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ANALYSIS AND TESTING OF TWO-DIMENSIONAL VENTED COANDA EJECTORS WITH ASYMMETRIC VARIABLE AREA MIXING SECTIONS

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16. Abstract

The analysis of asmymmetric, curved (Coanda) ejector flow has been completed using a finite difference technique and a quasi-orthogonal streamline coordinate system. The boundary=layer-type jet mixing analysis accounts for the effect of streamline curvature in pressure gradients normal to the streamlines and on eddy viscosities. The analysis assured perfect gases, free of pressure discontinuities and flow separation. The analysis treats the three compound flows of supersonic and subsonic streams, those are: (1) primary flow of the driving nozzle, (2) secondary flow between the primary nozzle and the Coanda surface, (3) tertiary flow between the primary nozzle and the other surface of the mixing section.

A test program was completed to measure flow parameters and ejector performance in a vented Coanda flow geometry for the verification of the computer analysis. A primary converging nozzle with a discharge geometry of 0.003175 m x 0.2032 m was supplied with 0.283 m^3/sec of air at about 241.3 kPa absolute stagnation pressure and 82°C stagnation temperature.

One mixing section geometry was used with a 0.127 m constant radius Coanda surface. Eight tests were run at spacings between the Coanda surface and primary nozzle 0.01915 m and 0.318 m and at three angles of Coanda turning: 22.5°, 45.0°, and 75.0°.

The wall static pressures, the locii of maximum stagnation pressures, and the stagnation pressure profiles agree well between analytical and experimental results.

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Key Words:

Ejector
Coanda
Compressible Flow
Analysis
Finite Difference
Computer Program
Experimental

Section 1

INTRODUCTION

1.1 Background

The augmentor wing concept under investigation by NASA for STOL aircraft lift augmentation is powered by an air to air ejector. The wing boundary layer is drawn into the deflected double flap augmentor channel at the trailing edge of the wing and is pressurized by a high velocity slot jet which is oriented at an angle to the augmentor channel. To predict the performance and to optimize the design of the complete augmentor wing, an analytical method is needed to predict the performance of the air ejector which powers the augmentor flap section.

Under contract NAS2-5845 a computer analysis was developed for single nozzle axisymmetric ejectors with variable area mixing sections using integral techniques, Reference 1. The ejectors of primary interest in that program and earlier programs were high-entrainment devices using small amounts of supersonic primary flow to pump large amounts of low-pressure secondary flow. Good agreement was achieved between analytical and experimental results.

The integral analytical techniques used to analyze the axisymmetric ejector configurations are also valid for the analysis of two-dimensional ejectors. However, the augmentor wing configuration may include asymmetric geometries, inlet flow distortions, wall slots, and primary nozzles that are at large angles to the axis of the augmentor mixing section. The integral techniques are not easily adaptable to these more complex flows. Finite difference techniques can be used to analyze these more complex flow geometries at the expense of increased computer time.

Under contract NAS2-6660 a computer program (Reference 2) was developed for two-dimensional, symmetrical mixing sections using finite difference technique and rectangular coordinate system. When Coanda effect is used in two-dimensional ejectors the geometry is not symmetrical and the use of rectilinear coordinates becomes difficult. Flow computations have to account for the pressure gradient in the direction normal to the streamlines.

1.2 Objectives of the Program

The following objectives were defined for this investigation:

1. Develop a computer program for two-dimensional vented Coanda ejectors with non-symmetric variable area mixing sections, with variable Coanda turning, and with variable primary nozzle spacing.

2. Obtain test results with a variable nozzle position vented Coanda ejector configuration for the development and checking of the computer program.

Section 2

NOMENCLATURE

A_N	Nozzle discharge area (m^2; in^2)
A_{n-1}	Coefficient appearing in the finite difference equations (-)
B_{n-1}	Coefficient appearing in the finite difference equations (-)
C_p	Specific heat at constant pressure (J/kg·K; Btu/lbm°R)
C_N	Nozzle discharge coefficient (-)
C_{n-1}	Coefficient appearing in the finite difference equations (-)
D_{n-1}	Coefficient appearing in the finite difference equations (-)
d	Nozzle exit height (m; ft)
E	Dimensionless eddy viscosity, $\nu_T/u_o d$ (-)
k	Thermal conductivity of fluid (W/mK; Btu/hrft°F)
K	Curvature of streamline, 1/R (1/m; 1/ft)
g_o	Dimensional constant, (32.2 lbm-ft/lbf-sec^2)
ℓ	Prandtl mixing length (m; ft)
L	Dimensionless mixing length, ℓ/d (-)
m	Node points along a streamline (-)
n	Streamline coordinate (normal to streamlines) (m; ft)
n	Streamline designation (-)
P_b	Barometric pressure (kPa; psia; inch H_2O)

P	Static pressure (kPa; psig; inch H_2O)
P_{01}	Reference pressure, primary stagnation pressure (kPa; psia)
P_{rt}	Turbulent Prandtl number, ν_T/ε_H (-)
P_r	Prandtl number, $\mu C_p/k$ (-)
q_{eff}	Effective heat transfer (between streamlines) ($W/m^2 °C$; $Btu/ft^2 sec °F$)
R	Radius of curvature (streamline; wall) (1/m; 1/ft; 1/in)
R_g	Gas constant (Nm/kg°K; lbf-ft/lbm°R)
R_i	Richardson number $(2u/R)/(\partial u/\partial n)$ (-)
S	Streamwise coordinate (along streamlines)(m; in)
t	Nozzle coanda wall spacing (m; inch)
T_a	Atmospheric temperature (°C; °F)
T	Fluid temperature (°K; °R)
T_{max}	Maximum fluid temperature at an x = constant cross section (°K; °R)
T_{01}	Reference temperature, primary stagnation temperature (°K; °R)
u	Velocity in s direction (m/s; ft/sec)
u_o	Reference velocity, $u_o = \sqrt{R_g T_{01}}$ (m/s; ft/sec)
$u_{2,n}$	Unknown velocity at the nth grid point (m/s; ft/sec)
U	Velocity in x direction (m/s; ft/sec)
U_{CL}	Centerline velocity (m/s; ft/sec)
U_{max}	Maximum fluid velocity (m/s; ft/sec)

U_{sec}	Secondary velocity (m/sec; ft/sec)
W_m	Mixing section total flow (kg/sec m; lbm/sec in)
W_n	Nozzle flow rate (kg/sec-m; lbm/sec in)
W_s	Secondary flow rate (kg/sec-m; lbm/sec in)
W_t	Tertiary flow rate (kg/sec-m; lbm/sec in)
x	Space coordinate in the axial direction (m; in)
X	Dimensionless space coordinate in the axial direction, x/d (-)
ΔX; dX	Step size in x- direction (-)
y	Space coordinate perpendicular to axial direction (m; in)
Y	Dimensionless space coordinate perpendicular to axial direction, y/d (-)
Δy	Dimensionless distance from wall (-)
β	Nozzle angle (coanda turning) (degrees)
γ	Ratio of specific heats, C_p/C_v (-)
Ψ	Stream function (kg m^2/sec; lbm/ft sec)
ρ	Fluid density (kg/m^3; lbm/ft^3)
ρ_{01}	Fluid density evaluated at a reference temperature, T_{01}, and pressure, P_{01} (kg/m^3; lbm/ft^3)
μ	Dynamic viscosity (Ns/m^2; lbm/ft sec)
τ_{eff}	Total effective shear stress (Pa; psi)
τ_w	Local wall shear stress (Pa; psi)

ν_T Turbulent eddy conductivity, $\ell^2 \partial u/\partial n$ (m^2/s; ft^2/sec)

ε_H Eddy coefficient of heat transfer (m^2/s; ft^2/sec)

ν Kinematic viscosity at local temperature (m^2/s; ft^2/sec)

δ Local wall boundary layer thickness or jet half width (m; in)

Φ Dissipation function ($N/m^2 s$; $lbm/ft\ sec^3$)

Section 3

ANALYSIS OF VENTED COANDA FLOWS IN AUGMENTOR DUCTS

3.1 Introduction

This section presents the analysis of asymmetric curved augmentor flows which are steady two-dimensional and compressible and for which the duct geometry is general. The method extends the work presented in reference (2) but a new analysis has been performed and a new program has been developed. The analysis employs the finite-difference technique for representing the equations of motion for compressible flow. It is essentially a boundary-layer-type jet mixing analysis, written in streamline coordinates for ease of computation of curvature effects. The effects of streamline curvature on pressure gradients normal to streamline and on eddy viscosities are computed. Magnitudes of streamline curvature effects are estimated by using a quasi-orthogonal coordinate system and assumed variation of curvature with distance in the normal coordinate direction.

The analysis treats the mixing of three compressible flows of the same perfect gas under the assumption that initial conditions are known and that pressure discontinuities and flow separation are absent. The nozzle exit flow may be supersonic but it is assumed that expansion or recompression outside the nozzle, if needed, will bring the nozzle stream to the local ambient pressure so that shocks and expansion waves at the nozzle exit plane are avoided. Previous work has shown that augmentor performance is little affected by moderate degrees of departure from conditions of correct nozzle expansion. The flows considered include compound flows of supersonic and subsonic streams; however no provision is made for compound choking which may occur with an appropriate transverse distribution of Mach number. Such a condition is amenable to analytical treatment under simplified circumstances, but has not been encountered in experimental tests carried out so far.

To retain the simplicity and speed of the boundary layer approach to augmentor calculation, while incorporating approximate curvature effects, it is necessary to assume an approximate starting line. In the present work the starting line is comprised of two circular-arcs (See Fig. 1) which are tangent to each other, and perpendicular to the nozzle axis at the nozzle exit plane; one arc is normal to the upper wall and one to the lower wall of the duct. In the absence of detailed experimental information on velocity profiles in the wall boundary layers and in the jet shearing layers at the initial plane, the initialization condition has assumed uniform stagnation pressure in the nozzle flow and, separately, for the secondary and tertiary flows. In addition, for the examples worked out in this report, the secondary and tertiary flows have been assumed to have the same stagnation conditions. In computing initial conditions around the circular arcs, the effect of curvature on normal pressure gradient has been taken into account; the initialization satisfies the continuity equation separately for primary, secondary, and tertiary streams under the constraints of local duct width (along the assumed circular arc starting lines), location of the nozzle centre line, and angle of the jet axis with respect to the coordinate system of the duct walls. This initialization is of course approximate but is reasonable to use in the absence of better information on flow starting conditions. If better information is available the initialization procedure adopted in this analysis may readily be replaced to use more detailed or exact information.

3.2 Equations of Motion

The momentum, normal pressure gradient, and energy equations in streamwise coordinates are:

$$\rho u \frac{\partial u}{\partial s} = - \frac{\partial P}{\partial s} + \frac{\partial}{\partial n} (\tau_{eff}) \tag{1}$$

$$\rho u^2 K = \frac{\partial P}{\partial n} \qquad K = \frac{1}{R} \tag{2}$$

$$\rho u \frac{\partial (C_p T)}{\partial s} = u \frac{\partial P}{\partial s} + \frac{\partial}{\partial n} (q_{eff}) + \Phi \tag{3}$$

in which

$$\tau_{eff} = (\mu + \rho \nu_T) \frac{\partial u}{\partial n}$$

$$q_{eff} = (k + \rho C_p \varepsilon_H) \frac{\partial T}{\partial n}$$

$$\Phi = (\mu + \rho \nu_T) \left(\frac{\partial u}{\partial n}\right)^2$$

In these equations s and n measure distance along and normal to the streamlines, respectively, and u is the velocity component in the stream direction; p is the static pressure, ρ the density and T the temperature of the fluid, τ_{eff} is the total effective shear stress and ν_T the eddy viscosity of the fluid with μ being the dynamic viscosity. Correspondingly, q_{eff} is the effective heat transfer between streamlines with ε_H be... eddy coefficient of heat transfer and k the fluid conductivity. Constant values of laminar and turbulent Prandtl numbers have been assumed in the analysis. The term Φ is the dissipation function, included in the energy equation. The first order effects of curvature on static pressure are included through the normal pressure gradient equation (2).

Stream Function

The stream function Ψ is defined by

$$\frac{\partial \Psi}{\partial n} = \rho u \qquad (4)$$

With (4) equations (1), (2), and (3) become:

$$u \frac{\partial u}{\partial s} = - \frac{1}{\rho} \frac{\partial P}{\partial s} + u \frac{\partial}{\partial \Psi} \left[\rho u(\mu + \rho \nu_T) \frac{\partial u}{\partial \Psi}\right]$$

$$uK = \frac{\partial P}{\partial \Psi}$$

$$u \frac{\partial (C_p T)}{\partial s} = \frac{u}{\rho} \frac{\partial P}{\partial s} + u \frac{\partial}{\partial \Psi} \left[\rho u(k + C_p \rho \varepsilon_H) \frac{\partial T}{\partial \Psi}\right] + \left(\frac{\mu + \rho \nu_T}{\rho}\right) \left[\rho u \frac{\partial u}{\partial \Psi}\right]^2$$

3.3 Dimensionless Parameters

Each variable in the equations of motion is normalized by use of the following reference variables:

$$u^* = \frac{u}{u_o} \qquad P^* = \frac{P}{P_{01}} \qquad T^* = \frac{T}{T_{01}} \qquad \rho^* = \frac{\rho}{\rho_{01}}$$

$$s^* = \frac{s}{d} \qquad n^* = \frac{n}{d} \qquad R^* = \frac{R}{d} \qquad K^* = \frac{d}{R}$$

$$\mu^* = \frac{\mu}{\rho_{01} u_o d} \qquad E = \frac{\nu_T}{u_o d} \qquad P_{rt} = \frac{\nu_T}{\varepsilon_H} \qquad P_r = \frac{\mu C_p}{k}$$

$$\Psi^* = \frac{\Psi}{\rho_{01} u_o d} \qquad \gamma = \frac{C_p}{C_v}$$

in which

$$u_o = \sqrt{R_g T_{01}}$$

$$\rho_{01} = P_{01}/(RT_{01})$$

d = nozzle exit height (the small dimension)

P_{01} = primary stagnation pressure

T_{01} = primary stagnation temperature

Introducing these dimensionless groups into the equation of motion yields the following results:

$$u^* \frac{\partial u^*}{\partial s^*} = -\frac{1}{\rho^*}\frac{\partial P^*}{\partial s^*} + u^* \frac{\partial}{\partial \Psi^*}\left[\rho^* u^* (\mu^* + \rho^* E)\frac{\partial u^*}{\partial \Psi^*}\right]$$

$$u^* K^* = \frac{\partial P}{\partial \Psi^*}$$

$$u^* \frac{\partial T^*}{\partial s^*} = \left(\frac{\gamma - 1}{\gamma}\right)\frac{u^*}{\rho^*}\frac{\partial P^*}{\partial s^*} + u^* \frac{\partial}{\partial \Psi^*}\left[\rho^* u^* \left(\frac{\mu^*}{P_r} + \frac{E}{P_{rt}}\right)\frac{\partial T^*}{\partial \Psi^*}\right]$$
$$+ \left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{\mu^* + \rho^* E}{\rho^*}\right)\left[\rho^* u^* \frac{\partial u^*}{\partial \Psi^*}\right]^2$$

From henceforth we omit the superscript * for convenience so that the following are the equations of motion in dimensionless form:

$$u \frac{\partial u}{\partial s} = -\frac{1}{\rho}\frac{\partial P}{\partial s} + u \frac{\partial}{\partial \Psi}\left[\rho u(\mu + \rho E)\frac{\partial u}{\partial \Psi}\right] \qquad (5)$$

$$uK = \frac{\partial P}{\partial \Psi} \tag{6}$$

$$u \frac{\partial T}{\partial s} = \left(\frac{\gamma - 1}{\gamma}\right) \frac{u}{\rho} \frac{\partial P}{\partial s} + u \frac{\partial}{\partial \Psi} \left[\rho u \left(\frac{\mu}{P_r} + \frac{E}{P_{rt}}\right) \frac{\partial T}{\partial \Psi}\right]$$

$$+ \left(\frac{\gamma - 1}{\gamma}\right) \left(\frac{\mu + \rho E}{\rho}\right) \left[\rho u \frac{\partial u}{\partial \Psi}\right]^2 \tag{7}$$

3.4 Evaluation of the Eddy Viscosity

In general the eddy viscosity is defined by:

$$\nu_T = \ell^2 \frac{\partial u}{\partial n} \tag{8}$$

Writing this in dimensionless form, using the dimensionless parameters specified previously and the stream function, the eddy viscosity expression becomes:

$$E = L^2 \rho u \frac{\partial u}{\partial \Psi} \tag{9}$$

in which

$$L = \frac{\ell}{d} \text{ and } E = \frac{\nu_T}{u_o d}$$

The effect of streamline curvature is taken into account by use of the Richardson Number correction in the following approximate form:

$$L = L_o \exp(-3R_i) \qquad R_i > 0 \tag{10}$$

$$L = L_o \left[2 - \exp(3R_i)\right] \qquad R_i < 0 \tag{11}$$

in which L_o is the dimensionless mixing length in the absence of stream curvature and R_i is the Richardson Number defined by:

$$R_i = \frac{2u}{R} \Big/ \frac{\partial u}{\partial n}$$

or

$$R_i = \frac{2K}{\rho \frac{\partial u}{\partial \Psi}} \tag{12}$$

For small values of $|R_i|$ the above dependence of L on R_i is approximately in accord with the linear relationship derived by Bradshaw (3). For large values of $|R_i|$ an empirical correlation is not available, and the exponential relationship has been assumed.

In the absence of curvature the mixing lengths L_o are defined as follows.

Boundary Layer

In the inner part of the layer the Van Driest approximation is used.

$$L_o = 0.41\ \Delta y[1 - \exp(-y^+/26)] \tag{13}$$

in which Δy is the dimensionless distance from the wall. The variable

$$y^+ = \frac{\Delta y}{\nu}\sqrt{\frac{\tau_w}{\rho}} \tag{14}$$

is evaluated using

$$\tau_w = \frac{\mu u_2}{\Delta y_2} - \frac{\Delta y_2}{2}\frac{\partial P}{\partial s} \tag{15}$$

in which the subscript 2 denotes the streamline coordinate point closest to the wall.

In the outer part of the layer the mixing length is evaluated by

$$L_o = 0.09\left(\frac{\delta}{d}\right) \tag{16}$$

in which δ is the boundary layer 99% thickness and d is the nozzle exit width.

In the middle part of the layer the smaller of the values of L_o provided by Equations (13) and (14) is used.

Jet Shear Layers

In the shear layer adjacent to the potential-core zone of the primary jet the mixing length is evaluated from

$$L_o = 0.07 \left(\frac{\Delta}{d}\right)\left(1 + 0.6 \frac{U_{sec}}{U_{CL}}\right) \quad (17)$$

in which Δ is the shear layer width (including the zone between 1% and 99% of the total velocity difference between primary and secondary streams) divided by the nozzle width at its exit plane. The term in parentheses takes approximate account of the effects of the secondary velocity, U_{sec}, of a co-flowing outer stream, on the mixing of a jet whose centerline velocity is U_{CL}.

For a "fully-rounded" portion of the jet flowing co-axially with a secondary potential stream, the mixing strength has been calculated from

$$L_o = 0.09 \left(\frac{\Delta}{d}\right)\left(1 + 0.6 \frac{U_{sec}}{U_{CL}}\right) \quad (18)$$

in which Δ is the half-width of the jet (evaluated from centerline to the point at which difference between local and secondary velocity is only 1% of the difference between centerline and secondary velocity), divided by the nozzle width at its exit plane.

Developing Pipe Flow Region

For the region downstream at the point where the jet spreads to intersect the edge of the boundary layer the mixing is evaluated, as a first approximation only, from

$$L_o = \frac{w}{d}\left(0.14 - 0.08\left[1 - \frac{\Delta y}{w}\right]^2 - 0.06\left[1 - \frac{\Delta y}{w}\right]^4\right) \quad (19)$$

in which w is half the total width, and Δy the distance from the wall. This formula is due to Nikuradse and is cited by Schlichting (3) for fully developed flow in round tubes. Near the wall the mixing length is evaluated by the Van Driest approximation cited earlier, provided the local mixing length so calculated is less than that given by the Nikuradse formula.

The laminar dynamic viscosity is evaluated from (1):

$$\mu = \frac{\mu_{ref}}{P_{01}\sqrt{R_g T_{01}}\ d} \left[\frac{T_{01}T}{T_{ref}}\right]^{1/2} \left[\frac{T_{ref} + 198.7°R}{T_{01}T + 198.7°R}\right] \qquad (20)$$

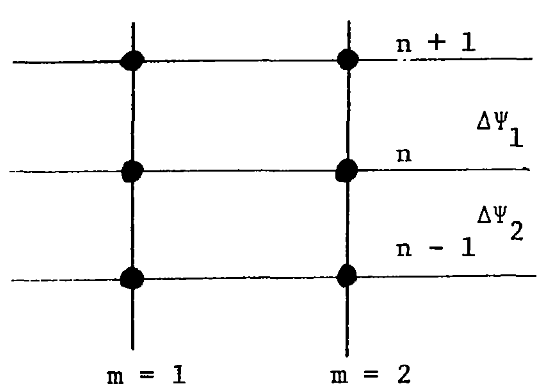
in which μ_{ref} is the laminar dynamic viscosity at a temperature T_{ref} and a pressure P_{ref} and T is the dimensionless temperature (normalized by the inlet stagnation temperature of the primary stream T_{01}.

The wall shear stress (normalized by the inlet stagnation pressure P_{01}) is given by Equation (15). For this equation to be realistic, the grid spacing must be chosen so that y^+ at node point 2 is not greater than 3 - 4. The associated wall friction velocity (normalized by the reference velocity u_o) is given by:

$$u^* = \sqrt{\left(\frac{u_2}{\Delta y} - \frac{\Delta y}{2\mu}\frac{dP}{dX}\right)\frac{\mu}{P}} \qquad (21)$$

3.5 Finite Difference Procedure

By the finite difference technique the derivatives in the differential equations of motion are replaced by differences either along a streamline between two neighboring points X and X + dX or normal to it between two neighboring points Ψ and Ψ + dΨ. The finite difference equivalence of the equations of motion are obtained as follows with reference to the following grid lines:



Writing the velocities in terms of a taylor expansion:

$$u_{n+1} = u_n + \left.\frac{\partial u}{\partial \Psi}\right|_n \Delta\Psi_1 + \left.\frac{\partial^2 u}{\partial \Psi^2}\right|_n \frac{\Delta\Psi_1^{\,2}}{2!}$$

$$u_{n-1} = u_n - \left.\frac{\partial u}{\partial \Psi}\right|_n \Delta\Psi_2 + \left.\frac{\partial^2 u}{\partial \Psi^2}\right|_n \frac{\Delta\Psi_2^{\,2}}{2!}$$

Eliminating $\left.\frac{\partial^2 u}{\partial \Psi^2}\right|_n$

$$\frac{\Delta\Psi_2^{\,2}}{2} u_{n+1} - \frac{\Delta\Psi_1^{\,2}}{2} u_{n-1} = \frac{\Delta\Psi_2^{\,2}}{2} u_n - \frac{\Delta\Psi_1^{\,2}}{2} u_n$$

$$+ \left.\frac{\partial u}{\partial \Psi}\right|_n \left[\Delta\Psi_1 \frac{\Delta\Psi_2^{\,2}}{2} + \Delta\Psi_2 \frac{\Delta\Psi_1^{\,2}}{2}\right]$$

Dividing by $\Delta\Psi_1 \Delta\Psi_2$

$$\frac{\Delta\Psi_2}{\Delta\Psi_1} u_{n+1} - \frac{\Delta\Psi_1}{\Delta\Psi_2} u_{n-1} = \frac{\Delta\Psi_2}{\Delta\Psi_1} u_n - \frac{\Delta\Psi_1}{\Delta\Psi_2} u_n + \left.\frac{\partial u}{\partial \Psi}\right|_n \left[\Delta\Psi_1 + \Delta\Psi_2\right]$$

This equation leads to:

$$\left.\frac{\partial u}{\partial \Psi}\right|_n = \frac{\Delta\Psi_2}{\Delta\Psi_1(\Delta\Psi_1 + \Delta\Psi_2)} (u_{n+1} - u_n) + \frac{\Delta\Psi_1}{\Delta\Psi_2(\Delta\Psi_1 + \Delta\Psi_2)} (u_n - u_{n-1})$$

or $\left.\frac{\partial u}{\partial \Psi}\right|_n = S_5(u_{n+1} - u_n) + S_4(u_n - u_{n-1})$

and $\left.\frac{\partial T}{\partial \Psi}\right|_n = S_5(T_{n+1} - T_n) + S_4(T_n - T_{n-1})$

in which $S_4 = \frac{\Delta\Psi_1}{\Delta\Psi_2(\Delta\Psi_1 + \Delta\Psi_2)}$ and $S_5 = \frac{\Delta\Psi_2}{\Delta\Psi_1(\Delta\Psi_1 + \Delta\Psi_2)}$

Using another Taylor Series expansion with a general coefficient:

$$S\left(\frac{\partial u}{\partial \Psi}\right)\bigg|_{n+1/2} = S\,\frac{\partial u}{\partial \Psi}\bigg|_{n} + \frac{\partial}{\partial \Psi}\left(S\,\frac{\partial u}{\partial \Psi}\right)\bigg|_{n}\frac{\Delta\Psi_1}{2} + - - - -$$

$$S\left(\frac{\partial u}{\partial \Psi}\right)\bigg|_{n-1/2} = S\,\frac{\partial u}{\partial \Psi}\bigg|_{n} - \frac{\partial}{\partial \Psi}\left(S\,\frac{\partial u}{\partial \Psi}\right)\bigg|_{n}\frac{\Delta\Psi_2}{2} + - - - -$$

Solving for $\frac{\partial}{\partial \Psi}\left(S\,\frac{\partial u}{\partial \Psi}\right)\bigg|_{n}$ we obtain:

$$\frac{\partial}{\partial \Psi}\left(S\,\frac{\partial u}{\partial \Psi}\right)\bigg|_{n} = \left(\frac{2}{\Delta\Psi_1 + \Delta\Psi_2}\right)\left[S\left(\frac{\partial u}{\partial \Psi}\right)\bigg|_{n+1/2} - S\left(\frac{\partial u}{\partial \Psi}\right)\bigg|_{n-1/2}\right]$$

$$= \frac{1}{(\Delta\Psi_1 + \Delta\Psi_2)}\left[(S_{n+1} + S_n)\,\frac{(u_{n+1} - u_n)}{\Delta\Psi_1} - (S_n + S_{n-1})\,\frac{(u_n - u_{n-1})}{\Delta\Psi_2}\right]$$

The terms in Equation (5) become:

$$u\,\frac{\partial u}{\partial s} = u_{1,\,n}\,\frac{u_{2,\,n} - u_{1,\,n}}{\Delta S_n}$$

$$-\frac{1}{\rho}\,\frac{\partial P}{\partial s} = -\frac{1}{\rho_{1,\,n}}\left[\frac{1}{2}\left(\frac{\partial P}{\partial s}\right)_{m=1} + \frac{1}{2}\left(\frac{\partial P}{\partial s}\right)_{m=2}\right]$$

and $u\,\frac{\partial}{\partial \Psi}\left(S\,\frac{\partial u}{\partial \Psi}\right) = \frac{u_{1,\,n}}{\Delta\Psi_1 + \Delta\Psi_2}\left[\left(\frac{S_{n+1} + S_n}{\Delta\Psi_1}\right)u_{n+1} - \left(\frac{S_{n+1} + S_n}{\Delta\Psi_1} + \frac{S_n + S_{n-1}}{\Delta\Psi_2}\right)u_n + \left(\frac{S_n + S_{n-1}}{\Delta\Psi_2}\right)u_{n-1}\right]$

in which $S = \rho u(\mu + \rho E)$

With these finite-difference equivalents for the derivative terms Equation (5) may be written in the form:

$$A_{n-1} u_{2,n} + B_{n-1} u_{2,n+1} + C_{n-1} u_{2,n-1} = D_{n-1} \tag{22}$$

in which

$$A_{n-1} = \frac{u_{1,n}}{\Delta S_n} + Y_8 + Y_9 \tag{23}$$

$$B_{n-1} = -Y_8 \tag{24}$$

$$C_{n-1} = -Y_9 \tag{25}$$

$$D_{n-1} = -\frac{1}{\rho_{1,n}} \left(\frac{\partial P}{\partial x}\right)_{m+1/2} + \frac{u_{1,n}^2}{\Delta S_n} \tag{26}$$

$$Y_8 = \frac{u_{1,n} (S_{n+1} + S_n)}{(\Delta\Psi_1 + \Delta\Psi_2) \Delta\Psi_1} \tag{27}$$

$$Y_9 = \frac{u_{1,n} (S_n + S_{n-1})}{(\Delta\Psi_1 + \Delta\Psi_2) \Delta\Psi_2} \tag{28}$$

$$S = \rho u(\mu + \rho E) \tag{29}$$

Similarly, Equation (7) may be written in the form:

$$A_{n-1} T_{2,n} + B_{n-1} T_{2,n-1} + C_{n-1} T_{2,n-1} = D_{n-1} \tag{30}$$

in which

$$A_{n-1} = \frac{u_{1,n}}{\Delta S_n} + Y_8' + Y_9' \tag{31}$$

$$B_{n-1} = -Y_8' \tag{32}$$

$$C_{n-1} = -Y_9' \tag{33}$$

$$D_{n-1} = \frac{u_{1,n}}{S_n} T_{1,n} + \frac{\gamma - 1}{\gamma} \frac{u_{1,n}}{\rho_{1,n}} \left(\frac{\partial P}{\partial x}\right)_{m+1/2}$$

$$+ \frac{\gamma - 1}{\gamma} u_{1,n} S_n \left[S_5 (u_{1,n+1} - u_{1,n}) + S_4 (u_{1,n} - u_{1,n-1}) \right]^2 \tag{34}$$

$$Y_8' = u_{1,n} \frac{S'_{n+1} + S'_n}{\Delta\Psi_1 (\Delta\Psi_1 + \Delta\Psi_2)} \tag{35}$$

$$Y_9' = u_{1,n} \frac{S'_n + S'_{n-1}}{\Delta\Psi_2 (\Delta\Psi_1 + \Delta\Psi_2)} \tag{36}$$

$$S' = \rho u \left(\frac{\mu}{P_r} + \frac{\rho E}{P_{rt}}\right) \tag{37}$$

Coefficient Matrix

The general equation

$$A_{n-1} X_n + B_{n-1} X_{n+1} + C_{n-1} X_{n-1} = D_{n-1}$$

with boundary conditions

$$u_1 = 0$$

$$T_1 = T_2$$

$$U_n = 0$$

$$T_n = T_{n-1}$$

can be written for each of the n grid points with the result:

$$
\begin{bmatrix}
1 & -\delta & & & & \\
C_1 & A_1 & B_1 & 0 & 0 & - \\
 & C_2 & A_2 & B_2 & 0 & - \\
 & & & & & \\
 & & C_{n-3} & A_{n-3} & B_{n-3} & 0 \\
 & & & C_{n-2} & A_{n-2} & B_{n-2} \\
 & & & & -\delta & 1
\end{bmatrix}
\begin{bmatrix}
X_1 \\ X_2 \\ X_3 \\ \\ X_{n-2} \\ X_{n-1} \\ X_n
\end{bmatrix}
=
\begin{bmatrix}
0 \\ D_1 \\ D_2 \\ \\ D_{n-3} \\ D_{n-2} \\ 0
\end{bmatrix}
$$

where $\delta = 0$ in momentum equation

$\delta = 1$ in energy equation

The first and second equations are:

$$X_1 - \delta X_2 = 0$$

$$C_1 X_1 + A_1 X_2 + B_1 X_3 = D_1$$

$$A'_1 X_2 + B_1 X_3 = D_1$$

in which

$$A'_1 = C_1 \delta + A_1$$

The last two equations are:

$$C_{n-2} X_{n-2} + A_{n-2} X_{n-1} + B_{n-2} X_n = D_{n-1}$$

$$-\delta X_{n-1} + X_n = 0$$

Combining these

$$C_{n-2} X_{n-2} + A'_{n-2} X_{n-1} = D_{n-1}$$

in which

$$A'_{n-2} = A_{n-2} + B_{n-2} \delta$$

With these two results the order of the matrix can be reduced to $n - 2$.

$$\begin{bmatrix} A'_1 & B_1 & 0 & - & - & \\ C_2 & A_2 & B_2 & 0 & - & \\ & & & & & \\ & & C_{n-3} & A_{n-3} & B_{n-3} & \\ & & & C_{n-2} & A'_{n-2} & \end{bmatrix} \begin{bmatrix} X_2 \\ X_3 \\ \\ X_{n-2} \\ X_{n-1} \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \\ \\ D_{n-3} \\ D_{n-2} \end{bmatrix}$$

This is solved by the Thomas Algorithm as indicated in References (2) and (5).

3.6 Boundary Conditions

The boundary conditions of the walls are:

$y = y_w(x)$ (upper and lower walls)

$K = K_w(x)$ (wall curvature)

$\Psi = \text{const}$

$u = 0$

$\frac{\partial T}{\partial \Psi} = 0$ (adiabatic wall)

In the program provision is made for the calculating wall curvatures from wall coordinates x, y using a least-squares smoothing procedure. Wall coordinates must be specified with sufficient precision to estimate realistic values of wall curvature.

3.7 Quasi-Orthogonal Coordinate System

To preserve orthogonality of the coordinate system the step sizes ΔS must be adjusted according to:

$$\frac{\partial \Delta s}{\partial n} = \frac{\Delta s}{R}$$

or

$$\frac{\partial \Delta s}{\partial \Psi} = \frac{\Delta s}{\rho u R} \qquad (38)$$

Starting with an arbitrarily chosen step size at the nozzle centre line the step size for adjacent streamlines is calculated with the finite-difference equivalent of this formula, using mean values of curvature, density, velocity and step size between the neighboring streamlines.

Streamline curvature ($K = 1/R$) is assumed to decay exponentially with distance from the wall according to:

$$K = K_1\left(1 - \frac{\Delta n_1}{w}\right)\exp(-\alpha|K_1|\Delta n_1) + K_2\,\frac{\Delta n_2}{w}\exp(-\alpha|K_2|\Delta n_2) \qquad (39)$$

in which K_1(positive) and K_2(negative) are the curvatures of the lower and upper walls, respectively. Δn_1 and Δn_2 are the distances along the streamline orthogonal to the lower and upper walls, respectively and w is the sum of the two. The decay constant α appears to have a value 2 for potential flow around a cylinder, but in these boundary layer flows a value of 4 seems more appropriate, from comparisons of measured and calculated pressure distributions. The contribution of the first term to streamline curvature in the vicinity of the upper wall is negligible, and vv.

3.8 Solution Procedure

The first step in the solution is to determine all flow properties on the initial line. First the radii of curvature of the two circular arcs are determined iteratively to satisfy the conditions of mutual tangency at the nozzle centerline, and orthogonality to the nozzle centerline and to the two walls. The initial stagnation conditions are given and assumed the same for secondary and tertiary flows (See Fig. 1). The primary mass flow and the sum of primary and secondary mass flow rates are also given. The initialization then solves iteratively for the split between secondary and tertiary flow rates, and the values of all properties along the initial line under the requirement of mass conservation, and assuming isentropic flow up to the initial line. The solution procedure accommodates substantial effects of streamline curvature, but will not succeed if the total ingested flow rate is so small that the local velocity e.g. at the lower side of the nozzle (Fig. 1) is negative. The initialization procedure also selects the location of streamline node points, with very close spacing required near the walls, and moderately close spacing in the initially-thin shear layers of the jet.

For a set of n streamlines and known boundary conditions, equations (22) and (30) each provide a set of n - 2 conditions to solve for the unknown velocities and temperatures. Each set of equations can be solved simultaneously if the pressures at downstream node points are known or assumed. For calculation of flow between curved channel walls, the pressure gradient along one streamline is assumed and the downstream pressure on that streamline determined from this gradient and the arbitrary step size. Pressures at corresponding node points on adjacent streamlines are determined by use of the finite-difference form of Equation (6). With the downstream pressures determined, equations (22) and (30) are solved to provide downstream velocities and temperatures, and subsequently all other properties at downstream points. If the calculated value of the outer boundary location does not agree satisfactorily with the actual wall geometry, a new value of the pressure gradient is chosen.

Section 4

TEST PROGRAM

A two dimensional experimental rig was designed, fabricated, and installed in our laboratory. The purpose of the experimental work was to obtain test data for verification and adjustment of the computer analysis. The experimental program is described in this section.

4.1 Experimental Apparatus

4.1.1 Two Dimensional Ejector

The two-dimensional ejector consists of a slot type primary nozzle and a two dimensional mixing section. The arrangement of the ejector system for the four nozzle positions tested is shown in Figures 2 and 3 and the positions are listed below.

Test #	Nozzle Angle (β)	Spacing (t) (inches)
1 & 2	22.5°	0.80
3 & 4	45.0°	0.80
5 & 6	45.0°	1.31
7 & 8	67.5°	0.70

The nozzle angle (β) is defined as the angle measured between the vertical and the line running from the center of the coanda arc to the centerline of the nozzle at the throat while the spacing (t) is defined as the perpendicular distance of closest approach between the nozzle and the coanda surface.

A picture of the primary nozzle is shown in Figure 4. The discharge slot is 0.1215" ± .0005" by 8.00" with rounded corners. The side walls are made from one quarter inch carbon steel. Four internal supports prevent substantial widening of the discharge slot when the nozzle is pressurized. Dial indicator measurements performed in previous tests revealed that the slot opened up by about 0.0008 inches in the center of the nozzle, about 0.0004" at the quarter width location, and zero near the ends of the slot. This corresponds to an increase in nozzle slot area of 0.33% when pressurized. Stagnation pressure measurements were made with a kiel probe from side to side at two different axial locations and were found to be uniform across the 8" width of the slot (See Appendix A).

Aluminum pieces on which the nozzle pattern had been cut at the desired angle were bolted to the inside of the side plates to position the nozzle in the inlet to the mixing section (Figs. 6, 7, 8). The nozzle was held firmly in place by threaded rods connecting brackets on each side of the nozzle to brackets attached to busses welded to the side plates of the mixing section (See Figs. 6, 7, 8).

The mixing section, as shown in Figure 2, consists of a rectangular variable area channel formed by two identically contoured aluminum plates, two flat side plates and two dissimilar curved inlet pieces. The upper bellmouth, the coanda surface, is a constant radius (r = 5.00 inch). The nozzle is positioned close to this surface and relies upon the coanda effect to turn the primary flow smoothly into the test section. The lower inlet consists of a bellmouth piece and a straight section. The pictures in Figures 6, 7, and 8 show three views of the mixing section. The two contoured plates were positioned in symmetrical locations about the centerline to form the channel tested (throat height of 1.875"). The width of the mixing section is a constant 8.00 inches along the entire length. The variation of channel height with distance from the nozzle discharge is given in Table 1. Three plexiglass windows were installed along each side of the mixing section so that tufts of wool mounted inside could be observed for indications of flow separations and unsteadiness.

The screened mixing section inlet is shown on Figure 9. Earlier tests (Ref. 2) without the extended inlet showed that highly swirling corner vortices were formed in the four corners of the bellmouth and extended into the test section. The extended inlet eliminated the corner vortices and improved the stability of the ejector flow and static pressures. Four sets of screened inlets were used in the testing to accommodate the four nozzle positions.

4.1.2 Facilities for Ejector Tests

Three subsystems are required for the operation, control and measurement of the air flow through the two dimensional ejector (Figure 9). These three subsystems referred to as the primary flow, the mixed flow, and the boundary layer suction systems are discussed below.

The primary air flow is supplied by a 900 scfm non lubricated screw compressor at 100 psig and an equilibrium operating temperature between 100°F and 140°F. The primary air flow rate and pressure are controlled by an automatic pressure regulator capable of maintaining pressure to within $\pm$ .1 psi of a set value. The mass flow is measured by a standard 3 inch Danial orifice system. A flexible hose connects the primary orifice system to the nozzle.

The mixed flow system consists of a plenum chamber and an 8 inch orifice system. Two different operating flow rates were achieved by the following equipment combinations.

1. Maximum Flow Rate at Atmospheric Discharge - Mixed flow discharges directly into the laboratory.

2. Reduced Flow Rate at Back Pressure - The plenum and orifice are connected to the mixing section discharge.

Mixed orifice flow rates were obtained only for the reduced flow rate conditions. The plenum is shown connected to the mixing section by flexible hose in Figure 10.

The suction system removes the boundary layer flow from each of the four corners of the mixing section to prevent wall boundary layer separation in the ejector. Figure 11 shows six 3/4 inch tubes connected to the top corners of the mixing section. A total of 12 tubes (top and bottom) collect the boundary layer flow from the four corner suction slots which are 0.060 inches wide and are machined into the sides of the contoured plates (See Figs. 12 and 13). The four tubes at one X location are connected to a single large tube under the mounting table (Fig. 14). The three large tubes are each connected to a large tank plenum through a separate throttle valve. A Roots blower draws the air through the suction system and through a three inch orifice system. The suction system is capable of removing about 1% and 2% of the mixing section flow rate. During the operation of the ejector rig, the boundary layer suction system was used to prevent flow separation in the mixing section diffuser.

The ejector system was operated by starting the primary air flow at low pressure and flow rate. The primary nozzle pressure was increased to 22 psig and the suction was then turned on. Approximately 1/2 hour of warm up time was allowed before testing.

4.2 Instrumentation and Data Reduction

4.2.1 Instrumentation

The following instrumentation was used in the test facility.

Primary Flow System

Flow Rate - Standard 3" orifice system
Nozzle Pressure - Bourdon Pressure Gage accurate to ± .10 psig
Nozzle Temperature - Copper Constantan thermocouple with digital readout

Mixed Flow System

Flow Rate - 8" orifice system for reduced flow rate conditions (Tests 2, 3, 6 and 7)

Static Pressures - 77 wall static pressure taps located throughout the mixing section on both top and bottom contoured plates and on both bellmouth pieces (See Figs. 12 and 13). As shown in Figure 10, the static taps were connected to a valving system which permitted easy determination of individual static pressures without the need for a large manometer bank. Tygon tubing was used to connect the taps to four pressure sampling valves each capable of handling 24 inputs. These pressure valves were in turn connected through a switching network to any of 3 well type manometers which permitted the accurate determination of pressures over the range of + 25 inches of water gage to - 75 inches of water gage.

Traverse Data - Stagnation pressure and temperature profiles were measured at up to 11 axial locations using a 1/8" diameter stem kiel pressure-temperature probe. Both a mercury manometer and a pressure transducer coupled with a direct digital readout were used for pressure measurements. A direct digital readout was used to indicate total temperatures.

Suction Flow System

Flow Rate - 3" orifice system

Suction Pressure - Bourdon type pressure gage

4.2.2 Data Reduction Procedures

Three types of data reduction calculations were needed in this program.

1. Standard orifice calculations
2. Velocity profile calculations
3. Integration of velocity profiles to calculate flow rate

All of these calculations were programmed on a time sharing computer. The orifice calculations were programmed as a subroutine to the main data reduction program using standard orifice equations and ASME orifice coefficients. The equations used in the determinations of velocity were included in the main program and are the standard compressible flow relationships which can be found in most fluid mechanics text books.

The integrated mass flow rate for each traverse was computed by integrating the product of the local velocity and local density over a two dimensional section of unit width. The program also calculated the "mass-momentum" stagnation pressure at each traverse section using the equations presented on pages 52 and 53 of Reference 6. The mass momentum method determines the flow conditions for a uniform velocity profile which has the same integrated values of mass flow rate, momentum, and energy as the non-uniform velocity profile actually present.

4.2.3 Experimental Uncertainty

Orifice Calculations

The techniques presented in Reference 7 were applied to the primary flow orifice calculations and the mixed flow orifice calculations. The following uncertainty results were obtained:

Orifice	Pressure (psig)	Uncertainty
Primary Nozzle	22 psig	$\pm$ 0.8%
Mixed	slightly above atmospheric	$\pm$ 1.3%

Static Pressure

Uncertainty in the wall static pressures occur mainly because of fluctuations in the manometer liquid columns caused by unsteadiness in the flow. The degree of these fluctuations may therefore be used as an indication of the uncertainty of the pressure readings. For the unrestricted maximum flow rate condition the wall static pressure fluctuation reached a maximum of $\pm$ 1.0 inch of water, while for the reduced flow rate condition the maximum reached only $\pm$ 0.4 inch of water.

Integrated Mass Flow Rate

The mass flow rate calculated by integrating the results of the stagnation pressure and temperature traverses is influenced by many items and is therefore very difficult to estimate. The following items all contribute to the uncertainty in integrated mass flow rate:

1. unsteady wall static pressures
2. unsteady traverse stagnation pressures
3. instrument accuracy of the pressure transducer and digital readout
4. inaccuracies due to the effect of steep velocity gradients on sensed pressure
5. inaccuracies due to probe effect near the mixing section walls
6. inaccuracy in probe position
7. assumptions and inaccuracies associated with the data reduction computer program
8. data recording errors or computer data input errors
9. errors caused by loose connections in the pneumatic sensing tube between the probe and the transducer
10. non-two-dimensional flow distribution across the width of the 8 inch mixing section

All of these effects could combine to give both a ± uncertainty band and a fixed error shift.

One measure of the uncertainty due to these effects is obtained from the limits of individual integrated mass flows for each test run. These values are listed on Table 2 for all of the test runs with traverse data. The results presented on Table 2 show an average variation of +4.4 and -3.0 or a total spread of 7.4%. These values only include the effect of variable uncertainty and exclude the uncertainty due to probe errors in steep gradients and near walls and integration assumptions. Both of the excluded errors probably cause the integrated mass flows to be too large because the probe tends to measure too high near the wall and the integration program neglects wall boundary layers.

