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Impulsive Solar X-Ray Bursts.  
III. Polarization and Directivity  
of Bremsstrahlung Radiation  
from a Beam of Electrons  
Directed Toward the Photosphere

by

Steven H. Langer  
Vahé Petrosian

July 1976

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National Aeronautics and  
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ABSTRACT

We present the spectrum, directivity and state of polarization of the bremsstrahlung radiation expected from a beam of high energy electrons spiraling along radial magnetic field lines toward the photosphere. The results from this paper are used in the accompanying paper for calculation of the characteristics of the reflected plus direct flux.

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## I. INTRODUCTION

The observed properties of impulsive x-ray bursts have led us to a model in which the bulk of the x-rays are emitted in the form of bremsstrahlung from a beam of high energy electrons directed toward the photosphere (c.f. Petrosian 1973, Paper I, and 1975). In Paper I we calculated the spectrum and angular distribution of the hard x-rays ( $> 10$  keV) produced by an electron beam directed radially into the photosphere.

The purpose of this paper is to extend these earlier calculations to include the effect of the electrons spiraling around magnetic field lines (non-zero pitch angles) and to calculate the polarization of the x-rays. This problem has previously been considered by Haug (1972) and Brown (1972). Haug's results are for a thin target model and Brown only published results for the polarization of x-rays directed away from the sun. However, to calculate the spectrum, angular dependence and polarization of the x-rays reflected by the photosphere we need the characteristics of the bremsstrahlung x-rays emitted in all directions. We briefly discuss the procedure used in this calculation (Sec. II) and present the results (Sec. III). These results are then used in the accompanying paper to calculate the characteristics of the radiation reflected from the photosphere and the total observable flux (direct plus reflected).

## II. CALCULATIONS

The bremsstrahlung cross sections needed in our calculations have been derived by May (1951) and by Gluckstern and Hull (1953) [to which we apply Elwert's (1939) Coulomb correction]. The cross sections  $d\sigma_1(E, k, \theta)$

and  $d\sigma_{\parallel}$  ( $E, k, \Theta$ ) are the cross sections for the emission of photons polarized perpendicular and parallel to the plane of the electron-photon momenta as a function of initial electron energy  $E$ , photon energy  $k$  and the angle  $\Theta$  between the electron and photon momenta  $\vec{p}$  and  $\vec{k}$ . However, for non-zero pitch angles this plane is constantly changing as the electron spirals around the magnetic field lines, which are assumed to be uniform, fixed and radial (the coordinate system is chosen so that the magnetic field vector  $\vec{H}$  is in the  $z$ -direction). It is convenient to fix the direction of observation with respect to the field lines in which case we need cross sections  $d\sigma_{\perp}'$  and  $d\sigma_{\parallel}'$  for emission of photons with polarization perpendicular and parallel to the plane of the magnetic field and the photon momentum  $\vec{k} = k(\sin\theta, 0, \cos\theta)$ . As shown by Haug (1972)

$$\begin{aligned} d\sigma_{\perp}' + d\sigma_{\parallel}' &= d\sigma_{\perp} + d\sigma_{\parallel} \\ d\sigma_{\perp}' - d\sigma_{\parallel}' &= \cos 2\psi (d\sigma_{\perp} - d\sigma_{\parallel}) \end{aligned} \quad (1)$$

where  $\psi$ , the angle between the two planes, is given by:

$$\cos \psi = \frac{|\vec{p} \times \vec{k}|}{|\vec{p} \times \vec{k}|} \cdot \frac{|\vec{H} \times \vec{k}|}{|\vec{H} \times \vec{k}|} = \frac{(\sin\theta \cos\eta - \cos\theta \sin\eta \cos\varphi)^2 - \sin^2 \eta \sin^2 \varphi}{(\sin\theta \cos\eta - \cos\theta \sin\eta \cos\varphi)^2 + \sin^2 \eta \sin^2 \varphi} \quad (2)$$

Here,  $\vec{p}$ , the initial momentum of an electron with pitch angle  $\eta$ , is  $\vec{p} = p(\sin\eta \cos\varphi, \sin\eta \sin\varphi, \cos\eta)$  where  $\varphi$  runs from 0 to  $2\pi$  for one revolution. With this geometry, which is the same as that described by

Brown (1972) ;

$$\cos \Theta = \frac{\vec{p} \cdot \vec{k}}{|\vec{p}| \cdot |\vec{k}|} = \cos \eta \cos \theta + \cos \phi \sin \eta \sin \theta \quad (3)$$

In paper I (equation 20) we showed that for a power law electron spectrum ( $dj_e = KE^{-\delta-1}dE$ ) the integral and differential spectra of the photon energy flux (all energies expressed in units of  $m_e c^2$  and  $K$  has unit of  $\text{sec}^{-1}$ ) are given by

$$J(k) = \frac{K}{\delta} \left( \frac{\alpha}{2\pi \ln \Lambda} \right) \int_k^\infty E'^{-\delta} \beta'^2 F(E', k) dE' \quad (4)$$

$$J(k, \theta) = \frac{K}{\delta} \left( \frac{\alpha}{2\pi \ln \Lambda} \right) \int_k^\infty E'^{-\delta} \beta'^2 f(E', k, \theta) dE'$$

where

$$f(E, k, \theta) = \frac{1}{\alpha r_0^2} \int_0^{2\pi} \frac{d\Omega}{2\pi} k \left[ \frac{d\sigma_{\perp}(E, k, \Theta)}{dk d\Omega} + \frac{d\sigma_{\parallel}(E, k, \Theta)}{dk d\Omega} \right] \quad (5)$$

$$F(E, k) = \frac{1}{2} \int_0^{\pi} f(E, k, \theta) \sin \theta d\theta .$$

To calculate the degree of polarization we need

$$Q(k, \theta) = \frac{K}{\delta} \left( \frac{\alpha}{2\pi \ln \Lambda} \right) \int_k^\infty E'^{-\delta} \beta'^2 g(E', k, \theta) dE' \quad (6)$$

where

$$g(E, k, \theta) = \frac{1}{\alpha r_0^2} \int_0^{2\pi} \frac{d\phi}{2\pi} k \left[ \frac{d\sigma_{\perp}(E, k, \theta)}{dk d\Omega} - \frac{d\sigma_{\parallel}(E, k, \theta)}{dk d\Omega} \right] \cos 2\psi \quad (7)$$

In the above equations  $\beta$  is the electron velocity in units where the speed of light  $c = 1$ ,  $\alpha$  is the fine structure constant,  $r_0$  is the classical electron radius, and  $\Lambda$  is given by equations (2) and (3) of paper I.

