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GREENBELT, MARYLAND

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ABSTRACT

The orbit of the 1967-92A satellite has been studied to ascertain the extent to which tidal forces contribute to orbital perturbations. This study has permitted an estimation of parameters describing the ocean tide potential—in particular for the M_2 and S_2 constituents. Since the ocean tide potential is less well known than the solid Earth tide, the ocean tide parameter estimation is based upon the use of a value of 0.3 for the solid Earth tide Love number in the orbit determination procedure. These tidal parameter values are in good agreement with those appearing in numerical models of the M_2 and S_2 tides derived from surface data—the results are as follows:

Tidal Constituent	C_{22} Coefficient (cm)	ϵ_{22} Phase Angle
M_2	3.0 ± 0.8	319° ± 14°
S_2	2.25 ± 0.06	286.7° ± 1.4°

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INTRODUCTION

The tidal deformations of the Earth can result in significant perturbations on the orbits of close Earth satellites, a fact which was mentioned some time ago by Kaula (1962). Indeed, these perturbations can be as large as several hundred meters in the orbits. Early work in attempting to detect and analyze these perturbations was performed by Kozai (1965, 1968) and Newton (1965, 1968). These efforts were aimed at deriving values for the solid Earth tide Love number k_2 and associated phase lag ϵ_2. These values were considerably different for the orbits of different satellites, a fact which indicated either a flaw in the tidal models or inadequate orbital data.

More recent studies with more precise orbital data by Anderle (1971), Smith et al. (1973), and Douglas et al. (1974) resulted in k_2 values varying from 0.22 to 0.31, which again exhibited a dependence upon the orbit of the particular satellite being analyzed. It was pointed out by Lambeck and Cazenave (1973) that these inconsistent results were caused by neglect of the ocean tidal effects in the studies. They demonstrated that neglecting the ocean tide could introduce errors of up to 15-20% in k_2 determinations and several degrees in the corresponding phase angle (depending upon the satellite's orbit and the ocean tide constituent). Therefore, the "solid-Earth" Love numbers which have been solved for in the several works previously mentioned are, in actuality, "composite"

Love numbers incorporating both solid Earth and ocean tidal effects. Hence, a more comprehensive tidal perturbation theory is necessary if one is to derive meaningful tidal parameters from satellite orbital data.

Kaula (1969) developed a theory based upon the dependency of the Love numbers and phase angles on latitude because of ocean tides. However, Lambeck et al. (1974) pointed out that the ocean tides are frequency dependent, and so would be Kaula's "combined" Love numbers. Hence, Kaula's treatment is judged to be too unwieldy. Lambeck et al. (1974) presented an explicit perturbation theory for the ocean tides alone which is more amenable to tidal parameter recovery, and they also gave a review of tidal effects on close Earth satellite orbits. This theory was used by Felsentreger et al. (1976) to estimate ocean tidal parameters for the K_1, S_2, and P_1 ocean tides from the orbits of the GEOS-1 and GEOS-2 satellites. A. Cazenave, S. Daillet, and K. Lambeck (personal communication from A. Cazenave) used a modification of this theory to recover parameters for the S_2 and M_2 ocean tides from the GEOS-1 and 1967-92A satellites. In addition, they attempted combined solutions for M_2 tidal parameters using the satellite results and the numerical models of Bogdanov and Magarik (1967) and Zahel (1970).

Studies of the tidal effects on satellite orbits for tidal parameter recovery are important for a number of reasons. Since the tidal perturbations can amount to as much as a few seconds of arc in, say, the angle of inclination and longitude of ascending node, they must be modelled in the orbit computations so that smaller effects (e.g., polar motion) can be studied. Also, satellites sense the tidal effects on a global basis, whereas surface techniques provide information on local tidal variations. Thus, we would have two complementary sets of data to consider in modelling of the tides. In addition, the tides cause secular perturbations in the moon's orbit, which can be calculated using the tidal parameters recoverable from satellite orbital data and numerical models (Lambeck, 1975).

In this paper we present a study of the tidal perturbations on the orbit of 1967-92A, which is a U.S. Navy navigation satellite tracked by a worldwide Doppler network. The global tracking coverage and excellent precision of this tracking data combined with improved analytical techniques has enabled us to detect the M_2 ocean tidal effect on this orbit as well as the S_2 and others. In most previous works, the magnitude of the M_2 perturbation was below the precision of the data, even though this tide is actually most important on the surface of the Earth. Other tides of lesser interest actually cause larger satellite orbit perturbations and thus are easier to detect.