4.3 Test Schedule and Results

A total of eight ejector tests were carried out on one mixing section configuration (1.875" height) and at one nozzle pressure, 22 psig. Four different nozzle positions were tested each at atmospheric discharge and at a back pressure condition. Figure 3 summarizes the conditions for each of the tests. In each test, readings were taken from the wall static pressure taps, orifice system instruments, and from the total pressure and temperature taps on the traversing probe. The number and location of traverses varied from test to test. The traverse locations and a summary of the data presented for each test is given in Table 3.

The data presented in this report falls into the following categories:

Test Conditions and Mass Flows
Static Pressures
Maximum Local Pressures
Velocity Profiles
Richardson Number Coefficient Sensitivity
Streamline Curvature Decay Sensitivity

A summary of the figures and tables used to present data from each test run is presented in Table 3. Discussion of the data will be taken up in the next section.

A sample of the static pressure data as taken is tabulated in Appendix A for Tests 7 and 8.

Section 5

COMPARISON OF ANALYTICAL AND TEST RESULTS

5.1 Test Conditions and Mass Flows

Table 5 and Figure 15 show a comparison of

a) the total flow rate measured by an orifice downstream of the diffuser

b) total flow rates determined by integration of measured velocity profiles at various sections in the test section

c) flow rates inferred by use of the computer program with best fit to the static pressure in the throat region

In general the flow rates determined by methods (a) and (c) agreed satisfactorily within approximately 6%. Flow rates estimated by integration of experimental velocity profiles at various stations were consistent with one another within approximately 8% but disagreed with the results of (a) and (c) up to 15%. Previous experience (Ref. 2) also showed that integration of experimental velocity profiles yielded too high a mass flow. In that case the discrepancy was of the order of 6%. The velocities were determined experimentally by the use of a Kiel probe to determine a local stagnation pressure coupled with the assumption that the local static pressure was equal to the wall static pressure. The velocities were determined in this way only for stations downstream of high wall curvature. The disagreement between total flows determined by integrating velocity profiles and those obtained from orifice measurements may be due to the effect of high shear and turbulence level in these flows upon the apparent stagnation pressure reading of the Kiel probe. In view of these discrepancies reliance was placed in these tests upon the mass flows determined by methods (a) and (c).

The accuracy of the primary flow measurements determined by orifice readings for the primary flow are of the order of 3%. The secondary flow determinations by orifice have an apparent uncertainty level of + or - 1.5%.

5.2 Wall Static Pressures

Figure 16 shows a comparison between the static pressures predicted using this computer program and those measured with the symmetrical diffuser employed in the NAS-50 program (Reference 2) for which wall curvatures were very small and had little effect upon the axial static pressure profile. This comparison which is included as a check point in this discussion shows that the curvature program predicts the wall static pressures reasonably well when the nozzle is located at the mid-plane of a symmetrical test section.

Figures 17 through 20 show experimental values of the wall static pressures measured for eight experimental cases with the unsymmetrical test section and with the nozzle located at various distances from the curved wall and at various angles with respect to the test section axis. In general there is a considerable difference in static pressure between top and bottom walls with the lowest pressure being near the top wall which had the highest curvature and near which the nozzle was located. These low pressures between the top wall and the nozzle were accompanied by velocities considerably higher than those in the region between the nozzle and the lower wall. Downstream of the region of considerably high wall curvature the measured wall static pressures were nearly the same on top and bottom walls.

Figures 17 through 20 also show the computed static pressure distributions on the top and bottom walls. In general the agreement between analytical measured results is considerably better with high coanda effect, i.e., large turning angle (up to 67.5°) and small spacing between the nozzle and the wall. The greatest discrepancies between analytical and experimental results are associated with those cases where the calculation method indicates that the flow is on the verge of separation, for example, cases 1 and 2. In cases 5 and 6 the static pressure distribution on the top wall near the nozzle is quite different from the calculated value. Here the spacing between nozzle and wall is large at 1.10 inches. This large venting of the flow between the nozzle and the wall appears to have substantially diminished the Coanda effect lessening the tendency of the flow to cling to the upper wall and increasing the possibility of separation in the region immediately downstream of the nozzle. In the region of separation the flow calculation becomes somewhat uncertain and experimental details were insufficient to ascertain whether the flow were actually separated in that region. However the degree of agreement between measured and calculated results was substantially poorer for cases 5 and 6 with large venting between the nozzle and the wall.

In general the reasons for differences between the analytical and calculated results are as follows:

1. Flow Separation - The experimental results for cases 1 and 2 and case 6 appear to be on the verge if not actually past the margin of flow separation, as indicated by the calculated values of the wall shear stress or the velocities near the wall for those cases.

2. Initialization Approximations - As explained in the earlier section of the report, the initialization process assumes a circular arc starting line for each of the two regions between the nozzle and the upper and lower walls. The initialization process is assumed to have isentropic flow up to the starting line where the nozzle has zero thickness, so an approximation is used for estimating the decay of streamline curvature with distance away from the wall. In the initialization process it was found that the calculated wall static pressure distribution immediately downstream of the nozzle was very sensitive to the ratio of the flows between nozzle and upper and lower walls respectively. This flow ratio was not available experimentally and was determined in the initialization process by requiring smooth continuity of static pressure along the circular arc starting lines. The initialization process was also sensitively dependent upon decay of wall curvature away from the upper and lower walls, especially in the jet zone.

3. The Use of a Quasi-Orthogonal Coordinate System - In the calculation method curvatures were estimated by the use of

equation 39. In principle this estimation could have been used for a first approximation to determine the entire velocity field then subsequent iterations could have utilized the calculated velocities to determine a second approximation for streamline curvature. However this approach was considered excessively time-consuming for the present problem and not sufficiently justified by the requirements of computing vented Coanda flows where the nozzle spacing is not large and the wall curvature is substantial. The major curvature effects are experienced in a region close to the wall itself.

4. The Effect of Streamline Curvature on Jet Mixing Turbulent Shear Stresses - As pointed out earlier, the first-order effects of curvature on turbulent mixing length have been estimated by Bradshaw. An approximate expression for the dependence of mixing length upon Richardson number is included in the calculation method. However this approximation is not well validated by experimental data and includes curvature effects significantly larger than those considered by Bradshaw, hence, this adds another element of uncertainty to the flow calculation.

Figure 21 shows the effect of varying the Richardson number coefficient (from 3 to 10) upon a computed static pressure distribution along the wall for case 6. This range of Richardson coefficient may be thought to represent the uncertainty in the magnitude of the effect but suggests very little alteration on computed wall static pressure distributions. Increasing the value of the Richardson number coefficient tends to decrease the turbulent shear stresses in the upper part of the jet mixing zone and to increase them on the lower side.

Figure 22 shows the sensitivity of the calculation of wall static pressures for case 6 upon the assumed value of the streamline curvature decay coefficient used in equation 39. The effects of variation of this coefficient are naturally unimportant in the downstream region where curvatures are small but can be quite significant in the region for a less than 0, i.e., close to the zones of high wall curvature.

5.3 Locus of Maximum Stagnation Pressure

Figures 23 through 30 show the computed location of the line of maximum stagnation pressure from the nozzle to a point far downstream in the test section. Also shown are test data points taken from the maximum stagnation pressure in the upstream zone and from the maximum velocity, i.e., maximum stagnation pressure for the downstream region in which curvature effects are negligible.

As with the static pressure comparisons the best agreement between calculated and measured values of the locations of maximum stagnation pressure correspond to those experimental cases in which there was the largest degree of Coanda turning and the smallest spacing between the nozzle and the wall. This is shown particularly by the comparison for cases 7 and 8. In other cases, for example, cases 5 and 6 (with large venting between the large spacing between the nozzle and the wall) the computed location of maximum stagnation pressure shows a substantial deviation from the experimental results. These two cases as pointed out earlier appear to show a substantially diminished Coanda effect. The jet clearly does not cling as closely to the wall as the computer model predicts. In general the differences between computed and experimental results may be ascribed to the reasons mentioned earlier for the static pressure discrepancies.

Figures 31 and 32 show the total pressure profiles across the mixing section. The analytical prediction shown with continuous lines shows a small deviation from the experimental results in the tertiary flow that is near the bottom wall.

5.4 Velocity Profiles

Figures 33 through 40 show comparisons of non-dimensional velocity profiles at various locations throughout the test section for each of the eight cases investigated. Owing to the uncertainty in velocity determination as evidenced by the integrated mass flow of discrepancy being up to 15% the large uncertainty level must be attached to each velocity determination. Hence, the rather substantial discrepancies between experimental and computed velocities are not conclusive indications of the degree of reliability of the analytical method. The experimental velocity profiles for Runs 5 and 6 show that the maximum velocity region has been shifted towards the bottom wall indicating a reduced coanda effect at large nozzle spacing.

Section 6

GENERAL CONCLUSIONS

1. An approximate method has been developed for calculation of vented Coanda flows in ducts. A method has been confirmed by experimental data with angles of turning up to 67.5° and for close spacing between the nozzle and the curved wall. At larger spacing the model indicated flow separation which limits the availability of the model to represent the flow profile in the downstream zone.

2. A quasi-orthogonal method of computation, which is more rapid than an iterative solution of the elliptic boundary value problem, appears best suited to ducted Coanda flows with low venting and large curvature. It requires approximate specification of streamline curvature decay with distance from the wall, and thus is best suited to cases in which the jet sheet is located close to the wall. It is desirable to extend the use of the method to non-vented Coanda flows.

3. Though the effects of streamline curvature on mixing length are known only for small curvature, and perhaps uncertain within a factor of 3, a simple correction for mixing length in terms of Richardson number appears to provide a reasonable estimate for jet curvatures d/R of the order of 0.02.

4. The flow model developed provides good agreement with secondary and primary mass flows measured with orifice plates. Integration of velocity profiles failed to provide satisfactory agreement with orifice measurements of mass flow apparently due to the effects of a high turbulence and high shear in the mixing zone on the stagnation pressure readings of a Kiel probe.

5. The flow model predictions were in good agreement with measured wall static pressures except in the immediate region of the nozzle apparently due to upstream boundary layer and nozzle thickness effects, and due to incipient flow separation in certain of the tests.

6. A sensitive measure of the degree of agreement between flow model and experimental results is the location of the maximum stagnation pressure line in these highly curved flows.

APPENDIX A

Sample Tabulation of Static Pressures (Tests 7 and 8)

Tap Position inch	Wall Static Pressure - inches of water gage			
	Run 7		Run 8	
	Top Wall	Bottom Wall	Top Wall	Bottom Wall
- 5.50	-	- 1.15	-	- 2.10
- 5.00	-	- 1.35	-	- 2.60
- 4.50	-11.60	- 1.50	-12.30	- 2.95
- 4.00	-12.90	- 1.80	-13.70	- 3.65
- 3.50	-15.05	- 2.00	-16.10	- 4.15
- 3.00	-17.45	- 2.05	-18.95	- 4.30
- 2.50	-19.10	- 2.12	-21.30	- 4.63
- 2.00	-19.90	- 2.20	-22.95	- 4.90
- 1.50	-20.15	- 2.50	-24.30	- 5.85
- 1.00	-19.85	- 2.90	-25.50	- 6.95
- 0.50	-21.75	- 3.35	-28.97	- 8.45
0.00	-26.93	- 4.00	-36.99	-10.45
0.50	-22.63	- 4.85	-33.76	-13.15
1.00	-18.35	- 5.60	-31.62	-15.80
1.50	-15.90	- 6.85	-30.80	-19.80
2.00	-15.20	- 7.85	-32.30	-23.30
2.50	-13.50	- 8.55	-31.96	-25.70
3.00	-11.05	- 8.50	-30.06	-26.59
3.50	-	- 8.90	-	-28.29
4.00	-10.90	- 8.65	-32.44	-29.17
4.50	-	- 8.40	-	-30.23
5.00	- 9.40	- 8.30	-32.30	-30.94
5.50	- 8.90	- 8.30	-32.16	-31.62
6.50	- 8.82	- 8.07	-33.15	-32.91
7.50	-10.25	- 9.25	-37.06	-35.43
8.50	- 9.80	- 9.35	-37.06	-36.04
10.00	-	- 9.55	-	-36.96
11.50	- 5.75	- 6.00	-31.48	-31.42
12.50	- 2.85	- 2.55	-26.59	-25.88
14.50	+ 2.85	+ 2.85	-17.85	-17.60
16.50	+ 7.00	+ 7.00	-	-11.18
18.50	+ 9.75	+ 9.75	- 7.65	- 7.05
20.50	+11.90	+11.90	- 4.10	- 3.75
22.50	+13.65	+13.65	- 1.95	- 1.30

—— denotes average pressure for 2 or 3 static taps located across width of test section (see Figs. 12 and 13).

APPENDIX B

Finite Difference Equations

This Appendix provides the detailed derivations of the finite difference equivalents of the momentum and energy conservation equations (5) and (7) respectively. For convenience the following definitions are introduced:

$$S = \rho u(\mu + \rho E)$$

and

$$S' = \rho u\left(\frac{\mu}{P_r} + \frac{\rho E}{P_{rt}}\right)$$

These definitions permit the momentum and energy equations to be expressed as:

$$u\,\frac{\partial u}{\partial X} = -\,\frac{1}{2\rho}\,\frac{dP}{dX} + u\,\frac{\partial}{\partial\psi}\left[S\,\frac{\partial u}{\partial\psi}\right] \tag{B-1}$$

$$u\,\frac{\partial T}{\partial X} = \frac{\gamma - 1}{2\rho\gamma}\,u\,\frac{dP}{dX} + \frac{\gamma - 1}{\gamma}\,uS\left[\frac{\partial u}{\partial\psi}\right]^2 + u\,\frac{\partial}{\partial\psi}\left[Q\,\frac{\partial T}{\partial\psi}\right] \tag{B-2}$$

Before approximating these equations with finite difference relations a system of grid lines parallel to the X and ψ axes must be introduced. As illustrated in Figure B-1, a nodal point coincides with each intersection of these lines. Lines parallel to the ψ axis are termed m-lines and those parallel to X axis n-lines. Each node is given a double subscript, the first being the number of the m-line passing through it, and the second the n-line number.

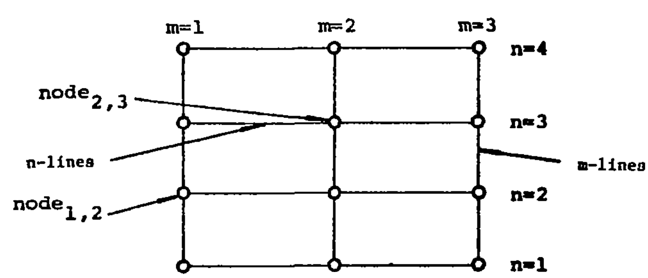


Figure B-1 Definition of Grid Lines for Finite Difference Solution

The values of the variables on the m = 1 line are the known initial conditions. The conservation equations express for each node on the m = 2 line its interrelation with other nodes on the m = 2 line and nodes on the m = 1 line. If m = 2 line nodes are only related to nodes which lie on the m = 1 line, the finite difference scheme is termed explicit. If an m = 2 node is also related to a number of other m = 2 nodes, the scheme is termed implicit (See Figure B-2).

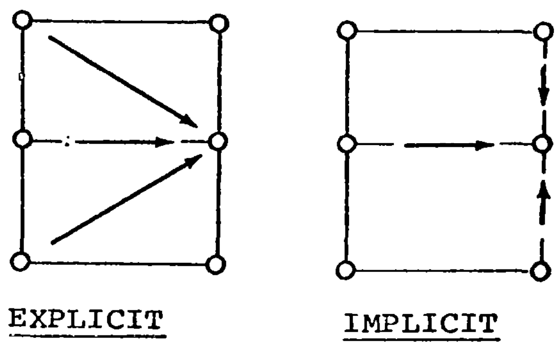


Figure B-2

Diagrams of Explicit and Implicit Solutions

The implicit form of finite difference schemes leads to a series of N simultaneous algebraic equations relating the known initial conditions on the m = 1 line and the unknown variables on each of the N nodes on the m = 2 line. After solution of these simultaneous equations, the variables on the m = 3 line are expressed in terms of the known values on the m = 2 line. Proceeding in this manner, a solution to the complete flow field is marched out. Although simpler to program, the explicit scheme shows unstable characteristics if the m-lines are widely spaced relative to the n-line spacing. Implicit schemes show much more stable characteristics and therefore allow much larger m-line spacings, thus reducing computation times. The computer procedure presented in this report employs a system of implicit finite difference approximations which are defined using the notation described in Figure B-3.

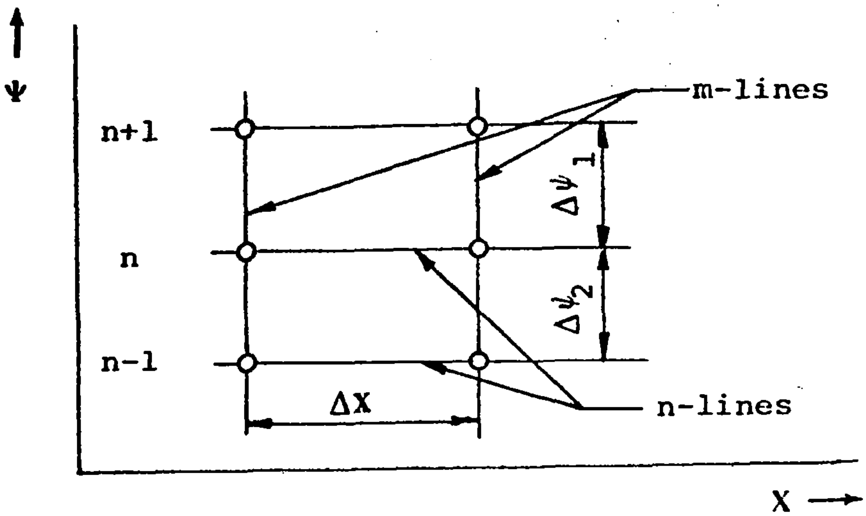


Figure B-3

Implicit Finite Difference Term Definition

The velocity at nodes n + 1 and n - 1 can be expressed in terms of a Taylor Series expanded about node n, on the same m-line,

$$u_{n+1} = u_n + \Delta\psi_1 \left.\frac{\partial u}{\partial \psi}\right|_n + \frac{(\Delta\psi_1)^2}{2} \left.\frac{\partial^2 u}{\partial \psi^2}\right|_n + \text{higher order terms} \qquad \text{(B-3)}$$

$$u_{n-1} = u_n - \Delta\psi_2 \left.\frac{\partial u}{\partial \psi}\right|_n + \frac{(\Delta\psi_2)^2}{2} \left.\frac{\partial^2 u}{\partial \psi^2}\right|_n + \text{higher order terms} \qquad \text{(B-4)}$$

Combining these equations to eliminate $\left.\frac{\partial^2 u}{\partial \psi^2}\right|_n$ yields,

$$\frac{(\Delta\psi_2)^2}{2} u_{n+1} - \frac{(\Delta\psi_1)^2}{2} u_{n-1} = \frac{u_n}{2} (\Delta\psi_2^2 - \Delta\psi_1^2) + \left.\frac{\partial u}{\partial \psi}\right|_n \frac{1}{2} (\Delta\psi_1\Delta\psi_2^2 + \Delta\psi_2\Delta\psi_1^2)$$

$$+ \text{higher order terms}$$

Neglecting terms of the order $(\Delta\psi)^3$ and higher, yields

$$\left.\frac{\partial u}{\partial \psi}\right|_n = \frac{\left(\frac{\Delta\psi_2}{\Delta\psi_1}\right) u_{n+1} - \left(\frac{\Delta\psi_1}{\Delta\psi_2}\right) u_{n-1} - \left(\frac{\Delta\psi_2}{\Delta\psi_1} - \frac{\Delta\psi_1}{\Delta\psi_2}\right) u_n}{\Delta\psi_2 + \Delta\psi_1}$$

Defining $S_5 = \dfrac{\Delta\psi_1}{\Delta\psi_2(\Delta\psi_2 + \Delta\psi_1)}$

and $S_4 = \dfrac{\Delta\psi_2}{\Delta\psi_1(\Delta\psi_2 + \Delta\psi_1)}$

yields

$$\left.\frac{\partial u}{\partial \psi}\right|_n = S_4(u_{n+1} - u_n) + S_5(u_n - u_{n-1}) \qquad \text{(B-5)}$$

Similarly,

$$\left.\frac{\partial T}{\partial \psi}\right|_n = S_4(T_{n+1} - T_n) + S_5(T_n - T_{n-1}) \qquad \text{(B-6)}$$

The second derivative term in the momentum equation is approximated using the following Taylor Series expansions,

$$\left(S\,\frac{\partial u}{\partial \psi}\right)_{n+1/2} = S\left(\frac{\partial u}{\partial \psi}\right)_n + \frac{\Delta\psi_1}{2}\,\frac{\partial}{\partial \psi}\left[\left(S\,\frac{\partial u}{\partial \psi}\right)_n\right] + \frac{\Delta\psi_1^{\,2}}{4}\,\frac{\partial^2}{\partial \psi^2}\left[\left(S\,\frac{\partial u}{\partial \psi}\right)_n\right]$$

$$+ \text{ higher order terms} \qquad \text{(B-7)}$$

$$\left(S\,\frac{\partial u}{\partial \psi}\right)_{n-1/2} = \left(S\,\frac{\partial u}{\partial \psi}\right)_n - \frac{\Delta\psi_2}{2}\,\frac{\partial}{\partial \psi}\left[\left(S\,\frac{\partial u}{\partial \psi}\right)_n\right] + \frac{\Delta\psi_1^{\,2}}{4}\,\frac{\partial^2}{\Delta \psi^2}\left[\left(S\,\frac{\partial u}{\partial \psi}\right)_n\right]$$

$$+ \text{ higher order terms} \qquad \text{(B-8)}$$

Neglecting terms of the order of $\frac{\Delta\psi^2}{4}$ and higher yields,

$$\frac{\partial}{\partial\psi}\left(S\frac{\partial u}{\partial\psi}\right)_n = \left[\left(S\frac{\partial u}{\partial\psi}\right)_{n+1/2} - \left(S\frac{\partial u}{\partial\psi}\right)_{n-1/2}\right]\left[\frac{2}{\Delta\psi_1 + \Delta\psi_2}\right]$$

$$= \frac{1}{\Delta\psi_1 + \Delta\psi_2}\left[\frac{(S_{n+1} + S_n)(u_{n+1} - u_n)}{\Delta\psi_1} - \frac{(S_n + S_{n-1})(u_n - u_{n-1})}{\Delta\psi_2}\right] \tag{B-9}$$

Similarly,

$$\frac{\partial}{\partial\psi}\left[S'\frac{\partial T}{\partial\psi}\right]_n = \frac{1}{\Delta\psi_1 + \Delta\psi_2}\left[\frac{(S'_{n+1} + S'_n)(T_{n+1} - T_n)}{\Delta\psi_1} - \frac{(S'_n + S'_{n-1})(T_n - T_{n-1})}{\Delta\psi_2}\right] \tag{B-10}$$

The velocity at a node located at the intersection of the downstream m-line and any n-line $u_{2,n}$ can be expressed in terms of the following Taylor Series,

$$u_{2,n} = u_{1,n} + \left.\frac{\partial u}{\partial X}\right|_n \Delta X + \left.\frac{\partial^2 u}{\partial X^2}\right|_n (\Delta X)^2 + \text{higher order terms} \tag{B-11}$$

Use of the boundary layer equations implies that gradients in the X- direction are much smaller than those in the ψ- direction. Therefore it is permissible to use a simpler approximation of the X- direction derivatives.

Neglecting terms of $(\Delta X)^2$ and higher yields,

$$\left.\frac{\partial u}{\partial X}\right|_n = \frac{u_{2,n} - u_{1,n}}{\Delta X} \tag{B-12}$$

This approximation is termed "backward-difference".

Similarly,

$$\left.\frac{\partial T}{\partial X}\right|_n = \frac{T_{2,\,n} - T_{1,\,n}}{\Delta X} \tag{B-13}$$

The only terms in the energy and momentum equations which cannot be approximated using the preceding equations are those containing the pressure gradient dP/dX. Assuming this gradient varies linearly throughout the ΔX interval yields:

$$\frac{dP}{dX} = \frac{1}{2}\left(\left.\frac{dP}{dX}\right|_{m = 1} + \left.\frac{dP}{dX}\right|_{m = 2}\right) \tag{B-14}$$

Momentum Equation

Combining equations (B-1), (B-9), (B-12) and (B-14) yields:

$$u_{1,\,n}\frac{(u_{2,\,n} - u_{1,\,n})}{\Delta X} = -\frac{1}{4\rho_{1,\,n}}\left[\left.\frac{dP}{dX}\right|_{m = 1} + \left.\frac{dP}{dX}\right|_{m = 2}\right] + \frac{u_{1,\,n}}{2\psi_n}\left(\frac{1}{\Delta\psi_1 + \Delta\psi_2}\right)$$

$$\left[\frac{(S_{n + 1} + S_n)(u_{2,\,n + 1} - u_{2,\,n})}{\Delta\psi_1} - \frac{(S_n + S_{n - 1})(u_{2,\,n} - u_{2,\,n - 1})}{\Delta\psi_2}\right] \tag{B-15}$$

This equation can be expressed in the form

$$A_{n - 1}u_{2,\,n} + B_{n - 1}u_{2,\,n + 1} + C_{n - 1}u_{2,\,n - 1} = D_{n - 1} \tag{B-16}$$

in which the coefficients are defined by equations (23) through (28) of the main text.

Energy Equation

Combining equations (B-2), (B-5), (B-10), (B-13) and (B-14) yields

$$\frac{u_{1,n}(T_{2,n} - T_{1,n})}{\Delta X} = \frac{\gamma - 1}{\gamma} u_{1,n} S_{1,n} \left[S_4(u_{2,n+1} - u_{2,n}) + S_5(u_{2,n} - u_{2,n-1}) \right]^2 + u_{1,n} \left[\frac{1}{\Delta\psi_1 + \Delta\psi_2} \right] \left[\frac{(S'_{n+1} + S'_n)(T_{2,n+1} - T_{2,n})}{\Delta\psi_1} - \frac{(S'_n + S'_{n-1})(T_{2,n} - T_{2,n-1})}{\Delta\psi_2} \right] + \left(\frac{\frac{\gamma - 1}{\gamma} u_{1,n}}{4\rho_{1,n}} \right) \left[\left.\frac{dP}{dX}\right|_{m=1} + \left.\frac{dP}{dX}\right|_{m=2} \right] \tag{B-17}$$

This equation can be expressed in the form:

$$A_{n-1} \cdot T_{2,n} + B_{n-1} \cdot T_{2,n+1} + C_{n-1} \cdot T_{2,n-1} = D_{n-1} \tag{B-18}$$

in which the coefficients are defined by equations (31) through (36) of the main text.

APPENDIX C

Solution Procedure

The calculation procedure starts at the upstream flow boundary, where the values of all flow variables must be known or assumed. Specification of the velocity and temperature distribution, dimensionless eddy viscosity, duct and nozzle inlet dimensions, and working fluid, defines all initial conditions.

The known initial conditions, m = 1 line, are related to the unknown conditions, m = 2 line, by the previously derived equations, and assumed boundary conditions. These inter-relations form a set of n - 2 simultaneous algebraic equations, where n is the number of n-lines, and the equations are shown in Appendix B. The resultant matrix of coefficients is tridiagonal in form except for the initial and final rows which only contain two terms. Rapid, exact solutions to this type of matrix are obtained using the Thomas Algorithm, a successive elimination technique, which is described in this Appendix.

The solution for the variables on the m = 2 line is iterative, because of the presence of the unknown pressure in the momentum equation. The procedure adopted was to estimate the pressure gradient, and solve the equations, using the algorithm. The equations automatically satisfy conservation of mass, momentum, and energy, but only one pressure gradient yields the correct wall geometry. The duct dimension corresponding to the estimated pressure gradient was calculated from the m = 2 line variables. The pressure gradient was then incremented by a small percentage of its initial estimated value, and the calculation process repeated for a new duct dimension. A third estimate of the pressure gradient was obtained by interpolation between the two calculated, and the actual duct dimension. In almost all the calculations performed to date, this value has been acceptably close, within 0.001%, to the actual duct dimension. If this criterion is not met, a further iteration is applied, and a fourth solution obtained.

The now known variables on the m = 2 line become the new m = 1 line variables and the procedure is repeated for another set of m = 2 line variables. Thus a solution to the complete flow field is marched out.

The difference form of the momentum and energy equation is:

$$A_{n-1}X_n + B_{n-1}X_{n+1} + C_{n-1}X_{n-1} = D_{n-1} \tag{C-1}$$

where X is either u or T. If the number of n-lines is n, there are n - 2 equations of the form (1) and two equations expressing the boundary conditions. The first and the last equations represent the boundary conditions, which are:

$$u_1 = u_n = 0 \tag{C-2}$$

and

$$\left.\frac{\partial T}{\partial \psi}\right|_1 = \left.\frac{\partial T}{\partial \psi}\right|_n = 0 \tag{C-3}$$

Equation (C-2) can be written in terms of X as follows:

$$X_1 = X_2 = 0 \tag{C-4}$$

Equation (C-3) correspondingly becomes:

$$X_1 = X_2 \tag{C-5}$$

and

$$X_{n-1} = X_n \tag{C-6}$$

Equations (C-4), (C-5) and (C-6) can be written in terms of X as follows:

$$X_n = KX_{n-1} \tag{C-7}$$

$$X_1 = KX_2 \tag{C-8}$$

where K is 0 for the momentum equation and unity for the energy equation. Thus, the matrix form of the equation (C-1) is shown on the following page (Table C-1).

Table C-1

Matrix Form of Equation C-1 Designated as Equation C-1

$$
\begin{bmatrix}
1 & -K & 0 & 0 & 0 & - & 0 & 0 & 0 & - & 0 & 0 & 0 \\
C_1 & A_1 & B_1 & 0 & 0 & - & 0 & 0 & 0 & - & 0 & 0 & 0 \\
0 & C_2 & A_2 & B_2 & 0 & - & 0 & 0 & 0 & - & 0 & 0 & 0 \\
0 & 0 & C_3 & A_3 & B_3 & - & 0 & 0 & 0 & - & 0 & 0 & 0 \\
- & - & - & - & - & - & - & - & - & - & - & - & - \\
0 & 0 & 0 & 0 & 0 & - & C_{n-1} & A_{n-1} & B_{n-1} & - & 0 & 0 & 0 \\
- & - & - & - & - & - & - & - & - & - & - & - & - \\
0 & 0 & 0 & 0 & 0 & - & 0 & 0 & 0 & - & C_{n-2} & A_{n-2} & B_{n-2} \\
0 & 0 & 0 & 0 & 0 & - & 0 & 0 & 0 & - & 0 & -K & 1
\end{bmatrix}
\begin{bmatrix}
X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_{n-1} \\ X_n \\ X_{n+1} \\ X_{n-1} \\ X_n
\end{bmatrix}
=
\begin{bmatrix}
0 \\ D_1 \\ D_2 \\ D_3 \\ D_{n-2} \\ D_{n-1} \\ D_n \\ D_{n-2} \\ 0
\end{bmatrix}
$$

The second equation is:

$$C_1X_1 + A_1X_2 + B_1X_3 = D_1 \tag{C-9}$$

Substituting equation (C-4) into this equation yields:

$$A'_1X_2 + B_1X_3 = D_1 \tag{C-10}$$

where $A'_1 = C_1 + A_1$

The $n^{th} - 1$ equation is:

$$C_{n-2}X_{n-2} + A_{n-2}X_{n-1} + B_{n-2}X_n = D_{n-2} \tag{C-11}$$

Substituting equation (C-7) into this equation yields:

$$C_{n-2}X_{n-2} + A'_{n-2}X_{n-1} = D_{n-2} \tag{C-12}$$

where $A'_{n-2} = A_{n-2} + KB_{n-2}$

Thus the n equations (C-8) can be reduced to the n - 2 equations shown on Table C-2.

The Thomas Algorithm

Starting with the first equation, X_2 can be expressed in terms of X_3. The second equation gives X_3 in terms of X_4. Continuing through all the equations until the n^{th} -3 equation gives X_{n-2} in terms of X_{n-1}. Combining this with the last equation gives X_{n-1}. Working backwards through the equations then allows the remaining unknowns to be found. This procedure is most easily applied by defining the following:

$$W_1 = A'_1 \qquad g_1 = \frac{D_1}{W_1}$$

$$Q_{n-1} = \frac{B_{n-1}}{W_{n-1}} \qquad n = 2, 3 --- (n-2) \tag{C-14}$$

$$W_n = A_n - C_nQ_{n-1} \qquad n = 2, 3 --- (n-2)$$

$$g_n = D_n - \frac{C_ng_{n-1}}{W_n} \qquad n = 2, 3 --- (n-2)$$

Table C-2

Matrix Form of Equation C-8 with Simplified Terms Designated as Equation C-13

$$
\begin{bmatrix}
A_1 & B_1 & 0 & 0 & - & 0 & 0 & 0 & - & 0 & 0 & 0 \\
C_2 & A_2 & B_2 & 0 & - & 0 & 0 & 0 & - & 0 & 0 & 0 \\
0 & C_3 & A_3 & B_3 & - & 0 & 0 & 0 & - & 0 & 0 & 0 \\
- & - & - & - & - & - & - & - & - & - & - & - \\
0 & 0 & 0 & 0 & - & C_{n-1} & A_{n-1} & B_{n-1} & - & 0 & 0 & 0 \\
- & - & - & - & - & - & - & - & - & - & - & - \\
0 & 0 & 0 & 0 & - & 0 & 0 & 0 & - & C_{n-3} & A_{n-3} & B_{n-3} \\
0 & 0 & 0 & 0 & - & 0 & 0 & 0 & - & 0 & C_{n-2} & A'_{n-2}
\end{bmatrix}
\begin{bmatrix}
X_2 \\ X_3 \\ X_4 \\ X_{n-1} \\ X_n \\ X_{n+1} \\ X_{n-2} \\ X_{n-1}
\end{bmatrix}
=
\begin{bmatrix}
D_1 \\ D_2 \\ D_3 \\ D_{n-2} \\ D_{n-1} \\ D_n \\ D_{n-3} \\ D_{n-2}
\end{bmatrix}
$$

Equations (C-13) then reduce to:

$$X_{n-1} = g_{n-2} \text{ and } X_n = g_{n-1} - Q_{n-1}X_{n+1} \quad n = (n-2), (n-3), \text{---} 2 \tag{C-15}$$

If the values of W, Q and g are calculated in order of increasing n using euqations (C-14), then equations (C-15) can be used to calculate the values of X in order of decreasing X starting with X_{n-1}. To clarify this procedure, the method is now used to solve the following four simultaneous equations:

$$\begin{bmatrix} A'_1 & B_1 & 0 & 0 \\ C_2 & A_2 & B_2 & 0 \\ 0 & C_3 & A_3 & B_3 \\ 0 & 0 & C_4 & A'_4 \end{bmatrix} \begin{bmatrix} X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} \qquad \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix}$$

$$A'_1X_2 + B_1X_3 = D_1$$

$$W_1 = A'_1$$

$$Q_1 = \frac{B_1}{W_1}$$

$$g_1 = \frac{D_1}{W_1}$$

hence $X_2 = g_1 - Q_1X_3$

$$A_2X_3 + B_2X_4 + C_2X_2 = D_2$$

$$W_2 = A_2 - C_2Q_1$$

$$Q_2 = \frac{B_2}{W_2}$$

$$g_2 = \frac{D_2 - C_2g_1}{W_1}$$

hence $X_3 = g_2 - X_4Q_2$ (C-16)

$$A_3X_4 + B_3X_5 + C_3X_3 = D_3$$

$$W_2 = A_3 - C_3Q_2$$

$$Q_3 = \frac{B_3}{W_3}$$

$$g_3 = \frac{D_3 - C_3g_2}{W_3}$$

hence $X_4 = g_3 - Q_3X_5$ (C-17)

$$A'_4X_5 + C_4X_4 = D_4$$

$$W_4 = A_4 - C_4Q_3$$

$$g_4 = \frac{D_4 - C_4g_3}{W_4}$$

hence $X_5 = g_4$ (C-18)

Substituting in equation (C-16) yields X_3. Equations (C-17) and (C-18) are special forms of equations (C-15) for n = 6 and n = 4.

Appendix D

COMPUTER PROGRAM

The computer program is designed to analyze the flow in a duct where geometry is shown in Figure D-1.

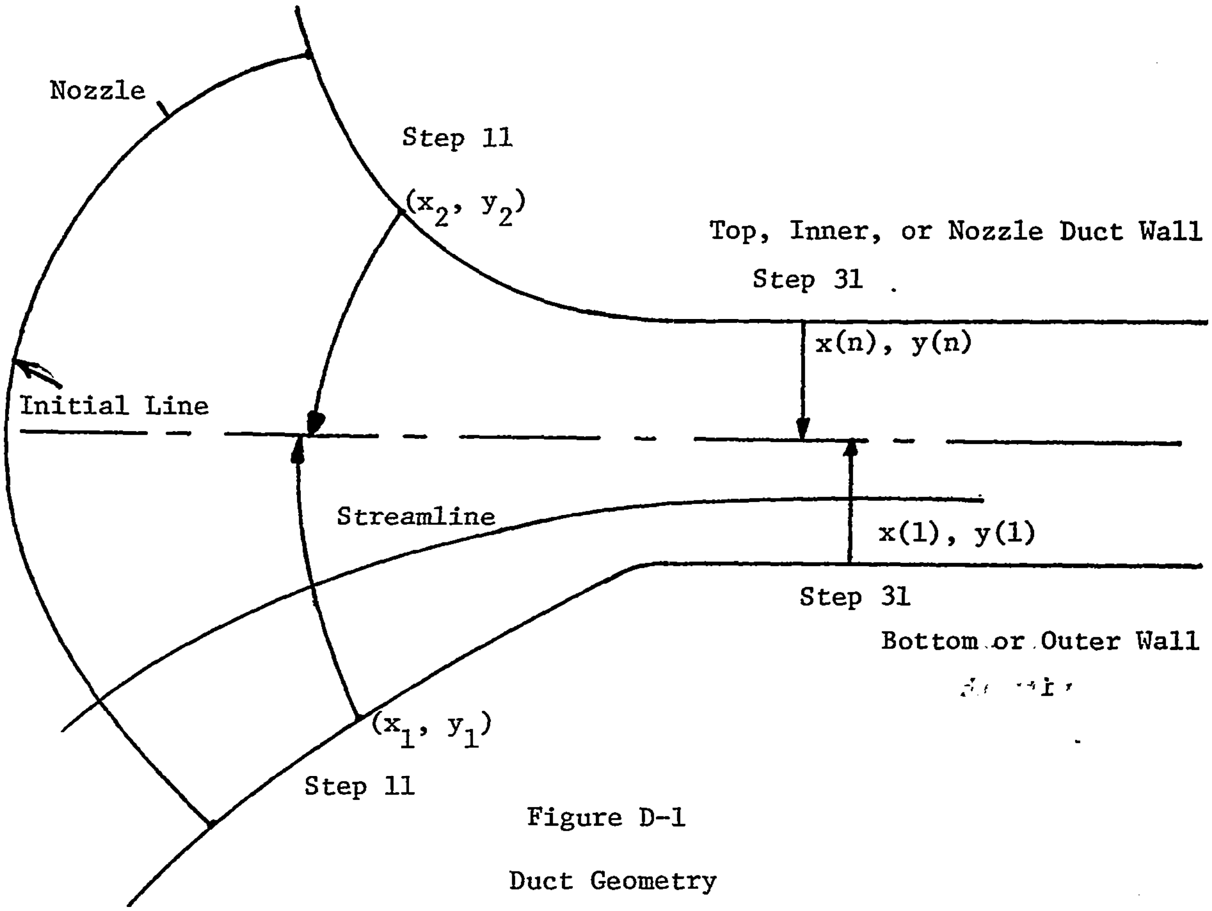


Figure D-1

Duct Geometry

The computation proceeds from the initial line by moving along the lower and upper wall a specified distance and then defining two arcs from the new wall locations to the duct midpoint. Since only a rough approximation is available for the wall slopes the two arcs do not necessarily meet resulting in the wall points becoming slightly unsynchronized as the computation proceeds.

The program organization consists of a main program (NAS) and sixteen subroutines and function subroutines. The functions of the main program and subroutines are:

PROGRAM NAS

The main program is divided as follows:

Input and Data Initialization: Cards $NA10to $NA2200

This section initializes the constants of the program, reads and prints the computation conditions and duct geometry, and puts this data in nondimensional form.

Initial Conditions: Cards $NA2210 to $NA2810

The subroutine INCOND is called to define the starting conditions. The initial flow conditions are then put in dimensional form and printed.

Main Body of Program: Cards $NA2820 to $NA4810

The computation proceeds down the duct in a sequence of steps. Values of pressure, temperature, velocity, density etc. are computed which are consistent with the previous step values and the geometry of the duct. The process stops when the end of the duct is reached.

Eddy Viscosity: Card $NA3010

The dimensionless eddy viscosity is calculated in subroutine EDDY using data from the preceding step.

Streamline Step Size: Cards $NA3100 to $NA3400

An appropriate step size along the duct is determined for the streamline of maximum velocity. Consistent step sizes are then determined for all other streamlines.

Pressure Gradient Approximation: Cards $NA3730 to $NA4540

The pressure gradient is determined by selecting a value such that the computed duct widths minus the actual duct width equals 0 ± .0001.

Flow Boundaries: Cards $NA4550 to $NA4700

At each step the boundaries of the flow regions are checked to determine when shear layers vanish.

Output Section: Cards $NA4820 to $NA5750

The flow variables are presented in dimensional form at preselected intervals.

Figure D-2 is a flow chart of the main progran NAS.

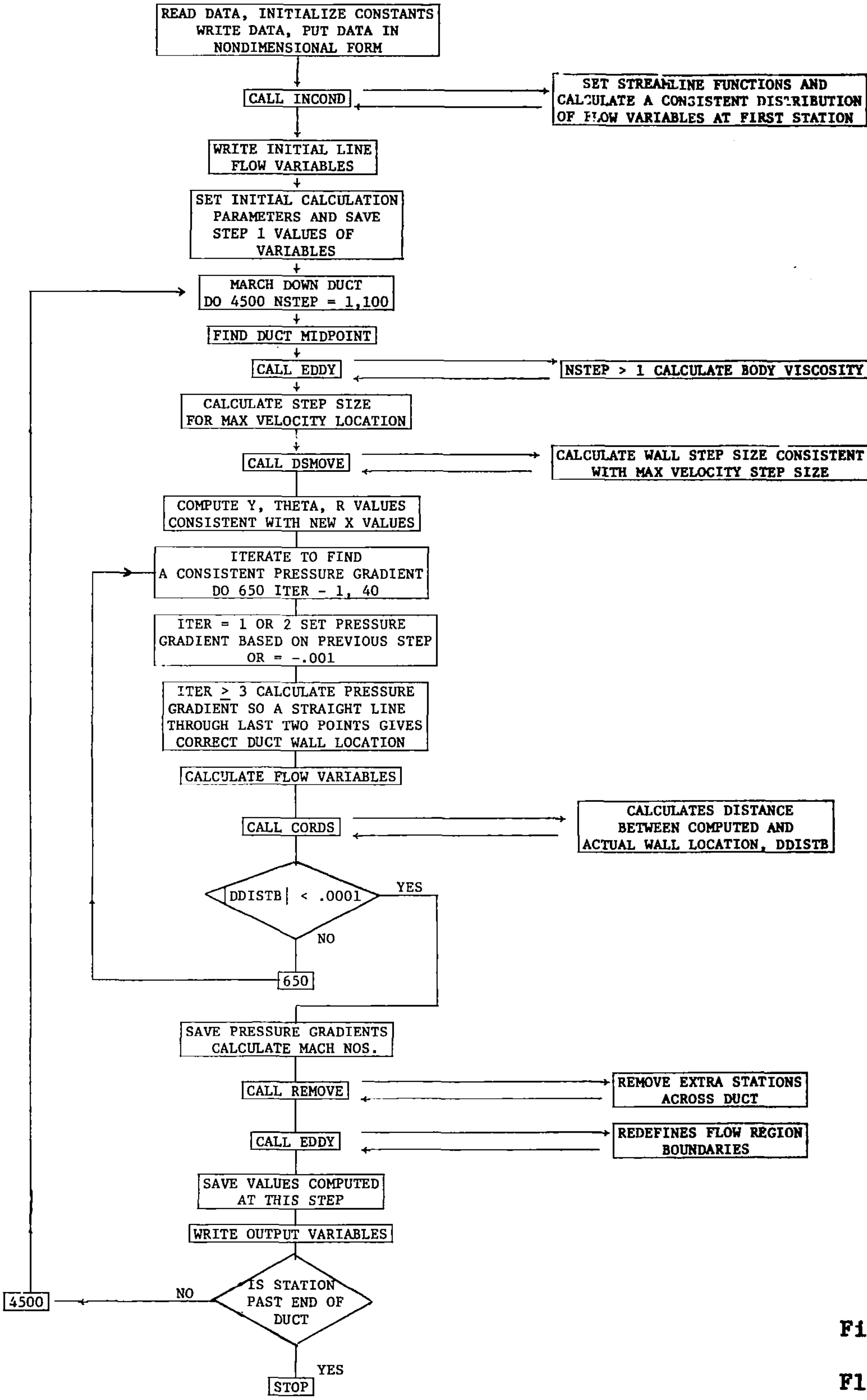


Figure D-2

Flow Chart

SUBROUTINES

INCOND

This subroutine contains the computation which defines the flow conditions at the initial station (Fig. D-3). The steps used in the process are:

1. Points on the boundaries (x_1, y_1), (x_2, y_2) are defined such that two circular arcs are normal to the wall at (x_1, y_1) and (x_2, y_2) and pass through the nozzle (x_{noz}, y_{noz}).

2. A flow split is determined.

3. The subroutine OMGSET is called to set the streamline locations.

4. The subroutine TMPSET is called to determine the temperature distribution.

5. The remaining flow variables are calculated.

6. The location of the nozzle streamline checked to see if it is within tolerance of its specified location. If it is then the computation is returned, if not a new flow split is determined and the calculation is repeated.

