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DETERMINATION OF THE HEAT BALANCE OF THE EARTH: INTERPRETATION OF RADIATION MEASUREMENTS FROM SATELLITES

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16. Abstract

Radiometric measurements made with horizon-to-horizon areas in view at practical satellite altitudes generally cannot be taken as directly representative of the radiation from significantly smaller areas. The central problem to satellite measurements of the Earth's radiation budget can be viewed either as a problem involving spatial resolution or as a problem involving height rectification. In either case, deterministic relationships are not possible, making methods of statistical parameter estimation essential.

A method is developed for estimating the mean and mean-square variation of the flux at an altitude arbitrarily chosen to represent the top of the atmosphere. When applied to practical satellite measurements, the method is shown to be optimum in that the estimated mean is unbiased and the mean square variation of the estimates converges to that of the true flux. Data from the Meteor-1 and -2 satellites support the essential assumptions and provide a quantitative indication of the performance that can be expected.

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INTRODUCTION

Only a part of the radiant energy of the Sun that falls on the Earth is absorbed by the Earth's surface and atmosphere, while the remainder is reflected back to space. The portion that is absorbed accounts for the bulk of the Earth/atmosphere system's thermal energy which ultimately returns to space as infrared radiation. These are the major components of the planetary radiation budget that must be in balance to maintain thermal equilibrium. Measurements of these components are the basic observations needed to monitor the climatic state of the planet.

The complex absorption, scattering, and emission processes connected with radiation from a given area of the Earth and throughout the vertical extent of the atmosphere above that area, account for both the temporal and spatial variations in weather and climate. A localized positive net radiation value (more energy absorbed from the Sun than is returned to space) means a localized warming, whereas a negative value means cooling. Together they represent energy sources and sinks that drive the circulations of the atmosphere and oceans.

By net radiation is meant the residual or resulting flux of electromagnetic radiation that represents either a net loss or gain. To be specific, the net flux density (F_n) is defined as the flux through an elemental unit area parallel to the mean surface of the Earth at any specified altitude outside of the Earth's atmosphere. Specifically, the three basic components that determine the net radiation are:

- The incoming solar flux (F_s)—While the total solar radiation remains relatively constant, the solar flux through the unit area depends upon time through its coordinates of position.
- The outgoing reflected flux (F_r)—This is by far the most variable component, since it is both temporally and spatially dependent upon such factors as Sun angle, cloud cover, turbidity, and surface characteristics.

- The outgoing thermally emitted flux (F_e)—Variations in intensity of this component with aspect angle are not large in the case of a clear sky or dense cloud cover, but can be large under conditions of partial cloudiness.

Since the only major source of heat is the excess of the solar energy over the portion that is reflected, given by ($F_s - F_r$), and the only way of losing heat is by flow of infrared radiation, F_e, to space, the net flux, F_n, for the elemental area is

$$F_n = (F_s - F_r) - F_e \quad (1)$$

Replacing the symbol F with Q to designate the planetary net, it follows that the mean planetary net flux is

$$\bar{Q}_n = (1 - A)\bar{Q}_s - \bar{Q}_e \quad (2)$$

where the planetary albedo, A, has been written for the ratio, $\bar{Q}_r/\bar{Q}_s$, and the bars indicate temporal averages.

While the mean temperature of our planet undergoes variations over a wide temporal scale, it must on the whole receive as much energy from the Sun as it sends to space so that the mean planetary net flux coverges toward zero over a sufficiently long time interval. In addition to the well-known annual cycle, there are shorter time variations over a few weeks, as well as longer variations ranging from periods of a few years, through centuries, to still longer geological glacial ages that require millions of years for completion. Yet we know from meteorological records that the mean temperature of our planet remains nearly constant over periods of many years so that the net energy change per year is quite small.

Beyond the climatological aspects, the large temporal and spatial variations in the net radiation are of fundamental value in understanding laws governing the general atmospheric circulation, since transformations of the solar radiation by the Earth/atmosphere system largely determine the energetics of large-scale atmospheric processes. Because most modern methods for numerical weather prediction use information initially interpolated on the points of a regular grid, it is essential that Earth radiation budget measurements can be interpreted on a commensurate scale. The purpose of this document is to explore the problem of interpreting the actinometric measurements obtainable at practical satellite altitudes to produce estimates of measurements that would be obtained at lower altitudes for which the spatial resolution would be correspondingly reduced. The approach used is to consider what is observable at the given altitude with minimum a priori assumptions concerning specifics of the scene in view.

GENERAL

In considering the situation depicted in figure 1, assume a remote observation point at 0′ and take another point P on Earth such that P belongs to the set of points making up area A; that is, take P:ϵ{A}. Now, if P results in some measurable influence, I, at 0′, that influence can be represented as being directed along a single ray P 0′ and can be described as I (θ,ϕ). The total influence, I_t, of {A} on 0′ is the sum of the influence over all points P; that is, $I_t = \int_A I \, dA$.

