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ANALYTICAL PROCEDURE FOR EVALUATING SPECKLE-EFFECT INSTRUMENTATION

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16. Abstract

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SUMMARY

A general analysis suitable for developing speckle-effect instruments and a simplified analysis suitable for evaluating laser-speckle instrumentation are presented. The simplified analysis is summarized as a list of equations. Existing speckle instrumentation and techniques can be understood by an application of the principles of geometrical optics and the orderly use of these equations. To demonstrate the use of the simplified analysis, several sample applications are described. Two forms of speckle interferometry are analyzed: one form is used to measure strain, and the other form to display the nodes of vibration of an object vibrating in one of its eigenmodes. A simple speckle viewer and the propagation of spatially coherent light from a rough surface are also evaluated. The properties often stated for speckle and the assumptions that warrant these statements are reviewed.

INTRODUCTION

In the future, the speckle effect probably will be applied in aeronautics research and development as a valuable supplement of holography. "Speckle effect" refers to the granular appearance of a rough surface illuminated with spatially coherent light. The large variation in intensity observed from speckle to speckle is caused by the random interference of the coherent light reflected from the rough surface. The apparent typical size of the speckles is approximately equal to that of a resolution element of the optical system used to review the rough surface. Speckle can be used in the measurement of surface strain (ref. 1), in the measurement of surface roughness (ref. 2), and in the measurement of the amplitude distribution and node locations on a vibrating object (ref. 3). In the measurement of surface strain, the speckle effect can be used to determine strain in the plane normal to the viewing direction. Holographic techniques can yield strain approximately along the viewing direction. Hence, the speckle techniques and the holographic techniques are definitely complementary for the measurement of strain. In determining the node locations on a vibrating object, using a speckle interferometer eliminates the need for the intermediate photographic step required in real-time holographic interferometry. Furthermore, as reviewed in this report, speckle interferometry can be used to yield time-average fringes for measuring the amplitude distribution of a vibrating object. These fringes appear within the integration time of an eye or a television vidicon and do not require the photographic step required for time-average holography. However, the visibility of the fringes formed in speckle interferometry is not nearly as good as the visibility of the fringes formed in time-average holography.

A guide is needed for analyzing the speckle phenomenon that can be used by those who wish to design, apply, or simply purchase speckle instrumentation for use with a variety of surfaces. Unfortunately, simple mathematical descriptions of the speckle phenomenon and its applications are not as easily generated as are descriptions of the holographic process. Indeed, a substantial part of speckle-effect research is devoted to determining the field and intensity statistics of spatially coherent light reflected from rough surfaces described with different roughness models (refs. 4 to 6).

This report evolves a guide for a general analysis of speckle-effect instrumentation based on standard coherent optics techniques (ref. 7). Then, the analysis is cast into a form that is not sensitive to surface properties and that yields some of the conclusions frequently quoted for speckle. These conclusions include a speckle size that is determined by the resolution of the optical system and a distribution of intensity that is a negative exponential. Where simplifying assumptions are required, they are carefully stated. Thus, the more general analysis can be referenced when the assumptions are not valid. Three examples of the application of the analysis are presented. First, the results for simple propagation of radiation between a rough surface and a detecting plane originally derived by Goodman (ref. 8) are derived. Next, the analysis is applied to a simple speckle viewer. Finally, a speckle interferometer of the type described by Stetson is analyzed (ref. 3). The intention is that this report can be used by those who wish merely to evaluate speckle instruments and techniques. However, more complex effects of instrument components or deviations of the speckle effect from the norm can also be analyzed.

GENERAL ANALYSIS OF A SPECKLE INSTRUMENT

The speckle instrument is treated as a linear system that transforms the light originating at a rough reflecting surface to a distribution of electric field in a detecting plane. The light is assumed to be spatially and temporally coherent. The rectangular components of the electric field are assumed to be uncoupled: They satisfy separate wave equations and separate transformation equations. If $\bar{E}_x(x, y)$ and $\bar{E}_y(x, y)$ are the phasor amplitudes of the x- and y-components immediately after reflection from the rough surface and $\bar{E}_{\tilde{x}}(\tilde{x}, \tilde{y})$ and $\bar{E}_{\tilde{y}}(\tilde{x}, \tilde{y})$ are the phasor amplitudes in the detecting plane, the linear transformation relating the fields is (ref. 7)

$$\left.\begin{aligned}\bar{E}_{\tilde{x}}(\tilde{x}, \tilde{y}) &= \iint_{-\infty}^{\infty} h_x(\tilde{x}, \tilde{y}; x, y)\bar{E}_x(x, y)dx\,dy \\ \bar{E}_{\tilde{y}}(\tilde{x}, \tilde{y}) &= \iint_{-\infty}^{\infty} h_y(\tilde{x}, \tilde{y}; x, y)\bar{E}_y(x, y)dx\,dy\end{aligned}\right\} \tag{1}$$

where h_x and h_y are the impulse response functions for the x- and y-polarizations of the field. For complex optical systems, h_x and h_y can be constructed from a sequence of linear transformations. These transformations are easily written for thin lenses with aberrations and finite dimensions and for simple propagation of light. Polarizations are specifically noted since the impulse response function may depend upon polarization. Frequently, only a single polarization is admitted (ref. 3). Then, either $h_x = 0$ or $h_y = 0$.

The field reflected from the rough surface is treated as a stochastic variable. Polarization, magnitude, and phase at a point x, y are random quantities characterized by a probability density function $f(\bar{E}_x, \bar{E}_y)$. Only the statistics of the field are predictable. As stated in the INTRODUCTION, the specification of the probability density function f is the subject of continuing investigation.

The goal of this report is to develop an analysis that will apply to a variety of density functions. Typically, $\langle \bar{E}_{\tilde{x}} \rangle = 0$ and $\langle \bar{E}_{\tilde{y}} \rangle = 0$, where $\langle \rangle$ denotes an ensemble average. In defining $\langle \rangle$, one imagines a set of an infinite number of macroscopically identical surfaces where the local microstructure varies from one surface to another. Then, the symbol $\langle \rangle$ refers to an arithmetic mean over all members of the set. The first-order statistics of the field are of no value in understanding speckle effect. However, the second-order statistics in the field and in the intensity are useful (ref. 8). In particular, the intensity correlation or the magnitude of the field correlation tends to be high within a speckle. Using the notation of reference 8, the correlations in the detecting plane are

$$
\left.
\begin{aligned}
\Gamma_{\widetilde{x}}(\widetilde{x}_2,\widetilde{y}_2;\widetilde{x}_1,\widetilde{y}_1) &= \langle \overline{E}^*_{\widetilde{x}}(\widetilde{x}_2,\widetilde{y}_2)\overline{E}_{\widetilde{x}}(\widetilde{x}_1,\widetilde{y}_1)\rangle = \iiiint_{-\infty}^{\infty} h^*_x(\widetilde{x}_2,\widetilde{y}_2;x_1,y_1) \\
&\quad \times h_x(\widetilde{x}_1,\widetilde{y}_1;x_2,y_2)\langle \overline{E}^*_x(x_1,y_1)\overline{E}_x(x_2,y_2)\rangle \, dx_1 \, dy_1 \, dx_2 \, dy_2 \\
\Gamma_{\widetilde{y}}(\widetilde{x}_2,\widetilde{y}_2;\widetilde{x}_1,\widetilde{y}_1) &= \langle \overline{E}^*_{\widetilde{y}}(\widetilde{x}_2,\widetilde{y}_2)\overline{E}_{\widetilde{y}}(\widetilde{x}_1,\widetilde{y}_1)\rangle = \iiiint_{-\infty}^{\infty} h^*_y(\widetilde{x}_2,\widetilde{y}_2;x_1,y_1) \\
&\quad \times h_y(\widetilde{x}_1,\widetilde{y}_1;x_2,y_2)\langle \overline{E}^*_y(x_1,y_1)\overline{E}_y(x_2,y_2)\rangle \, dx_1 \, dy_1 \, dx_2 \, dy_2
\end{aligned}
\right\} \qquad (2)
$$

Goodman in reference 8 calls these correlations the coherence functions. When nearly monochromatic radiation is used, these functions are a useful extension of the concept of coherence from the temporal domain to the spatial domain (ref. 9). Coherence theory has traditionally considered time averages or time as a parameter in evaluating an ensemble average (refs. 9 and 10). However, the nature of speckle instrumentation and the availability of highly monochromatic laser radiation make these spatial coherence functions more useful. Ensemble averages are evaluated with position as a parameter.

The actual relation between $\Gamma_{\widetilde{x}}$ and $\Gamma_{\widetilde{y}}$ depends on the details of polarization. The ensemble average $\langle \overline{E}^*_x(x_1,y_1)\overline{E}_x(x_2,y_2)\rangle$ depends on a density function $f_x\left(\overline{E}_{x_1},\overline{E}_{x_2}\right)$ and, similarly, the ensemble average $\langle \overline{E}^*_y(x_1,y_1)\overline{E}_y(x_2,y_2)\rangle$ depends on a density function $f_y\left(\overline{E}_{y_1},\overline{E}_{y_2}\right)$, where 1 and 2 in the subscripts refer to the positions (x_1,y_1) and (x_2,y_2). For completely random polarization, $\Gamma_{\widetilde{x}}$ does not depend on the y-polarization and $\Gamma_{\widetilde{y}}$ does not depend on the x-polarization. Two completely independent speckle patterns ensue and are superimposed.

