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Efficient Detection, Analysis and Classification of Lightning Radiation Fields

Principle Investigator:
R. O. Harger

Research Associate
S. A. Tretter

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DEPARTMENT OF
ELECTRICAL ENGINEERING

UNIVERSITY OF MARYLAND

COLLEGE PARK, MARYLAND 20742

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Progress Report

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Efficient Detection, Analysis and Classification of Lightning Radiation Fields

by

S. A. Tretter

August 23, 1976

1. Introduction

What we observe visually as a single lightning flash actually has a complicated internal structure. The initial portion of a flash is called a stepped leader. The leader is formed when the cloud potential becomes sufficiently large to cause an electrical breakdown in the atmosphere. In a cloud-to-ground flash, the breakdown progresses in steps until it reaches the ground. When the leader reaches the gound a large current flows in the ionized channel caused by the breakdown and is called a return stroke. A sequence of several leaders and return strokes can follow. An entire sequence of associated leaders and return strokes is called a flash. A comprehensive statistical model for the observed electromagnetic radiation generated by lightning must describe (1) the large scale flash structure, that is, the times between flashes and the amplitudes and durations of flashes and (2) the internal structure of a single flash including the portion generated by leaders and the portion generated by return strokes.

The major effort during this reporting period has been devoted to modeling the large scale flash structure. Large scale flash data has been measured manually from strip charts of storms of August 5, August 26, and September 12, 1975. The data is being processed by a computer program called SASEV to estimate the large scale flash statistics. The program, experimental results, and conclusions for the large scale flash structure are described in Sections 2 and 3.

Some progress has been made in examining the internal flash structure. This consists mainly of developing the software required to process the NASA digital tape data on the University of Maryland's UNIVAC 1108 computer and is described in Section 4.

Section 5 lists plans for future research.

2. Description of the Point Process Analysis Program SASEV

P.A.W. Lewis, A.M. Katcher, and A.H. Weis have written a FORTRAN program called SASEIV for the statistical analysis of series of events [1]. The program implements the techniques described in the book, <u>The Statistical Analysis of Series of Events</u>, by D.R. Cox and P.A.W. Lewis [2]. We have modified this program to run on the University of Maryland UNIVAC 1108 computer in the batch or demand modes and have called the modified program SASEV. The principal modifications are the reduction of the length of the sequence of events that can be handled from 1999 to 1024 and the elimination of double precision arithmetic. Otherwise, the capabilities of SASEV and SASEIV are identical and are described in detail in Reference 1. The statistics computed and tests performed by SASEV that have been found to be particularly useful in the analysis of lightning data are summarized in this section.

2.1. Sample Moments

Let $x_1, \ldots, x_N$ be a sequence of N positive numbers. Normally, these numbers would be the times between events in a point process. SASEV computes the following sample moments and normalized moments.

(a) Sample mean

$$\hat{\mu} = \frac{1}{N} \sum_{n=1}^{N} x_n$$

(b) Sample variance

$$\hat{\sigma}^2 = \frac{1}{N-1} \sum_{n=1}^{N} (x_n - \hat{\mu})^2$$

(c) Sample standard deviation

$$\hat{\sigma} = \sqrt{\hat{\sigma}^2}$$

(d) Coefficient of variation

$$c = \hat{\sigma} / \hat{\mu}$$

(e) Third central moment

$$\hat{\mu}_3 = \frac{N}{(N-1)(N-2)} \sum_{n=1}^{N} (x_n - \hat{\mu})^3$$

(f) Coefficient of skewness

$$\hat{\gamma} = \hat{\mu}_3 / \hat{\sigma}^3$$

(g) Fourth central moment

$$\hat{\mu}_4 = \frac{N(N^2 - 2N + 3)}{(N-1)(N-2)(N-3)} \sum_{n=1}^{N} (x_n - \hat{\mu})^4 - \frac{3(N-1)(2N-3)}{(N-1)(N-2)(N-3)} \hat{\sigma}^4$$

(h) Coefficient of Kurtosis

$$\hat{k} = \hat{\mu}_4/\hat{\sigma}^4 - 3$$

Many physically observed point processes are accurately modeled as homogeneous Poisson processes. A reasonable first step in the analysis of a point process is to see if the Poisson model applies. A simple check is to compare the sample moments defined above with the theoretical statistical moments. The times between events in a homogenous Poisson process are independent, identically distributed random variables with an exponential probability density function, say $f(x) = \lambda e^{-\lambda x} u(x)$ where $u(x)$ is the unit step function. In this case the ideal moments are

(a) $\mu = E\{X\} = \lambda^{-1}$

(b) $\sigma^2 = E\{(X - \mu)^2\} = \lambda^{-2}$

(c) $\sigma = \lambda^{-1}$

(d) $c = \sigma/\mu = 1$

(e) $\mu_3 = E\{(X - \mu)^3\} = 2\lambda^{-3}$

(f) $\gamma = \mu_3/\sigma^3 = 2$

(g) $\mu_4 = E\{(X - \mu)^4\} = 9\lambda^{-4}$

(h) $k = \mu_4/\sigma^4 - 3 = 6$

2.2. The Sample Distribution Function, Log Survivor Function, and Histogram

SASEV computes the sample distribution function for the sequence $\{x_n\}$ as

$$F_N(x) = \frac{\text{number of } x_n\text{'s} \leq x}{N}$$

This is an estimate of the actual distribution function $F(x) = P\{X \leq x\}$.

The function $R(x) = 1 - F(x)$ is called the survivor function. SASEV estimates $R(x)$ as

$$R_N(x) \quad 1 - F_N(x)$$

It computes and plots the log survivor function

$$G_N(x) = \ln R_N(x)$$

For the Poisson process, $G(x) = \ln R(x) = -\lambda x$. Thus deviations from linearity in the plot of $G_N(x)$ give insight into how the observed processes differs from an ideal Poisson process. Another closely related function is the hazard function

$$h(x) = -\frac{d}{dx} G(x) = \frac{f(x)}{1 - F(x)}$$

The hazard function can be estimated from the plot of the log survivor function. For the ideal Poisson process, $h(x) = \lambda$.

SASEV makes a direct estimate of the probability density function for the random variables $\{X_n\}$ by computing and plotting a histogram for the observations $\{x_n\}$.

2.3. Goodness-of-Fit Tests for the Poisson Process

SASEV performs several goodness-of-fit tests to check if the homogenous Poisson process hypothesis is statistically reasonable. All of these tests are based on the fact that if a homogeneous Poisson process is observed over the interval (0, T), the observed normalized arrival times

$$y_i = t_i/T = \sum_{n=1}^{i} x_n/T \quad , \quad i = 1, \ldots, N$$

are the order statistics of a random sample of size N from a population uniformly distributed over (0,1). Tests based on this fact are called uniform conditional tests. These tests are particularly convenient for the Posson hypothesis because the rate parameter λ need not be estimated and no grouping of data is required.

Let the sample distribution function for the normalized arrival times be

$$F_N(y) = \frac{\text{number of } y_i \leq y}{N}$$

For the homogeneous Poisson process hypothesis, the theoretical distribution function would be

$$F(y) = \begin{cases} 0 & \text{for} \quad y \leq 0 \\ y & \text{for} \quad 0 \leq y \leq 1 \\ 1 & \text{for} \quad y \geq 1 \end{cases}$$

SASEV computes the following four statistics.

(a) The One-Sided Kolmogorov-Smirnov Statistics

$$KS+ = D_N^+ = \sqrt{N} \sup_{0 \leq y \leq 1} [F_N(y) - y] = \sqrt{N} \max_{1 \leq i \leq N} [\frac{i}{N} - y_i]$$

$$KS- = D_N^- = \sqrt{N} \sup_{0 \leq y \leq 1} [y - F_N(y)] = \sqrt{N} \max_{1 \leq i \leq N} [y_i - \frac{i-1}{N}]$$

(b) The Two-Sided Kolmogorov-Smirnov Statistic

$$KS = D_N = \sqrt{N} \sup_{0 \le y \le 1} |F_N(y) - y| = \max \{D_N^+ , D_N^- \}$$

(c) The Anderson-Darling Statistic

$$WN2 = W_N^2 = N \int_0^1 \frac{[F_N(y) - y]^2}{y(1-y)} dy$$

$$= -N - \frac{1}{N} \sum_{i=1}^{N} \{ (2i-1) \ln y_i + [2(N-i)+1] \ln (1-y_i) \}$$

Each of these statistics gives a measure of the deviation of $F_N(y)$ from the hypothesized distribution $F(y) = y$. The Anderson-Darling statistic emphasizes deviations near 0 and 1 as a result of the factor $y(1-y)$ in the denominator.

These statistics are used by selecting a threshold z and rejecting the hypothesis H_0, that the process is a homogeneous Poisson process, if the threshold is exceeded. The threshold is selected so that the test has a desired level α, that is, so that

$$P \{ \text{statistic} > z \mid H_0 \} = \alpha$$

Asymptotic formulas for the levels of the Kolmogorov-Smirnov statistics are derived in Kendall and Stuart [3]. The asymptotic formula for the two-sided Kolmogorov-Smirnov statistic is

$$\lim_{N \to \infty} P \{D_N > z \} = 2 \sum_{r=1}^{\infty} (-1)^{r-1} \exp (-2 r^2 z^2)$$

The approximation is satisfactory for $N \geq 80$. Values of this expression for a range of z are given in Table 3 of Appendix 1. A table of asymptotic levels for the Anderson-Darling statistic is given by Lewis [4].

The power of a test is defined to be the probability that the null hypothesis H_0 is rejected given that an alternative hypothesis H_1 is true. Tests for the null Poisson hypothesis based on the uniform conditional property are not very powerful against a variety of alternatives. For example, suppose that the normalized arrival times $\{y_i\}$ are equally spaced in the interval (0,1). Then $F_N(y)$ remains close to $F(y) = y$. Durbin [5, 6] has suggested a modification of the uniform conditional tests that gives a large increase in power over the uniform conditional tests for a broad class of alternatives. The first step in Durbin's modification is to order the sequence of times between events $\{x_n\}$ to obtain the observed order statistics

$$0 < x_{(1)} \leq x_{(2)} \leq \ldots \leq x_{(N)}$$

The next step is to compute the sequence

$$w_i = \frac{x_{(1)}}{t_N} + \frac{x_{(2)}}{t_N} + \ldots + \frac{x_{(i-1)}}{t_N} + (N + 2 - i) \frac{x_{(i)}}{t_N} \quad \text{for } i=1, \ldots, N-1$$

where

$$t_N = \sum_{n=1}^{N} x_n \quad .$$

It can be shown that under the null Poisson hypothesis, the sequence $\{w_i\}$ has the same distributional properties as the sequence $\{y_i\}$. Thus, the Kolmogorov-Smirnov and Anderson-Darling tests described above can be applied to $\{w_i\}$.

2.4. Serial correlation coefficients

SASEV computes the serial correlation coefficients

$$\hat{\rho}_j = \frac{N}{N-j} \frac{\sum_{i=1}^{N-j} (x_i - \hat{\mu})(x_{i+j} - \hat{\mu})}{\sum_{i=1}^{N} (x_i - \hat{\mu})}$$

for $j = 1, 2, \ldots, \min(N/2, 100)$ where $\hat{\mu}$ is the sample mean. This sequence is an estimate of the autocovariance function

$$\rho_j = \mathrm{cov}(X_i, X_{i+j})/\mathrm{var}(X_i)$$

Under the hypothesis that the x_n's are uncorrelated, that is, $\rho_j = 0$ for $j \neq 0$, the $\hat{\rho}_j$'s are approximately normally distributed with zero mean and variance $(N-j)^{-\frac{1}{2}}$. This approximation is reasonable for $N \geq 100$ if the skewness is moderate. SASEV plots $(N-j)^{\frac{1}{2}} \hat{\rho}_j$ as a function of j.

A renewal process is a point process in which the times between events are independent identically distributed random variables. The homogeneous Poisson process is a special type of renewal process. A set of independent random variables are always uncorrelated. However, a set of uncorrelated random variables may or may not be independent. Thus, the serial correlation coefficients can indicate whether or not a renewal process model is appropriate.

2.5. Spectrum of the Intervals

SASEV estimates the spectral density of the sequence of times between events as

$$\hat{S}(w) = \frac{1}{\pi} \left\{ 1 + 2 \sum_{n=1}^{M} \hat{\rho}_n a_n \cos(nw) \right\}$$

where a_n is the Parzen window, that is,

$$a_n = \begin{cases} 1 - 6\left(\frac{n}{M}\right)^2 + 6\left(\frac{|n|}{M}\right)^3 & \text{for } |n| \leq M/2 \\ 2\left(1 - \frac{|n|}{M}\right)^3 & \text{for } \frac{M}{2} \leq |n| \leq M \\ 0 & \text{elsewhere} \end{cases}$$

It also computes the discrete Fourier transform

$$B_k = \sum_{n=1}^{N} x_n e^{-j2\pi(n-1)k/N} \qquad \text{for } k=0,\ldots,N-1$$

and the periodogram

$$C_k = |B_k|^2/N \qquad \text{for } k=0,\ldots,N-1$$

It can be shown that if the times between events are independent identically distributed random variables, then the values of C_k for $0 < k < N/2$ are asymptotically independent identically distributed random variables with the exponential density. Consequently, the goodness-of-fit tests described in Section 2.3 can be applied to the periodogram to check the renewal process hypothesis. SASEV appies the Kolmogorov-Smirnov and Anderson-Darling uniform conditional tests to the periodogram.

2.6. Spectrum of the Point Process

SASEV estimates the spectrum of the point process in the following way

Let

$$D = (t_N - t_1)/(N-1)$$

$$A(k) = \sum_{i=2}^{N} \exp [j\ B\ (t_i - t_1)/D]$$

and

$$I(k) = 2\,|A(k)|^2 /(N-1)$$

for $k = 1, 2, \ldots, P$. B is normally chosen as $2\pi/(N-1)$. P should be chosen to be larger than N. Usually all the important features of the spectrum will be shown if $P = 2N$. SASEV smooths I(k) over 5,10, and 20 points using a rectangular window and plots the results.

Theoretically, the spectrum for a homogeneous Poisson process would be flat.

3. Experimental Results and Conclusions on the Large-Scale Flash Structure

Experimental data for storms of August 5, August 26, and September 12, 1975 has been supplied by Goddard Space Flight Center in the form of strip charts. The time scale on the charts is highly compressed so that an individual lightning flash has a length in the order of one centimeter. Thus the envelopes but not the fine structure of the flashes can be observed on the charts. We are manually measuring the time between flashes, the peak amplitude of the flashes and the durations of the flashes from the strip charts and punching the data on cards in the format required by SASEV. A signal is judged to be a flash if its envelope exceeds 5 strip chart amplitude units on channel 1 and it also appears in several channels. The storm of August 5 has been completely measured and processed by SASEV. The storm of September 12 has been completely measured and punched on cards but not yet processed. The storm of August 26 is approximately 95% measured. Some of the results of SASEV for the storm of August 5 are presented in Tables 1 and 2 in Appendix 1 and in Figures 1 to 60 in Appendix 2. They are discussed in this section.

3.1. Time Between Flashes for the Storm of August 5, 1975

The results of SASEV for the time between flashes are summarized in Tables 1(a) and 2(a) and Figures 1 to 22. By the time between two flashes, we mean the time from the beginning of one flash to the beginning of the next flash.

Table 1(a) shows the computed sample moments defined in Section 2.1. The coefficients of variation are reasonably close to the ideal value of unity for a homogeneous Poisson process. The coefficients of skewness, for the most part, are close to the ideal value of 2.0 for the Poisson process. The coefficients of kurtosis vary moderately from the ideal value of 6.0 for the Poisson process but are still "in the ball park." Looking at the means, we see that the rate of flashes increases from tape 1 to tape 4 and decreases in tape 5. Thus, as expected, the flash rate is small at the beginning of the storm, increases in the middle, and decreases at the end. Clearly, the homogeneous Poisson process hypothesis is not valid on a long time scale since trends are present.

Histograms of the times between flashes for tapes 1 through 5 are shown in Figures 1, 6, 11, 15, and 19, respectively. The corresponding log survivor functions are shown in Figures 2, 7, 12, 16, and 20. The histograms for tapes 1, 3, and 4 appear as though they could be reasonably fit with an exponential.

The histogram for tape 2 is light at the low end while the histogram for tape 5 seems heavy at the low and high ends. The deviations from an exponential density can be clearly observed in the log survivor functions as deviations from linearity. For the most part, the log survivor functions are reasonably linear.

Table 2(a) displays the computed Kolmogorov-Smirnov and Anderson-Darling statistics defined in Section 2.3. The column labelled KS shows the two-sided Kolmogorov-Smirnov statistic for the uniform conditional test. From Table 3 it can be seen that, using KS, the homogeneous Poisson process hypothesis is accepted at about the 80% level for tape 1, the 27% level for tape 2, the 71% level for tape 3, and the 39% level for tape 4. The hypothesis is clearly rejected for tape 5. The column labelled DN shows the two-sided Kolmogorov-Smirnov statistic when Durbin's modification is used. Using DN, the homogeneous Poisson process hypothesis is accepted at about the 3.9% level for tape 1, the 0.012% level for tape 4, and the 0.06% level for tape 5. It is clearly rejected for tapes 2 and 3. The uniform conditional test with Durbin's modification is picking up definite deviations from the Poisson hypothesis. We conjecture that these deviations may be caused by trends and quantization of the measured data.

The serial correlation coefficients for tapes 1 through 5 are shown in Figures 3, 8, 13, 17, and 21, respectively. The corresponding spectra are shown in Figures 4, 9, 14, 18, and 22. These plots indicate that, except in tape 5, the times between flashes are reasonably uncorrelated. There is definite correlation out to a lag of about 50 in Figure 21 for tape 5. This correlation causes the low-frequency peak in the spectrum in Figure 22. We conjecture that the correlation was caused by a trend.

The Kolmogorov-Smirnov and Anderson-Darling uniform conditional tests were performed on the periodograms of the sequences of times between flashes as described in Section 2.5. The results are presented in the last two columns of Table 2(a). The hypothesis that the times between flashes are independent is clearly accepted for tapes 1 through 4. It is accepted at about only the 0.05% level by the Kolmogorov-Smirnov test and at about only the 0.07% level by the Anderson-Darling test for tape 5. This is consistent with the conclusions from the plots of the serial correlation coefficients.

The spectra of the point processes for tapes 1 and 2 are shown in Figures 5 and 10 respectively. The plots are very wild indicating that the smoothing in SASEV is insufficient. The points marked by x's are the most smoothed. Smoothing the plots further by eye, we conclude that the spectra are consistent with the hypothesis of a flat spectrum for the homogeneous Poisson case. These spectra were not computed for the other tapes since the computation time is long and no unusual results were expected on the basis of the other computations.

Taken as a whole, the results from SASEV indicate that the sequence of flashes on the large scale can be modeled as a homogeneous Poisson process over a time period that is moderate with respect to the duration of the storm.

3.2. Peak Amplitudes of the Flashes for the Storm of August 5, 1975

The peak amplitudes of the flashes were recorded in strip chart amplitude units and processed by the appropriate subroutines of SASEV.

The various sample moments for tapes 1 through 5 are summarized in Table 1(b). Notice that the coefficients of variation, skewness, and kurtosis differ significantly from the theoretical values for an exponential distribution.

The Kolmogorov-Smirnov and Anderson-Darling statistics are presented in Table 2(b). Notice that the Kolmogorov-Smirnov statistic KS or Anderson-Darling statistic WN2 does not reject the exponential distribution hypothesis significantly but that with Durbin's modification the Kolmogorov-Smirnov statistic DN strongly rejects this hypothesis. These results are interesting in that they show the lack of power of the ordinary uniform conditional tests against some alternative hypothesis.

Histograms, log survivor functions, serial correlation coefficients, and spectra of the amplitudes are shown in Figures 23 to 41 for tapes 1 through 5. The histograms and, consequently, the log survivor functions differ significantly from tape to tape. Therefore, we propose no specific distribution function for the amplitudes at this time. However, it is clear from the plots that the exponential distribution is not appropriate. Except for tape 2, the serial correlation coefficients, spectra, and goodness-of-fit tests applied to the periodograms indicate that the amplitudes are uncorrelated.

3.3. Durations of the Flashes for the Storm of August 5, 1975

The sample moments for the durations of the flashes in tapes 1 through 5 are summarized in Table 1(c).

Histograms, log survivor functions, serial correlation coefficients, and spectra of the durations are shown in Figures 42 to 60. The histograms do not seem to be fit by any well known densities. The log survivor functions have significant deviations from linearity which rules out the exponential density. In particular, the histograms seem to be quite large for the small durations. The uniform conditional test statistics with Durbin's modification given in Table 2(c) also strongly reject the exponential hypothesis. The correlation coefficients, spectra, and goodness-of-fit tests applied to the periodogram accept the hypothesis that the durations are uncorrelated.

4. Progress in Investigating the Internal Flash Structure

The major effort to date in investigating the internal flash structure has been the development of software to process the NASA digital lightning tapes on the University of Maryland's UNIVAC 1108 computer.

The FORTRAN program CONVERT shown in Appendix 3 converts the 16 bit two's complement format words on the NASA digital tapes to 36 bit one's complement format UNIVAC 1108 integers and then to 36 bit UNIVAC 1108 floating-point words. The converted data can be plotted on the line printer using the plot routines in SASEV.

The FORTRAN program PEAK shown in Appendix 3 detects peaks in the sampled lightning data. The following three conditions must be satisfied before a point is declared a peak (1) an amplitude threshold must be exceeded, (2) the point must be greater than the adjacent two points, i. e., it is a local maximum, and (3) the difference between the point and at least one of the adjacent points must exceed a threshold. PEAK was applied to a record of approximately 6,500 points and the detected peaks were compared with the plotted data. For this record, PEAK detected the peaks with 100% accuracy.

The data on the NASA digital tape was selected to display the internal structure of a flash. The signals on the tape consist of a series of similarly shaped pulses with varying amplitudes and spacing. It is believed that the pulses are caused by the individual steps in the leaders. No indication of return strokes could be found in the file examined. A very preliminary analysis of the intervals between pulses has been made using PEAK and SASEV. For the small segment of data processed, the results of SASEV indicated that the homogeneous Poisson process model may not be appropriate for the sequence of pulses in the leader signal. Much more data must be analyzed before any definite conclusions are drawn.

5. Future Research

Research ideas to be pursued during the remainder of this grant and in the future, if funds are available, are outlined in this section.

(1) We will complete measuring the large scale flash data from the strip charts for the storms of August 26 and September 9, 1975 and process the data with SASEV.

(2) The large scale flash statistics for new storm data supplied by NASA will be processed with SASEV.

(3) The instantaneous flash rate, that is, the flashes per minute, will be estimated for the storms on which we have data and plotted as a function of time. A program to perform this task is nearly completed. In addition to providing a history of the storm intensity for NASA, the results will indicate to us if the tests performed by SASEV were influenced by trends.

(4) A significant effort will be devoted to investigating the internal flash structure. Statistical models for the times between pulses and amplitudes of the pulses in the leader process will be developed using the results of SASEV. The sequences of return strokes will be investigated similarly.

(6) If time and computer funds permit, we will perform a spectral analysis on the digitized storm data supplied by NASA. Spectral analysis can often uncover interesting signal characteristics.

(7) We will investigate modeling the entire lightning signal by a branching renewal process. A branching renewal process is a point process in which each event in a primary sequence of events spawns a secondary sequence of events. The primary events would be the beginnings of flashes and the secondary events would be the pulses within a flash. Methods for estimating process parameters will be investigated.

(8) The leader process contains a sequence of similarly shape pulses with random amplitudes and spacing. We believe that the shape of these pulses is the system impulse response. The pulses often overlap and are corrupted by noise. We would like to investigate the problem of optimally estimating the impulse response of a system driven by a continuous-time point process when the observed data is the system output corrupted by additive noise and sampled at a uniform rate. To our knowledge, this problem has not been previously solved.

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APPENDIX 1

TABLES

TABLE 1

Storm of August 5, 1975

Some Statistics Generated by SASEV

(a) Time Between Flashes (Seconds)

TAPE	N	MEAN μ	STANDARD DEVIATION σ	COEFF. OF VARIATION σ/μ	COEFF. OF SKEWNESS μ_3/σ^3	COEFF. OF KURTOSIS $\mu_4/\sigma^4 - 3$
1	97	3.4876	3.0484	0.8741	2.0935	8.9313
2	410	2.9318	2.3470	0.8005	1.3940	4.9723
3	506	1.7802	1.6425	0.9226	1.9734	7.9215
4	240	1.4079	1.3991	0.9937	1.9350	6.8081
5	228	6.1395	6.8915	1.1225	1.6741	5.6559

(b) Peak Amplitude of Flashes (In Strip Chart Amplitude Units)

TAPE	N	MEAN μ	STANDARD DEVIATION σ	COEFF. OF VARIATION σ/μ	COEFF. OF SKEWNESS μ_3/σ^3	COEFF. OF KURTOSIS $\mu_4/\sigma^4 - 3$
1	97	14.9897	5.0796	0.3389	0.6523	2.3001
2	410	11.7829	5.6624	0.4806	0.7579	2.5375
3	506	10.3945	6.5425	0.6294	1.3719	3.5116
4	240	8.8500	5.1823	0.5856	1.9760	5.9478
5	228	10.2939	6.1319	0.5957	1.0990	2.9002

(c) Duration of Flashes (Seconds)

TAPE	N	MEAN μ	STANDARD DEVIATION σ	COEFF. OF VARIATION σ/μ	COEFF. OF SKEWNESS μ_3/σ^3	COEFF. OF KURTOSIS $\mu_4/\sigma^4 - 3$
1	97	0.4299	0.2732	0.6354	0.8323	2.8771
2	410	0.3624	0.2400	0.6622	0.9291	3.3349
3	506	0.2544	0.2288	0.8990	1.9212	7.1981
4	240	0.2033	0.2263	1.1128	3.3910	16.9794
5	228	0.2272	0.2001	0.8809	2.0493	7.0931

Storm of August 5, 1975
Kolmogorov-Smirnov and Anderson-Darling Goodness-of-Fit Tests
for Exponential Density Based on Uniform Conditional Property

(a) Time Between Flashes

TAPE	N	KS	WN2	WITH DURBIN'S TRANSFORMATION DN	WN2	APPLIED TO PERIODOGRAM KS	WN2
1	97	0.6321029	0.383976	1.401567	3.550858	0.8107939	0.9660473
2	410	1.045668	2.098625	2.639786	16.57763	0.6290504	0.4334488
3	506	0.6995085	0.7348518	3.317009	12.56082	0.3746214	0.1418037
4	240	0.8894655	1.157148	2.196101	4.452011	0.5797793	0.2736702
5	228	2.877865	26.99329	2.004672	7.86273	2.034177	6.590207

(b) Peak Amplitude of Flashes

1	97	0.2439943	0.0568504	5.440877	60.25202	---------	---------
2	410	----------	----------	---------	---------	1.558849	4.433937
3	506	0.7116409	0.7573547	10.82038	---------	0.8282011	0.8978100
4	240	0.5034376	0.3427868	8.734251	---------	0.5788312	0.2426233
5	228	0.5596259	0.4229412	7.318207	---------	0.5745411	0.3834915

(c) Duration of Flashes

1	97	0.7276520	0.7565508	2.622042	---------	----------	---------
2	410	0.8409247	1.455425	5.579909	---------	0.7827463	0.9495754
3	506	1.632041	4.418961	8.840851	56.30646	0.9280081	1.249138
4	240	1.015736	0.8823013	7.603098	48.96658	0.7042681	0.7879419
5	228	0.9044824	0.9611149	6.631598	32.84324	0.6494161	0.5272541

TABLE 3

Asymptotic Significance Levels for the

Kolmogorov-Smirnov Statistic

z	$P(D_N > z)$
.10	.10000000+01
.15	.10000000+01
.20	.10000000+01
.25	.99999999+00
.30	.99999070+00
.35	.99969714+00
.40	.99719233+00
.45	.98741063+00
.50	.96394525+00
.55	.92281682+00
.60	.86428262+00
.65	.79201308+00
.70	.71123525+00
.75	.62716711+00
.80	.54414248+00
.85	.46531929+00
.90	.39273078+00
.95	.32748555+00
1.00	.26999973+00
1.05	.22020562+00
1.10	.17771825+00
1.15	.14195991+00
1.20	.11224971+00
1.25	.87866448-01
1.30	.68092250-01
1.35	.52241911-01
1.40	.39681899-01
1.45	.29841489-01
1.50	.22217975-01
1.55	.16377412-01
1.60	.11952054-01
1.65	.86356858-02
1.70	.61774354-02
1.75	.43749856-02
1.80	.30676239-02
1.85	.21295343-02
1.90	.14636061-02
1.95	.99591177-03
2.00	.67092590-03
2.05	.44749213-03
2.10	.29549713-03
2.15	.19318713-03
2.20	.12504322-03
2.25	.80130746-04
2.30	.50638800-04
2.35	.31933641-04
2.40	.19859059-04
2.45	.12227169-04
2.50	.74533286-05
2.55	.44981264-05

APPENDIX 2

FIGURES

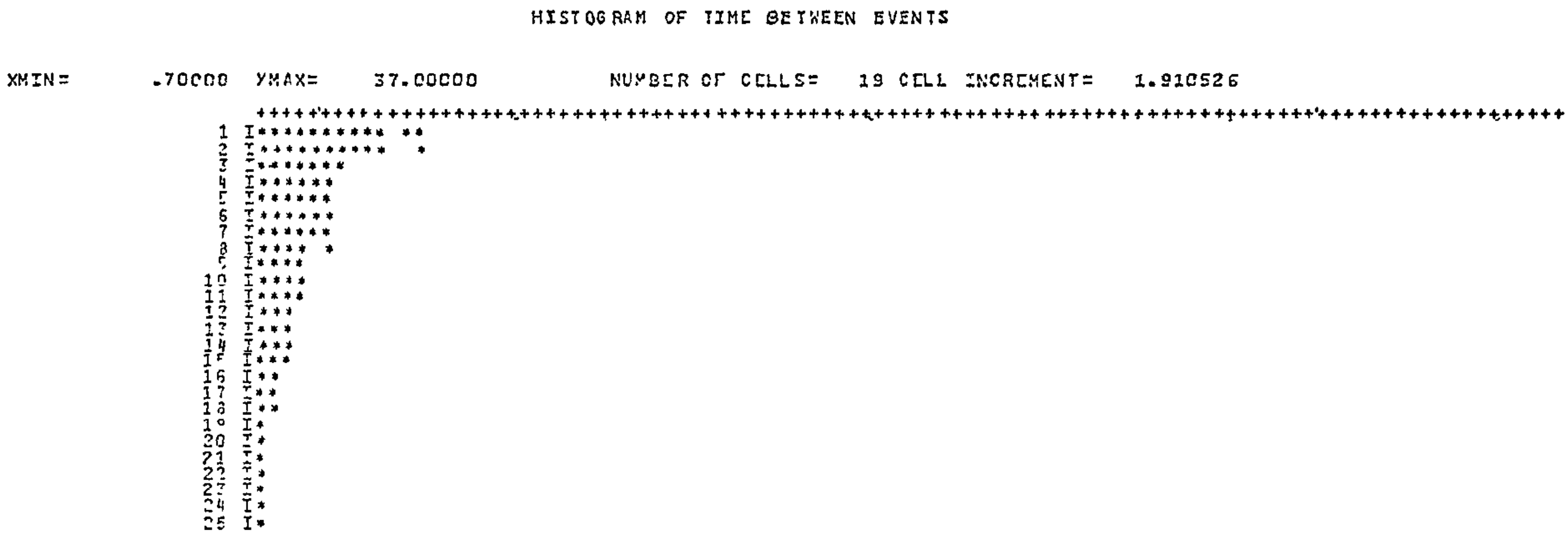


FIG.1 HISTOGRAM OF TIME BETWEEN EVENTS (TAPE 1)

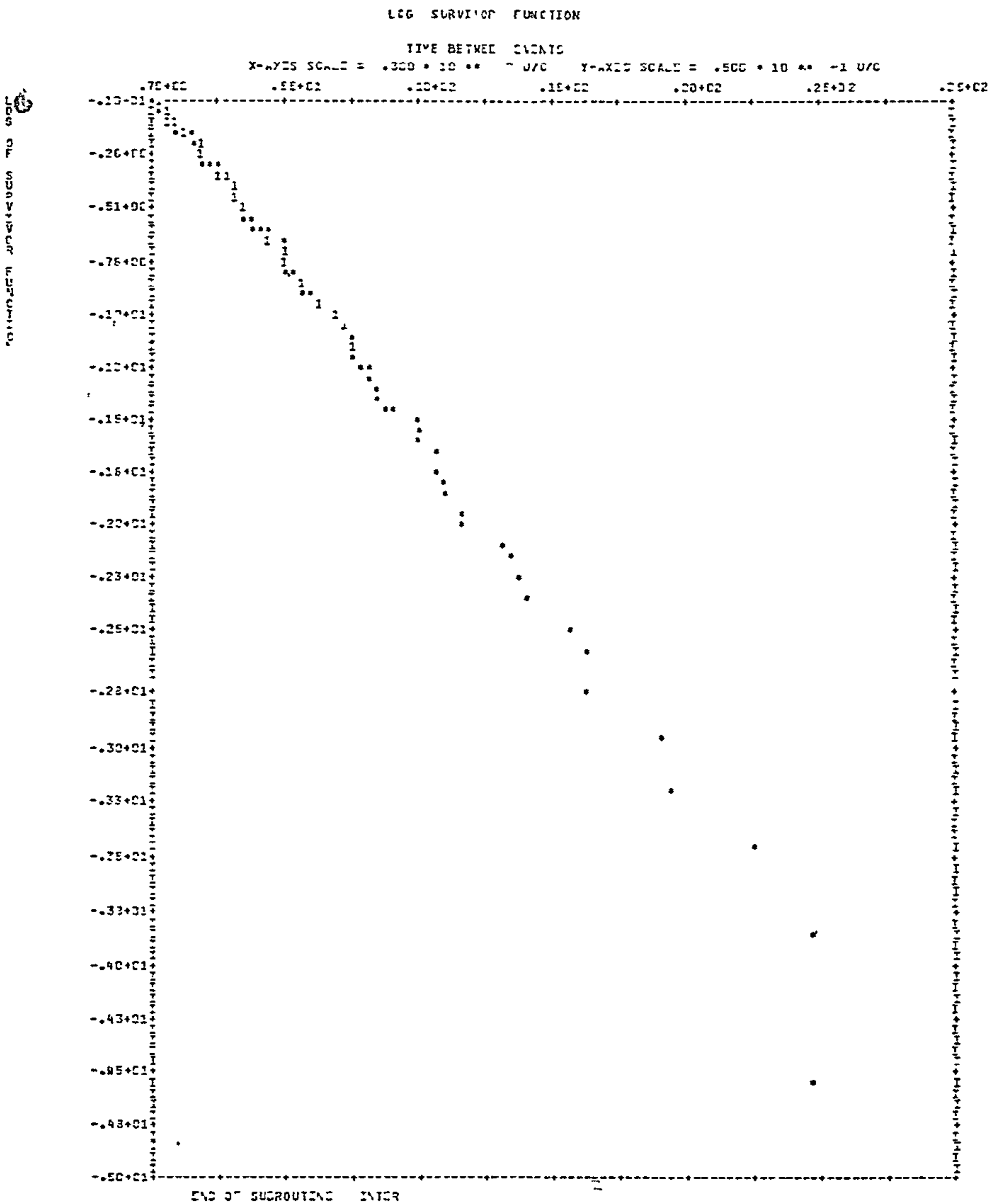


FIG.2 LOG SURVIVOR FUNCTION (TAPE 1)

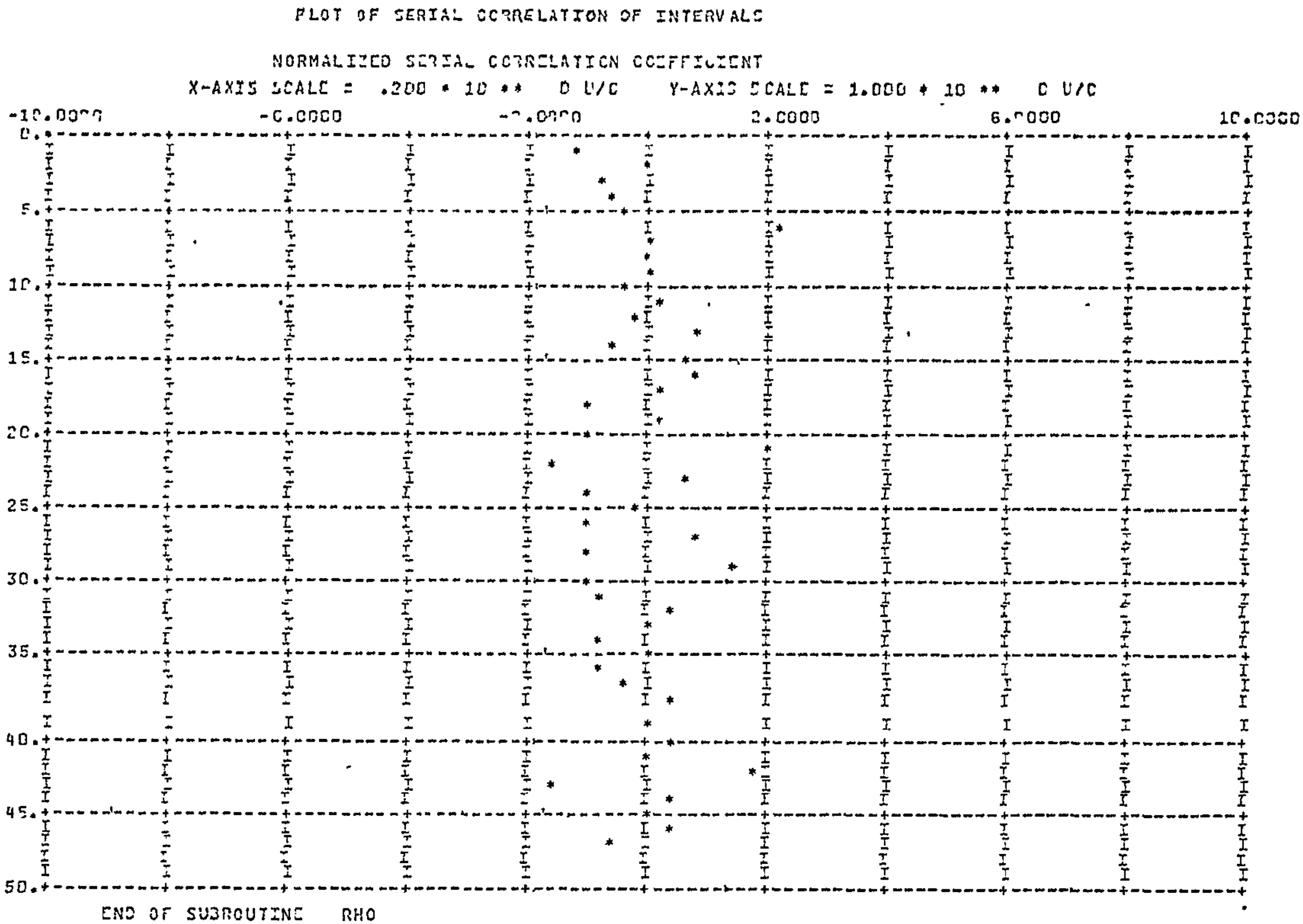


FIG.3 SERIAL CORRELATION OF INTERVALS (TAPE 1)

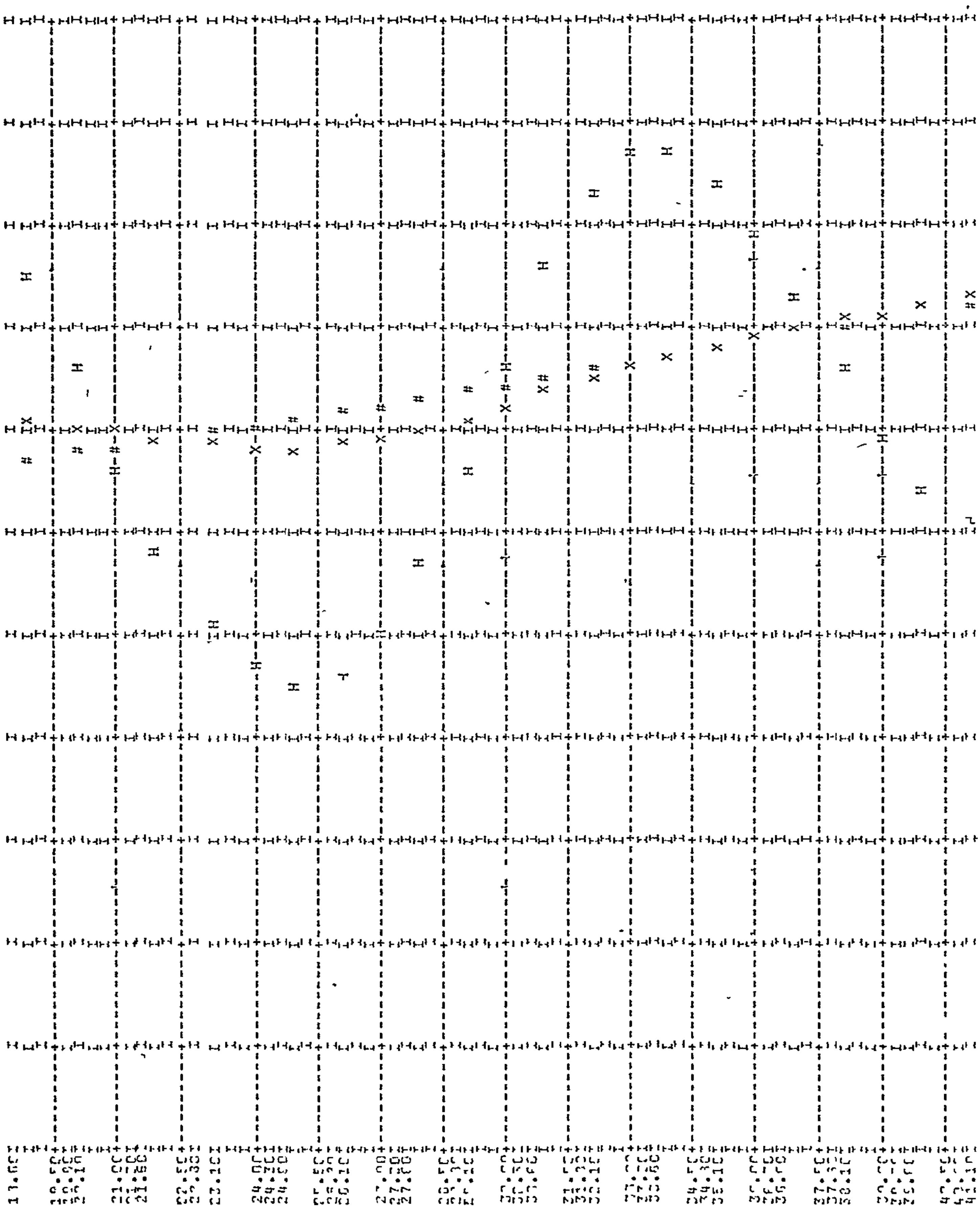


FIG.4 (CONT.)

PRECEDING PAGE BLANK NOT FILMED

FIG.4 (CONT.)

