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A TECHNIQUE FOR GENERATING ARBITRARILY SHAPED CURVED APPROACH PATHS

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The technique developed utilizes straight lines and elliptical segments to connect a series of data points into a continuous approach path. Digital computer software was generated to implement the technique in the NASA VALT system. Software was also created to allow the generation of arbitrarily shaped altitude and speed profiles in conjunction with the lateral curved paths.

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by

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SUMMARY

This report describes a technique for creating and using, in conjunction with either automatic or manual guidance, arbitrarily curved flight paths. This technique was developed for the NASA VTOL Approach and Landing Technology (VALT) program. One of the objectives of this program is to investigate operating techniques for VTOL aircraft in the terminal area environment. In order to accomplish this objective, it was necessary to develop a technique that would allow the creation and investigation of virtually any flight path, not just those defined by simple mathematical forms.

The technique developed utilizes straight lines and elliptical segments to connect a series of data points into a continuous approach path. Digital computer software was generated to implement the technique in the NASA VALT system. Software was also created to allow the generation of arbitrarily shaped altitude and speed profiles in conjunction with the lateral curved paths.

INTRODUCTION

In June of 1973, NASA/LaRC, Hampton, Virginia awarded contract NAS1-12365 to Sperry Flight Systems for the design and fabrication of a Digital Navigation System to be used in conjunction with a modified CH-47B helicopter. The work performed under this contract is a part of the NASA/ARMY VTOL Approach and Landing Technology (VALT) program designed to investigate the operating envelope and piloting procedures for VTOL aircraft in the terminal area.

The operation of VTOL aircraft in the urban environment requires complex landing approach trajectories that ensure adequate clearance from other traffic and obstructions and provide the most direct routing for efficient operations. It has become increasingly evident from previous research that coping with trajectories of this type will require both significant improvements in handling qualities through the development and application of advanced control and display technology and operating techniques that rely heavily on automation.

Many years of research have led to the ability to perform automatic landings in Conventional Take-off and Landing (CTOL) aircraft. There are, however many fundamental differences in both the vehicles and their terminal environments that preclude direct transfer of CTOL automatic landing techniques to VTOL aircraft. Furthermore, VTOL aircraft possess many unique characteristics that could be utilized to deal effectively with adverse wind conditions and enhance integration with the air traffic control system. In order to develop the navigation, guidance, control and flight-management technology base needed to establish system design concepts and operating procedures for VTOL short-haul transportation systems, the Langley Research Center has initiated the VALT program. As part of this effort, a flight investigation of the steep, decelerating, curved path, approach task will be undertaken as a continuation of the work presented in Reference 1.

In order to conduct such an investigation a technique was needed that could be used to generate arbitrarily shaped lateral curved paths and provide the necessary guidance information required to fly such a path. This report will describe the technique that will be used to generate such paths for the VALT system. This effort was carried out as part of a larger project to develop a system that will be used in conjunction with the NASA VALT CH-47B to investigate the VTOL approach problem. Reference 2 is a complete report on the development of that system.

VALT SYSTEM DESCRIPTION

The NASA VALT System is an integrated hardware and software package designed to provide a tool for investigating the problems associated with terminal area operations of VTOL aircraft. The system is built around a modified CH-47B helicopter and contains research oriented control, display, navigation and guidance subsystems. A block diagram of the VALT system is shown in Figure 1.

Research Aircraft

The NASA research aircraft is a highly modified CH-47B helicopter. The aircraft contains a research pilot station in the right side cockpit and a safety pilot station on the left side. The research pilot position instrument panel incorporates two video monitors for use with a ground-based, flight display research subsystem. The VALT engineers station is located in the main cabin area and provides the research engineer the capability to monitor and control the entire system. The aircraft is equipped with monitored, full authority, actuators in the pitch, roll, yaw and collective axes.

Control System

The VALT CH-47B control system is a hybrid combination of analog and digital components. The analog elements of the system include the sensors, the actuation system and the attitude stabilization portion of the control system. The digital elements include the digital navigation system and the air/ground telemetry data link. The digital navigation system contains the following components:

Digital Computer.- The primary hardware component in the Digital Navigation System is the Sperry Flight Systems 1819A Digital Computer. The computer performs all of the computations, data formatting, and logic decision making for the navigation system. In addition, the computer controls and directs the flow of digital data to and from the remaining hardware elements of the system.

Analog Interface.- The interface between the 1819A navigation computer and the analog sensors, displays, and actuation components is provided by the Digital Interface Unit (DIU). The DIU is configured to provide 30 channels of analog-to-digital conversion and 30 channels of digital-to-analog conversion. In addition, the DIU provides the capability to input 12 discretes into the computer and to accept 12 discretes from the computer.

Control and Display.- The man/machine interface function within the navigation system is provided by two Navigation Guidance Control Panels (Nav/Guidance). The two panels are identical in both appearance and operation and allow simultaneous interrogation of the computer by either the cockpit personnel or the flight test engineer. The primary functions of the Nav/Guidance panel are mode selection and indication, parameter insertion, and in-flight programming of the digital computer.

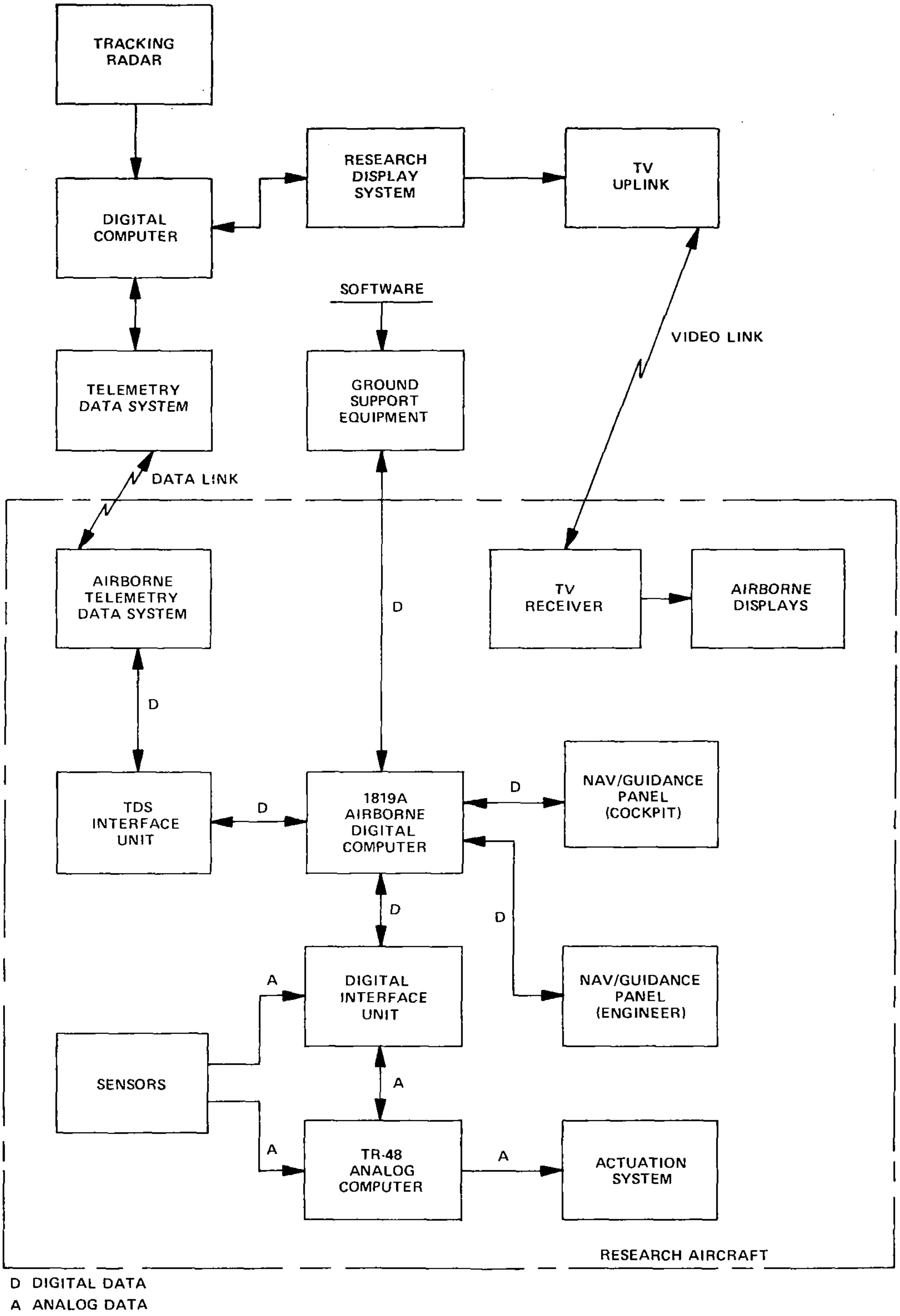


Figure 1
VALT Research System

Flight Software.- The flight software consists of a group of special purpose subroutine modules, each of which performs a specific computational or logic task. Control of the subroutines is maintained by a master logic executive routine that decides which subroutines should be used. This decision is based on data that is entered into the system through either of the Nav/ Guidance control panels.

Support Equipment.- A set of Ground Support Equipment (GSE) provides the means to modify existing software or to generate new software for the digital computer. The GSE provides the capability for interactive communication between the programmer and the computer as well as the capability to read in or punch out paper tapes.

Ground Support Software.- The ground support utility package consists of five major programs and is the primary tool for the system programmer. This software package, when used in conjunction with the ground support equipment, provides the programmer with the capability to write, debug, edit, and assemble programs for the 1819A computer.

Radar Tracking and Data Link

The primary navigational position information for the VALT system is provided through the use of an FPS-16 tracking radar combined with a laser ranging system located at the Wallops Research Center. Radar information is processed on the ground with a digital computer in order to generate a three-dimensional coordinate fix. The X position, Y position and Z position information is transmitted to the aircraft through the Telemetry Data System (TDS) data link. Position data update rates are variable, and will be set at 10 updates per second initially.

A TDS Interface Unit (TIU) in the aircraft provides the interface between the data link and the 1819A digital computer. A computer program combines the raw position data with aircraft acceleration data to produce inertially smoothed position information for guidance computations. Reference 3 contains a description of the technique used.

LATERAL PATH PROBLEM

The selection of the geometry to be used for generation of the lateral curved path involves a tradeoff between versatility and reasonable programming requirements. Ideally, the ability to synthesize any arbitrary path in the horizontal and vertical planes, with no reprogramming, is desired. However, this flexibility is limited by certain practical constraints. The path geometry must be expressible in a mathematical form suitable for numeric computation techniques. Also, aircraft position, with respect to the desired position on the path, must be definable such that error signals can be generated. In addition, the engineering work load in generating the input parameters for the computer to specify paths and path variations must be reasonable.

At first, this seems a reasonably simple problem, particularly to one familiar with the widespread utilization of interactive graphic displays for generating and displaying arbitrarily shaped objects. However, the problem contemplated here is not the simple one of data display, but rather the utilization of raw data in a real-time flight control environment. To solve the latter problem, it must be possible for the flight control computer to determine at a given instant, not only where the aircraft is, but also where it should be at that instant, and the necessary corrective maneuver required to eliminate any errors. To accomplish this, present position and desired position must be expressible in a mathematical form which allows unambiguous determination of errors. Further, the computational load must be such that the resultant throughput allows iteration rates consistent with stable closed-loop control operation. As an added complexity, vertical and speed profiles must be expressed as a function of distance-to-go along the desired curved path. Therefore, it must be possible to compute this distance on a continuous basis.

It is possible to represent a desired path as a series of straight line segments. But for the resultant curve to approach continuity, the number of straight line segments so used must approach infinity. This results in an untenable programming load. If the number of segments used is reduced to provide a reasonable programming level, the magnitude of the path discontinuities becomes objectionable even for paths of relatively mild curvature. In addition, if the number of straight line segments is large, the problem of projecting the aircraft position onto the path can produce ambiguities in the desired position determination. It is possible to project the present aircraft position onto a straight line; the problem is to select the correct straight line. This problem may be visualized by reference to Figure 2. Here, a portion of the desired path is approximated by straight line segments a, b, c, d and e, and the aircraft position, at some time, t_n, is shown as P_{t_n}. Assuming desired progress along the path is shown by the arrows, the aircraft should correct the position error and proceed to $P_{t_{n+y}}$ at some t_{n+y}.