Tidal perturbations were easily observed in the angle of inclination and longitude of ascending node of 1967-92A. Only in the angle of inclination was there sufficient separation of the constituent effects to afford parameter estimation for the M_2 and S_2 ocean tides; however, we are able to show that the perturbations in the longitude of ascending node can be attributed to a combination of tidal forces of several selected frequencies.

ORBIT COMPUTATIONAL PROCEDURES

The tracking data used in these analyses consisted of range difference observations of the U. S. Navy Navigation satellite 1967-92A provided by Continuous Count Integrated Doppler (CCID) equipment at the U.S. Navy Doppler tracking sites. This equipment measures the time required to count a preset number of beat cycles over a period of about 20 seconds (Anderle and Tanenbaum, 1974). The random error on the data is about 10 cm; however, actual satellite position errors are about a meter in the inclination and several meters along track due primarily to gravity model errors.

The data covered the time period from May 17, 1973 to March 25, 1974 and were contributed by the following eight stations: Las Cruces, New Mexico; McMurdo, Antarctica; Thule, Greenland; Howard County, Maryland; Anchorage, Alaska; Tafuna, American Samoa; and Pretoria, Republic of South Africa. The data consisted of about 30 passes per day. Orbital solutions based upon two-day arcs were computed using the GEODYN orbit and geodetic parameter estimation computer program (Martin et al., 1974) which employs an 11th-order Cowell numerical integration procedure. Satellite disturbing forces modelled in the analyses were the Earth's gravity field-GEM-7 model (Wagner et al., 1976), luni-solar gravitation with the lunar and solar positions obtained from the Jet Propulsion Laboratory tape DE-69, luni-solar induced solid earth tides—$k_2 = 0.30$ and lag = 0°, atmospheric drag with densities modelled by the Jacchia model atmosphere (Jacchia, 1965, 1970, 1971; Jacchia et al., 1968), and direct solar radiation pressure.

The GEODYN osculating orbital elements were converted to mean elements by C. Goad and B. Douglas of the National Oceanic and Atmospheric Administration using a refinement of the technique employed earlier by Douglas et al., (1973) for GEOS-1 and GEOS-2 data. This new technique basically consists of first of all, analytically removing first order effects due to about 600 terms in Kaula's (1966) formulation of the geopotential, and then passing these Keplerian elements through an ideal low-pass filter with the mean elements taken to be the average of the middle day of the output of the filter. In this process, the direct lunar perturbations were modelled prior to the averaging in order to eliminate problems due to the absorption of a damped lunar effect into the mean elements (Goad and Douglas, 1976).

The mean orbital elements were subsequently analyzed using the ROAD program (Wagner et al., 1974). This program uses an Adams multi-step integrator to solve the first-order system of Lagrange's equations which describe the long period and secular satellite motion.

In the ROAD orbit computations a radiation pressure coefficient was solved for using a constant area to mass ratio for the satellite. A measure of the ability to model solar radiation pressure is indicated by the residuals remaining in the semi-major axis since only drag and radiation pressure produce long periodic effects in this element. The amplitude of the unmodelled effects is about 2 meters with an apparent solar frequency. These effects are most likely due to problems in modelling solar radiation pressure. This unmodelled effect is probably significant enough to affect any solar tidal parameters derived from the data.

OCEAN TIDAL PERTURBATION ANALYSIS

Figures 1 and 2 show the resulting residuals for the angle of inclination and longitude of ascending node, resp., with no tidal effects modelled in the orbit determination. Presumably, these residuals still contain all solid Earth, ocean, and atmospheric tidal perturbations, in addition to any remaining solar radiation pressure effects resulting from possible inadequate modelling. Figure 1 quite clearly shows 13.6 and 170 day period perturbations caused by the M_2 and S_2 tides, resp.

The residuals in Figures 3 and 4 are the results of processing the same data, but now "removing" the solid Earth tidal effects by setting the Love number $k_2 = 0.3$ (a value established from seismic data, Earth tide measurements, and Earth rotation observations) and the lag angle $\sigma = 0°$. In addition, a solar atmospheric tidal perturbation was removed from the data by using the coefficient and phase angle reported in Chapman and Lindzen (1970) ($C_{22} = 0.352$ mbar, $\epsilon_{22} = 158°$). The perturbation equations are as follows:

$$(\delta I)_{atm} = 0.''04393 \sin(\theta + \epsilon_{22}) \quad (1)$$

$$(\delta\Omega)_{atm} = 0.''1203 \cos(\theta + \epsilon_{22}) \quad (2)$$

where

$\theta = 2(\Omega - \lambda_{\odot}) = 325^{\circ}.1512 + \dot{\theta}t$

$\lambda_{\odot}$ = mean ecliptic longitude of sun

$\dot{\theta} = -2^{\circ}.1208$/day

t = time in days from 5/17/73, 0 hrs U.T.

