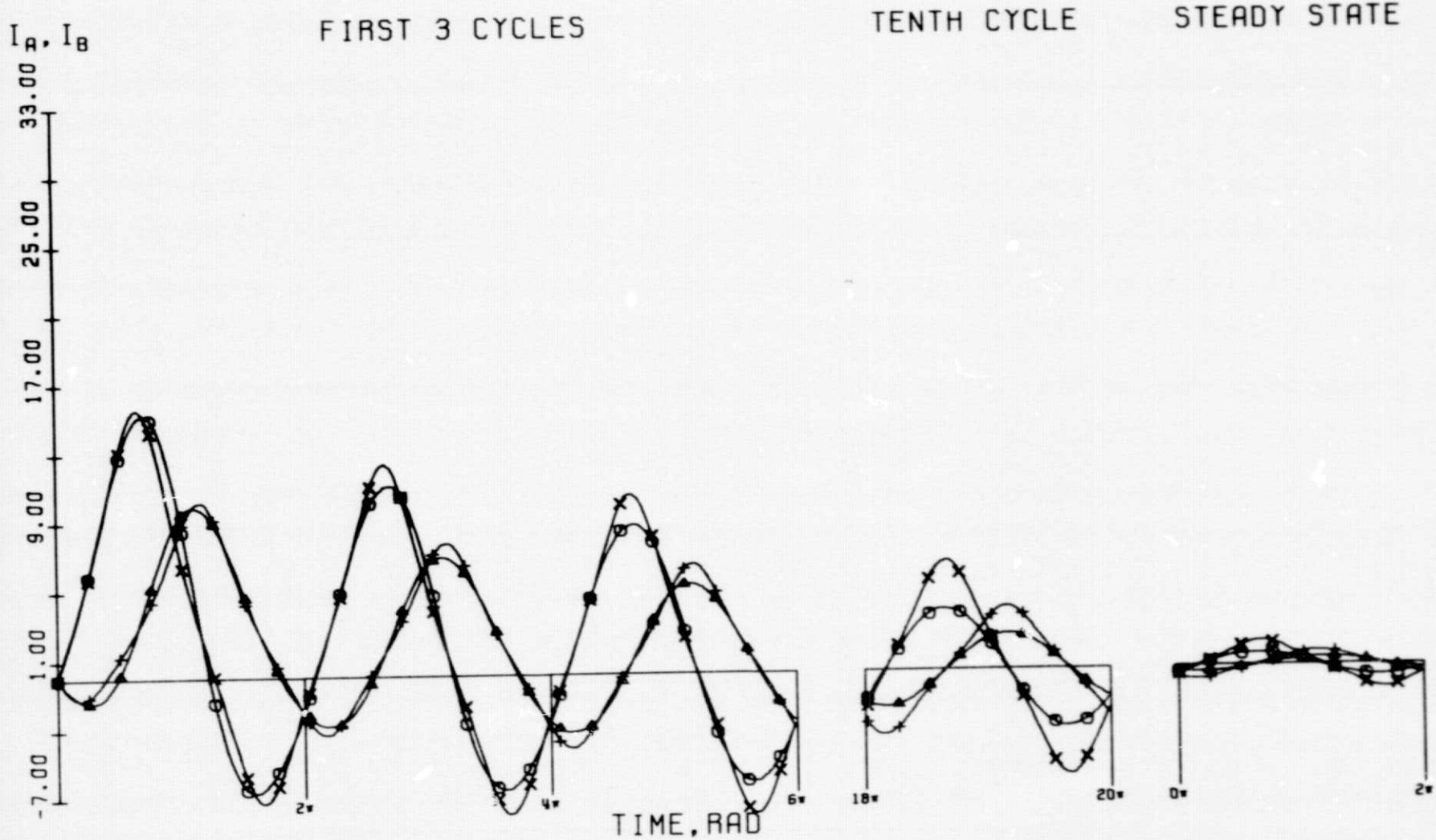
SHORT CIRCUIT ARMATURE CURRENTS
Fig. 6

OPEN PHASE VOLTAGES

Fig. 7

FIRST 3 CYCLES   TENTH