A LINEARIZED THEORY METHOD OF CONSTRAINED OPTIMIZATION FOR SUPERSONIC CRUISE WING DESIGN

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SUMMARY

A linearized theory wing design and optimization procedure which allows physical realism and practical considerations to be imposed as constraints on the optimum (least drag due to lift) solution is discussed and examples of application are presented. In addition to the usual constraints on lift and pitching moment, constraints can also be imposed on wing surface ordinates and wing upper surface pressure levels and gradients. The design procedure also provides the capability of including directly in the optimization process the effects of other aircraft components such as a fuselage, canards, and nacelles.

INTRODUCTION

Because of their versatility, speed, and convenience, linearized theory computer methods have been widely employed for aerodynamic design and analysis. The close correspondence between the small disturbance assumptions of the theory and the small disturbance requirements for high aerodynamic efficiency makes the methods particularly attractive for supersonic cruise vehicles. The greatest advantage of the linearized methods lies in a unique capability for direct design and optimization made possible by the provision for linear addition and superposition of basic solutions; similar design and optimization capability for nonlinear methods does not presently exist.

Foremost among the linearized theory design methods are those which provide for the definition of wing lifting surface shapes for minimization of drag due to lift, typically the largest contribution to cruise vehicle inviscid drag. The first computerized wing design procedure (ref. 1) calculated the optimum (least drag for a given lift) combination of three simple analytic loadings (uniform, linear chordwise, and linear spanwise) and the resulting wing surface shape for an arbitrary planform wing. The original procedure was extended (ref. 2) to include constraints on pitching moment and root chord z-ordinate; with these additional two constraints, the original set of three component loadings was increased to eight to allow for effective optimization. In rather extensive applications of the method certain numerical deficiencies were revealed and techniques to overcome them were incorporated into the procedure (ref. 3). In order to integrate the resulting optimum wing designs into the complete airplane environment, other design procedures such as wing-nacelle reflexing (ref. 4) and fuselage camber shaping (refs. 5 and 6) were developed.

Wing design and configuration studies conducted in the past (refs. 7, 8, and 9) have indicated that the theoretical benefits predicted by linear theory design and optimization procedures are generally not fully realized. It became apparent that there was a need for considerations of additional restraints in the design process to avoid highly distorted surface shapes as well as local pressure levels and gradients which depart from physical realism and from the linear theory assumptions.

As a part of a computational system for the aerodynamic design and analysis of supersonic airplanes (ref. 10), a wing design procedure has been developed which employs a constrained optimization process for determining the wing surface shape to support a minimum drag pressure distribution. In addition to the usual constraints of desired lift and pitching moment, other optional constraints are available to impose physical realism and practical design considerations on the linearized theory solution. These new options include the application of equality constraints on surface ordinates at specified planform locations and inequality constraints on upper surface pressure levels and gradients. Special attention has been given to include directly in the optimization process the major effects of other aircraft components such as a fuselage, canards, and nacelles.

A general description of the new constrained optimization procedure is presented along with examples of calculated results to illustrate the design capability of the system. Particular attention is given to the control over the design afforded by the various constraint options.

SYMBOLS

ΔC_D — incremental drag coefficient due to lift

C_L — lift coefficient

$C_{L, design}$ — lift coefficient for which the warped wing surface is designed to produce minimum drag

$C_{m,o}$ — pitching-moment coefficient at zero lift

C_p — pressure coefficient

c_r — wing root chord length (see fig. 7)

$(L/D)_{max}$ — maximum lift-drag ratio

M — Mach number

z wing camber surface ordinate

z^* wing camber surface ordinate constraints (see fig. 7)

DISCUSSION

The Aerodynamic Design and Analysis System

The wing design and optimization capability described herein is provided by one element of a set of computer program modules contained in an integrated supersonic aerodynamic design and analysis system shown in figure 1. Details of the system are given in reference 10. The fundamental concept of the system is that complete airplane solutions can be assembled by superposition of individual contributions evaluated by means of linear theory and the supersonic area rule. An executive "driver" controls the execution and sequencing of several basic aerodynamic programs which perform analysis or design functions shown in the figure. The major role of the analysis programs is to produce loading information and total integrated aerodynamic forces for a given aircraft; however, several of these same programs are used to provide input to the wing design program.

The remainder of this paper will describe the wing design and optimization method and present examples of typical results.

Wing Design and Optimization Procedure

The wing design solution determines an optimum (least drag due to lift) pressure distribution and the corresponding mean camber surface shape. The procedure can be divided into two computational tasks.

