A PLANE STRAIN ANALYSIS OF THE BLUNTED CRACK TIP USING SMALL STRAIN DEFORMATION PLASTICITY THEORY*

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SUMMARY

This paper presents a deformation plasticity analysis of the tip region of a blunted crack in plane strain. The power hardening material is incompressible both elastically and plastically, in order to simulate behavior of a stress freezing material above critical temperature. The study represents a full field, finite difference solution to the Mode I problem. Stress and displacement fields surrounding the crack tip are presented. The results of this study indicate that the maximum stress seen at the crack tip is indeed limited and is determined by the tensile properties; however, the scale over which the stresses act is dependent on the loading. Comparisons are good between the forward crack tip displacement and micro-fractographic measurements of "stretch" zones observed in plane strain fracture toughness tests.

INTRODUCTION

In recent years Cherepanov (ref. 1), Rice (ref. 2,3), Hutchinson (ref. 4, 5), and Rice and Rosengren (ref. 6) have shown the asymptotic behavior of stress and strain fields surrounding sharp crack tips in plane strain. Using these studies as a guide, full field solutions with finite elements have been obtained by Levy, Marcal, Ostergren and Rice (ref. 7) and Hilton and Hutchinson (ref. 8). These two studies give accurate near and far field behavior due to the inclusion of singular elements reflecting plasticity at the crack tip. Other numerical solutions by Marcal and King (ref. 9), Mendelson (ref. 10), Swedlow and coworkers (ref. 11,12) and Tuba (ref. 13) show qualitative features of the near field, but may not yield accurate stress field definition due to the large gradients there.

In order to have an accurate description of the near field surrounding crack tips, and hence a good understanding of the mechanisms of failure, Rice and Johnson (ref. 14) have pointed out that crack tip blunting must also be included. Their analysis accounted in an approximate manner for the blunting at the crack tip and for strain hardening in the plastic zone. As a result they showed that the stresses near the crack tip are indeed finite and that the maximum σ_{yy} stress occurred at some small distance from the deformed crack tip. A finite deformation analysis by McGowan and Smith (ref. 15) of blunted cracks in a linear (stress-strain) incompressible material shows the same general behavior. The maximum σ_{yy} stress occurs in front of the blunted crack tip and the magnitude is independent of the remote loading.

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The purpose of the present study is to gain a full field solution around a blunted crack tip in a strain hardening incompressible material under Mode I loading. This work will provide an accurate description of the stress and deformation fields immediately surrounding the blunted tip, and thereby gain insight to fracture behavior. Deformation theory of plasticity with a Mises yield condition is used. The resulting set of equations is solved for the blunted crack tip in the deformed state under load by finite differences. The linear theory of Inglis (ref. 16) gives the necessary asymptotic boundary conditions.

An initial goal of the present study was to gain a more complete understanding of the near field behavior of stress freezing photoelastic materials above critical temperature; however, this study should also give considerable insight to the general behavior of engineering materials under Mode I loading.

SYMBOLS

c	One-half crack length	ε_o	Initial yield strain
E	Young's Modulus	ε_p	Effective plastic strain
K	Stress intensity factor	ε_T	Effective total strain = $\varepsilon_p + \sigma_e/E$
n	Strain hardening exponent	ρ	Deformed crack root radius
r,θ	Cylindrical coordinates measured from crack tip	ν	Poisson's ratio
T	Secant modulus = σ_e/ε_T	σ_{ij}	Stress tensor
u_i	Displacement vector	σ_{yy}	Hoop stress
U	Strain energy density	σ_e	Effective stress
X	Distance in front of deformed crack tip	σ_o	Tensile yield stress
Y	Distance perpendicular to deformed crack tip	Φ	Airy stress function
ε_{ij}	Strain tensor	α	Constant in eq. (2)