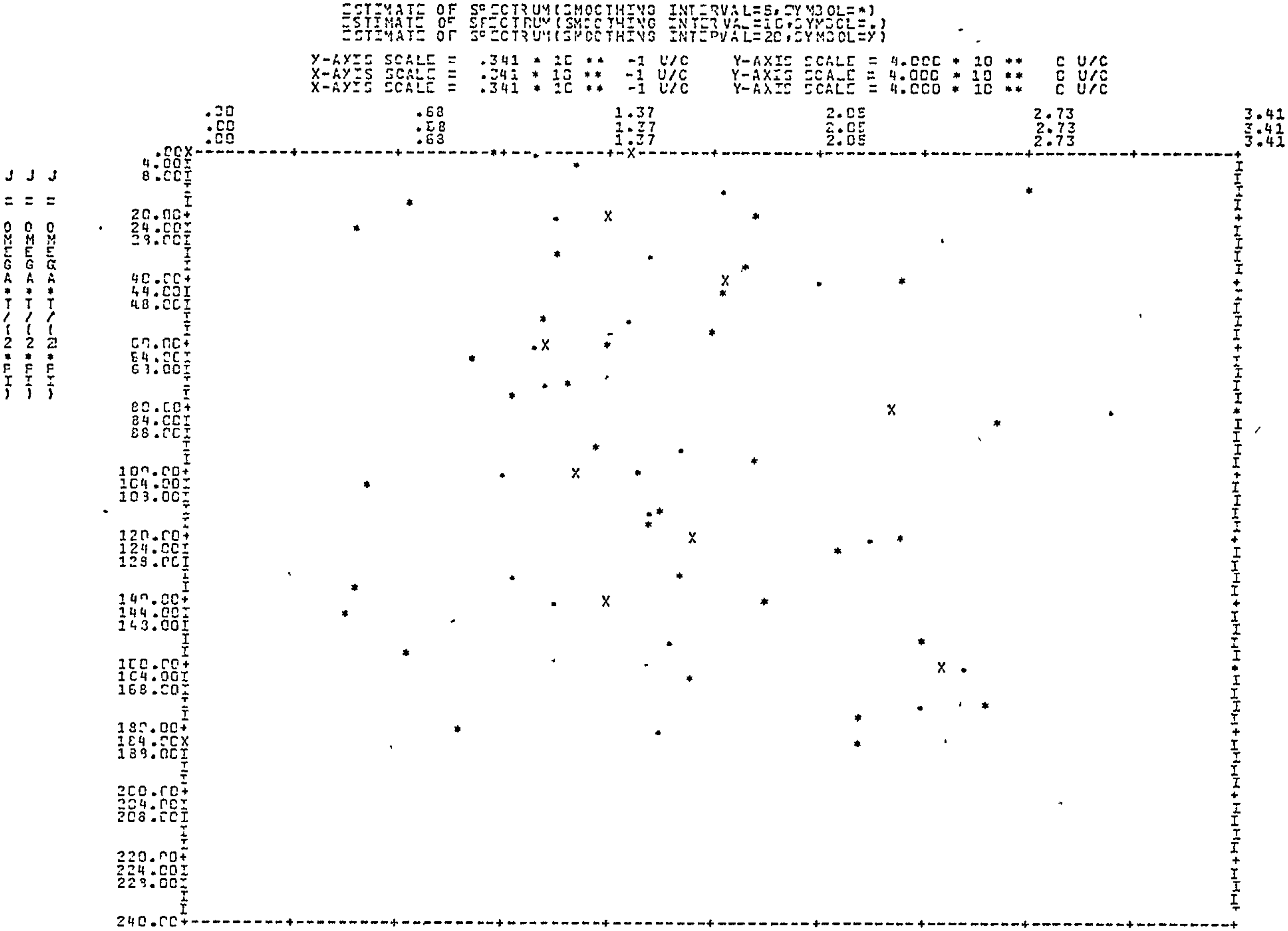


FIG.5 SPECTRUM OF THE COUNTING PROCESS (TAPE 1)

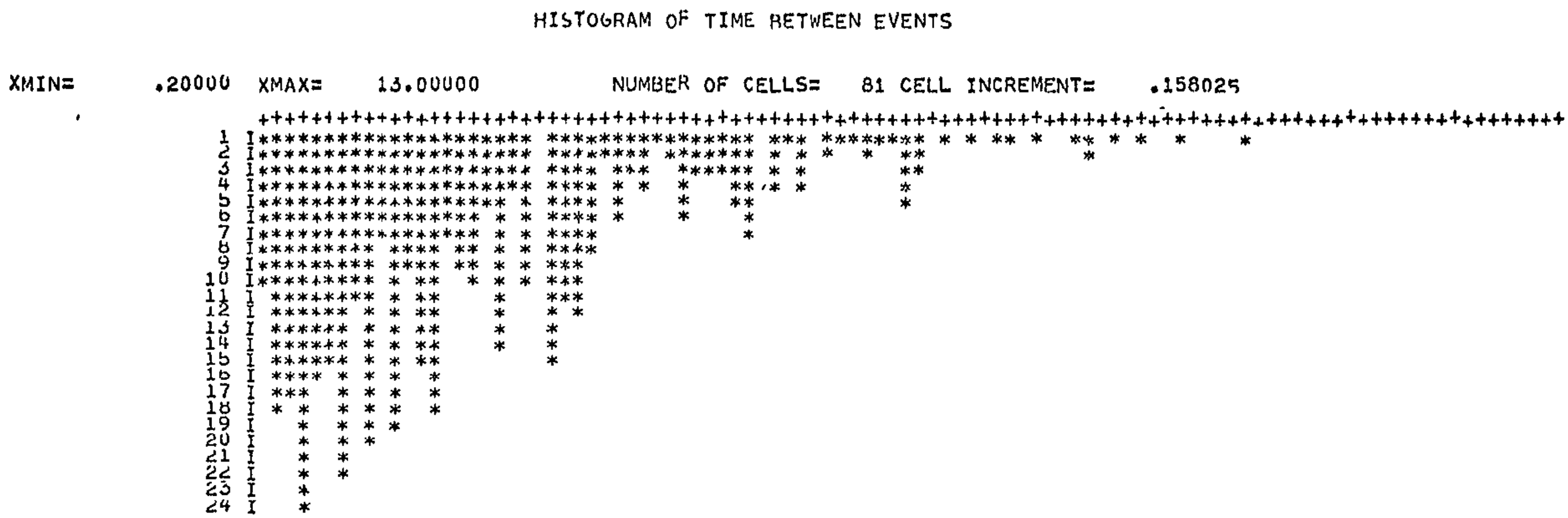


FIG.6 HISTOGRAM OF TIME BETWEEN EVENTS (TAPE 2)

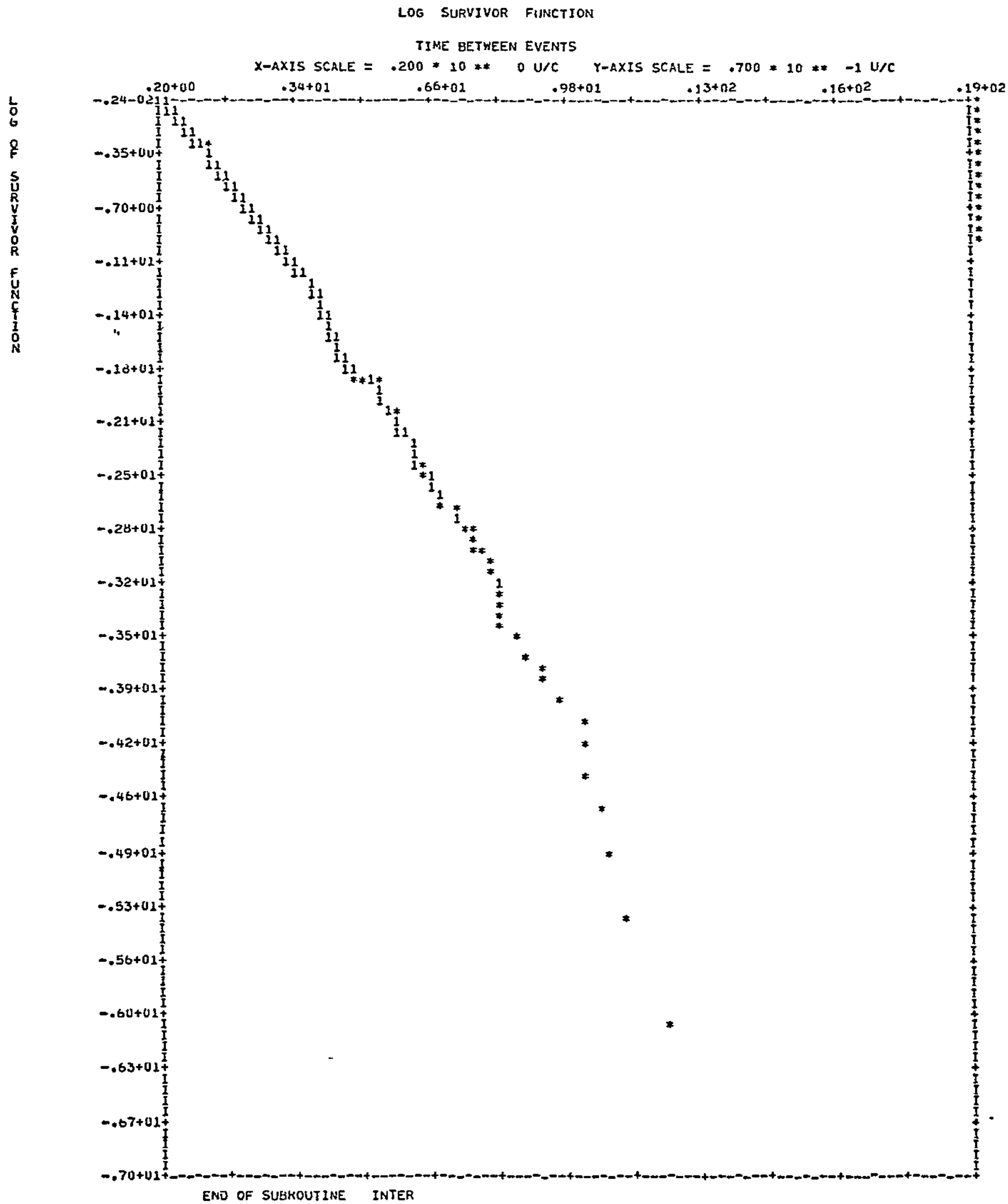


FIG.7 LOG SURVIVOR FUNCTION (TAPE 2)

PLOT OF SERIAL CORRELATION OF INTERVALS

NORMALIZED SERIAL CORRELATION COEFFICIENT

X-AXIS SCALE = .200 * 10 ** 0 U/C Y-AXIS SCALE = 1.000 * 10 ** 0 U/C

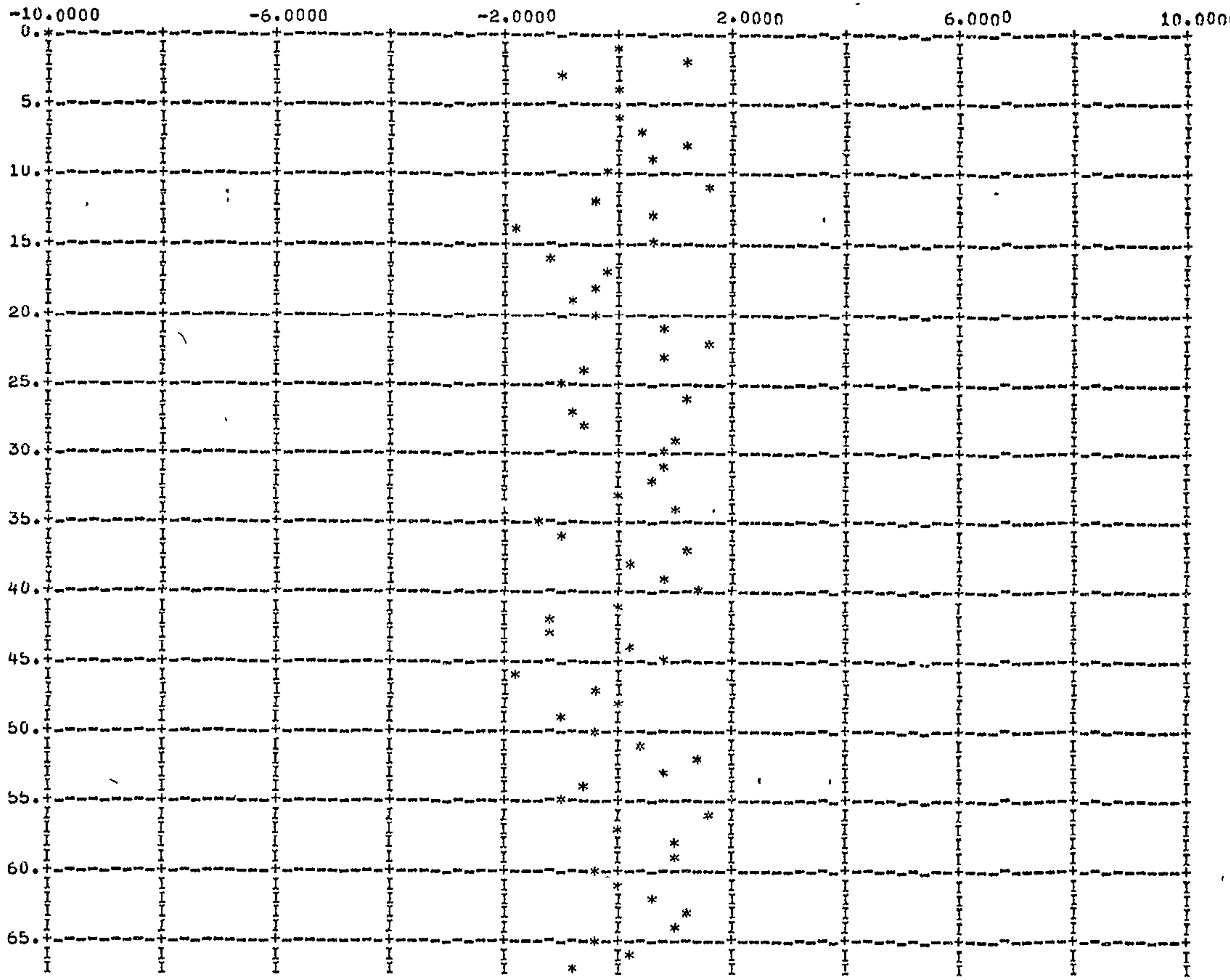


FIG.8 SERIAL CORRELATION OF INTERVALS (TAPE 2)

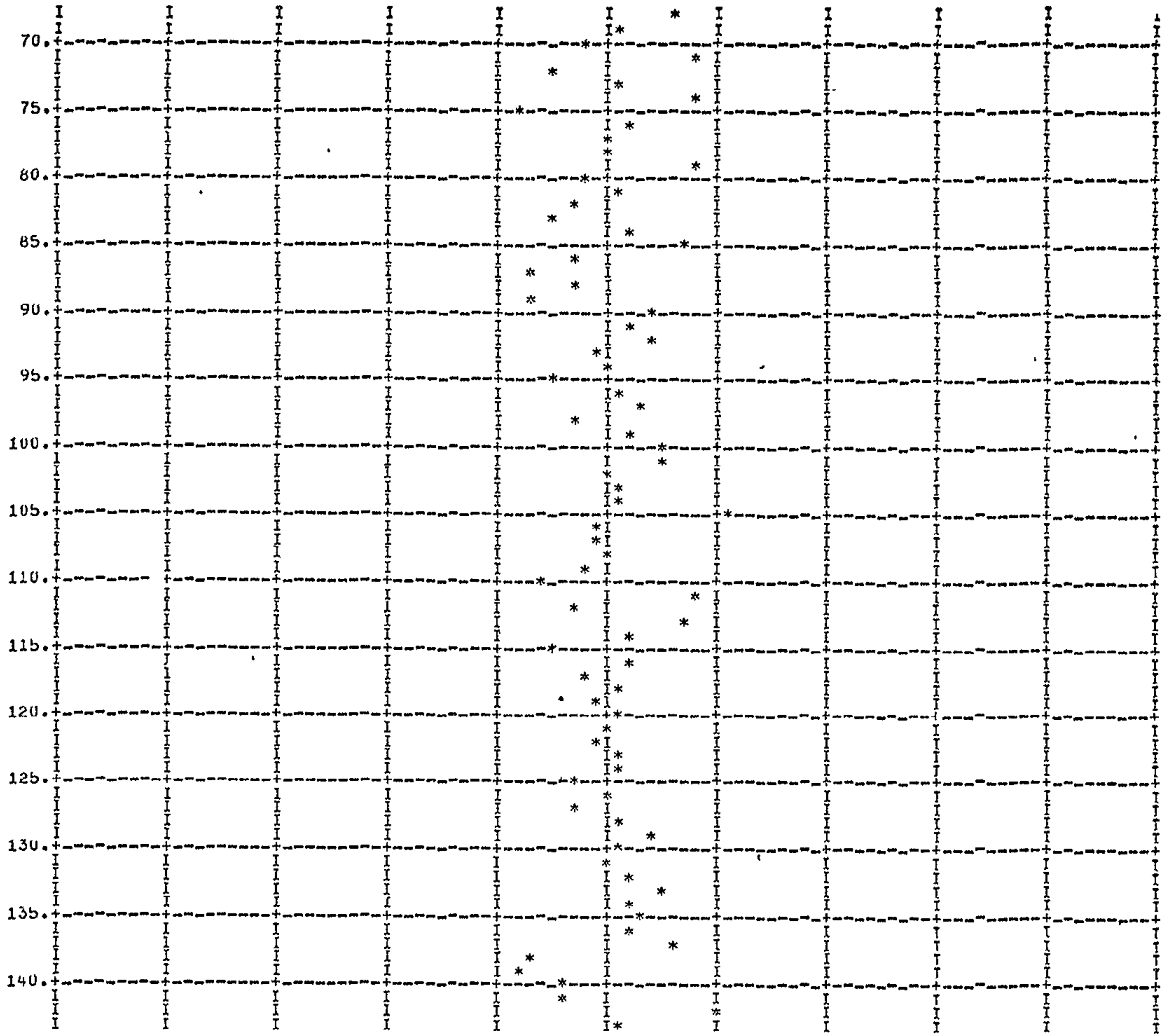
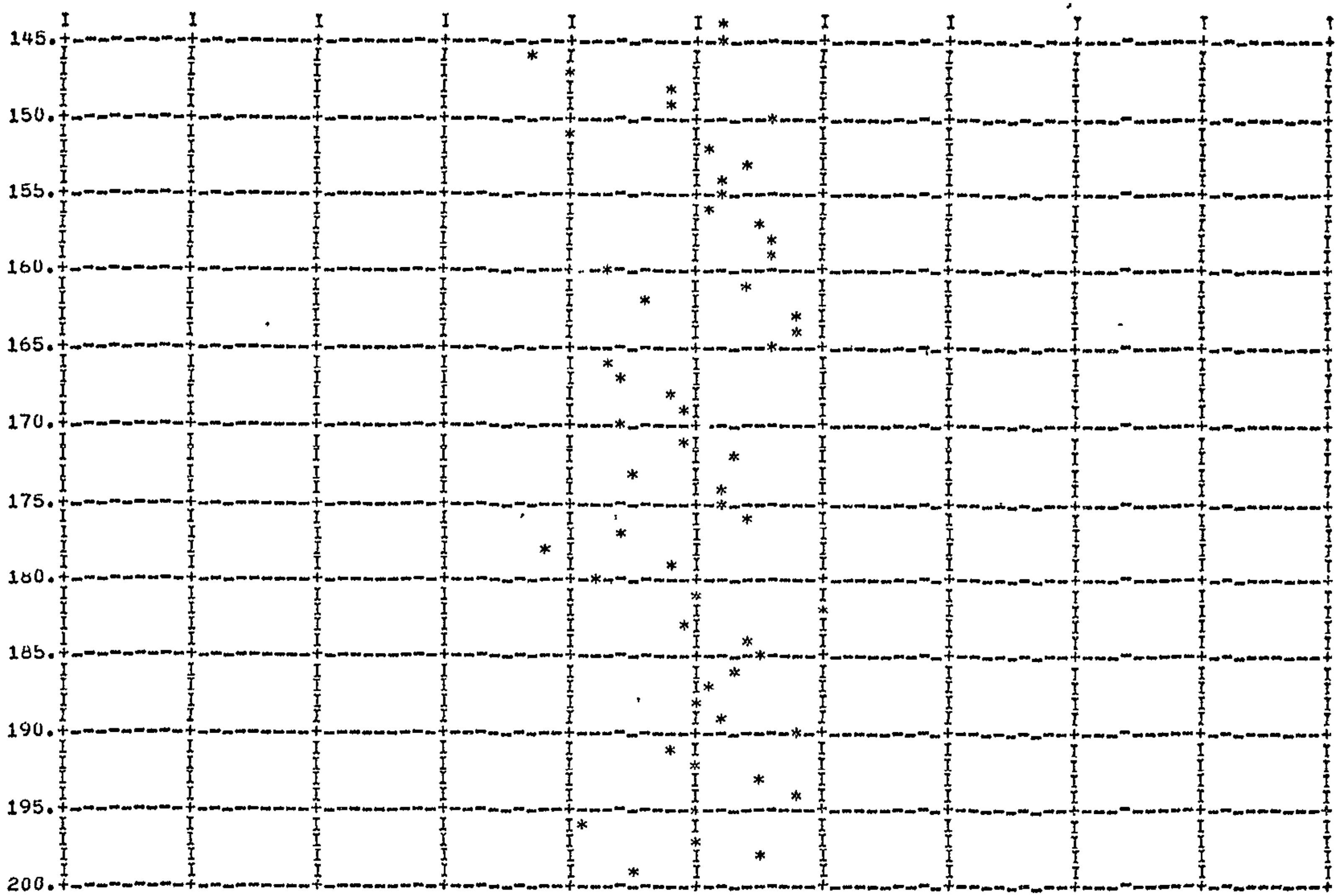


FIG.8 (CONT.)



END OF SUBROUTINE RHO

FIG.8 (CONT.)

SMOOTHED SPECTRUM OF INTERVALS FOR THREE VALUES OF M

MAGNITUDE OF SPECTRUM FOR M1(SYMBOL=#)
MAGNITUDE OF SPECTRUM FOR M2(SYMBOL=X)
MAGNITUDE OF SPECTRUM FOR M3(SYMBOL=H)

X-AXIS SCALE = .439 * 10 ** -2 U/C   Y-AXIS SCALE = 2.000 * 10 ** 0 U/C
X-AXIS SCALE = .439 * 10 ** -2 U/C   Y-AXIS SCALE = 2.000 * 10 ** 0 U/C
X-AXIS SCALE = .439 * 10 ** -2 U/C   Y-AXIS SCALE = 2.000 * 10 ** 0 U/C

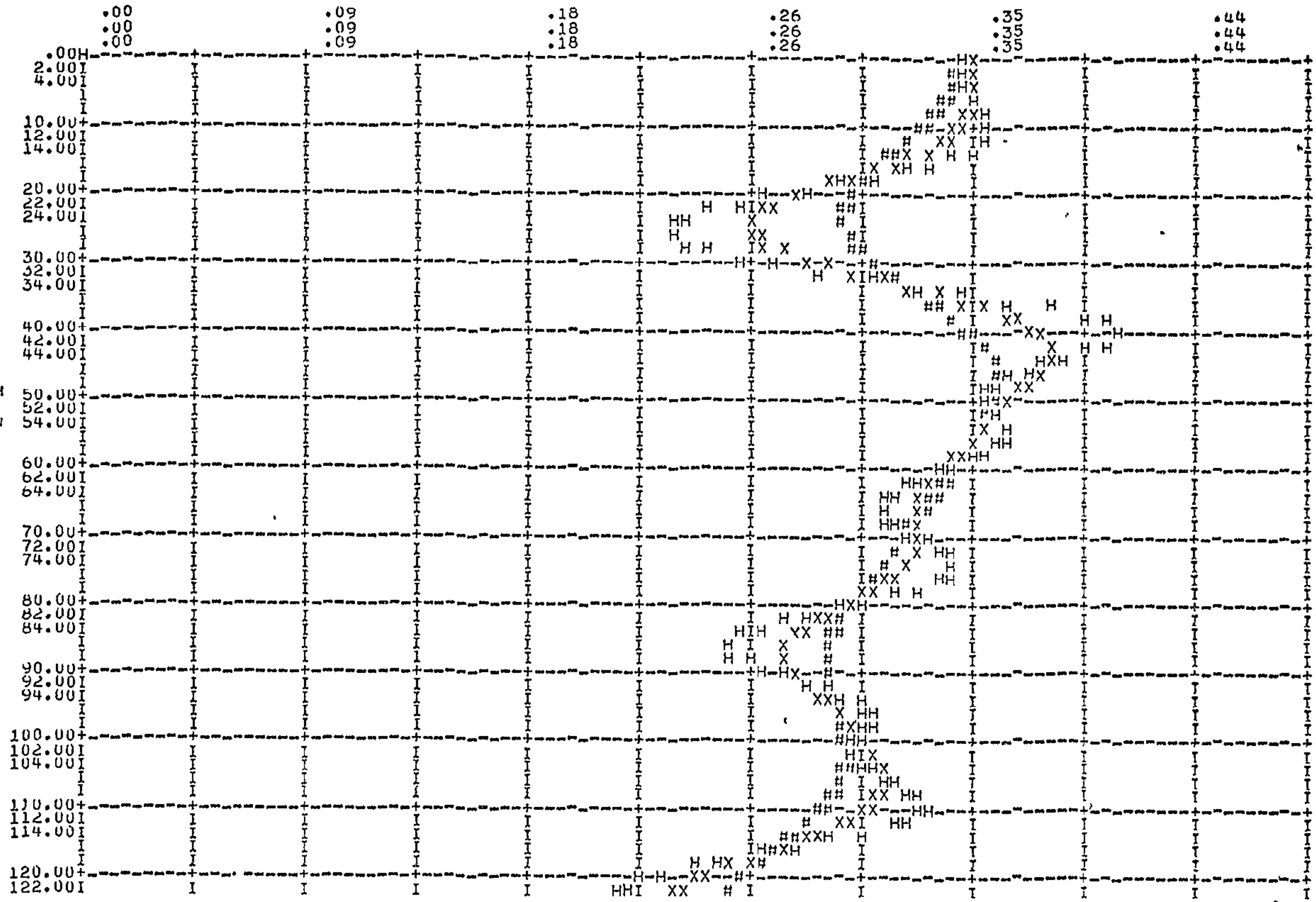


FIG.9 SPECTRUM OF INTERVALS (TAPE 2)

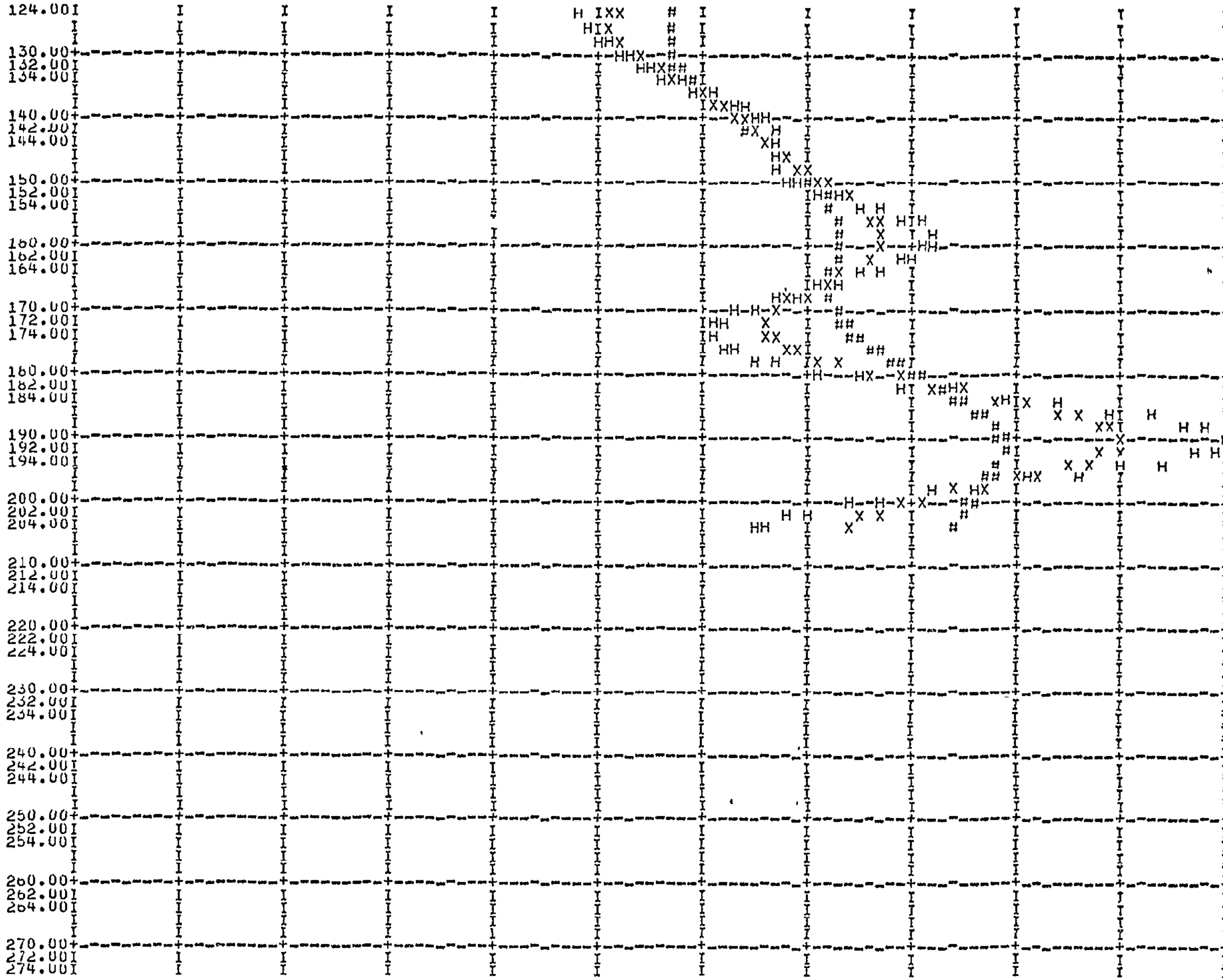
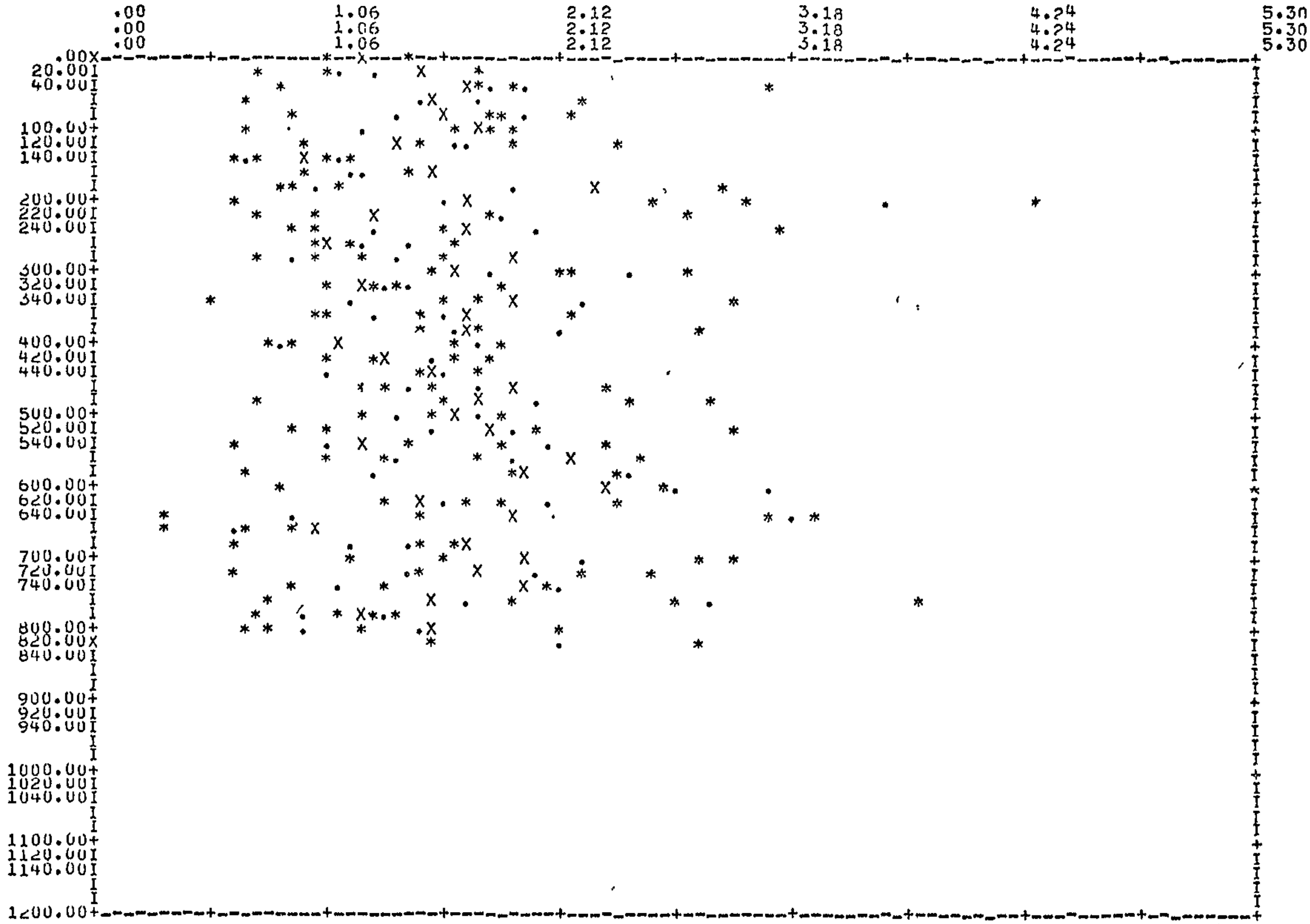


FIG.9 (CONT.)

ESTIMATE OF SPECTRUM(SMOOTHING INTERVAL=5,SYMBOL=*)
ESTIMATE OF SPECTRUM(SMOOTHING INTERVAL=10,SYMBOL=.)
ESTIMATE OF SPECTRUM(SMOOTHING INTERVAL=20,SYMBOL=X)

X-AXIS SCALE = .530 * 10 ** -1 U/C   Y-AXIS SCALE = 2.000 * 10 ** 1 U/C
X-AXIS SCALE = .530 * 10 ** -1 U/C   Y-AXIS SCALE = 2.000 * 10 ** 1 U/C
X-AXIS SCALE = .530 * 10 ** -1 U/C   Y-AXIS SCALE = 2.000 * 10 ** 1 U/C



J = OMEGA*T/(2*PI)
J = OMEGA*T/(2*PI)
J = OMEGA*T/(2*PI)

FIG.10 SPECTRUM OF THE COUNTING PROCESS (TAPE 2)

HISTOGRAM OF TIME BETWEEN EVENTS

XMIN= .20000 XMAX= 11.30000 NUMBER OF CELLS= 100 CELL INCREMENT= .111000

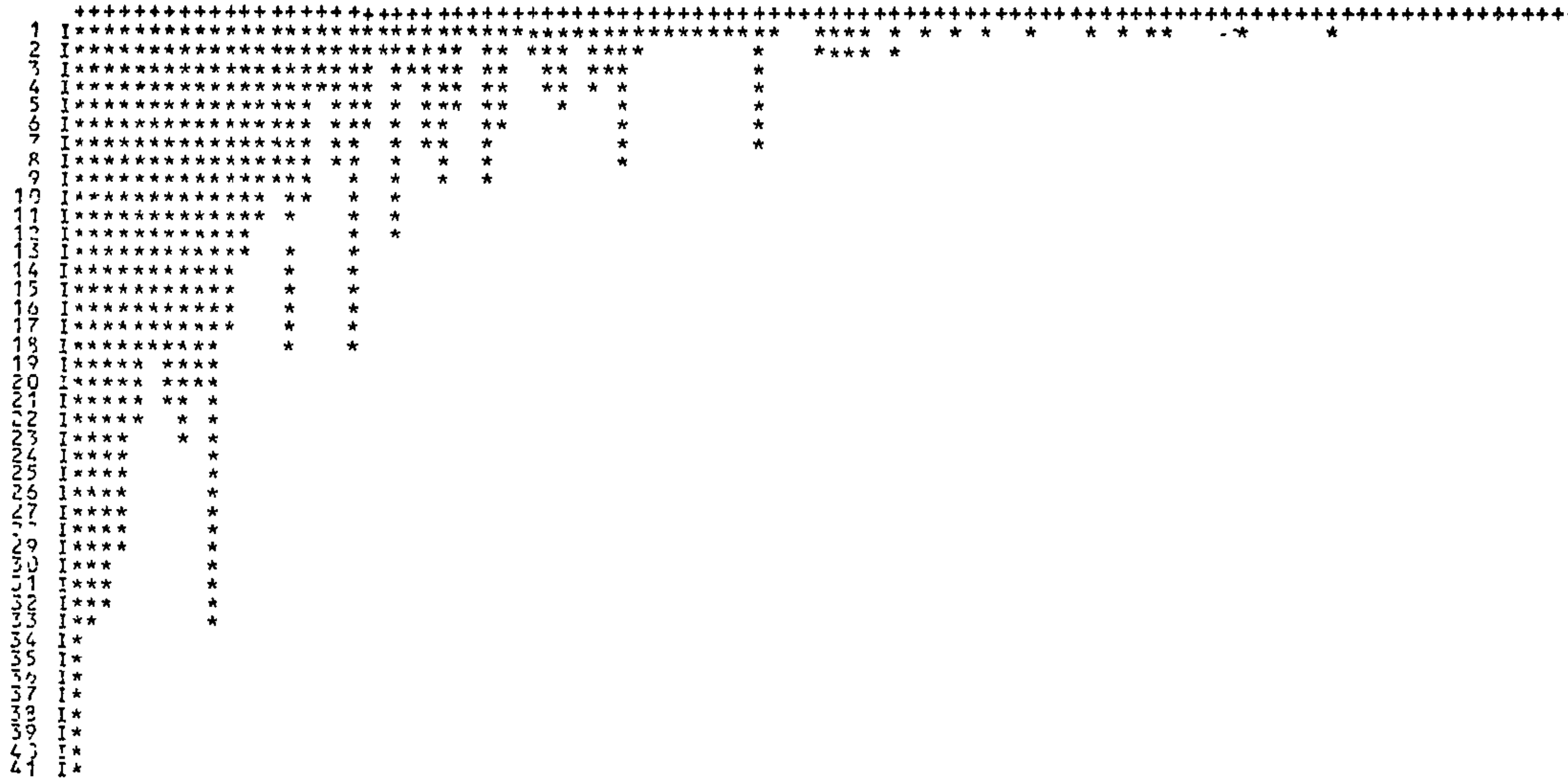


FIG.11 HISTOGRAM OF TIME BETWEEN EVENTS (TAPE 3)

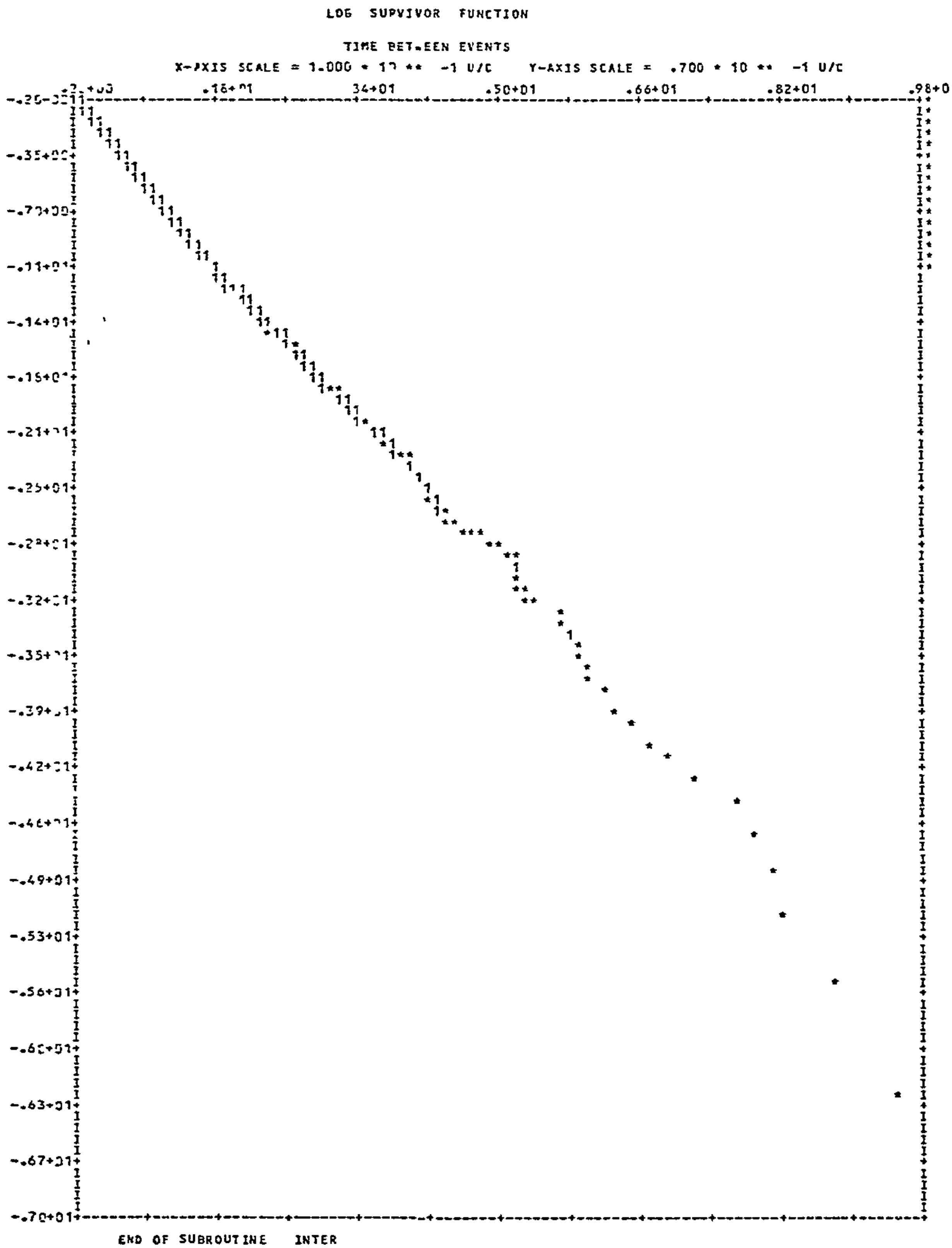


FIG.12 LOG SURVIVOR FUNCTION (TAPE 3)

FIG.13 PLOT OF SERIAL CORRELATION OF INTERVALS (TAPE 3)

NORMALIZED SERIAL CORRELATION COEFFICIENT

X-AXIS SCALE = .200 * 10 ** 0 U/C   Y-AXIS SCALE = 1.000 * 10 ** 0 U/C

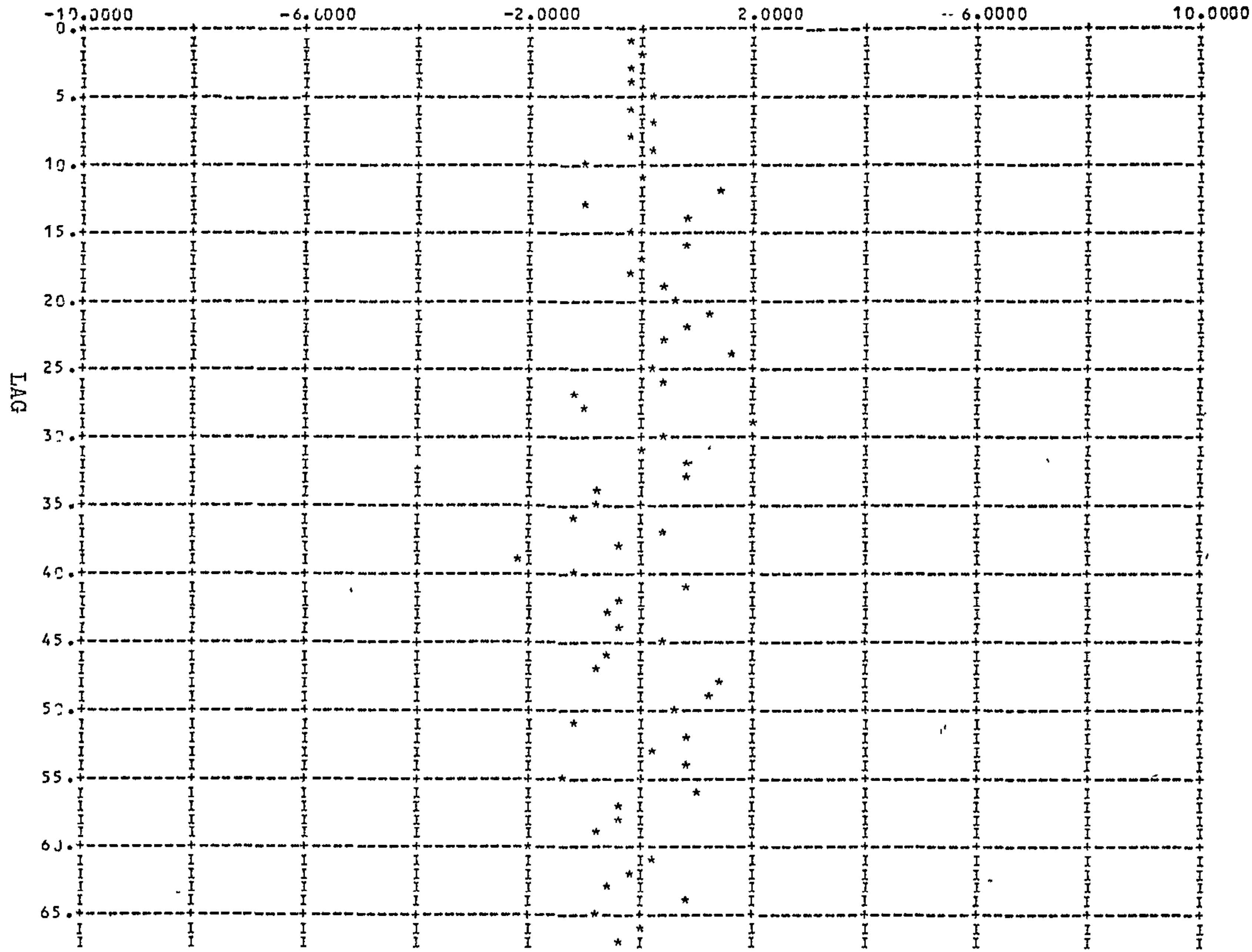


FIG.13 SERIAL CORRELATION OF INTERVALS (TAPE 3)

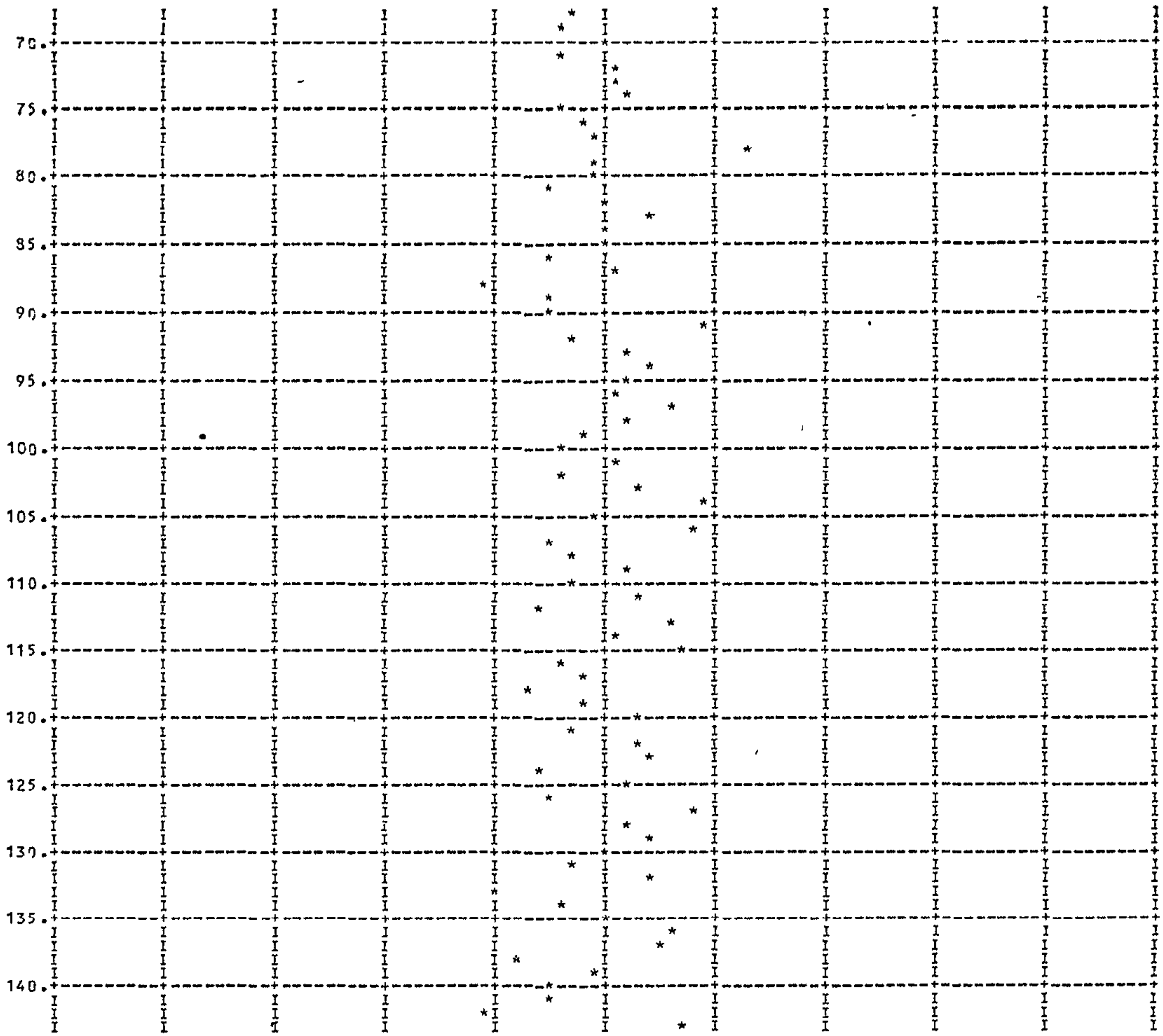
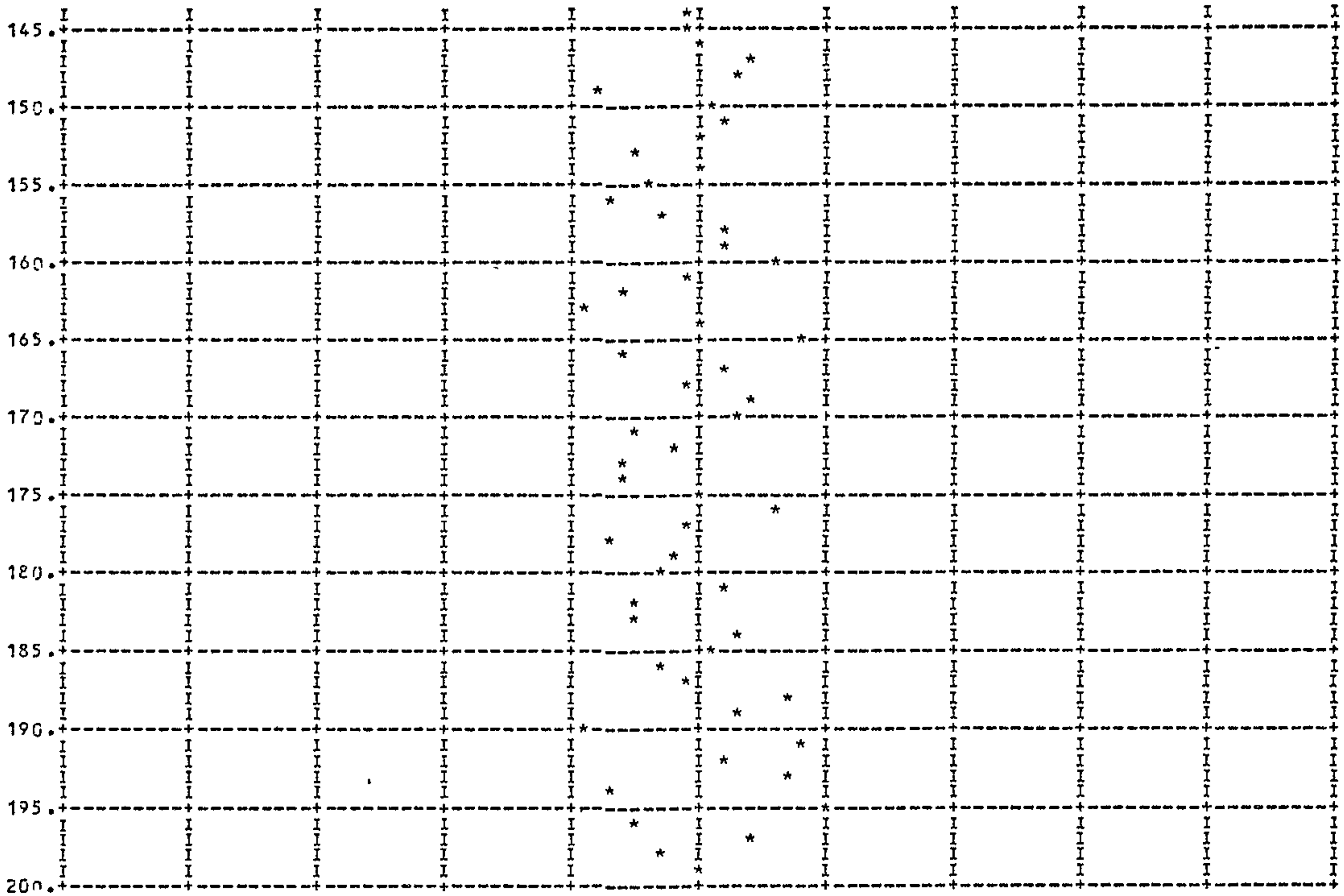


FIG.13 (CONT.)



END OF SUBROUTINE RHO

FIG.13 (CONT.)