Obviously, each straight line segment can be represented mathematically, as can the aircraft projection onto each line and the distance to each line. The problem is, then, which projection represents the true "desired" position, and which distance represents the correct "cross-track" error. If the shortest distance were used, a recursive solution would be required each computational cycle, and it is possible that the aircraft desired position could jump from

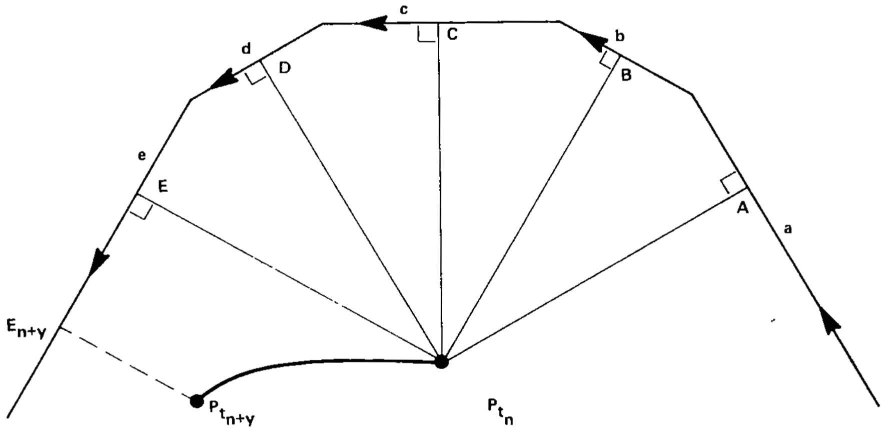


Figure 2
Straight Line Segment Approximation

segment to segment. The former produces high computational loads, and potential ambiguity. The latter could produce objectionable discontinuities in the "distance-to-go" computation with resultant discontinuities in the desired speed and altitude profiles. Other straight line approximation techniques suffer similar limitations.

Because of the requirement to generate curved flight paths or flight paths with one or more curved segments, it is desirable to utilize some type of curved line mathematical equation to formulate the flight path. Many mathematical forms could be used, but few have a closed form solution for the projection of actual position onto the desired path. Recursive solutions could be defined, but they imply high computational loads and may suffer from convergence limitations. Schemes could be postulated where the path is predefined as a series of mathematically defined curves, but these would require an excessive programming load which could seriously limit the versatility of the system.

A relatively obvious technique for generating curved paths involves the use of straight lines combined with some portion of a simple geometric curve, such as a circle, to generate an arbitrarily shaped path in a piecewise manner. This type of path construction is attractive since it reduces the number of geometrical forms that must be handled by the computer down to two. In addition, the configuration of a particular path can be readily described and easily visualized by a series of data points without the use of formulae or equations. If the parameters that define two consecutive data points are carefully selected and if the type of curved path segment is properly selected, a unique path can be specified by a simple data table.

A natural way to define the data points is to use the rectangular coordinate system to represent the desired x, y position of the aircraft at a series of points on the path. In addition, the desired slope or path heading can be used to specify the direction of the path of each data point. Once the data points are specified in this manner, it then becomes necessary to determine what type of mathematical curved segment can be used to connect the various data points.

The form of the curved segment used to join the data points will be dictated by the requirement to provide a continuous path between the data points. This requirement eliminates asymptotic conic sections such as hyperbolas and parabolas since the use of these types of curves will produce small, but significant, path discontinuities. Circular segments could be used; however, the selection of circular segments will require the use of a straight line extension from one of the two data points in all but a few special cases. In addition, this technique produces paths that are somewhat difficult to visualize and which require extra data point information in order to define where the straight line extension should be placed. The ellipse, however, provides a curved segment that readily fits the requirements outlined above. Given two data points that each specify an x, y position and a slope or path heading, it is possible to define a unique elliptical section that will fit the points and that will not result in path discontinuities when joined to an adjacent path segment.

In summary, the problem to be solved required that a technique be developed that considers the following four factors. First, the technique used to describe or define the path should be such that only one path can be generated from a given set of data. Second, the technique should generate a continuous path in order to reduce the possibility of projection ambiguity by eliminating any areas of redundancy that are defined by the normal projections of the end points of any path segment. Third, any mathematical forms used to define a path or a segment of a path should be amendable to closed-form solution of the cross-track error. And finally, the technique used must rely on computations that can be iterated at rates suitable for stable closed-loop control operation.

LATERAL PATH DESCRIPTION

Definition of the Path

Path Model.- The mathematical model used to represent the desired path could be in the form of an equation or it could be in the form of a series of data points that are to be connected together using some type of curve fitting technique. A curve fitting technique was selected for the VALT system in order to achieve a very flexible system for research applications. The flight path consists of a series of data points in the X-Y plane that are connected together with combinations of straight line segments and elliptically curved segments. This technique has the following advantages: first, good approximations of crosstrack error, path distance to go and desired heading can be obtained; second, since the circle is a special case of the ellipse, circular paths are inherently included and third, the specification of the path is relatively simple, requiring only a series of data points consisting of an X-coordinate, a Y-coordinate and a desired heading at that point.

For a given pair of data points defined by X_1, Y_1, ψ_1, and X_2, Y_2, ψ_2, it is possible to determine the equation of the ellipse that contains the two data points by appropriate rotation and translation of the coordinate system. Given the ability to determine an appropriate elliptical segment, it is then possible to fit a smooth, continuous path to a set of X, Y data points using a combination of elliptical and straight line segments. An example of such a path is shown in Figure 3. Note that if the two data points are to be connected with a straight line segment, the headings at the two data points are determined and only the X, Y position is required to specify the segment. If a data point is the junction of two elliptical segments however, then it is necessary to specify the heading of the path at that data point in addition to the X, Y position data.

Data Format and Path Restrictions.- A digital computer routine calculates the parameters of the elliptical or straight line segment that would fit between two data points while maintaining the desired entry and exit headings at those points. The path is then constructed by sequentially joining each of these segments into a continuous path. The coordinate system used to specify the data points is shown in Figure 4. The X and Y position data is inserted into the digital computer in signed decimal format. Heading information is inserted in degrees of magnetic heading. Data points to describe a particular flight path are entered sequentially starting at the desired path termination point and working backwards. An end of path code is entered last to indicate the point where the path should start. The selection of a straight line or an elliptical section to join a pair of data points is indicated by the third entry for each data point. If two data points are to be joined by a straight line, then a code number is inserted along with the X, Y data for these points. If a data point is the junction of a straight line and an elliptical section, then the same code is inserted. If a data point is the junction of two elliptical sections, then the third entry is the magnetic heading, in degrees, that the aircraft should have as it passes over that data point. In order to simplify the geometry and the logic decisions necessary to generate the lateral path, some restrictions on the data used to specify the path are necessary.