For a completely deterministic polarization,

$$
f_x\left(\overline{E}_{x_1},\overline{E}_{x_2}\right) = f'_x\left(\overline{E}_{y_1},\overline{E}_{y_2}\right)
$$

$$
f_y\left(\overline{E}_{x_1},\overline{E}_{x_2}\right) = f'_y\left(\overline{E}_{y_1},\overline{E}_{y_2}\right)
$$

The function $f'_x\left(\overline{E}_{y_1},\overline{E}_{y_2}\right)$ is used to express the probability density function for an x-polarized field $\overline{E}_{x_1}$ at position (x_1,y_1) and a x-polarized field $\overline{E}_{x_2}$ at position (x_2,y_2) in terms of a function of the corresponding y-polarizations. Similarly, the function

$f_y\left(\bar{E}_{x_1}, \bar{E}_{x_2}\right)$ is used to express the probability density function for a y-polarized field $\bar{E}_{y_1}$ at position (x_1, y_1) and for a y-polarized field $\bar{E}_{y_2}$ at position (x_2, y_2) in terms of the corresponding x-polarizations.

The quantity $\langle \bar{E}_x^*(x_1, y_1)\bar{E}_x(x_2, y_2)\rangle$ is proportional to $\langle \bar{E}_y^*(x_1, y_1)E_y(x_2, y_2)\rangle$. If h_x is proportional to h_y (simple attenuation and phase shift of one polarization relative to the other), $\Gamma_{\tilde{x}}$ is proportional to $\Gamma_{\tilde{y}}$. For circular polarization and $h_x = h_y$, $\Gamma_{\tilde{x}} = \Gamma_{\tilde{y}}$. In any case, for a completely deterministic polarization, a single speckle pattern is created and either $\Gamma_{\tilde{x}}$ or $\Gamma_{\tilde{y}}$ suffices for evaluating speckle size.

The analysis that immediately follows assumes that the polarization is deterministic or that the instrument has a polarizer that sets $h_y = 0$. A polarizer is easily added to a speckle viewer and is required for the speckle interferometer discussed under EXAMPLES (ref. 3). The case of random polarization is discussed in the section Intensity Probability Density and Overlapping Speckle Patterns.

Speckle Instrument: Single Polarization or Light Deterministically Polarized

The subscript x will be dropped in referring to $\bar{E}_x$ and h_x. The subscript $\tilde{x}$ will be dropped in referring to $\Gamma_{\tilde{x}}$. The field correlation or spatial coherence function in the detecting plane is then

$$\Gamma(\tilde{x}_2, \tilde{y}_2; \tilde{x}_1, \tilde{y}_1) = \iiiint_{-\infty}^{\infty} h^*(\tilde{x}_2, \tilde{y}_2; x_1, y_1)h(\tilde{x}_1, \tilde{y}_1; x_2, y_2) \times \langle \bar{E}^*(x_1, y_1)\bar{E}(x_2, y_2)\rangle \, dx_1 \, dy_1 \, dx_2 \, dy_2 \tag{3}$$

The entire space between the rough surface and the detecting plane is called the optical system. This system is divided into three parts. The key part is a physical aperture stop that defines both the exit and entrance pupils. This stop may be, for example, an iris of adjustable size, a lens, a rough surface of finite extent, or a detector of finite extent at the detecting plane. The stop is defined by the equations

$$\left.\begin{aligned} P(x_a, y_a) &= 1 \quad \text{when } (x_a, y_a) \text{ is a point in the aperture} \\ P(x_a, y_a) &= 0 \quad \text{otherwise} \end{aligned}\right\} \tag{4}$$

The optical system between the physical stop and the rough surface is described by the impulse response function

$$h_f(x_a, y_a; x, y)$$

where f refers to the front section. The optical system between the physical stop and the detecting surface is described by the impulse response function

$$h_b(\tilde{x}, \tilde{y}; x_a, y_a)$$

where b refers to the back section. The impulse response function for the entire system can be shown to be

$$h(\tilde{x}, \tilde{y}; x, y) = \iint_{-\infty}^{\infty} h_b(\tilde{x}, \tilde{y}; x_a, y_a) P(x_a, y_a) h_f(x_a, y_a; x, y) dx_a\, dy_a \tag{5}$$

The quantity $h_b \cdot P(x_a, y_a)$ will be primarily responsible for typical speckle size. Before writing Γ again, make the following changes of variable.

$$\begin{array}{ll}
x_1 = x & y_1 = y \\
x_2 = x + \Delta x & y_2 = y + \Delta y \\
x_a = x_a & y_a = y_a \\
x_{a_2} = x_a + \Delta x_a & y_{a_2} = y_a + \Delta y_a \\
\tilde{x}_1 = \tilde{x} & \tilde{y}_1 = \tilde{y} \\
\tilde{x}_2 = \tilde{x} + \Delta\tilde{x} & \tilde{y}_2 = \tilde{y} + \Delta\tilde{y}
\end{array}$$

These variable changes and equation (5) are substituted in equation (3) to give

$$\Gamma(\tilde{x} + \Delta\tilde{x}, \tilde{y} + \Delta\tilde{y}; \tilde{x}, \tilde{y}) = \langle \bar{E}^*_{\tilde{x}}(\tilde{x} + \Delta\tilde{x}, \tilde{y} + \Delta\tilde{y}) \bar{E}_{\tilde{x}}(\tilde{x}, \tilde{y}) \rangle$$

$$\begin{aligned}
= \iiiint\!\!\!\iiiint_{-\infty}^{\infty} & \Big\{ h_b^*(\tilde{x} + \Delta\tilde{x}, \tilde{y} + \Delta\tilde{y}; x_a, y_a) \\
& \times h_b(\tilde{x}, \tilde{y}; x_a + \Delta x_a, y_a + \Delta y_a) P(x_a, y_a) P(x_a + \Delta x_a, y_a + \Delta y_a) \\
& \times h_f^*(x_a, y_a; x, y) h_f(x_a + \Delta x_a, y_a + \Delta y_a; x + \Delta x, y + \Delta y) \\
& \times \langle \bar{E}^*(x, y) \bar{E}(x + \Delta x, y + \Delta y) \rangle \Big\} dx\, dx_a\, d\,\Delta x\, d\,\Delta x_a\, dy\, dy_a\, d\,\Delta y\, d\,\Delta y_a
\end{aligned} \tag{6}$$

The pairs of variables x and y, $\tilde{x}$ and $\tilde{y}$, and $\approx{x}$ and $\approx{y}$ and their subscripted and incremented forms appear symmetrically in many of the equations that follow. To shorten the equations, only a single variable is written to stand for both variables in the pair.

Define the correlation in the aperture stop as

$$\langle \bar{E}_a(x_a + \Delta x_a)\bar{E}_a^*(x_a)\rangle = P(x_a)P(x_a + \Delta x_a)$$

$$\times \iint_{-\infty}^{\infty} \left\{h_f^*(x_a; x)h_f(x_a + \Delta x_a; x + \Delta x)\langle \bar{E}^*(x)\bar{E}(x + \Delta x)\rangle\right\} dx\, d\,\Delta x \tag{7}$$

where x only is written to stand for x, y. When equation (7) is substituted into equation (6),

$$\Gamma(\tilde{x} + \Delta\tilde{x}; \tilde{x}) = \iint_{-\infty}^{\infty} h_b^*(\tilde{x} + \Delta\tilde{x}; x_a)h_b(\tilde{x}; x_a + \Delta x_a)\langle \bar{E}_a(x_a + \Delta x_a)\bar{E}_a^*(x_a)\rangle\, dx_a\, d\,\Delta x_a \tag{8}$$

Again $dx_a\, d\,\Delta x_a$ is taken to mean $dx_a\, dy_a\, d\,\Delta x_a\, d\,\Delta y_a$. With a finite aperture, the limits of Δx_a depend on x_a.

The expression for the spatial coherence function is seen to be quite general. The coherence function in the detecting plane is a transformation of the coherence function in the aperture. The transformation is four-dimensional depending on x_a, y_a, Δx_a, and Δy_a. Two assumptions are required if a simple interpretation of the speckle characteristic size is to be made. The first assumption is that

$$\frac{\langle \bar{E}_a(x_a + \Delta x_a)\bar{E}_a^*(x_a)\rangle}{\langle \bar{E}_a(x_a)\bar{E}_a^*(x_a)\rangle}$$

is at least locally wide-sense stationary. That is, the normalized aperture correlation depends only on Δx_a but not on x_a at least over the dimensions of the aperture. The second assumption is that $h_b^*(\tilde{x} + \Delta\tilde{x}; x_a) \times h_b^*(\tilde{x}; x_a + \Delta x_a)$ can be broken approximately into three factors:

$$h_b^*(\tilde{x} + \Delta\tilde{x}; x_a)h_b(\tilde{x}; x_a + \Delta x_a) \approx D(\tilde{x}; \Delta\tilde{x})T_c(\Delta x_a; \tilde{x})T_a(x_a; \Delta\tilde{x}) \tag{9a}$$

The reason for this factorization is made apparent later. This assumption may occasionally be a poor assumption; however, it simplifies the results and facilitates the understanding of speckle phenomena. The coherence function becomes

$$\Gamma(\tilde{x} + \Delta\tilde{x}; \tilde{x}) = \int_{-\infty}^{\infty} \frac{\langle \bar{E}_a(x_a + \Delta x_a)\bar{E}_a^*(x_a)\rangle}{\langle \bar{E}_a^*(x_a)\bar{E}_a(x_a)\rangle} T_c(\Delta x_a; \tilde{x}) d\, \Delta x_a$$

$$\times \int_{-\infty}^{\infty} \langle \bar{E}_a^*(x_a)\bar{E}_a(x_a)\rangle\, T_a(x_a; \Delta\tilde{x}) dx_a\, D(\tilde{x}; \Delta\tilde{x}) \tag{9b}$$

In equation (9b), Γ is expressed as a product of a two-dimensional transformation of the normalized correlation or the spatial coherence function of the aperture field, a two-dimensional transformation of the aperture intensity, and a factor that depends on $\tilde{x}$ and $\Delta\tilde{x}$. This factor $D(\tilde{x}, \Delta\tilde{x})$ usually consists of phase factors if the Fresnel diffraction approximation is made.

