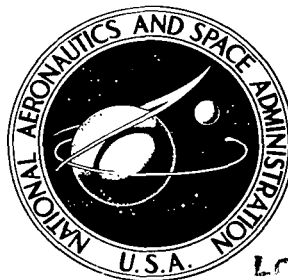


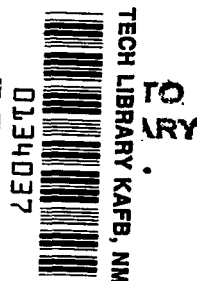
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**PRESSURE MODEL OF A FOUR-WAY  
SPOOL VALVE FOR SIMULATING  
ELECTROHYDRAULIC CONTROL SYSTEMS**

*Vernon D. Gebben*

*Lewis Research Center*

*Cleveland, Ohio 44135*



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16. Abstract A new equation that relates the pressure-flow characteristics of hydraulic spool valves was developed. The dependent variable is valve output pressure, and the independent variables are spool position and flow. This causal form of equation is preferred in applications that simulate the effects of hydraulic line dynamics. Results from this new equation are compared with those from the conventional valve equation, whose dependent variable is flow. A computer program of the new valve equation includes spool stops, leakage through spool clearances, and dead-zone characteristics of overlap spools.		13. Type of Report and Period Covered Technical Note
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# PRESSURE MODEL OF A FOUR-WAY SPOOL VALVE FOR SIMULATING ELECTROHYDRAULIC CONTROL SYSTEMS

by Vernon D. Gebben

Lewis Research Center

## SUMMARY

Two programs for modeling the spool-valve portion of electrohydraulic servovalves are described. One computer model represents the valve as a flow device where spool position and load reaction pressure are input signals. It uses the underlap-spool-valve equation to define leakage through spool clearances. This flow model is a conventional representation for applications where fluid compressibility is the immediate system element loading the valve. The other computer model simulates a pressure source by using the opposite causality; that is, pressure is the dependent variable and flow is an independent variable determined by load reactions. It uses a new equation that provides the pressure-flow characteristics for spool leakage. This pressure model was designed for use in simulations that include hydraulic line dynamics. These models are described and defined by their equations, computer flow diagrams, and FORTRAN expressions. They have demonstrated characteristics that are realistic for time-domain simulation of electrohydraulic control systems.

## INTRODUCTION

Electrohydraulic servovalves made their appearance in the late 1940's to satisfy aerospace needs for compact, high-power, fast-response control systems. Analyses of these valves have remained involved and tedious. Difficulties are evidenced by the fact that articles on analysis techniques for estimating dynamic performance of hydraulic control systems are still being published in technical journals.

Problems in valve simulation result from inherent nonlinearities. For example, the quadratic pressure-flow relation of orifices presents one problem source. Analysis problems also arise from nonlinear characteristics caused by spool overlap, leakages through spool clearances, and limited spool displacement.

Analyses of hydraulic control systems have been structured around standard techniques of linear approximations and describing functions. Linear approximations obtained from the small-signal theory (theory of small perturbations) have provided solutions that proved particularly effective in evaluating system stability in the frequency domain (refs. 1 and 2). Stability studies in the frequency domain for large signals also have been accomplished by using describing functions that simplify the nonlinear problem by excluding harmonic components from signals (refs. 3 to 6). Although these techniques have demonstrated successful applications, they become inaccurate when the system contains multiple nonlinearities such as a combination of coulomb friction and backlash.

Analog and digital computer simulations are more direct and flexible. Computer models can represent most nonlinear electrohydraulic control systems with sufficient accuracy for time-domain and frequency-domain design analyses and provide the convenience of permitting direct changes to parameters.

For one example, a computer simulation of a hydraulic control system was chosen for evaluating dynamic effects of long lines connecting the valve to the actuator. Servovalves usually are simulated with a "flow model" that relates output flow as a function of both spool position and output pressure, where output pressure is determined by load reactions. The flow model for this application produced a program that used differentiators to simulate line dynamics. Differentiators, however, should be avoided whenever possible, especially in analog circuits, where they often saturate computer elements with amplified circuit noise. The "pressure model" described in this report provides an advantage by allowing line dynamics to be implemented with integrators instead of differentiators. It is a model whose output pressure depends on the combination of spool position and output flow.

A pressure model that includes leakage through spool clearances was not available in the literature. Flow models seem to be the only kind reported. Furthermore, the nonlinear equations of the flow model could not be transformed into a pressure model. It was therefore necessary to invent a pressure model that closely approximates the pressure-flow characteristics produced by the conventional flow model. The new computer model is described in this report.

Mathematical models for the spool portion of electrohydraulic servovalves are the subject of this report. The first section describes the pressure-flow characteristics of four-way spool valves and gives the flow-model equation. The second demonstrates the need for a new model. The new pressure model is then presented and compared with the conventional flow model. In the last section, the models are used to derive differences between the output characteristics of a servovalve connected to an inertial load and those of one connected to a capacitive load. Both models are defined in the appendix by their equations, computer flow diagrams, and FORTRAN expressions for digital programs such as the "Continuous System Modeling Program" (ref. 7).

## SYMBOLS

A	amplitude of spool motion, dimensionless
B	blocked-line pressure gain ( $\partial p/\partial x$ )
C	normalized hydraulic capacitance or compliance
$C_1$	hydraulic capacitance or compliance
D	computer program variable
E	electrical potential
F	computer program variable
G(s)	transfer function
H	computer program variable
I	electrical current
j	imaginary unit, $\sqrt{-1}$
K	computer program variable
L	dimensionless displacement parameter
$M_1$	inertia or mass
N	no-load flow gain ( $\partial q/\partial x$ )
P	normalized differential output pressure, dimensionless
p	pressure
Q	normalized output flow, dimensionless
q	flow
R(X)	modulated resistance, dimensionless
$R_1$	resistance
s	Laplace operator
t	time
U	normalized spool center region, dimensionless
V	normalized spool overlap, dimensionless
W	computer program variable
X	normalized spool position, dimensionless
x	spool position

Y	computer program variable
Z	computer program variable
$\omega$	angular frequency

## ELECTROHYDRAULIC SERVOVALVES

A typical servovalve is shown in figure 1. An input electrical-power signal operates a torque motor. The torque motor operates a flapper valve which drives the spool to position X. Spool position controls the output pressure-flow relation. The main background information for this report is given in reference 8, which describes four-way valves, the pressure-flow characteristics of four-way valves, and the frequency-response characteristics of electrohydraulic servovalves.

For analysis, the operating characteristics of servovalves can be separated into dynamic and static components as shown in figure 2. The first block represents the combined dynamics of the torque motor, flapper valve, and spool. Generally, the dynamic portion is assumed to be a linear third- or fourth-order system. The linear frequency-response characteristics that define  $G(s)$  can be obtained from the manufacturer of the servovalve. In certain applications, it may be necessary to include flapper-valve nonlinearities which can significantly influence the dynamic performance capabilities of fast-response systems. This limitation has been investigated and formulated into general design criteria (ref. 9).

Function  $R(X)$  is the subject of this report. It is a load-dependent nonlinear function comprised of orifice equations that relate the output pressure-flow characteristics to input spool position. Except for the overlap case, the present analysis assumes a symmetrical spool with port areas proportional to X. Also, as is commonly done to obtain a simplified analytical representation of  $R(X)$ , it will be assumed that the flow in one output line always equals the flow in the other output line. Equal flows require identical dynamic loads on the two output ports, which is a conservative estimate for stability analysis (ref. 10). The output flow can then be represented by a single variable, Q.

Symbols Q, P, and X denote normalized flow, normalized pressure, and normalized spool position. Variable Q is equal to the average value of the flows through the two output ports divided by the maximum output flow at zero-differential output pressure. Variable P is equal to the differential output pressure divided by the maximum differential output pressure that the valve produces for a blocked output line. Variable X is normalized spool position defined as a fraction of maximum displacement.

Ideal critical-center (critical-lapped) four-way spool valves have characteristics that are displayed in figure 3 and defined by

$$Q = X \sqrt{1 - P} \quad \text{if } 0 \leq X \leq 1 \quad (1)$$

$$Q = X \sqrt{1 + P} \quad \text{if } -1 \leq X \leq 0 \quad (2)$$

The characteristics at null ( $X = 0$ ) are very important to basic performance. Figure 4 shows that the no-load ( $P = 0$ ) flow sensitivity is linear and has a slope value of 1. Figure 5 shows the blocked-line ( $Q = 0$ ) pressure sensitivity. Pressure  $P$  is either +1 or -1, which means that it is impossible to maintain zero pressure across the load. The ideal critical-center valve would thus be impractical for use in position-control systems containing coulomb friction.