FORMULATION OF THE PROBLEM

Using small strain deformation theory of plasticity for an incompressible (ν= 1/2) material the governing equation for the field can be shown to be:

$$\frac{1}{T}(\Phi_{,2222} + \Phi_{,1111} + 2\Phi_{,1122}) + 2\left(\frac{1}{T}\right)_{,1}(\Phi_{,111} + \Phi_{,122})$$

$$+ 2 \left(\frac{1}{T}\right)_{,2} (\Phi_{,222} + \Phi_{,112}) + (\Phi_{,22} - \Phi_{,11}) \left[\left(\frac{1}{T}\right)_{,22} - \left(\frac{1}{T}\right)_{,11}\right] \tag{1}$$

$$+ 4 \left(\frac{1}{T}\right)_{,12} \Phi_{,12} = 0 \quad .$$

(The details of this analysis are given in ref. 17)
For this study a Ramberg-Osgood material will be used

$$\frac{E}{\sigma_o} \varepsilon_T = \sigma_e/\sigma_o + b\alpha[(\sigma_e/\sigma_o)^{1/n} - 1]$$

$$\text{or } \frac{E}{T} = 1 + b\alpha[(\sigma_e/\sigma_o)^{(1-n)/n} - \sigma_o/\sigma_e] \tag{2}$$

where $b = 0$ if $\sigma_e < \sigma_o$

$b = 1$ if $\sigma_e \geq \sigma_o$.

Thus the governing equation (1) will be solved subject to the constitutive laws (eq. (2)).

The geometry of the blunted cracks in the deformed state under Mode I loading will resemble small elliptical perforations as shown in figure 1. The size of the deformed crack tip root radius will be determined through integration of the strain displacement relationships

$$u_{i,j} + u_{j,i} = 2\varepsilon_{ij}$$

The affected strain hardening region will be divided into a small grid utilizing elliptical coordinates and the governing set of equations will be solved through the method of finite differences. At some distance from the deformed crack tip the linear solution of Inglis (ref. 16) will apply. The stress at the outer boundary of the inner strain hardening region will be then matched to the Inglis solution. The outer boundary will be enlarged until there is no change in the inner stress field. (A detailed description of the solution procedure is given in ref. 18.)

PRESENTATION OF RESULTS

The stress and displacement fields in the field surrounding a deformed crack tip in a strain hardening material which is incompressible in both the elastic and plastic regions are examined. Strain hardening exponents of 0.2 through 0.01 are presented. The range of initial yield strain values is from 0.01 through 0.0001. The value of α in the effective stress-effective strain relationship, equation (2), is taken to be 1.0 in this study. (The authors have found that small changes in α and ν do not influence the solution significantly.) The "linear" results reported here are those of Inglis (ref. 16) for a deformed crack tip in a linear material. The "singular" results are those corresponding to a crack which has no root radius in a linear material.

The plastic zone shape for the smallest ellipse investigated ($\rho = 0.0018 (K/\sigma_o)^2$) is shown in figure 2. Note that with decreasing hardening ($n \to 0$)

the plastic zone grows in maximum extent and leans progressively in the direction of crack propagation. For comparison, the singular plastic zone from McClintock and Irwin (ref. 19) and that of Levy et al (ref. 7) for a non-hardening ($n = 0$) material are also shown. As shown by figure 2, the plastic zone shape predicted by McClintock and Irwin (ref. 19) is approached by the present study as $n \to \infty$. The difference between the plastic zone shape predicted by Levy et al (ref. 7) and the present study for $n = 0.01$ is primarily due to the inclusion of blunting effects and use of ν of 0.5 in the latter; the difference should be negligible as $\varepsilon_o \to 0$.

The effective stress σ_e is shown versus the distance ahead of the deformed crack tip in figure 3. This figure shows that the effective stress varies as $(r^{n/n+1})^{-1}$ in the plastic zone ahead of the tip. (The behavior for other values of θ is similar). It can be shown that the strain energy has the form:

$$\frac{2EU}{\sigma_o^2} = \left[\frac{\sigma_e}{\sigma_o}\right]^2 + \frac{2}{1+n}\,\alpha\, b \left[\left[\frac{\sigma_e}{\sigma_o}\right]^{(1+n)/n} - 1\right] \quad \text{for a power hardening material}$$

Therefore, the strain energy varies approximately as $1/r$ in the plastic zone. This was a key assumption in the analysis of Rice and Rosengren (ref. 6) and Hutchinson (ref. 4).

The σ_{yy} stress in front of the crack tip is shown for various values of yield strain for $n = 0.01$ in figure 4. As shown in this figure, this stress is substantially reduced near the crack tip because of blunting and strain hardening, with the maximum value developed at some distance forward of the crack tip. (The σ_{yy} stress distributions for other values of n is quite analogous.) The analysis of Rice and Johnson (ref. 14) gives the same qualitative behavior; the correlation is believed to be quite reasonable in view of the several approximations involved. For a non-hardening material Rice (ref. 2) has shown that the maximum σ_{yy} stress is 2.97 σ_o. This stress, as predicted by Levy et al (ref. 7), approaches this limit at the crack tip as shown in figure 4. The σ_{yy} stress distribution of the present study in this figure reflects the presence of blunting and should coincide with the work of Levy et al (ref. 7) as $\varepsilon_o \to 0$.