SMOOTHED SPECTRUM OF INTERVALS FOR THREE VALUES OF M

MAGNITUDE OF SPECTRUM FOR M1(SYMBOL=#)
MAGNITUDE OF SPECTRUM FOR M2(SYMBOL=X)
MAGNITUDE OF SPECTRUM FOR M3(SYMBOL=H)

X-AXIS SCALE = .368 * 10 ** -2 U/C   Y-AXIS SCALE = 2.000 * 10 ** 0 U/C
X-AXIS SCALE = .368 * 10 ** -2 U/C   Y-AXIS SCALE = 2.000 * 10 ** 0 U/C
X-AXIS SCALE = .368 * 10 ** -2 U/C   Y-AXIS SCALE = 2.000 * 10 ** 0 U/C

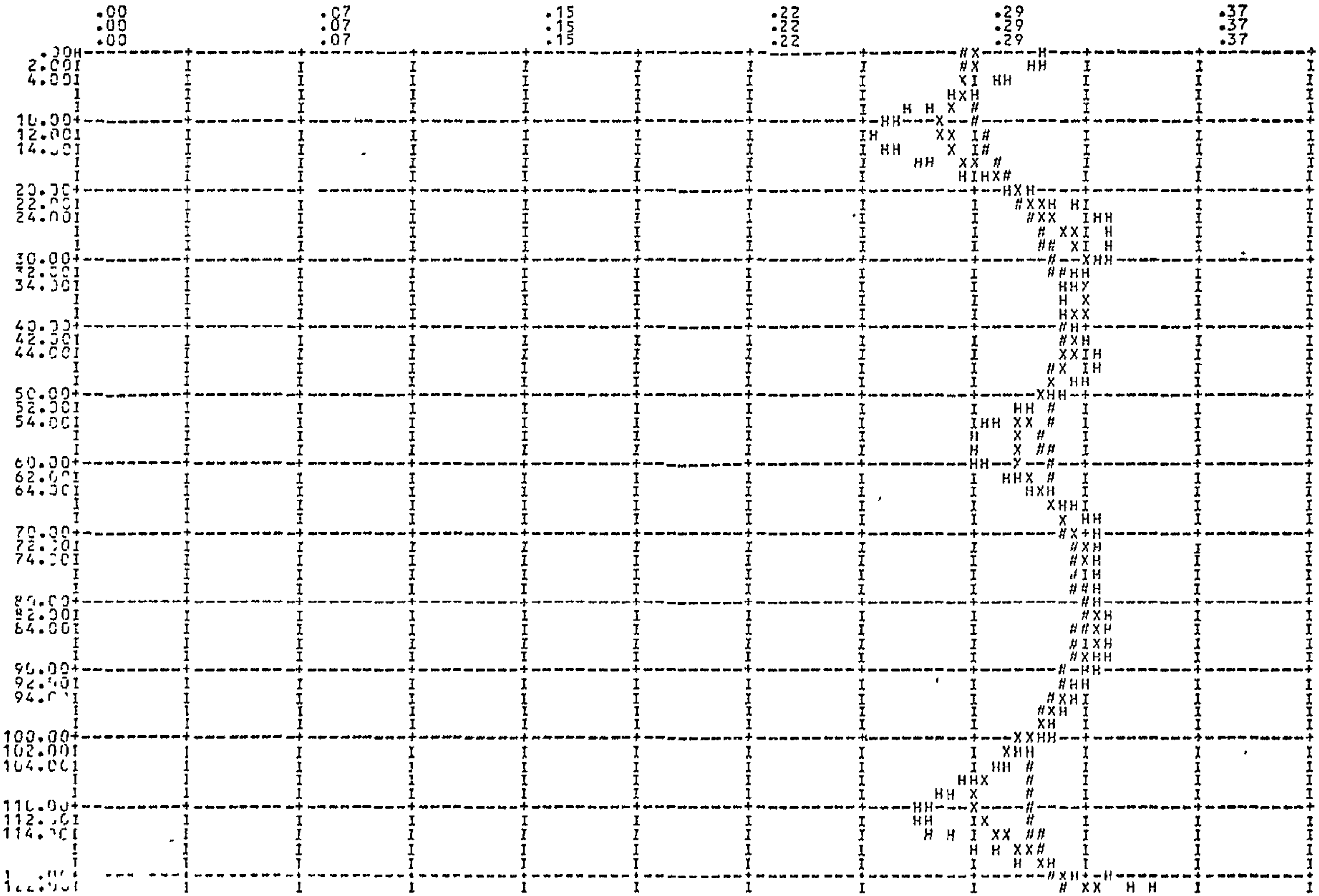


FIG.14 SPECTRUM OF INTERVALS (TAPE 3)

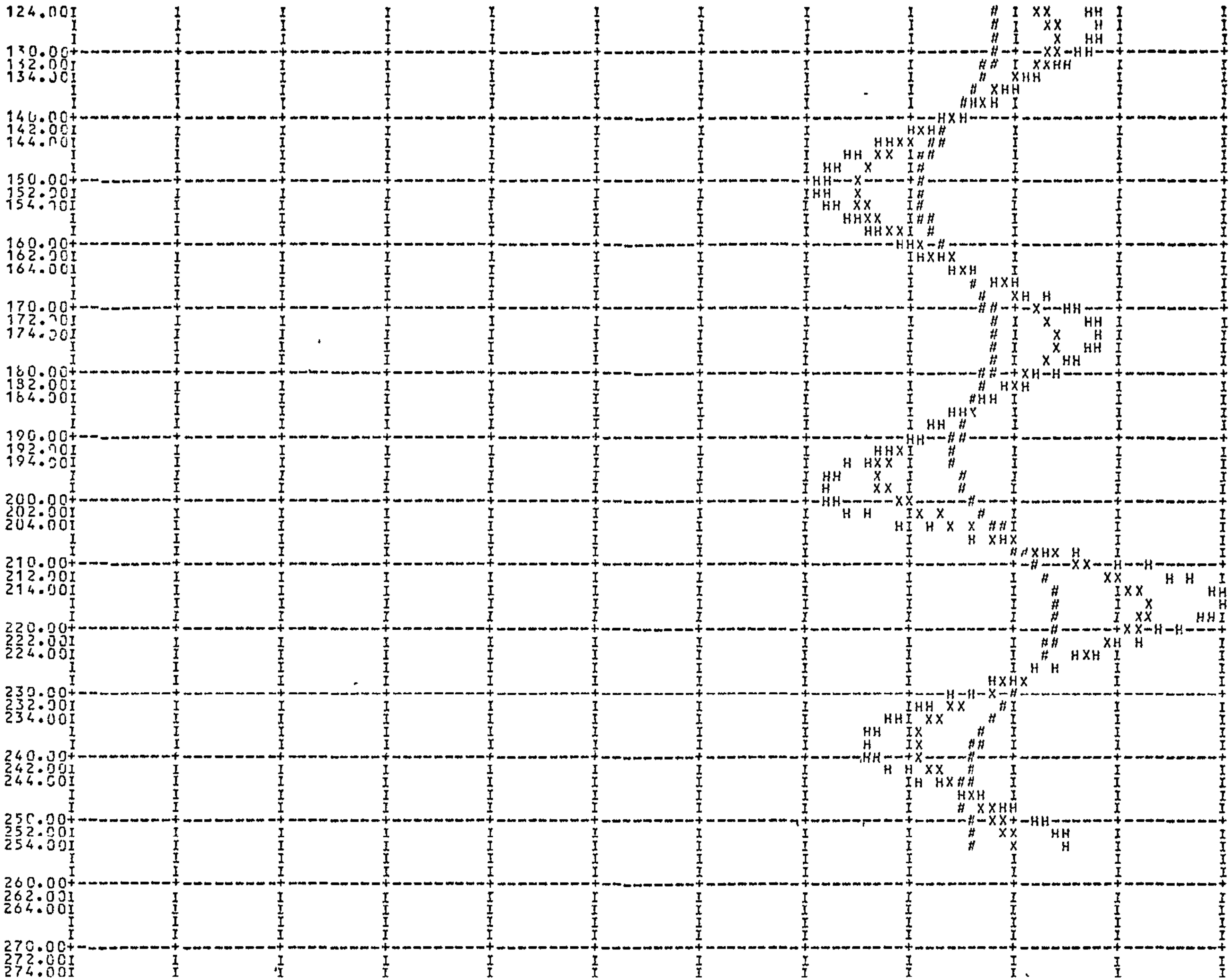

124.00
130.00
132.00
134.00
140.00
142.00
144.00
150.00
152.00
154.00
160.00
162.00
164.00
170.00
172.00
174.00
180.00
182.00
184.00
190.00
192.00
194.00
200.00
202.00
204.00
210.00
212.00
214.00
220.00
222.00
224.00
230.00
232.00
234.00
240.00
242.00
244.00
250.00
252.00
254.00
260.00
262.00
264.00
270.00
272.00
274.00

FIG.14 (CONT.)

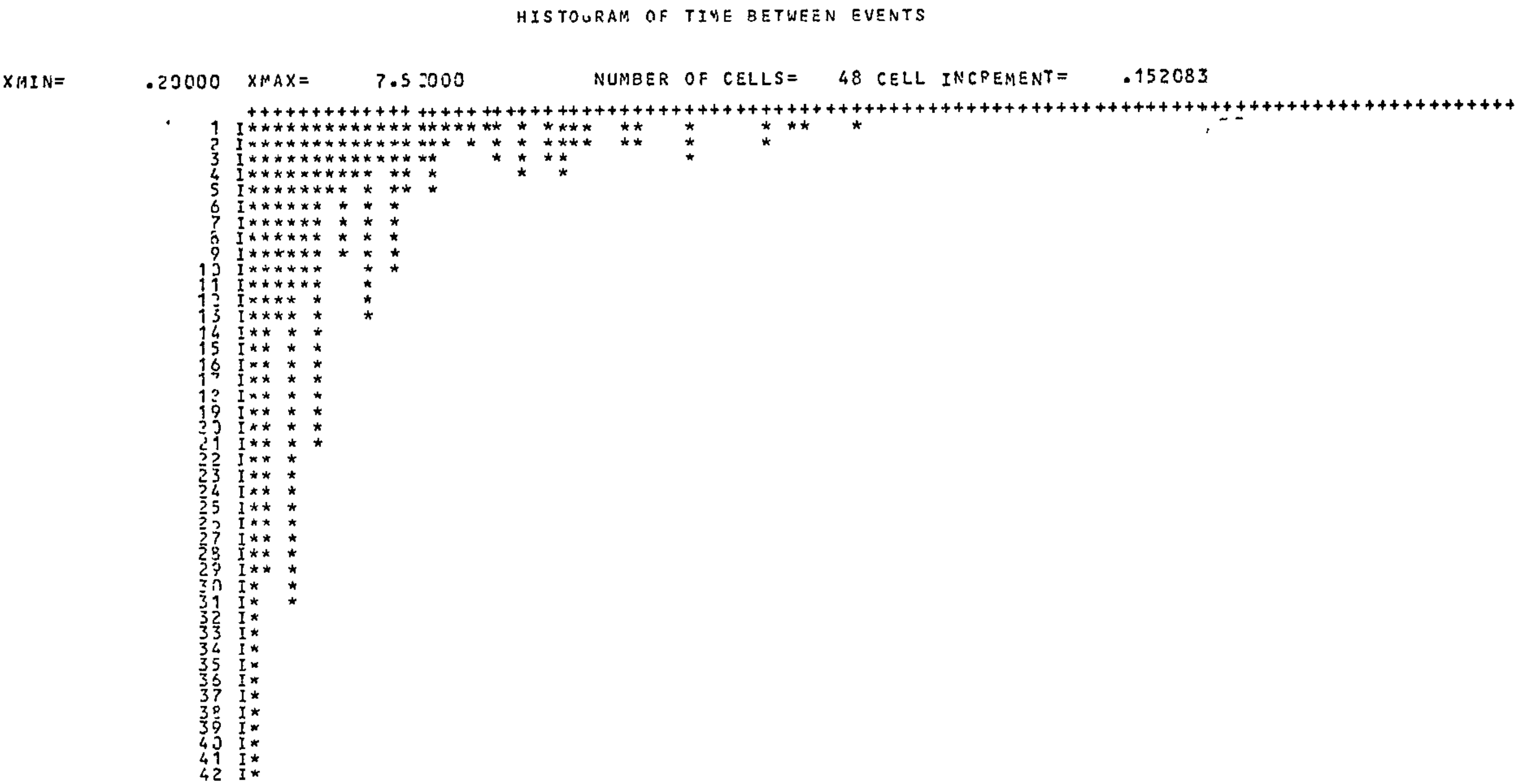


FIG.15 HISTOGRAM OF TIME BETWEEN EVENTS (TAPE 4)

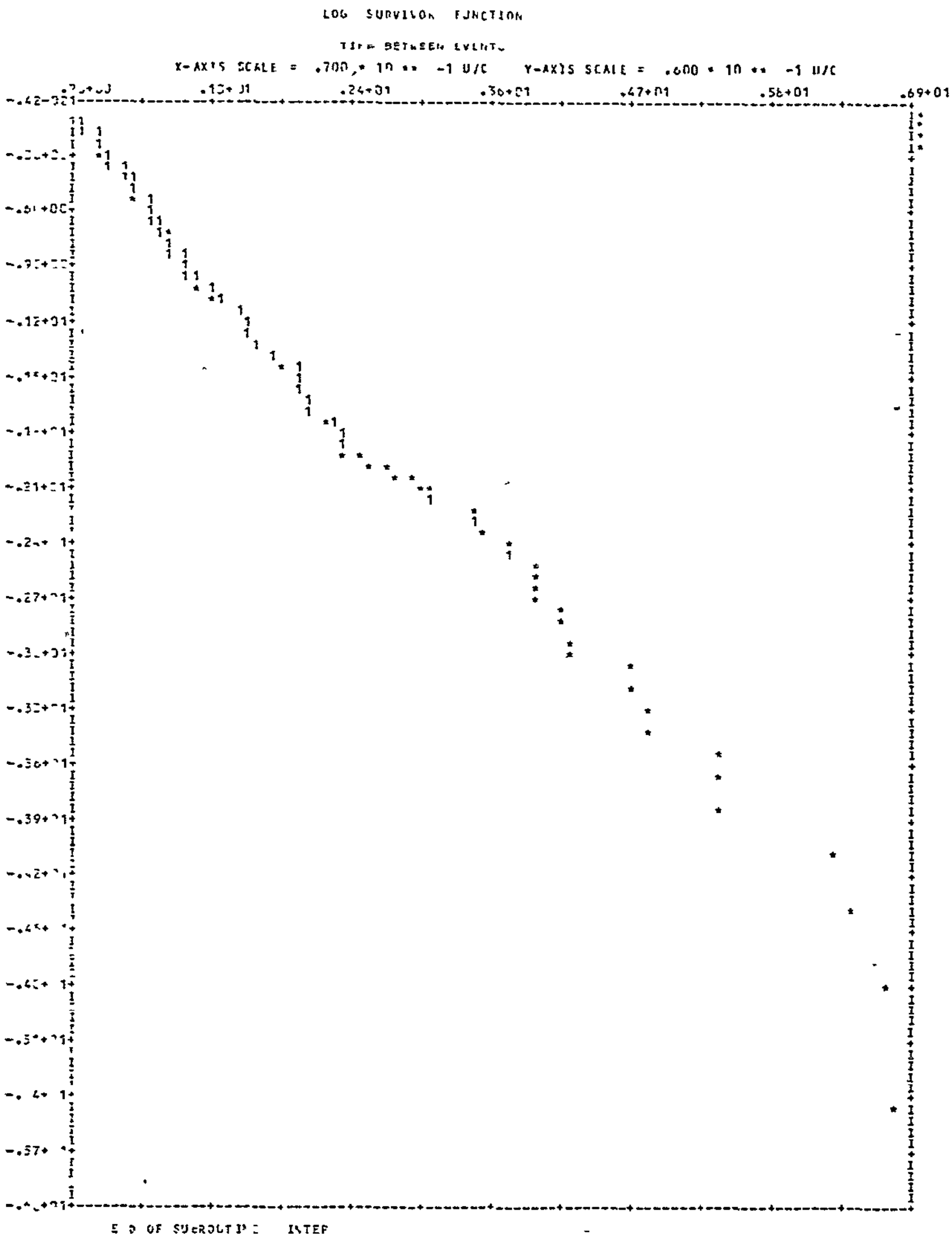


FIG.16 LOG SURVIVOR FUNCTION (TAPE 4)

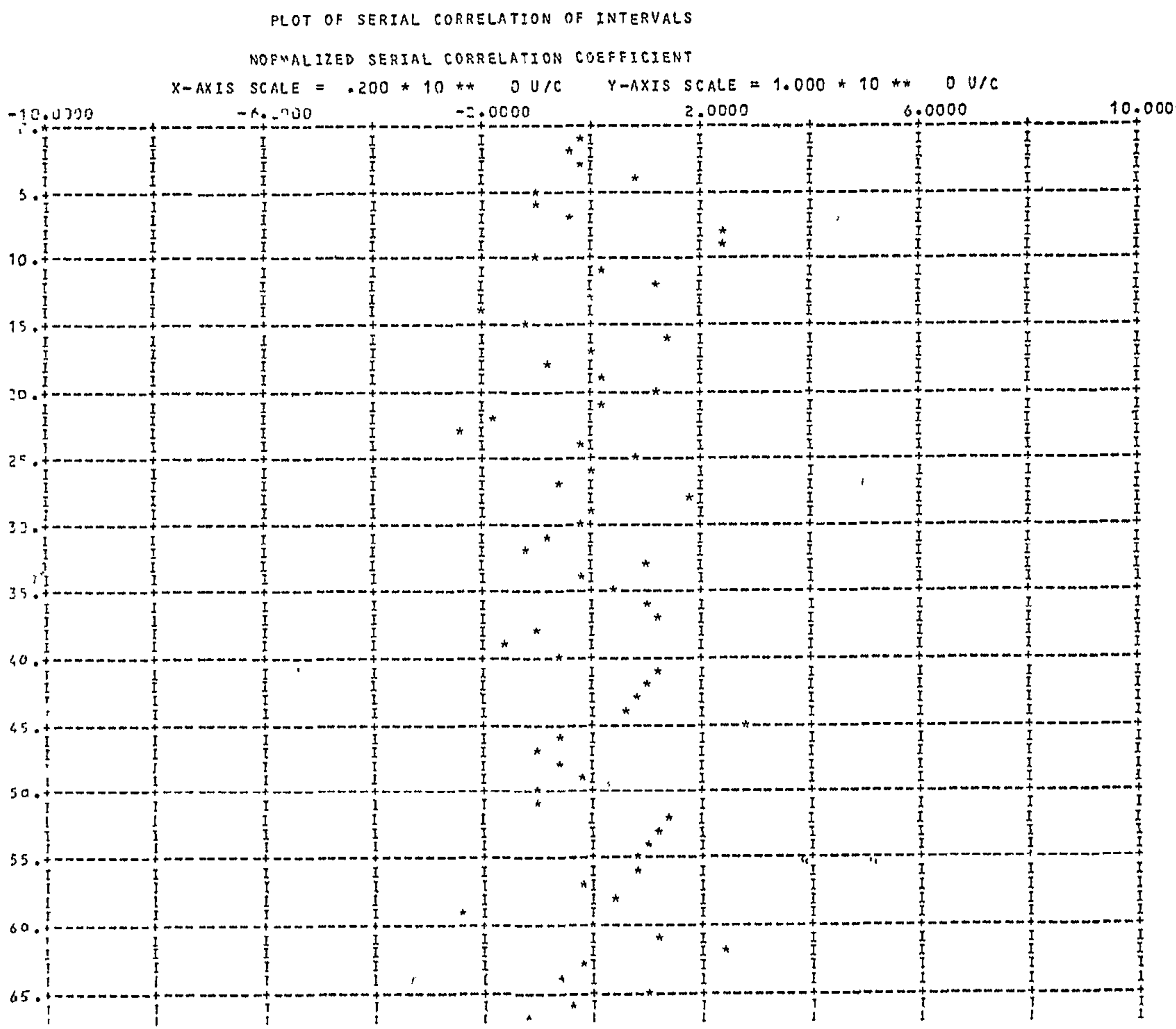
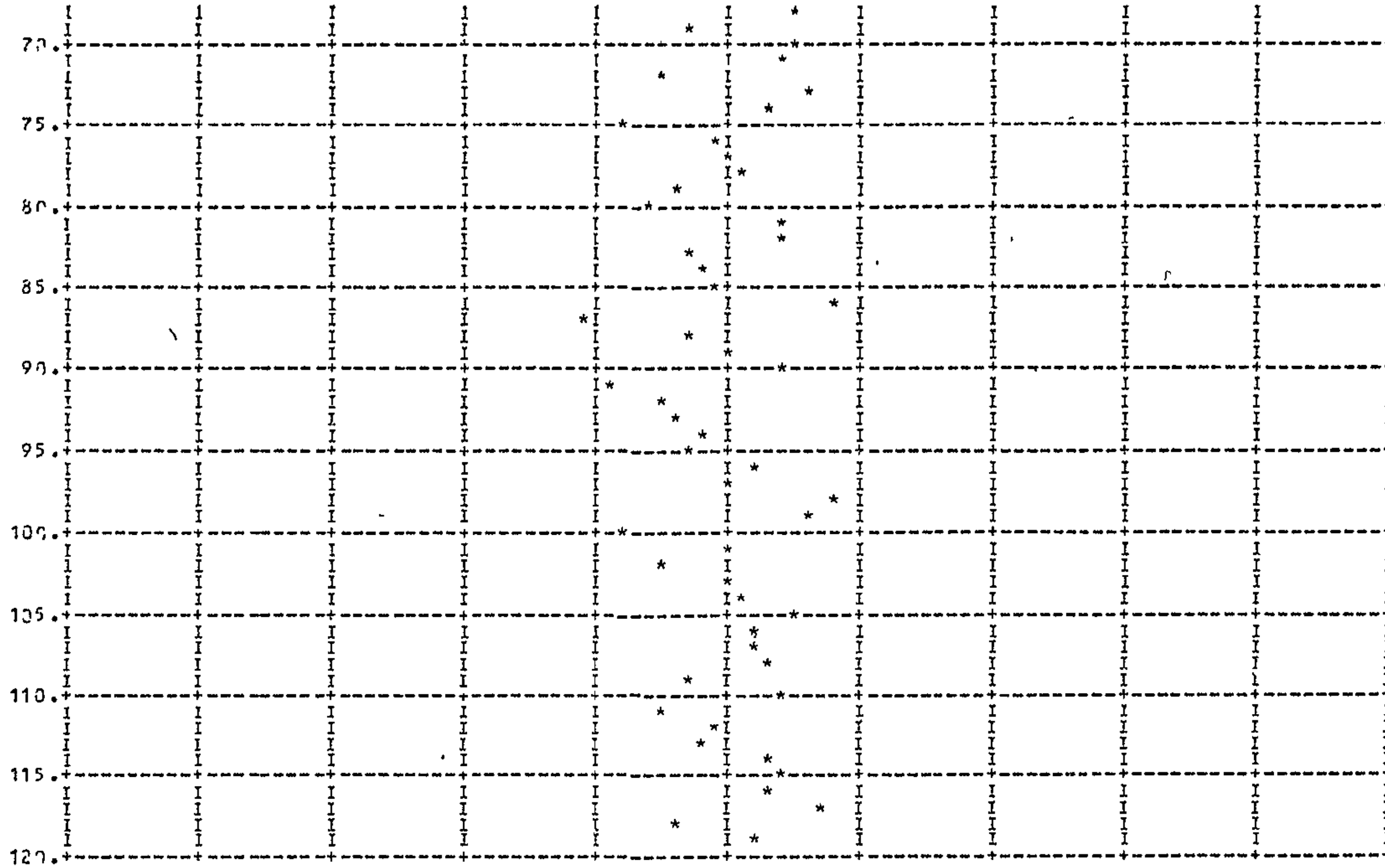


FIG.17 SERIAL CORRELATION OF INTERVALS (TAPE 4)



END OF SUBROUTINE RHO

FIG.17 (CONT.)

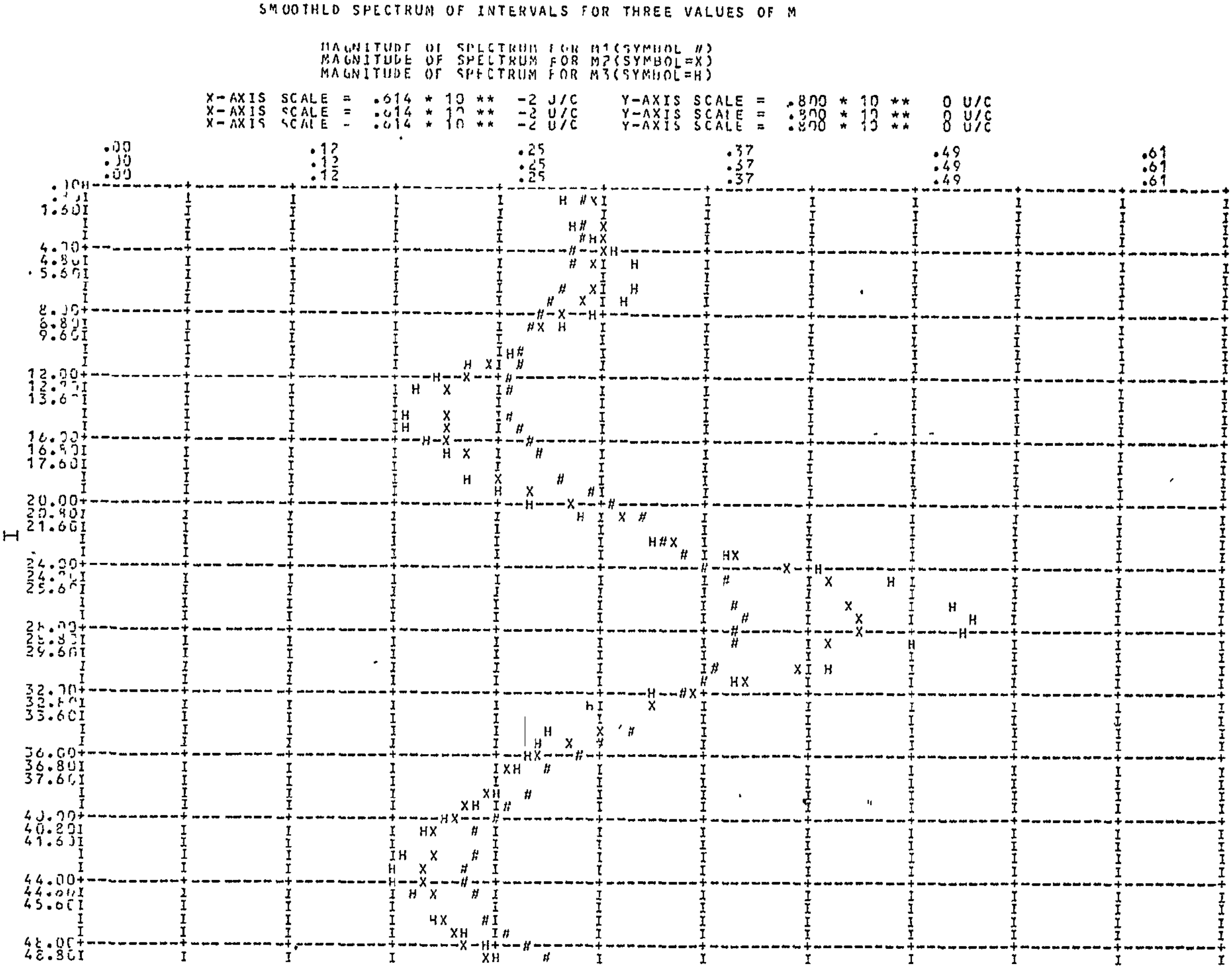


FIG.18 SPECTRUM OF INTERVALS (TAPE 4)

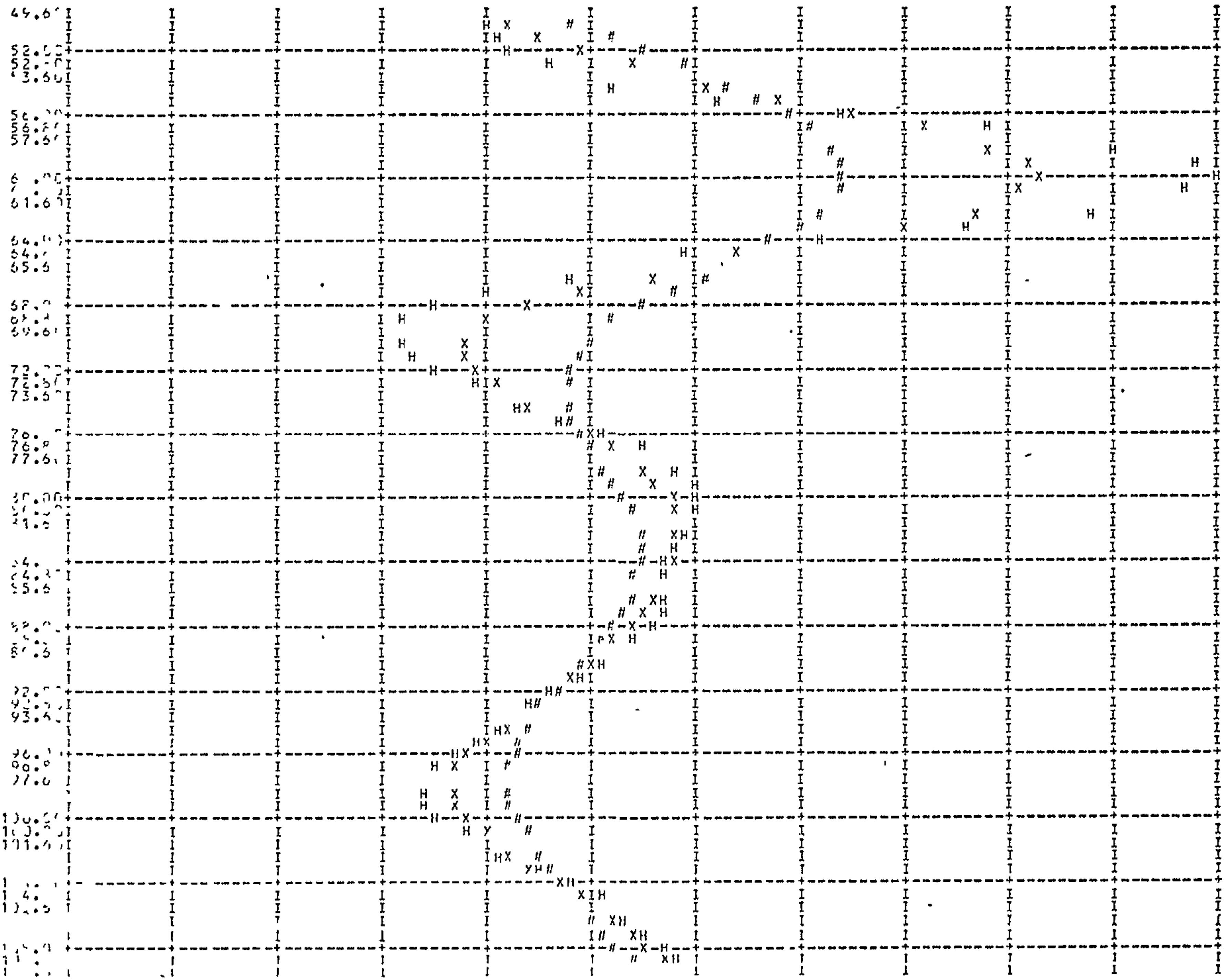


FIG.18 (CONT.)

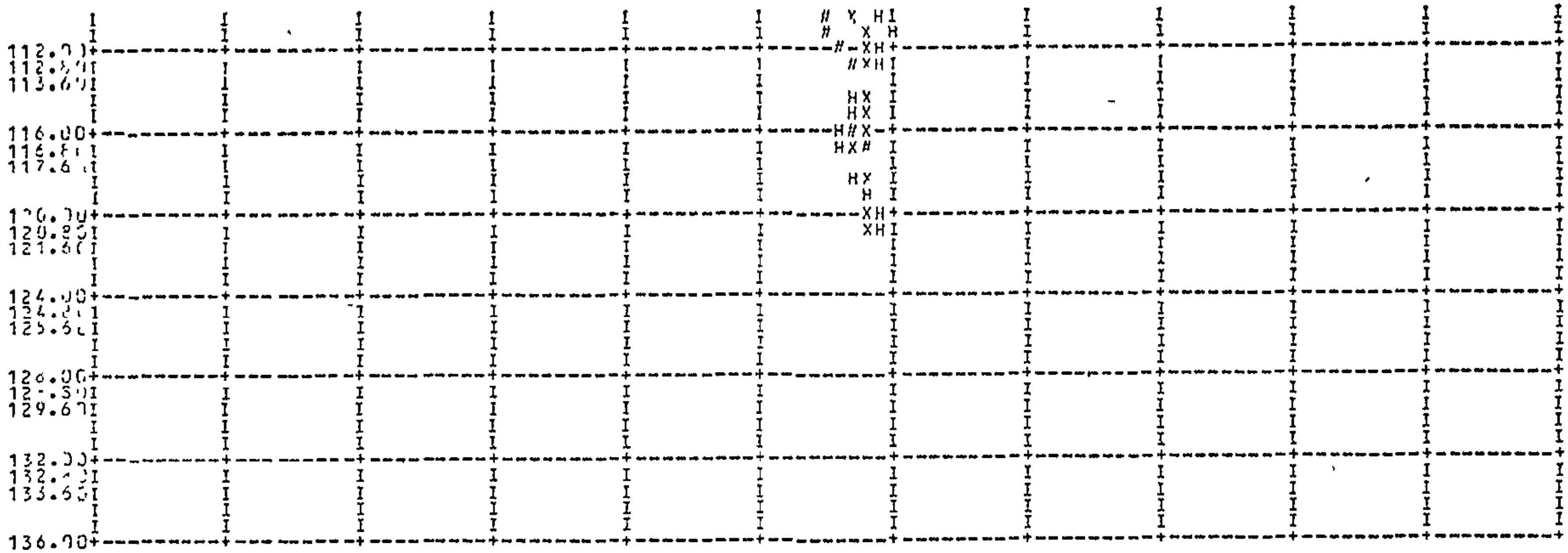


FIG.18 (CONT.)

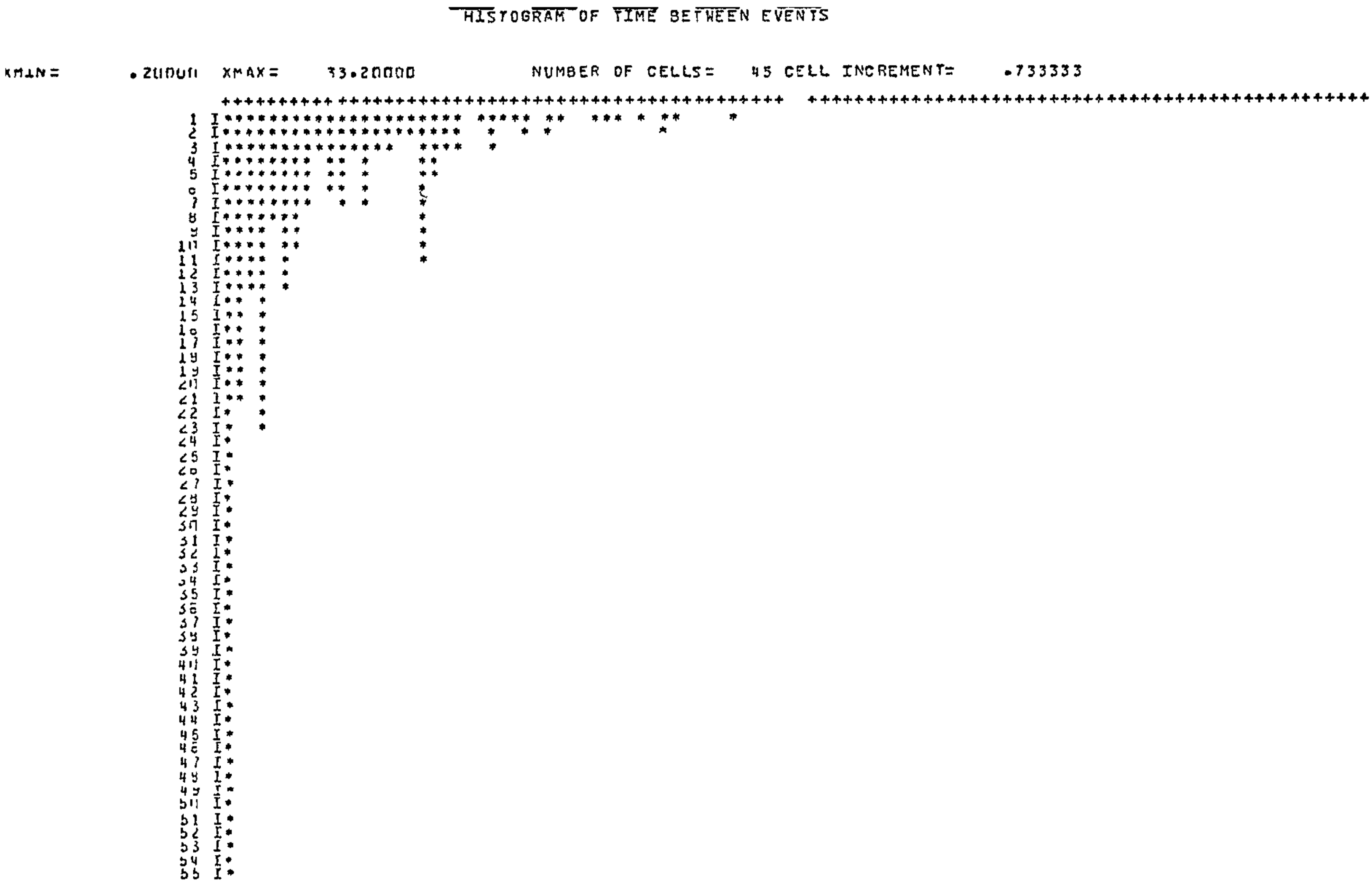


FIG.19 HISTOGRAM OF TIME BETWEEN EVENTS (TAPE 5)

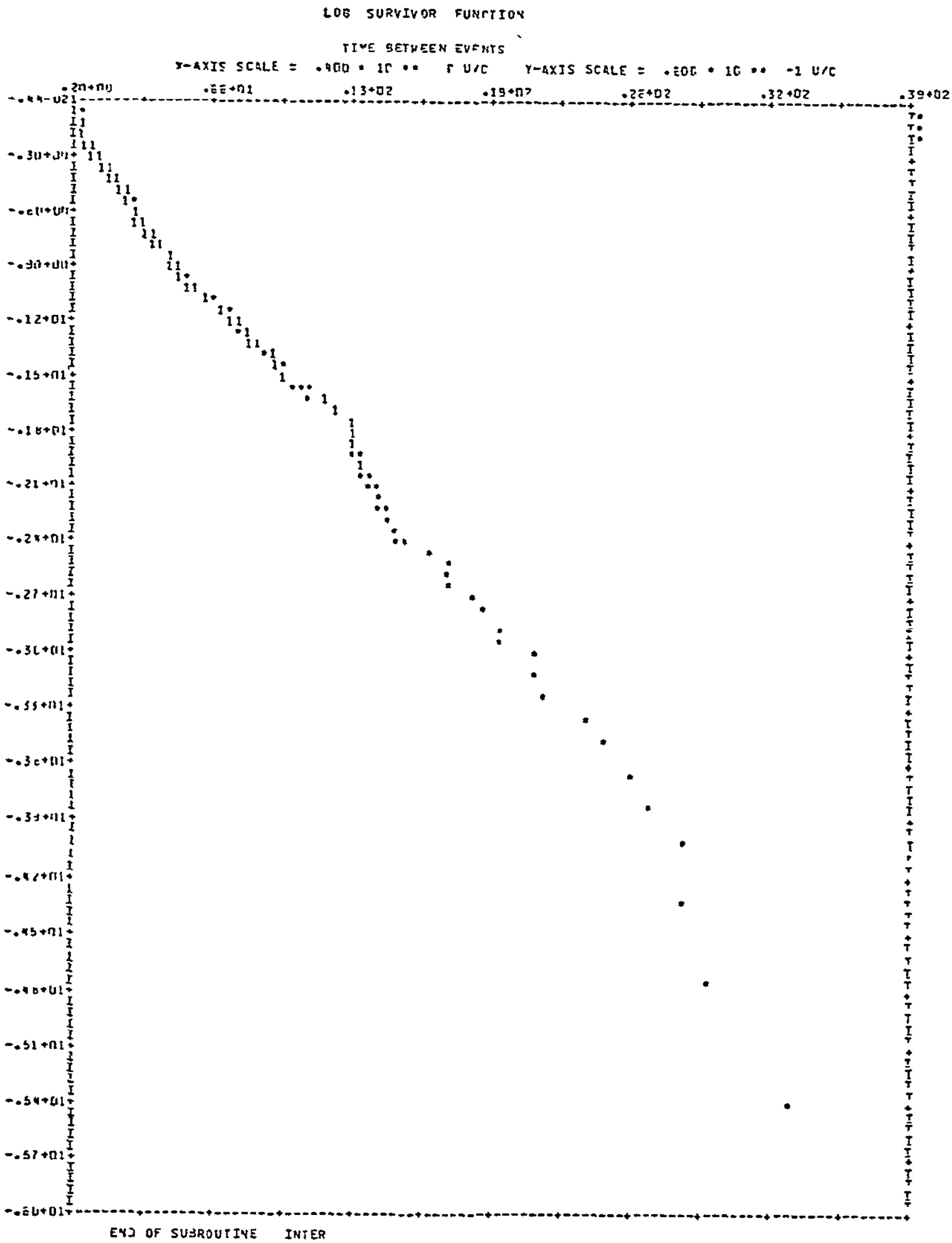


FIG.20 LOG SURVIVOR FUNCTION (TAPE 5)

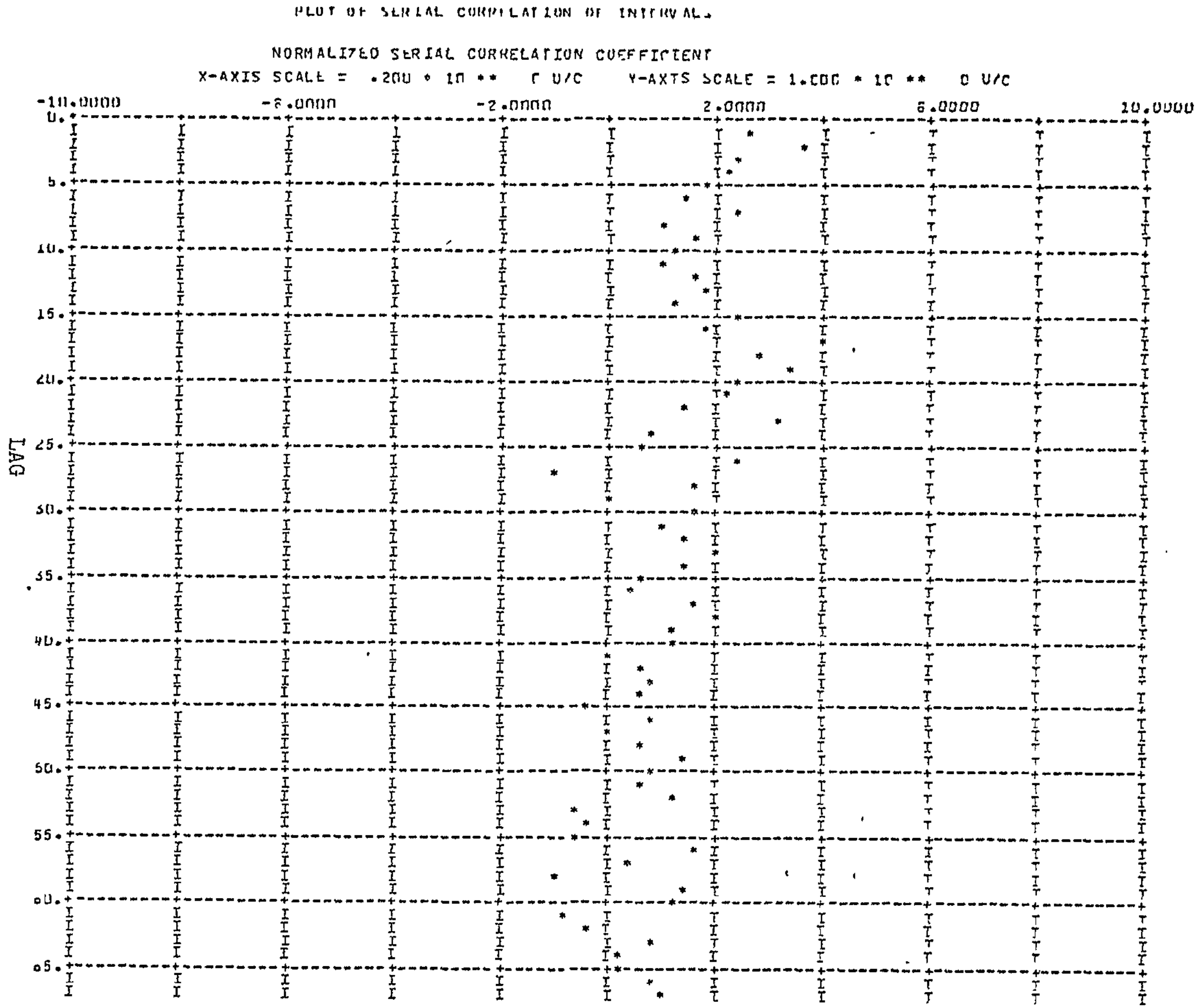
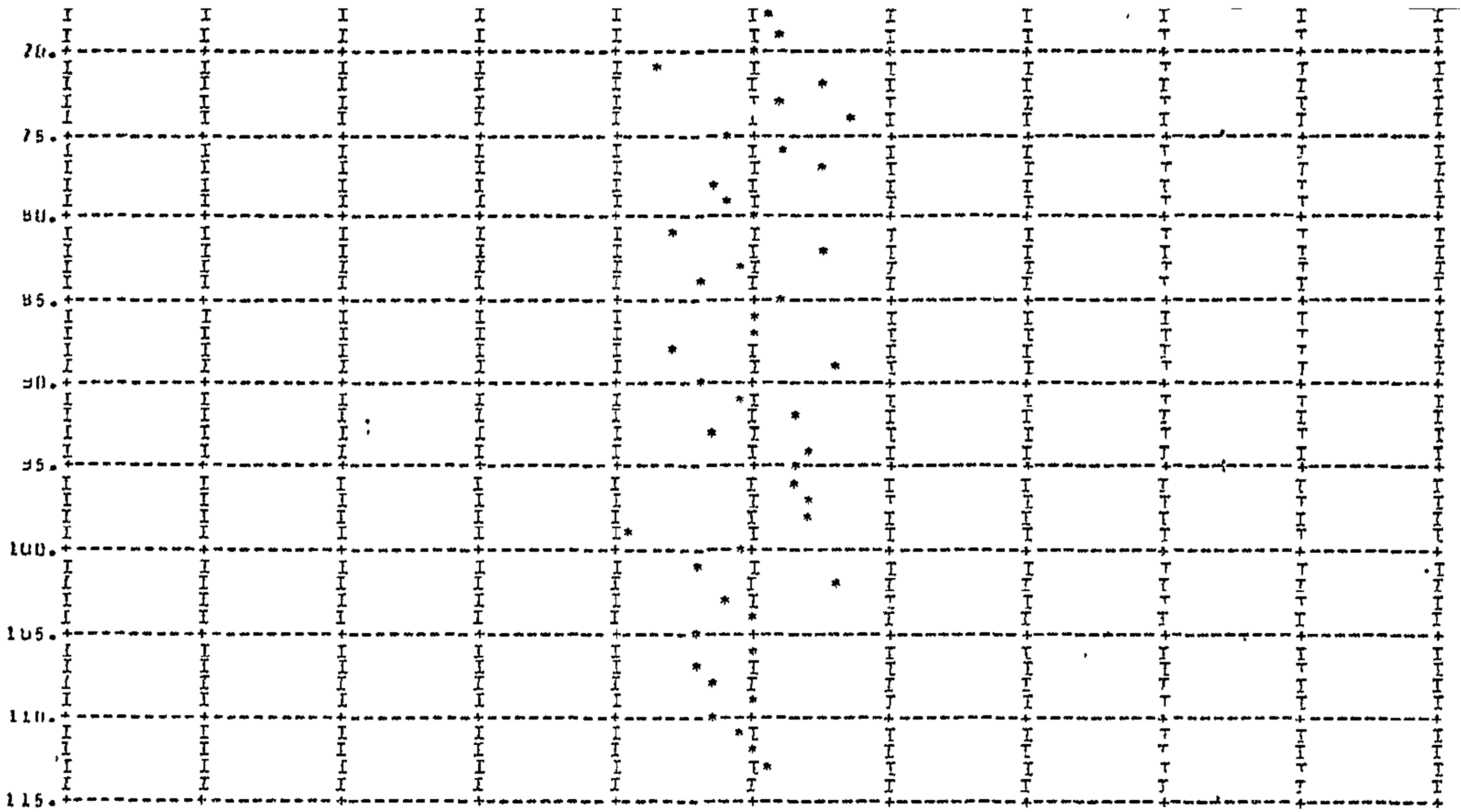


FIG.21 SERIAL CORRELATION OF INTERVALS (TAPE 5)



END OF SUBROUTINE RHO

FIG.21 (CONT.)

