# R-9808 <br> ORBITAL MANEUVERING ENGINE <br> FEED SYSTEM COUPLED STABILITY INVESTIGATION <br> COMPUTER USER'S MANUAL 

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## Rockwell International



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PREPARED FOR
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## ABSTRACT

This report is an operating manual for the Feed System Coupled Stability Model. It is submitted as partial fulfillment of an ll-month program designed to develop, verify, and document a digital computer model that can be used to analyze and predict engine/feed system coupled instabilities in pressure-fed storable propellant propulsion systems over a frequency range of 10 to 1000 Hz .

The first section describes the analytical approach to modeling the feed system hydrodynamics, combustion dynamics, chamber dynamics, and overall engineering model structure, and presents the governing equations in each of the technical areas. This is followed by the Program User's Guide, which is a complete description of the structure and operation of the computerized model. Last, appendixes provide an alphabetized FØRTRAN symbol table, detailed program logic diagrams, computer code listings, and sample case input and output data listings.
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## INTRODUCTION

Historically, during the development of pressure-fed propulsion systems, feed system/engine coupled instabilities have been frequently encountered. Resolution of these problems usually included increasing injector pressure drop to decouple the feed system from the combustor, the result being substantial system weight penalties. A dynamic computer model would be a useful tool in obviating coupled stability problems during the development of the Space Shuttle Orbit Maneuvering System (SS/OMS). A model could be used both as a system design tool to optimize component location and pressure profile (minimize system weight) and a system development tool to define test programs for assessing stability margins of the OMS.

This document is an operating manual for the Feed System Coupled Stability Model (FSCSM) and is submitted as partial fulfillment of an 11 -month program conducted by Rocketdyne to develop and verify an engineering digital computer model for the NASA/JSC which can be used to analyze feed system/engine coupled instabilities in pressure-fed, storable propellant, propulsion systems over a frequency range of 10 to 1000 Hz (frequencies lower than the chamber transverse frequencies). The model is sufficiently general so that it may be readily applicable to present and future engine and propulsion programs. For scaling purposes, the baseline configuration chosen is the OMS engine. The model has been written for use on the NASA/ JSC Univac 1110, EXEC-8 computer system, and provides NASA a tool which can be used to:

1. Conduct preliminary design tradeoff for feasibility studies prior to propulsion concept selection.
2. Guide the design of propulsion systems to ensure stability at all operating ranges and with minimum penalties.
3. Guide testing programs by predicting the least stable operating regimes thereby reducing the number of stability tests required.
4. Provide stability verification in the event system changes are made and hot-fire verification is impractical.
5. Diagnose problems on existing systems and evaluate potential solutions.

The work performed in completing the requirements of the program's technical effort is described in a separate companion document, entitled OME Feed SystemCoupled Stability Model, Final Report (Ref. 1). This includes the mathematical formulation of the model, development of the model into an overall engineering structure, and verification of the model's operation and capabilities by comparing the model's theoretical predictions with experimental data from an OMS engine and test rig with known feed system/engine chugging history.

The present document contains a detailed description of the structure and operation of the FSCSM. In the first section, the mathematical formulation of the model is reviewed. The analytical approach to modeling the feed system hydrodynamics, combustion dynamics, chamber dynamics, and overall engineering structure is described and the equations utilized by the model in each of the technical areas are presented. The reader may consult Ref. 1 for more details pertaining to the derivation of the equations.

The Program User's Guide section contains the instructions necessary to operate the computer model and interpret the results. First, the structure and logic of the main program and all subroutines are described, followed by a description of the input data required to run FSCSM. The input is divided into four major sections: (1) main control, (2) nozzle admittance control, (3) hydrodynamics control, and (4) combustion dynamics control. The format, content, and description of each input data card is clearly specified for each control section. The output of the FSCSM computer program is then discussed in terms of each tabular page of printout. Finally, additional details on program operation are presented, including program size, overlay structure, computer time, and program input/output data set file information. Appendixes provide an alphabetized FORTRAN symbol table, detailed program logic diagrams, computer code listings, and sample case input and output data listings.

## MATHEMATICAL FORMULATION OF MODEL

## INTRODUCTION

During certain periods of a rocket engine's operation, conditions within the combustion chamber and feed system are time variant, i.e., the operation is not steady with respect to time. Prime interest of this computer model is focused on abnormal transient operation during unstable combustion, i.e., pressure oscillations in a combustion device which are driven by the feed system and sustained by the combustion process. Start and stop transients are not considered.

The deviations from steady-state combustion which occur durina unstable burning depend upon the kind of instability experienced. Liauid rocket instabilities are classified according to their dominant time-varying processes. They may be divided initially into two categories, depending upon whether the instability oscillation wave length is long or short compared with the chamber dimensions.

If the instability wave length is considerably longer than the chamber length and diameter, pressure disturbances propagate rapidly through the combustion space compared with rates of change due to the instability. As a result, wave motion in the chamber may be neglected and chamber pressure can be considered to vary only with time but not to vary spatially (i.e., $P_{c}$ is a lumped parameter). These instabilities depend upon a fluid mechanical coupling between the propellant feed system(s) dynamics (fluctuating injection rates), the propellant combustion rates (delay times), and the combustion gas exhaust rates (pressure relaxation). Such instabilities can be further subdivided into various categories depending on the extent of wave motion in the feed system.

The breakpoint at which chamber wave metion becomes important is not abrupt. In reality, chamber wave motion is always present and, in effect, lumped chambe instabilities are really "zero order mode" limits of more general wave motion instabilities. In practice, it is found that the chamber gases
can be considered to act as a lump until the frequency of oscillation exceeds roughly one-fourth of the frequency of the lowest chamber acoustic resonance mode. At and above such frequencies wave motion becomes important and cannot be neglected in analysis. Chamber wave motion instabilities are characterized by the wave-length of the oscillatory motion being comparable to the chamber dimensions. As with lumped chamber instabilities, the driving energy comes from oscillatory snray combustion. With wave motion instabilities, however, in addition to the effects of injection rate fluctuations, there is the combustion response of burning propellant sprays as they are disturbed by passage of a pressure wave through them. Wave motion may increase local buraing rates by any of several mechanisms: (1) a pressure effect on the drop vapor gas phase burning rates: (2) enhanced mixino between gases and between sprays and gases; and (3) increased spray oasification rates. Increased spray gasification may be due to transient increases in convective flow velocities, to increased temperature or concentration gradients, and/or spray droplet shattering. The instability amplitude depends upon the magnitude of the response, and vice versa; typically, the interacting processes are driven to a limit represented by abrupt, essentially complete consumption of the propellant sprays. This direct response can be so great that injection rate fluctuations may be of secondary importance. As a result this class of instability can also be further subdivided as to the importance of feed system coupling. In the absence of feed system coupling, the instability is referred to as "classical acoustic instability." Only longitudinal chamber modes with feed system coupled instabilities are considered in this program.

FEED SYSTEM DYNAMICS
Development of the Waterhammer Equations
Consider the differential control volume of a fluid element in a duct shown in Fig. 1.


Fig. 1. Differential Pressures Developed Across the Incremental Length of a Fluid Element

Fluid compressibility and Newton's second law leads to the following pair of differential equations:

$$
\begin{align*}
& \frac{\partial p}{\partial t}=-\beta \frac{\partial v}{\partial x}=-c^{2} \rho \frac{\partial v}{\partial x}  \tag{1}\\
& \frac{\partial p}{\partial x}=-\rho \frac{\partial v}{\partial t}=-\frac{\beta}{c^{2}} \frac{\partial v}{\partial t}, \tag{2}
\end{align*}
$$

where

$$
\begin{aligned}
& p=\text { fluid pressure } \\
& v=\text { fluid velocity } \\
& B=\text { fluid bulk modulus } \\
& \rho=\text { fluid density } \\
& C=\text { acoustic velocity }=(B / \rho)^{\frac{1}{2}}
\end{aligned}
$$

There are several ways in which to solve these equations. The solution method presented here follows that of Ezekiel (Ref. 2). The general form of the solution that satisfies either of equations 7 ) and (2) is

$$
\begin{equation*}
p=F_{1}\left(t+\frac{x}{c}\right)+F_{2}\left(t-\frac{x}{c}\right) \tag{3}
\end{equation*}
$$

where $F_{1}$ and $F_{2}$ are arbitrary functions.

Taking the partial derivative of $p$ with respect to $x$ and $t$ separately and substituting the results in equations (1) and (2) gives:

$$
\begin{align*}
& \frac{\partial v}{\partial x}=-\frac{1}{\beta} \frac{\partial p}{\partial t}=-\frac{1}{B}\left[F_{1}^{\prime}\left(t+\frac{x}{c}\right)+F_{2}^{\prime}\left(t-\frac{x}{c}\right)\right]  \tag{4}\\
& \frac{\partial v}{\partial t}=-\frac{1}{\rho} \frac{\partial p}{\partial x}=-\frac{1}{\rho c}\left[F_{1}^{\prime}\left(t+\frac{x}{c}\right)-F_{2}^{\prime}\left(t-\frac{x}{\varepsilon}\right)\right] \tag{5}
\end{align*}
$$

where

$$
F^{\prime}(\xi)=\frac{\partial F(\xi)}{\partial \xi} .
$$

The expression for $v$ is obtained from either equation (4) or (5):

$$
z v=-F_{1}\left(t+\frac{x}{c}\right)+F_{2}\left(t-\frac{x}{c}\right)
$$

where

$$
\begin{equation*}
z \equiv(\rho \beta)^{\frac{1}{2}} . \tag{7}
\end{equation*}
$$

Letting the subscript $o$ denote $x=0$, the upstream position, and the subscript $L$ denote $x=L$, the downstream position, and defining $\tau=L / C$ as the signal propagation time between the two positions, equations (.3) and (.6) become

$$
\begin{align*}
& P_{0}=F_{1}(t)+F_{2}(t)  \tag{8}\\
& P_{L}=F_{1}(t+\tau)+F_{2}(t-\tau)  \tag{9}\\
& z v_{0}=-F_{1}(t)+F_{2}(t)  \tag{10}\\
& z v_{L}=-F_{1}(t+\tau)+F_{2}(t-\tau) \tag{11}
\end{align*}
$$

Combining Eqs. (8) and (10), and Eqs. (9) and (11) separately, yields four additional relations:

$$
\begin{align*}
& p_{0}+2 v_{0}=2 F_{2}(t)  \tag{12}\\
& p_{0}-2 v_{0}=2 F_{1}(t) \tag{13}
\end{align*}
$$

$$
\begin{align*}
& P_{L}+z v_{L}=2 F_{2}(t-\tau)  \tag{14}\\
& p_{L}-z v_{L}=2 F_{1}(t+\tau) . \tag{15}
\end{align*}
$$

Eliminating the functions $F_{1}$ and $F_{2}$ gives the final result as:

$$
\begin{align*}
& {\left[p_{0}+z v_{0}\right]_{(t-\tau)}=p_{L}+z v_{L}}  \tag{16}\\
& {\left[p_{L}-z v_{L}\right]_{(t-\tau)}=p_{0}-z v_{0}} \tag{17}
\end{align*}
$$

Consider now Fig. 2, which depicts a generalized line segment forming a portion of a feed system with many such segments.


Figure 2. Generalized Line Segment

The equations which describe the pressure and flows as functions of time and of each other for the generalized line segment are obtained from Eqs. (16) and (17):

$$
\begin{align*}
& p_{n}-\left(\frac{v_{n}}{A_{n} g}\right) \dot{w}_{n}=\left[p_{n+1}^{\prime}-\left(\frac{v_{n}}{A_{n} g}\right) \dot{w}_{n+1}\right]_{\left(t-\tau_{n}\right)}-  \tag{18}\\
& P_{n+1}^{\prime}+\left(\frac{v_{n}}{A_{n} g}\right) \dot{w}_{n+1}=\left[p_{n}+\left(\frac{v_{n}}{A_{n} g}\right) \dot{w}_{n}\right]_{\left(t-\tau_{n}\right)} \tag{19}
\end{align*}
$$

The expression, $t-\tau_{n}$, indicates values at $\tau_{n}$ seconds before, and

$$
\begin{align*}
& p_{n+1}^{\prime}=p_{n+1}+R_{n}\left|\dot{w}_{n}\right| \dot{w}_{n}  \tag{20}\\
& \dot{w}_{n}=\rho_{n} A_{n} v_{n} . \tag{21}
\end{align*}
$$

Equations (18) and (19) are solutions of the wave equation, and equation ( 20 ) is the flow through a nonlinear fluid resistance. Letting

$$
\begin{equation*}
\alpha_{n}=\frac{v_{n}}{A_{n} g} \tag{22}
\end{equation*}
$$

these equations can be combined to give:

$$
\begin{align*}
& p_{n}-\alpha_{n} \dot{w}_{n}=\left[p_{n+1}+R_{n}\left|\dot{w}_{n+1}\right|\left(\dot{w}_{n+1}-\alpha_{n}\right)\right]_{\left(t-\tau_{n}\right)}  \tag{23}\\
& p_{n}+R_{n-1}\left|\dot{w}_{n}\right| \dot{w}_{n}+\alpha_{n-1} \dot{w}_{n}=\left[p_{n-1}+\alpha_{n-1} \dot{w}_{n-1}\right]_{\left(t-\tau_{n-1}\right)} . \tag{24}
\end{align*}
$$

Eliminating $p_{n}$ and rearranging into quadratic form results in

$$
\begin{align*}
& R_{n-1} \dot{w}_{n}^{2}+\left(\alpha_{n-1}+\alpha_{n}\right) \dot{w}_{n}-\left[p_{n-1}+\alpha_{n-1} \dot{w}_{n-1}\right]_{\left(t-\tau_{n-1}\right)}  \tag{25}\\
& +\left[p_{n+1}+R_{n}!\dot{w}_{n+1} \mid\left(\dot{w}_{n+1}-\alpha_{n}\right)\right]_{\left(t-\tau_{n-1}\right)}=0
\end{align*}
$$

which can be solved for the appropriate solution using the quadratic formula. The tank end parameters are obtained using a solution of Eq. ( 23 ) only. The injector end solution is obtained using the quadratic formula for equation (25).

The lineār model incorporated in the Hydrodynamics subprōram utilizes the same basic equations, (23) and (24), but in the following linearized form:

$$
\begin{gather*}
\left(\delta p_{n}\right)-a_{n}\left(\delta \dot{w}_{n}\right)=\left[\left(\delta p_{n+1}^{\prime}\right)-a_{n}\left(\delta \dot{w}_{n+1}\right)\right]_{\left(t-\tau_{n}\right)}  \tag{26}\\
\cdots\left(\delta p_{n+1}^{\prime}\right)+\alpha_{n}\left(\delta \dot{w}_{n+1}\right)=\left[\left(\delta p_{n}\right)+\alpha_{n}\left(\delta \dot{w}_{n}\right)\right]_{\left(t-\tau_{n}\right)}, \tag{27}
\end{gather*}
$$

where

$$
\begin{equation*}
\left(\delta p_{n+1}^{\prime}\right)=\left(\delta p_{n+1}\right)+2 R_{n} \overline{\dot{w}}_{n+1}\left(\delta \dot{w}_{n+1}\right) . \tag{28}
\end{equation*}
$$

These equations are then combined, resulting in

$$
\begin{align*}
& a_{n}\left(\delta \dot{w}_{n}\right)-\left(\delta p_{n}\right)+\left[\left(\delta p_{n+1}\right)+\left(\bar{R}_{n}-\alpha_{n}\right)\left(\delta \dot{w}_{n+1}\right)\right]_{\left(t-\tau_{n}\right)}=0  \tag{29}\\
& \left(\bar{R}_{n}+a_{n}\right)\left(\delta \dot{w}_{n+1}\right)+\left(\delta p_{n+1}\right)-\left[\left(\delta p_{n}\right)+a_{n}\left(\delta \dot{w}_{n}\right)\right]_{\left(t-\tau_{n}\right)}=0, \tag{30}
\end{align*}
$$

where

$$
\begin{equation*}
\bar{R}=2 R_{N} \overline{\dot{w}}_{n+1} \tag{31}
\end{equation*}
$$

At the tank end, the term $\delta p_{n}$ is zero for constant tank pressure. At the injector end, $\delta p_{n+1}$ is the independent variable.

## Injector Dynamics

The injector dynamics are included by treating the injector as a lumped compressible volume as shown in the figure below.


Figure 3. Schematic of the Injector as a Lumped Compressible Volume

The pressure in the injector manifold, $p_{I}$, is related to the entering flow, $\dot{w}_{n}$, from the upstream pipe segment and the injector flow, $\dot{w}_{I}$, as follows:

$$
\begin{equation*}
\frac{d p_{I}}{d t}=\frac{c_{I}^{2}}{V_{I} g}\left(\dot{w}_{n}-\dot{w}_{I}\right) \tag{32}
\end{equation*}
$$

where $V_{I}$ is the injector volume and $c_{I}$ is the fluid sonic velocity. The injector flow is controlled by the differential pressure across the injector as well as by the resistance and inertia of the injector orifices. Thus,

$$
\begin{equation*}
p_{I}-p_{C}=R_{I} \dot{w}_{I}^{2}+\frac{\ell}{A g} \frac{d}{d t} \dot{w}_{I}, \tag{33}
\end{equation*}
$$

where $P_{C}$ is the thrust chamber pressure, $R_{I}$ is the injector hydraulic resistance and $\ell / A g$ is the equivalent inertance of all the injector orifices combined, i.e.,

$$
\begin{equation*}
\frac{1}{\ell / A g}=g \quad \sum_{i=1}^{n} \frac{1}{\ell_{i} / A_{i}} . \tag{34}
\end{equation*}
$$

In the preceding equation, $\ell_{i}$ and $A_{i}$ are the length and area, respectively, of an individual injector orifice.

An additional factor which can have a significant effect on the response of the feed system to chamber pressure oscillations is injector face flexibility. This effect can be expressed as a change in injector volume proportional to a change in injector pressure drop:

$$
\begin{equation*}
\frac{d v_{I}}{d t}=k\left(\frac{d p_{I}}{d t}-\frac{d p_{c}}{d t}\right) \tag{35}
\end{equation*}
$$

Also,

$$
\begin{equation*}
\frac{d p}{d t}=\frac{c^{2}}{g} \frac{d}{d t}\left(\frac{w}{V}\right) \tag{36}
\end{equation*}
$$

which can be rewritten as

$$
\begin{equation*}
\frac{d p}{d t}=\frac{c^{2}}{V g} \dot{w}-\frac{c^{2} \rho}{V g} \dot{V} . \tag{37}
\end{equation*}
$$

Combining Eqs. (35) and (37) gives

$$
\begin{equation*}
\frac{d p_{I}}{d t}=\frac{c_{I}^{2}}{V_{I} g}\left(\dot{w}_{n}-\dot{w}_{I}\right)-\frac{c_{I}^{2} \rho_{I}}{v_{I} g}\left[k\left(\frac{d p_{I}}{d t}-\frac{d p_{c}}{d t}\right)\right] \tag{38}
\end{equation*}
$$

which can be rewritten as

$$
\begin{equation*}
\left(1+\frac{c_{I}^{2} \rho_{I} k}{V_{I} g}\right) \quad \frac{d p_{I}}{d t}=\frac{c_{I}^{2}}{V_{I} g}\left(\dot{w}_{n}-\dot{w}_{I}\right)+\frac{c_{I}^{2} \rho_{I} k}{V_{I} g} \frac{d p_{c}}{d t} \tag{39}
\end{equation*}
$$

This expression reduces to Eq. (32) if no injector flexibility exists ( $K=0$ ).

Two-Phase Flow Acoustic Velocity
In the waterhammer equations the acoustic velocity, $c$, of the fluid appears in two places; (1) directly in the constant relating flow to pressure, and (2) indirectly in the time delay value, $\tau$, which equals $\ell / C$ seconds, where $\ell$ is the pipe segment length. The acoustic velocity of a fluid is a property of that fluid. However, its effective value can be reduced by the elastic walls of the flow passage or by the entrainment of gas and vapor in the liquid (two phase flow). Gas in the liquid can appear from two sources. One source is direct entrainment from mixing of gas and liquid in the propellant tank, while the other can result from the evolution of dissolved gas as the pressure drops along the feed system.

Given the steady-state pressure at each point in the feed system and data on the solubility of the pressurant gas in the propellant as a function of pressure and temperature, the amount of gas in the fluid can be determined for each feed system segment. Then, knowing the amount of gas in the liquid, the effective acoustic velocity of the mixture may be calculated.

Assuming isentropic compression of the gas, the change in volume of the gas is

$$
\begin{equation*}
d V_{g}=-\frac{V_{g}}{K p} d p \tag{40}
\end{equation*}
$$

and for the liquid

$$
\begin{equation*}
d V_{\ell}=\frac{V_{\ell}}{\beta} d p \tag{41}
\end{equation*}
$$

Defining a constant, $\alpha \equiv \frac{V_{g}}{V_{\ell}}$
the following relation is obtained:

$$
\begin{equation*}
\frac{d V_{t}}{V_{t}}=\frac{-d p}{\left[\frac{1+\alpha}{\frac{1}{B}+\frac{\alpha}{K p}}\right]} \tag{42}
\end{equation*}
$$

The bracketed term is the compressibility of the mixture. The density of the mixture can be shown to be

$$
\begin{equation*}
\rho_{m}=\frac{\alpha \rho_{g}+\rho_{\ell}}{(1+\alpha)} . \tag{43}
\end{equation*}
$$

The acoustic velocity of a liquid in an elastic pipe is

$$
\begin{equation*}
c=\sqrt{\frac{1}{\frac{\rho}{g}\left(\frac{1}{\beta}+\frac{D c_{f}}{e E}\right)}} \tag{44}
\end{equation*}
$$

Using the above expressions for density and compressibility, the acoustic velocity, can be written as

$$
\begin{equation*}
\left.c=\left[\frac{1}{\frac{\rho_{m}}{1+\alpha}\left(\frac{\alpha}{\rho_{\ell} c_{\ell}{ }^{2}}+\frac{1}{\rho_{g} c_{g}{ }^{2}}+\frac{1+\alpha}{g} \frac{D c_{f}}{E e}\right.}\right)\right]^{\frac{1}{2}} \tag{45}
\end{equation*}
$$

This expression can be used to define the acoustic velocity of a feed system segment with two phase flow. For an all liquid system, $\alpha=0$ and the same equation can be used.

In the Hydrodynamics subprogram the effect of the wall compressibility term, $\frac{\mathrm{C}_{f}}{\mathrm{Ee}}$, on the fluid acoustic velocity is handled automatically (assuming input value of $\frac{\mathrm{UC} f}{\mathrm{Ee}}$ are provided for each feed system segment). However, the program does not compute the effects of two-phase flow. If such flow occurs in the feed system being modeled, an effective fluid acoustic velocity must be pre-calculated for each affected segment. Equation (45) above, with the $\frac{\mathrm{D}_{\mathrm{f}}}{\mathrm{Ee}}$ term set equal to zero can be used for this calculation.

## Simulation of Branch Lines

In the Hydrodynamics subprogram, branched lines are handled by assuming that each branch has zero internal volume and that its flows are incompressible. Thus, the pressures at the end of all segments which meet at a branch are set equal. The continuity of flow is then used to provide the additional equations in combination with the waterhammer equations to solve for the overall feed system dynamic response.

## Generalized Feed System Model

A schematic of the generalized feed system which is modeled by the hydrodynamics subprogram is shown in Fig. 4. The configuration chosen is based on design and operating mode data for the OMS, PBK, and RCS feed systems obtained from McDonnell Douglas/St. Louis. The system is comprised of 30 individual line segments, each denoted in Fig. 4, as the lines between the black dots. A continuous parameter representation of each line segment is obtained through the use of separate sets of waterhammer equations. Each line segment can have a different line length, area, wall compliance, fluid acoustic velocity and resistance, and hence can model a wide variety of feed system components by merely choosing the appropriate values from these parameters. Also included in the generalized model are lumped parameter descriptions of two injectors (designated "0" and "F" on Fig. 4). Parameters for the injectors are volume, resistance, inertance, fluid acoustic velocity and face flexibility.

The system of 57 equations describing the generalized Fig. 4 feed system is listed in Table 1. The equations are shown in the linearized, LaPlace transformed format required by the frequency response subroutine.


Figure 4. Generalized OME Feed System Schematic

$$
\begin{align*}
& P_{1}+\left(R_{1}+a_{1}\right) \dot{w}_{1}+\left[p_{1}+\left(R_{1}-a_{1}\right) \dot{w}_{1}\right] e^{-2 T_{1} s}=0 \\
& p_{1}-a_{3} \dot{w}_{3}-\left[p_{2}+\left(R_{3}-a_{3}\right) \dot{w}_{4}\right] e^{-T_{3} s}=0 \\
& P_{2}+\left(R_{3}+a_{3}\right) \dot{w}_{4}-\left[P_{1}+a_{3} \dot{w}_{3}\right] e^{-T_{3} s}=0 \\
& P_{1}-\left(R_{2}+a_{2}\right)\left(\dot{w}_{1}-\dot{w}_{3}\right)-\left[p_{1}-\left(R_{2}-a_{2}\right)\left(\dot{w}_{1}-\dot{w}_{3}\right)\right] e^{-2 T_{2} 5}=0 \\
& P_{2}-a_{5} \dot{w}_{6}-\left[P_{3}+\left(R_{5}-a_{5}\right) \dot{w}_{7}\right] e^{-T_{5} s}=0 \\
& P_{2}-a_{4}\left(\dot{w}_{4}-\dot{w}_{6}\right)-\left[P_{8}+\left(R_{4}-a_{4}\right) \dot{w}_{15}\right] e^{-T_{4} s}=0 \\
& P_{8}+\left(R_{4}+a_{4}\right) \dot{w}_{15}-\left[P_{2}+a_{4}\left(\dot{w}_{4}-\dot{w}_{6}\right)\right] e^{-T_{4} s}=0 \\
& P_{3}+\left(R_{5}+a_{5}\right) \dot{w}_{7}-\left[P_{2}+a_{5} \dot{w}_{6}\right] e^{-T_{5} s}=0 \\
& P_{3}-a_{6} \dot{w}_{7}-\left[P_{4}+\left(R_{6}-a_{6}\right) \dot{w}_{8}\right] e^{-T_{6} s}=0 \\
& P_{4}-\left(R_{7}+a_{7}\right)\left(\dot{w}_{8}-\dot{w}_{10}\right)-\left[F_{4}-\left(R_{7}-a_{7}\right)\left(\dot{w}_{8}-\dot{w}_{10}\right)\right] e^{-2 T_{7} s}=0 \\
& P_{4}-a_{8} \dot{w}_{10}-\left[P_{5}+\left(R_{8}-a_{8}\right) \dot{w}_{11}\right] e^{-T} 8^{s}=0 \\
& P_{5}\left(R_{8}+a_{8}\right) \dot{w}_{11}-\left[P_{4}+a_{8} \dot{w}_{10}\right] e^{-T_{8} s}=0 \\
& P_{5}-a_{9} \dot{w}_{17}-\left[P_{6}+\left(R_{9}-a_{9}\right) \dot{w}_{12}\right] e^{-T_{9} s}=0
\end{align*}
$$

$$
\begin{align*}
& P_{6}+\left(R_{9}+a_{9}\right) \dot{w}_{12}-\left[P_{5}+a_{9} \dot{w}_{11}\right] e^{-T_{9} s}=0 \\
& P_{6}-a_{10} \dot{w}_{12}-\left[P_{7}+\left(R_{10}-a_{10}\right) \dot{w}_{13}\right] e^{-T} 10^{S}=0 \\
& P_{7}+\left(R_{10}+a_{10}\right) \dot{w}_{13}-\left[P_{6}+a_{10} \dot{w}_{12}\right] e^{-T} 10^{S}=0 \\
& P_{7}-Z_{0} s \dot{w}_{14}-R_{0} \dot{w}_{14}=P_{0} \\
& s P_{7}-\left(\frac{c^{2}}{g V}\right)_{0} \dot{w}_{13}+\left(\frac{c^{2}}{g V}\right)_{0} \dot{w}_{14}+\left(\frac{c^{2}}{g V}\right)_{0} K_{0} s P_{7}=\left(\frac{c^{2}}{g V}\right)_{0} K_{0} s P_{0} \\
& P_{8}+a_{14} \dot{w}_{17}-\left[P_{12}-\left(R_{14}-a_{14}\right) \dot{w}_{24}\right] e^{-T_{14} 4^{5}}=0 \\
& P_{12}-\left(R_{14}+a_{14}\right) \dot{w}_{24}-\left[P_{8}-a_{14} \dot{w}_{17}\right] e^{-T} 14^{S}=0 \\
& P_{8}-a_{11}\left(\dot{w}_{15}+\dot{w}_{17}\right)-\left[P_{9}+\left(R_{11}-a_{11}\right) \dot{w}_{18}\right] e^{-T} 11^{s}=0 \\
& P_{9}+\left(R_{11}+a_{11}\right) \dot{w}_{18}-\left[P_{8}+a_{11}\left(\dot{w}_{15}+\dot{w}_{17}\right)\right] e^{-T} 11^{s}=0 \\
& P_{9}-\left(R_{13}+a_{13}\right)\left(\dot{w}_{18}-\dot{w}_{19}\right)-\left[P_{9}-\left(R_{13}-a_{13}\right)\left(\dot{w}_{18}-\dot{w}_{19}\right)\right] e^{-2 T_{13} s}=0 \\
& \left.P_{9}-a_{12} \dot{w}_{19}-\left[x+R_{12}-a_{12}\right) \dot{w}_{20}\right] e^{-T} 12^{s}=0 \\
& p_{11}+\left(R_{15}+a_{15}\right) \dot{w}_{22}+\left[p_{11}+\left(R_{15}-a_{15}\right) \dot{w}_{22}\right] e^{-2 T_{15} s}=0 \\
& P_{11}-a_{16} \dot{w}_{22}-\left[P_{12}+\left(R_{16}-a_{16}\right)\left(\dot{w}_{24}+\dot{w}_{25}\right)\right] e^{-T_{16} s}=0
\end{align*}
$$

TABLE 1. (Continued)

$$
\begin{align*}
& P_{12}+\left(R_{16}+a_{16}\right)\left(\dot{w}_{24}+\dot{w}_{25}\right)-\left[P_{11}+a_{16} \dot{w}_{22}\right] e^{-T} 16^{S}=0 \\
& P_{13}+\left(R_{22}+a_{22}\right) \dot{w}_{34}+\left[p_{13}+\left(R_{22}-a_{22}\right) \dot{w}_{34}\right] e^{-2 T_{22} s}=0 \\
& P_{13}-a_{24} \dot{w}_{36}-\left[P_{17}+\left(R_{24}-a_{24}\right) \dot{w}_{37}\right] e^{-T_{24} s}=0 \\
& P_{13}-\left(R_{23}+a_{23}\right)\left(\dot{w}_{34}-\dot{w}_{36}\right)-\left[P_{13}-\left(R_{23}-a_{23}\right)\left(\dot{w}_{34}-\dot{w}_{36}\right)\right] e^{-2 T_{23} s}=01-30 \\
& P_{12}-\left(R_{17}+a_{17}\right) \dot{w}_{25}-\left[P_{14}-a_{17} \dot{w}_{26}\right] e^{-T} 17^{S}=0 \\
& P_{14}+a_{17} \dot{w}_{26}-\left[P_{12}-\left(R_{17}-a_{17}\right) \dot{w}_{25}\right] e^{-T_{17} S}=0 . \\
& P_{17}-a_{21}\left(\dot{w}_{37}-\dot{w}_{38}\right)-\left[P_{14}+\left(R_{21}-a_{21}\right) \dot{w}_{27}\right] e^{-T_{21} 5}=0 \\
& P_{14}+\left(R_{21}+a_{21}\right) \dot{w}_{27}-\left[P_{17}+a_{21}\left(\dot{w}_{37}-\dot{w}_{38}\right)\right] e^{-T_{21} s}=0 \\
& P_{14}-a_{18}\left(\dot{w}_{26}+\dot{w}_{27}\right)-\left[P_{15}+\left(R_{18}-a_{18}\right) \dot{w}_{29}\right] e^{-T} 18^{S}=0 \\
& P_{15}+\left(R_{18}+a_{18}\right) \dot{w}_{29}-\left[P_{14}+a_{18}\left(\dot{w}_{26}+\dot{w}_{27}\right)\right] e^{-T} 18^{S}=0 \\
& P_{15}-\left(R_{19}+a_{19}\right)\left(\dot{w}_{29}-\dot{w}_{31}\right)-\left[P_{15}-\left(R_{19}-a_{19}\right)\left(\dot{w}_{29}-\dot{w}_{31}\right)\right] e^{-2 T} 19^{s}=0 \\
& P_{15}-a_{20} \dot{w}_{31}-\left[Y+\left(R_{20}-a_{20}\right) \dot{W}_{32}\right] e^{-T_{20} s}=0 \\
& P_{16}+\left(R_{20}+a_{20}\right) \dot{w}_{32}-\left[P_{15}+a_{20} \dot{w}_{31}\right] e^{-T_{20}}=0
\end{align*}
$$

$$
\begin{align*}
& \mathrm{P}_{17}-\mathrm{a}_{25} \dot{w}_{38}-\left[\mathrm{P}_{18}+\left(\mathrm{R}_{25}-\mathrm{a}_{25}\right) \dot{w}_{39}\right] \mathrm{e}^{-\mathrm{T}_{25} \mathrm{~S}}=0 \\
& P_{18}+\left(R_{25}+a_{25}\right) \dot{w}_{39}-\left[P_{17}+a_{25} \dot{w}_{38}\right] e^{-T_{25^{s}}^{s}}=0 \\
& P_{18}-a_{26} \dot{w}_{39}-\left[P_{19}+\left(R_{26}-a_{26}\right) \dot{w}_{40}\right] e^{-T_{26} s}=0 \\
& P_{19}+\left(R_{26}+a_{26}\right) \dot{w}_{40}-\left[P_{18}+a_{26} \dot{w}_{39}\right] e^{-T_{26} \mathbf{s}}=0 \\
& P_{19}-\left(R_{27}+a_{27}\right)\left(\dot{w}_{40}-\dot{w}_{42}\right)-\left[P_{19}-\left(R_{27}-a_{27}\right)\left(\dot{w}_{40}-\dot{w}_{42}\right)\right] e^{-2 T_{27} s}=0 \\
& P_{19}-a_{28} \dot{w}_{42}-\left[P_{20}+\left(R_{28}-a_{28}\right) \dot{w}_{43}\right] e^{-T_{28} s}=0 \\
& P_{20}+\left(R_{28}+a_{28}\right) \dot{w}_{43}-\left[P_{19}+a_{28} \dot{w}_{42}\right] e^{-T_{28} s}=0 \\
& P_{20}-a_{29} \dot{w}_{43}-\left[P_{21}+\left(R_{29}-a_{29}\right) \dot{w}_{44}\right] e^{-T_{29} s}=0 \\
& P_{21}+\left(R_{29}+a_{29}\right) \dot{w}_{44}-\left[P_{20}+a_{29} \dot{w}_{43}\right] e^{-T_{29} s}=0 \\
& P_{21}-a_{30} \dot{w}_{44}-\left[P_{22}+\left(R_{30}-a_{30}\right) \dot{w}_{45}\right] e^{-T_{30} s}=0 \\
& P_{22}+\left(R_{30}+a_{30}\right) \dot{w}_{45}-\left[P_{21}+a_{30} \dot{w}_{44}\right] e^{-T} 30^{s}=0 \\
& P_{22}-Z_{F} s \dot{w}_{46}-R_{F} \dot{w}_{46}=P_{F} \\
& s P_{22}=\left(\frac{c^{2}}{g V}\right)_{F}\left(\dot{w}_{45}-\dot{w}_{46}\right)+\left(\frac{c^{2}}{g V}\right)_{F} K_{F} s P_{22}=\left(\frac{c^{2}}{g V}\right)_{F} K_{F} s \bar{P}_{F}
\end{align*}
$$

## TABLE 1. (Concluded)

$$
\begin{array}{ll}
P_{4}+\left(R_{6}+a_{6}\right) \dot{w}_{8}-\left[P_{3}+a_{6} \dot{w}_{7}\right] e^{-T_{6} s}=0 & 1-53 \\
P_{10}+\left(R_{12}+a_{12}\right) \dot{w}_{20}-\left[P_{9}+a_{12} \dot{w}_{19}\right] e^{-T} 12^{S}=0 & 1-54 \\
P_{17}+\left(R_{24}+a_{24}\right) \dot{w}_{37}-\left[P_{13}+a_{24} \dot{w}_{36}\right] e^{-T_{24} s}=0 & 1-55 \\
X=P_{10} & 1-56 \\
Y=P_{16} & 1-57
\end{array}
$$

NOTE: $\quad a_{N}=\frac{c_{N}}{A_{N} g}$

It should be noted that the hydrodynamics subprogram solves the complete system of 57 equations (describing the complete Fig. 4 feed system) each time it is called. Thus the frequency response of the entire system is calculated each time. It has been shown, however, that simpler feed systems, representing only a portion of the Fig. 4 schematic, can be modeled by merely assigning values to the parameters of the unneeded line segments which will exclude them from any effect on the system frequency response. This is accomplished automatically by the hydrodynamics subprogram via the assignment of very large resistances and very short lengths to all line segments for which no data is entered.