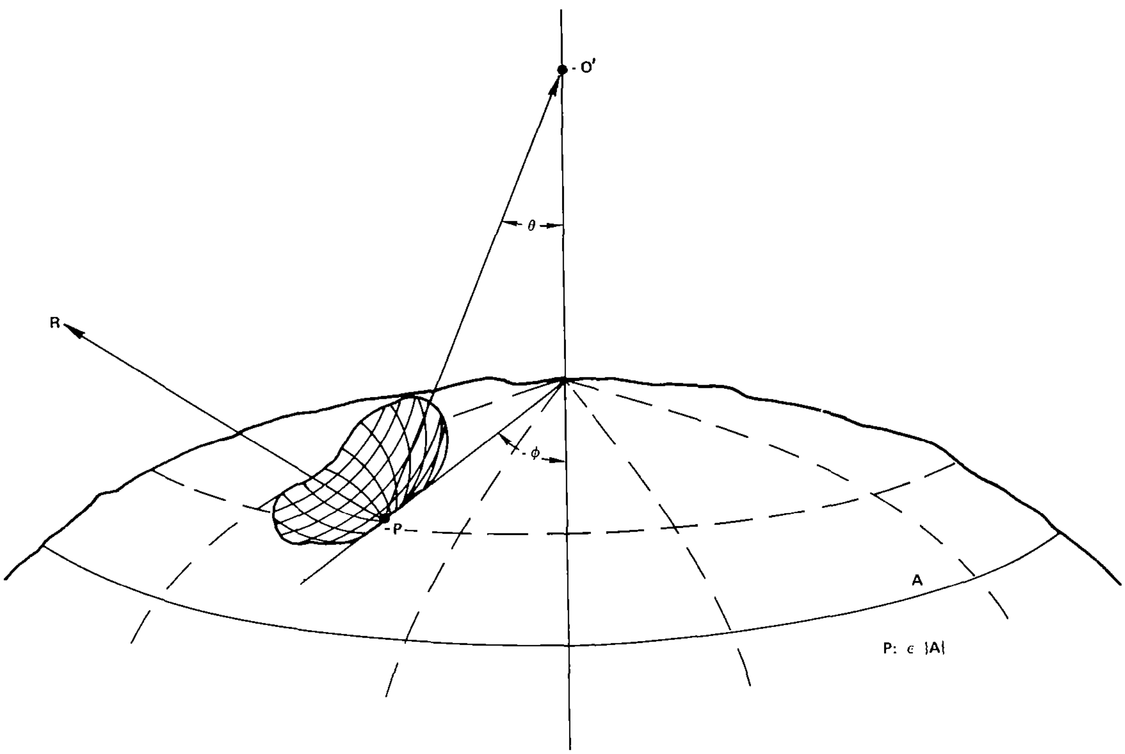


Figure 1. Directional nature of the radiation.

Consider any other ray, for example $\overrightarrow{PR}$ in some direction not towards 0′. The magnitude of this ray represents the influence of P on points in the direction of $\overrightarrow{PR}$ which does not include 0′. Therefore, knowledge of the angular distribution of relative influence about P provides no additional information concerning what is measurable at 0′ from A.

Next, consider figure 2 with P:ϵ{A} as before and Q:ϵ{B} with the set {C} being the union of {A} and {B}. Assume that a radiometer is located at 0′, and has a horizon-to-horizon field of view, with the objective being to determine the outgoing radial flux from {A}. This requires that the radiometer respond only to the radially directed component of all flux incident upon it, and that it have a known orientation (presumably normal to nadir).

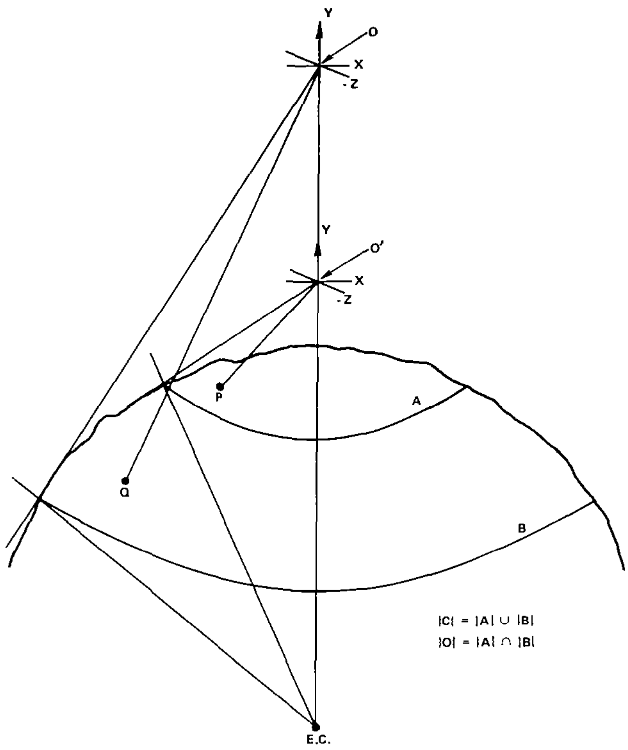


Figure 2. Relationships at different altitudes with different areas in view.

If the altitude of the radiometer is now increased so as to be at point 0, elements of $\{B\}$ would come into view. Since the objective is to measure flux from only $\{A\}$, it is clear that admitting radiation from points Q provides no additional information concerning $\{A\}$. Therefore the acceptance angle of the radiometer at 0 must be narrowed to include only area A (or possibly some alternative equivalent scheme) as a first requisite to the objective.

It is evident that the measurement made by the radiometer with a field of view constrained to $\{A\}$ positioned at 0 will not be the same as the measurement of a horizon-to-horizon field-of-view radiometer positioned at 0′. It is also evident that widening the field of view of 0 to include points Q does not add to what is observable at 0 concerning $\{A\}$; in fact, when the influences of all points Q are removed from such a measurement (by any means whatever), what must remain is the same as that which is directly observable at 0 with a field of view constrained to $\{A\}$.

The purpose of the discussion that follows is to examine in detail the situation just described. Specifically, the discussion will include the relationships between two observations at different altitudes, as well as a means for interpreting radiation measurements at a given altitude to provide estimates of radially-directed flux at another arbitrarily selected altitude that can be taken as defining the top of the atmosphere. An optimum estimation procedure will be developed, and a means for establishing confidence bounds will be indicated.

