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PROBABILISTIC FRACTURE MECHANICS AND OPTIMUM FRACTURE CONTROL OF THE SOLID ROCKET MOTOR CASE OF THE SHUTTLE

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16. ABSTRACT

Development of a procedure for the reliability analysis of the solid rocket motor case of the space shuttle has been described in this report. The analysis is based on probabilistic fracture mechanics and consideration of a probability distribution for the initial flaw sizes. The reliability analysis can be used to select design variables, such as the thickness of the SRM case, projected design life and proof factor, on the basis of minimum expected cost and specified reliability bounds. Effects of fracture control plans such as the non-destructive inspections and the material erosion between missions can also be considered in the developed methodology for selection of design variables. The reliability-based procedure that has been developed in this report can be easily modified to consider other similar structures and different fracture control plans.

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Introduction

All structural components of the solid rocket motor case of the space shuttle are considered to be fracture critical. It is also the present plan to reuse the solid rocket motor case for a designated number of missions. The expected number of missions and operations such as the tests on the case between the missions are accounted in the projected design life of the structure. A fracture control plan is necessary because fracture critical components are being reused.

In particular, this report is concerned with the fracture control of the membrane of the six cylindrical segments that are considered to be the most critical of all structural components of the case. The developed procedure can, however, be used for all similar structures. During each mission, significant loads are applied to these six cylindrical segments during the flight and "slap down" operations. The applied stresses from all other events during the mission are considered not significant enough to result in cyclic or time dependent crack growth. If the test or analysis indicate the possibility of other critical loading events they can be included in the fracture control plan by extending the reported analysis. Before each mission, the cylindrical segments are also subjected to a proof test. The loads applied during the proof tests can result in significant amount of crack growth. As a preventive measure to reduce the effective depth of cracks, the thickness of the membrane is reduced by a selected amount between two missions. While the effective depth of crack is reduced, the operation has the effect of increasing the applied stresses. This necessitates a larger initial thickness of the membranes than that would be designed without this particular plan for fracture control.

Therefore, any design of the membrane of the six cylindrical segments of the solid rocket motor case must arrive at an initial wallthickness "t", the thickness 'Δt' that will be decreased between each mission and the proof load factor 'K_p'. For example, a large value of initial wall thickness results in increased reliability, but results in the need for increased propellant, increased cost of operation and reduced pay load capability. On the other hand, a small initial wall thickness increases the probability of failure and the resulting loss of the shuttle vehicle and the pay load. Therefore, there is a need for optimizing the initial wall thickness. Similar arguments can be presented to explain the need for selecting the other design variables such as 'ΔL' and 'K_p' by optimizing the desired objective function of cost and weight.

In general, these design variables depend on the probability distribution for the initial flaw sizes present in the membrane, applied stresses during the use of the vehicle, crack growth characteristics of the material, fracture control plans, specified reliability bounds, weight and cost considerations. The report describes a reliability-based procedure that can be used to select the design variables of SRM by using probabilistic fracture mechanics and cost or weight considerations.

Method of Approach

As discussed in reference 1, careful NDI techniques can detect initial cracks greater than the surface length of $c_0 = 0.1$ inch and surface depth of $a_o = 0.5\ C_o$ with 100% success. It has been claimed that cracks corresponding to surface length $c_o = 0.1$ inch can be identified 100% of the time. If the corresponding maximum depth is 0.05 inch there is no possibility of any initial cracks of depth larger than 0.05 inch. Such an initial crack depth distribution can be analytically represented[2] by Johnson S_b distribution. The density function for the probabilistic model is written as follows

$$f_{a_o}(a_o) = \frac{\eta}{\sqrt{2\pi}} \frac{\lambda}{(a_o - \varepsilon)(\lambda - a_o + \varepsilon)} \exp\left[-\frac{1}{2}\left\{ \gamma + \eta\ \ell n \left(\frac{a_o - \varepsilon}{\lambda - a_o + \varepsilon} \right) \right\}^2 \right] ,$$

$$\varepsilon \leq a_o \leq \varepsilon + \lambda \quad , \quad \eta > 0 \quad , \quad -\infty < \gamma < \infty \quad , \quad \lambda > 0 \quad , \quad -\infty < \varepsilon < \infty \ ; \qquad (1)$$

The four parameters of the distribution are η, λ, ε and γ.