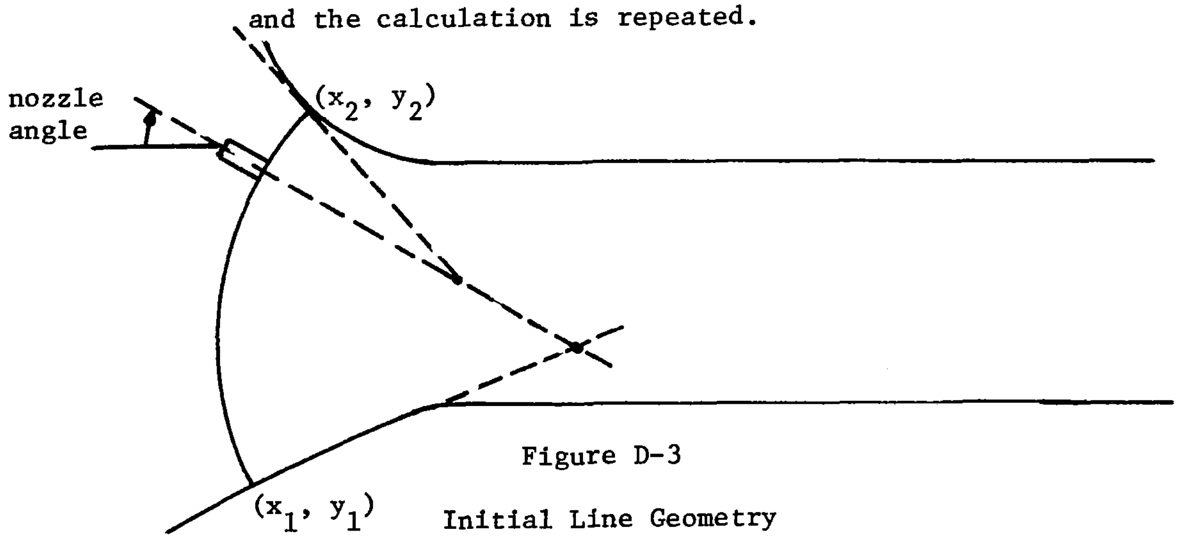


Figure D-3

Initial Line Geometry

OMGSET

This subroutine sets the distribution of streamlines. The flow is divided into regions of width DOMG1, DOMG2, DOMG3 (Fig. D-4 and D-5). Given an initial spacing at each flow edge (DMWALL and DMJET) and a specific number of points (KWALL, KJET) the subroutine fills in the spaces close to the edges.

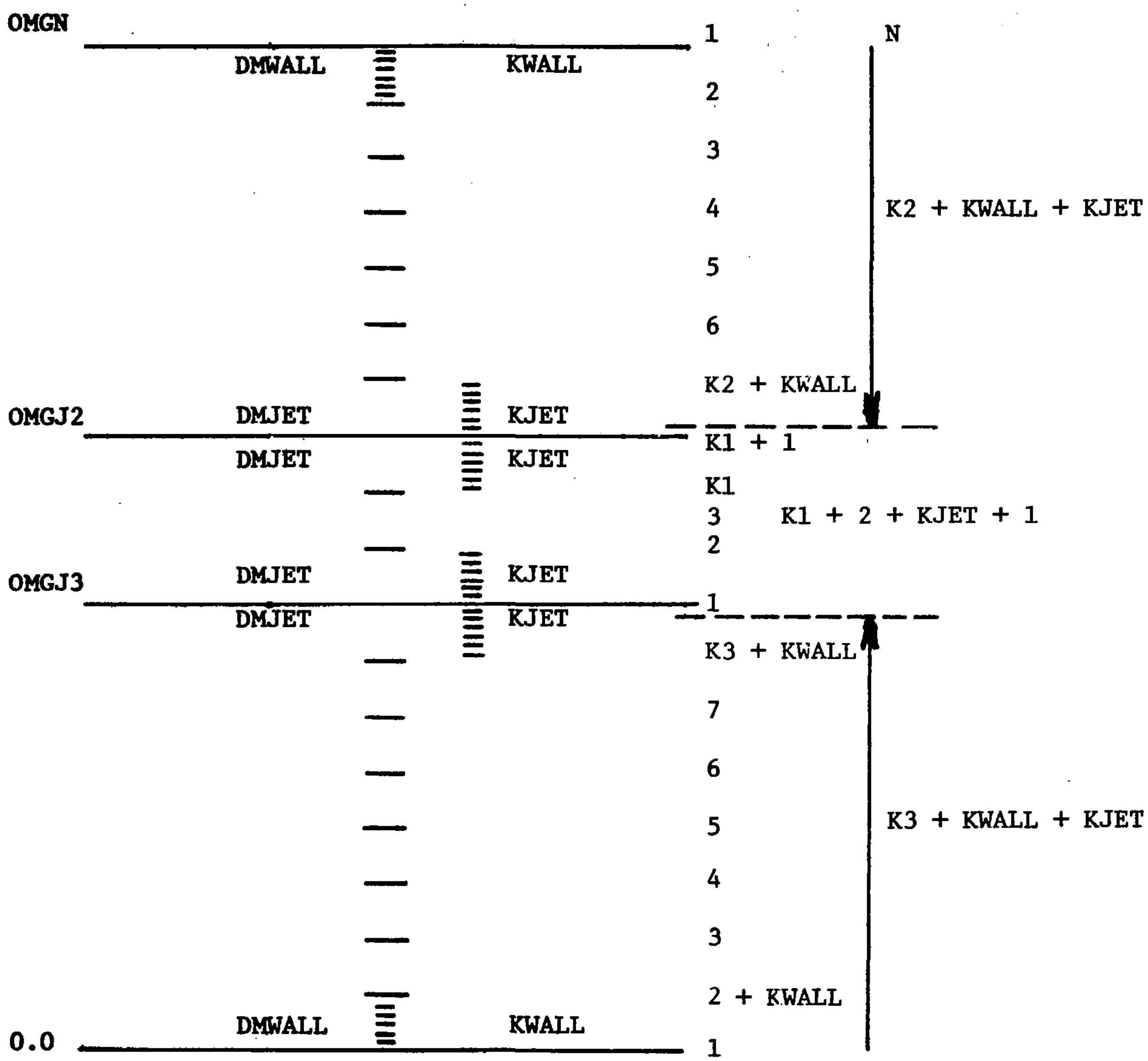


Figure D-4

Streamline Locations

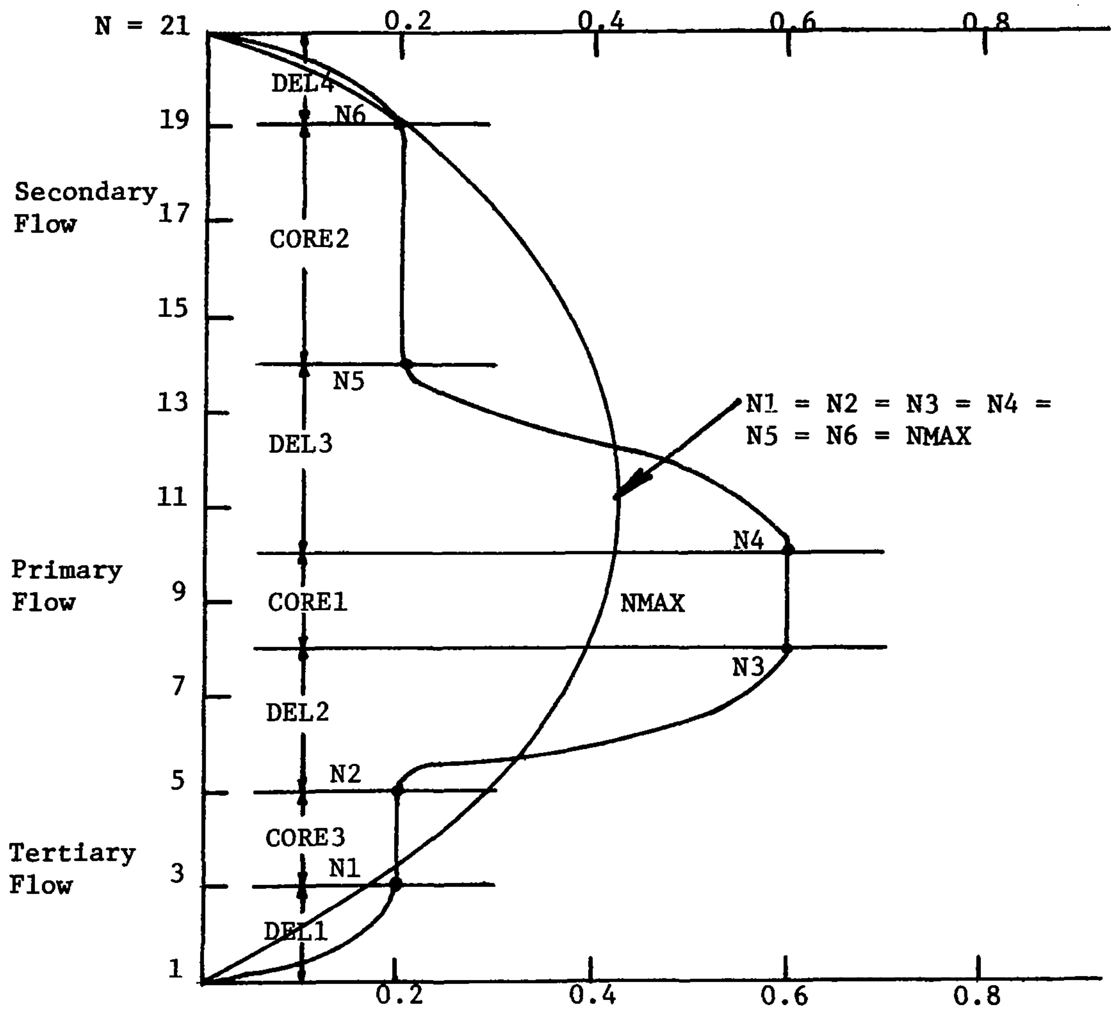


Figure D-5

Flow Structure

TMPSET

An initial value of static temperature at the nozzle, TS(NMID), is selected. Values of the temperature are then computed at the outer wall consistent with the wall curvature. The distance between the wall and nozzle consistent with TS(NM1D) is returned to the calling program.

WALLS

Wall curvatures are calculated from the wall geometry data. This data is then used to interpolate values of wall location slope and curvature for specific axial locations. The computer calculates curvatures assuming the data is segmented with at least three points per segment. When five or more points are available a least squares parabola using the data points and the two points to either side is used to smooth the data. Subsequent interpolations are parabolic.

LSQ

This subroutine calculates the least squares parabolas for walls.

EDDY

This subroutine contains the eddy viscosity calculation.

DSMOVE

This subroutine contains the calculation of the distance moved along the walls in each step.

SDWE

This subroutine contains the calculation of temperatures and velocities at a computing station.

CALC

This is a simultaneous linear equation solution subroutine.

CORDS

The distance between streamlines and the x, y coordinates of the streamlines are computed in this subroutine. The value DDIST, the difference between the y coordinates of the two arcs at the middle duct point, is returned.

LOOK

The algorithm identifies an edge of a core region and then checks to see if a core region that existed in the previous step has disappeared. If this has occurred that core width is set to 0.0. Core regions may disappear in any order.

REMOVE

This subroutine is used to remove grid points as the computation proceeds in order to reduce computing time. This is accomplished by every tenth step scanning the velocity array to see if the velocity gradients in the shear regions are less than a tolerance.

ARCDIS

This subroutine defines wall coordinates such that the arc is normal to the wall and nozzle.

FUNCTION SUBROUTINES

CENTRE

This is used in conjunction with ARCDIS. Given a nozzle and wall location it projects the two tangents, finds their intersection and returns the difference in length.

CURVDS

Used in DSMOVE to calculate distances along an arc.

SERIES

Used in OMGSET to determine streamline spacing in the boundary and shear layers.

INPUT DATA

Card 1 - Format I1,18A4

NUNIT - units indicator

NUNIT = 0 English units

NUNIT ≠ 0 S.I. units

TITLE(I) - title, printed at start of first output page

Card 2 - Format 6F10.0

P01 - primary stagnation pressure - psia or pascal

T01 - primary stagnation temperature - Rankine or Kelvin

P02 - secondary stagnation pressure - psia or pascal

T02 - secondary stagnation temperature - Rankine or Kelvin

MASS1 - primary mass flow rate - lbm/sec-in or kg/sec-m

MASS2 - secondary + tertiary flow - lbm/sec-in or kg /sec-m

Card 3 - Format 5I5

K1 - no. of grid points in primary flow (even)

K2 - no. of grid points in secondary flow (even)

K3 - no. of grid points in tertiary flow (even)

NPCYCL - print cycle, (e.g. NPCYCL = 10 causes print every ten steps)

NQUICK - 0 = full print; 1 = partial printout

Card 4 - Format 4F10.0

XNOZ - x coordinate of nozzle center - in. or meters

YNOZ - y coordinate of nozzle center - in. or meters

TNOZ - nozzle angle - degrees

N - nozzle slot width - in. or meters

Card 5 - Format 11I5

MST - number of segments used to describe lower wall geometry,

$10 \geq MST \geq 1$

NDI(I, 1) - MST values of the data point number of the segment end point. NDI(MST,1) = NPAIR1, the total number of points needed to describe the wall.

$NDI(I + 1,1) - NDI(I, 1) \geq 2$

Cards 6 - Format 2F10.0 - NPAIR1 Cards

XW(I, 1) - x lower wall coordinate

YW(I, 1) - y lower wall coordinate

Card 7 - Format 11I5

MSS - number of segments used to describe upper wall geometry

$10 \geq MSS \geq 1$

NDI(I,2) - MSS values of the data point number of the segment end point, NDI(MSS, 1) = NPAIR2 the total number of points needed to describe the wall.

$NDI(I + 1,2) - NDI(I, 2) \geq 2$

Cards 8 - Format 2F10.0 - NPAIR2 Cards

XW(I, 2) - x upper wall coordinate

YW(I, 2) - y upper wall coordinate

Card 9 - Format I5,10F5.0

NFULL - number of values of x for which full output is required

XPRØF(I) - NFULL values of x for which full output is required

The input data K1, K2, and K3 (Card 3) denote the number of grid points in the primary, secondary and tertiary flow. These are arbitrary numbers chosen so as to give the desired spacing of output data in the three regions. A total K1 + K2 + K3 of between 20 and 30 should give good results.

The data numbers MST, MSS (Cards 5 and 7) denote the number of segments needed to describe the wall geometry. For walls with continuous curvature values, a value of 1 is sufficient. Values of MST and MSS greater than 1 allow the user to describe walls with discontinuous slopes and curvatures. The data NDI(I,2) and NDI(I,1) are the data point numbers at the boundary segment ends.

A sample of input data is shown in Table D-1.

Table D-1

Sample of Input Data

```
1       TEST CASE  5   METRIC
253934.   322.      102249.   301.       1.696      7.393
    8    10   10     5    0
-.07366   .04023     -35.        .0030861
    6    12   16    26    31    35    43
-.1524    -.10396
-.1511    -.1024
-.1499    -.10033
-.1397    -.08694
-.1270    -.07450
-.1143    -.06535
-.1016    -.05837
-.0889    -.05339
-.0762    -.05004
-.0635    -.04712
-.0597    -.04678
-.0587    -.04671
-.0508    -.04517
-.0254    -.04023
0.0000    -.03523
.01905    -.03160
.02159    -.03115
.02032    -.03137
.02549    -.03048
.03810    -.02870
.05080    -.02736
.06350    -.02647
.07620    -.02596
.08636    -.02564
.08763    -.02560
.08890    -.02558
.10160    -.02536
.12700    -.02492
.15240    -.02448
.17780    -.02404
.19050    -.02383
.20320    -.02383
.22860    -.02383
.25400    -.02383
.26670    -.02383
.27940    -.02449
.30480    -.02582
.35560    -.02848
.40640    -.03115
.45720    -.03381
.50800    -.03649
.55880    -.03913
.58420    -.04038
```

Table D-1 (Concluded)

5 16 26 31 35 43

-.0845	.156410
-.0838	.143230
-.0826	.134200
-.0794	.120680
-.0762	.111240
-.0699	.097190
-.0635	.086450
-.0572	.077670
-.0508	.070240
-.0381	.058260
-.0254	.049090
-.0127	.042030
0.0000	.036730
.01270	.032960
.01588	.032240
.01905	.03160
.02032	.03137
.02159	.03115
.02549	.03048
.03810	.02870
.05080	.02736
.06350	.02647
.07620	.02596
.08636	.02564
.08763	.02560
.08890	.02558
.10160	.02536
.12700	.02492
.15240	.02448
.17780	.02404
.19050	.02383
.20320	.02383
.22860	.02383
.25400	.02383
.26670	.02383
.27940	.02449
.30480	.02582
.35560	.02848
.40640	.03115
.45720	.03381
.50800	.03649
.55880	.03913
.58420	.04038

1 .038

OUTPUT DATA

The program printed output consists of:

1. Title

2. Input flow and geometry (Cards 2, 3, and 4)

3. Wall Geometry

   X, Y - wall coordinates (smoothed)
   CURV - negative of wall curvature

4. Values of X at which full output is required (values of XPROF(I) from Card 9)

5. Flow split

   SPLIT - ratio of tertiary flow to the sum of secondary and tertiary flow

6. Initial conditions along the computing station through the nozzle

ISENTROPIC NOZZLE THRUST PER UNIT WIDTH - lb/in or N/m

AMASS1, AMASS2, AMASS3 - primary, secondary and tertiary flow rates - lb/s/in or kg/s/m

J - station number

X, Y - coordinates of computing station streamline intersection - in or m

DN - distance from lower wall - in or m

THETA - streamline angle - degrees

K - negative of streamline curvature - 1/in or 1/m

OMG - streamline function

PO, PS - total and static pressure - psi or pascal

TS - static temperature - degrees Rankine or Kelvin

RHO - density - lb/f**3 or kg/m**3

U - speed - f/s or m/s

M - Mach number

7. Flow description downstream - partial output

NSTEP - step number

XI(XO) - coordinate along lower (upper) wall - in or m

PI(PO) - static pressure at lower (upper) wall - in of H_2O or pascal

USTARI(USTARØ) - friction velocity at lower (upper) wall - f/s or m/s

KI(KO) - negative of wall curvature at XI(XO) - 1/in or 1/m

RNI(RNØ) - Richardson number at lower (upper) wall

UMAX - maximum velocity, f/s or m/s

NUMBER OF ITERATIONS

DELTA X - step size at lower wall - in or m

DELTASI(DELTASO) - distance increment of lower (upper) wall - in or m

SI(SO) - cumulative distance along lower (upper) wall - in or m

SHRIN(SHROR) - shear stress at lower (upper) wall - psi or pascal

TOTAL AXIAL MOMENTUM PER UNIT WIDTH - lbf/in or N/m

THRUST RATIO -

8. Flow description downstream - full output
Same as partial output plus

N1 ... N6 - streamline numbers at edges of boundary and shear layers

CORE1, CORE2, CORE3 - widths of primary, secondary, and tertiary regions - in or m

DEL1, DEL2, DEL3, DEL4 - widths of boundary and shear layers - in or m

J - streamline number

1/R - negative of curvature - 1/in or 1/m

THETA - angle of streamline - degrees

X, Y - computing mode location - in or m

YREL - relative y coordinate

UREL - relative velocity (UMAX is normalizing quantity)

POREL - relative total pressure

PS - static pressure - psia or pascal

E - Eddy viscosity

TOTEMP - total temperature - degrees Rankine or Kelvin

9. In addition several warnings may be printed.

NO CONVERGENCE, DPDSA = ... DPDSB = ... DDISTB ... STEP 4

Convergence was not achieved in establishing the pressure gradient. The criteria for convergence is that |DDISTB| the distance between the computed and actual will location be <.0001. DPDSA and DPDSB are pressure gradient increments.

NEGATIVE SHEAR STRESS AT THIS STATION. THIS INDICATES POSSIBLE SEPARATION. SUBSEQUENT RESULTS SHOULD BE USED WITH CAUTION.

The message is self-explanatory. When this occurs the calculation may be unstable and may stop for a variety of reasons. When this occurs the full output at that station is printed.