We define the directivity<sup>1</sup> and degree of polarization as

$$D(k, \theta) = \frac{J(k, \theta)}{J(k)} \quad (8)$$

$$P(k, \theta) = \frac{Q(k, \theta)}{J(k, \theta)}$$

The total flux,  $J(k)$ , is independent of the pitch angle  $\eta$  because the amount of radiation emitted by an electron is independent of the direction of travel as long as the thick target assumption holds. The quantity  $\frac{\delta}{k} \frac{2\pi \ln \Lambda}{\alpha} k^{\delta-1} J(k)$  is shown in figure 3 of paper I. The x-ray spectrum is essentially a power law (but with an index about one smaller than the electron spectral index  $-\delta$ ).

The directivity is shown in figures 1a, b, and c for a  $\delta = 4$  electron spectrum and three values of the pitch angle  $\eta$ . In all cases the radiation is strongly beamed into the photosphere. The shape of the beam changes with pitch angle with the maximum directivity dropping and

<sup>1</sup>D is the same as the function R, of paper I, the new notation is adopted to be consistent with the work of Brown (1972), Henoux (1975) and others.



the peak moving away from the magnetic field direction as  $\eta$  increases. The fraction of the radiation beamed into the photosphere decreases from about 85% at  $\eta = 0$  to 80% at  $\eta = 30^\circ$  for a photon energy of 46.4 keV and radial field lines.

The degree of polarization is shown in figures 2a, b and c for a  $\delta = 4$  power law and three pitch angles. At low energies the radiation is polarized in the plane containing the observer, the flare site and the center of the sun. As the energy increases the polarization decreases and finally changes sign at high enough energies, so that the plane of polarization is normal to the plane found at lower energies. The magnitude of the polarization decreases as the pitch angle increases. Figures 3a and b show the variation of the degree of polarization with pitch angle at two different photon energies. The variation is slow and smooth.

Figures 4a and b show the variation of the polarization with the power law index at zero pitch angle. The degree of polarization is a monotonically increasing function of the power law index.

In the accompanying paper we use these results to calculate the characteristics of the radiation which is reflected from the photosphere via Compton scattering. The reflected radiation is then added to that which escapes directly from the production region to yield a prediction for the characteristics of the observed x-ray flux.

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#### FIGURE CAPTIONS

**Figure 1** Directivity of the x-ray energy flux for an electron spectrum with index  $\delta = 4$ . Curves are labelled by the value of the photon energy in keV.  $\cos\theta = 1$  for photons directed radially into the photosphere. (a) Pitch angle  $\eta = 0$ ; (b)  $\eta = 30^\circ$ ; (c)  $\eta = 45^\circ$ .

**Figure 2** Same as figure 1 except for degree of linear polarization: (a)  $\eta = 0$ ; (b)  $\eta = 30^\circ$ ; (c)  $\eta = 45^\circ$ .

**Figure 3** Degree of polarization for  $\delta = 4$  and various values of the pitch angle in degrees as indicated. (a) Photon energy  $k = 21.5$  keV; (b) Photon energy  $k = 100$  keV.

**Figure 4** Same as figure 3 except  $\eta = 0$  and for  $\delta = 2, 3, 4, 5$  as labelled. (a)  $k = 21.5$  keV; (b)  $k = 100$  keV.