SMOOTHED SPECTRUM OF INTERVALS FOR THREE VALUES OF M

MAGNITUDE OF SPECTRUM FOR M1(SYMBOL=#)
MAGNITUDE OF SPECTRUM FOR M2(SYMBOL=X)
MAGNITUDE OF SPECTRUM FOR M3(SYMBOL=H)

X-AXIS SCALE = .163 * 10 ** -1 U/C   Y-AXIS SCALE = .700 * 10 ** 0 U/C
X-AXIS SCALE = .163 * 10 ** -1 U/C   Y-AXIS SCALE = .700 * 10 ** 0 U/C
X-AXIS SCALE = .163 * 10 ** -1 U/C   Y-AXIS SCALE = .700 * 10 ** 0 U/C

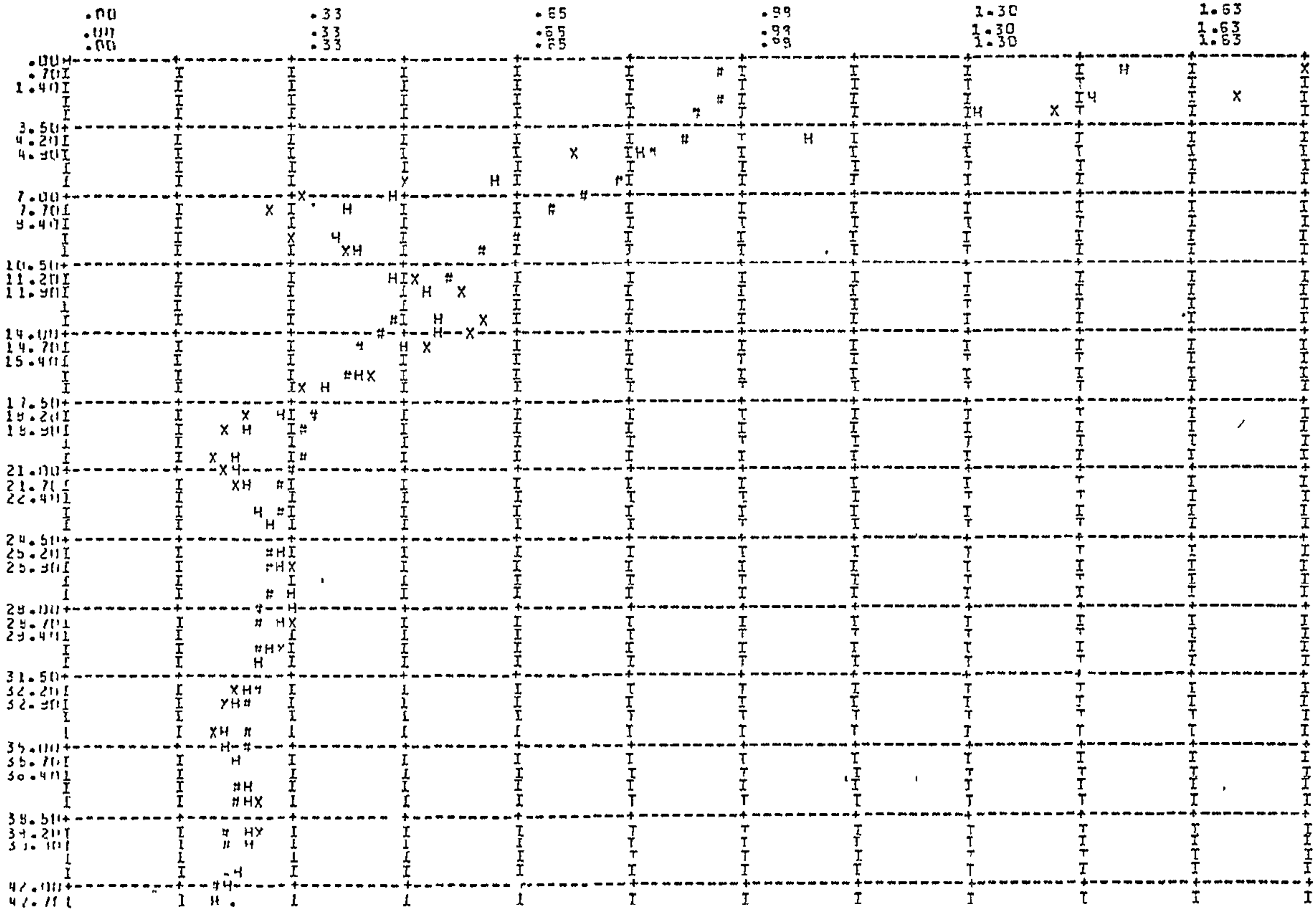


FIG.22 SPECTRUM OF INTERVALS (TAPE 5)

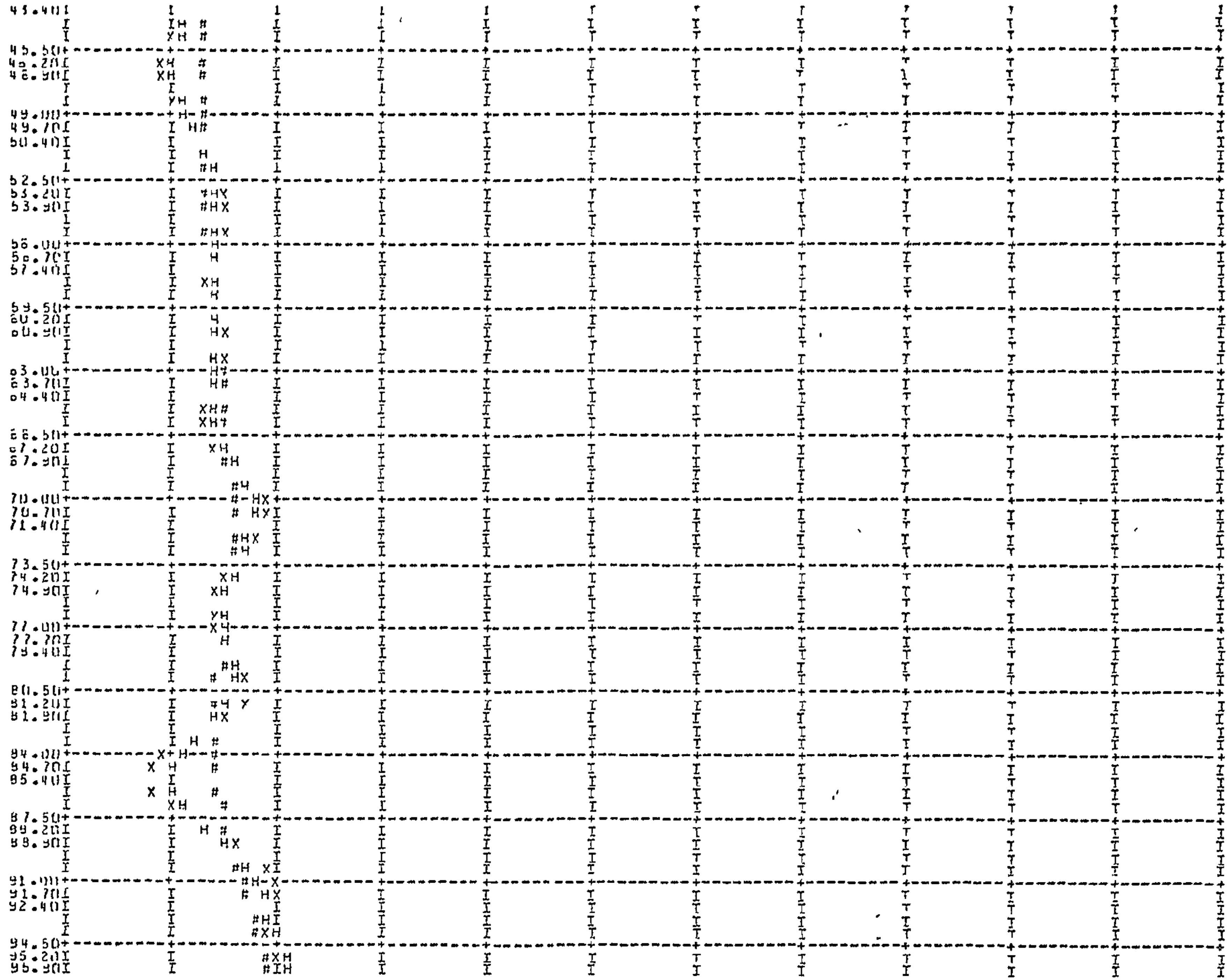


FIG.22 (CONT.)

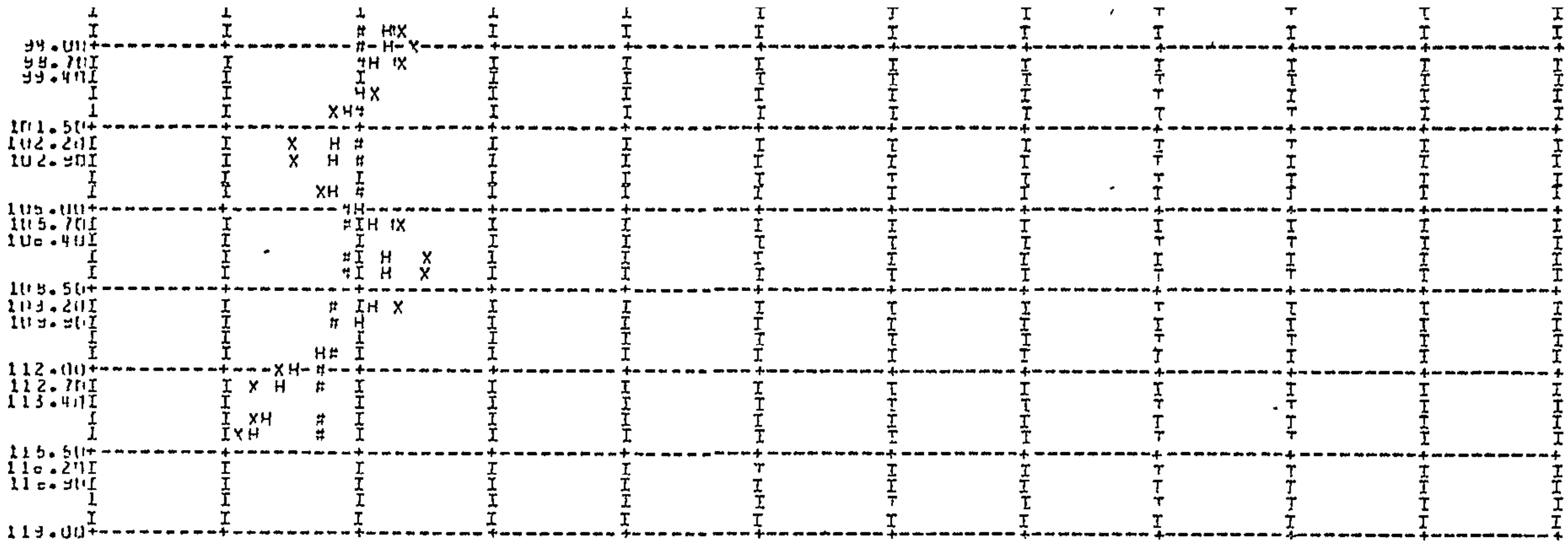


FIG.22 (CONT.)

HISTOGRAM OF AMPLITUDES

XMIN= 7.00000 XMAX= 20.00000 NUMBER OF CELLS= 13 CELL INCREMENT= 1.00000

FIG.23 HISTOGRAM OF AMPLITUDES (TAPE 1)

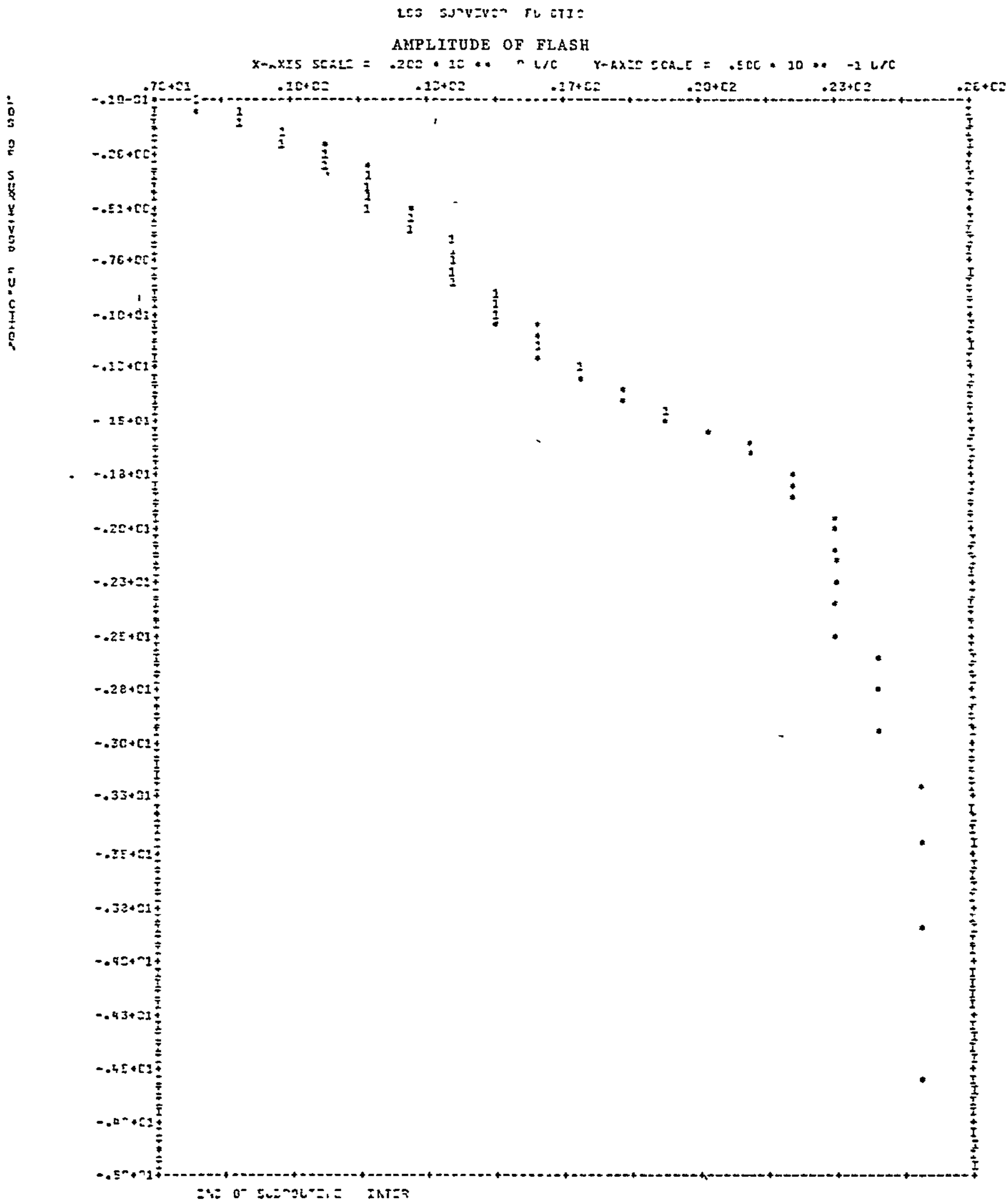
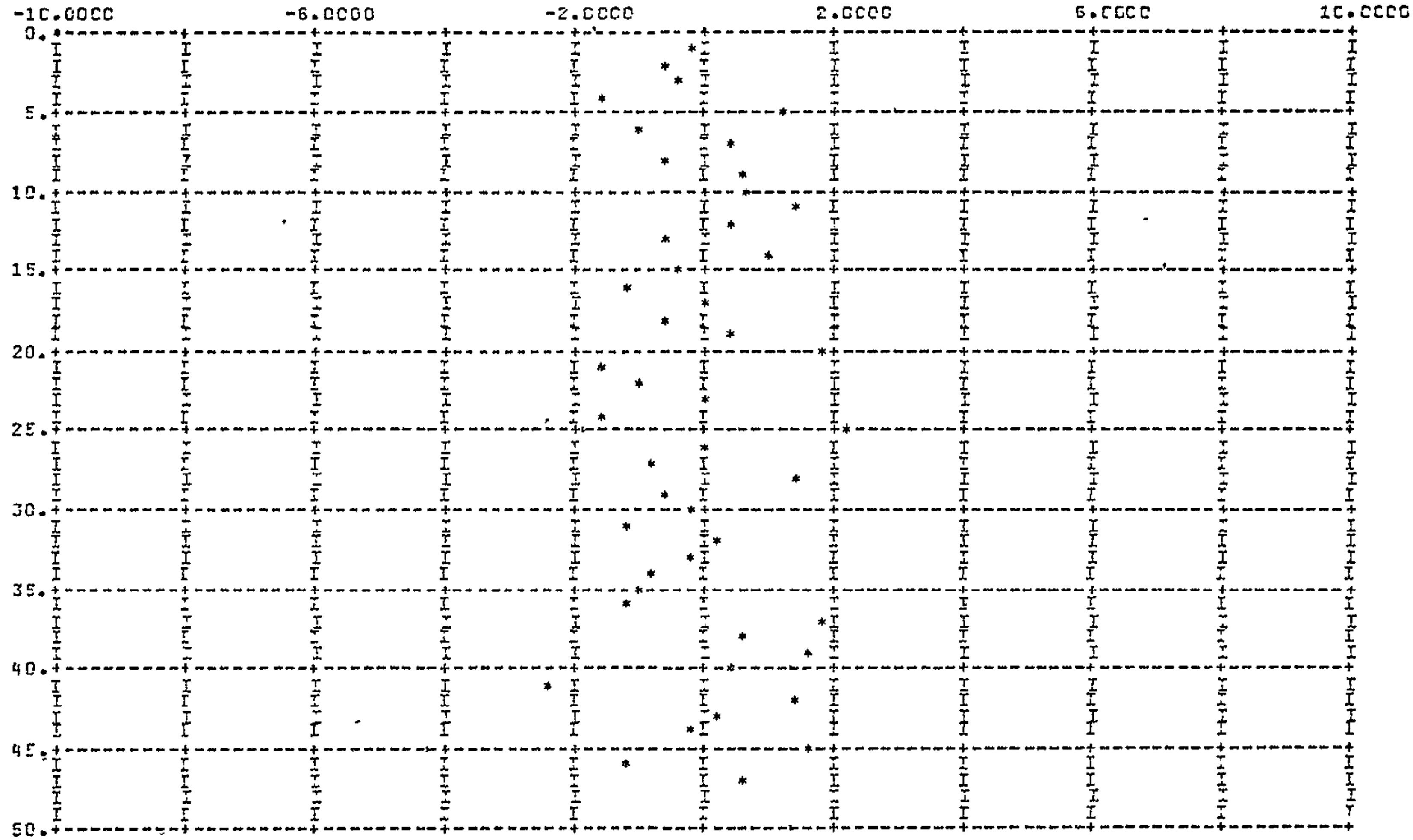


FIG.24 LOG SURVIVOR FUNCTION (TAPE 1)

PLOT OF SERIAL CORRELATION OF AMPLITUDES

NORMALIZED SERIAL CORRELATION COEFFICIENT

X-AXIS SCALE = .200 * 10 ** 0 U/C   Y-AXIS SCALE = 1.000 * 10 ** 0 U/C



END OF SUBROUTINE RHO

FIG.25 SERIAL CORRELATION OF AMPLITUDES (TAPE 1)

HISTOGRAM OF AMPLITUDES

XMIN= 5.00000 XMAX= 25.00000 NUMBER OF CELLS= 82 CELL INCREMENT= .243902

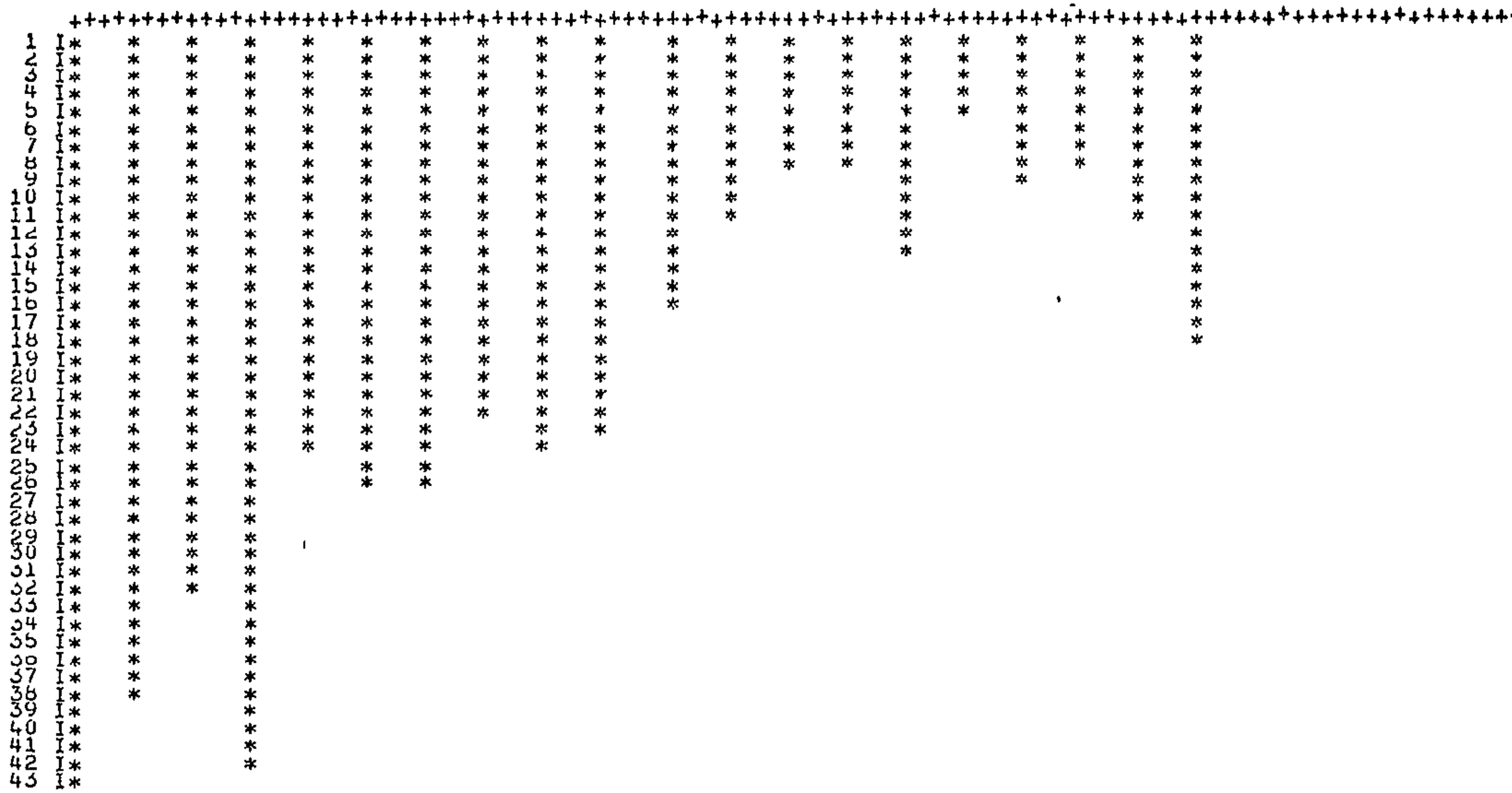


FIG.26 HISTOGRAM OF AMPLITUDES (TAPE 2)

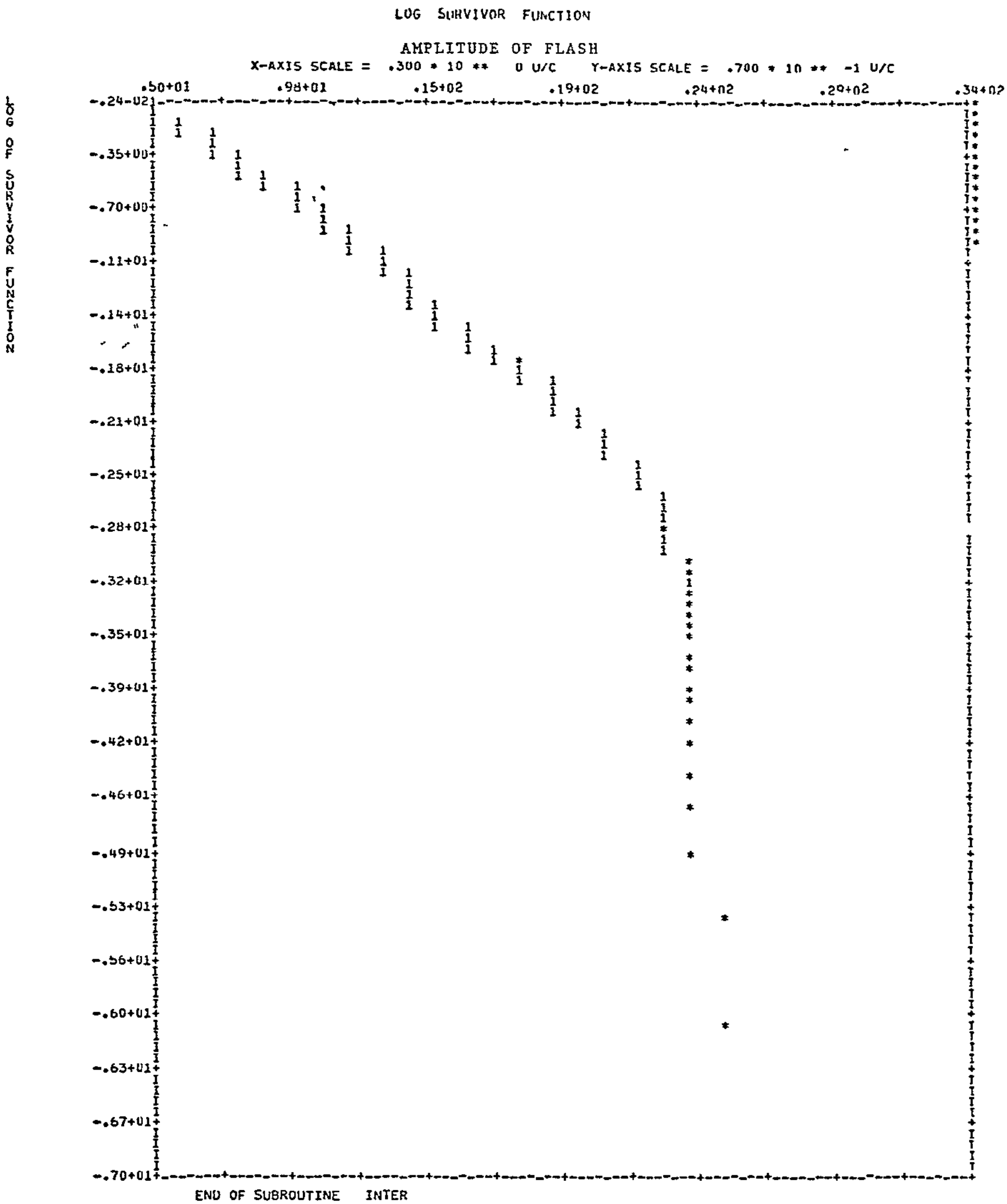


FIG.27 LOG SURVIVOR FUNCTION (TAPE 2)

PLOT OF SERIAL CORRELATION OF AMPLITUDES

NORMALIZED SERIAL CORRELATION COEFFICIENT

X-AXIS SCALE = .200 * 10 ** 0 U/C   Y-AXIS SCALE = 1.000 * 10 ** 0 U/C

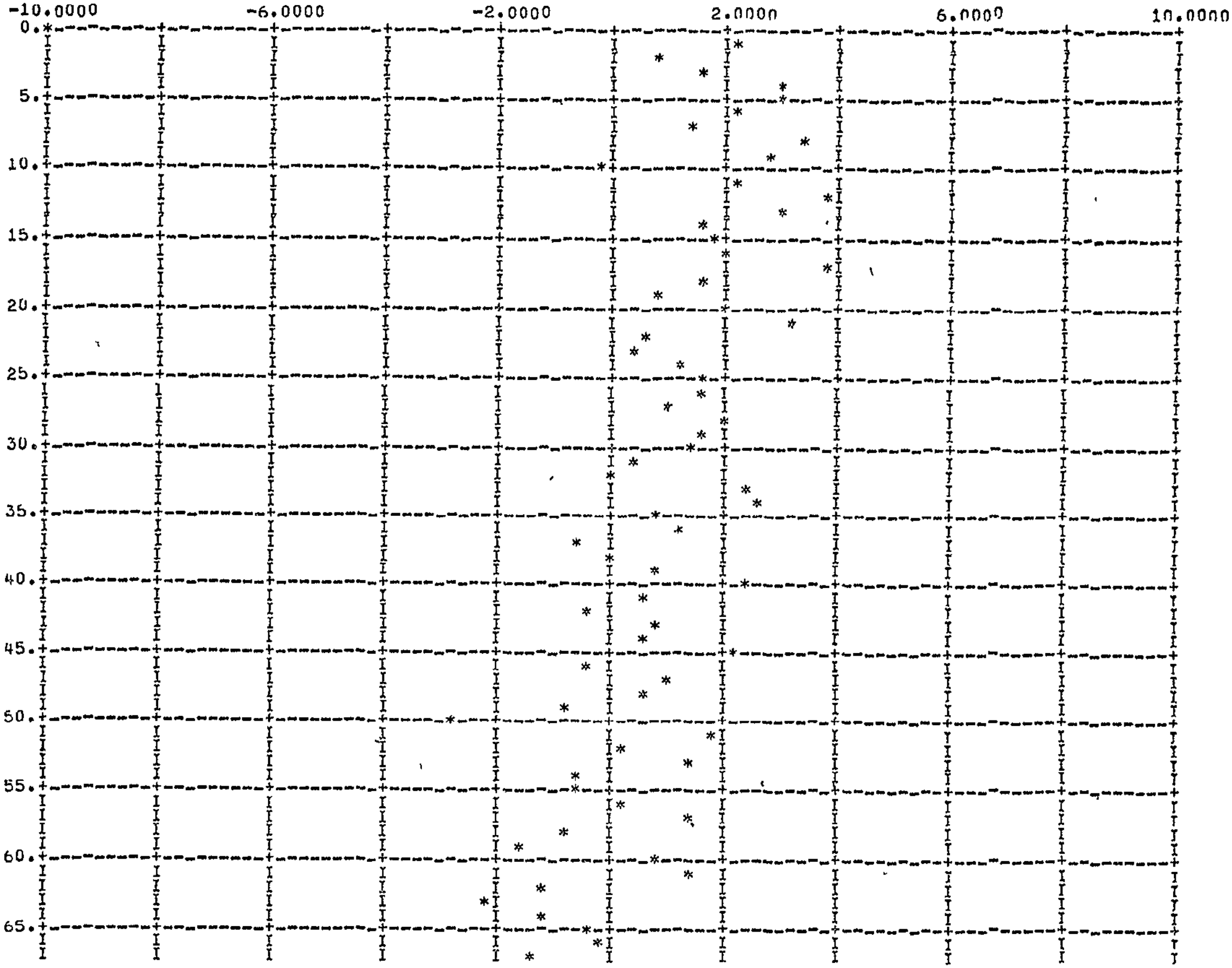


FIG.28 SERIAL CORRELATION OF AMPLITUDES (TAPE 2)

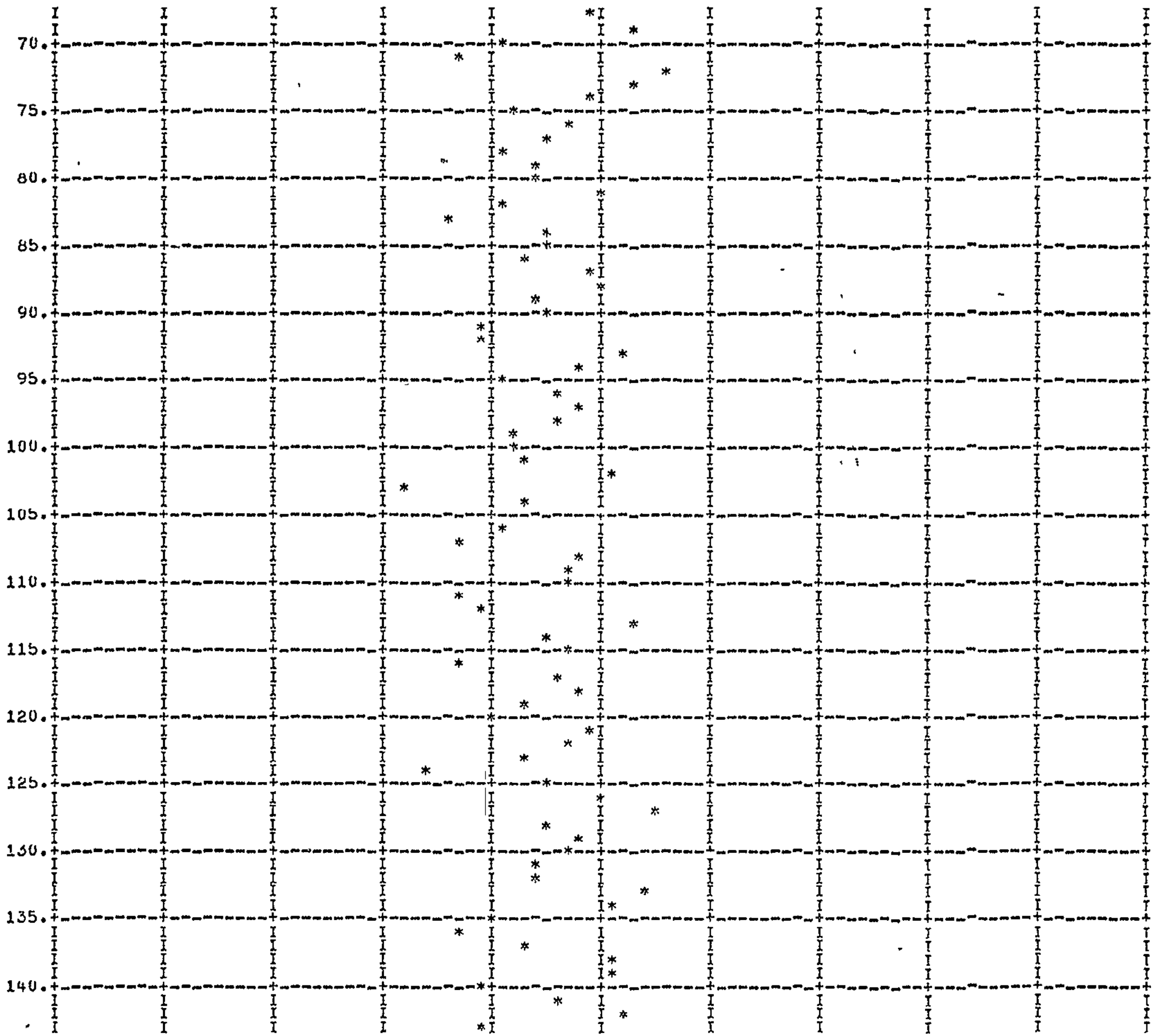
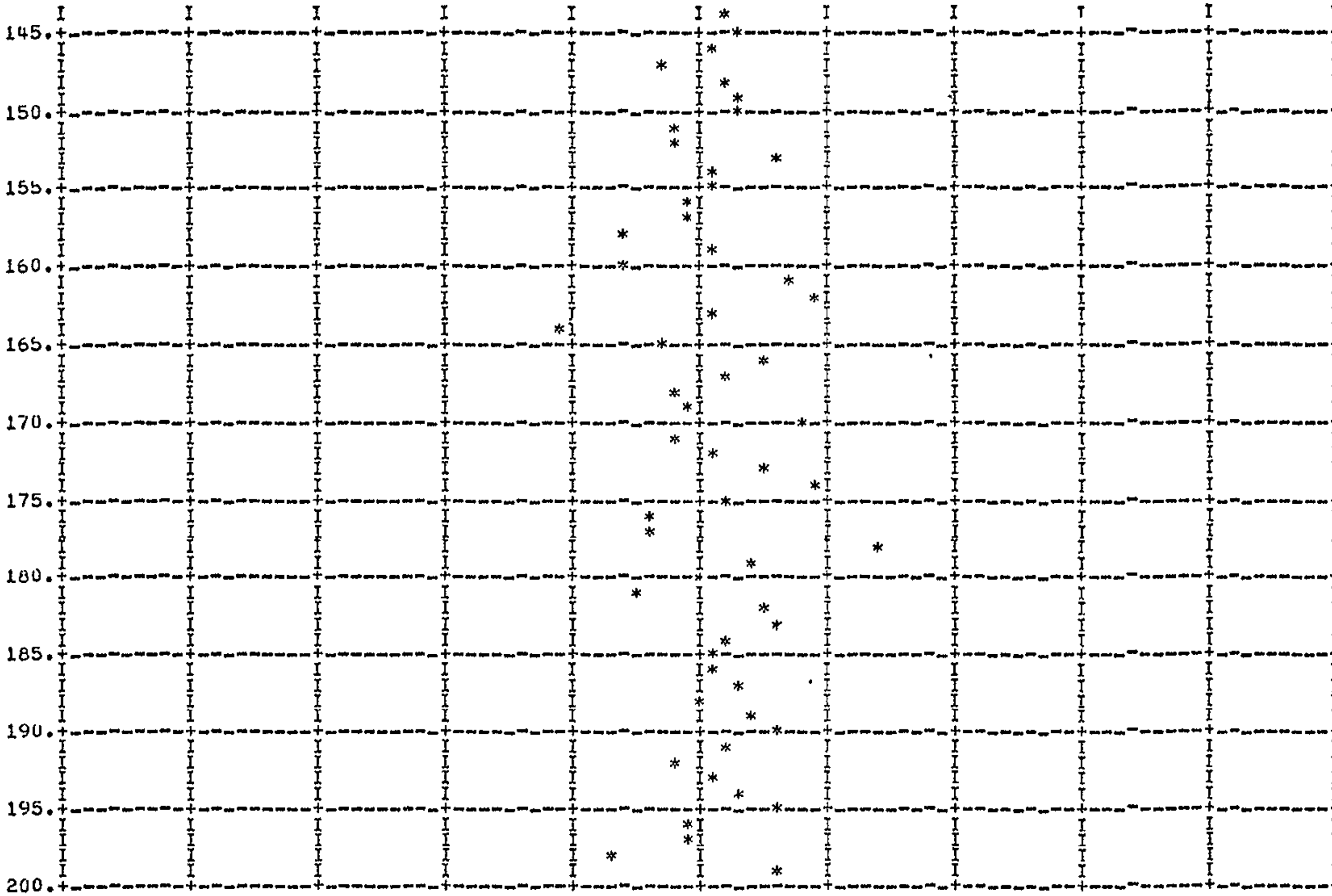


FIG.28 (CONT.)



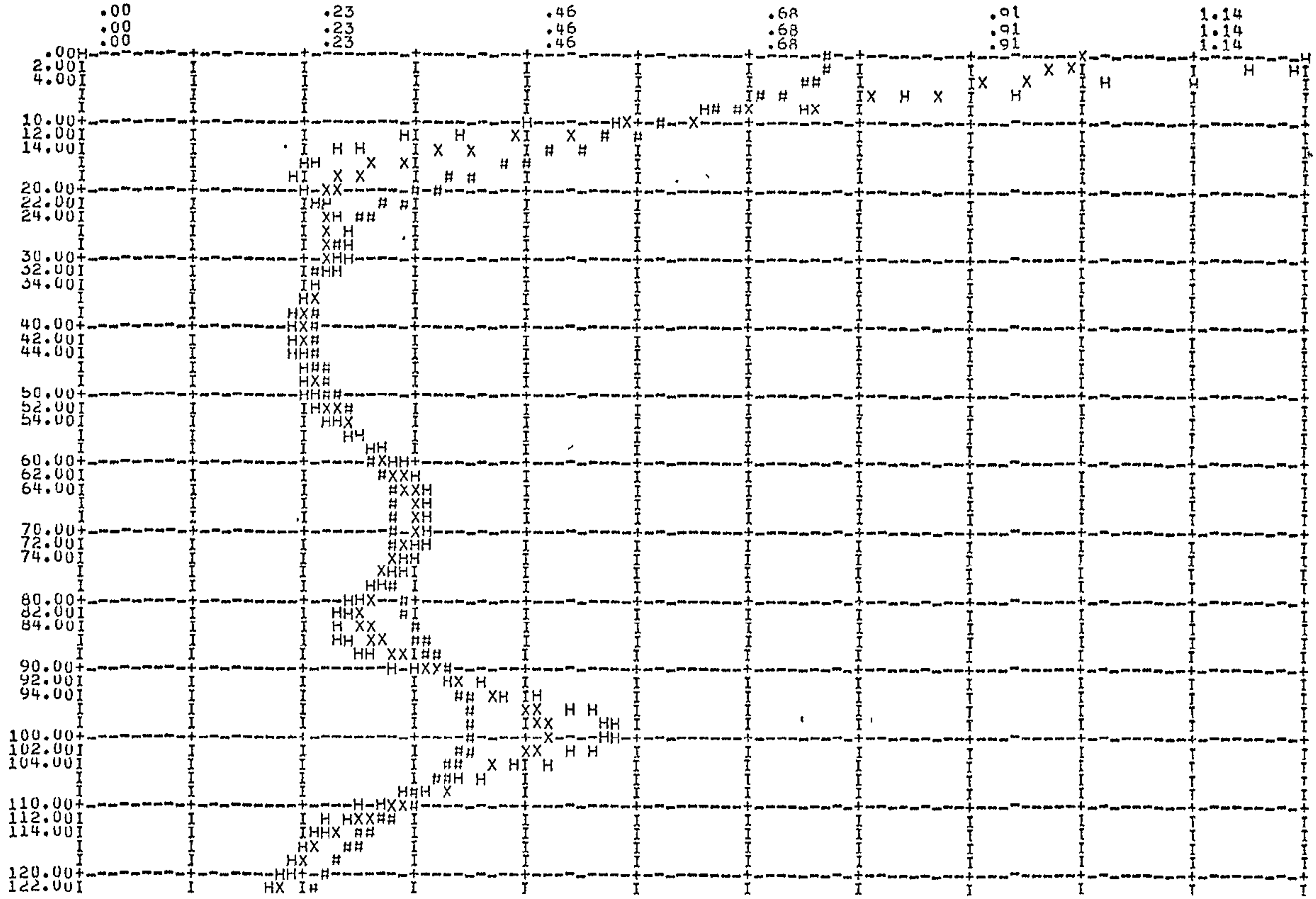
END OF SUBROUTINE RHO

FIG.28 (CONT.).

SPECTRUM OF AMPLITUDES

MAGNITUDE OF SPECTRUM FOR M1(SYMBOL=#)
MAGNITUDE OF SPECTRUM FOR M2(SYMBOL=X)
MAGNITUDE OF SPECTRUM FOR M3(SYMBOL=H)

X-AXIS SCALE = .114 * 10 ** -1 U/C Y-AXIS SCALE = 2.000 * 10 ** 0 U/C
X-AXIS SCALE = .114 * 10 ** -1 U/C Y-AXIS SCALE = 2.000 * 10 ** 0 U/C
X-AXIS SCALE = .114 * 10 ** -1 U/C Y-AXIS SCALE = 2.000 * 10 ** 0 U/C



I

FIG.29 SPECTRUM OF AMPLITUDES (TAPE 2)

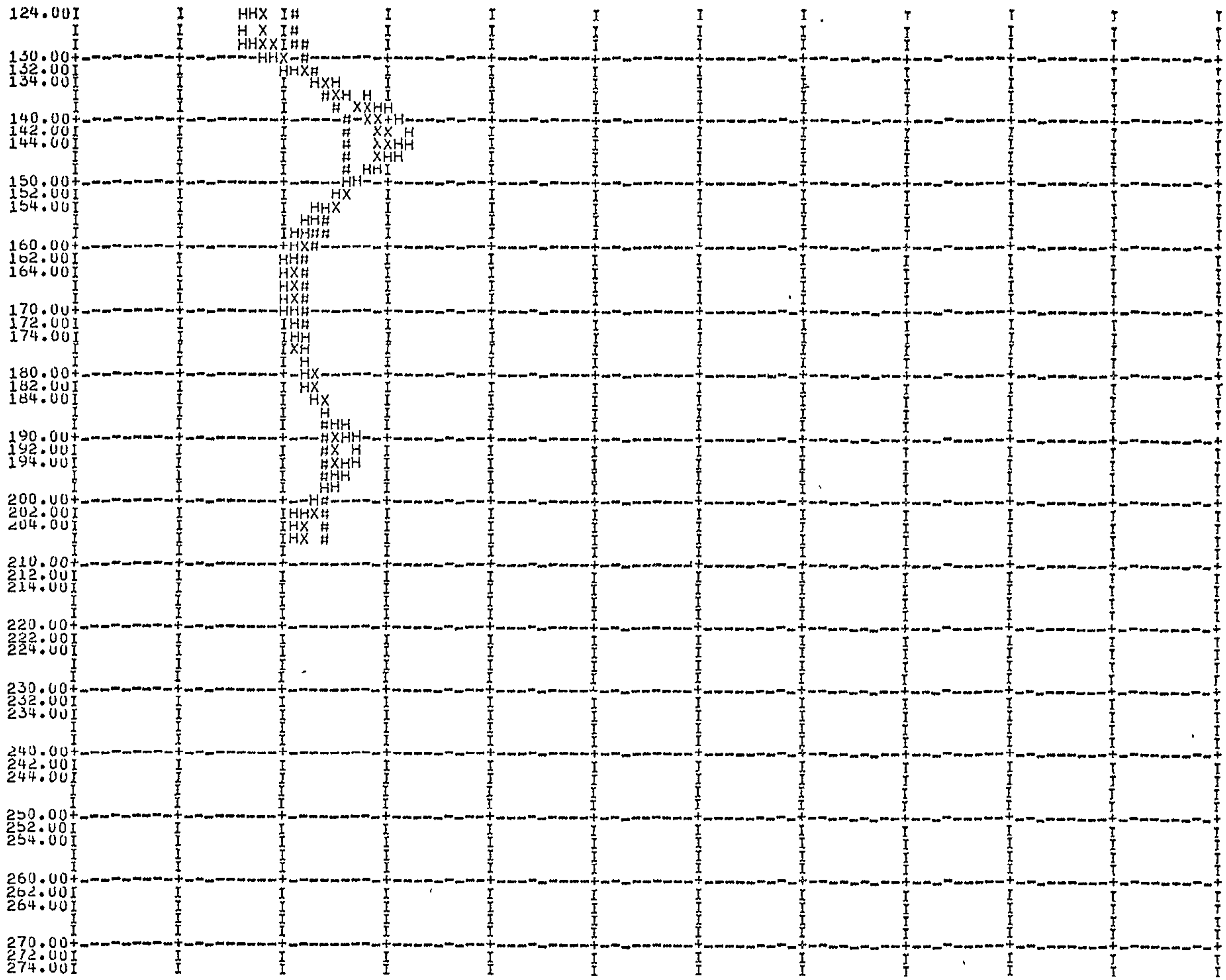


FIG.29 (CONT.)