OPTIMIZATION CRITERIA

Consider a plane-flux measuring radiometer at satellite altitude, h, with its view angle constrained so as to include only the area A as shown in figure 3. The goal is to use the radiometer measurements to form an estimate of the plane flux that would have been measured by the radiometer had it viewed the same area from altitude h′ where h′ corresponds to the arbitrarily defined top of the atmosphere. To better illustrate the problem under consideration, imagine the satellite and Earth/Sun system to be stationary in position for the moment so that both the plane flux F at h, and the plane flux F′ at h′, can be written as

$$F(t) = \int_0^{\theta_m} \frac{\sin 2\theta}{2} \int_0^{2\pi} I(\theta, \phi, t)\, d\phi\, d\theta \tag{3}$$

$$F'(t) = \int_0^{\theta'_m} \frac{\sin 2\theta}{2} \int_0^{2\pi} I'(\theta, \phi, t)\, d\phi\, d\theta \tag{4}$$

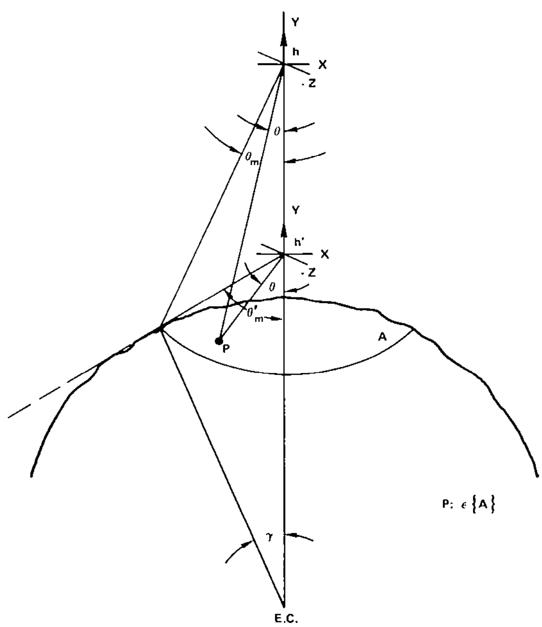


Figure 3. Relationships at different altitudes with the same area in view.

where θ is as indicated in figure 3 (θ_m and θ'_m are the corresponding maximum view angles from local vertical), and ϕ is the azimuth angle (Reference 1).

At any time, t, the actual measurement made at h cannot be put into one-to-one correspondence with the measurement that would have been made at h′. Thus, an analytic mapping transformation (functional relationship) that will permit a deterministic solution simply does not exist.

To substantiate this assertion, it can be noted that at either h or h′, the flux is the total flux as contributed by all of the points, P, belonging to A, and that each P can radiate with a different intensity in each of the directions toward h and h′, as expressed by the kernels of equations 3 and 4. Now consider the following proposition: If the angular distribution of the radiation for each P (as shown in figure 1) were known, then the relative intensities for each P could be specified, so that $I'(\theta, \phi, t)$ could also be specified through these distributions given $I(\theta, \phi, t)$. Thus, $F'(t)$ could be uniquely determined.

This proposition does not take into account the fact that with the given measurements, we have no way of knowing $I(\theta, \phi, t)$. In fact, the integrals of equation 3 are definite integrals and the flux is a function of time only, so that while its value depends upon $I(\theta, \phi, t)$, it is not unique to any particular $I(\theta, \phi, t)$. Indeed, there is an unbounded number of distributions that will produce the same $F(t)$, thereby precluding a deterministic solution.

The indeterminancy involved here is a feature that commonly occurs in natural processes. Very often, a complete record of measurements of a given phenomenological process does not provide a one-to-one indication of the measurements that will result from another record of the same process even under the same conditions. In such circumstances, the best that one can do is to collect an ensemble (or set) of records and determine their statistical parameters. In practice, the mean and mean-square fluctuations are most often taken as the two pieces of statistical information that will permit reasonable statements about the phenomenon under study.

In the present situation, hypothesize that the true mean, $m'(t)$, and the true mean square fluctuation, $\sigma'(t)^2$, of $F'(t)$ valid over some desired time interval, T, (such as 30 days) are known. Now, assume that a time sequence of a number, n, of equally spaced satellite measurements taken from a time record of $F(t)$ are given along with some procedure for operating on this sequence to produce estimates of $F'(t)$, given by $\{\hat{F} n(t)\}$. Then, a reasonable requirement for assuring the quality of the estimation procedure would be to insist that the expected mean of the set, $\{\hat{F}(nt)\}$, be equal to the true mean, m'_t, so that the estimates are unbiased in a statistical sense.

However, the property of being unbiased alone is an insufficient guarantee of the quality of an estimation procedure, because individual estimates could fluctuate widely and still produce the correct mean.

One measure of the fluctuation of $\hat{F}(t)$ is its variance

$$\sigma^2_{\hat{F}} = E\left[(\hat{F} - E[\hat{F}])^2\right] \tag{5}$$

where E [ ] means the expected value of [ ].

For an unbiased estimator, equation 5 reduces to

$$\sigma^2_{\hat{F}} = E\left[(\hat{F} - m_t)^2\right] \tag{6}$$

An ideal estimator is usually taken to mean an unbiased estimator for which $\sigma^2_{\hat{F}} = \sigma^2_F$, that is, the variance of the estimates is the same as the true variance. Such an estimator is called a minimum-variance-bound estimator (or a sufficient and efficient estimator), since no other estimator can be found that will produce a smaller variance than the true variance.*

It is common practice to refer to a statistical processing method that meets such criteria established for the first and second moment statistics (the mean and variance) as an optimum method according to those criteria. The purpose of the following discussion is to develop an estimation procedure and to show that it is an optimum procedure that produces unbiased and minimum-variance-bound estimates.

