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FIRST-ORDER-HOLD INTERPOLATION DIGITAL-TO-ANALOG CONVERTER WITH APPLICATION TO AIRCRAFT SIMULATION

William B. Cleveland

Ames Research Center
Moffett Field, Calif. 94035

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16. Abstract

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The First-Order-Hold Interpolation (FOHI) Digital-to-Analog Converter (DAC) is presented as a device which offers smooth output, regardless of cycle time — a significant improvement over the conventional Zero-Order-Hold (ZOH) DAC and the First-Order-Hold Extrapolation (FOHE) DAC. The Laplace transforms of these three conversion types are developed and their frequency response characteristics and output smoothness are compared. The FOHI DAC exhibits a pure one-cycle delay. Whenever the FOHI DAC input comes from a second-order (or higher) system, a simple computer software technique can be used to compensate for the DAC phase lag. When so compensated, the FOHI DAC has (1) an output signal that is very smooth, (2) a flat frequency response in frequency ranges of interest, and (3) no phase error. When the input comes from a first-order system, software compensation may cause the FOHI DAC to perform as an FOHE DAC, which, although its output is not as smooth as that of the FOHI DAC, has a smoother output than that of the ZOH DAC.

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SYMBOLS

j	imaginary operator, $\sqrt{-1}$
s	Laplace variable
$\mathscr{L}$	Laplace transform operator
t	time, sec
T	computer step size, "frame time," sec
$u(t)$	unit step at time t
x	continuous input variable
X	Laplace transform of x
x^*	sampled input variable
X^*	Laplace transform of x^*
y	piecewise continuous DAC output variable
Y	Laplace transform of y
ε	error between x and y
τ	continuous time variable in interval T
ω	frequency, rad/sec
$(\dot{\ })$	time derivative
$\triangleq$	definition

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WITH APPLICATION TO AIRCRAFT SIMULATION

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SUMMARY

Those who design piloted aircraft simulations must contend with the finite size and speed of the available digital computer and the requirement for simulation reality. With a fixed computational plant, the more complex the model, the more computing cycle time is required. While increasing the cycle time may not degrade the fidelity of the simulated aircraft dynamics, the larger steps in the pilot cue feedback variables (such as the visual scene cues), may be disconcerting to the pilot.

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INTRODUCTION

Over the last decade, the field of real-time, man-in-the-loop aircraft simulation has undergone some basic changes. The parallel computing ability of analog computers has given way to the serial digital computer with its obvious computational advantages and some not so obvious disadvantages. Early digital computers had insufficient speed to simulate an aircraft, control cockpit simulators, and acquire data (see fig. 1), all without producing a "ratchetlike" response in the simulation. Now, however, digital computers are so fast that the serial computation of the kinematic equations in real-time is no longer a serious problem, but discreteness in the outputs remains a problem for simulation. From a psychological standpoint, the discreteness of the data output from the computer can be distracting to the pilot attempting to fly a simulated aircraft. He expects to see smooth continuous changes in his flight information, just as he would in the real aircraft, but the discreteness of

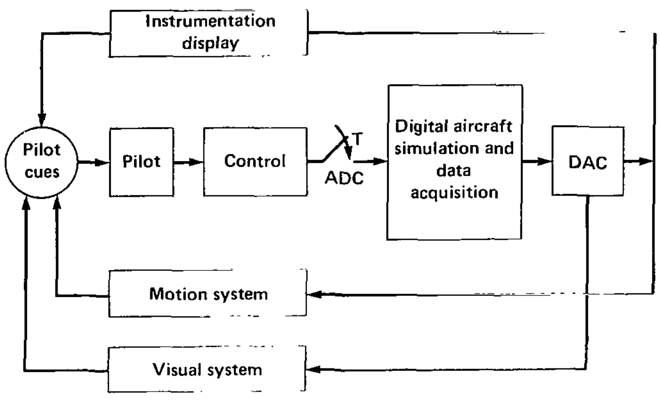


Figure 1.- Elements of the piloted digital aircraft simulation.

the computer output may degrade the realism of the simulator to the point where he "flies the simulator" rather than the aircraft. Experienced pilots can accommodate to or compensate for many of the indications that they are flying a simulated aircraft rather than a real aircraft; however, it is distracting to the pilot to contend with spurious flying cues that are due to output data discreteness and not to the simulated aircraft characteristics.