## COMBUSTION DYNAMICS

## Analytical Approach

In the past, the combustion response has been modeled with a simple time delay(s) (Ref. 3 through 9). This time delay reoresents the time required for the propellants to travel at their injected velocity from the point where they are injected to another point where they burn, and implies the burning is concentrated at a fixed nlane some arbitrary distance from the injector face. The procedure outlined above is obviously an oversimplification of the burning process which is distributed in some fashion throughout the combustion chamber.

Steady-state combustion models (Ref. 10 and 11 for example) provide insight to determine the droplet burning distribution as well as additional information required to relate the distribution to a combustion response as a function of frequency. Combustion models are designed to march incrementally down the combustion chamber from a set of specified initial conditions. In so doing, the model calculates the rate at which the propellants are consumed as a function of the axial position in the combustion chamber (burning rate profile).

The analytical technique selected to describe the combustion dynamics is based on employing the mathematical expressions used in the steady-state combustion models (in particular the JANNAF DEP program, Ref. 11). These mathematical expressions are expanded into time averace and oscillatory components and are described in the following sections.

## Atomization Process

A very essential part of the combustion field initialization is the assignment of propellant spray droplet sizes and flowrates. Analytical descrintions of the atomization process are not available but empirical correlations that relate droplet diameter to injector geometry and flow conditions are available (Refs. 12, 13, and 14). For like-doublets, one empirical relationship is (Ref. 12).

$$
\begin{equation*}
D_{d}=4.85 \times 10^{4} v_{j}^{-0.75}\left(p_{c} / p_{j}\right)^{-0.52}{d_{j}}^{0.57} \tag{46}
\end{equation*}
$$

where $v_{j}$ is the liquid jet velocity and $d_{j}$ is the liquid jet diameter at the atomization plane. (For steady-state analysis, the velocity is the injection velocity and the diameter is the orifice diameter.)

For purposes of the current analysis, the atomization process is described by:

$$
\begin{equation*}
D_{d}=k\left(d_{j}\right)_{x=x_{i m p}}^{a} \quad\left(v_{j}\right)_{x=x_{i m p}}^{b} \tag{47}
\end{equation*}
$$

where $x_{i m p}$ is the location of the atomization plane or the impingement point. Expanding Eq. 47 into time-averaged and oscillatory parts, yields the oscillatory droplet diameter

$$
\begin{equation*}
\frac{\tilde{D}_{d}}{\bar{D}_{d}}=a\left(\frac{\tilde{d}_{j}}{\bar{d}_{j}}\right)_{x=x_{i m p}}+b\left(\frac{\tilde{v}_{j}}{\bar{v}_{j}}\right)_{x=x_{i m p}} \tag{48}
\end{equation*}
$$

In order to evaluate the oscillatory droplet diameter, the oscillatory liquid jet diameter and velocity (and therefore the jet flowrate) are required at the atomization plane. Therefore, the dynamics of the fluid from the injector to the atomization plane is reauired and outlined in the following section.

## Klystron Effect

The dynamics of the liquid propellant jet from the injector face to any location in the chamber are described by the continuity and momentum equations:

$$
\begin{align*}
& \frac{\partial}{\partial t}\left(A_{j} \rho_{j}\right)+\frac{\partial}{\partial x}\left(A_{j} \rho_{j} v_{j}\right)=0  \tag{49}\\
& \frac{\partial}{\partial t}\left(A_{j} \rho_{j} v_{j}\right)+\frac{\partial}{\partial x}\left(A_{j} \rho_{j} v_{j}^{2}\right)=-A_{j} \frac{\partial p}{\partial x} \tag{50}
\end{align*}
$$

Assuming

$$
\begin{align*}
& \rho_{\mathbf{j}}=\text { constant }  \tag{51}\\
& \frac{\partial p}{\partial x}=0  \tag{52}\\
& \phi=\bar{\phi}+\Phi \quad \text { ( } \phi \text { any variable }), \tag{53}
\end{align*}
$$

where

$$
\begin{array}{ll}
\bar{\phi}=f(x) & \text { (time average value } \\
\Phi=\phi^{\prime} e^{-i \omega t} & \text { (oscillatory value) } \\
\phi^{\prime}=g(x), & \tag{56}
\end{array}
$$

the preceding equations can be expanded into time average and oscillatory parts and integrated between the injector face and any location in the chamber to yield:

$$
\begin{align*}
& \left(\frac{\tilde{v}_{j}}{\bar{v}_{j}}\right)=e^{i \omega x / \bar{v}_{j}}\left(\frac{\tilde{\dot{m}}_{j}}{\overline{m_{j}}}\right)_{i n j}  \tag{57}\\
& \left(\frac{\tilde{A}_{j}}{\overline{A_{j}}}\right)=\frac{-i \omega x}{\bar{v}_{j}} e^{i \omega x / \bar{v}_{j}}\left(\frac{\tilde{\dot{m}}_{j}}{\overline{\dot{m}_{j}}}\right)_{i n j}  \tag{58}\\
& \left(\frac{\tilde{\dot{m}}_{j}}{\overline{m_{j}}}\right)=e^{i \omega x / \bar{v}_{j}}\left[1-\frac{i \omega x}{\bar{v}_{j}}\right]\left(\frac{\tilde{\dot{m}}_{j}}{\bar{m}_{j}}\right)_{i n j} \tag{59}
\end{align*}
$$

where $\omega$ is the angular frequency and the oscillatory injection rate, ( $\check{m}_{j}$ ) ${ }_{\mathrm{inj}}$ is determined by the feed system dynamics. Equation 59 is the oscillatory jet flowrate at $x$ and is usually referred to as the Klystron effect (Ref. 15). The Klystron time delay, $\tau_{K}$, is therefore given by

$$
\begin{equation*}
{ }^{\tau} K_{j}=\frac{{ }^{K_{K}}}{\bar{v}_{j}} \tag{60}
\end{equation*}
$$

Considerable amplification of the injector face flow oscillations are possible when the klystron effect is present and could explain the periodic burst of acoustic resonances called resurging and the steep-fronted waves seen in low and intermediate frequency instabilities.

## Droplet Vaporization

Theories of droplet combustion (Refs. 10, 16, 17) are available which may be used to evaluate the extent of coupling between drodlet burning rate and local pressure and velocity fluctuations. In general, droplet burning is enhanced by increased turbulence levels or by periodic directional variations in velocity, because droplets are relatively heavy and resist following gas streamlines.

Calculation of the spray heating and vaporization is usually accomplished through specification of the corresponding individual droplet processes and summation over all the droplets that constitute the spray(s) being analyzed. The calculation of single droplet evaporation is usually based on a spherically symmetric model of simultaneous heat transfer and mass transfer across the gas side boundary, or film, separating the liquid droplet from the surrounding hot combustion gas. Forced convection and resultant nonspherical transfer processes are accounted for through empirical Nusselt number correlations for both heat and mass transfer.

For the fuel or oxidizer spray, the droplet continuity equation is

$$
\begin{equation*}
\frac{d}{d x}\left(A \rho_{k} v_{k}\right)=-A N_{k} \dot{m}_{v a p_{k}} \tag{61}
\end{equation*}
$$

and the vaporization rate is (Ref. 10)

$$
\begin{equation*}
\dot{m}_{v a p_{k}}=\frac{\pi D_{k} k_{f_{k}} N u_{H_{k}} Z_{k}}{C_{p_{v_{k}}}} \tag{62}
\end{equation*}
$$

where $\rho_{k}$ is the spray density (mass of spray per unit chamber volume), $N_{k}$ is the number of droplets per unit chamber volume, and

$$
\begin{equation*}
z_{k} \equiv \frac{{ }_{p_{v_{k}}} N u_{m_{k}}{ }^{\rho M W_{v_{k}}} \mathscr{D}_{v_{k}}}{k_{f_{k}} N u_{H_{k}}{ }^{R T_{f_{k}}}} \ln \left(\frac{p}{p-p_{v_{k}}}\right) \tag{63}
\end{equation*}
$$

Noting that

$$
\begin{equation*}
\rho_{k}=N_{k} m_{k}=N_{k} \rho_{\ell_{k}} \frac{\pi D_{k}^{3}}{6} \tag{64}
\end{equation*}
$$

the droplet number flowrate can be written as

$$
\begin{equation*}
\dot{N}_{k}=v_{k} A N_{k}=\frac{A v_{k} \rho_{k}}{m_{k}} \tag{65}
\end{equation*}
$$

Therefore, Eq. 61 can be written as

$$
\begin{equation*}
\frac{d}{d x}\left(m_{k} \dot{N}_{k}\right)=-\frac{\dot{N}_{k}}{v_{k}} \dot{m}_{\operatorname{vap}_{k}} \tag{66}
\end{equation*}
$$

For steady-state combustion models, the preceding equation (along with Eq. 62) is numerically integrated allowing the droplet diameter, $D_{k}$, to vary along the length of the combustion and maintaining constant droplet number flowrate ( $\dot{N}_{k}$ ). Combs (Ref. 18) has shown that changing from a variable droplet diameter to a variatle droplet number flowrate yields approximately the same results for steady-state vaporization. Therefore, in order to simplify the integration for stability analysis, the droplet diameter was held constant and the droplet number flowrate was assumed to vary.

Summing Eq. 61 over all fuel or oxidizer droplet size groups yields

$$
\begin{equation*}
\sum_{k} \frac{d}{d x}\left(A \rho_{k} v_{k}\right)=-A \sum_{k} \rho_{k} \frac{\dot{m}_{v a p_{k}}}{m_{k}}=-A \sum_{k} \frac{\rho_{k}(6) z_{k} k_{f_{k}} N u_{H_{k}}}{\rho_{\ell k} D_{k}^{2} c_{p_{v_{k}}}-} \tag{67}
\end{equation*}
$$

which can be written as

$$
\begin{equation*}
\frac{d}{d x}\left(A \rho_{s} v_{s}\right)=-\frac{A \rho_{s}}{\tau_{s}}=-A \dot{m}_{v a p_{s}} \tag{68}
\end{equation*}
$$

where

$$
\begin{align*}
& \rho_{s} \equiv \sum_{k} \rho_{k}  \tag{69}\\
& v_{s} \equiv \frac{1}{\rho_{s}} \sum_{k}\left(\rho_{k} v_{k}\right)  \tag{70}\\
& \frac{1}{\tau_{s}} \equiv \frac{1}{\rho_{s}} \sum_{k} \frac{\rho_{k}(6) Z_{k} k_{f}{ }^{N u_{H_{k}}}}{\rho_{\ell_{k}} D_{k}^{2} c_{p_{v_{k}}}} \tag{71}
\end{align*}
$$

Letting $Z_{k}, k_{f_{k}}, c_{p_{v_{k}}}$ be independent of $k$ and assuming

$$
\begin{equation*}
\tau_{s}=f(t) \tag{72}
\end{equation*}
$$

yields:

$$
\begin{equation*}
\tau_{s}=\frac{\rho_{\ell_{s}}{ }^{c_{p_{v}}}{ }^{D_{s}^{2}}}{(6) Z_{s} k_{f_{s}} N u_{H_{s}}} \tag{73}
\end{equation*}
$$

where

$$
\begin{equation*}
D_{s}^{2}=\left(\dot{m}_{s}\right)_{i n j} / \sum_{k}\left(\frac{\dot{m}_{k}}{D_{k}^{2}}\right)_{i n j} \tag{74}
\end{equation*}
$$

From Eq. 68

$$
\begin{equation*}
\frac{d\left(A \rho_{s} v_{s}\right)}{A p_{s} v_{s}}=-\frac{d x}{\tau_{s} v_{s}} \tag{75}
\end{equation*}
$$

Integrating Eq. 75 between $x_{0}$, the start of vadorization olane, and any location x yields

$$
\begin{equation*}
A_{s} v_{s}=\left(\dot{m}_{x}\right)_{x=x_{0}} \quad \exp \left[-\int_{x_{0}}^{x} \frac{d x}{\tau_{s} v_{s}}\right] \tag{76}
\end{equation*}
$$

substituting Eq. 76 into Ea. 68 yields the fuel or oxidizer spray vaporization rate:

$$
\begin{equation*}
\dot{m}_{v a p}=\frac{\left(\dot{m}_{s}\right)_{x=x_{0}}}{A \tau_{s} v_{s}} \exp \left[-\int_{x_{0}}^{x} \frac{d x}{\tau_{s} v_{s}}\right] \tag{77}
\end{equation*}
$$

Using perturbation techniques, the time average vaporization rate can be written as

$$
\begin{equation*}
\overline{\dot{m}}_{v a p_{s}}=\frac{\left(\overline{\dot{m}}_{s}\right)_{x=x_{0}}}{A \bar{\tau}_{s} \bar{v}_{s}} \quad \exp \quad\left[-\frac{\left(x-x_{0}\right)}{\bar{\tau}_{s} \bar{v}_{s}}\right] \tag{78}
\end{equation*}
$$

and the oscillatory vaporization rate can be written as

$$
\begin{align*}
& \check{\tilde{m}}_{\text {vap }}=\overline{\tilde{m}}_{\text {vap }}\left\{\begin{array}{l}
\left(\dot{m}_{s}\right)_{x=x_{k}} \\
\frac{\tilde{\tau}_{s}}{\tilde{m}_{x=x_{0}}}-\frac{\tilde{v}_{s}}{\bar{\tau}_{s}}-\frac{\tilde{v}_{s}}{\bar{v}_{s}}
\end{array}\right. \\
& \left.+\int_{x_{0}}^{x}\left(\frac{\tilde{\tau}_{s}}{\overline{\tau_{s}}}+\frac{\tilde{v}_{s}}{\bar{v}_{s}}\right) \frac{d x}{\bar{\tau}_{s} \bar{v}_{s}}\right\} \tag{79}
\end{align*}
$$

Assuming

$$
\begin{equation*}
v_{\mathrm{s}} \approx\left(v_{\mathrm{s}}\right)_{\mathrm{x}=\mathrm{x}_{\mathrm{k}_{\mathrm{s}}}} \tag{80}
\end{equation*}
$$

yields

$$
\begin{equation*}
\frac{\tilde{v}_{s}}{\bar{v}_{s}}=\left(\frac{\tilde{v}_{s}}{\bar{v}_{s}}\right)_{x=x_{k_{s}}} \tag{81}
\end{equation*}
$$

Letting $\rho_{\ell_{s}}, c_{p_{v_{s}}}$, and $k_{f_{s}}$ be constant, the oscillatory time delay can therefore be expressed as

$$
\begin{equation*}
\frac{\tilde{\tau}_{s}}{\bar{\tau}_{s}}=2 \frac{\tilde{D}_{s}}{\bar{D}_{s}}-\frac{\tilde{Z}_{s}}{\bar{Z}_{s}}-\frac{\widetilde{N u}_{H_{s}}}{\cdot \overline{N u}_{H_{s}}}+\left(\frac{\partial \tau}{\partial M R}\right) \frac{\widetilde{M R}}{\bar{\tau}_{s}} \tag{82}
\end{equation*}
$$

The oscillatory spray droplet diameter $\left(D_{s}\right)$ is given by Eq. 48 and the oscillatory flowrate is given by Eq. 59. The above formulation results in a linear oscillatory vaporization model similar to, but more realistic than Crocco's n-t model (Ref. 4). The formulation includes the effects of: (1) distributed energy release, (2) oscillations in the injection rate, (3) oscillations in droplet diameter, (4) oscillations in droplet temperature, (5) ọas pressure and velocity oscillations, and (6) oscillations in the local mixture ratio.

Nusselt Number. It may be observed that one of the dominant terms in both the expressions for the average and oscillatory time delay is the Nusselt number. The Nusselt number, for longitudinal modes, is (Ref. 19).

$$
\begin{equation*}
N{u_{H}}=2.0+0.6 \operatorname{Pr}_{s} 1 / 3\left[\frac{\rho D_{s}}{\mu}\left|v-v_{s}\right|\right]^{\frac{1}{2}} \tag{83}
\end{equation*}
$$

In or der to evaluate the oscillatory Nusselt number, the oscillatory droplet spray velocity is required. The droplet spray velocity can be obtained from the drag equation.

$$
\begin{equation*}
m_{s} \frac{d v_{s}}{d t}=\frac{\pi}{8} \rho D_{s}^{2}\left|v-v_{s}\right|\left(v-v_{s}\right) C_{D_{s}} \tag{84}
\end{equation*}
$$

## Letting

$$
\begin{equation*}
{ }^{\tau} \operatorname{drag}_{s} \equiv \frac{\rho_{\ell} D_{s}^{2}}{(18) \alpha_{s}{ }^{\mu}} \tag{85}
\end{equation*}
$$

where

$$
\begin{equation*}
\alpha_{s} \equiv \frac{C_{D_{s}}}{24}\left(\frac{D_{s} \rho}{\mu}\left|v-v_{s}\right|\right), \tag{86}
\end{equation*}
$$

the oscillatory droplet spray velocity can be written as

$$
\begin{equation*}
\tilde{v}_{S}=\left[\frac{1+i \omega \tau_{d r a g_{S}}}{1+\left(\omega \tau_{d r a g_{S}}\right)^{2}}\right] \tilde{v}=R_{v_{S}} \tilde{v} \tag{87}
\end{equation*}
$$

Defining

$$
\begin{equation*}
F_{\rho} \equiv\left(\frac{\rho}{\bar{\rho}}\right)^{\frac{3}{2}}, F_{v_{s}}=\left[\frac{\left|v-v_{s}\right|}{c \Delta M_{s}}\right]^{1 / 2} \tag{88}
\end{equation*}
$$

where

$$
\begin{equation*}
\Delta M_{s} \equiv\left[\frac{\left|v-v_{s}\right|}{c}\right]_{\text {steady state }}, \tag{89}
\end{equation*}
$$

the Nusselt number can be written as

$$
\begin{equation*}
N u_{H_{s}}=2.0+0.6 \operatorname{Pr}_{s}^{1 / 3}\left[\frac{\bar{\rho}_{s} \bar{D}_{s}}{\mu} \quad c \Delta M_{s}\right]^{1 / 2} \quad F_{\rho} F_{v_{s}}\left(\frac{D_{s}}{\bar{D}_{s}}\right)^{1 / 2} \tag{90}
\end{equation*}
$$

Expanding the preceding equation into time average and oscillatory parts yields

$$
\begin{align*}
& \overline{N u}_{H_{s}}=2.0+0.6 \operatorname{Pr}_{s} 1 / 3\left[\frac{\bar{\rho}_{s}}{\mu} c \Delta M_{s}\right]^{\frac{1}{2}} \bar{F}_{\rho} \bar{F}_{v_{s}}  \tag{91}\\
& \frac{\widetilde{N u}_{H_{s}}}{\bar{N}_{H_{s}}}=\left(\frac{\overline{N u}_{H_{s}}^{-2}}{\overline{N u}_{H_{s}}}\right)\left[\frac{1}{2}\left(\frac{\tilde{D}_{s}}{\bar{D}_{s}}\right)+\frac{\tilde{F}_{\rho}}{\bar{F}_{\rho}}+\frac{\widetilde{F}_{v_{s}}}{\bar{F}_{v_{s}}}\right] \tag{92}
\end{align*}
$$

Letting

$$
\begin{align*}
& \frac{\tilde{F}_{v_{s}}}{\bar{F}_{v_{s}}}=R_{F_{v_{s}}} \frac{\tilde{v}}{\bar{c}}, \text { and }  \tag{93}\\
& \widetilde{F}_{\rho}  \tag{94}\\
& \frac{\bar{F}_{\rho}}{}=R_{F_{\rho}} \frac{\tilde{\rho}^{\prime}}{\bar{\rho}}
\end{align*}
$$

the oscillatory Nusselt number is

$$
\begin{equation*}
\frac{\tilde{N} u_{H_{s}}}{\bar{N} u_{H_{s}}}=\frac{\tilde{N} u_{H_{s}}-2}{\tilde{N} u_{H_{s}}}\left[\frac{1}{2}\left(\frac{\tilde{D}_{s}}{\bar{D}_{s}}\right)+R_{F_{\rho}}\left(\frac{\tilde{\rho}}{\bar{\rho}}\right)+R_{F_{v_{s}}}\left(\frac{\tilde{v}}{\bar{c}}\right)\right] \tag{95}
\end{equation*}
$$

For small perturbations in the pressure, the linear response factors and the time average values for $F_{p}$ and $F_{v}$ are

$$
\begin{align*}
& \bar{F}_{\rho}=1, \bar{F}_{v_{s}}=1  \tag{96}\\
& R_{F_{\rho}}=\frac{1}{2}, R_{F_{v_{s}}}=\frac{1}{2}\left(\frac{1-R_{v_{s}}}{\Delta M_{s}}\right) \tag{97}
\end{align*}
$$

Calculations have been made which indicate that, for large droplet diameters, the average and oscillatory Nusselt numbers are auite sensitive to pressure and velocity oscillations. Therefore, the Nusselt number can have a significant effect on engine stability.

Droplet Heat Transfer Blockage Term. The oscillatory combustion time delay given by Eq. 82 requires the evaluation of the heat transfer blockage term $\left(Z_{s}\right)$ which is related to the combustion gas and liquid vapor properties bv Eq. 63. Because the vapor pressure ( $\mathbf{P}_{\mathbf{v}_{s}}$ ) at the droplet surface is related to the droplet temperature, the blockage term also depends on the oscillatory droplet surface temperature inside the droplet which is given by:

$$
\begin{equation*}
\frac{\partial}{\partial t}\left(\rho_{\ell} c_{v_{\ell}} T_{\ell}\right)=\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} k_{e f f} \frac{\partial T_{\ell}}{\partial r}\right) \tag{98}
\end{equation*}
$$

Therefore, the oscillatory heat transfer rate to the droplet can be related to the oscillatory droplet surface temperature by

$$
\begin{equation*}
\tilde{Q}_{S}=R_{T} \tilde{T}_{S} \tag{99}
\end{equation*}
$$

The droplet heating rate can also be written as (Ref. 10)

$$
\begin{equation*}
Q_{s}=Z_{s} k_{f_{s}} N u_{H_{s}}\left[\frac{\left(T-T_{s}\right)}{\left(e^{Z_{s}}-1\right)}-\frac{\Delta H_{v a d_{s}}}{c_{p_{v_{s}}}}\right] \quad\left(\pi D_{s}\right) \tag{100}
\end{equation*}
$$

Assuming that

$$
\begin{equation*}
\left(\frac{d T_{s}}{d t}\right)=0 \quad \text { (droplet at "wet bulb" temperature) } \tag{101}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{\tilde{p_{v}}}{\overline{p_{v}}}=R_{p_{v}} \frac{\tilde{p}}{\bar{p}} \tag{102}
\end{equation*}
$$

etc. for the other variables, the response factor for the heat transfer blockage term can be related to droplet and gas properties and flow conditions.

Examination of the response factor for the heat transfer blockage term indicates that this term is not important at low frequencies (also see Ref. 20 and 21). Therefore, the oscillatory heat blockage term has not been included in the computer program. Detailed equations for this term are presented in Appendix B of Ref. 1.

## Generalized Vaporization Rate Expression

In order to maintain generality in representing the combustor dynamics, the spray vaporization rates (fuel and oxidizer) were written as:

$$
\begin{align*}
& \dot{m}_{\text {vap }}=\overline{\dot{m}}_{\text {vap }}\left\{c_{1_{s}}\left(\frac{\tilde{p}}{\bar{p}}\right)_{x=0}+c_{2_{s}}\left(\frac{\tilde{p}}{\bar{p}}\right)+c_{3_{s}}\left(\frac{\tilde{\tilde{p}}}{\bar{p}}\right)_{x=0}\right. \\
& +c_{4}\left(\frac{\tilde{\nu}}{-}\right)+c_{5_{s}}(\widetilde{M R})_{x=0}+c_{6_{s}}(\widetilde{M R}) \\
& +c_{7}\left(\frac{\tilde{v}}{\bar{c}}\right)_{x=0}+c_{8}\left(\frac{\tilde{v}}{\bar{c}}\right)+\int_{x_{0}}^{x}\left[c_{g_{s}}\left(\frac{\tilde{p}}{\bar{p}}\right)_{x=0}\right. \\
& +c_{10_{s}}\left(\frac{\tilde{p}}{\bar{p}}\right)+c_{11}\left(\frac{\tilde{n}}{\bar{r}}\right)_{x=0}+c_{12_{s}}\left(\frac{\tilde{n}}{\bar{p}}\right)+c_{13}(\tilde{M R})_{x=0} \\
& \left.\left.+c_{14}(\widetilde{M} \dot{R})+c_{15_{s}}\left(\frac{\tilde{v}}{\bar{c}}\right)_{x=0}+c_{16_{s}}\left(\frac{\tilde{v}}{\bar{c}}\right)\right] \frac{d x}{\bar{\tau}_{s} \bar{v}_{s}}\right\} \tag{103}
\end{align*}
$$

Combining the expressions of the preceding sections with this generalized vaporization rate expression yielded the combustion coefficients:

$$
\begin{align*}
& c_{T_{s}}=R_{m_{s}}-R_{v_{s}}-2 R_{D_{s}}+\left(\frac{\overline{N u_{s}}-2}{\overline{N u_{s}}}\right) \frac{R_{D_{s}}}{2}  \tag{104}\\
& c_{4_{s}}=R_{F_{P_{s}}}\left(\frac{\overline{N U_{s}}-2}{\overline{N u_{s}}}\right) \tag{105}
\end{align*}
$$

$$
\begin{align*}
& c_{6_{s}}=-\left(\frac{\partial \tau_{s}}{\partial \cdot M R}\right) \frac{1}{\bar{\tau}_{s}}  \tag{106}\\
& c_{b_{s}}=R_{F_{v_{s}}}\left(\frac{\overline{N u}_{s}-2}{\overline{N u}_{s}}\right)  \tag{107}\\
& c_{g_{s}}=2 R_{D_{s}}-\left(\frac{\overline{N u}_{s}-2}{\overline{N u}_{s}}\right) \frac{R_{D_{s}}}{2}+R_{v_{s}}  \tag{108}\\
& c_{12}=-c_{4}  \tag{109}\\
& c_{14_{s}}=-c_{6_{s}}  \tag{110}\\
& c_{16}=-c_{8_{s}}  \tag{111}\\
& c_{2}=c_{3}=c_{5}=c_{c_{7}}=c_{10}=c_{11}=c_{13}=c_{15}=0 \tag{112}
\end{align*}
$$

where the subscript $s$ denotes the fuel or oxidizer and

$$
\begin{align*}
& R_{m_{s}}=G_{i n j_{s}}\left(1-\frac{i \omega x_{k_{s}}}{\bar{v}_{s}}\right)  \tag{113}\\
& R_{u_{s}}=G_{i n j_{s}}  \tag{114}\\
& R_{D_{s}}=\left[b_{s}-a_{s} \frac{i \omega x_{i m p_{s}}}{\bar{v}_{s}}\right] G_{i n j_{s}} \tag{115}
\end{align*}
$$

In the above expressions $G_{i n j}$ is the oscillatory injection rate divided by the oscillatory pressure at the injector face and is calculated by the hydrodynamics subbrogram.

The main function of the combustion dynamics subprogram is the calculation of the combustion coefficients. The general spray vaporization rate expressions.are used in the chamber dynamic subprogram whichis discussed in the following section.

## CHAMBER DYNAMICS

## Analytical Approach

Two methods of approach were considered for solving the chamber dynamics. The first method used a linear lump chamber coefficient. This method is valid only at low frequencies (less than 500 Hz ) and results in a set of nonlinear algebraic equations to be solved.

The second method employed a first-order perturbation model to define the chamber frequency and growth coefficient along with the oscillatory pressure distribution in the chamber. This method is valid for all frequencies of interest in the present program ( 10 to 1000 Hz ). For the oscillatory variables, solutions of the form $\Phi=\phi^{\prime} e^{-i \omega t}$, where $\omega$ is the complex frequency, were assumed. These forms yielded a set of nonlinear differential equations which were numerically integrated between the injector face and the nozzle inlet plane. Using iteration techniques and the requisite boundary conditions at the injector and nozzle inlet plane, the chamber frequency and growth coefficient are obtained.

Consideration of the degree of complexity in solving the governing equations by each of the above methods as well as the range of validity of each approach resulted in choosing the first-order perturbation models as the best method for describing the chamber dynamics. In the following paragraphs, the derivation and solution to the first-order perturbation model stability equations are presented.

## First-Order Perturbation Model

In this section, chamber model equations are stated without showing their detail derivations. Complete derivation of the basic equations is presented in Ref. 1. Assumptions used in the derivation of the basic equations are: (1) ideal gas flow is a valid state equation; (2) dilute sprays occupy a negligible fraction of chamber volume; (3) the spray can be represented by a finite number of dropsize groups; each dropsize group contains a large number of locally identical drops; and, each size group constitutes a separate liquid phase and exchange terms between liquid phases are not included; (4) drag contributes only kinetic energy to the spray energy equation; (5) secondary "shear" breakup of
drops is not included; (6) negligible coupling between diffusion and thermal gradients; and (7) no body forces.