This probability distribution for initial crack depth changes after each mission, proof test and the material removal from the wall thickness. The change in distribution after each mission and proof test is due to the crack growth resulting from the applied stresses. This crack growth also depends on the lengths of the crack that are already present and the material properties that are responsible for the crack growth. In this analysis, the applied stresses and material properties are assumed to be known deterministically. If the initial crack length were also known deterministically the crack length after each use can be determined from equations such as Paris' equation[3], Foreman's equation[4] or Collipriests[5] equations. Because initial crack lengths are not known deterministically, crack length after each use of the vehicle is again another probabilistic distribution that has to be determined.

The cumulative density function for crack length after 'n' uses is denoted by $F(a_n)$. This represents the probability that $a_n \leq A$ after n uses. Each use is defined as one flight, slap down, proof test and material removal. In this analysis "slap down" effects have not been considered. The crack growth due to "slap down" effects can be considered in a similar way. Also, crack growth due to time related effects such as stress corrosion have also been neglected.

If $F(a_n)$ is known the probability distribution for the stress intensity factor (K) can be obtained from the knowledge of the applied stresses. The probability distribution $F(K_n)$ for stress intensity factor can be used to estimate the probability failure (P_f) which is the probability of stress intensity factor K_n greater than or equal to the critical stress intensoty factor during the projected design life of the structure. The critical stress intensity factor is denoted by K^c. In this analysis, stresses and the material properties are assumed to be known deterministically. However, the applied stress changes after each use due to material removal. Therefore, the probability of failure can be expressed as the probability of $a_n \geq a^c$. In this expression a^c is the critical crack depth that can be obtained from the critical stress intensity factor and the applied stress. This relationship between the stress intensity and the applied stress is discussed in the next section.

Stress Intensity Factor

For the analysis of the stress intensity factor in the membrane, an infinite plate model with elliptical surface flaws that are oriented perpendicular to the applied stress has been assumed. The relationship between the stress intensity, the applied tensile stress and crack depth is given by

$$K = \sqrt{\frac{1.2\ \pi\sigma^2 a}{Q\left\{\frac{a}{c}\right\}}} \tag{2}$$

where

$$Q\left\{\frac{a}{c}\right\} = \phi^2 - 0.212\,\frac{\sigma^2}{\sigma y^2} \tag{3}$$

In this equation, σY is the yield stress and ϕ is a function of the ratio of crack depth to crack length (a/c). Variation ϕ^2 with (a/c) is given in reference 1.

Because the crack depth (a) is a random variable the stress intensity factor K is also a random variable. In general, both crack depth a and crack length c are random variables and there is a need for a joint distribution for a and c. In this analysis, only the crack depth is considered as the random variable. It is also assumed that the probability distribution for crack depth 'a' is known initially and is given by a Johnson S_b distribution. The density function for the distribution is given in equation (1). This probability distribution for crack depth changes with use. The next step will be to determine the change and the new probability distribution after each flight and proof test.

Probability Distributions for Crack Depth After Use

The following symbols are used to properly account for the changes in probability distributions.

$f(a_o)$: Probability density function for the initial crack depth

$F(a_o)$: Cumulative distribution function for initial crack depth

$F(a_{op})$: Cumulative distribution function for initial crack depth after the first proof test

$F(a_n)$: Cumulative distribution function after N flights and (N+1) tests

$F(a_{np})$: Cumulative distribution function after N flights and N proof proof tests.

$F(a_n)$: Cumulative distribution function after material removal from the wall thickness.

Similarly, density functions are denoted by lower case 'f'. As discussed before, 'slap down' effects are not considered in the analysis but can be included by following a similar procedure.

The rate at which crack depth increases is assumed to be given by Paris' equation. Then

$$\frac{da}{dn} = C \quad (\Delta K)^n$$

where C and n are empirical constants. Alternately, the rate of crack growth can be assumed to be given by Foreman's equation of Collipriest's equation if they are found to represent the situation more accurately. For example, Collipriest's equation can be written as follows:

$$\frac{da}{dn} = \exp\left[n \frac{\ln K_c - \ln \Delta K_o}{2} \text{arc tan h} \left\{ \frac{\ln \Delta K - \frac{\ln K_c(1-R) + \ln \Delta K_o}{2}}{\frac{\ln K_c(1-R) - \ln K_o}{2}} \right\} \right.$$
$$\left. + \ln\left\{ c \exp\left(\frac{\ln K_c + \ln K_o}{2} n \right) \right\} \right] \qquad (4)$$