Figure D-6

COMPUTER PROGRAM LISTING

```
      PROGRAM NAS(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C     THIS PROGRAM CALCULATES VENTED COANDA JET IN A CURVED DUCT
C        BY A FINITE-DIFFERENCE METHOD
C
C        THE PROGRAM CALCULATES THE SPLIT BETWEEN SECONDARY
C          AND TERTIARY MASS FLOW
C
C     READ VARIABLES
C        NUNIT...UNITS INDICATOR--NUNIT=0..ENGLISH UNITS
C                                 NUNIT GT 0 ..SI UNITS
C        P01...PRIMARY STAGNATION PRESSURE--PSIA OR PASCAL
C        T01...PRIMARY STAGNATION TEMPERATURE--RANKINE OR KELVIN
C        P02...SECONDARY STAGNATION PRESSURE--PSIA OR PASCAL
C        T02...SECONDARY STAGNATION TEMPERATURE--RANKINE OR KELVIN
C        MASS1...PRIMARY MASS FLOW RATE--LBM/SEC-IN. OR KG/SEC-M
C        MASS2...TOTAL OF SECONDARY PLUS TERTIARY FLOW RATE--LBM/SEC-IN.
C                                                   OR KG/SEC-M
C        K1...NO. OF GRID PTS. IN PRIMARY FLOW (EVEN NUMBER)
C        K2...NO. OF GRID PTS. IN SECONDARY FLOW (EVEN NUMBER)
C        K3...NO. OF GRID PTS. IN TERTIARY FLOW (EVEN NUMBER)
C
C        NPCYCL...  PRINT CYCLE
C        NQUICK...  0 - FULL PRINT, 1 - PARTIAL PRINT OUT
C        XNOZ...    X COORD OF CENTRE OF NOZZLE ...IN. OR METERS
C        YNOZ...    Y COORD OF CENTRE OF NOZZLE .. IN. OR METERS
C        TNOZ...    NOZZLE ANGLE    DEGREES
C        D...       NOZZLE SLOT WIDTH     INCHES  OR METERS
C
C        NPAIR1...  NUMBER OF DUCT WALL COORDS, LOWER WALL
C        NPAIR2...  NUMBER OF DUCT WALL COORDS, UPPER WALL
C        XW...      ARRAY OF DUCT WALL  X COORDS .. IN. OR METERS
C        YW...      ARRAY OF DUCT WALL  Y COORDS .. IN. OR METERS
C
C        NFULL...   NUMBER OF PROFILE PRINTOUT LOCATIONS
C        XPROF...   ARRAY OF LOCATIONS WHERE PROFILES REQUIRED IN. OR M.
C
C        IT IS ASSUMED THAT P02=P03,  T02=T03
C
C     DATA VARIABLES
C        RG...GAS CONSTANT--FT.LMF/LBM.R OR N-M/KG-K
C        G...SPECIFIC HEAT CONSTANT
C        NUREF...REFERENCE KIN. VISCOSITY--FT**2/SEC OR M**2/SEC
C        TREF...REFERENCE TEMPERATURE--RANKINE  OR KELVIN
C        RHOREF...REFERENCE DENSITY--LBM/FT**3 OR KG/M**3
C       'GC...CONSTANT--LBM.FT/LBF.SEC**2 OR KG-M/N-SEC**2
C        PR...PRANDTL NUMBER
C        PRT...TURBULENT PRANDTL NUMBER
C        TW...WALL TEMPERATURE--RANKINE OR KELVIN
C
C     DIMENSIONLESS VARIABLES
C        OMG(I)          STREAMLINE  FUNCTIONS
C        S1,S2,S3,S4,S5(I)     OMG DIFFERENCES
C        DS(I)           STREAMLINE STEP SIZES
C        R(I,2)          CURVATURES
C        THETA(I,2)      ANGLES
C        PS(I,2)         PRESSURES
C        U(I,2)          VELOCITIES
C        TS(I,2)         TEMPERATURES
C        RHO(I,2)        DENSITIES
C        VIS(I)          VISCOSITIES
C        DN(I)           DISTANCE TO INNER WALL
C        X(I)            X COORDINATE
```

```
C         Y(I)              Y COORDINATE OF STEAMLINES
C         M(I)              MACH NUMBERS
C         PO(I)             STAGNATION PRESSURES
C         E(I)              EDDY VISCOSITIES
C
C         PRESSURES NORMALIZED BY...P01
C         TEMPERATURES NORMALIZED BY...T01
C         VELOCITIES NORMALIZED BY...UREFP
C         LENGTHS NORMALIZED BY ...D
C         DYNAMIC VISCOSITES NORMALIZED BY...RHO01*UREFP*D
C         KINEMATIC EDDY VISCOSITY NORMALIZED BY...UREFP*D
C         STREAM FUNCTION NORMALIZED BY...SQRT(RHO01*UREFP*D)
C         MASS FLOW RATE NORMALIZED BY...RHO01*UREFP*D
C            R(I) IS THE INVERSE OF THE DIMENSIONLESS RADIUS OF CURVATURE
C
      REAL NUREF,KS,LM,MASS1,MASS2,M(190)
      DIMENSION XW(99,2),YW(99,2),S1(190),S2(190),S3(190), S4(190),
     1          S5(190),E(190),E1(190),DS(190),X(190),Y(190),VIS(190),
     2          PO(190),H(190),THETA(190,2),R(190,2),U(190,2),
     3          TS(190,2),RHO(190,2),DIP(190),NX(6),XPROF(10)
      DIMENSION OMG(190),DN(190),PS(190,2)
C     DOUBLE    OMG(190),DN(190),PS(190,2)
        DIMENSION PSI(180)
C       DOUBLE PSI(190)
          DIMENSION NDI(10,2),CURV(99,2) ,TITLE(18)
       COMMON/KURV/MST,MSS,NDI,CURV
       COMMON/SOLV/ S1,S2,S3,E,E1,H,S4,S5
       COMMON/INEDY/PSI,RHO,VIS
       COMMON/INCD/ U,PS,TS,M,PO,THETA,R,X,Y,DN
       COMMON  /WALL/  NPAIR1, NPAIR2, XW, YW
       COMMON /CONST/  P01,P02,P03, T01,T02,T03, X1, XC, YC
        DATA RG,G,GC,TREF,RHOREF,TW/53.2,1.4,32.2,530.,0.075,530.0/
        DATA NUREF,PR,PRT/0.00014,0.7,0.9/
          DATA PR1,PRT1/1.0,1.0/
       DATA A1,A2,A3/144.,198.7,.03611/
C     ---------------------------------------
C
C     INPUT DATA SECTION
C
C     ------------------------------------------
       READ(5,800) NUNIT,(TITLE(I),I=1,18)
       READ(5,801)P01,T01,P02,T02,MASS1,MASS2
       READ(5,802) K1,K2,K3,NPCYCL,NQUICK
       READ(5,801) XNOZ,YNOZ,TNOZ,D
       READ(5,802) MST,(NDI(I,1),I=1,MST)
      NPAIR1=NDI(MST,1)
       READ(5,803) (XW(I,1),YW(I,1),I=1,NPAIR1)
       READ(5,802) MSS,(NDI(I,2),I=1,MSS)
      NPAIR2=NDI(MSS,2)
      READ(5,803) (XW(I,2),YW(I,2),I=1,NPAIR2)
       READ(5,804) NFULL,(XPROF(J),J=1,NFULL)
800    FORMAT(I1,18A4)
801    FORMAT(6F10.0)
802    FORMAT(11I5)
803    FORMAT(2F10.0)
804    FORMAT(I5,10F5.0)
       XPROF(NFULL+1)=  XW(NPAIR1,1)
       WRITE(6,900)(TITLE(I),I=1,18)
       IF(NUNIT.GT.0) GO TO 9
       WRITE(6,901) P01,T01,P02,T02,MASS1,MASS2,D
       WRITE(6,902) XNOZ,YNOZ,TNOZ
       GO TO 16
9      WRITE(6,914) P01,T01,P02,T02,MASS1,MASS2,D
       WRITE(6,915) XNOZ,YNOZ,TNOZ
       RG=284.
       GC=1..
```

```
      TREF=295.
      RHOREF=1.201
      TW=295.
      NUREF=1.3E-5
      A1=1.
      A2=110.
      A3=1.0
16    WRITE(6,903) K1,K2,K3,NPCYCL
      WRITE(6,904)
     CALL WALLS(XW,YW,1000.,0.,T1,R1,NPAIR1,1)
     CALL WALLS(XW,YW,1000.,0.,T2,R2,NPAIR2,2)
     NPAIR=  NPAIR1
     IF (NPAIR1 .GT. NPAIR2)  NPAIR=  NPAIR2
      WRITE(6,905) (J,XW(J,1),YW(J,1),CURV(J,1),XW(J,2),YW(J,2),
    1 CURV(J,2),J=1,NPAIR)
      WRITE(6,906) (XPROF(I),I=1,NFULL)
900   FORMAT(1H1,39X,43(1H*)/40X,1H*,41X,1H*/40X,1H*,2X,
    1         40HCOANDA EFFECTS IN A PLANE CURVED DUCT  */
    2         40X,1H*,41X,1H*/40X,43(1H*)//18A4//)
901   FORMAT(25X,29HPRIMARY STAGNATION PRESSURE =,F6.2,5H PSIA/
    1         25X,26HPRIMARY STAGNATION TEMP. =,F7.2,16H DEGREES RANKINE
    2/        25X,31HSECONDARY STAGNATION PRESSURE =,F6.2,5H PSIA/
    3         25X,28HSECONDARY STAGNATION TEMP. =,F7.2,16H DEGREES RANKI
    4NE//     25X,24HPRIMARY MASS FLOW RATE =,F8.5,12H LBM/SEC-IN./
    5         25X,44HTOTAL OF SECONDARY PLUS TERTIARY FLOW RATE =,F8.5,
    612H LBM/SEC-IN.//25X,19HNOZZLE SLOT WIDTH =,F6.3,4H IN./)
902   FORMAT(25X,21HNOZZLE X COORDINATE =,F10.4,4H IN./
    1         25X,21HNOZZLE Y COORDINATE =,F10.4,4H IN./
    2         25X,14HNOZZLE ANGLE =,F10.3,8H DEGREES/)
903   FORMAT(25X,39HNUMBER OF GRID POINTS IN PRIMARY FLOW =,I4/
    1         25X,41HNUMBER OF GRID POINTS IN SECONDARY FLOW =,I4/
    2         25X,40HNUMBER OF GRID POINTS IN TERTIARY FLOW =,I4/
    3         25X,12HPRINT CYCLE ,I4)
904   FORMAT(1H1,10X,25HMIXING SECTION DIMENSIONS////
    1         17X,10HLOWER WALL,35X,10HUPPER WALL/
    2         6X,1HJ,10X,1HX,9X1HY,11X,4HCURV,19X,1HX,9X,1HY,11X,4HCURV
    3/)
905   FORMAT(I7,3X,2F10.4,F15.5,10X,2F10.4,F15.5)
906   FORMAT(/7X,12HPROFILES AT,10F10.5/)
914   FORMAT(25X,29HPRIMARY STAGNATION PRESSURE =,E12.5,7H PASCAL/
    1         25X,26HPRIMARY STAGNATION TEMP. =,F7.2,15H DEGREES KELVIN/
    2         25X,31HSECONDARY STAGNATION PRESSURE =,E12.5,7H PASCAL/
    3         25X,28HSECONDARY STAGNATION TEMP. =,F7.2,15H DEGREES KELVI
    4N//      25X,24HPRIMARY MASS FLOW RATE =,F8.5, 9H KG/SEC-M/
    5         25X,44HTOTAL OF SECONDARY PLUS TERTIARY FLOW RATE =,F8.5,
    69H KG/SEC-M    //25X,19HNOZZLE SLOT WIDTH =,F9.6,7H METERS/)
915   FORMAT(25X,21HNOZZLE X COORDINATE =,F10.6,7H METERS/
    1         25X,21HNOZZLE Y COORDINATE =,F10.6,7H METERS/
    2         25X,14HNOZZLE ANGLE =,F10.3,8H DEGREES/)
      XNOZ= XNOZ/D
      YNOZ= YNOZ/D
      TNOZ=  TNOZ / 180.0 * 3.1416
      DO 601 J=1,NPAIR1
      XW(J,1)= XW(J,1) / D
      YW(J,1)= YW(J,1) / D
      CURV(J,1)=CURV(J,1)*D
601   CONTINUE
      DO 602 J=1,NPAIR2
      XW(J,2)= XW(J,2) / D
      YW(J,2)= YW(J,2) / D
      CURV(J,2)=CURV(J,2)*D
602   CONTINUE
      RHO01=A1*P01/RG/T01
       UREFP=SQRT(  GC*RG*T01)
C
C     KJET IS NUMBER OF POINTS CLOSE TO JET NOZZLE WALLS
```

```
C     KWALL IS NUMBER OF POINTS CLOSE TO DUCT WALLS                  $NA1950
C                                                                    $NA1960
      KJET=  6                                                       $NA1970
      KWALL=  20                                                     $NA1980
      N=  K3 + K1 + K2 + 2 * KWALL + 4 * KJET + 1                    $NA1990
       NN=N-1                                                        $NA2000
     NX1=1                                                           $NA2010
     NX2=K3+KWALL+KJET+1                                             $NA2020
     NX3=NX2                                                         $NA2030
     NX4=N-K2-KWALL-KJET                                             $NA2040
     NX5=NX4                                                         $NA2050
     NX6=N                                                           $NA2060
     NX(1)=NX1                                                       $NA2070
     NX(2)=NX2                                                       $NA2080
     NX(3)=NX3                                                       $NA2090
     NX(4)=NX4                                                       $NA2100
     NX(5)=NX5                                                       $NA2110
     NX(6)=NX6                                                       $NA2120
      L1=  NX(3)                                                     $NA2130
      L2=  NX(4)                                                     $NA2140
      NMAX=  (L1 + L2) / 2                                           $NA2150
      NF=  1                                                         $NA2160
       P03=P02                                                       $NA2170
       T03=T02                                                       $NA2180
      MASS1=A1*MASS1/RHO01/UREFP/D                                   $NA2190
      MASS2=A1*MASS2/RHO01/UREFP/D                                   $NA2200
```

```
C                                                                        $NA2210
C     SUBROUTINE INCOND IS USED TO CALCULATE THE INITIAL CONDITIONS      $NA2220
C                                                                        $NA2230
      CALL        INCOND(N,NN,K1,K2,K3,KWALL,KJET,L1,L2,                 $NA2240
     1              MASS1,MASS2,AMASS1,AMASS2,AMASS3,                    $NA2250
     2              RG,GC,G,XNOZ,YNOZ,TNOZ,XW,YW,NPAIR1,NPAIR2)          $NA2260
      AMA1=AMASS1*UREFP*D*RHO01/A1                                       $NA2270
      AMA2=AMASS2*UREFP*D*RHO01/A1                                       $NA2280
      AMA3=AMASS3*UREFP*D*RHO01/A1                                       $NA2290
      WRITE(6,921)                                                       $NA2300
      G2=G/(G-1.)                                                        $NA2301
      VNOZI=UREFP*SQRT(2.*G2*(1.-(P02/P01)**(1./G2)))                    $NA2302
      TNOZI=AMA1*VNOZI                                                   $NA2303
      WRITE(6,927) TNOZI                                                 $NA2304
      WRITE(6,922) AMA1,AMA2,AMA3                                        $NA2310
      WRITE(6,923)                                                       $NA2320
      DO 28 J=1,N                                                        $NA2330
      XS=X(J)*D                                                          $NA2340
      DNS=DN(J)*D                                                        $NA2350
      YS=Y(J)*D                                                          $NA2360
      TX=THETA(J,1)*57.2958                                              $NA2370
      XR=R(J,1)/D                                                        $NA2380
   28 WRITE(6,924)J,XS,YS,DNS,TX,XR,PSI(J)                               $NA2390
      WRITE(6,925)                                                       $NA2400
      DO 29 J=1,N                                                        $NA2410
      P0D=P0(J)*P01                                                      $NA2420
      PSD=PS(J,1)*P01                                                    $NA2430
      T0=TS(J,1)*T01                                                     $NA2440
      RHOD=RHO(J,1)*RHO01                                                $NA2450
      UV=U(J,1)*UREFP                                                    $NA2460
   29 WRITE(6,926) J,P0D,PSD,T0,RHOD,UV,M(J)                             $NA2470
  921 FORMAT(1H1,///,25X,18HINITIAL CONDITIONS//)                        $NA2480
  922 FORMAT(7X,8HAMASS1 =F10.5,5X,8HAMASS2 =F10.5,5X,8HAMASS3 =F10.5/)  $NA2490
  923 FORMAT(7X,1HJ,9X,1HX,9X,1HY,8X,2HDN,6X,5HTHETA,8X,1HK,7X,3HOMG/)   $NA2500
  924 FORMAT(5X,I3,2X,4F10.3,F10.6,F10.4)                                $NA2510
  925 FORMAT(///7X,1HJ,8X,2HP0,8X,2HPS,8X,2HTS,7X,3HRHO,9X,1HU,9X,1HM/)  $NA2520
  926 FORMAT(5X,I3,2X,2E10.3,F10.1,F10.5,F10.1,F10.3)                    $NA2530
  927 FORMAT(7X,41HISENTROPIC NOZZLE THRUST PER UNIT WIDTH =,E10.3)      $NA2531
     DO 30 J=1,N                                                         $NA2540
      THETA(J,2)=THETA(J,1)                                              $NA2550
      PS(J,2)=PS(J,1)                                                    $NA2560
      TS(J,2)=TS(J,1)                                                    $NA2570
     RHO(J,2) =RHO(J,1)                                                  $NA2580
     U(J,2)=U(J,1)                                                       $NA2590
      R(J,2)=R(J,1)                                                      $NA2600
```

```
 30   E(J) = 0.
        DEL1=0.0
        DEL2=0.0
        DEL3=0.
        DEL4=0.
      CORE3=DN(NX2)-DN(NX1)
      CORE1=DN(NX4)-DN(NX3)
      CORE2=DN(NX6)-DN(NX5)
      DPDSA= -0.001
       USTARI  = 0.0
       USTARO  = 0.0
       DGO=0.
       DGI=0.0
       NGRID=0
       NPRINT= 0
       DSA=  .02 * DN(N)
       CC= .07
       NCORE=1
       NSEP=0
       RDECAY=4.
       ICOUNT=0
C
C     -------------------------------------------------
C
C        BEGINNING OF MARCHING CALCULATION
C
C     -------------------------------------------------
C
       DO 4500 NSTEP = 1,100
       ICHS = 0
C
C      FIND MIDDLE OF DUCT
C
       DO 527 J=1,N
       IF (DN(J) .LT. DN(N)*0.5)  NDUCT=  J
527    CONTINUE
       IF (NSTEP .EQ. 1) GO TO 77
C
C        SUBROUTINE EDDY CALCULATES THE EDDY VISCOSITY FOR SUB. SOLV
C
       CALL           EDDY(N,NN,NX,U,PS,CC,DS,E,E1,RHO,VIS,R,DN,
     1          S4, S5,  DEL1,DEL2,DEL3,DEL4,CORE1,CORE2,CORE3)
 77     CONTINUE
      E1(1) = 0.
       DO 40 J=2,NN
      E1(J) = RHO(J,1)*U(J,1)
  40   CONTINUE
       E1(N)=0.
C
C     MOVE TO NEXT POINT ON WALL.....
C
       DS(NMAX)=  DSA * (1.04) ** (NSTEP-1)
       IF (DS(NMAX) .LT. .02 * DN(N) )  DS(NMAX)=  .02 * DN(N)
C
C      CALCULATE  DS(I)  VALUES
C      (MIDDLE DS VALUE CALCULATED FIRST
C      THEN DS VALUES CALCULATED OUT TO BOTH WALLS
C
      NPRR=NMAX+1
      DO 50 J=NPRR,N
          JM= J-1
          C1=RHO(J,2)+RHO(JM,2)
          C2=U(J,2)+U(JM,2)
          C3=R(J,2)+R(JM,2)
           C4=  PSI(J) - PSI(J-1)
          C5 =         C4*C3/(C1*C2)
```

```
         DS(J) = (1.+C5)/(1.-C5)*DS(JM)
 50    CONTINUE
       NPRR=NMAX-1
       DO 52 MM=1,NPRR
        J=  NMAX - MM
        JM= J+1
           C1=RHO(J,2)+RHO(JM,2)
           C2=U(J,2)+U(JM,2)
            C3=  R(J,2) + R(JM,2)
            C4=  PSI(J) - PSI(JM)
           C5 =           C4*C3/(C1*C2)
         DS(J) = (1.+C5)/(1.-C5)*DS(JM)
 52     CONTINUE
C
C       MOVE TO NEW POINT X(1) AND X(N) ALONG WALL SURFACES
C           AT A DISTANCE SPECIFIED BY DS VALUES
C
        DX1=  X(1)
        CALL DSMOVE(XW,YW,X(1),Y(1),DS(1),THETA(1,1), R(1,1), NPAIR1,1)
        CALL DSMOVE(XW,YW,X(N),Y(N), DS(N), THETA(N,1), R(N,1), NPAIR2,2)
        IF(X(1) .GT. XW(NPAIR1  ,1))  STOP
        IF (X(N) .GT. XW(NPAIR2  ,2) )  STOP
        DX1=  X(1) - DX1
C
C       COMPUTE Y, THETA, AND R VALUES CORRESPONDING TO NEW X VALUES
C
        CALL WALLS(XW,YW, X(1), Y(1), THETA1, R1, NPAIR1, 1)
        CALL WALLS(XW,YW, X(N), Y(N), THETA2, R2, NPAIR2, 2)
         Y2=  Y(N)
C
C       COMPUTE CURVATURE  R(J,2)
C
        DO 400 J=1,N
        DW1=  DN(J)
        DW2=    DN(N) - DN(J)
        R(J,2)=R1*(1.-DW1/DN(N))*EXP(-RDECAY*DW1*ABS(R1))+R2*DW1/DN(N)*
     1         EXP(-RDECAY*DW2*ABS(R2))
400     CONTINUE
C
C       COMPUTE  THETA(J,2)
C
        DO 51 J=2,NN
        THETA(J,2)=  THETA1 + (THETA2-THETA1) * DN(J)/DN(N)
 51     CONTINUE
        THETA(1,2)=  THETA1
        THETA(N,2)=  THETA2
C
C      ---------------------------------------
C
C      THIS SECTION ATTEMPTS TO SATISFY CONTINUITY
C      LOOKS FOR A PS(1,2) SUCH THAT  Y2 - Y(N) = 0.0
C      DPDSA, DPDSB ARE PRESSURE GRADIENTS
C
C      ---------------------------------------
C
        IF(ABS(DPDSA) .LT. 1.E-08)  DPDSA=-.001
       DPDSB=  DPDSA * 0.9
       DDISTB= 1.0
       DO 650  ITER= 1,40
       IF (ITER .EQ. 1)  PS(1,2)=  PS(1,1) + DPDSA
C
C      AT THIRD ITERATION GUESS NEW DPDSB
C          USING EXTRAPOLATION THROUGH  DPDSA AND PREVIOUS DPDSB
       IF (ITER .GE. 3)
     1    DPDSB=  (DDISTB*DPDSA - DDISTA*DPDSB) / (DDISTB-DDISTA)
       IF (ITER .GE. 2)  PS(1,2)=  PS(1,1) + DPDSB
```

```
          DO 60 J=2,N
          JM= J-1
          C2=U(J,2)+U(JM,2)
          C3=R(J,2)+R(JM,2)
           C4=  PSI(J) - PSI(J-1)
          PS(J,2)=PS(JM,2)+       C2*C3*C4/4.0
60        CONTINUE
C
C      SUBROUTINE SOLV  IS USED TO SOLVE U,T ON M=2 LINE
C
         CALL SOLV(DS,N,NN,PR1,PRT1,G)
          U(1,2) = 0.
          DO 70 J = 2,NN
          U(J,2) = H(J-1)
70        CONTINUE
          U(N,2) = 0.
         CALL SOLV(DS,N,NN,PR,PRT,G)
          DO 80 J = 2,NN
          TS(J,2) = H(J-1)
80        CONTINUE
          TS(1,2) = TS(2,2)
          TS(N,2) = TS(N-1,2)
          DO 90 I = 1,N
          RHO(I,2) =  PS(I,2)/TS(I,2)
        . RHO(I,1) =RHO(I,2)
       VIS(I)=(TREF+A2)/(T01*TS(I,2)+A2)
          VIS(I)=VIS(I)*(T01*TS(I,2)/TREF)**0.5
          VIS(I)=VIS(I)*RHOREF*NUREF/(P01*SQRT(GC/RG/T01))
90        CONTINUE
C
C     GIVEN PS(1,2), CALCULATE UPPER WALL COORDINATE  Y(N)
C
       CALL   CORDS(N,X,Y,THETA,U,RHO,PSI,DN,NDUCT,DDIST)
C
C     IF Y2 .EQ. Y(N)  THEN THIS PS(1,2) SATISFIES CONTINUITY
C
           IF (ITER .EQ. 1)  DDISTA=  DDIST
           IF (ITER .GE. 2)  DDISTB=  DDIST
C
C     IF SUFFICIENTLY SMALL INTERVAL, THEN EXIT
C
      IF (ABS(DDISTB)  .LE. 0.1 ** 4) GO TO 660
650   CONTINUE
       WRITE(6,916) DPDSA,DPDSB,DDISTB,NSTEP
916    FORMAT(/7X,23HNO CONVERGENCE, DPDSA =,F10.5,8H DPDSB =,F10.5,
     1 9H DDISTB =,F10.5,6H STEP=I4)
660   CONTINUE
C
C      SAVE PRESSURE GRADIENT THIS STEP
C
      DPDSA=  DPDSB
       G1=G-1.
       G2=G/G1
C
C      COMPUTE MACH NUMBERS  M(I)
C
       DO 220 I=1,N
       M(I)=U(I,2)/SQRT(G*TS(I,2))
      G4 = 1. + G1/2.*(M(I)*M(I))
      P0(I)    = PS(I,2)*G4**G2
 220   CONTINUE
C
C     ---------------------------------------
C     REMOVE EXTRA GRID POINTS, FIND EDGES OF FLOWS
C     ---------------------------------------
```

```
      CALL REMOVE(N,NN,NGRID,NX,NMAX,PSI,S1,S2,S3,S4,S5,DS,VIS,DN,
     1 X,Y,M,P0,H,THETA,R,PS,U,TS,RHO)
      CALL LOOK(N,NN,K3,NCORE,DEL1,DEL2,DEL3,DEL4,CORE1,CORE2,CORE3,
     1  NX,P0,DN,U,DIP,NMAX)
      SHRIN=U(2,2)*VIS(2)/(DN(2)-DN(1))    - .5*(PS(1,2)-PS(1,1))/DS(1)*
     1 (DN(2)-DN(1))
      SHROR=U(NN,2)*VIS(NN)/(DN(N)-DN(NN))   -.5*(PS(N,2)-PS(N,1))/
     1 DS(N)*(DN(N)-DN(NN))
      IF((SHRIN.LE.0.).OR.(SHROR.LE.0.)) NSEP=1
      IF(NSEP.EQ.1) NPRINT=NPCYCL-1
      IF(NSEP.EQ.1) ICOUNT=1
C
C     ----------------------------------------
C     SAVE VALUES COMPUTED THIS STEP
C     ----------------------------------------
C
      DO  222   I=1,N
      PS(I,1) = PS(I,2)
      U(I,1) =U(I,2)
      TS(I,1) = TS(I,2)
      THETA(I,1)=THETA(I,2)
 222  R(I,1) = R(I,2)
C
C     ----------------------------------------
C     OUTPUT SECTION
C     ----------------------------------------
C
      DS1Q=  DS(1) * D
      DS2Q=  DS(N) * D
      DSUM1=  DSUM1 +DS1Q
      DSUM2=  DS2Q + DSUM2
      NPRINT= NPRINT+1
      IF ( (X(1)*D) .GE. XPROF(NF) )  NPRINT=  NPCYCL
      IF(X(1)*D.GE.XPROF(NF)) ICOUNT=1
      IF ( (X(1)*D) .GE. XPROF(NF) )  NF=  NF + 1
      IF (NSTEP .EQ. 1)  NPRINT= NPCYCL
      IF (NPRINT .LT. NPCYCL)  GO TO 45
      NPRINT= 0
      DXQ=  DX1 * D
      XI=  X(1) *D
      XO=  X(N) * D
      PH2OI=(PS(1,2)*P01-P02)/A3
      PH2OO=(PS(N,2)*P01-P02)/A3
      IF (SHRIN .GT. 0.0)
     1    USTARI=SQRT(SHRIN/RHO(2,2))*UREFP
      IF (SHROR .GT. 0.0)
     1    USTARO=SQRT(SHROR/RHO(NN,2))*UREFP
      RI=  R(1,2) / D
      RO=  R(N,2) / D
      RNI=  2.0 * (R(2,1) / RHO(2,1))
     RNI=RNI/(S5(2)*(U( 3,1)-U(2,1))+S4(2)*(U(2,1)-U(1,1)))
      RNO=  2.0 * (R(NN,1) / RHO(NN,1))
     RNO=RNO/(S5(NN)*(U( N,1)-U(NN,1))+S4(NN)*(U(NN,1)-U(NN-1,1)))
      UMAX=  U(NMAX,2) * UREFP
      XTSI=  TS(1,2) * T01 + 0.5/G2 * T01 * (U(1,2))**2
      XTSO=  TS(N,2) * T01 + 0.5/G2 * T01 * (U(N,2))**2
      WRITE(6,908) NSTEP,XI,PH2OI,USTARI,RI,RNI,XO,PH2OO,USTARO,RO,RNO,
     1        UMAX,ITER
 908  FORMAT(1H1,5X,5HNSTEP,6X,2HXI,8X,2HPI,4X,6HUSTARI,8X,2HKI,7X,
     1 3HRNI,8X,2HXO,8X,2HPO,4X,6HUSTARO,8X,2HKO,7X,3HRNO,6X,4HUMAX/
     2        5X,I3,2(F10.4,E11.3,2F10.4,F10.5),F10.1//7X,
     3        22HNUMBER OF ITERATIONS =,I3/)
      WRITE(6,907)DXQ,DS1Q,DS2Q,DSUM1,DSUM2
 907  FORMAT( 7X, 9HDELTA X =,F10.5,12H  DELTA SI =,F10.5,
     1       12H  DELTA SO =,F10.5,6H  SI =F10.5,6H  SO =,F10.5/)
      SHRIN=SHRIN*P01
```

```
      SHROR=SHROR*P01                                                   $NA5250
      WRITE(6,912) SHRIN,SHROR                                          $NA5260
      TMOMX=0.                                                          $NA5261
      NNY=N-1                                                           $NA5262
      DO 95 J=1,NNY                                                     $NA5263
 95   TMOMX=TMOMX+(RHO(J,1)*U(J,1)**2*COS(THETA(J,1))+RHO(J+1,1)*       $NA5264
     1     U(J+1,1)**2*COS(THETA(J+1,1)))/2.*(Y(J+1)-Y(J))              $NA5265
      TMOMX=TMOMX*P01*D                                                 $NA5266
      CT=TMOMX/TNOZI                                                    $NA5267
      WRITE(6,931) TMOMX,CT                                             $NA5268
 912  FORMAT(  7X,7HSHRIN =E12.5,5X,7HSHROR =E12.5/)                    $NA5270
 931  FORMAT(/7X,37HTOTAL AXIAL MOMENTUM PER UNIT WIDTH =,E10.3//7X,    $NA5272
     1          14HTHRUST RATIO =,E10.3)                                $NA5272
      IF (NQUICK .GT. 0)  GO TO 45                                      $NA5280
      IF(ICOUNT.EQ.0) GO TO 45                                          $NA5290
      ICOUNT=0                                                          $NA5300
      CORE1Q=  CORE1 * D                                                $NA5310
      CORE2Q=  CORE2 * D                                                $NA5320
      CORE3Q=  CORE3 * D                                                $NA5330
      DEL1Q=  DEL1 * D                                                  $NA5340
      DEL2Q=  DEL2 * D                                                  $NA5350
      DEL3Q=  DEL3 * D                                                  $NA5360
      DEL4Q=  DEL4 * D                                                  $NA5370
      WRITE(6,909) (NX(I),I=1,6),CORE1Q,CORE2Q,CORE3Q,DEL1Q,DEL2Q,      $NA5380
     1      DEL3Q,DEL4Q                                                 $NA5390
 909  FORMAT(7X,2HN1,5X,2HN2,5X,2HN3,5X,2HN4,5X,2HN5,5X,2HN6/5X,I4,     $NA5400
     1       5(3X,I4)// 7X,5HCORE1,5X,5HCORE2,5X,5HCORE3,/4X,3F10.5//   $NA5410
     2        7X,4HDEL1,6X,4HDEL2,6X,4HDEL3,6X,4HDEL4/4X,4F10.5//)      $NA5420
      WRITE(6,910)                                                      $NA5430
 910  FORMAT(7X,1HJ,7X,3HI/R,5X,5HTHETA,9X,1HX,9X,1HY,6X,4HYREL,6X,     $NA5440
     1    4HUREL,5X,5HPOREL,8X,2HPS,9X,1HE,5X,6HTOTEMP/)                $NA5450
      DO 100 J=1,N                                                      $NA5460
      THEIO=THETA(J,2)*180./3.1416                                      $NA5470
      XR=R(J,2)/D                                                       $NA5480
      YS=Y(J)*D                                                         $NA5490
      YREL=          Y(J) / Y(N)                                        $NA5500
      XS= X(J)*D                                                        $NA5510
      UV=U(J,2) / U(NMAX,2)                                             $NA5520
      POD=PO(J)                                                         $NA5530
      PSD=PS(J,2)*P01                                                   $NA5540
      TO=  TS(J,2) * T01 + 0.5/G2 * T01 * (U(J,2))**2                   $NA5550
      TOMAX=  TS(NMAX,2) * T01 + 0.5/G2 * T01 * (U(NMAX,2))**2          $NA5560
      XTS=  (TO - XTSO) / (TOMAX - XTSO)                                $NA5570
      XT=E(J)*UREFP*D/SQRT(A1)                                          $NA5580
      DND=DN(J)*D                                                       $NA5590
      WRITE(6,911)J,XR,THEIO,XS,YS,YREL,UV,POD,PSD,XT,TO                $NA5600
 911  FORMAT(5X,I3,2X,F10.5,3F10.4,3F10.5,E11.3,F10.5,F10.2)            $NA5610
 100  CONTINUE                                                          $NA5620
 45   CONTINUE                                                          $NA5630
      IF(NSEP.EQ.1) GO TO 300                                           $NA5640
 4499 CONTINUE                                                          $NA5650
 4500 CONTINUE                                                          $NA5660
      STOP                                                              $NA5670
 300  WRITE(6,930)                                                      $NA5680
 930  FORMAT(7X,24(1H*)/7X,38HNEGATIVE SHEAR STRESS AT THIS STATION./   $NA5690
     1        7X,36H THIS INDICATES POSSIBLE SEPARATION./               $NA5700
     2        7X,48H SUBSEQUENT RESULTS SHOULD BE USED WITH CAUTION.//  $NA5710
     3        7X,24(1H*))                                               $NA5720
      NSEP=0                                                            $NA5730
      GO TO 4499                                                        $NA5740
      END                                                               $NA5750
```

```
      SUBROUTINE INCOND(N,NN,K1,K2,K3,KWALL,KJET,L1,L2,
     1                MASS1,MASS2,AMASS1,AMASS2,AMASS3,
     2                RG,GC,G,XNOZ,YNOZ,TNOZ,XW,YW,NPAIR1,NPAIR2)
C
C     ************************************************
C     SUBROUTINE INCOND
C     CALCULATES INITIAL CONDITIONS
C     INPUT VARIABLES
C         NPAIR1:    NUMBER OF POINTS ON LOWER WALL
C         NPAIR2:    NUMBER OF POINTS ON UPPER WALL
C         XW(I,1), YW(I,1):    DATA POINTS SPECIFYING LOWER WALL
C         XW(I,2), YW(I,2):    DATA POINTS SPECIFYING UPPER WALL
C         X1:        STARTING POINT ON LOWER WALL
C         XC,YC:     CENTRE OF INITIAL RADIUS OF CURVATURE
C     ***********************************************
C
      REAL MASS1,MASS2,NUD,M(190)
      DIMENSION XW(99,2),YW(99,2),S1(190),S2(190),S3(190), S4(190),
     1          S5(190),E(190),E1(190),DS(190),X(190),Y(190),
     2          P0(190),H(190),THETA(190,2),R(190,2),U(190,2),
     3          TS(190,2),RHO(190,2),VIS(190)
      DIMENSION OMG(190),DN(190),PS(190,2)
C     DOUBLE    OMG(190),DN(190),PS(190,2)
       DIMENSION PSI(190)
C      DOUBLE PSI(190)
C     DOUBLE XT,P3,RY,PP,DY,PSI,DPSI,ZY,DSQRT,DPS,Z
       COMMON/SOLV/ S1,S2,S3,E,E1,H,S4,S5
       COMMON/INEDY/PSI,RHO,VIS
       COMMON/INCD/ U,PS,TS,M,P0,THETA,R,X,Y,DN
       COMMON /CONST/  P01,P02,P03, T01,T02,T03, X1, XC, YC
       DIST(XX,YY,XXC,YYC)=SQRT((ABS(XX-XXC)**2)+(ABS(YY-YYC)**2))
C
C      LOOK FOR ARC TANGENT TO BOTH WALLS WHICH PASSES THROUGH NOZZLE
C          X1 AND X2 ARE STARTING POINTS ON LOWER AND UPPER DUCT WALL
C
C      XNOZ, YNOZ IS MIDDLE OF NOZZLE
C          FIND COORDS OF EACH EDGE
C      NOZZLE WIDTH IS 1.0
C
       XNOZ1=  XNOZ + (    (SIN(TNOZ)) * 0.5)
       XNOZ2=  XNOZ - (    (SIN(TNOZ)) * 0.5)
      YNOZ1=  YNOZ - (ABS(COS(TNOZ)) * 0.5)
      YNOZ2=  YNOZ + (ABS(COS(TNOZ)) * 0.5)
C
C      COMPUTE ARC LENGTH ACROSS DUCT (DN2)
C      ARC LENGTHS TO NOZZLE (DNI, DNO)
C
      CALL ARCDIS(XNOZ1,YNOZ1,TNOZ,X1,RC1,XW,YW,NPAIR1,1)
      CALL WALLS(XW,YW,X1,Y1,T1,R1,NPAIR1,1)
      IF (RC1 .LT. 100.0)  DNI=  RC1 * ABS(TNOZ-T1)
      IF (RC1 .GE. 100.0)  DNI=  DIST(XNOZ1,YNOZ1,X1,Y1)
      CALL ARCDIS(XNOZ2,YNOZ2,TNOZ,X2,RC2,XW,YW,NPAIR2,2)
      CALL WALLS(XW,YW,X2,Y2,T2,R2,NPAIR2,2)
      IF (RC2 .LT. 100.0)  DNO=  RC2 * ABS(TNOZ-T2)
      IF (RC2 .GE. 100.0)  DNO=  DIST(XNOZ2,YNOZ2,X2,Y2)
      DN2=  DNI + DNO +1.0
      X(1)=X1
      Y(1)=Y1
      X(N)= X2
      Y(N)=  Y2
      THETA(1,1) = T1
      THETA(N,1) = T2
```

```
C       SOLVE FOR MASS SPLIT (NOZZLE EXIT CORRESPONDS TO DN(L1) )
C            DUCT COMPUTED MUST CORRESPOND TO ACTUAL DUCT WIDTH
C            THUS DN(1) SHOULD EQUAL 0.0
C
C           FIND LOCATION OF NOZZLE
C           (DIFFERENT CALCULATIONS FOR SPLMAX DEPENDING ON
C            NOZZLE LOCATION)
C
        IF (DNI .LT. DNO)  NZONE= 0
        IF (DNI .GE. DNO)  NZONE=  1
        IF (NZONE .EQ. 0)  NMID=  L2
        IF (NZONE .EQ. 1)  NMID=  L1
        G1=  1.0 / (G-1.0)
        G2=  G/(G-1.0)
        RDECAY=4.
        A=DNI+1.+DNO
        IF(NZONE.EQ.0) RTEST=R1*(1.-(DNI+1.)/A)*EXP(-RDECAY*(DNI+1.)*
     1    ABS(R1))+R2*(DNI+1.)/A*EXP(-RDECAY*DNO*ABS(DNO))
        IF(NZONE.EQ.1) RTEST=R1*(1.-DNI/A)*EXP(-RDECAY*DNI*ABS(R1))+
     1    R2*DNI/A*EXP(-RDECAY*(DNO+1.)*ABS(R2))
        TMAX=  (T02/T01 - .0001*((G-1.0)/(G*2.0)) )
     1      * (P03/P01) ** ((G-1.0)/G) * (T01/T02)
        M(NMID)=  SQRT(2.0 * G1 * (1.0/ TMAX      - 1.0) )
        PSNMID=  TMAX ** G2
        RHO(NMID,1)=  P02 / P01 * T01 / T02
        U2MIN=ABS(.0001-2.*G*PSNMID*(M(NMID))**2*RTEST/RHO(NMID,1))
        U2MIN=  SQRT(U2MIN)
        SPLMAX=  1.0 - ( P02 / P01 * T01 / T02 * U2MIN * DNO / MASS2)
        SPLMAX=  ABS(SPLMAX)
        IF (SPLMAX .GT. 0.99)  SPLMAX=  0.99
C
C       THIS IS ENTRY POINT FOR LOWERING SPLMAX IF TMAX IS EXCEEDED
C
555     CONTINUE
        SPLMIN=  1.0 - SPLMAX
        SPLTST=  DNI / (DNI + DNO)
        XA=  SPLTST
        IF ((NZONE .EQ. 0).AND.(XA .LT. SPLMIN*1.11)) XA=  SPLMIN*1.11
        IF ((NZONE .EQ. 1).AND.(XA .GT. SPLMAX* 1.0))  XA=  SPLMAX* 1.0
        XB=  XA * 0.9
        DIFFB= 1.0
        NMID=  (L1 + L2) / 2
        DO 40 ITER= 1,50
        IF (ITER .EQ. 1)  SPLIT=XA
        IF (ITER .GE. 3)  XB=  (DIFFB*XA - DIFFA*XB) / (DIFFB-DIFFA)
        IF (ITER .GE. 2)  SPLIT=  XB
        AMASS1=MASS1
        AMASS3=  SPLIT * MASS2
        AMASS2=  (1.0-SPLIT) * MASS2
C
C         STREAM FUNCTIONS
C
        OMGN=  AMASS1 + AMASS2 + AMASS3
        OMGJ3=  AMASS3
        OMGJ2=  AMASS3 + AMASS1
C
C       DMJET IS THE SMALL DELTA OMG AT JET WALLS
C       DMWALL IS THE SMALL DELTA OMG STARTING IN FROM DUCT WALLS
C
        DMWALL=  .00001 * OMGN
        DMJET=  DMWALL * 100.0
        DOMG3=  OMGJ3 / FLOAT(K3)
        DOMG2=  (OMGN - OMGJ2) / FLOAT(K2)
```

```
      DOMG1=  (OMGJ2 - OMGJ3) / FLOAT(K1)
      PSI(1)=  0.0
      PSI(L1)=  OMGJ3
      PSI(L2)=  OMGJ2
      PSI(N)=   OMGN
      CALL  OMGSET(PSI, 1,L1,KWALL,KJET,DMWALL,DMJET,DOMG3,RST3,RFIN3)
      CALL  OMGSET(PSI,L1,L2,KJET,KJET,DMJET,DMJET,DOMG1,RST1,RFIN1)
      CALL  OMGSET(PSI,L2,N,KJET,KWALL,DMJET,DMWALL,DOMG2,RST2,RFIN2)
      CALL          TMPSET(N,NN,RG,GC,G,DN2,R1,R2,L1,L2,DNI,DNO,
     1                PSI, R, M, TS, PS, P0, RHO, U, DN,MAXERR)
      IF ((MAXERR .EQ. 1).AND.(ITER .EQ. 1))  GO TO 589
      IF ((MAXERR .EQ. 1).AND.(ITER .NE. 1))  GO TO 599
      IF (ITER .EQ. 1)  DIFFA=  DN(1)
      IF (ITER .GE. 2)  DIFFB=  DN(1)
C
C     CHECK FOR TERMINATION
C
      IF (ABS(DIFFB) .LE. 0.001) GO TO 42
40    CONTINUE
42    WRITE(6,902)SPLIT,ITER
902   FORMAT(/7X,7HSPLIT =,F10.5/
     1       7X,36HNUMBER OF ITERATIONS TO FIND SPLIT =,I4//)
C
C     COMPUTE OMG DIFFERENCE ARRAYS  S1... S5
C
      DO 20  J= 2,NN
      JP = J+1
      JM = J-1
      S1(J)=  PSI(JP) - PSI(JM)
      S2(J)=  PSI(JP) - PSI(J)
      S3(J)=  PSI(J) - PSI(JM)
      S4(J) = S2(J)/S3(J)/S1(J)
      S5(J) = S3(J)/S2(J)/S1(J)
 20    CONTINUE
C
C     COMPUTE ANGLES  THETA (I)
C
      DO 410 J=2,NN
      IF ( (J .GE. L1).AND.(J .LE. L2)) THETA(J,1)=  TNOZ
      IF(J.LT.L1) THETA(J,1)=THETA(J-1,1)-(DN(J)-DN(J-1))/RC1
      IF(J.GT.L2) THETA(J,1)=THETA(J-1,1)-(DN(J)-DN(J-1))/RC2
410   CONTINUE
      DO 2 J=2,NN
      JM=J-1
      D=.5*(THETA(J,1)+THETA(JM,1))
       X(J)=X(JM)    -(DN(J)-DN(JM))*SIN(D)
       Y(J) = Y(JM)  +(DN(J)-DN(JM))*COS(D)
 2    CONTINUE
      RETURN
C
C     THIS SPLMAX GIVES A TS EXCEEDING TMAX
C     SO TRY LOWERING SPLMAX 1 PERCENT
C
589   CONTINUE
      SPLMAX=  SPLMAX * 0.99
      GO TO 555
C
C     NO SOLUTION FOR SPLIT
C
599   CONTINUE
      WRITE(6,911)
911   FORMAT(/7X,21HNO SOLUTION FOR SPLIT)
      STOP
      END
```

```
      SUBROUTINE  TMPSET(N,NN,RG,GC,G,DN2,R1,R2,L1,L2,DNI,DNO,
     1                  PSI, R, M, TS, PS, P0, RHO, U, DN, MAXERR)
C    -------------------------------------------
C     SETS AN INITIAL TS IN MIDDLE OF NOZZLE,
C         DETERMINES CORRESPONDING M(I), TS(I),  DN(I), ETC VALUES
C         CHECKS DIFFERENCE BETWEEN DN(N) AND DN2
C    -------------------------------------------
C
      REAL M(190)
      DIMENSION P0(190),THETA(190,2),R(190,2),U(190,2),TS(190,2),
     1          RHO(190,2)
      DIMENSION OMG(190),DN(190),PS(190,2)
C     DOUBLE    OMG(190),DN(190),PS(190,2)
       DIMENSION PSI(180)
C      DOUBLE PSI(190)
      COMMON /CONST/  P01,P02,P03, T01,T02,T03, X1, XC, YC
C
C     THE STARTING ITERATION POINT IS AT PRESENT DETERMINED BY THE
C         LOCATION OF THE NOZZLE IN THE DUCT
C     IF NOZZLE IS CLOSE TO UPPER WALL, THE TMAX  CALCULATION IS
C         VALID ONLY FOR TS(L1,1), AND SO NMID SHOULD EQUAL L1
C     IF NOZZLE IN MIDDLE OF DUCT  NMID SHOULD = (L1 + L2) / 2
C     IF NOZZLE CLOSE TO LOWER WALL NMID SHOULD = L2
C      AND CHANGE COMPUTATION OF R(J,NMID), AND DN(NMID)
C
      IF (DNI .LT. DNO)  NZONE= 0
      IF (DNI .GE. DNO)  NZONE=  1
      MAXERR= 0
      G1=1./(G-1.)
      G2=G/(G-1.)
      RDECAY=4.
      IF (NZONE .EQ. 0)  NMID=  L2
      IF (NZONE .EQ. 0)  R(NMID,1)=  R1 * EXP(-RDECAY*R1*(DNI+1.0))
     1                            + R2 * EXP( RDECAY*R2*DNO)
      IF (NZONE .EQ. 0)  DN(NMID)=  DNI + 1.0
      IF (NZONE .EQ. 1)  NMID=  L1
      IF (NZONE .EQ. 1)  R(NMID,1)=  R1 * EXP(-RDECAY*R1*DNI)
     1                            + R2 * EXP( RDECAY*R2*(DNO+1.0))
      IF (NZONE .EQ. 1)  DN(NMID)=  DNI
C
C     TMAX IS THE TEMPERATURE WHICH PREVENTS VELOCITY FROM
C         FALLING BELOW .01  (IE GOING NEGATIVE)
C
      TMAX=  (T02/T01 - .0001*((G-1.0)/(G*2.0)) )
     1     * (P03/P01) ** ((G-1.0)/G) * (T01/T02)
  595 CONTINUE
      TSA=. TMAX
      TSB=  TSA - (5.0/T01)
      DIFFB= 1.0
C
C     DO LOOP WHICH SOLVES FOR TSB SUCH THAT DN2-DN(N)=  0.0
C         GUESS NEW TSB USING FIXED TSA AND PREVIOUS TSB VALUE
C
      DO 40  J=1,100
      IF (J .EQ. 1)  TS(NMID,1) = TSA
      IF (J .GE. 3)  TSB=  (DIFFB*TSA-DIFFA*TSB) / (DIFFB-DIFFA)
      IF (J .GE. 2)  TS(NMID,1)=  TSB
      IF (TS(NMID,1) .GT. TMAX)  GO TO 599
```

```
                                                                        $TM 570
C         SOLVE FOR TEMPFRATURES,MACH NUMBERS,ETC. AT NODE POINTS       $TM 580
C           WORKING FROM NOZZLE MIDDLE (NMID) OUT TO BOTH WALLS         $TM 590
C                                                                       $TM 600
C           NOZZLE CENTRE LINE VALUES                                   $TM 610
C                                                                       $TM 620
        M(NMID)=  SQRT(2.0 * G1 * (1.0/TS(NMID,1) - 1.0) )              $TM 630
        PS(NMID,1)=  TS(NMID,1) ** G2                                   $TM 640
        RHO(NMID,1)=  PS(NMID,1) / TS(NMID,1)                           $TM 650
        U(NMID,1)=  M(NMID) * SQRT(G * TS(NMID,1))                      $TM 660
        PO(NMID)=  PS(NMID,1) * (1.0 + 0.5 / G1* M(NMID)**2) **G2       $TM 670
C                                                                       $TM 680
C       REGION FROM NOZZLE MIDDLE TO OUTER WALL                         $TM 690
C           SET M, TS, PS                                               $TM 700
C           (L2 IS JUMP FROM NOZZLE TO SECONDARY STREAM)                $TM 710
C                                                                       $TM 720
        PK=1.                                                           $TM 730
        QK=1.                                                           $TM 740
        RK=1.                                                           $TM 750
      NPRR=NMID+1                                                       $TM 760
      DO 42 I=NPRR,N                                                    $TM 770
        IM=I-1                                                          $TM 780
C                                                                       $TM 790
C           JUMP POINT TO SECONDARY STREAM                              $TM 800
C                                                                       $TM 810
        IF (I .EQ.L2+1)        PK=  P02/P01                             $TM 820
        IF (I .EQ.L2+1)             QK=P02/P01*(T01/T02)**G2            $TM 830
        IF (I .EQ.L2+1)             RK=T02/T01                          $TM 840
        SX=RK*R(IM,1)*(PSI(I)-PSI(IM))                                  $TM 850
        M(I)= M(IM) - SX/(G**.5 * QK * TS( IM,1)**(G2+0.5) )            $TM 860
            B1= ((PK/PS( IM,1)) **(1./G2)-1.)*2.*G1                     $TM 870
        IF((I.EQ.L2+1).AND.(B1.LE.0.0))  TMAX=TMAX*0.995                $TM 872
        IF((I.EQ.L2+1).AND.(B1.LE.0.0))  GO TO 595                      $TM 873
            IF (I .EQ. L2+1)  M(I)=  SQRT(B1)                           $TM 880
C                                                                       $TM 890
C       SET TS, PS, RHO, U, PO, DN, R   ACROSS STREAM  FROM  M(I)       $TM 900
C                                                                       $TM 910
        TS(I,1)=RK/(1.+0.5/G1*(M(I))**2.)                               $TM 920
        PS(I,1)=QK*(TS(I,1))**G2                                        $TM 930
        RHO(I,1)=PS(I,1)/TS(I,1)                                        $TM 940
        U(I,1)=M(I)*SQRT(G*TS(I,1))                                     $TM 950
        PO(I)=PS(I,1)*(1.+0.5/G1*(M(I))**2.)**G2                        $TM 960
        Z=  2.0 / (RHO(IM,1)*U(IM,1)+RHO(I,1)*U(I,1))                   $TM 970
        DN(I)=  DN(IM) + Z * (PSI(I) - PSI(IM))                         $TM 980
        DW1=  DN(I)                                                     $TM 990
        DW2=   DN2   - DN(I)                                            $TM1000
        IF (DW2 .LT. 0.0)  DW2= 0.0                                     $TM1010
42      R(I,1)=R1*(1.-DW1/DN2)*EXP(-RDECAY*DW1*ABS(R1))+R2*DW1/DN2*     $TM1020
     1        EXP(-RDECAY*DW2*ABS(R2))                                  $TM1030
C                                                                       $TM1040
C      SEARCH FOR TSA WHERE  DN2-DN(N) .EQ. 0.0                         $TM1050
C                                                                       $TM1060
        IF (J .EQ. 1)  DIFFA=  DN2-DN(N)                                $TM1070
        IF (J .GE. 2)  DIFFB=  DN2-DN(N)                                $TM1080
C                                                                       $TM1090
C          IF SUFFICIENTLY SMALL INTERVAL, THEN EXIT                    $TM1100
C                                                                       $TM1110
       IF (ABS(TSB-TSA) .LE. 0.0000001) GO TO 41                        $TM1120
        IF (ABS(DIFFB) .LE. 0.0001)  GO TO 41                           $TM1130
40     CONTINUE                                                         $TM1140
       WRITE(6,900) DN2,DN(N),TSA,TSB                                   $TM1150
900    FORMAT(/7X,22HNO CONVERGENCE   DN2 =,F10.5,5X,7HDN(N) =,F10.5,5X $TM1160
     1      5HTSA =,F10.5,5X,5HTSB =,F10.5/)                            $TM1170
41     CONTINUE                                                         $TM1180
```

```
C
C      REGION FROM NOZZLE MIDDLE TO INNER DUCT WALL
C
       PK=1.
       QK=1.
       RK=1.
      NPRR=NMID-1
      DO 46 KK=1,NPRR
       I=NMID-KK
       IM=I+1
C
C          JUMP POINT TO TERTIARY STREAM
C
       IF(I .EQ. L1-1) PK=P03/P01
       IF(I .EQ. L1-1) QK = P03/P01*(T01/T03)**G2
       IF(I .EQ. L1-1) RK = T03/T01
       SX=RK*R(IM,1)*(PSI(I)-PSI(IM))
       M(I)= M(IM) - SX/(G**.5 * QK * TS( IM,1)**(G2+0.5) )
       B1=     ((PK/PS( IM,1)) **(1./G2)-1.)*2.*G1
       IF((I.EQ.L1-1).AND.(B1.LE.0.0))  TMAX=TMAX*0.995
       IF((I.EQ.L1-1).AND.(B1.LE.0.0))  GO TO 595
       IF (I .EQ. L1-1) M(I)=SQRT(B1)
C
C      SET TS, PS, RHO, U, P0, DN, R   ACROSS STREAM  FROM  M(I)
C
       TS(I,1)=RK/(1.+0.5/G1*(M(I))**2.)
       PS(I,1)=QK*(TS(I,1))**G2
       RHO(I,1)=PS(I,1)/TS(I,1)
       U(I,1)=M(I)*SQRT(G*TS(I,1))
       P0(I)=PS(I,1)*(1.+0.5/G1*(M(I))**2.)**G2
       Z=  2.0 / (RHO(IM,1)*U(IM,1)+RHO(I,1)*U(I,1))
       DN(I)=  DN(IM) + Z * (PSI(I) - PSI(IM))
       DW1=  DN(I)
       IF (DW1 .LT. 0.0)  DW1= 0.0
       DW2=   DN2    - DN(I)
46     R(I,1)=R1*(1.-DW1/DN2)*EXP(-RDECAY*DW1*ABS(R1))+R2*DW1/DN2*
     1        EXP(-RDECAY*DW2*ABS(R2))
       RETURN
599    CONTINUE
       MAXERR= 1
       RETURN
      END
```

```
      SUBROUTINE    EDDY(N,NN,NX,U,PS,CC,DS,E,E1,RHO,VIS,R, Y,
     1         S4, S5,  DEL1,DEL2,DEL3,DEL4,CORE1,CORE2,CORE3)
C     ***************************************************
C     SUBROUTINE EDDY
C
C     CALCULATES EDDY VISCOSITY VALUES - ARRAY E(N)
C     REQUIRED PARAMETERS:
C          ARRAYS  OMG, RHO, VIS, Y, PS, DS, S1...S5
C          VARIABLES  N, NN, N1...N6, CORE1, CORE2, CORE3
C     METHOD
C          CALCULATES REFERENCE VARIABLES (DP, DELI, DELO,
C                                          UREFI, UREFO, CC)
C          FOR EACH J FROM 1 TO N
C               DETERMINES THE REGION BY CHECKING N1...N5
C               CALCULATES EDDY VISCOSITY:   E(J)
C     ***************************************************
C
      REAL LZ,LZR,LM
      DIMENSION S1(190),S2(190),S3(190),S4(190),S5(190),E(190),E1(190),
     1          DS(190),VIS(190),THETA(190,2),R(190,2),U(190,2),
     2          TS(190,2),RHO(190,2),NX(6)
      DIMENSION OMG(190), Y(190),PS(190,2)
C     DOUBLE    OMG(190), Y(190),PS(190,2)
C
C     CALCULATE REFERENCE VARIABLES
C
      DP=(PS(1,2)-PS(1,1))/DS(1)
       DELI=Y(2)-Y(1)
       DELO=Y(N)-Y(NN)
      UREFI=SQRT((U( 2,2)/DELI-DELI*DP/(2.*VIS(2)))  / RHO( 2,2) *
     1  VIS( 2))
      DP=(PS(N,2)-PS(N,1))/DS(N)
      UREFO=SQRT((U(NN,2)/DELO-DELO*DP/(2.*VIS(NN))  )/RHO(NN,2)*
     1  VIS(NN))
       IF (CORE1 .EQ. 0.0)  CC= 0.08
      SCORE = CORE1 + CORE2 + CORE3
C
C     DO LOOP WHICH COMPUTES EDDY VALUES E(J)
C          YS:   DIMENSIONLESS DIST TO WALL
C          LZR:  CURVATURE EFFECT
C          RN:   RICHARDSON NUMBER
C
C     DIFFERING CALCULATIONS FOR EACH REGION (N1...N6)
C
      DO 10 J=2,NN
      JM=J-1
      JP=J+1
      IF (J. .LT. NX(1) ) GO TO 20
      IF (J .GT. NX(6) ) GO TO 21
      IF (J .LE. NX(2) ) GO TO 30
      IF (J .GE. NX(5) ) GO TO 30
      IF (J .LT. NX(3) ) GO TO 29
      IF (J .GT. NX(4) ) GO TO 40
      GO TO 30
C
C     INNER WALL
C
 20    DRN=Y(J)-Y(1)
```

```
      YS=DRN*UREFI*RHO(J,1)/VIS(J)
      LZ=.41*(1.-EXP(-YS/26.))*DRN
      AZ1=0.09*DEL1
      DDZ=  1.0 - 2.0 * DRN / Y(N)
      IF(SCORE.EQ.0.0) AZ1=(.14-.08*DDZ**2-.06*DDZ**4)*Y(N)*.5
      LZ=  LZ * 1.2
      IF(LZ .GT. AZ1) LZ= AZ1
      GO TO 25
C
C     CORE REGIONS
C
 30   E(J)=0.
      GO TO 10
C
C     DEL2  REGION
C
29    NX3=NX(3)
      NX2=NX(2)
      LZ=CC*DEL2*(1.+.6*U(NX2,2)/U(NX3,2))
       GO TO 25
C
C     DEL3 REGION
C
40    NX5=NX(5)
      NX4=NX(4)
      LZ=CC*DEL3*(1.+.6*U(NX5,2)/U(NX4,2))
       GO TO 25
C
C      OUTER WALL
C
21     CONTINUE
       DRN=   Y(N)-Y(J)
       YS=DRN*UREFO*RHO(J,1)/VIS(J)
       LZ=0.41*(1.-EXP(-YS/26.))*DRN
       AZ4=0.09*DEL4
      DDZ=1.0-2.0*DRN/Y(N)
       IF  (SCORE.EQ.0.0) AZ4=(0.14-0.08*DDZ**2-0.06*DDZ**4)*Y(N)*0.5
        IF(LZ .GT. AZ4) LZ=AZ4
       LZ=  LZ * 1.2
 25    CONTINUE
       RN=  2.0 * (R(J,1) / RHO(J,1))
       RN=RN/(S5(J)*(U(JP,1)-U(J,1))+S4(J)*(U(J,1)-U(JM,1)))
         IF (RN .LE. 0.0)  LZR=  LZ*(2.0 - EXP(3.0*RN))
         IF (RN .GT. 0.0)  LZR=  LZ*EXP(-3.0*RN)
        EI=Y(J)-Y(JM)
        EJ=Y(JP)-Y(J)
        EK=Y(JP)-Y(JM)
      DUY=ABS(EI*(U(JP,2)-U(J,2))/(EK*EJ)+EJ*(U(J,2)-U(JM,2))/(EK*EI))
        E(J)=DUY*LZR*LZR
10     CONTINUE
       E(1)= 0.0
       E(N)= 0.0
        RETURN
      END
      SUBROUTINE WALLS(XW,YW,XX,YY,T,CUR,N,MQ)
C  *********
C    SUBROUTINE WALLS SMOOTHS THE BOUNDARY  DATA USING A LEAST
C     SQUARES PROCEDURE.  IT ALSO INTERPOLATES THE SMOOTHED
C     DATA TO GET THE CURVATURE AND SLOPE AT ANY POINT
C    ***********
      DIMENSION XW(99,2),YW(99,2),YP(99,2),CURV(99,2),NDI(10,2),
     1       YB(99,2)
```

```
      COMMON/KURV/MST,MSS,NDI,CURV
      IF(MQ.EQ.1) M=MST
      IF(MQ.EQ.2) M=MSS
      IF(XX.LT.999.) GO TO 60
      N=N+M-1
      NX=N
      MP=M-1
      DO 4 J=1,MP
      JK=M-J
      JK1=JK+1
      NP=NDI(JK1,MQ)-NDI(JK,MQ)+1
      DO 3 I=1,NP
      K=NX+1-I
      L=K-JK
      XW(K,MQ)=XW(L,MQ)
3     YW(K,MQ)=YW(L,MQ)
      NDI(JK1,MQ)=NX
4     NX=NX-NP
      DO 5 I=1,N
5     YB(I,MQ)=YW(I,MQ)
      KM=1
      DO 198 MP=1,M
      NDA=NDI(MP,MQ)
      KL=NDA-KM+1
      IF(KL.LT.3)  GO TO 700
      IF(KL.EQ.3)  GO TO 30
      IF(KL.EQ.4) GO TO 40
      L=5
16    DO 20 KS=KM,NDA
      IF(L.EQ.4)  GO TO 17
      CALL LSQ(KS,KM,NDA,L,A1,A2,A3,XW(1,MQ),YB(1,MQ))
17    YW(KS,MQ)=A1+A2*XW(KS,MQ)+A3*XW(KS,MQ)**2
      YP(KS,MQ)=A2+2.*A3*XW(KS,MQ)
      CURV(KS,MQ)=-2.*A3/(1.+YP(KS,MQ)**2)**1.5
      IF(ABS(CURV(KS,MQ)).LT..001) CURV(KS,MQ)=0.
20    CONTINUE
      GO TO 198
30    C1=XW(KM,MQ)-XW(KM+1,MQ)
      C2=XW(KM,MQ)-XW(NDA,MQ)
      C3=XW(KM+1,MQ)-XW(NDA,MQ)
      YPP=2.*(YW(KM,MQ)/C1/C2-YW(KM+1,MQ)/C1/C3+YW(NDA,MQ)/C2/C3)
      DO 31 K=KM,NDA
      YP(K,MQ)=(2.*XW(K,MQ)-XW(NDA,MQ)-XW(KM+1,MQ))*YW(KM,MQ)/C1/C2
     1         -(2.*XW(K,MQ)-XW(KM,MQ)-XW(NDA,MQ))*YW(KM+1,MQ)/C1/C3
     2         +(2.*XW(K,MQ)-XW(KM,MQ)-XW(KM+1,MQ))*YW(NDA,MQ)/C2/C3
      CURV(K,MQ)=-2.*YPP/(1.+YP(K,MQ)**2)**1.5
      IF(ABS(CURV(K,MQ)).LT.0.001) CURV(K,MQ)=0.
31    CONTINUE
      GO TO 198
40    L=4
      CALL LSQ(KS,KM,NDA,L,A1,A2,A3,XW(1,MQ),YB(1,MQ))
      GO TO 16
198   KM=NDA+1
      RETURN
60    NDA=1
      DO 70 I=1,M
      J=NDI(I,MQ)
      IF((XX.GE.XW(NDA,MQ)).AND.(XX.LE.XW(J,MQ)))  GO TO 71
      IF(I.EQ.M) GO TO 71
70    NDA=J+1
71    JJ=J-1
      DO 72 K=NDA,JJ
```

```
      IF((XX.GE.XW(K,MQ)).AND.(XX.LE.XW(K+1,MQ)))   GO TO 73
72     CONTINUE
73     IF(K.EQ.NDA) K=NDA+1
      C1=XW(K+1,MQ)-XW(K,MQ)
      C2=XW(K+1,MQ)-XW(K-1,MQ)
      C3=XW(K,MQ)-XW(K-1,MQ)
      S1=XX-XW(K+1,MQ)
      S2=XX-XW(K,MQ)
      S3=XX-XW(K-1,MQ)
      YY=S3*S2*YW(K+1,MQ)/C1/C2-S1*S3*YW(K,MQ)/C1/C3+S1*S2*YW(K-1,MQ)/
     1  C2/C3
      CUR=S2*S3*CURV(K+1,MQ)/C1/C2-S1*S3*CURV(K,MQ)/C1/C3
     1   +S1*S2*CURV(K-1,MQ)/C2/C3
       T=S3*S2*YP(K+1,MQ)/C1/C2-S1*S3*YP(K,MQ)/C1/C3+S1*S2*YP(K-1,MQ)
     1  /C2/C3
      T=ATAN(T)
      RETURN
700    WRITE(6,900)
900   FORMAT(//4X,34H LESS THAN THREE POINTS IN SEGMENT)
       STOP
      END
       SUBROUTINE LSQ(KS,KM,NDA,L,A1,A2,A3,C,B)
       DIMENSION B(99),C(99),S(2,4)
       K=KS
       IF((KS.EQ.NDA).OR.(KS.EQ.NDA-1)) K=NDA-2
       IF((KS.EQ.KM).OR.(KS.EQ.KM+1)) K=KM+2
       IF(L.EQ.4) K=KM+2
       DO 5 I=1,2
       DO 5 J=1,4
       N=I-1
       S(I,J)=0.
       DO 4 M=1,L
       KK=K+M-3
       IF(C(KK).EQ.0.) C(KK)=1.E-06
       IF(B(KK).EQ.0.) B(KK)=1.E-06
4      S(I,J)= S(I,J)+B(KK)**N*C(KK)**(J-N)
5      CONTINUE
       A=L
       D=A*(S(1,2)*S(1,4)-S(1,3)**2)-S(1,1)*(S(1,1)*S(1,4)-S(1,2)*S(1,3)
     1  )+S(1,2)*(S(1,1)*S(1,3)-S(1,2)**2)
       A1=(S(2,1)*(S(1,2)*S(1,4)-S(1,3)**2)-S(2,2)*(S(1,1)*S(1,4)-
     1  S(1,2)*S(1,3))+S(2,3)*(S(1,1)*S(1,3)-S(1,2)**2))/D
       A2=(S(2,1)*(S(1,2)*S(1,3)-S(1,1)*S(1,4))+S(2,2)*(A*S(1,4)-
     1  S(1,2)**2)+S(2,3)*(S(1,1)*S(1,2)-A*S(1,3)))/D
       A3=(S(2,1)*(S(1,1)*S(1,3)-S(1,2)**2)-S(2,2)*(A*S(1,3)-
     1  S(1,1)*S(1,2))+S(2,3)*(A*S(1,2)-S(1,1)**2))/D
       RETURN
      END
       SUBROUTINE SOLV(DS,N,NN,PR,PRT,G)
      REAL M(190)
      DIMENSION S1(190),S2(190),S3(190),S4(190),S5(190),E(190),E1(190),
     1          DS(190),VIS(190),X(190),Y(190),P0(190),H(190),
     2          THETA(190,2),R(190,2),U(190,2),TS(190,2),RHO(190,2),
     3          A(190),B(190),C(190),D(190)
      DIMENSION OMG(190),DN(190),PS(190,2)
C     DOUBLE    OMG(190),DN(190),PS(190,2)
        DIMENSION PSI(180)
C       DOUBLE PSI(190)
       COMMON/INCD/ U,PS,TS,M,P0,THETA,R,X,Y,DN
       COMMON/SOLV/ S1,S2,S3,E,E1,H,S4,S5
       COMMON/INEDY/PSI,RHO,VIS
      CL=(G-1.)/G
      C0 = 0.
```

```
      IF(PR .EQ. .7) C0 = 1.
      DO 10 J=2,NN
      JP=J+1
      JM= J-1
      Y1 =E1(JP)*(VIS(JP)/PR + RHO(JP,1)*E(JP)/PRT)
      Y2=E1(JM)*( VIS(JM)/PR + RHO(JM,1)*E(JM)/PRT)
      Y3 =E1(J) *(VIS(J )/PR + RHO(J ,1)*E(J )/PRT)
      Y4 = (Y1+Y3)/S2(J)
      Y5 =(Y3+Y2)/S3(J)
       Y6=  Y4 / S1(J)
       Y7=  Y5 / S1(J)
       Y8=  Y6 * U(J,1)
       Y9=  Y7 * U(J,1)
       A(JM) = U(J,1)/DS(J) + Y8 + Y9
      B(JM) = -Y8
      C(JM) = -Y9
       IF(PR.NE.0.7) D(JM)=U(J,1)**2/DS(J)-(PS(J,2)-PS(J,1))/DS(J)*(1./
     1  (RHO(J,1)))
       IF(PR.EQ.0.7) D(JM)=TS(J,1)*U(J,1)/DS(J)+(CL*U(J,1)/(RHO(J,1)))
     1 *(PS(J,2)-PS(J,1))/DS(J)+RHO(J,1)*CL*(VIS(J)+RHO(J,1)*E(J))*(U
     2 (J,1)**2*(S5(J)*(U(JP,2)-U(J,2))+S4(J)*(U(J,2)-U(JM,2))))**2
 10    CONTINUE
        A(1) =A(1) + C0*C(1)
       A(N-2) = A(N-2) + C0*B(N-2)
       CALL CALC(A,B,C,D,H,N)
 413   RETURN
      END
      SUBROUTINE CALC(A,B,C,D,H,J)
       DIMENSION A(190),B(190),C(190),D(190),W(190),G(190),H(190),Q(190)
       N2 = J-2
       N1= J-2
       W(1) = A(1)
      G(1)  = D(1)/W(1)
      DO 1 K = 2,N2
       K1 = K-1
       Q(K1) = B(K1)/W(K1)
       W(K) = A(K)-C(K)*Q(K1)
  1    G(K) = ( D(K)-C(K)*G(K1))/W(K)
       H(N2) = G(N2)
       N3 =J-3
       DO 2 K = 1,N3
       KK = N2-K
  2    H(KK) = G(KK) -Q(KK)*H(KK+1)
       RETURN
      END
       SUBROUTINE DSMOVE(XW,YW,XX,YY,DSX,THETA,RR,NPAIR,J)
C
C      *************************************************
C
C      SUBROUTINE DSMOVE
C      FINDS NEW X VALUE AT A GIVEN DISTANCE DSX ALONG WALL SURFACE
C      METHOD:
C              GUESSES VALUES OF DX UNTIL (DSX - DS COMPUTED) = 0.0
C      *************************************************
C
       DIMENSION XW(99,2),YW(99,2)
C
C     APPROXIMATE POINT   X(N), Y(N)
C          AT DISTANCE DS(N)  ALONG UPPER WALL
C      (TWO METHODS OF APPROXIMATING, DEPENDING ON R1)
```

```
      DXSTAR=  DSX * ABS(COS(THETA))
      XSTAR=  DXSTAR + XX
      CALL WALLS(XW,YW, XSTAR, YSTAR, TSTAR, RSTAR,NPAIR, J)
      DDSTAR=  DSX - CURVDS(DXSTAR, RR,RSTAR, THETA,TSTAR,YY,YSTAR)
      DXB= DSX * ABS(COS((THETA + TSTAR)*0.5))
      XB=  DXB + XX
      CALL WALLS(XW,YW,XB, YB, TB, RB, NPAIR, J)
      DDIFFB=  DSX - CURVDS(DXB, RR, RB, THETA, TB,YY, YB)
      DO 510 I=1,50
      IF (ABS(DDIFFB-DDSTAR) .LE. 0.1 **8) GO TO 512
      DXB=(DDIFFB*DXSTAR - DDSTAR*DXB) / (DDIFFB-DDSTAR)
      IF (DXB .LT. 0.0)  DXB=  DXSTAR * 0.1
      XB=  DXB + XX
      CALL WALLS(XW, YW, XB, YB, TB, RB, NPAIR, J)
      DDIFFB=  DSX - CURVDS(DXB, RR, RB, THETA, TB,YY, YB)
C
C    CHECK FOR TERMINATION
C
      IF (ABS(XB-XSTAR).LE. 0.1 **6)  GO TO 512
      IF (ABS(DDIFFB) .LE. 0.1 ** 7)  GO TO 512
510   CONTINUE
      WRITE(6,900) XB, DDIFFB
900   FORMAT(/7X,21HNO CONVERGENCE DSMOVE,5X,4HXB =F10.4,5X,8HDDIFFB =,
     1       F10.5/)
512   CONTINUE
      XX=  XB
      RETURN
      END
      REAL FUNCTION CURVDS(DX,RA,RB,TA,TB,YA,YB)
      IF (ABS(TA-TB) .LE. 0.0001) GO TO 500
      DR=  ABS(RA+RB)
      IF (DR .LE. 0.02  ) GO TO 500
      CURVDS=  (2.0/DR) * ABS(TB-TA)
      RETURN
500   CURVDS=  SQRT(DX ** 2 + (ABS(YB-YA)**2) )
      RETURN
      END
      SUBROUTINE  LOOK(N,NN,K3,NCORE,DEL1,DEL2,DEL3,DEL4,
     1                 CORE1,CORE2,CORE3, NX, P0, DN, U, DIP, NMAX)
C    ******************************************
C    SUBROUTINE LOOK
C
C    CALCULATES
C         EDGES OF POTENTIAL FLOWS - N1... N6
C         WIDTH OF FLOWS - DEL1, DEL2, DEL3, DEL4
C         WIDTH OF CORES - CORE1, CORE2, CORE3
C    REQUIRED PARAMETERS:
C         ARRAYS  DN, P0
C         VARIABLES  N, NN, NSTEP, K3, P01, P02, P03
C    METHOD
C         CALCULATES TOLERANCES  TOL1, TOL2, TOL3, TOL4
C         CALCULATES  ARRAY DIP  -  RATE OF CHANGE OF P0
C
C         FOR EACH J FROM 2 TO NN
C              SCAN ARRAY DIP(J)
C              FOR REGION WHERE DIP GT TOLERANCE
C                   SET FLOW REGION EDGE  NX
C                   SET FLOW REGION WIDTH DELX
C              FOR REGION WHERE DIP(J) LT TOLERANCE
C                   SET FLOW REGION EDGE NX
C                   COMPUTE CORE REGION AREA COREX
C    **************************************************
```

```
C
      DIMENSION X(190),Y(190),U(190,2),DIP(190),NX(6),P0(190),H(190)
      DIMENSION DN(190)
C       DOUBLE DN(190)
        COMMON/CONST/   P01,P02,P03, T01,T02,T03, X1, XC, YC
C
C     SET TOLERANCES
C
      TOL1=0.01*P03/DN(N)     / P01
      TOL2=0.01*(P01-P03)/DN(N)    / P01
      TOL3= 0.01*(P01-P02)/DN(N)   / P01
      TOL4= 0.01*P02/DN(N)         / P01
      IF(K3.EQ.0)CORE3=0.
C
C     THIS LOOP FINDS A VALUE FOR NMAX
C
      DO 11 J=2,N
      DIP(J)=(P0(J)-P0(J-1))/(DN(J)-DN(J-1))
       DIP(J)=  ABS(DIP(J))
11    CONTINUE
       NMAX=1
       DO 12 J=2,N
       IF (U(J,2) .GT. U(NMAX,2) )  NMAX=  J
12     CONTINUE
C
C       THIS LOOP COMPUTES EDGE VALUES FOR THE PRIMARY,,SECONDARY
C                 AND TERTIARY POTENTIAL FLOWS
C      TERTIARY FLOW REGION
C
       DO 50 I=2,N
       IF   (DIP(I) .LT. TOL1)  GO TO 52
50     CONTINUE
52     CONTINUE
       NX(1)=  I-1
C
C      SECONDARY FLOW REGION
C
       DO 60 M=2,N
       I= N + (2-M)
       IF   (DIP(I) .LT.  TOL4)  GO TO 62
60     CONTINUE
62     CONTINUE
       NX(6)= I
       IF (NX(1) .GT. NX(2) )  CORE3= 0.0
       IF (NX(6) .LT. NX(5) )  CORE2= 0.0
       IF (CORE3 .EQ. 0.0)  GO TO 102
C
C      TERTIARY CORE REGION
C
       NPRR=NX(1)+1
       DO 70 I=NPRR,N
       IF (DIP(I) .GT. TOL2)  GO TO  72
70     CONTINUE
72     CONTINUE
       NX(2)=  I-1
       NPRR=NX(2)+1
       DO 75 I=NPRR,N
       IF (DIP(I) .LT. TOL2)  GO TO 77
75     CONTINUE
77     CONTINUE
       NX(3)=  I-1
102    CONTINUE
       IF (CORE2 .EQ. 0.0)  GO TO 101
```

```
C
C       SECONDARY CORE REGION
C
        NPRR=NX(6)
        DO 80 M=2,NPRR
        I= NX(6) + (2-M)
        IF (DIP(I) .GT. TOL3)  GO TO 82
 80     CONTINUE
 82     CONTINUE
        NX(5)=  I
        NPRR=NX(5)
        DO 85 M=2,NPRR
        I= NX(5) + (2-M)
        IF (DIP(I) .LT. TOL3)  GO TO 88
 85     CONTINUE
 88     CONTINUE
        NX(4)=  I
 101    CONTINUE
C
C       CHECK IF PRIMARY CORE EXISTS
C
        IF (CORE1 .EQ. 0.0)  GO TO 110
        IF (CORE3 .NE. 0.0)  GO TO 120
        DO 130  I=2,N
        IF (DIP(I) .LT. TOL2)  GO TO 132
 130    CONTINUE
 132    CONTINUE
        NX(3)= I-1
 120    CONTINUE
        IF (CORE2 .NE. 0.0)  GO TO 140
        DO 135 M=2,N
        I= N + (2-M)
        IF (DIP(I) .LT. TOL3)  GO TO 137
 135    CONTINUE
 137    CONTINUE
        NX(4)= I
 140    CONTINUE
        IF (NX(3) .GE. NX(4) )   CORE1=0.0
 110    CONTINUE
C
C      CHECK VALUES OF      , CORE1, CORE2, CORE3
C      NMAX IS MAXIMUM U( ) VALUE
C
       IF (CORE1 .NE. 0.0 )  GO TO 670
C
C      CORE1 = 0.0
C
       NX(3)=NMAX
       NX(4)=NMAX
 670   IF (CORE3 .NE. 0.0)  GO TO 680
C
C      CORE3 = 0.0
C
       NX(1)=NX(3)
       NX(2)=NX(3)
 680   IF (CORE2 .NE. 0.0)  GO TO 90
C
C      CORE2 = 0.0
C
       NX(5)=NX(4)
       NX(6)=NX(4)
 90     CONTINUE
       NX1=NX(1)
```

```
      NX2=NX(2)
      NX3=NX(3)
      NX4=NX(4)
      NX5=NX(5)
      NX6=NX(6)
      CORE1=DN(NX4)-DN(NX3)
      CORE2=DN(NX6)-DN(NX5)
      CORE3=DN(NX2)-DN(NX1)
      DEL1=DN(NX1)
      DEL3=DN(NX5)-DN(NX4)
      DEL2=DN(NX3)-DN(NX2)
      DEL4=DN(N)-DN(NX6)
      IF ((CORE1.EQ.0.).AND.(CORE2.EQ.0.).AND.(CORE3.EQ.0.)) NCORE= 0
      RETURN
      END
       SUBROUTINE CORDS(N,X,Y,THETA,U,RHO,PSI,DN,NDUCT,DDIST)
C
C      ****************************************
C      SUBROUTINE CORDS
C      CALCULATES
C          DISTANCE BETWEEN STREAM LINES  (ARRAY DN)
C          DRAWS ARCS FROM TOP AND BOTTOM WALL TO MIDDLE OF DUCT
C          COMPARES  Y COORD OF BOTH ARCS AT MIDDLE POINT
C      ****************************************
C
      REAL M(190)
      DIMENSION X(190),Y(190),PO(190),H(190),THETA(190,2),R(190,2),
     1          U(190,2),TS(190,2),RHO(190,2)
      DIMENSION OMG(190),DN(190)
C     DOUBLE    OMG(190),DN(190)
        DIMENSION PSI(180)
C       DOUBLE PSI(190)
       DN(1) = 0.
      DO 2 J=2,N
      JM=J-1
       Z=2./(RHO(JM,2)*U(JM,2)+RHO(J,2)*U(J,2))
       DN(J)=  DN(JM)  +  Z * (PSI(J) - PSI(JM))
2      CONTINUE
       DO 50  J=2,NDUCT
       JM=J-1
       X(J)= X(JM)-(DN(J)-DN(JM))*SIN(THETA(J,2))
       Y(J)= Y(JM)+(DN(J)-DN(JM))*COS(THETA(J,2))
50     CONTINUE
       NPAR=NDUCT+1
       NPARR=N-1
       DO 52 MM=NPAR,NPARR
       J=N-MM+NDUCT
       JM=J+1
       X(J)= X(JM)-(DN(J)-DN(JM))*SIN(THETA(J,2))
       Y(J)= Y(JM)+(DN(J)-DN(JM))*COS(THETA(J,2))
52     CONTINUE
       YCOMP=  Y(NDUCT+1) + (DN(NDUCT) - DN(NDUCT+1)) * COS(THETA(J,2))
       DDIST=  (Y(NDUCT) - YCOMP) / DN(N)
       RETURN
      END
       SUBROUTINE REMOVE(N,NN,NGRID,NX,NMAX,OMG,S1,S2,S3,S4,S5,DS,VIS,
     1  DN,X,Y,M,PO,H,THETA,R,PS,U,TS,RHO)
C
C     ****************************************
C     SUBROUTINE REMOVEPTS
```

```
C      REMOVES GRID POINTS AS PROGRAM ITERATES
C           (TO REDUCE COMPUTING TIME)
C      REQUIRED PARAMETERS:
C           N, NGRID, ARRAY  U(90)
C      (ALL OTHER ARRAYS)
C      METHOD:
C           FOR EVERY TENTH NSTEP
C                SCANS ARRAY U
C                IF   (U(I-1) - U(I) / U(I) ) LESS THAN TOLERANCE,
C                     THEN GRID POINT IS REMOVED
C           CORE REGIONS ARE NOT SCANNED
C      ARRAYS COPIED ARE    (OMG,S1,S2,S3,S4,S5,DS,VIS,DN,X,Y,M,P0,H,
C                           THETA,R,PS,U,TS,RHO)
C      ********************************************
C
       REAL M(190)
       DIMENSION S1(190),S2(190),S3(190),S4(190),S5(190),E(190),E1(190),
      1          DS(190),VIS(190),X(190),Y(190),P0(190),H(190),
      2          THETA(190,2),R(190,2),U(190,2),TS(190,2),RHO(190,2),
      3          NXSAVE(6),NX(6)
       DIMENSION OMG(190),DN(190),PS(190,2)
C      DOUBLE    OMG(190),DN(190),PS(190,2)
        NGRID=  NGRID+1
       IF (NGRID .LT. 10)  RETURN
       NGRID= 0
        DO 300 I=1,6
        NXSAVE(I)= NX(I)
300     CONTINUE
       J= 1
       DO 24 I=2,N
C
C      DO NOT REMOVE GRID POINTS FROM CORE REGIONS
C
        IF ((I.GT. (NXSAVE(1)-3)) .AND. (I.LE.(NXSAVE(2)+3))) GO TO 51
        IF ((I.GT. (NXSAVE(3)-3)) .AND. (I.LE.(NXSAVE(4)+3))) GO TO 51
        IF ((I.GT. (NXSAVE(5)-3)) .AND. (I.LE.(NXSAVE(6)+3))) GO TO 51
       IF (I .EQ. N) GO TO 51
       UR=  ABS((U(I,2) - U(J,2)) / U(I,2))
        DNN=   ABS( (DN(I) - DN(J) ) / DN(N) )
        IF  ( (UR .LT. 0.02).AND.(DNN .LT. 0.015))  GO TO 50
C
C      GO TO STATEMENT 50 WILL REMOVE GRID POINT
C
51     J= J + 1
50     OMG(J) = OMG(I)
       DS(J) = DS(I)
       VIS(J) = VIS(I)
       DN(J) = DN(I)
       X(J) = X(I)
       Y(J) = Y(I)
       M(J) = M(I)
       P0(J) = P0(I)
       H(J) = H(I)
       THETA(J,2) = THETA(I,2)
       R(J,2) = R(I,2)
       PS(J,2) = PS(I,2)
       U(J,2) = U(I,2)
       TS(J,2) = TS(I,2)
       RHO(J,2) = RHO(I,2)
        IF (NX(1) .EQ. I)  NX(1) = J
        IF (NX(2) .EQ. I)  NX(2) = J
        IF (NX(3) .EQ. I)  NX(3) = J
```

```
      GO TO 320
350   CONTINUE
      XXA=XX
      DRCA=DRC
      IF(FW*DRC .GT.0.0) DRCB= DRCB*0.5
320   CONTINUE
      FW= DRC
C
C        CHECK FOR TERMINATION
C
      IF(ABS(XXB-XXA) .LE. 1.E-6)  GO TO 310
      IF(ABS(DRC) .LE. 1.E-9 ) GO TO 310
300   CONTINUE
       WRITE(6,900)
900   FORMAT(/7X,35HNO CONVERGENCE IN FINDING XC AND YC/)
310   CONTINUE
      RETURN
     END
      REAL FUNCTION CENTRE(XNOZ,YNOZ,MNOZ,BNOZ,XX,RC,XW,YW,NPAIR,J)
C
C     ******************************************
C     FUNCTION CENTRE
C          AIDS IN FINDING A SUITABLE TANGENT TO UPPER WALL
C     GIVEN XNOZ, XX - PROJECTS TWO TANGENTS
C          FINDS INTERSECTION XC, YC
C          RETURNS DIFFERENCE BETWEEN LENGTH OF RADIUS
C     ******************************************
C
      REAL MNOZ, MM
     DIMENSION XW(99,2),YW(99,2)
      DIST(XX,YY,XXC,YYC)=SQRT((ABS(XX-XXC)**2)+(ABS(YY-YYC)**2))
      CALL WALLS(XW,YW,XX,YY,TT,RR,NPAIR,J)
      MM=  TAN(TT)
      BB=  YY - (MM* XX)
      IF (ABS(MNOZ-MM) .GT. 0.00001)  XC=  (BB-BNOZ) / (MNOZ-MM)
      IF (ABS(MNOZ-MM) .LE. 0.00001)  XC=  (XNOZ+XX) * 0.5
      YC=  MNOZ*XC + BNOZ
      RC=  DIST(XNOZ,YNOZ,XC,YC)
      CENTRE=  DIST(XNOZ,YNOZ,XC,YC) - DIST(XX,YY,XC,YC)
      RETURN
     END
      SUBROUTINE OMGSET(OMG,JSTART,JFIN,JSTSER,JFNSER,DMST,DMFN,DMMID,
    1                   RST,RFIN)
C
C     ******************************************
C     SUBROUTINE OMGSET
C     SETS OMG DISTRIBUTION  ARRAY OMG
C     FOR EACH CORE REGION, OMGS ARE CONSTANT IN MIDDLE
C                           CLOSE TOGETHER AT EDGES
C     OMG(JSTART) AND OMG(JFIN)   MUST ALREADY BE SPECIFIED
C     ******************************************
C
     DIMENSION OMG(190)
C    DOUBLE    OMG(190)
      JCONST=  JSTART + JSTSER + 1
      OMG(JCONST)=  OMG(JSTART) + DMMID
     NPRR=JCONST+1
     NPRRR=JFIN-JFNSER-1
     DO 500 J=NPRR,NPRRR
500   OMG(J)=OMG(J-1)+DMMID
C     SOLVE FOR RST IN SERIES  A+AR+AR**2+...
      XA=  1.5
      FXA=  SERIES(DMMID,DMST,JSTSER,XA)
```

```
      IF (NX(4) .EQ. I)  NX(4) = J
      IF (NX(5) .EQ. I)  NX(5) = J
      IF (NX(6) .EQ. I)  NX(6) = J
      IF (NMAX .EQ. I)  NMAX= J
 24   CONTINUE
      N= J
      NN = N - 1
       DO 20  J=2,NN
       JP=  J+1
       JM=  J-1
       S1(J) = OMG(JP) - OMG(JM)
       S2(J)=  OMG(JP) - OMG(J)
       S3(J)=  OMG(J) - OMG(JM)
       S4(J)=  S2(J) / S3(J) / S1(J)
       S5(J)=  S3(J) / S2(J) / S1(J)
 20    CONTINUE
      RETURN
      END
       SUBROUTINE ARCDIS(XNOZ,YNOZ,TNOZ,XX,RC,XW,YW,NPAIR,J)
C
C     ******************************************
C      SUBROUTINE ARCDIS
C      FINDS XX  (ON WALL SPECIFIED) SUCH THAT
C          ARC RADIUS RC IS TANGENT AT XX
C          AND PASSES THROUGH NOZZLE AT ANGLE TNOZ
C      **********************************************
C
C      FIND EQUATION OF LINE TANGENT TO X1
C
       REAL MNOZ
      DIMENSION XW(99,2),YW(99,2)
       DIST(XX,YY,XXC,YYC)=SQRT((ABS(XX-XXC)**2)+(ABS(YY-YYC)**2))
       MNOZ=  TAN(TNOZ)
       BNOZ=  YNOZ - MNOZ * XNOZ
C
C      CHOOPPING TECHNIQUE TO FIND CENTRE(X2A) .LT. 0.0
C                                AND CENTRE(X2B) .GT. 0.0
C
       DO 200 K =1,NPAIR
       XXB= XW(K,J)
       DRCB=  CENTRE(XNOZ,YNOZ,MNOZ,BNOZ, XXB,RC,XW,YW,NPAIR,J)
       IF (DRCB .GT. 0.0) GO TO 210
 200   CONTINUE
 210   CONTINUE
       XXA=  XW(K-2,J)
       DRCA=  CENTRE(XNOZ,YNOZ,MNOZ,BNOZ,XXA,RC,XW,YW,NPAIR,J)
       FW=  DRCA
C
C      MODIFIED REGULA FALSI ALGORITHM
C
       DO  300 K=1,50
C
C      GUESS NEW X2
C
       XX=      (DRCB*XXA - DRCA*XXB) / (DRCB-DRCA)
       DRC=  CENTRE(XNOZ,YNOZ,MNOZ,BNOZ,XX,RC,XW,YW,NPAIR,J)
C
C          CHANGE APPROPRIATE ENDPOINT TO MIDPOINT
C
       IF (DRCA*DRC .GT.0.0) GO TO 350
       XXB= XX
       DRCB= DRC
       IF (FW*DRC.GT. 0.0) DRCA=DRCA*.5
```

```
      XB=  1.3
      FXB=  SERIES(DMMID,DMST,JSTSER,XB)
      DO 505 J=1,50
      XB=      (FXB*XA - FXA*XB)/(FXB-FXA)
      IF (XB .GT. 2.0)   XB= 2.0
      FXB=     SERIES(DMMID,DMST,JSTSER,XB)
C
C     CHECK FOR TERMINATION
C
      IF (ABS(FXB) .LE. 1.E-04)  GO TO 510
505   CONTINUE
      WRITE(6,900) JSTART
900   FORMAT(/7X,22HNO SOLUTION   JSTART =,I5/)
510   CONTINUE
      RST= XB
C
C     SOLVE FOR RFIN IN SERIES
C
      XA=  1.5
      FXA=  SERIES(DMMID,DMFN,JFNSER,XA)
      XB=  1.3
      FXB=  SERIES(DMMID,DMFN,JFNSER,XB)
      DO 525 J=1,50
      XB=  (FXB*XA - FXA*XB) / (FXB-FXA)
      IF (XB .GT. 2.0)   XB= 2.0
      FXB=  SERIES(DMMID,DMFN,JFNSER,XB)
C
C     CHECK FOR TERMINATION
C
      IF (ABS(FXB) .LE. 1.E-04 )  GO TO 520
525   CONTINUE
      WRITE(6,901)JFIN
901   FORMAT(/7X,20HNO SOLUTION   JFIN =,I5/)
520   CONTINUE
      RFIN=  XB
      DO 550  J=1,JSTSER
550   OMG(JSTART+J)=      OMG(JSTART+J-1) + DMST * (RST**(J-1))
      DO 560 J=1,JFNSER
560   OMG(JFIN-J)=      OMG(JFIN-J+1) - DMFN * (RFIN**(J-1))
      RETURN
     END
      REAL FUNCTION SERIES(SUM,A,N,R)
      IF (R .NE. 1.0)  SERIES= (R**(N+1)-1.0) / (R-1.0) - (SUM/A)
      IF (R .EQ. 1.0) SERIES=   N+1 - (SUM/A)
      RETURN
     END
```