The following equations can be formulated for the gas phase:

Gas Continuity

$$
\begin{equation*}
\frac{\partial \rho}{\partial t}+\nabla \cdot(\rho \vec{u})=\sum_{n} \sum_{j}\left(N_{j}^{n} \dot{m}_{\text {val }_{j}}^{n}\right) \tag{116}
\end{equation*}
$$

Gas Momentum

$$
\begin{align*}
& \frac{\partial}{\partial t}(\rho \vec{u})+\nabla \cdot(\rho \vec{u} ; \quad \vec{u})=-\nabla p+\nabla \cdot \underline{\underline{\tau}} \\
& \quad-\sum_{n} \sum_{j}\left[N_{j}^{n}\left(\vec{F}_{j}^{n}-\dot{m}_{v^{n}} \vec{u}_{j}^{n}\right)\right] \tag{117}
\end{align*}
$$

Equation of State

$$
\begin{equation*}
p=\rho R T \tag{118}
\end{equation*}
$$

Shear Stress

$$
\begin{equation*}
\underline{\underline{t}}=-\mu_{\text {eff }}\left[\nabla \vec{u}+(\nabla \vec{u})^{t}-\frac{2}{3}(\nabla \cdot \vec{u}) \underline{\underline{I}}\right] \tag{119}
\end{equation*}
$$

Gas Energy

$$
\begin{align*}
\frac{\partial}{\partial t} & {\left[\rho\left(h+\frac{u^{2}}{2}\right)\right]+\nabla \cdot\left[\rho \vec{u}\left(h+\frac{u^{2}}{2}\right)\right] } \\
& =-\nabla \cdot \vec{q}+\nabla \cdot(\vec{u} \underline{\underline{\tau}})+\frac{\partial p}{\partial t} \\
& +\sum_{n} \sum_{j}\left\{N _ { j } ^ { n } \left[\dot{m}_{\operatorname{vap}_{j}}\left(n_{j}+\frac{\left(u_{j}^{n}\right)^{2}}{2}\right)\right.\right. \\
& \left.\left.-Q_{j}^{n}-\vec{u}_{j}^{n} \cdot \vec{F}_{j}^{n}\right]\right\} \tag{120}
\end{align*}
$$

Gas Mixture Ratio

$$
\begin{align*}
& \frac{\partial}{\partial t}(\rho M R)+\nabla \cdot(\rho \vec{u} M R) \\
& -\rho D_{\text {eff }}\left[\nabla^{2} M R-\frac{2|\nabla M R|^{2}}{M R+1}\right] \\
& -(\nabla M R) \cdot \nabla\left(\rho \mathscr{D}_{e f f}\right)= \\
& (2 M R+1)\left[\sum_{n}^{o x} \sum_{j} N_{j}^{n} \dot{m}_{v a p_{j}}^{n}\right] \\
&  \tag{121}\\
& -(M R)^{2}\left[\sum_{n}^{f} \sum_{j}^{u} N_{j}^{n} \dot{m}_{v a p_{j}}^{n}\right]
\end{align*}
$$

## Heat Transfer Rate

$$
\begin{equation*}
\stackrel{\rightharpoonup}{q}=-k_{e f f} \nabla T-\sum_{i}\left(0 D_{e f f}\right) h_{i} \nabla y_{i} \tag{122}
\end{equation*}
$$

Drag Force

$$
\begin{equation*}
\vec{F}_{j}^{n}=\frac{\pi}{8}\left\{\rho\left(D_{j}^{n}\right)^{2}\left|\vec{u}-\vec{u}_{j}^{n}\right|\left(\vec{u}-\vec{u}_{j}^{n}\right) C_{D_{j}}^{n}\right\} \tag{123}
\end{equation*}
$$

Assuming
(1) Diffuser, thermal and viscous gradients are negligible,
(2) Droplet drag forces and heat transfer to the droplets are negligible,
(3) Croplet velocities are approximately equal to the gas velocity, ana letting

$$
\begin{align*}
& h=\left(\frac{c_{p}}{R}\right)_{\phi} R T+\left(h_{r e f}\right)+\left(\frac{\partial h}{\partial M R}\right) \quad\left(M R-M R_{\phi}\right),  \tag{124}\\
& \left(\frac{c_{p}}{R}\right)_{\phi}=\frac{\gamma_{\phi}}{\left(\gamma_{\phi}-1\right)},  \tag{125}\\
& R=R_{\phi}+\left(\frac{\partial R}{\partial M R}\right)_{\phi} \quad\left(M R-M R_{\phi}\right),  \tag{126}\\
&
\end{align*}
$$

$$
\begin{align*}
\dot{m}_{v a p_{o x}} & \equiv \sum_{n}^{o x} \sum_{j} N_{j}^{n} \dot{m}_{v a p_{j}}^{n}  \tag{127}\\
\dot{m}_{v^{n a p}} \equiv & \equiv \sum_{n}^{f u} \sum_{j} N_{j}^{n} \dot{m}_{v a p_{j}}^{n} \tag{128}
\end{align*}
$$

where the subscript $\phi$ denotes that the properties are evaluated based on the overall injection mixture ratio during steady-state operation, the preceding equations can be simplified for longitudinal modes to

Gas Continuity
$A \frac{\partial p}{\partial t}+\frac{\partial}{\partial x}\left(A_{\rho} v\right)=A\left(\dot{m}_{v a p_{o x}}+\dot{m}_{v a p}\right)$
Gas Momentum
$\rho \frac{\partial v}{\partial t}+\rho v \frac{\partial v}{\partial x}+\frac{\partial p}{\partial x}=0$
Equation of State
$p=\rho T\left[R_{\varnothing}+\left(\frac{\partial R}{\partial M R}\right)_{\rho}\left(M R-M R_{\rho}\right)\right]$
Gas Energy

$$
\begin{align*}
& A \frac{\partial p}{\partial t}+A v \frac{\partial p}{\partial x}+\gamma_{\phi} p \frac{\partial}{\partial x}(A v)= \\
& \quad\left(r_{\phi}-1\right) A\left\{\dot{m}_{v a p} \quad\left[\Delta h_{o x}-\left(\frac{\partial h}{\partial M R}\right)_{\phi}(2 M R+1)\right]\right. \\
& \left.\quad+\dot{m}_{v a p_{f u}}\left[\Delta h_{f u}+\left(\frac{\partial h}{\partial M R}\right)_{\phi}(M R)^{2}\right]\right\} \tag{132}
\end{align*}
$$

Gas Mixture Ratio

$$
\begin{equation*}
\dot{\rho} \frac{\partial M R}{\partial t}+\rho v \frac{\partial M R}{\partial x}=(M R+1)\left[\dot{m}_{v a p_{o x}}-(M R) \dot{m}_{v a p}\right] \tag{133}
\end{equation*}
$$

Because of the complexity in solving nonlinear partial differential equations, perturbation techniques were used to simplify the governing dynamic equations. Assuming

$$
\begin{equation*}
\phi=\bar{\phi}+\tilde{\phi} \quad(\phi \text { any variable }), \tag{134}
\end{equation*}
$$

where

$$
\begin{equation*}
\bar{\phi}=f(x) \tag{135}
\end{equation*}
$$

and

$$
\begin{equation*}
\tilde{\phi}=g(x, t), \tag{136}
\end{equation*}
$$

each perturbation quantity was taken to he of order $(\varepsilon)$, where $(\varepsilon)$ is a small ordering parameter that is a measure of the wave amplitude. The perturbation expressions for each of the independent variables were substituted back into the nonlinear partial differential equations, where all terms of the order $\left(\varepsilon^{2}\right)$ or higher were neglected. The resulting time-averaged equations were solved for the time-averaged variables and the oscillatory equations were solved by assuming solutions of the form

$$
\begin{equation*}
\tilde{\phi}=\phi^{\prime} e^{-i \omega t} \tag{137}
\end{equation*}
$$

where $\phi^{\prime}=f(x)$ and $\omega$ is the complex frequency. The resulting equations form a system of ordinary differential equations in terms of the variables $\phi^{\prime} \quad$ and can be numerically integrated by employing boundary conditions and iteration techniques.

Following this approach the perturbation equations were expressed as:

$$
\begin{align*}
& \rho \equiv \bar{\rho}\left[1+\rho^{\prime} e^{-i \omega t}\right]  \tag{138}\\
& v \equiv \bar{v}+c_{\phi} v^{\prime} e^{-i \omega t}  \tag{139}\\
& T=\bar{T}\left[1+T^{\prime} e^{-i \omega t}\right] \tag{16.3}
\end{align*}
$$

$$
\begin{align*}
& p=\bar{p}\left[1+p^{\prime} e^{-i \omega t}\right]  \tag{141}\\
& M R=\overline{M R}+M R \prime e^{-i \omega t}  \tag{142}\\
& \dot{m}_{v a p_{o x}}=\overline{\dot{m}}_{v a p_{o x}}+\dot{m}_{v a p_{o x}^{\prime}} e^{-i \omega t}  \tag{143}\\
& \dot{m}_{v a p_{f u}}=\overline{\dot{m}}_{v a p_{f u}}+\dot{m}_{v a p_{f u}^{\prime}}^{\prime} e^{-i \omega t} \tag{144}
\end{align*}
$$

The time-averaged equations were determined to be:
(a) Gas Continuity

$$
\begin{equation*}
\frac{d}{d x}(A \bar{\rho} \bar{v})=A\left(\overline{\dot{m}}_{v a p}+\overline{\dot{m}}_{v a p}\right) \tag{145}
\end{equation*}
$$

(b) Gas Momentum

$$
\begin{equation*}
\bar{\rho} \bar{v} \frac{d \bar{v}}{d x}+\frac{d \bar{p}}{d x}=0 \tag{146}
\end{equation*}
$$

(c) Equation of State

$$
\begin{equation*}
\bar{p}=\bar{\rho} R_{\phi} T\left[1+\frac{1}{R_{\phi}}\left(\frac{\partial R}{\partial M R}\right)_{\phi}\left(\overline{M R}-M R_{\phi}\right)\right] \tag{147}
\end{equation*}
$$

(d) Gas Energy

$$
\begin{align*}
& A \bar{v} \frac{d \bar{p}}{d x}+\gamma_{p} \bar{p} \frac{d}{d x}(A \bar{v})= \\
& \quad\left(\gamma_{\phi}-1\right) A\left\{\overline{\dot{m}}_{v a p_{o x}}\left[\Delta h_{o x}-\left(\frac{\partial h}{\partial M R}\right)_{\phi}(2 \overline{M R}+1)\right]\right. \\
& \left.\quad+\overline{\dot{m}}_{v_{v a p}}\left[\Delta h_{f u}+\left(\frac{\partial h}{\partial M R}\right)(\overline{M R})^{2}\right]\right\} \tag{148}
\end{align*}
$$

(e) Gas Mixture Ratio

$$
\begin{equation*}
\bar{\rho} \bar{v} \frac{d \overline{M R}}{d x}=(\overline{M R}+1)\left[\overline{\dot{m}}_{v_{0 p}}-\overline{M R}{\stackrel{\dot{m}_{v a p}}{ }}^{v_{f u}}\right] \tag{149}
\end{equation*}
$$

and the oscillatory equations were determined to be:
(a) Gas Continuity

$$
\begin{align*}
& \rho^{\prime}\left(\frac{-i \omega}{c_{\dot{\rho}}}\right)+\frac{d v^{\prime}}{d x}+\frac{v^{\prime}}{A_{\rho}^{\prime}} \frac{d\left(A_{\rho}\right)}{d x} \\
& \quad+\left(\frac{\bar{v}}{c_{\phi}}\right) \frac{d \rho^{\prime}}{d x}+\frac{\rho^{\prime}}{A_{\rho} c_{\phi}} \frac{d}{d x}(A \bar{\rho} \bar{v})= \\
& \quad \frac{\left(\dot{m}_{v a p_{o x}^{\prime}}^{\prime}+\dot{m}_{v a p_{f u}^{\prime}}^{\prime}\right)}{\bar{\rho} c_{\phi}} \tag{150}
\end{align*}
$$

(b) Gas Momentum

$$
\begin{align*}
& v^{\prime}\left(\frac{-i \omega}{c_{\phi}}\right)+\frac{v^{\prime}}{c_{\phi}} \frac{d \bar{v}}{d x}+\frac{\bar{v}}{c_{\phi}} \frac{d v^{\prime}}{d x}-\frac{\rho^{\prime}}{\bar{\rho} c_{\phi}{ }^{2}} \frac{d \bar{p}}{d x} \\
& \quad+\frac{p^{\prime}}{\bar{\rho} c_{\phi}{ }^{2}} \frac{d \bar{p}}{d x}+\frac{\bar{p}}{\bar{\rho} c_{\phi}{ }^{2}} \frac{d p^{\prime}}{d x}=0 \tag{151}
\end{align*}
$$

(c) Equation of State

$$
\begin{equation*}
p^{\prime}=\rho^{\prime}+T^{\prime}+\frac{\bar{\rho} \bar{T}}{\bar{p}}\left(\frac{\partial R}{\partial M R}\right)_{\varnothing} M R^{\prime} \tag{152}
\end{equation*}
$$

(d) Gas Energy

$$
\begin{aligned}
& p^{\prime}\left(\frac{-i \omega}{c_{\phi}}\right)+\left(\frac{\bar{v}}{c_{\phi}}\right)\left[\frac{d p^{\prime}}{d x}+\frac{p^{\prime}}{\bar{p}} \frac{d \bar{p}}{d x}\right] \\
& \quad+\frac{v^{\prime}}{\bar{p}} \frac{d \bar{p}}{d x}+\gamma_{\phi}\left[\frac{d v^{\prime}}{d x}+\frac{v^{\prime}}{A} \frac{d A}{d x}\right] \\
& \\
& \quad+\frac{\gamma_{\phi} p^{\prime}}{A c_{\phi}} \cdot \frac{d}{d x}(A \bar{v})=\frac{\left(\gamma_{\phi}-1\right)}{\bar{p} c_{\phi}}\left\{\dot { m } _ { \operatorname { v a p } _ { o x } } \left[\Delta h_{o x}\right.\right.
\end{aligned}
$$

$$
\begin{align*}
& \left.-\left(\frac{\partial h}{\partial M R}\right)_{\phi}(2 \overline{M R}+1)\right]+\dot{m}_{v a p}^{\prime} \quad\left[\Delta h_{f u}\right. \\
& \left.+\left(\frac{\partial h}{\partial R}\right)_{\phi}(\overline{M R})^{2}\right]-2 \overline{\dot{m}}_{v^{2}}\left(\frac{\partial h}{\partial M R}\right)_{\phi} M R^{\prime} \\
& \left.+2 \overline{\dot{m}}_{v^{\prime} p_{f u}} M R\left(\frac{\partial h}{\partial M R}\right)_{\phi} M R^{\prime}\right\} \tag{153}
\end{align*}
$$

(e) Gas Mixture Ratio

$$
\begin{align*}
M R^{\prime} & \left(\frac{-i \omega}{c_{\phi}}\right)+\left(\frac{\bar{v}}{c_{\phi}}\right) \frac{d M R^{\prime}}{d x}+\left[\left(\frac{\bar{v}}{c_{\phi}}\right) \rho^{\prime}+v^{\prime}\right] \frac{d M R}{d x} \\
& =\frac{(\overline{M R}+1)}{\bar{\rho} c_{\phi}}\left[\dot{m}_{v a p_{o x}}^{\prime}-\overline{M R} \dot{m}_{v a p}^{\prime}\right] \\
& +\frac{1}{\bar{\rho} c_{\phi}}\left[\overline{\dot{m}}_{v^{\prime}}\right.  \tag{154}\\
& \left.-(2 \overline{M R}+1) \overline{\dot{m}}_{v a p}\right] \quad\left(M R^{\prime}\right)
\end{align*}
$$

In the computer model, the preceding set of ordinary differential equations are numerically integrated between the injector face and the nozzle inlet plane. The method of calculating the complex frequency for the perturbation morel, based on nozzle admittances calculated from upstream and downstream variables, is discussed in the Engineering Model section.

## Steady-State Solution

The boundary conditions for the steady-state differential equations are

$$
\begin{align*}
& 0=x_{0}  \tag{155}\\
& \bar{p}_{x_{0}}=p_{c}  \tag{156}\\
& \bar{v}_{x_{0}}=\bar{v}_{x=0}  \tag{157}\\
&-\quad \overline{M R}_{x_{0}}=M R_{x=0} \quad\left(\text { if } \bar{v}_{x=0} \neq 0\right)  \tag{158}\\
&\left(A \bar{p} \bar{v}_{x_{0}}=(\bar{m})_{x=0} \quad\left(\text { if } \bar{v}_{x=0} \neq 0\right)\right.  \tag{159}\\
& R-9808 / 42
\end{align*}
$$

Assuming small Mach numbers, i.e., $M^{2} \ll 1$, the steady-state differential equations can be integrated between the start plane for vaporization ( $x_{0}$ ) and any location ( $x$ ) to yield

$$
\begin{align*}
& \bar{p}=\text { constant }=p_{c}  \tag{160}\\
& \overline{M R}=\frac{\left(\frac{\overline{M R}}{1+\overline{M R}}\right)_{x=0}(A \rho \bar{v})_{x=0}+\left(\bar{m}_{o x}\right)_{i n j}\left(1-\phi_{o x}\right)}{\left(\frac{1}{1+\overline{M R}}\right)_{x=0}\left(A_{\rho} \bar{v}\right)_{x=0}+\left(\bar{m}_{f u}\right)_{i n j}\left(1-\phi_{f u}\right)}  \tag{161}\\
& \bar{v}=\frac{\left(A_{v}\right)_{x=0}}{A}+\frac{\left(\gamma_{\phi}-1\right)}{\gamma_{\phi} \bar{p} A}\left\{\left(\bar{m}_{o x}\right)_{i n j}\left(1-\phi_{o x}\right) \Delta h_{o x}\right. \\
& +\left(\bar{\Pi}_{f u}\right)_{i n j}\left(1-\phi_{f u}\right) \Delta h_{f u}+(A \bar{\rho} \bar{v} \overline{M R})_{x=0}\left(\frac{\partial h}{\partial M R}\right)_{\phi} \\
& -\overline{M R}\left(\frac{\partial h}{\partial M R}\right)\left[\left(A_{\rho} \bar{v}\right)_{x=0}+\left(\overline{\dot{m}}_{o x}\right)_{i n j}\left(1-\phi_{0 x}\right)\right. \\
& \left.\left.+\left(\bar{m}_{f u}\right)_{i n j}\left(1-\phi_{f u}\right)\right]\right)  \tag{162}\\
& \bar{\rho}=\frac{1}{A \bar{v}}\left\{(A \bar{\rho} \bar{v})_{x=0}+\left(\bar{m}_{o x}\right)_{i n j}\left(1-\phi_{o x}\right)\right. \\
& \left.+\left(\bar{m}_{f u}\right)_{i n j}\left(1-\phi_{f u}\right)\right\}  \tag{163}\\
& \bar{T}=\frac{\bar{p}}{\bar{\rho} R_{\phi}\left[1+\frac{1}{R_{\phi}}\left(\frac{\partial R}{\partial M R}\right)_{\phi}\left(M R-M R_{\phi}\right)\right]} \tag{164}
\end{align*}
$$

where

$$
\begin{equation*}
\varphi_{s}=e^{-\left(x-x_{0}\right) / \tau_{s}} \bar{v}_{s} \tag{165}
\end{equation*}
$$

If the gaseous injection velocity is equal to zero ( $\bar{v}_{x=0}=0$ ), the steads state mixture ratio and density at $x=x_{0}$ are determined by

$$
\begin{align*}
& \overline{M R}_{x_{0}}=\left(\frac{\bar{m}_{o x}}{\bar{m}_{f u}}\right)_{i n j}\left(\frac{\bar{\tau}_{f u} \bar{v}_{f u}}{\bar{\tau}_{o x} \bar{v}_{o x}}\right)  \tag{166}\\
& (\bar{\rho})_{x_{0}} \quad \frac{\left(\gamma_{\phi}-1\right)}{\gamma_{\phi} \bar{p}}\left\{\overline{M R}_{x_{0}}\left[\Delta h_{o x}-\overline{M R}_{x_{0}}\left(\frac{\partial h}{\partial M R}\right)_{\phi}\right]\right. \\
&  \tag{167}\\
& +\Delta h_{f u}-\overline{M R}_{x_{0}}\left(\frac{\partial h}{\partial \overline{M R}_{\phi}}\right)=\overline{M R}_{x_{0}}+1
\end{align*}
$$

These equations were developed by taking the limit as $x \rightarrow x_{0}$ from a downstream distance.

## Oscillatory Solution

The boundary conditions for the oscillatory differential equations are

$$
\begin{align*}
x & =0  \tag{168}\\
p^{\prime} & =\Delta p  \tag{169}\\
v^{\prime} & =\left(v^{\prime}\right)_{x=0} \tag{170}
\end{align*}
$$

From these boundary conditions and the oscillatory differential equations the oscillatory conditions at the start plane for vaporization ( $x_{0}$ ) can be determined and are:

$$
\begin{gather*}
P_{x_{0}-}^{\prime}=\left[\Delta p \cos \left(n \frac{\omega x_{0}}{c_{\phi}}\right)+\right.  \tag{171}\\
\left.\quad i \gamma_{\phi} n\left(v^{\prime}\right)_{x=0} \sin \left(\eta \frac{\omega x_{0}}{c_{\phi}}\right)\right] e^{-\frac{i \omega}{c_{\phi}} n^{2} \cdot \frac{\bar{v}_{x=0}}{c_{\phi}} x_{0}} \\
P-3808 / 44
\end{gather*}
$$

$$
\begin{align*}
v_{x_{0}}^{\prime}= & {\left[\begin{array}{lll}
\frac{i}{\gamma_{\phi}} & \frac{\Delta p}{\eta} & \sin \left(n \frac{\omega x_{0}}{c_{\phi}}\right)+ \\
& \left.\left(v^{\prime}\right)_{x=0} \cos \left(n \frac{\omega x_{0}}{c_{\phi}}\right)\right] e^{-\frac{i \omega}{c_{\phi}}} \eta^{2} \frac{\bar{v}_{x=0}}{c_{\phi}} x_{0} \\
\rho_{x_{0}}^{\prime}= & \frac{F_{x_{0}}^{\prime}}{\gamma_{\phi}} \\
M R_{x_{0}}^{\prime}= & \left(M R^{\prime}\right)_{x=0} e^{\left(\frac{i \omega}{c_{\phi}}\right)\left(\frac{c_{\phi}}{\bar{v}_{x=0}}\right) x_{0}} \\
T_{x_{0}}^{\prime}= & p_{x_{0}}^{\prime}-\rho_{x_{0}}^{\prime}-\frac{1}{\bar{R}_{x_{0}}}\left(\frac{\partial R}{\partial M R}\right)_{\phi} M R_{x_{0}}^{\prime}
\end{array}, l\right.}
\end{align*}
$$

where

$$
\begin{equation*}
n \equiv \sqrt{\frac{\bar{\rho}_{x_{0}} c_{\phi}^{2}}{\gamma_{\phi} \bar{p}}} \tag{176}
\end{equation*}
$$

If the gaseous injection velocity is equal to zero ( $\bar{v}_{x=0}=0$ ), the oscillatory mixture ratio at $x_{0}$ is determined by

$$
\begin{align*}
& M_{x_{0}}^{\prime}\left[\frac{\left(\bar{m}_{f u}\right) i n j}{A \bar{\rho}_{x_{0}} c_{\phi}}\left(1+\overline{M R}_{x_{0}}\right)-\frac{i \omega}{c_{\phi}}\left(\bar{\tau}_{f u} \bar{v}_{f u}\right)\right] \\
& =\frac{\left(\overline{M R}_{x_{0}}+1\right)}{A \bar{\rho}_{x_{0}} c_{\phi}}\left(\bar{m}_{f u}\right)_{i n j} \overline{M R}_{x_{0}}\left(\begin{array}{ll}
\frac{\dot{m}_{v a p}^{\prime}}{\prime} & \frac{\dot{m}_{0 x}^{\prime}}{\bar{m}_{v a p_{0 x}}} \\
\overline{\dot{m}_{v a p}} \\
\bar{m}_{f u}
\end{array}\right) \\
& \left.-\quad-\left(v_{x_{0}}^{\prime}\right) \overline{M R}_{x_{0}} \frac{\left(\overline{M R}_{x_{0}}+1\right)}{\left(1-\frac{\overline{M R}}{\overline{M R}_{x_{0}}}+2\right)}\right)= \tag{177}
\end{align*}
$$

This equation was developed by taking the limit of the mixture ratio equation as $x \rightarrow x_{0}$ from a downstream distance.

The ordinary differential equations describing the oscillatory solution are solved using a second order implicit finite difference method. This method has the advantage of being simple to implement and modify, as well as being unconditionally stable for systems of equations which do not have exponentially growing solutions. The method as applied to the first order systen

$$
\begin{equation*}
Y^{\prime}=A Y+g \tag{178}
\end{equation*}
$$

where $Y$ and $g$ are $n x l$ vectors and $A$ is an $n \times n$ matrix is as follows:

$$
\begin{equation*}
y_{i+1}=y_{i}+\frac{\Delta x}{2} A_{i+\frac{1}{2}}\left(y_{i}+y_{i+1}\right)+g_{i+\frac{1}{2}} \tag{179}
\end{equation*}
$$

Here, the subscript $i$ refers to the $i$ 'th mesh point in the finite difference scheme, e.g., $x_{i}=x_{0}+i \Delta x$. The $y_{i}$ approximate the $\gamma$ vector at $x_{i}$. That is $y_{i} \sim Y_{i}=Y\left(x_{i}\right)$. The subscript $i+\frac{1}{2}$ refers to evaluation at $x_{i}+\Delta x / 2$; e.g., $A_{i+\frac{1}{2}}=A\left(x_{i}+\frac{\Delta x}{2}\right)$.

That the above method leads to a second order approximation (error is provortional to $\Delta x^{3}$ ) can be shown as follows:

Solving for $y_{i+1}$ yields

$$
\begin{equation*}
y_{i+1}=\left(I-\frac{\Delta x}{2} A_{i+\frac{1}{2}}\right)^{-1}\left(I+\frac{\Delta x}{2} A_{i+\frac{1}{2}}\right) y_{i}+\Delta x\left(I-\frac{\Delta x}{2} A_{i+\frac{1}{2}}\right)^{-1} g_{i+3 / 2} \tag{180}
\end{equation*}
$$

Without loss of generality, assume $\mathbf{i}=0$.

From the two expansions

$$
\begin{align*}
& Y_{1}=Y_{\frac{1}{2}}+\frac{\Delta x}{2} Y_{\frac{1}{2}}^{\prime}+\frac{\Delta x^{2}}{8} Y_{\frac{1}{2}}^{\prime \prime}+o\left(\Delta x^{3}\right)  \tag{181}\\
& Y_{0}=Y_{\frac{1}{2}}-\frac{\Delta x}{2} Y_{\frac{1}{2}}^{\prime}+\frac{\Delta x^{2}}{8} Y_{\frac{1}{2}}^{\prime \prime}+o\left(\Delta x^{3}\right) \tag{c}
\end{align*}
$$

the following are obtained

$$
\begin{equation*}
Y_{1}=Y_{0}+\Delta x Y_{\frac{1}{2}}^{\prime}+o\left(\Delta x^{3}\right) \tag{183}
\end{equation*}
$$

and

$$
\begin{equation*}
Y_{\frac{1}{2}}=\left(Y_{0}+Y_{1}\right) / 2+o\left(\Delta x^{2}\right) \tag{184}
\end{equation*}
$$

Let $y_{0}=Y_{0}$; it is necessary to show $y_{1}=Y_{1}+o\left(\Delta x^{3}\right)$ in order to demonstrate second-order accuracy.

From the differential equation

$$
\begin{equation*}
G_{\frac{1}{2}}=Y_{\frac{1}{2}}^{\prime}-A_{\frac{1}{2}} Y_{\frac{1}{2}} \tag{185}
\end{equation*}
$$

Substituting(185)into(180) and noting $y_{0}=Y_{0}$,

$$
\begin{align*}
y_{1} & =\left(I-\frac{\Delta x}{2} A_{\frac{1}{2}}\right)^{-1}\left(I+\frac{\Delta x}{2} A_{\frac{1}{2}}\right) Y_{0}+\Delta x\left(I-\frac{\Delta x}{2} A_{\frac{1}{2}}\right)^{-1}\left(Y_{\frac{1}{2}}^{\prime}-A_{\frac{1}{2}} Y_{\frac{1}{2}}\right) \\
& =\left(I-\frac{\Delta x}{2} A_{\frac{1}{2}}\right)^{-1}\left\{Y_{0}+\Delta x Y_{\frac{3}{2}}^{\prime}+A_{\frac{1}{2}} \Delta x\left(\frac{1}{2} Y_{0}-Y_{\frac{1}{2}}\right)\right\} \tag{186}
\end{align*}
$$

Using (183) and (134), gives the result

$$
\begin{align*}
y_{1} & =Y_{1}+\left(I-\frac{\Delta x}{2} A_{\frac{1}{2}}\right)^{-1} o\left(\Delta x^{3}\right) \\
& =Y_{1}+o\left(\Delta x^{3}\right) \tag{187}
\end{align*}
$$

Consider nov the stabilitv of the finite difference formula(179) for systems which do not have exponentially increasing solutions; that is, the real part of each of the eigenvalues of $A$ is negative. To prove that they are stable for this situation, define the error $\varepsilon_{i}=y_{i}-y_{i}$ and consider the two equations given by (179) and

$$
\begin{array}{r}
Y_{i+1}=\left(I-\frac{\Delta x}{2} A_{i+\frac{1}{2}}\right)^{-1}\left(I+\frac{\Delta x}{2} A_{i+\frac{1}{2}}\right) Y_{i}+ \\
\Delta x\left(I-\frac{\Delta x}{2} A_{i+\frac{1}{2}}\right)^{-1} g_{i+\frac{1}{2}}+o\left(\Delta x^{3}\right) \tag{188}
\end{array}
$$

the latter resulting from (187). Subtracting (179) from (188)

$$
\begin{equation*}
\varepsilon_{i+1}=\left(I-\frac{\Delta x}{2} A_{i+\frac{1}{2}}\right)^{-1}\left(I+\frac{\Delta x}{2} A_{i+\frac{1}{2}}\right) \varepsilon_{i}+o\left(\Delta x^{3}\right) \tag{189}
\end{equation*}
$$

Let $B=\frac{\Delta x}{2} A_{i+\frac{1}{2}}$. The method is stable if and only if the matrix $(I-B)^{-1}$ $(I+B)$ has a spectral radius less than one, for this would produce (Ref. 22)

$$
\begin{equation*}
\lim _{n \rightarrow \infty}\left[(I-B)^{-1}(I+B)\right]^{n}=0 \tag{190}
\end{equation*}
$$

Since the eigenvalues of $(I-B)^{-1}(I+B)$ are just equal to $(1+B) /(1-B)$, where $\varepsilon$ is an eigenvalue of $B$, the spectral radius of $(I-B)^{-1}(I+B)$ is just

$$
\begin{equation*}
\max _{\beta}|(1+\beta) /(1-\beta)| \tag{191}
\end{equation*}
$$

For this to be less than one,

$$
\begin{equation*}
|1+B|<|1-B| \tag{192}
\end{equation*}
$$

for all B. This implies

$$
\begin{equation*}
1+\beta+\bar{\beta}+\beta \bar{B}<1-\beta-\bar{\beta}+\bar{\beta} \bar{\beta} \tag{193}
\end{equation*}
$$

or

$$
\begin{equation*}
\beta+\bar{\beta}<-(\beta+\bar{\beta}) \tag{194}
\end{equation*}
$$

Real $(\beta)<0$
Since $\beta=\frac{\Delta x}{2 \alpha}$, where $\alpha$ is an eigenvalue of $A$, the method will be stable if all the eigenvalues of $A$ have real parts less than zero, that is, the solutions to (178) are not exponentially increasing.

## Nozzle Admittance

The nozzle admittance based on downstream conditions is calculated based on the following analysis.

The gas flowrate of the nozzle inlet plane is

$$
\begin{equation*}
\dot{m}=\frac{p A_{t} g}{c^{*}}=A \rho v \tag{196}
\end{equation*}
$$

where the characteristic velocity is

$$
\begin{equation*}
c^{*}=\frac{\sqrt{g \gamma R T}}{\gamma\left[\frac{2}{\gamma+1}\right]^{(\gamma+1) / 2(\gamma-1)}} \tag{197}
\end{equation*}
$$

For short nozzles, the oscillatory mass flowrate can be written as

$$
\begin{equation*}
\frac{\dot{m}^{\prime}}{\overline{\dot{m}}}=\frac{\rho^{\prime}}{\bar{p}}+\frac{v^{\prime}}{\bar{v}}=\frac{p^{\prime}}{\bar{p}}-\frac{1}{2} \frac{T^{\prime}}{\bar{T}}-\left(\frac{\partial \overline{c^{\star}}}{\partial \overline{M R}}\right) \frac{M R^{\prime}}{\overline{c^{\star}}} \tag{198}
\end{equation*}
$$

Assuming

$$
\begin{equation*}
\frac{\rho^{\prime}}{\rho}=\frac{1}{\gamma} \frac{p^{\prime}}{\bar{p}}, \frac{T^{\prime}}{\bar{T}}=\left(\frac{\gamma-1}{\gamma}\right) \frac{p^{\prime}}{\bar{p}}, \tag{199}
\end{equation*}
$$

the nozzle admittance for a short nozzle can be written as

$$
\begin{equation*}
A_{N_{S}}=\bar{\rho} \bar{c} \bar{v}\left[\frac{(\gamma-1)}{2 \gamma \bar{p}}-\left(\frac{\partial \overline{c^{\star}}}{\partial \overline{M R}}\right) \frac{M R^{\prime}}{\overline{c^{\star} p^{\prime}}}\right] \tag{200}
\end{equation*}
$$

Assuming

$$
\begin{equation*}
A_{N_{D}}=A_{N_{S}}\left(\frac{A_{N_{D}}}{A_{N_{S}}} \quad M R=\right.\text { constant } \tag{201}
\end{equation*}
$$

the nozzle admittance based on downstream conditions becomes

$$
\begin{equation*}
A_{N_{D}}=\left[1-\left(\frac{\partial \bar{c}^{\star}}{\partial \overline{M R}}\right) \frac{M R^{\prime} \bar{p}}{\overline{c^{\star}} p^{\prime}}\left(\frac{2 \gamma}{\gamma-T}\right)\right]_{X=\ell} A_{N_{M R}}=\text { constant } \tag{202}
\end{equation*}
$$

where $A_{N_{M R}}=$ constant is calculated using the admittance program developed by Rell (Ref. 23).

The nozzle admittance based on upstream conditions is

$$
\begin{equation*}
A_{N_{U}}=\gamma_{\phi}\left(\frac{v^{\prime}}{p^{\prime}}\right)_{x=\ell} \tag{203}
\end{equation*}
$$

where $\left(v^{\prime} / p^{\prime}\right)_{x=\ell}$ is calculated from the oscillatory solution. For solutions to the chamber dynamic equations, the nozzle admittance based on upstream and downstream conditions must be equal. The method of calculating the complex frequency which satisfies this condition is discussed in the Engineering Model section which follows.

ENGINEERING MODEL

Analytical Approach
The overall model structure had the greatest variety of factors influencing its nature. Some of these factors were related to the overall confidence in the success of the effort. Factors relating to cost included the solution time and numerical stability, which bears on the number of runs which will be required for a solution. Still other factors were related to the JSC Univac 1110 capabilities. The remaining factors concerning the overall model structure reflect on its conversion cost applicability to the resolution of propulsion system problems. Its accuracy has direct bearing on the design margins involved. The type of impact and the obtainability of characterization parameters could not limit the accuracy and useability. The type and useability of the output was also given due consideration as well as the degree of generalization such that the model can be applied to a range of systems.

The structure of the Engineering Program was based on a trade-off of setup time, storage capabilities, and solution time. General input data to the program includes geometric factors, engine operating conditions and propellant properties. An equilibrium gas properties program similar to NASA ODE computer program, and the DER combustion model program are executed external to the stability program. The control program then executes the nozzle admittance and hydrodynamics programs to calculate the admittance and oscillatory injector flowrate as a function of frequency and stores the results on tapes. Steady-state distributed combustion parameters calculated from the DER Model are inputs to the combustion dynamics subprogram which are iterated with the chamber dynamic subprogram until the nozzle and injection admittance conditions are satisfied. The solution method for obtaining solutions for the complex frequency is outlined in the following section.