where n is an empirical constant. These equations can be used to obtain crack depth after N+1 uses if the crack depth after N uses and N proof tests are known deterministically, i.e.,

$$a_{N+1} = a_{N+1} \{a_{NP}\} \tag{5}$$

Similarly, crack depth after the proof test can be determined from equation (3) or (4) if the crack depth before the proof test is known deterministically, i.e.,

$$a_{NP} = a_{NP} \{a_N\} \tag{6}$$

These functions represented by equations (5) or (6) can be determined analytically or in the form of quadratures from equation (3) or (4). From equation (5), a_{N+1} can be obtained for every known value of a_{NP}. Similarly, a_{NP} can be obtained for every known value of a_N from equation (6). However, both a_{NP} and a_N are random variables in the present analysis. In this case equation (5) can be used to obtain the probability distribution for a_{n+1} if the probability distribution for a_{np} is known by using the principle of transformation of random variables. It should be noted that all equations similar to (5) or (6) involving crack depths are increasing functions. This property is useful in transforming the random variables.

For example, the probability density function for a_{n+1} can be written as follows

$$f(a_{n+1}) = f[a_{n+1}\{a_{np}\}] \left| \frac{da_{np}}{da_{n+1}} \right| \tag{7}$$

similarly

$$f(a_{np}) = f[a_{np}\{a_n\}] \left| \frac{da_n}{da_{np}} \right| \tag{8}$$

Equations (7) and (8) can be written for every value of n from zero to the projected number of uses.

Details of obtaining these equations for the membrane of the SRM with the expression for stress intensity given by equation (2) and Paris' equation for crack growth is discussed in Appendix I.

The next step is to obtain a tool for change of probability distribution due to the material removal from the wall thickness.

Material Removal and the Change of Probability Distribution

Due to material removal after each use the effective crack depth is reduced by 'Δt'. Thus new crack depth is

$$\bar{a}_n = a_n - \Delta t \tag{9}$$

It is assumed that Δt is a constant. Thus, by using the principles of transformation of random variables (2), the probability density function for a_n can be written as follows.

$$p(\bar{a}_n) = f(\bar{a}_n + \Delta t) \tag{10}$$

In this equation, $p(\bar{a}_n)$ represents the density function for $\bar{a}_n$ and f represents the functional form of the probability density function for a_n.

Probability of Failure

By following the method discussed in the preceding two sections, probability density function for crack depth can be obtained after every flight, proof test and material removal. From the density function, cumulative probabilities can be obtained by integration. Integration after the transformation of variables as discussed in equations (7), (8) and (10) needs the determination of appropriate limits of integration consistent with the transformation of variables. This is also discussed in the Appendix I. If $F(a_n)$ represents the CDF after n flights and n proof tests the probability of failure is given by the probability of $a_n \geq a_c$.

It is to be noted that the probability of failure changes with different selections of the initial wall thickness t, increased loading due to proof test, the material removed Δt and the number of designated missions. The increased loading due to proof tests is denoted by a factor K_p. A cost function or a weight function can be formulated from this knowledge of probability of failure and other related unit-cost or weight. Such a cost or weight function depends on t, K_p, Δt and number of missions N. It is possible to select these design variables by minimizing the cost or weight function subject to appropriate reliability bounds. The effect of NDI is indirectly related to initial flaw distribution. Additional NDI effects such as the rejection of structures are not considered in the analysis. However, they can be included as cost units related to the probability of failure. A numerical example is illustrated in the next section to illustrate the developments of the report.

Numerical Example and the Computer Program

For the numerical example, it is assumed that the 'Johnson S_b' distribution for the initial crack depth is such that the minimum crack depth is zero and the maximum crack depth is 0.1 inch. Different possible ratios (a/c) are considered. Paris' equation for crack growth is assumed with $c = 0.847 \times 10^{-18}$ and exponent equal to 3.0. The variation of ϕ^2 with (a/c) is approximated by a quadratic relation.