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2. Gerald B. Gilbert and Philip G. Hill "Analysis and Testing of Two-Dimensional Slot Nozzle Ejectors with Variable Area Mixing Sections" NASA CR-2251, May 1973.

3. P. Bradshaw "Effects of Streamline Curvature on Turbulence Flow" AGARDograph No. 169 August 1973 Available from Report Distribution and Storage Unit, NASA, Langley Field, Virginia 23365.

4. H. Schlichting "Boundary Layer Theory" McGraw Hill Book Company, New York, 1968.

5. K.R. Hedges and P.G. Hill "A Finite Difference Method for Compressible Jet Mixing in Converging - Diverging Ducts" Queen's University Thermal Sciences Report No. 3/72 June 1, 1972 Department of Mechanical Engineering, Kingston, Ontario, Canada.

6. Hickman, K.E., Gilbert, G.B., and Carey, J.H., "Analytical and Experimental Investigation of High Entrainment Jet Pumps", NASA CR-1602, July 1970.

7. Kline, S.J., and McClintock, F.A., "Describing Uncertainties in Single Sample Experiments", Mechanical Engineering, 1953.

Table 1

Mixing Section Dimensions

(Inches)

J	LOWER WALL X	LOWER WALL Y	CURV	UPPER WALL X	UPPER WALL Y	CURV
1	-6.0000	-4.1028	.11979	-3.3270	6.0206	-.03137
2	-5.9500	-4.0266	.12608	-3.3000	5.7680	-.04041
3	-5.9000	-3.9523	.13277	-3.2500	5.3572	-.06887
4	-5.5000	-3.4324	.17553	-3.1250	4.7764	-.11339
5	-5.0000	-2.9342	.21607	-3.0000	4.3953	-.13194
6	-4.5000	-2.5697	.22500	-2.7500	3.8304	-.17358
7	-4.0000	-2.2988	.22416	-2.5000	3.4011	-.20405
8	-3.5000	-2.1047	.18402	-2.2500	3.0633	-.17665
9	-3.0000	-1.9628	.15850	-2.0000	2.7705	-.16650
10	-2.5000	-1.8637	.13904	-1.5000	2.2963	-.18696
11	-2.3500	-1.8403	.14043	-1.0000	1.9315	-.20381
12	-2.3125	-1.8349	.14075	-.5000	1.6544	-.20305
13	-2.3125	-1.8390	.00000	.0000	1.4449	-.21235
14	-2.0000	-1.7783	.00000	.5000	1.2971	-.21607
15	-1.0000	-1.5840	.00000	.6250	1.2689	-.22019
16	.0000	-1.3897	.00000	.7500	1.2444	-.22389
17	.7500	-1.2440	.00000	.7500	1.2443	-.08962
18	.7500	-1.2443	.08962	.8000	1.2350	-.08984
19	.8000	-1.2350	.08984	.8500	1.2260	-.09007
20	.8500	-1.2260	.09007	1.0000	1.2009	-.07313
21	1.0000	-1.2009	.07313	1.5000	1.1298	-.07108
22	1.5000	-1.1298	.07108	2.0000	1.0773	-.06664
23	2.0000	-1.0773	.06664	2.5000	1.0423	-.05432
24	2.5000	-1.0423	.05432	3.0000	1.0205	-.04063
25	3.0000	-1.0205	.04063	3.4000	1.0095	-.01971
26	3.4000	-1.0095	.01971	3.4500	1.0082	-.01971
27	3.4500	-1.0082	.01971	3.5000	1.0069	-.01971
28	3.5000	-1.0069	.01971	3.5000	1.0070	.00000
29	3.5000	-1.0070	.00000	4.0000	.9984	.00000
30	4.0000	-.9984	.00000	5.0000	.9811	.00000
31	5.0000	-.9811	.00000	6.0000	.9639	.00000
32	6.0000	-.9639	.00000	7.0000	.9466	.00000
33	7.0000	-.9466	.00000	7.5000	.9380	.00000
34	7.5000	-.9380	.00000	7.5000	.9380	.00000
35	7.5000	-.9380	.00000	8.0000	.9380	.00000
36	8.0000	-.9380	.00000	9.0000	.9380	.00000
37	9.0000	-.9380	.00000	10.0000	.9380	.00000
38	10.0000	-.9380	.00000	10.5000	.9380	.00000
39	10.5000	-.9380	.00000	10.5000	.9380	.00000
40	10.5000	-.9380	.00000	11.0000	.9642	.00000
41	11.0000	-.9642	.00000	12.0000	1.0166	.00000
42	12.0000	-1.0166	.00000	14.0000	1.1214	.00000
43	14.0000	-1.1214	.00000	16.0000	1.2262	.00000
44	16.0000	-1.2262	.00000	18.0000	1.3310	.00000
45	18.0000	-1.3310	.00000	20.0000	1.4358	.00000
46	20.0000	-1.4358	.00000	22.0000	1.5406	.00000
47	22.0000	-1.5406	.00000	23.0000	1.5930	.00000

Table 2

Variation of Individual Integrated Traverse Mass Flows for Each Test Run

Run	Variation of Integrated Taverse Mass Flow Rate Around An Average Value	Number of Traverses
1	-4.1%, +7.3%	4
2	-3.1%, +5.1%	10
3	-2.1%, +2.2%	3
4	-3.7%, +5.9%	4
5	-2.7%, +3.7%	4
6	-1.6%, +3.0%	3
7	-3.5%, +4.6%	5
8	-3.4%, +3.5%	5
Average	-3.0%, +4.4%	

Table 3

Traverse Locations and Data Summary

	Maximum Flow - Atmospheric Discharge				Reduced Flow - Back Pressure at Discharge			
Test Number	2	3	6	7	1	4	5	8
Nozzle Position (See Fig. 3)	1	2	3	4	1	2	3	4
Nozzle Angle	22.5°	45°	45°	67.5°	22.5°	45°	45°	67.5°
Nozzle Spacing meter	0.020	0.020	0.034	0.018	0.020	0.020	0.034	0.018
Nozzle Spacing inch	0.80	0.80	1.32	0.70	0.80	0.80	1.32	0.70
Traverse Location ↓								
s = -0.070 meters, - 2.75 inches				P_{max}				P_{max}
s = -0.024 meters, - 0.95 inches		P_{max}	P_{max}	P, P_{max}		P_{max}	P_{max}	P, P_{max}
x = +0.013 meters, + 0.50 inches	P_{max}	P_{max}	P_{max}	P_{max}	P_{max}	P_{max}	P_{max}	P_{max}
x = +0.038 meters, + 1.50 inches					P_{max}	P_{max}	P_{max}	
x = +0.064 meters, + 2.50 inches								
x = +0.114 meters, + 4.50 inches	U, P_{max}	U, P_{max}	U, P_{max}	U, P_{max}	U, P_{max}	U, P_{max}	U, P_{max}	U, P_{max}
x = +0.165 meters, + 6.50 inches				U, P_{max}	U, P_{max}	U, P_{max}	U, P_{max}	U, P_{max}
x = +0.254 meters, +10.00 inches	U, P_{max}							
x = +0.318 meters, +12.50 inches		U, P_{max}	U, P_{max}	U, P_{max}	U, P_{max}	U, P_{max}	U, P_{max}	U, P_{max}
x = +0.419 meters, +16.50 inches				U, P_{max}				
x = +0.521 meters, +20.50 inches	U, P_{max}	U, P_{max}	U, P_{max}	U, P_{max}	U, P_{max}	U, P_{max}	U, P_{max}	U, P_{max}

Axial Wall Static Pressure Profiles (Figures 16 to 19)

P = Vertical Total Pressure Profile (Figures 21 and 22)

P_{max} = Maximum Pressure Location (Figures 23 to 30)

U = Vertical Velocity Profile (Figures 31 to 38)

Table 4

Summary of Experimental Test Conditions and Flow Rates

Nozzle Throat Area = .9688 in^2

Mixing Section Throat Size = 1.875 inch

Nozzle Pressure = 36.70 psia (constant)

Run No	Nozzle Temp °R T_N	Nozzle Throat Coefficient C_N	Barometric Pressure in Hg P_b	Atmospheric Temp °R T_a	Measured Nozzle Flow Rate lb/(sec-in) W_N	Mixing Section Flow lb/(sec-in) W_m	Secondary Flow Rate lb/(sec-in) W_s	Flow Ratio W_s/W_N
1	556	.959	30.02	527	.0967	.4995	.4028	4.17
2	575	.966	29.96	540	.0952	.4332	.3380	3.55
3	564	.961	29.89	536	.0952	.4627	.3675	3.86
4	561	.957	29.91	536	.0955	.5238	.4283	4.48
5	576	.959	30.16	544	.0889	.5208	.4319	4.86
6	561	.959	30.00	545	.0958	.4220	.3262	3.41
7	576	.960	30.13	543	.0955	.4390	.3435	3.60
8	582	.954	30.04	548	.0932	.5242	.4310	4.62

Table 5

Comparison of Experimental and Analytical Mass Flow Rates

Run No.	Mixing Section Mass Flow Rate From Traverse Data lb/(sec-m) (1)	Mixing Section Mass Flow Rate From Orifice Data lb/(sec-in) (2)	Percent Difference in Measured Data $\frac{(1)-(2)}{(1)}$	Analytical Mass Flow for Best Static Pressure Match lb/(sec-m) (3)	Comparison of Traverse to Analytical Mass Flow $\frac{(1)-(3)}{(1)}$	Comparison of Orifice to Analytical Mass Flow $\frac{(2)-(3)}{(2)}$
1	.4995	-	-	.4905	+ 1.8%	-
2	.4332	.3776	+ 12.8%	.4086	+ 5.7%	- 8.2%
3	.4627	.3954	+ 14.5%	.4175	+ 9.8%	- 5.6%
4	.5238	-	-	.4930	+ 5.9%	-
5	.5208	-	-	.5100	+ 2.1%	-
6	.4220	.3886	+ 7.9%	.4117	+ 2.4%	- 5.9%
7	.4390	.3846	+ 12.4%	.3960	+ 9.8%	- 3.0%
8	.5242	-	-	.4840	+ 7.7%	-

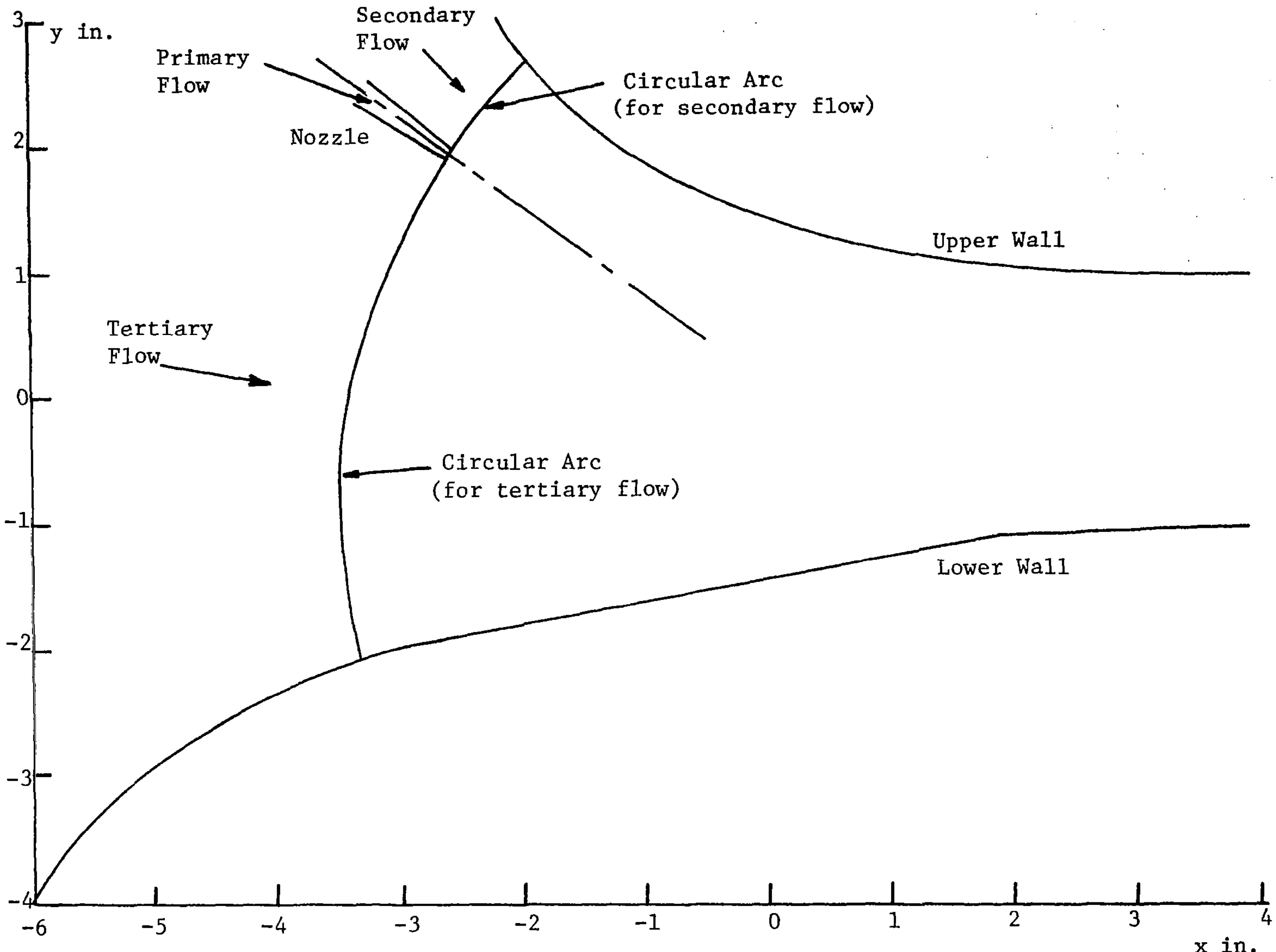
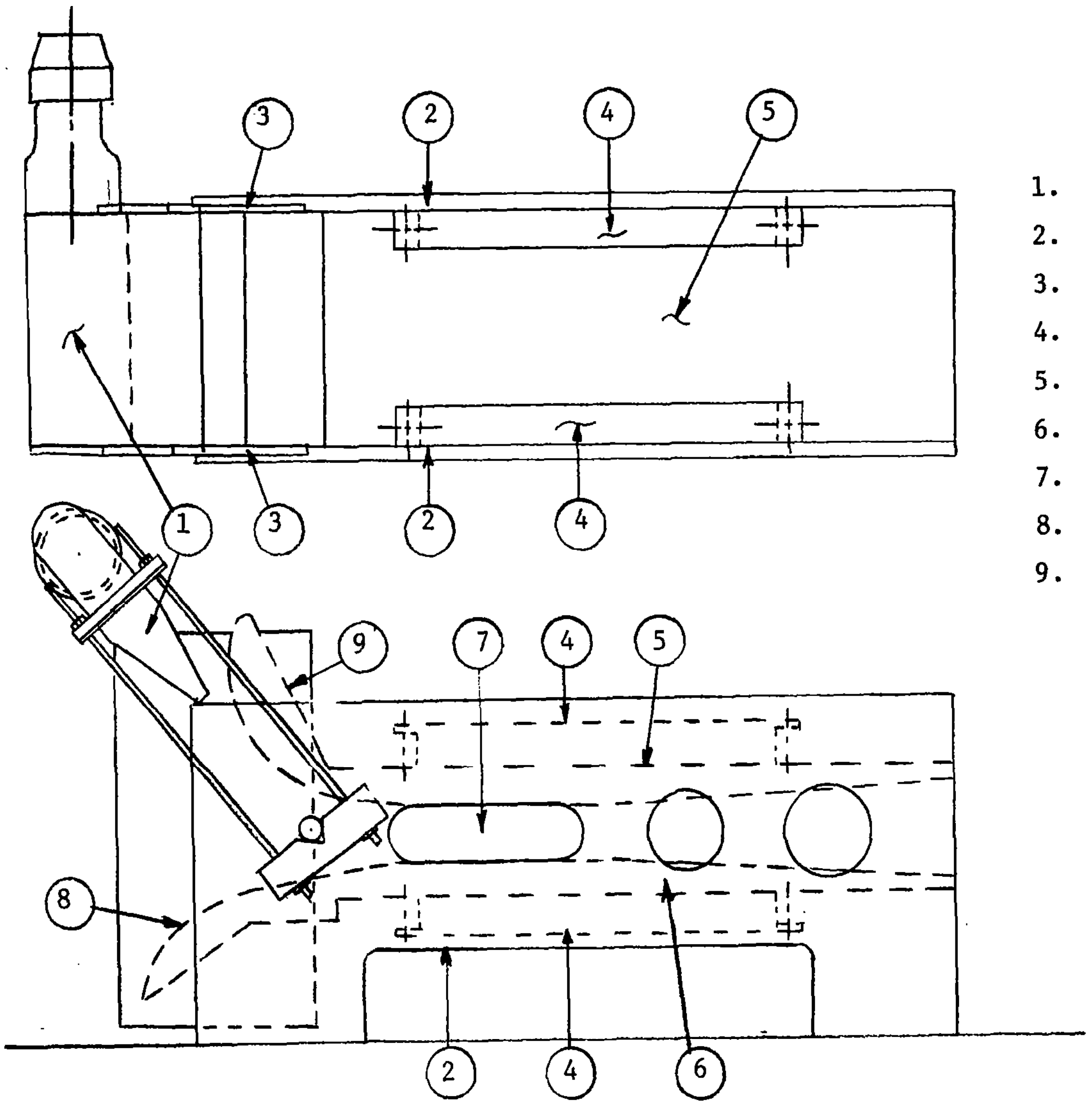


Figure 1

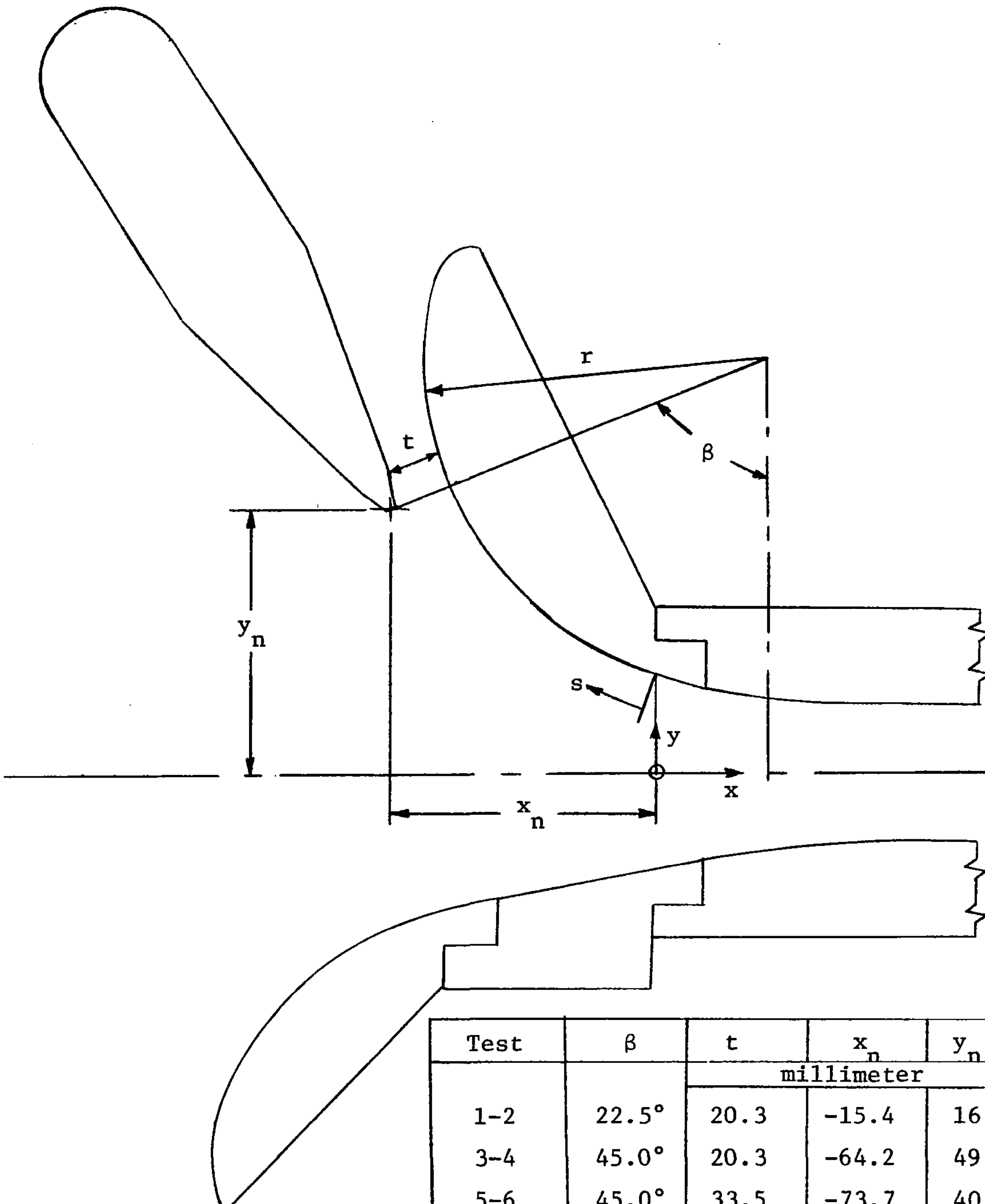
Initial Line for Flow Calculation



1. Primary Nozzle
2. Side Plates
3. Nozzle Positioning Plates
4. Suction Plenum
5. Upper Contoured Plate
6. Lower Contoured Plate
7. Plexiglass Windows
8. Inlet Bellmouth
9. Coanda Surface

Figure 2

Assembly Sketch of Two Dimensional Ejector Test Rig



Test	β	t	x_n	y_n
		millimeter		
1-2	22.5°	20.3	-15.4	16.6
3-4	45.0°	20.3	-64.2	49.7
5-6	45.0°	33.5	-73.7	40.2
7-8	67.5°	17.8	-96.0	109.2