If the back optical system consists only of Fraunhofer diffraction from the aperture, the decomposition into three factors is exact. The coherence function Γ is then the product of the spatial power spectral density of the normalized correlation of the aperture field and the Fourier transform of the intensity in the aperture.

A normalized coherence function is more useful for evaluating speckle properties. It is assumed that

$$\int_{-\infty}^{\infty} \frac{\langle \bar{E}_a(x_a + \Delta x_a)\bar{E}_a^*(x_a)\rangle}{\langle \bar{E}_a^*(x_a)\bar{E}_a(x_a)\rangle} T_c(\Delta x_a; \tilde{x}) d\, \Delta x_a$$

is approximately constant over the detecting surface. This assumption is generally equivalent to assuming that

$$\frac{\langle \bar{E}_a(x_a + \Delta x_a)\bar{E}_a^*(x_a)\rangle}{\langle \bar{E}_a^*(x_a)\bar{E}_a(x_a)\rangle}$$

is a narrow function of Δx_a. Normalizing with respect to $\Delta\tilde{x} = 0$,

$$\gamma(\tilde{x} + \Delta\tilde{x};\tilde{x}) = \frac{\Gamma(\tilde{x} + \Delta\tilde{x};\tilde{x})}{\Gamma(\tilde{x};\tilde{x})}$$

$$= \frac{D(\tilde{x};\Delta\tilde{x})}{D(\tilde{x};0)} \frac{\int_{-\infty}^{\infty} \langle \bar{E}_a^*(x_a)\bar{E}_a(x_a)\rangle\, T_a(x_a;\Delta\tilde{x})dx_a}{\int_{-\infty}^{\infty} \langle \bar{E}_a^*(x_a)\bar{E}_a(x_a)\rangle\, T_a(x_a;0)dx_a} \qquad (10)$$

Equation (10) is a primary result of this analysis. The equation expresses the degree of spatial coherence of the field in the detecting plane. In the case of constant intensity in the aperture, the degree of spatial coherence depends only on a transformation $T_a(x_a;\Delta\tilde{x})$ of the pupil function $P(x_a, y_a)$ defining the physical aperture stop. A number of assumptions have been required to reach this point in the report. The correctness of the assumptions must be determined on a case-by-case basis.

A number of properties are to be noted.

$$\frac{\gamma(\tilde{x} + \Delta\tilde{x};\tilde{x})D(\tilde{x};0)}{D(\tilde{x};\Delta\tilde{x})}$$

or $|\gamma|$ depends only on $\Delta\tilde{x}$ and is, therefore, stationary in the wide sense (ref. 11). If $D(\tilde{x};\Delta\tilde{x})$ is a phase factor involving $\tilde{x}$ and $\Delta\tilde{x}$, the following properties will characterize $|\gamma|$:

$$\left.\begin{aligned} &|\gamma| \leq 1 \\ &|\gamma(\tilde{x} + \Delta\tilde{x};\tilde{x})| = |\gamma(\tilde{x} - \Delta\tilde{x};\tilde{x})| = |\gamma(\Delta\tilde{x})| \\ &|\gamma(0)| \geq |\gamma(\Delta\tilde{x})| \\ &|\gamma(0)| = 1 \end{aligned}\right\} \qquad (11)$$

Intensity Correlations and Speckle Area

The quantity actually detected in the detecting plane is the intensity rather than the field. Speckles actually detected are regions of high correlation between intensity fluctuations rather than regions of high correlation between fields. That is, if $I(\tilde{x}) - \langle I(\tilde{x})\rangle$ is an intensity fluctuation, that quantity should be consistently positive (bright speckle) or consistently negative (dark speckle) within a speckle. The field has been used since the optical system is represented as a linear transformation on the field rather than on the intensity. Fortunately, the typical extent of the region in which fields are well correlated is identical with the typical extent of the region in which intensity fluctuations are well correlated (refs. 8 and 10). The correlation between intensity fluctuations is

$$\left\langle \left[I(\tilde{x} + \Delta\tilde{x}) - \langle I(\tilde{x} + \Delta\tilde{x})\rangle\right]\left[I(\tilde{x}) - \langle I(\tilde{x})\rangle\right]\right\rangle = \langle I(\tilde{x} + \Delta\tilde{x}) I(\tilde{x})\rangle - \langle I(\tilde{x})\rangle \langle I(\tilde{x} + \Delta\tilde{x})\rangle$$

A normalized intensity fluctuation correlation is

$$|\gamma|^2 = \frac{\langle I(\tilde{x} + \Delta\tilde{x}) I(\tilde{x})\rangle - \langle I(\tilde{x} + \Delta\tilde{x})\rangle \langle I(\tilde{x})\rangle}{\langle I^2(\tilde{x})\rangle - \langle I(\tilde{x})\rangle^2} \tag{12a}$$

provided that the statistics of the field are Gaussian (refs. 8 to 10). The statistics are discussed later.

A typical speckle dimension at $(\tilde{x}, \tilde{y})$ should be approximately the interval $\Delta\tilde{x}$ or $\Delta\tilde{y}$ over which $|\gamma|$ is significant. However, $|\gamma|$ generally will be nonzero over the entire detecting surface. For example, a square rough surface of side l viewed at a distance d in the Fraunhofer diffraction region results in

$$|\gamma| = \left| \operatorname{sinc} \frac{l\,\Delta\tilde{x}}{\lambda d} \operatorname{sinc} \frac{l\,\Delta\tilde{y}}{\lambda d} \right|$$

where $\operatorname{sinc} x = \sin \pi x/\pi x$. This result is derived later.

One estimate of speckle size is that it is approximately given by the first zero of the sinc function: $\Delta x_s \approx \lambda d/l$. This estimate is confirmed if a definition of speckle area is used that is derived from a physical argument given in reference 8. The physical argument assumes Gaussian statistics. Speckle area is defined as

$$A_s = \frac{1}{A_d} \iint_{A_d} d\tilde{x}\, d\tilde{y} \iint_{A_d} d\,\Delta\tilde{x}\, d\,\Delta\tilde{y} \left|\gamma(\tilde{x} + \Delta\tilde{x}, \tilde{y} + \Delta\tilde{y}; \tilde{x}, \tilde{y})\right|^2 \tag{12b}$$

where A_s refers to the area of a typical speckle and A_d is the area of the detector at the detecting surface. For purposes of instrument evaluation and this report, A_d is assumed to be large when compared with A_s. Equation (10) is used for the evaluation of γ.

Then,

$$A_s = 4 \int\int_0^\infty d\,\Delta\tilde{x}\, d\,\Delta\tilde{y} |\gamma(\Delta\tilde{x}, \Delta\tilde{y})|^2 \tag{13}$$

where $|\gamma(\Delta\tilde{x}, \Delta\tilde{y})|^2$ is simply used as a weighting function. If $|\gamma| = 1$, the whole detector area is covered with a single speckle (no granular appearance). If $|\gamma| \approx 0$ except very near $\Delta\tilde{x} = 0$ and $\Delta\tilde{y} = 0$ (low spatial coherence), A_s will be very small and granulation will not be resolved. Intermediate cases will give an A_s smaller than the detector area but large enough to be resolved.

Equations (10) and (13) are the most important for instrument evaluation.

Intensity Probability Density and Overlapping Speckle Patterns

The intensity at the detecting plane is characterized by a probability density function. Although there is a high-intensity correlation within a speckle, the intensity from speckle to speckle will vary at random. Only a large variance in intensity yields the visual high-contrast speckle pattern. The exact probability density function will depend on detailed surface statistics. Application of the Central Limit Theorem is assumed in order to avoid having to know the detailed surface statistics.