The primary objective of reusing the SRM case is to reduce the cost of operation of the shuttle. However, as the number of uses (or cycles) is increased probability of failure increases because of large crack depths associated with more use. The probability of failure also increases with higher proof factors because of higher stresses. Thus, smaller number of cycles and small proof factors, result in higher reliability. However, small number of cycles increase the cost of the SRM case because it has to be replaced after relatively smaller number of uses. Then the total cost function consists of (a) the cost due to number of uses and proof factor and (b) the expected cost of failure, i.e.,

$$C_{total} = C(N,K^P) + C_3 P_f$$

In the equation $C(N,K^P)$ is the cost due to number of uses N, and proof factor K_p. The cost of failure of SRM case is denoted by C_3 and the probability of failure by P_f. The cost $C(N,K^P)$ can be expressed as

$$C(N,K^P) = c_1 N^a + c_2 (K^P)^b$$

It is to be noted that the expression is only for the purpose of illustration in this report and can be changed to reflect the figures more accurately.

The power 'a' is negative to reflect the fact that the effective investment cost is lower if more number of uses can be obtained from the same vehicle. Similarly the power 'b' is also negative. This is to reflect the fact that the capability of vehicle to withstand higher proof load usually indicates larger available margin of safety and increased confidence in the success of the next mission. This also includes intangible cost due to confidence. It is to be noted that t and 'Δt' are not varried in the numerical example. Therefore, there is no cost associated directly with t or Δt.

Initial thickness of the case is assumed to be 0.686 inch and it is assumed that 1% of the thickness is reduced after each use. The flight loading is assumed to be 936 psi. For the purposes of the illustrative example, the problem posed is to select the number of use cycles and proof factor for minimum expected cost. A reliability restraint can be imposed. However, the numerical example has not been considered such a restraint. Arbitrarily, the following values have been used for C_1, C_2 and C_3, $c_3 = c_1 = 1000.00$ units, $c_2 = 180$ units, $a = -0.3$ and $b = 4.0$ have been used.

The general procedure can be summarized in the following steps. A computer program has been written to carry out the needed computations.

1. Obtain the parameters of the Johnson S_b distribution for the initial flaw size.

2. Obtain the stress in the membrane from the known geometry of the case and wall thickness

$$\sigma = K^p \frac{PR}{t}$$

In the equation K_p is the proof stress factor. During flight K^p is equal to one. Pressure P is the MEOP pressure and R is the radius of the SRM case equal to 72.5 inches.

3. Obtain the new CDF and density function for the crack depth after the proof test. A value of K^p close 1.0 is assumed to start the calculation.

4. Obtain the new CDF for the crack depth during the flight following the proof test.

5. Estimate the probability of failure.

6. Compute the cost function parameters.

7. Obtain the new CDF after the material removal.

8. Repeat steps 2 to 7 for the new thickness and the next mission until the total number of missions are complete.

9. Change 'Δt', t, , N and repeat the calculations as necessary.

10. Select the design variables for the minimum value of the objective function subject to reliability constraints.

A computer program has been written to carry out these steps. Only N is varied in step number 9. The program is listed in Appendix II.

Figure 1 illustrates the variation of cost with number of cycles and proof factor in the range 1.02 to 1.20. From the assumed arbitrary cost figures minimum expected cost occurs for 16 cycles and proof factor of 1.12. The corresponding reliability is only 0.9. Lower proof factor need to be used for higher reliability. In the numerical example presented in this report, t and Δt have not been varied.

Conclusions and Recommendations

This report has demonstrated that the reliability analysis based on probabilistic fracture mechanics can be used to optimize the selection of the design variables of the SRM case. In particular, basic design variables such as the thickness and projected design life as well as the fracture control variables such as the proof factor and material erosion can be included. Accuracy in estimation of the initial flaw size distribution is reflected in the assessment of the risks involved in the design. By knowing the risks involved in the design, weight and cost can be reduced from those obtained by deterministic analysis and use of arbitrary safety margins.

This report is only a first step in the development of procedures based probabilistic fracture mechanics. Additional work that is necessary can be listed as follows:

1. A more accurate analysis can be obtained by considering the joint distribution for the crack depth and crack length along the surface.
2. Accurate methods of estimating the probability distribution for the initial flaw size distribution should be developed.
3. In particular, effects of slap down and time dependent crack growth including stress corrosion should be considered in the SRM analysis.
4. Uncertainties in external loads and material properties should be considered.

5. Accuracy of the different models for crack growth (in the point of view of probabilistic fracture mechanics) should be evaluated.
6. Alternate fracture control plans and more accurate stress intensity measures based on cylindrical geometry can be considered.
7. Cost of NDI efforts in relation to the cost that will be incurred by additional safety factor should be evaluated in the point of view of improved reliability.