The first task requires some method of computing the wing surface shape which would support a prescribed wing loading. Any reliable supersonic lifting surface method could be employed; a vortex lattice method is presently being used. As illustrated in figure 2, the wing planform is divided into a large number of grid elements and a matrix of aerodynamic influence functions relating wing loading and wing surface is calculated. For a given set of wing loadings, a set of corresponding wing camber surfaces is readily obtained. The lift, pitching moment, and drag are then calculated for each of the loadings. Using superposition principles, the individual loadings and camber surfaces are combined assuming that each loading has an unknown strength to be determined. This formulation gives the total wing loading, camber surface, lift, and pitching moment as linear functions of the unknown strengths and the total drag due to lift as a quadratic function of these unknowns. Refer to reference 3 for a detailed description of the formulation.

The second task is to determine values for the unknown strengths which will give minimum drag and satisfy certain constraints. The drag to be minimized is quadratic in terms of the unknown strengths, and the constraints are linear; thus, application of Lagrange's method of undetermined multipliers reduces the solution of the constrained optimization problem to the solution of a set of linear algebraic equations for determining the unknown loading strengths. The imposition of inequality constraints is achieved by automated successive applications of the optimization process as set forth in reference 10.

The present capabilities of the computer implemented wing design procedure are shown in figure 3. The set of loadings available for optimization are of two types: Basic lifting surface loadings and configuration related loadings. The basic loadings are analytic functions of planform position and have unknown overall strengths to be determined. The second type of loading is configuration related because it accounts for the effect of another configuration component interacting with the wing. Each of these configuration dependent loadings has two contributions: One which is generated by the configuration component and is of fixed distribution and strength, and one which is generated by an incremental wing camber surface (reflexing) to have the same distribution but unknown strength. The strength of the second contribution of the pair is determined in the optimization process to provide whatever degree of counteraction or augmentation of the component induced loadings that may be required for drag minimization. Presently, three configuration related loadings are available to account for the effects of fuselage upwash, fuselage volume, and nacelles. An example of wing-nacelle interference will be presented later to illustrate this capability.

The use of linear theory methods to design wings which have reasonable camber surface shapes, produce flows that are physically attainable, and yield certain desired aerodynamic characteristics requires that restrictions be placed on the theoretical solution. The restraints available are shown in figure 3 and are imposed mathematically as equality or inequality constraints. Desired aerodynamic characteristics of lift and pitching moment may be specified and are treated as equality constraints. For the wing upper surface, values for minimum pressure level and maximum pressure gradient may be specified. These pressure values, which need not be constant but may vary over the wing planform, are not to be exceeded by the solution and thus are treated as inequality constraints. In almost every practical wing design, regions exist where surface ordinates must be specified such as at wing-body junctions or hinge lines. To insure these criterions can be satisfied, wing surface ordinate values (which may be specified at up to five planform locations) are treated as equality constraints.

Application to Lifting Surface Design

The benefits which may be obtained from employment of linearized theory methods for wing camber surface design are perhaps best illustrated in data obtained more than a decade ago in an experimental program which first demonstrated a truly successful application of the design concepts. Data from this study are presented in figure 4. For this investigation, an arrow wing with a 70° leading-edge sweep angle was constructed with a camber surface corresponding to a simple two-component loading (uniform and linear chordwise) designed to minimize drag at a Mach number of 2.05 and a lift coefficient of 0.16. For comparison purposes, a flat wing of the same planform and an additional twisted and cambered wing with one half the camber surface severity (a design lift coefficient of 0.08) of the first were included in the test program.

In figure 4, the maximum lift-drag ratio achieved for each wing as well as the moment coefficient at zero lift is shown as a function of the design lift coefficient which serves as an index of the camber surface severity illustrated in the inset sketches. The test results indicated not only that improvements in lift-drag ratio approaching a value of 1 (or about 12.5 percent) might be attainable but also that a substantial additional reduction in trim drag might be afforded by the self-trimming moment provided.

The data obtained in this study also point out the need for the application of appropriate constraints in the design process. Note that the experimental $(L/D)_{max}$ does not continue to increase up to the optimum design lift coefficient of 0.16 indicated by the theory. This discrepancy is probably due to a camber surface severity which violates to too large a degree the assumptions of linearized theory. In this investigation, consideration was given to placement of restraints on the pressure distribution by the simple expedient of limiting lifting pressures to those permitted by the simple two-component loading. An indication that this rather arbitrary constraint is not the most appropriate and probably is too restrictive is given by the fact that greater benefits were achieved by a wing with a considerable flat-plate loading contribution, a loading with high peaks and large gradients.