THE ESTIMATION PROCEDURE

For a radiometer with an unrestricted aperture located at h', chosen to correspond to an arbitrarily defined height of the atmosphere, $\theta m'$ of equation 4 becomes $\pi/2$. The true flux as given by equation 4 can then be broken into separate parts at a given time t as follows:

$$F_T = \int_0^{\theta_m} \frac{\sin 2\theta}{2} \int_0^{2\pi} I_T(\theta, \phi)\, d\phi\, d\theta + \int_{\theta_m}^{\pi/2} \frac{\sin 2\theta}{2} \int_0^{2\pi} I_T(\theta, \phi)\, d\phi\, d\theta \tag{7}$$

where θm is the same as in equation 3 and

*For discussions on statistical estimation, see References 2 and 3.

where the subscript T has replaced the prime to emphasize that this represents what a true measurement would be. The random variation with time has also been suppressed to simplify the notation. In searching for a means of forming an estimate, we know we have available the satellite measurement of equation 3 which is repeated below for convenience

$$F_M = \int_0^{\theta_m} \frac{\sin 2\theta}{2} \int_0^{2\pi} I_M(\theta,\phi)\, d\phi\, d\theta \tag{8}$$

where the subscript M has been added to emphasize that this is the measurement available, and the variation with time again been suppressed.

Note that while the kernel of the first right-hand term of equation 7 is different from that of equation 8, they do have the same limits of integration. A first thought is to substitute equation 8 into equation 7 to form an estimate $\hat{F}$ as

$$\hat{F} = F_M + \int_{\theta_m}^{\pi/2} \frac{\sin 2\theta}{2} \int_0^{2\pi} I_T(\theta,\phi)\, d\phi\, d\theta \tag{9}$$

We still need an estimate for the second right-hand term of equation 9 that represents the range of $\theta_m \leqslant \theta \leqslant \pi/2$ not included at the satellite point. A reasonable estimate for the angular distribution over this range might be taken as the average or mean value of the angular distribution $I_M(\theta,\phi)$ which is observable at h, that is, over the range $0 \leqslant \theta \leqslant \theta_m$. This mean value is

$$\overline{I_M(\theta,\phi)} = \frac{\int_0^{\theta_m} \frac{\sin 2\theta}{2} \int_0^{2\pi} I_M(\theta,\phi)\, d\phi\, d\theta}{\int_0^{\theta_m} \frac{\sin 2\theta}{2} \int_0^{2\pi} d\phi\, d\theta}$$

$$= F_m \Big/ \int_0^{\theta_m} \frac{\sin 2\theta}{2} \int_0^{2\pi} d\phi\, d\theta \tag{10}$$

Returning to equation 9 and replacing $I_T(\theta,\phi)$ with $I_M(\theta,\phi)$

$$\hat{F} = F_M + \overline{I_M(\theta,\phi)} \int_0^{\pi/2} \frac{\sin 2\theta}{2} \int_0^{2\pi} d\phi\, d\theta$$

and using the right-hand side of equation 10

$$\hat{F} = \chi F_M \tag{11}$$

where

$$\chi = \int_0^{\pi/2} \frac{\sin 2\theta}{2} \int_0^{2\pi} d\phi\, d\theta \Bigg/ \int_0^{\theta_m} \frac{\sin 2\theta}{2} \int_0^{2\pi} d\phi\, d\theta$$

$$= 2/(1 - \cos 2\theta_m) \tag{12}$$

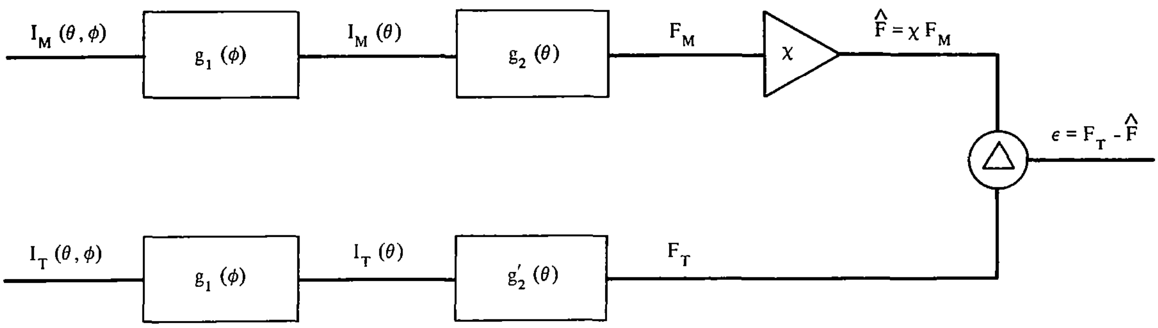
In summary, the estimate $\hat{F}$ has been taken to be the measurement F_M made at the satellite altitude, corrected by a term that accounts for the wider beam-width that would be filled by the Earth at h'. Note that here we are dealing only with the question of the angular distribution and that the simpler problem of normalization for the spreading loss (inverse ratio of the distances squared) has been ignored.

ERROR ANALYSIS

As yet, a basis has not been put forth for judging the quality of this estimation procedure over any other that might be proposed, and indeed, there are other possibilities. An analysis will now be presented to demonstrate that no other procedure can be found that will produce better results in the mean and mean-square sense.

The operation performed by a radiometer on the radiation field at either altitude can be represented by the double linear operations indicated in figure 4. This provides an electrical network analogy in which the square boxes would be perfect integrators, the triangle would be a multiplier to form the estimate, and the circle would be a differencing network to form the output error signal. An established procedure for analyzing such networks is to express the statistics of the output in terms of the statistics of the two inputs as determined by the weighting functions of the networks (References 2, 3, and 4). A discussion of the nature of the inputs follows.

Consider a time record of a radiometer output at either the satellite altitude, or at the top of the atmosphere as indicated in figure 5a. Two alternatives are available for determining the statistical properties of F (t). One is to work with time averages of the given single record F (t) on the assumption that it is sufficiently long so as to be statistically representative of the ensemble of possible records. The other alternative is to work with averages over an ensemble generated from F (t), namely $\{{}^kF(t)\}$, k = 1, 2 . . . (as indicated in figure 5b). Appropriate ensemble averages can be computed, which can then be related both to the larger ensemble to which F (t) is statistically equivalent, and to direct time averages of F (t).



$$g_1(\phi) \cdot I = \int_0^{2\pi} I(\theta,\phi)\, d\phi$$

$$g_2(\theta) = \int_0^{\theta_m} \frac{\sin 2\theta}{2}\, g_1(\phi) \cdot I_M\, d\theta$$

$$g_2'(\theta) = \int_0^{\pi/2} \frac{\sin 2\theta}{2}\, g_1(\phi) \cdot I_T\, d\theta$$

Figure 4. An electrical network analogy.