APPENDIX I

Estimation of the new CDF of crack depth after use from a knowledge of the old CDF and probability density before use.

Crack Growth Rate

The rate at which the crack depth increases is given by Paris equation as follows.

$$\frac{da}{dN} = C(\Delta K)^n = 0.847(\Delta K)^n \times 10^{-16}$$

For subsequent convenience in algebra, the value of 'n' is taken to be 3.0. The suggested value from current state of art is 2.48 (C = 0.847 x 10^{-18}). Now substituting for ΔK

$$\frac{da}{dN} = 0.847 \left[C_4 \left\{ \frac{a}{C_5 + C_2(\frac{a}{c}) + C_3(\frac{a}{c})^2} \right\}^{\frac{1}{2}} \right]^3$$

Simplifying this further,

$$\frac{da}{dN} = C_6 \left\{ \frac{a}{C_5 + C_2(\frac{a}{c}) + C_3(\frac{a}{c})^2} \right\}$$

where $C_6 = 0.847 \times (C_4)^3 \times 10^{-18}$

Integration of $(\frac{da}{dN})$

Separating the variables a and N in $\frac{da}{dN}$, it follows that

$$dN = \frac{1}{C_6} \left\{ \frac{C_5 + C_2(\frac{a}{c}) + C_3(\frac{a}{c})^2}{a} \right\}^{1.5} da$$

Integrating both sides between state (1) and state (2)

$$[N]_{N_1}^{N_2} = \frac{1}{C_6} \int_{a_1}^{a_2} \left\{ \frac{C_5 + C_2(\frac{a}{c}) + C_3(\frac{a}{c})^2}{(a)^{1.5}} \right\}^{1.5} da$$

In order to evaluate the integral on the right hand side, it is found necessary to expand the numerator of the integrant binomially .

Now consider the numerator of the integrand with $C_5 - 1$. Neglecting terms of higher order than $(\frac{a}{c})^3$, it follows that

$$\left\{ 1 + C_2(\frac{a}{c}) + C_3(\frac{a}{c})^2 \right\}^{1.5}$$

$$= 1.0 + 1.5\ C_2(\frac{a}{c}) + \left\{ 1.5\ C_3 + 1.5(0.25) \right\} (\frac{a}{c})^2$$

$$+ \left\{ 1.5(0.5)\ C_2 C_3 - 0.25(0.5)^2\ C_2^{\ 3} \right\} (\frac{a}{c})^3$$

Letting

$$P_1 = \frac{1}{c}\, 1.5\ C_2$$

$$P_2 = \frac{1}{c^2} \left\{ 1.5\ C_3 + 1.5(0.25)\ C_2^{\ 2} \right\}$$

and

$$P_3 = \frac{1}{c^3} \left\{ 1.5(0.5)\ C_2 C_3 - (0.25)^2\ C_2^{\ 3} \right\}$$

it follows that

$$\left\{1 + C_2\left(\frac{a}{c}\right) + C_3\left(\frac{a}{c}\right)^2\right\} = 1.0 + P_1\, a + P_2\, a^2 + P_3\, a^3.$$

Substituting in the integral

$$[N]_{N_1}^{N_2} = \frac{1}{C_6}\left[-\frac{1}{0.5}(a)^{-0.5} + \frac{P_1}{0.5}(a)^{0.5} + \frac{P_2}{1.5}(a) + \frac{P_3}{2.5}(a)^{2.5}\right]_{a_1}^{a_2}$$

<u>Solution of a_1 as a function of a_2</u>

Substituting the limits

$$C_6(N_2 - N_1) = -2(a_2)^{-0.5} + 2\,P_1(a_2)^{0.5} + \frac{2}{3}P_2(a_2)^{1.5} + \frac{2}{5}P_3(a_2)^{2}$$

$$+ 2(a_1)^{-0.5} - 2\,P_1(a_1)^{0.5} - \frac{2}{3}P_2(a_1)^{1.5} - \frac{2}{5}P_3(a_1)^{2.5}$$