## Determination of Complex Frequency

The complex frequency, $\omega$, is determined such that the boundary condition in the nozzle is satisfied. Specifically, the admittance is required to be continuous across the interface between the combustion zone and the zone immediately downstream of the combustion zone. In the downstream zone, the nozzle admittance, $A_{N_{D}}$, is computed from a nozzle admittance program. In the upstream combustion zone, the nozzle admittance, $A_{N_{U}}$, is computed from the oscillatory flow parameters determined by the ${ }^{N}$ chamber dynamics. The complex frequency must be such that


Let $\omega=x+i y$ and $F=A_{N_{U}}-A_{N_{D}}=u+i v$. The numerical problem is to find $x$ and $y$ such that

$$
\begin{align*}
& u(x, y)=0  \tag{205}\\
& v(x, y)=0 \tag{206}
\end{align*}
$$

Several methods were considered for solving this system of equations. Because $F$ is not an analytic function of $\omega$, the complex form of the NewtonRaphson method may not always work. On the other hand, one could use the two-dimensional form of Newton-Raphson (Ref. 24), but since the derivatives of $u$ and $v$ with respect to $x$ and $y$ must be computed numerically, the twodimensional Newton-Raphson method will require three functional evaluations of $F$ at each $\omega$, i.e., $(x, y),(x+\Delta x, y)$, and $(x, y+\Delta y)$. Alternatively, a far more efficient method is to use the two-dimensional form of the secant method (Ref. 24) since this does not require the evaluation of any derivatives. Specifically, this method approximates the $u$ and $v$ surfaces with linear functions $u_{L}$ and $v_{L}$ (planes) based on three previous guesses for $\omega$, $\left(x_{1}, y_{1}\right),\left(x_{2}, y_{2}\right),\left(x_{3}, y_{3}\right)$. The next guess for $\omega,\left(x_{4}, y_{4}\right)$, is determined from the equations $u_{L}\left(x_{4}, y_{4}\right)=v_{L}\left(x_{4}, y_{4}\right)=0$. The new value of $\omega$ then replaces one of the previous three values, normally the one with the largest error as measured by the absolute value of $F\left(x_{j}, y_{j}\right)$, and the iteration
is continued until convergence is reached. The actual equations for the above process take the following form. Let $w_{j}=x_{j}+i y_{j}, u_{j}=$ $u\left(x_{j}, y_{j}\right)$, and $v_{j}=v\left(x_{j}, y_{j}\right)$ for $j=1,2,3$.

1. Determine $\pi_{j}, j=1,2,3$, such that

$$
\begin{align*}
& \pi_{1}+\pi_{2}+\pi_{3}=1  \tag{207}\\
& \pi_{1} u_{3}+\pi_{2} u_{2}+\pi_{3} u_{1}=0  \tag{208}\\
& \pi_{1} v_{3}+\pi_{2} v_{2}+\pi_{3} v_{1}=0 \tag{209}
\end{align*}
$$

2. Compute $\omega_{4}=\pi_{1} \omega_{3}+\pi_{2} \omega_{2}+\pi_{3} \omega_{1}$.
3. Compute $u_{4}$ and $v_{4}$ based on $x_{4}$ and $y_{4}$.
4. Test for convergence, i.e., require that $\min \left|\omega_{j}-\omega_{4}\right| / \omega_{4} \mid<\varepsilon_{1}$ and $\left|F_{4}\right|<\varepsilon_{2}$.
If the process has not converged, continue with steps 5,6 , and 7 .
5. Determine the $j$ between 1 and 3 such that $u_{j}^{2}+v_{j}^{2}$ is maximum.
6. Replace $\omega_{j}, u_{j}$, and $v_{j}$ by $\omega_{4}, u_{4}$, and $v_{4}$, respectively.
7. Go to 1.

Operationally, steps 4 and 5 may be altered to replace the $\omega$ 's cyclically, i.e., $w_{i} \rightarrow w_{i-1}, u_{i} \rightarrow u_{i-1}, v_{i} \rightarrow v_{i-1}$. In fact the computer program as written alternates between these two procedures every three iterations in order to avoid any possible cycling that may occur.

The above algorithm has been found to be very efficient when the first three guesses are relatively near an actual solution. The difficult problem was to develop a searching algorithm which determines the regions in the $\omega$ plane where solutions exist.

One possible procedure would be to utilize the fact that the surface $u^{2}+v^{2}$ has an absolute minimum at each solution. Using any reasonable value of $\omega$ as a first guess, one might be tempted to employ a gradient, or modified gradient, method to march along the surface until one comes near a relative minimum. Unfortunately, this procedure fails because the surface $u^{2}+v^{2}$ has many relative minima which are not actual solutions. The reason for the large number of relative minima (and maxima) for this surface is undoubtedly due to the coupling between the combustion processes and the feed system oscillation in conjunction with the very rapid change of the feed system response as a function of frequency. The searching algorithm must be able to discriminate between those relative minima that are not solutions and those that are. Such a procedure was developed for this program. It takes advantage of the fact that a large portion of the computations required are only a function of the real part of $\omega$, i.e., they use $x$ as an independent variable and do not depend upon $y$. Thus, $y$ may be changed without having to redo many of the calculations within the program.

Intuitively, the idea is to increment $x$ through a range of values, while determining $y$ at each $x$ according to the criterion mentioned below, until it is determined that a solution has been crossed. This determination employs the use of a test function which changes sign when a root is crossed in the same manner that a single equation in one unknown changes sign as it goes through a zero. The task of developing a defining criterion for $y$ and a test function for $x$ would be easy if, for example, $v$ were a strong function of $y$ and $u$ were a strong function of $x$. Then, each $x, y$ could be chosen such that $v(x, y)=0$ and, as $x$ is incremented, a solution would be crossed when $u[x, y(x)]$ changes sign. Unfortunately, neither $u$ nor $v$ behaves this way.

To develop functions that do behave this way, the following procedure was developed. First, for each $x$, choose $y$ such that the absolute value of $F$ is minimized. This can be done in several ways. The program uses a method that always guarantees finding a value if one exists. Essentially, the absolute
value of $F$ squared and its gradients are computed. The value of $y$ is altered in the direction indicated by the gradient until either the gradient changes sign or is so close to zero that convergence has been reached. Once the gradient changes sign, Muller's method (Ref. 25) is used to converge on the root. This is essentially a bisection methoc followed by inverse parabolic interpolation. For this searching process, it is not necessary to make the convergence criteria very tight, since only rough estimates are eventually needed in order to start the two-dimensional secant method described earlier.

Now that a criterion for $y$ has been established, it is only necessary to find a test function that will change sign when a solution is crossed while incrementing x. Such a function is given by

$$
\begin{equation*}
u u_{x}+v v_{x} \tag{211}
\end{equation*}
$$

This function acts as a very good test function because it represents the . coordinate direction in the $u, v$ plane along which the vector ( $u, v$ ) changes most with $x$. When this coordinate changes sign as one goes from, say, $x_{1}$ to $x_{2}$ with $y_{1}$ and $y_{2}$ chosen so that the length of the vector ( $u, v$ ) is minimized, then it is very likely that a solution has been crossed. Exceptions to this rule occur when one is near relative minima of the surface $u^{2}+v^{2}$ that are not zero. To see this, consider the actual equations that are being solved. In order that the vector ( $u, v$ ) is minimum for each $y$, it is necessary that $\partial\left(u^{2}+v^{2}\right) / \partial y=0$. That is $u u_{y}+v v_{y}=0$. Combining this with the above equation, we see we are finding an $x$ and $y$ such that the matrix equation

is satisfied. The matrix is just the transpose of Jacobian of $u$ and $v$ with respect to $x$ and $y$.

This equation can be satisfied if either $u$ and $v$ are zero, or the Jacobian is singular. The Jacobian is necessarily singular at all relative minima of the surface $u^{2}+v^{2}$ except those at $u=v=0$. In order to differentiate between those solutions to (212) that are due to singularities of the Jacobian and those that are due to $u$ and $v$ vanishing, we employ two different tests. First of all, when a singularity point is crossed, the determinant of the Jacobian should change sign. If this occurs, then the program rejects this point as a possible solution. Sometimes, however, the determinant does not change sign because either the convergence criterion used in the searching algorithm is too loose or because the singularity has a double root. In either case, the procedure is to test the condition number* of the transpose of the Jacobian matrix in the region near the suspected solution. If the condition number does not exceed a given input limit (e.g., around 80), then the point in question is usually a solution.

Once it is determined that a potential solution has been crossed between $x_{1}$ and $x_{2}$, for example, the procedure is to (a) determine $x_{3}$ based on the method of false position using the test function given in (211), (b) determine $y_{3}$ to minimize $|F|$, and (c) use $\left(x_{1}, y_{1}\right),\left(x_{2}, y_{2}\right)$, and $\left(x_{3}, y_{3}\right)$ as the required first three guesses for the two-dimensional secant method.

The above procedure has been found to be most satisfactory for the conditions tested in this program. The search algorithm described above has several salient features. First, as mentioned earlier, the search method takes advantage of the fact that many of the computations are not a function of the imaginary part of $\omega$, namely $y$ : This allows the minimization of $|F|$ with respect to $y$ to proceed with high efficiency. Secondly, and more importantly,
*The condition number of a matrix, $A$, is a measure of how sensitive a solution to the system $A b=c$ is to perturbations in $c$. It is equal to the square root ratio of the absolute value of the largest eigenvalue of $A^{\prime} A$ to the smallest eigenvalue of $A^{\prime} A$. For singular matrices, the condition number is infinite. For matrices that are nearly singular, the condition number will be quite large.
the procedure has been automated to the extent that the user only has to specify a frequency range and a maximum number of roots desired in that range. The algorithm will start at the lower end of the frequency range and will increment through it until either the maximum number of roots are found or the upper end of the frequency range is reached. This is a very powerful property since it does not require the user to have a clear knowledge of the location of any of the roots in the $\omega$ plane.

## PROGRAM USER'S GUIDE

## FSCSM MAIN PROGRAM

The main program for the FSCSM computer model performs much of the input/output activity of the model and controls the sequencing through the various major subprogram blocks of the model. After writing the header page (Fig. 5), the main program reads in and writes out the data described in Table 2 of the input section of this report under the heading Main Control Section Input. The program then computes the area profiles through a call to subroutine AREA. It then beains its main do loop controlling the number of solutions to the nozzle admittance boundary equation that are desired. During the iterations for each solution to be found, the main program proceeds with successive calls to subroutines NQZADM, HYDRDY, CQMBDY, and SOLVW in order to compute the downstream nozzle admittance factor, the feed system response parameters, the combustion dynamics coefficients, and performance calculations necessary to solve the nozzle admittance boundary equations respectively. Subroutine SQLVW also causes a call to subroutine CHAMDY which computes the oscillatory profiles. Further, during the first iteration in the do loop for the main program for the first solution, the main program also calls subroutine STEADY in order to obtain the steadv-state profiles.

The variable ISCNT is the FØRTRAN variable set by the main program and altered in S $\varnothing$ LVW which controls the type of iteration being performed. When ISCNT equals one or four, the search algorithm described on pages 54-56 is called out. This is the initial condition at the beginning of each set of iterations to solve the nozzle admittance boundary equation. When ISCNT equals five, the two-dimensional secant method is being performed in SøLVW.

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Figure 5. Logic Diagram for FSCSM Main Program


The variable KWHERE, which is set by subroutine SQLVW, controls the logical flow in the main program subsequent to a call to SøLVW. When ISCNT equals one or four and KWHERE equals one, the control is returned to that portion of the main program which starts the calculation for the next value of $\omega$ being tried by the search algorithm. When KWHERE equals two and ISCNT equals one or four, control is passed to that portion which will perform calculations for a perturbed $\omega$ in order to compute the derivative of $A_{N U}-A_{N_{D}}$ with respect to $\omega_{R}$. When ISCNT equals 5 and convergence has not been reached, the normal exit from SøLVW also sets KWHERE equal to two. For this case however, no derivatives are calculated. The program will Just proceed with successive calls to NØZADM, HYDRDY, and CØMBDY in order to compute the downstream nozzle admittance factor, the feed system response, and the combustion dynamic coefficients, respectively. It then proceeds to SøLVW in order to obtain an updated estimate of $\omega$ using the two-dimensional secant method.

When KWHERE equals three or four, convergence on a solution to the nozzle admittance boundary equation has been attained. For this case, control is passed to that portion of the main program which prints the final results for that solution.

When KWHERE equals five, control is passed to the end of the main do loop in the main program. The output portion is bypassed. This only occurs when an error was detected by subroutine S@LVW.

## SØLVW

This subroutine performs many of the calculations and controls most of the logical flow required to match the downstream boundary condition on the nozzle admittance. The FORTRAN variable ISCNT controls the logical flow within subroutine SøLVW (Fig. 6). If ISCNT equals one, then that portion of the subroutine used for searching the $\omega-p l a n e$ for possible solutions to the nozzle admittance boundary equation is used. Two calls to SøLVW are used for this purpose. During the first call, the imaginary part of $\omega$ is determined so that the absolute value of the upstream nozzle admittance minus the downstream nozzle admittance is minimized. The second call is made in order to complete the computation of the Jacobian of this difference with respect to the real portion of $\omega$. (The derivatives of the difference with respect to the imaginary part of $\omega$ are computed during the first call.) When ISCNT equals one, tests are also made to determine if a solution is nearby. The actual test function and the logic employed is described on pages 54-56.

Once it is determined that a possible solution is bracketed by two successive frequencies, the variable ISCNT is set equal to four. Subroutine SøLVW performs the same calculations for this value of the variable ISCNT as it does when ISCNT equals one. The only difference occurs at the end of the second call to SøLVW. At that point, checks are made to ensure that the potential solution is in fact an actual solution and not due to a singularity in the Jacobian. If the error passes certain criteria and at least three iterations have been performed with ISCNT equal to four, then ISCNT is set equal to five for subsequent calls to SøLVW. Between each iteration for ISCNT equal to four, as well as for the first iteration for ISCNT equal to four, the real part of the frequency is modified using the method of false position or the bisection method, depending upon the value of the iteration counter, KSCNT4.

When ISCNT equals five, no derivatives are computed. For this situation, subroutine SOLVW only computes the difference between the upstream and downstream nozzle admittances based on the current value of $\omega$. It then checks to determine whether convergence has been obtained. If not, the


Figure 6. Logic Diagram for Subroutine SøLVW

value of $\omega$ is updated according to the two-dimensional secant method described on page 53.

The variable which controls the flow in the main program subsequent to a call to SØLVW is KWHERE and is described in the section of this report dealing with the main program.

CHAMDY
This subprogram is called by XIMAGF in order to compute the oscillatory pressure, temperature, velocity, mixture ratio, and density profiles. From these quantities (Fig. 7), it determines the upstream nozzle admittance. It solves for the oscillatory profiles by solving the linearized set of differential equations presented on pages 41 and 42 . This is done using a second order implicit finite difference scheme. Those integrals appearing in the vaporization expression whicn cannot be integrated analytically are numerically integrated using the trapezoidal rule.

Because the differential equations represent a linear initial value problem, the finite difference equations are also linear and one can "march off" the solution from the initial plane. Since the four differential equations are coupled, replacing them at each axial position by their finite difference apprcximation results in a four by four system of complex linear equations. Because of the nature of the differential equations, the resulting matrix equations are essentially diagonally dominant and can therefore be solved very quickly using Gaussian elimination with the diagonal element used for pivoting.

## XIMAGF

This subroutine is called by SøLVW and ZERø. Its main function is to compute the difference between the upstream and downstream nozzle admittances (Fig. 8). When the program is still performing the search algorithm, this routine computes the derivative of this difference with respect to the imaginary part of $\omega$ as well as the derivative of the absolute value of this difference squared with respect to the imaginary part of $\omega$.


Figure 7. Logic Diagram for Subroutine CHAMDY


Figure 8. Logic Diagram for Subroutine XIMAGF

## CHMCØN

This routine is called by CHAMDY and calculates certain parameters necessary for determining the coefficients used in CHAMDY.

## CøMMAT

This routine solves the four by four complex system of linear equations discussed in the section describing subroutine CHAMDY. It uses Gaussian elimination with pivoting on the diagonal.

## ZERD

This routine is called by subroutine CHAMDY. Its function is to find the zero of a given functional when that zero is bracketed both above and below. The functional in this case is the derivative with respect to the imaginary part of $\omega$ of the absolute value squared of the difference between the upstream and downstream nozzle admittances. Finding the zero of this functional is done in order to minimize the error in the difference between the nozzle admittances with respect to the imaginary part of $\omega$. The method used by subroutine ZERO is due to Muller (Ref. 25). It essentially involves a bisection step followed by inverse parabolic interpolation to determine the next guess.

## STEADY

This routine (Fig. 9) is called by the main program to determine the time independent solution to the set of differential equations given on page 40 . These equations have been analytically integrated on pages 42-44. This subroutine uses these latter equations to determine the steady state profiles. Also, several parameters which are a function of these steady state variables are computed and saved for subsequent use by the chamber dynamics subprogram, CHAMDY. If the FORTRAN variable IPRSTE is greater than zero, a printout of the steady state profiles will be given.

## CØMBDY

This subprogram (Fig. 10) calculates the fuel and oxidizer combustion coupling coefficients required for the determination of the time oscillatory vaporization rates needed to solve the chamber dynamics. The equations for these parameters


Figure 9. Logic Diagram for Subroutine STEADY


Figure 10. Logic Diagram for Subprogram C $\quad$ MBDY
are given on pages 33-34. During the first pass into this program, the combustion dynamic input variables are read in from input device 5 and written out onto device 6. A discussion of these variables is given in the Model Input Section. The logical flow in CØMBDY is controlled by the FØRTRAN variable INPCØM. If this variable is less than or equal to one, the combustion coefficients are computed for the current frequency only. If INPCDM is greater than one, these coefficients are computed for the entire frequency table, FREQT (e.g., from 10 to 1000 Hz ), and saved on tape/disk ITAPE for subsequent use by the main program.

AREA

This subroutine is called by the main program. It computes the area profiles and axial distance profiles necessary for solution of the steady-state and transient profiles.

LOCFAC

This routine is used to determine the subscript, I, within an ordered array, $T X$, such that the input argument, $X$, is in the interval $T X(I), T X(I+1)$.
This routine also returns the interpolating factor $F X=(X-T X(I) /(T X(I+1)$ $T X(I))$ for subsequent use in linear interpolation.

HEAD

This subroutine is called by the main program to print the heading page which gives the title of the program, by whom and where it was developed, and the program sponsor.

HYDRDY

Subroutine HYDRDY (Fig. 11) is called by the main program to calculate the frequency domain characteristics of the feed system. Functions performed by HYDRDY are (1) reading of input data describing the physical attributes


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Figure 11. Logic Diagram for Subprogram HYDRDY
of the feed system components, (2) generation of a matrix of linear differential equations representing the complete feed system, (3) solution of the feed system equations to yield the amplitude and phase response of all feed system pressures and flowrates as a function of chamber pressure oscillation amplitude and frequency, and (4) generation of tabulated output of injector flowrate frequency response for use by the main program.

A basic assumption of subroutine HYDRDY is that the feed system being modeled can be represented by the generalized schematic of Fig. 4 (or by some portion of this schematic). This assumption is necessary because HYDRDY sets up and solves the complete set of simultaneous equations representing the Fig. 4 schematic. By assigning very high resistance and very short length attributes to any of the 30 numbered line segments of the generalized schematic, whose segment can effectively be excluded from having any effect on the frequency response characteristics of the rest of the system. With this approach a wide variety of feed systems can be modeled with no changes to the program other than the input data.

Figure 11 shows the functional block diagram for subroutine HYDRDY. When called, the subroutine initially zeroes the values of all of the elements of the coefficient and time delay matrices $C$ and TD in labeled conmon block $F$. The values of various fixed input arguments required by the frequency response subroutine (FRESP) are then initialized.

Input argument IR directs the reading of subroutine HYDRDY input data. If IR is zero or less, the program assumes that all required data has previously been read and the data read function is bypassed. If $I R=1$, the program assumes that no hydrodynamic data has been read and proceeds to initialize all hydrodynamic input variables to values which will exclude all 30 line segments and both injectors of the generalized Fig. 4 feed system schematic. Control is then passed to statement 45 for reading of input data. If
$I R=2$, the assumption is made that most input data is already set up (such as from a previous case during the same program run). Control is passed directly to statement 45 for reading any changes to the input data.

Input data reading for HYDRDY is in the NAMELIST format (NAMELIST name HYD) and is normally in the form of card input on logical unit number 5. However, if the program is run in a timesharing environment, an option is provided for reading data from a timesharing terminal. This option is controlled by variables IRFLAG and ITERM. Both of these variables are stored in labeled common block/F/ and can be changed by input to NAMELIST HYD. Variable IRFLAG is tested in statement 45 and if non-zero specifies reading NAMELIST HYD data from unit 5 . If timesharing terminal input is desired, variable ITERM is set (by input data read or block data initialization of labeled common block/F/) to the logical unit number to the timesharing terminal. If variable ITERM is non-zero, statement 46 sets IRFLAG to 1 . Thus, once terminal data input has been specified, all subsequent data reads will default to the terminal. Card input can be respecified (for a subsequent data case) by entering IRFLAG $=0$ in the terminal data input.

Input variables for subroutine HYDRDY are described in detail in the Hydrodynamic Input Section. The variables include the length ( $L$ ), cross-sectional area ( $A$ ), propellant sonic velocity ( $V$ ), propellant density (RHOL), hydraulic resistance (R), and wall compliance (CW) for each of the 30 numbered waterhammer segments in the generalized Fig. 4 feed system schematic. Input variables for the left ("す") injector of Fig. 4 are resistance ( $\mathrm{R} \|$ ), inertance ( $Z \emptyset$ ), volume (VOL $\emptyset$ ), propellant sonic velocity (V $\downarrow$ ), and injector deflection constant ( $K \|$ ). The corresponding input variables for the right ("F") injector are RF, $Z F, V \not \subset L F, V F$ and $K F$. The designation of the two injectors as " $\emptyset$ " and "F" is a notational convenience for cases in which the feed system being modeled has only one injector and sufficiently simple flow paths so that both oxidizer and fuel systems can simultaneously be laid out on the Fig 4 schematic. Such cases have the advantage of reduced computer time because the
frequency response of both fuel and oxidizer feed systems is obtained with a single call to subroutine HYDRDY. Of the fuel and oxidizer feed systems overlap when laid out on the Fig. 4 schematic, subroutine HYDRDY must be called twice - once for each feed system.

When data input is complete, a value of 2 (or greater) for variable IWRITE specifies a printout of all input data on logical unit 6. IWRFTE $=0$ is the default and specifies no printout of input data.

Next, control is passed to the $D \emptyset$ loop at statement 100 in which the input values for propellant density, propellant acoustic velocity and segment wall compliance for each of the 30 waterhammer segments of the Fig. 4 schematic are combined to yield an effective acoustic velocity for each segment. The subsequent statements, up to statement 400 , combine the input variables as required to yield the constant coefficients of the 57 linear waterhammer and injector equations describing the complete Fig. 4 feed system. Simultaneous solution of these 57 equations, at each specified input frequency, yields the oscillatory amplitude and phase response of all pressures and flowrates in the feed system to inputs via chamber pressure oscillations at that freauency.

At statement 500 a call to subroutine FRESP is made to obtain the frequency response solution of the feed system equations. Initially, however, at statement 400 the value of input argument INPHYD is tested to determine the desired output from FRESP. If INPHYD is greater than 1, HYDRDY will call FRESP to calculate the feed system frequency response for each of the frequencies in array $\operatorname{FREQT}$. The total number of frequencies is given by variable NFREQT and may range from 1 to 100 . Both the variable, NFREQT, and the array, FREQT are in labeled common block/COMTAP/. If the value of INPHYD is less than or equal to 1 HYDRDY will call FRESP to calculate the feed system frequency response for the single frequency given by input argument, FRE.

Outputchata from subroutine HYDRDY consists of a pair of complex numbers for each specified input frequency. If INPHYD was specified as $\leq 1$ then the output numbers GINJ $\$ x$ and GINJFU are returned in labeled common block/F/ and also in the HYDRDY argument list as GIN
and imaginary parts of complex number GINJ $\emptyset X$ (GIN $\emptyset$ ) represent the amplitude and phase angle respectively of $\frac{\Delta W Q}{W Q I N} / \frac{\Delta P C}{P C I N}$ at frequency FREQ (FRE). Similarly, the complex number GINJFU (GINF) represents the amplitude and phase angle of $\frac{\Delta W D}{W F I N} / \frac{\Delta P C}{\text { PCIN }}$ at frequency FREQ (FRE), WФIN, WFIN and PCIN are the input normal values for the oxidizer injector flowrate, fuel injector flowrate and chamber pressure, respectively, from the HYDRDY argument list.

If INPHYD was specified as $>1$, then rather than a single pair of complex numbers representing oscillatory injection flowrates, HYDRDY returns two arrays, GINJゆT and GINJFT, containing the oscillatory injection flowrate amplitude and phase data for each of the NFREQT frequencies in array FREQT. The output arrays GINJOT and GINJFT are stored in labeled common block/CФMTAP/ and are also written out on the output device whose logical unit number is designated by variable ITAPH in labeled common block/CØMTAP/. The order of storage on the output device is GINJ $\emptyset$ T(I), GINJFT(I), for I values from 1 through NFREOT. After writing the GINJ $\varnothing$ T and GINJFT arrays on the output device HYDRDY sets the value of variable INPHYD to 3. Also, before returning control to the main program, HYDRDY tests the value of variable IWRITE. If IWRITE is non-zero, each specified frequency and the corresponding value of GINJ $\emptyset T$ and GINJFT are written out on logical unit 6. If only one frequency was specified (INPHYD $\leq 1$ ), then only the single point values of FREQ, GINJゆX and GINJFU are written out.

It should be noted that although output from a single call to HYDRDY contains values for both "oxidizer" and "fuel" oscillatory injection flowrates (at one or more frequencies), the output values actually refer to the " $\phi$ " and "F" injectors of the Fig. 4 schematic. Thus, unless both oxidizer and fuel feed systems can simultaneously be modeled with the Fig. 4 layout, it is necessary to call HYDRDY twice - once for the oxidizer feed system and once for the fuel feed system.

## FRESP

Subroutine FRESP (Fig. 12) is used to obtain the frequency domain characteristics of the feed system indirectly from input data that describes the physical


Figure 12. Logic Diagram for Subroutine FRESP
characteristics of the feed system. (The actual input to FRESP is generated by the subroutine HYDRDY which orders the physical characteristics of the system into specific matrices of coefficients that FRESP can use as input data.)

FRESP merely solves for the variables $X_{i}$ in the following relationship:

$$
C \quad\{X\}=a \quad Y
$$

where $Y$ is a single input variable that represents a unit value of the injector end combustion chamber pressures. The matrix a then relates the specific pressure input to each applicable equation that contains combustion chamber pressure (a may contain both static and dynamic terms.) The matrix $C$ is simply the coefficients of the linear differential equations that represent the physical system. The values of the coefficients for the $a$ and $C$ matrices are computed by the subroutine HYDRDY.

The FRESP matrices can be expressed as:

$$
c_{i j k} s^{k-1} \cdot\left\{x_{j}\right\}=a_{i k} s^{k-1} \cdot Y
$$

with the differential operator defined as $S=J \omega$, where $J=\sqrt{-1}$ and $\omega$ is the frequency. The matrices may be broken down to provide real matrices and imaginary matrices.

$$
\begin{align*}
& c_{i j 1}-c_{i j 3} \omega^{2}+c_{i j 5} \omega^{4}-\ldots+J+c_{i j 2} \omega-c_{i j 4} \omega^{3}+\ldots \\
& x_{j}=a_{i 1}-a_{i 3} \omega^{2}+\ldots+J a_{i 2} \omega-a_{i 4}-A_{i 4} \omega^{3}+\ldots . Y \tag{213}
\end{align*}
$$

Since the time delay coefficients used in the differential equations are of the form $e^{-\tau S} \cdot x$, which is equivalent to $e^{-\tau j \omega} \cdot x$, and since $e^{-j y}=$ $\cos (y)+j \sin (y)$, these terms may be added to the previous $\rceil \bar{y}$ formed real and imaginary matrices to give:

$$
\begin{align*}
& \left\{\left[c_{i j 1}-c_{1 j 3} \omega^{2}+\ldots .+\cos \left(\tau_{i j} \omega\right)\right]\right. \\
& \left.\quad+\left[J c_{i j 2} \omega-c_{i j 4} \omega^{3}+\ldots \ldots+\sin \left(\tau_{i j} \omega\right)\right]\right\} \cdot\left[x_{j}\right]= \\
& \\
& \quad\left\{\begin{array}{l}
{\left[a_{i 1}=a_{i 3} \omega^{2}+\ldots \ldots \cos \left(\tau_{i} \omega\right)\right]} \\
\\
\left.+J\left[a_{i 2} \omega-a_{i 4} \omega^{3}+\ldots .+\sin \left(\tau_{i} \omega\right)\right]\right\} Y
\end{array}\right. \tag{214}
\end{align*}
$$

and solved for $\left[x_{i}\right]$ :

$$
\begin{align*}
{\left[x_{i}\right]=} & \left\{\left[\begin{array}{c}
\left.c_{i j 1}-c_{i j 3} \omega^{2}+\ldots+\cos \left(\tau_{i j} \omega\right)\right]+ \\
\left.J\left[c_{i j 2} \omega-c_{i j 4} \omega^{3}+\ldots \ldots+\sin \left(\tau_{i j} \omega\right)\right]\right\}
\end{array}\right.\right. \\
& \left\{\left[\begin{array}{l}
\left.i 1-a_{i 3} \omega^{2}+\ldots+\cos \left(\tau_{i} \omega\right)\right]+ \\
\left.J\left[a_{i 2} \omega-a_{i 4} \omega^{3}+\ldots+\sin (\tau \omega)\right]\right\}
\end{array}\right.\right.
\end{align*}
$$

The matrices are multiplied and then solved for $\left|x_{i}\right|$ in the subroutine C $\$ G A E L$ which employs the standard Gaussian elimination procedure for solving linear equations. The $\left\{x_{i} \mid\right.$ solution is still separated into real and imaginary components, and are simply combined to form a vector for each variable. The procedure is repeated for each frequency being considered.

CDGAEL
This subroutine is called by the hydrodynamic frequency response subroutine, FRESP, to triangularize the complex matrix of feed system equations. Back substitution into the triangular system of equations is subsequently performed by subroutine FRESP to yield the real and imaginary portions of each feed system variable.

The conventional method of Gaussian elimination is employed by CØGAEL to triangularize the system of equations. The reduction process proceeds in column order from left to right. First the complex element with the largest absolute value in the current ("nivot") column at or below the diagonal is located. Then the rows are interchanged if required to move this maximum element ( ivot element) to the diagonal. The row interchange serves to minimize the round-off errors from the subsequent reduction process. The pivot row (row containing the oivot element) is then divided by the pivot element yielding 1.0 from the pivot element. Finally, the elements in the pivot column below the diagonal are eliminated by subtracting the appropriate multidle of the pivot row from each row below it. The subtraction is not actually performed on the elements below the diagonal since these elements do not enter into the subsequent back substitution process performed by subroutine FRESP.

It should be noted that the above discussion refers to the complex matrix as if the elements were single numbers. The actual elements are stored as two numbers in each row, the real portion to the left of the constant term, and the imaginary portion on the right. This distinction does not alter the elimination orocess except that two separate numbers must be operated on at each step.

## TDPLDT

If the value of ICRT is greater than 0 this subroutine is called by the frequency response routine FRESP to generate CRT plots of the gain and phase of the output variables as a function of frequency.

The infut arguments to TDPLDT are, W, an array of up to 101 frequencies; $Y$, an array of gain or phase values; NFP, the number of data points in arrays $W$ and $Y$; TL, the lowest desired frequency grid line; $X R$, the highest desired frequency grid line: and $L L$, a flag indicating a gain plot if 1 or a phase plot if 2.

Initially TDPL $\emptyset$ T scans the $Y$ array for the maximum and minimum values and generates values for the $Y$ axis grid scaling. The first value of $Y$ is not included in this scan. This allows an initial very low value of frequency to be used to approximate the system $D C$ frequency response without upsetting the plot frequency scaling. TDPL $\emptyset T$ uses the standard graphics package routines for the SC-4020 to generate the plot arids and plot the data points. If the value of $L L$ is 1 the CRT frame is advanced and a plot of $Y(I)$ versus $W(I)$ is made on the bottom half of the page. If LL is 2 , the frame is not advanced and the plot is made on the top half of the page.

In addition to the plots, TDPL $\varnothing \mathrm{T}$ prints the numeric value of the first $Y$ array element immediately above the corresponding plot. This element typically corresponds to a very low frequency value (default value of .001 cPs in subroutine FRESP) which is well below the frequency range desired for the plot and approximates the $D C$ value of the output variable.

TDPLDT does not generate any titles or identificating information on the plots.

N@ZADM, (RKTDIF, RKTZ, RKZDIF, TADAMS, ZADAMS)

This routine and its support routines, is called by the main program to determine the nozzle admittance based on downstream conditions. The programs were developed and programmed by Georgia Institute of Technology and the user is referred to Ref. 23 for a complete description of these routines. The main nozzle admittance program was modified by Rocketdyne so that input data could be read if required and also the nozzle admittance saved on a tape unit ITAPN (Fig. 13).

Figure 13. Logic Diagram for Subroutine NØZADM

## FSCSM COMPUTER MODEL INPUT

This section, and the Hydrodynamic Input section, describes the input necessary to run the FSCSM computer model. The input is broken up into four major control sections. These are the main control section input, the nozzle admittance control section input, the hydrodynamics control section input, and the combustion dynamics control section input. Table 2 lists all the variables that are to be input for each control section. This input is in the usual 80 character card form. Listed in Table 2 for each control section are each card's number and format, the variable names appearing on each card, and a brief description of each variable appearing in the list.