Again, in Figure 3 the M_2 and S_2 effects are easily observable, while the frequencies present in Figure 4 are not so apparent. The assumption was now made, however, that Figures 3 and 4 represented only ocean tidal effects, subject to the limitations of the solid Earth tide and solar radiation pressure modelling.

The ocean tide potential and resulting general perturbation equations have been derived by Lambeck et al. (1974) and presented again by Felsentreger et al. (1976), so the derivation will not be repeated here. Based upon this theory, Table 1 presents the expected perturbation amplitudes for some of the major ocean tide constituents, in terms of the principal second degree harmonic coefficient. This table indicates that one could reasonably expect to determine the M_2 and S_2 effects in the inclination data, but not so in the node data where the O_1 and P_1 perturbations cannot be neglected. In addition, the constituents with periods between 300-400 days (T_2, S_1, and ψ_1) cannot be separated in the 312 day data span. However, it can be shown that a choice of constituents with periods of 13.6, 116, 170, and 317 days fits the node data very well. For both the inclination and node, the K_2 perturbation will appear as a secular, or linear, effect. Therefore, the observation equations for Figures 3 and 4 are the following:

$$\delta I - (\delta I)_{atm} = (\delta I)_0 + \alpha t + \sum_{i=1}^{2} A_i \sin(\omega_i t + \epsilon_i) \tag{3}$$

$$\delta\Omega - (\delta\Omega)_{atm} = (\delta\Omega)_0 + \beta t + \sum_{i=1}^{4} B_i \cos(\omega_i t + \epsilon_i) \tag{4}$$

where

ω_1 = 1 rev./13.58 days

ω_2 = 1 rev./169.3 days

ω_3 = 1 rev./115.9 days

ω_4 = 1 rev./317.2 days

ϵ_i (i = 1, ..., 4) = phase angles

$(\delta I)_0$, $(\delta\Omega)_0$ = constants

α, β, A_i, B_i (i = 1, ... , 4) = coefficients

Figures 3 and 4 also show the least squares fits and the resulting amplitudes for these equations. The rms of fit for Figures 3 and 4 were 0.045 and 0.068 arc sec, resp. Figures 5 and 6 show the residuals remaining after the least squares fits.

For the inclination, we can now set (using Equations 3 and 4 and Table 1), for the M_2 tide,

$$A_1 \sin(\omega_1 t + \epsilon_1) = 0''03059 \sin(\omega_1 t + \epsilon_1) = \frac{0''01003}{cm} C_{22} \sin(2\Omega - 2\lambda_{☾} + \epsilon_{22}) \quad (5)$$

and for the S_2 tide,

$$A_2 \sin(\omega_2 t + \epsilon_2) = 0''2826 \sin(\omega_2 t + \epsilon_2) = \frac{0''1253}{cm} C_{22} \sin(2\Omega - 2\lambda_{\odot} + \epsilon_{22}) \quad (6)$$

where

$\lambda_{☾}$ = mean longitude of moon.

From these, we can derive the parameters for M_2 and S_2 shown in Table 2.

A number of numerical solutions for the M_2 and S_2 ocean tide parameters are in the literature, and the principal 2nd and 4th degree parameters for these models are shown in Table 3. The fact that only 2nd degree parameters were solved for using the satellite data must be taken into account before Tables 2 and 3 can be compared. In essence, Equations 5 and 6 should read

$$A_1 \sin(\omega_1 t + \epsilon_1) = \frac{0''01003}{cm} C_{22} \sin(2\Omega - 2\lambda_{☾} + \epsilon_{22}) + \frac{0''009612}{cm} C_{42} \sin(2\Omega - 2\lambda_{☾} + \epsilon_{42}) + \ldots \quad (7)$$

$$A_2 \sin(\omega_2 t + \epsilon_2) = \frac{0''1253}{cm} C_{22} \sin(2\Omega - 2\lambda_{\odot} + \epsilon_{22}) + \frac{0''1201}{cm} C_{42} \sin(2\Omega - 2\lambda_{\odot} + \epsilon_{42}) + \ldots \quad (8)$$