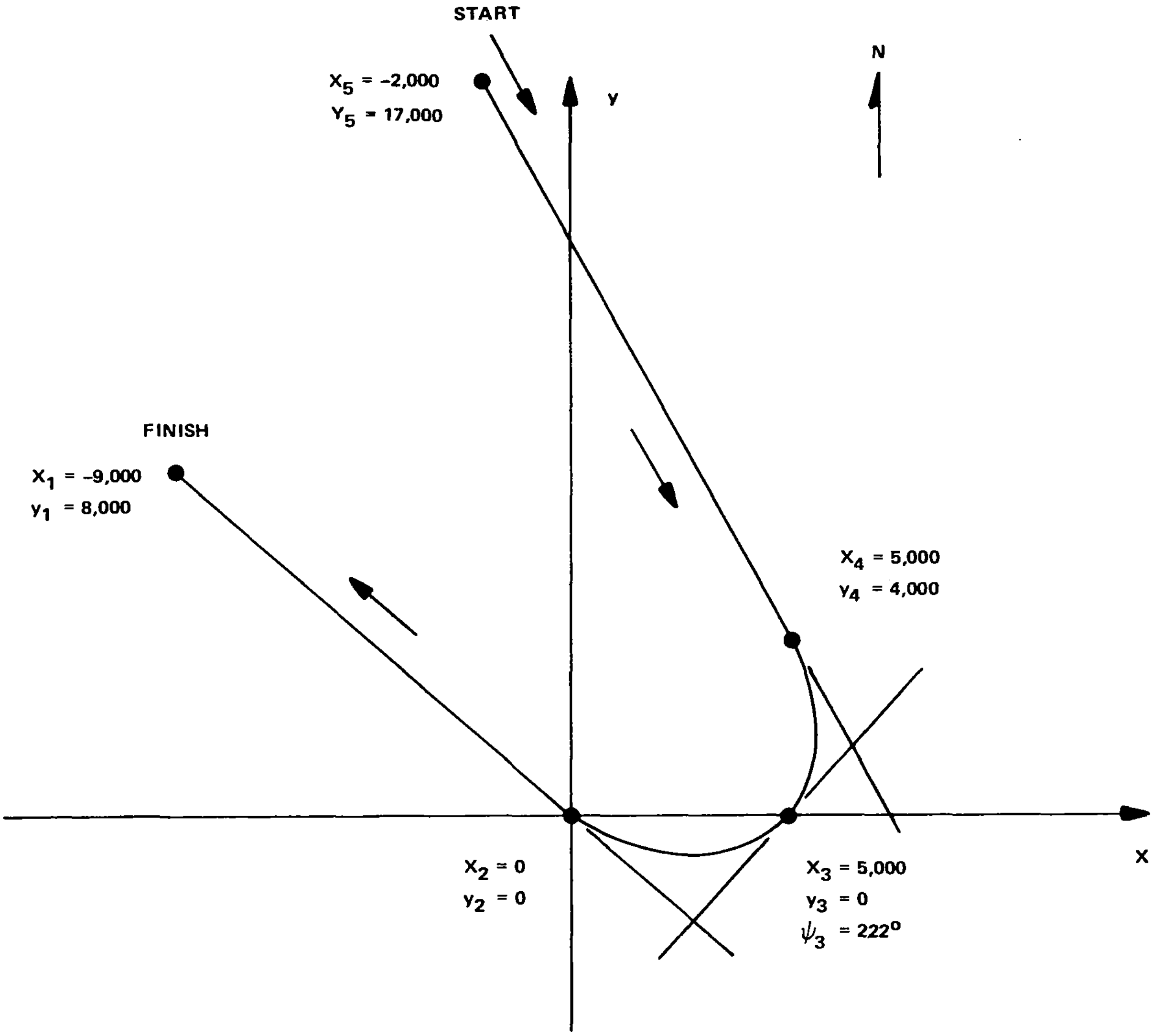


Figure 3
Simple Curved Path

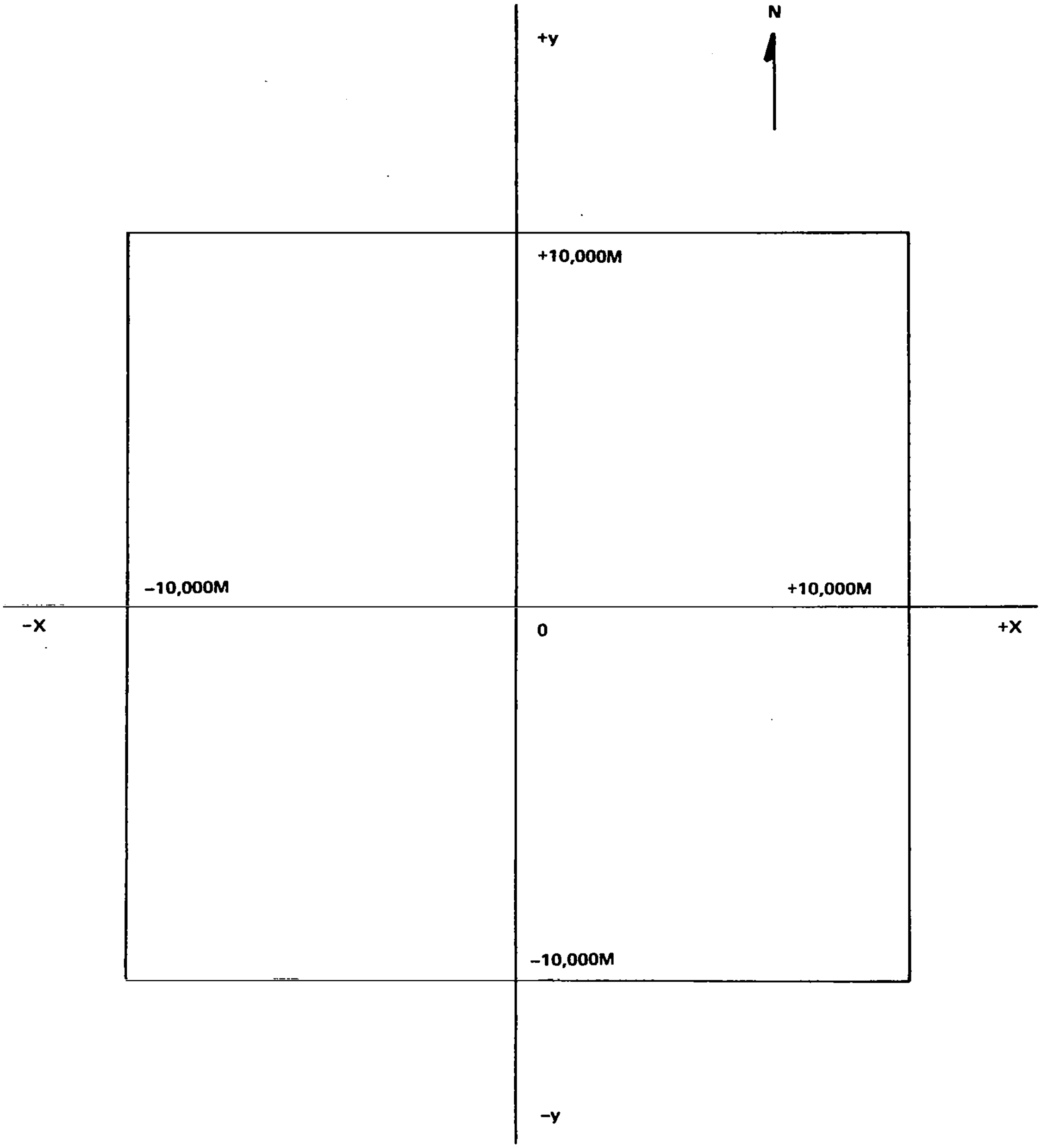


Figure 4
Lateral Path Coordinate System

- The path cannot have two consecutive straight line segments.
- A single elliptical section cannot be used to change the flight path heading by an angle greater than 90 degrees.
- Two consecutive data points cannot describe a path that contains an inflection point. At least three data points are required in this case.

The form of the data entry for a sample path is shown in Table 1 and the path described by this data is shown in Figure 5.

TABLE 1

LATERAL PATH DATA

Input	Data Point	Entry	Notes
1	1	0	X_1 Position
2		0	Y_1 Position
3		77776	Code indicating straight line
4	2	0	
5		-3000	
6		77776	
7	3	-600	
8		-4600	
9		77776	
10	4	-2000	
11		-6000	
12		77776	
13	5	-3000	X_5 Position
14		-8000	Y_5 Position
15		0	Desired heading at point 5
16	6	-2000	
17		-10000	
18		77776	

TABLE 1 (cont)

LATERAL PATH DATA

Input	Data Point	Entry	Notes
19	7	0	
20		-12000	
21		77776	
22	8	2000	
23		-13000	
24		77776	
25	9	4000	
26		-13000	
27		77776	
28	10	6000	
29		-16000	
30		0	
31	11	3000	
32		-20000	
33		90	
34	12	1000	
35		-19000	
36		77776	
37	13	-600	
38		-17400	
39		77776	
40	14	-3600	
41		-17400	
42		45	

TABLE 1 (cont)

LATERAL PATH DATA

Input	Data Point	Entry	Notes
43	15	-3600	
44		-20400	
45		-45	
46	16	-600	
47		-20400	
48		-135	
49	17	-600	
50		-17400	
51		135	
52	18	-3600	
53		-17400	
54		45	
55	19	-3600	
56		-20400	
57		-45	
58	20	-600	
59		-20400	
60		-135	
61	21	-600	
62		-17400	
63		77776	
64	22	-4000	
65		-14000	
66		77776	
67	--	77777	Code indicating end of path data

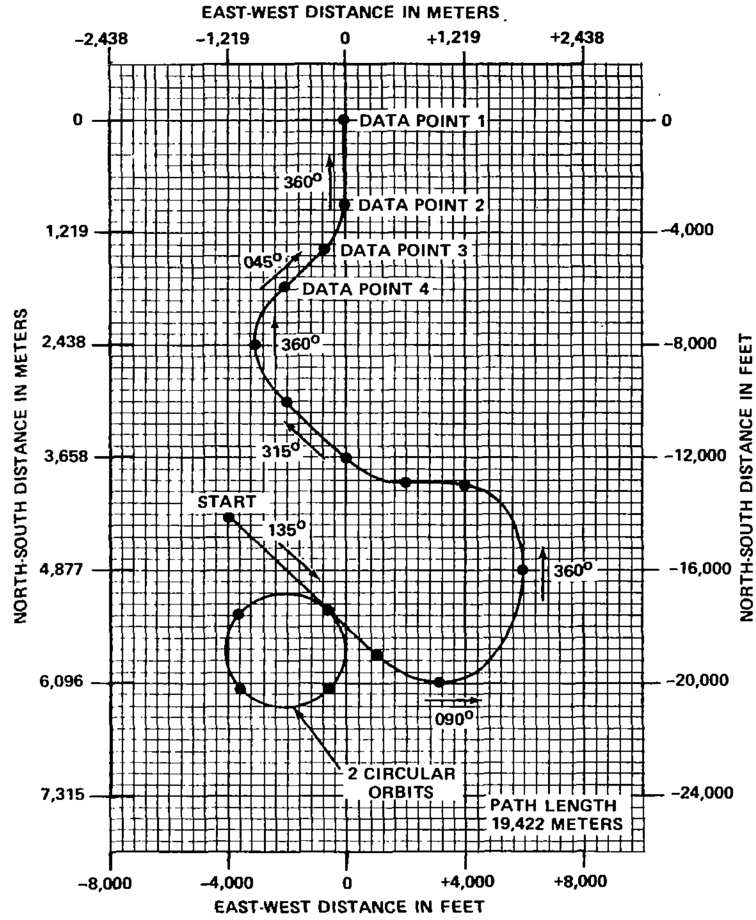


Figure 5
NASA VALT Baseline Path

Straight Line Segment

Consider the line segment defined by X_1, Y_1, and X_2, Y_2, and the point off the segment, X_A, Y_A shown in Figure 6a. If the segment and the point are rotated and translated to a coordinate system as shown in Figure 6b. then:

$$X_1' = 0$$

$$Y_1' = 0$$

$$X_2' = 0$$

$$Y_2' = (Y_2 - Y_1) \cos \theta - (X_2 - X_1) \sin \theta$$

$$X_A' = (X_A - X_1) \cos \theta + (Y_A - Y_1) \sin \theta$$

$$Y_A' = (Y_A - Y_1) \cos \theta - (X_A - X_1) \sin \theta$$

The perpendicular distance from the point to the line (crosstrack error) is X_A'. The length of the segment is Y_2'. The projection of the point on the segment is $(0, Y_A')$ and the length remaining on the segment past the projection point is $Y_2' - Y_A'$.

Elliptical Segment

Coordinate System.- The curved sections of the lateral path are generated by fitting an elliptical segment between two data points. In order to simplify the geometry and the calculation of crosstrack error, each elliptical segment is oriented in a coordinate system such that the X axis bisects the ellipse and the origin of the coordinate system lies on one of the end points of the ellipse. This standard orientation is shown in Figure 7. The restrictions on the form of the data used to specify the path, result in an elliptical section that is located entirely within the first or second quadrant of the reference coordinate system. The coordinate transformations required to achieve this standard orientation are a part of the digital computer program. The elliptical section used is defined such that the origin of the coordinate system is always a point of maximum or minimum curvature on the ellipse and the length of the segment is equal to or less than one quarter of the length of the perimeter of the ellipse.