The main control section requires either eight or nine cards depending upon the input value of INPNØZ. (If INPNQZ is less than or equal to three, the last card in this section should be input). The first two cards, cards 1 and 2, should contain title and case identification information. These are read in alpha numeric format and printed at the top of almost every page of output to identify the case being considered.

The third card contains control information for various run options, file numbers for the auxiliary storage of datasets used by the program, print codes, and the number of mesh points to be used in the analysis. The control flags are INPHYD, INPCQM, and INPNQZ. These flags allow the user to save the results from the current case or use the results from a previous case for the current case. It is recommended that these datasets be set up as permanent files in order to use them for subsequent job submittals.

The first of these flags, INPHYD, controls the hydrodynamics feed system dataset. If it is input as one, no action will be taken with respect to saving or reading information on or from a dataset. The hydrodynamics coupling terms will be recomputed each time the frequency changes. If INPHYD is input as two, a table of hydrodynamic coupling terms will be generated for the frequency range specified by the input variables NFREQ, FREQMI, and FREQMX. This table will be saved on file ITAPH (also input on this card) and used to linearly interpolate in each time the frequency changes. If INPHYD is input as three, the program assumes a

TABLE 2. FSCSM INPUT DATA


| 1,2 | $((\operatorname{TITLE}(I, J)$, |
| :--- | :--- |
| $(18 A 4)$ | $I=1,18), J=1,2)$ |

3
(12I6) INPHYD

INPCOM

INPNOZ

Title information is input on the first two cards in the input dataset. These should be used to identify the case being run.

Code used for hydrodynamic calculations.
1: Hydrodynamic coupling terms (HCT) will be computed each time the frequency changes.
2: A table of HCT's will be computed using frequency table, FREQT, and saved on file ITAPH. Each time the frequency changes in trying to satisfy the nozzle admittance boundary condition, the HCT will be interpolated for in that table.
3: Table of HCT's already resides on file ITAPH from an earlier run. When the frequency changes, the HCT will be interpolated for in that table.

Code used for the combustion dynamics coefficients (CDC). It can take on values from one to three. It has the same meaning with respect to the CDC as INPHYD does with respect to the HCT. The tables are saved on file ITAPC.

Code used for the downstream nozzle admittance term (DNAT).
1: The DNAT is combuted each time the frequency changes.
2: A table of DNT's versus frequency using frequency table, FREQT, is computed and saved internally. Each time the frequency hanges in trying to satisfy the nozzle admittance boundary condition, the DNT will be interpolated for in that table.

TABLE 2. (Continued)

| CONTROL | CARD NO. | VARIABLE |
| :--- | :--- | :--- |
| SECTION | FORMAT |  |

Main Program Input (Cont.)

TARLE 2. (Continued)

| CONTROL <br> SECTION | CARD NO./ <br> FORMAT | VARIABLE <br> Main Program <br> Input (Cont.) |
| :--- | :--- | :--- |
|  |  | IPRSTE |
| NAME |  |  |

Code used for the steady-state profiles printout.

0 : Profiles will not be printed.
1: Profiles will be printed.
Number of axial positions to be used between the XO plane (start of vaporization) and the nozzle inlet plane. (Both the $X O$ and nozzle inlet planes must be counted.)

Axial coordinate of start of vaporization plane, inches.

Nozzle inlet plane, inches
Radius of combustion chamber at injector face, inches.

Ratio of specific heats of combustion gas ( $C_{p} / C_{v}$ ) evaluated at overall mixture ratio, unitless.

Sonic velocity evaluated at overall mixture ratio, ft/sec.

Oscillatory non-dimensional pressure amplitude at injector face, unitless.

Absolute value of this variable is the number of frequency solutions that will be searched for starting at the frequency specified by the real part of 9 MEGA and ending at FROMAX. This variable may be input as either positive or negative. (See input of 9 MEGAR for explanation.)

IWRT
Intermediate output dump code used to write the oscillatory profiles solved for in CHAMDY for each iteration. 0 : Oscillatory profiles will not be printed out between iterations.
1: Oscillztory profiles will be printed for each iteration.

TABLE 2. (Continued)

| $\begin{aligned} & \text { CONTROL } \\ & \text { SECTION } \\ & \hline \end{aligned}$ | CARD NO./ FORMAT | VARIABLE NAME | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| Main Program Input (Cont.) |  | IWSKP | Intermediate output dump code used in subroutine SøLVW. <br> 0 : No intermediate output will be printed from SØLVW during iterations. <br> 1: Limited intermediate output will be printed by SøLVW <br> 2: Extended intermediate output will be printed by SøLVW. |
|  |  | KNTMX | Maximum number of iterations allowed to minimize the error in the nozzle admittance boundary equation with respect to the imaginary part of $\omega$. |
|  |  | KNTRMX | Maximum number of times the frequency will be allowed to be changed by DELFRO during the searching algorithm between each solution. |
|  |  | KNTSMX | Maximum number of iterations allowed for the convergence of the two-dimensional secant method used in S@LVW. |
|  | $\begin{aligned} & 6 \\ & (6 \mathrm{El2.}) \end{aligned}$ | QMEGAR | Starting value for the real part of complex frequency of NROOT>O. This should be input in units of Hertz times $2 \pi$. If $N R O O T<0$, this should be input in Hertz. |
|  |  | QMEGAI | Starting value for the imaginary part of complex frequency. This should be input as the growth coefficient if NROOT>O. It should be input as the decrement if NROOT<0. |
|  |  | FRQMAX | Maximum freauency above which no solutions to the nozzle admittance boundary equation will be sought, Hertz. |
|  |  | DELFRQ | Increment used to adjust the frequency during the searching portion of the algorithm to sclve the nozzle admittance boundary equation, Hertz. |

TABLE 2. (Continued)

| CONTROL <br> SECTION | CARD NO./ FORMAT | VARIABLE NAME |
| :---: | :---: | :---: |
| Main Program Input (Cont.) |  | DELMX |

7 EPSF
(4E12.8)

## DESCRIPTION

Maximum allowable change in the growth coefficient between two successive iterations in the portion of the program that minimizes the error in the nozzle boundary condition equation with respect to the growth coefficient, $\mathrm{sec}^{-1}$.

Upper bound on the condition number of the transpose of the Jacobian of the difference between the upstream and downstream nozzle admittances with respect to the complex frequency. If the condition number of that matrix exceeds CTEST for a given frequency, then it is assumed the Jacobian is singular near that frequency and hence a solution will not be sought at that point. Unitless.

Relative error criterion used during the search algorithm/or the portion of the program that minimizes the eror, $H N$, in the nozzle admittance boundary equation with respect to the growth coefficient, wi. To obtain convergence, it is necessary that

$$
\frac{\partial|H N|^{2}}{\partial \omega_{I}} /\left|N_{D}\right|<E P S F .
$$

Unitless.
EPSX

Relative error criterion used during the search algorithm for the portion of the program that minimizes the error, HN , in the nozzle admittance boundary equation with respect to the growth coefficient, $\omega_{I}$. To obtain convergence, it is necessary that

$$
\left|\omega_{I_{1}}-\omega_{I_{2}}\right| /\left|\omega_{I_{2}}\right|<E P S X
$$

where the subscripts 1 and 2 refer to two successive iterations. Unitless.

TABLE 2. (Continued)

| CONTROL  <br> SECTION CARD NO./ <br> Main Program  <br> Input (Cont.)  |  | VARIABLE <br> NAME |
| :--- | :--- | :--- |
|  |  | EPSFS |

DESCRIPTION
Tightened relative error criterion used to determine if convergence has been obtained while iterating to solve the nozzle admittance boundary equation. To obtain convergence, it is necessary that


Unitless.
EPSXS Tightened relative error criterion used to determine if convergence has been obtained while iterating to solve the nozzle admittance boundary equation. To obtain convergence, it is necessary that

$$
\left|\omega_{1}-\omega_{2}\right| /\left|\omega_{2}\right|<E P S X S
$$

where the subscripts 1 and 2 refer to successive iterations. Unitless.

Steady-state chamber pressure, psia. Oxidizer injection flowrate, $1 \mathrm{bm} / \mathrm{sec}$ Fuel injection flowrate, 1bm/sec

$$
\text { INPUT THIS CARD ONLY IF INPNפZ } \leq 3
$$

| (I12,2E12.8) | NFREOT | Number of points in frequency table. |
| :--- | :--- | :--- |
|  | FREOMI | Minimum frequency in frequency table, |
|  | Hertz. |  |
|  | FREQMX | Maximum frequency in frequency table, <br> Hertz. |

TABLE 2. (Continued)

| CONTROL <br> SECTION | CARD NO./ <br> FORMAT |  | VARIABLE <br> NAME |
| :--- | :--- | :--- | :--- |
| Nozzle <br> Admittance <br> Program |  | INPUT THE CARD IN THIS CONTROL SECTION <br> ONLY IF INPNQZ $\leq 3$ |  |

Hydrodynamics
Program

| (4E12.8) | RCCX |
| :--- | :--- |
|  | RCTX |

ANGLEX Nozzle convergence half angle, degrees, (see Fig.15).

CRR Contraction ratio, cross-sectional area of chamber/throat area, unitless (see Fig.15)

THIS CONTROL SECTION READS IN ITS INPUT DATA IN NAMELIST FORMAT. THE NAMELIST NAME IS /HYD/. INPUT THIS DATA ONLY IF INPHYD $\leq 2$.

| \&HYD* | Input these characters starting in column <br> two of the first card of the <br> input. |
| :--- | :--- |
| NAMELIST | See Table 3 for a listing of the NAMELIST <br> data input names. The accompanying text <br> Variables <br> in any <br> order |
| describes the meaning of these variables. |  |

*For Univac $1 T 10$ systems, use \$HYD and \$END

TABLE 2. (Continued)

| CONTROL <br> SECTION | CARD NO./ FORMAT | VARIABLE $\qquad$ | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| Combustion <br> Dynamics <br> Program |  | INPUT THE CARDS IN THIS CONTROL SECTION ONLY IF INPC甲M $\leq 2$ |  |
|  | $\begin{aligned} & 1 \\ & (6 \mathrm{E} 12.8) \end{aligned}$ | XKOX | Klystron constant for oxidizer jet, inches. |
|  |  | taubgx | Steady-state oxidizer vaporization time delay, sec. |
|  |  | Vbøx | Steady-state oxidizer injection velocity, ft/sec. |
|  |  | DELHOX | Pseudo energy term for oxidizer, Btu/lbm. |
|  |  | TDRAGD | Steady-state oxidizer drag time delay, sec. |
|  |  | ADV@X | Velocity exponent for the oxidizer atomization process, unitless. |
|  | $\begin{aligned} & 2 \\ & (5 \mathrm{E} 12.8) \end{aligned}$ | ADD®X | Oxidizer liquid jet diameter exponent, unitless. |
|  |  | DELVOX | Steady-state velocity difference between oxidizer droplets and gas stream normalized to the sonic velocity at the overall mixture ratio, unitless. |
|  |  | NUBQX | Steady-state oxidizer Nusselt number used in vaporization expression, unitless. |
|  |  | DTOXDM | Partial derivative of oxidizer vaporization time delay with respect to mixture ratio, holding the vaporization blockage term, drop diameter, and Nusselt number constant, sec. |
|  |  | XIMPDX | 0xidizer jet impinaement point, inches. |
|  | $\begin{aligned} & 3 \\ & (6 \mathrm{E} 12.8) \end{aligned}$ | XKFU | Klystron constant for fuel jet, inches. |
|  | - | TAUBFU | Steady-state vaporization time delay, sec |
|  |  | VBFU | Steady-state fuel injection velocity, $\mathrm{ft} / \mathrm{sec}$. |

TABLE 2. (Concluded)

| CONTROL | CARD NO./ | VARIABLE |
| :--- | :--- | :--- |
| SECTION | FORMAT |  |

Combustion Dynamics Program (Cont.)

DELHFU
TDRAGF

ADVFU

ADDFU
(5E12.8)

5 MWG
(5E12.8)

DESCRIPTION

Pseudo energy term for fuel, Btu/lbm. Steady-state fuel drag time delay, sec.

Velocity exponent for fuel atomization process, unitless.

Fuel liquid jet diameter exponent, unitless.

Steady-state velocity difference between fuel droplets and gas stream normalized to the sonic velocity at the overall mixture ratio, unitless.

Average steady-state fuel Nusselt number used in vaporization expression, unitless.

Partial derivative of fuel vaporization time delay with respect to mixture ratio, holding the vaporization blockage term, droplet diameter, and Nusselt number constant, sec.

Fuel jet injection impingement point, inches.

Steady-state molecular weight of the gas at the overall mixture ratio, lbm/lbm-mole.

Characteristic velocity at the overall mixture ratio, ft/sec.

Partial derivative of gas constant with respect to mixture ratio evaluated af the overall mixture ratio, $\mathrm{ft}-\mathrm{lb} / \mathrm{lb} / \mathrm{O}_{\mathrm{R}}$.

Partial derivative of characteristic velocity with respect to mixture ratio evaluated at the overall mixture ratio, $\mathrm{ft} / \mathrm{sec}$.

DHDMR
Partial derivative of gas reference enthalpy with respect to mixture ratio averaged over the mixture range during steady-state oderation, Btu/lbm.
table of hydrodynamic coupling terms versus frequency already resides on file ITAPH in the format used during generation of such a table when INPHYD is input as two. The program will interpolate in this table in order to obtain the hydrodynamic coupling terms each time the frequency changes. The use of interpolation, once a table has been generated, substantially reduces the computer run time for each case run.

The other two control flags input on the third card control the datasets for the combustion dynamics coefficients and the nozzle admittance factors. These flags are similar to INPHYD. Their description is given in Table 2.

Also input on the third card of the main control section input are the file numbers of the datasets discussed above and print control flags for the various forms of output one can obtain. These are all self-explanatory. The user need only refer to Table 2 in order to determine the values that should be input for the case being considered.

The final entry on the third card is for the variable NXP, the number of points to be used for the axial distance and area arrays. This controls the step size that will be taken during the integration of the chamber dynamics equations; i.e., step size $=(X N \varnothing Z-X 0)(N X P-1)$ where $X 0$ is the axial location of the start of vaporization plane and $X N \varnothing Z$ is the axial location of the nozzle inlet plane. The values of $X O$ and $X N \emptyset Z$ are both input on the very next card read in (card 4 in Table 2).

The start of vaporization plane ( XO ) is calculated by plotting the percent unburned of both fuel and oxidizer that is calculated by the DER program (or equivalent steady-state combustion model) as a function of distance from the injector face (Fig. 14). These plots are then extrapolated back to $100 \%$ unburned and the axial location of this point is XO .

Also input on card 4 are the radius of the combustion chamber at the injector face, RINJ, the ratio of specific heats $\left(C_{p} / C_{v}\right)$, GAMD, the sonic velocity, $C \emptyset$, and the oscillatory non-dimensional pressure amplitude desiredat the injector face $(\triangle P / P)$, DELP. The variables GAM $\varnothing$ and $C \varnothing$ should be evaluated at the overall


Figure 14. Percent Unburned Propellant as a Function of Distance from Injector Face
mixture ratio. The variable DELP will scale the amplitude of the oscillatory waves solved for in subroutine CHAMDY. A value of 0.1 is recommended.

The first variable input on the fifth card of the MAIN program control section input is NRPDT. The absolute value of this variable controls the maximum number of frequency solutions the program will try to find. The program begins its search at the frequency implied by the input variable QMEGAR. It will stop looking once it has found |NRDD| solutions or if the frequency is above FRQMAX. The variables IWRT and IWSKP are the next two variables input on this card. They control the amount of intermediate output one desires. Their exact function is described in the Program Operation Section of this manual.

The last three variables input on the fifth card are KNTMX, KNTRMX, and KNTSMX. Their meaning is explained in Table 2 . Recommended values for these variables are 50,100 , and 20 respectively.

The sixth card of the MAIN program control section input contains the variables QMEGAR, QMEGAI, FRQMAX, DELFRQ, DELMX, and CTEST. The first two of these variables specify the starting guess in the w plane for solution. No solutions will be sought below the frequency implied by OMEGAR. Note that 9 MEGAR and QMEGAI can be input as the frequency in Hertz times $2 \pi$ and the growth coefficient or the frequency in Hertz and the decrement depending upon whether NROOT was input as positive or negative. The variable FRQMAX, as mentioned earlier, is the maximum frequency allowed for the search algorithm to find solutions. The variable DELFRQ specifies the "stepsize" used by the search algorithm. Since there are sometimes many areas in the $\omega$ plane which contain solutions, a fairly small stepsize is recommended, e.g., 5 Hz . The variable DELMX controls the maximum allowable change in the growth coefficient during successive iterations to minimize the error in the nozzle admittance boundary condition as a function of the imaginary part of $\omega$. A recommended value for this variable is $50 \mathrm{sec}^{-1}$. The last variable on this card, CTEST, is the upper bound on the condition number of the transposed Jacobian used to solve the nozzle admittance boundary condition. If the calculated condition number exceeds CTEST, then the search algorithm assumes that there is a singularity near the current value of $\omega$ and hence, does not proceed further in that area to try and find a solution. A value of 50 to 80 is recommended.

The seventh card of the MAIN Program Control Section input contains error tolerances used in solving the nozzle admittance boundary condition. The first two, EPSF and EPSX, are used during the search algorithm and should be fairly large, e.g. 0.01 to 0.05 ( $1 \%$ to $5 \%$ error). The last two, EPSFS and EPSXS, control the final stages of iteration and should be fairly tight, e.g. 0.0005 (0.05\%).

The eighth card of this control section contains the variables PC, MBOXI, and MBFUI. The first is the steady state chamber pressure, in PSIA, and the next two are the oxidizer and fuel injection flowrates, respectively ( $1 \mathrm{bm} / \mathrm{sec}$ ).

The last card, card number 9, in the MAIN Program Control Section input should be input only if the variable INPN@Z is less than or equal to three. If this is the case, the program needs to know the size and range of the frequency table it will use to generate tables for linear interpolation as discussed in the section describing the input variables INPHYD, INPCDM, and INPN@Z. The input variables on this ninth card are NFREQT, FREQMI, and FREQMX. Their meaning is described in Table 2.

The next control section to read data after the MAIN program is the Nozzle . Admittance Program. The data for this control section should be input only if INPN $\varnothing Z \leq 3$. Otherwise, the information is not needed since the nozzle admittance information will be on tape ITAPN. Even when INPN@Z $\leq 3$, there is only one card input. This card contains information describing the nozzle geometry. Refer to Table 2 to determine the meaning of the variables on this card. Figure 15 shows exactly what portion of the nozzle each variable is applicable to.

The next control section that requires data is the Hydrodynamics Program section. This control section uses namelist input. The data for this control section are only input if INPHYD $\leq 2$. Otherwise, the hydrodynamics information will be on tape ITAPH. The Hydrodynamic Input section (page 102) describes the meaning of the variables to be input for the control section.

The last $\overline{\text { control }}$ section to require input data is the Combustion Dynamics Program. This input is contained on five cards. It should be omitted if INPC\&M is greater than or equal to three, since then the combustion dynamics information will reside on tape ITAPC.


Figure 15. Chamber and Nozzle Geometry
$-$

The first two cards for this section contain information specific to the oxidizer; the next two cards contain information specific to the fuel; the last card contains information specific to the combustion gas. Much of the information required in this section is obtainable from the output of the DER program (Ref. 11). This program solves for the steady state behavior of the chamber under consideration. The single stream tube option of the DER program is sufficient for this application.

As mentioned above, the variables input on the first two cards of the Combustion Dynamics Program control section are applicable to the oxidizer. These variables
 XIMPDX. The first variable XKøX, is the Klystron constant for the oxidizer jet. This variable controls the distance downstream from the injector face that the Klystron effect will be allowed to occur. The exact method of calculating XKøX has not been determined but it is recommended that a distance corresponding to approximately $45 \%$ of the oxidizer vaporized be used.

The next variable, TAUBOX, is the steady state oxidizer vaporization time delay. Figure 14 gives the percent unburned for the sample input case of liquid oxidizer and fuel remaining in the chamber as a function of axial distance plotted on semi log graph paper. This output was derived from a DER computer run. The reciprocal of the average slope of the oxidizer curve during the initial burning phases should be taken as the oxidizer vaporization distance delay. For Figure 14, this is 2.318 inches. The time delay is obtained by dividing this by the average injector velocity, which, for the sample case presented in Figure 14 is $28.859 \mathrm{ft} / \mathrm{sec}$. The result is 0.00669 seconds. The next variable, VBøX, is the average oxidizer injection velocity. This is also given by the DER program from injector orifice and steady state $\Delta P$ considerations.

The variable DELHDX is the pseudo energy of the oxidizer droplets and is discussed at the end of this section. The variable TDRAGD is the steady-state oxidizer drag time delay and a value of zero is currently recommended. If this variable is different from zero, the computed pressure and velocity profiles calculated in the chamber are unrealistic (see Conclusions and Recommendations in Ref. 1).

ADVDX and ADDØX are the velocity and orifice diameter exponents for the oxidizer atomization process, i.e.,

$$
\begin{equation*}
\bar{D}_{o x}-V^{A D V \emptyset X} D_{\text {orif }} A \tag{216}
\end{equation*}
$$

These variables are obtained from cold flow tests and for like-doublets -0.75 and 0.57 are recommended. It should be noted that these variables are on different input cards. The next input variable is DELVQX which is the steadystate velocity difference between the oxidizer droplets and the combustion gas stream normalized to the sonic velocity. Since the vaporization rate is highest near the injector face, this variable should be evaluated near the injector face. Because the gas and the droplet velocities are approximately equal to each other at this location, a value of 0.01 is recommended based on turbulence levels in a rocket chamber (Ref. 26).

The next variable, NUB $\emptyset X$, is the steady state oxidizer Nusselt number used in the vaporization expression. This variable should be computed from the relation

$$
\begin{equation*}
N_{u_{o x}}=8 / 6 \frac{\bar{D}_{o x}^{2}}{k_{o x}^{\prime} \bar{\tau}_{o x}} \tag{217}
\end{equation*}
$$

where $\bar{D}_{o x}$ is the average oxidizer droplet diameter near the injector face, $k_{o x}^{\prime}$ is the vaporization coefficient used in the $k^{\prime}$-model evaluated at a mixture ratio near the injector face and $\bar{\tau}_{o x}$ is the oxidizer time delay. Both $\bar{D}_{o x}$ and $k_{o x}^{\prime}$ are obtainable from a DER computer run. For the sample case input, "near the injector face" was taken as the first axial step printed by the DER program.

The variable DTOXDM is the change in the oxidizer vaporization time delay with respect to mixture ratio. At the present time, a value of zero is recommended based on model verification cases (Ref. 1).

The last oxidizer variable input is XIMPDX. This is the impingement point in inches for the oxidizer jets.

The next two cards contain data for the fuel. These data are obtained the same way as they were for the oxidizer and are input in the same order.

The last card for this control section contains combustion gas data. The first variable input on this card, MWG, is the steady state molecular weight evaluated at the overall mixture ratio. This may be obtained from tables of molecular weight versus mixture ratio printed by the DER program. The next variable, CS, is the characteristic velocity evaluated at the overall mixture ratio. This is also obtainable from DER table output. The last three variables on this card are DRGDMR, DCSDMR, and DHDMR. These are the partial derivatives with respect to mixture ratio of the gas constant, the characteristic velocity, and the reference enthalpy respectively. DRGDMR and DCSDMR can be calculated from equilibrium calculations but a value of zero is recommended for DCSDMR based on model verification cases (Ref. 1).

The variable DHDMR, and also the variables DELH $\emptyset X$ and DELHFU, is calculated by curvfitting the steady-state energy equation with stagnation temperature/mixture ratio data calculated by an equilibrium program. The steady-state energy equation can be written as

$$
\begin{align*}
& \left(\frac{\gamma_{\phi}}{\gamma_{\phi}-1}\right) R_{\phi}\left\{T\left[1+\frac{1}{R_{\phi}}\left(\frac{\partial R}{\partial M R}\right)_{\phi}\left(M R_{-M R_{\phi}}\right)\right]\right. \\
& \left.-\left(1+M R_{\phi}\right) T_{\phi}\right\}=\left(M R_{-M R_{\phi}}\right)\left(\Delta h_{o x}\right) \\
& -\left[M R^{(1+M R)}-M R_{\phi}\left(1+M R_{\phi}\right)\right]\left(\frac{\partial h}{\partial M R}\right)_{\phi}  \tag{218}\\
& \text { where } \\
& \qquad \begin{aligned}
\left(\Delta h_{f u}\right) & =\left(\frac{\gamma_{\phi}}{\gamma_{\phi}-1}\right)\left(1+M R_{\phi}\right) R_{\phi} T_{\phi}-M R_{\phi}\left(\Delta h_{o x}\right) \\
& +\left(M R_{\phi}\right)\left(1+M R_{\phi}\right)\left(\frac{\partial h}{\partial M R}\right)_{\phi}
\end{aligned} \tag{219}
\end{align*}
$$

and the subscript $\phi$ indicates that the variable is to be evaluated based on the overall mixture ratio.

HYDRODYNAMIC INPUT

This section describes data needed by the hydrodynamics subroutine, HYDRDY, to simulate the various feed system components. It is assumed that the feed system being modeled has been laid out on the generalized feed system schematic of Fig. 4 with an appropriate segment number assigned to each feed system component (or combination of components).

## Basic Feed-System Data

To describe the basic feed system it is necessary to know the length, area, resistance and wall compliance of each of the numbered segments of Fig. 4 which are being used. Also, the acoustic velocity and density of the fluid in each segment must be known. If there is dissolved an entrained gas in the system, then a preliminary calculation must be made for each feed system section to account for the effect of the gas on the fluid acoustic velocity.

Specific parameters required for the numbered segments are:
A - Segment cross-sectional area - in. ${ }^{2}$
CW - Segment wall compliance ( $\Delta V / \Delta P / V-i n .{ }^{2} / 1 \mathrm{~b}$
L - Segment length - in.
$R$ - Segment linearized hydraulic resistance $(\Delta P / \dot{W})-\sec /$ in. ${ }^{2}$
$V$ - Segment fluid acoustic velocity - in./sec
RHOL - Segment fluid density - 1b/in. ${ }^{3}$
Valves, Fittings, Orifices, Screens, Flowmeters, etc.
These components can each be described in the model simply as lumped resistance at the end of a line segment. Rather than using all the attributes of one of the numbered segments (length, area, wall compliance, etc.) for one of these "resistance only" components, it is suggested that its resistance merely be added to that of the adjacent upstream pipe segment. The combination can-then be entered as one of the numbered segments with the length, area and wall compliance values being primarily those of the pipe segment.

## Accumulators

A feed system accumulator can be represented as one of the side branch lines of the Fig. 4 schematic by specifying an appropriate length, area, acoustic velocity and fluid density for the fluid volume of the accumulator and also specifying an appropriate connecting resistance. The spring rate of the accumulator piston can be specified in terms of the segment wall compliance value.

## Propellant Tanks

A large tank will have the effect of constant fluid pressure at its outlet and can be represented simply as the input to segment number 1 , 15, or 22 . No descriptive parameters are required for these inputs. Small tanks can be represented as one of the side branch lines in a manner similar to an accumulator. Ullage volume in a small tank is represented by a reduced value for the segment acoustic velocity.

## Cavitating Venturies

The steady-state effect of a cavitating venturi is to have constant flow through the venturi as a function of variations in downstream pressure. For an oscillatory system, the vapor bubble downstream of the venturi throat makes the venturi look like a constant pressure boundary for small amplitude oscillations. Tosimulate this effect the effective acoustic velocity for the segment downstream of the cavitating venturi should be made very small ( $=10$ inches $/ \mathrm{sec}$ would be appropriate). The steady-state hydraulic resistance of the cavitating venturi can be lumped with that of the upstream pipe segment as described above for valves, fittings, etc.

## Regeneratively-Cooled Thrust Chamber

Regeneratively-cooled thrust chamber jackets can be represented as one or more of the numbered Fig. 4 segments. Because in most thrust chambers the coolant flow area changes continuously with length, as many seqments as possible should be devoted to the jacket so as to improve the simulation
accuracy. The fluid temperature also may change significantly along the chamber. length thereby necessitating the use of several segments with different acoustic velocities to achieve accurate simulation.

## Lines, Ducts, Bends, Bellows, and Flex Lines

These components are described in the model in terms of the basic numbered segment input parameters of length, area, fluid acoustic velocity, fluid density, wall compliance and linearized hydraulic resistance. For a duct or line of constant diameter, $D$, wall thickness, $h$, and wall material bulk modulus, $E$, the program input wall compliance value, $C W$, is simply D/LE. For a bellows or flux line of volume, $V$, the wall compliance value, $C W$, may be calculated from $\Delta V / \Delta P / V$ where $\Delta V / \Delta P$ is the volume change per psi at the operating pressure.

## Injectors

The hydrodynamics subprogram employs a separate set of equations to describe the hydrodynamic characteristics of the two injectors in the Fig. 4 generalized feed system schematic. The specific input parameters for the two injectors are the volume, linearized hydraulic resistance, orifice inertance (1/A.g), fluid acoustic velocity, and a structural parameter defining the change in injector volume per psi of injector $\Delta P$. In terms of the program variable names the required injector parameters are:


## Tees, Splitters and Capped Lines

No provision is made in the model for completely generalized input of tees and branched lines. However, a system of considerable complexity can be modeled by laying out an appropriate flow path on the generalized Fig. 4 schematic. For example, a feed system with up to seven side branch lines can be simulated by choosing the main flow path through segments 1, 3, 4, 14, 17, 21, 25, 26, 28, 29, 30 and the" $F$ "injector in series.

## Input Variables

Data input to the hydrodynamics subprogram is from three sources: (1) Via the argument list in the CALL HYDRDY statement, (2) Through labeled common block/CDMTAP/and (3) By use of the NAMELIST data read routine.

The argument list variables, in order, are:
IR - Data read flag - dimensionless
INPHYD- Program function flag - dimensionless
FRE - Single frequency for feed system frequency response calculation - Hz
GIND - Output value of oscillatory oxidizer injector flowrate for input frequency FRE - dimensionless
GINF - Output value of oscillatory fuel injector flowrate for input frequency FRE - dimensionless
PCIN - Injector end thrust chamber pressure - psia
WØIN - Steady-state oxidizer injector flowrate - lb/sec
WFIN - Steady-state fuel injector flowrate - lb/sec
Several HYDRDY input variables are transmitted via labeled common block/ CQNTAP/. ITAPH is the logical unit number of the output device on which subroutine HYDRY tabulates output values of oscillatory injection flowrates for the specific frequencies (up to 100 separate values) given in the array FREQT in common block/CONTAP/. The value of the FREQT in common block/CØNTAP/ is the total number of frequencies stored in the array FREQT.

All other data required by HYDRDY, including all the feed system descriptive data, is read in by use of the NAMELIST routine. The local rules for using this routine should be checked to verify that the correct card or terminal format is being used. Table 3 shows a list of allowable F $\emptyset R T R A N$ names, the maximum values of subscripts, and a definition of the names. The name of the NAMELIST block is HYD.

TABLE 3. NAME LIST/HYD/DATA INPUT NAMES

| NAME | DEFINITION |
| :---: | :---: |
| A(30) | Array containing segment cross-sectional area values |
| $\mathrm{CW}(30)$ | Array containing segment wall compliance values |
| FREQ | Frequency at which HYDRDY will compute feed system frequency response if INPHYD < 1 |
| FREQT (100) | Array containing frequencies at which HYDRDY will compute feed system frequency response of INPHYD > 1 . |
| ICRT | ICRT $=1$; injector flowrate gain and phase will be plotted vs frequency. ICRT $=0$; no plot (default) |
| ID | Dummy name to allow for data card sequence numbers |
| IH(126) | Array containing control flags used by subroutine FREQD. Can be used to obtain printouts and plots of feed system frequency response for other variables in addition to the injector flowrates. |
| IRFLAG | $\begin{aligned} & \text { IRFLAG }=0 \text {; read data from unit } 5 \text { (default) } \\ & \text { IRFLAG } \neq 0 \text {; read data from unit ITERM } \end{aligned}$ |
| ITERM | ```ITERM = 0; no terminal data input (default) ITERM > 0; read data from terminal (unit ITERM)``` |
| ITYPE | ITYPE $=1$; both oxidizer and fuel feed systems are modeled simultaneously (default) <br> ITYPE = 2; oxidizer feed system modeled on first pass of frequency response routine: fuel feed system modeled on second pass. |
| IWRITE | IURITE $=0$; HYDRDY input printed on unit 6 (default) <br> IWRITE = 2; extensive printout of all HYDRDY input, intermediate output and final output on unit 6 IWRITE = 1; printout of HYDRDY input and final output on unit 6 |
| KF | Injector face flexibility constant for "F" injector |
| $K \emptyset$ | Injector face flexibility constant for "中" injector. |
| L(30) - | Array containing segment length values |

TABLE 3．（Concluded）

| NAME | DEFINITION |
| :---: | :---: |
| QMI | Lowest frequency for injector flowrate gain／phase plot； default to FREQT（1）if not entered |
| QMFL | Highest frequency for injector flowrate gain／phase plot； default to FREQT（NFREQT）if not entered |
| $R(30)$ | Array containing segment hydraulic resistance values |
| RF | Hydraulic resistance for＂F＂injector |
| RHOL（30） | Array containing segment propellant density values |
| $R D$ | Hydraulic resistance for＂中＂injector |
| $V(30)$ | Array containing segment acoustic velocity values |
| VF | Acoustic velocity for＂F＂injector |
| vo | Acoustic velocity for＂中＂injector |
| VØLF | Volume of＂F＂injector |
| VOLD | Volume of＂中＂injector |
| ZF | Inertance of＂F＂injector |
| ZØ | Inertance of＂中＂injector |

Required input in the NAMELIST/HYD/data is a value of $A, C W, L, R, R H \nmid L$, and $V$ (see Table 3 for descriptions) for each numbered segment being included in the feed system. Values of KF and/or KD, RF and/or RD, VF and/ or V $V$, V $\emptyset L F$ and/or $V \emptyset L \emptyset$ and $Z F$ and/or $Z \emptyset$ are also required. It should be noted that, when possible, both oxidizer and fuel feed systems should simultaneously be laid out on the Fig. 4 schematic with the injector labeled " $\emptyset$ " being used for the oxidizer side (data values $K \emptyset, R \emptyset, V \emptyset, V \emptyset L \emptyset$ and $Z \emptyset$ ) and the injector labeled "F" being used for the fuel side (data values, KF, $R F, V F, V \emptyset L F$ and $Z F$ ). If this can be done, a single call to subroutine FRESP will generate frequency response data for both oxidizer and fuel feed systems. If feed system complexity requires that the oxidizer and fuel feed systems be laid out separately on the Fig. 4 schematic, then two sets of input data must be read and subroutine HYDRDY must call FRESP twice - first for the oxidizer feed system calculations and second for the fuel feed system calculations. To specify this option, variable ITYPE must be set equal to 2.