Figure 3

Drawing of Nozzle Coordinate System
and Positioning Data for Eight Tests

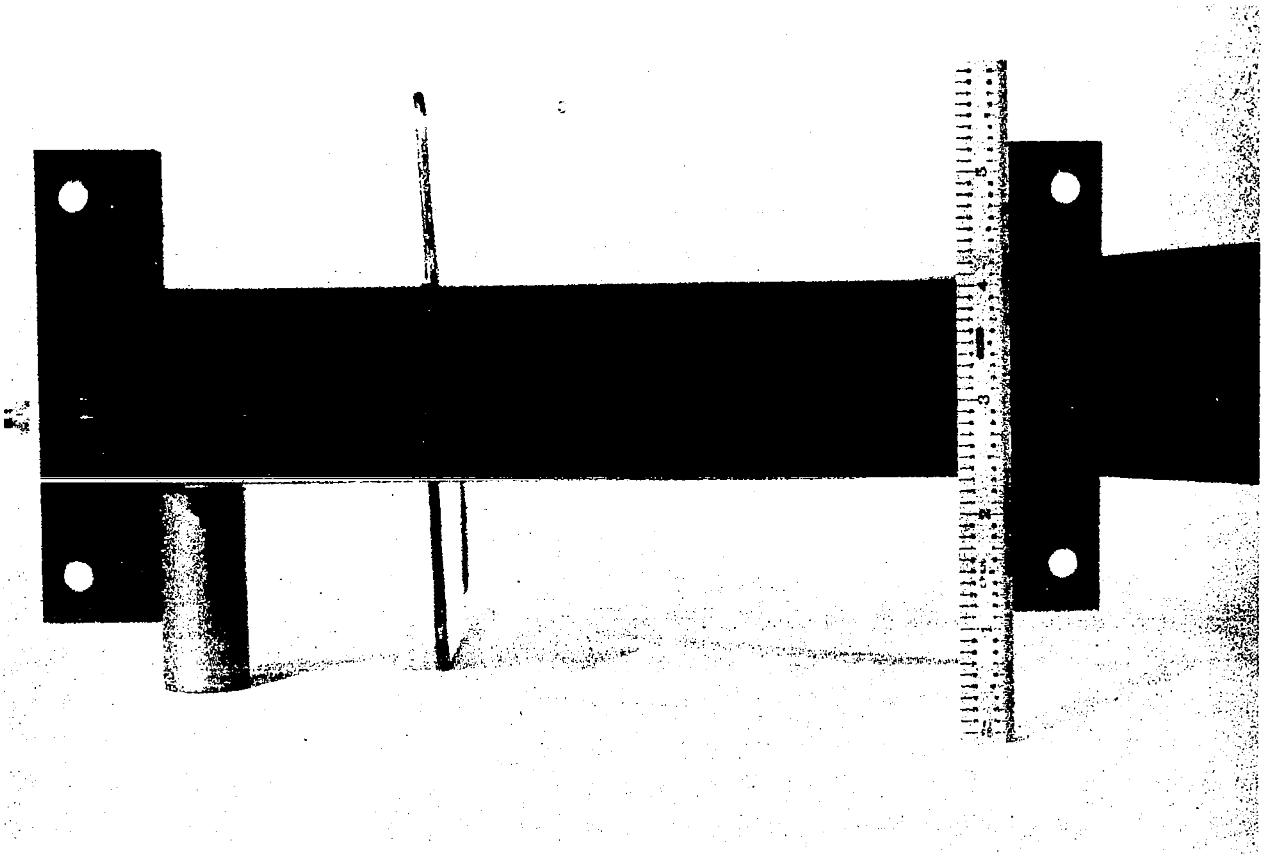


Figure 4

Primary Nozzle

1. Nozzle
2. Mixing Section Side Plate
3. Top Contoured Plate
4. Bottom Contoured Plate
5. Table Top
6. Screen
7. Solid Side Plates
8. Nozzle Positioning Plates

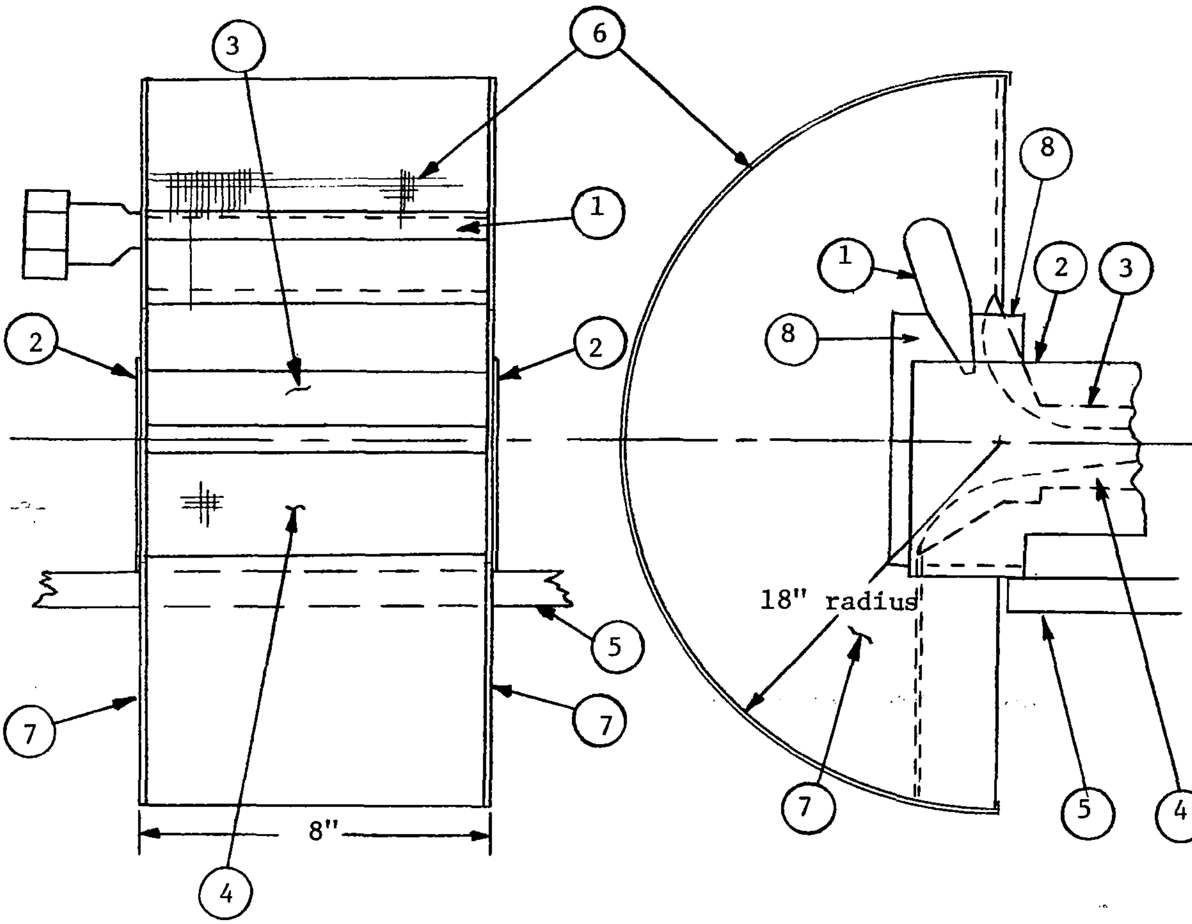


Figure 5

Extended Inlet on Ejector Test Rig

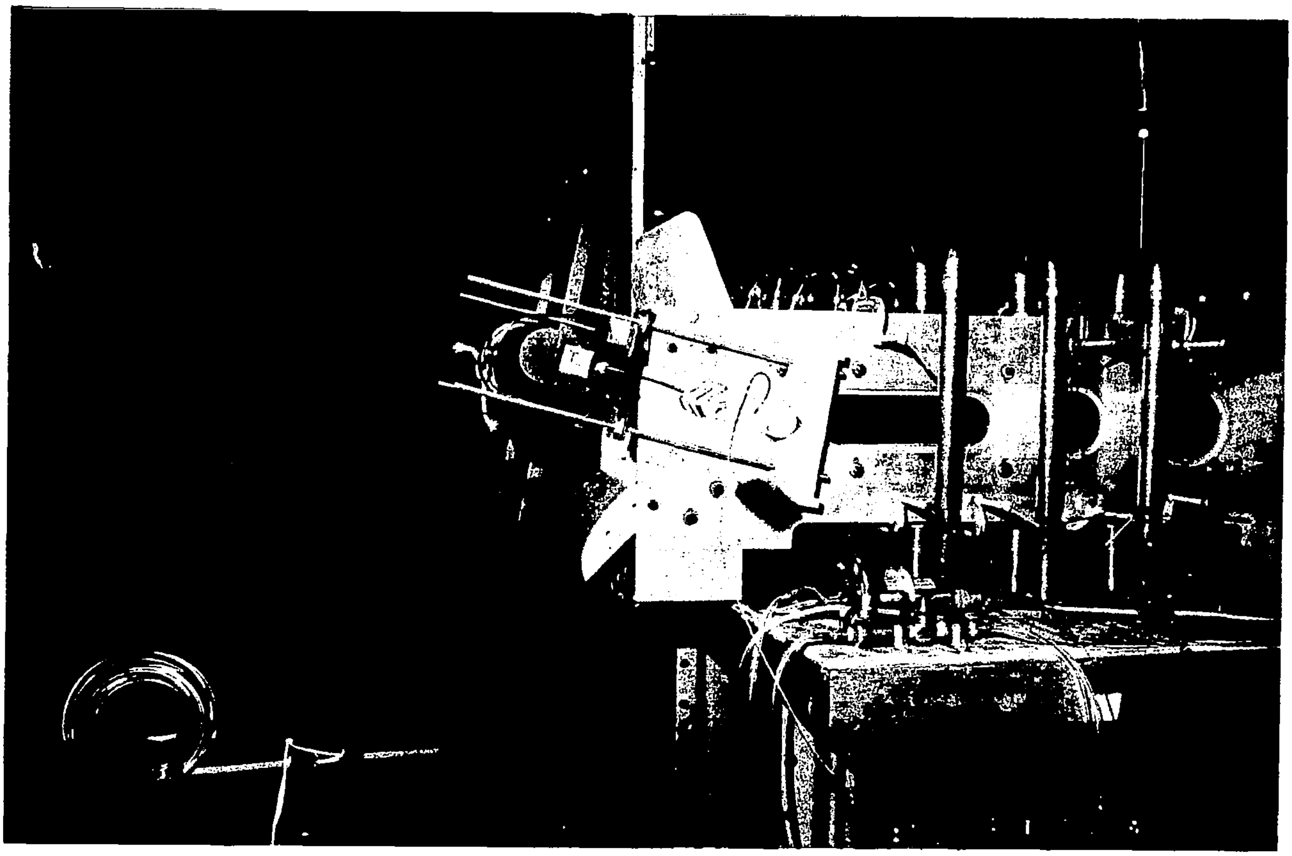


Figure 6

Side View of Test Section Showing
Nozzle Positioned at
22.5° and 0.020 meters (0.80 inches) spacing

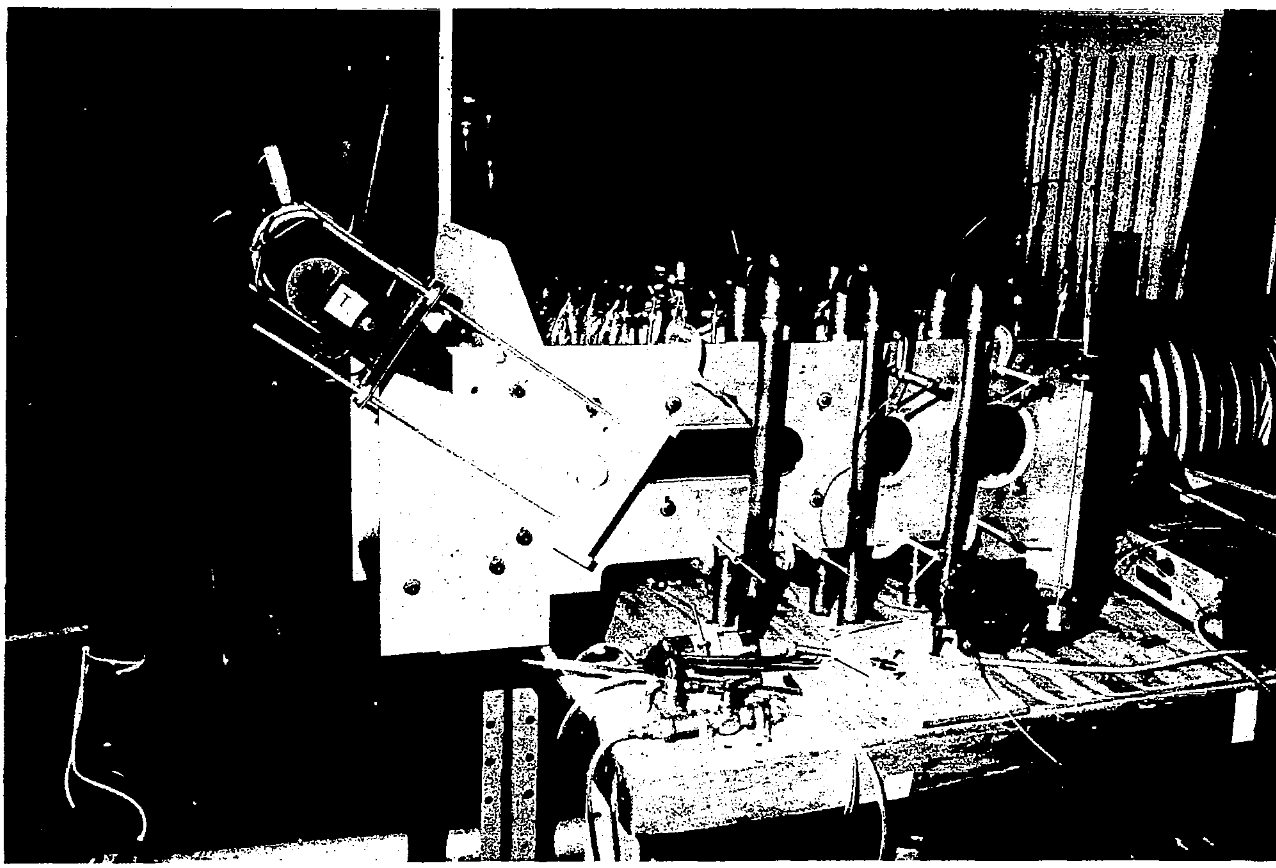


Figure 7

Side View of Test Section Showing
Nozzle Positioned at
45° and 0.034 meters (1.32 inches) spacing

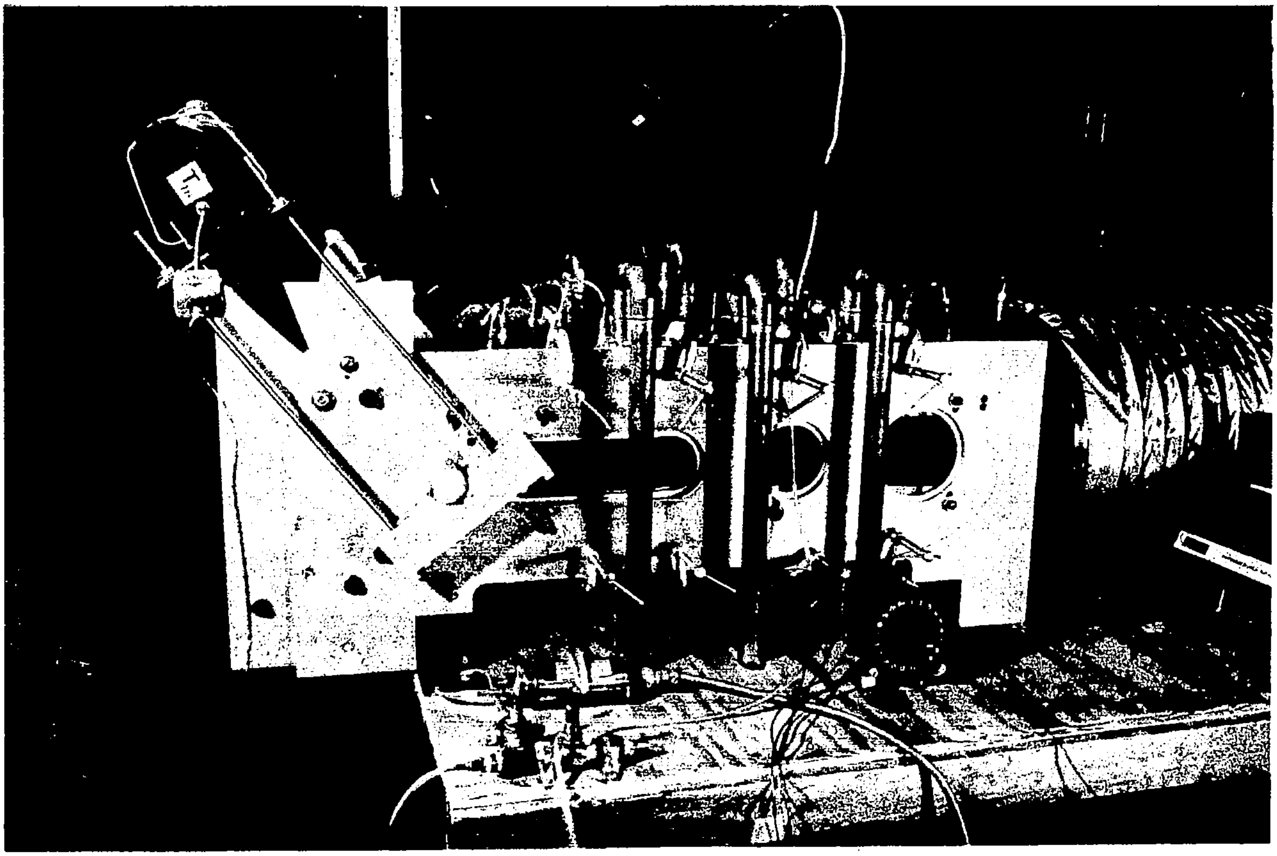


Figure 8

Side View of Test Section Showing
Nozzle Positioned at
67.5° and 0.018 meters (0.7 inches) spacing

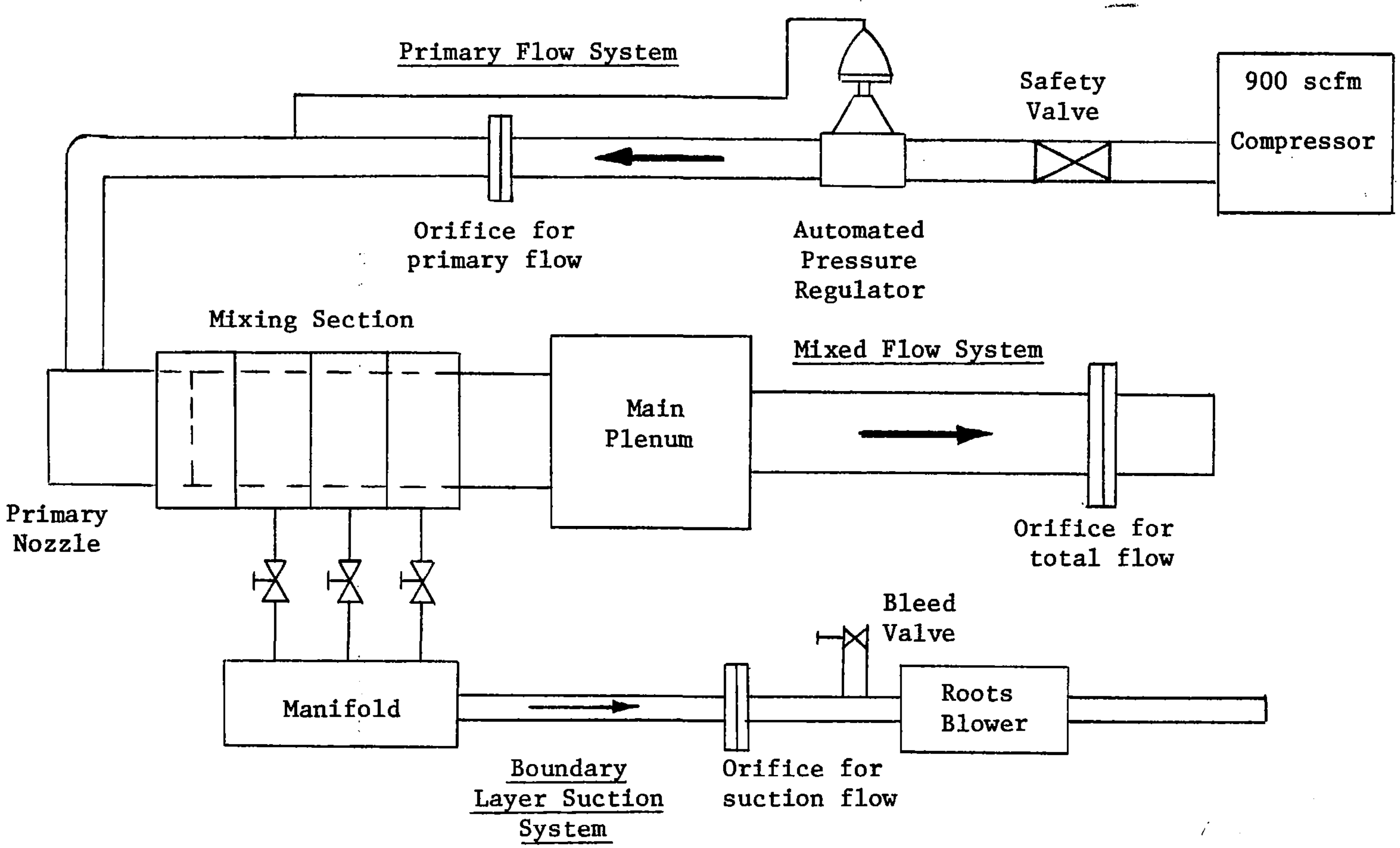


Figure 9

Schematic of Experimental Layout
for Reduced Flow Conditions

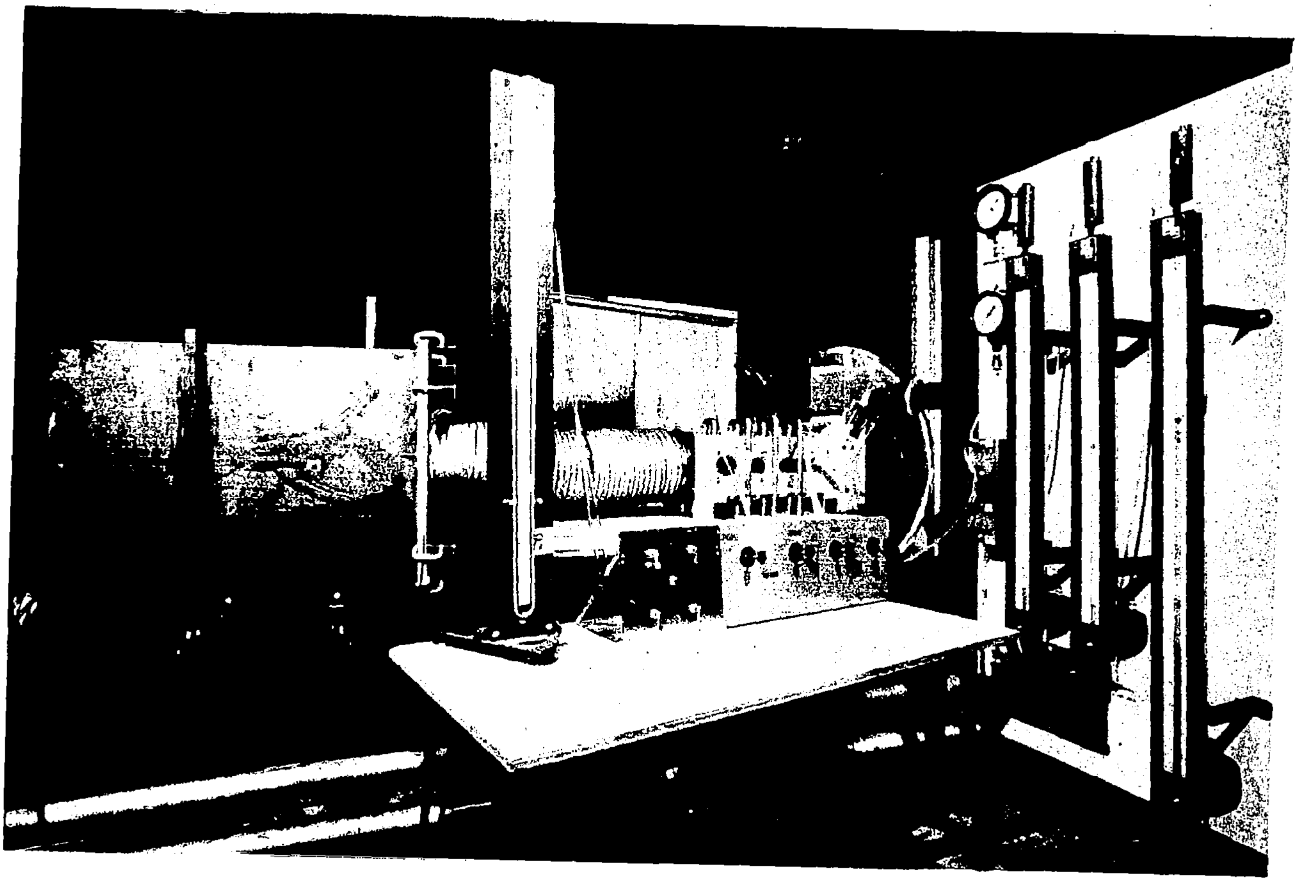


Figure 10

View of Test Facility Showing Plenum, Mixing Section, Manometer Board, and Pressure Sampling Valves

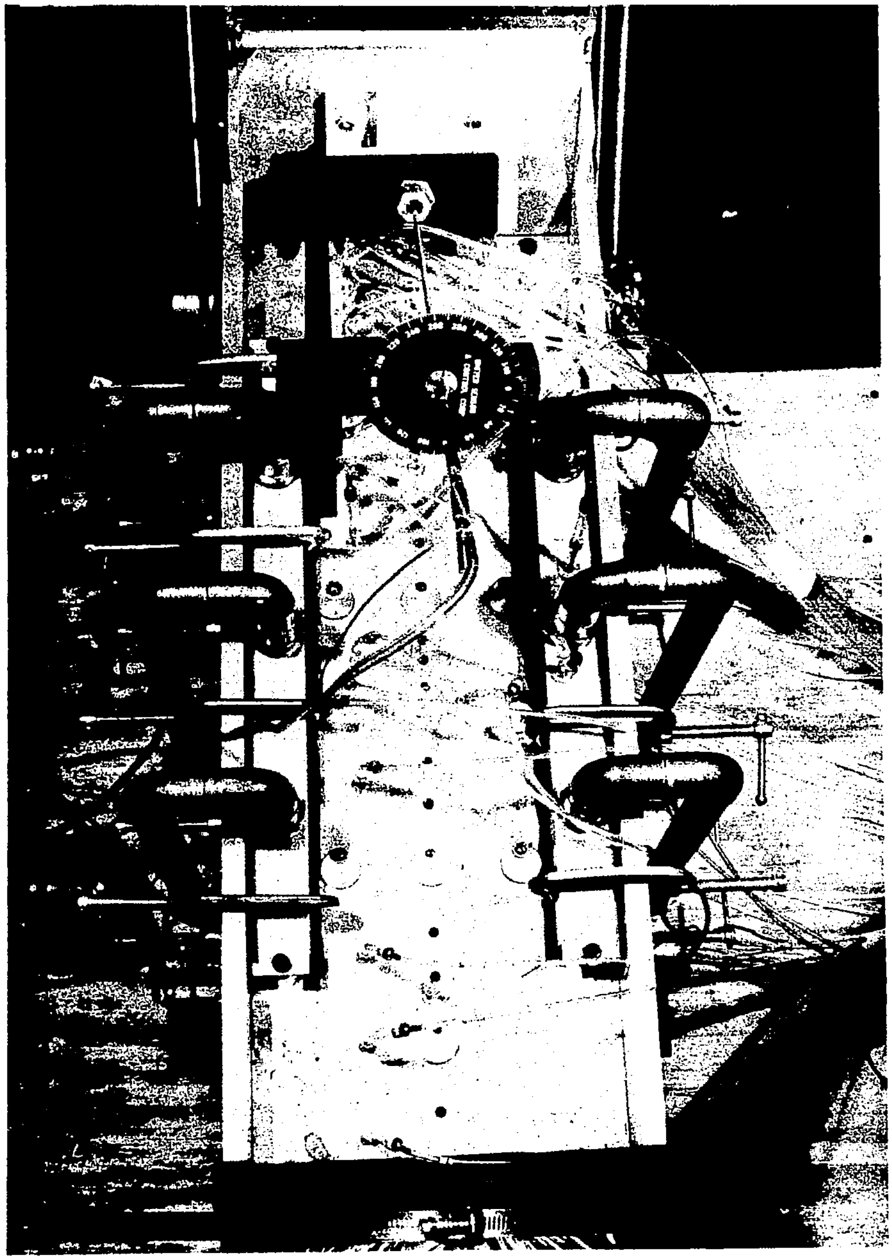


Figure 11

Top View of Mixing Section Showing
Static and Traversing Tap Locations
Traversing Probe is Positioned at
"A" Location on Coanda Surface

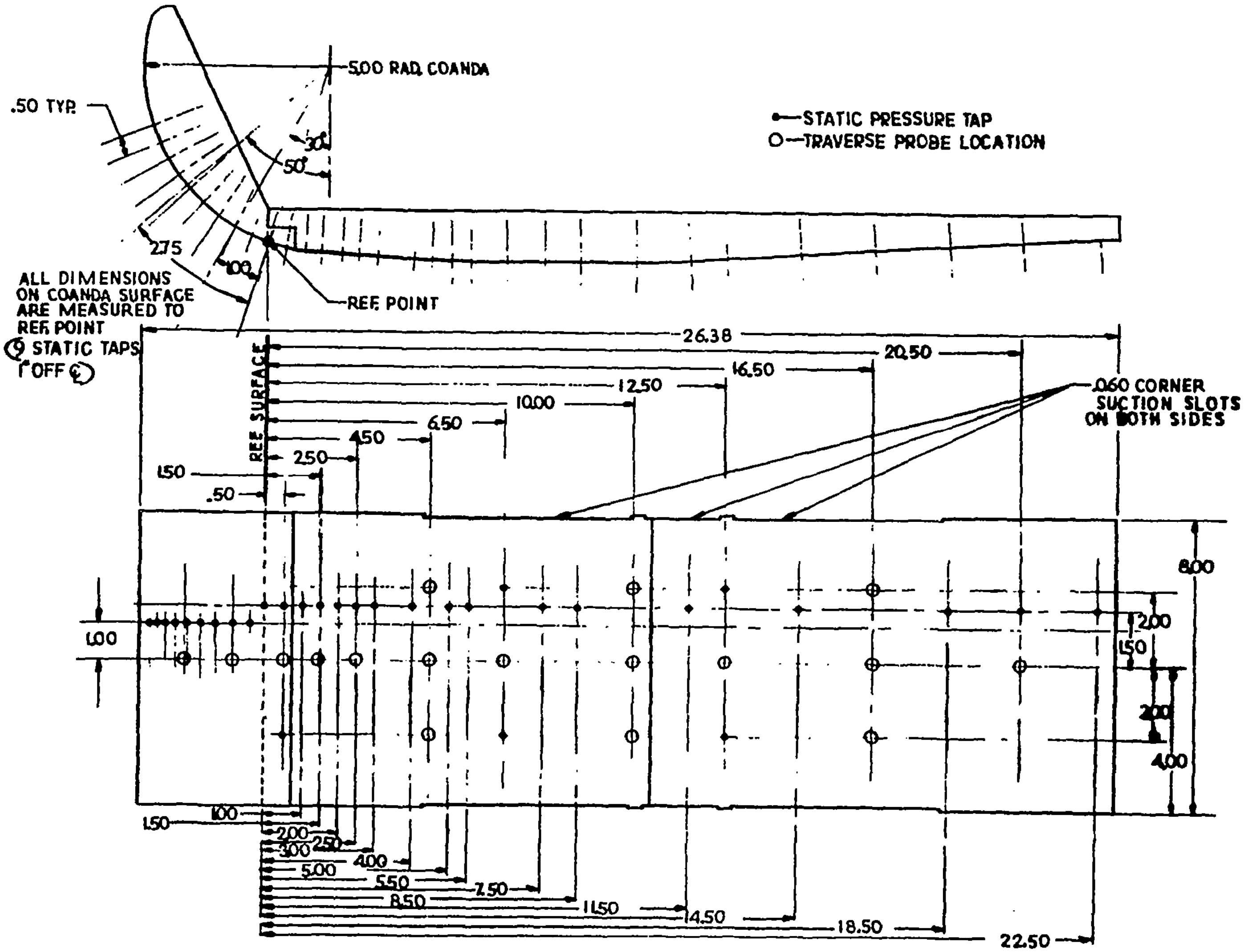


Figure 12

Mixing Section Traverse and Static Pressure Tap Locations

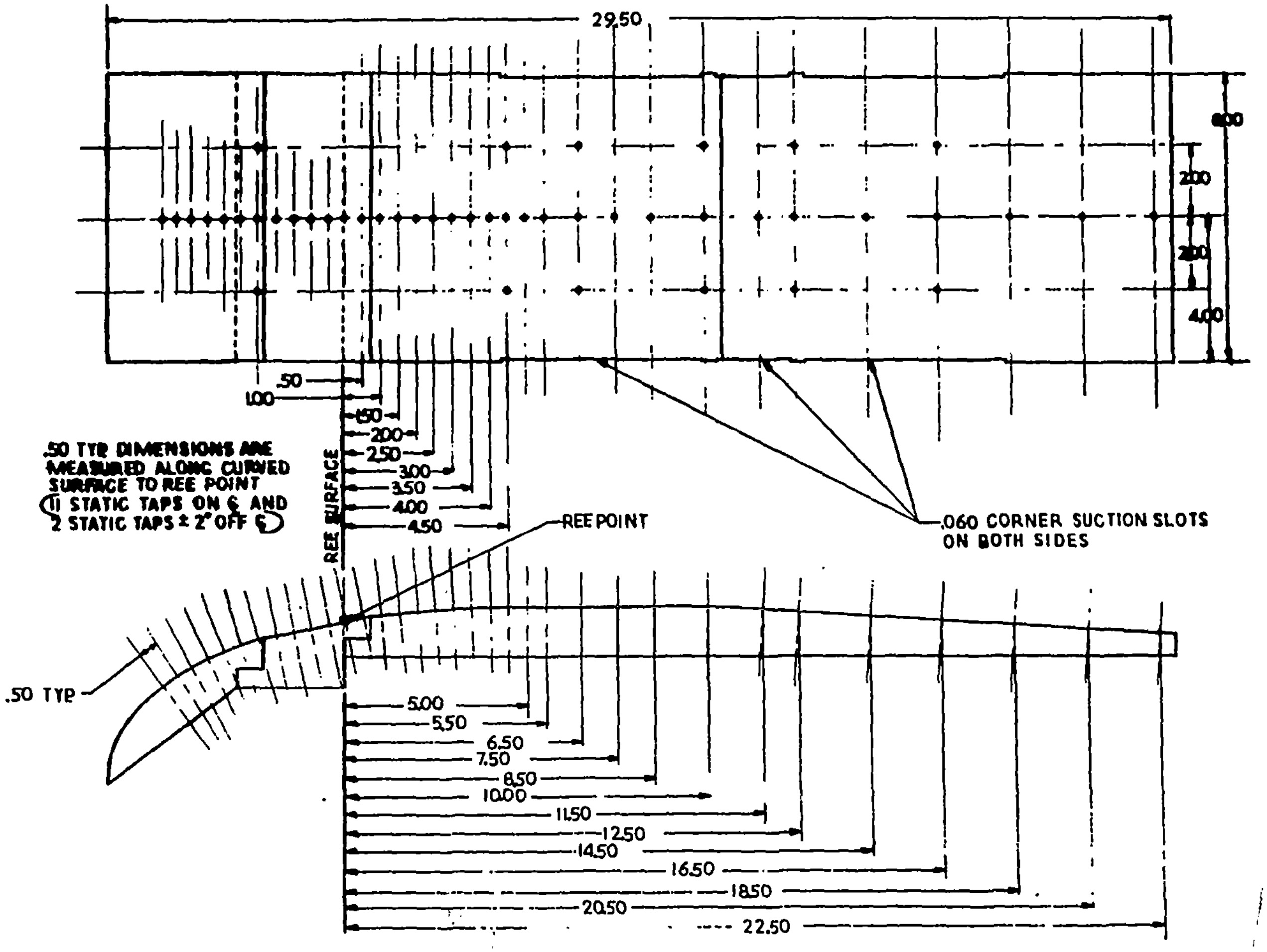


Figure 13

Mixing Section Static Pressure Tap Locations

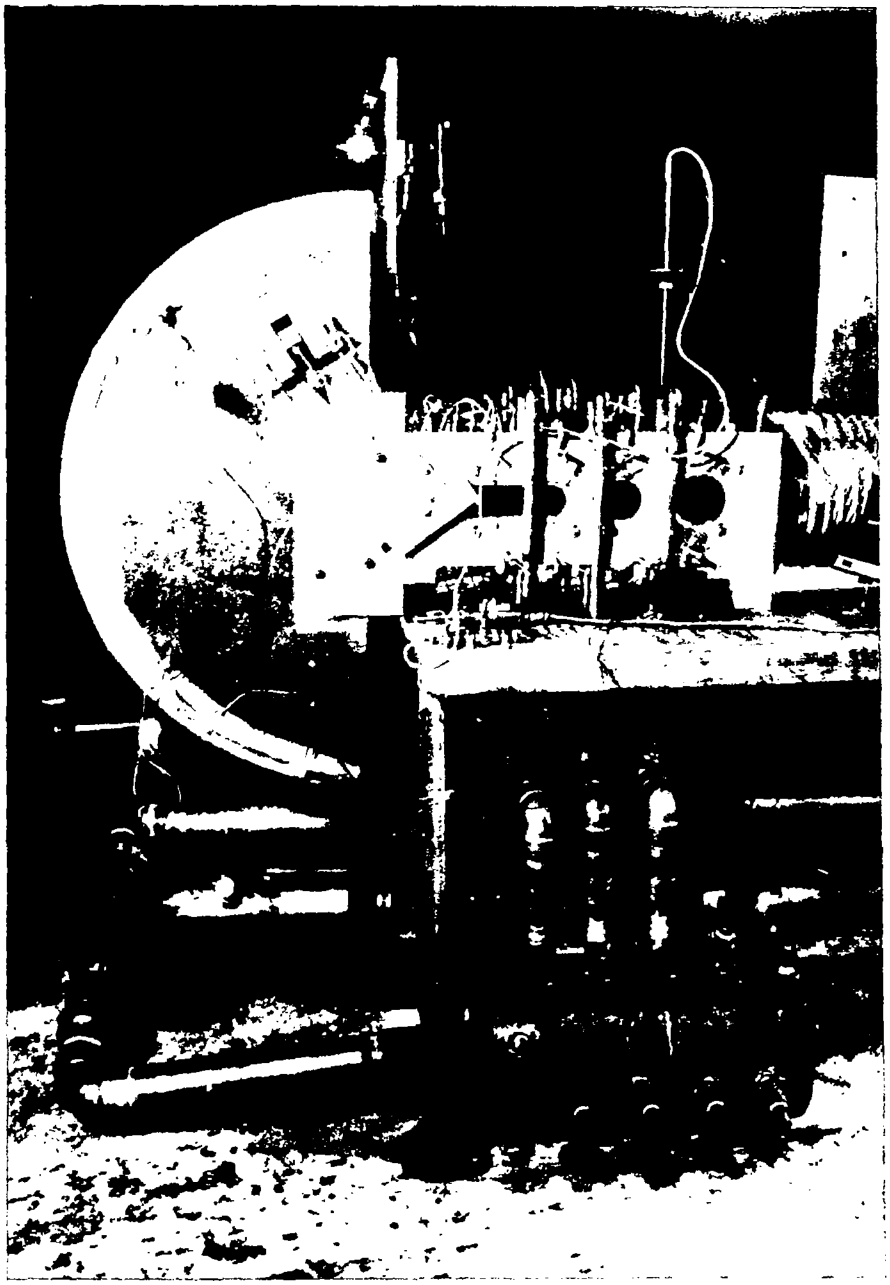


Figure 14

Side View of Test Section Showing
Extended Inlet, Mixing Section and Suction
System Piping and Manifold

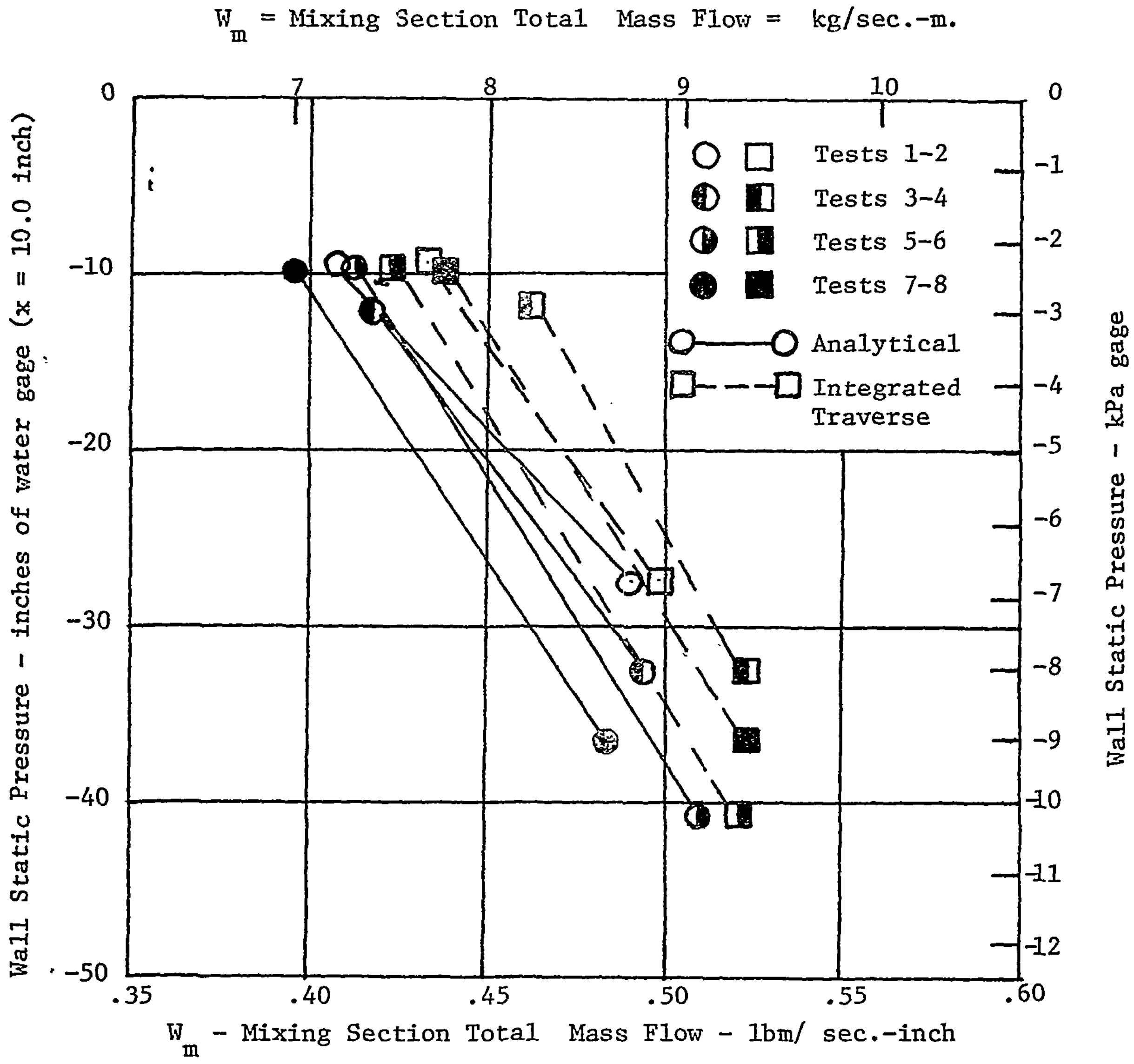


Figure 15

Comparison of Experimental and Analytical Flow Rates

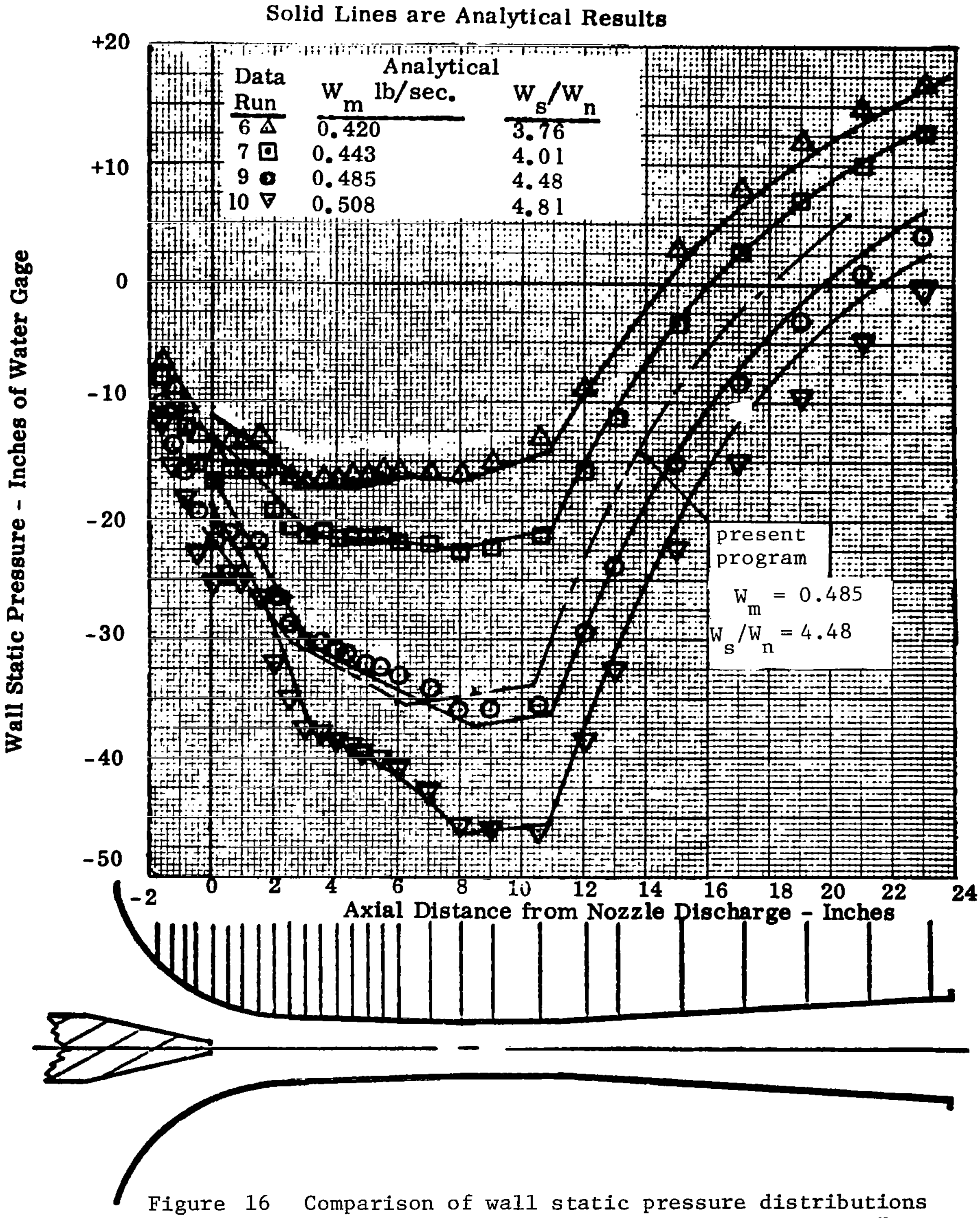


Figure 16 Comparison of wall static pressure distributions in a symmetrical mixing section with a 1.875" throat computed with the present streamline coordinate program and with the program from CR-2251