CYCLE   STEADY STATE
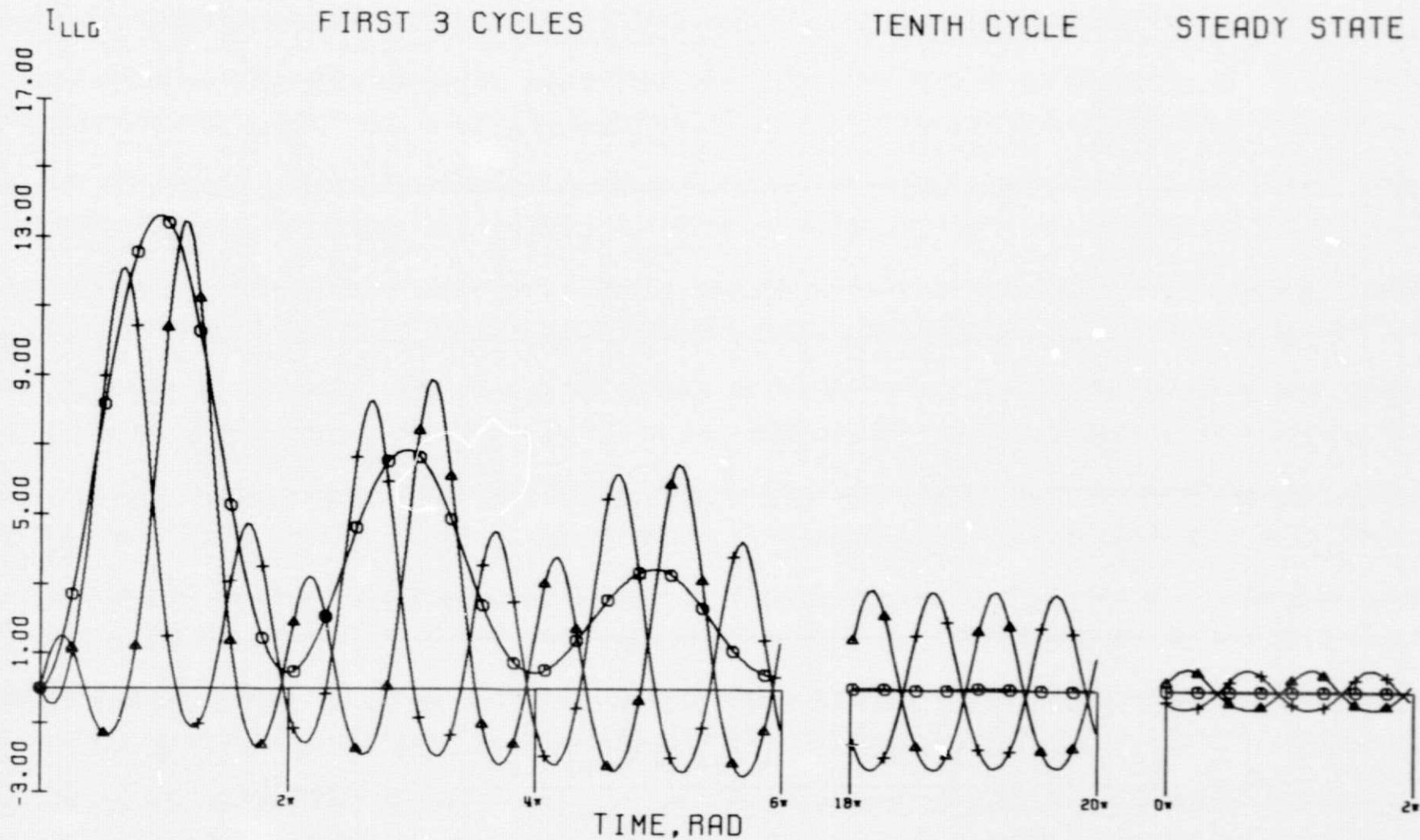
DIRECT-AXIS DAMPER CURRENT

Fig. 8

⊕ SIMULTANEOUS FAULTS
▲ LINE-TO-LINE FAULT