As pointed out previously, the newer design methods reported in reference 10 now provide the designer with a wide range of choices for application of needed restraints and at the same time offer a large enough number of candidate loadings to maintain a viable optimization process. Some indication of the theoretical potential for lifting efficiency improvement offered by these expanded capabilities is given in figure 5. The drag-due-to-lift factor, $\Delta C_D/C_L^2$, a measure of the degree of optimization achieved, is shown for a series of wing designs in which the design loadings vary from a simple uniform distribution to a complex 10-term optimum. A design Mach number of 2.05 and a design lift coefficient of 0.16 were imposed for all results presented. The order in which the loadings are added is shown in the inset sketches. For any given design, the loading shown directly above the bar has been considered in addition to all the loadings to the left in arriving at an optimum combination.

Data are shown for an unrestrained solution and for a solution to which both pressure and camber surface ordinates restraints are imposed. The pressure level and gradient restraints were determined from an assessment of attainable values based on an examination of pressure data obtained in an experimental investigation (ref. 11). The ordinate restraints imposed are, in the belief of the authors, a reasonable compromise between indicated theoretical efficiencies and practical aircraft design realities. Levels of these restraints applicable to a given design problem are, of course, a matter subject to the judgment of the program user. For both unrestrained and restrained solutions, rather dramatic improvements over the uniform load case are shown for only a few additional loadings. Beyond that, the benefits increase more gradually. It will be noted that as the number of loadings is increased the imposition of restraints exerts a greater influence on the solution. The more loadings there are, the greater will be the opportunity for pressure peaks, steep gradients, and severe camber shapes to arise in the theoretical solution, and thus the greater will be the need for imposition of realistic restraints. These data indicate that even with the restraints applied, there is a potential for a 32-percent improvement over the uniform load case and about a 15-percent improvement over the two-loading case used in the experimental program previously discussed. The amount of this additional theoretical potential which can actually be achieved in practice remains to be determined by experimentation.

Theoretical levels of lifting efficiency which may be approached through application of the new design methods may be placed in better perspective by comparison with known standards as has been done in figure 6. Here, the theoretical goals for a 10-loading cambered wing with and without restraints is compared with theoretical and experimental data for a flat wing and for the two-loading wing. Flat wing theoretical values are given for full leading-edge suction and for no leading-edge suction. In this and other experiments, little evidence of leading-edge suction is found. Theoretical results for the two-loading wing indicate that the use of wing twist and camber can more than make up for the loss of suction. However, only about half of this gain is realized experimentally. The 10-loading theoretical data indicate a further potential gain. Only a relatively small penalty is predicted for imposition of what are believed to be realistic restraints. There is clearly a need for further investigation of this subject.

Application to Configuration Integration

The problem under discussion up to this point, that of optimizing the lifting efficiency as measured by the drag-due-to-lift factor or the untrimmed lift-drag ratio, does not take into account all the factors that must be considered in a real configuration design process. Another major consideration is that of providing the pitching-moment characteristics necessary to trim the aircraft for steady level flight without excessive drag penalties. Earlier in this paper, it was pointed out that for one example of the application of wing design methods, an increase in $C_{m,o}$ as well as an increase in untrimmed L/D resulted. This increase in moment coefficient at zero lift, which acts to reduce the control surface deflection required for trimmed flight at supersonic speeds and thus reduce trim drag, was in that case a byproduct of the drag optimization.

The new wing design methods allow moment considerations to be included as a fundamental part of the optimization process. An example of the use of the design program for the definition of wing surfaces providing for drag minimization subject to specified moment constraints is shown in figure 7. Because of the close interrelationship, camber surface ordinate restraints as well as moment restraints are treated in a single illustration.

The plot on the left of the figure shows the effect on drag-due-to-lift factor, $\Delta C_D/C_L{}^2$, of constraining the pitching moment at zero lift, $C_{m,o}$, to values between 0 and 0.06. No z-ordinate constraint is applied and, as depicted in the wing surface sketches, the root chord region exhibits large shape changes across the range of $C_{m,o}$ values. It should be noted that the more reasonable surface shapes and the near minimum values of drag due to lift both occur in the vicinity of $C_{m,o}$ values estimated to be desirable for acceptable trimming characteristics.

The plot on the right of figure 7 illustrates the effect on drag due to lift and on surface shape of varying the value of a single z-ordinate constraint while maintaining a $C_{m,o}$ of 0.04. The ordinate constraint is applied on the root chord at 67 percent of its length and surface ordinate, z^*, values range from -20 percent to 10 percent of the root chord length. An examination of the results indicates that a z^*/c_r constraint value of about -0.1 probably provides a reasonable compromise between a practical wing shape and a minimization of drag due to lift.