There are two causes of the discreteness of data output from the digital computer via the ordinary DAC. These are quantization effects that are due to the number of bits in the DAC word and the effects that are due to the discrete nature of the output. Figure 2 illustrates the latter. The roughness due to DAC quantization is alleviated by increasing the number of bits in the DAC word length. However, the roughness due to time discreteness is a complex phenomenon which involves computer hardware and software. The problem for the simulator pilot is most acute when the instrument dynamics can closely follow the stairstep-type signal or when the steps are magnified in some way, such as with angular displacements in a visual scene. A typical visual scene is modeled to scale and scanned by a television camera on a servo-driven gantry which responds to an angular pitch "up" command by shifting the scene downward on the television monitor. The angular discreteness is translated into a positional discreteness on the visual scene monitor through a multiplication of the distance from pilot's eye to the monitor. Reduction of these jumps is desirable to avoid pilot distraction in a research environment.

Many roughness problems can be cured by increasing computation speed, but most complex real-time programs are already running near the maximum speed of the computer. This conflicts directly with the need to lengthen the cycle time to allow more complex (and therefore more realistic) models to be simulated. The FOHI DAC, analyzed within this report, copes with this problem by eliminating or reducing the discreteness in DAC outputs to the simulator.

DAC's are a rather small component in a total piloted simulation system which includes a computer, a cockpit, and simulators. However, they are the important link within the pilot's closed-loop control system. In the analysis of any control loop, the model of each element is needed to assess the whole. Therefore, the frequency response and the discreteness effects of two well-known types of DAC's, the ZOH and FOHE DAC's, are compared with those of the FOHI DAC in the following discussion.

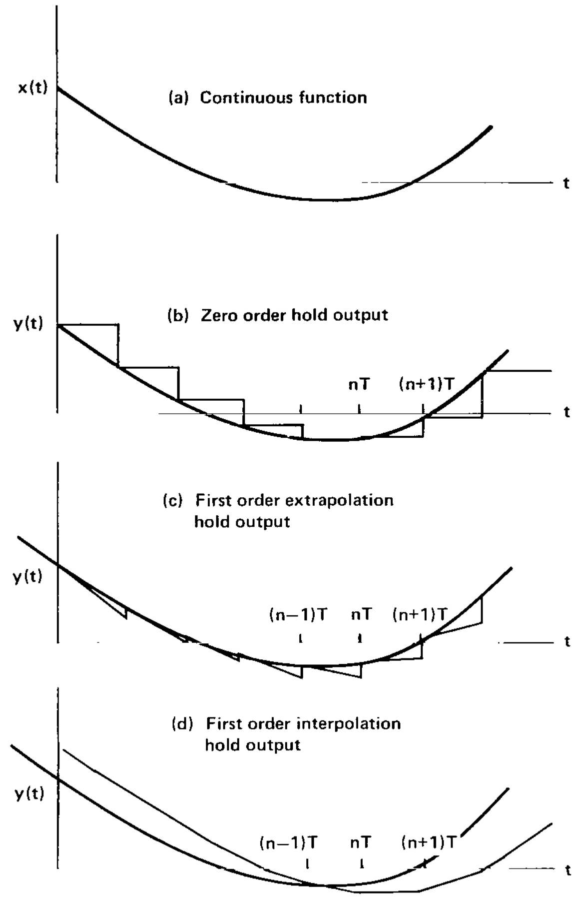


Figure 2.- Comparison of various DAC outputs with a corresponding continuous signal.

DIGITAL-TO-ANALOG-CONVERTER FREQUENCY RESPONSE

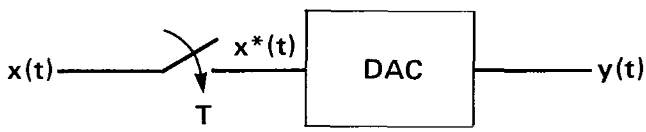
The function of the DAC is to change the form of a variable from a pattern of bits in the digital word into a continuous (normally piecewise continuous) analog voltage signal. Figure 2 compares the ZOH, FOHE, and FOHI DAC output signals versus a continuous counterpart signal. By far the most common type in use today is the ZOH DAC. Figure 2(b) shows the way a value is held constant between sample points which occur at interval T.

In an effort to obtain a smoother output from the digital computer, First-Order Hold (FOH) DAC's may be employed. Figure 2(c) illustrates the time history of an FOHE DAC, which uses one past in addition to the present value of discrete input data to extrapolate a more representative output than that of the ZOH DAC. Inspection of figures 2(b) and (c) shows that, as the frequency content of the continuous signal increases and as T is increased, the discontinuous jump in the outputs also increases. If interpolation, instead of extrapolation, between sample points is used on the FOH DAC, there are no discontinuous jumps, since the linear portions of the output start and end on sampled values, as seen in figure 2(d).