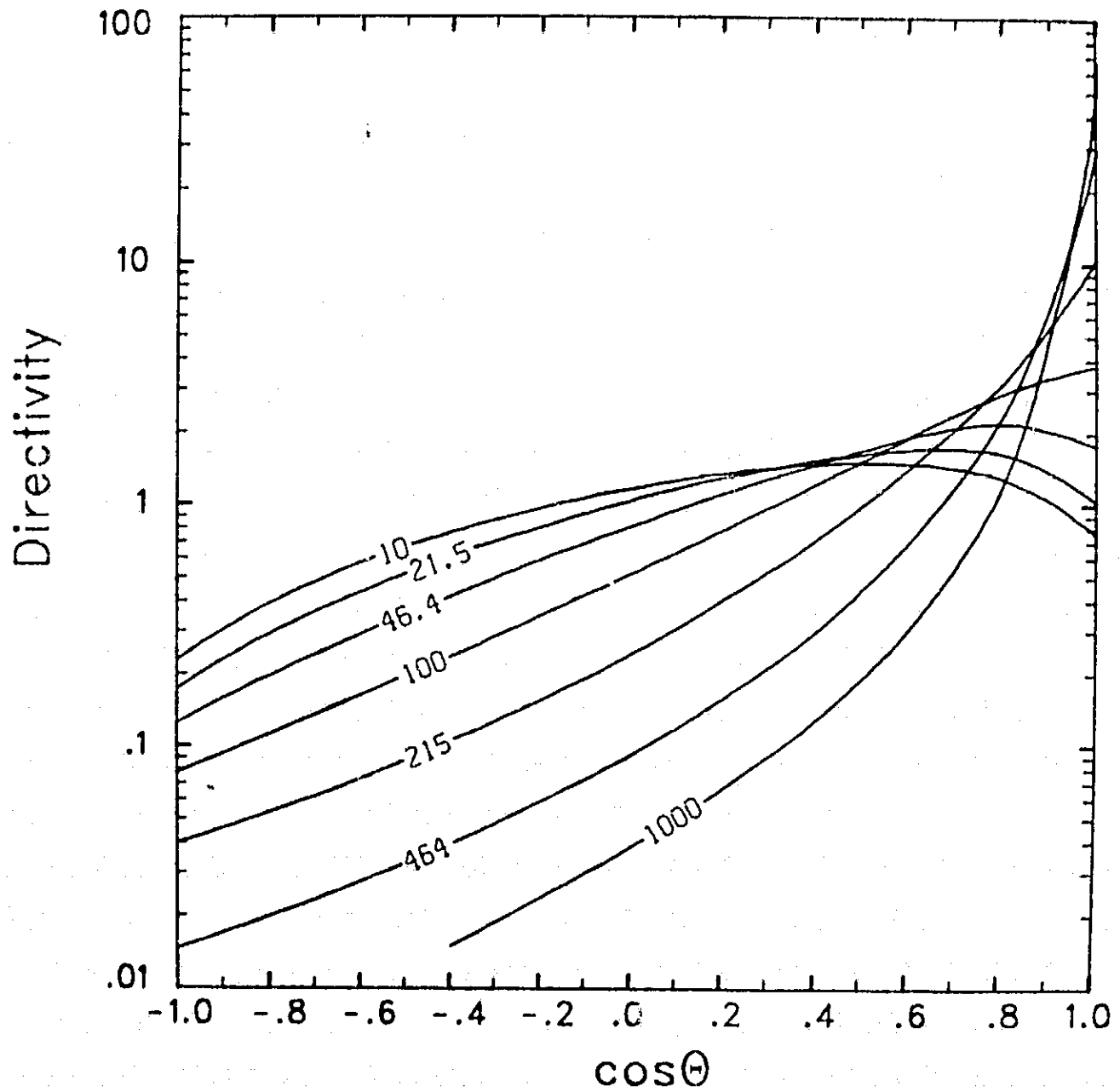


Figure 1a

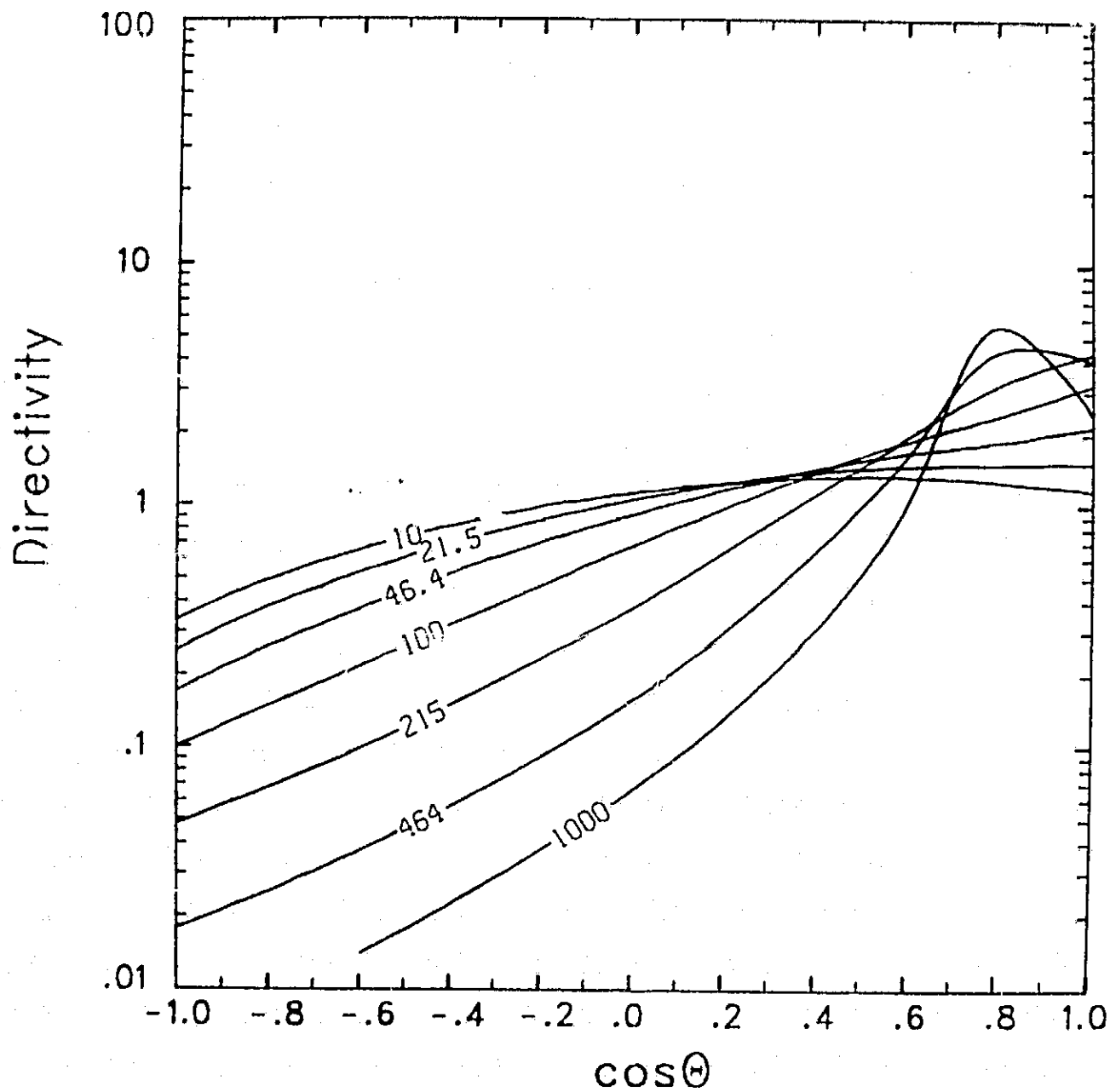