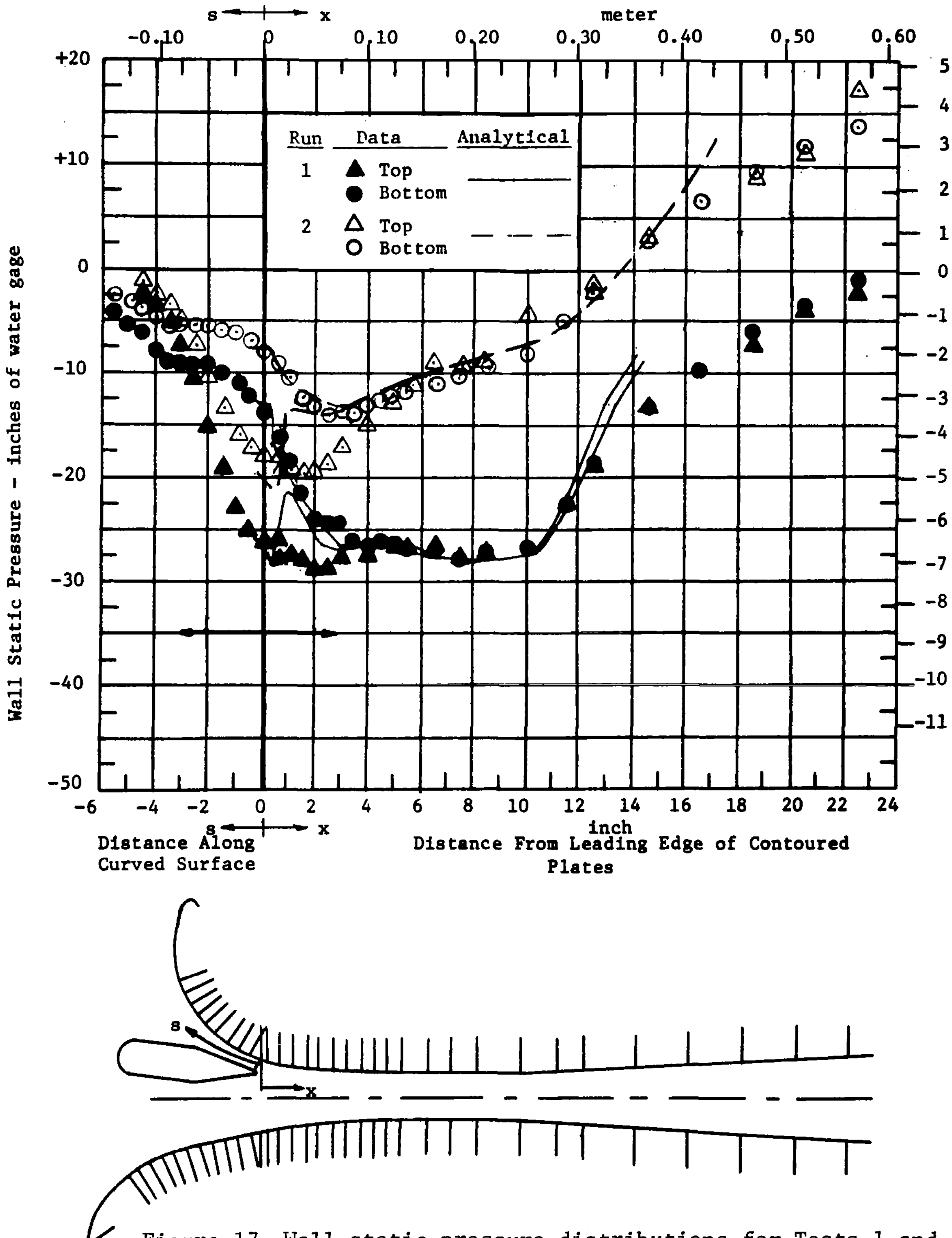


Figure 17 Wall static pressure distributions for Tests 1 and 2. Nozzle positioned at 22.5° and 0.020 meters (0.80 inches) spacing

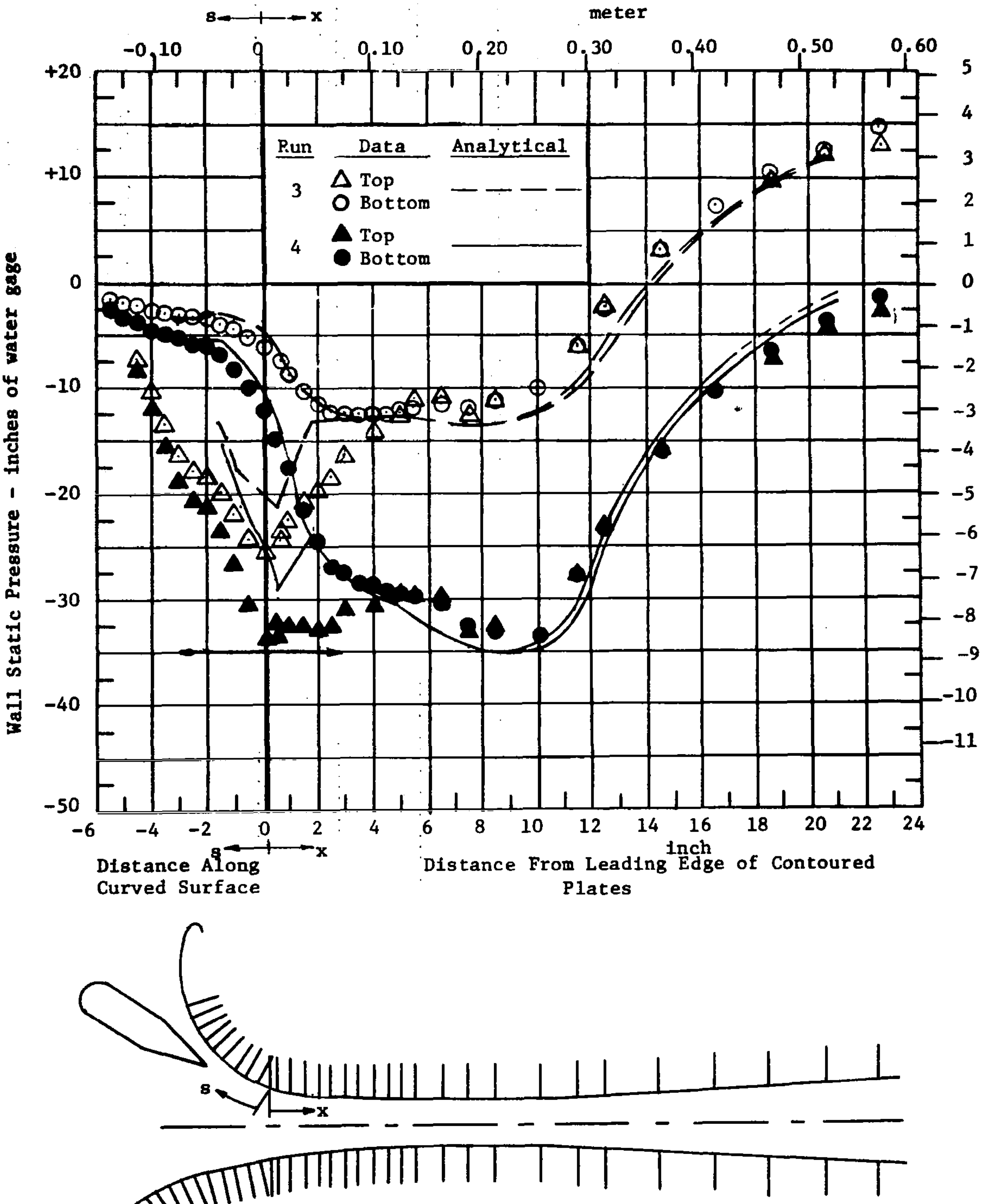


Figure 18 Wall static pressure distributions for Tests 3 and 4. Nozzle positioned at 45° and 0.020 meters (0.80 inches) spacing

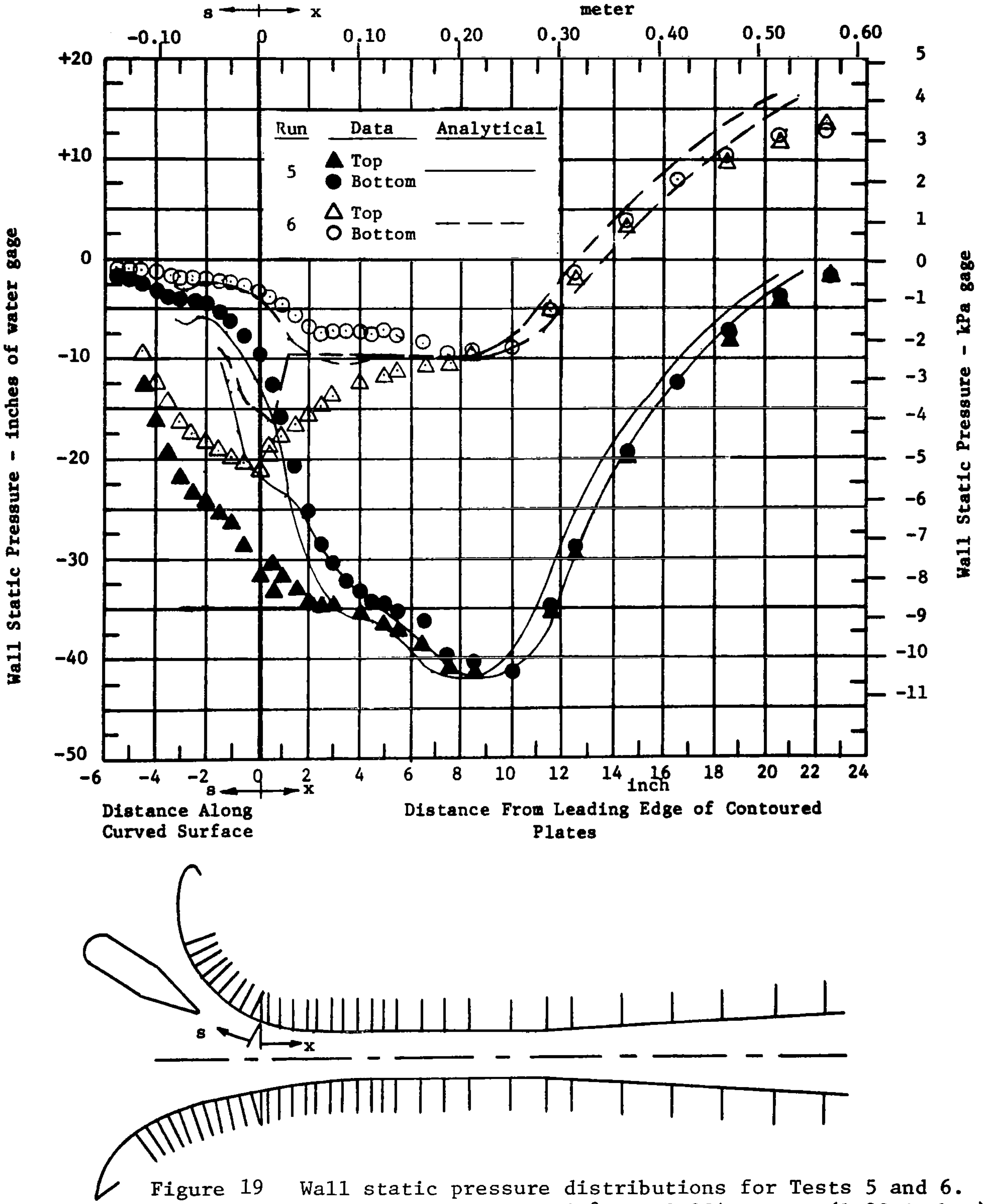


Figure 19 Wall static pressure distributions for Tests 5 and 6. Nozzle positioned at 45° and 0.034 meters (1.32 inches) spacing

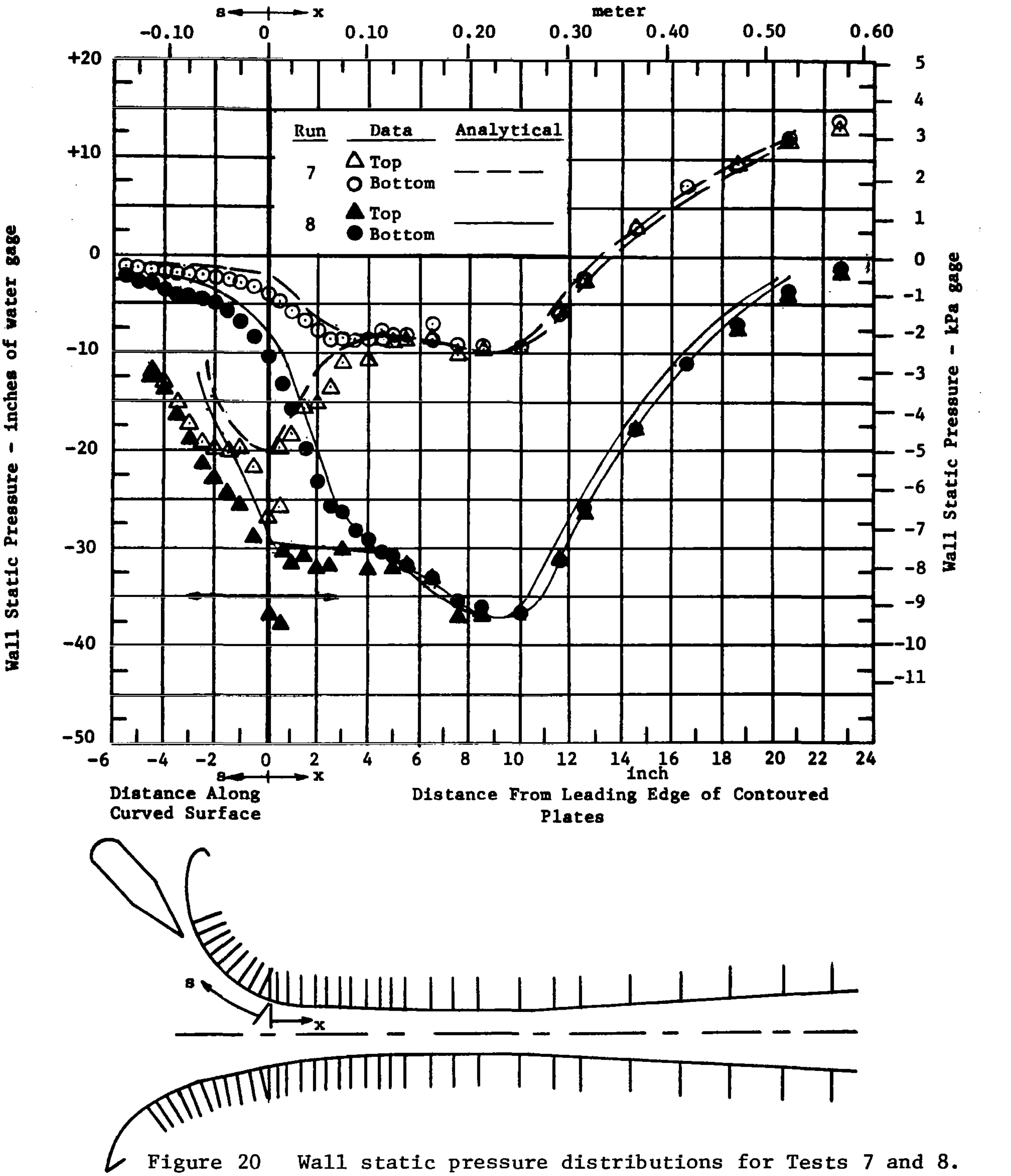


Figure 20 Wall static pressure distributions for Tests 7 and 8. Nozzle positioned at 67.5° and 0.018 meters (0.70 inches) spacing

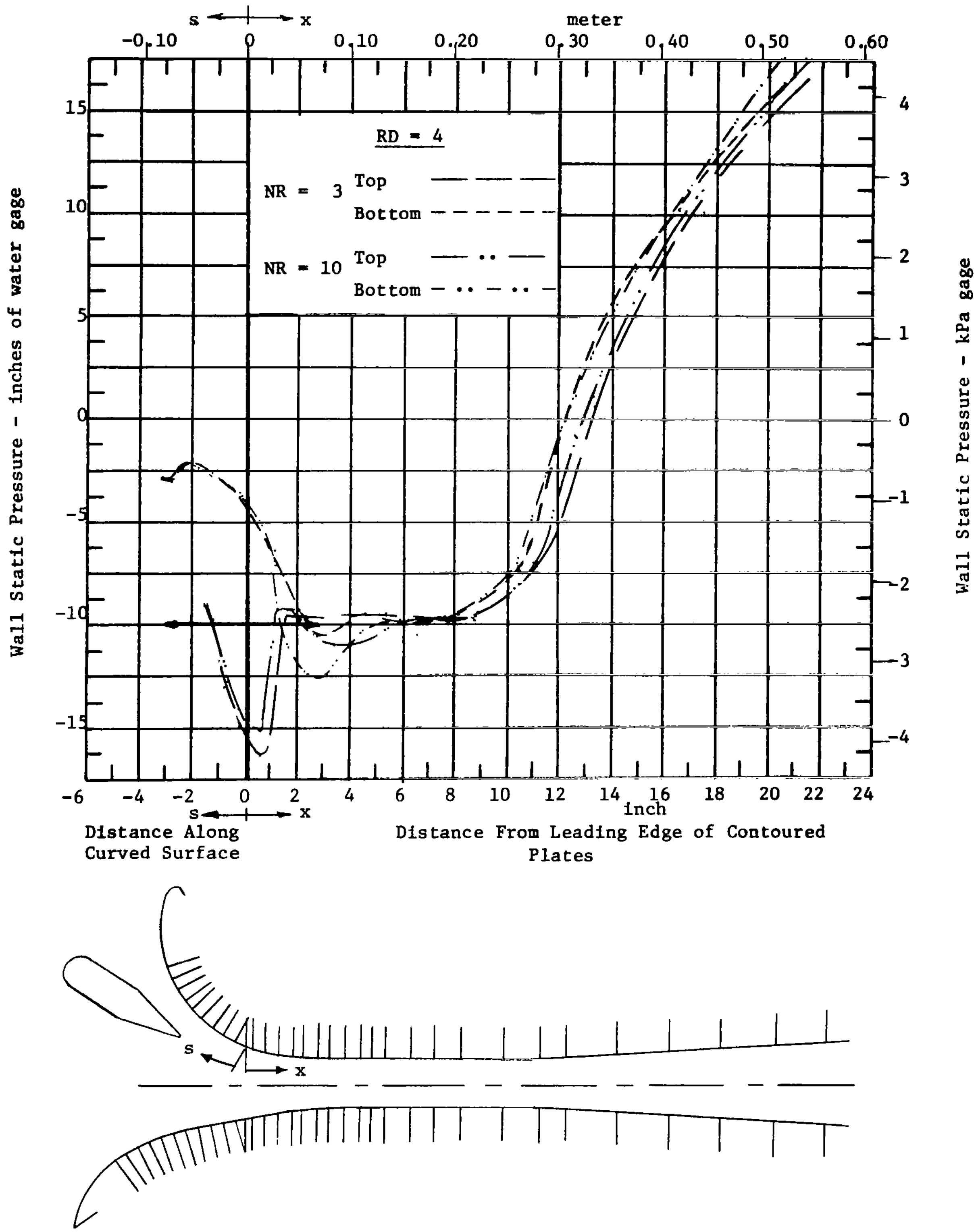


Figure 21 Wall static pressure sensitivity to the Richardson number coefficient (NR) for Test 6

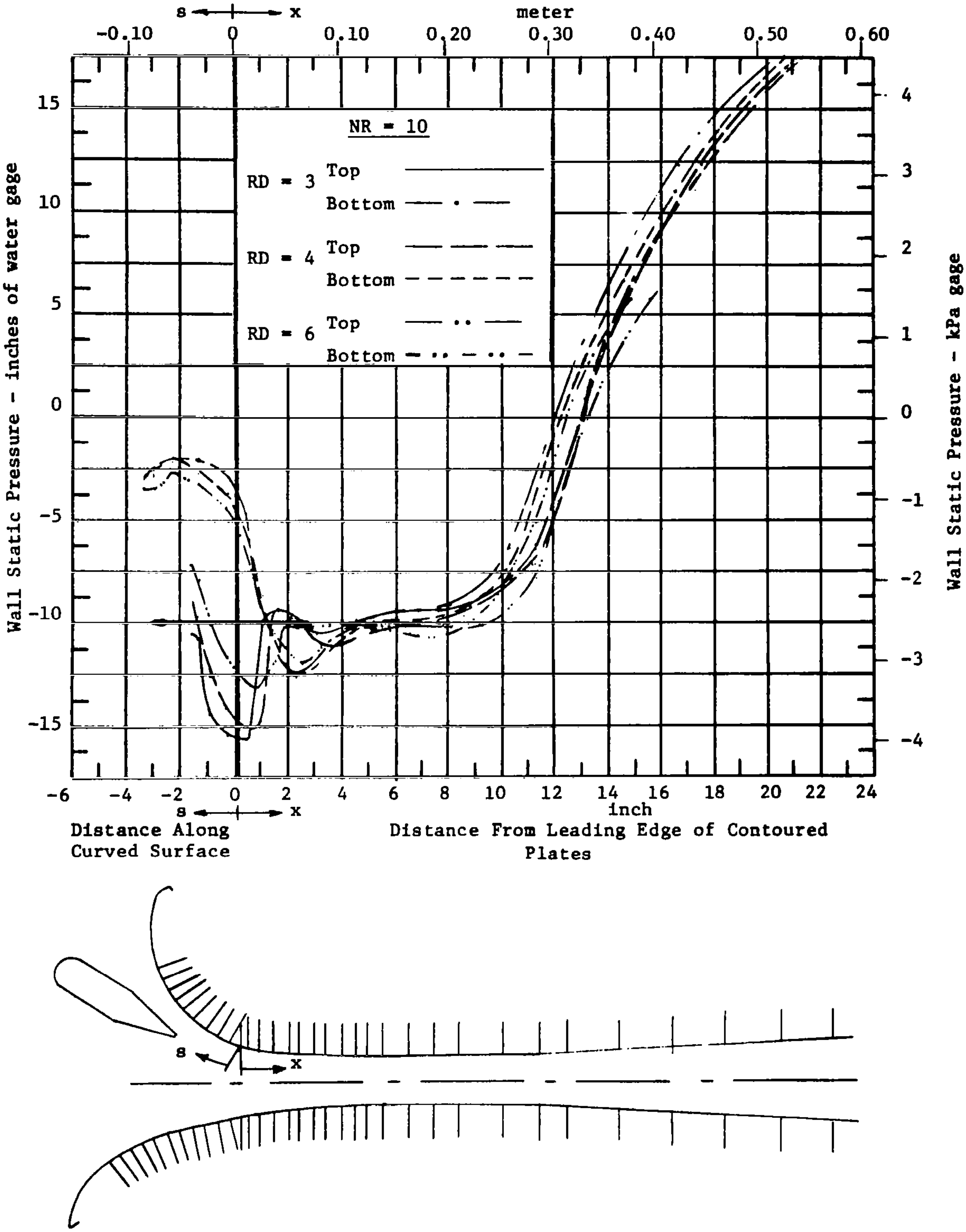


Figure 22 Wall static pressure sensitivity to the rate of streamline curvature decay (RD) for Test 6

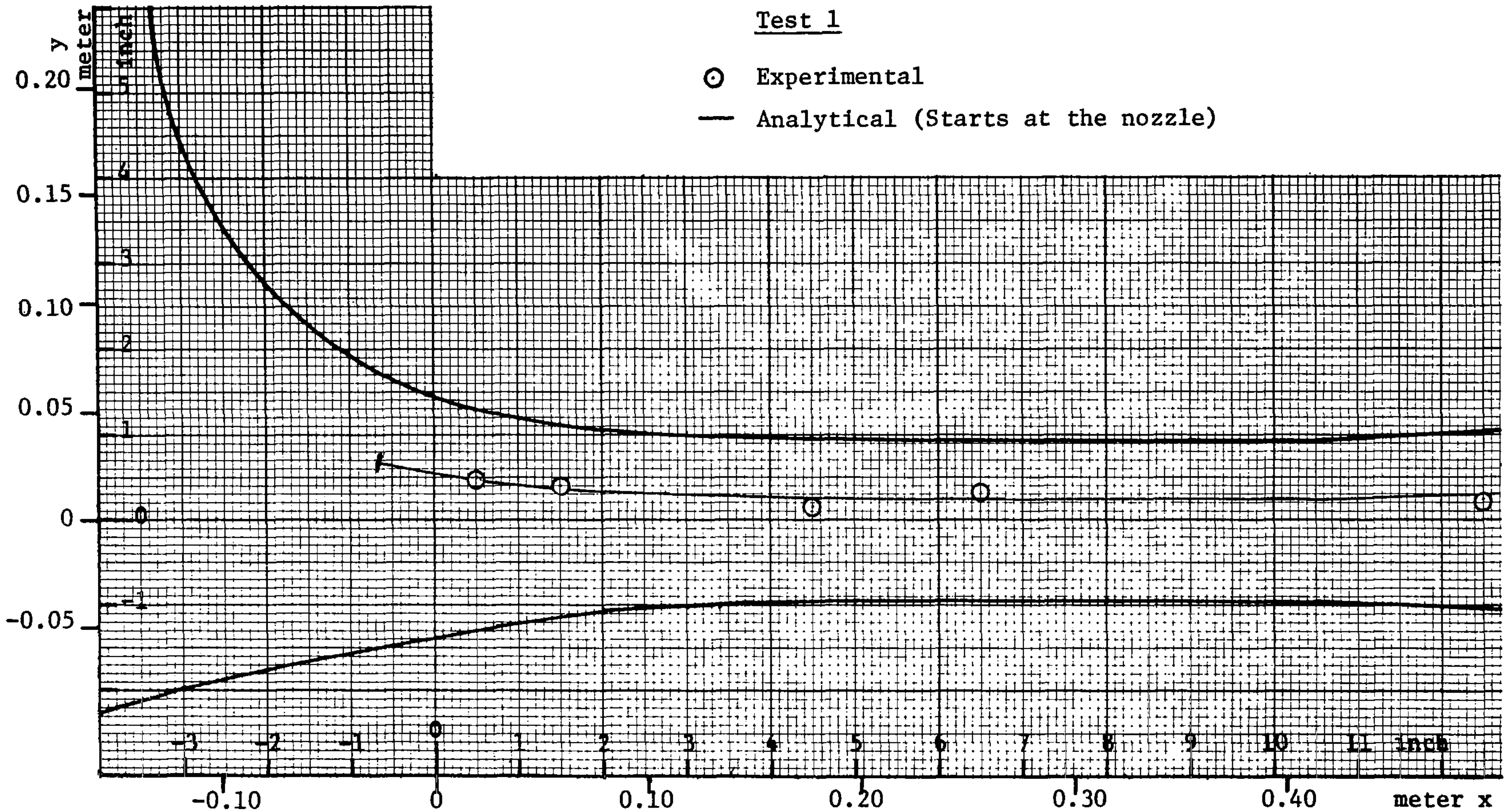


Figure 23

Locii of Maximum Stagnation Pressures ($P_{o\ max}$) in the Mixing Section

Nozzle Positioned at 22.5° and 0.020 meters (0.80 inches) spacing

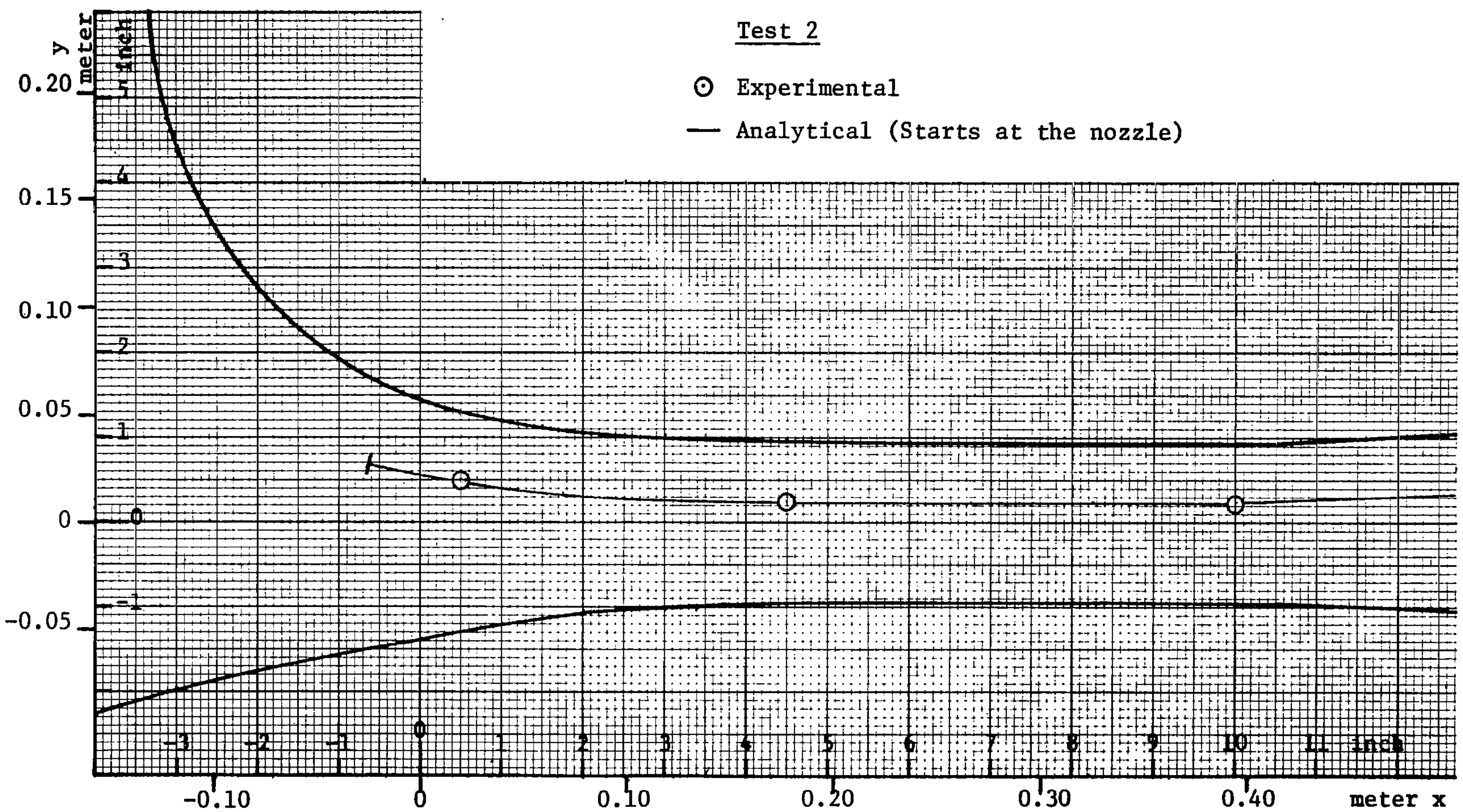


Figure 24

Locii of Maximum Stagnation Pressure ($P_{o\ max}$) in the Mixing Section
Nozzle Positioned at 22.5° and 0.020 meters (0.80 inches) spacing

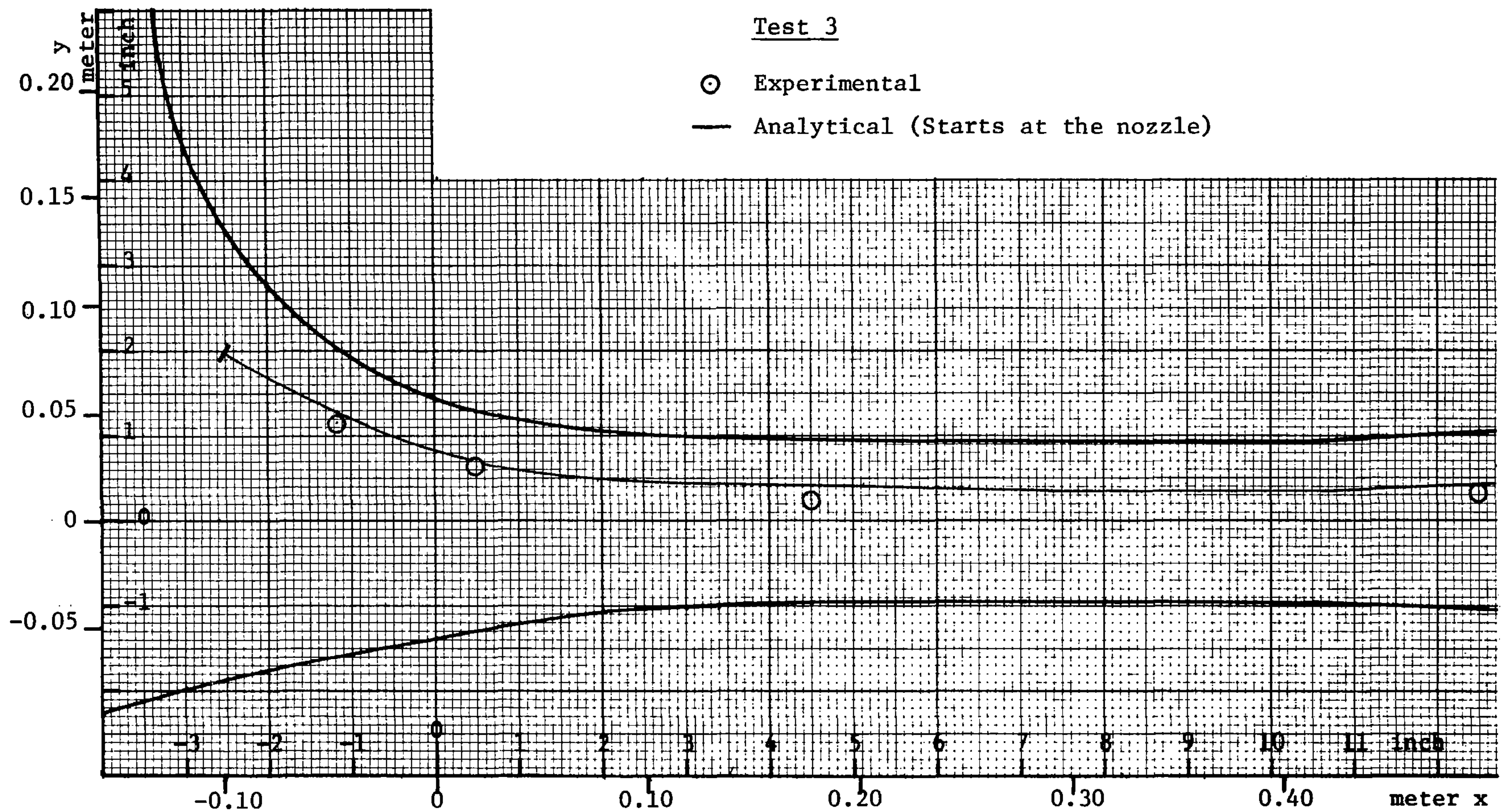
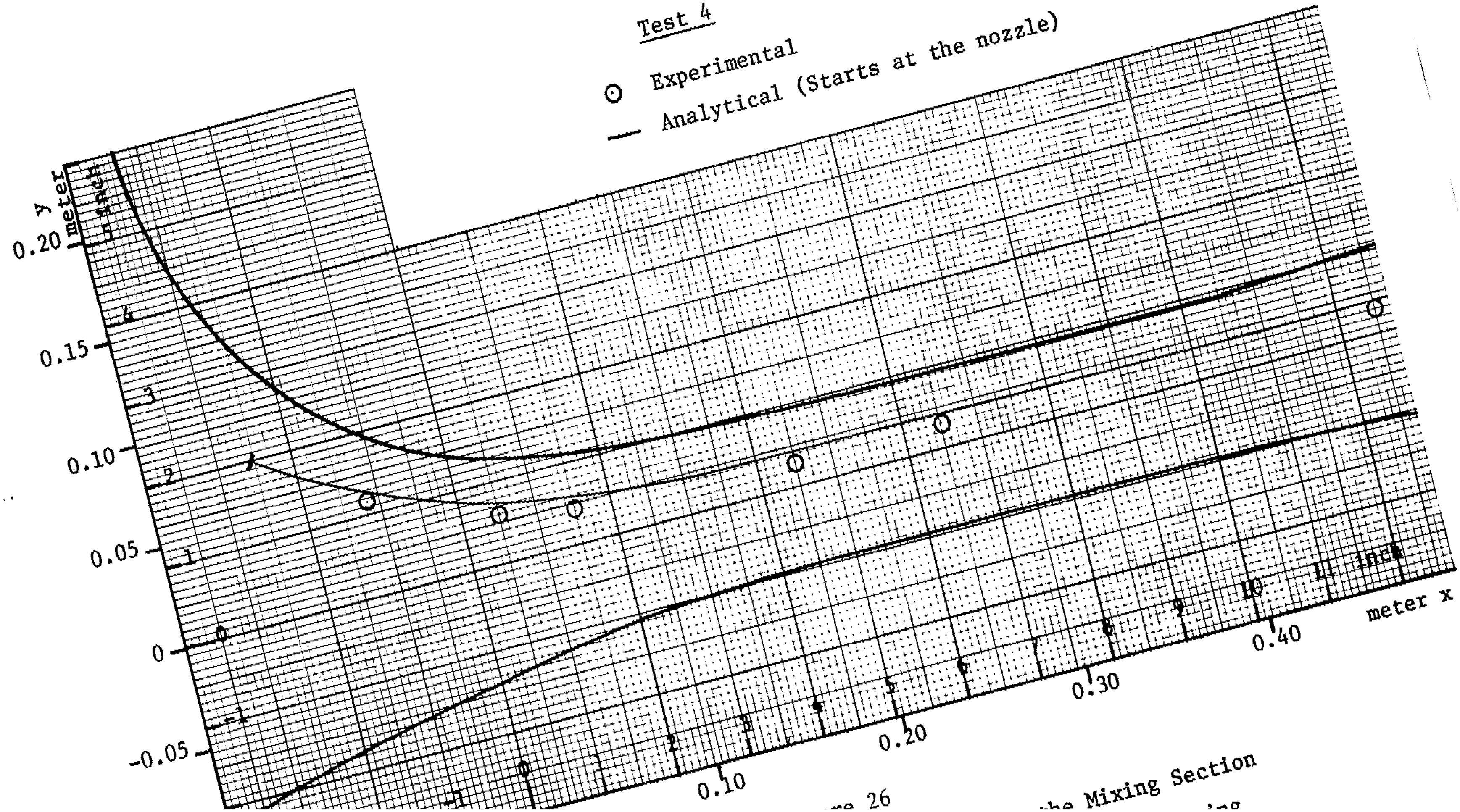