Rearranging and neglecting terms of order higher than three, it reduces to the following

$$(a_1)^3 + P(a_1)^2 + q\,a_1 + r = 0$$

where

$$P = \frac{1.0}{\left[\frac{8}{3}P_1P_2 - \frac{8}{5}P_3\right]} \cdot \left[4\,P_1^{\,2} - \frac{8}{3}P_2\right]$$

$$q = \frac{-1.0}{\left[\frac{8}{3} P_1 P_2 - \frac{8}{5} P_3\right]} \cdot [8 P_1 + (C_1)^2]$$

and

$$r = \frac{4}{\left[\frac{8}{3} P_1 P_2 - \frac{8}{5} P_3\right]}$$

Now, the three roots of this cubic equation, $(a_1)^i$ are given by the following [CRC tables 17th edition P. 105]

$$a_1^{(1)} = A + B - \frac{P}{3}$$

$$a_1^{(2)} = -\frac{A + B}{2} + \frac{A - B}{2} \sqrt{-3} - \frac{P}{3}$$

$$a_1^{(3)} = -\frac{A + B}{2} - \frac{A - B}{2} \sqrt{-3} - \frac{P}{3}$$

where

$$A = \sqrt[3]{-\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{\bar{a}^3}{27}}}$$

$$B = \sqrt[3]{-\frac{b}{2} - \sqrt{\frac{b^2}{4} + \frac{\bar{a}^3}{27}}}$$

$$\bar{a} = \frac{1}{3} (3q - P^2)$$

$$b = \frac{1}{27}(2P^3 - 9Pq + 27r)$$

Transformation

Probability density of a_2 is given by

$$f_{a_2}(a_2) = \frac{da_1}{da_2} f_{a_1}(a_1)$$

CDF of a_2 is then

$$\int_{\bar{a}_1}^{\bar{a}_2} f_{a_2}(a_2)\, da_2 = \int_0^{a_1(a_2)} f_{a_1}(a_1)\, da_1$$

$$= \left[F_{a_1}(a_1)\right]_0^{a_1(a_2)}$$

where $F_{a_1}(a_1)$ is the CDF of Johnson S_B distribution.

Now, it is needed to obtain a_1 as a function of a_2, No. of cycles etc. This can be done by solving the polynomial equation obtained previously in terms of a_1 and treating a_2, N_1 and N_2 as constants. The infinite degree polynomial equation is truncated at the 3rd degree for convenience.

Of the three roots only one will be the real root because of the physical nature of the problem, say $\hat{a}_1(a_2)$

Then, substituting in the expression for the CDF of a_2

$$F_{a_2}(a_2) = \int_0^{\hat{a}_1(a_2)} f_{a_1}(a_1)\, d(a_1)$$

or if the CDF of a_1 is known,

$$F_{a_2}(a_2) = \left[F_a(a_1)\right]_0^{\hat{a}_1(a_2)}$$

Thus $F_{a_2}(a_2)$ is a function of the parameters of the initial flaw distribution i.e. λ , ϵ , γ and η , the proof test factor K_p and the number of uses $(N_2 - N_1)$.

The effect of each of these parameters can be studied by calculating $F_{a_2}(a_2)$ for various cases, by means of a computer.

Parabolic Fit to $\phi^2(\frac{a}{c})$

Consider the range $\phi \leq (\frac{a}{c}) \leq 1.0$. In this range it is attempted to fit a parabolic curve for $\phi^2(\frac{a}{c})$ such as follows.

$$\phi^2(\frac{a}{c}) = C_1 + C_2 \cdot (\frac{a}{c}) + C_3 (\frac{a}{c})^2$$

In order to determine the three constants C_1, C_2 and C_3 three points are considered on the given curve.

(i) $\frac{a}{c} = 0$ $\qquad \phi^2(\frac{a}{c}) = 1.0$

(ii) $(\frac{a}{c}) = 0.5$ $\qquad \phi^2(\frac{a}{c}) = 1.5$

(iii) $(\frac{a}{c}) = 1.0$ $\qquad \phi^2(\frac{a}{c}) = 2.5$

Substituting the values for point (i),

$$C_1 = 1.0$$

Substituting the values for point (ii)

$$1.0 + C_2(0.5) + C_3(0.25) = 1.5$$

or

$$2C_2 + C_3 = 2.0$$

Substituting the values for point (iii)