Figure 1b

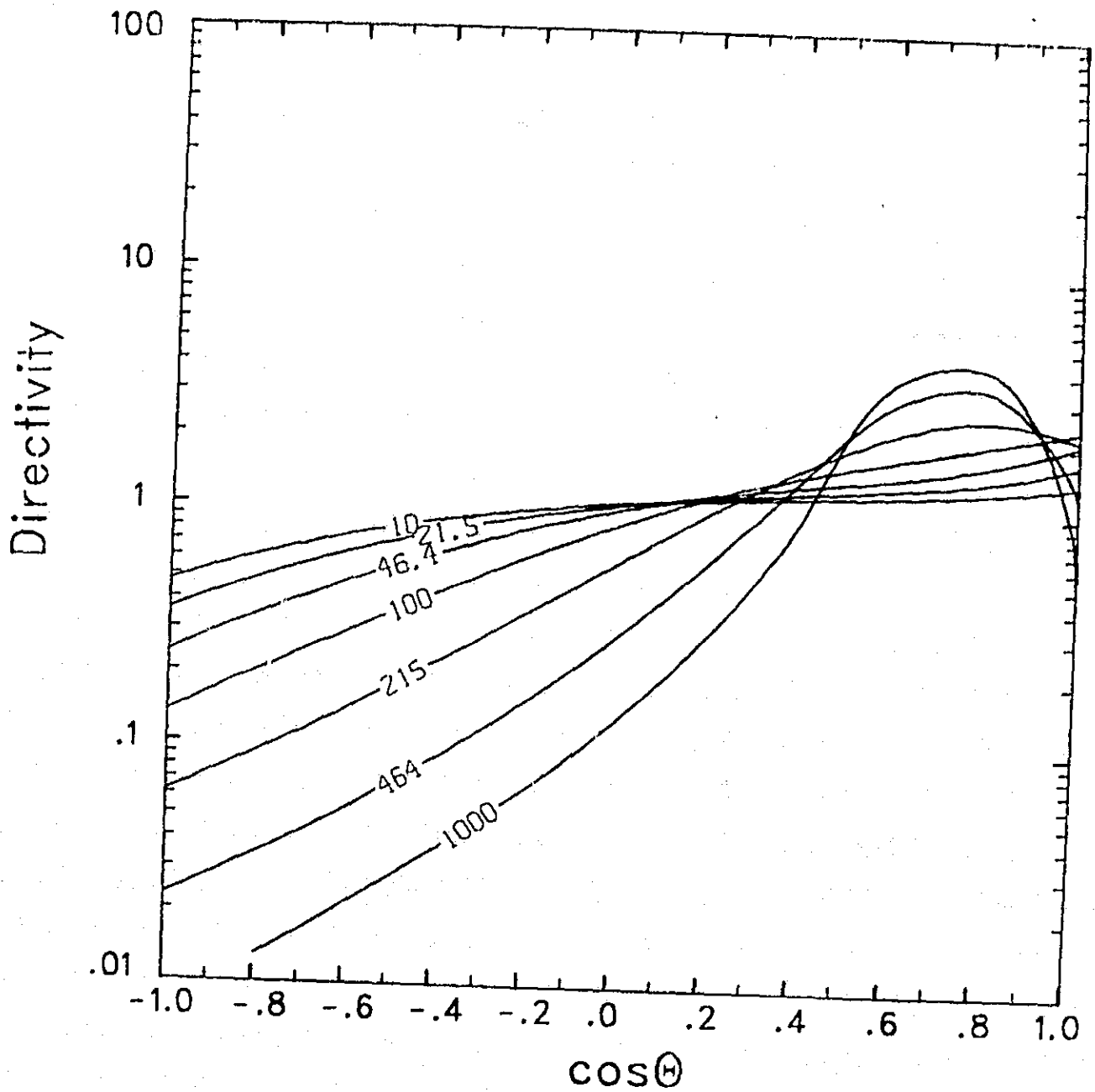


Figure 1c