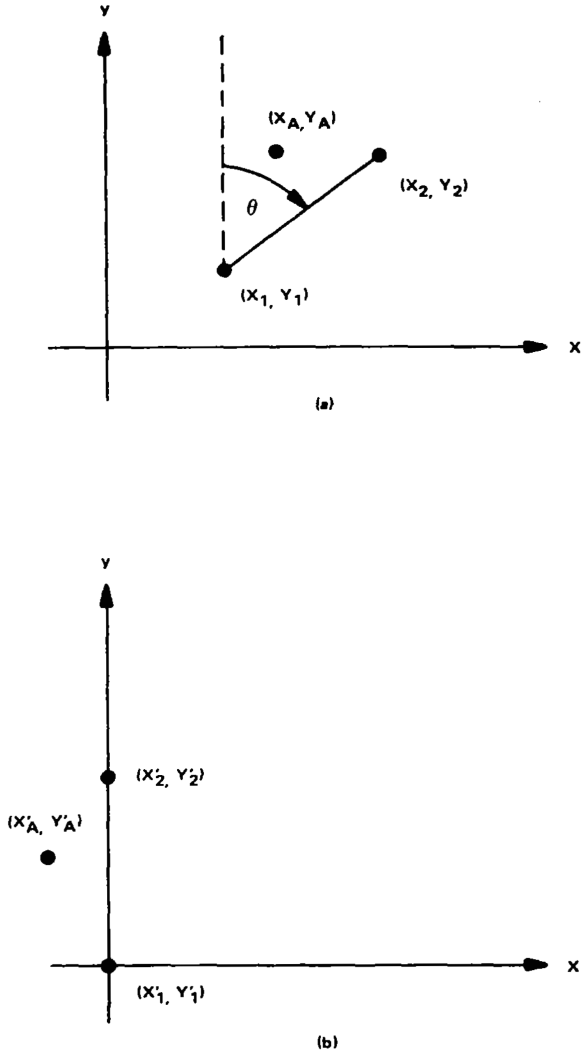


Figure 6
Straight Line Geometry

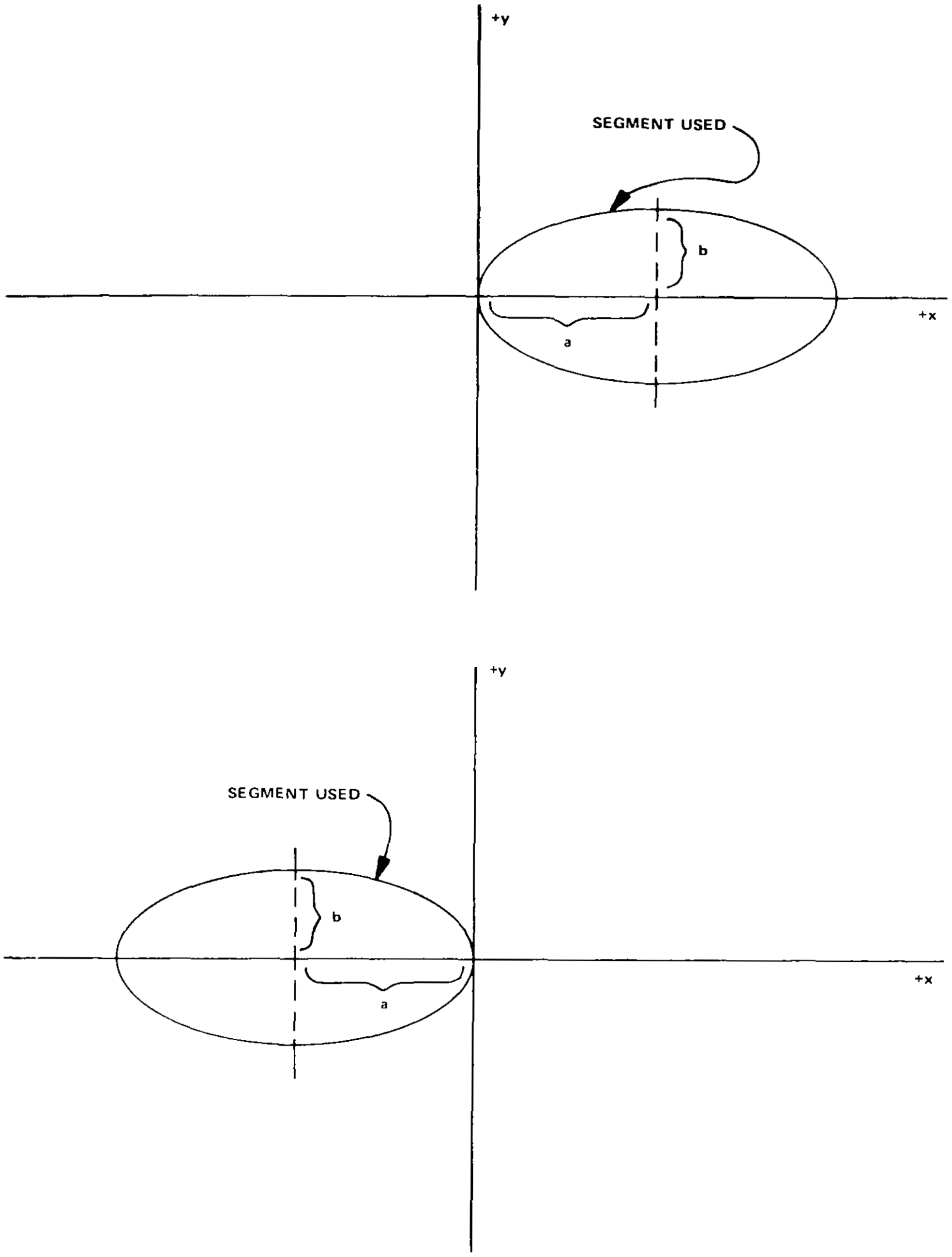


Figure 7
Standard Ellipse Orientations

Determination of the Ellipse.- Consider the section of the ellipse and the coordinate system shown in Figure 8. The equation for the ellipse shown is:

$$\frac{(x - a)^2}{a^2} + \frac{y^2}{b^2} = 1$$

The slope, m, of this ellipse at point (x, y) is:

$$m = - \frac{(x - a)}{a^2} \cdot \frac{b^2}{y}$$

If this slope and point (x, y) are given, then a and b are determined:

$$a = \frac{yx - mx^2}{y - 2\ xm}$$

$$b^2 = \frac{ma^2 y}{a - x}$$

For the elliptical section shown in Figure 8, $a > 0$. To satisfy this condition, the following inequality must hold:

$$\frac{y}{x} > 2m$$

Similarly, for the case in which $a < 0$, the inequality that must hold is:

$$\frac{y}{x} < 2m$$

The restrictions $y/x > 2m$ for $a > 0$ and $y/x < 2m$ for $a < 0$ may appear too limiting at first glance. It should be noted, however, that for a given set of data points, the restriction encountered by placing the origin of the coordinate system on one of the points is not present if the origin is placed on the other data point. This is illustrated in Figures 9 and 10. The elimination of these path restrictions in this manner leaves only one major restriction remaining. For a given set of data points connected by an elliptical segment, the slope of the path at the exit point must be greater than the slope of a straight line joining the points for $a < 0$. For $a > 0$, the slope of the path at the exit point must be less than the slope of a straight line joining the points. This is illustrated in Figure 11. This restriction precludes the introduction of a point of inflection in the curved path unless the inflection point is specified as a data point.

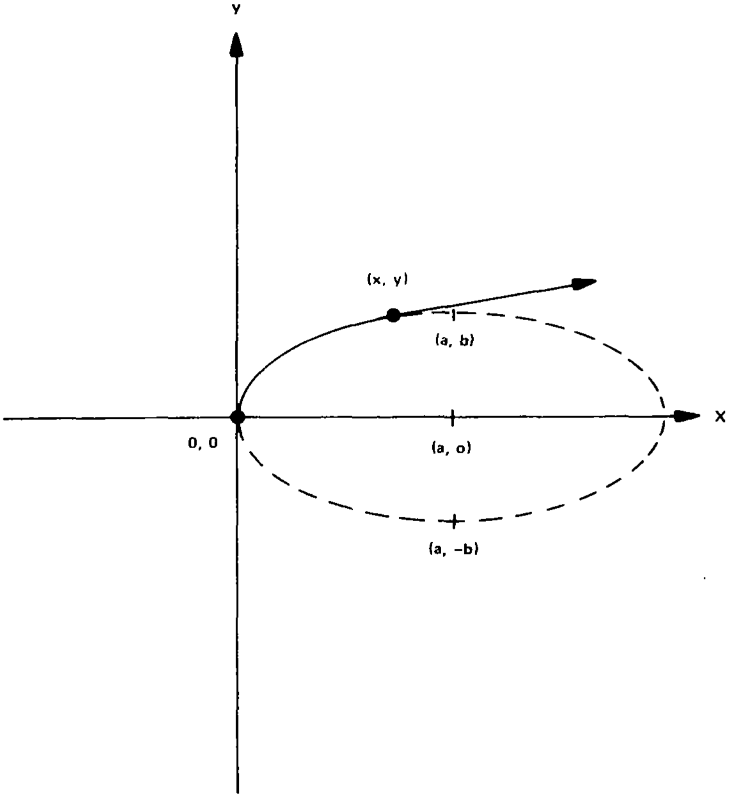


Figure 8
Elliptical Segment Coordinate System

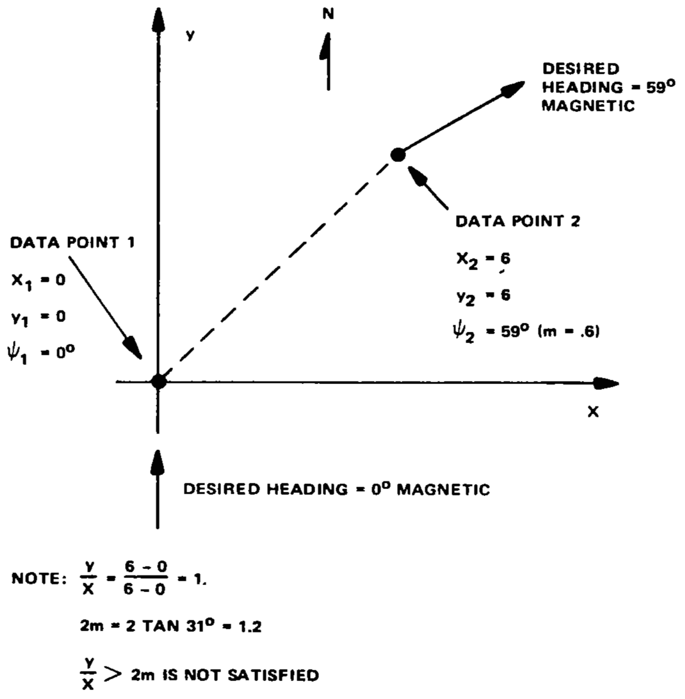
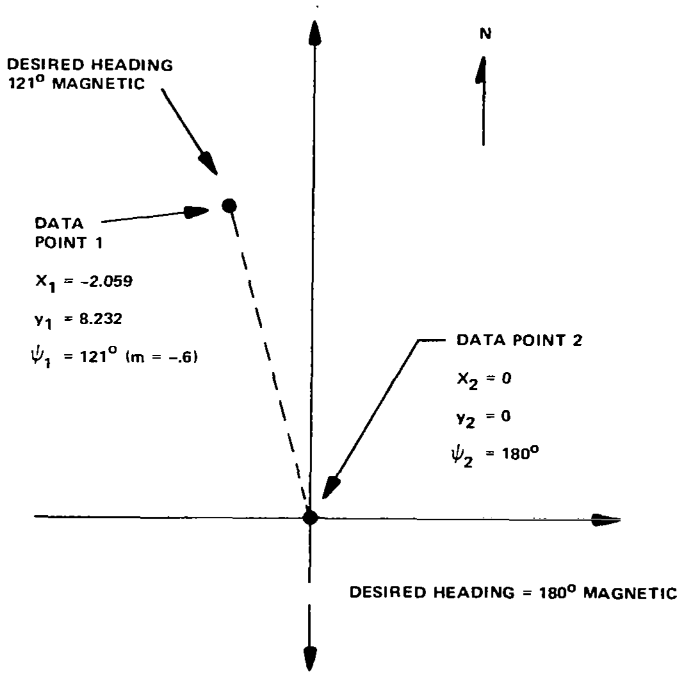


Figure 9
Elliptical Segment Path Restriction



NOTE: $\frac{Y}{X} = \frac{8.232 - 0}{-2.059 - 0} = -3.998$

$2m = 2 \tan(-31^\circ) = -1.2$

$\frac{Y}{X} < 2m$ IS SATISFIED

Figure 10
Rotated Elliptical Segment

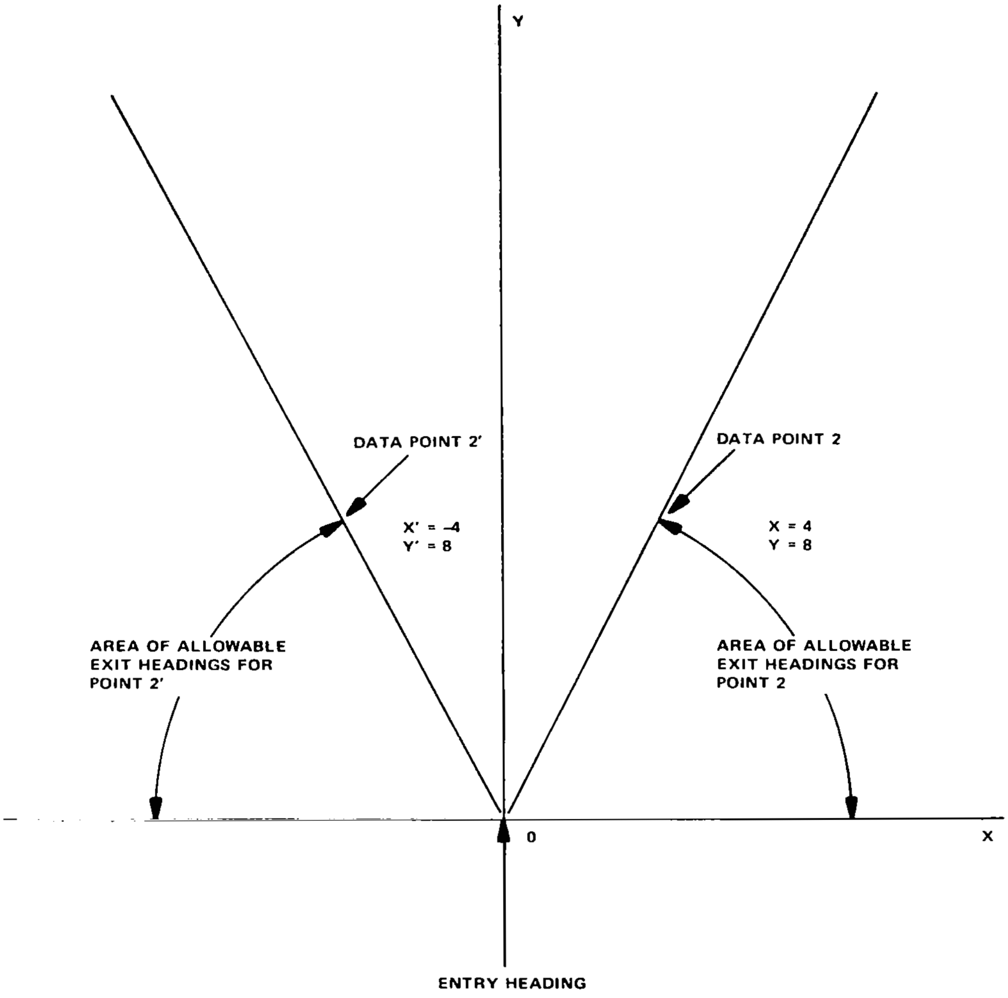


Figure 11
Area of Allowable Heading Changes

Determination of Crosstrack Error.- In order to determine the crosstrack error while flying along an elliptical section, it is necessary to determine the perpendicular distance from a given point to the ellipse. Consider the ellipse shown in Figure 12:

$$\frac{(x - a)^2}{a^2} + \frac{y^2}{b^2} = 1$$

and a point A near the ellipse defined by:

$$x = X_A$$

$$y = Y_A$$

The center of the ellipse is located at (a, 0) and the distance from the center of the ellipse to the two foci is given by:

$$c = \sqrt{a^2 - b^2}$$

The coordinates of the foci are then given as:

$$F_1 = (a - c),\ 0$$

$$F_2 = (a + c),\ 0$$

If X_A, Y_A were located on the ellipse, then the normal (perpendicular) to the ellipse at that point is the bisection of the angle defined as $F_1\ AF_2$.