Figure 5 shows the variation of maximum σ_{yy} stress with initial yield strain for varying hardening. As shown in the figure, blunting alone (the "linear" curve) forces the peak σ_{yy} stress to be finite and the inclusion of finite deformations (ref. 15) reduces the magnitude somewhat. However, the effects of blunting and plasticity taken together are significant: the peak σ_{yy} stress is reduced by a factor of 10 from that with blunting alone. From figure 5, one observes the peak σ_{yy} stress to be $3\sigma_o$ to $7\sigma_o$ depending upon n and σ_o/E. The peak σ_{yy} stress increases with n and decreases with σ_o/E. (The large value of peak σ_{yy} stress compared to the uniaxial yield stress, σ_o, is believed due to the presence of triaxiality in the crack tip region.)

The crack tip displacement in the direction of propagation (which is also the deformed crack root radius, ρ) is shown in figure 6 for varying initial yield strain and hardening exponent. The present study predicts that the forward crack tip displacement increases with σ_o/E and decreases with n. For comparison one-half the crack tip opening displacement calculated by Levy (ref. 7) is shown. The forward crack tip displacement as predicted by the present study and the work of Levy et al (ref. 7) show parallel behavior, although they are separated by some distance. This disagreement is believed due to the shape of the crack tip being elliptical in the present study instead of cylindrical.

Included also on figure 6 is the width of the "transition" or "stretch" zone which exists on the fracture surface between the cracked and the overload regions in fatigue. As Broek (ref. 20) has discussed, the depth of this transition zone is the crack tip opening displacement, and, therefore the width is the tip forward displacement.

Examination of the figure shows the correlation between the forward tip displacement and failure. The measurements of the stretch zone fall close to $n = 0.2$. For the steels and aluminums shown values of n around 0.05 have been reported in references 20, 21 and 22. However, it is known that for this class of materials the value of n varies with plastic strain (ref. 23). For large plastic strain ($\varepsilon_p > 10\%$), the strain hardening exponent is close to 0.2 as shown by Jones and Brown (ref. 24) for 4340 steel. The strains in the tip region are clearly greater than 10% so that the agreement between the measurements and the analysis appears quite reasonable. The scatter band shown on the figure is an indication of the span of actual measurements (authors typically report a 40% variation).

DISCUSSION

Previously McGowan and Smith (ref. 15) performed a finite deformation analysis of the region surrounding deformed crack tips for a linear (stress-strain) material. The results of the finite deformation work showed that the maximum σ_{yy} stress occurred in front of the deformed crack tip. It was determined that the stress distribution around the crack tip was "similar", in the sense that one stress distribution could be used to describe the response of the material under load. The size of the affected zone would depend upon the load and crack length through K. The self-similarity of the stress field was a direct result of the blunting process, and would be expected to remain as long as the affected zone stayed small with respect to the crack length, thickness, or any other in-plane dimension.

The behavior is quite similar for a power hardening material. The stress field is self-similar with the size of the affected zone varying with K. The maximum σ_{yy} stress will only then be a function of the material properties E, n, and σ_o. The stretching of the similar stress distribution will depend upon K as well as the other material properties. One may conjecture that failure would depend upon the growth in size of a critical dimension, such as plastic zone size, which increases with K.

Wells (ref. 25) and others have used the crack opening displacement as a fracture criterion. Broek (ref. 20) has used this concept to correlate the depth of transition zones in aluminum with fracture toughness. The present study shows good correlation of fracture toughness and transition zone width. Krafft (ref. 26), Hahn and Rosenfield (ref. 27) and Rice and Johnson (ref. 14) have all shown good correlation of plane strain fracture toughness with some minute particle size or process zone size for specific cases.