Practical spools have radial clearances and can have minute underlap allowed by machining tolerances. The result is bypass leakage for spool positions where  $X$  is less than position  $U$ . A typical value for  $U$  would be 0.03; that is, 3 percent of maximum spool displacement. The center region defined by  $U$  is small, but important to accurate simulations. A small amount of leakage along the spool is beneficial, for it reduces blocked-line pressure sensitivity and provides intermediate output pressures, as shown in figure 6. Spools can also be manufactured with overlap. In this case, the pressure-flow characteristics at null will be a combination of a dead zone (threshold) caused by overlap and bypass leakage through radial clearances.

Figure 7 has the pressure-flow characteristics divided into three zones. The ideal critical-center spool-valve equations (eqs. (1) and (2)) fit the principal regions very well. The center region generally has characteristics similar to those of underlap (open-center) four-way spool valves. The constitutive relation for the center region would then be described by the following relation, derived from reference 8, section 5-4:

$$Q = (0.5)(X + U) \sqrt{1 - P} + (0.5)(X - U) \sqrt{1 + P} \quad \text{if } -U \leq X \leq U \quad (3)$$

Equations (1), (2), and (3) are limited to a range of values of  $P$  between 1 and -1. Inertial loads, however, can produce pressure transients that put  $P$  outside of this range (ref. 11). High pressure peaks occur when, for example, a valve switches from fully open ( $X = 1$ ) to closed ( $X = 0$ ) in an effort to stop a fast-moving mass. High differential pressure then develops across the actuator as the result of high resistance of the closed valve and high flow maintained by load momentum. Since  $P$  is not physically limited between 1 and -1, a new set of relations between  $X$ ,  $P$ , and  $Q$  are needed for simulating system transient response.

Pressure limits in equations (1) to (3) result from incorporating the following ori-

ifice equation:

$$q = D \sqrt{p_1 - p_2} \quad (4)$$

where

q flow

D conductance coefficient

$p_1$  upstream pressure

$p_2$  downstream pressure

This equation provides no information concerning flow direction, except that upstream pressure is higher than downstream pressure. Conventional network equations of the four-way spool valve have fixed directions of flows (ref. 8, p. 79).

A different set of equations describing the constitutive relations of four-way valves results when flow polarity is added to the orifice network equations (ref. 12); namely,

$$Q = [\text{sign}(1 - P)](X) \sqrt{|1 - P|} \quad \text{if } U \leq X \leq 1 \quad (5)$$

$$Q = [\text{sign}(1 - P)](0.5)(X + U) \sqrt{|1 - P|} + [\text{sign}(1 + P)](0.5)(X - U) \sqrt{|1 + P|} \quad \text{if } -U \leq X \leq U \quad (6)$$

$$Q = [\text{sign}(1 + P)](X) \sqrt{|1 + P|} \quad \text{if } -1 \leq X \leq -U \quad (7)$$

Equations (5), (6), and (7) can be combined and expressed by a single constitutive relation:

$$Q = [\text{sign}(1 - P)](0.5)(X + L) \sqrt{|1 - P|} + [\text{sign}(1 + P)](0.5)(X - L) \sqrt{|1 + P|} \quad (8)$$

where

$$L = U \quad \text{if } |X| \leq U$$

$$L = |X| \quad \text{if } |X| \geq U$$

Figure 8 shows the pressure-flow characteristics obtained from equation (8) for pressures in the extended range where  $P$  is greater than 1. The scales of figure 8 are noticeably different from those of figure 6. To make the curves of equation (8) visible



in the center region, the graph was stretched in the Q-axis direction by multiplying variable Q by constant 1/U. Lines of X, designating spool position, were changed to lines of X/U.

Although these equations extend the pressure range, they are considered valid only if cavitation does not occur. This restraint comes from the definition of Q which assumes that the two output ports have equal flows. The output of models described in this report should therefore be limited to conditions where P does not exceed 1. Equation (8) still provides the advantage of allowing simulations to operate in the extended pressure range ( $P > 1$ ) without program interrupts. The effects of excessively high values of P can then be qualified (ref. 12).

Equation (8) is a flow model, designated "model Q1." The appendix gives a computer program for equation (8) that provides the same pressure-flow characteristics used in other published computer programs (refs. 11 to 15). Also included in the program is a dead zone that simulates the characteristics of overlap spools. A program that solves for Q, however, is not always best for system simulation, as will be demonstrated in the next section.

#### MODEL OPTIONS

Load elements of typical actuation systems shown in figure 9 include line inertia, fluid compliance, actuator leakage, actuator inertia, and friction. Not all of these elements, however, may be necessary for an accurate system analysis. For example, line dynamics generally can be neglected if the valve is mounted on the actuator. Fluid compliance can also be neglected if the resonant frequency due to compliance is much higher than the system bandwidth. It will be demonstrated that the type of valve model needed for a simulation depends on the immediate element loading the valve.

Dynamics of capacitive and inertial load elements can be expressed mathematically in one of two forms: either integral causality -

$$p = \frac{1}{C_1} \int q \, dt \quad (9)$$

$$q = \frac{1}{M_1} \int p \, dt \quad (10)$$

or derivative causality -

$$q = C_1 \frac{dp}{dt} \quad (11)$$

$$p = M_1 \frac{dq}{dt} \quad (12)$$

Interdependence of valve model and load equations can be demonstrated with linear load elements and a linear valve model. Let the valve be defined by the following equation:

$$x - \frac{p}{B} - \frac{q}{N} = 0 \quad (13)$$

where  $B$  is the blocked-line pressure gain, and  $N$  is the no-load flow gain. The causal form of the valve flow model is written

$$q = N \left( x - \frac{p}{B} \right) \quad (14)$$

For the valve pressure model,

$$p = B \left( x - \frac{q}{N} \right) \quad (15)$$

Two different programs result when these equations are applied to the load shown in figure 9. Figure 10 describes the simulation with the valve flow model, and figure 11 describes the simulation with the valve pressure model. The simulation of figure 10 requires a differentiator ( $M_1s$ ) to compute the pressure that accelerates the fluid in the line. Algebraic manipulation of figure 10 will not eliminate the differentiator. Differentiation disappears only when line inertia is zero. Then fluid compliance is the immediate element loading the valve.

The simulation of figure 11 has the advantage of using an integrator ( $1/M_1s$ ) to compute line flow. This causal form is preferred over that of figure 10, because computers are more accurate at integration than differentiation. Also, digital computer programs are difficult to stabilize when a set of simultaneous equations (algebraic loop) contains differentiation. Therefore, the valve pressure model should be used in applications where the nearest load element to the valve is inertia.

Likewise, the valve flow model is preferred when line inertia is not included in the simulation. Actuator pressure, determined by fluid accumulated in the cylinder, is then

related to the time integration of the difference between valve flow and flow due to actuator motion.

Valve models with flow for their dependent variable seem to be the only kind reported. The author is unaware of any previous descriptions of pressure models (i. e., valve models that use output pressure as the dependent variable). Hence, a pressure model representing four-way valves was developed for analyzing systems involving line dynamics and other systems where flow models are unusable.

### VALVE MODEL P1

The pressure model defines  $P$  as a function of  $X$  and  $Q$ . The direct method for deriving this causal form would be to express the flow model in terms of  $P$ . Unfortunately, this approach is obstructed by square-root terms in equation (8). Equation (8) cannot be solved explicitly in terms of  $P$ . Therefore, a new constitutive relation approximating equation (8) is needed for modeling four-way spool valves.

To provide realistic valve characteristics, the new relation should have the following features:

- (a) Identical equations for the principal operating region ( $|X| \geq U$ )
- (b) Pressure gain ( $\partial P/\partial X$ ) that does not change abruptly during transition from principal operating region to center region
- (c) Realistic slopes ( $\partial P/\partial Q$ ,  $\partial P/\partial X$ ,  $\partial Q/\partial X$ ) around the origin ( $X = 0$ ,  $P = 0$ ,  $Q = 0$ )
- (d) Equation that is tangent to equation (8) at the origin

These conditions were satisfied by an equation that was invented, rather than having been derived from a set of theoretical equations. The new relation, designated as "model P1," is

$$P = \frac{X}{L} \left( 2 - \frac{|X|}{L} \right) - \frac{Q|Q|}{L^2} - \frac{2Q}{L} \left( 1 - \frac{|X|}{L} \right) \quad (16)$$

where

$$L = U \quad \text{if } |X| \leq U$$

$$L = |X| \quad \text{if } |X| \geq U$$

Models Q1 and P1 are identical in the principal operating regions. For example, in the first quadrant of the P-Q diagram, where  $P > 0$ ,  $Q > 0$ , and  $X > 0$ , equations (8) and (16) reduce to the following identical relations:

$$Q = X \sqrt{1 - P} \quad (17)$$

$$P = 1 - \frac{Q^2}{X^2} \quad (18)$$

Similar results for equations (8) and (16) are obtained by examining the principal regions in the second, third, and fourth quadrants of the P-Q diagram.