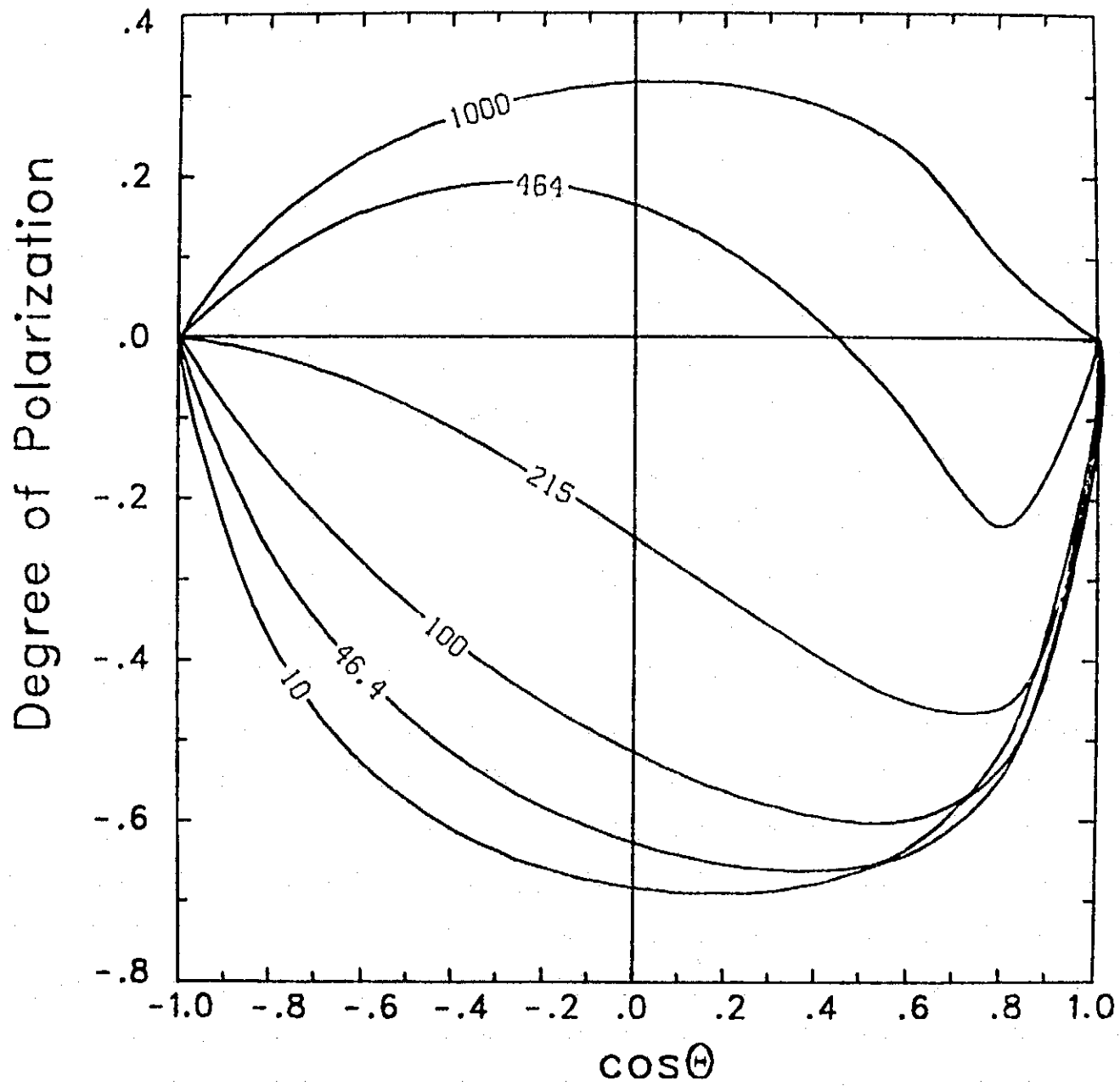


Figure 2a



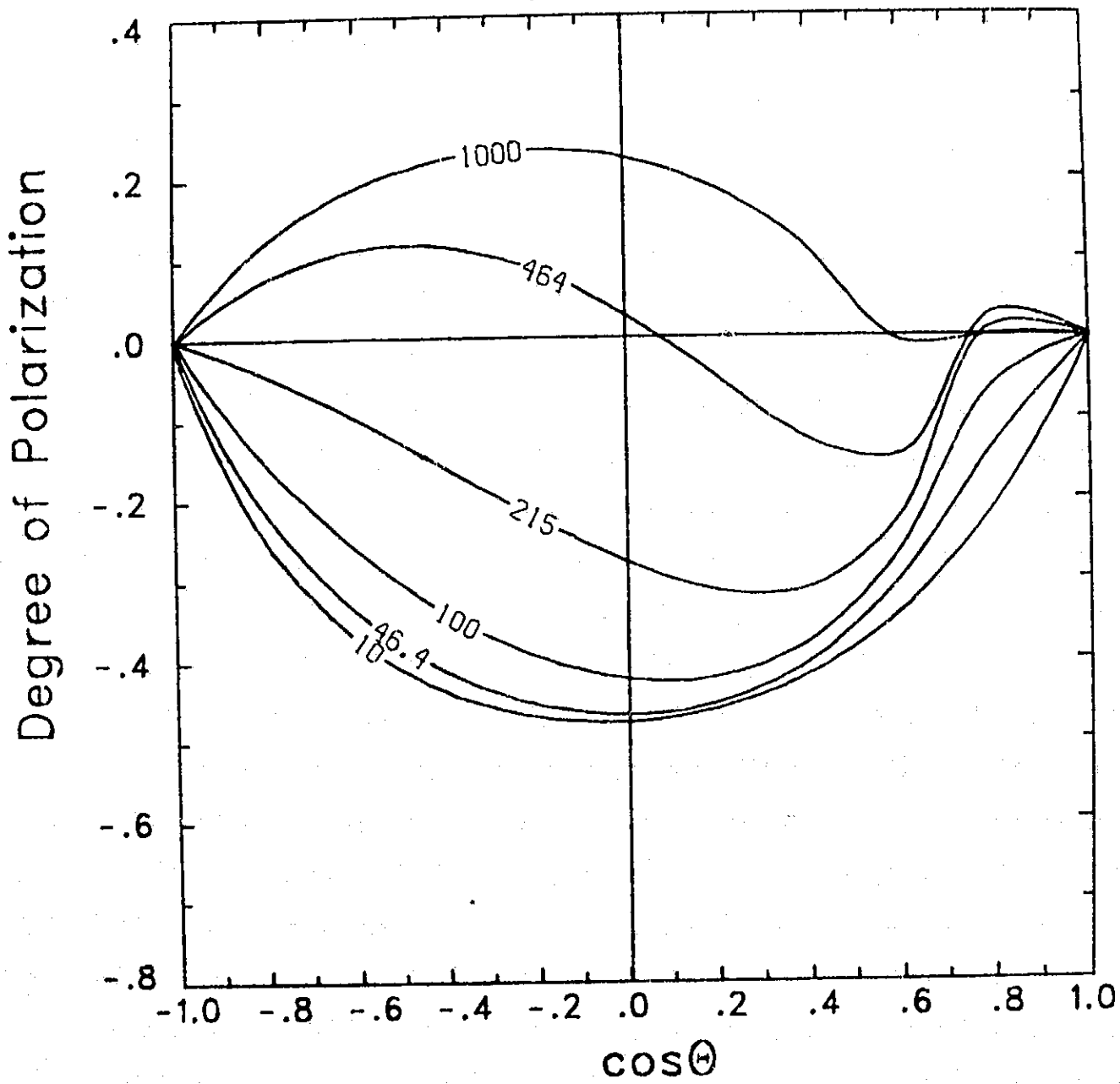


Figure 2b