Variable INPHYD in the HYDRDY argument list controls the HYDRDY calculation process. If INPHYD $\leq 1$ the oscillation injection flowrates are calculated for a single frequency, specified by variable FRE in the HYDRDY argument list. If INPHYD > 1, HYDRDY calculates oscillatory injector flowrates for the number of frequencies, NFREQT, which are contained in array FREQT. Both NFREQT and the array $\operatorname{FREQT}$ are stored in labeled common block/C@MTAP/ prior to calling HYDRDY.

NAMELIST variable ICRT controls the option for generating CRT plots of the oscillatory injection flowrate gains and phase values as a function of frequency. If ICRT $=0$ (the default value) no plots are made. If ICRT $\geq 1$ plot output is written to the output file named SYSCRT.

NAMELIST variable, ID, is a dummy name which can be used on each input card to provide an identification number field without violating the NAMELIST restriction that the entire card is read as data. For example, $I D=00000010$ could be in columns $70-80$ of a HYDRDY data card and ID $=00000020$ in columns 70-80 of the next card. The NAMELIST routine would then interpret each card's sequence number as a new value for the dummy variable, ID. The value of ID is not used in any way by subroutine HYDRDY.

NAMELIST variable, ITYPE, is used to indicate to HYDRDY the format of the feed system modeling. If ITYPE $=1$ (the default value) it is assumed that both oxidizer and fuel feed systems are modeled simultaneously with only one set of HYDRDY input values (for the 30 segments and 2 injectors of the Fig. 4 schematic). If ITYPE $=2$, HYDRDY will send two consecutive sets of input data; the first set will be assigned to the oxidizer feed system and the second set to the fuel feed system. For either value of ITYPE the program will assume that the injector labeled " $\emptyset$ " on Fig. 4 is the oxidizer flow outlet and the injector labeled " $F$ " is the fuel flow outlet. Therefore, this convention must be followed when laying out the feed system model.

NAMELIST variables IRFLAG and ITERM are optional HYDRDY inputs which indicate that data input will be provided from a timesharing terminal. If IRFLAG $=0$ (the default value), input data will be read only from FORTRAN logic unit number, ITERM. It should be noted that the default values for IRFLAG and ITERM are set up so that the initial data input will always be card input on unit 5 .

After reading the initial NAMELIST data on unit 5, HRDRDY checks the value of ITERM and, if non-zero, proceeds to read additional first case data from the terminal on unit ITERM. Thus, the first case card NAMELIST input could consist of the single item ITERM $=N$, where $N$ is the terminal logical unit number. If terminal input only is desired, block data program/F/ can be recompiled with the IRFLAG default value changed from 0 to 1 and the ITERM default value changed from 0 to the desired unit number.

NAMELIST variable IWRITE (main program control variable IPRHYD) controls the printed output from HYDRDY. If IWRITE $=0$ (the default value), only the NAMELIST input to HYDRDY is printed on logical unit 6. If IWRITE < 0 the NAMELIST input is printed and the normal HYDRDY output is printed as well as being saved on an output device in binary form. If IWRITE $=1$ both HYDRY input and normal output are printed. If IWRITE > 1 extensive printouts of subroutine FRESP intermediate calculations are printed in addition to the normal HYDRDY input and output.

## PROGRAM OUTPUT

The output of the FSCSM computer program is provided as the usual tabular printout. A sample case is included in Appendix $E$ which corresponds to the input dataset listed in Appendix $D$. As is also mentioned in the Program Operation section of this manual, the input case listed in Appendix $D$ consists of two cases being run back to back. The output from the first case is given in Appendix E from pages E-2 through E-14. The first page of output consists of a title page identifying the current version of the FSCSM comnuter program. The input data are printed out as they are read in. This permits both a full documentation of the computer run conditions for later analysis as well as a convenient method to check for input errors if unusual results are calculated. Page E-3 of the listing in Appendix E gives the two alphanumeric cards identifying the case at the top of the page riaht under the proaram title. Subsenuent to these two cards, the information on the cards read in by the main control section and the nozzle admittance control section are printed out. After reading and writing these cards, and since INPNQZ $=3$, the program proceeds to the nozzle admittance table calculations. Information pertinent to these calculations is printed on page E-4. The freauency table goes from 150 Hz to 400 Hz as specified by the input variables FREOMI and FREOMX.

Since INPHYD $=2$, the program proceeds to the hydrodynamic subroutines right after the nozzle admittance calculations. Input for this routine is in the form of NAMELIST data. The NAMELIST is output on pages E-5 and $E-6$. A printout of the feed system response table computed by subroutine HYDRDY and saved on file ITAPH is given on page E-7.

The next set of input required is used in subroutine CDMBDY and STEADY. This is output on page E-8. The steady-state profiles are then computed and printed on pages E-9 and E-10.

The program then begins its search for solutions to the nozzle admittance boundary condition. The first one it finds is at 210.42 Hz . The program then outputs the combustion dynamic coefficients, the frequency and decrement, and the feed system response for this solution on page E-14. On page E-12, the oscillatory profiles correspond to this solution are given. The program then proceeds to the next case.

Since the second case does not generate the data on files ITAPN and ITAPH (it only reads this information), these tables are not printed. The first page of output, page E- 15, in Appendix E, consists entirely of the data read by the Main Control Section and the Combustion Dynamics Control Section. Since the STEADY Contrcl Section print code is zero (IPRSTE $=0$ ), the program skips over the steadv-state output (although of course, it still computes it) and proceeds directly to the section which solves the nozzle admittance boundary condition. The first root it finds above the input frequency of 265 Hz (given by the variable @MEGAR), is at 280.62 Hz . It prints out the frequency, decrement, nozzle admittances, and feed system response for this solution. Output of the combustion coefficients and oscillatory profiles is bypassed because the input flags IPRC§M and IPRCHM were set to zero.

The final page of output is the title page. This indicates normal termination of the job.

## PROGRAM OPERATION

The FSCSM computer program is designed to read in an input case sequentially, perform the calculations for that case, and output the results. The program then transfers back to its beginning to read in the next case. In this manner, running jobs back-to-back is quite straightforward. The sample input case listed in Appendix $D$ provides an example of two such cases run back to back. The first case, given by the first 24 cards, is run with no prior information residing on the hydrodynamic feed system, the combustion dynamics, or the nozzle admittance datasets. Since INPHYD $=2$ and INPNOZ $=3$, this case generates tables of the hydrodynamic feed system response and nozzle admittance versus frequency and saves them on files ITAPH and ITAPN, respectively. The subseauent case (the last 13 cards in Appendix D ) will use the information stored on these datasets. Although these two cases were run back to back, this was by no means necessary. The second case is self-contained and could be submitted separately. Of course, if this were the situation, the user must be sure there are datasets on files ITAPH and ITAPN which are adplicable to that second case.

For the sample dataset run, the two input cases found solutions to the nozzle admittance boundary equation at 210.42 Hz and 280.62 Hz , respectively. If there are no other solutions between these two frequencies, the same effect could have been obtained by setting the input variable NROQT equal to -2 for the first case instead of -1 . The program would have then looked for the first two roots above the input frequency 190 Hz ( $9 M E G A R$ ) and found both solutions automatically. The second case would not be input for this situation.

Program Size, Overlay Structure, and Timing
Without overlay, the FSCSM computer program load module reauires 262.4 K Bytes of computer storage on the IBM 370 Model 165 computer. This storage does not count the buffers needed for input/output. If one allocates a 1 K Byte of buffer size for each of the three date sets used to store the feed system, combustion dynamic, and nozzle admittance data (which are all unformatted input/output), uses two buffers for each data set, and adds in the buffer reauirements for his card input, printed output, and CRT output, then the total buffer space should be well under 10 K bytes on a $370 / 165$ computer. With the overlay structure specified in Fig. 16 , the total program reouirement is 220 K bytes of storage on an IBM 370/165 computer, including two buffers for each of the three unformatted datasets at 1 K bytes each.

Computer run time has only been checked for an IBM 370/165 computer where the subroutines were compiled using the IEM procedure AFØRTRAN with the optimizing parameter, ØPT, equal to one. For this situation, each iteration during the search algorithm portion of the program (when ISCNT equals one or four) averaged 3.7 CPU seconds. When ISCNT=5, each iteration is about twice as fast. For the cases run during model verification a five Hz step size for the search algorithim was used ( $D E L F R O=5$ ). For these cases, each solution to the nozzle admittance boundary equations averaged 0.85 minutes of CPU.

## Program Input/Output Dataset File Information

The case input dataset file number used by the FSCSM computer prooram is 5 . The printed output dataset file number is 6 . There are three auxiliary files used by the program. These are specified by the input parameters, ITAPC, ITAPH, and ITAPN, corresponding to the combustion dynamics, hydrodynamic feed system, and nozzle admittance datasets. Control of the reading from or writing on to these respective datasets is specified by the three input flags INPCOM, INPHYD, and INPNQZ. The
program uses unformatted input/output statements for transmitting information to and from these datasets. A convenient blocksize to use is 1 K bytes.


Figure 16. FSCSM Program Overlay Structure

## Diagnostics

The Feed System Coupled Stability Model computer program has been designed to operate as straightforward as possible with a minimum amount of user interaction for each case being run. There may be times however, when the program's results appear questionable or the algorithm used to find solutions in the frequency space to the nozzle admittance boundary equation runs into difficulty or does not find solutions that were expected. Many diagnostic messages are coded into the program to warn the user of such problems. Also, there are certain dump codes which enable the user to obtain intermediate output in order to debug most problems that may arise.

One of these dump codes is the variable IWSKP. When this is set to zero, no intermediate output is obtained. When it equals one, a certain amount of limited output will be generated. This output comes in two forms depending upon whether or not the program is within its search algorithm portion or its two-dimensional secant portion. For the first case, the variable ISCNT has the value of one or four. In the second case it has the value 5 . When ISCNT equals one or four and IWSKP equals one, subroutine SOLVW will print the following variables in the order given: the iteration counter (KNTR), the control flag (ISCNT), the counter (KSCNT4), the current values of omega $(\omega)$, the upstream and downstream nozzle admittances (CNQZA and N@ZA), the absolute value of the error in the nozzle admittance equation ( $H N$ ), the value of the test function (FTST2), and the determinant and condition number of the transposed Jacobian (DET2 and COND2). This printing will be performed every time the real part of $\omega$ is incremented by $2 \pi$ *DELFRQ right after the imaginary part of $\omega$ has been chosen to minimize $|F|$, the absolute value of the error in the nozzle admittance boundary equation. The user can employ this output to determine if there is a region in the $\omega$-plane where a possible solution may have existed (e.g., the error became small but the test function did not change sign). He can then rerun his case while taking smaller frequency steps through the narrowed range where he suspects a solution may exist. Also, the program may jump over a solution if there is a singularity within DELFRQ of that solution. If this is the case, the program will sense the singularity and not proceed any further in its search in that range. Rerunning the case with a smaller value of DELFRQ will solve this problem.

When ISCNT equals five and IWSKP equals one, subroutine SøLVW prints after each two-dimensional secant method, these variables in the following order: ISCNT, KNTS, ØMEGA, CNØZA, NØZA, and HN. Although it did not happen for any of the cases performed during the checkout of the computer model, the two-dimensional secant method may diverge. The above computer output would be useful in determining the cause of the problem.

When the variable IWSKP equals two, all the above output is printed plus the following:

1. When ISCNT equals one or four, intermediate output is obtained during the iterations to minimize $|F N|$ with respect to Imag $(\omega)$. For this case, one obtains the variables KNT, IER, X1, X2, F1, F2, ØMEGA, FN, GN, and HN. These are all described in Appendix A. This output may be useful in seeing how the error is changing as a function of the decrement when the real part of the frequency is held fixed. Further, when ISCNT equals one or four, the variables FN, DFRDX, DFIDX, DFRDY, and DFIDY are printed along with the output obtained when IWSKP $=1$.
2. When ISCNT equals five and IWSKP equals two, one obtains the output for the case IWSKP equals one for the two-dimensional secant method plus the following variables in order: XRI, XII, FRI, FII, XR2, XI2, FR2, FI2, XR3, XI3, FR3, FI3, XR4, XI4, FR4, FI4, and FN. These variables correspond to the current values of $\omega$ and FN being used by the two-dimensional secant method. They can be used to trace which points the algorithm is replacing as the iteration proceeds as well as how the error is behaving.

Another input variable which controls intermediate output is the FDRTRAN variable IWRT. This variable is input as zero, no intermediate output is obtained. If this variable is input as a positive number, then intermediate output from subroutine CHAMDY is obtained. This output consists of the oscillatory profiles for the variables $P$ (pressure), RH $\emptyset$ (density), MR (mixture
ratio), and $T$ (temperature) along with the current value of the complex frequency, w. This output is printed everytime subroutine CHAMDY is entered.

The diagnostic messages that are coded within the FSCSM computer program may be printed for several reasons.

Within subroutine SøLVW, there are three diagnostic messages coded which will appear when certain iteration counters are exceeded. The first is

WARNING, POSSIBLE ROOT IN FREQUENCY RANGE: --

When this message appears, it means that a potential root was bracketed but the error did not decrease sufficiently within ten additional iterations to warrant the program proceeding further with its search in that range. Moreover, the determinant did not change sign and the condition number remained less than CTEST in that range. Rerunning the case over the specified frequency range given in the message with IWSKP equal to one or two may prove beneficial if the user suspects there may be an actual solution $i_{n}$ that range.

The second diagnostic message printed by subroutine SQLVW is
**** UNABLE TØ FIND RØØT FØR IMAG PART $9 F F$ ****

Along with this message, the variables X1, F1, K2, F2, X3, F3, ANS, FANS, KNT, IER, and QMEGA are printed in the order listed. When this message appears, it means the algorithm to minimize $|F N|$ with respect to Imag ( $\omega$ ) has failed. If this messace appears, it usually means something is wron with the input parameters. The only occurrence that the proarammers are aware of when this is not the case is when the error attains a minimum as $\mid$ Imag $(\omega)\rfloor \rightarrow \infty$. Since this happens only in the most extraordinary situations, the procedure should be to rerun the case and not include the frequency range where that anomally is occurring.

The third diagnostic produced by subroutine S甲LVW is **** EXCEEDED CONVERGENCE LIMIT ****

Along with this message, the variables IER, KNTS, KNTR, ISCNT, XRI, XII, FR1, FII, ......, XR4, XI4, FR4, FI4, are printed.

This message will appear if KNTR is greater than KNTRMX or KNTS is areater than KNTSMX. In the former case, the usual error is that the user input too small a DELFRO to cover the range between sclutions to the nozzle admittance boundary equation in KNTRMX stens or too small a KNTRMX to allow that range to be covered in steps of length DELFRN.

In the case where KNTS is areater than KNTSMX, it would probably mean that the two-dimensional secant method is diverging. The job should be rerun with IWSKP equal to one or two to obtain more information concerning the problem.

There is also a diagnostic message printed from subroutine CØMMAT. This is the subroutine that solves the four by four system of linear equations for subroutine CHAMDY. If any of the diagonal elements of the associated matrix are zero, then the message
***** DIVIDE CHECK IN CQMMAT *****
will appear along with a printout of the row number of the zero diagonal as well as the complex matrix being solved. If this error messade appears, then there must be something very wrong with the case being run, e.g, the innut data is in error, or a dimension has been exceeded. One should recheck his input carefully and then, if necessary, rerun the case with IWRT equal to one and IWSKP equal to one or two.

Two similar messages as the one above are printed by subroutine CDGAEL. The first of these messages is
****ERROR IN CQGAEL SUBROUTINE, J AND JMAX EQUAL, RESPECTIVELY****
and the second is
****MATRIX IS SINGULAR, EXIT FROM CØGAEL. THE PIVOT ELEMENTS ARE...****
The reasons for these errors are similar to the CDMMAT error message. Conversion to UNIVAC

The following cards must be changed to execute the FSCSM program on a UNIVAC computer (see Appendix $C$ for code listing):

1. Change C@MPLEX*16...to C@MPLEX....

| Routine | Card Number |
| :---: | :---: |
| CHAMDY | 80 |
| COMMAT | 150 |

2. In subroutine CDMMAT, change CDABS to CABS on card No. 24.
3. In subroutine TDPLøT, replace card 11310 with

31 CØNTINUE..... 0011310
4. In subroutine NØZADM, replace card 1730 with

8 NØZA $=\operatorname{CMPLX}(S Y R, S Y I) \ldots 00001730$
5. In the main program, change ATAND( ) to 57.296*ATAN( ) on card numbers $3370,3410,3450$, and 3490.

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APPENDIX A
FORTRAN SYMBOL TABLE

R-9808/A-1

| FORTRAN VARIABLE NAME | $\begin{aligned} & \text { ENGINEERING } \\ & \text { VARIABLE } \\ & \text { SYMBOL } \\ & \hline \end{aligned}$ | TYPE | $\begin{aligned} & \text { CONTROL } \\ & \text { SECTION } \\ & \hline \end{aligned}$ | DESCRIPTION |
| :---: | :---: | :---: | :---: | :---: |
| A(100) | A | R | CDMARE | Axial distance array, $X(I)=X O+(X-1) * D E L X, m$ |
| ADDFU | ${ }^{\text {a }}$ fu | R | CDMCBM | Fuel liquid jet diameter exponent, unitless |
| ADD@X | ${ }^{\square}{ }_{\phi x}$ | R | СøмСВM | Oxidizer liquid jet diameter exponent, unitless |
| ADVFU | $\mathrm{b}_{\mathrm{fu}}$ | R | CDMCBM | Velocity exponent for fuel atomization process, unitless |
| ADVgX | $b_{\phi x}$ | R | CDMCBM | Velocity exponent for the oxidizer atomization |
| AINJ | $A_{i n j}$ | R | CPMARE | Cross sectional area at injector face, $\mathrm{m}^{2}$ |
| $\operatorname{AMA}(4,5)$ | -- | C | CHAMDY | Array used to store coefficients of finite difference equations approximating the oscillatory differential equations |
| ANGLEX | -- | R | COMN ${ }^{\text {ch }}$ | Nozzle convergence half angle, degrees (see Fig. 15) |
| ANS | -- | R | SøLVW | Solution to minimization of error in nozzle admittance boundary equations. Returned to SØLVW from subroutine ZERD. |
| CFU1,...., CFU16 | , ${ }^{c} 1_{\mathrm{fu}}, \ldots$, | C | CDMCBM | Fuel combustion coefficients, unitless |
| CMA (4) | ${ }^{c}{ }_{16} \mathrm{fu}$ | C | CHAMDY | Right hand side of finite difference equation approximating the oscillatory differential equations. In equivalence with CMA $(1,5)$. |
| CN@ZA | $\mathrm{A}_{\mathrm{H}_{u}}$ | C | C®MCHM | Upstream nozzle admittance |
| CD | ${ }^{\text {c }}$ | R | C®MCBM | Sonic velocity evaluated at overall mixture ratio, $\mathrm{m} / \mathrm{sec}(\mathrm{ft} / \mathrm{sec})$ |
| COND | -- | R | S®LVW | Condition number of the transpose of the Jacobian of the difference between the upstream and downstream nozzle admittances with respect to $\omega$. |


| FORTRAN VARIABLE $\qquad$ | ENGINEERING VARIABLE SYMBOL | TYPE | $\begin{aligned} & \text { CONTROL } \\ & \text { SECTION } \\ & \hline \end{aligned}$ | DESCRIPTION |
| :---: | :---: | :---: | :---: | :---: |
| COND2 | -- | R | SØLVW | Condition number of the transpose of the Jacobian of the difference between the upstream and downstream nozzle admittances with respect to $\omega$. |
| $\begin{aligned} & \operatorname{cox} 1, \ldots \\ & \operatorname{cox} 16 \end{aligned}$ | ${ }^{c_{1}}$ | c | CDMCBM | Oxidizer combustion coefficients, unitless |
| CRR |  | R | camngz | Contraction ratio, cross-sectional area of chamber/throat area, unitless (see Fig. 15) |
| CS | $c^{*}$ | R | CDMCBM | Characteristic velocity evaluated at the overall mixture ratio, $\mathrm{m} / \mathrm{sec}(\mathrm{ft} / \mathrm{sec}$ ) |
| CTEST | -- | R | S@LVE | Upper bound on the condition number of the transpose of the Jacobian of the difference between the upstream and downstream nozzle admittances with respect to the complex frequency. If the condition number of that matrix exceeds CTEST for a given frequency, then it is assumed that Jacobian is singular near that frequency and hence a solution will not be sought at that point, unitless. |
| DA(100) | $\partial A / \partial x$ | R | COMARE | Slope of area of chamber at XM(I), mo |
| DCSDMR | $\partial c^{\star} / \partial M R$ | R | сомСВМ | Partial derivative of the characteristic velocity with respect to mixture ratio holding, $\mathrm{m} / \mathrm{sec}$. |
| DELFRQ | -- | R | SøLVE | Frequency increment used during the search procedure to solve the nozzle admittance boundary equation, Hz . |
| DELHFU | $\Delta h_{f u}$ | R | СøМСВМ | Pseudo energy term for fuel, J/kg (Btu/lbm) |
| DELHOX | $\Delta h_{\text {ox }}$ | R | С9MCBM | Pseudo energy term for oxidizer, $\mathrm{J} / \mathrm{kg}$ (Btu/lbm) |
| DELMX | -- | R | søLVE | Maximum change in $\omega$ allowed between iterations |
| DELP | $\Delta P$ | R | C®MCHM | Oscillatory pressure at injector face, dimensionless |


| FORTRAN VARIABLE NAME | ENGINEERING VARIABLE SYMBOL | TYPE | CONTROL SECTION | DESCRIPTION |
| :---: | :---: | :---: | :---: | :---: |
| DELVFU | -- | R | C®MCBM | Steady-state velocity difference between fuel droplets and gas stream normalized to the sonic velocity at the overall mixture ratio, unitless |
| DELVDX | -- | R | C®MCBM | Steady-state velocity difference between oxidizer droplets and gas stream normalized to the sonic velocity at the overall mixture ratio, unitless |
| DELX | $\Delta x$ | R | COMARE | Axial distance between successive $X(I)$, m |
| DET 1 | -- | R | SøLVW | Determinant of the Jacobian of the difference between the upstream and downstream nozzle admittances with respect to $\omega$. |
| DET2 | -- | R | SøL.VW | Determinant of the Jacobian of the difference between the upstream and downstream nozzle admittances with respect to $\omega$. |
| DFIDX | -- | R | S@LVW | Derivative of Imag (FN) with respect to real ( $\omega$ ). |
| DFIDY | -- | R | S@LVW | Derivative of real (FN) with respect to real ( $\omega$ ). |
| DHDMR | $(\partial h / \partial M R)_{\phi}$ | R | СФМСВМ | Partial derivative of gas reference enthalpy with respect to mixture ratio averaged over the mixture ratio range during steady-state operation, J/kg (Btu/l bm). |
| DM1 | -- | c | ADARND | Used to store intermediate values needed to compute boundary conditions and coefficients of finite difference equations approximating the oscillatory differential equations. |
| DM2 | -- | R | ADARND | Used to store intermediate values needed to compute boundary conditions and coefficients of finite difference equation approximating the oscillatory differential equations. |


| $\begin{aligned} & \text { FORTRAN } \\ & \text { VARIABLE } \\ & \end{aligned}$ NAME | ENGINEERING VARIABLE SYMBOL | TYPE | CONTROL SECTION | DESCRIPTION |
| :---: | :---: | :---: | :---: | :---: |
| DM3 | -- | C | ADARND | Used to store intermediate values needed to compute boundary conditions and coefficients of finite difference equations approximating the oscillatory differential equations. |
| DM4 | -- | C | ADARND |  |
| DM5 | -- | C | ADARND |  |
| DM6 | -- | C | ADARND |  |
| DM7FU | -- | C | ADARND |  |
| DM70X | -- | C | ADARND |  |
| DM8FU | -- | C | ADARND |  |
| DM80X | -- | C | ADARND |  |
| DM9FU | -- | C | ADARND |  |
| DM90x | -- | C | ADARND |  |
| DM22 | -- | R | ADARND | $\dagger$ |
| DMRB(100) | $2 \overline{\mathrm{MR}} / \mathrm{x}$ | R | CONSTS | Derivative of steady-state mjxture ratio with respect to distance, $\mathrm{m}^{-1}$ |
| DRGDMR | $2 \mathrm{R} / 8 \mathrm{x}$ | R | CФMCBM | Partial derivative of gas constant with respect to mixture ratio evaluated at the overall mixture ratio, $\mathrm{J} / \mathrm{kmole} /{ }^{\circ} \mathrm{K}$ (ft-lb/lb/ ${ }^{\circ} \mathrm{R}$ ) |
| DRHQB(100) | ) $\partial \bar{\rho} / \partial x$ | R | CONSTS | Derivative of steady-state density with respect to distance, $\mathrm{kg} / \mathrm{m} 3 / \mathrm{m}$ |
| DTFUDM | $\partial \bar{\tau}_{\mathrm{fu}} / \partial \mathrm{MR}$ | R | CPMCBM | Partial derivative of fuel vaporization time delay with respect to mixture ratio, holding the vaporization blockage term, droplet diameter, and Nusselt number constant, sec |


| FORTRAN VARIABLE NAME | $\begin{aligned} & \text { ENGINEERING } \\ & \text { VARIABLE } \\ & \text { SYMBOL } \\ & \hline \end{aligned}$ | TYPE | CONTROL <br> SECTION | DESCRIPTION |
| :---: | :---: | :---: | :---: | :---: |
| DT0XDM | $\partial \bar{T}_{0 X} / \partial M R$ | R | СøМСВМ | Partial derivative of oxidizer vaporization time delay with respect to mixture ratio, holding the vaporization blockage term, drop diameter, and Nusselt number constant, sec |
| EPSF | -- | R | SØLVE | Relative error criterion used during the search algorithm for the portion of the program that minimizes the error, HN , in the nozzle admittance boundary equation with respect to the growth coefficient, $\omega_{I}$. To obtain convergence, it is necessary that <br> $\frac{\partial\|\mathrm{HN}\|^{2}}{\partial \omega_{I}}<E P S F S$ |
|  |  |  |  | Unitless. |
| EPSFS | -- | R | SØLVE | Tightened relative error criterion used to determine if convergence has been obtained while iterating to solve the nozzle admittance boundary equation. To obtain convergence, it is necessary that |
|  |  |  |  | $\left\|N_{A_{U}}-N_{A_{D}}\right\| /\left\|N_{A_{D}}\right\|<E P S F S .$ |
| EPSX | -- | R | SOLVE | Relative error criterion used during the search algorithm for the portion of the program that minimizes the error, $H N$, in the nozzle admittance boundary equation with respect to the growth coefficient, $\omega_{I}$. To obtain convergence, it is necessary that |
|  |  |  |  | $\left\|\omega_{I_{1}},-\omega_{I_{2}}\right\| /\left\|\omega_{I_{2}}\right\|<E P S X$ |
|  |  |  |  | where the subscripts 1 and 2 refer to two successive iterations, unitless. |


| FORTRAN <br> VARIABLE <br> NAME | ENGINEERING <br> VARIABLE <br> SYMBOL |  |  |
| :--- | :--- | :--- | :--- |
| EPSXS |  |  |  |


| FORTRAN VARIABLE $\qquad$ | $\begin{gathered} \text { ENGINEERING } \\ \text { VARIABLE } \\ \text { SYMBOL } \\ \hline \end{gathered}$ | TYPE | CONTROL <br> SECTION | DESCRIPTION |
| :---: | :---: | :---: | :---: | :---: |
| FR2 | -- | $R$ | SøLVW | Used to store successive values of Real (FN) during the 2 -dimensional secant method. |
| FR3 | -- | R | SøLVW |  |
| FR4 | -- | R | SøLVW | 1 |
| FREQ | $2 \pi \omega_{R}$ | R | SOLVE | Frequency, i.e., real ( $\omega$ ), Hz |
| FREQMI | -- | R | CØMN@Z | Minimum frequency, used for generation of frequency table FREQT, Hz |
| FREQMX | -- | R | CøMNØZ | Maximum frequency used for generation of frequency table FREQT, Hz |
| FREQT (100) | ) | $R$ | COMTAP | Table of terms used for computation of downstream nozzle admittance. |
| FRQMAX | -- | $R$ | S¢LVE | Maximum frequency above which no solutions to the nozzle admittance boundary equation will be sought, Hz |
| FTSTI | -- | R | S®LVW | Test function used to determine if a solution to the nozzle admittance boundary equation has been bracketed. |
| FTST2 | -- | R | SøLVW | Test function used to determine if a solution to the nozzle admittance boundary equation has been bracketed. |
| GIFU | -- | C | ADARND | Coefficient of oscillatory pressure in fuel oscillatory vaporization expression. |
| G10x | -- | C | ADARND | Coefficient of oscillatory pressure in oxidizer oscillating vaporization expression. |
| G2FU | -- | C | ADARND | Coefficient of oscillatory density in fuel oscillatory vaporization expression. |
| G29x | -- | C | ADARND | Coefficient of oscillatory density in oxidizer oscillating vaporization expression. |


| FORTRAN EN VARIABLE NAME | ENGINEERING VARIABLE SYMBOL | TYPE | $\begin{aligned} & \text { CONTROL } \\ & \text { SECTION } \end{aligned}$ | DESCRIPTION |
| :---: | :---: | :---: | :---: | :---: |
| G3FU | -- | C | ADARND | Coefficient of oscillatory mixture ratio in fuel oscillatory vaporization expression. |
| G30X | -- | C | ADARND | Coefficient of oscillatory mixture ratio in oxidizer oscillating vaporization expression. |
| G4FU | -- | C | ADARND | Coefficient of oscillatory velocity in fuel oscillatory vaporization expression. |
| G40x | -- | C | ADARND | Coefficient of oscillatory velocity in oxidizer oscillating vaporization expression. |
| GAMD | $\gamma_{\phi}$ | R | CDMCBM | Specific heat ratio evaluated at the overall mixture ratio, unitless. |
| GINJFT (100) | ) Ginjfu | C | COMTAP | Fuel feed system response table. Real (GINJFT) is the amplitude of the response and Imag (GINJFT) is the phase angle of the response. |
| GINJOT(100) | 0) Ginjox | $c$ | COMTAP | Oxidizer feed system response table. Real (GINJØT) is the amplitude of the response and Imag (GINJDT) is the phase angle of the response. |
| GN | -- | C | FZER $\emptyset$ | Variable used to store the value of the derivative of CNØZA-NØZA with respect to $\omega$ I or the value of $C N \nsupseteq Z A-N \emptyset Z A$ itself. |
| HN | -- | R | FZERD | FN*FN |
| I | -- | I | - | Used throughout the program as a do loop index. |
| INPNQZ | -- | I | CQMN®Z | Code used for the downstream nozzle admittance term calculation. |
| INRT | -- | I | DUMP | Code used to determine whether or not intermediate output from CHAMDY is desired. |
| IPASS | -- | I | SøLVE | Internal code no longer in use. |


| FORTRAN VARIABLE $\qquad$ | $\begin{aligned} & \text { ENGINEERING } \\ & \text { VARIABLE } \\ & \text { SYMBOL } \\ & \hline \end{aligned}$ | TYPE | $\begin{aligned} & \text { CONTROL } \\ & \text { SECTION } \\ & \hline \end{aligned}$ | DESCRIPTION |
| :---: | :---: | :---: | :---: | :---: |
| IPRN@Z | -- | R | CøMNøZ | Code used for downstream nozzle admittance term printout |
| IR | -- | I | MAIN | Flag set by MAIN program to indicate the first pass through it. After reading in a new case. |
| ISCNT | -- | I | FZERø | Code used to determine logical flow in subroutine SøLVW. |
| ISLP | -- | I | FZERD | Code used to determine whether or not the derivative of FN with respect to Imag ( $\omega$ ) is needed. |
| ISTRT | -- | I | SøLVE | Code used to indicate the first iteration after a solution to the nozzle boundary equation. |
| II |  | c | ADARND | The imaginary number $\boldsymbol{i}$. |
| ITAPC | -- | I | camtap | File number used to save combustion coefficients table. |
| ITAPH | -- | I | COMTAP | File number used to save hydrodynamic coupling term table. |
| ITAPN | -- | I | COMTAP | File number used to save nozzle admittance term table. |
| IWSKP | -- | I | S@LVE | Intermediate output dump code used in subroutine SøLVW. |
| 〕 | -- | I | - | Used throughout the program as a do loop index. |
| KNTMX | -- | I | SgLVE | Maximum number of iterations allowed to minimize the error in the nozzle admittance boundary equation with respect to the imaginary part of $\omega$. |
| KNTR | -- | I | S@LVE | Counter used to control the number of iterations used during the search algorithm between solutions. |
| KNTRMX | -- | I | S@LVE | Maximum number of times the frequency will be allowed to be changed by DELFRQ during the searching algorithm between each solution. |