The transfer function of a DAC is derived next in order to make meaningful comparisons between the three types of DACs discussed.

General Transfer Function of Linear Hold DACs

For zero- and first-order DAC's, the output $y(t)$ is a piecewise linear function of the input $x^*(t)$ and t.



The output function is a constant or ramp in the nth T period and can be represented by

$$y_n(\tau) = A_n\tau + B_n \qquad 0 \leq \tau < T \tag{1}$$

so that

$$y(t) = \sum_{n=-\infty}^{\infty} (A_n\tau + B_n) \tag{2}$$

where

$$A_n = a_1 x[(n - 1)T] + a_2 x(nT) \tag{3}$$

$$B_n = b_1 x[(n - 1)T] + b_2 x(nT) \tag{4}$$

and where a_i and b_i are constants determined by the particular DAC. For example, the ZOH output has a constant value equal to $x(nT)$ throughout the interval τ. Thus $a_1 = a_2 = b_1 = 0$ and $b_2 = 1.0$.

Taking the Laplace transforms, one obtains

$$Y(s) = \mathscr{L}[y(t)] = \sum_{n=-\infty}^{\infty} \int_{nT}^{(n+1)T} (A_n\tau + B_n)e^{-st}\, dt \tag{5}$$

By letting $\tau = t - nT$, the above equation becomes

$$Y(s) = \sum_{n=-\infty}^{\infty} \left(A_n e^{-snT} \int_0^T \tau e^{-s\tau}\, d\tau + B_n e^{-snT} \int_0^T e^{-s\tau}\, d\tau \right)$$

$$= \sum_{n=-\infty}^{\infty} (S_1 A_n + S_2 B_n) e^{-snT}$$

where

$$S_1 = \frac{1}{s}\,(1 - e^{-sT} - sTe^{-sT})$$

and

$$S_2 = \frac{1}{s}\,(1 - e^{-sT})$$

By using the shifting properties (discussed in the appendix), for S_1, one gets

$$\sum_{n=-\infty}^{\infty} S_1 A_n e^{-snT} = S_1 \sum_{n=-\infty}^{\infty} \{a_1 x[(n - 1)T] + a_1 x(nT)\} e^{-snT}$$

$$= X^*(s) S_1 (a_1 e^{-sT} + a_2)$$

Similarly, for S_2, we have

$$\sum_{n=-\infty}^{\infty} S_2 B_n e^{-snT} = X^*(s) S_2 (b_1 e^{-sT} + b_2)$$

The desired transfer function then becomes

$$\frac{Y(s)}{X^*(s)} = \frac{1}{s^2}\,(a_1 e^{-sT} + a_2)(1 - e^{-sT} - sTe^{-sT}) + \frac{1}{s}\,(b_1 e^{-sT} + b_2)(1 - e^{-sT}) \qquad (6)$$

The specific cases of the ZOH, FOHE, and FOHI DAC's may be obtained by proper selections of the a and b constants.

Zero-Order-Hold DAC

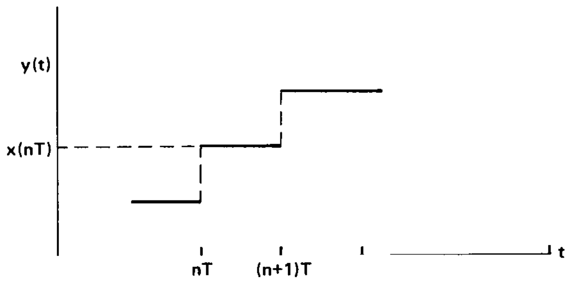
The time function $y(t)$ is seen, from sketch (a), to be a constant over the period T. In which case,

$$y(t) = x(nT) \text{ for } nT \leq t < (n+1)T \; . \qquad (7)$$

Thus, comparing equation (7) with (1), (3), and (4) it follows that

$$a_1 = a_2 = b_1 = 0 \text{ and } b_2 = 1 \; .$$

By substituting the derived values for a_i and b_i into the general transfer function, we have



Sketch (a)

$$\frac{Y(s)}{X^*(s)} = \frac{1 - e^{-sT}}{s} \qquad (8)$$

As shown in the appendix, given $X(s)$ is the Laplace transform of $x(t)$, then the transform of the sampled variable $x^*(t)$ is

$$X^*(s) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X\left(s - j\,\frac{n2\pi}{T}\right) \tag{9}$$

The transfer function from x to y for the ZOH DAC may then be written

$$\frac{Y(s)}{X(s)} = \frac{1 - e^{-sT}}{sT} \sum_{n=-\infty}^{\infty} X\left(s - j\,\frac{n2\pi}{T}\right)$$

While the transfer function from x to y is complicated by the repeating spectrum, the multiplier $1/T$ renders the transfer function nondimensional and allows determination of the magnitude of the frequency response, as a function of the nondimensional product ωT.