Let: θ = bisection of angle $F_1\ AF_2$.

The angle from A to F_1 is defined by:

$$\theta_1 = \tan^{-1} \frac{Y_{F_1} - Y_A}{X_{F_1} - X_A}$$

where X_{F_1} and Y_{F_1} are coordinates of focus F_1.

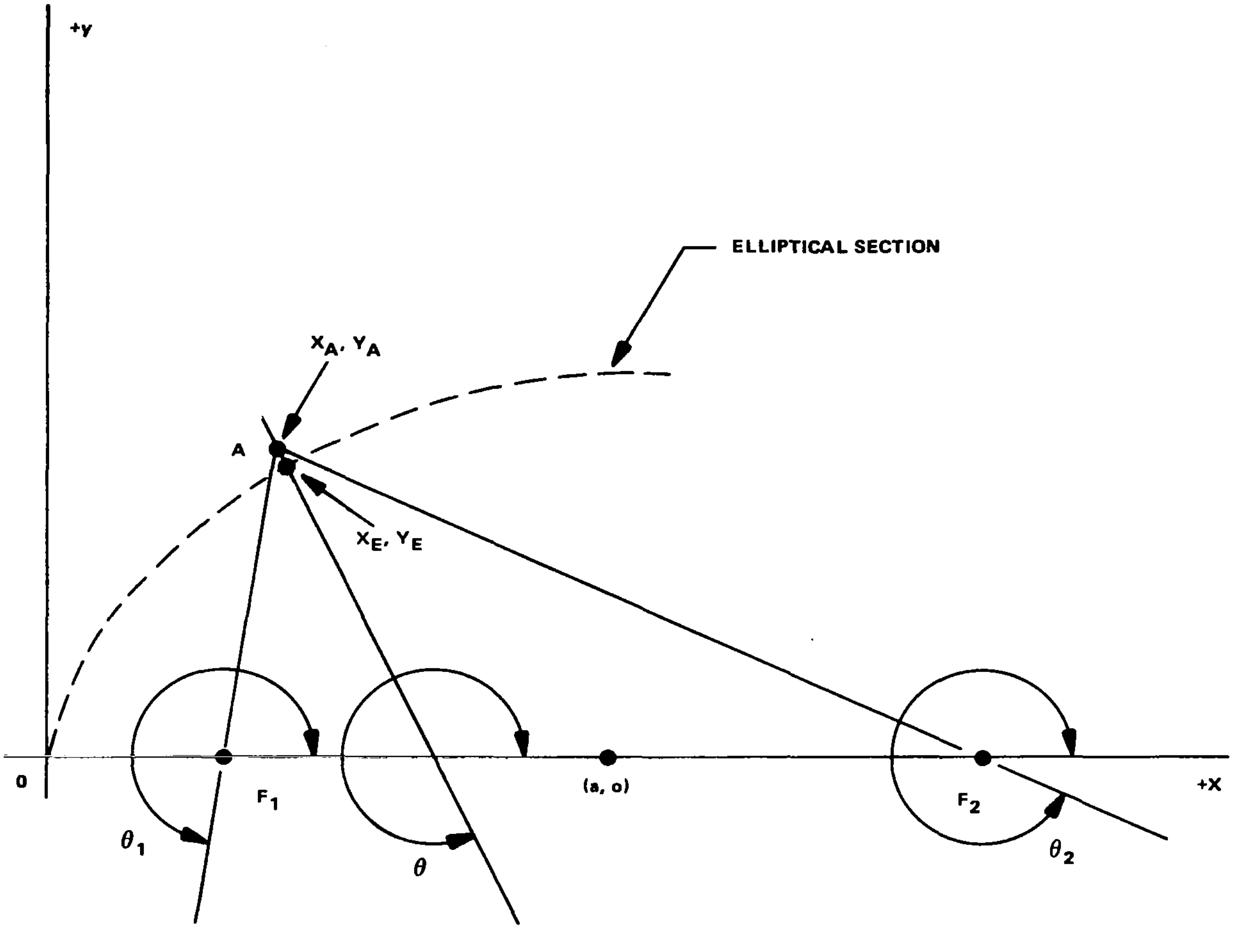


Figure 12
Normal Projection for a > b

for the orientation shown, $Y_{F_1} = 0$

$$\theta_1 = \tan^{-1} \frac{-Y_A}{X_{F_1} - X_A}$$

Similarly:

$$\theta_2 = \tan^{-1} \frac{-Y_A}{X_{F_2} - X_A}$$

θ is given as:

$$\theta = \frac{\theta_1 + \theta_2}{2}$$

Consider an ellipse of the same form as described, except that $b > a$. For this case, shown in Figure 13, the distance from the center of the ellipse to the foci is given by:

$$c = \sqrt{b^2 - a^2}$$

and the coordinates of the focal points are:

$$F_1 = a, c$$

$$F_2 = a, -c$$

In a manner similar to that previously presented, the angle θ is determined to be

$$\theta = \frac{\theta_1 + \theta_2}{2}$$

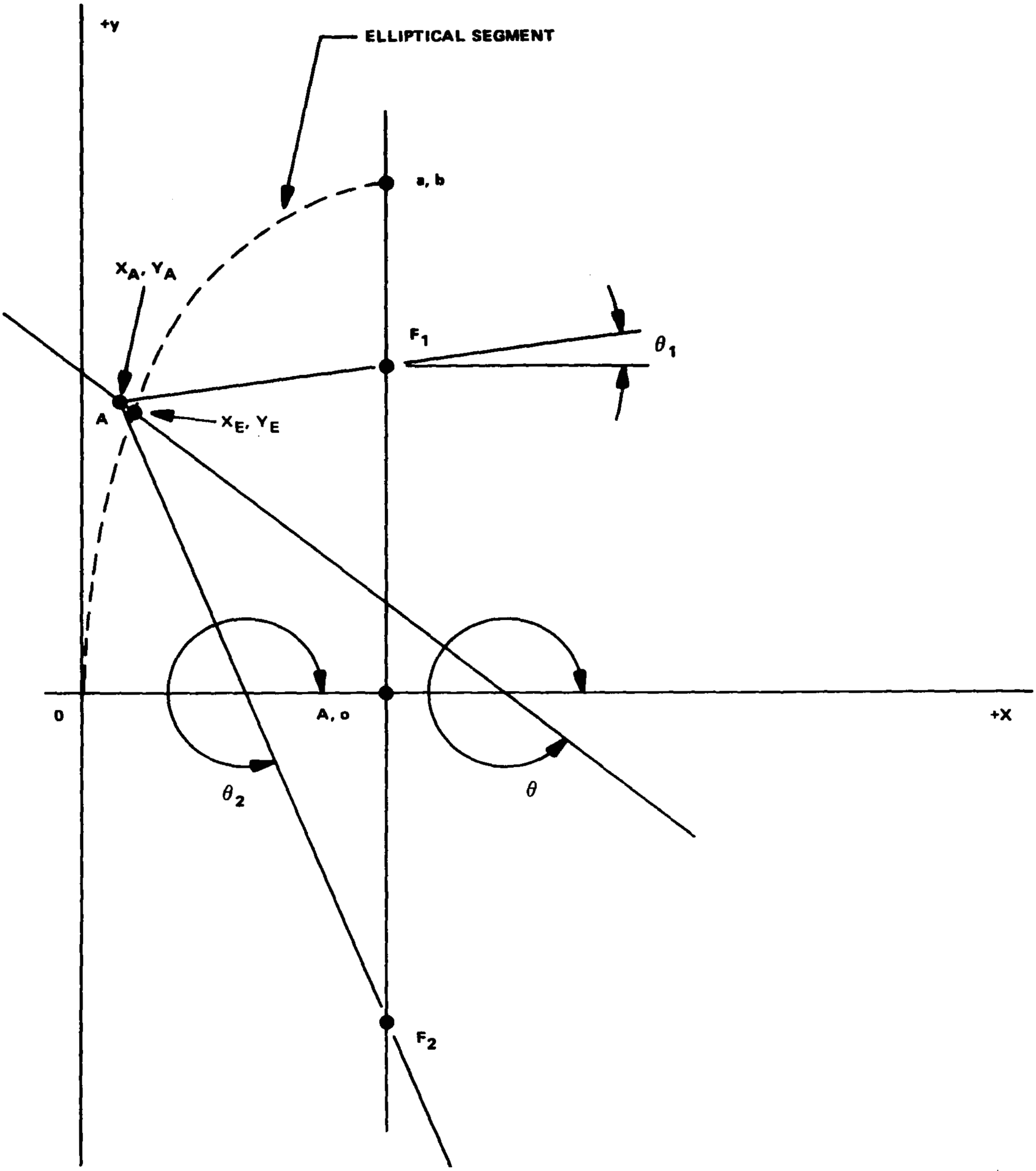


Figure 13
Normal Projection for b > a

where

$$\theta_1 = \tan^{-1} \frac{c - Y_A}{a - X_A}$$

$$\theta_2 = \tan^{-1} \frac{-c - Y_A}{a - X_A}$$

For small deviations of (X_A, Y_A) off the ellipse, assume θ still defines the normal to the ellipse. Then the projection (X_E, Y_E) of (X_A, Y_A) onto the ellipse is defined by θ. The slope of the ellipse at the point (X_E, Y_E) is the slope of a line perpendicular to the normal:

$$m_E = \tan (\theta + 90°)$$

The equations for the ellipse and the slope of a point on the ellipse can be solved to yield X_E and Y_E in terms of a, b, and m_E:

$$X_E = a_{\pm} \frac{a^2 m_E}{\sqrt{b^2 + a^2 \; m_E{}^2}}$$

$$Y_E = \frac{b^2}{\sqrt{b^2 + a^2 \; m_E{}^2}}$$

Given the capability to determine the projection of the aircraft position onto a specified elliptical section, it is then possible to compute the magnitude of the crosstrack error as:

$$\text{XTRACK} = \left| \sqrt{(X_A - X_E)^2 + (Y_A - Y_E)^2} \right|$$

where X_A, Y_A is the actual aircraft position and X_E, Y_E is the projection of this position onto the ellipse. The direction of the error is given by θ as previously defined.

Determination of Path Distance.- In order to determine the distance to go from any point on the lateral path it is necessary to determine the length of an elliptical segment. Determination of this length involves the use of elliptical integrals for which there are no explicit solutions. For this reason, a piecewise linear approximation of the elliptical segment is used to calculate distance data. Consider the approximation shown in Figure 14 where:

$$S = \sum_{i = 1}^{n} \Delta S_i$$

and

$$\Delta S_1 = \sqrt{(y_i - y_{i - 1})^2 + (x_i - x_{i - 1})^2}$$

If the x axis is divided into n equal segments then:

$$(x_n - x_{n - 1}) = \frac{x_T}{n}$$

where x_T = x coordinate where the elliptical segment ends.