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| FORTRAN VARIABLE NAME | ENGINEERING VARIABLE SYMBOL | TYPE | $\begin{aligned} & \text { CONTROL } \\ & \text { SECTION } \\ & \hline \end{aligned}$ | DESCRIPTION |
| :---: | :---: | :---: | :---: | :---: |
| KNTSMX | -- | I | s@LVE | Maximum number of iterations allowed for the convergence of the two-dimensional secant method used in SøLVW. |
| KSCNT4 | -- | I | SOLVE | Counter used to control the number of iterations used when ISCNT $=4$. |
| KWHERE | -- | I | MAIN | Flag to control logical flow in the MAIN program after a call to subroutine SøLVW. |
| MBFUI | $\left(\bar{m}_{f u}\right)_{i n j}$ | R | COMCBM | Fuel injection mass flowrate, $\mathrm{kg} / \mathrm{sec}$ ( $\mathrm{lb} / \mathrm{sec}$ ). |
| MBQXI ( | $\left(\overline{\dot{m}}_{0 x}\right)_{i n j}$ | R | CDMCBM | Oxidizer injection mass flowrate, $\mathrm{kg} / \mathrm{sec}$ ( $1 \mathrm{~b} / \mathrm{sec}$ ). |
| MGI | $\overline{\dot{m}}_{i n j}$ | R | C@NSTS | Steady-state gas flowrate at injector face, kg/sec. |
| MR (100) | MR ${ }^{-}$ | R | COMCHM | Oscillatory mixture ratio, dimensionless. |
| $\operatorname{MRB}$ (100) | $\overline{M R}$ | R | CONSTS | Steady-state mixture ratio, unitless. |
| MRGI | $\overline{M R}_{i n j}$ | R | CONSTS | Steady-state gas mixture ratio at injector face, unitless. |
| MRNTFU | -- | c | ADARND | Mixture ratio integral in fuel oscillatory vaporization expression. |
| MRNTDX | -- | C | ADARND | Mixture ratio integral in oxidizer oscillatory vaporization expression. |
| MWG | $M W_{\emptyset}$ | R | CDMCBM | Molecular weight of the gas evaluated at the overall mixture ratio, $\mathrm{kg} / \mathrm{kmole}$ (lbm/lb-mole). |
| NFREQT | -- | I | COMTAP | Number of points in frequency table. |
| NøZA | $A_{N_{0}}$ | C | FZERD | Downstream nozzle admittance |
| NøZAMR | $A_{\text {NMR }}=$ constant | C | FZER® | Term used in computation of downstream nozzle admittance |
| NøZAT(100 | ) -- | C | CDMTAP | Table of terms used for computation of downstream nozzle admittance. |


| FORTRAN VARIABLE $\qquad$ | ENGINEERING VARIABLE SYMBOL | TYPE | CONTROL SECTION | DESCRIPTION |
| :---: | :---: | :---: | :---: | :---: |
| NRØøT | -- | 1 | MAIN | Number of solutions to the downstream nozzle admittance boundary equation being sought. |
| NRT | -- | I | MAIN | Used as do loop index for MAIN program. Counts the number of solutions to the nozzle admittance boundary equation. |
| NUBFU | $\overline{N u}_{\text {fu }}$ | R | C®MCBM | Average steady-state fuel Nusselt number used in vaporization expression, unitless. |
| NUBøX | $\bar{N}_{u_{\phi x}}$ | R | COMCBM | Steady-state oxidizer Nusselt number used in vaporization expression, unitless. |
| NXP | -- | I | CDMARE | Number of points in axial distance array, inclusion between $X 0$ and the start of nozzle inlet. |
| NXPM 1 | -- | I | CHAMDY | NXP-1 |
| gMEGA | $\omega$ | C | C®MCHM | Complex frequency. |
| $P(100)$ | $0^{-}$ | R | CQMCHM | Oscillatory pressure, dimensionless. |
| PI | $\pi$ | R | - | pi (3.141593) |
| PC | $\bar{p}$ | R | COMCBM | Steady-state chamber pressure, N/in. ${ }^{2}$ (psia). |
| PINTFU | -- | C | ADARND | Pressure integral in fuel oscillatory vaporization expression. |
| PINTOX | -- | C | ADARND | Pressure integral in oxidizer oscillatory vaporization expression. |
| RBS@X | -- | C | ADARND | Collection of terms used in oxidizer oscillatory vaporization expression. |
| RCCX | -- | R | camngz | Ratio of the radius of curvature at the nozzle inlet to the chamber radius at nozzle inlet, unitless (see Fig. 15). |
| RCTX | -- | R | camnaz | Ratio of the radius of curvature upstream of the throat to the chamber radius at nozzle inlet, unitless (see Fig. 15). |


| FORTRAN VARIABLE NAME | ENGINEERING VARIABLE SYMBOL | TYPE | CONTROL <br> SECTION | DESCRIPTION |
| :---: | :---: | :---: | :---: | :---: |
| RGØ | $\mathrm{R}_{\varnothing}$ | R | CФMCBM | Gas constant evaluated at the overall mixture ratio, $\mathrm{J} / \mathrm{kmole}{ }^{\circ}{ }^{\circ} \mathrm{K}$ (Btu/lb mole/ ${ }^{\circ} \mathrm{R}$ ). |
| RHNTFU | -- | C | ADARND | Density integral in fuel oscillatory vaporization expression. |
| RHNTQX | -- | C | ADARND | Density integral in oxidizer oscillatory vaporization expression. |
| RHO (100) | $\rho^{\prime}$ | R | C@MCHM | Oscillatory density, dimensionless. |
| RH®B (100) | $\bar{\rho}$ | R | CONSTS | Steady-state density, $\mathrm{kg} / \mathrm{m}^{3}$. |
| RHØGI | $\rho_{\text {inj }}$ | R | CONSTS | Steady-state gas density at injector face, $\mathrm{kg} / \mathrm{m}^{3}$ |
| RH@INJ | $\rho^{\prime} \mathrm{inj}$ | R | ADARND | Oscillatory density at injector face, unitless. |
| RINJ | -- | R | COMN®Z | Radius of the chamber at the injector, in. |
| RPSFU | -- | C | ADARND | Collection of terms used in fuel oscillatory vaporization expression. |
| SSV1(100) | -- | R | CONSTS | Steady-state parameters computed in subroutine STEADY for use by subroutine CHAMDY |
| SSV2(100) | ) | R | CQNSTS |  |
| SSV3(100) | ) | R | consts |  |
| SSV4(100) | ) | R | CONSTS |  |
| SSV5(100) | ) | R | CONSTS |  |
| SSV6(100) | ) | R | CONSTS |  |
| SSV7(100) | ) -- | R | CONSTS |  |
| SSV8(100) | ) -- | R | CONSTS |  |
| SSV9FU(100) | 00) -- | R | CONSTS |  |
| Ssv99x (100) | 00) -- | R | CgNSTS | $\pm$ |



| FORTRAN VARIABLE NAME | ENGINEERING VARIABLE SYMBOL | TYPE | CONTROL <br> SECTION | DESCRIPTION |
| :---: | :---: | :---: | :---: | :---: |
| VINTFU | -- | C | ADARND | Velocity integral in oxidizer oscillatory vaporization expression. |
| VINTOX | -- | C | ADARND | Velocity integral in oxidizer oscillatory vaporization expression. |
| VX® | $v^{\prime}{ }_{x}=0$ | C | CDMCHM | Oscillatory velocity at $X=0$, dimensionless. |
| $x(100)$ | $x$ | R | CDMARE | Axial distance array, $X(I)=X 0+(X-1) * \text { DELX, } m .$ |
| XO | $x_{0}$ | R | CDMARE | Start of vaporization point, m |
| X1 | -- | R | SøLVW | Used to store successive values of Imag (w) during the iteration to minimize the error in the nozzle admittance boundary equation with respect to Imag ( $\omega$ ). |
| X2 | -- | R | SøLVH |  |
| X3 | -- | R | SøLVW | $\dagger$ |
| XIMPFU | $\operatorname{ximp}_{f u}$ | R | С9мСВМ | Fuel jet injection impingement point, m (in.). |
| XIMPDX | $\mathrm{ximp}_{0 x}$ | R | C®MCBM | Oxidizer jet injection impingement point, m(in.). |
| XII | -- | R | SDLVW | Used to store successive values of Imag ( $\omega$ ) during the 2-dimensional secant method. |
| XI2 | -- | R | SøLVW |  |
| XI3 | -- | R | S@LVW |  |
| XI4 | -- | R | SøLVW | $\gamma$ |
| XKFU | $\mathrm{xk}_{\text {fu }}$ | R | CDMCBM | Fuel Klystron distance, m(in.). |
| XKøX | $\mathrm{xk}_{0 \times}$ | R | СФмСВМ | Oxidizer Klystron distance, m(in.). |
|  | - |  |  | - |


| FORTRAN VARIABLE NAME | $\begin{aligned} & \text { ENGINEERING } \\ & \text { VARIABLE } \\ & \text { SYMBOL } \\ & \hline \end{aligned}$ | TYPE | CONTROL <br> SECTION | DESCRIPTION |
| :---: | :---: | :---: | :---: | :---: |
| XM(100) | -- | R | CØMARE | Axial distance midpoints, $X M(I)=(X(I)+X(I-T)) / 2, m$. |
| XNØZ | -- | R | CgMARE | Nozzle inlet point, m. |
| XR1 | -- | $R$ | SØLVW | Used to store successive values of Real ( $\omega$ ) during the 2 -dimensional secant method. |
| XR2 | -- | $R$ | SDLVW |  |
| XR3 | -- | $R$ | SØLVW |  |
| XR4 | -- | R | SØLVW | 1 |

APPENUIX 3

## PROGRAM FLOIV CHARTS

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| Curo 10 | Pacc/box | nure |  | reterences isa |  | Scouence no. mo pace/80x) |  |  |  |  |
| 00007450 | 30.1439 |  |  |  |  |  |  |  |  |  |
| 06007400 | 30.1540 | 00007040 | 30.13 |  |  |  |  |  |  |  |
| 00007490 | $30.15 \quad 41$ | 0000740 | 30.14 |  |  |  |  |  |  |  |
| 00007500 | 30.1242 |  |  |  |  |  |  |  |  |  |
| 00007520 | 31.01 -4 | 0000749 | 30.16 | 00007680 | 31.14 |  |  |  |  |  |
| 0000730 | 31.04 | 00007520 | 31.01 |  |  |  |  |  |  |  |
| 00007560 | 31.06 | $0000 \% 670$ | 31.13 |  |  |  |  |  |  |  |
| 00007570 | 31.07301 |  |  |  |  |  |  |  |  |  |
| 000075e0 | 31.08 48 |  |  |  |  |  |  |  |  |  |
| 00007620 | 31.10300 | 00007560 | 31.06 |  |  |  |  |  |  |  |
| 00007630 | 31.11302 |  |  |  |  |  |  |  |  |  |
| 00007670 | $31.13 \quad 47$ | 00007570 | 31.07 | 00007610 | 31.09 | 00005s20 | 31.10 |  |  |  |
| 00507680 | 31.1449 |  |  |  |  |  |  |  |  |  |
| 00007590 | 31.1560 | 0000790 | 31.00 | 20007610 | 31.09 | 00007860 | 31.12 |  |  |  |
| 00007740 | 31.18 | 00007710 | 31.16 |  |  |  |  |  |  |  |
| 00007750 | 31.20 | d0007no | 31.10 |  |  |  |  |  |  |  |
| 0000 mo | 31.23 | 00007730 | 31.20 |  |  |  |  |  |  |  |
| 00007780 | 31.24111 |  |  |  |  |  |  |  |  |  |
| 00007790 | 31.24 | 00007780 | 31.27 |  |  |  |  |  |  |  |
| 0000780 | 31.27 | 00007780 | 31.24 |  |  |  |  |  |  |  |
| 00007820 | 31.30 | 00007800 | 31.28 |  |  |  |  |  |  |  |
| 00007830 | 31.31311 |  |  |  |  |  |  |  |  |  |
| 00007e30 | 31.3: | 00007830 | 31.34 |  |  |  |  |  |  |  |
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| 000080 10 | 32.0: 69 | 900079e0 | 32.19 |  |  |  |  |  |  |  |
| 000080e0 | $32.03 \mathrm{s5}$ | 00008:70 | 32.07 |  |  |  |  |  |  |  |
| 00058110 | 32.0556 |  |  |  |  |  |  |  |  |  |
| 00008180 | 32.0857 | 00008100 | 32.04 |  |  |  |  |  |  |  |
| рооотего | 32.09309 |  |  |  |  |  |  |  |  |  |
| 00007860 | 32.1275 | 00007e40 | 31. 35 | 00009690 | 35.20 |  |  |  |  |  |
| 0000760 | 32.12 | 00007e50 | 32.09 |  |  |  |  |  |  |  |
| 00007870 | 32.13 | 00000585 | 33.0 cm |  |  |  |  |  |  |  |
| 00007920 | 30.14 6 | 000085:0 | 33.03 |  |  |  |  |  |  |  |
| 00007940 | 32.1662 |  |  |  |  |  |  |  |  |  |
| 00007950 | 32.1765 |  |  |  |  | . |  |  |  |  |
| 00007970 | 32.1867 | 00008000 | 32.2 |  |  |  |  |  |  |  |
| 00008460 | 32.20 , | 00308100 | 32.24 | 00000:70 | 32.07 | 00007930 | 32.15 | 00008350 32.ze | 00000-20 35.3 |  |
| 00008210 | 32.2164 | 00007900 | 32.:6 | 00009>40 | 32.23 |  |  |  |  |  |
| 00009230 | 32.2371 |  |  |  |  |  |  |  |  |  |
| 00007990 | 32.2. 60 |  |  |  |  |  |  |  |  |  |
| 00008zso | 32.50 | 00008220 | 32.22 |  |  |  |  |  |  |  |
| c0009350 | 32.27772 | 00008420 | 32.31 |  |  |  |  |  |  |  |
| 00008360 | 32.29773 |  |  |  |  |  |  |  |  |  |
| 00009*30 | 32.3263 | 00000150 | 32. 28 |  |  |  |  |  |  |  |
| 00008190 | 33.01510 | 00007930 | 32.15 |  |  |  |  |  |  |  |
| 10000500 | 33.0261 | 00009200 | 32.08 | 00000 50 | 32.32 |  |  |  |  |  |
| 0000e5310 | 33.03320 |  |  |  |  |  |  |  |  |  |
| 00009520 | 33.04568 |  |  |  |  |  |  |  |  |  |
| 0000esso | 33.07 | 00000650 | 33.14 |  |  |  |  |  |  |  |
| 00000560 | 33.08 | 00008650 | 33.13 |  |  |  |  |  |  |  |
| 00000590 | $33.10 \quad 316$ |  |  |  |  |  |  |  |  |  |
| 00008610 | 33.11313 | -9,00esbo | 33.09 |  |  |  |  |  |  |  |
| 00000650 | $33.13 \quad 313$ | 00000580 | 33.09 |  |  |  |  |  |  |  |
| 00009680 | 33.17 | 00000740 | 33.2: |  |  |  |  |  |  |  |
| 00009700 | 33.1972 |  |  |  |  |  |  |  |  |  |
| 00088740 | 33.2173 | 00008690 | 33.18 |  |  |  |  |  |  |  |
| 0000em70 | 33.204303 |  |  |  |  |  |  |  |  |  |
| 00008790 | 33.25 | 05000770 | 33.24 |  |  |  |  |  |  |  |
| 20008790 | 33.27 | ооловвос | 33.3: |  |  |  |  |  |  |  |
| c0008800 | $33.28 \quad 304$ |  |  |  |  |  |  | $\bigcirc$ |  |  |
| 00008800 | 33.31 | 0000esco | 33.28 |  |  |  |  | -•• |  |  |
| 00009810 | 33.32305 | 00008760 | 33.23 |  |  |  |  | $2$ |  |  |
| 00008870 | 33.35 | 00008910 | 33.39 |  |  |  |  |  | $\left.Q_{i}\right]$ |  |
| 00008500 | 33.37 | 00006910 | 33.38 |  |  |  |  |  |  |  |
| 00009910 | 33.30202 |  |  |  |  |  |  |  |  |  |
| $00009950$ | 33.42 | 00008900 | 33.43 |  |  |  |  |  |  |  |
| 00000960 | 33.43203 |  |  |  |  |  |  |  |  |  |
| 00009140 | 34.04 | 00009860 | 34,11 |  |  |  |  |  |  |  |
| 00009190 | 33.06 | 00009220 | 34.08 |  |  |  |  |  |  |  |
| 00009220 | 34.08 206 |  |  |  |  |  |  |  |  |  |
| 00009850 | 34.10205 |  |  |  |  |  |  |  |  |  |
| 00009890 | 34.13 82 |  |  |  |  |  |  |  |  |  |




| 00000030 | 50.01 | LOCFAC | 00002150 | 3.23-x | 00002320 | 4.09-x |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 00000180 | 50.06 |  | 00000170 | 50.04 |  |  |  |  |  |  |
| 00000610 | 50.10 | 200 | 00000150 | 50.02 | 00000440 | 51.05 | 00000400 | 51.10 | 00000540 | 51. 16 |
| 00000270 | 30.11 | 30 | 00000220 | 50.09 |  |  |  |  |  |  |
| 00000290 | 50.12 |  | 00000290 | 50.13 |  |  |  |  |  |  |
| 00000eso | 50.13 | 40 |  |  |  |  |  |  |  |  |
| 00000200 | 50.15 |  | 00000250 | 50. 16 |  |  |  |  |  |  |
| 00000250 | 50.16 | 20 |  |  |  |  |  |  |  |  |
| 00060320 | 50.17 | 50 | 000002e0 | 30.:2 | 00000240 | 50.15 |  |  |  |  |
| 0000030 | 50.18 | 70 | 00000550 | 51.:7 |  |  |  |  |  |  |
| 00000000 | $5 t .01$ | 90 | 00000150 | 50.07 | 00000300 | 50.13 | D0000260 | 50.16 |  |  |
| 00000-30 | 51.04 |  | 00000450 | 51.07 |  |  |  |  |  |  |
| 000000450 | 51.06 | 100 |  |  |  |  |  |  |  |  |
| 00000*60 | 51.08 | 110 | 00000-10 | 51.02 | 00000440 | 51.05 |  |  |  |  |
| 00000070 | 51.09 |  | 00000490 | 51.11 |  |  |  |  |  |  |
| 00000490 | 51.11 | 120 |  |  |  |  |  |  |  |  |
| 00000500 | 51.12 | 130 | 00000480 | 51.10 |  |  |  |  |  |  |
| 000005i0 | 51.24 |  | 08000500 | 51.12 |  |  |  |  |  |  |
| 00000560 | 51.17 | 150 | 00000520 | 51.15 | 00000530 | 51.16 |  |  |  |  |

OMPT IITLE - NOH-PROCEDURL STATEMENIS

OWRT 1ITLE - SUEROUTINE NOZAOM: IR,GNOX.CO,fREO,MOZAI

| 00000030 | 53.01 | nozam | 00002230 | 3.25-x |
| :---: | :---: | :---: | :---: | :---: |
| 00000290 | 53.03 |  | 00000270 | 53.0: |
| 00000410 | 53.00 | 2 | 00000290 | 53.03 |
| 00000470 | 53.12 |  | 00000460 | 53.09 |
| 00000s+0 | 53.15 | 2000 | 00000490 | 53.13 |
| 00003570 | 53.16 | 2010 | 00000530 | 53.14 |
| 00000620 | 53.20 | 5 | 00000560 | 53.22 |
| 00000670 | 53.23 | 7 | 60000600 | 53.18 |
| 00000700 | 53.27 |  | 0000050 | 53.24 |
| 00000710 | 53.29 | 20 |  |  |
| 000007t0 | 53.28 |  | 00001810 | 58.24 |
| 00000720 | 53.29 | 25 |  |  |
| 00001110 | 91.09 |  | 00001220 | 54.13 |
| 00001280 | 54.12 | 30 |  |  |
| 00001400 | 55.01 | 3 | 00001390 | 54.19 |
| 00001490 | 5.03 | 40 | 00001430 | 54.20 |
| 100001940 | 38.05 | 45 |  |  |







#  pacosin acyeloped or mocketome. A oIvision or mocionl <br> INTE*UTIONNL, CMOCA PABK, CAL!F 91304 <br> procenved or M. D. scramw, Rooct Pome, may 1975 

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R-9808/8-16



R-9808/B-18




```
    E1!.4
800 FорM"tile,g[lz.8
```



```
    Fredex = , E|, (1)
```




```
    ,*.E11.4,'.'.E11.41
```



```
        accrement =.,50.3./l
```



```
        'FEEO SKSTEH RESPONGE', .,20X.'OX!DIZEN -',F9.5.'.',2x,ro.5.
        /.24x.'fuLL -'.rg.5.'.`,2x,re.5./1/)
2010 FOMUTIIOX.4111X.'DSCILLATORY'1./,5X, DISIMCE'.EX.
        pressume RAIIO .7M., velocity RATIO'.EX.
```



```
        15x.'APPITLOE PMUSE 1,1,
```




## 

 xo. 3002, hins


```
CMPLDE:16 NU(4,5), COM141
COMPLEX OFEBA, P, AOO, Y, Mh, T, ONOZA, vxO.
```



```
    cox:0, cox11, coxi2, cox13, cox14, cox15, caxis,
```




```
COMLEX DM, DN3, DN4, ONS, OHE, OMTOX, DNFU, Dreox.
    DMEN, G10x, GIFU, QZOx, CarN, G30x, OFY, Gu0x, GHF,
    mesox, rasfu, pintox, pintiy, rentox, bantfu, maniox,
    imatru, vintox. VINTFU, 11
    .0mgox,0mery
```




```
    VENU, GNO, gGO, DELMOX, DELFFU. PC, CO.
```



```
    cox11, coxi2, coxi3, cox14, cox15, coxi6, crul, crue.
    crus, crif, crus, CFus, crun, crve, cfug, cruio, crult,
    CFUIE, CFUI3. CFUI4, CTUIS, CFLIS, MGG, XIPFOU, XIMPOX.
    CS, DCSDMR, DMONR, DAGOMR. NOYOX. NDOOX. TPRAGO, DELVOX,
    NUEOX, DTOXDM, AOMFU, ADDFU. TDPAGF, OELVFU, MLEFU. DIFLOM
```




```
    S5v111001, SSVE1,1001, 55v3(100). S5N+100),
    S5v51100), 55%51,100). 55V7t1001. 55ve(1001. 55v90x(100),
```



```
    S5V1411001, S5v15(100), S5v16(100),
    #NOL, VG1, HAG1, MGI
```



```
    vxo, ONEGA. CNOZA. DELP
CORTON TCOMARE/ NOP, X(100), xM11001, A(1001, DA(100). DELX,
    xo. xo02. AlNL
COMTON/MONRND/GIOX,GEOX,GSOX,GUCX,G:FU,GEOU,GSU,GHFU,PINTOX
        FWTOX, PNNTOX.VINTOX, PINTFU, RWTFU, HPNTFU,VINTTU, RESOX
```



```
        argy,argu, ll,ome.amee.raiNJ
    COHON /DOPS, IWPT
    EOUIVALENCE TANAIT,51.CMA1!1)
```



```
    20X.'HR'.20X.'V'.2IX.'T'//IX.13, 1PIOEII.4;
FOPGAT(IX.13.1PIOEII.41
```



R-9e08/B-27

COPREX OEOA. P, MOD, V. MR. T, OMOZA, vxo.

coxio, coxit. coxit. cowi3. coxit, coxis. cox 16,

crula, वrull, arule, aful3. Crul4, cruls, crule

 masox. pestu, pintox, pintiv, gantox, mentru, mintox. MPNTFU, vintox. vintru. It . Desocx. oursin

comon rcorcemf mocx, wocl, reoxt, Merul, theox, taleru. veox.

 coxil. Coxi2, coxis, coxim, coxis, coxis, cFul, Crue, crus. CFIn, Crus, ows, orn, crue, crug, truio, crull,

 MEOX, OTOMCM, MONU. MOOFU. TORAGF, DELVFU. MEFU, DTFINM





S5vi4iloon, SSvis(100). SSV151100).
atocl. vol. Mat. Mal
 vxj, orega. ovaza, dap
 XD. XNOZ. AIN


 arefu.dreru.11.De.dres.frolins
awet tink - seraitine cocilith.n



R-9008/8-30

OWRT TITE - MOW-mROCELIRL STATEPENTS


#### Abstract

OITENSION A(BE, IES)  ver. .2112)  TS MRE, द्E14.61




OART TITLE - SUEROUTINE COAOYCIR,FREO. OINOX.GINFU, IPRCOM, INFCOMI


```
COPPLEX GINNOX, DINFU, DUM, MOZAT, GINNT, GINET,
```



```
    cox:0, cow11, conv12, cox:3, cox14. cox15, covi6:
    ovi, crue, crus, oru, crus. Crus, crut, orus. crus.
    orulo, CPIL, CFUl2, Cruls. Crut4, Cruls, cruls,
    COXT(161. CFUTI161, RM, RN, RO, %U
```



```
CONHON /CONTAP/ NFREOT, FREOT(1001, NOZATIIOOI, OLNJOT:100),
    GINFTIIOO), ITAPN, ITAPC, ITAPM
```



```
    verv, GMD. RGO. DELHOX, DEIFU, PG. CO.
    cox1, coxe, cox3, cox4. coxs, coxs, cox7, coxs, cox9, cox,10,
    cox11, cox12. coxi3, cox14, cox15. coxib, cFu1. crue.
```



```
    CFUI2, CFUI3, CFU14. CFUI5, CFUIE, HNG, XIMFF. xIPPOX.
    CS. DCSOPR, DIOMP, DPGORA, AOVOX, NDOOX, TDRACO. DELVOX.
```



```
cauIvalemce (coxt(1),COX1), (CruT{'1),CFU1)
OuTA PI/3.14159,
TORMAT(EE12.日)
TOPGATI/SX,'xKCOX =',IPE11.4.5x.'TNUEOX =',
    E:1,4,3x.vEOX =',E11.4.5x, DELMOX ='.E11.4,'.27X
```





```
FORMY/1,5x, %OTU -',TPE:1,4,5x, 'TALEFU ='.
    E11.4,3x, Yerv =',E11.4,5x, DELHFU -.,E!1.4,1,27x,
```




```
    E11.4,\,27x, X!|िएU '',E11.41
```




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    8.1,4)
```



# Dwant 

## COMPLDXP18 A(NAA, 1). H

## 

IA - '.l10,' mitixall.j) = •,
(5X.216.1fe15.6.' . . Ife:13.6)


R-9808/8-37


OWAT TITLE - SMROUTINE RRESP:ICRT,ILA!TE,IXI


CHRT IINE - SLAROUIIE FRESP(ICRT,IWR!TE,IX)



2-sene/t-41

a-seners-42




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$\operatorname{cosen} y t-45$



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\(06 / 25 / 75\)


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 '5ION. MYY 1975',/1/.2SX. ©CVELOFED EV. M. D. SOMWN.'. J. K. MNT INC. NO K. H. FERTIG' .I.4EX, 'AOVNCED PROCRHMS.

 ESX. 'SPONSERED BY. MESARL NOON B. JOHESN SPACE CENTER',.


W.vo.var.valo. 2r. zo.0save (IEB). FSAVE (IBe)








\(R-9808 / 8-60\)
\begin{tabular}{|c|c|c|}
\hline \multicolumn{3}{|l|}{} \\
\hline \multicolumn{3}{|l|}{REAL x (100)} \\
\hline \multicolumn{3}{|l|}{EOUTVALCNCE (Xil), All \({ }^{\text {a }}\)} \\
\hline \multicolumn{3}{|l|}{COMONTINH.NCTO, NP. MEO, TRIG, IAPS. ONI, OFL, H, IH,C.TD, ATO.} \\
\hline \multicolumn{3}{|l|}{} \\
\hline \multicolumn{3}{|l|}{} \\
\hline \multicolumn{3}{|l|}{Gino.gin} \\
\hline \multicolumn{3}{|l|}{} \\
\hline \multicolumn{3}{|l|}{ITAPW} \\
\hline \multicolumn{3}{|l|}{FENC L(301,R(30),A(30),V(30),C(62,128),TD(E2,63),ATD(128),N(101)} \\
\hline \multicolumn{3}{|l|}{.CW1301.mot 301.100 .16} \\
\hline \multicolumn{3}{|l|}{INTECORT4 IMIIE61} \\
\hline \multicolumn{3}{|l|}{\begin{tabular}{l}
 \\

\end{tabular}} \\
\hline \multicolumn{3}{|l|}{} \\
\hline \multicolumn{3}{|l|}{} \\
\hline \multicolumn{3}{|l|}{} \\
\hline \multicolumn{3}{|l|}{ropentl eno or amo inputil} \\
\hline \multicolumn{3}{|l|}{} \\
\hline \multicolumn{3}{|r|}{5x, FREOUENCY, WX, OXIOIZER INUECTION RATE , 4X,} \\
\hline \multicolumn{3}{|r|}{} \\
\hline \multicolumn{3}{|r|}{} \\
\hline \multicolumn{3}{|c|}{OPF9.2.Sx, IPE11.4.OPr9.231} \\
\hline
\end{tabular}


CART TITLE - SUEROUTINE LOCFACINK, X, FX,NX, XX,FX)


DIMENSION RX(1)

FORMTIIMI 2SX OUERROR - EXTRAPCATION OF TAELE IS BETONO \(R\) Ensomalie LIM! TS



R-5008/B-66

08/2375



```

corplex noza, NOZAT. GINDOT, OINET

```

```

    frEOMI. FREOT, FRED, FRECO
    DIRENSION OV(5,4).G15), CP(5), Y(5)
COMOON /XI/GNM,5WW,MGEF,RCT.ACC /XE/T.RT.D.RI.RE.LC,IP
X3/2IR.2I! mewom
COHFON/COMTAP/ NFREOT. FREOTIIOOI. NOZATCIOO1, GINLOTCIOOI.
GINFTIIOOI. ITNPN. ITAPC. ITAPM
COHON/COMNO/ RCCX, RCTX. WOLEX. CRR. RINJ, INPNOZ, FREOMX,
FREOH1, IPPNOZ
000 FOFMarlgie.0)

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```

    E1:.4.3X, CRP =', C11.41
    ```





```

    6x.5tw_PTM,5x, weETA,/1
    ```

1005

OURT IITE - SEMONTINE EXTDIFIP,0.0.


ITHLICIT REM-B(A-H.O-Z
 DITENSION G(5). ©p(5)

FAPMTISX, •PRINTING FROM CNRD W570., 3X, RO E15.8.
3X, 'R1-', E15. 日. 3X, 'RT"'.E:5.e)



\section*{Ownt tite - MON-mocsoull statients}

\section*{}

CONHON MCe/T.RT,O,R1,Re,MC,IP Difension ve(5). A(5), U(15), F2(4,5), OM(5)

```

MPLICIT RCN-8IA-H.O-2
COHTOW /XI/ONM,SWN,NOE,RCT.RCC /X2/T,RT,O,RI,RZ.WC.IP
/X3/ZIR.Z:1
OIMENSION G(5), GPIS)
16 FOPMATISX.'PRINTING FRON CNRO 5000',I.JX.'R='.EIS.B.
3X.'RI=',E15.G.3K.'RT=',EIS.G)

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n-9808/B-78

\section*{OWAT YITLE - SUBROUTINE SAWHIGAEREI}



08/23/75


```

COPLEX ONESA, P. MOD, V. M., I. ONOZA, VXO.
Noza, arzoo.
OCN. 21. 22. 23. FN, NOZANR. ON.
ONOZD, NOZO. NNO, OND, OECOS
COMON/COMCH/P P1:00), RHO(1001, V(100), WR1100), T1:00),
von, areca, croza, delp
COMTON /FZERC/ NDZA.NOZNR,ON,FN,FNA.FNI.IN,ISONT,ISRP
EXTEPNM XImag
DATA P1/3.141593;
CONHON/SOLVE/ FREO, OEURRO, DEIMX, EPSF. EPSX, EPSIS. EPSXS.
FROMUX, ETEST, IPASS, KNTR, ISTRT, KSCNTM, ILSKP
havilox, wutsed, nentrat

```

```

0601 FOPMAT%/. .... UNREE TO FINO ROCT FOR IMAG PNRT OFF.....
NM X1,FI,XC,F2,X3,F3.MG,FNS,KNT, IER,ORECOR =',
3X,1PEE13.5/3x,2110/3X,1PE13.5.' . .,1PE|3.5)
8500 FOPMATI/315.3IIPE14.5,' .,1PE12.51/2X,!P4E13.5)

```


```

    HFNING. POSSIELE ROOT IN fREOUENCY RANCE.
        1PZE!5.6/, **.0....................../1/1
    ES15 FOPMUT//215.3(!PE14.5. .',1PE12.5),!PE:4.5)

```

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        IPE12.5%/2X,1PE14.5.' . .,IPE:2.5)
    ```

```

    IER,NOTS,NNTA,ISCNT - ',4110/ X,F FMOM 1-> = '
    ```


09/2.