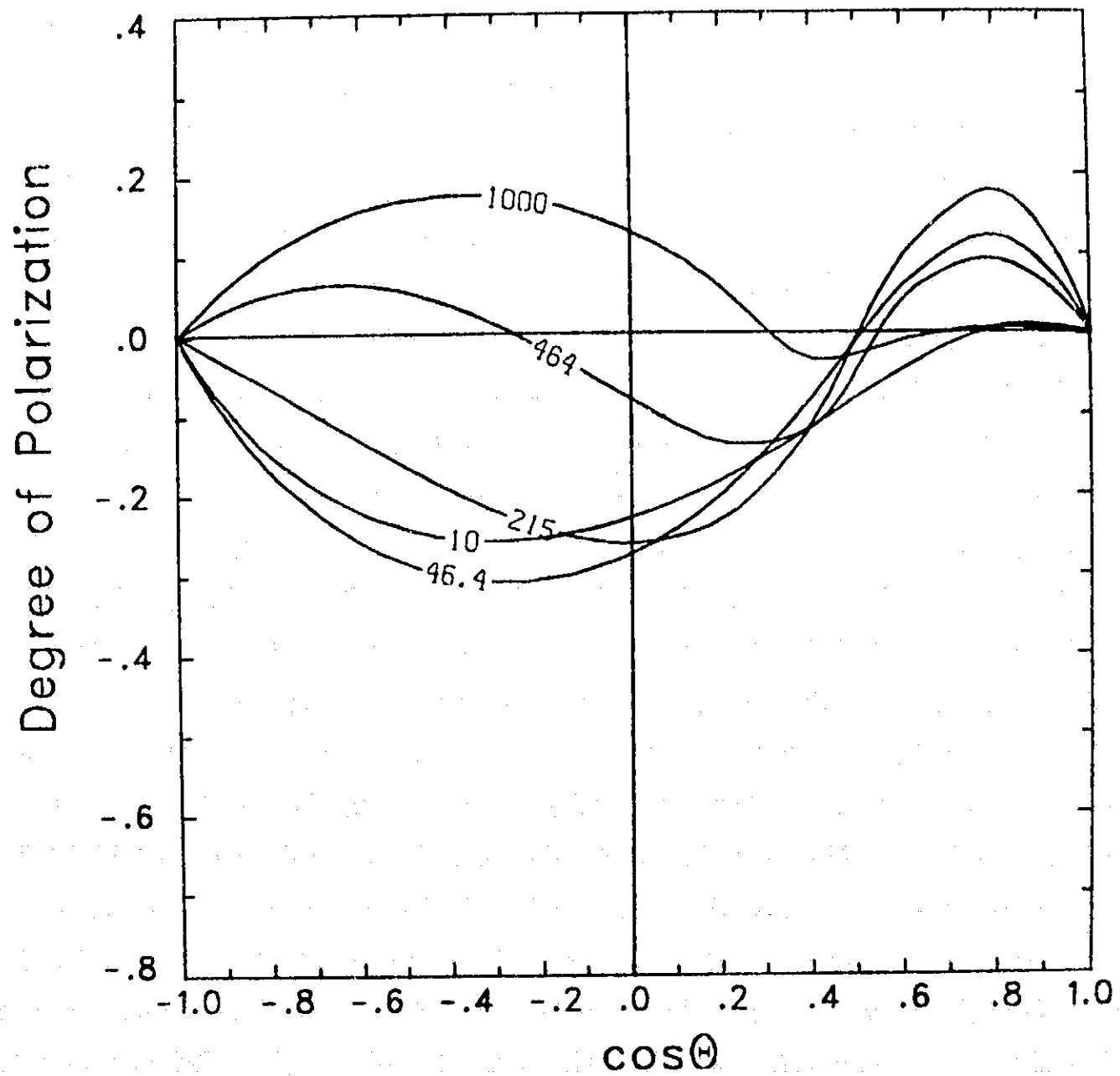


Figure 2c

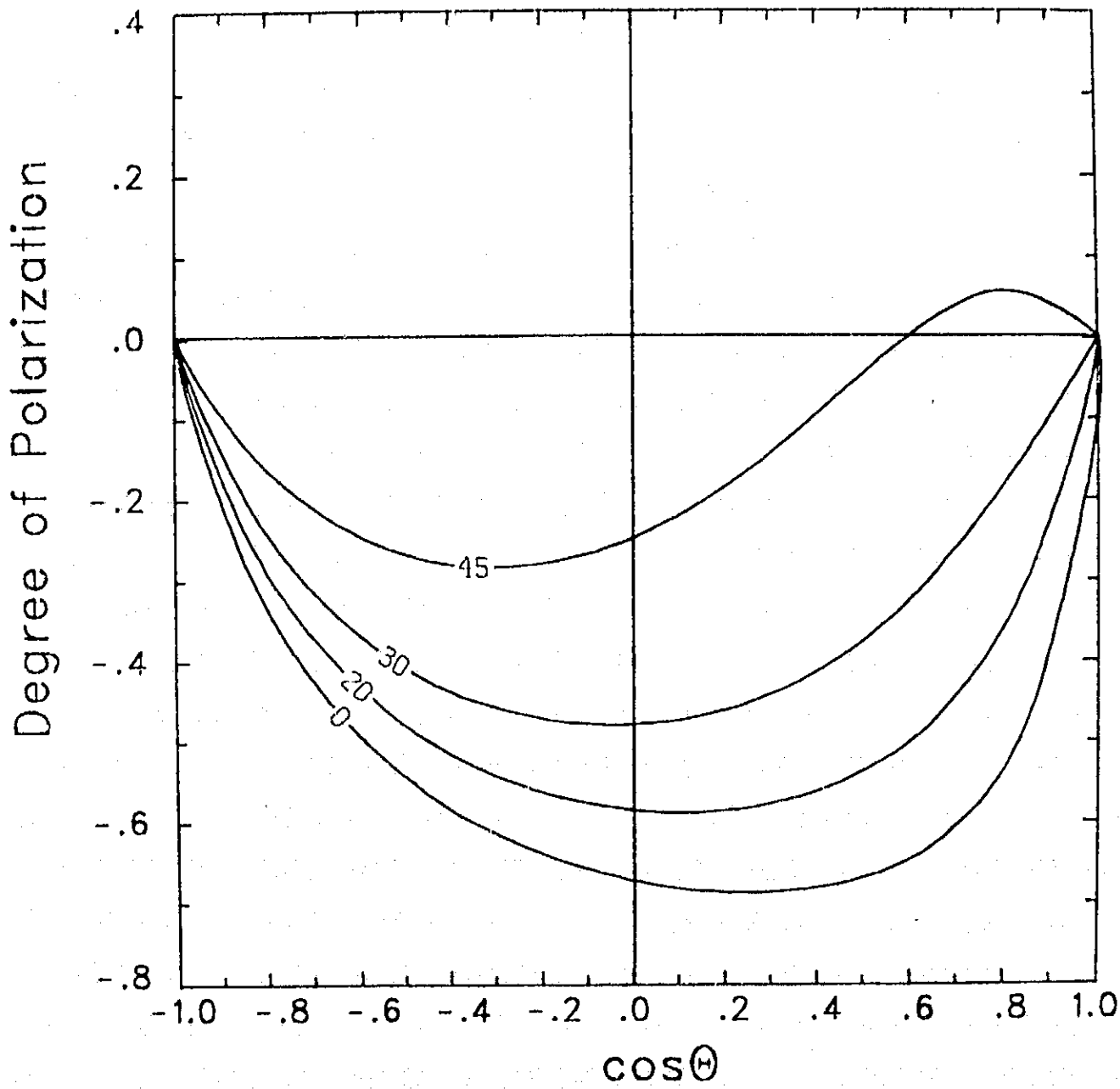