Then:

$$S = \sum_{i = 1}^{n} \left[(y_i - y_{i - 1})^2 + \frac{x_T^2}{n^2} \right]^{1/2} \quad \text{for } 0 \leqslant x \leqslant x_T$$

where:

$$y_i = \left[b^2 - \frac{b^2}{a^2} (x_i - a)^2 \right]^{1/2}$$

The variable n can be readily changed and will be set equal to 8 initially.

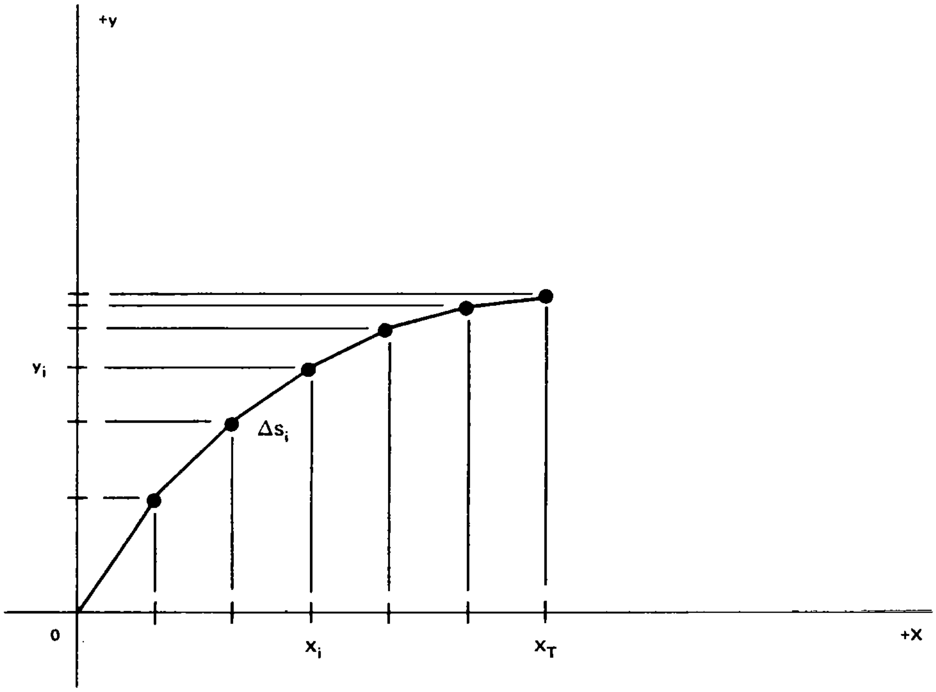


Figure 14
Path Length Approximation

The total path distance-to-go is then determined by computing the distance-to-go on the particular segment being traversed and adding it to the sum of the path lengths of the remaining segments. This information is used in conjunction with the speed and altitude profiles that are discussed in the next section. In addition, the distance-to-go can be presented to the pilot or the test engineer through a digital readout on the navigation/guidance control panel.

ALTITUDE AND VELOCITY PROFILES

In addition to the general purpose lateral path, the VALT system has the capability to generate and track arbitrarily shaped altitude and speed profiles. The lateral, altitude and speed profiles can be combined to produce curved, decelerating descents to a hover or a touchdown.

Altitude Profile

The altitude profile is specified by a series of data points. Each data point consists of a distance-to-go along the lateral flight path and a desired altitude at that distance. The altitude change between two data points is a linear function of the distance between those points.

Consider a portion of the altitude profile as shown in Figure 15. Between data point 3(S_3, H_3) and data point 2(S_2, H_2) the change in altitude as a function of distance is:

$$\Delta H = \frac{H_3 - H_2}{S_3 - S_2} \cdot \Delta S$$

and the altitude for any S ($S_2 \leqslant S \leqslant S_3$) is:

$$H = H_2 + \frac{(H_3 - H_2)}{(S_3 - S_2)} \cdot (S - S_2)$$

Data to describe the altitude profile is entered into the computer in the form of distance-to-go and desired altitude at that distance for each data point. Up to 25 data points may be used to define an altitude profile. Storage is provided for five different sets of data points in the computer. Distance-to-go and altitude are entered using a decimal data format. Data points to describe the altitude profile are entered in order, starting at the hover position and working backwards. A termination code (77777) is entered last to indicate the data point where the profile should start. The form of the data entry for a sample path is shown in Table 2. It should be noted that Table 2 does not show the data for Figure 15.

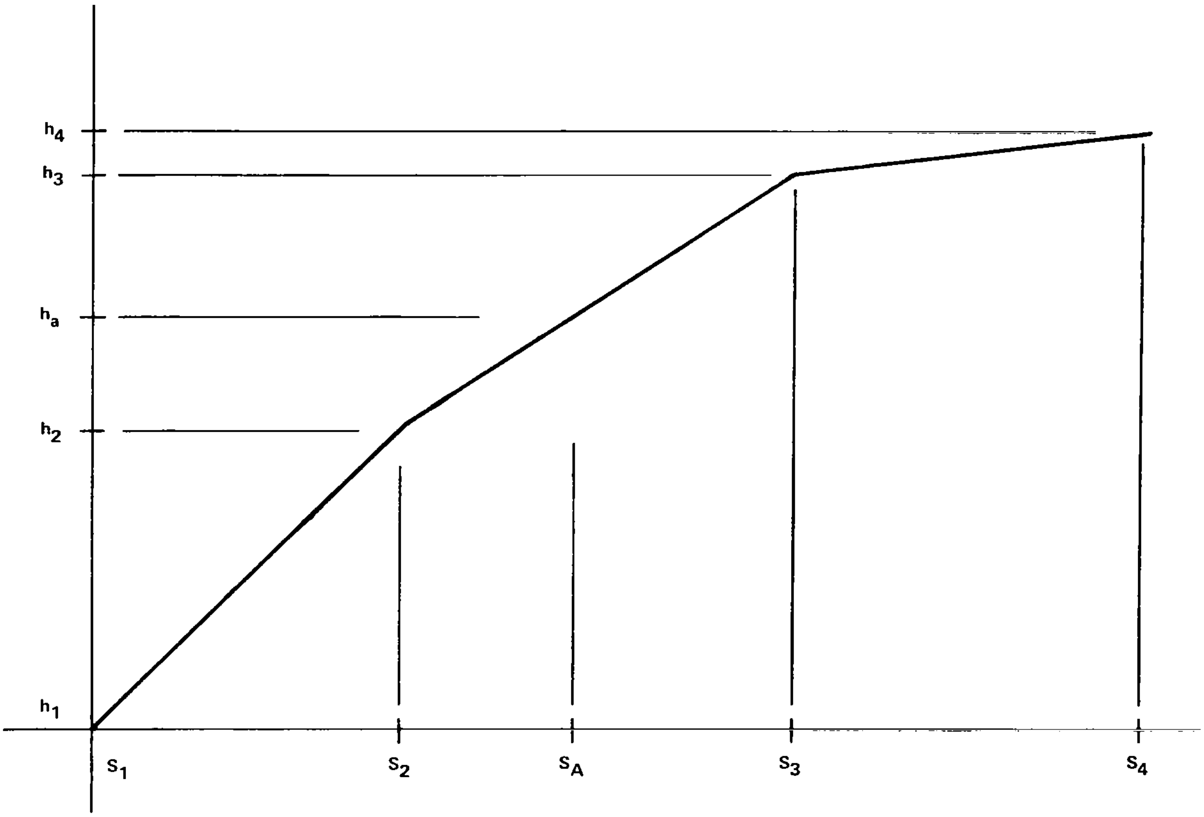


Figure 15
Altitude Profile Geometry

TABLE 2
ALTITUDE PROFILE DATA

Input	Data Point	Entry
1	1	0 Distance-to-go
2		0 Desired Altitude
3	2	200
4		25
5	3	300
6		35
7	4	1000
8		200
9	5	5000
10		1000
11	6	77777 Termination Code

Speed Profile

The speed profile is specified by a series of data points. Each data point consists of a distance-to-go along the flight path and a desired velocity at that distance. The velocity change between two data points is a linear function of the distance between those points.

Consider a portion of the velocity profile as shown in Figure 16. Between data point 3(S_3, V_3) and data point 2(S_2, V_2) the change in velocity as a function of distance is:

$$\Delta V = \frac{V_3 - V_2}{S_3 - S_2} \cdot \Delta S$$

and the velocity for any S ($S_2 \leqslant S \leqslant S_3$) is:

$$V = V_2 + \frac{(V_3 - V_2)}{(S_3 - S_2)} \cdot (S - S_2)$$

Data to describe the speed profile is entered into the computer in the form of distance-to-go and desired velocity at that distance for each data point. Up to 25 data points may be used to define a velocity profile. Storage is provided for five different sets of data points in the computer. Distance-to-go and desired velocity are entered using a decimal data format. Data points to describe the speed profile are entered in order, starting at the hover position and working backwards. A termination code (77777) is entered last to indicate the data point where the profile should start. The form of the data entry for a sample path is shown in Table 3. It should be noted that Table 3 does not show the data for Figure 16.

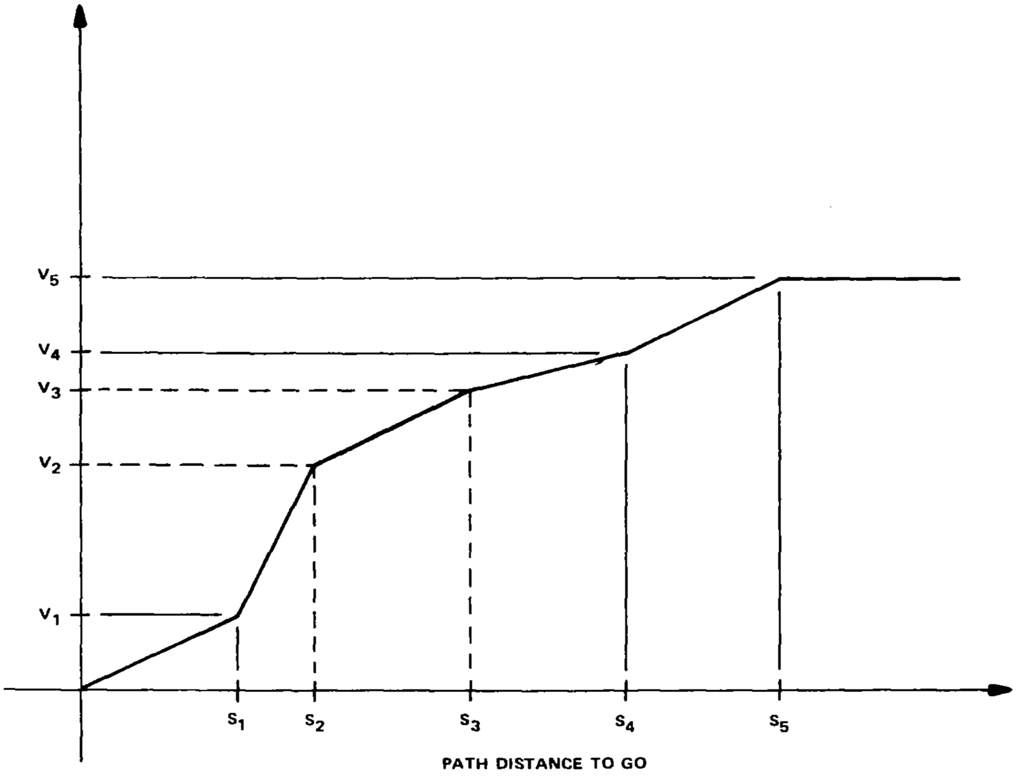


Figure 16
Speed Profile Geometry

TABLE 3

VELOCITY PROFILE DATA

Input	Data Point	Entry
1	1	0 Distance to go
2		0 Desired Velocity
3	2	500
4		10
5	3	1000
6		25
7	4	2500
8		50
9	5	6000
10		60
11	6	10000
12		60
13	7	77777 Termination Code

Combining Paths and Profiles

The techniques that have been developed for generating lateral paths and speed and altitude profiles can be combined to create a great variety of three dimensional flight trajectories. Since the technique used to generate each path or profile is essentially independent of the other path or profile, some consideration must be given to the problems that could occur when these various elements are combined into a trajectory. For example, a speed profile that calls for a constant speed over some portion of the lateral profile may not, due to aircraft power limitations, be compatible with an altitude profile that calls for a rapid altitude change over the same portion of the lateral profile.