The primary point of the example presented here lies, however, not in the specific results obtained but in the demonstration of the improved design capability.

As previously discussed, the present wing design procedure has the capability of including the effects of other aircraft components directly in the optimization process. To illustrate this capability, two approaches for designing a wing-nacelle combination will be discussed. One approach utilizes the well-known method of wing-nacelle reflexing and the other approach employs the present wing design procedure. For illustrative purposes, both procedures are applied to the design of a delta wing in the presence of two nacelles and results for a design Mach number of 2.0 and a design lift coefficient of 0.10 are shown in figure 8.

The method of wing-nacelle reflexing is described in detail in reference 4 and will only be outlined here. The process of reflexing begins with the selection of an optimum wing alone design which has desired aerodynamic characteristics (open circle on solid line). The addition of the nacelles introduces a pressure field acting on the wing which requires a reduction in other loadings and corresponding changes in the camber surface to preserve the design lift. This results in a decrease in both $C_{m,o}$ and $\Delta C_D/C_L{}^2$ (open circle on dashed line). Wing reflexing is then applied to alter the basic wing surface in the region influenced by the nacelle pressure field to cancel or augment to varying degrees the nacelle effects. A wing reflexing which cancels all the nacelle induced loading, restores the original loading distribution, and reproduces the original value of $C_{m,o}$ is referred to as 100 percent reflexing (shaded circle on dashed line). As indicated in the figure, although drag penalties above the minimum point may be small for this 100-percent reflexing, the use of larger values of positive reflexing to achieve moments in the desirable trim range may bring about large penalties.

The present design procedure differs from the simple reflexing procedure because the entire wing is redesigned with the fixed nacelle loading and reflex (camber induced) loading as well as all the other loadings included in the optimization process. In the range of desirable pitching-moment characteristics, the present procedure provides substantial improvements in drag due to lift over the simple reflexing method.

Constraint Selection Considerations

As previously discussed, a new broad range of controls over the wing design process is now provided in a linearized theory method for drag-due-to-lift minimization. The constraints provided have been identified, and examples of their application given, but little has been said about the establishment of these constraints. It is this aspect of the problem which most severely taxes the knowledge and skill of the program user, calling on the art as well as the science of aerodynamics.

Some of the considerations involved in the selection of restraints applicable to a wing designed for a given Mach number, lift coefficient, and pitching moment may be discussed with the aid of figure 9. Near the wing leading edge in the vicinity of the fuselage juncture, it may be necessary to impose pressure level limitations to prevent a strong sidewash directed toward the fuselage which could create an inboard shock on being redirected by the fuselage surface. Pressure level constraints may also be required along the leading edge to avoid flow separation due to the large flow turning angles associated with high-pressure levels. At the trailing edge, some measure of C_p control may be needed to alleviate trailing-edge shock strength which could, under some circumstances, lead to flow separation well ahead of that location. In addition, pressure level and gradient constraints may be imposed on the entire wing surface or on suspected trouble spots as may be desired. A more complete discussion of the pressure restraint application in the design process and of criteria for establishment of specific restraint levels is given in reference 12. Some degree of control over fuselage floor angle and ground clearance may be afforded by strategic placement of ordinate constraints beginning at the wing-fuselage juncture. Ordinate constraints may also be useful in providing for straight hinge lines, although much of this can be accomplished by wing surface shearing (discussed in ref. 5).

Other Design Factors

Although the wing design procedure discussed in this paper is probably the most versatile and most comprehensive of its type, all aspects of wing design are by no means encompassed. Some of the more obvious aspects worthy of considerations are listed in figure 10. Linear theory methods provide no means of accounting for shock waves; thus, for example, to detect and control the strength of a forward shock (shown in the sketch) one must resort to empirical information, nonlinear solutions (see ref. 13), and, in many instances, wind-tunnel testing. Several researchers have proposed the utilization of vortex lift as an efficient means of producing low drag lift (see ref. 14); however, more knowledge and better computational methods must be acquired to design for vortex lift. The present procedure has no capability for taking advantage of various flow-control devices or accounting for propulsion-system integration effects which have been demonstrated to be beneficial if properly applied. These are but a few of the items which should be incorporated into future wing design procedures.

CONCLUDING REMARKS

A linearized theory wing design and optimization procedure which allows physical realism and practical considerations to be imposed as constraints on the optimum (least drag due to lift) solution has been presented and discussed. In addition to the usual constraints on lift and pitching moment, constraints can also be imposed in wing surface ordinates and wing upper surface pressure levels and gradients. The design procedure also provides the capability of including directly in the optimization process the effects of other aircraft components such as a fuselage, canards, and nacelles.