Referring to equation (1), an impulse response function of a typical optical system superimposes the fields from many correlation elements on the rough surface. Again, a correlation element is a region with respect to $\Delta x, \Delta y$ over which $\langle \overline{E}_x(x + \Delta x, y + \Delta y)\, \overline{E}_x^*(x, y)\rangle$ has a significant value. It can be shown that a minimum correlation interval is 1 wavelength of the light used. Outside a correlation region, $\langle \overline{E}_x(x + \Delta x, y + \Delta y)\, \overline{E}_x^*(x, y)\rangle \approx 0$. This report goes one step further and assumes that the fields are effectively statistically independent outside a correlation interval. Also, $\langle \overline{E}_x(x, y)\rangle = 0$. In general, the integral in equation (1) is divided into a sum of integrals, each integral evaluated over a correlation region of the rough surface. These integrals are taken to be zero mean and statistically independent. The Central Limit Theorem (ref. 11) can then be invoked to show that in the limit of an increasing number of correlation elements, the integral in equation (1) will have a normal distribution. An exact proof of this conclusion for a random phase diffuser is given in reference 12. If $\langle I \rangle$ is the mean intensity in the detecting plane, the intensity will have the probability density function (ref. 8)

$$P(I) = \frac{1}{\langle I \rangle} e^{-I/\langle I \rangle} \tag{14}$$

The variance of this distribution is $\langle I \rangle^2$. Hence, the standard deviation equals the mean intensity and the high speckle contrast is easily understood. Equation (14) is included with equations (10) and (13) for the evaluation of an instrument.

Various applications and manifestations of the speckle effect involve a superposition of two speckle patterns. As stated previously, if a rough surface that reflects randomly polarized light is viewed with a speckle instrument that admits both polarizations, two independent speckle patterns are superimposed. If two identical rough-surface images are viewed superimposed and with slight relative displacement, this displacement or strain can be measured by using the right instrument (ref. 1).

First, consider the case of randomly polarized light. Write the intensity fluctuations as

$$\Delta I_x = I_x(\tilde{x}) - \langle I_x(\tilde{x}) \rangle$$

$$\Delta I_y = I_y(\tilde{x}) - \langle I_y(\tilde{x}) \rangle$$

where I_x refers to the intensity in the x-polarized pattern and I_y refers to the intensity in the y-polarized pattern. From equation (12a) and equation (14),

$$\langle \Delta I_x(\tilde{x} + \Delta\tilde{x}) \Delta I_x(\tilde{x}) \rangle = \langle I_x(\tilde{x}) \rangle^2 |\gamma_x|^2$$

$$\langle \Delta I_y(\tilde{x} + \Delta\tilde{x}) \Delta I_y(\tilde{x}) \rangle = \langle I_y(\tilde{x}) \rangle^2 |\gamma_y|^2$$

The fluctuation of the total intensity is $\Delta I = \Delta I_x + \Delta I_y = I_x(\tilde{x}) + I_y(\tilde{y}) - \langle I_x(\tilde{x}) + I_y(\tilde{x}) \rangle$. Since ΔI_x and ΔI_y are independent, zero-mean, random variables, the correlation of the fluctuation of the total intensity is the sum of the correlations of ΔI_x and ΔI_y:

$$\langle \Delta I(\tilde{x} + \Delta\tilde{x}) \Delta I(\tilde{x}) \rangle = \langle I_x(\tilde{x}) \rangle^2 |\gamma_x|^2 + \langle I_y(\tilde{x}) \rangle^2 |\gamma_y|^2$$

Similarly, $\langle I^2(\tilde{x}) \rangle - \langle I(\tilde{x}) \rangle^2 = \langle I_x(\tilde{x}) \rangle^2 + \langle I_y(\tilde{x}) \rangle^2$. Hence, the normalized correlation of the total fluctuation is

$$|\gamma_t|^2 = \frac{\langle I_x(\tilde{x}) \rangle^2 |\gamma_x|^2 + \langle I_y(\tilde{y}) \rangle^2 |\gamma_y|^2}{\langle I_x(\tilde{x}) \rangle^2 + \langle I_y(\tilde{x}) \rangle^2} \tag{15}$$

Equation (15) is used in equation (13) to determine typical speckle size when the polarization is random. If the average intensity in the two patterns is the same and if the optical system affects both polarizations equally, $|\gamma_t|^2 = |\gamma_x|^2 = |\gamma_y|^2$ and the speckle size is the same as for deterministically polarized light.

Next, consider the two identical but slightly displaced speckle pattern images. This case occurs with one of the more popular forms of speckle pattern interferometry (ref. 1) and is included for completeness. The intensity in one of the patterns is designated $I(\tilde{x})$ and the intensity in the other pattern is designated $I(\tilde{x} + \delta)$, where δ is the displacement. The optical process, which may include photographic steps, is chosen to superimpose the two speckle patterns so as to produce an output field proportional to $I(\tilde{x} + \delta) + I(\tilde{x})$. The Fraunhofer diffraction pattern of this field is then viewed at some distance d. The field viewed is

$$E_F(\tilde{\tilde{x}}) \propto \int_{-\infty}^{\infty} \left[I(\tilde{x}) + I(\tilde{x} + \delta)\right] e^{-j(2\pi\tilde{\tilde{x}}\tilde{x}/\lambda d)} d\tilde{x}$$

$$\propto \int_{-\infty}^{\infty} I(\tilde{x}) e^{-j(2\pi\tilde{\tilde{x}}\tilde{x}/\lambda d)} d\tilde{x} \left[1 + e^{j(2\pi\tilde{\tilde{x}}/\lambda d)\delta}\right] \tag{16}$$

The intensity detected in the Fraunhofer diffraction pattern is

$$I_F(\tilde{\tilde{x}}) \propto \left| \int_{-\infty}^{\infty} I(\tilde{x}) e^{-j(2\pi\tilde{\tilde{x}}\tilde{x}/\lambda d)} dx \right|^2 \left(2 + 2\cos\frac{2\pi\delta\tilde{\tilde{x}}}{\lambda d}\right) \tag{17}$$

The viewed intensity consists of a factor containing the Fourier transform of the speckle pattern intensity multiplied by a sinusoidal factor whose frequency is proportional to the displacement δ. The only effect of the stochastic variable $I(\tilde{x})$ is to create a field with a broadband Fourier transform. With a broadband transform, there will be illumination over many cycles of $\left[2 + 2\cos(2\pi\delta/\lambda d)\tilde{\tilde{x}}\right]$ in the Fraunhofer diffraction pattern. Thus, an accurate measurement of displacement δ is possible. The transform

$$\int_{-\infty}^{\infty} I(\tilde{x}) e^{-j(2\pi\tilde{\tilde{x}}\tilde{x}/\lambda d)} d\tilde{x}$$

will change from one sample function $I(\tilde{x})$ to another. Thus, the Fraunhofer diffraction pattern $I_F(\tilde{\tilde{x}})$ will change from one sample of the rough surface to another. In order to establish a unique transform, $\langle I_F(\tilde{\tilde{x}})\rangle$ is usually dealt with.

If the sensitive area of the detector at the detecting plane is large,

$$\langle I_F(\tilde{\tilde{x}})\rangle \propto \int_{-\infty}^{\infty} \langle I(\tilde{x} + \Delta\tilde{x})I(\tilde{x})\rangle\, e^{-j(2\pi\tilde{\tilde{x}}\,\Delta\tilde{x}/\lambda d)} d\,\Delta\tilde{x} \left(2 + 2\cos\frac{2\pi\delta\tilde{\tilde{x}}}{\lambda d}\right) \tag{18}$$

From this result comes the confusing statement that this type of speckle interferometry displays the Fourier transform of the autocorrelation function of a speckle pattern. The physically useful fact is that the rapidly varying intensity in a speckle pattern assures uniform illumination in the Fraunhofer diffraction pattern for ease of measuring displacement.

The general analysis is concluded at this point.

SUMMARY OF EQUATIONS

Before the analysis is applied to examples, the essential equations are summarized:

(1) Decomposition of the optical system into a front part designated f, a physical aperture stop $P(x_a, y_a)$, and a back part designated b:

$$h(\tilde{x}, \tilde{y}; x, y) = \iint_{-\infty}^{\infty} h_b(\tilde{x}, \tilde{y}; x_a, y_a)P(x_a, y_a)h_f(x_a, y_a; x, y)dx_a\, dy_a \tag{5}$$

(2) Approximate factorization of $h_b^*(\tilde{x} + \Delta\tilde{x}; x_a)h_b(\tilde{x}; x_a + \Delta x_a)$ into three factors:

$$h_b^*(\tilde{x} + \Delta\tilde{x}; x_a)h_b(\tilde{x}; x_a + \Delta x_a) \approx D(\tilde{x}, \Delta\tilde{x})T_c(\Delta x_a; \tilde{x})T_a(x_a; \Delta\tilde{x}) \tag{9a}$$

where $\tilde{x}$ is a shorthand for $(\tilde{x}, \tilde{y})$ and x_a is a shorthand for (x_a, y_a)

(3) Field correlation or coherence function in the detecting plane expressed in terms of the field correlation in the physical aperture:

$$\Gamma(\tilde{x} + \Delta\tilde{x}; \tilde{x}) = \int_{-\infty}^{\infty} \frac{\langle \bar{E}_a(x_a + \Delta x_a)\bar{E}_a^*(x_a)\rangle}{\langle \bar{E}_a^*(x_a)\bar{E}_a(x_a)\rangle}\, T_c(\Delta x_a; \tilde{x})d\,\Delta x_a$$

$$\times \int_{-\infty}^{\infty} \langle \bar{E}_a^*(x_a)\bar{E}_a(x_a)\rangle\, T_a(x_a; \Delta\tilde{x})dx_a\, D(\tilde{x}, \Delta\tilde{x}) \tag{9b}$$

(4) Spatial degree of coherence in the detecting plane:

$$\gamma(\tilde{x} + \Delta\tilde{x};\tilde{x}) = \frac{D(\tilde{x};\Delta\tilde{x})}{D(\tilde{x};0)} \frac{\int_{-\infty}^{\infty} \langle \overline{E}_a^*(x_a)\overline{E}_a(x_a)\rangle T_a(x_a;\Delta\tilde{x})dx_a}{\int_{-\infty}^{\infty} \langle \overline{E}_a^*(x_a)\overline{E}_a(x_a)\rangle T_a(x_a;0)dx_a} \tag{10}$$

(5) Intensity fluctuation correlation for Gaussian statistics:

$$|\gamma|^2 = \frac{\langle I(\tilde{x} + \Delta\tilde{x})I(\tilde{x})\rangle - \langle I(\tilde{x} + \Delta\tilde{x})\rangle\langle I(\tilde{x})\rangle}{\langle I(\tilde{x})\rangle^2}$$

which is equation (12a) rewritten with a slight change since $\langle I^2(\tilde{x})\rangle = 2\langle I(\tilde{x})\rangle^2$.