The most common potential conflict of this type results when a velocity profile and a lateral profile are combined. The VALT system is normally constrained so as not to exceed a programmable bank-angle limit. If a speed profile is defined such as to require a certain speed at some distance-to-go for any path, and if a bank-angle limit is imposed on the system, then the minimum radius turn that could be flown at that point would be defined by the equation:

$$R = \frac{V^2}{G \tan \phi}$$

where

ϕ = bank angle

V = speed

G = acceleration due to gravity

R = radius

Since the lateral path technique used in the VALT system utilizes either straight line segments or elliptical line segments, it is possible to determine the radius of curvature at every point along the path. If the radius of curvature is smaller at any point than the minimum radius allowed at that point due to speed and bank angle constraints, then the lateral path and the speed profile are incompatible. The digital computer program that fits the lateral path to the data points has the capability to detect this type of incompatibility and to indicate which particular data points contribute to the problem. In order for this checking to be valid, however, it is necessary to add one more restriction to the list of restrictions previously presented:

- A speed profile must be stored in the computer before a lateral path curve fit is attempted in order to ensure that the nominal aircraft bank angle limit is not exceeded. The profile is defined in terms of speed versus along-track distance-to-go in the $Z = 0$ plane.

CONCLUDING REMARKS

Conclusions

A technique has been developed that can be used to generate arbitrarily shaped, curved path approach trajectories for the NASA VALT research helicopter. Although developed for the VALT Program, the technique is general purpose in nature and could be used in other applications. The method presented utilizes a combination of straight line segments and elliptical sections to fit a smooth, continuous, path to a series of data points. Altitude and speed profiles can be combined with the lateral path to create a curved, decelerating, descending approach to a hover or touchdown. Digital computer software has been created to implement the path-forming techniques in the NASA VALT system.

Recommendation

The final evaluation of the technique that was developed during this contract must await subsequent flight tests in the NASA VALT research aircraft. Preliminary evaluation of the technique in a fixed based simulator, however, indicates that consideration should be given to improving the transitions from straight line segments to elliptical segments and from the elliptical to the straight line segments. The technique presented in this report does produce paths that are continuous, i.e., no heading discontinuities are present. However, the junction between a curved segment and a straight line segment generally results in a bank-angle-command discontinuity. The curve fitting technique should be expanded to incorporate some form of transitional curve in order to provide smoother turn entry and exit and to reduce crosstrack error at those points.

APPENDIX

COMPUTER FLOW CHARTS

Flow charts describing the sequential operations that are performed by the digital computer during the curve-fitting and real-time guidance data processing programs are shown in Figures 17 and 18. Comments that relate to specific parts of the flow charts are keyed to reference numbers on the charts.

Comments Related to the Path Initialization Flow Chart

1. Compute straight line headings.- Process data table, converting all straight-segment flags to magnetic headings.

$$\text{Magnetic Heading:}\quad 90° - \tan^{-1}\frac{Y_2 - Y_1}{X_2 - X_1}$$

The following error checks are also made:

a) No path specified

b) Unpaired straight line point

c) Inflection point (two consecutive points have same heading. Straight segments must be identified with a straight-segment flag).

2. Compute segment length

$$\text{Segment Length} = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$$

3. Translate and rotate data points.- Transform points into new coordinate system having point 1 as the origin and the point 1 heading as the +Y direction.

4. Are data points valid:- Points are valid if new point 2 (X_2', Y_2') has slope M such that:

$$|M| > \left|\frac{Y_2}{2X_2}\right|$$

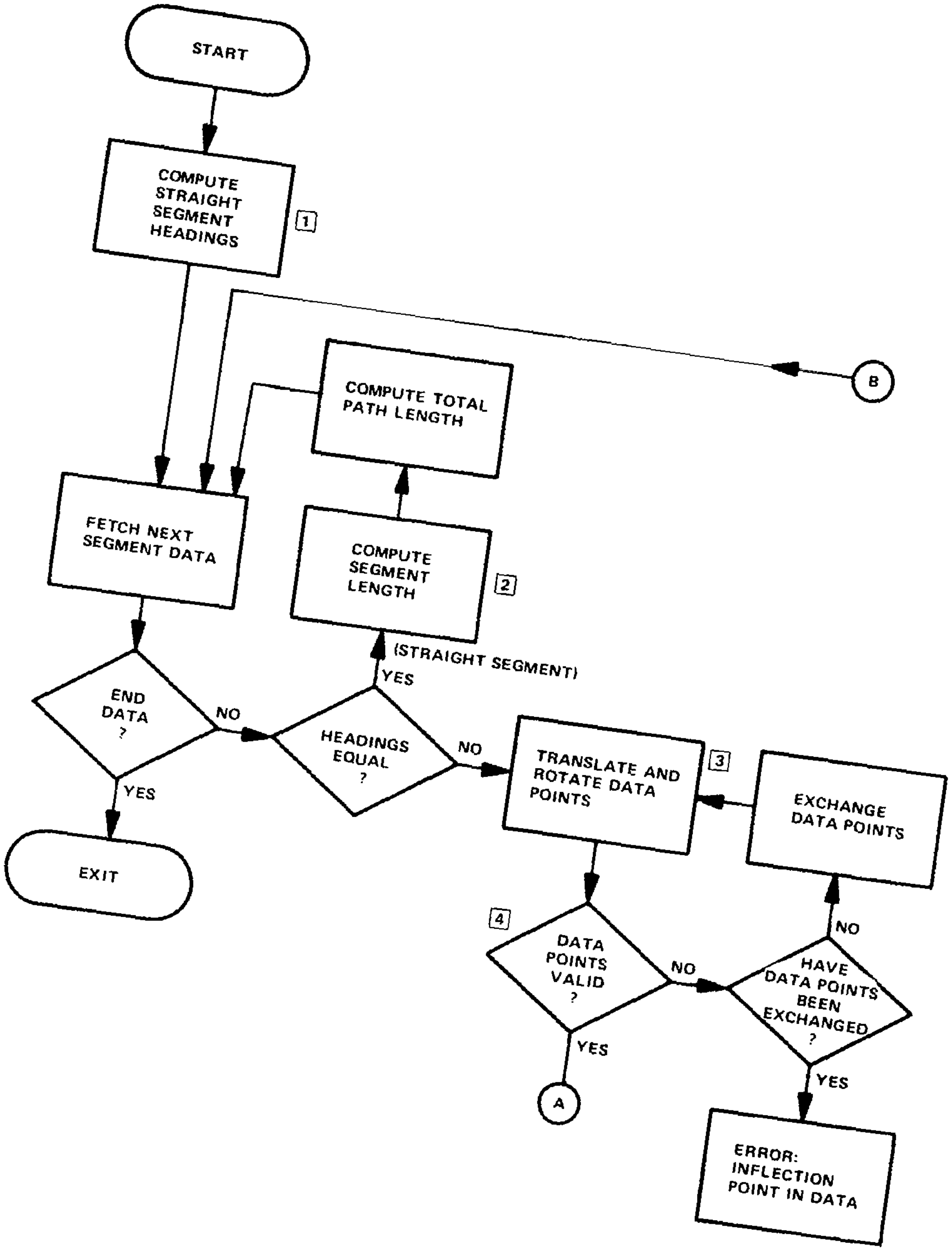


Figure 17
Path Initialization Flow Chart
(Sheet 1 of 2)

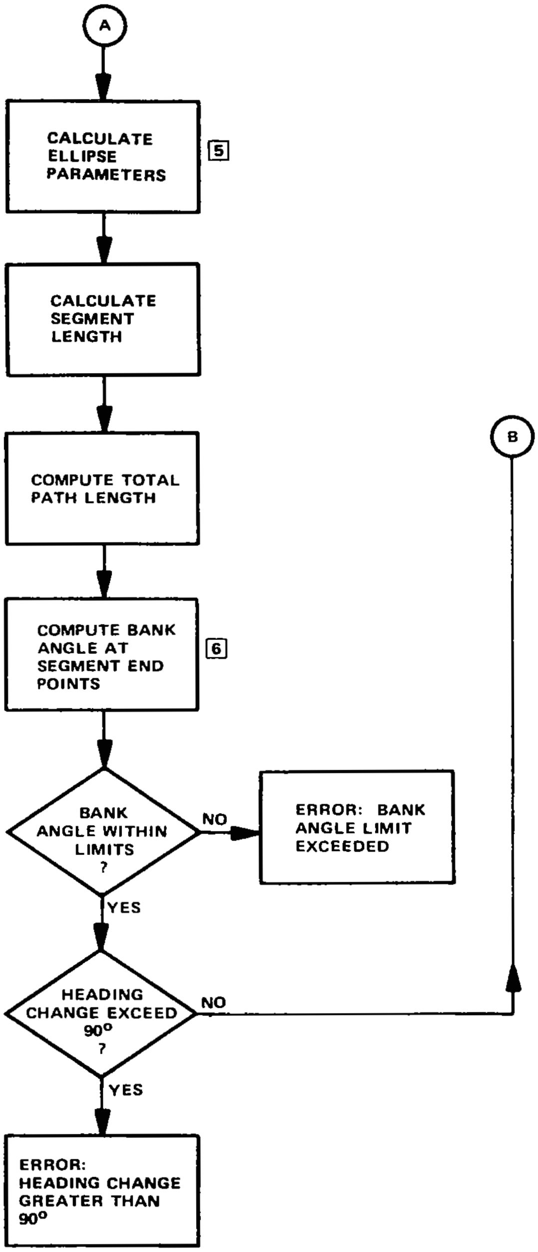


Figure 17
Path Initialization Flow Chart
(Sheet 2 of 2)

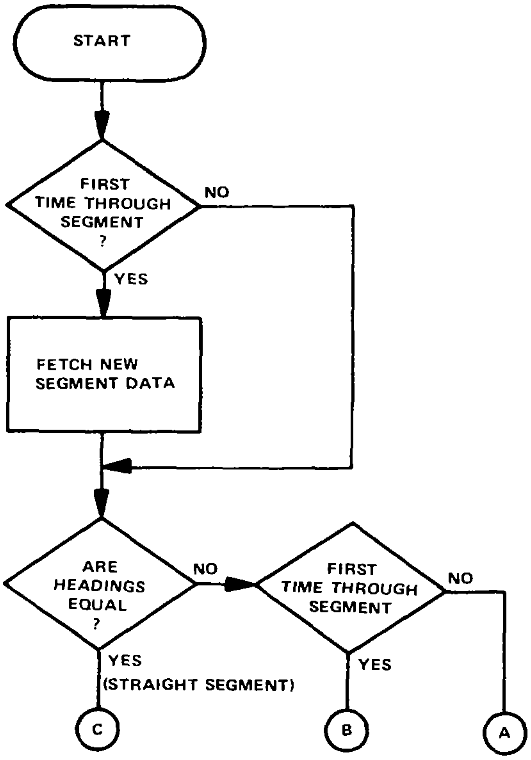


Figure 18
Real Time Computation Flow Chart
(Sheet 1 of 4)