The capability and versatility of the design and optimization procedure have been illustrated by examples of its application to an arrow planform wing and a delta wing nacelle configuration. The results indicate a substantial theoretical potential for further gains in supersonic cruise vehicle aerodynamic performance. Experimental studies, however, are required to determine achievable levels and to provide empirical design criteria needed in more refined implementations of the method.

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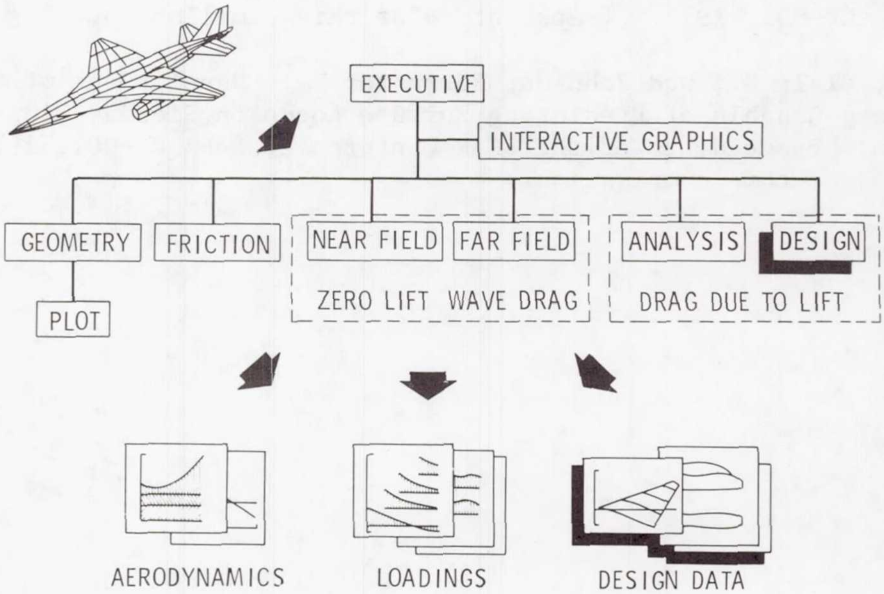

Figure 1.- Integrated supersonic design and analysis system.

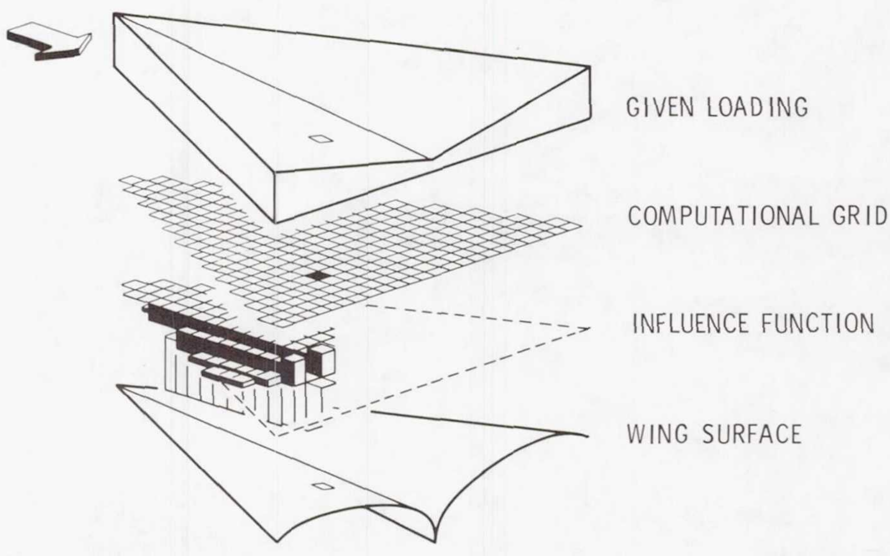

Figure 2.- Fundamental approach for wing surface design.