$$1.0 + C_2 + C_3 = 2.5$$

or

$$C_2 + C_3 = 1.5$$

Solving equations (2) and (3) simultaneously

$$C_2 = 0.5$$

and $$C_3 = 1.0$$

Thus the chosen parabolic fit is as follows

$$\phi^2(\frac{a}{c}) = 1.0 + 0.5(\frac{a}{c}) + (\frac{a}{c})^2$$

<u>Limits of Integration for the CDF of 'a_2"</u>

By hypothesis, the initial flas 'a_1' has a Johnson - S_B distribution. Also, there is a functional relationship between the initial flaw size 'a_1' and the subsequent flaw size 'a_2' after N cycles. This relationship renders 'a_2' a random variable because 'a_1' is a random variable by hypothesis. Having known the range space of 'a_1' the range space of 'a_2' can be derived from the functional relationship between 'a_1' and 'a_2'. Thus, if the lower limit of 'a_1' is zero, it follows from the functional relationship between 'a_1 and 'a_2' that the lower limit of a_2 is also zero. Next, if the upper limit of 'a_1' is a_1 the corresponding upper limit for 'a_2' can be obtained by solving the cubic relation between a_1 and a_2, as a function of the number of cycles $N = N_2 - N_1$.

APPENDIX II

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      PROGRAM MAIN (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C     ************************************************************************
C     THIS PROGRAM CALCULATES THE RELIABILITY OF THE SRM CASING AT THE END OF
C     EACH PROOF TEST AND USE CYCLE GIVEN THE INITIAL FLAW DISTRIBUTION FOR
C     VARIOUS PROOF-TEST FACTORS.
      DIMENSION F(30,1),Y(30,1)
      COMPLEX AA,BB,A1HAT
      READ(5,*)ALAMDA,EATA,GAMA
      EPS=0.0
      WRITE(6,100) ALAMDA,EATA,GAMA,EPS
      ENDFILE6
  100 FORMAT(1H1,/,5X,"PARAMETERS OF THE JOHNSON SB DISTRIBUTION ARE
     1:LAMBDA =",F4.1,2X,"EATA =",F5.2,2X,"GAMA =",F5.2,2X,"EPSILON =",
     1F5.2)
      DO 5000 I=1,10
      AKP=1.0+(0.02*I)
C     "AKP"  IS THE PROOF-TEST FACTOR
      WRITE(6,102)AKP
  102 FORMAT(/////,5X," PROOF - TEST FACTOR =",F15.3)
      DO 4000 N=1,25
C     "N" IS THE NUMBER OF THE CYCLE CONSISTING OF ONE PROOF-TEST AND ONE USE.
      DO 3010 L=1,2
      IF(L.EQ.1) WRITE(6,104) N
      IF(L.EQ.2) WRITE(6,105) N
  104 FORMAT(///,5X,"PROOF TEST NUMBER =",I13)
  105 FORMAT(5X,"LAUNCH NUMBER OF SRM CASE =",I7)
C     ************************************************************************
C     THIS SECTION CALCULATES THE THICKNESS AT THE END OF EACH CYCLE.
      THIK=0.486-((0.0048*(N-1)))
      WRITE(6,106) THIK
  106 FORMAT(5X,"THICKNESS OF SRM CASE=",F15.4)
C     ************************************************************************
C     THIS SECTION CALCULATES THE APPLIED STRESS  (SIGMP)
      SIGMP=AKP*950.7*72.5/THIK
      ACR=((93500/SIGMP)**2.0)/(1.2*3.147)
      IF(L.EQ.2) SIGMP=SIGMP/AKP
      WRITE(6,108)SIGMP
  108 FORMAT(5X,"PROOF-STRESS =",F26.1)
C     ************************************************************************
C     THIS SECTION  CONSIDERS THE CUBIC APPROXIMATION.
      C=0.4
      C1=1.0
      C2=0.5
      C3=1.0
      C4=(SQRT(1.2*3.147))*SIGMP
      C6=0.847*(C4**3.0)
      C6=1.0E-18*C6
      P1=1.5*C2/C
      P2=((1.5*C3)+(1.5*0.25*C2*C2))/(C*C)
      P3=((1.5*0.5*C2*C3)-(0.0625*(C2**3.0)))/(C**3.0)
      A2=ACR
      Z1=2.0*P3*(A2**2.5)/5.0
      Z2=2.0*P2*(A2**1.5)/3.0
```