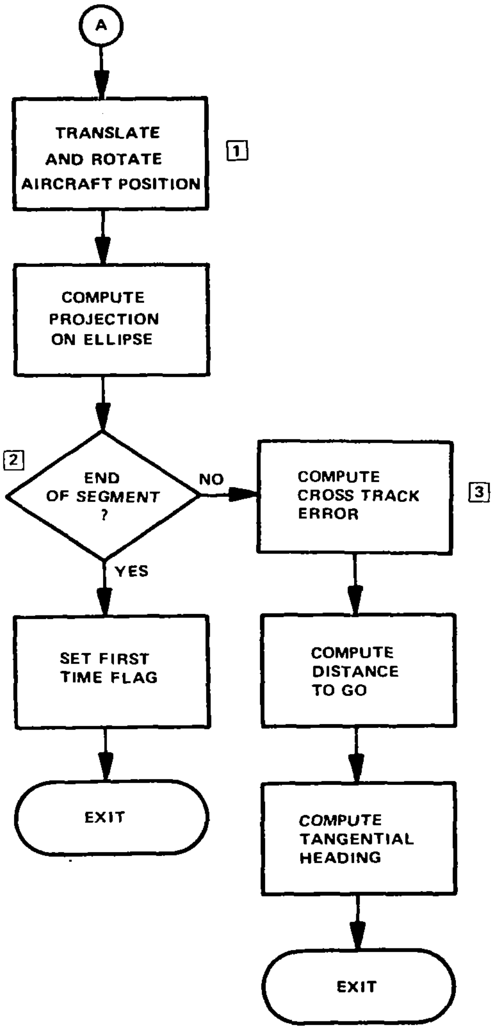


Figure 18
Real Time Computation Flow Chart
(Sheet 2 of 4)

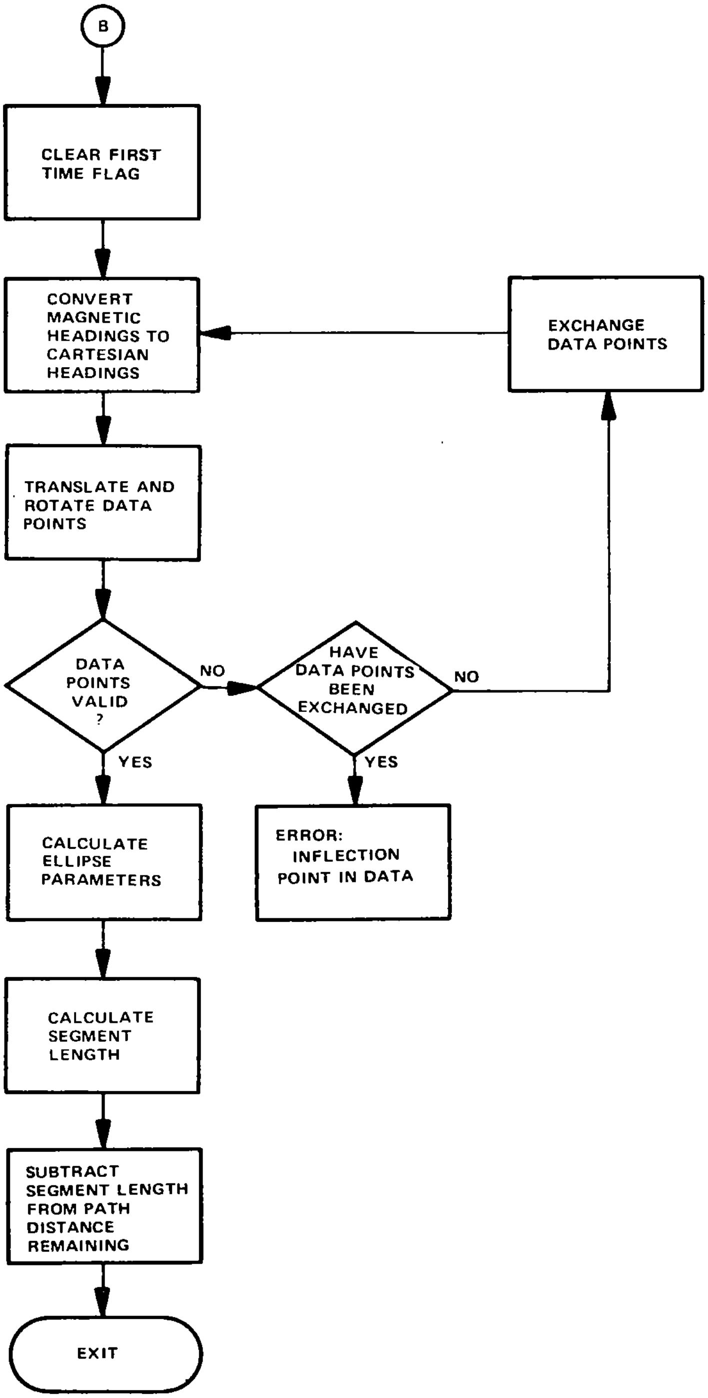


Figure 18
Real Time Computation Flow Chart
(Sheet 3 of 4)

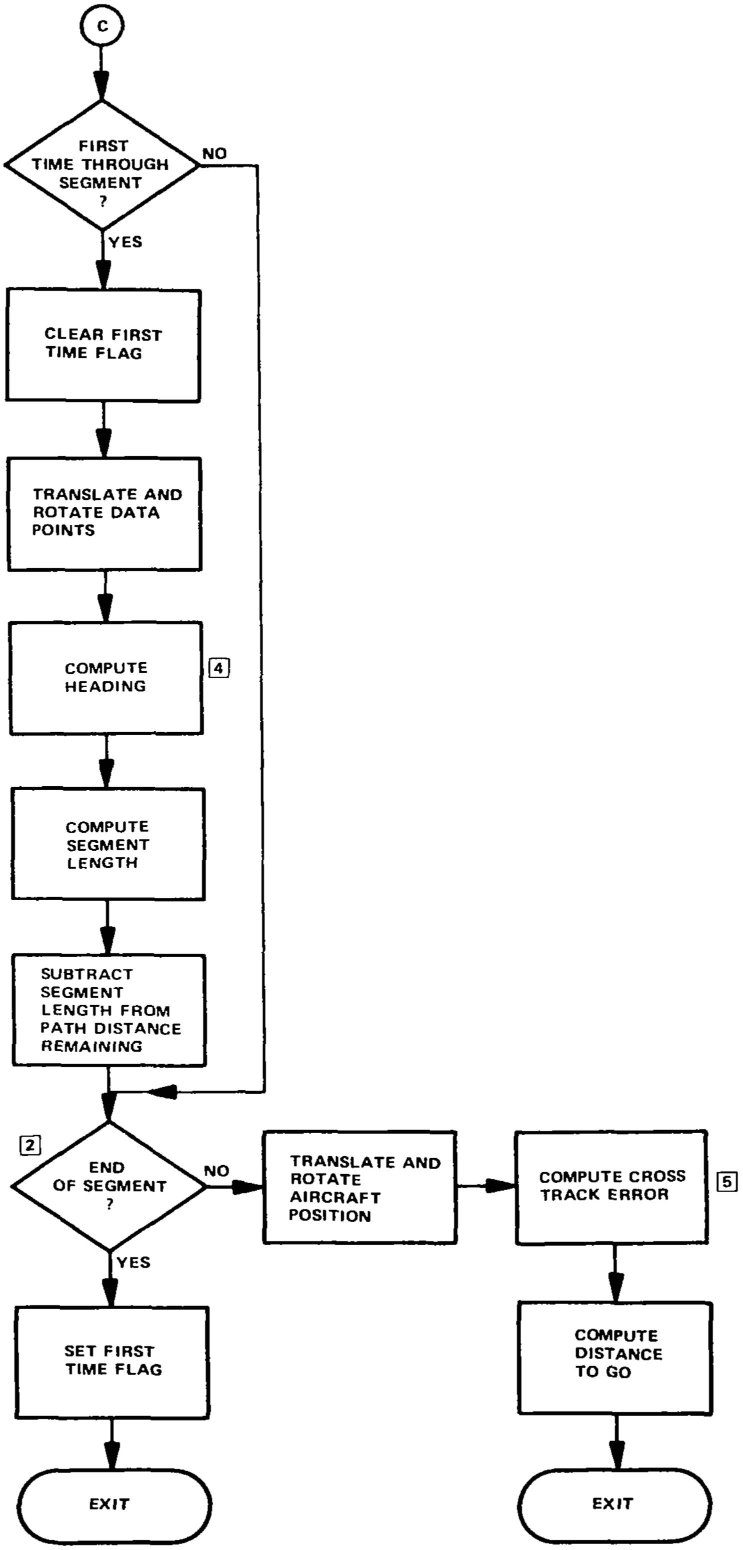


Figure 18
Real Time Computation Flow Chart
(Sheet 4 of 4)

5. Calculate ellipse axes and foci.

$$a = \frac{Y_2 X_2 - M X_2^{\,2}}{Y_2 - 2X_2 M}$$

$$b = \frac{Y_2}{\sqrt{1 - \frac{(X_2 - a)^2}{a^2}}}$$

where

M = slope of point (X_2, Y_2)

(X_2, Y_2) = point 2 in new coordinate system.

The distance from the center of the ellipse to the two foci is:

$$c = \sqrt{|a^2 - b^2|}$$

6. Compute bank angle at segment-end points and check against limit. Calculate the radius of curvature at the end points of the quarter ellipse of which the current segment is a part. If the bank-angle limit for the given lateral path is not exceeded at the end points of the quarter ellipse, then it is not exceeded anywhere on the segment. The velocity information required for this calculation is obtained by calculating distance-to-go for each segment end point and then calling the speed profile program twice to obtain the velocity desired for each of these points. If the limit is exceeded at the origin, signal an error. If the limit is exceeded at the other quarter-ellipse end point, then calculate the radius of curvature at the actual segment-end point rather than the quarter-ellipse end point.

Comments Related to the Real-Time Computation Flow Chart

1. Take the aircraft (X, Y) position and transform it to the current segment coordinate system.

2. Transition to next segment?- The initial decision to transition is made if the projection of the actual position onto the extended segment is beyond the segment proper. Once a transition is made, it is not possible to go back to a previous segment. Therefore, a filter is provided to prevent spurious data from causing a false transition. A count is kept in FCOUNT. The initial decision to transition must be made FCOUNT cycles in a row before a transition will occur.

3. $$\text{Crosstrack Error} = \pm\sqrt{(X_A - X_E)^2 + (Y_A - Y_E)^2}$$

where

$$\left.\begin{aligned}(X_A, Y_A) &= \text{aircraft position}\\(X_E, Y_E) &= \text{projection point}\end{aligned}\right\}\quad\text{in transformed coordinate system}$$

The crosstrack error is positive if the aircraft is to the right of the path; otherwise it is negative. To determine where the aircraft is with respect to the ellipse, do a horizontal or vertical projection onto the ellipse, and compare this projection point with the aircraft position. In Quadrant I, the aircraft is to the right of the path if it is on the "inside" of the ellipse. In Quadrant IV the aircraft is to the right of the path if it is on the "outside" of the ellipse. If the data points of the segment have been exchanged, change the sign of the crosstrack error determined above.

4. The desired heading at any point on a straight segment is the heading at the end points.

5. Calculate Cross Track Error.- For the straight segment case, crosstrack error is simply the X-coordinate (X_T) of actual position in the new coordinate system. This is because the translated and rotated segment lies on the Y-axis. Cross-track error is positive if the aircraft is to the right of the path and negative otherwise. Since the segment heading is along +Y direction, the sign of X_T is the correct sign for the cross-track error.

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1. Kelly, James R.; Niessen, Frank R.; Thibodeaux, Jerry J.; Yenni, Kenneth R.; and Garren, John F. Jr.; Flight Investigation of Manual and Automatic VTOL Decelerating Instrument Approaches and Landings. NASA TM D-7524, 1974.

2. McConnell, Walter J. Jr.; Skutecki, Edmund R.; and Calzado, Alfonso J.: Development of the NASA VALT Digital Navigation System. NASA CR 144894, September 1975.

3. Niessen, Frank R: A Low-Cost Inertial Smoothing System for Landing Approach Guidance. NASA TM D-7271, June 1973.

NASA-Langley, 1976 CR-2734