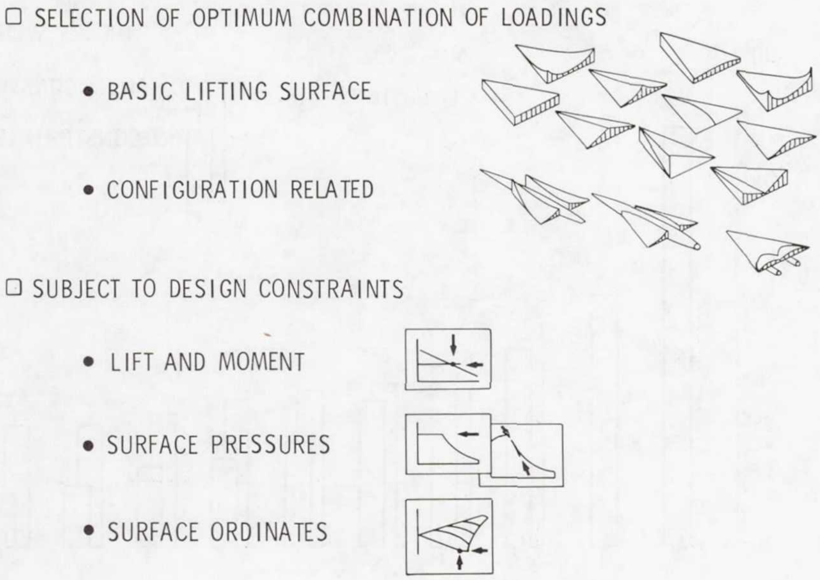

Figure 3.- System for optimization of configuration lifting efficiency.

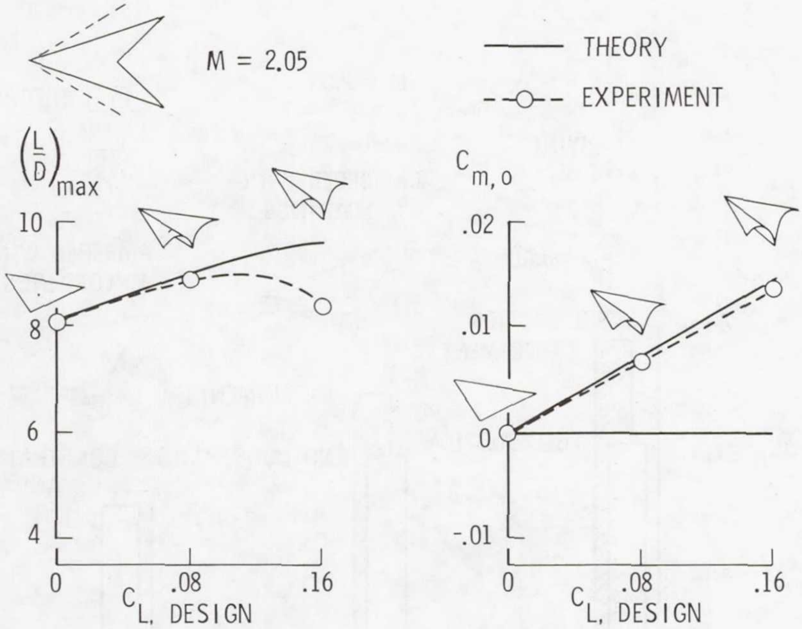

Figure 4.- Example of wing design for drag minimization.

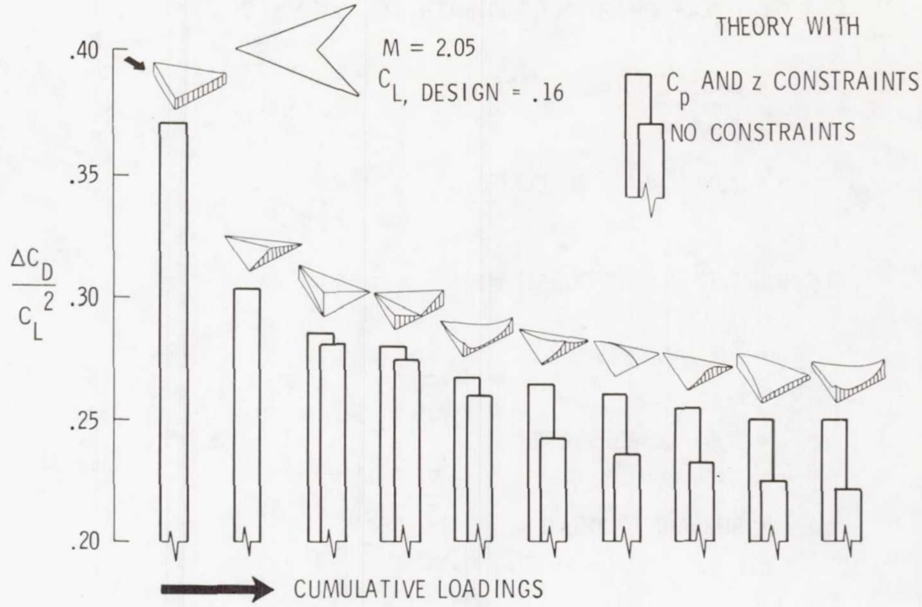

Figure 5.- Theoretical effect of optimized loadings on drag due to lift.