```
      Z3=2.0*P1*(SQRT(A2))
      Z4=-(2.0/SQRT(A2))-(C6*N)
      C7=Z1+Z2+Z3+Z4
      Z1=(4.0*P1*P1)-(8.0*P2/3.0)
      Z2=(8.0*P1*P2/3.0)-(8.0*P3/5.0)
      P=Z1/Z2
      Z1=-((8.0*P1)+(C7*C7))
      Q=Z1/Z2
      R=4.0/Z2
      ABAR=((3.0*Q)-(P*P))/3.0
      Z1=2.0*(P**3.0)
      Z2=-9*P*Q
      Z3=27*R
      B=(Z1+Z2+Z3)/27.0
      Z1=(B*B/4.0)+((ABAR/3.0)**3.0)
112   FORMAT(/," DISCREMINENT SQUARE =",E15.5)
      AA=((-B/2.0)+CSQRT(CMPLX(Z1,0.0)))
      BB=((-B/2.0)-CSQRT(CMPLX(Z1,0.0)))
      Q1=REAL(AA)
      Q2=AIMAG(AA)
      RR=(SQRT((Q1**2.0)+(Q2**2.0)))**(1.0/3.0)
      THET=ATAN(Q2/Q1)
      AR1=RR*COS(THET/3.0)
      AI1=RR*SIN(THET/3.0)
      AR2=RR*COS((THET+(2.*3.147))/3.0)
      AI2=RR*SIN((THET+(2.*3.147))/3.0)
      AR3=RR*COS((THET+(4.*3.147))/3.0)
      AI3=RR*SIN((THET+(4.*3.147))/3.0)
      Q1=REAL(BB)
      Q2=AIMAG(BB)
      RR=(SQRT((Q1**2.0)+(Q2**2.0)))**(1.0/3.0)
      THET=ATAN(Q2/Q1)
      BR1=RR*COS(THET/3.0)
      BI1=RR*SIN(THET/3.0)
      BR2=RR*COS((THET+(2.*3.147))/3.0)
      BI2=RR*SIN((THET+(2.*3.147))/3.0)
      BR3=RR*COS((THET+(4.*3.147))/3.0)
      BI3=RR*SIN((THET+(4.*3.147))/3.0)
      A1HAT=AR1+BR1-(P/3.0)
      AA=REAL(A1HAT)
C     ***********************************************************************
C     THIS SECTION CONSIDERS THE QUADRATIC APPROXIMATION.
      Z1=(4.0*P1*P1)-(8.0*P2/3.0)
      Z2=-((8.0*P1)+(C7*C7))
      P=Z2/Z1
      Q=4.0/Z1
      A1=(-P+SQRT((P*P)-(4.0*Q)))/2.0
      WRITE(6,114)A1
114   FORMAT(5X,"UPPER LIMIT OF A1 =",E24.6)
      A1HA1=A1
      IF (A1HA1.GE.ALAMDA) Y(N1,1)=1.0
      IF (A1HA1.GE.ALAMDA) GO TO 3005
```

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RAM MAIN        74/74    OPT=1                             FTN 4.6+420          76/08/17

C       THIS SECTION CALCULATES THE RELIABILITY
        N1=15
        DO 3000 K=2,N1
        A1=A1HA1*(K-1)/FLOAT(N1-1)
        Z1=EATA*ALAMDA/SQRT(2.0*3.147)
        Z2=1.0/(A1*(ALAMDA-A1))
        Z4=EATA*ALOG(A1/(ALAMDA-A1))
        Z3=EXP(-((GAMA+Z4)**2.0)/2.0)
        F(K,1)=Z1*Z2*Z3
3000    CONTINUE
        F(1,1)=0.0
        CALL INTGRL(1,A1HA1,F,Y,N1)
3005    WRITE(6,116)Y(N1,1)
116     FORMAT(5X,"PROBABILITY OF NO FAILURE =",E14.5)
C       ****************************************************************************
C       THIS SECTION CALCULATES THE TOTAL COST FUNCTION .
        C8=1000.0
        C10=0.18/(AKP**4.0)
        C11=(1.0/FLOAT(N))*0.30
        ZZ1=Y(N1,1)
        CTOT=C8*(C10+C11+1.0-ZZ1)
        WRITE(6,118)CTOT
118     FORMAT(5X,"********************* TOTAL COST FUNCTION =",F15.5)
3010    CONTINUE
4000    CONTINUE
5000    CONTINUE
999     STOP
        END
```