Models Q1 and P1 have different equations in the center region ( $|X| \leq U$ ). An overlay of Q1 and P1 is shown in figure 12. The differences between equations are most noticeable for high output pressures (e. g.,  $|P| > 0.4$ ). The empirical relation of P1 may be more realistic than that of Q1, because model Q1 assumes turbulent flow through sharp-edge orifices. In practice, leakage passages resulting from spool clearances do not resemble ideal orifices. The passage surface area, in effect, is large in comparison to the flow area. Hence, the more linear curves of P1 should provide a better representation.

Differences between models are probably insignificant relative to estimated values often used for viscous friction, coulomb friction, backlash, and other system parameters. Also, the center region represents only a small portion of the spool's operating range. A typical value for the center region would be 3 percent of the maximum spool displacement. If duration in the center region is short with respect to the total transient time, and if  $|P|$  is less than 1, then both models should provide essentially the same pressure-flow characteristics.

Impedance and gain of P1 at the origin are derived directly from equation (16). At  $X = 0$ ,  $P = 0$ , and  $Q = 0$ , we obtain

$$\text{Output impedance} = \frac{\partial P}{\partial Q} = -\frac{2}{U} \quad (19)$$

$$\text{Pressure gain} = \frac{\partial P}{\partial X} = \frac{2}{U} \quad (20)$$

$$\text{Flow gain} = \frac{\partial Q}{\partial X} = 1 \quad (21)$$

The small-signal equation at the origin then becomes

$$X - Q - \frac{U}{2} P = 0 \quad (22)$$

The same linear small-signal equation results from equation (8) for Q1. Therefore, equations (8) and (16) are tangent to each other at the origin. Both models will produce identical system dynamics for small signal disturbances at the origin.

The appendix gives a computer program for equation (16). Also included in the program is a dead zone, or threshold, that simulates the characteristics of overlap spools.

### EXAMPLES USING MODELS Q1 AND P1

Models Q1 and P1 were tested with the digital Continuous System Modeling Program (ref. 7) by using either pure capacitive or pure inertial loads. Pure loads were used to simplify the demonstration; they are not valve model requirements. Model Q1 was tested by using sinusoidal spool motion, a center region (U) of 0.03, zero overlap, and a shunt capacitive load. The input equation was

$$X = A \sin \omega t \tag{23}$$

The integral causal relation for the capacitor is

$$P = \frac{1}{C} \int Q dt = \frac{Q}{Cs} \tag{24}$$

For a linear system, the P and Q waveforms are sinusoidal. Equation (24) then reduces to

$$\frac{Q}{P} = j\omega C \tag{25}$$

Capacitance was set to give the linear case a circular load line on the P-Q diagram. This load line is obtained by choosing  $C = 1/\omega$ . Equation (25) then becomes

$$\frac{Q}{P} = j \tag{26}$$

According to equation (26), Q and P have equal amplitudes, and Q leads P by a 90° phase angle. As a result, the load line on the P-Q diagram is circular and is generated by a point that moves in a clockwise direction.

The load equation obtained from equation (24) for testing Q1 then takes the form

$$P = \omega \int Q dt \quad (27)$$

Two input amplitudes were used. The smaller signal moved the spool only to the edge of the center region. The other signal oscillated the spool to its maximum limits. The resultant load lines for steady-state conditions are shown in figure 13 with scales that are normalized to amplitude  $A$ .

The curve for the small-amplitude signal ( $A = 0.03$ ) is circular with flow and pressure amplitudes of 0.03. Deviations from a perfect circle were small. The maximum change in the radial direction with respect to a circumscribed circle was only 1.2 percent. The circular load line indicates a linear input-output relation. Good linearity for small pressure amplitudes is also implied by the characteristics of the curves at the  $Q$ -axis in figure 12 - they are equally spaced and have equal slopes.

The curve for  $A = 1$  has an elliptic shape, with its major axis in the second and fourth quadrants. This peculiar shape results from variations in flow gain during the cycle. Equation (8) indicates that flow gain ( $\partial Q/\partial X$ ) is a function of  $P$ . For example, if  $X > U$ , then flow gain is 1.413 at  $P = -1$ , 1.000 at  $P = 0$ , and 0 at  $P = +1$ . Likewise, if the spool displacement is negative and  $X < -U$ , then flow gain is 1.413 at  $P = +1$ , 1.000 at  $P = 0$ , and 0 at  $P = -1$ . Therefore, a large gain change occurs for  $A = 1$  when the spool moves through its null position.

The load curve is generated in a clockwise direction. Consequently, when the spool moves through null, for example, the flow gain increases. The result is a sudden rate increase in  $Q$ . Then, gain gradually decreases as the load curve proceeds through the second and first quadrants. The waveforms corresponding to figure 13 are shown in figure 14.

The same two tests were conducted with model P1, except an inertial load was used instead of the capacitive load. The load equation was similar to equation (27); that is,

$$Q = \omega \int P dt \quad (28)$$

The resultant load lines are shown in figure 15.

The curve for  $A = 0.03$  is again circular but evolves in a counter clockwise direction. The curve deviates from a circumscribed circle by less than 0.01 percent, except at zero flow, where it changed 0.6 percent. This transient at  $Q = 0$  is more pronounced in the  $A = 1$  curve.

Abrupt changes in pressure occur when load momentum diminishes to zero. Then  $P$  is unaffected by load and is a function of  $X$  only. If  $X$  and  $Q$  went to zero at the same time,  $P$  would become zero. Zero pressure, however, did not happen, because a small phase lag existed between  $Q$  and  $X$ .

Pressure transients noted in figure 15 did not appear in figure 13. Equation (24) explains this behavior. According to equation (24), when  $Q = 0$ , then  $dP/dt$  is zero. Hence, the pressure in an accumulator is constant, as expected, when its flow is zero.

The waveforms corresponding to figure 15 are displayed in figure 16. Signal distortions produced by valve nonlinearities are entirely different from the capacitive load curves. A major difference is the shape of the pressure wave, which reaches an extra-high amplitude as the flow approaches zero. High pressure amplitude results from a pressure gain ( $\partial P/\partial X$ ) that increases as the load decelerates in the second and fourth quadrants of the P-Q diagram.

A Fourier series of the flow waveform in figure 16 was obtained for the  $A = 1$  case. The amplitude of the fundamental component is 89 percent of the amplitude obtained with the linear valve equation (eq. (22)). The phase angle of the fundamental component lagged the linear valve equation by  $11^\circ$ .

Attempts were made to use model P1 with a capacitive load and to use model Q1 with an inertial load. Solutions for derivative-causality loads could not be obtained from the digital Continuous System Modeling Program for either case. This problem supports the claim that when the digital computer is used, model Q1 is essential for analyzing the effects of fluid compressibility, and model P1 is essential for analyzing inertial effects.

## CONCLUDING REMARKS

Programs simulating standard four-way spool valves were developed for two different applications. One was designed for driving capacitive loads, and the other, for inertial loads. Both models have demonstrated characteristics that are realistic for time-domain simulation of electrohydraulic control systems.

Model Q1 is a flow-source representation. Input signals are spool position and load reaction pressure. Flow is the output signal acting on the load. An underlap spool-valve equation defines the leakage through spool clearances. This model is essentially the same as other published computer programs that have been successfully used to simulate valves. Model Q1 should be used in simulations where the capacitive effect of entrapped air is the immediate element loading the valve.

Model P1 is a pressure-source representation. Input signals are spool position and load reaction flow. Pressure is the output signal acting on the load. This model is

based on a new constitutive relation for spool leakage. Model P1 should be used when the immediate load element is either fluid inertia or motor inertia.

Lewis Research Center,

National Aeronautics and Space Administration,

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778-24.



## APPENDIX - COMPUTER PROGRAMS FOR MODELING FOUR-WAY SPOOL VALVES

The following two relations were developed for modeling the pressure-flow characteristics of four-way spool valves:

$$Q = [\text{sign}(1 - P)](0.5)(X + L) \sqrt{|1 - P|} + [\text{sign}(1 + P)](0.5)(X - L) \sqrt{|1 + P|} \quad (8)$$

$$P = \frac{X}{L} \left( 2 - \frac{|X|}{L} \right) - \frac{Q|Q|}{L^2} - \frac{2Q}{L} \left( 1 - \frac{|X|}{L} \right) \quad (16)$$

where

$$L = U \quad \text{if } |X| \leq U$$

$$L = |X| \quad \text{if } |X| \geq U$$

Symbols  $Q$ ,  $P$ ,  $X$ , and  $U$  have normalized values. Variable  $Q$  is equal to the flow through the output port divided by the maximum output flow at zero differential output pressure. Variable  $P$  is equal to the differential output pressure divided by the maximum differential output pressure that the valve produces for a blocked output line. Variable  $X$  is normalized spool position, defined as a fraction of maximum displacement. Symbol  $U$  denotes a center operating region of  $X$  where spool leakage occurs. A typical value for  $U$  would be 0.03; that is, 3 percent of maximum spool displacement.