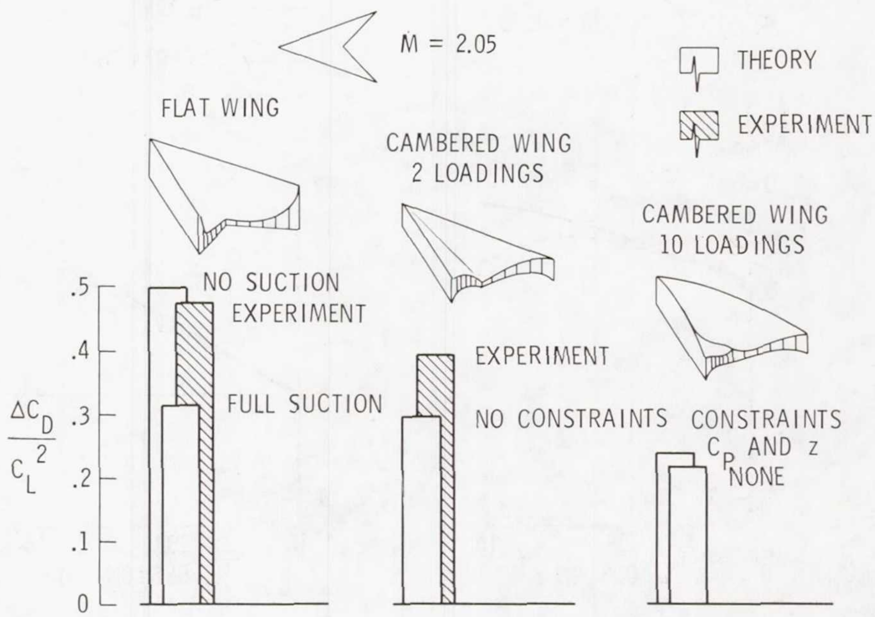

Figure 6.- Theoretical potential for wing design improvement.

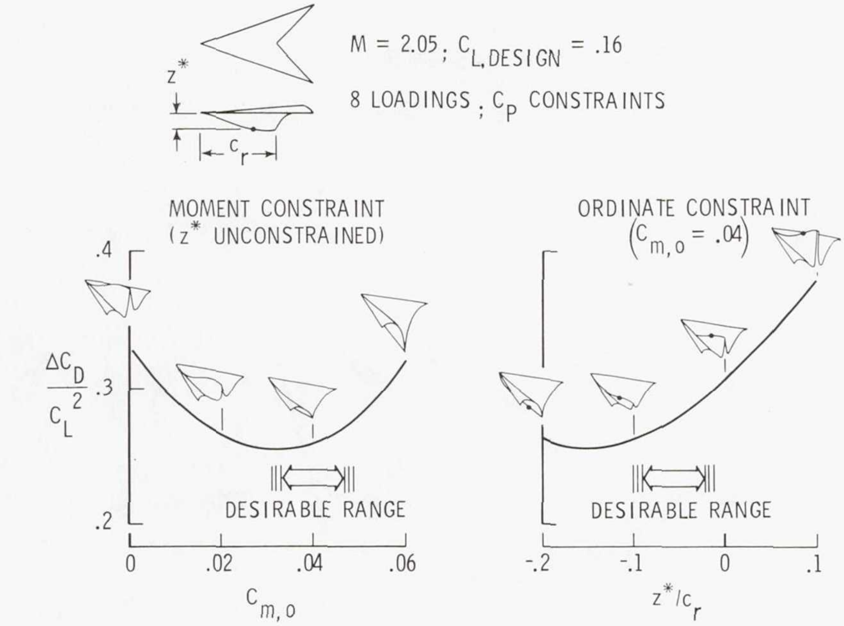

Figure 7.- Influence of moment and surface ordinate constraints.