(6) Typical area of a speckle on a large detector:

$$A_s = 4 \iint_0^{\infty} d\,\Delta\tilde{x}\; d\,\Delta\tilde{y}\, |\gamma(\Delta\tilde{x}, \Delta\tilde{y})|^2 \tag{13}$$

(7) Probability density of intensity when central limit theorem applies:

$$P(I) = \frac{1}{\langle I\rangle} e^{-I/\langle I\rangle} \tag{14}$$

(8) Normalized correlation of intensity fluctuations for randomly polarized light:

$$|\gamma_t|^2 = \frac{\langle I_x(\tilde{x})\rangle^2 |\gamma_x|^2 + \langle I_y(\tilde{x})\rangle^2 |\gamma_y|^2}{\langle I_x\rangle^2 + \langle I_y\rangle^2} \tag{15}$$

(9) Ensemble average of intensity in the Fraunhofer diffraction pattern of overlapping, identical, but displaced speckle patterns:

$$\langle I_F(\tilde{\tilde{x}})\rangle \propto \int_{-\infty}^{\infty} \langle I(\tilde{x} + \Delta\tilde{x})I(\tilde{x})\rangle e^{-j(2\pi\tilde{\tilde{x}}\,\Delta\tilde{x}/\lambda d)} d\,\Delta\tilde{x} \left(2 + 2\cos\frac{2\pi\delta}{\lambda d}\tilde{\tilde{x}}\right) \tag{18}$$

EXAMPLES

Propagation of Light between a Rectangular Rough Surface and an Infinite Detecting Plane

Figure 1 shows a rectangular surface of size L_x by L_y separated by distance D from a detecting surface of infinite extent. In this case, the aperture stop is the object itself and is specified by the function

$$P(x_a, y_a) = \text{rect}\,\frac{x}{L_x}\ \text{rect}\,\frac{x}{L_y}$$

Rect x = 1 when $|x| \leq 1/2$ and rect x = 0 when $|x| > 1/2$. The factors in equations (5) are easy to write down.

$$h_f(x_a, y_a; x, y) = 1$$

If Fresnel diffraction is used to describe propagation (ref. 7)

$$h_b(\tilde{x}, \tilde{y}; x_a, y_a) = \frac{1}{j\lambda D} \exp\left\{\frac{j\pi}{\lambda D}\left[\left(x_a - \tilde{x}\right)^2 + \left(y_a - \tilde{y}\right)^2\right]\right\}$$

A phase factor has been dropped. In preparation for the factorization of equation (9a),

$$h_b^*(\tilde{x} + \Delta\tilde{x}; x_a)h_b(\tilde{x}; x_a + \Delta x_a) = \left\{\frac{\exp\left[-\frac{j\pi}{\lambda D}\left(\Delta\tilde{x}^2 + \Delta\tilde{y}^2\right)\right]\exp\left[-\frac{j2\pi}{\lambda D}\left(\tilde{x}\,\Delta\tilde{x} + \tilde{y}\,\Delta\tilde{y}\right)\right]}{\lambda^2 D^2}\right.$$

$$\times \exp\left[-\frac{j2\pi}{\lambda D}\left(\tilde{x}\,\Delta x_a + \tilde{y}\,\Delta y_a\right)\right]\exp\left[\frac{j2\pi}{\lambda D}\left(x_a\,\Delta\tilde{x} + y_a\,\Delta\tilde{y}\right)\right]$$

$$\left.\times \exp\left[\frac{j2\pi}{\lambda D}\left(\Delta x_a^2 + \Delta y_a^2\right)\right]\exp\left[\frac{j2\pi}{\lambda D}\left(x_a\,\Delta x_a + y_a\,\Delta y_a\right)\right]\right\}$$

Referring to equation (9a), clearly

$$D(\tilde{x};\Delta\tilde{x}) = \frac{\exp\left[-\frac{j\pi}{\lambda D}(\Delta\tilde{x}^2 + \Delta\tilde{y}^2)\right]\exp\left[-\frac{j2\pi}{\lambda D}(\tilde{x}\,\Delta\tilde{x} + \tilde{y}\,\Delta\tilde{y})\right]}{\lambda^2 D^2}$$

$$T_c(\Delta x_a;\tilde{x}) = \exp\left[-\frac{j2\pi}{\lambda D}(\tilde{x}\,\Delta x_a + \tilde{y}\,\Delta y_a)\right]$$

$$T_a(x_a;\Delta\tilde{x}) = \exp\left[\frac{j2\pi}{\lambda D}(x_a\,\Delta\tilde{x} + y_a\,\Delta\tilde{y})\right]$$

The other factors must be set approximately equal to unity if the factorization implied by equation (9a) is to be possible. The simple interpretation of speckle outlined in this report requires the factorization implied by equation (9a). Assume

$$\exp\left[\frac{j\pi}{\lambda D}\left(\Delta x_a^2 + \Delta y_a^2\right)\right] \approx 1$$

$$\exp\left[\frac{j2\pi}{\lambda D}(x_a\,\Delta x_a + y_a\,\Delta y_a)\right] \approx 1$$

These approximations will be valid provided that the field correlation at the aperture is a narrow function of $\Delta x_a, \Delta y_a$.

From equation (10) the spatial degree of coherence is

$$\gamma(\tilde{x} + \Delta\tilde{x};\tilde{x}) = \exp\left[-\frac{j\pi}{\lambda D}(\Delta\tilde{x}^2 + \Delta\tilde{y}^2)\right]\exp\left[-\frac{j2\pi}{\lambda D}(\tilde{x}\,\Delta\tilde{x} + \tilde{y}\,\Delta\tilde{y})\right]$$

$$\cdot \frac{\iint_{-\infty}^{\infty}\langle \bar{E}_a^*(x_a,y_a)\bar{E}_a(x_a,y_a)\rangle \exp\left[\frac{j2\pi}{\lambda D}(x_a\,\Delta\tilde{x} + y_a\,\Delta\tilde{y})\right]dx_a\,dy_a}{\iint_{-\infty}^{\infty}\langle \bar{E}_a^*(x_a,y_a)\bar{E}_a(x_a,y_a)\rangle\,dx_a\,dy_a}$$

The spatial coherence function γ simply involves the Fourier transform of the intensity expectation at the object.

For a uniform ensemble average of intensity, the correlation of the intensity fluctuations expressed by equation (10) is

$$|\gamma|^2 = \frac{1}{L_x^2 L_y^2}\left|\iint_{-\infty}^{\infty} \mathrm{rect}\,\frac{x_a}{L_x}\,\mathrm{rect}\,\frac{y_a}{L_y}\,\exp\left[\frac{j2\pi}{\lambda D}\left(x_a\,\Delta\tilde{x} + y_a\,\Delta\tilde{y}\right)\right]dx_a\,dy_a\right|^2$$

Hence, speckle size will be determined by a transform of the shape function of the object.

$$|\gamma|^2 = \left(\mathrm{sinc}\,\frac{\Delta\tilde{x}\,L_x}{\lambda D}\right)^2\left(\mathrm{sinc}\,\frac{\Delta\tilde{y}\,L_y}{\lambda D}\right)^2 \tag{19a}$$

Referring to equation (13) for a calculation of the area of a typical speckle,

$$A_s = 4\iint_{-\infty}^{\infty} d\,\Delta\tilde{x}\,d\,\Delta\tilde{y}\,|\gamma(\Delta\tilde{x},\Delta\tilde{y})|^2$$

$$= 4\iint_{-\infty}^{\infty} d\,\Delta\tilde{x}\,d\,\Delta\tilde{y}\left(\mathrm{sinc}\,\frac{\Delta\tilde{x}\,L_x}{\lambda D}\right)^2\left(\mathrm{sinc}\,\frac{\Delta\tilde{y}\,L_y}{\lambda D}\right)^2 \tag{19b}$$

$$= \left(\frac{\lambda D}{L_x}\right)\left(\frac{\lambda D}{L_y}\right)$$

This formula simply expresses the typical area of a speckle, according to one definition of area, in terms of the wavelength of light λ, the separation D, and the rough-surface size L_x by L_y. The actual speckle pattern will appear to consist of irregularly shaped speckles of varying size and intensity.