+ LINE-TO-GROUND FAULT
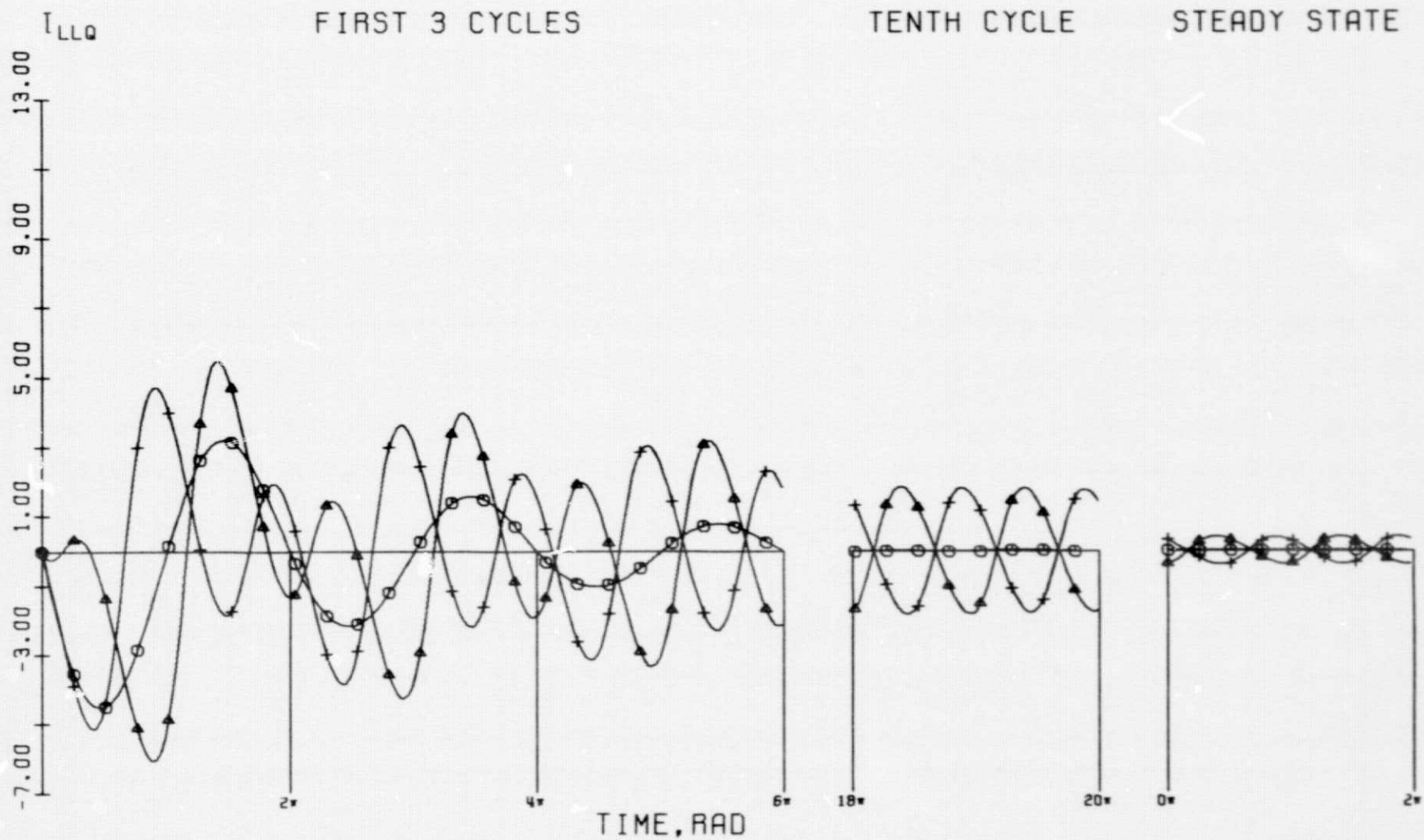
QUADRATURE-AXIS DAMPER CURRENT

Fig. 9

FIELD CURRENT

Fig. 10