HISTOGRAM OF AMPLITUDES

5.00000 XMAX= 25.00000 NUMBER OF CELLS= 100 CELL INCREMENT= .200000

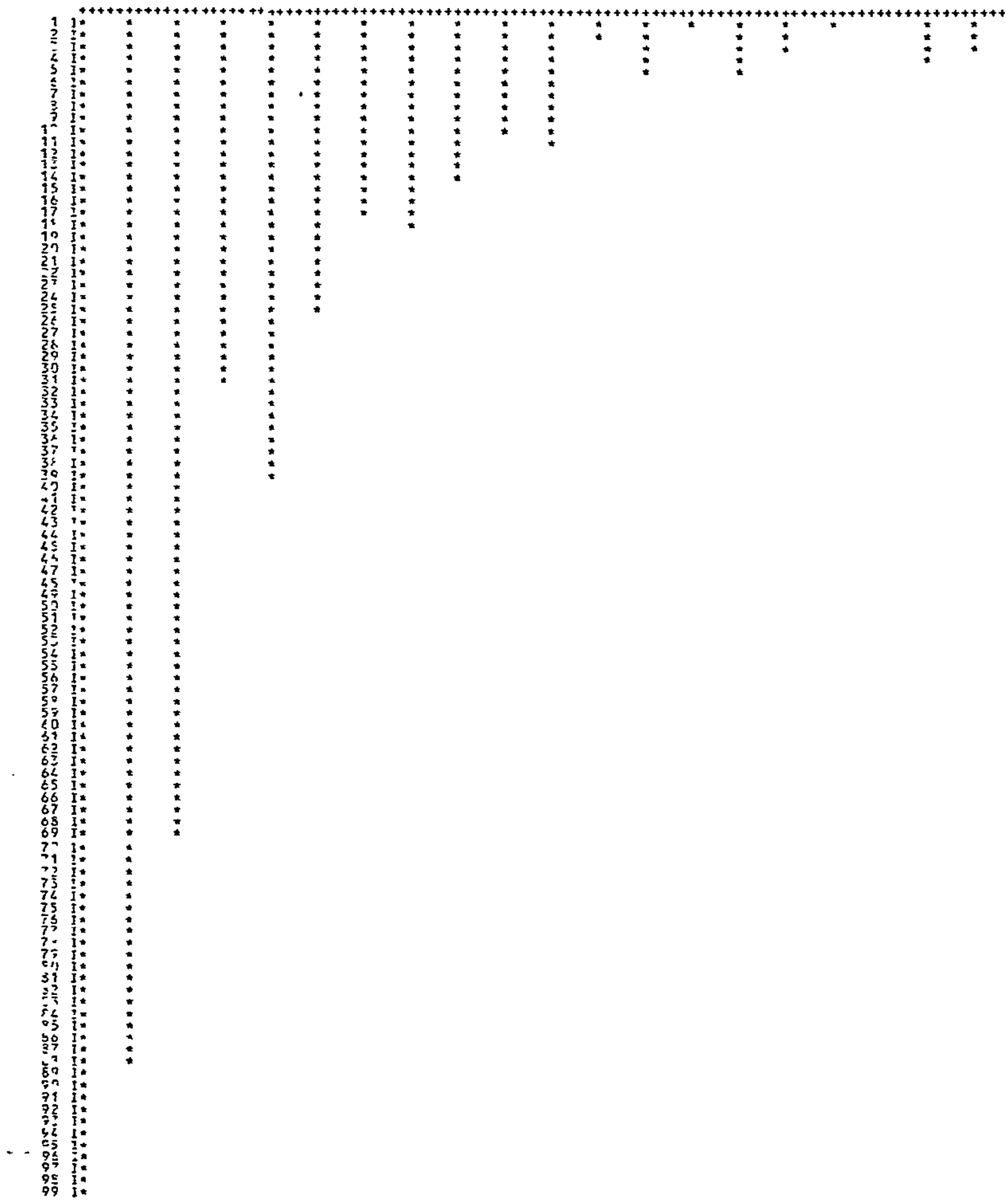


FIG.30 HISTOGRAM OF AMPLITUDES (TAPE 3)

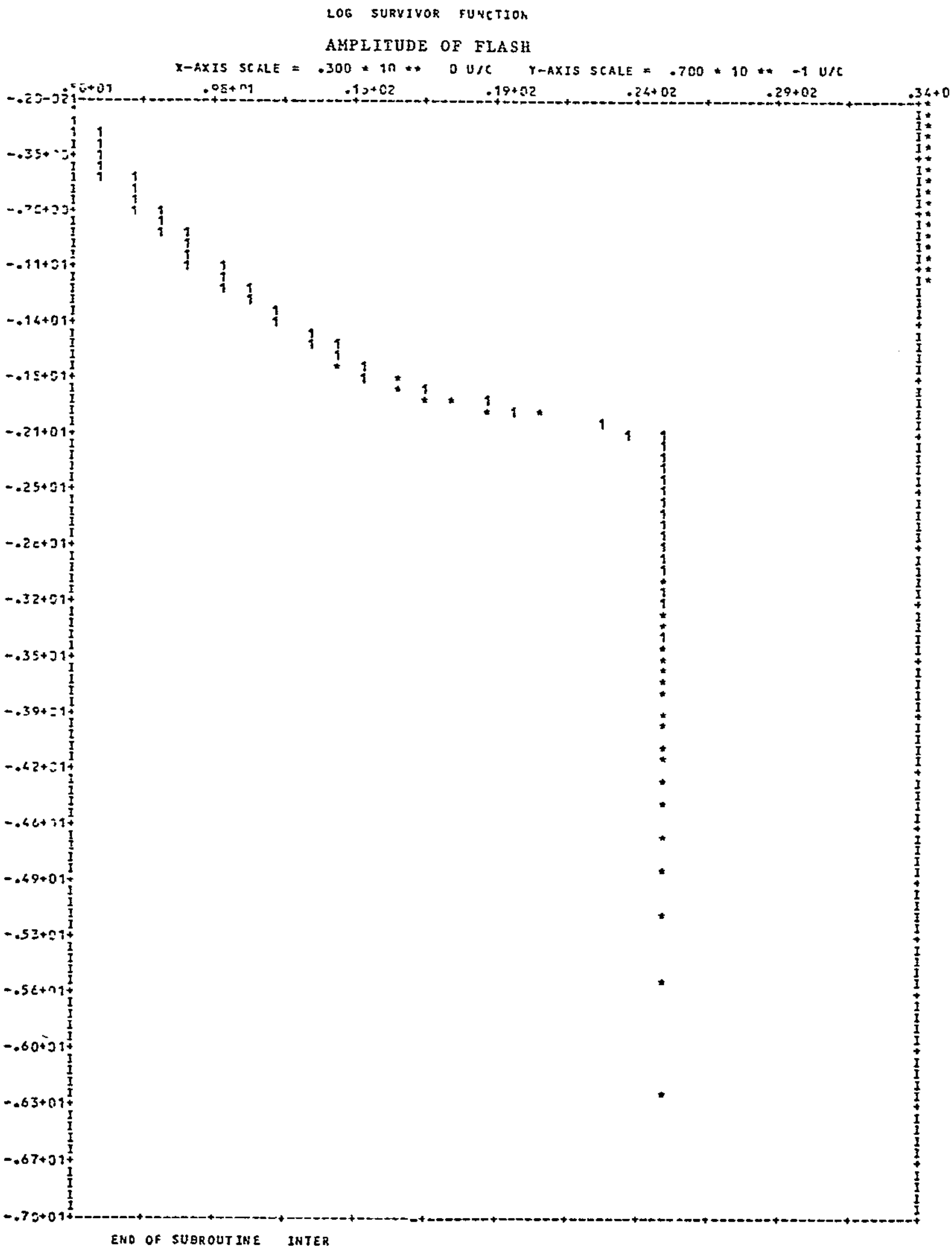


FIG.31 LOG SURVIVOR FUNCTION (TAPE 3)

PLOT OF SERIAL CORRELATION OF AMPLITUDES

NORMALIZED SERIAL CORRELATION COEFFICIENT

X-AXIS SCALE = .200 * 10 ** 0 U/C   Y-AXIS SCALE = 1.000 * 10 ** 0 U/C

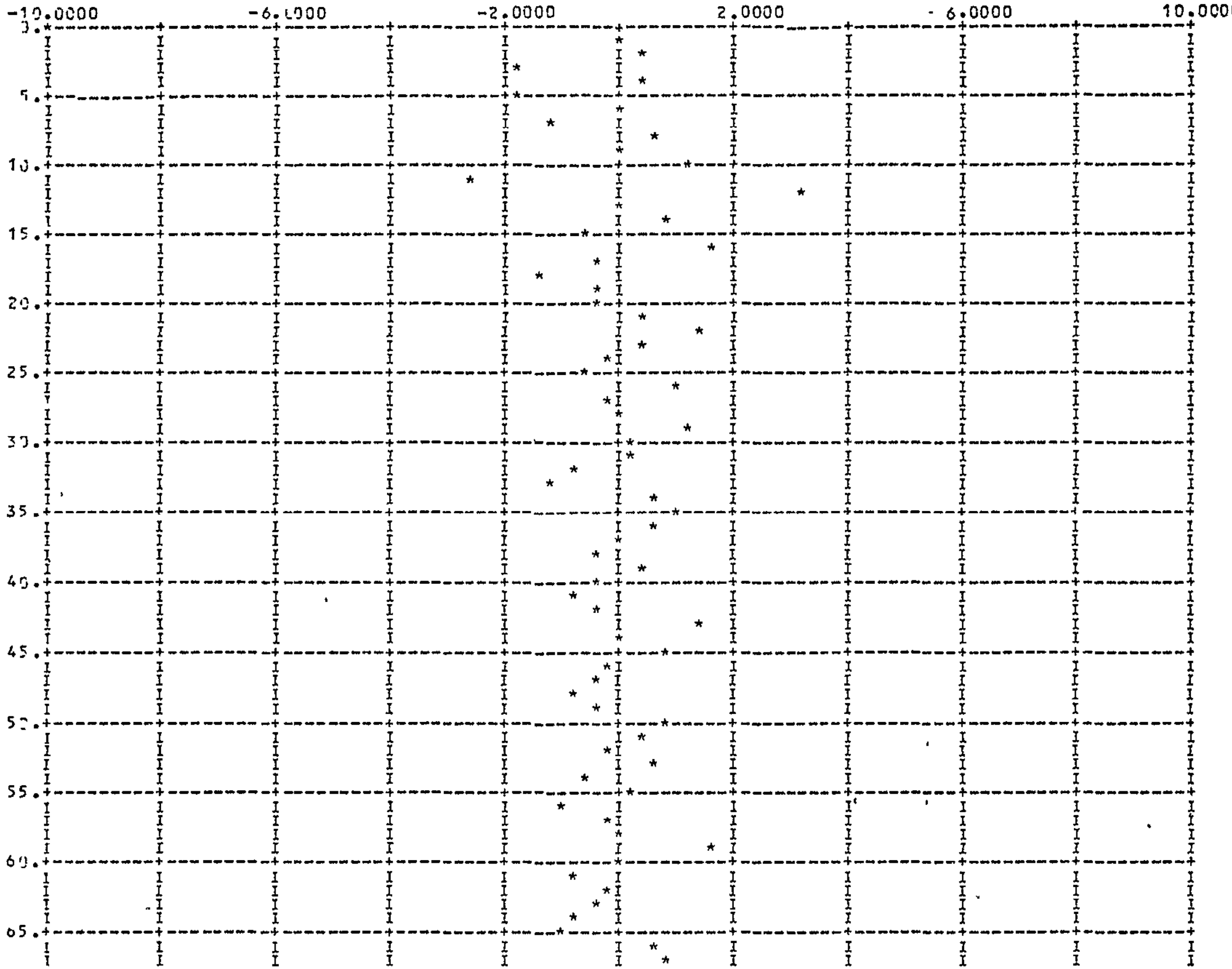


FIG.32 SERIAL CORRELATION OF AMPLITUDES (TAPE 3)

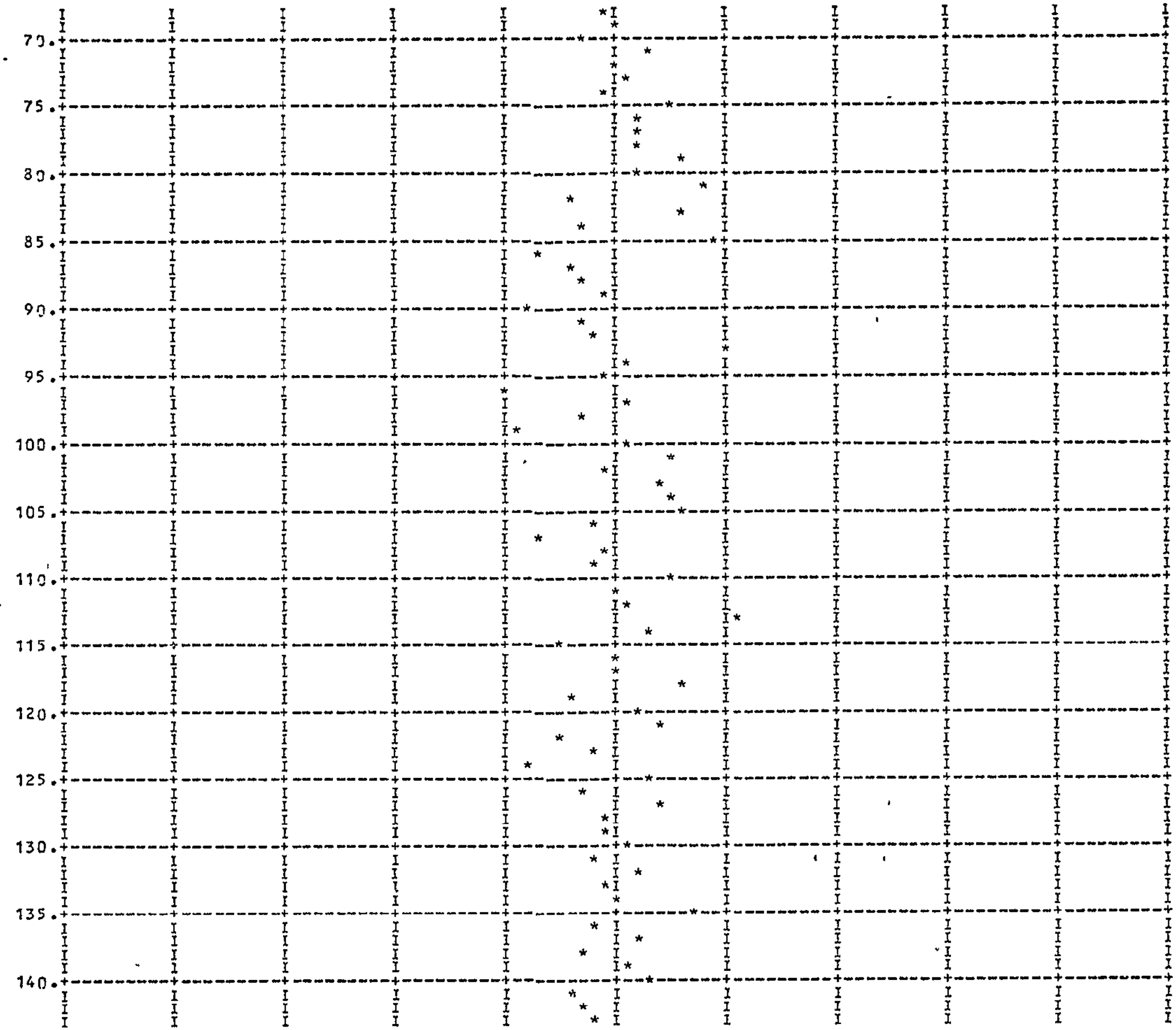
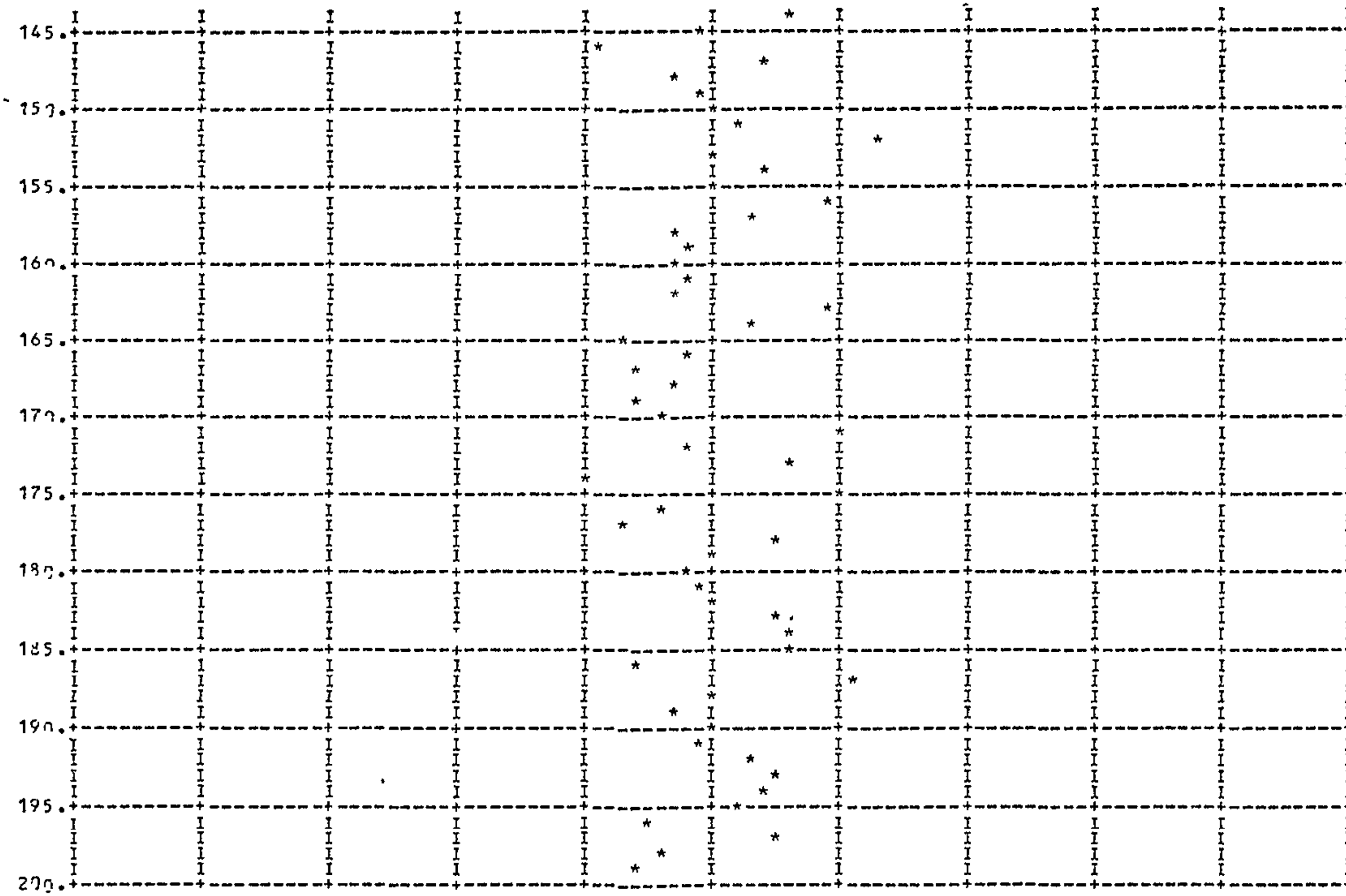


FIG.32 (CONT.)



END OF SUBROUTINE RHO

FIG.32 (CONT.)

SPECTRUM OF AMPLITUDES

MAGNITUDE OF SPECTRUM FOR M1(SYMBOL=#)
MAGNITUDE OF SPECTRUM FOR M2(SYMBOL=X)
MAGNITUDE OF SPECTRUM FOR M3(SYMBOL=H)

X-AXIS SCALE = .587 * 10 ** -2 U/C   Y-AXIS SCALE = 2.000 * 10 ** 0 U/C
X-AXIS SCALE = .587 * 10 ** -2 U/C   Y-AXIS SCALE = 2.000 * 10 ** 0 U/C
X-AXIS SCALE = .587 * 10 ** -2 U/C   Y-AXIS SCALE = 2.000 * 10 ** 0 U/C

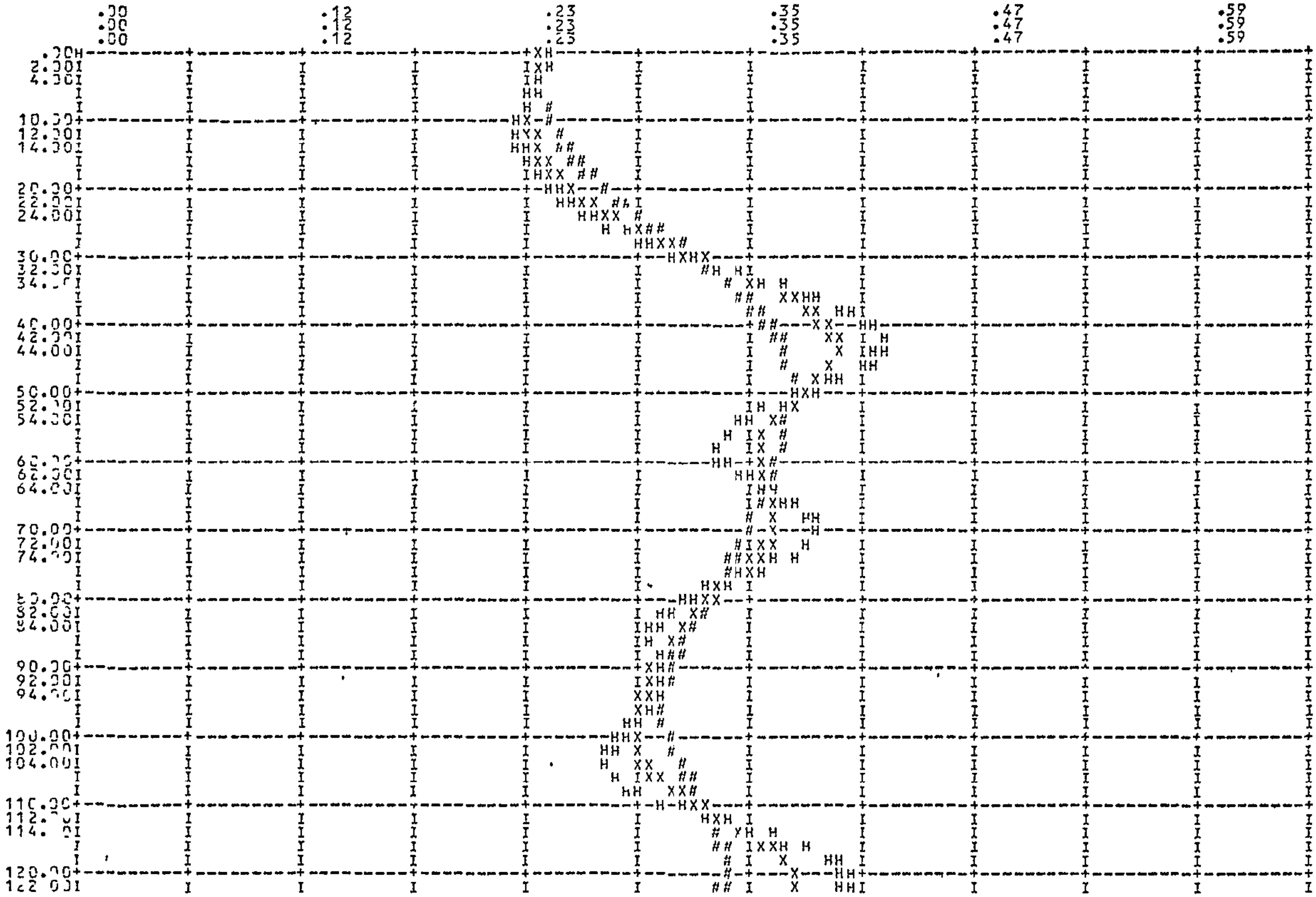
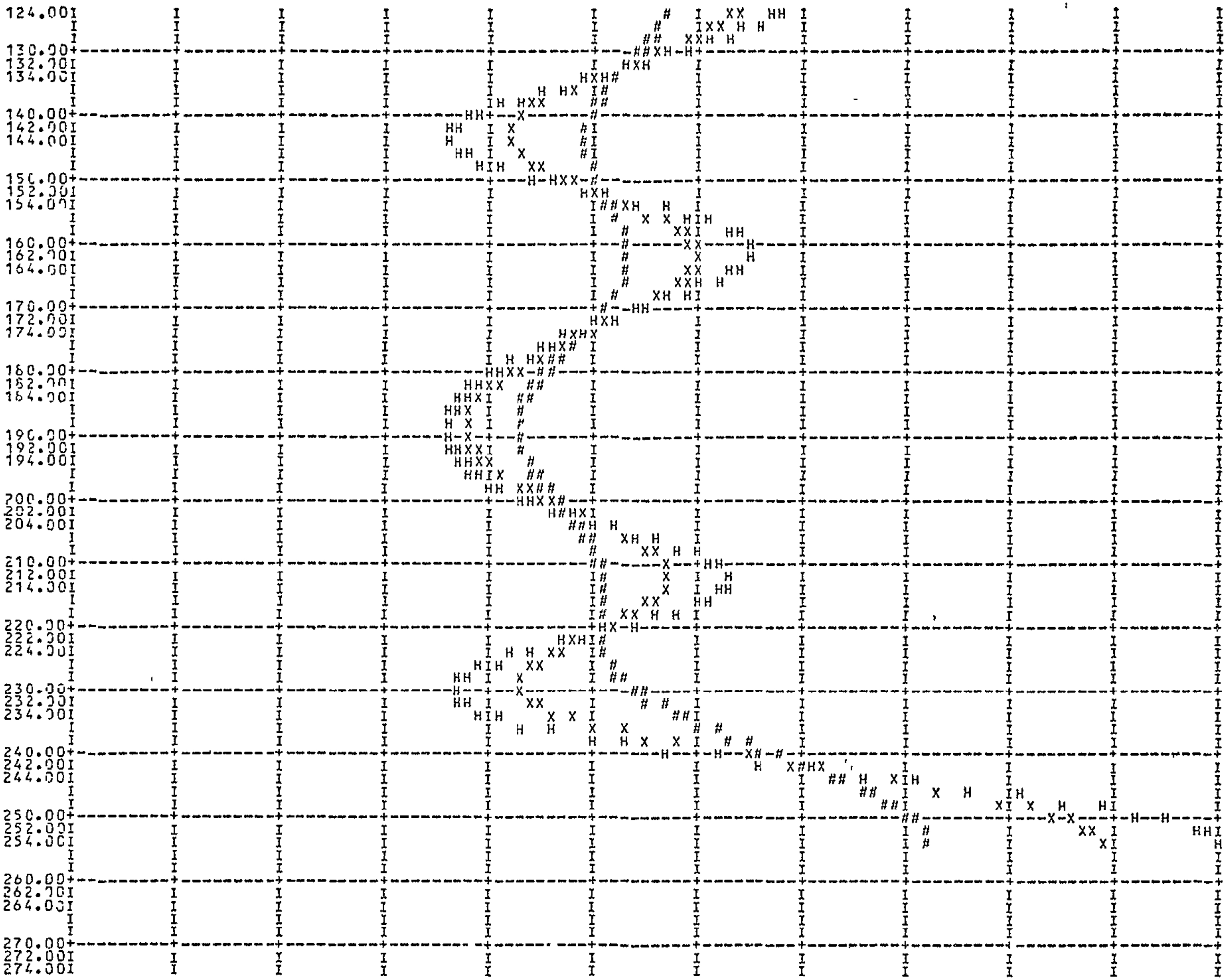


FIG.33 SPECTRUM OF AMPLITUDES (TAPE 3)

I



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272.00
274.00

FIG.33 (CONT.)

HISTOGRAM OF AMPLITUDES

XMIN= 5.00000 XMAX= 24.00000 NUMBER OF CELLS= 48 CELL INCREMENT= .395833

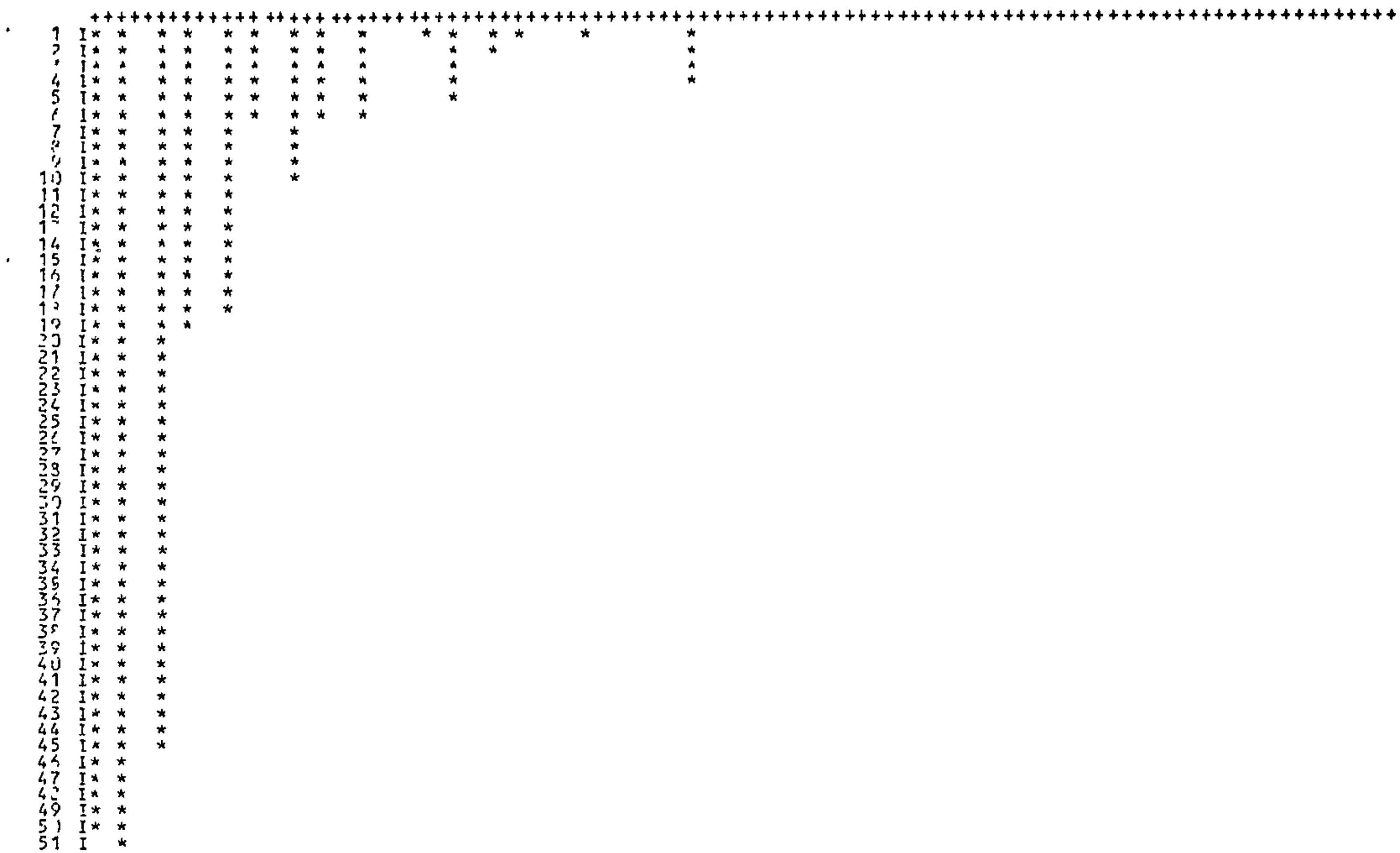


FIG.34 HISTOGRAM OF AMPLITUDES (TAPE 4)

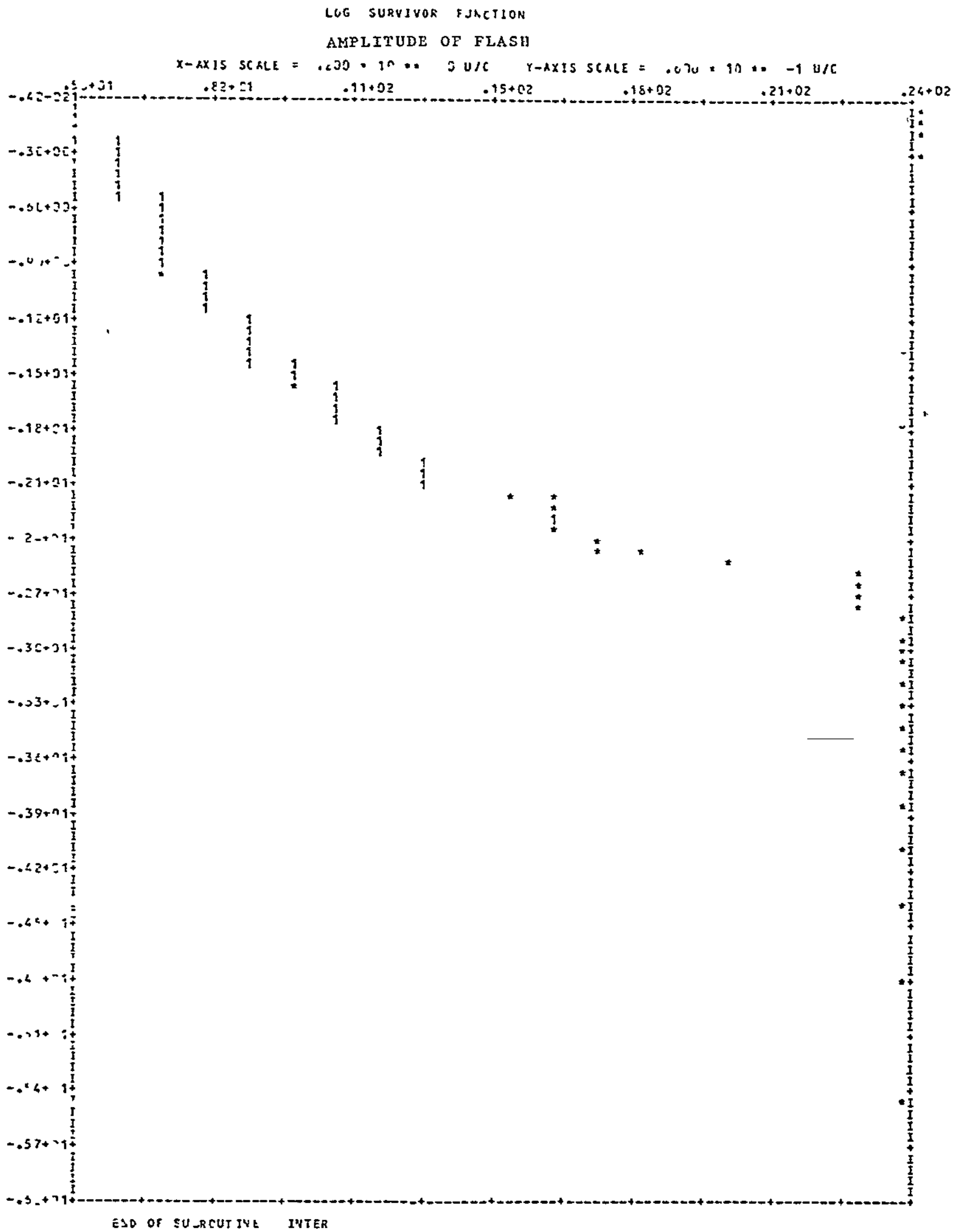


FIG.35 LOG SURVIVOR FUNCTION (TAPE 4)

PLOT OF SERIAL CORRELATION OF AMPLITUDES

NORMALIZED SERIAL CORRELATION COEFFICIENT

X-AXIS SCALE = .200 * 10 ** 0 U/C   Y-AXIS SCALE = 1.000 * 10 ** 0 U/C

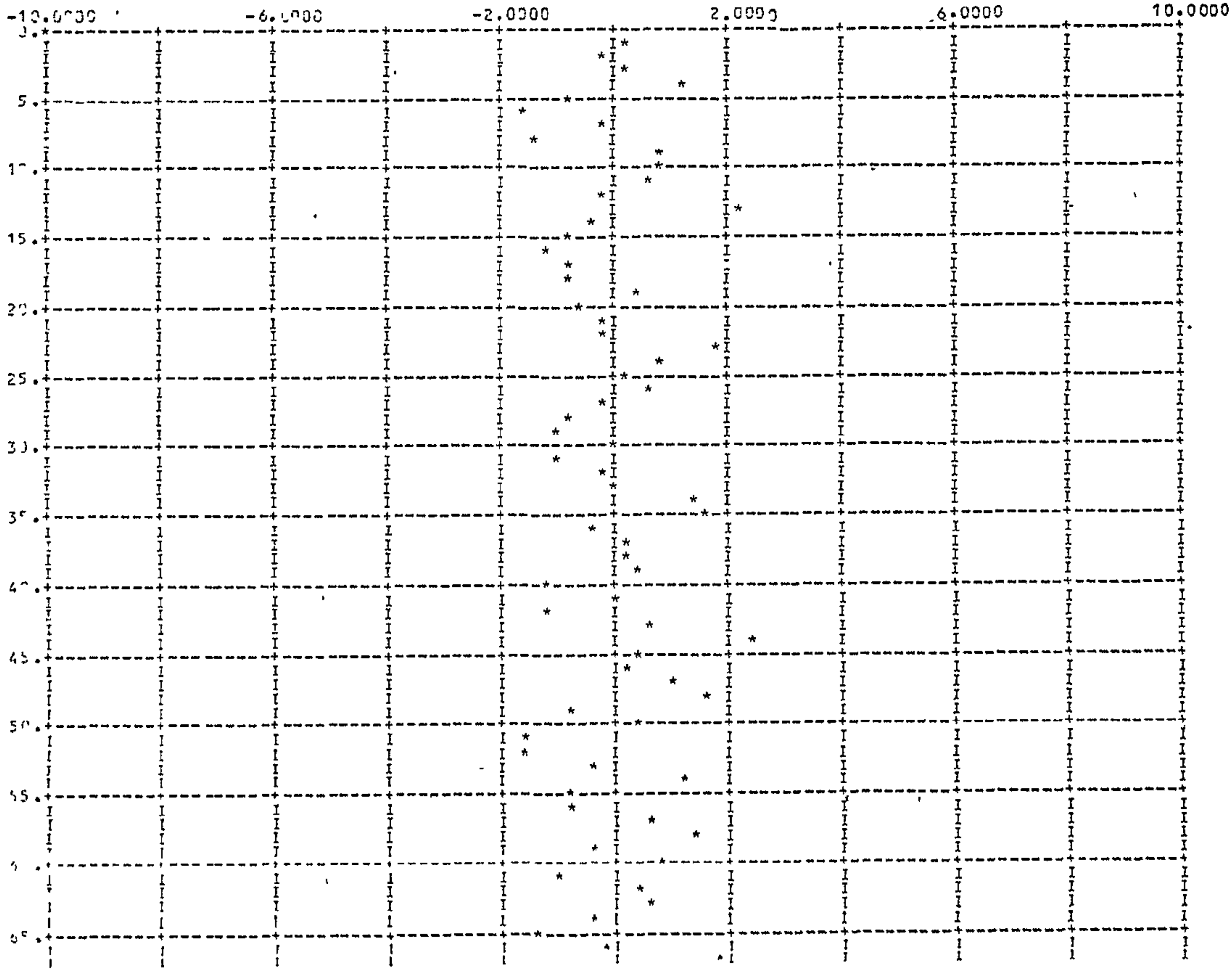
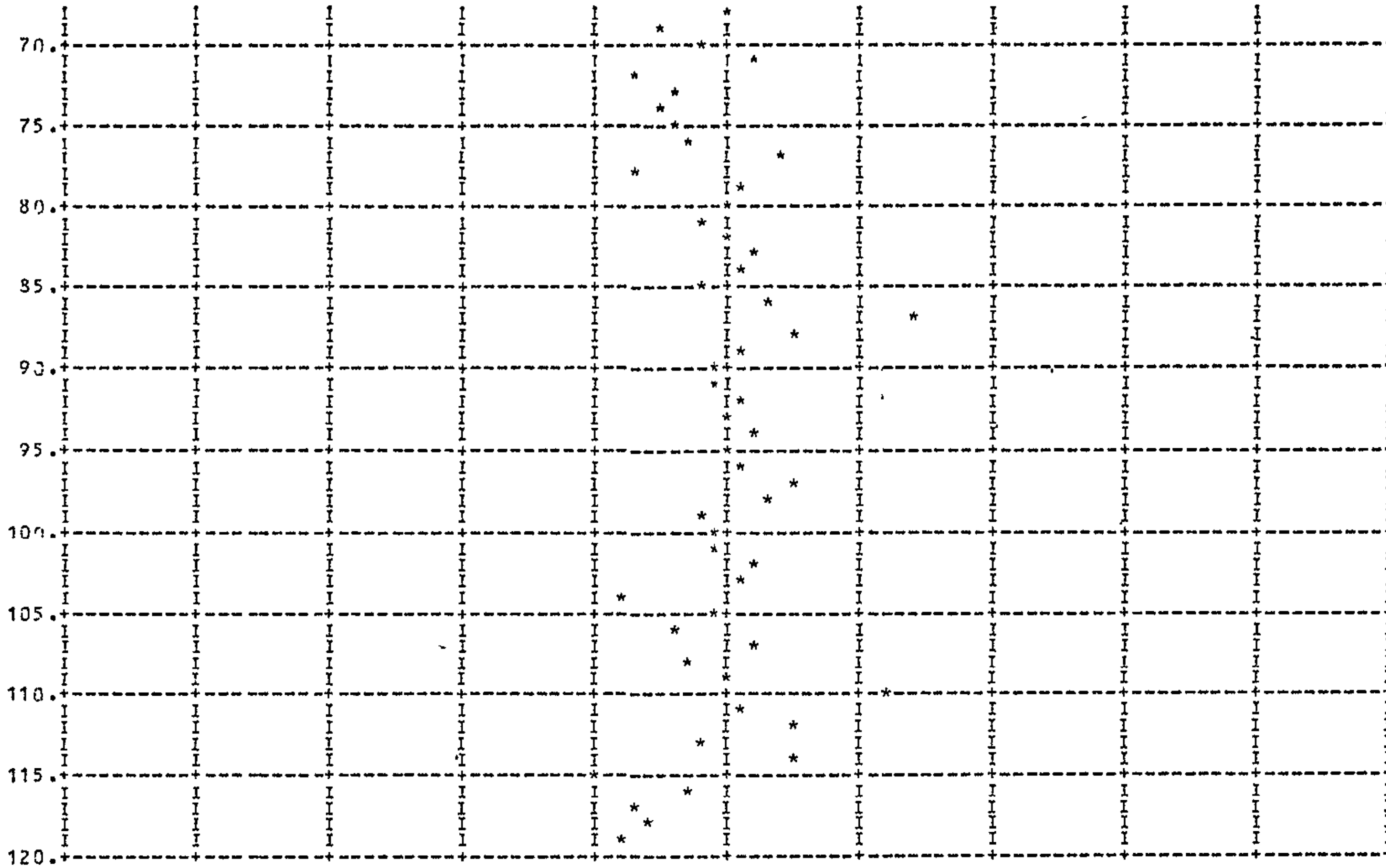


FIG.36 SERIAL CORRELATION OF AMPLITUDES (TAPE 4)



END OF SUBROUTINE RHO

FIG.36 (CONT.)

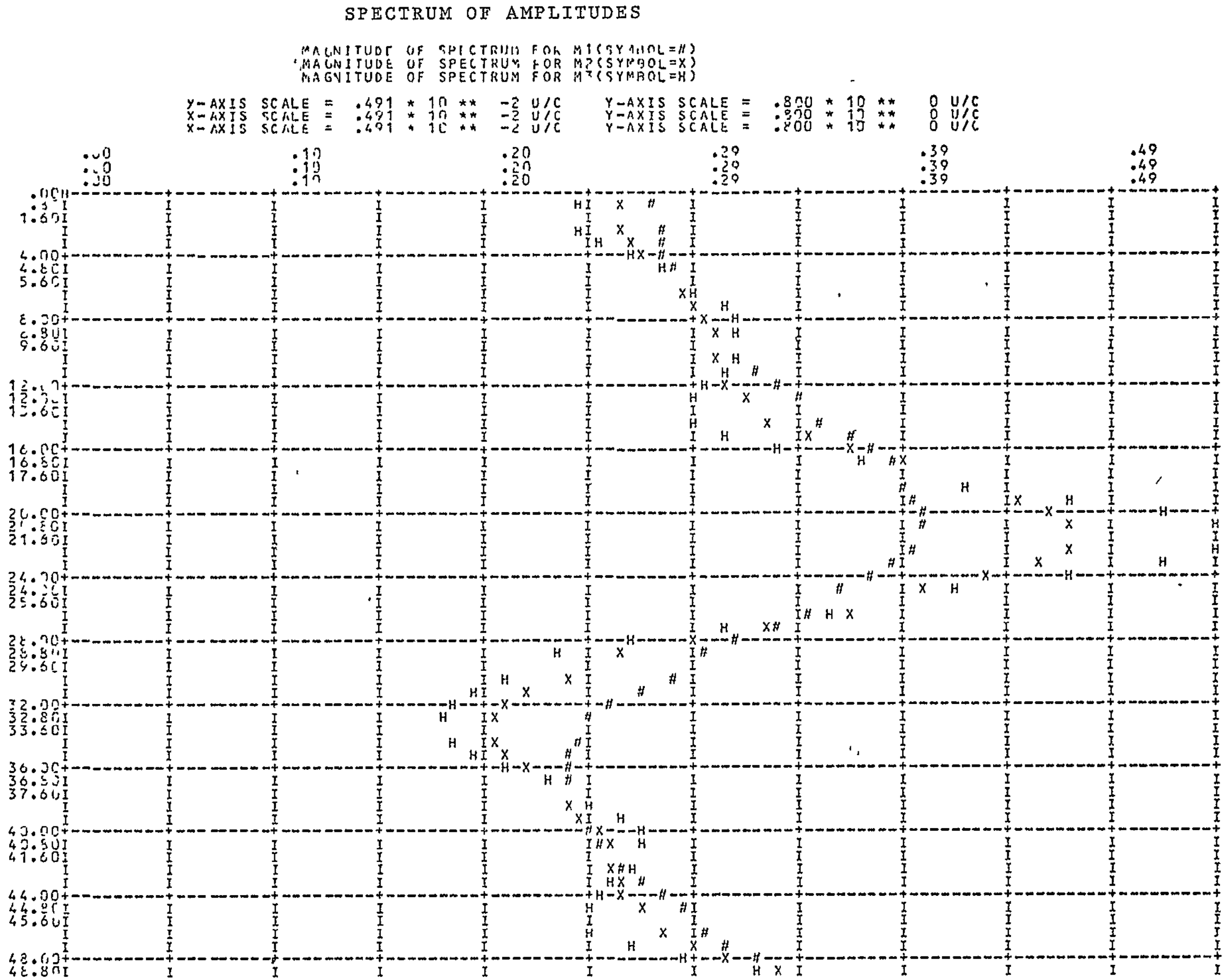


FIG.37 SPECTRUM OF AMPLITUDES (TAPE 4)

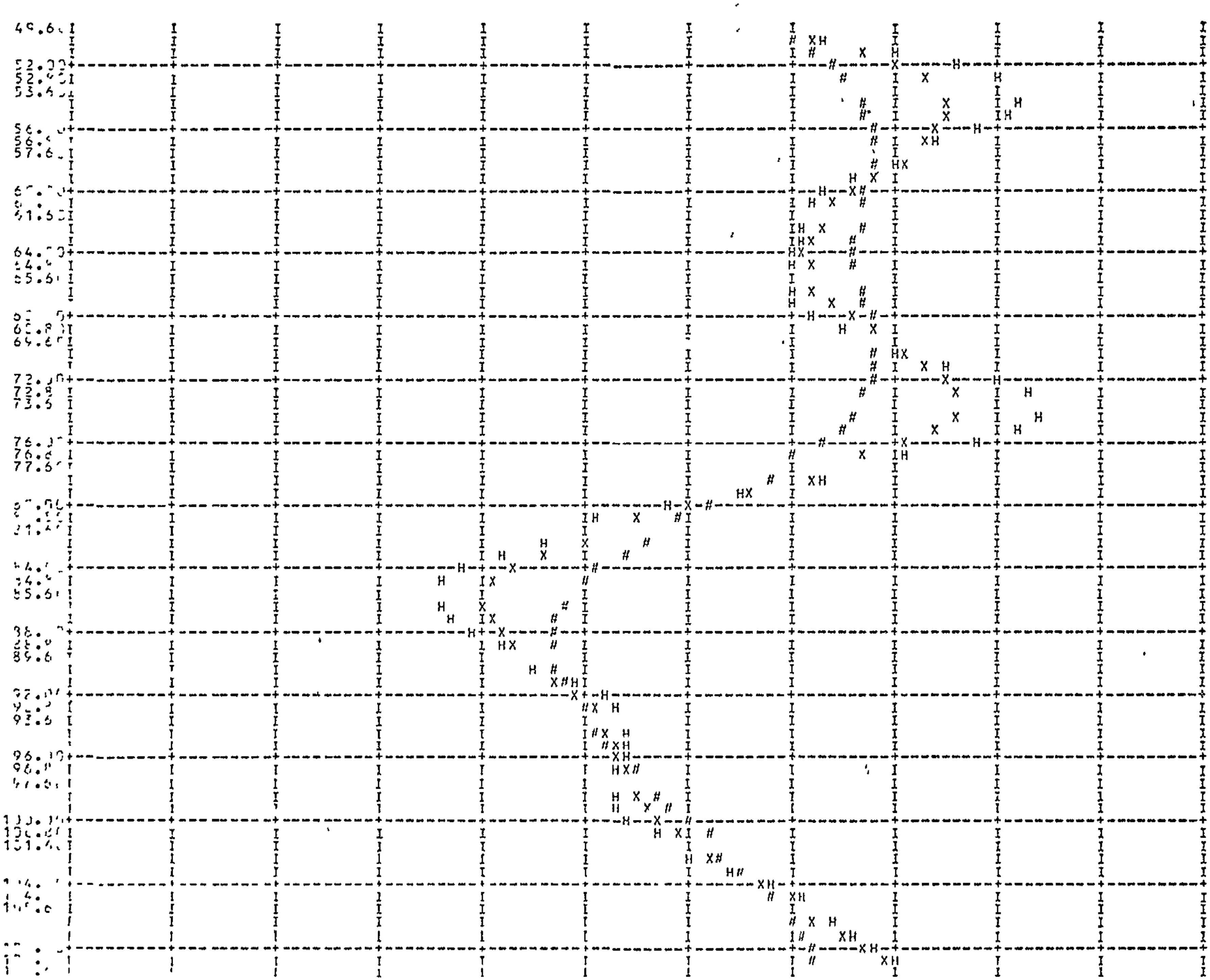


FIG.37 (CONT.)

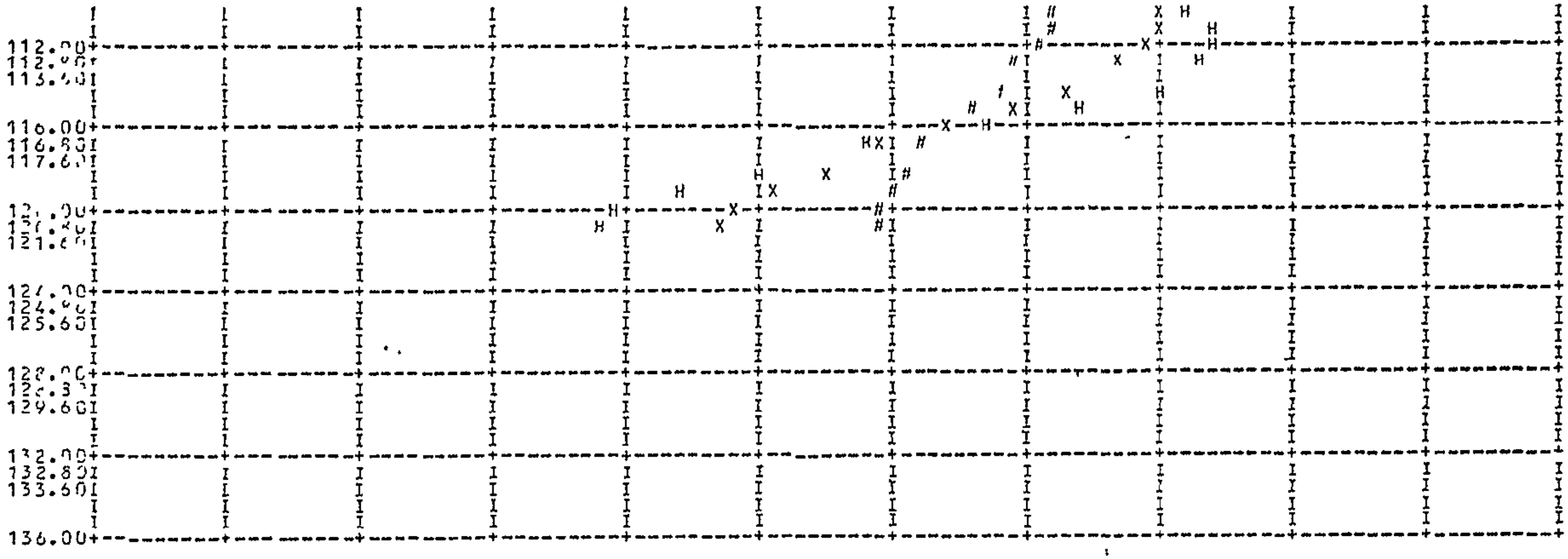


FIG.37 (CONT.)