```

    cox10. cox11, comi2, com13. coxi4, cox15. cox15.
    ```




```

    VESU, GNO. RSO. LEMOK, delfl, PC. co.
    Cox1, coxe, cox3, comy. cox5, coxs, cox7, coxs, coxs. coxio
    cox11, coxi2, cox13, com,14, comi5, cox16. Crul, crue.
    crus, oru, ors, ofus, Cru7. cFus, crus, crulo. Crul:
    ```

```

    CS. DCSOPR, DHOMR, DNGDNR, AOVON, ADDOX, TORACO, DEIVOX,
    NMEOX, DTOXCM, NOWU, NDOFU, TDRACF, OELWU, NRFU. DTFUDM
    ```

```

    OPRE(100). ORHOB(100), OVE(1001. VAPBOX:100), VAPBFU(100)
    5Sv1:1001. 55ve(100), 55v3:100). S5w(100).
    ```

```

    SSvgru(100), 5SV10(100). SSV111100), SSvi2(100), sSvi3(100).
    S5v14(100), 5Sv151100). S5v:6:100).
    mWOC!, VGI. mRGI, mEI
    COHTON TCOMARE/ NNP. X:1001. XT(100), A(100), Da:100). DELX.
xo, SovZ. AiNu

```

```

    7x. DISTANCE-.
    4x. 'remperature'.Ex, veloc:ty'.3x, mixtufe'
    GX.'OCNSITY'.TX, 'FERCENT VAPORIZED'./.7X.', INCESSI'.5X
        (RANCIEE)'.4X.'IFT/SI'.5x. RATIO'.5x,
        'LEM/FT\cdot03'.,EX. FULL', 3X. '0x10:ZER',')
    FOPOMT(6X,F9.4.2X.2F11.2.F11.4,1PE15.5.0PF11.2.F9.e%

```



R-9808/B-86

OWRT TITLE - SGBROUTINE TAONSIN,H,X,Y,OV.IOZ.IO)


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IMLICIT REN \(-8(A-H, 0-2)\)

COTHON/X4/OM





1000




Dimosion witil, Y(1013

```

COPLIEX ORBA,NOZA,GNOZA,P.MNO,V,MR,T,VEN,TN,NOZNR,ON,

```

```

    coxi0. coxit, coxie. coxi3. comi4. cowis, coxis.
    ```

```

    cruto, ovil. Orvie, crul3, crul4. cruls. orvic
    Oreo
    fral meoxl, MEFU!, nloox. NuEV, mo

```

```

    VEFU, OND, RCO. DELHOX, DELNU. PC, CO.
    ```

```

    cox:1, coxi2. cox13, cox14, cox15, cox15. cru1, crue.
    orus. crue, crus, crus, cruf. crua, crus, crulo, cruls,
    ```

```

    CS, DCSMRA. DHORR. DACDRR, NDVOX. ADOOX, TDRNCO, DCLVOX.
    ```

```

COPCON/ETEROU, NOZA.NOZNPR.ON.FN.FNR,FNT.WN,ISONY.ISLP
COMON /CONCH/P P(100),R00(100),V(100),RR1100), T(100).
VXO,OREGA.CNOZA,DELP

```

```

x0.0007.AINU

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JHPLICIT ReAl－日IA \(+\mathrm{H}, \mathrm{O}-21\)
 1x＋1CM
DITERSION COR（5），DP（5），OY（5，41，PRED（5），Y（5），O15），CP（5）
16
FOREATISX，＇PRINTINO FROM CARO 3430＇，J，3X．＇R＂＇．EI5．B．


3X．＇RI＝＇，E15．日．3X．＇RT＊＇，E15．日）
1000
FORMAT IUEX，F6．4，IX，F10．5，3X，F10．5）

OREGNAL
Dr POOR IAGE IS







06/2575
statdent fuction definition, F(X) - Svefilxi

\section*{APPENDIX C}

\section*{COMPUTER CODE LISTINGS}
\begin{tabular}{|c|c|}
\hline INDEX & PAGE \\
\hline MAIN & C-1 \\
\hline AREA & C-12 \\
\hline CHAMDY & C-13 \\
\hline CHMCDN & C-20 \\
\hline COKAEL & C-24 \\
\hline COMBDY & C-27 \\
\hline COMMAT & C-32 \\
\hline FRESP & C-34 \\
\hline HEAD & C-48 \\
\hline HY (Block Data) & C-49 \\
\hline HYDRDY & C-50 \\
\hline LøCFAC & C-67 \\
\hline NØZADM & C-68 \\
\hline RKTDIF & C-75 \\
\hline RKTZ & C-77 \\
\hline RKZDIF & C-79 \\
\hline SQLVIN & C-81 \\
\hline STEADY & C-94 \\
\hline TADAMS & C-98 \\
\hline TDPLDT & C-103 \\
\hline XIMAGF & C-105 \\
\hline ZADAMS & C-108 \\
\hline ZERØ & C-112 \\
\hline
\end{tabular}






4


WVyכOMd NIVW W S \(2 \mathrm{~S} \pm\)


1

55 CONTINUE
6O GINJIX \(=\) GINJCT(II)+FI*(GINJITIII+1)-GINJUT(II))
GINJFU \(=\) GINJFT(II)+FI*(GINJFT(II+I)-GINJFT(II))
70 CONTINUE.
CUMPUTE COMBUSTIUN COEFFICIENTS. (THROUGH STATEMENT ILO)
UU
COMBUSTIUN COEFFICIENTS. (THROUGH STATEMENT 110)
WH Clidd

PROLR

\(\operatorname{orccoz8} 30\)
 oncoj3100 61.003110


\[
2010
\]

\[
1
\]
\[
\begin{aligned}
& \text { OUU } 1=1, N X \\
& \text { AP }=C A B S 1 P 11
\end{aligned}
\]
\[
\times P
\]
\[
\begin{aligned}
& P N=A T A N D(A I M A C \\
& A V=C A H S I V I I))
\end{aligned}
\]

OSCILLATORY•), \(1.5 X, 0 I S T A N C E \cdot 6 X\) RATIO••TX. VELOCITY RATIOV.GX.

IFIAP.LE.O.O.ANU.I.LT.NYPI PII)=P(I+1)*I.E-1G
\[
I+(A P \cdot L E \cdot O \cdot O \cdot A N D \cdot I \cdot G E \cdot N X P) P(I)=P(I-1) * 1 \cdot E-10
\]
\[
P N=A T A N D(A I M A G \mid P(I)), R E A L(P(I))
\]
\[
I F(A V \cdot L E \cdot U \cdot G \cdot A N D . I \cdot L T \cdot N X P) V(I)=V(I+1) * 1 \cdot E-10
\]
\[
\begin{aligned}
& \text { IF(AV.LE•U.C.AND.I•LT.NXP) V(I)=V(I+1)*1.E-10 } \\
& 1 F(A V \cdot L E \cdot O \cdot C \cdot A N D . I \cdot G E \cdot N X P) \\
& V(I)=V(I-I) * 1 \cdot E-1 U
\end{aligned}
\]
\[
\begin{aligned}
& P V=A T A N D(A I M A C \\
& A T=C A B S(T(I))
\end{aligned}
\]
\[
I F(A T \cdot L E \cdot H . U, A N D . I . G E, N X P) T(I)=T(I-I) * I \cdot E-10
\]
\[
I F(A T \cdot L I \cdot O \cdot O \cdot A N D \cdot I \cdot L T \cdot N X P) T(I)=T(I+I) * I \cdot E-1 C
\]
PT = ATANU(AIMAG(TII)),REALIT(I)))
TUDE PHASE 1,1

IF (AMR.LE.U.O.AND.I.LT.NXP) \(\operatorname{MR}(I)=M R(I+1) * 1 . E-1 C\) \(A M R=C A B S(M R(I))\)





FSCSM SUBRDUTINES

F \(S\) C \(S M\) SUBROUTINES


COMMUN /COMCHM/ P(1OC), RHU(1GO), VILUO), MK(IOO), T11001.
1 VXO, OMEGA, CNOZA, DELP
CUMMCIN /LOMARE/ NXP, X(1OG), XM(10O), A(10O), DAIIOOI, UELX.
XC. XNOZ. AINJ RESFU, DMI, DM3, DM4, UMS, DM6, DM7OX, DMBUX, DM9OX, DM7FU,
CMEFU, DM9FU,II,UM2,DM22,RHOINJ

-
\(\pm\)

\section*{sヨNIINOqgns w s J S}

INITIALILE INTEGRALS USEU IN VAPGRIZATIUN [XPRESSION. PINTIXX \(=(0 ., 0\).


\footnotetext{
IFIIWRT.GT.(i) WRITE 6,9000 ) CMEGA,I,P(1),RHC(1), MR(1),V(1),T(1) FURMATI///' OMEGA \(=, 1 P 2 E 13,5 / / 3 X, I^{\prime}, 11 X,{ }^{\prime} P \cdot, 19 X\), RHO!.
}



F S C S M SUBROUTINES
\(\underset{\sim}{2}\)
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\(u^{-}\)

\title{
SUBROUTINE CHMCON(I)
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\section*{}


F C S M Sugruutines


RETUR'V
END
FSCSM SUBROUTINES

find maximum amplitude in column and below the olagonal．


\footnotetext{
INTERCHANGE ROWS JF REQUIKED TO GBTAIN MAXIMUM PIVOTAL ELEMENT．
}
IF（J－JMAX）4，5，6 WRITE（ 6.12\()\) J，JMAX
FÜMAT（OIHIE GKOR IN






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\(06 I 21000\)
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F S C S M SUBROUTINES
\begin{tabular}{|c|c|}
\hline \multicolumn{2}{|l|}{RFU \(=0.5 / D E L V O X *(D 1 * O 1-(0.0,1.0) * D 1) /(1 .+01 * D 1)\)
\(C O X 1=R M-R U-2 . * R D+(N U B O X-2) * 0.5 * R. D / N U B O X\)} \\
\hline \multicolumn{2}{|l|}{\(\operatorname{COX2} 2=0.0\)} \\
\hline \multicolumn{2}{|l|}{\(\operatorname{cox} 3=0.0\)} \\
\hline \multicolumn{2}{|l|}{COX4 \(=\) RFR*(NUBOX-2.)/NUBOX} \\
\hline \multicolumn{2}{|l|}{coxs \(=0.0\)} \\
\hline \multicolumn{2}{|l|}{COX6 \(=-\) DTOXDM/TAUBOX} \\
\hline \multicolumn{2}{|l|}{\(\operatorname{cox} 7=0.0\)} \\
\hline \multicolumn{2}{|l|}{COXE \(=\) RFU* (NUBOX-2.)/NUBOX} \\
\hline \multicolumn{2}{|l|}{} \\
\hline \multicolumn{2}{|l|}{\(\operatorname{cox} 10=0.0\)} \\
\hline \multicolumn{2}{|l|}{\(\operatorname{cox} 11=0.0\)} \\
\hline \multicolumn{2}{|l|}{\(\operatorname{cox} 12=-\cos 4\)} \\
\hline \multicolumn{2}{|l|}{\(\operatorname{cox13}=0.0\)} \\
\hline \multicolumn{2}{|l|}{\(\operatorname{cox} 14=-\mathrm{CO} \times 6\)} \\
\hline \multicolumn{2}{|l|}{\(\operatorname{cox} 15=0.0\)} \\
\hline \multicolumn{2}{|l|}{\(\operatorname{cox} 16=-\operatorname{coxs}\)} \\
\hline \multicolumn{2}{|l|}{UUH \(=(0.0,1.0) * 2 . * P 1 * F R E * X K F U / V B F U\)} \\
\hline \multicolumn{2}{|l|}{RM \(=(1 .-D U M) *(; I N J F U\)} \\
\hline \multicolumn{2}{|l|}{\(R U=G I N J F U\)} \\
\hline \multicolumn{2}{|l|}{DUM \(=\) UU:9*XIMPFU/XKFU} \\
\hline \multicolumn{2}{|l|}{\(R D=(A D V F U-0.5 * A D D F U * D U M) * G I N J F U\)} \\
\hline \multicolumn{2}{|l|}{RFR \(=0.5\)} \\
\hline \multicolumn{2}{|l|}{D1 = 2.*PI*FRE*TORAGF} \\
\hline \multicolumn{2}{|l|}{RFU \(=0.5 / 0 E L V F U *(01 * 01-(0.0 .1 .0) * D 1) /(1 .+01 * D 1)\)} \\
\hline & CFU1 \(=\) KM-RU-2.*RD+(NUBFU-2.)*0.5*RD/NUbFU \\
\hline & CFU2 \(=0.0\) \\
\hline & CFU3 \(=0.0\) U \\
\hline & CFU4 \(=\) RFR*(NUBFU-2.)/NUBFU \\
\hline & CFUS \(=0.0\) \\
\hline & CFUG \(=-\) DTFUDM/TAUBFU \\
\hline & CFU7 = 0.0 \\
\hline & CFUB \(=\) KFU*(NUBFU-2.)/NUBFU \\
\hline
\end{tabular}

SヨNIInO४gis w S J S 」


SヨNIAMO甘日GS W S J S y


> \(\begin{array}{llll}n 0 & 0 & 0 & 0 \\ N & 0 & N & N\end{array}\)
FSCSM SUBROUTINES



FS C S M SUBRDUTINES


F S C S M SUBROUTINES

FSCSM SUBROUTINES
\begin{tabular}{|c|c|c|}
\hline \multicolumn{3}{|l|}{312 ANGLE=AUS(TD(1,J)*OMS(IW) -6.283165*IFIX(ABS(TD(I,J)*UMS(IW)/6.283CC00861C} \\
\hline 1 & \(\operatorname{CP}(1, J)=C P(I, J)+C\) TU* \(\operatorname{Cos}(\) ANGLE \()\) & cccor63C \\
\hline & CP(1,KTD) \(=\) CP(1,KTD)-CTD*SIN(ANGLE) & OCCU8640 \\
\hline \multirow[t]{5}{*}{313} & CONTINUE & 00068650 \\
\hline & K=KEQ+1 & OCCOHG60 \\
\hline & \(00731=1\), KEU & ccuobs 70 \\
\hline & \(\mathrm{L}=\mathrm{KECO}+\mathrm{I}\) & ancie 8680 \\
\hline & 1F( \(\mathrm{CD}(1, \mathrm{~K})\) )7く,73,72 & c0008690 \\
\hline \multirow[t]{4}{*}{} &  & 0cce8700 \\
\hline & 1851) & c.008710 \\
\hline & \(0(1)=0(1)+\operatorname{COS}(\) ANGLE \()\) & cccori2c \\
\hline & \(U(L)=U(L)+S I N(A N G L E) * S I G N(1.0, T O(I, K))\) & CCOL8730 \\
\hline \multirow[t]{3}{*}{73} & coivtinue & CLCO8140 \\
\hline & NPH=2*KEW & ccoce \({ }^{\text {c }}\) \\
\hline & \(1+(1 w-1) 363,303,305\) & cocos 00 \\
\hline \multirow[t]{3}{*}{303} & IFIIWRITE.GT.1)WRITE (6,323) & 00008770 \\
\hline & DO \(3041=1, \mathrm{KEQ}\) & ccoug 080 \\
\hline & \(1 J=1+K E Q\) & 0cccis790 \\
\hline 304 & IFIIWRI SE.GT.1)WRITE (0,321)I,D(I), D(IJ), (CP(I, J), J=1,NPH) & cuccesco \\
\hline \multirow[t]{2}{*}{305} & Cuntinut & cccoselo \\
\hline & & crocy820 \\
\hline \multirow[t]{2}{*}{C} & ALD U-VECTUK TO CP-MATRIX. SHIFT COLUMNS OF IMAG CP BY 1 TO RIGHT. & orrici8830 \\
\hline & KEAL D INTO COL. KEQ+1. IMAG D INTO COL. \(2 \mathrm{KEG}+2\). & \(0 \mathrm{O}, 008840\) \\
\hline \multirow[t]{2}{*}{c} & & coccerso \\
\hline & UO \(202 \mathrm{I}=1, \mathrm{KEQ}\) & 000C880C \\
\hline \multirow[t]{4}{*}{} & IN = 2*KED - 1-1 & ccocer 10 \\
\hline & \(I N P I=I N+1\) & ccoursso \\
\hline & DO \(202 \mathrm{~J}=1 . \mathrm{KEQ}\) & 06008890 \\
\hline & CP(J,INPI) \(=\) CP(J,IN) & 00008900 \\
\hline \multirow[t]{3}{*}{202} & CONTINUE & coucirylo \\
\hline & \(1 \mathrm{~N}=\mathrm{KEW}+1\) & 00008420 \\
\hline & INN = 2*KEQ + 2 & U0068430 \\
\hline
\end{tabular}
4


CALL COGAEL(CH, KEQ)
C CALL COGAEL TO TRIANGULARIZE THE COMPLEX MATRIX BY GAUSS. ELIMIN.

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BACK SUBSTIIUTE TO LETERMINE REAL AND IMAGINARY PART OF VARIABLES, THEN DETERIIINE MAGNITUDE AND PHASE.

PO1 \(=K E Q+1\)
\[
\begin{aligned}
& \text { (1:1AG(KEW)=CP(KEG,NNP1) } \\
& \text { XMAG(KEW) }=\text { SUR }(\text { (XREAL(KEQ)**2 + XIMAG(KEU)**2) }
\end{aligned}
\]
\[
\begin{aligned}
& X M A G(K E W)=S U R Y(X R E A L(K E Q) * * 2+X I M A G(K E \\
& F A L E(K E W)=\text { THE }(A(X R E A L(K E U), X I M A G(K E Q))
\end{aligned}
\] CORT INUE

\[
\begin{aligned}
& X R E S L(K L(W)=C P(K E Q, N P 1) \\
& \times 1: A A G(K L(W)=C P(K E G, N N P 1)
\end{aligned}
\]
\[
\begin{aligned}
& \text { NNP } 1=2 * N H 1 \\
& \text { MRESL(KL(J) }=\text { CP(KEQ,NP1) }
\end{aligned}
\]
\[
\text { Uu } 205 j=2, K E 6
\]
\[
\begin{aligned}
& \text { UU } 205 J=2, K E G \\
& J J=K E H-J+i
\end{aligned}
\]
\[
J J I=1 J+1
\]
\[
\begin{aligned}
& \text { SUMR }=0.0 \\
& \text { SUMI }=\dot{U} .0
\end{aligned}
\]
\[
\begin{aligned}
& \text { SUM }=, U . U \\
& D O 206 K=J J P 1 . K E O
\end{aligned}
\]
\[
\text { SUMR }=\text { SUMR }+C P(J J, K) * X R E A L(K)-C P(J J, K N) * X I M A G(K)
\] SUMI \(=\) SUMI +CP(JJ,KN)*XREAL(K) +CP(JJ,K)*XIMAG(K

XMAG(JJ) \(=\) SQRT(XREAL(JJ)**2 + XIMAG(JJ)**2) FALE(JJ) \(=\) THETA(XREAL(JJ),XIMAG(JJ)) XREAL(JJ) \(=\) CP(JJ,NP1)-SUMR
XIMAG(JJ) \(=\) CP(JJ,NNP1)-SUMI

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F S C S M SUBRDUTINES
FSCSM SUBROUTINES
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 n9nojJon 0000970 IF（J．EQ．1．AND．IX．LE．I）GINJOX＝CMPLX（PC／WO＊（10．＊＊（GAINI 1）／20．）），（FAZCOC10000 \(0 c 0100151\)
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STOP
END

R-9808/C-48
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F S C S M SUGROUTINES

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\footnotetext{

F S C S M SUBROUTINES




 گ
SUBRDUTINES


\((8,7)=1\)
\((8, G)=R(5)+V(5) / G / A(5)\)
\(C(10,10)=-(R(7)+V(7) / G / A(7))\)
\(C(10,11)=R(7)+V(7) / G / A(7)\)
\(C(11,9)=1 \cdot\)
\(C(11,11)=-V(8) / G / A(8)\)
\(C(12,12)=1\)
\(C(12,13)=R(8)+V(8) / G / A(B)\)
\(C(13,22)=1\)
\(C(13,13)=-V(9) / G / A(9)\)
\(C(14,14)=1\)
\(C(14,27)=R(9)+V(9) / G / A(9)\)
\(C(15,14)=1\)
\(C(15,27)=-V(10) / G / A(10)\)
\(C(16,16)=1\)
\(C(16,17)=R(10)+V(1 C) / G / A(10)\)
\(C(17,16)=1\)
\(C(17,18)=-20\)
\(C(17,19)=-R C\)
\(C(18,15)=1\)
\(C(18,17)=-(V O * V O / G / V O L O)\)
\(C(18,19)=V O \neq V O / G / V O L O\)
\(C(18,62)=V O * V(* K O / G / V O L O\)
\(C(19,21)=1\)
\(C(19,22)=V(14) / G / A(14)\)
\(C(20,29)=1\)
\(C(20,28)=-(K(14)+V(14) / G / A(14))\)
\(C(21,21)=1\)
\(C(21,20)=-V(11) / G / A(11)\)
\(C(21,22)=-V(11) / G / A(11)\)


\section*{\(-(R(17)+V(17) / G / A(17))\)}

\footnotetext{
\(=1\)
\(=V(17) / G / A(17)\)

}


SUBRCUTINES
\(n\)
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\(u\)
\[
\begin{aligned}
& C(36,36)=1 \\
& C(36,37)=R(18)+V(18) / G / A(1 B) \\
& C(37,36)=1 \\
& C(37,37)=-(R(19)+V(19) / G / A(191) \\
& C(37,38)=R(19)+V(19) / G / A(19) \\
& C(38,36)=1 \\
& C(38,38)=-V(20) / G / A(20) \\
& C(39,40)=-(R(\angle C)+V(20) / G / A(20)) \\
& C(40,41)=1 \\
& C(40,46)=-V(25) / G / A(25) \\
& C(41,47)=1 \\
& C(41,48)=R(25)+V(25) / G / A(25) \\
& C(42,47)=1 \\
& C(42,48)=-V(26) / G / A(26) \\
& C(43,49)=1 \\
& C(43,50)=R(26)+V(26) / G / A(26) \\
& C(44,49)=1 \\
& C(44,5 C)=-(R(27)+V(27) / G / A(27)) \\
& C(44,51)=R(27)+V(27) / G / A(27) \\
& C(45,49)=1 \\
& C(45,51)=-V(28) / G / A(28) \\
& C(46,52)=1 \\
& C(40,53)=R(28)+V(28) / G / A(28) \\
& C(47,52)=1 \\
& C(47,53)=-V(29) / G / A(29) \\
& C(48,54)=1 \\
& C(48,39)=R(29)+V(29) / G / A(29) \\
& C(49,54)=1 \\
& C(49,39)=-V(30) / G / A(30) \\
& C(50,56)=1 \\
& C(50,57)=R(30)+V(30) / G / A(30) \\
& C(51,56)=1 \\
& C(51,58)=-Z F \\
& C(1)
\end{aligned}
\]


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\(8)\)

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SUBROUTINES

\[
A T D(4)=-(R(3)-V(3) / G / A(3))
\]


ATU(37)=-1.


SUBROUTINES \(\Sigma\)





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F S C S M SUBROUTINES


\section*{S3NIInO8日CS W S J S =}


F S C S M SUBROUTINES







F S C S M SUBROUTINES


F S C S M SUBROUTINES

\(00 C 02630\)

S C S M SUBROUTINES


\(u\)

SUBRDUTINES

F S C S M SUBROUTINES


F S C S M SUBRDUTINES

F S C S M SUBROUTINES
\[
X 1=A I M A G(O M E G A)
\] 00000340
00000350
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00000370
00000380
00000390 \(X 1\) = AIMAG(OMEGA)
\(X I M A G F\) COMPUTES FNR \(+(0,1)\) FFNI =CNOZA-NOZA. HN=CABS(CNOZA-NOZA).
XIMAGF COMPUTES FNR \(+(0,1)\) \#FNI =CNOZA-NOZA, HN=CABS(CNOZA-NOZA)
WHEN ISCNT \(=1\) OR 4 ANO ISLP \(=1, ~ X I M A G F=D(H N * * 2) / D(I M A G(O M E G A))\)

 IMAGIUMEGAI ANG COMPUTATION OF JACOBIAN OF FNR,FNI W•R•T• OMEGA. TO 195
IF(ISCNT.EQ.5) GO TO 195
IF ISLP=O, THEN (CNOZA-NOZA) AT OMEGA \& DELTA REALIOMEGA) HAS
JUST BEEN COMPUTED IN XIMAGF. GO TO 146 TO FINISH COMPUTATION
OF JACOBIAN AND THEN COMPUTE FTESTZ.
IF(ISLP.EO.O) GO TO 146
ISLP EQUALS I- PROCEEO TO MINIMIZE CABSICNOZA-NOZAI W.R.T.
IMAG(OMEGA). THIS IS DONE UP TO STATEMENT I45. 00600000
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\(\begin{array}{ll} & 115 \\ C \mid & \text { IFIFI＊FZ．LE．} O .1 \text { GO TO } 125 \\ C & \text { USE TIGHTENEQ CRITERIA TO } \\ C & \text { REACHED WITHCUT CHANGING }\end{array}\)
\[
\begin{array}{ll}
\text { C USE TIGHIENEQ CRITERIA TO CHECK TO SEE IF CONVERGENCE HAS BEEN } \\
\text { C } \\
\text { C }
\end{array}
\]


\[
x_{2}=\left(x_{2}+x_{2}\right) / 2
\]
F S C S M SUBROUTINES
\begin{tabular}{|c|c|c|}
\hline C & & 000001000 \\
\hline 120 & X3 = (F2*x1-F1**2)/(F2-F1) & 00001010 \\
\hline & GO TO 118 & 00001020 \\
\hline 125 & FANS \(=F 2\) & \(0 \mathrm{COC1} 030\) \\
\hline & ANS = AIMAG(OMEGA) & onccio40 \\
\hline 135 &  & 00002050 \\
\hline & EPSX) GO TO 145 & 00001060 \\
\hline &  & 00001670 \\
\hline & \(\times 3=\times 2\) & 00CC1080 \\
\hline & \(F 3=F 2\) & 00001090 \\
\hline & X2 \(2=x_{1}\) & 00001100 \\
\hline & \(F 2=F 1\) & cocoll10 \\
\hline & \(x_{1}=x_{3}\) & 00001120 \\
\hline & \(F 1=F 3\) & c0001130 \\
\hline 137 & \(K=0\) & ccuoll40 \\
\hline C & & 0c001150 \\
\hline c & ROOT IS BRACKETED BUT CONVERGENCE NOT YET REACHED. CALL ZERO & 00601160 \\
\hline \({ }^{\text {c }}\) & TO FIND ROOT WITHIN EPSX. & ococl170 \\
\hline C & & 000C1180 \\
\hline & CALL ZEROIXIMAGF, X1, X2,F1,F2,ANS,FANS, EPSFF, EPSX,AF, KNT, KNTMX, IER, & kuccol140 \\
\hline & 1 ) & coocl 200 \\
\hline & IF(IER.EQ.O) GO TO 145 & 00001210 \\
\hline 140 & WRITE(6,8601) \(\times 1, F 1, \times 2, F 2, \times 3, F 3, A N S, F A N S, K N T, I E R, O M E G A\) & coccl220 \\
\hline 8601 & FORMAT///' **** UNABLE TO FIND RDOT FOR IMAG PART OF F ***** & 00001230 \\
\hline & \(1 / / 1 \times 1, F 1, \times 2, F 2, \times 3, F 3, A N S, F A N S, K N T, I E R, O M E G A=1 /\) & OCCOLi40 \\
\hline & 2 3X,1P8E13.5/3X,2I10/3X,1PE13.5.0 : ', 1PE13.51 & ucoc 1250 \\
\hline & GO TO 5000 & c.001260 \\
\hline c & & 01001270 \\
\hline C & (CNOZA-NOZA) HAS BEEN MINIMIZED W.R.T. IMAG(OMEGA). SAVE VALUES & ococi280 \\
\hline C & FOK COMPUTATION OF DERIVATIVES W.R.T. REAL (OME (,A). & occcl290 \\
\hline C & & 00CC1300 \\
\hline 145 & CNOZO \(=\) CNOZA & 00001310 \\
\hline & NOZO \(=\) NOZA & 0c001220 \\
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\(\begin{aligned} & \text { COMMUN /CISMCHM/ P(100),RHO(2OU),V(100), MK(1CO),T(16O). } \\ & \text { VXO,OMEGA,CNOZA,LIELP }\end{aligned}\)


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APPENDIX D
SAMPLE CASE INPUT


APPENDIX E
SAMPLE CASE OUTPUT
ANALYTICAL DESCKIPTION
FEEO SYSTEM CUUPLEL
STAGILITY MUDEL

THEORETICAL NOLZLE AOMITTANCES
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline \multirow[t]{2}{*}{NOZZLE} & \multicolumn{4}{|l|}{MACH VUMBER \(=.31\) SVV \(=0.0\)} & \multicolumn{3}{|l|}{GAMMA \(=1.33 n\)} \\
\hline & ANGLE \(=1\) & 1Y．6 RADII & Cit CukVa & RE：THR & \(\mathbf{T}=\mathbf{L}\) & LNTEAVCE & \(=4.1100\) \\
\hline FC & YR & Y I & F & SVR & SYI & ALPHA & －¢ TA \\
\hline 150.0 & U．03305 & Cusycl6 & 150.85 & 2．03849 & 0.11432 & C．UC140 & 0.49704 \\
\hline 160.0 & 0.03334 & 0.1 .3477 & 164.90 & U．03883 & 0.12202 & i．OU107 & U．49663 \\
\hline 170.0 & 0． 0.3365 & 6.11124 & 17C．＂O & C． 3.5914 & 0.12972 & C．JC115 & 0.44620 \\
\hline 180.0 & 0.03398 & 0.11802 & 181.02 & 0.03957 & 0.13744 & 0．กn1：3 & \(\therefore .44573\) \\
\hline 190.0 & 0.03432 & C． 12466 & 191.07 & 0.03941 & U．14j18 & い．Un131 & 0.46524 \\
\hline 200．u & 0.03469 & v． 13132 & 201.13 & 0.04040 & \(0.1 \pm 243\) & C．00134 & 0.49473 \\
\hline 210.0 & 0.03508 & U．13749 & 211.19 & 0.040885 & 0.16070 & 0.00148 & C． 49418 \\
\hline 220．0 & \[
0.43544
\] & 0.14465 & 221.24 & 0.04133 & 0.16849 & ＊－U0157 & 0.44301 \\
\hline \(230 \cdot 0\) & 0.03592 & 0.15136 & 231.32 & 0.04192 & 0.1703 s & c．．．166 & C．4．4301 \\
\hline 240.0 & 0.03637 & 0.15810 & 241.36 & 0.04235 & 0.15413 & O．OO175 & 0.49212 \\
\hline 250.0 & 0.03684 & 0.15484 & 251.41 & 0.04293 & 0.19148 & cocrich & 0.40175 \\
\hline 260．0 & －．03733 & 0.17100 & 261.47 & 0.04548 & 0.14985 & U． 0195 & 0.49164 \\
\hline 270.0 & 0.03785 & C． 17838 & 271.52 & 0.04468 & C．2し774 & 0.0 .0245 & 0.44033 \\
\hline 280.0 & U．C3839 & U．15516 & \(<81.56\) & 0.04471 & 0.21500 & C．EU21t， & 0.48954 \\
\hline 290．0 & U． 03895 & い．192u & 291.04 & C．04537 & 0.22300 & コ．vへ227 & C．4P862 \\
\hline 300.0 & 0.03954 & 0.19884 & 301.69 & C． 04605 & 0.23157 & c．uc238 & C．48863 \\
\hline 310.0 & 0.04015 & U． \(205 \% 1\) & 311.75 & 0.04676 & 0.23957 & 0.00250 & C． 46720 \\
\hline 320.0 & 0.04078 & U． 21200 & 321.81 & 0.44750 & 0.24759 & C．U02ヵ？ & 0.48035 \\
\hline 330.0 & 0.054144 & 0.21931 & 331.86 & 0.04827 & 0.25545 & C．00274 & 0．49540 \\
\hline \[
340.0
\] & 0004213 & ن． 22645 & 341.92 & U．049．0 & 0.26373 & C．ここ287 & 0.4045 ： \\
\hline \[
35 C .0
\] & 01． 04284 & －． 23342 & 351.48 & 0.04989 & 0.27144 & 0.00300 & 0.48361 \\
\hline \[
360.0
\] & 0.044357 & 0.24041 & \(30<0.03\) & 0.05075 & 0.27499 & 0.04314 & 0.45263 \\
\hline \[
370.0
\] & 0.04434 & 0.24743 & \(372 . \cup 9\) & 0.05163 & U． 28 A16 & C．00328 & 0.48163 \\
\hline \[
380.0
\] & 0.04513 & i． 25446 & 382.15 & 0.05255 & 0.29638 & 0．00343 & \(0.4806 C\) \\
\hline \[
390.0
\] & 0.04594
0.04679 & C． 26156 & 392．20 & 0.05351 & 0.30462 & C． 2635 A & 0.47954 \\
\hline 400.0 & 0.04679 & 0.26868 & 402．26 & 0.05449 & 0.31241 & 0.00374 & 0.47845 \\
\hline
\end{tabular}








FEED SYSTEM COUPLED STABILITY MODEL
OME FEED SYSTEM COUPLED STABILITY INVESIIGATIUN - MODEL VERIFICATION
OK OMS ENGINE TECHNOLOGY PROGRAM - IEST NUMHER IZ - CASE NI


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ON - MUUEL VERIFICATION
VUMBIER I_ - CASE \&I
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& 0 \\
& 100
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PHASE OK OMS ENGINE
1 PRESSURE RATIO phast


\subsection*{0.69998} 0.10000 0.10003 0.10001
0.69989 0.09989 0.04934 0.69890 0.0982 0.04744 0.09775 0.09786 0.099868
0.09840 0.09877 0.09951 0.099980
0.09494 U. 10.06 8
8
0
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0 DISTANCE
(INCHES) \(C .8600\)
6.9000 \(C .8000\)
.9000
 1.1 u0 .2000 1.4000 1.500 1.600 1.800 1.9006
2.000 C 2.1000 2.2000 2.3000 30
0.8
0.8
0
0
0 .6000
2.700 C .8000 .9000
.0000 03
08
0.
0. 0.69978 0.09954
0.09927 \(0.0440 n\) 0.04876 0.09858
0.04846
\[
\begin{aligned}
& 0.33561 \\
& 0.26464 \\
& 0.26425 \\
& 0.25461 \\
& 0.25030 \\
& 0.24124 \\
& 0.25294 \\
& 0.21946 \\
& 0.20765 \\
& 0.19346 \\
& 0.14022 \\
& 0.10622 \\
& 0.15213 \\
& 0.13816 \\
& 0.12455
\end{aligned}
\]
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0.08815
\]
 \(\rightarrow 5960^{\circ} 0\)


\footnotetext{



}



ANALYIICAL EESCRIPTION
\[
\begin{aligned}
& \text { COMPUTER MODEL } \\
& \text { FSCSM, FIV VERSIIIN, MAY } 1975 \\
& \text { M. D. SCHUMAN, J. K. HUNTING, ANL K. W. FFRTIG } \\
& \text { AOVANCED PROGRAMS, ROCKEIOYVE } \\
& \text { OIVISION OF ROCKWELL INTERNATIUNAL } \\
& \text { CANOGA PARK, CALIF YIJU4 } \\
& \text { NASA/LYNOON G. JUHISSUN SHACE CEVTER } \\
& \text { HOUSTON, TEXAS 77U5K } \\
& \text { UNDEK CONTRACT NASY-14EI5 }
\end{aligned}
\]
PROT,RAM NAME:

SPONSERED OY:
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\[
\begin{aligned}
& \text { FEED SYSTEM COUPLED STABILITY MODEL } \\
& \text { OME FEED SYSTEM COUPLED STABILITY INVESTICATION - MCDEL VFRIFICATION } \\
& \text { OK OMS ENGINE TECHNOLOGY PROGRAM - TEST NUMGEF } 12 \text {-CASF }
\end{aligned}
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