Figure 3a

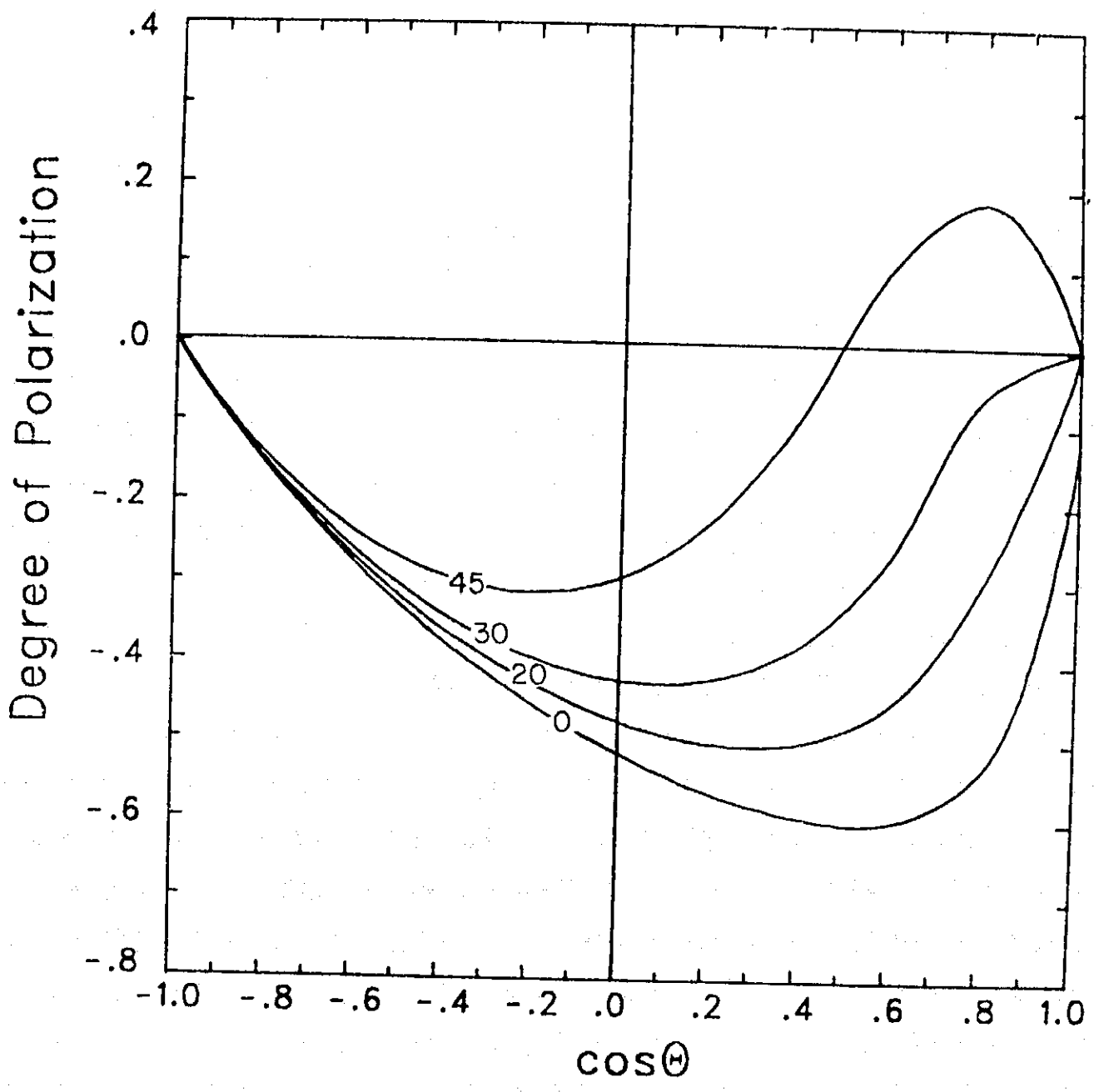


Figure 3b