HISTOGRAM OF AMPLITUDES

XMIN= 5.00000 XMAX= 24.00000 NUMBER OF CELLS= 45 CELL INCREMENT= .422222

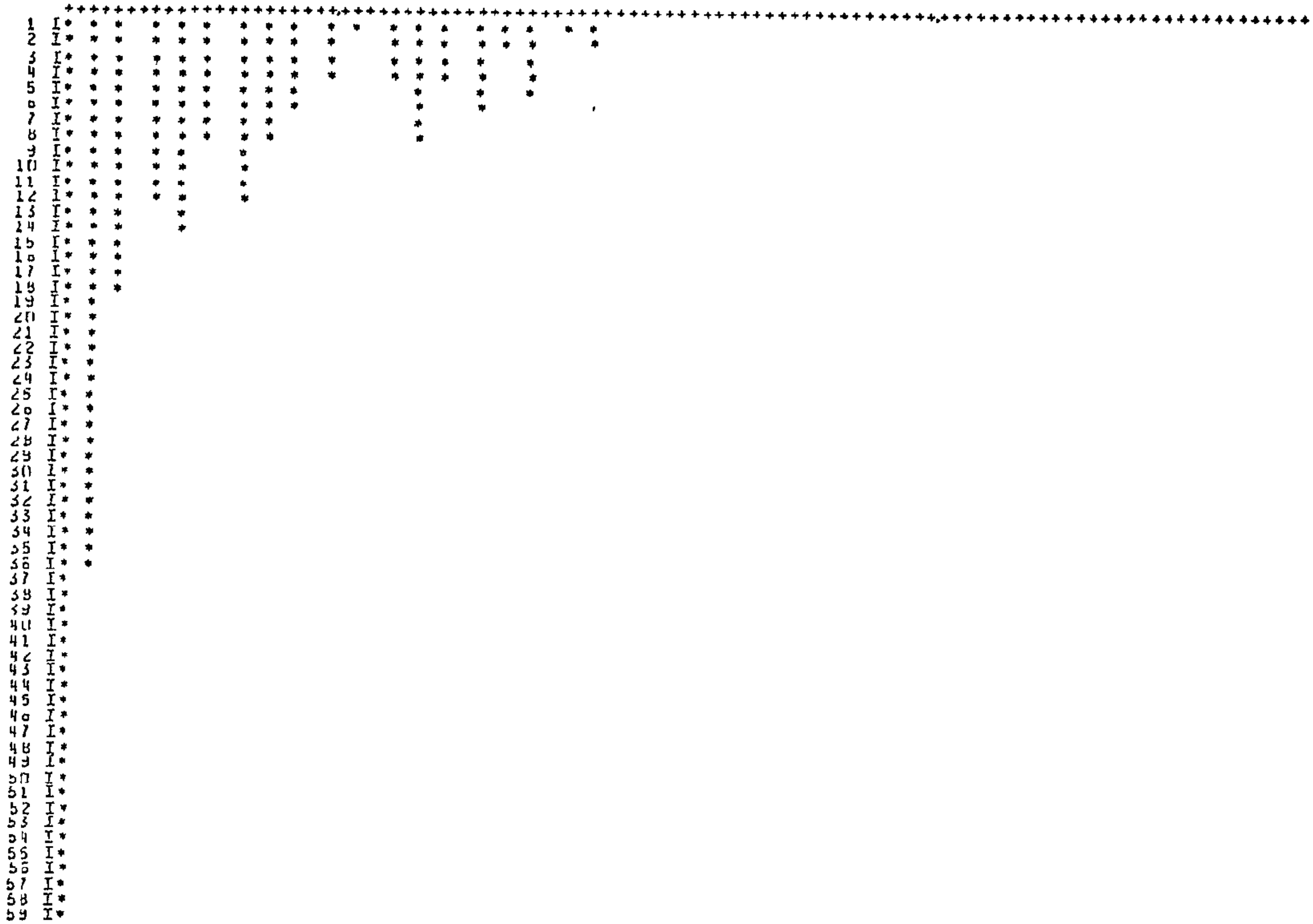


FIG.38 HISTOGRAM OF AMPLITUDES (TAPE 5)

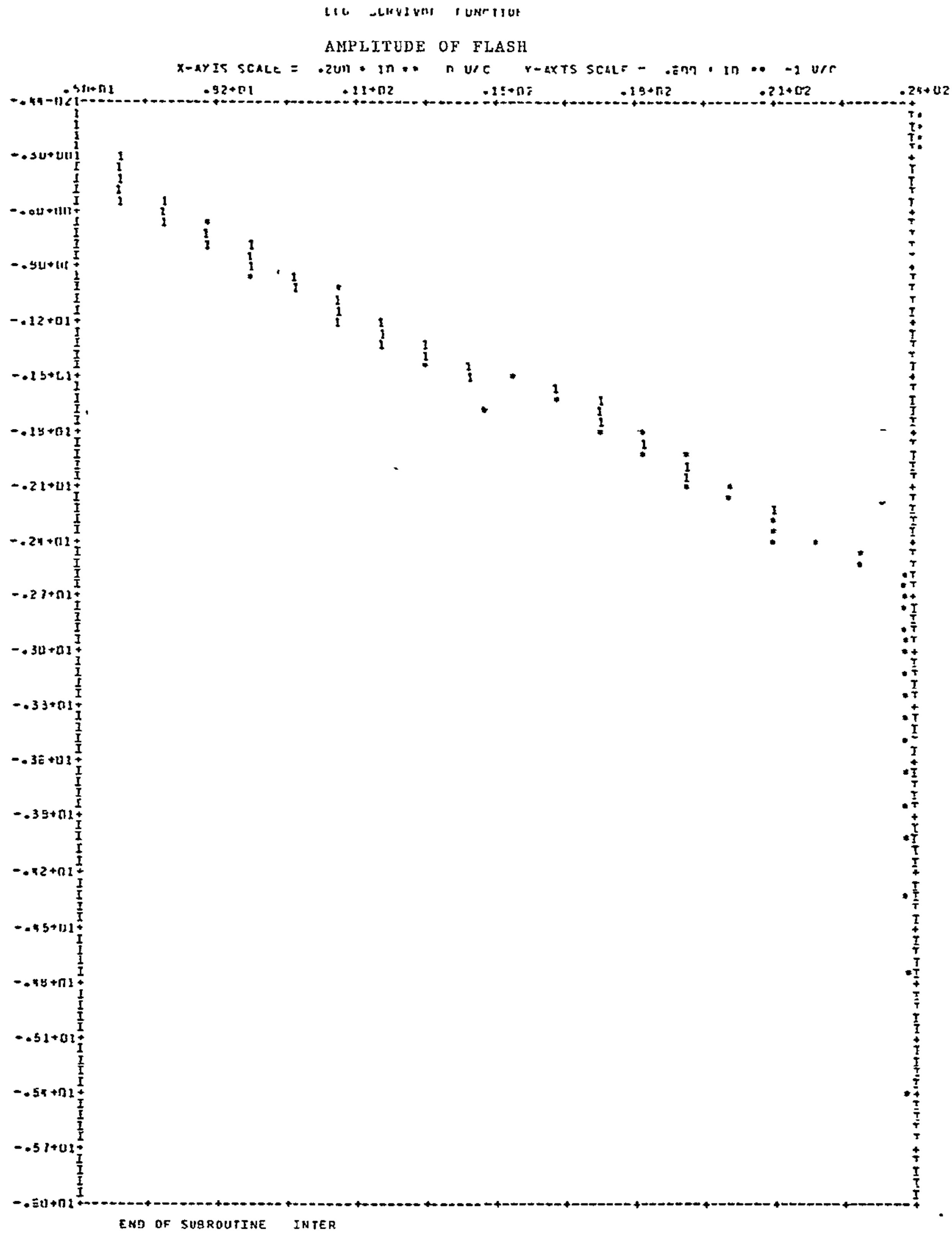


FIG.39 LOG SURVIVOR FUNCTION (TAPE 5)

PLOT OF SERIAL CORRELATION OF AMPLITUDES

NORMALIZED SERIAL CORRELATION COEFFICIENT

X-AXIS SCALE = .200 * 10 ** 0 U/C   Y-AXIS SCALE = 1.000 * 10 ** 0 U/C

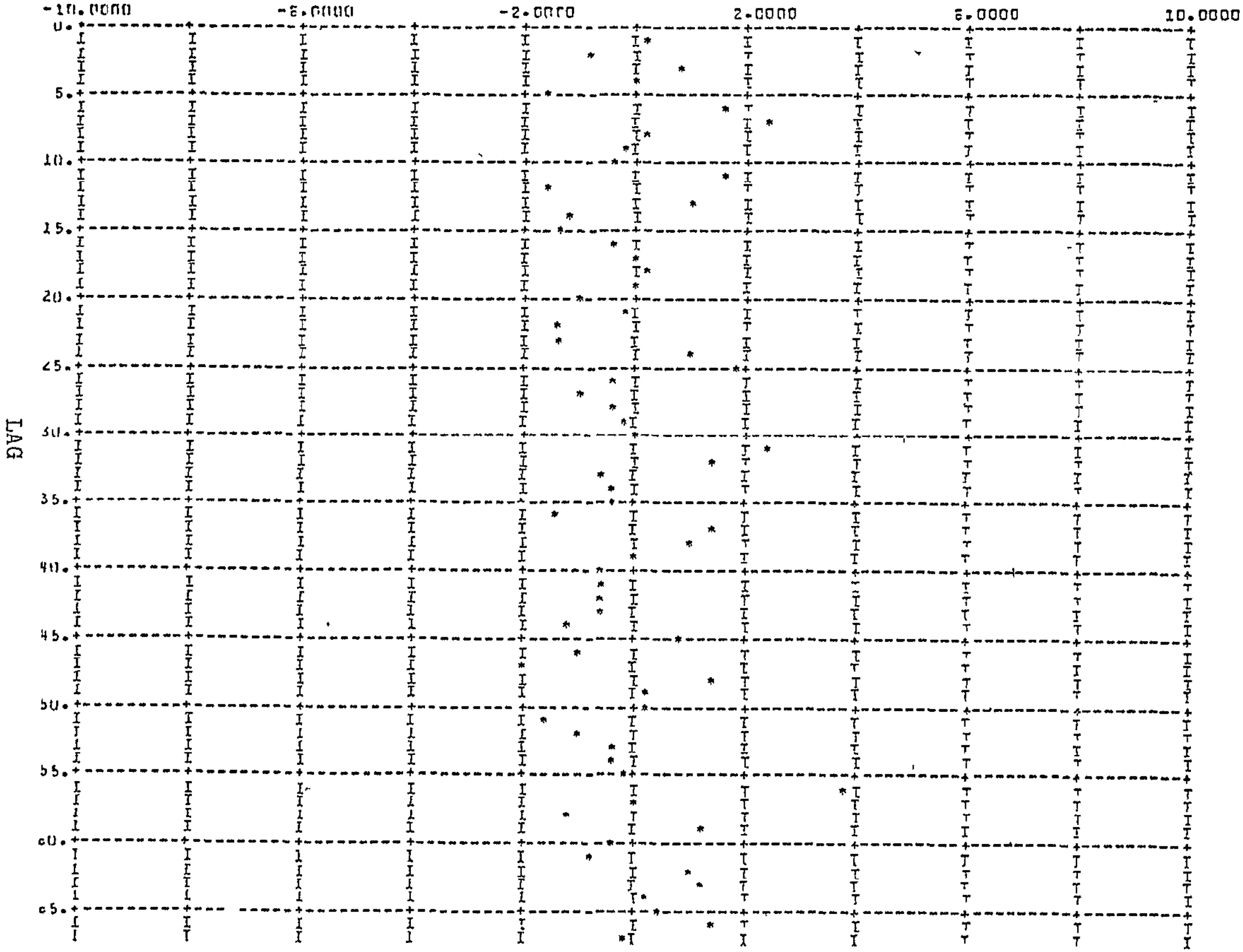
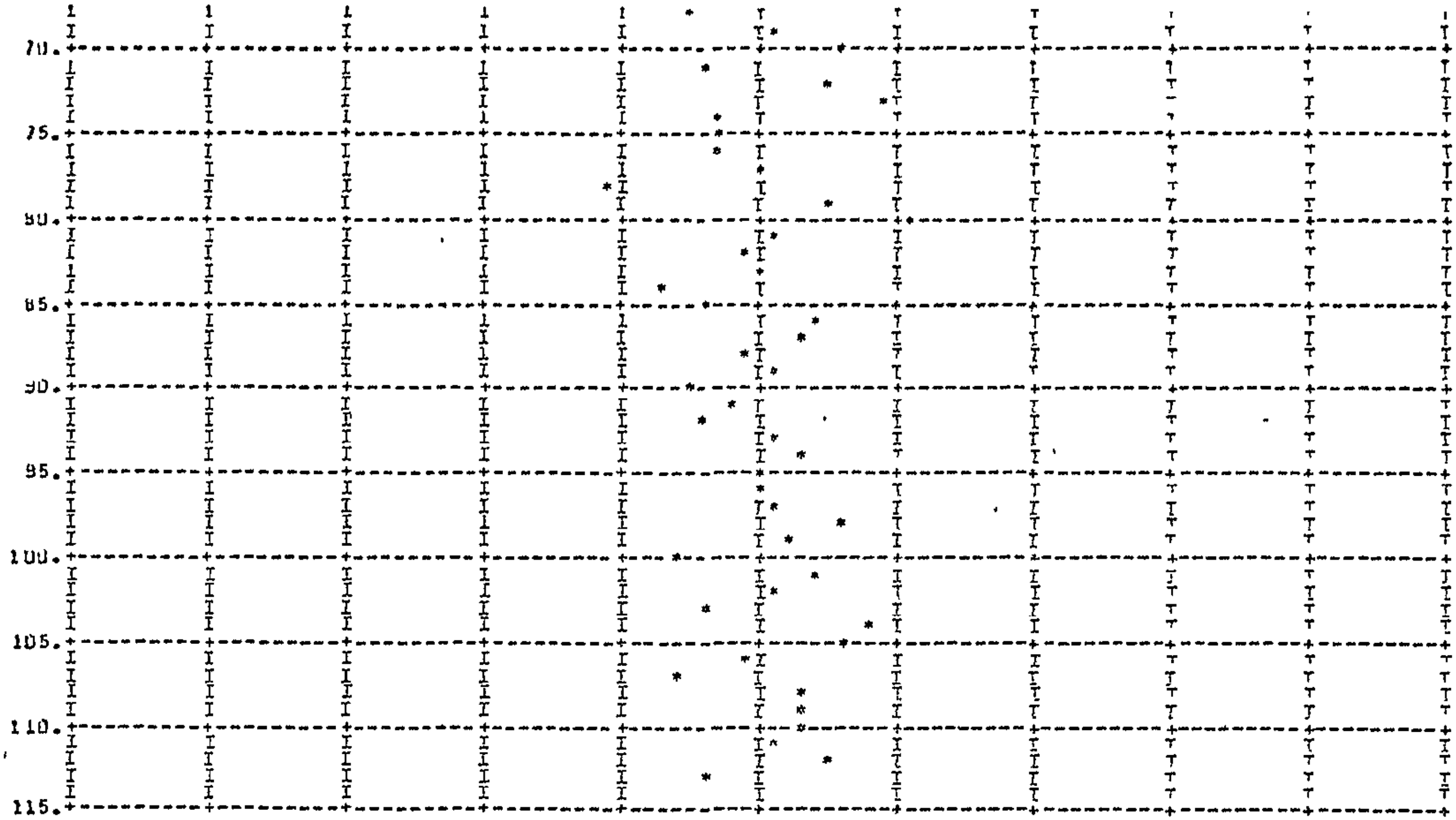


FIG. 40 SERIAL CORRELATION OF AMPLITUDES (TAPE 5)



END OF SUBROUTINE RHO

FIG. 40 (CONT.)

SPECTRUM OF AMPLITUDES

MAGNITUDE OF SPECTRUM FOR M1(SYMBOL=#)
MAGNITUDE OF SPECTRUM FOR M2(SYMBOL=X)
MAGNITUDE OF SPECTRUM FOR M3(SYMBOL=H)

X-AXIS SCALE = .585 * 10 ** -2 U/C   Y-AXIS SCALE = .700 * 10 ** 0 U/C
X-AXIS SCALE = .585 * 10 ** -2 U/C   Y-AXIS SCALE = .700 * 10 ** 0 U/C
X-AXIS SCALE = .585 * 10 ** -2 U/C   Y-AXIS SCALE = .700 * 10 ** 0 U/C

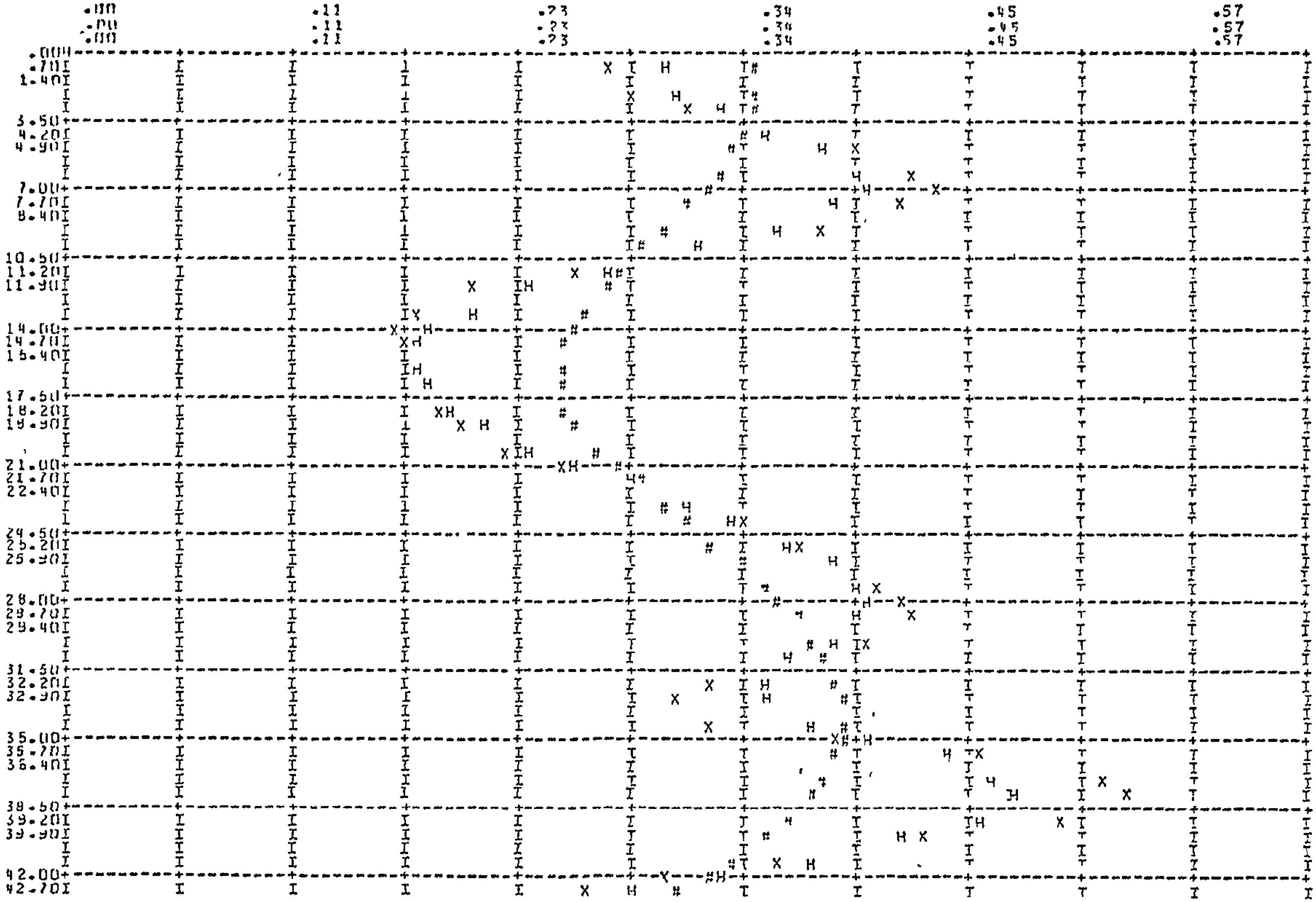


FIG.41 SPECTRUM OF AMPLITUDES (TAPE 5)

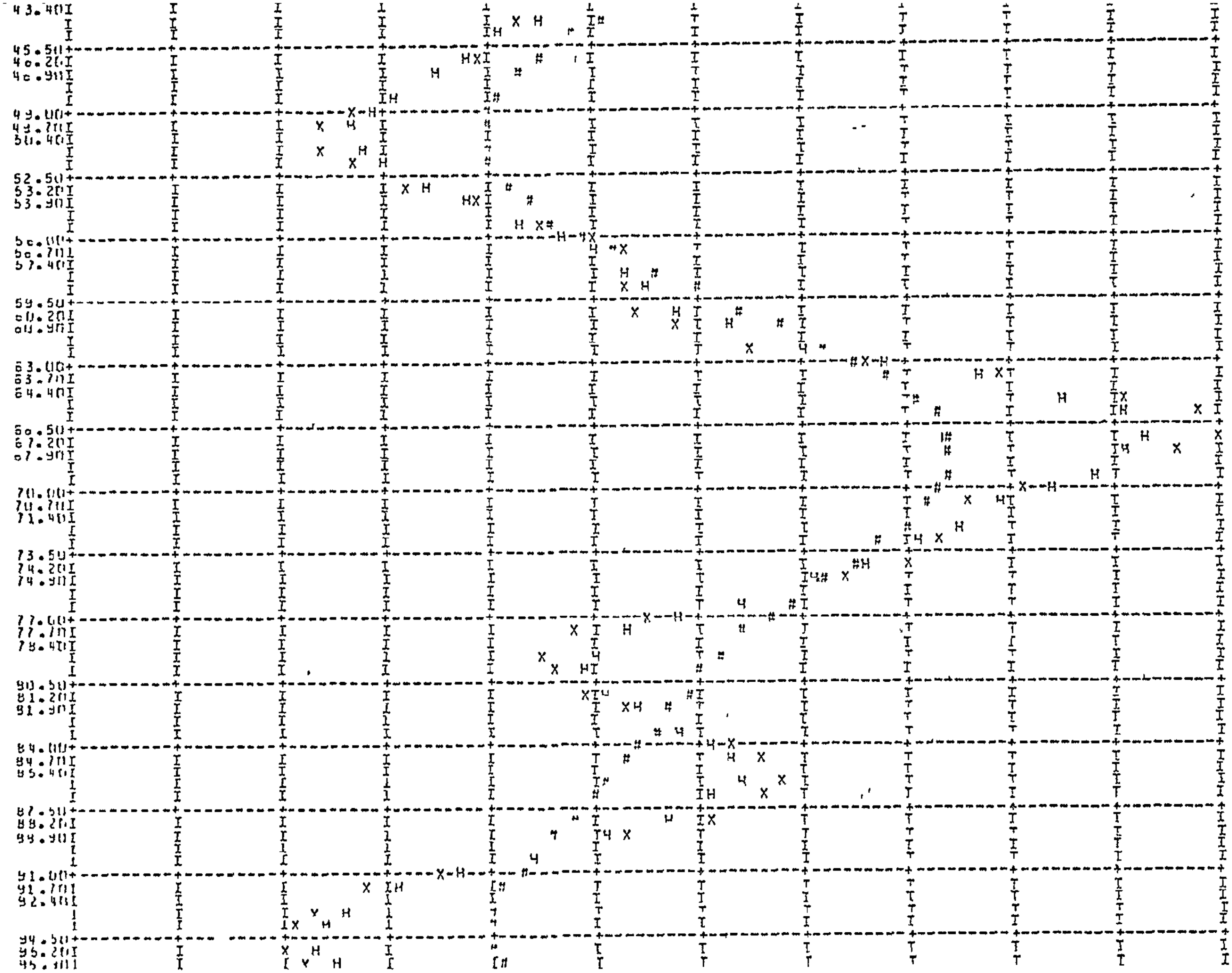


FIG. 41 (CONT.)

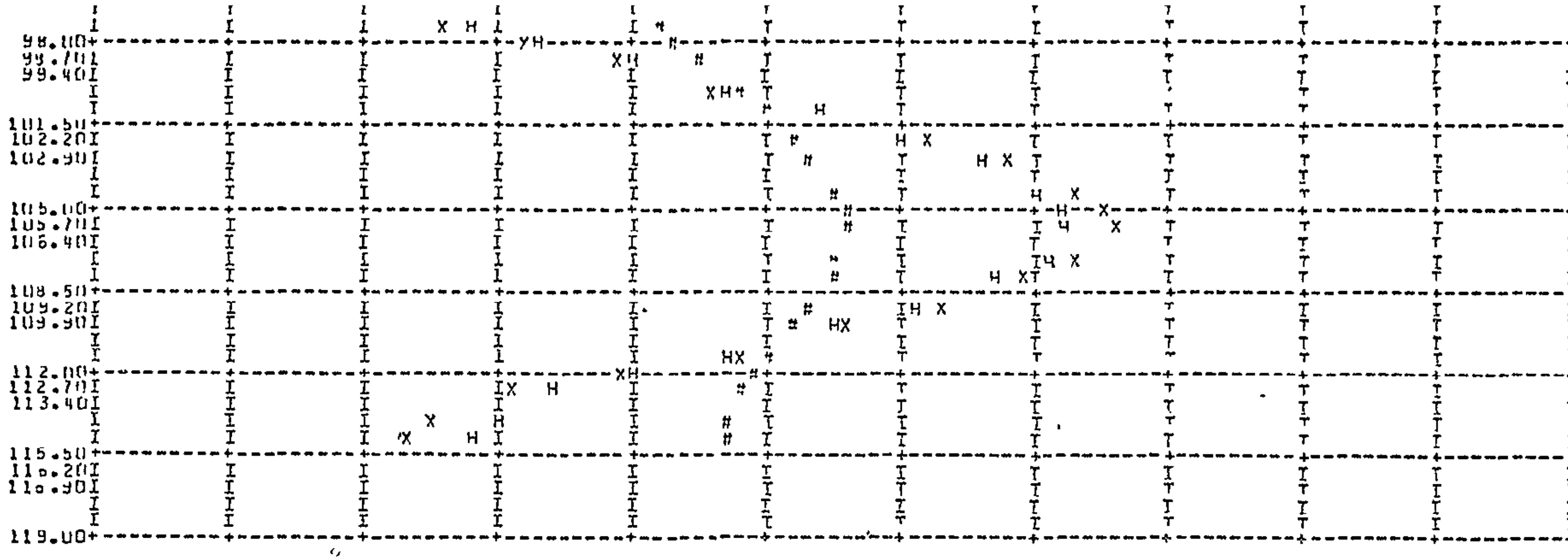


FIG. 41 (CONT.)

HISTOGRAM OF DURATIONS

```
XMIN=      .20000  XMAY=     2.30000         NUMBER OF CELLS=   19 CELL INCREMENT=    .110526

                  ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
                1 I*****************
                2 I********** ****
                3 I********** ** *
                4 I*****   **
                5 I** **   **
                6 I** **    *
                7 I** **    *
                8 I** **    *
                9 I** **    *
               10 I** *     *
               11 I** *     *
               12 I**       *
               13 I*
               14 I*
               15 I*
               16 I*
               17 I*
               13 I*
```

FIG.42 HISTOGRAM OF DURATIONS (TAPE 1)

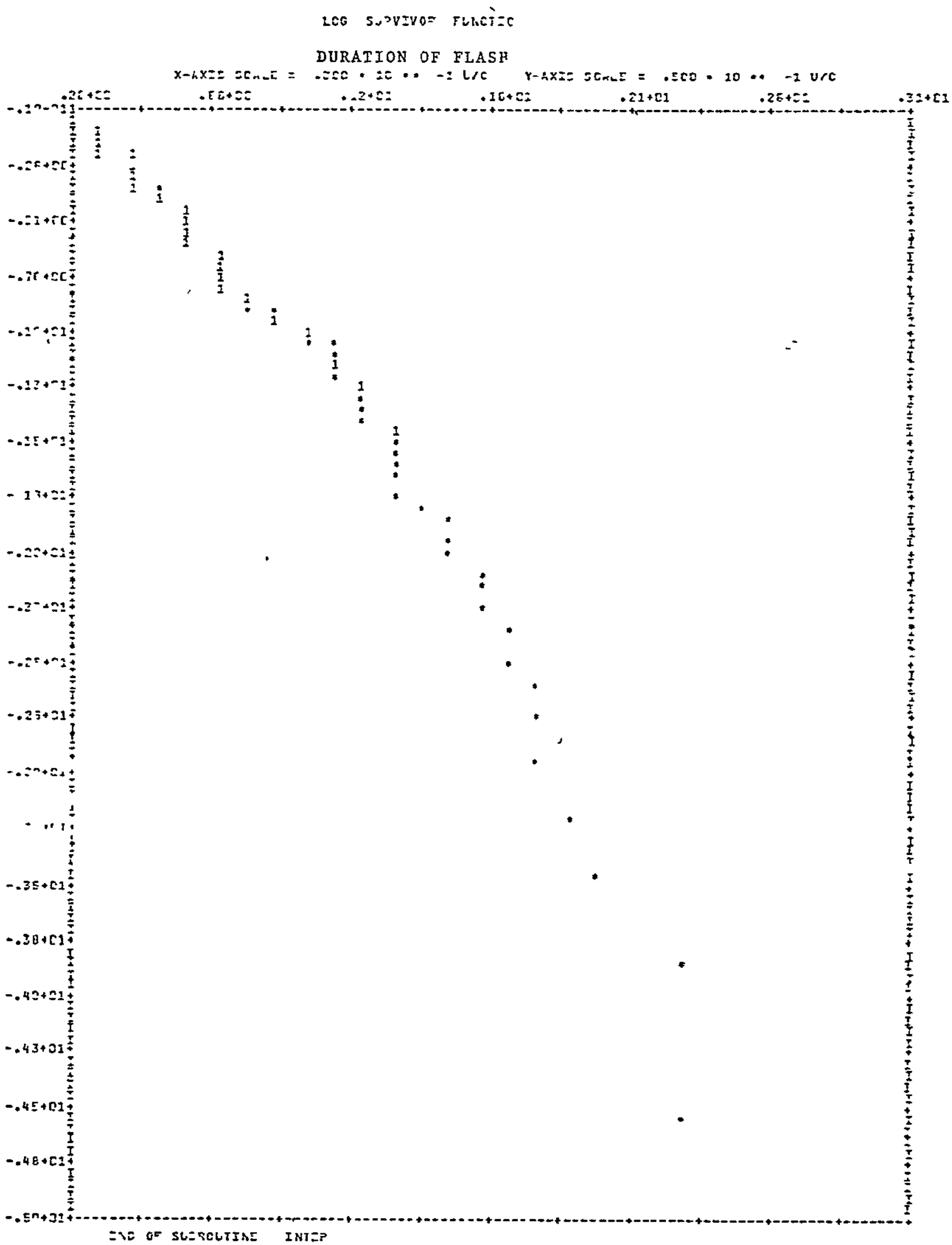


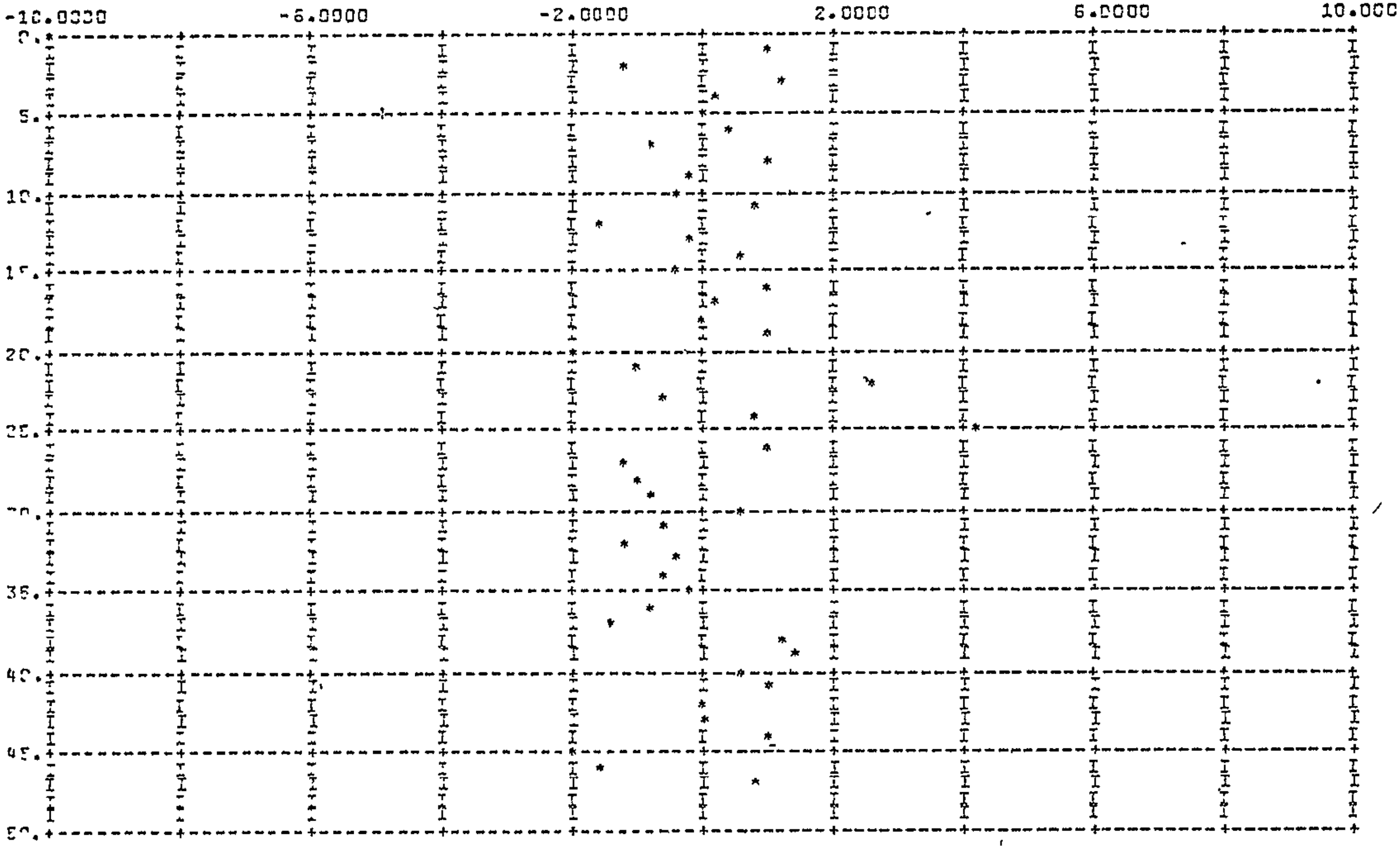
FIG.43 LOG SURVIVOR FUNCTION (TAPE 1)

C-2

PLOT OF SERIAL CORRELATION OF DURATIONS

NORMALIZED SERIAL CORRELATION COEFFICIENT

Y-AXIS SCALE = .200 * 10 ** 0 U/C   Y-AXIS SCALE = 1.000 * 10 ** 0 U/C



END OF SUBROUTINE RHO

FIG. 44 SERIAL CORRELATION OF DURATIONS (TAPE 1)

HISTOGRAM OF DURATIONS

XMIN= .10000 XMAX= 1.20000 NUMBER OF CELLS= 82 CELL INCREMENT= .013415

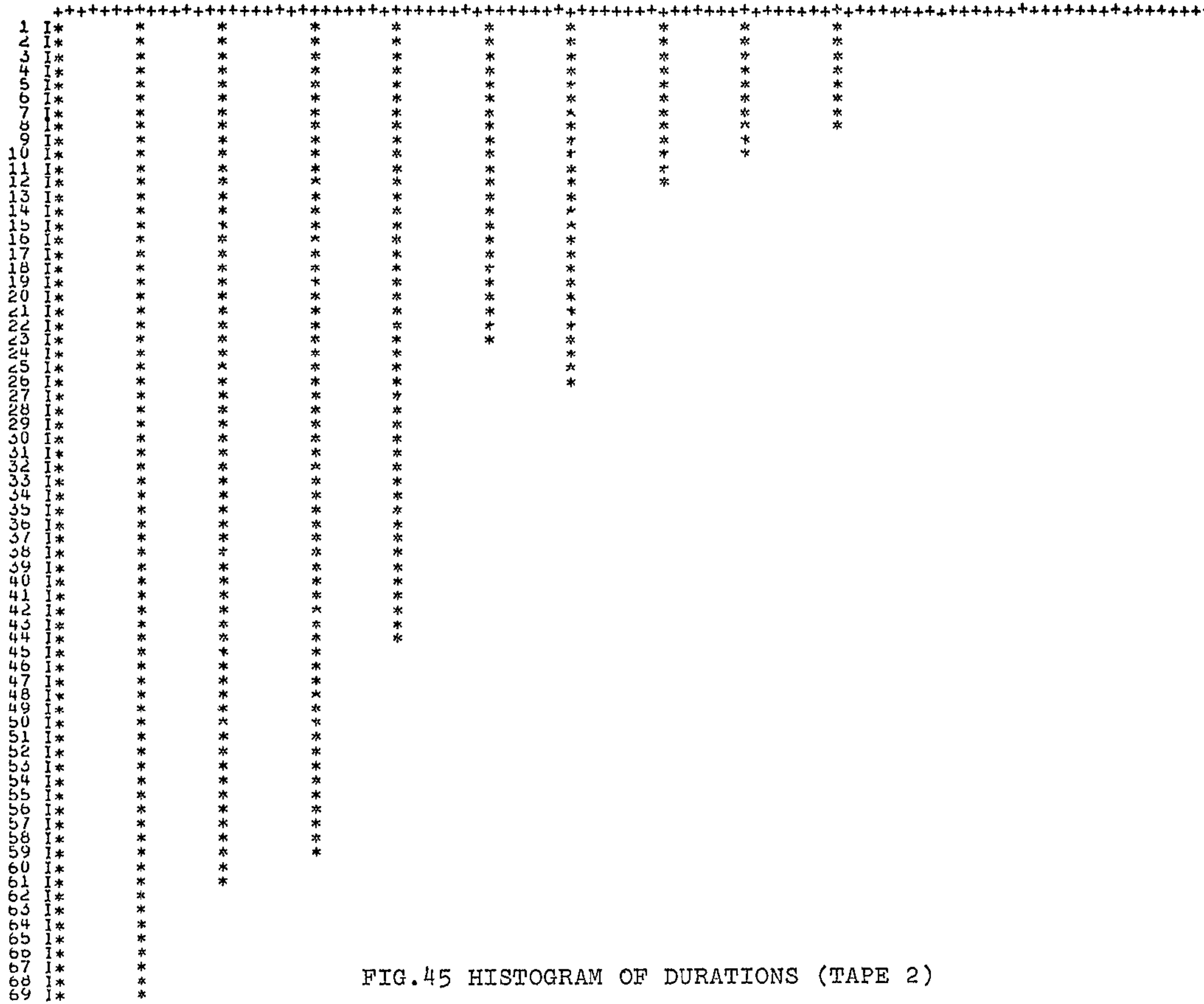


FIG.45 HISTOGRAM OF DURATIONS (TAPE 2)

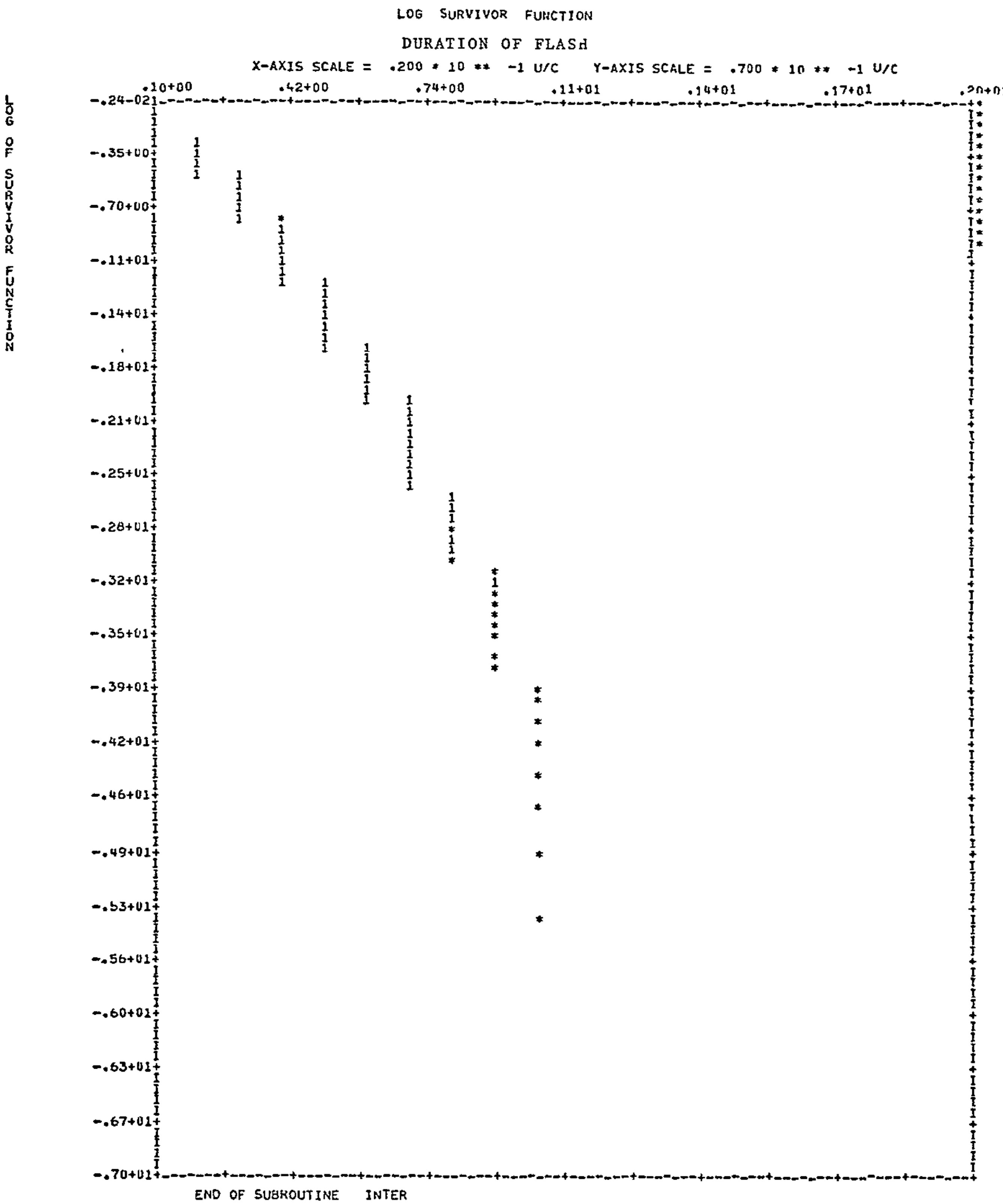


FIG.46 LOG SURVIVOR FUNCTION (TAPE 2)

PLOT OF SERIAL CORRELATION OF DURATIONS

NORMALIZED SERIAL CORRELATION COEFFICIENT

X-AXIS SCALE = .200 * 10 ** 0 U/C Y-AXIS SCALE = 1.000 * 10 ** 0 U/C

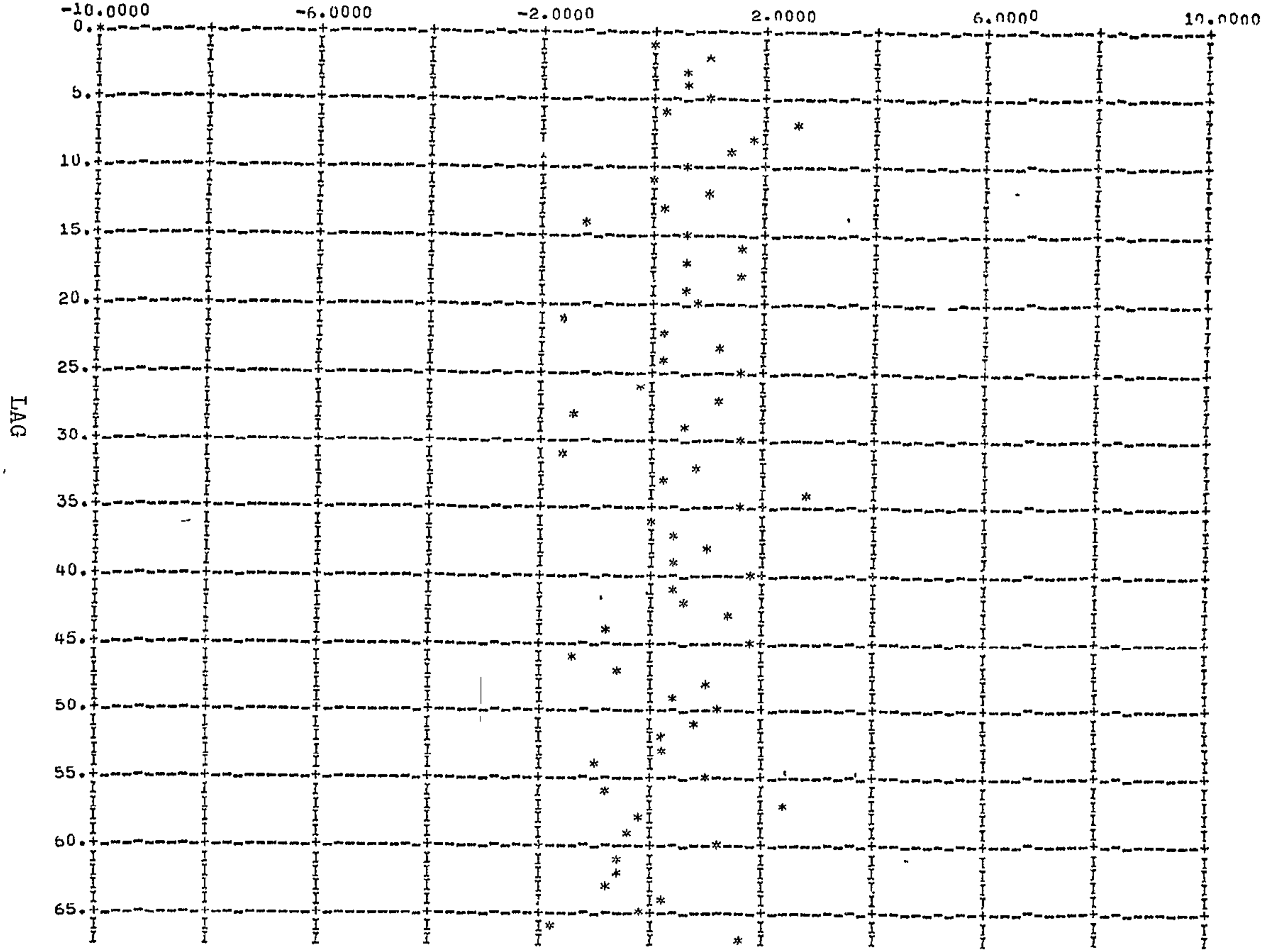


FIG. 47 SERIAL CORRELATION OF DURATIONS (TAPE 2)

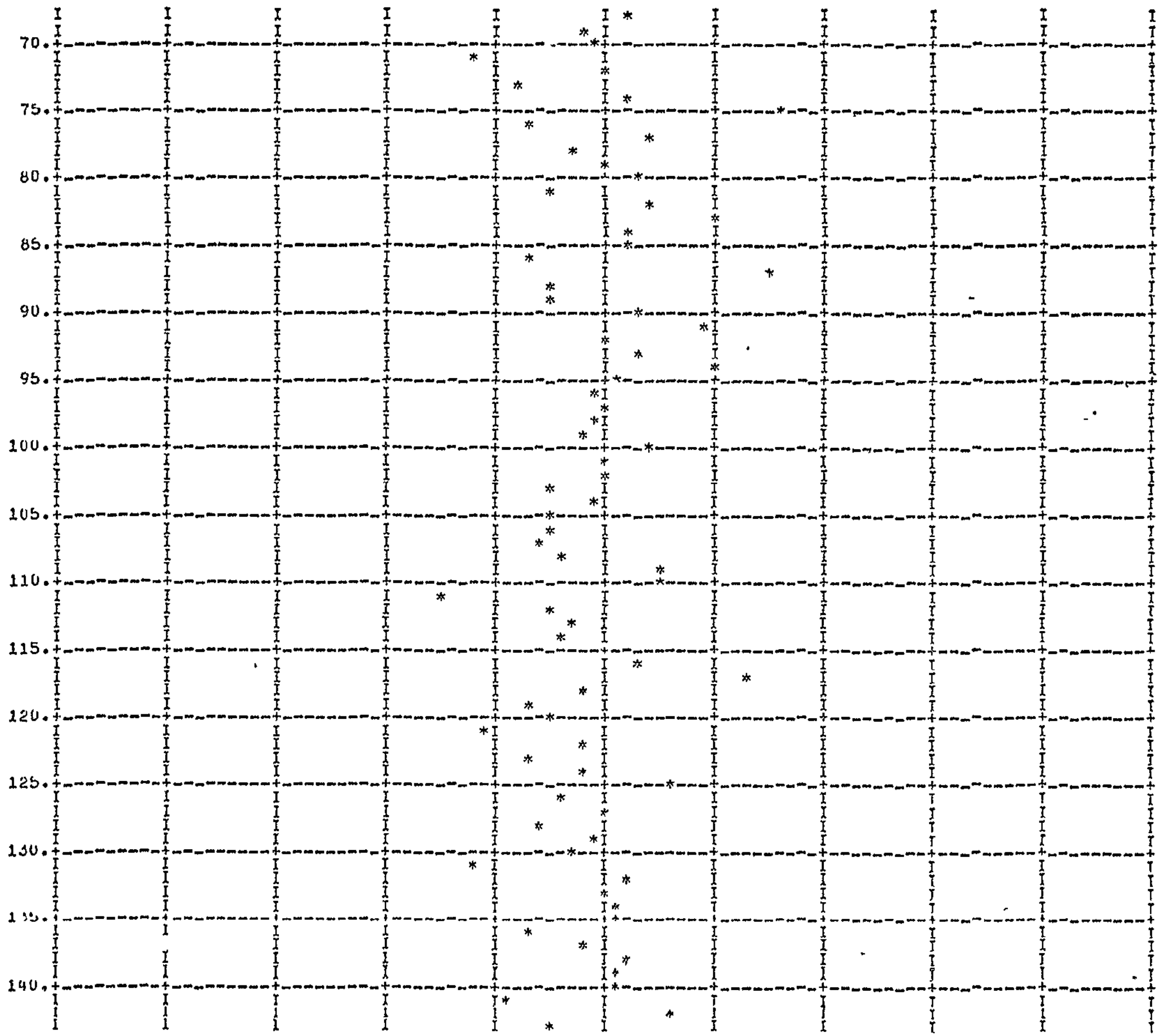


FIG.47 (CONT.)