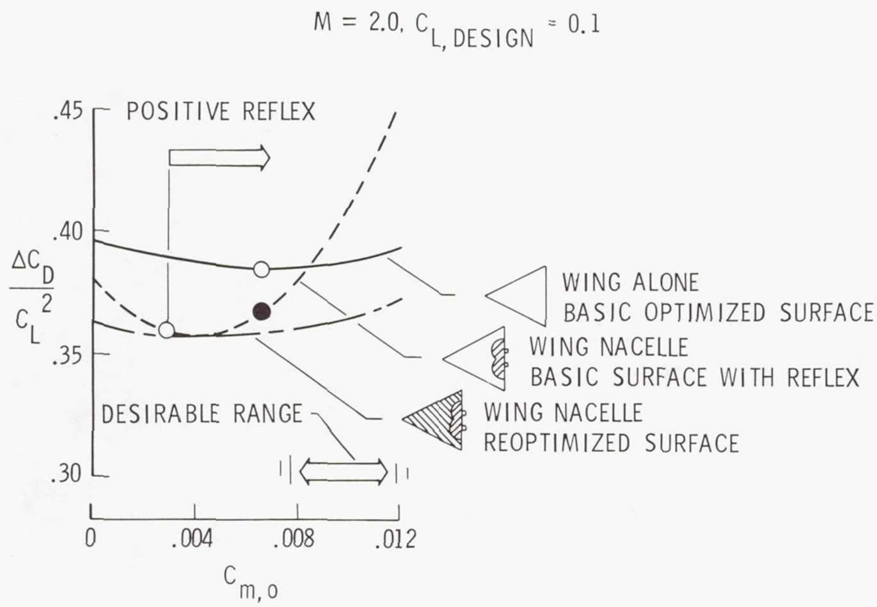

Figure 8.- Nacelle effects on wing design.

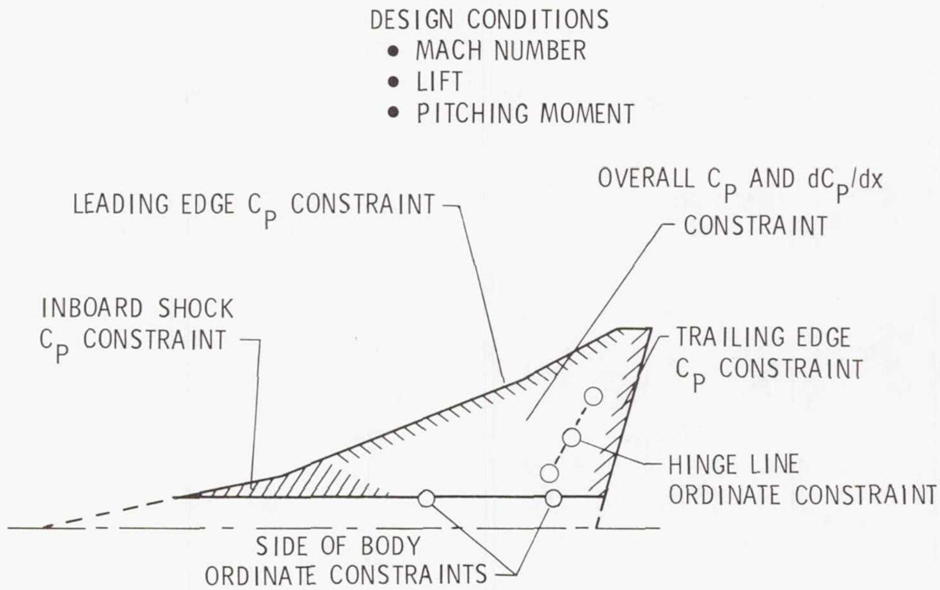

Figure 9.- Typical wing design constraint imposition.

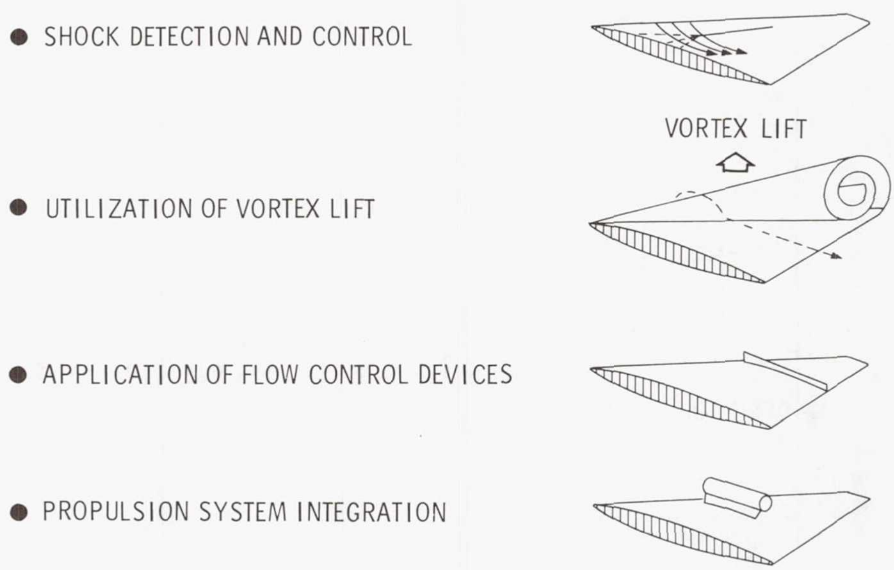

Figure 10.- Other design considerations.