Equations (8) and (16) assume that the flow in one output line always equals the flow in the other output line; this assumption is generally accepted as a conservative estimate for stability analysis. The equations also assume a symmetrical spool, with port areas proportional to  $X$ .

Spool stops and overlap characteristics can be included in analog and digital computer programs. Input signal  $X$  is then limited to  $-1 < X < 1$  and modified by a dead zone at its center position. The center leakage region remains with overlap spool models in order to account for leakage through radial clearances. FORTRAN statements for equations (8) and (16) that include spool stops and overlap are given in figures 17 and 18. Variable  $V$  represents spool overlap and is expressed as a fraction of maximum spool displacement. Letters  $D$ ,  $F$ ,  $H$ ,  $K$ ,  $W$ ,  $Y$  and  $Z$  are program variables. For the zero-overlap case, variable  $Y$  is replaced by  $X$ .

Simultaneity of action-reaction signals is an important consideration in the use of these models. Bilateral coupling between valve and load requires that the output signal and the load reaction signal correspond in time, in order to prevent errors or program instabilities. Digital programs produce correlation errors, inasmuch as they can only

represent continuous processes by using fixed values during the time-integration intervals. For accurate results, these intervals must be appreciably smaller than the load-time constants. In some cases, this integration interval is so small with respect to the overall system response, that the program becomes impractical for use in parametric studies. Bilateral coupling presents no problem in analog computers, because simultaneity is a natural product of continuous processes. The valve models are therefore more suited for the analog computer.

The equation preferred for simulating a valve in a control system will depend on the load, which may include line dynamics, fluid compliance, actuator leakage, actuator inertia, and friction. The valve equation that allows the load equations to have integral causality is the one most suited for computer simulations. For example, equation (8) is preferred when the effect of entrapped air is simulated as the immediate element loading the valve. In this case,  $P$  is determined by the fluid accumulated in the cylinders; that is, the time integral of the difference between  $Q$  and the actuator speed. Equation (16) is preferred when the immediate load element is fluid inertia in lines connecting the valve to the actuator. Then,  $P$  represents the input variable to the load, and  $Q$  becomes a function of the time integral of  $P$ .

Both valve equations have demonstrated characteristics that are realistic for time-domain simulations of electrohydraulic control systems. Each has special applications which provide the advantage of using integrators instead of differentiators.

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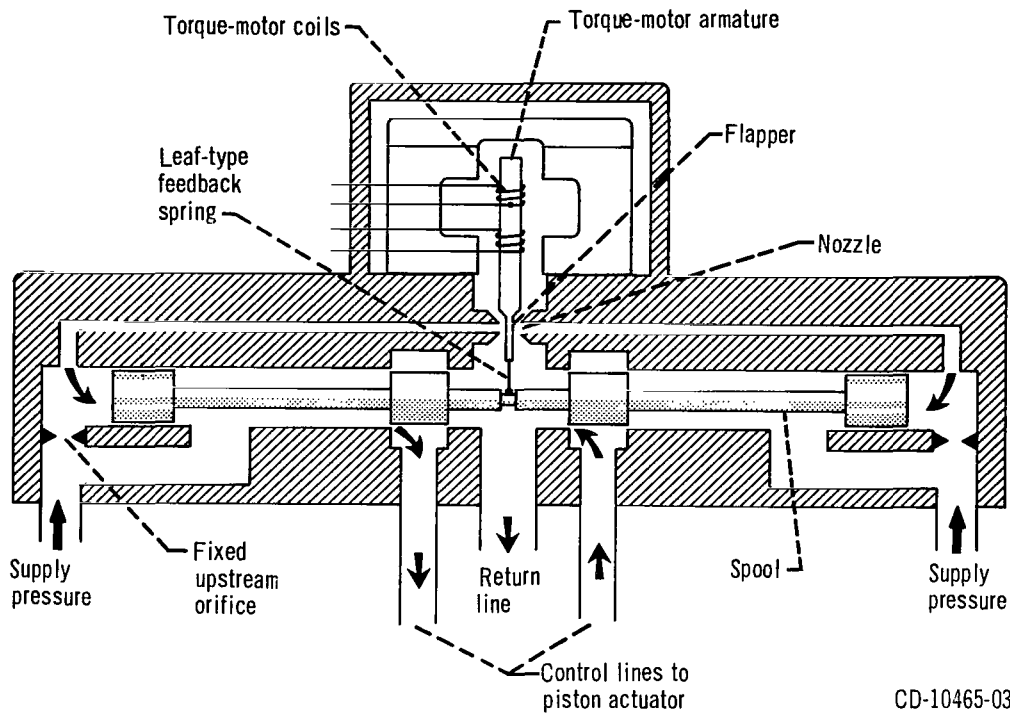


Figure 1. - Schematic representation of two-stage electrohydraulic servovalve.

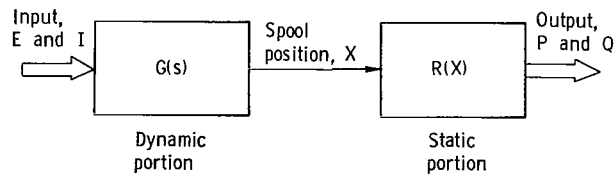


Figure 2. - Block diagram of electrohydraulic servovalve.

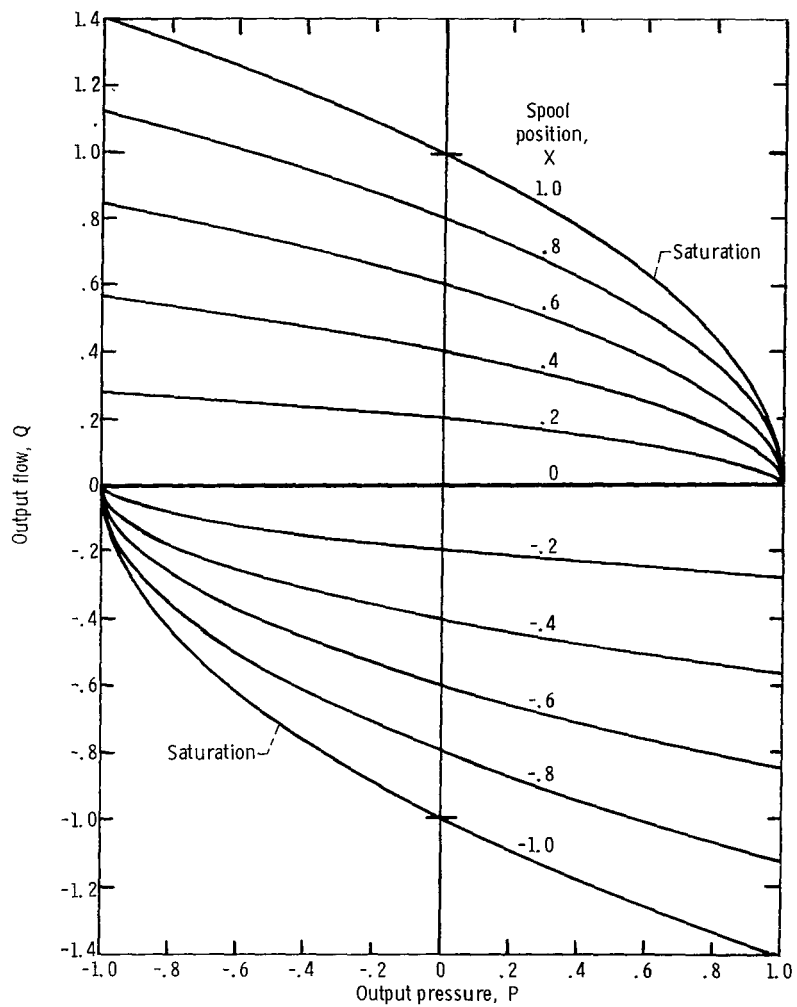


Figure 3. - Pressure-flow diagram showing ideal pressure-flow curves of critical-center (zero-overlap) four-way spool valves.