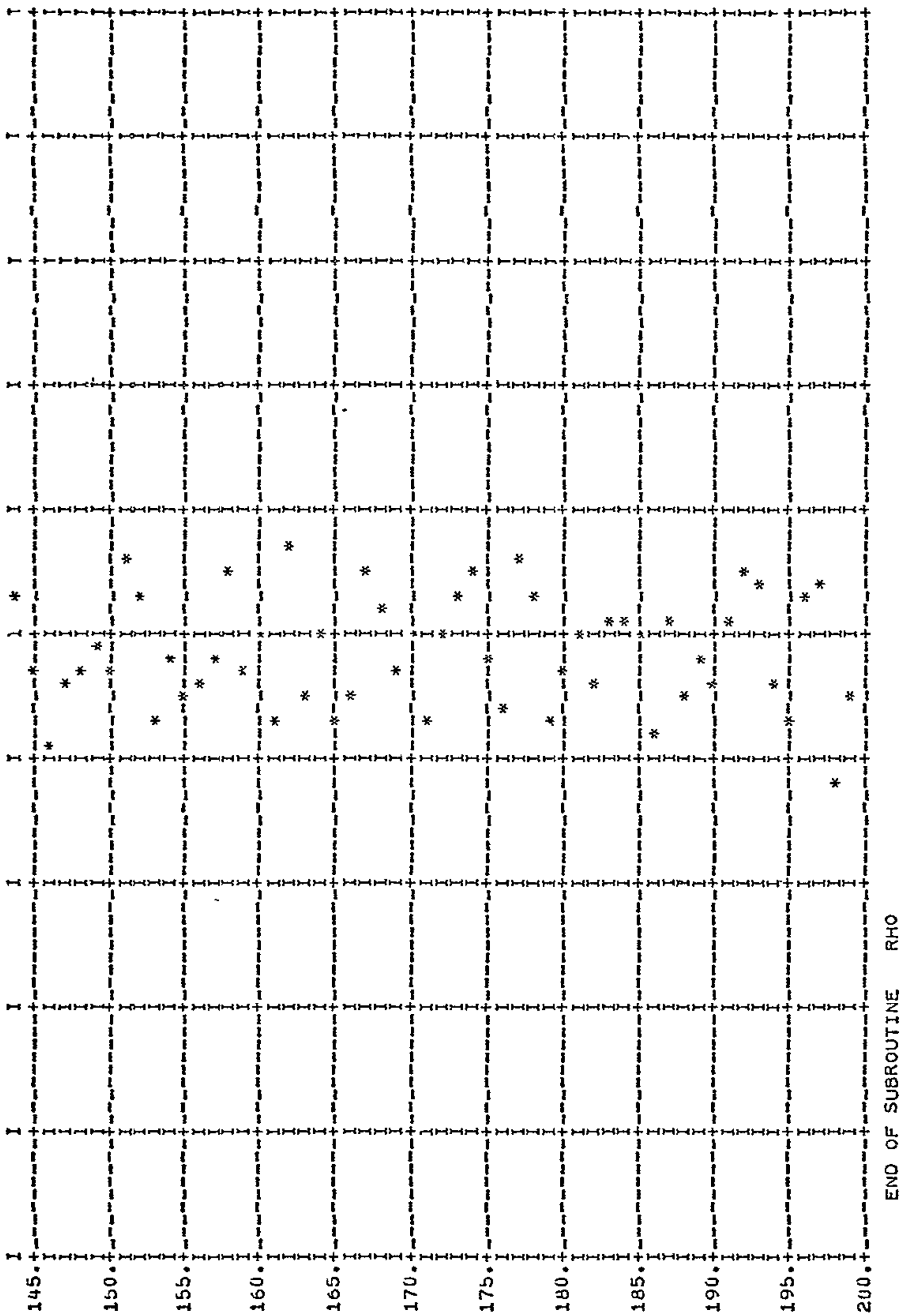


FIG.47 (CONT.) .

SPECTRUM OF DURATIONS

MAGNITUDE OF SPECTRUM FOR M1(SYMBOL=#)
MAGNITUDE OF SPECTRUM FOR M2(SYMBOL=X)
MAGNITUDE OF SPECTRUM FOR M3(SYMBOL=H)

X-AXIS SCALE = .588 * 10 ** -2 U/C   Y-AXIS SCALE = 2.000 * 10 ** 0 U/C
X-AXIS SCALE = .588 * 10 ** -2 U/C   Y-AXIS SCALE = 2.000 * 10 ** 0 U/C
X-AXIS SCALE = .588 * 10 ** -2 U/C   Y-AXIS SCALE = 2.000 * 10 ** 0 U/C

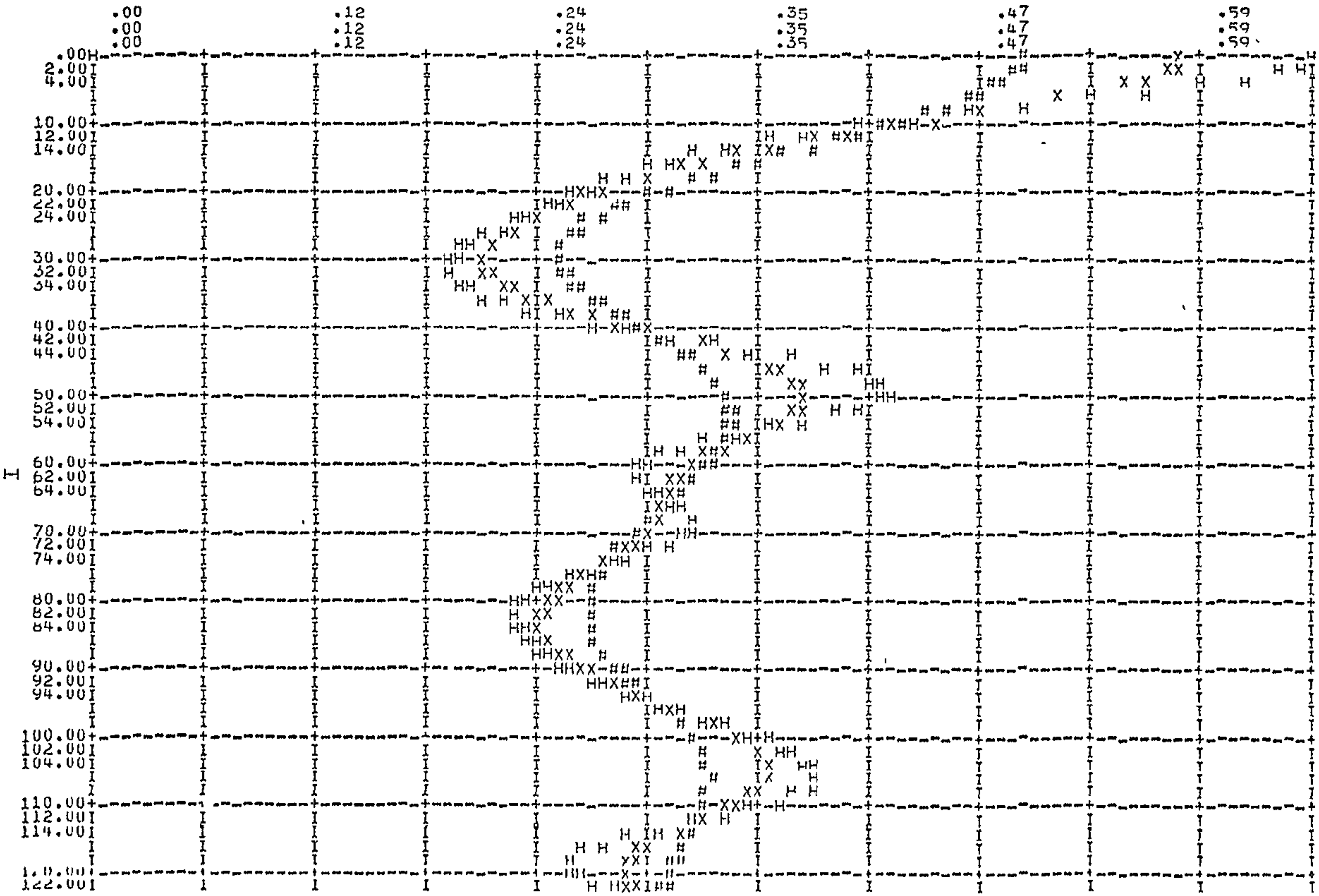
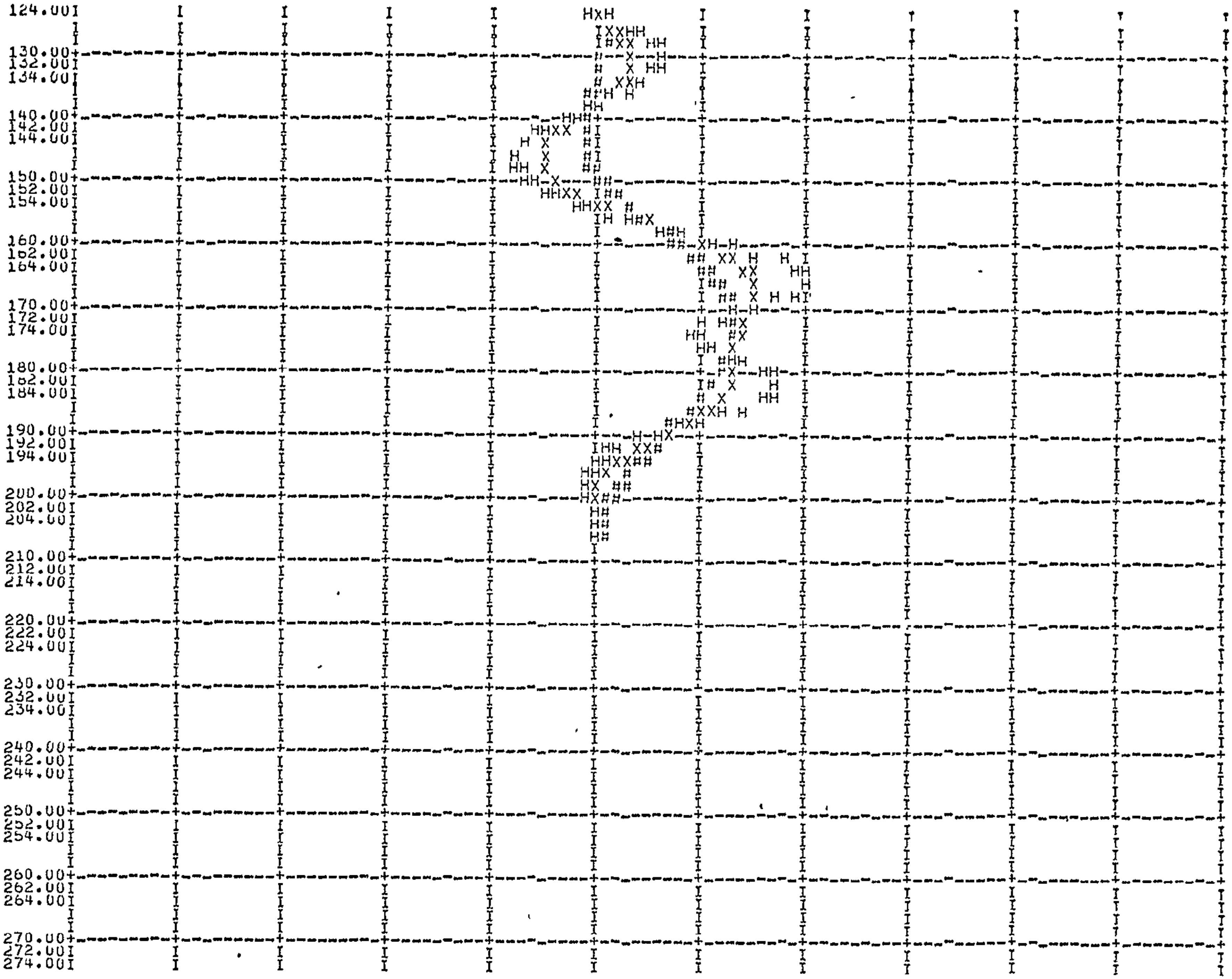


FIG.48 SPECTRUM OF DURATIONS (TAPE 2)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262.00
264.00
270.00
272.00
274.00

FIG. 48 (CONT.)

HISTOGRAM OF DURATIONS

XMIN= .10000 XMAX= 1.60000 NUMBER OF CELLS= 100 CELL INCREMENT= .015000

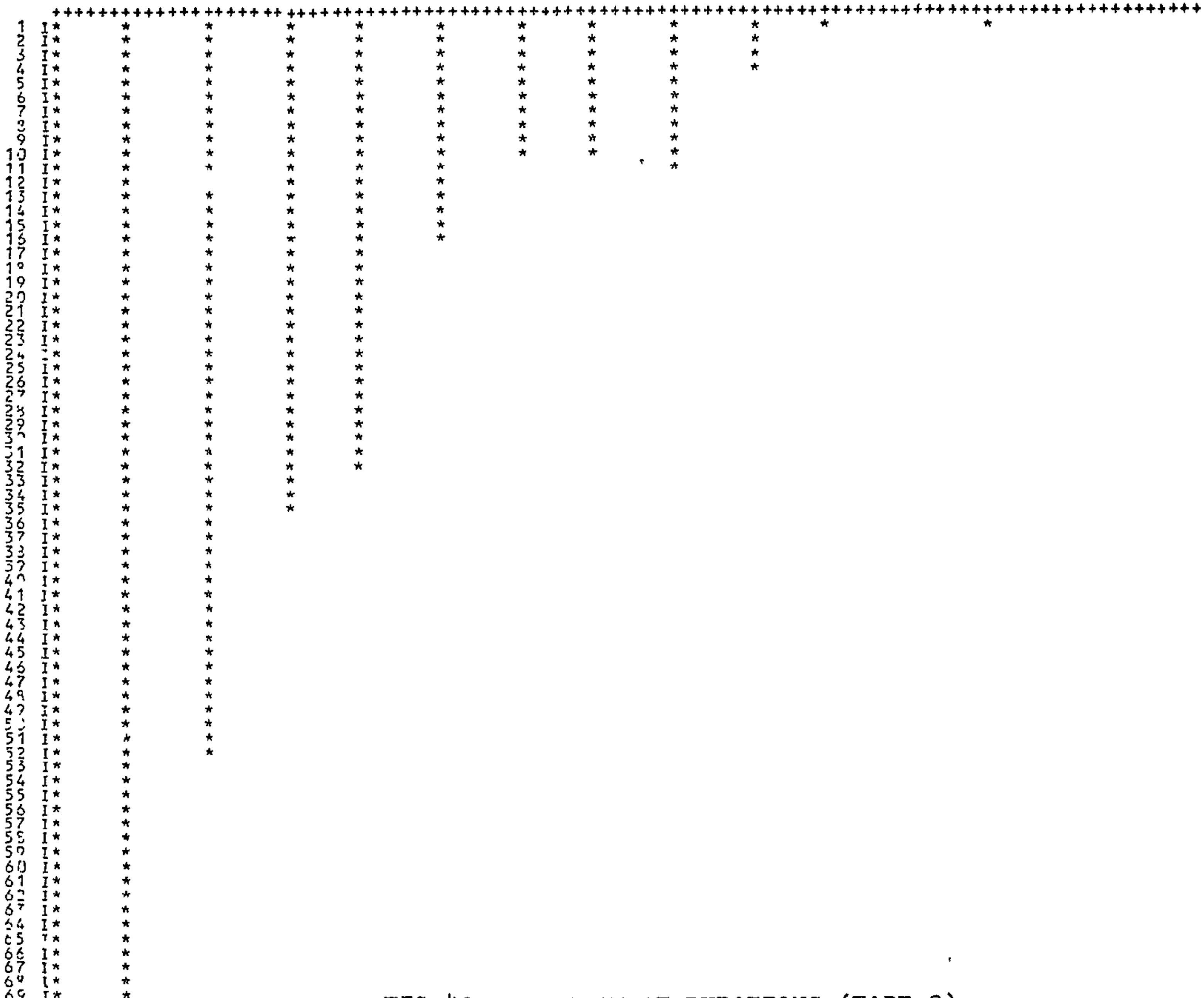

FIG.49 HISTOGRAM OF DURATIONS (TAPE 3)

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 70 I*       *
 71 I*       *
 72 I*       *
 73 I*       *
 74 I*       *
 75 I*       *
 76 I*
 77 I*
 78 I*
 79 I*
 80 I*
 81 I*
 82 I*
 83 I*
 84 I*
 85 I*
 86 I*
 87 I*
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 90 I*
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125 I*
126 I*
127 I*
128 I*
129 I*
130 I*
131 I*
132 I*
133 I*
134 I*
135 I*
136 I*
137 I*
138 I*
139 I*
140 I*
141 I*
142 I*
143 I*
144 I*
145 I*
```

FIG.49 (CONT.)

```
146 I*
147 I*
148 I*
149 I*
150 I*
151 I*
152 I*
153 I*
154 I*
155 I*
156 I*
157 I*
158 I*
159 I*
160 I*
161 I*
162 I*
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210 I*
211 I*
212 I*
213 I*
214 I*
215 I*
216 I*
217 I*
218 I*
219 I*
220 I*
221 I*
```

FIG.49 (CONT.)

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222 I*
223 I*
224 I*
225 I*
226 I*
227 I*
228 I*
229 I*
230 I*
231 I*
232 I*
233 I*
234 I*
235 I*
236 I*
237 I*
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240 I*
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247 I*
248 I*
249 I*
250 I*
251 I*
252 I*
253 I*
254 I*
255 I*
256 I*
257 I*
258 I*
259 I*
```

FIG.49 (CONT.)

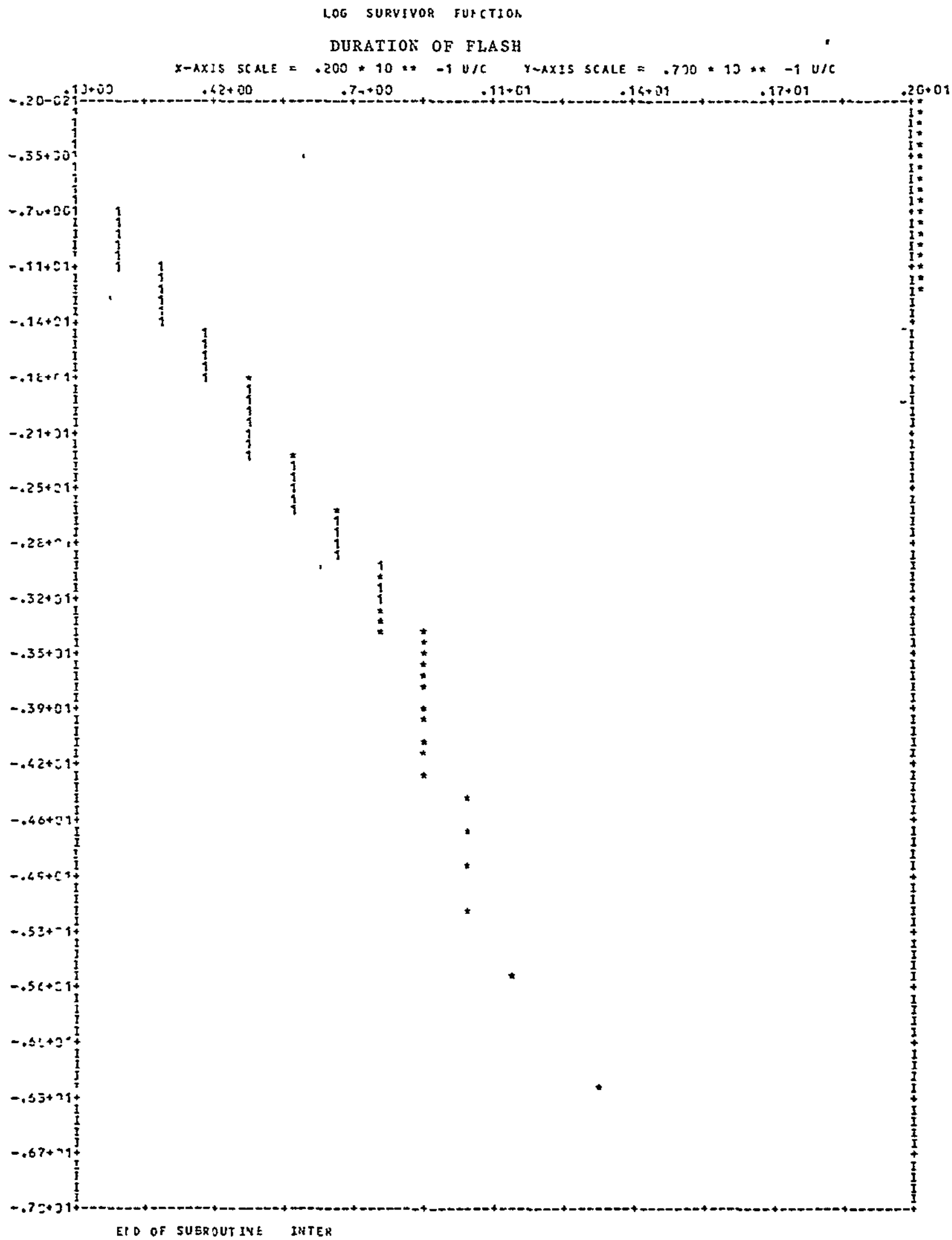


FIG.50 LOG SURVIVOR FUNCTION (TAPE 3)

PLOT OF SERIAL CORRELATION OF DURATIONS

NORMALIZED SERIAL CORRELATION COEFFICIENT

X-AXIS SCALE = .200 * 10 ** 0 U/C   Y-AXIS SCALE = 1.000 * 10 ** 0 U/C

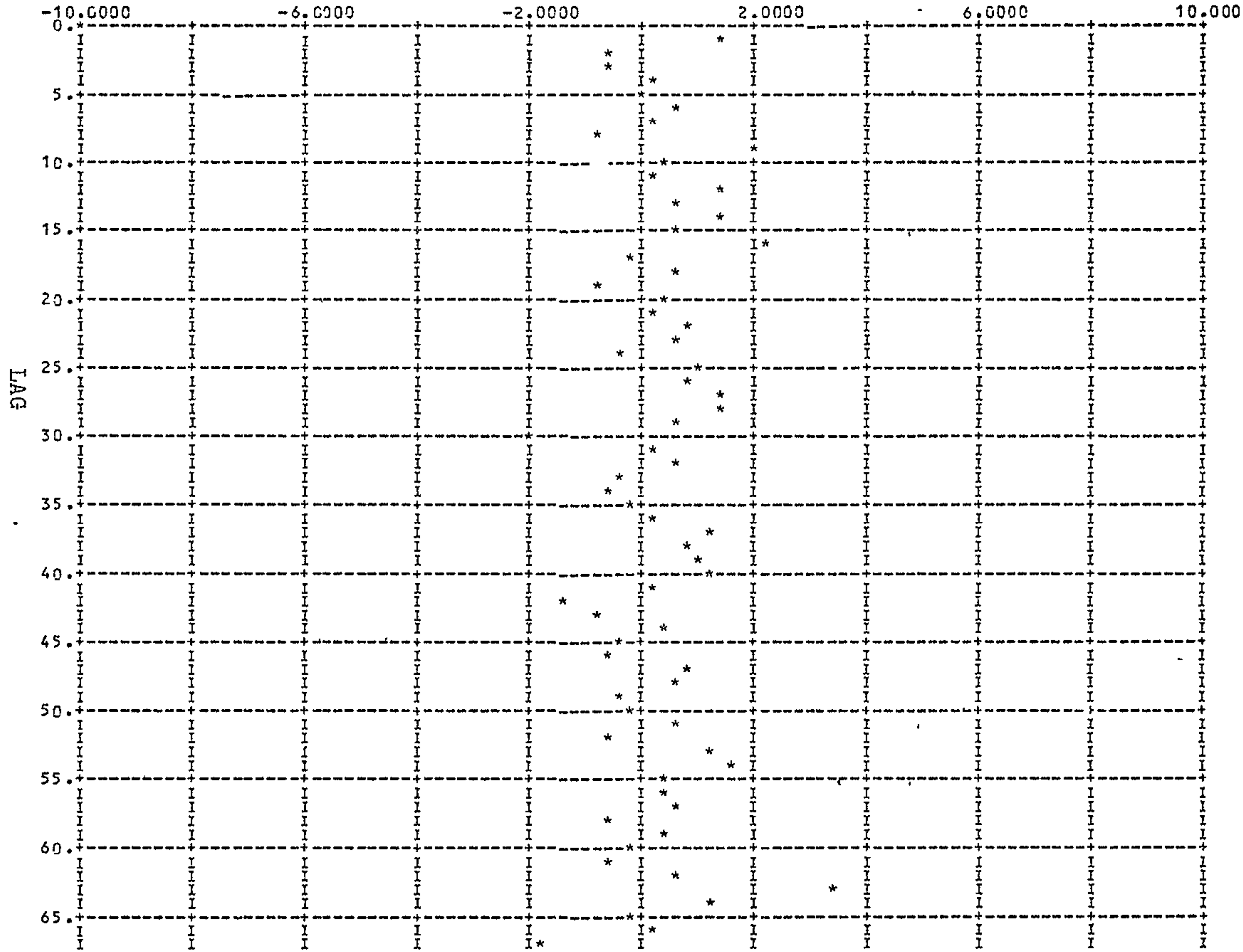


FIG.51 SERIAL CORRELATION OF DURATIONS (TAPE 3)

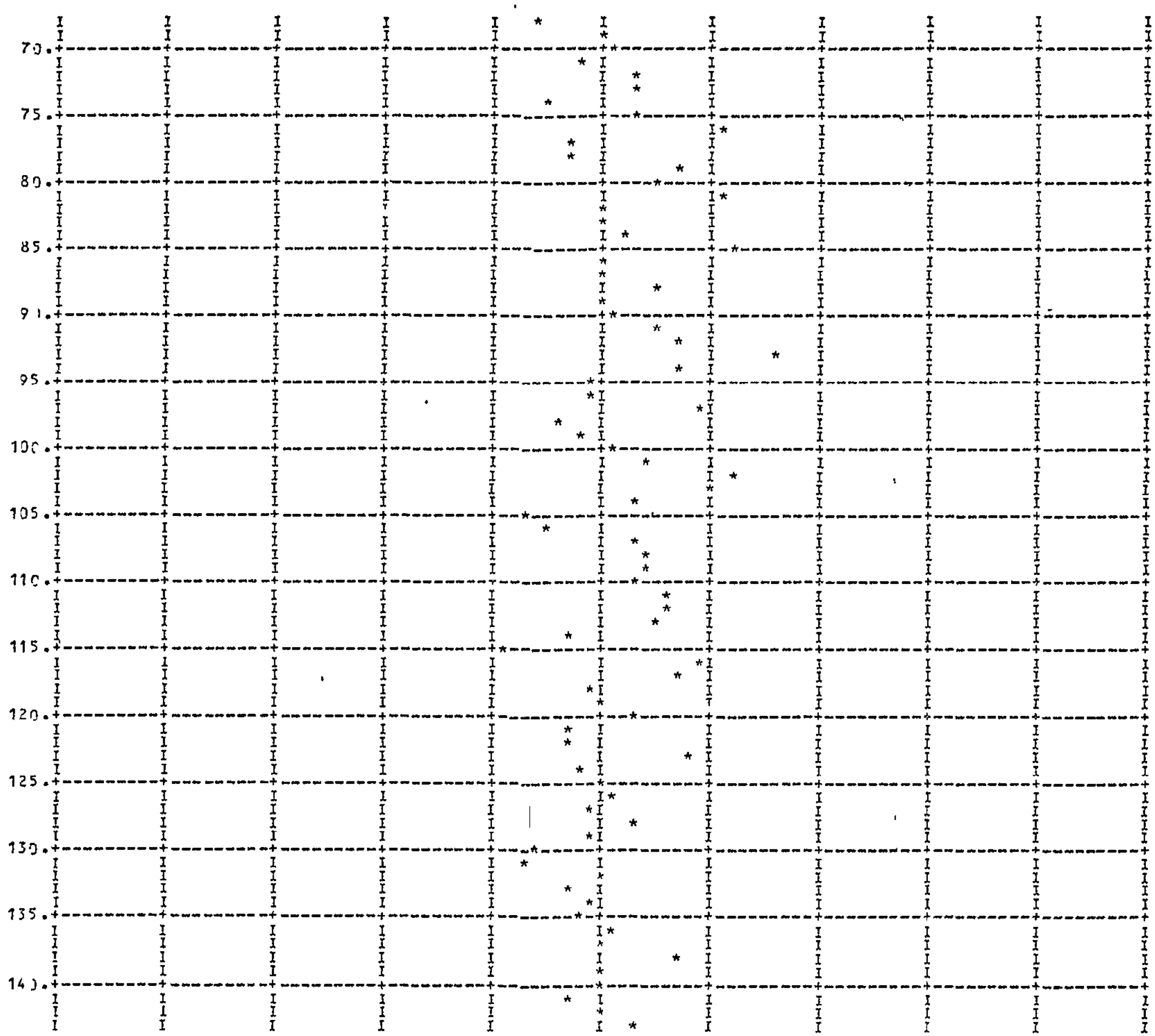
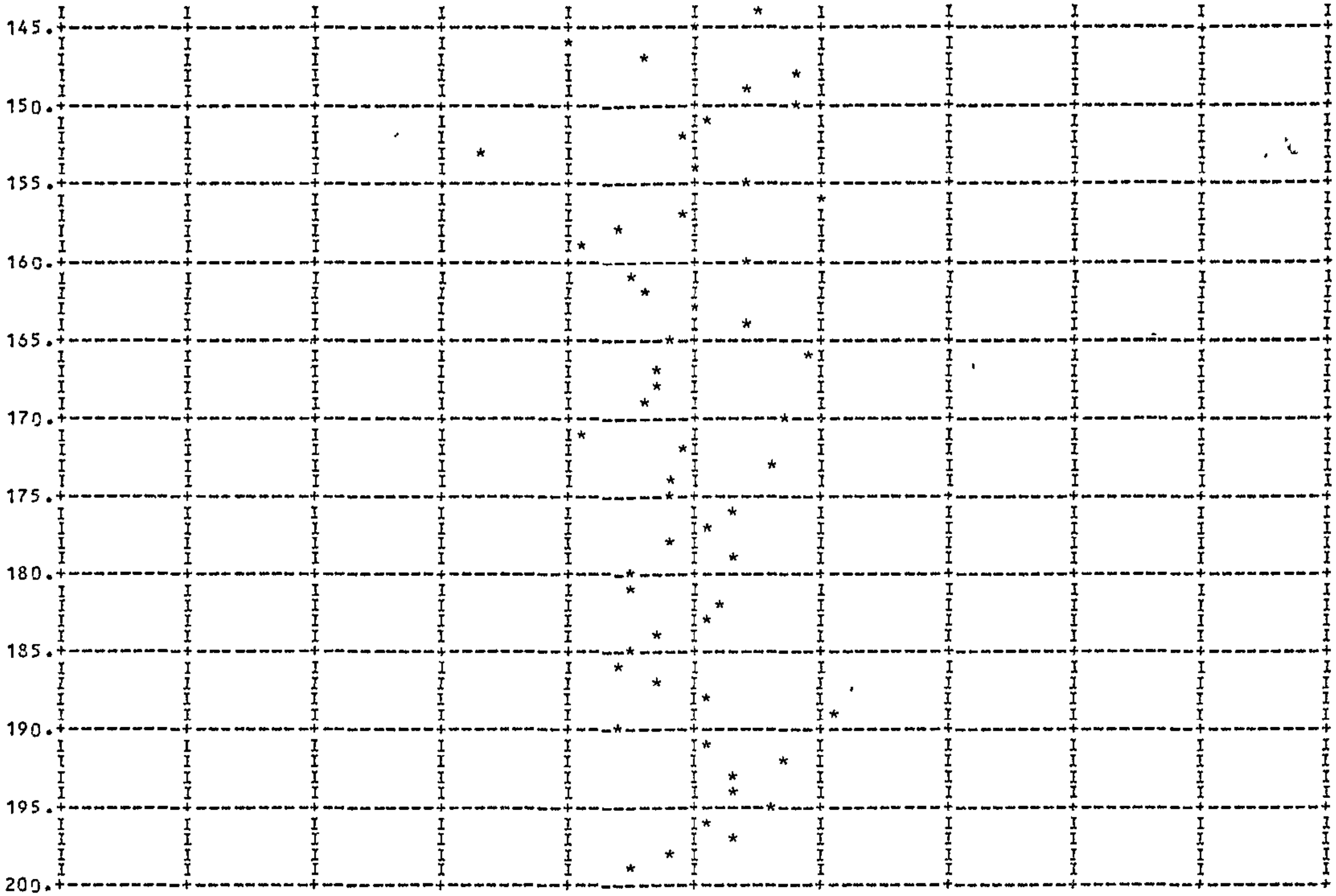


FIG.51 (CONT.)

FIG.51 (CONT.)



END OF SUBROUTINE RHO

SPECTRUM OF DURATIONS

MAGNITUDE OF SPECTRUM FOR M1(SYMBOL=#)
MAGNITUDE OF SPECTRUM FOR M2(SYMBOL=X)
MAGNITUDE OF SPECTRUM FOR M3(SYMBOL=H)

X-AXIS SCALE = .441 * 10 ** -2 U/C   Y-AXIS SCALE = 2.000 * 10 ** 0 U/C
X-AXIS SCALE = .441 * 10 ** -2 U/C   Y-AXIS SCALE = 2.000 * 10 ** 0 U/C
X-AXIS SCALE = .441 * 10 ** -2 U/C   Y-AXIS SCALE = 2.000 * 10 ** 0 U/C

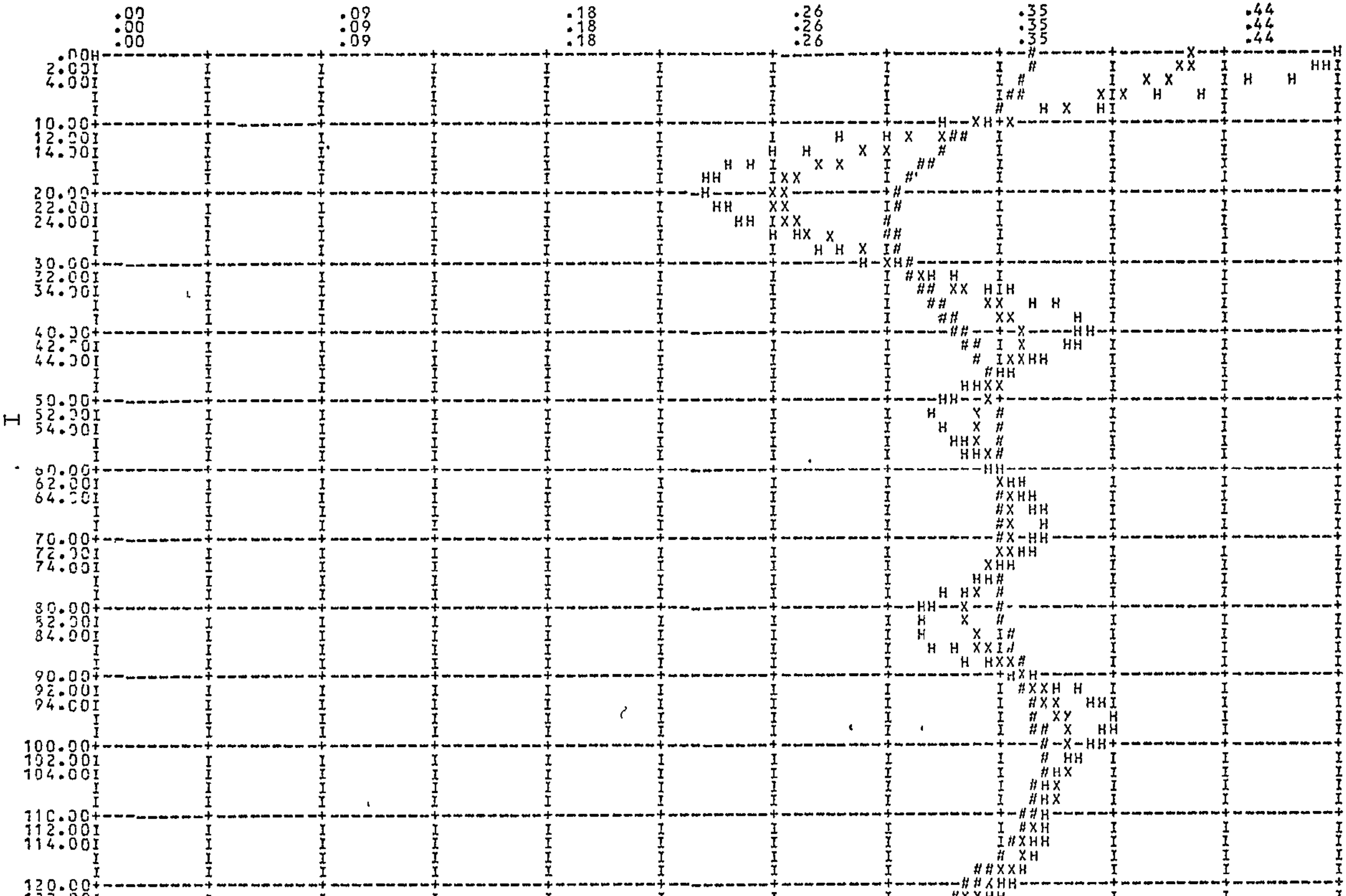
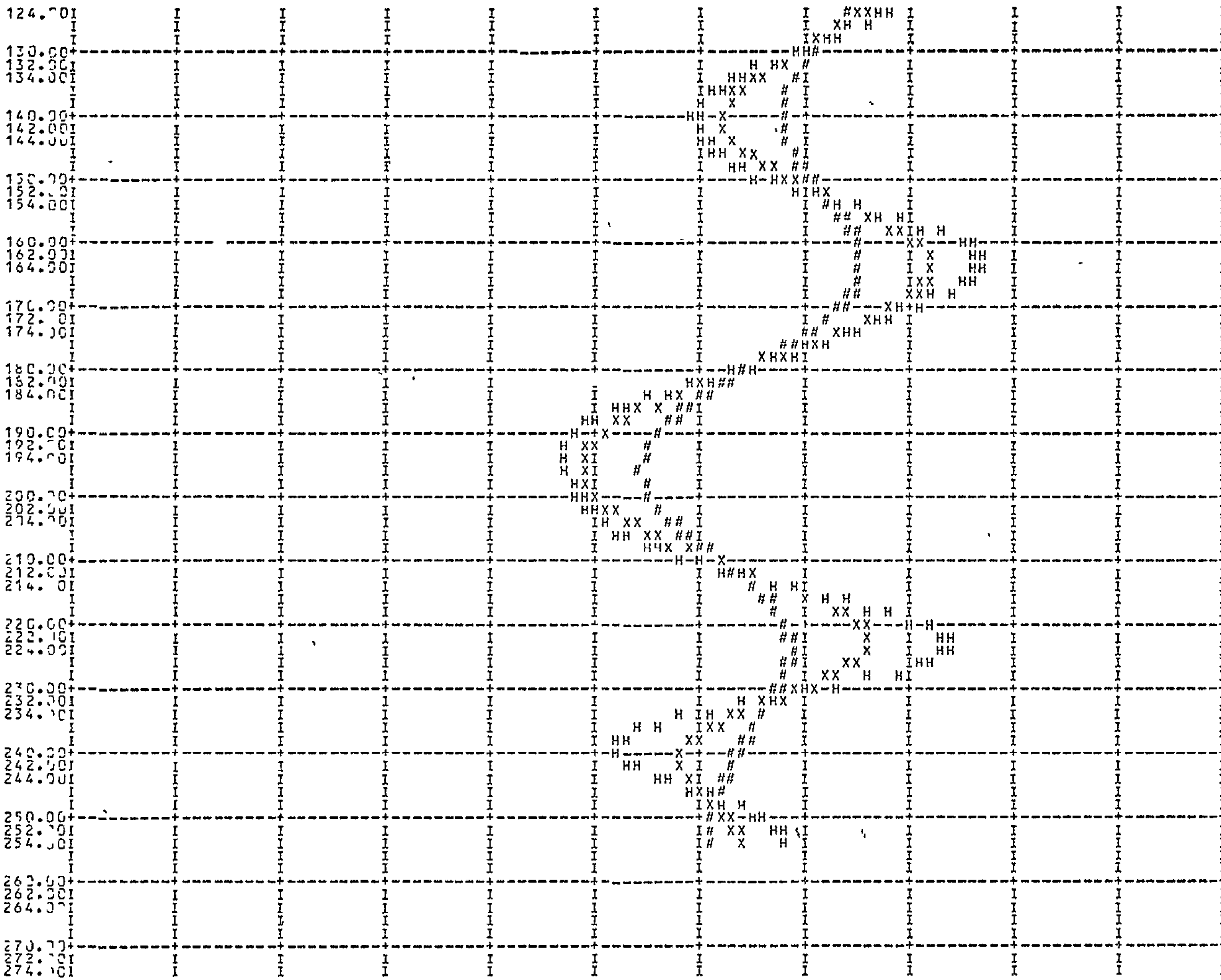


FIG.52 SPECTRUM OF DURATIONS (TAPE 3)



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240.00
242.00
244.00
250.00
252.00
254.00
260.00
262.00
264.00
270.00
272.00
274.00

FIG.52 (CONT.)

HISTOGRAM OF DURATIONS

XMIN= .10000 XMAX= 1.70000 NUMBER OF CELLS= 48 CELL INCREMENT= .033333

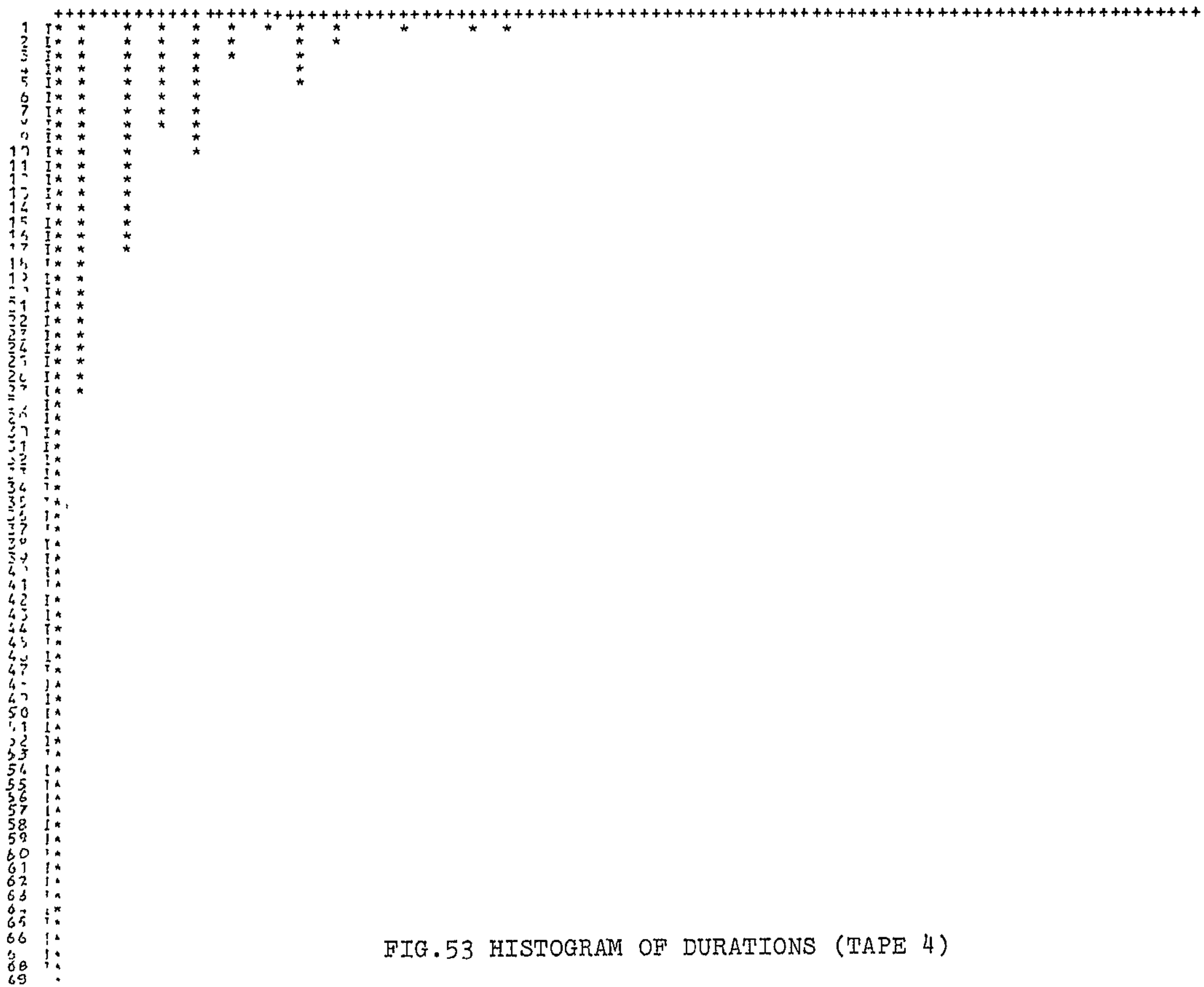

FIG.53 HISTOGRAM OF DURATIONS (TAPE 4)

```
 70  I*
 71  I*
 72  I*
 73  I*
 74  I*
 75  I*
 76  I*
 77  I*
 78  I*
 79  I*
 80  I*
 81  I*
 82  I*
 83  I*
 84  I*
 85  I*
 86  I*
 87  I*
 88  I*
 89  I*
 90  I*
 91  I*
 92  I*
 93  I*
 94  I*
 95  I*
 96  I*
 97  I*
 98  I*
 99  I*
100  I*
101  I*
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104  I*
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109  I*
110  I*
111  I*
112  I*
113  I*
114  I*
115  I*
116  I*
117  I*
118  I*
119  I*
120  I*
121  I*
122  I*
123  I*
124  I*
125  I*
126  I*
127  I*
128  I*
129  I*
130  I*
131  I*
132  I*
133  I*
134  I*
135  I*
136  I*
137  I*
138  I*
139  I*
140  I*
141  I*
142  I*
143  I*
144  I*
145  I*
```

FIG.53 (CONT.)

FIG.53 (CONT.)