That is, for arbitrary fixed time t, and averaging over the k different records from the ensemble $\overline{F(t)} = \langle {}^kF(t)\rangle_{ave}$ over k, and $\overline{F^2(t)} = \langle {}^kF(t)\rangle_{ave}$ over k. This is the usual ergodic hypothesis on which most statistical processing methods depend and which certainly seems appropriate to the problem at hand.

Taking the error ϵ from figure 4 as the difference between the true flux and the estimated flux we have

$$\epsilon = F_T - \chi F_M$$

and the mean error as

$$\bar{\epsilon} = \overline{F_T - X F_m} = \overline{F_T} - \chi \overline{F_m} = \langle {}^kF_T\rangle_{ave} \text{over k} - \chi \langle {}^kF_M\rangle_{ave} \text{over k} \tag{13}$$

Now select the k records so that they all line up with the same Earth scene in view (coincident subsatellite points) at some time t_o.

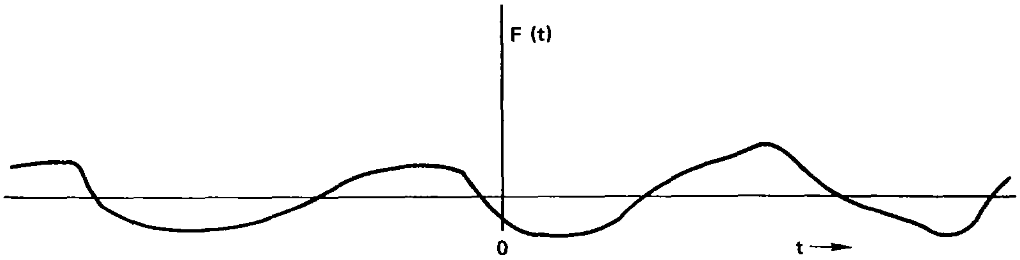


Figure 5a. A single time record of F (t).

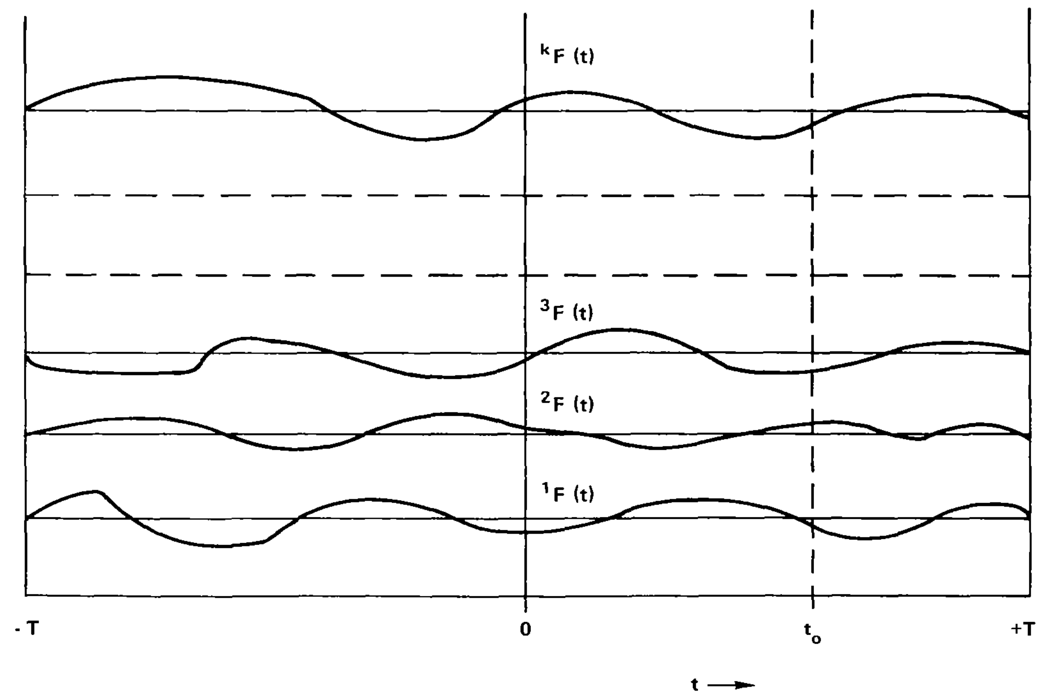


Figure 5b. An ensemble generated from a single time record of F (t).

Since equation 13 is true for an arbitrary fixed time t_o, the time variable has been suppressed and the averaging process over k has also been suppressed in the following equations.

Taking the k^{th} record, and using equation 4 with $\theta_m = \pi/2$ for an unrestricted aperture, we have

$$ {}^{k}F_{T} = \int_{0}^{\pi/2} \frac{\sin 2\theta}{2} \int_{0}^{2\pi} {}^{k}I_{T}(\theta,\phi)\, d\phi\, d\theta \qquad (14)$$

and for the first right-hand term of equation 13

$$\left\langle {}^k F_T \right\rangle = \int_0^{\pi/2} \frac{\sin 2\theta}{2} \int_0^{2\pi} \left\langle {}^k I_T(\theta,\phi) \right\rangle d\phi\, d\theta = \pi \left\langle {}^k I_T(\theta,\phi) \right\rangle \qquad (15)$$