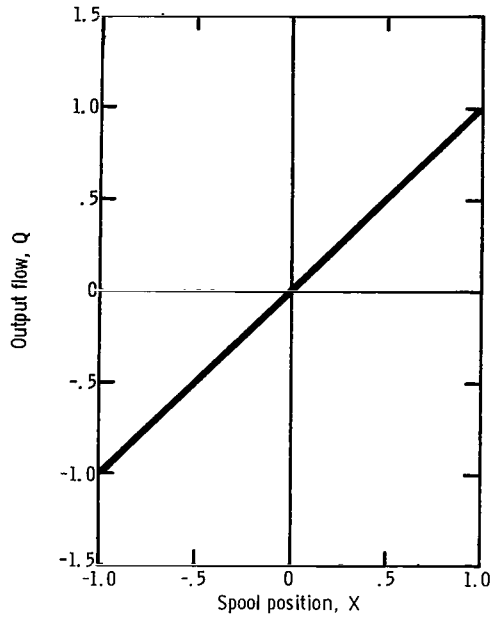


Figure 4. - No-load flow sensitivity ( $P = 0$ ) for critical-center, no-leakage valve.

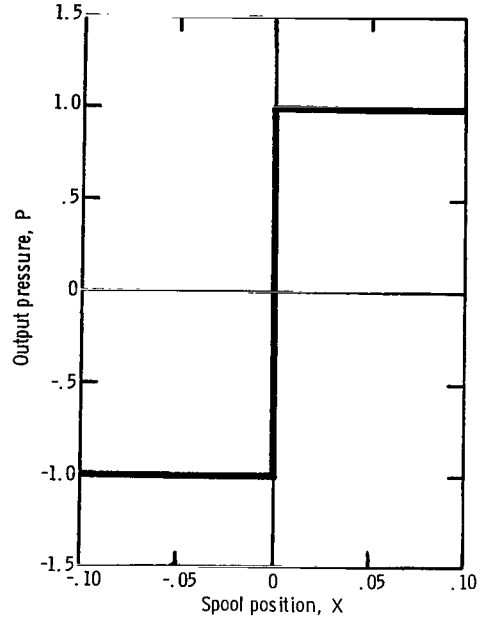


Figure 5. - Blocked-line pressure sensitivity ( $Q = 0$ ) for critical-center, no-leakage valve.

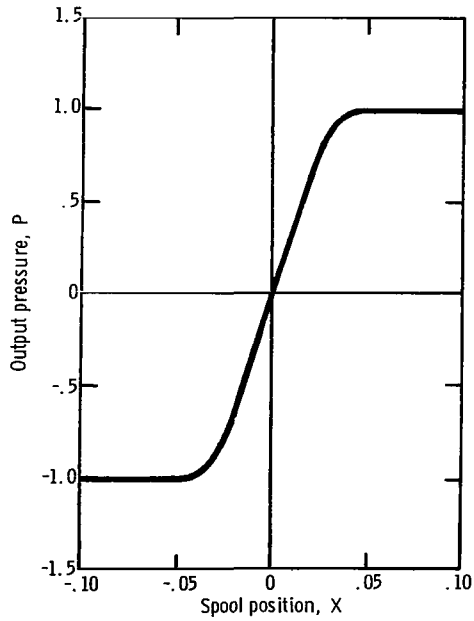


Figure 6. - Typical blocked-line pressure-sensitivity curve for practical spool valve.

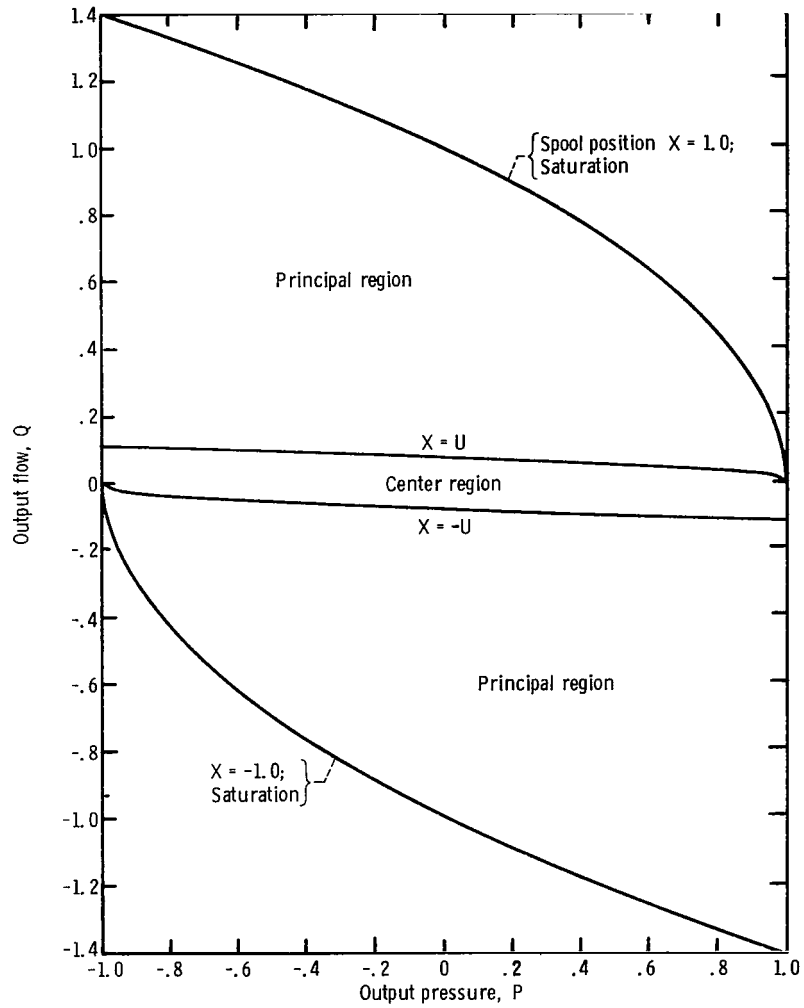


Figure 7. - Pressure-flow diagram showing distinct operating regions of practical, four-way spool valves.



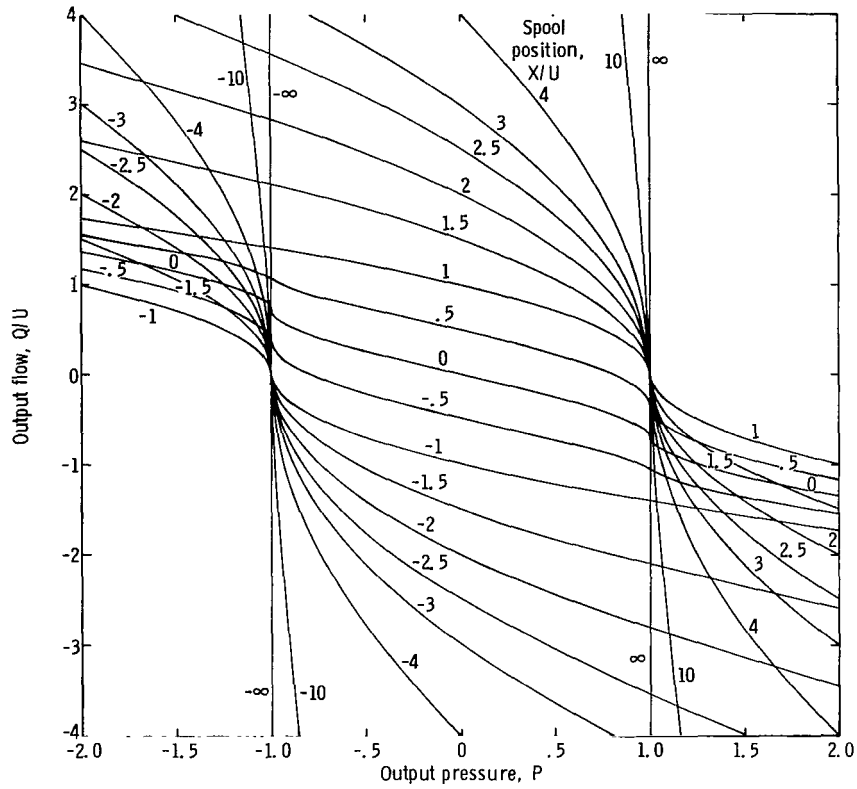


Figure 8. - Pressure-flow diagram of four-way spool valve with bypass leakage represented by characteristics of the open-center spool.

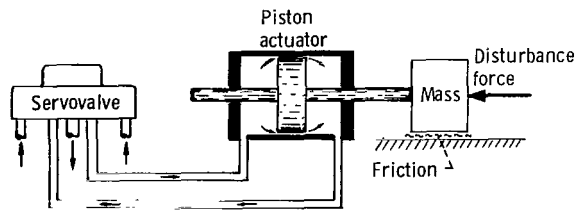


Figure 9. - Layout of typical hydraulic-power actuator system.