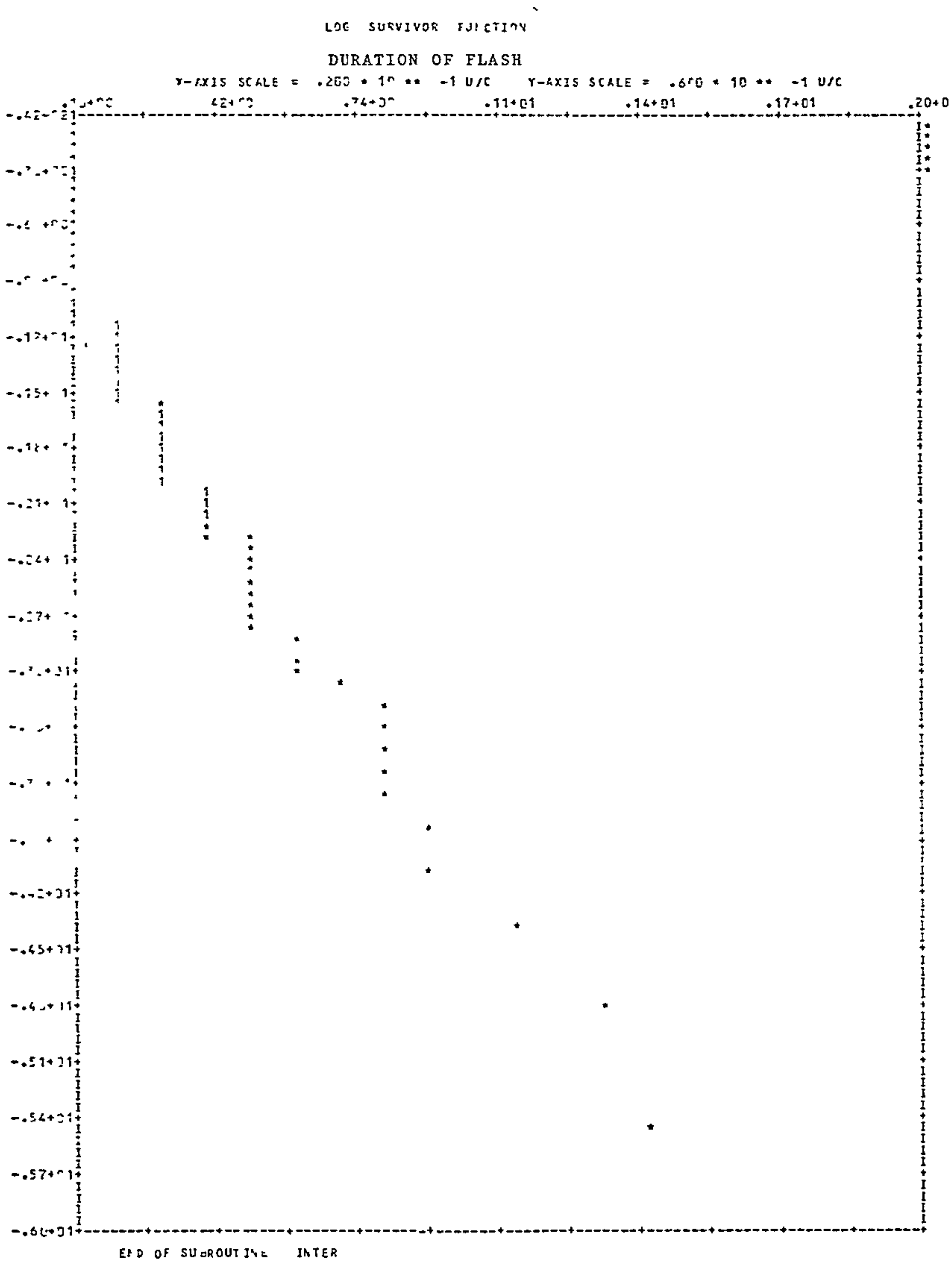


FIG.54 LOG SURVIVOR FUNCTION (TAPE 4)

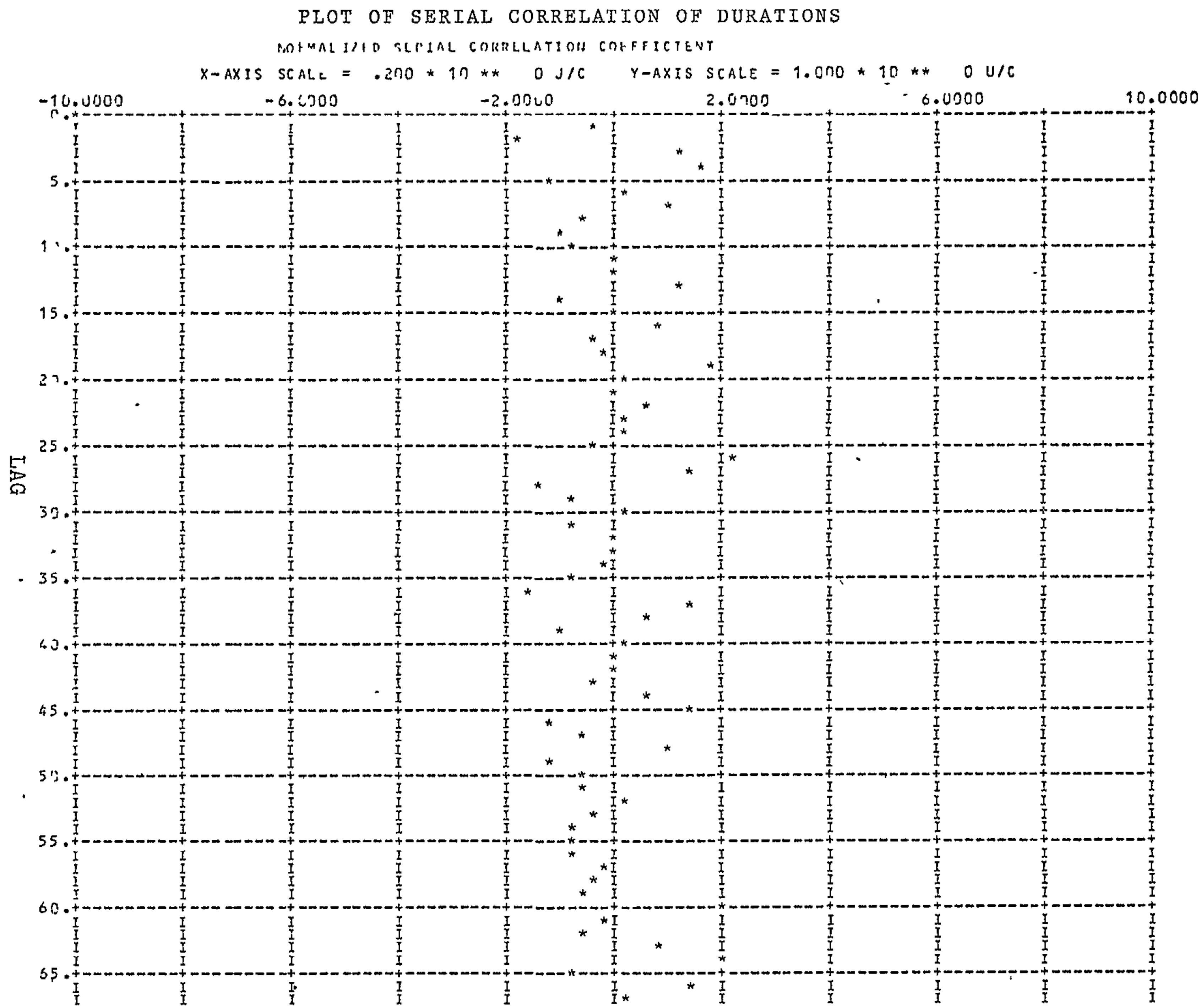
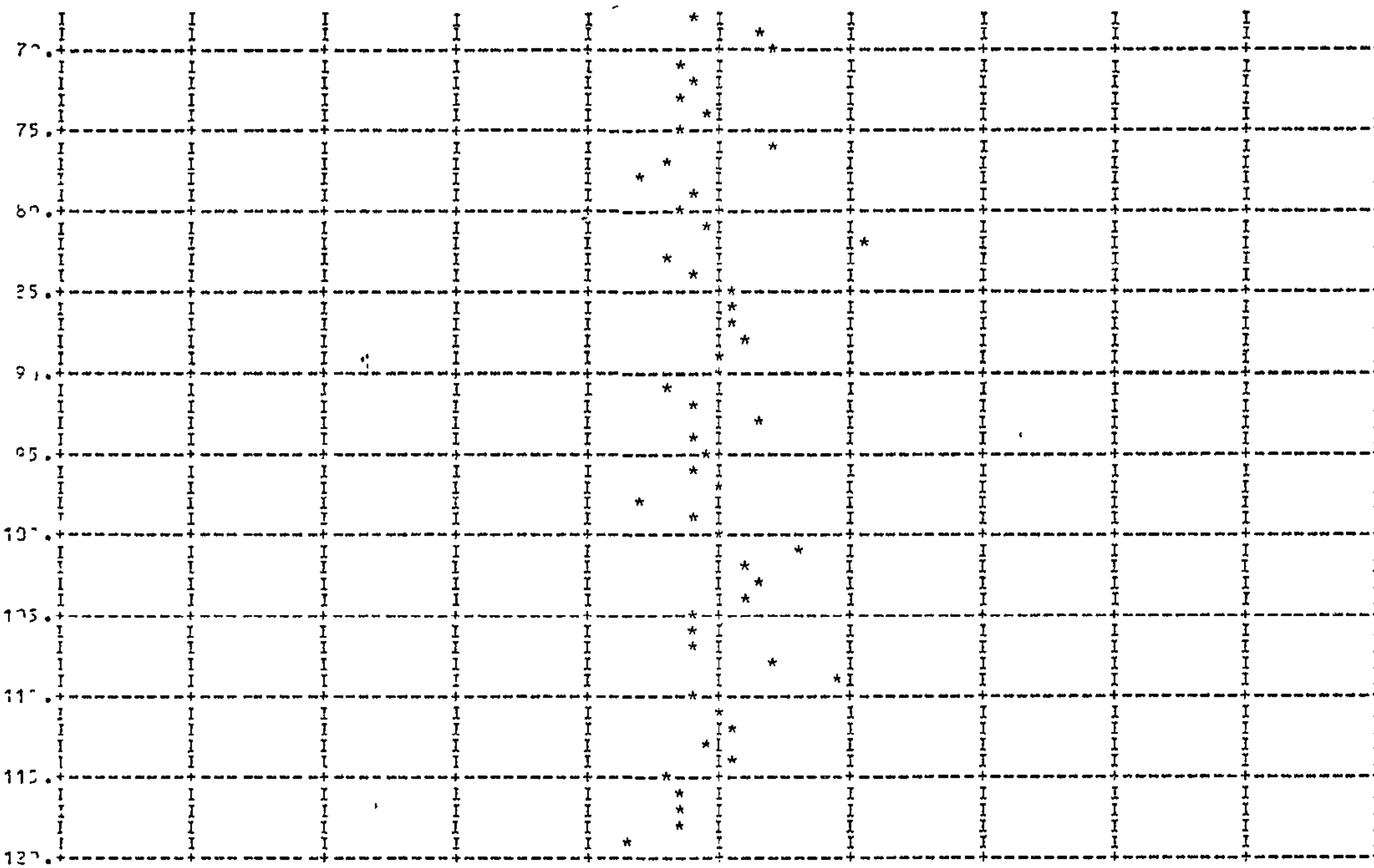


FIG.55 SERIAL CORRELATION OF DURATIONS (TAPE 4)



END OF SUBROUTINE RHO

FIG.55 (CONT.)

SPECTRUM OF DURATIONS

MAGNITUDE OF SPECTRUM FOR M1(SYMBOL=#)
MAGNITUDE OF SPECTRUM FOR M2(SYMBOL=X)
MAGNITUDE OF SPECTRUM FOR M3(SYMBOL=H)

X-AXIS SCALE = .510 * 10 ** -2 U/C   Y-AXIS SCALE = .800 * 10 ** 0 U/C
X-AXIS SCALE = .510 * 10 ** -2 U/C   Y-AXIS SCALE = .800 * 10 ** 0 U/C
X-AXIS SCALE = .510 * 10 ** -2 U/C   Y-AXIS SCALE = .800 * 10 ** 0 U/C

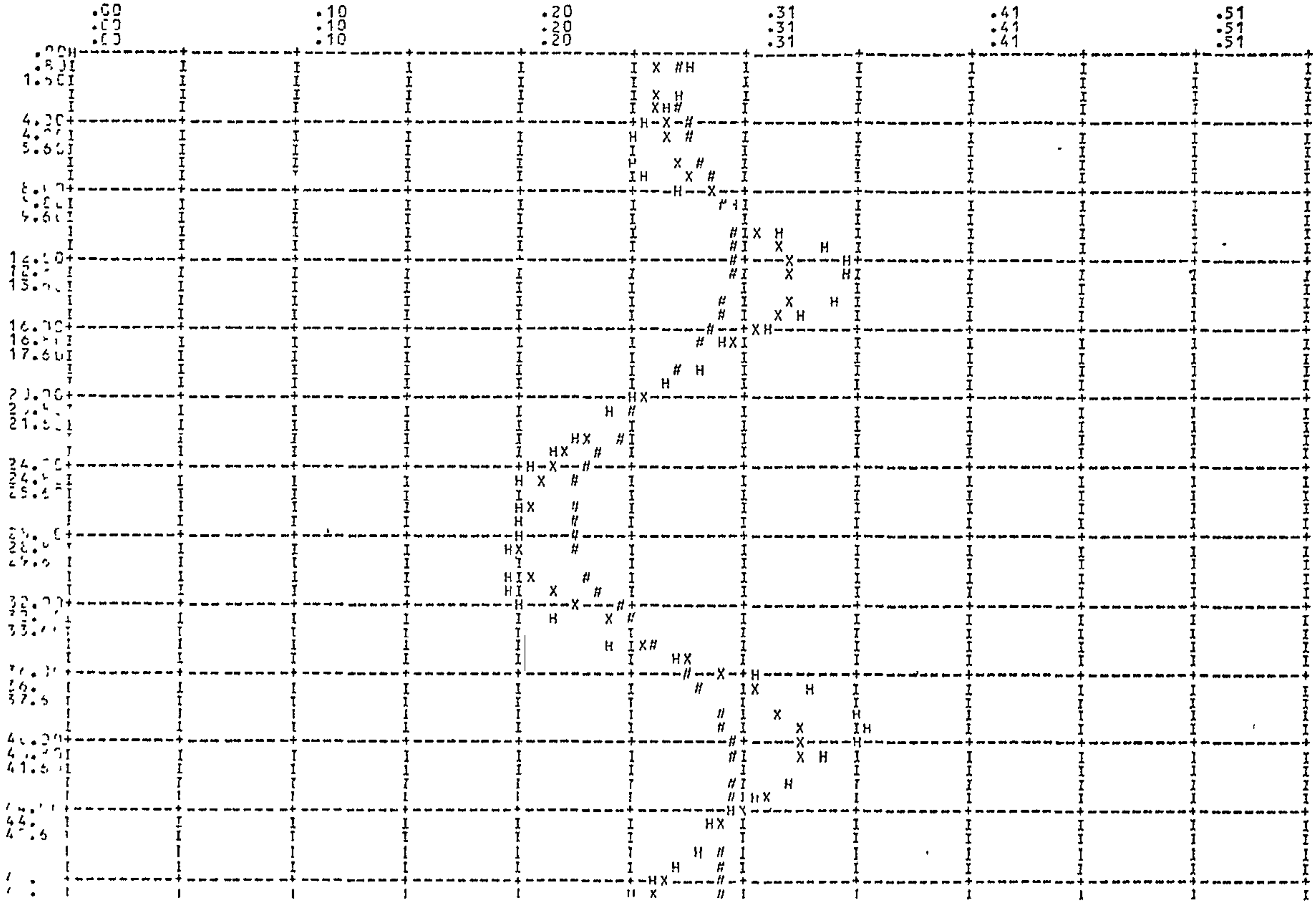


FIG.56 SPECTRUM OF DURATIONS (TAPE 4)

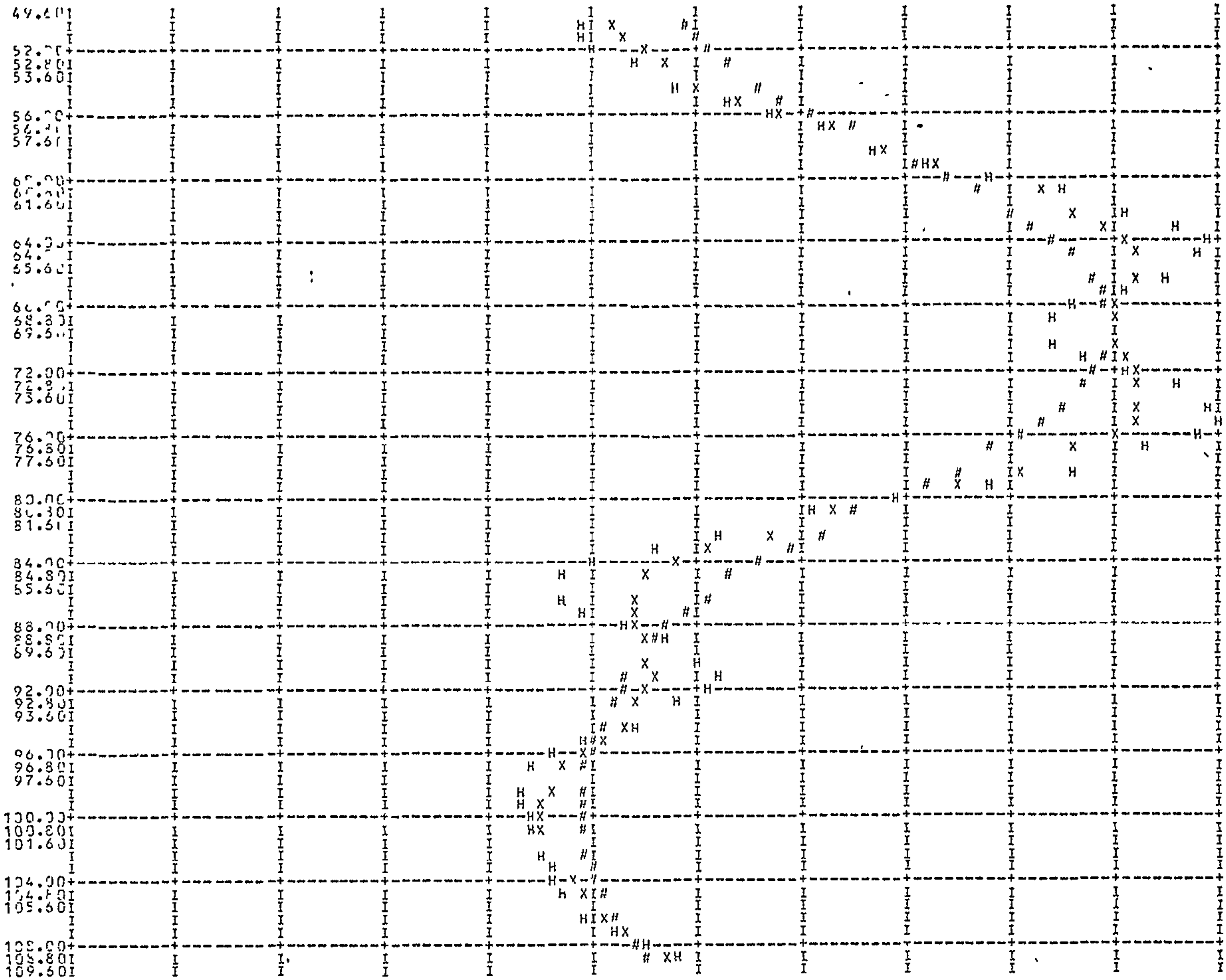


FIG.56 (CONT.)

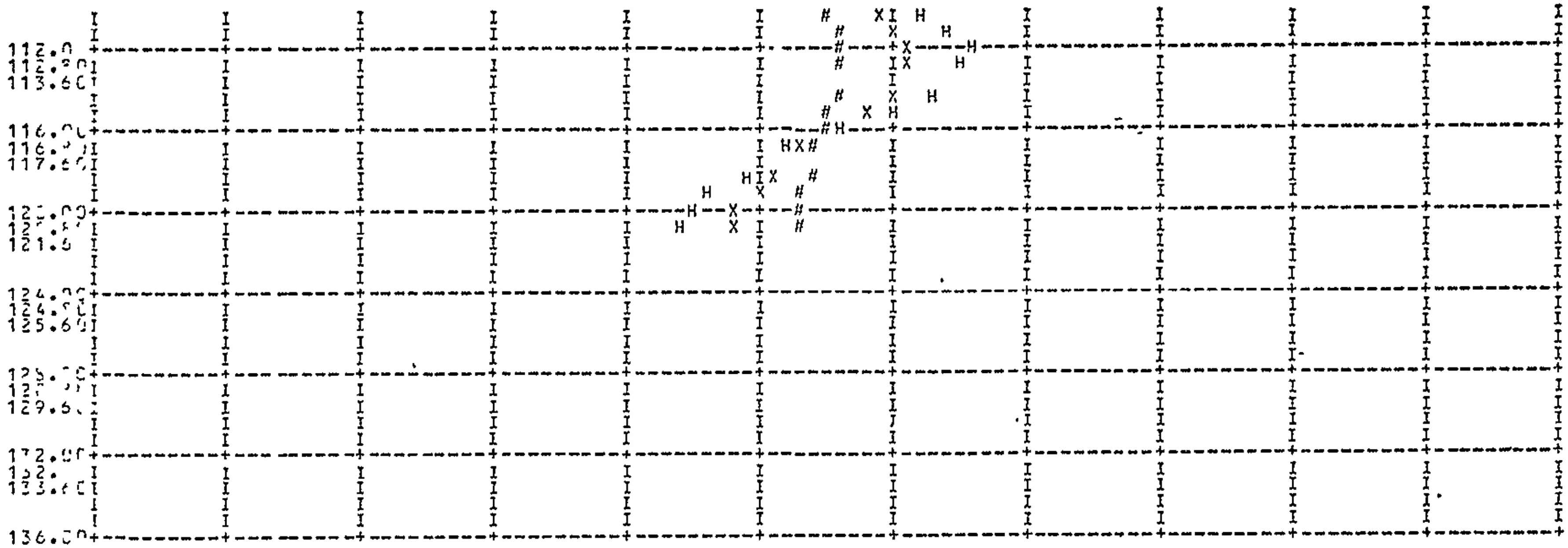


FIG.56 (CONT.)

HISTOGRAM OF DURATIONS

XMIN= .10000 XMAX= 1.10000 NUMBER OF CELLS= 45 CELL INCREMENT= .022222

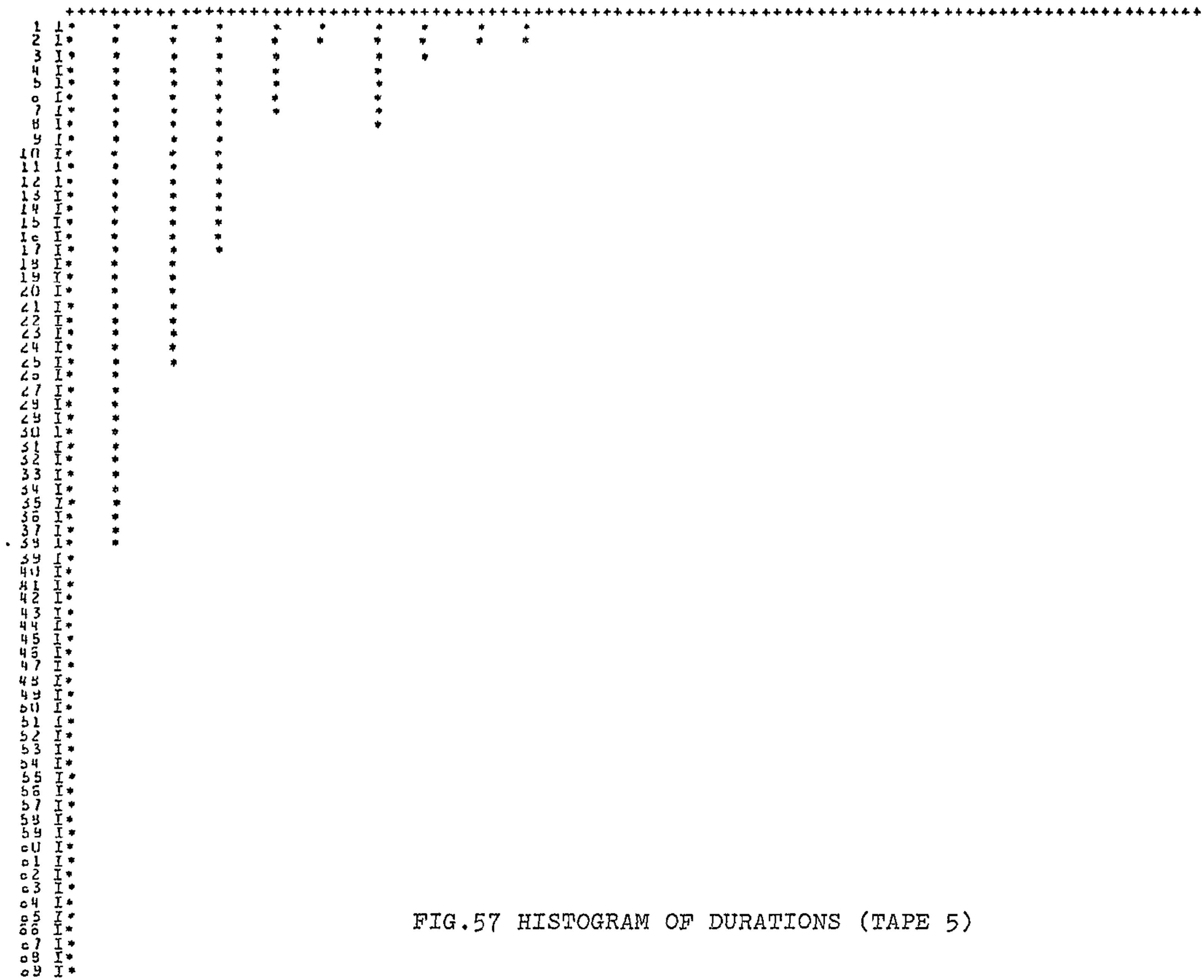


FIG.57 HISTOGRAM OF DURATIONS (TAPE 5)

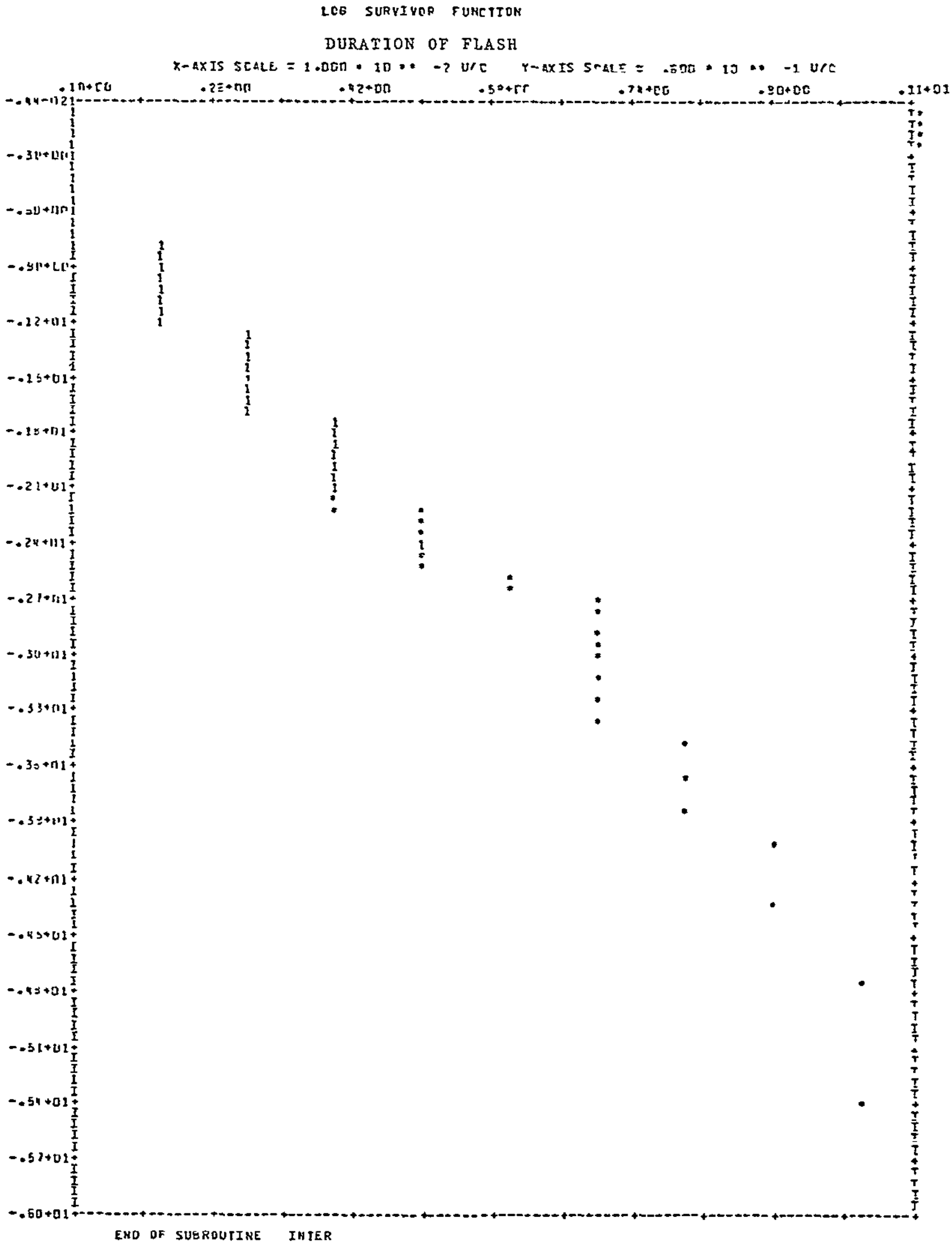


FIG.58 LOG SURVIVOR FUNCTION (TAPE 5)

PLOT OF SERIAL CORRELATION OF DURATIONS

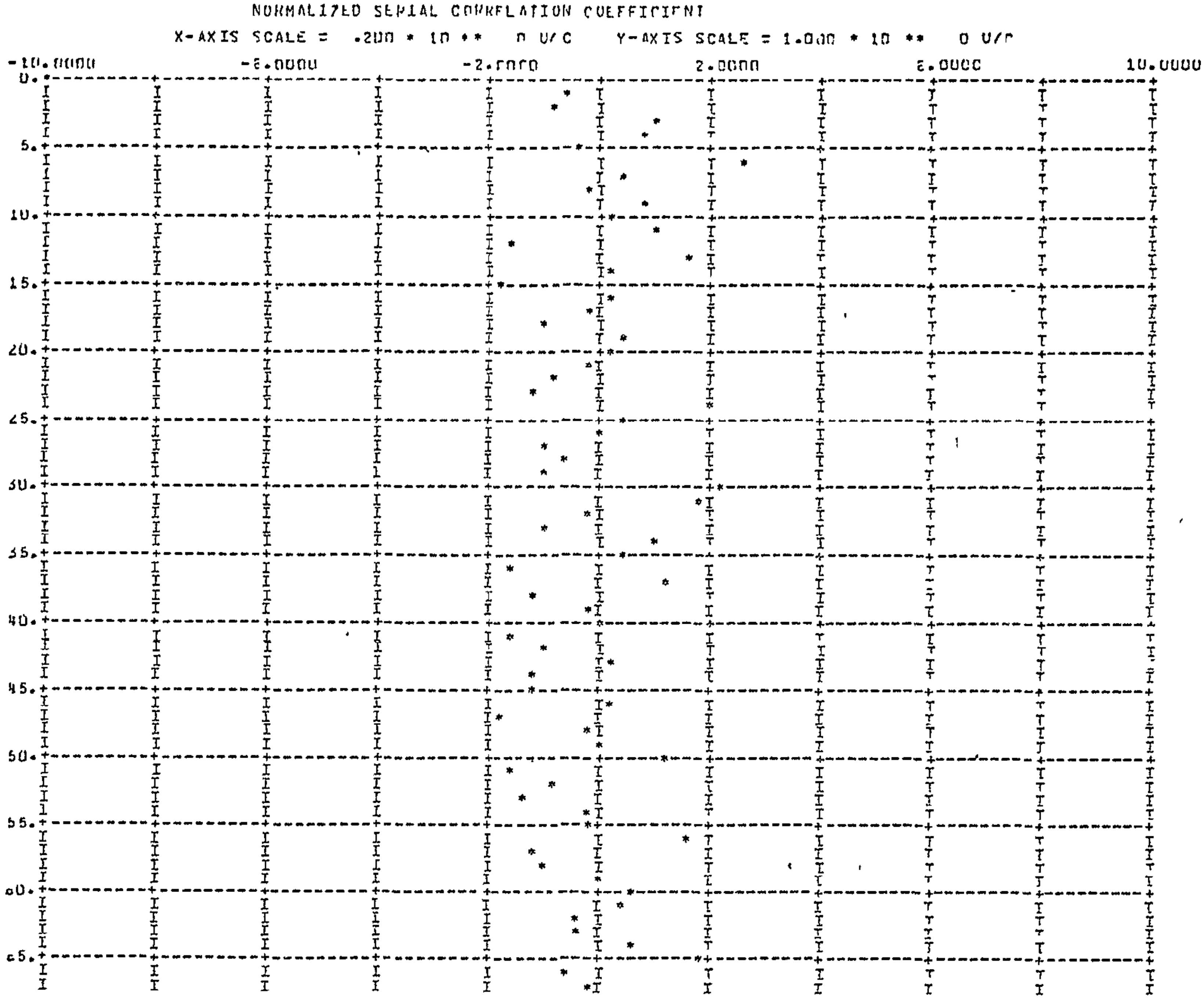
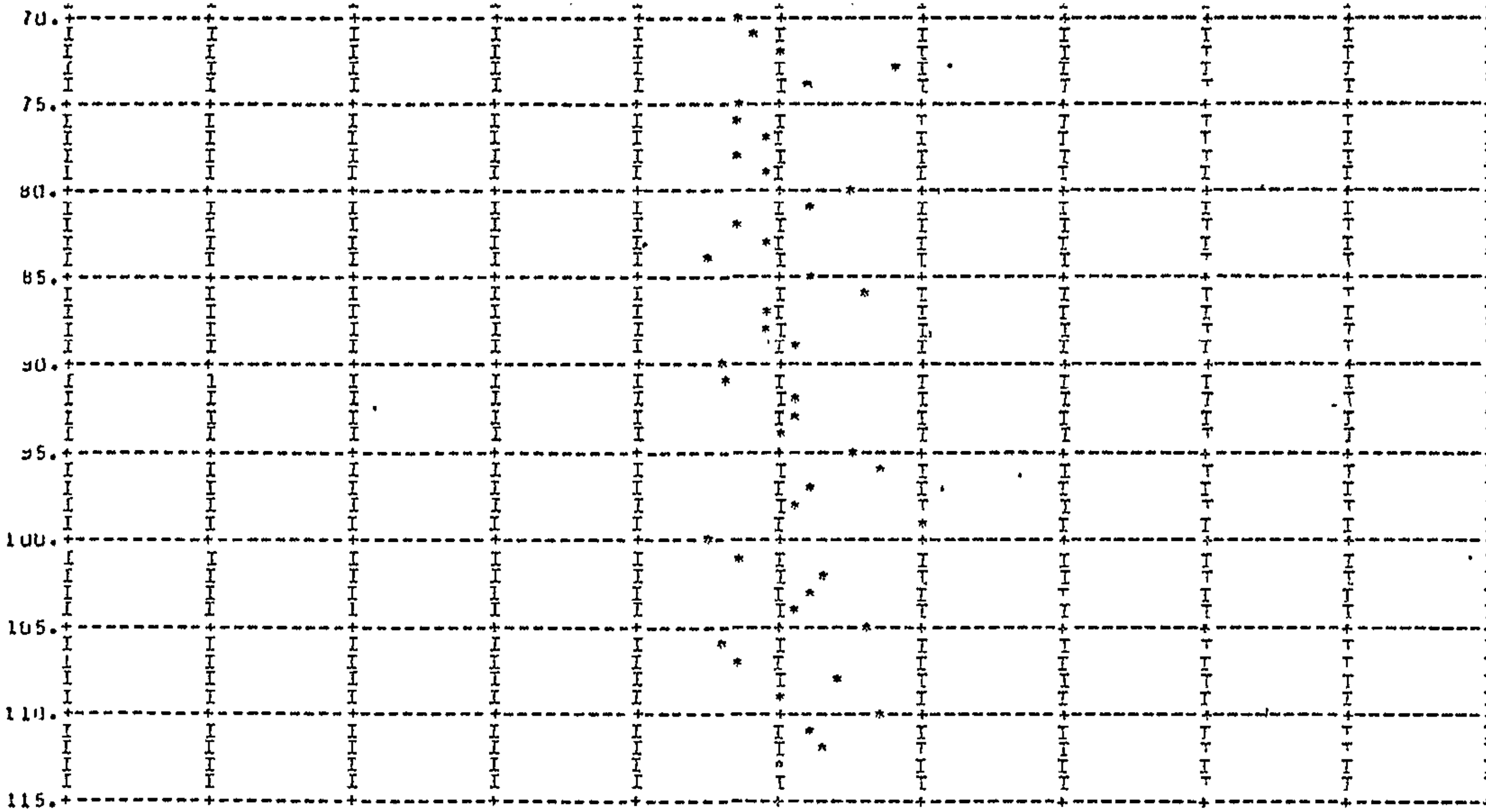


FIG.59 SERIAL CORRELATION OF DURATIONS (TAPE 5)



END OF SUBROUTINE RHO

FIG.59 (CONT.)

SPECTRUM OF DURATIONS

MAGNITUDE OF SPECTRUM FOR M1(SYMBOL=#)
MAGNITUDE OF SPECTRUM FOR M2(SYMBOL=X)
MAGNITUDE OF SPECTRUM FOR M3(SYMBOL=H)

X-AXIS SCALE = .547 * 10 ** -2 U/C   Y-AXIS SCALE = .700 * 10 ** 0 U/C
X-AXIS SCALE = .547 * 10 ** -2 U/C   Y-AXIS SCALE = .700 * 10 ** 0 U/C
X-AXIS SCALE = .547 * 10 ** -2 U/C   Y-AXIS SCALE = .700 * 10 ** 0 U/C

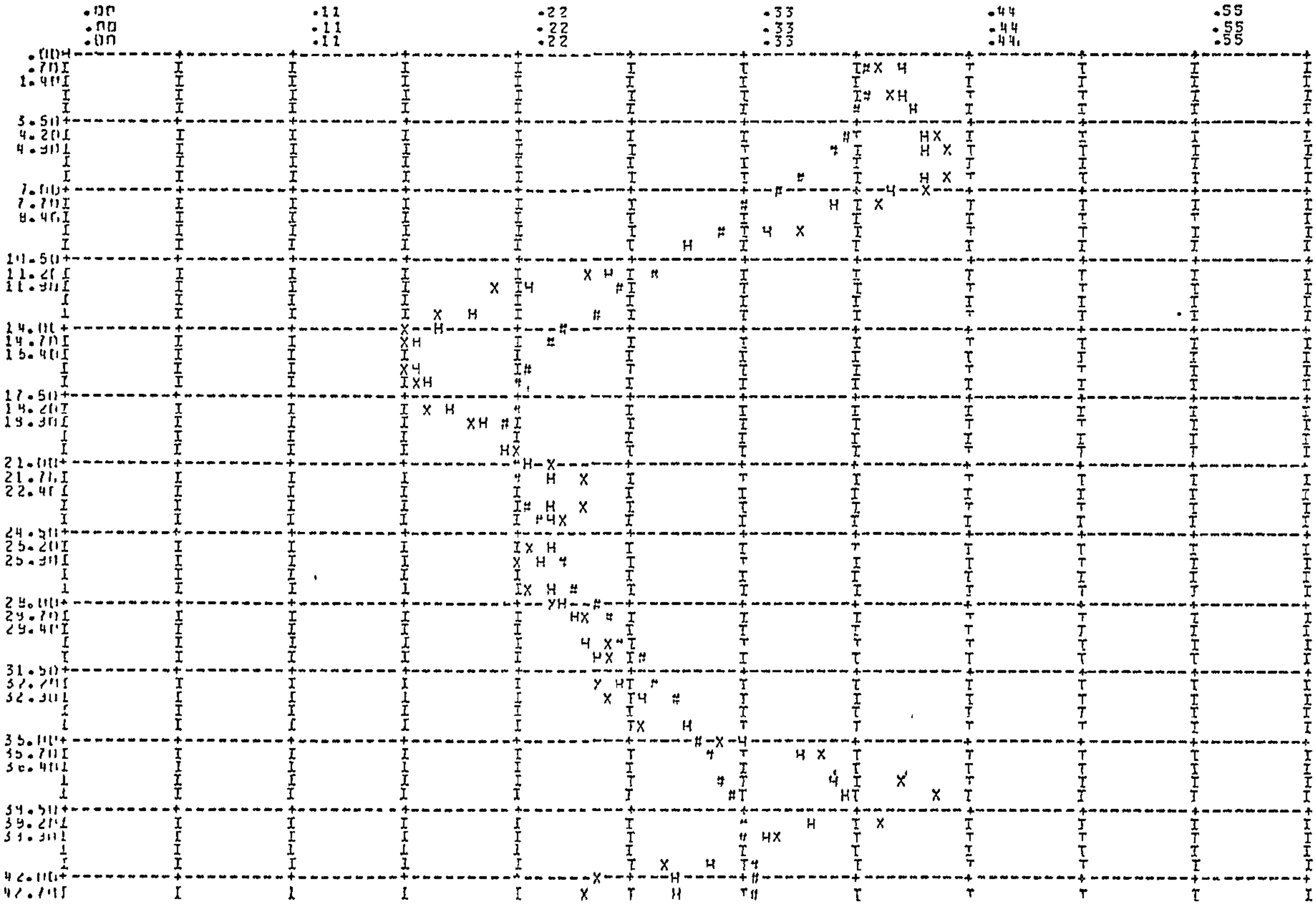


FIG.60 SPECTRUM OF DURATIONS (TAPE 5)

I

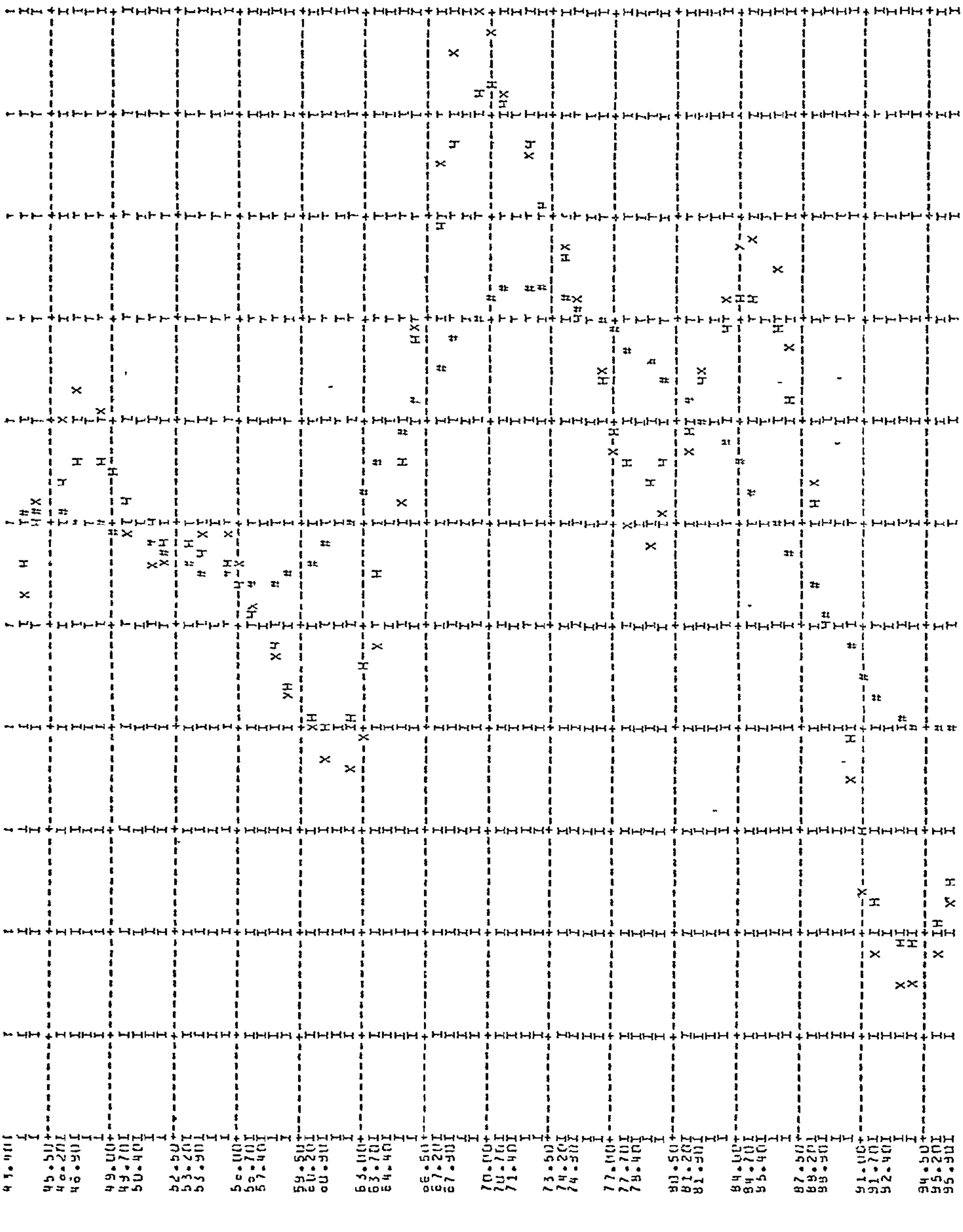


FIG.60 (CONT.)

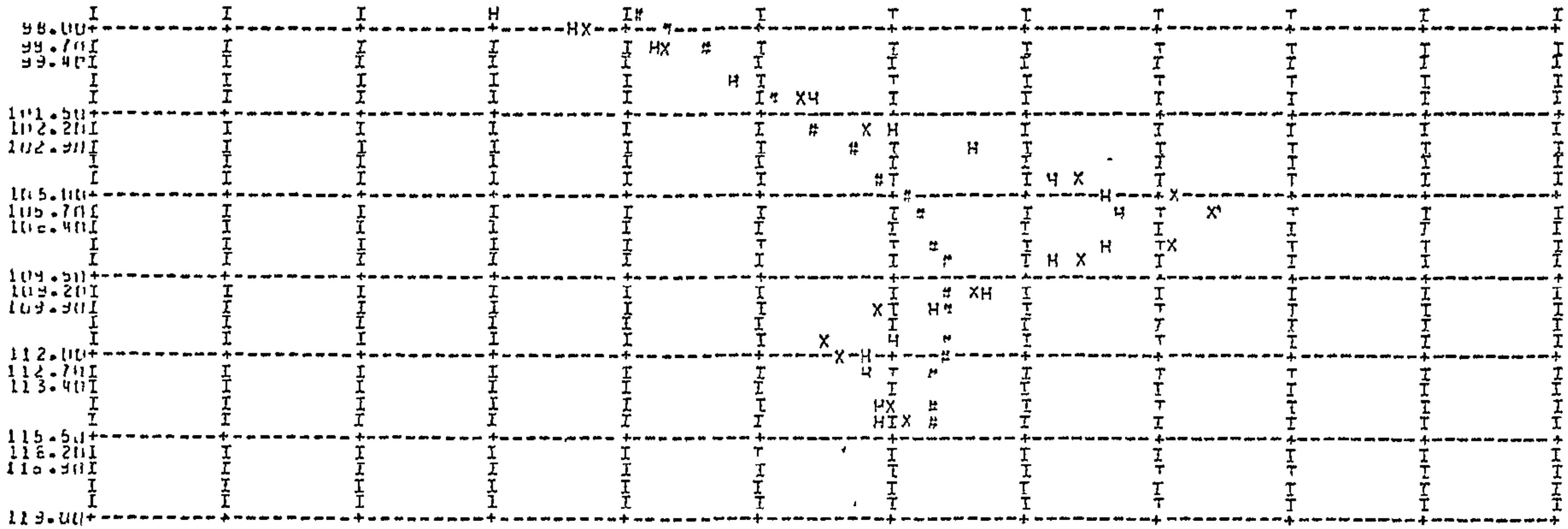


FIG.60 (CONT.)

APPENDIX 3

PROGRAMS

SASEV

A Program for the Statistical Analysis
of Series of Events

SASEV is a large FORTRAN program (approximately 3, 000 cards) for the statistical analysis of point processes. The program is documented in detail in Reference 1. A listing of SASEV will be provided on request.

CONVERT

A Program for Converting NASA Digital Lightning

Tapes to University of Maryland UNIVAC 1108 Format

```
      DIMENSION FDAT(12900),IDUM(18),FN(6000)
      EQUIVALENCE (DATA,FDAT)
       DATA  IDUM/18*'      '/
      INTEGER PARITY,DENS,RCOVR,STATUS,BUF(5700),DATA(12900)
      DATA NFILL/O/777776000000/
      STATUS=0
      PARITY=1
      DENS=1600
      RCOVR=0
      NWORDS=5689
      NSAMP=12600
      CALL XREAD(10,BUF,NWORDS,STATUS,PARITY,DENS,RCOVR)
      L=1
      DO 1 I=1,NWORDS,4
      DATA(L)=FLD(0,16,BUF(I))
      DATA(L+1)=FLD(16,16,BUF(I))
      N11=FLD(32,4,BUF(I))
      N12=FLD(0,12,BUF(I+1))
      DATA(L+2)=N11.LS.12.V.N12
      DATA(L+3)=FLD(12,16,BUF(I+1))
      N11=FLD(28,8,BUF(I+1))
      N12=FLD(0,8,BUF(I+2))
      DATA(L+4)=N11.LS.8.V.N12
      DATA(L+5)=FLD(8,16,BUF(I+2))
      N11=FLD(24,12,BUF(I+2))
      N12=FLD(0,4,BUF(I+3))
      DATA(L+6)=N11.LS.4.V.N12
      DATA(L+7)=FLD(4,16,BUF(I+3))
      DATA(L+8)=FLD(20,16,BUF(I+3))
      L=L+9
      DO2 I=1,NSAMP
      IF(FLD(20,1,DATA(I)).EQ.1) DATA(I)=(DATA(I)-1).V.NFILL
      DO 3 I=1,NSAMP
      FDAT(I)=DATA(I)
      EI='EI'
      GG='GG'
      ITAB=LN=1
      WRITE(2,4) GG,LN,ITAB,IDUM
      FORMAT(8X,A2,2I2,16A4)
      WRITE(2,4) EI,LN,ITAB,IDUM
      END FILE 2
      REWIND 2
      N=600
      N1=1
      DO 5 I=1,N
      FN(I)=I-1+N1
      CALL PLT(1,1,1,1,0,0,2,1,1,0,10,N/10,10,0,0,
     1    1.,0,0,0,0,2,FDAT(N1),FN,N,0,
     2    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
      END
```

PEAK

A Program for Detecting the Peaks in Sampled

Lightning Data

```
      DIMENSION X(6650)
      DIMENSION AMP(600)
      REWIND 3
      READ(3) X
      IA=100.
      ID=60.
      N=0
      DO 5 I=2,6649
      IF(X(I).LE.IA) GO TO 5
      IF(X(I-1).GE.X(I).OR.X(I).LT.X(I+1)) GO TO 5
      U1=X(I)-X(I-1)
      U2=X(I)-X(I+1)
      IF(U1.LT.ID.AND.U2.LT.ID) GO TO 5
      N=N+1
      I1=I+6150
      X(N)=I1
      AMP(N)=X(I)
5     CONTINUE
       WRITE(6,4) N
4      FORMAT(3H N=,I10)
       WRITE(4,2) (X(I),I=1,N)
       END FILE 4
2      FORMAT(12F6.0)
       WRITE(7,3) (AMP(I),I=1,N)
       END FILE 7
3      FORMAT(12F5.0)
      END
```