SUMMARY AND RECOMMENDATIONS

Following the pioneering studies of Hutchinson (ref. 4), Rice and Rosengren (ref. 6), Levy et al (ref. 7) and Hilton and Hutchinson (ref. 8), the authors have obtained a full field deformation plasticity finite difference solution to the Mode I plane strain problem including the effects of blunting. The material was incompressible in both the elastic and plastic regions, and followed a power hardening rule. Stress and displacement fields surrounding the deformed crack tip are presented, and are found to compare favorably both with the analysis of other investigators as well as experimental results. Because of the improved accuracy expected from a full field solution, it would be appropriate to incorporate such a solution into theories concerning void coalescence and final instability. Efforts are currently being devoted to such an approach.

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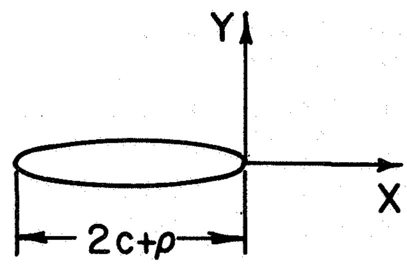
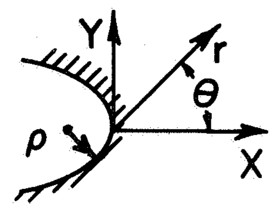


Deformed crack geometry



Enlarged view of deformed crack tip region

Figure 1.- Problem geometry.

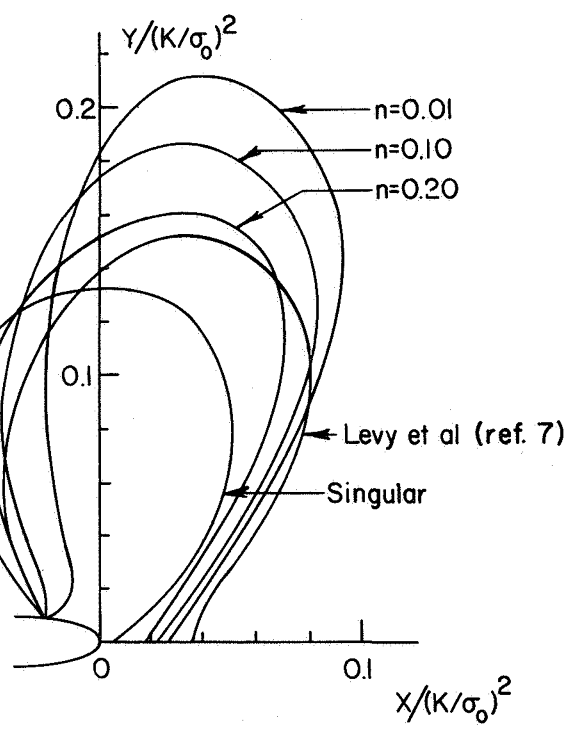


Figure 2.- Plastic zone shape.

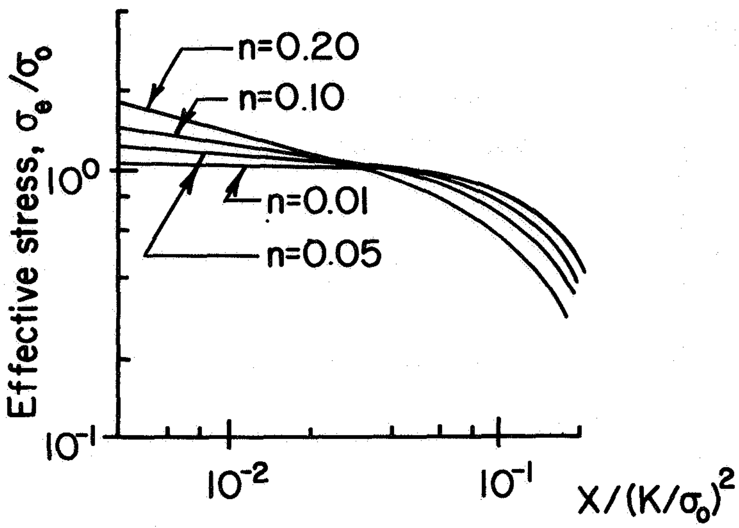


Figure 3.- Effective stress distribution forward of the blunted crack tip.