In a similar way, the second right-hand term of equation 15 becomes

$$\chi \left\langle {}^k F_M \right\rangle = \chi \int_0^{\theta_m} \frac{\sin 2\theta}{2} \int_0^{2\pi} \left\langle {}^k I_M(\theta,\phi) \right\rangle d\phi\, d\theta$$

$$= \pi\chi \left(\frac{1 - \cos 2\theta_m}{2} \right) \left\langle {}^k I_M(\theta,\phi) \right\rangle \qquad (16)$$

Substituting equation 15 and equation 16 into equation 13 and solving for χ under the condition of a mean error of zero, we have

$$\chi = \frac{2}{1 - \cos 2\theta_m} \cdot \left\langle \frac{{}^k I_T(\theta,\phi)}{{}^k I_M(\theta,\phi)} \right\rangle = \frac{2}{1 - \cos 2\theta_m} \cdot \frac{\overline{F}_T}{\overline{F}_M} \qquad (17)$$

where $\overline{F}_T$ is the mean value of the true flux (as can be measured by an unrestricted field-of-view radiometer at satellite altitude) and $\overline{F}_M$ is the mean value of the flux measured by the restricted field-of-view radiometer at satellite altitude. Thus, we are assured of an unbiased estimate over the whole globe by virtue of direct measurements as well as at any subsatellite point.

The mean-square error can be found from the statistical average over the ensemble of errors. That is,

$$\left\langle {}^k\epsilon^2 \right\rangle = \left\langle (\chi\, {}^k F_M - {}^k F_T)^2 \right\rangle$$

$$= \chi^2 \left\langle {}^k F_M{}^2 \right\rangle + \left\langle {}^k F_T{}^2 \right\rangle - 2\chi \left\langle {}^k F_M\, {}^k F_T \right\rangle \qquad (18)$$

all averaged over k. Take each term from the right-hand side of equation 18 one at a time.

$$\chi^2 \left\langle {}^k F_M{}^2 \right\rangle = \chi^2 \left\langle \left[\int_0^{\theta_m} \frac{\sin 2\theta}{2} d\theta \int_0^{\pi/2} {}^k I_M(\theta,\phi)\, d\phi \right]^2 \right\rangle$$

$$= \chi^2 \int_0^{\theta_m}\int_0^{\theta_m} \frac{\sin 2\tau}{2}\,\frac{\sin 2\mu}{2} \left\langle \int_0^{\pi/2} {}^k I_M(\tau,\phi)\, d\phi \int_0^{\pi/2} \cdot {}^k I_M(\eta,\phi)\, d\eta \right\rangle d\tau\, d\mu$$

$$= \chi^2 \int_0^{\theta_m}\int_0^{\theta_m} \frac{\sin 2\tau}{2}\,\frac{\sin 2\mu}{2} \left(\int_0^{\pi/2}\int_0^{\pi/2} \left\langle {}^k I_M(\tau,\eta)\, {}^k I_M(\mu,\xi) \right\rangle \cdot d\eta\, d\xi \right) d\tau\, d\mu$$

$$= \chi^2 \int_0^{\theta_m}\int_0^{\theta_m} \frac{\sin 2\tau}{2}\,\frac{\sin 2\mu}{2}\, R_{MM}(\tau,\mu)\, d\tau\, d\mu \tag{19}$$

where $R_{MM}\ (\tau, \mu)$ is the autocorrelation of the radiation field at the satellite given by

$$R_{MM}(\tau,\mu) = \int_0^{\pi/2}\int_0^{\pi/2} \left\langle {}^k I_M\ (\tau,\eta)\, {}^k I_M\ (\mu,\xi) \right\rangle d\eta\, d\xi \tag{20}$$

The only requirement concerning the above steps is that the integrals exist in the mean-square sense, and we can be assured of this since all functions involved are bounded (Reference 5).

Taking the second from the right-hand side of equation 18 and following the same procedure we have

$$\left\langle {}^k F_T{}^2 \right\rangle = \int_0^{\pi/2}\int_0^{\pi/2} \frac{\sin 2\tau}{2}\,\frac{\sin 2\mu}{2}\, R_{TT}(\tau,\eta)\, d\tau\, d\mu \tag{21}$$

where $R_{TT}\ (\tau, \mu)$ is the autocorrelation of the radiation field at the top of the atmosphere given by

$$R_{TT}(\tau,\mu) = \int_0^{\pi/2}\int_0^{\pi/2} \left\langle {}^k I_T(\tau,\eta)\, {}^k I_M(\mu,\zeta) \right\rangle d\eta\, d\zeta \tag{22}$$

For the third term

$$- 2\chi \left\langle {}^{k}I_{M}\ {}^{k}I_{T} \right\rangle = -2\chi \int_{0}^{\pi/2} \int_{0}^{\theta_m} \frac{\sin 2\tau}{2} \frac{\sin 2\mu}{\mu}$$

$$\cdot R_{TM}(\tau, \eta)\, d\tau\, d\eta \qquad (23)$$

where $R_{TM}(\tau, \mu)$ is the cross-correlation of the radiation field at the satellite with that at the top of the atmosphere given by

$$R_{TM}(\tau, \mu) = \int_{0}^{\pi/2} \int_{0}^{\theta_m} \left\langle {}^{k}I_{T}(\tau, \eta)\ {}^{k}I_{M}(\mu, \zeta) \right\rangle d\eta\, d\zeta \qquad (24)$$

To proceed further with equations 19, 21, and 23, the nature of the scene must be specified in some way that will permit evaluation of the indicated correlation functions.