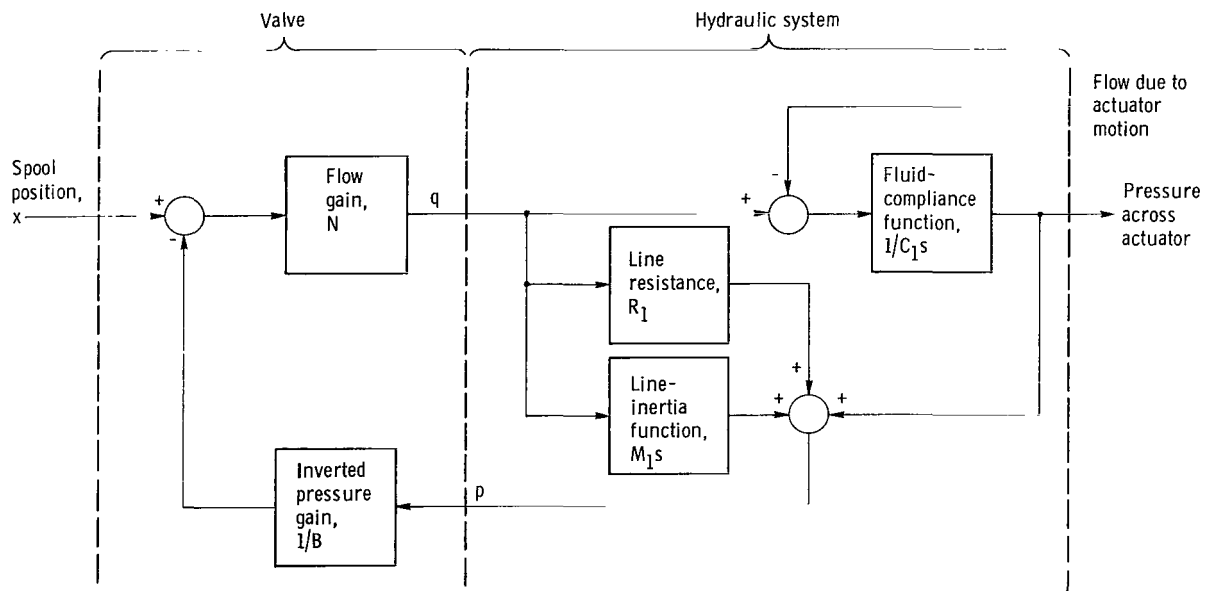


Figure 10. - Partial block diagram of figure 9 to show program that uses the flow model of the valve. Actuator leakage is zero.

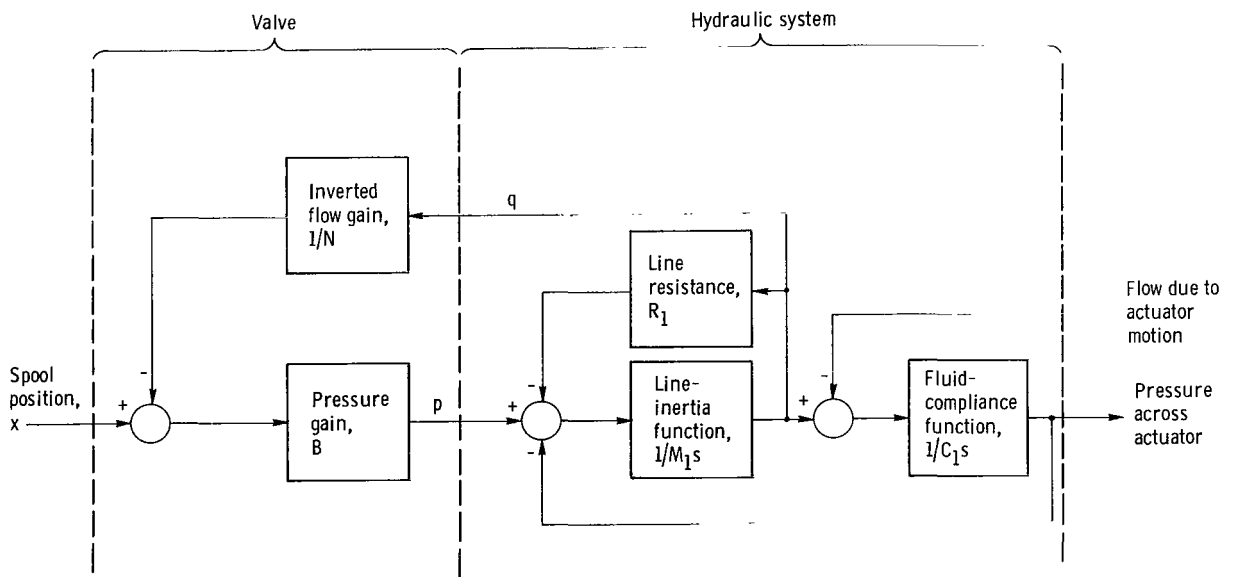


Figure 11. - Partial block diagram of figure 9 to show program that uses the pressure model of the valve. Actuator leakage is zero.

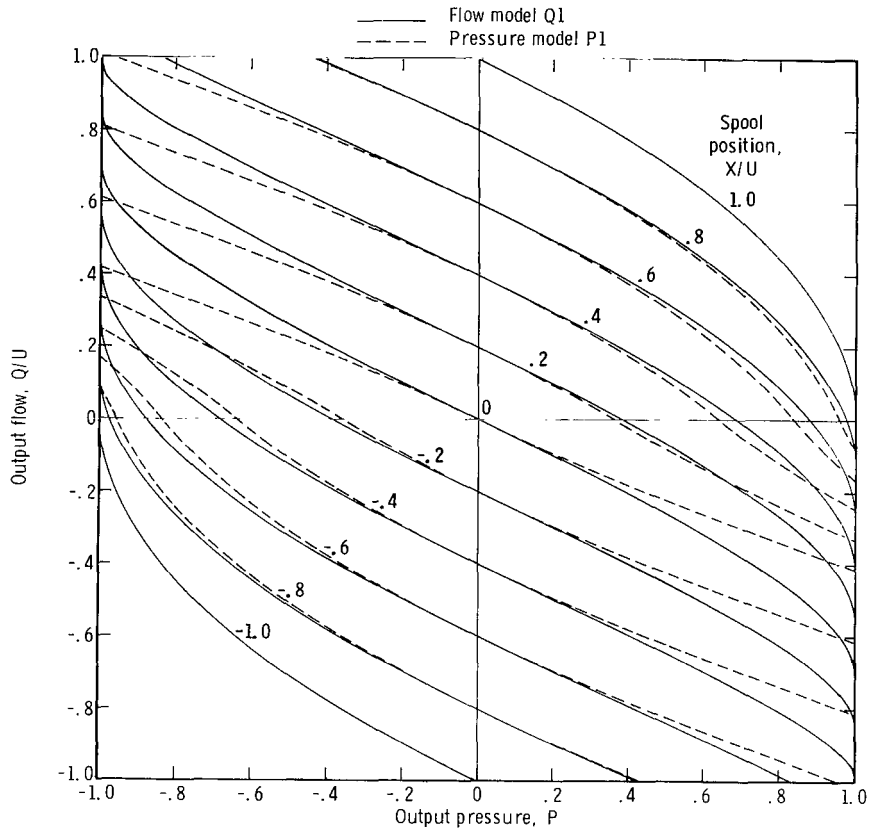


Figure 12. - Center regions of flow model Q1 and pressure model P1.