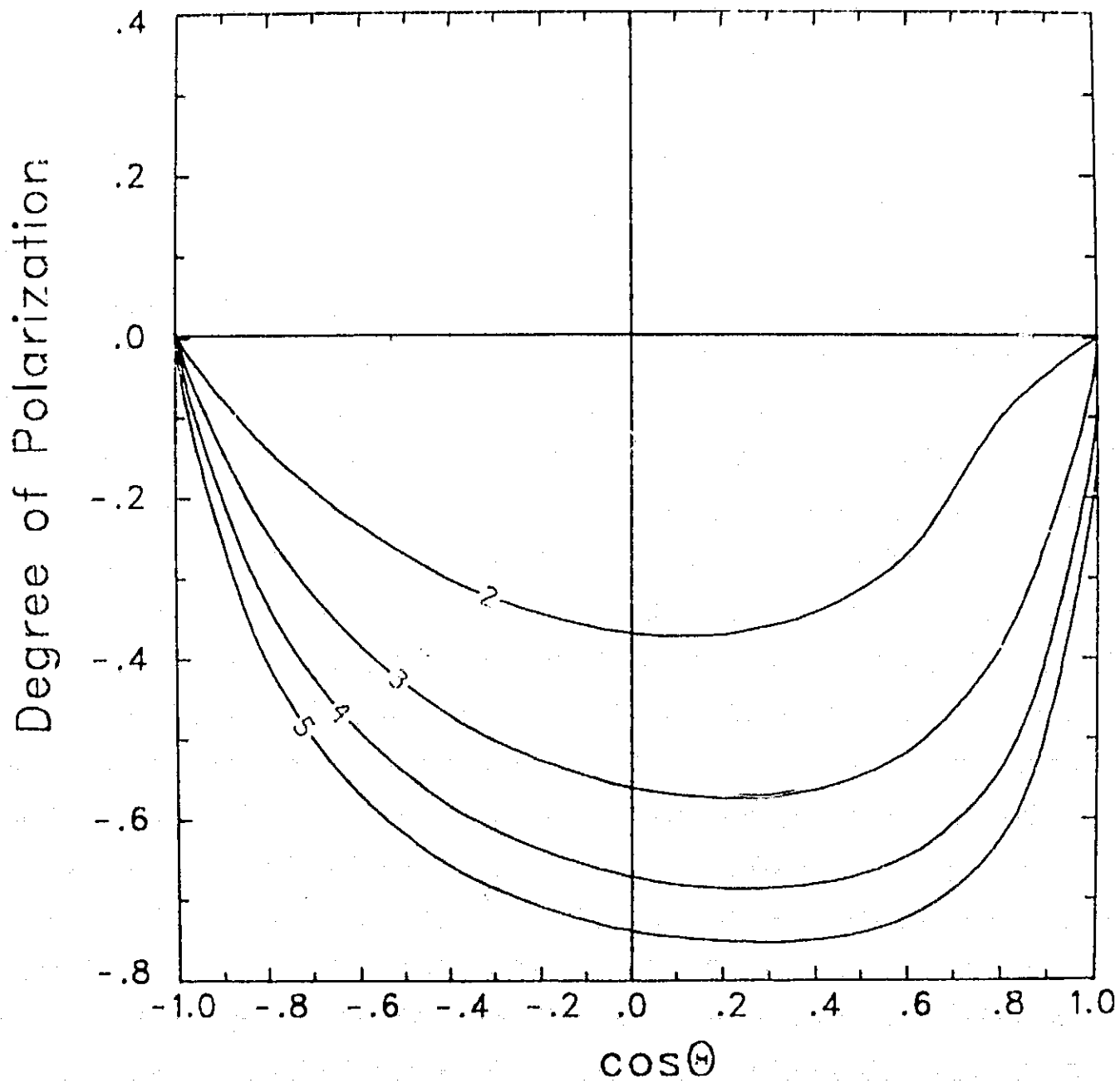


Figure 4a

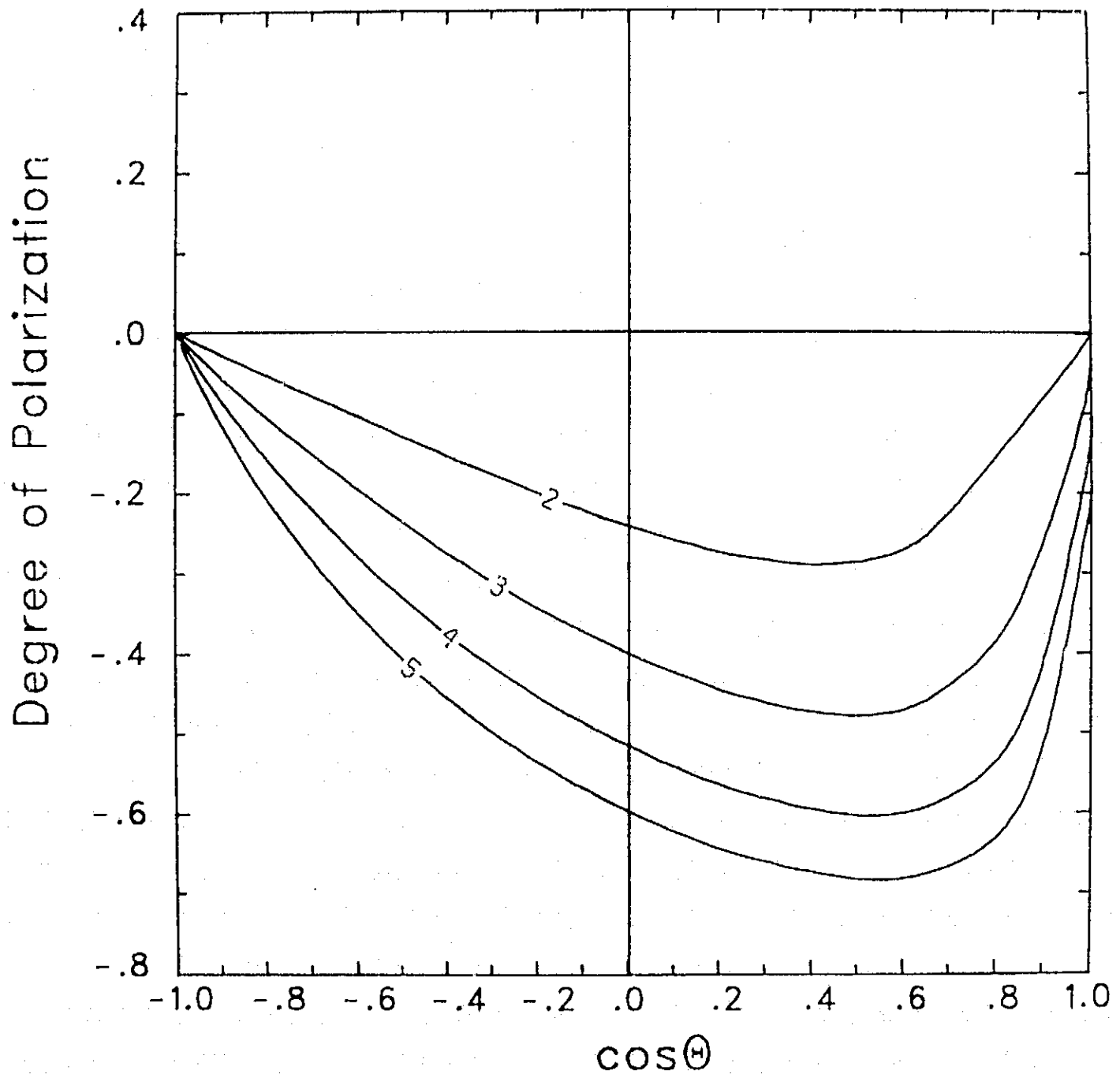


Figure 4b