Thus,

$$\frac{1}{T}\,G_z(s) = \frac{1}{T}\,\frac{Y(s)}{X^*(s)} = \frac{1 - e^{-Ts}}{sT} \tag{10}$$

By substituting $s = j\omega$, one may obtain the magnitude and phase of the sinusoidal frequency response, as follows:

$$\left|\frac{1}{T}\,G_z(j\omega)\right| = \frac{2}{\omega T}\left|\sin\frac{\omega T}{2}\right| \tag{11}$$

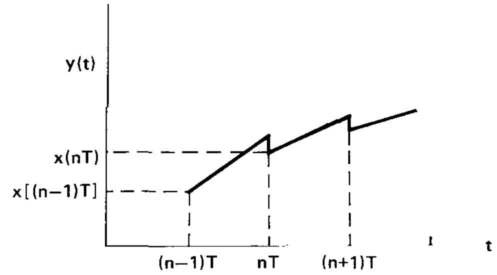
$$\measuredangle\, G_z(j\omega) = -\frac{\omega T}{2} \tag{12}$$

First-Order-Hold Extrapolation DAC

From sketch (b), one can see that

$$y(t) = \left\{x(nT) - x[(n-1)T]\right\}\frac{\tau}{T} + x(nT) \tag{13}$$

for $nT \leq t \leq (n+1)T$ and $0 \leq \tau < T$.



Sketch (b)

Comparison of equation (13) with (1), (3), and (4) yields

$a_2 = -a_1 = \frac{1}{T}$; $b_1 = 0$ and $b_2 = 1$

which may be substituted into the general transfer function, equation (6), to obtain

$$G_{FE}(s) = \frac{Y(s)}{X^*(s)} = \frac{1}{s^2T}\left(1 - e^{-sT}\right)^2 (1 + sT) \tag{14}$$

For the frequency response characteristics, the magnitude relationship is

$$\left|\frac{1}{T} G_{FE}(j\omega)\right| = \frac{4(1 + \omega^2T^2)^{1/2}}{\omega^2T^2} \sin^2 \frac{\omega T}{2} \tag{15}$$

and the phase is

$$\measuredangle\, G_{FE}(j\omega) = \tan^{-1} \omega T - \omega T \tag{16}$$

First-Order-Hold Interpolation DAC

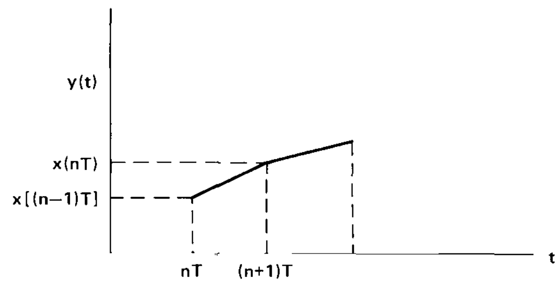
Sketch (c) illustrates the time shift between input and output, which yields the time function

$$y(t) = \{x(nT) - x[(n - 1)T\} \frac{\tau}{T} + x[(n - 1)T] \tag{17}$$

for $nT \leq t \leq (n + 1)T$ and $0 \leq \tau \leq T$.

Comparison of equation (17) with (1), (3), and (4) yields the constants

$$a_2 = -a_1 = \frac{1}{T}\ ,\ b_1 = 1 \text{ and } b_2 = 0$$



Sketch (c)

which, upon substitution into the general transfer function, equation (6), yields

$$G_{FI}(s) = \frac{Y(s)}{X^*(s)} = \frac{1}{s^2T}\left(1 - e^{-sT}\right)^2 \tag{18}$$