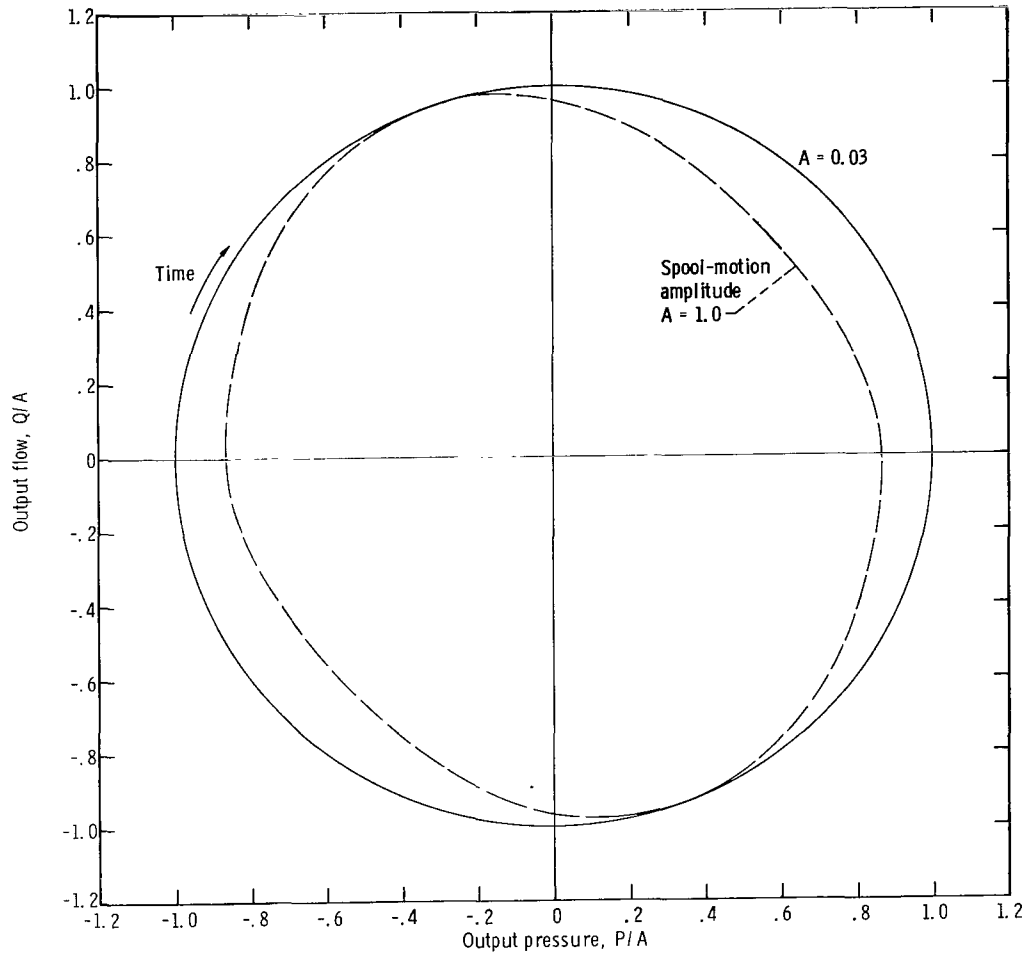


Figure 13. - Flow model Q1 load lines for capacitive load.

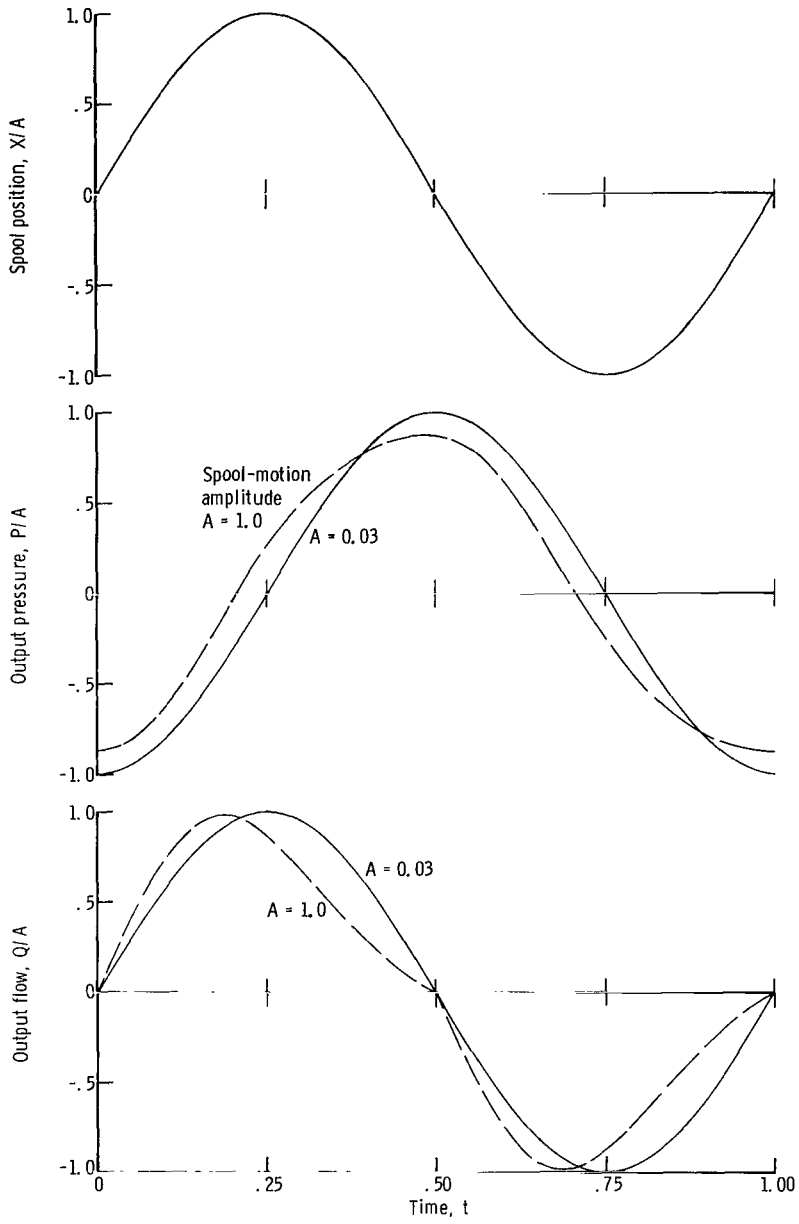


Figure 14. - Flow model Q1 signals for capacitive load.

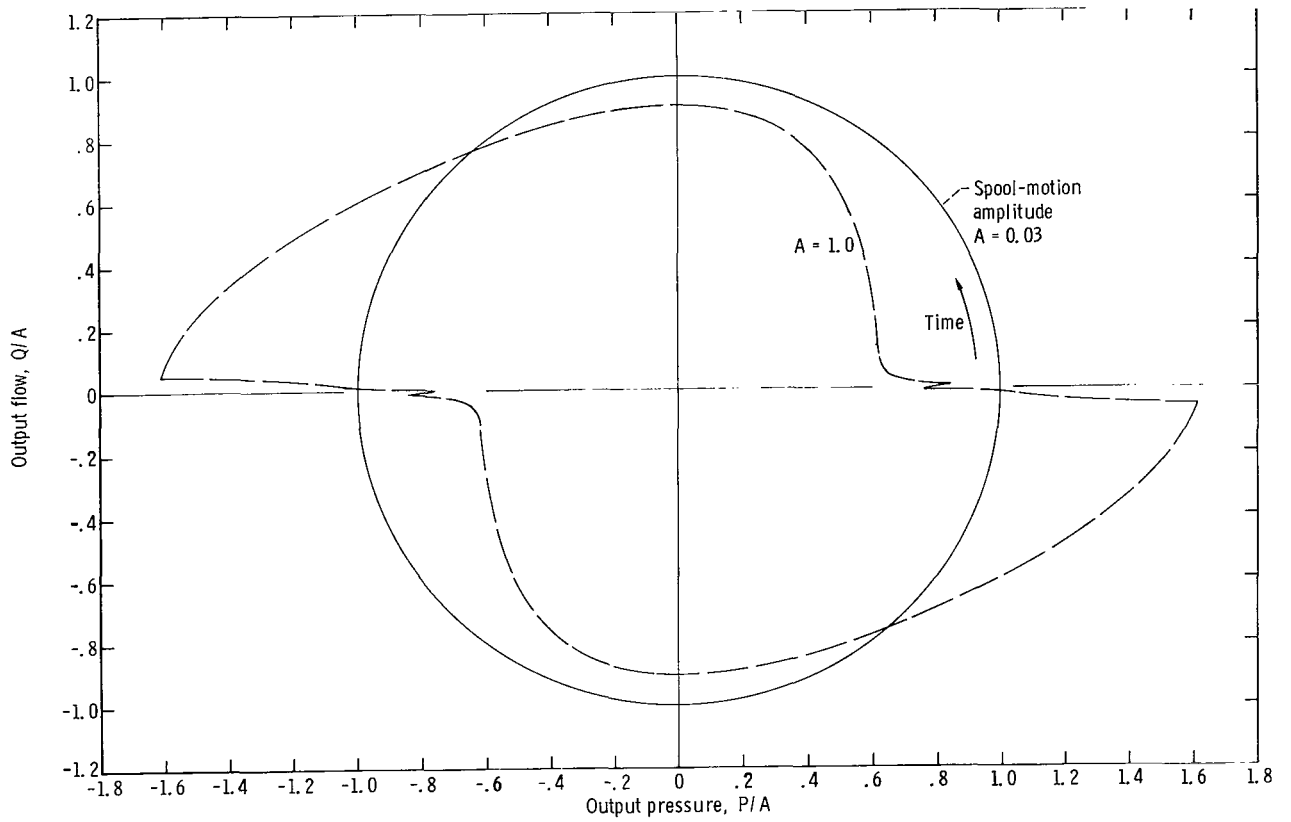


Figure 15. - Pressure model P1 load lines for inertial load.