These results originally derived by Goodman (ref. 8), were derived herein by a systematic application of the analysis developed in this report. The typical extent of a speckle corresponds to the first zeros of the sinc functions. The power spectral density of the intensity fluctuations can be gotten by evaluating the Fourier transform of $|\gamma|^2$.

Speckle Pattern Viewer

The next sample analysis treats the speckle pattern viewer shown in figure 2. Lens L placed two focal lengths from physical aperture stop a creates a unity magnification image of the stop on the television camera lens. The focal length f_L of lens L is considered to be large when compared with the focal length f_{TVL} of the television camera lens. The rough surface object is considered to be at a large distance from aperture a. Hence, the image of the surface is focused on the vidicon target at approximately f_{TVL} from the camera lens. The impulse response function $h_b(\tilde{x},\tilde{y};x_a,y_a)$ is evaluated in two steps. An impulse response function is first evaluated from the aperture to the television camera lens. This impulse response function is designated $h_L(x_{TV},y_{TV};x_a,y_a)$. Next, an impulse response function is evaluated from the television camera lens to the target. This impulse response function is designated $h_{TV}(\tilde{x},\tilde{y};x_{TV},y_{TV})$. The impulse response function from the aperture a to the target is then

$$h_b(\tilde{x},\tilde{y};x_a,y_a) = \iint_{-\infty}^{\infty} h_{TV}(\tilde{x},\tilde{y};x_{TV},y_{TV})h_L(x_{TV},y_{TV};x_a,y_a)dx_{TV}\,dy_{TV}$$

In the analysis that follows, the y-coordinate will be omitted prior to the final results since the y-coordinate enters symmetrically with the x-coordinate in the formulas.

Using reference 7, the impulse response function from aperture to television camera lens is

$$h_L(x_{TV};x_a) = \frac{1}{4\lambda^2 f_L^2} e^{j\pi x_{TV}^2/2\lambda f_L} e^{j\pi x_a^2/2\lambda f_L} \int_{-\infty}^{\infty} P_L(x_L)\exp\left[\frac{-j\pi}{\lambda f_L}(x_{TV}+x_a)x_L\right]dx_L$$

where x_L refers to positions on lens plane L. The pupil function $P_L(x_L) = e^{j\varphi_L(x_L)}$ when x_L is on the lens, and $P_L(x_L) = 0$ when x_L is outside the lens, where $\varphi_L(x_L)$ is a phase error due to aberrations of the lens L. For this calculation, neglect the effect of aberrations so that $P_L(x_L) = 1$ on the lens. Also, assume that the lens is large enough so that $P_L(x_L) = 1$ from $-\infty$ to ∞. Then

$$h_L(x_{TV};x_a) = \frac{1}{4} e^{j\pi x_{TV}^2/2\lambda f_L} e^{j\pi x_a^2/2\lambda f_L} \delta\left(\frac{x_a + x_{TV}}{2}\right)$$

where $\delta(x)$ is the Dirac delta function.

The impulse response function (ref. 7) from the television lens to the target is

$$h_{TV}(\tilde{x};x_{TV}) = \frac{1}{j\lambda f_{TVL}} e^{j\pi\tilde{x}^2/\lambda f_{TVL}} P_{TV}(x_{TV})e^{-j2\pi x_{TV}\tilde{\tilde{x}}/\lambda f_{TVL}}$$

Again, $P_{TV}(x_{TV}) = e^{j\varphi_{TV}(x_{TV})}$ on the television lens and $P_{TV}(x_{TV}) = 0$ off the television lens. As before, neglect aberrations and set $P_{TV}(x_{TV}) = 1$ from $-\infty$ to ∞. Because of $\delta(x_{TV} + x_a)/2$, h_b is easily calculated:

$$h_b(\tilde{x}; x_a) = \frac{1}{j\lambda f_{TVL}} e^{j\pi x_a^2/\lambda f_L} e^{j\pi \tilde{x}^2/\lambda f_{TVL}} e^{j2\pi x_a \tilde{x}/\lambda f_{TVL}}$$

Before performing the factorization implied by equation (9a), evaluate

$$h_b^*(\tilde{x} + \Delta\tilde{x}; x_a) h_b(\tilde{x}; x_a + \Delta x_a) = \frac{1}{\lambda^2 f_{TVL}^2} e^{j\pi\, \Delta x_a^2/\lambda f_L}$$

$$\times e^{j2\pi x_a\, \Delta x_a/\lambda f_L} e^{-j\pi\, \Delta\tilde{x}^2/\lambda f_{TVL}} e^{-j2\pi\tilde{x}\, \Delta\tilde{x}/\lambda f_{TVL}}$$

$$\times e^{-j2\pi x_a\, \Delta\tilde{x}/\lambda f_{TVL}} e^{j2\pi\tilde{x}\, \Delta x_a/\lambda f_{TVL}}$$

The factorization implied by equation (9a) can be performed if $e^{j\pi\, \Delta x_a^2/\lambda f_L} \approx 1$ and if $e^{j2\pi x_a\, \Delta x_a/\lambda f_L} \approx 1$ within the aperture or over the interval Δx_a, where the field correlation is significant. The factorization is

$$D(\tilde{x}, \Delta\tilde{x}) = \frac{1}{\lambda^2 f_{TVL}^2} e^{-j\pi\, \Delta\tilde{x}^2/\lambda f_{TVL}} e^{-j2\pi\tilde{x}\, \Delta\tilde{x}/\lambda f_{TVL}}$$

$$T_a(x_a; \Delta\tilde{x}) = e^{-j2\pi x_a\, \Delta\tilde{x}/\lambda f_{TVL}}$$

$$T_c(\Delta x_a; \tilde{x}) = e^{j2\pi\tilde{x}\, \Delta x_a/\lambda f_{TVL}}$$

Equation (10) then yields the results

$$\gamma(\tilde{x} + \Delta\tilde{x}; \tilde{x}) = e^{-j2\pi\tilde{x}\,\Delta\tilde{x}/\lambda f_{TVL}}\, e^{-j\pi\,\Delta\tilde{x}^2/\lambda f_{TVL}}$$

$$\times \frac{\int_{-\infty}^{\infty} \langle \bar{E}_a^*(x_a)\bar{E}_a(x_a)\rangle\, e^{-j2\pi x_a\,\Delta\tilde{x}/\lambda f_{TVL}}\, dx_a}{\int_{-\infty}^{\infty} \langle \bar{E}_a^*(x_a)\bar{E}_a(x_a)\rangle\, dx_a} \tag{20}$$

This result is identical with the expression for γ obtained for propagation of light from an object of finite size to a detecting plane. The plane separation D is replaced by the focal length f_{TVL} of the television camera lens. The quantity $\langle \bar{E}_a^*(x_a)\bar{E}_a(x_a)\rangle$ is the expected intensity over the aperture. For example, a circular aperture of radius R with approximately uniform intensity in the aperture has an intensity fluctuation correlation

$$|\gamma|^2 = 4\left|\frac{J_1\left(\frac{2\pi R}{\lambda f_{TVL}}\Delta\rho\right)}{\frac{2\pi R}{\lambda f_{TVL}}\Delta\rho}\right|^2 \qquad \text{where } \Delta\rho = \sqrt{\Delta\tilde{x}^2 + \Delta\tilde{y}^2}$$

The first zero of the correlation that provides an estimate of typical speckle size occurs at

$$\Delta\rho = \frac{1.22\,\lambda f_{TVL}}{2R} = \frac{1.22\,\lambda f_{TVL}}{D_a}$$

where $D_a = 2R$ is the diameter of the aperture. This dimension is identical with the separation of two barely resolved points in the focal plane of a diffraction-limited lens of diameter D_a and focal length f_{TVL} when that lens is used to image a distant object.

Speckle Interferometer

The next example treats the laser-speckle interferometer shown in figure 3 (ref. 3). This interferometer is but a slight modification of the speckle viewer shown in figure 2 and described in the previous section. The interferometer is discussed because it exemplifies an interesting property of speckle: adjacent bright speckles tend to have opposite phase relative to a reference beam. The single lens L in figure 2 is replaced by two lenses as shown in figure 3. A beam splitter is introduced between the two lenses to turn a reference beam toward the target. As shown in figure 3, this arrangement is equivalent to placing a point source in the aperture a. Again, the optical parts are assumed large enough to be considered of infinite lateral extent. As in the speckle viewer, the aperture is imaged onto the television camera lens. An analytically equivalent interferometer can be formed by placing a point source in the aperture of the speckle viewer shown in figure 2 and choosing a lens L with one-half the focal length of the lenses L used in the speckle interferometer. The light illuminating the rough surface object and the light from the point source are, of course, mutually coherent. The equivalent interferometer is shown in figure 4 and is used for the analysis that follows.