The principle of operation of a radiometer is based on the fact that any object at a temperature above absolute zero radiates electromagnetic waves. The radiation from any two objects from the set of objects that fills the beamwidth of a radiometer is incoherent even though both objects may be of the same physical makeup and temperature. According to the central-limit theorem (Reference 4) the sum total of all the incident radiation results in a noise-like signal with a normal probability distribution no matter what the probability distribution may be for the individual objects and no matter what the spatial distribution may be throughout the scene. These are the usual assumptions taken for analysis of radiometer systems and they appear extensively in the literature.*

For the present purpose, it is not necessary to assume that the sum of the incident radiation has a normal probability distribution. Instead, the assumption needed is only that the field consists of a large number of independent radiators. The term "independent" implies that, considering the ensemble of all possible points within the scene, the probability that any point radiates at a given intensity in the direction of the satellite at a given time is not conditioned by any of the other points. Invoking the ergodic hypothesis, we have

$$R_{MM}(\tau, \eta) = R_{MM}(0) = \sigma_M^2 \qquad (25)$$

$$R_{TT}(\tau, \eta) = R_{TT}(0) = \sigma_T^2 \qquad (26)$$

*See, for example, Ohlson and Swett (Reference 6) where the input signal to a radiometer follows the description set forth by S. O. Rice, of two quadrature phase sinusoids with independent random amplitudes, each having a normal distribution.

and

$$R_{TM}(\tau, \eta) = 0 \tag{27}$$

The assumption of independence does not imply anything concerning the mean temperature or mean reflectance of the individual points. In particular, they might all have the same mean temperature. Still, each point is a fluctuating source and its fluctuations are independent of all other points. Note that the assumption of independence is a condition of least constraint, for if the intensity distributions $I_M(\theta, \phi)$ and $I_T(\theta, \phi)$ were in any way correlated, then the two scenes would be similar in some ways and the error of estimation could thereby only be reduced.

Since the mean error is zero, equation 6 allows us to write

$$\left\langle k_\epsilon 2 \right\rangle = \sigma_\epsilon^2 \tag{28}$$

and substituting equations 25, 26, and 27 into equations 19, 21, and 23, respectively, and then the results into equation 18 we have

$$\begin{aligned} \sigma_\epsilon^2 &= \frac{\chi^2 \sigma_M^2}{4} \int_0^{\theta_m} \sin 2\tau \int_0^{\theta_m} \sin 2\mu \, d\mu \, d\tau \\ &\quad + \frac{\sigma_T^2}{4} \int_0^{\pi/2} \sin 2\tau \int_0^{\pi/2} \sin 2\mu \, d\mu \, d\tau \\ &= \chi^2 \sigma_M^2 + \sigma_T^2 \end{aligned} \tag{29}$$

Now, σ^2 will be minimum when $\chi^2 \sigma_m{}^2$ is minimum and from equation 11, $\chi^2 \sigma_m{}^2 = \sigma_F{}^2$ so that an equivalent condition is that $\sigma_F{}^2$ be minimum. This condition is assured by selecting χ so that the mean error is zero which necessarily means that $\chi^2 \sigma_m^2 = \sigma_T{}^2$. Therefore, the estimation procedure is an optimum procedure.

COMPARISON WITH ACTUAL MEASUREMENTS

Timofeyev (Reference 7) provides actual data from the Meteor-1 and -2 satellites (for April, July, and October 1969 and January 1970) that allow a satisfying confirmation of the above results. Figure 6 was derived from the data of Timofeyev's figure 1 (Reference 7). To obtain figure 6, averages were first taken over the four ranges of azimuth angle and then over the three ranges of Sun angle as given in Reference 8.

For case I, that is, clear to low cloud obsuration, figure 6 indicates a mean intensity of 0.061 $cal \cdot cm^{-2} \cdot min^{-1} \cdot sr^{-1}$ with a standard deviation about the mean of 0.013, measured in the same units. Case II, that is, considerable cloud obscuration, and case III, solid cloud cover are also illustrated. The means (m) and standard deviations (σ), are given in table 1 along with their ratios (σ/m) in percent.