Since

$$\frac{1}{T} G_{FI}(s) = \left[\frac{1}{T} G_z(s)\right]^2$$

the magnitude and phase relationships are

$$\left|\frac{1}{T} G_{FI}(j\omega)\right| = \frac{4}{\omega^2T^2} \sin^2 \frac{\omega T}{2} \tag{19}$$

$$\measuredangle\, G_{FI}(j\omega) = -\omega T \tag{20}$$

DAC SMOOTHNESS

A measure of smoothness is the maximum error between the DAC output and a corresponding continuous input sine wave signal $x = A \sin \omega t$. For the ZOH, the error is

$$\varepsilon = A \sin\left[\left(n + \frac{\tau}{T}\right)\omega T\right] - A \sin(n\omega T) \quad 0 \le \tau < T \tag{21}$$

which is a maximum near the zero crossing of the sine wave. If we let the time nT be at the crossing, the maximum error occurs when $\tau = T$. The maximum error is then given by

$$\varepsilon_{max} = A \sin \omega T$$

and, for small ωT, the maximum error is

$$\varepsilon_{max}/A \approx \omega T \tag{22}$$

The maximum error for FOHE occurs when ωt is near $\pi/2$. The FOHE error relation is

$$\varepsilon = A_{\sin}\left[\left(n + \frac{\tau}{T}\right)\omega T\right] - \left\{A_{\sin}(n\omega T) - A \frac{\tau}{T} \sin(n\omega T) + A \frac{\tau}{T} \sin[(n - 1)\omega T]\right\} \quad 0 \le \tau < T \tag{23}$$

If the time nT occurs at the peak of the sine wave, then $n = \pi/2\omega T$. The maximum error occurs at $t = (n + 1)T$. On substituting these values in the FOHE error relation, the solution for the maximum error is given by

$$|\varepsilon_{max}|/A \approx \omega^2 T^2 \tag{24}$$

The maximum error for FOHI also occurs when ωT is near $\pi/2$. Assuming the sample points are equally spaced about the peak amplitude, the maximum error occurs at the peak of the sine wave. Thus, the FOHI error relation is

$$\varepsilon = A_{\sin} \frac{\pi}{2} - \left\{A_{\sin}(n\omega T) + A \frac{\tau}{T} \sin[(n + 1)\omega T] - A \frac{\tau}{T} \sin(n\omega T)\right\} \quad 0 \le \tau < T \tag{25}$$

Under these conditions, nT occurs at $t = \pi/2\omega - T/2$, so that $n = \pi/2\omega T - 1/2$. The maximum error occurs when $\tau = 1/2$. On substituting these values in the FOHI error relation and assuming small ωT, the solution for maximum error provides

$$\frac{\varepsilon_{max}}{A} \approx \frac{\omega^2 T^2}{8} \tag{26}$$

The results are as one might expect from inspection of figure 2; the ZOH DAC exhibits the most error and the FOHI DAC the least. Table 1 gives a comparison of the relative DAC error for the ωT of interest in aircraft simulations.

TABLE 1.- RELATIVE SMOOTHNESS USING THE MAXIMUM ERROR IN THE DAC REPRESENTATION OF A SINE WAVE OF AMPLITUDE A AND FREQUENCY ω

ωT	Relative error ε_{max}/A		
	ZOH	FOHE	FOHI
0.1	0.1	0.01	0.00125
.2	.2	.04	.005
.3	.3	.09	.01125
.4	.4	.16	.02
.5	.5	.25	.03125
.6	.6	.36	.045

COMPARISON OF THE DAC CONFIGURATIONS

Figure 3 illustrates the magnitude and phase for each of the three DACs under consideration. The ZOH DAC appears to be superior in terms of less phase lag over the range shown and also in the way its magnitude is relatively flat at low values of ωT. In some control systems, ωT might range close to the maximum allowable of $\omega T = \pi$, whereas, in simulations of aircraft, such things as numerical instabilities of integration algorithms require $\omega T < 0.6$ in practical cases. For example, the second-order Adams-Bashforth predictor, an integration algorithm commonly used in simulation, is numerically unstable when $\omega T = 1.0$; the third-order Adams-Bashforth is unstable when $\omega T = 0.5$. In this case, ω is the natural frequency of the system under consideration. To obtain reasonable accuracy, ωT must be considerably smaller than the stability bound.