The point source representing the reference beam is represented as $\bar{E}_r \delta(x, y)$, where $\delta(x, y)$ is the Dirac delta function. On the television target in the detecting plane, the reference beam is

$$\bar{E}_{\tilde{r}}(\tilde{x}) = \bar{E}_r h_b(\tilde{x}; 0) = \frac{\bar{E}_r}{j\lambda f_{TVL}} e^{j\pi\tilde{x}^2/\lambda f_{TVL}}$$

With respect to both $\tilde{x}$ and $\tilde{y}$

$$\bar{E}_{\tilde{r}}(\tilde{x}, \tilde{y}) = \frac{\bar{E}_r}{j\lambda f_{TVL}} \exp\left[\frac{j\pi}{\lambda f_{TVL}} (\tilde{x}^2 + \tilde{y}^2)\right]$$

The formula is the usual parabolic approximation to a spherical wave. The function h_b was derived in the section on the speckle viewer. If the wave from the rough-surface object is designated $E(\tilde{x}, \tilde{y})$, the intensity on the target is

$$I(\tilde{x}, \tilde{y}) = \frac{|\bar{E}_r|^2}{\lambda^2 f_{TVL}^2} + |\bar{E}(\tilde{x}, \tilde{y})|^2 - 2\, \mathcal{R}e\, \frac{\bar{E}_r^*}{j\lambda f_{TVL}} \exp\left[-j \frac{\pi}{\lambda f_{TVL}} (\tilde{x}^2 + \tilde{y}^2)\right] \bar{E}(\tilde{x}, \tilde{y}) \qquad (21)$$

The third term in the intensity depends on the relative phase between the reference wave and the wave from the rough-surface object. Hence, this term is sensitive to a phase change in the object wave as a result of object motion. Because adjacent bright speckles tend to have opposite phase relative to the reference beam, the speckle size will appear to increase when a bright reference beam is present.

In order to understand these properties of speckle and the speckle interferometer, consider the square aperture:

$$P_a(x_a, y_a) = \text{rect}\, \frac{x_a}{l}\, \text{rect}\, \frac{y_a}{l}$$

Assuming that the intensity in the square aperture is approximately constant and using equation (20),

$$\gamma(\tilde{x} + \Delta\tilde{x}, \tilde{y} + \Delta\tilde{y}; \tilde{x}, \tilde{y}) = \left\{ \left[e^{-j2\pi\tilde{x}\,\Delta\tilde{x}/\lambda f_{TVL}}\, e^{-j2\pi\tilde{y}\,\Delta\tilde{y}/\lambda f_{TVL}} \right.\right.$$

$$\left.\left. \times\, e^{-j\pi\,\Delta\tilde{x}^2/\lambda f_{TVL}}\, e^{-j\pi\,\Delta\tilde{y}^2/\lambda f_{TVL}} \right] \left[\mathrm{sinc}\,\frac{\Delta\tilde{x}l}{\lambda f_{TVL}}\ \mathrm{sinc}\,\frac{\Delta\tilde{y}l}{\lambda f_{TVL}} \right] \right\} \tag{22}$$

From equation (13), the typical area of a speckle can be calculated to be

$$A_s = \left(\frac{\lambda f_{TVL}}{l}\right)^2$$

Figure 5 shows two typical bright speckles separated by a typical dark speckle. The separation along the x-axis between pairs of points, one point in each of the bright speckles, varies from $\lambda f_{TVL}/l$ to $3\lambda f_{TVL}/l$. The separation between the same pairs along the y-axis varies from 0 to $\lambda f_{TVL}/l$. Hence, the average value of the sinc function product will be negative in equation (22) when comparing points in adjacent bright speckles. The phase factors in the first square brackets in equation (22) are a consequence of Fresnel diffraction. These factors also occur in the reference beam so that they do not introduce a phase difference between the reference beam and the object beam. However, since the sinc function product is negative between adjacent bright speckles, adjacent bright speckles differ by a factor $e^{j\pi}$ when compared with the reference beam. Adjacent bright speckles therefore have opposite phase when compared with the reference beam.

Referring to equation (21), if one bright speckle results in a positive third term and adds intensity to the first two terms, the adjacent bright speckle will tend to contribute a negative third term. One bright speckle will tend to darken relative to the other.

Since the speckle interferometer is being used as an example, another property of the interference between the speckle pattern and a reference beam will be mentioned. If the third term in equation (21) for the intensity contains a time-varying phase factor, time averaging may be performed by the detector. Time-average holography, discussed for example in reference 7, describes the effect of time averaging on an interference pattern. The sum of the first term in equation (21) plus the third term yields an interference speckle effect. If the third term has a time-varying phase factor, a time-average envelope will be superimposed on the interference speckle. The envelope is a function of the amplitude of vibration if the time-varying phase is caused by object vibration. For sinusoidal vibration, that envelope is the same Bessel function that appears in time-average holography.

CONCLUDING REMARKS

A general description of the speckle effect requires analytical tools that can handle a light source of limited coherence; a reflecting object with arbitrary surface properties that manifest themselves in a reflected field of arbitrary spatial correlation, polarization, and time variation; an arbitrarily complex intervening optical system that may be changing in time; and a detector with arbitrary properties. This report begins with a more restricted general analysis that should be useful for guiding the design or development of a new speckle technique or instrument. The assumptions are a highly coherent light source, the validity of a linear systems approach, possible arbitrary polarization, and a time-invariant optical system. This development is presented in equations (1) to (8). This analysis is still too general for the simple understanding of, application of, or evaluation of existing speckle instrumentation. Since these activities are more common in aeronautics applications, the analysis is further developed subject to assumptions to give a simple set of equations. The application of equations (5), (9a), (9b), (10), (12a), and (13) to (15) more or less in order and the use of geometrical optics should be adequate to evaluate a speckle instrument or technique. To test this conclusion, a speckle viewer, a speckle interferometer, and propagation of light from a rough surface were evaluated.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, October 26, 1976,
505-04.

APPENDIX - SYMBOLS

The variables x, $\tilde{x}$, and $\tilde{\tilde{x}}$; subscripted forms of these variables; or increments of these variables, when appearing alone as arguments of functions, are understood to represent (x, y), $(\tilde{x}, \tilde{y})$, and $(\tilde{\tilde{x}}, \tilde{\tilde{y}})$; subscripted forms of these pairs of variables; or increments of these pairs of variables.

A_d	area of detector
A_s	typical area of speckle
D, d	propagation distances
$D(\tilde{x}, \Delta\tilde{x})$	function involving possible phase and attenuation factors in detecting plane
$E_{x,y}(x)$	x, y-component of electric field phasor at position (x, y)
f_L	focal length of lens L
f_{TVL}	focal length of television camera lens TVL
$f(\bar{E}_x, \bar{E}_y)$	probability density function for x-polarized field $\bar{E}_x$ and y-polarized field $\bar{E}_y$ at position (x, y)
$f_x\left(\bar{E}_{x_1}, \bar{E}_{x_2}\right)$	probability density function for x-polarized field $\bar{E}_{x_1}$ at position (x_1, y_1) and x-polarized field $\bar{E}_{x_2}$ at position (x_2, y_2)
$f'_x\left(\bar{E}_{y_1}, \bar{E}_{y_2}\right)$	for deterministically polarized light, probability density function for x-polarized field $\bar{E}_{x_1}$ at position (x_1, y_1) and x-polarized field $\bar{E}_{x_2}$ at position (x_2, y_2) expressed as function of corresponding y-polarizations
$f_y\left(\bar{E}_{x_1}, \bar{E}_{x_2}\right)$	for deterministically polarized light, probability density function for y-polarized field $\bar{E}_{y_1}$ at position (x_1, y_1) and y-polarized field $\bar{E}_{y_2}$ at position (x_2, y_2) expressed as function of corresponding x-polarizations
$f_y\left(\bar{E}_{y_1}, \bar{E}_{y_2}\right)$	probability density function for y-polarized field $\bar{E}_{y_1}$ at position (x_1, y_1) and y-polarized field $\bar{E}_{y_2}$ at position (x_2, y_2)
$f'_y\left(\bar{E}_{y_1}, \bar{E}_{y_2}\right)$	for deterministically polarized light, probability density function for y-polarized field $\bar{E}_{y_2}$ at position (x_2, y_2) and y-polarized field $\bar{E}_{y_1}$ at position (x_1, y_1) expressed as function of corresponding y-polarizations
$h_{x,y}(\tilde{x}; x)$	impulse response function or point spread function for light polarized in x, y-direction
$I(x)$	intensity at (x, y)
I_F	intensity of light in Fraunhofer diffraction pattern
$I_{x,y}$	intensity of light polarized in x, y-direction

j	imaginary part of complex number
L_x, L_y	lengths of sides of rectangular aperture
l	length
$P(I)$	probability density function for intensity I
$P(x)$	pupil function (In general, $P(x, y) = e^{j\varphi(x, y)}$ where (x, y) is a position in an aperture and $P(x, y) = 0$ for positions (x, y) outside aperture; $\varphi(x, y)$ is a phase function.)
R	radius of circular aperture
$T_a(x_a; \Delta\tilde{x})$	function that transforms intensity in limiting aperture to detecting plane
$T_c(\Delta x_a; \tilde{x})$	function that transforms wide-sense stationary function $\gamma(x_a + \Delta x_a; x_a)$ from limiting aperture to detecting plane
(x, y)	position on rough surface
$(\tilde{x}, \tilde{y})$	position on detector
$(\tilde{\tilde{x}}, \tilde{\tilde{y}})$	position in Fraunhofer diffraction pattern of superimposed speckle patterns
(x_a, y_a)	position in aperture
(x_L, y_L)	position on thin lens
(x_{TV}, y_{TV})	position on vidicon
$\Gamma(x_2; x_1)$	correlation or spatial coherence between fields at positions (x_1, y_1) and (x_2, y_2) when a single polarization is understood to be present
$\Gamma_{x,y}(x_2; x_1)$	correlation or spatial coherence between x, y-polarized fields at positions (x_1, y_1) and (x_2, y_2)
$\gamma(x + \Delta x; x)$	normalized correlation or spatial coherence function between fields at positions (x, y) and $(x + \Delta x, y + \Delta y)$; $\gamma(x; x) = 1$
δ	relative displacement of overlapping, identical speckle pattern
$\delta(x)$	Dirac delta function
λ	wavelength of light
$\Delta\rho$	$\Delta\rho = \sqrt{\Delta x^2 + \Delta y^2}$