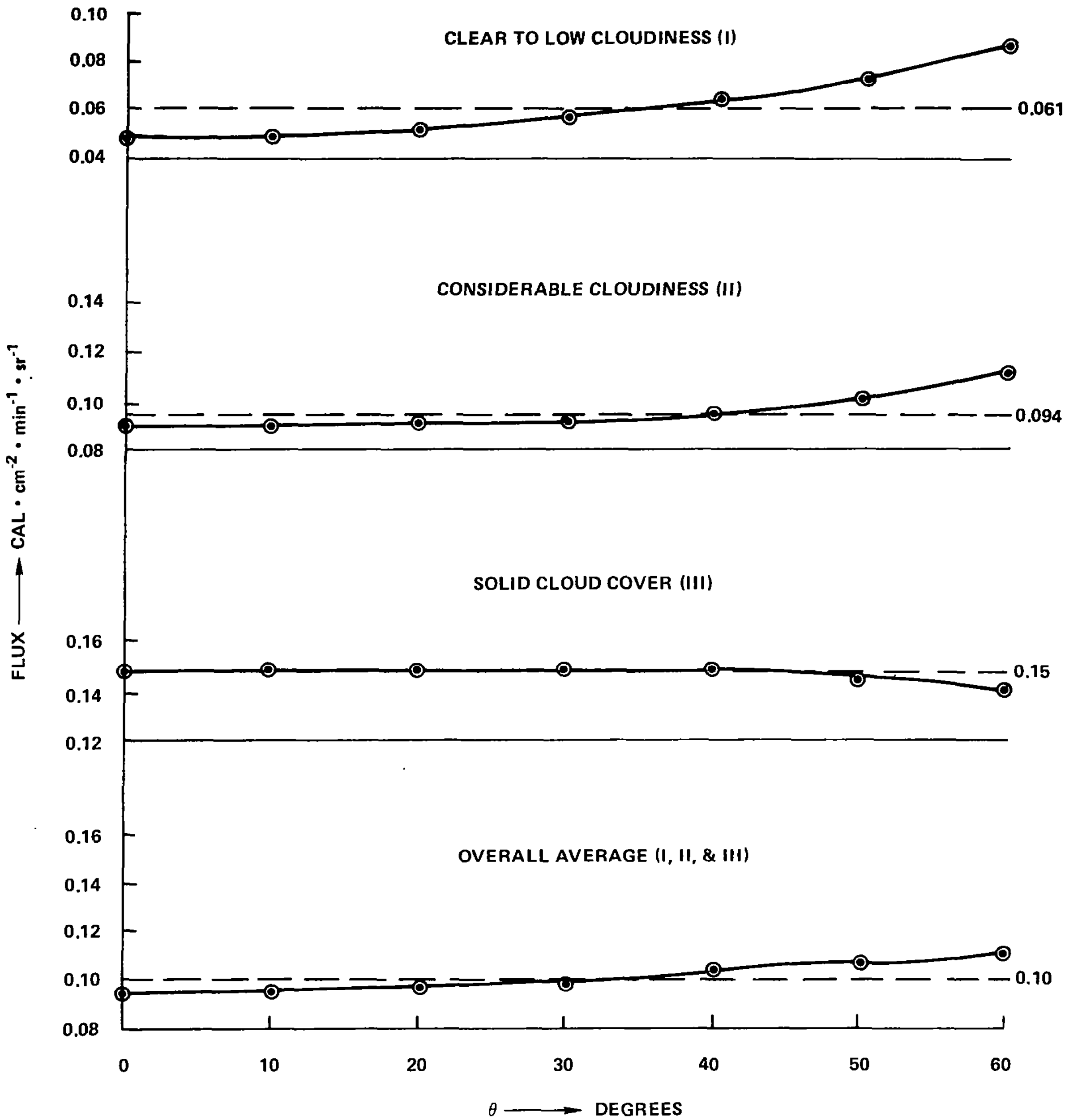


Figure 6. Average profiles of reflected intensities with satellite elevation angle θ (Reference 7).

Table 1
Parameters Pertaining to Figure 6

Cloud Cover	m* Mean	σ* Standard Deviation	σ/m Percent
I. Clear to low cloud observation	0.061	0.013	20.6
II. Considerable cloud observation	0.094	0.0076	8.0
III. Solid cloud cover	0.15	0.0039	2.7
I, II, III Overall	0.10	0.0052	5.2

* Values expressed in cal · cm^{-2} · min^{-1} · sr^{-1}

Figure 6 is an indication of what might be expected from satellite observations over ocean areas having each of the three states of cloudiness between the latitude belt ±60 degrees (averaged over all Sun angles) as a function of the elevation angle θ.

Note that the deviation from the mean is largest for the ocean surface without clouds and is least for solid cloud cover. For solid cloud cover, the mean is nearly three times larger than for slight cloud cover, as would be expected. It also seems reasonable to expect that solid cloud cover over land would produce similar results. However, the variation is probably smaller with slight cloud cover over land, than it is for the ocean under the same conditions. A solid cloud cover condition actually presents a situation of less variation with zenith angle than does a clear situation.

The curve of the overall average illustrated in figure 6 might be indicative of a 30-day average over which the three types of cloud cover occurred with more or less equal total duration (for instance 10 days of each). However, this curve also implies many observations each day, requiring an impractical number of satellites. Two observations a day (of a given region), spread over the range of Sun angles, for a 30-day period, would probably produce something between the three cases shown.

Based on figure 6, it is reasonable to expect a standard deviation of the flux for regions of 50-percent cloudiness of 8 percent with approximately 60 samples per region uniformly distributed over 30 days. The standard deviation of the flux might be as much as 21 percent over either ocean or land with little cloudiness under the same conditions.

Table 2 is taken from Faraponova (Reference 8) and shows an average standard deviation for the thermal interval of 0.05 cal • cm^{-2} • min^{-1} over five orbits of the KOSMOS-340 satellite. With something on the order of 60 samples per region, uniformly distributed over 30 days, it is reasonable to expect a standard deviation of about 0.0065 cal • cm^{-2} • min^{-1} or about 2 percent. (Note that the standard deviation about the mean over the five orbits is $0.02 \approx 0.05/\sqrt{5}$.)

Table 2
Mean and Standard Deviations
of Longwave Radiation for
Five Orbits of KOSMOS-340

Orbit	E_L Mean Outgoing Longwave Radiation*	σ_E Standard Deviation About the Mean E_L *
1	0.366	0.042
2	0.370	0.056
15	0.357	0.048
16	0.357	0.042
64	0.341	0.050
Average	0.359	0.050

*Values expressed in cal • cm^{-2} • min^{-1}

On the whole, the thermal flux is about twice the reflected flux, but the standard deviation of the thermal flux averaged over 30 days should be far less than the same for the reflected flux, and the accuracy of determining the net radiation balance should be limited by the variations in the reflected flux.

SUMMARY

The method described for relating flux measurements obtained at practical satellite altitudes to the actual flux at an altitude arbitrarily chosen to represent the top of the atmosphere was shown to be an optimum method in a statistical sense. That is, this method will produce estimates that are unbiased in the mean, and the variance of the estimates will converge to the true variance. Since the method described meets the selected criteria, no other method can be found that will produce better results according to those criteria, and it is therefore an optimum method.

The performance of the method can be checked in practice by direct measurements made with unrestricted field-of-view radiometers. In this way, the mean flux leaving the Earth over a given time interval at the satellite altitude is known. The derived flux estimates from the restricted field-of-view radiometers can be spatially averaged over the same time interval and the estimation process can be adjusted (without iteration) to assure agreement.

The method described is applicable to both shortwave and longwave Earth radiation. However, the notoriously large variations in reflected intensity with respect to aspect angle will more severely limit the accuracy to which the mean value of this component can be determined as compared to the thermal component.

Data from the Meteor-1 and -2 satellites indicate that for the shortwave radiation, an overall standard deviation of about 8 percent of the mean should be expected for approximately 60 observations of each region uniformly distributed over a 30-day interval. The corresponding standard deviation for the thermal radiation should be about 2 percent.

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