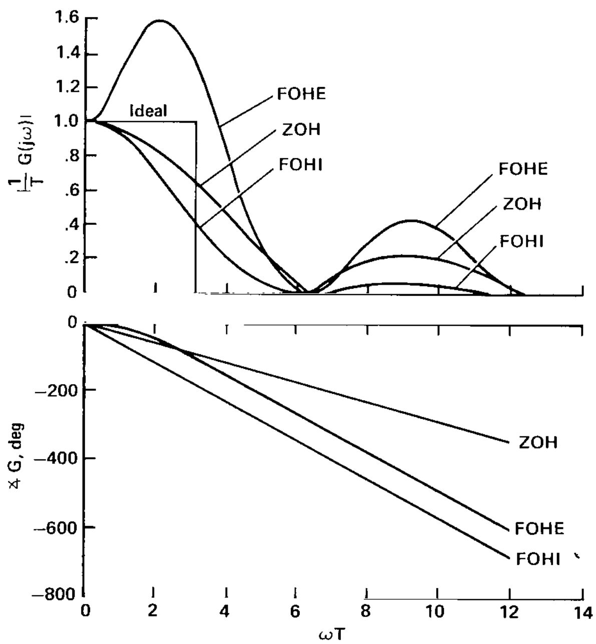


Figure 3.- Magnitude and phase characteristics of zero- and first-order DAC's.

A second rationale for limiting ωT to 0.6 concerns the aircraft itself. The value of ω in aircraft simulations depends on the aircraft type: a transport, such as the DC-8, has its highest short-period frequency at approximately 3.6 rad/sec; a high-performance aircraft, such as the A7, has its highest

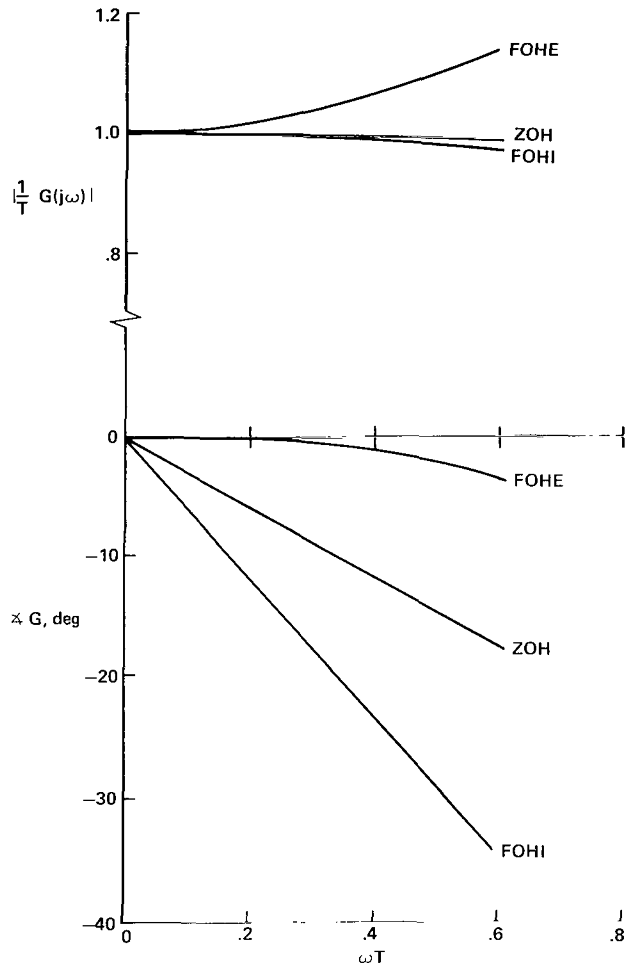


Figure 4.- Enlargement of magnitude and phase characteristics of the DAC's at low ωT.

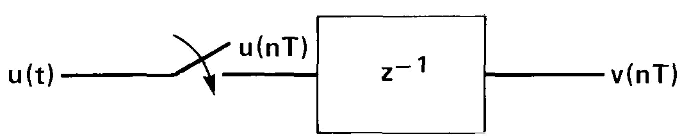
value at about 8.5 rad/sec; while a helicopter's flapping frequency, such as the CH-46, has a value of about 14 rad/sec. Using a T of 0.05, a representative value in aircraft simulations, ωT is 0.7 for the CH-46, but for the fixed wing aircraft, ωT is well below 0.6. Consequently, helicopter simulations present computation problems for digital computers and require special modelling techniques.

Figure 4 shows the phase and magnitudes of the DAC's over the lower range of ωT. It is interesting to note that the FOHI and ZOH DAC's are almost flat in magnitude over this range. The phase of the FOHE DAC is very good over this range, but near an ωT of 0.5, the DAC shows significant gain.

The phase of the FOHI DAC is that of a pure one-T lag. If, somehow, a pure one-T lead could be introduced, the combination of lead and FOHI DAC would: be very smooth, have flat frequency response over ωT ranges of interest, and have zero phase error.