Subscripts:

a aperture

b back part of optical system, or part between limiting aperture and detector

F Fraunhofer diffraction pattern

f front part of optical system, or the part between limiting aperture and rough surface

t total

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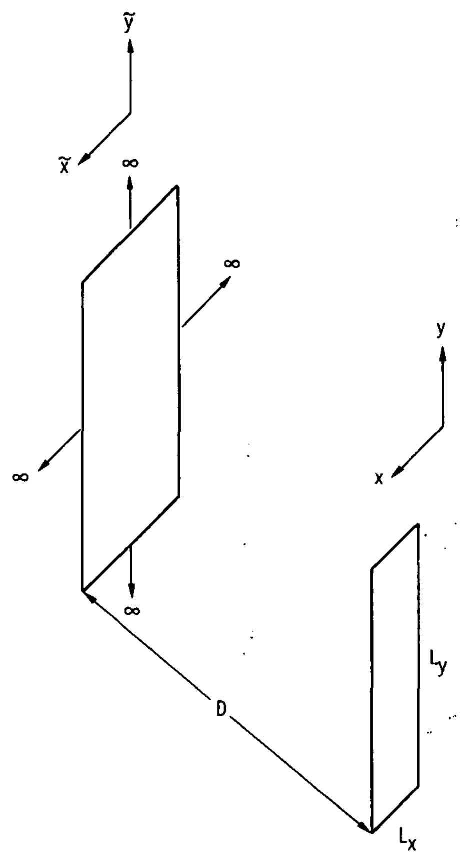

Figure 1. - Rectangular rough reflecting surface separated by distance D from an infinite detecting plane.

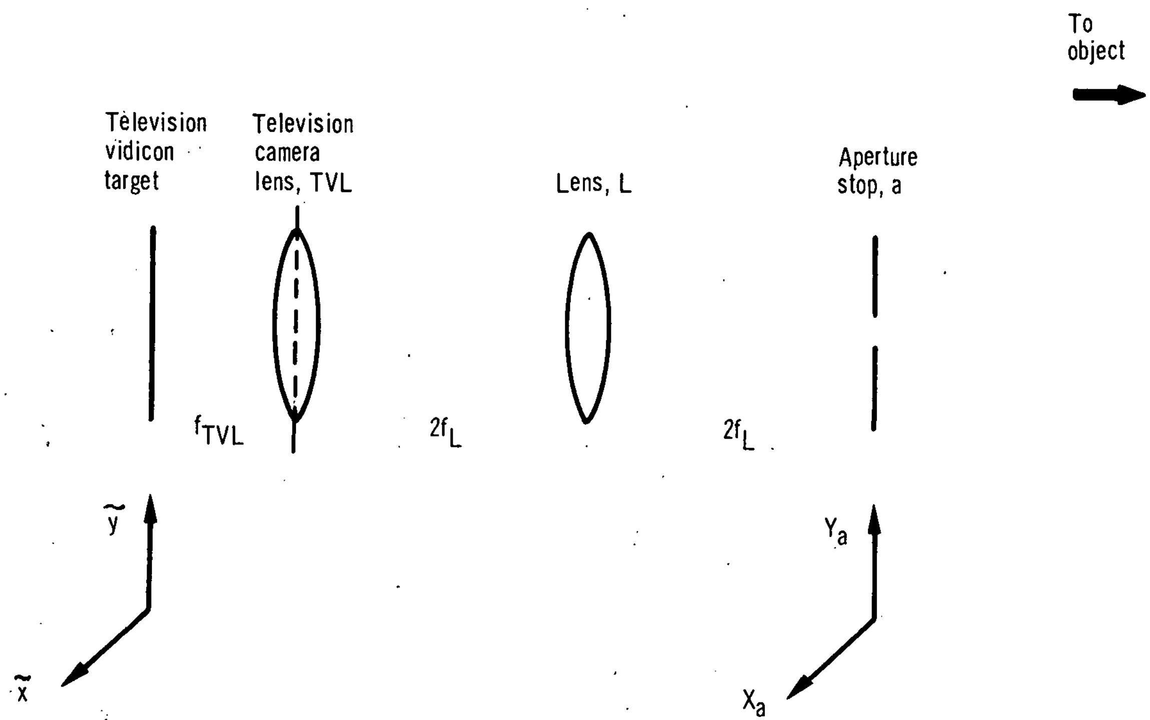

Figure 2. - Speckle viewer.

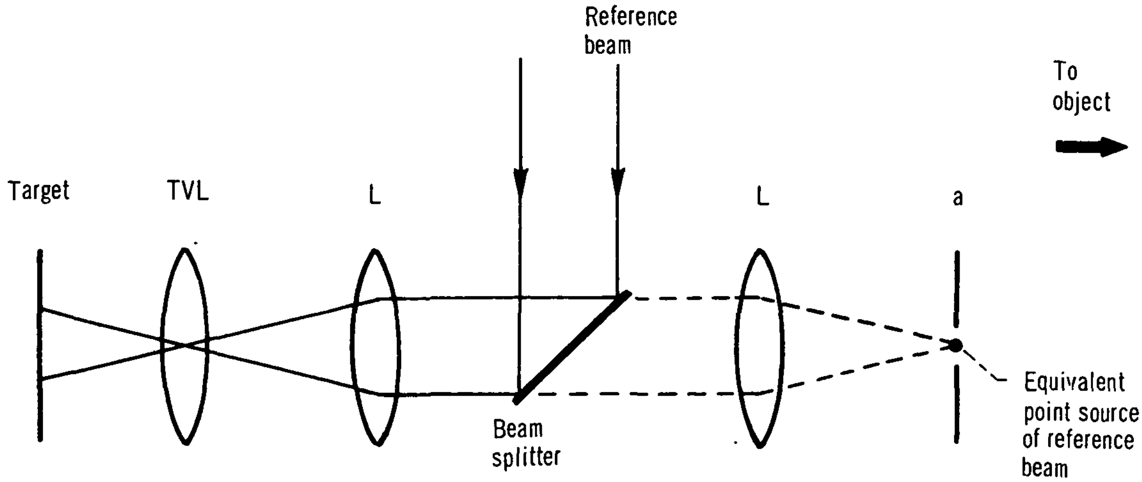

Figure 3. - Speckle interferometer.

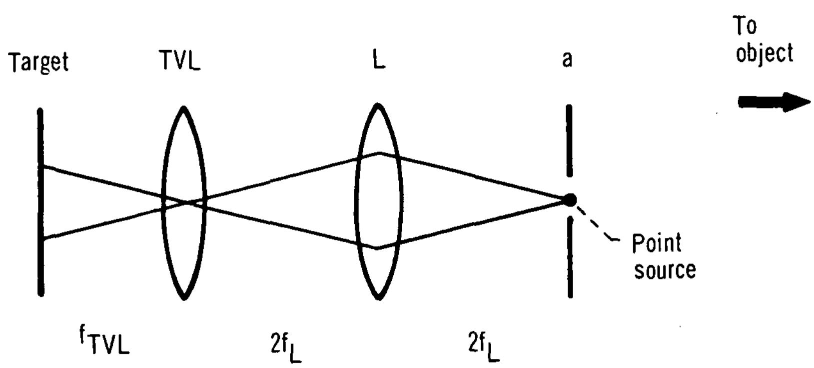

Figure 4. - Equivalent interferometer.

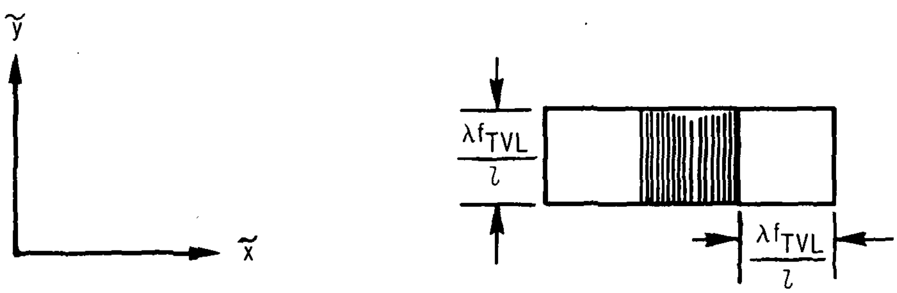

Figure 5. - Average speckles.

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