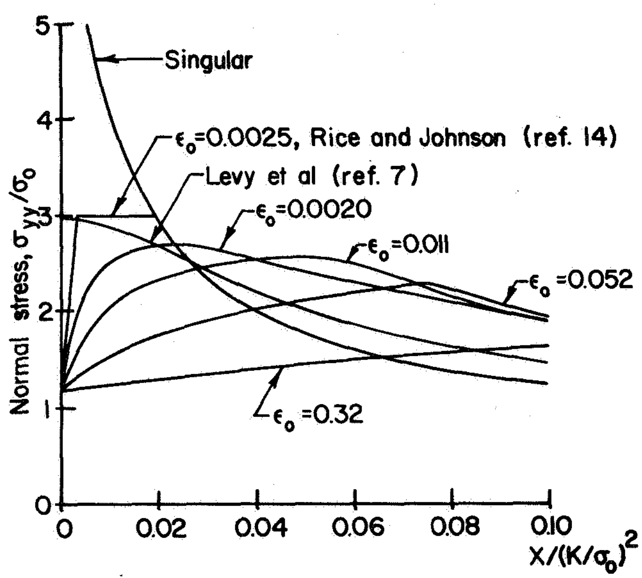


Figure 4.- Distribution of σ_{yy} stress forward of the blunted crack tip for n = 0.01.

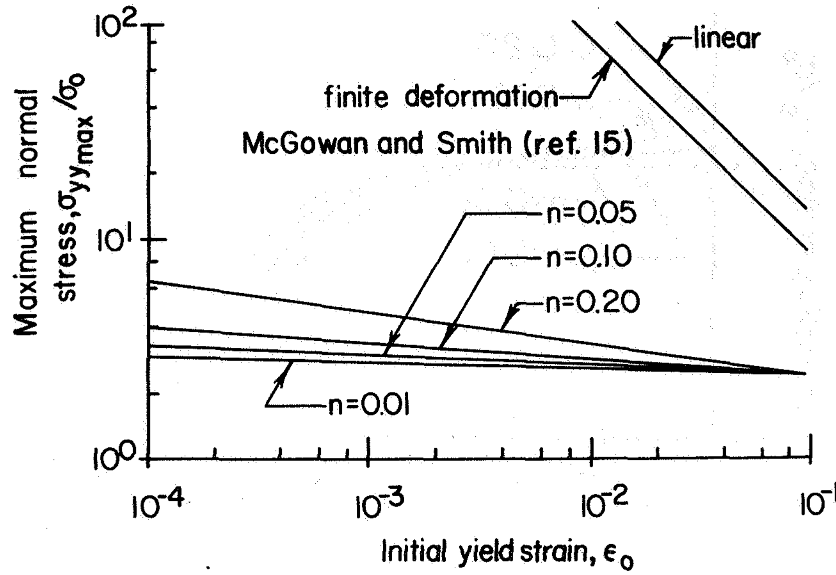


Figure 5.- Variation of σ_{yy} stress maximum with ε_0 and n.

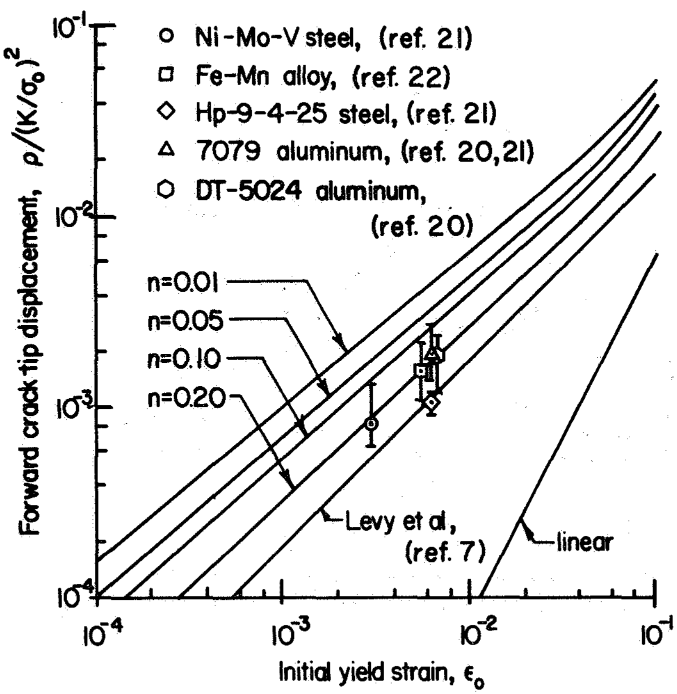


Figure 6.- Forward crack tip displacement and stretch zone variation with n and ε_0.