Phase Compensation of the FOHI DAC

To show that the phase lag of the FOHI DAC is equivalent to a one-T lag, consider a unit which might produce such a lag:



Between v and u is the T-lag device. In terms of Z-transforms,

$$V(z)/U(z) = Z^{-1} \tag{27}$$

Now

$$Z = e^{sT} = e^{j\omega T} = \cos\,\omega T + j\,\sin\,\omega T \tag{28}$$

so that upon substitution of equation (28) into (27) the magnitude and phase of the lag device are

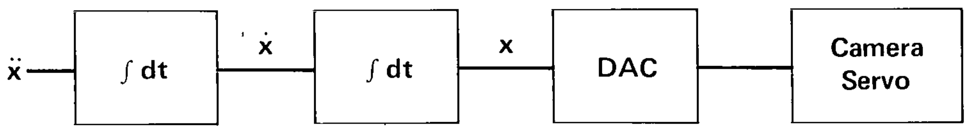
$$\left|\frac{V(j\omega)}{U(j\omega)}\right| = 1 \tag{29}$$

and

$$\measuredangle\ V(j\omega)/U(j\omega) = -\omega T \tag{30}$$

Thus, comparing equation (20) with (30) is seen that the FOHI DAC has the phase of a one-T time-lag device. A device which produces a pure one-T lead would be useful in compensating for the time lag, but, unfortunately, is not available. However, one can resort to a computer programming procedure to introduce the desired one-T lead in certain applications.

Consider the case in which a second-order system precedes the DAC. For example, in the visual display for an aircraft simulation, the camera servo is positioned in the translational direction X through a double integration of the X component of the aircraft's acceleration:



On the digital computer, the integrations are performed numerically with an integration algorithm. As an example, the Euler integration method illustrates how a one-T lead may be generated through the double integration process.

Applying the Euler algorithm in the double integration as illustrated above, we have

$$\dot{x}(nT) = \dot{x}[(n - 1)T] + T\ddot{x}[(n - 1)T] \tag{31}$$

$$x(nT) = x[(n - 1)T] + T\dot{x}[(n - 1)T] \tag{32}$$

The computational technique would normally be to solve equation (32) first for $x(nT)$ and then equation (31) for $\dot{x}(nT)$, in that order. The DAC output at the time nT is $\dot{x}(nT)$. Now, if the order of integration is reversed, the first calculation yields $x(nT)$; the second integration is performed by using $\dot{x}(nT)$ to obtain $x[(n + 1)T]$. When $x[(n + 1)T]$ is transmitted through the DAC at time nT, a full one-cycle lead has been obtained. When this technique is combined with the FOHI DAC, the phase lead of the computation cancels the phase lag of the DAC.

Where there is no second-order (or higher) system preceding the DAC, the application of the FOHI DAC is less advantageous. However, in cases where there is one integration preceding the DAC, it is an easy matter to add a simple lead term, thus converting the FOHI to a FOHE DAC as shown in figure 5. The input to the DAC is of the form $x + T\dot{x}$, where $T\dot{x}$ is the lead compensation.

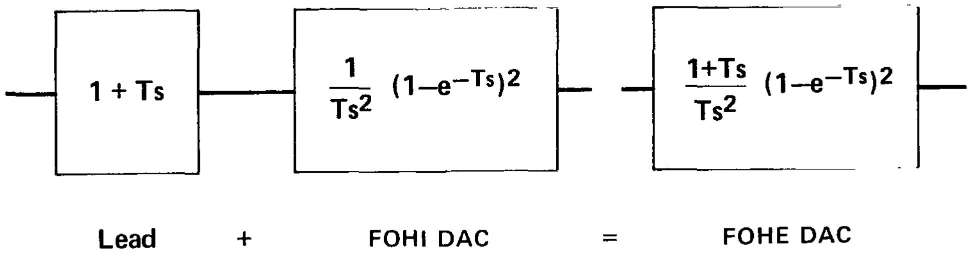
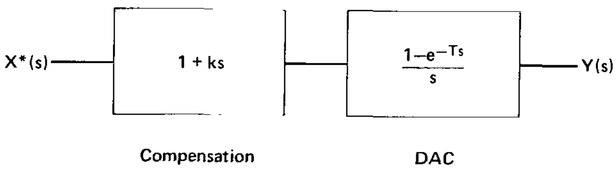


Figure 5.- Relationship between FOHI and FOHE DAC's.