Figure 25

Locii of Maximum Stagnation Pressure ($P_{o\ max}$) in the Mixing Section

Nozzle Positioned at 45° and 0.020 meters (0.80 inches) spacing



...re 26 ... the Mixing Section

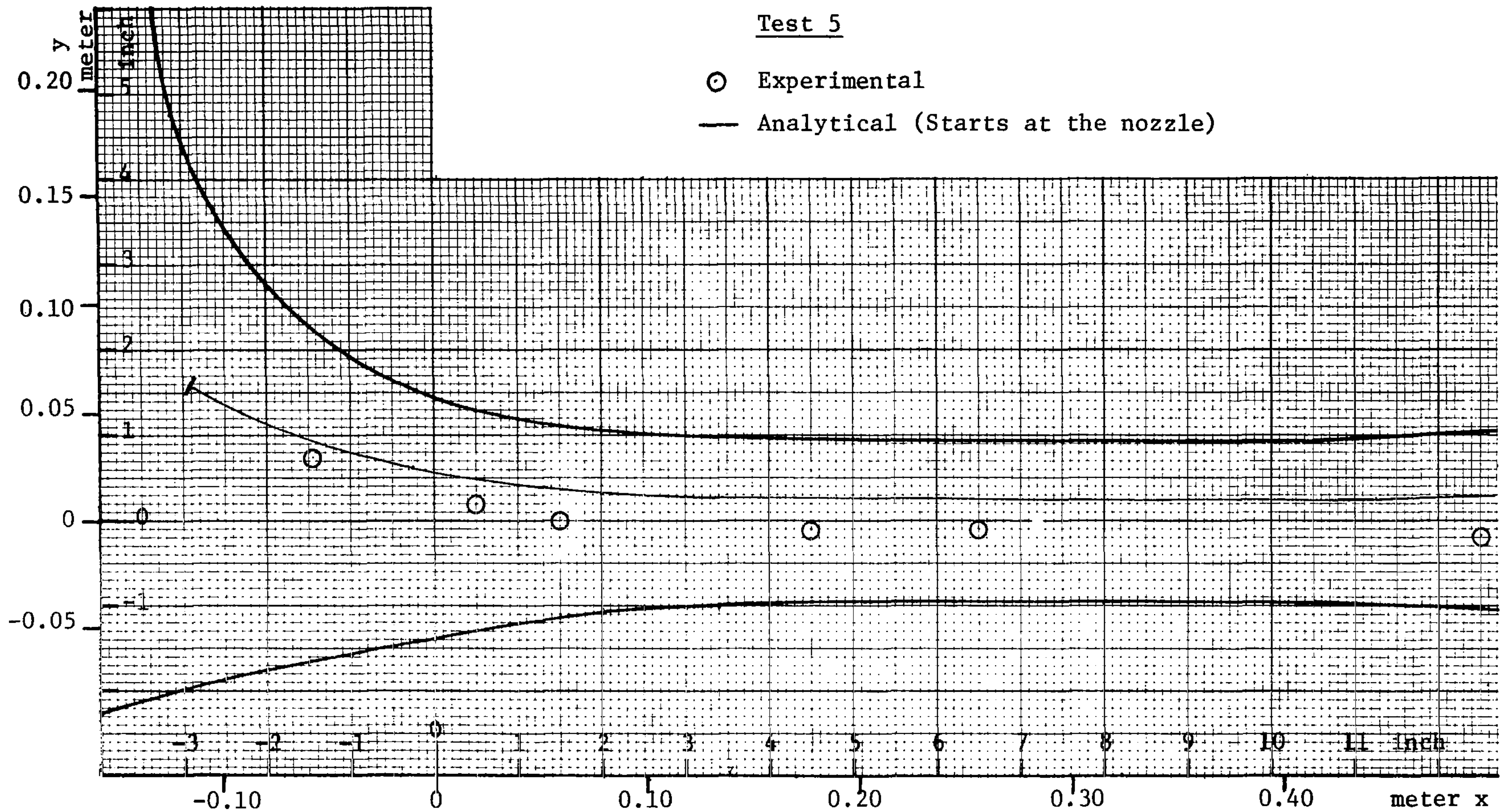


Figure 27

Locii of Maximum Stagnation Pressure ($P_{o\ max}$) in the Mixing Section

Nozzle Positioned at 45° and 0.034 meters (1.32 inches) spacing

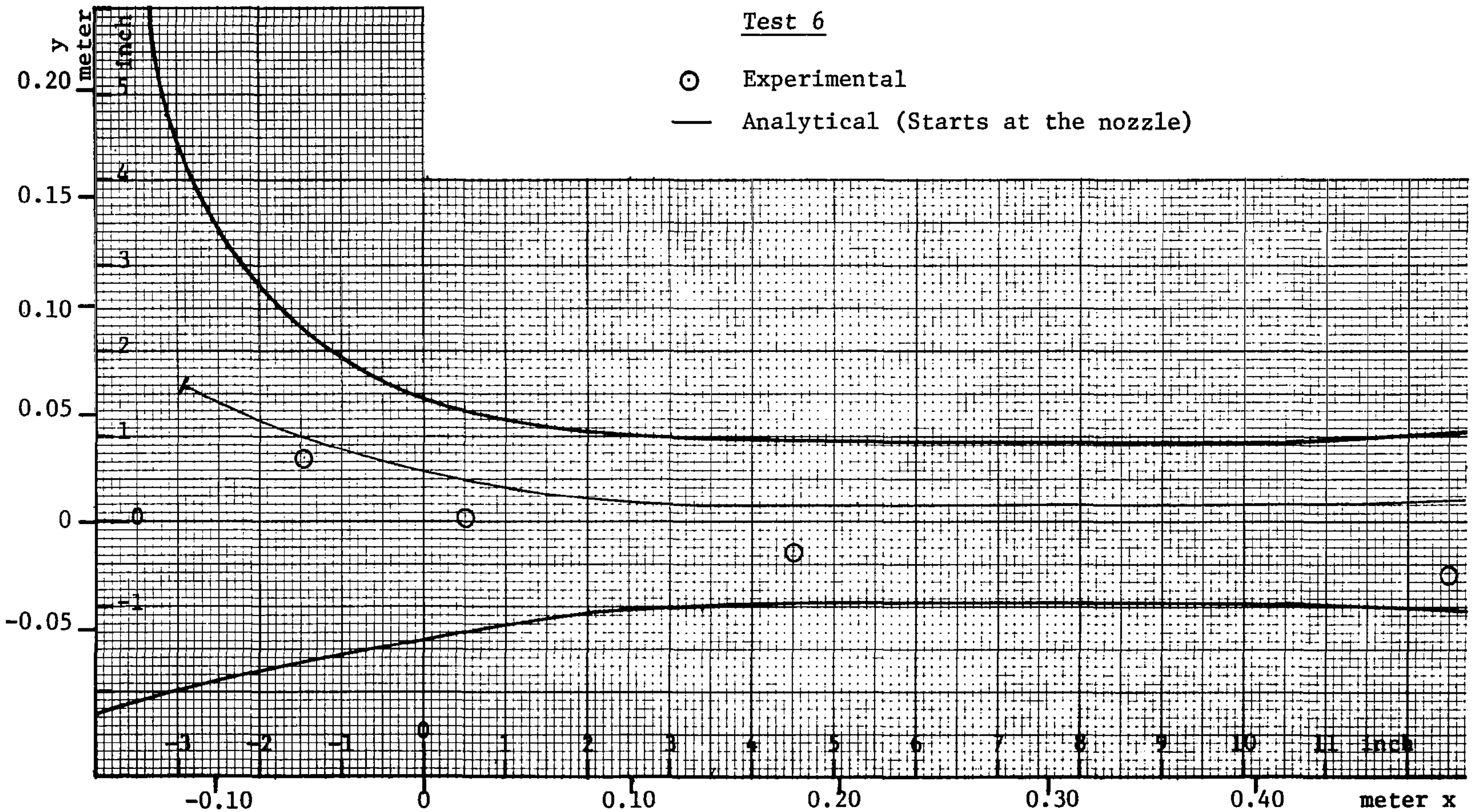


Figure 28

Locii of Maximum Stagnation Pressure ($P_{o\ max}$) in Mixing Section

Nozzle positioned at 45° and 0.034 meters (1.32 inches) spacing

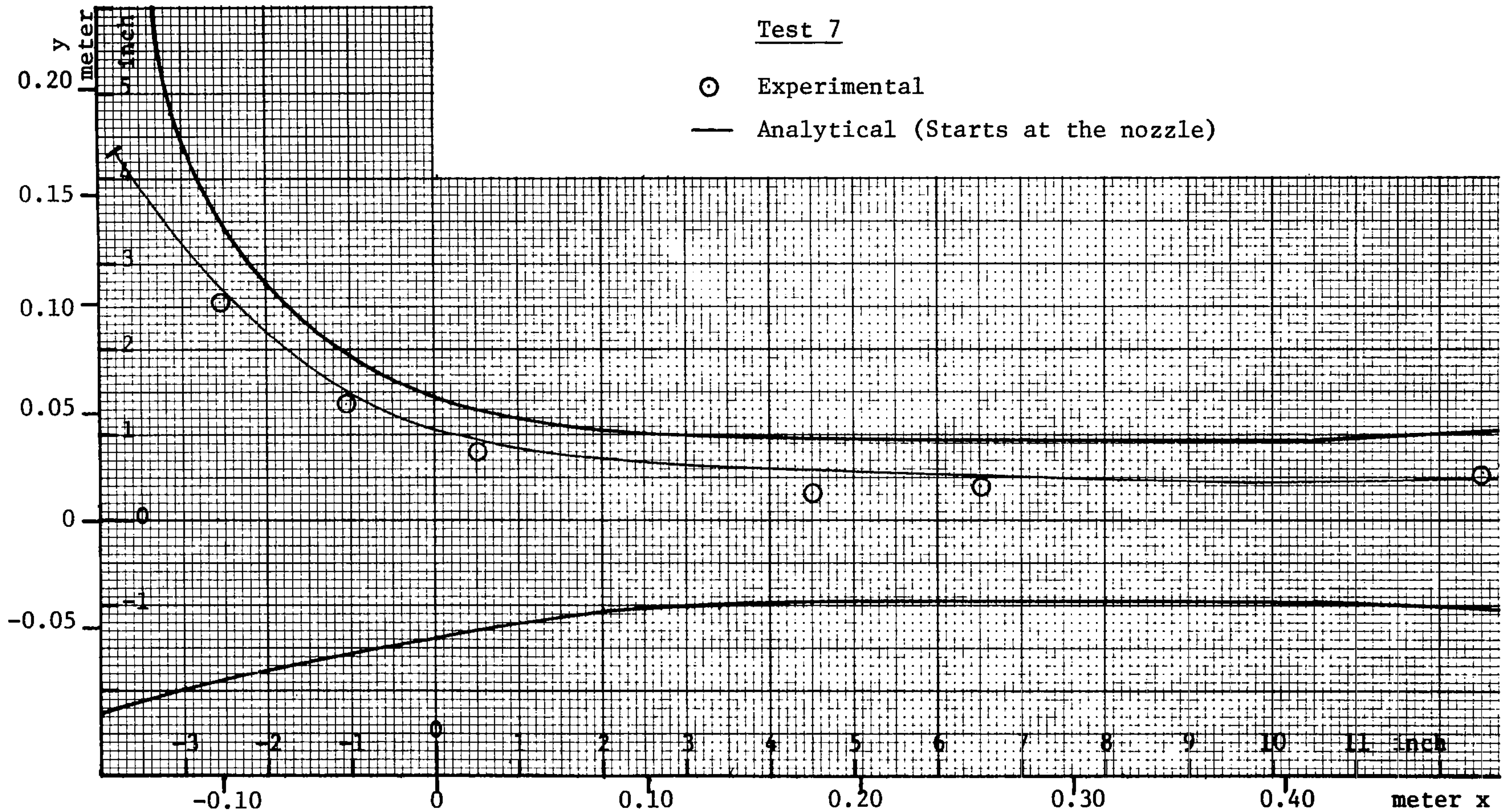


Figure 29

Locii of Maximum Stagnation Pressure ($P_{o\ max}$) in Mixing Section

Nozzle positioned at 67.5° and 0.018 meters (0.70 inches) spacing

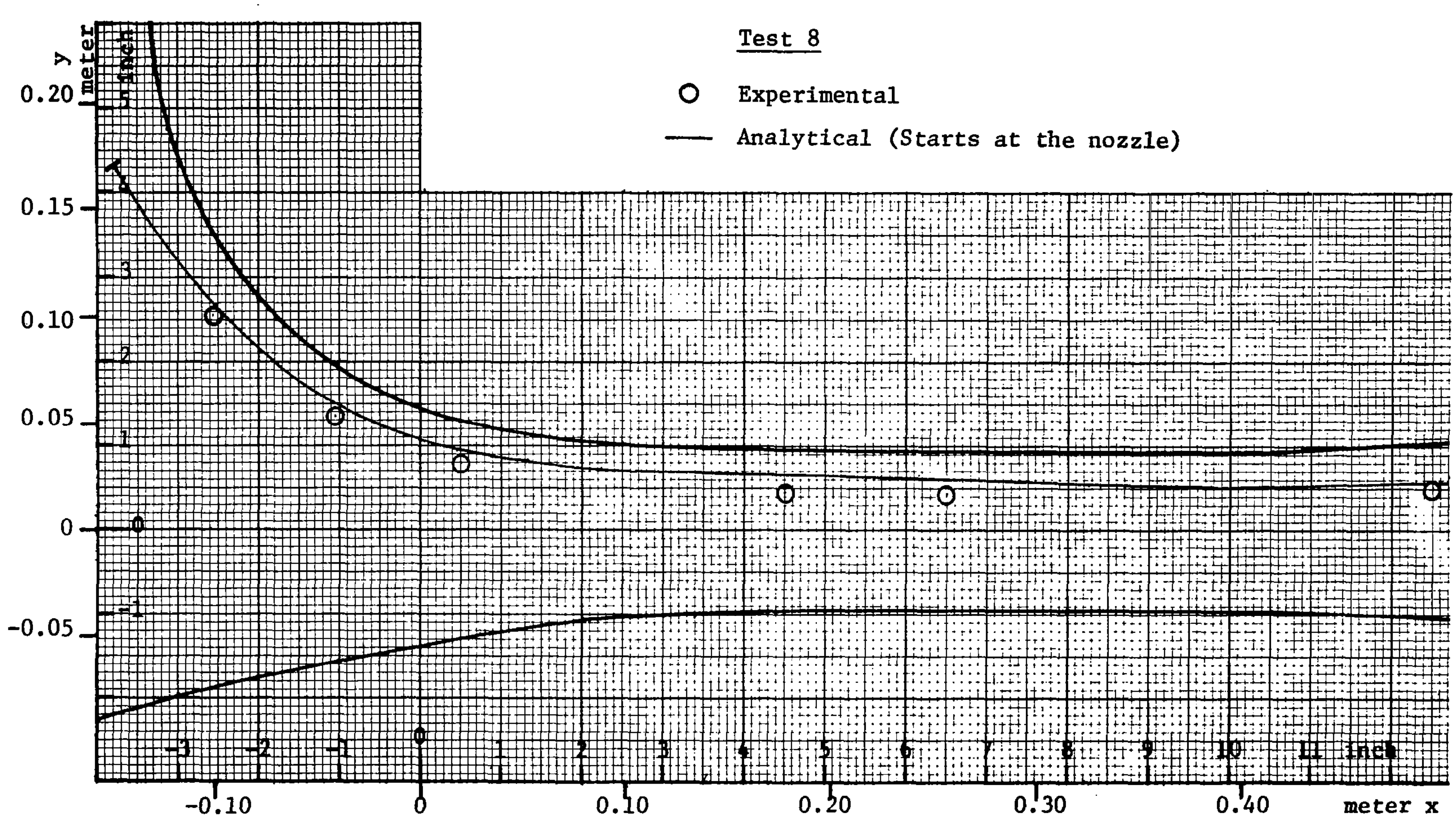


Figure 30

Locii of Maximum Stagnation Pressure ($P_{o\ max}$) in Mixing Section

Nozzle positioned at 67.5° and 0.018 meters (0.70 inches) spacing

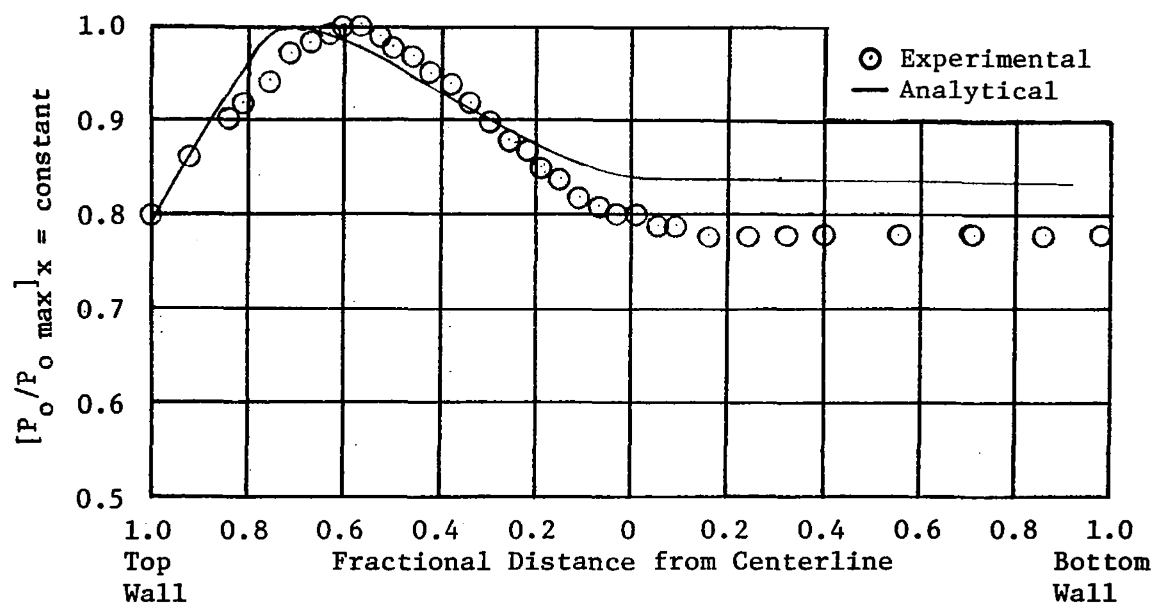


Figure 31 Total Pressure Profiles for Run 7 at x = +0.013 meters (+0.50 inches), Nozzle Positioned at 67.5°, 0.018 meters (0.70 inches) spacing

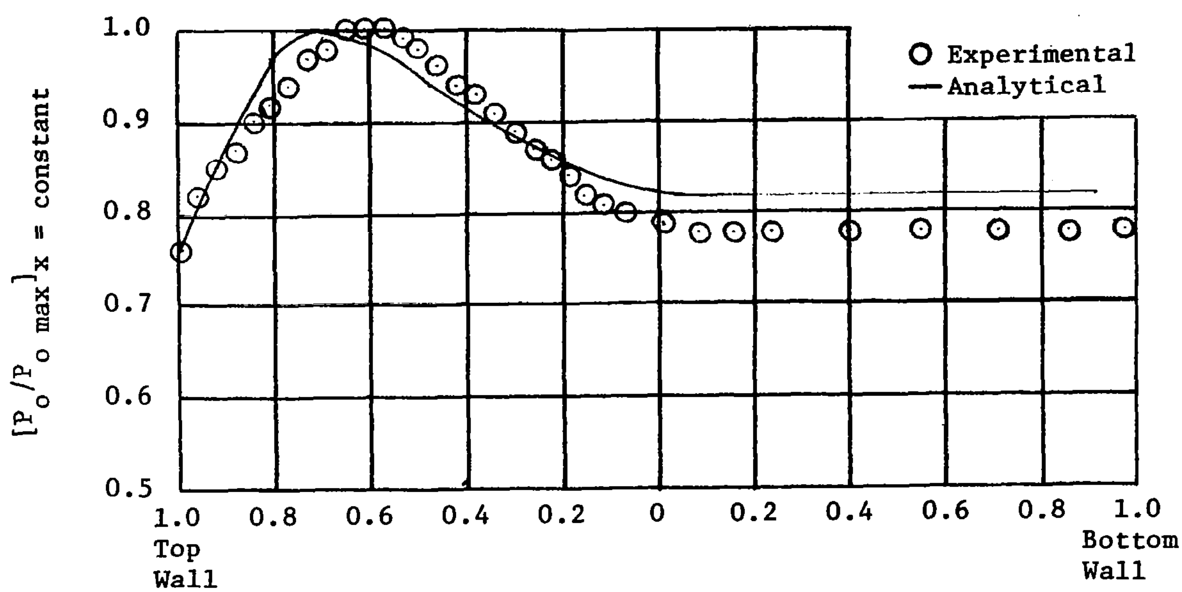


Figure 32 Total Pressure Profile for Run 8 at x = +0.013 meters (+0.50 inches), Nozzle Positioned at 67.5°, 0.018 meters (0.70 inches) spacing

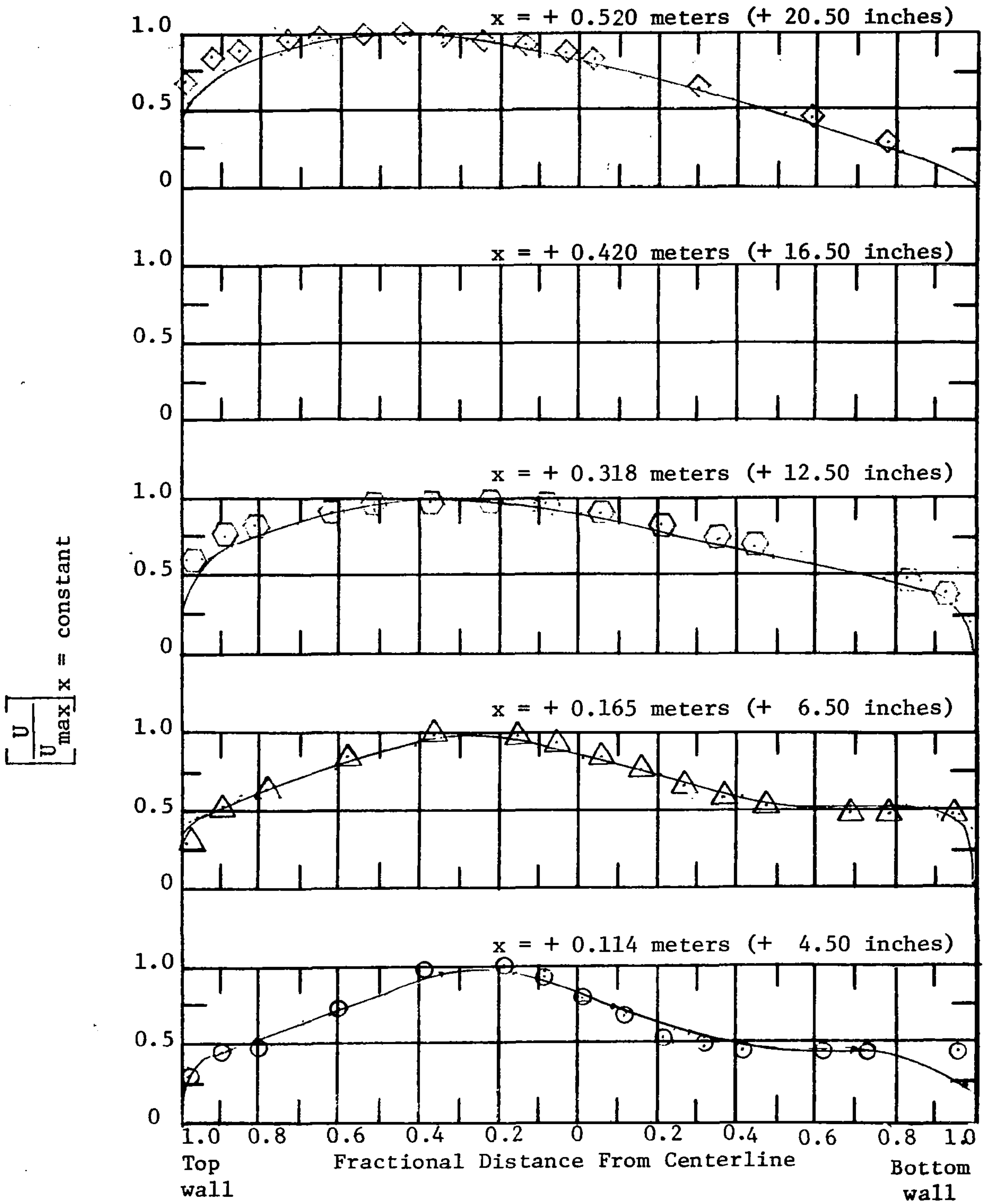


Figure 33 Velocity Profiles for Run 1, Nozzle Positioned at 22.5° and 0.020 meters (0.80 inches) spacing

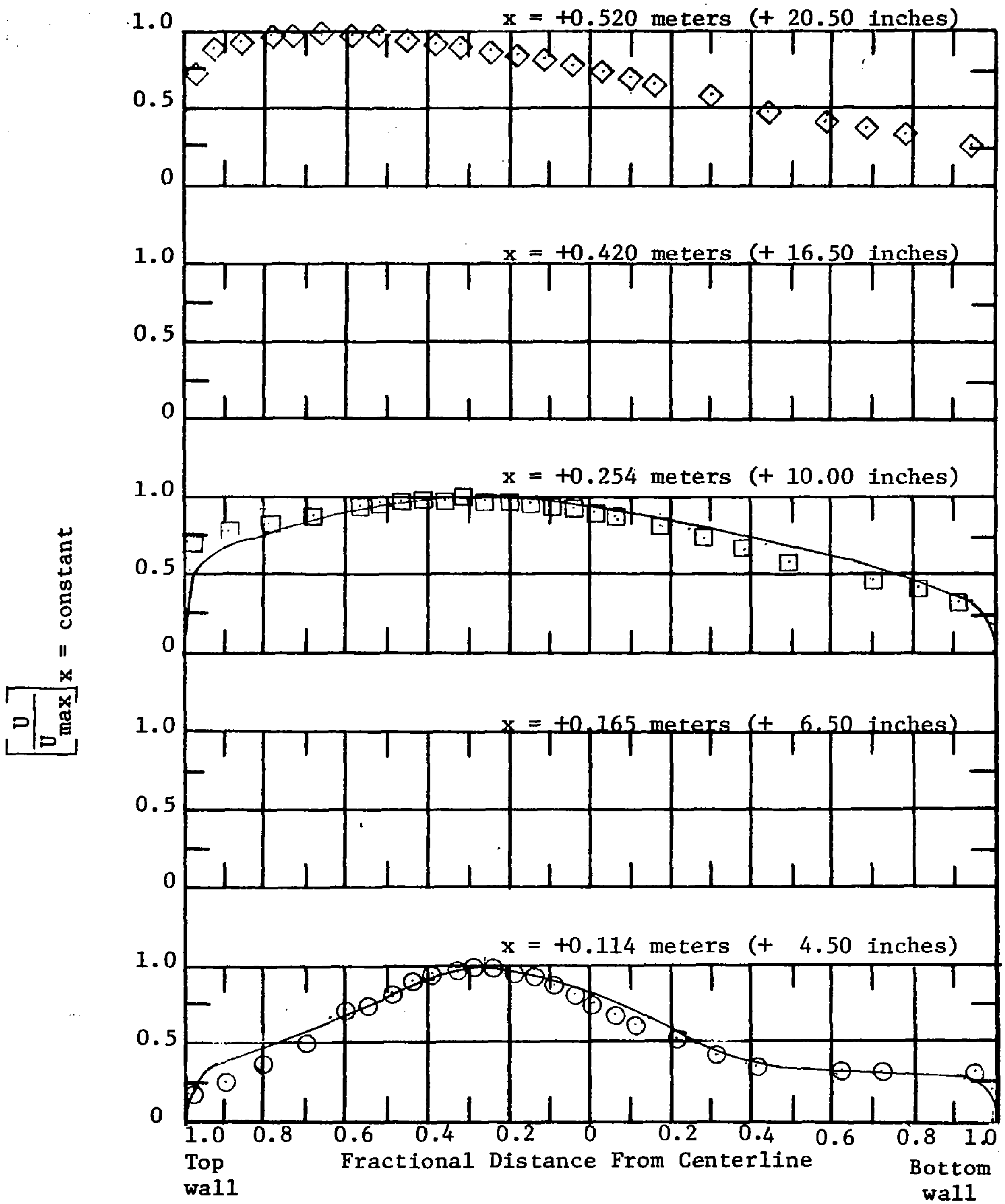


Figure 34 Velocity Profiles for Run 2, Nozzle Positioned at 22.5° and 0.020 meters (0.80 inches) spacing

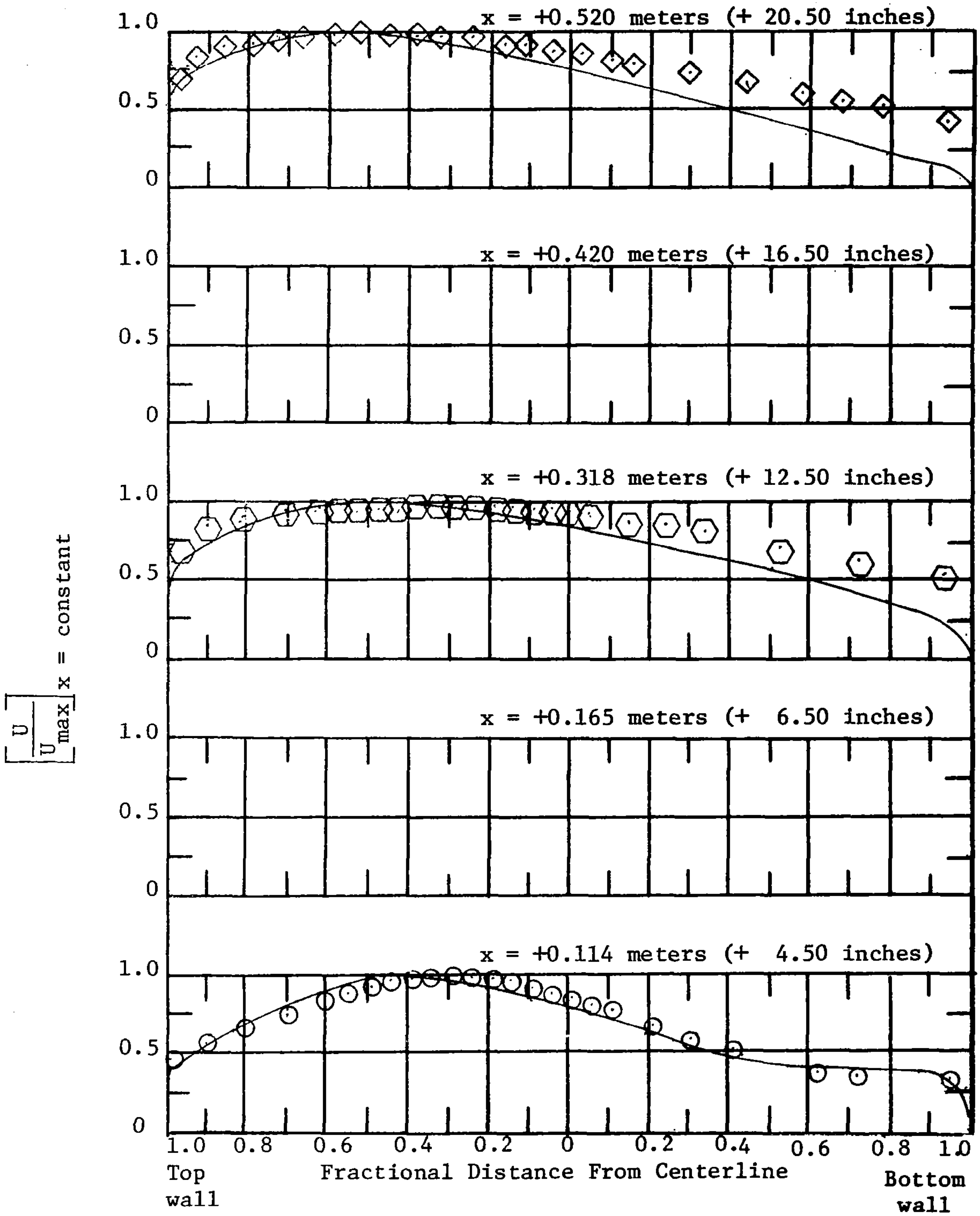


Figure 35 Velocity Profiles for Run 3, Nozzle Positioned at 45° and 0.020 meters (0.80 inches) spacing

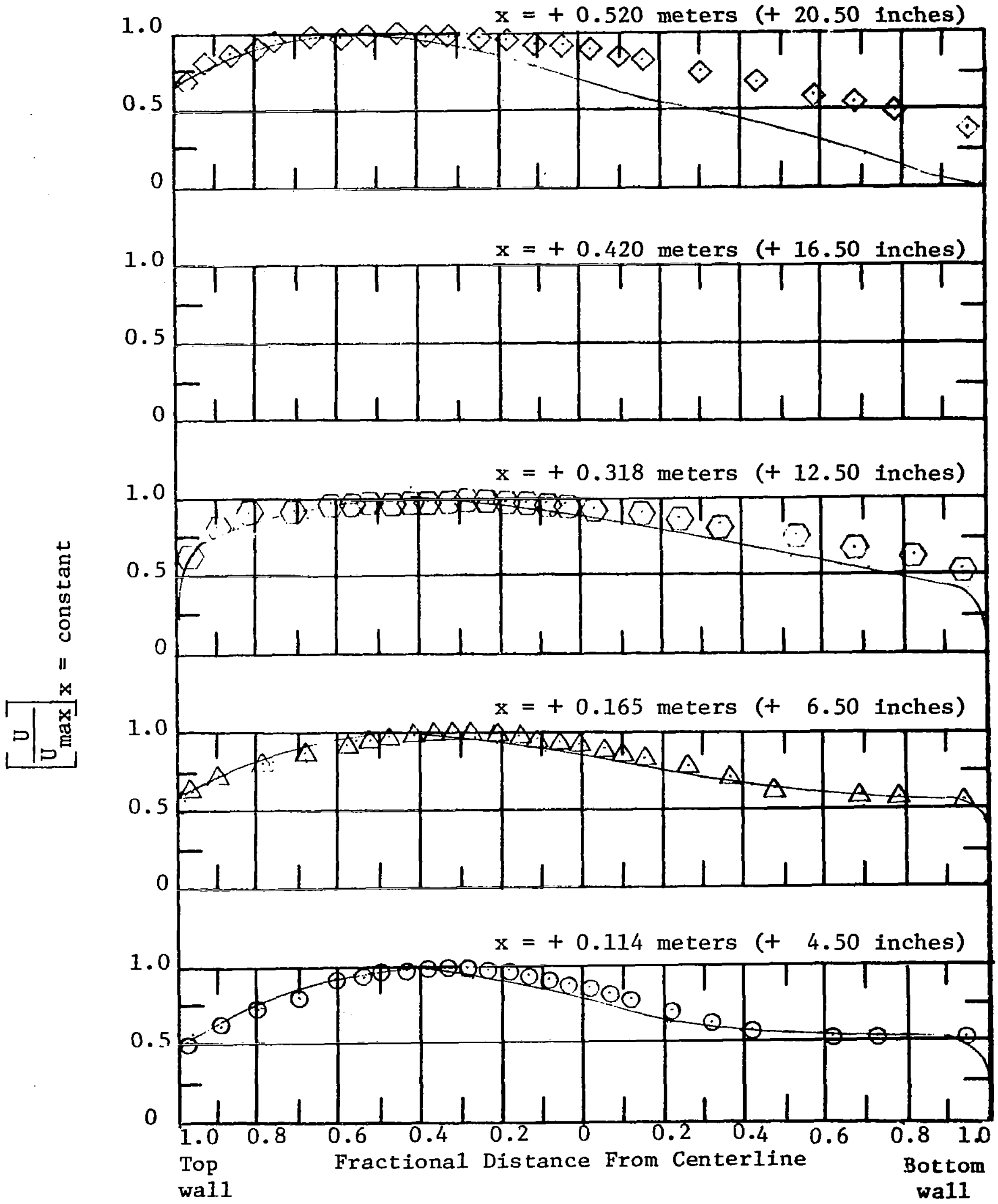


Figure 36 Velocity Profiles for Run 4, Nozzle Positioned at 45° and 0.020 meters (0.80 inches) spacing

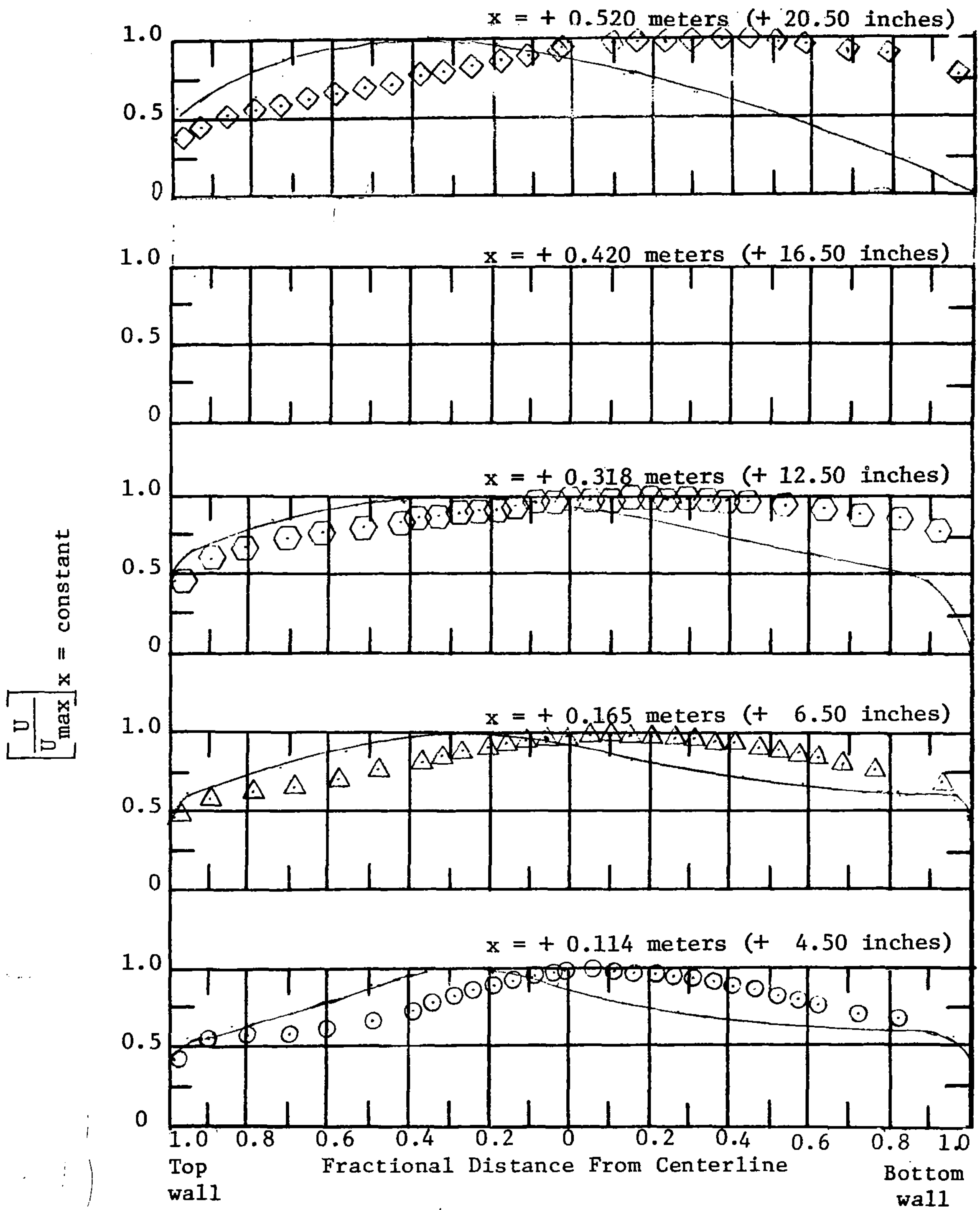


Figure 37 Velocity Profiles for Run 5, Nozzle Positioned at 45° and 0.034 meters (1.32 inches) spacing

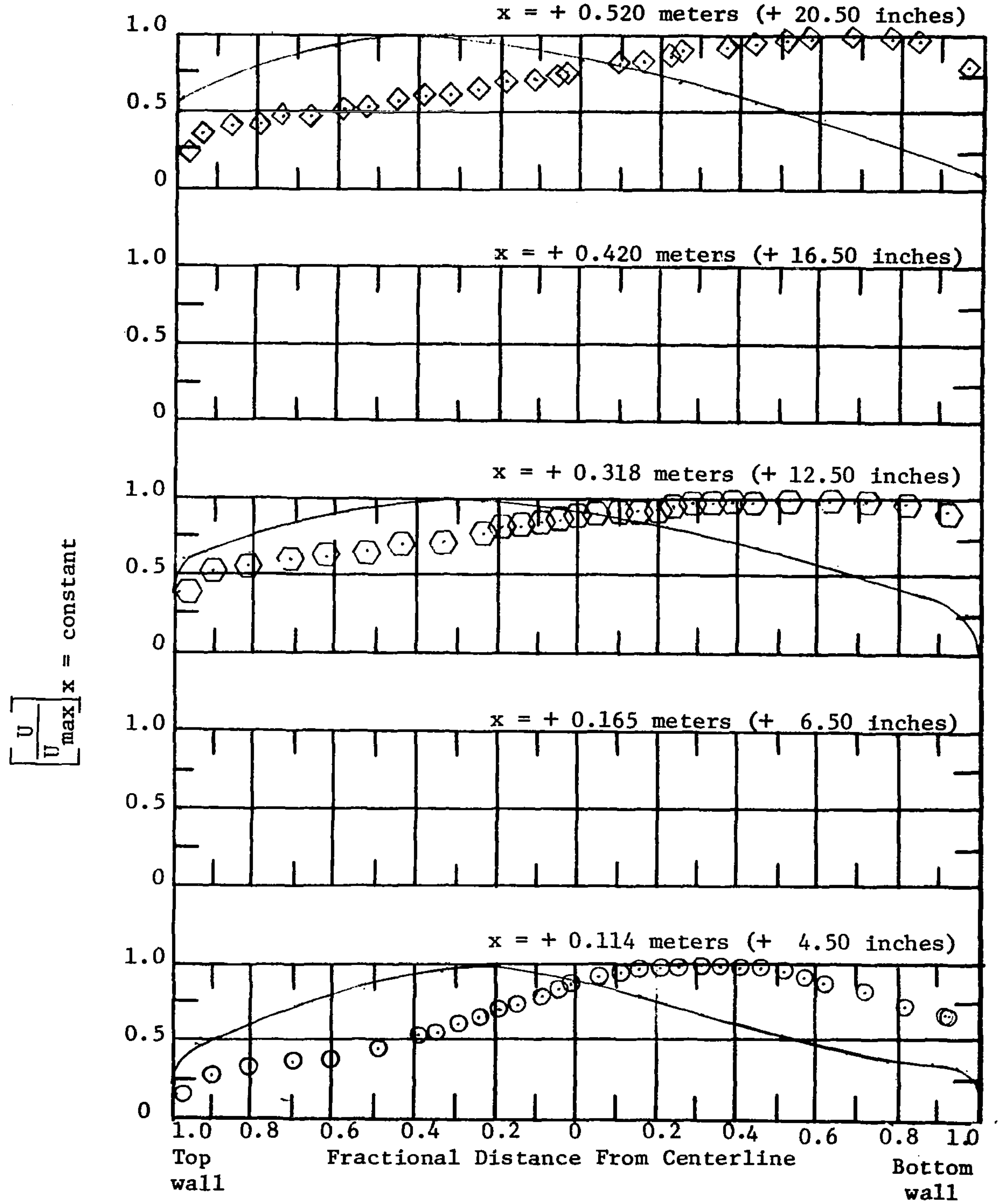


Figure 38 Velocity Profiles for Run 6, Nozzle Positioned at 45° and 0.034 meters (1.32 inches) spacing

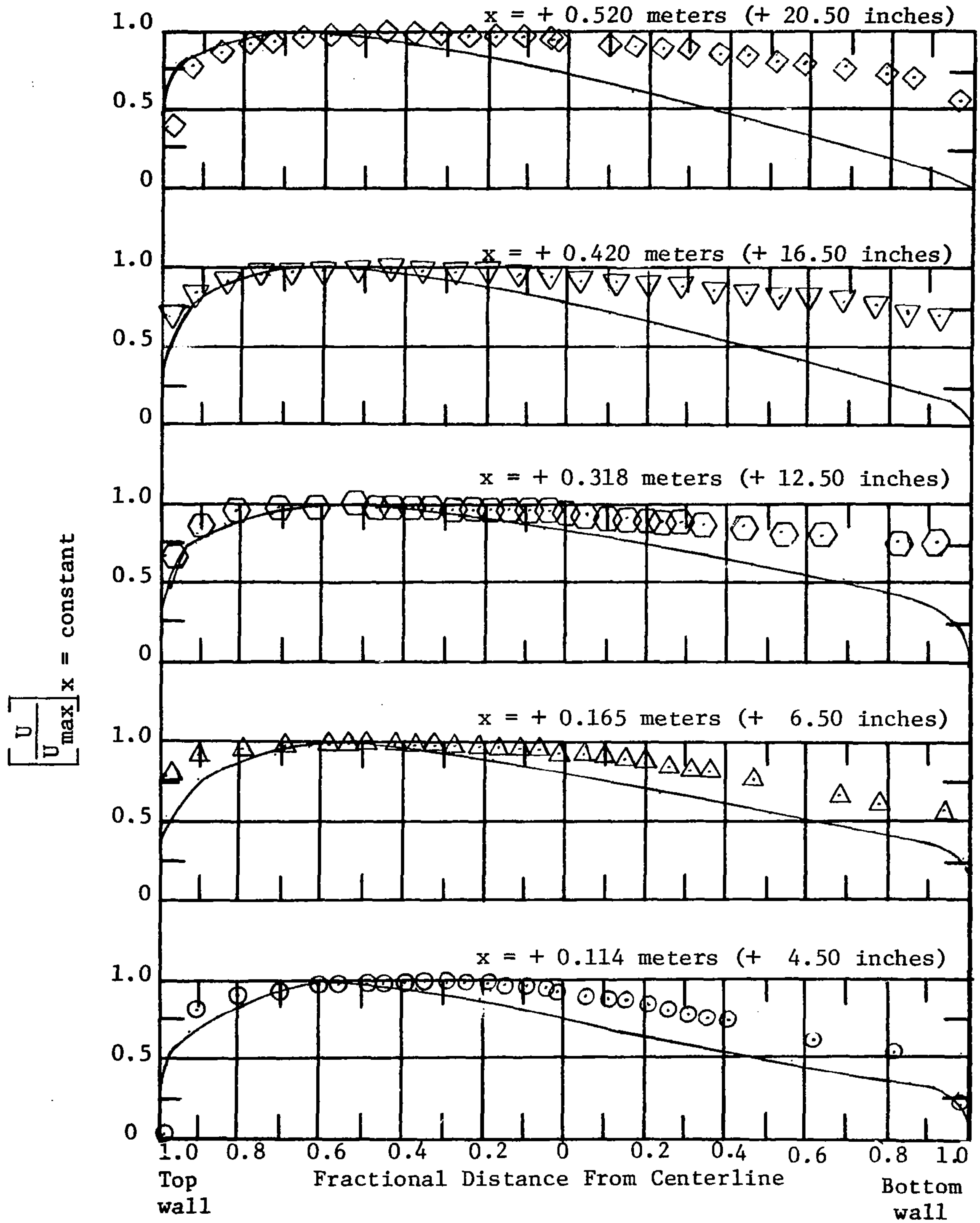


Figure 39 Velocity Profiles for Run 7, Nozzle Positioned at 67.5° and 0.018 meters (0.70 inches) spacing

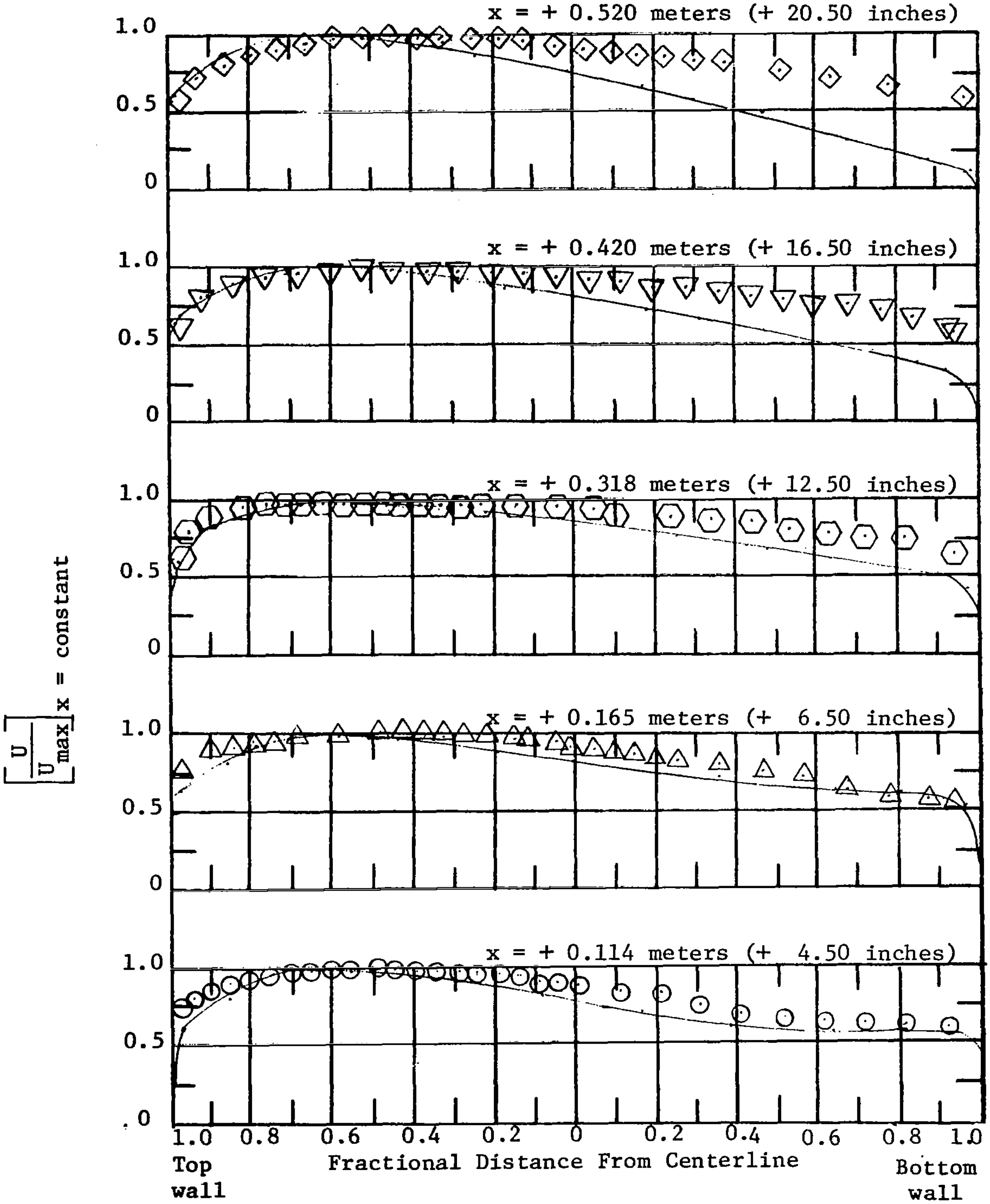


Figure 40 Velocity Profiles for Run 8, Nozzle Positioned at 67.5° and 0.018 meters (0.70 inches) spacing