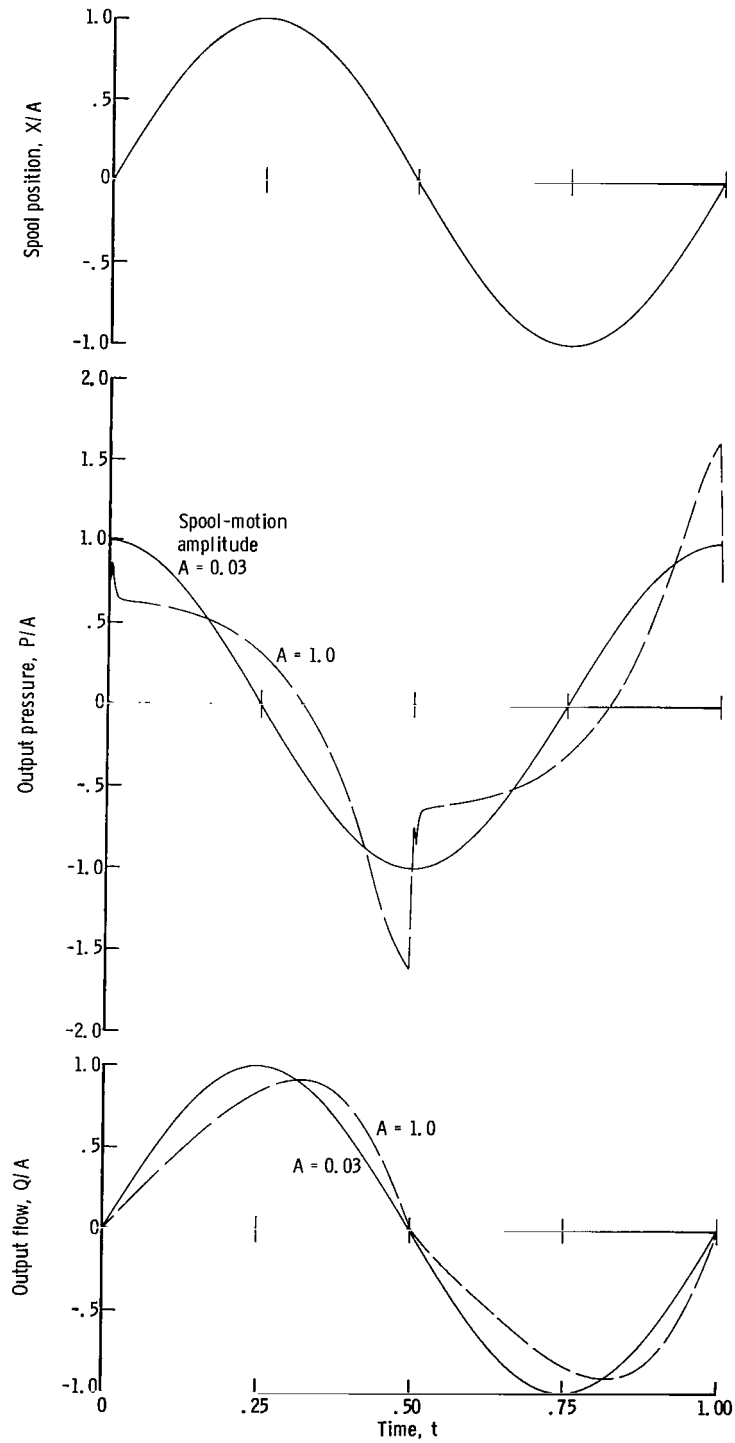
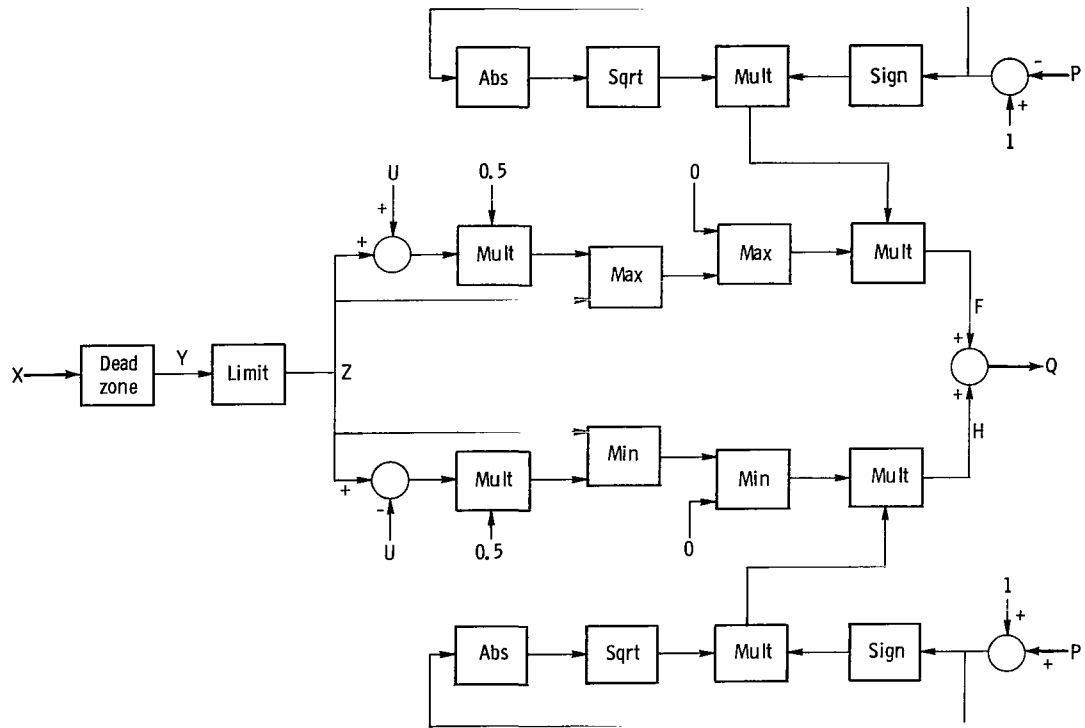


Figure 16. - Pressure model P1 signals for inertial load.



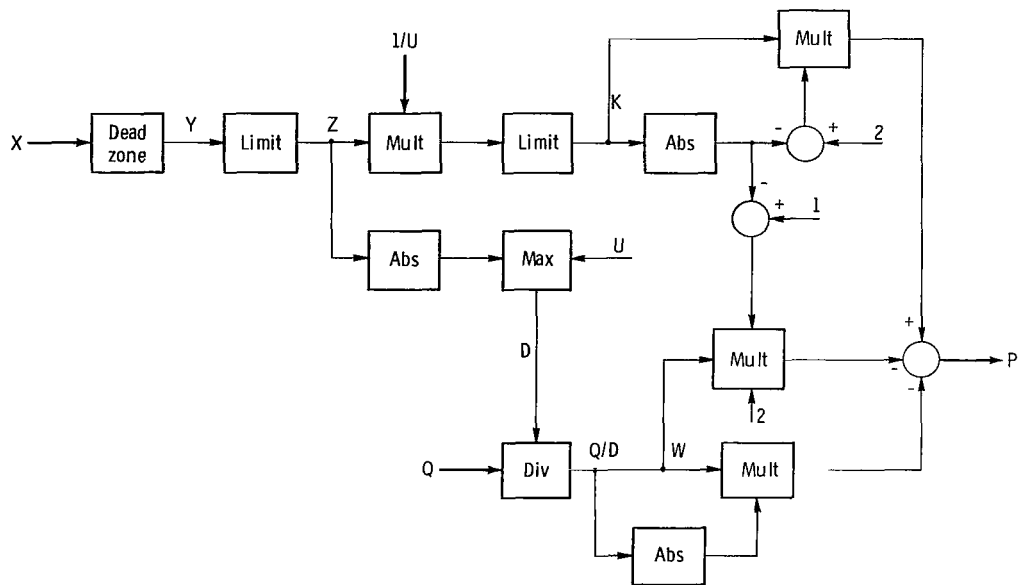
```

Y=AMAX1(0.,X-V)+AMIN1(0.,X+V)
Z=AMAX1(-1.,AMIN1(1.,Y))
F=SIGN(SQRT(ABS(1.-P)),(1.-P))*AMAX1(0.,Z,0.5*(Z+U))
H=SIGN(SQRT(ABS(1.+P)),(1.+P))*AMIN1(0.,Z,0.5*(Z-U))
Q=F+H

```

Figure 17. - Block diagram and FORTRAN statements of equation (8).





$Y = \text{AMAX1}(0., X - V) + \text{AMIN1}(0., X + V)$   
 $Z = \text{AMAX1}(-1., \text{AMIN1}(1., Y))$   
 $K = \text{AMAX1}(-1., \text{AMIN1}(1., Z/U))$   
 $D = \text{AMAX1}(U, \text{ABS}(Z))$   
 $W = Q/D$   
 $P = K * (2. - \text{ABS}(K)) - W * \text{ABS}(W) - 2. * W * (1. - \text{ABS}(K))$

Figure 18. - Block diagram and FORTRAN statements of equation (16).

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