Phase Compensation for the ZOH DAC

Since the ZOH DAC is widely used, it is worthwhile to describe one method of compensation for the phase characteristics of this DAC. If the first time derivative of the output is available within the digital computer, the addition of $k\dot{x}$ to the normal output x will result in phase lead, which may be used to offset the phase lag of the DAC. In block form, the system is



The input/output transfer function is

$$\frac{Y(s)}{X^*(s)} = \frac{1 + ks}{s}\left(1 - e^{-Ts}\right) \tag{33}$$

The phase of $Y(s)/X^*(s)$ is the phase of the compensation plus that of $G_Z(s)$

$$\phi = \tan^{-1} k\omega - \frac{\omega T}{2}$$

When $\phi = 0$,

$$k = \frac{1}{\omega} \tan \frac{\omega T}{2}$$

For $\omega T < 0.6$

$$\tan \frac{\omega T}{2} \approx \frac{\omega T}{2}$$

so that

$$k = \frac{T}{2}$$

While setting $k = T/2$ results in nearly zero phase, there is a minimal gain magnification with a value of 1.04 at an ωT of 0.6.

CONCLUSION

The FOHI DAC offers a method of replacing the stairstep output of the common ZOH DAC with a smooth signal. Where a DAC is associated with a second-order (or higher) system, the programming procedure described will result in an output which is smooth and has a flat frequency response without phase error. By itself, the smooth signal is useful for cases where signal phase angle is unimportant, as found in many nonreal-time applications. In the real-time case, the DAC whose output commands a position servo system is often associated with a second-order system simulated in the computer. In the field of aircraft simulation, the visual simulators, in which a servo positions a camera over a model runway scene, is such a system. Conventional ZOH DACs produce "ratchetlike" jumps in the scene when either the signal frequency content or the computer stepsize is large. Through the use of FOHI DAC's, the distracting ratchet response is eliminated, even for large step sizes. This technique effectively conserves computer power by eliminating the need to shorten computer step size to achieve needed smoothness. The time saved is therefore available to expand the simulation model for increased fidelity.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, CA 94035, May 27, 1976

APPENDIX

TRANSFER FUNCTION OF A SAMPLED SIGNAL

The sampled signal $x^*(t)$ of a continuous signal $x(t)$ can be represented by a series of spikes (delta functions) multiplied against $x(t)$:

$$x^*(t) = x(t) \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

where

$$\delta(t - kT) = \begin{cases} 1 & t = kT \\ 0 & \text{else} \end{cases}$$

The Fourier Series representation of this summation is

$$\sum_{k=-\infty}^{\infty} \delta(t - kT) = \sum_{n=-\infty}^{\infty} C_n \exp(j2\pi nt/T)$$

where

$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} \sum_{n=-\infty}^{\infty} \delta(t - nT)\exp(-j2\pi nt/T)dt = \frac{1}{T}$$

so that the Laplace transform is

$$\mathscr{L}[x^*(t)] = \int_{-\infty}^{\infty} x(t)\left[\frac{1}{T} \sum_{n=-\infty}^{\infty} \exp(j2\pi nt/T)\right] e^{-sT}\, dt$$

$$= \frac{1}{T} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} x(t)\exp[(-s + 2\pi nT^{-1})t]dt$$

Now if

$$x(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\, dt$$

then

$$x(s - j2\pi n/t) = \int_{-\infty}^{\infty} x(t)e^{-(s-j2\pi n/T)}\, dt$$

and so

$$\mathscr{L}[x^*(t)] = 1/T \sum_{n=-\infty}^{\infty} x(s - j2\pi n/T)$$

Alternative Version of the Transfer Function

Whereas the first version of the transfer function yields a result with a repeating spectrum, this alternative is of more use in the mathematical operations dealing with the discrete sampled signals. Again, the sampled signal is represented by a summation of the sampled signals, but the "sifting" properties of the delta function are utilized rather than the spectral properties of the Fourier Series approach.

$$\mathscr{L}[x^*(t)] = \int_{-\infty}^{\infty} x^*(t)e^{-st}\, dt$$

$$= \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} x(t)\delta(t - kT)e^{-st}\, dt$$

$$= \sum_{k=-\infty}^{\infty} x(kT)e^{-skT}$$

It is convenient in working with discrete signals to use the following shifting property of the transform:

$$\mathscr{L}\{x[(k + n)T]\} = \sum^{\infty} x[(k + n)T]e^{-skT}$$

$$= e^{nsT} \sum_{m=-\infty}^{\infty} x(mT)e^{-smT}$$

$$= e^{nsT}\, x^*(s)$$

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