which can be rewritten as

$$A_1 \sin(\omega_1 t + \epsilon_1) = \eta_1 \hat{C}_{22} \sin(2\Omega - 2\lambda_{☾} + \hat{\epsilon}_{22}) \quad (9)$$

$$A_2 \sin(\omega_2 t + \epsilon_2) = \eta_2 \hat{C}_{22} \sin(2\Omega - 2\lambda_\circ + \hat{\epsilon}_{22}) \tag{10}$$

where

$$\eta_1 \hat{C}_{22} \begin{bmatrix} \cos \\ \sin \end{bmatrix} \hat{\epsilon}_{22} = \frac{0''.01003}{cm} C_{22} \begin{bmatrix} \cos \\ \sin \end{bmatrix} \epsilon_{22} + \frac{0''.009612}{cm} C_{42} \begin{bmatrix} \cos \\ \sin \end{bmatrix} \epsilon_{42} \tag{11}$$

$$\eta_2 \hat{C}_{22} \begin{bmatrix} \cos \\ \sin \end{bmatrix} \hat{\epsilon}_{22} = \frac{0''.1253}{cm} C_{22} \begin{bmatrix} \cos \\ \sin \end{bmatrix} \epsilon_{22} + \frac{0''.1201}{cm} C_{42} \begin{bmatrix} \cos \\ \sin \end{bmatrix} \epsilon_{42} \tag{12}$$

The right hand sides of Equations 11 and 12 can be evaluated using Table 3, so the amplitudes $\eta_1 \hat{C}_{22}$ and $\eta_2 \hat{C}_{22}$ can be calculated along with the $\hat{\epsilon}_{22}$ phase angles. These are compared with the satellite derived values in Table 4. Appearing also in Table 4 are the results of Goad and Douglas (1976) for 1967-92A using the first 160 days of the 312 day data span.

Since Equations 9 and 10 are of the same form as Equations 5 and 6, one can compute values for the "combined" $\hat{C}_{22}$ coefficients by setting $\eta_1 = \frac{0''.01003}{cm}$ and $\eta_2 = \frac{0''.1253}{cm}$ in Equations 11 and 12. These are given in Table 5, and can be compared with the satellite derived values in Table 2.

CONCLUSIONS

The results for the M_2 tide from the satellite inclination data are in good agreement, within the error bounds of the solution, with the numerical models found in the literature and with the results of Goad and Douglas (1976). However, for the S_2 tide the satellite results give a somewhat larger effect than that indicated by the numerical model of Bogdanov and Magarik (1967). This is probably due to inadequate solar radiation pressure modelling in the orbit determination procedure.

While the node data for this satellite does not allow a solution for parameters of individual ocean tidal constituents, it has been shown that a combination of several tidal frequencies fits the data very well.

Another limitation on the meaningfullness of the parameters derived in this paper is the manner in which the solid Earth and atmospheric tidal effects were "removed" from the data apart from the ocean tides. Ideally, a comprehensive perturbation theory including both solid Earth and fluid tidal effects and their interactions is needed. Musen (1973) and Estes (1974; 1975) have done further work along this line.

In order to recover a sufficient number of parameters for a more complete tidal model, data from a number of satellites and/or more than one orbital element is necessary. Towards this end, in addition to GEOS-1, GEOS-2, and 1967-92A, the satellites GEOS-3 and STARLETTE are also being studied.

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Table 1

Theoretical Amplitudes of the Ocean Tide Perturbations on the Orbit of 1967-92A

Tidal Constituent	Period (days)	Inclination Perturbation (sec)	Node Perturbation (sec)
M_2	13.6	0.01003 C_{22}	0.0023 C_{22}
O_1	13.6	0.00006 C_{21}	0.0050 C_{21}
K_1	4817	0.0230 C_{21}	0
K_2	2409	1.777 C_{22}	68.8 C_{22}
S_2	170	0.1253 C_{22}	0.3433 C_{22}
P_1	176	0.0008 C_{21}	0.0626 C_{21}
R_2	116		0.1604 C_{22}
T_2	317		1.196 C_{22}
S_1	340		0.1166 C_{21}
ψ_1	395		0.1579 C_{21}
π_1	119		0.0428 C_{21}

Table 2

Ocean Tide Parameters Derived from 1967-92A Inclination Data

Tidal Constituent	C_{22} (cm)	ϵ_{22} (degrees)
M_2	3.0 ± 0.8	319 ± 14
S_2	2.25 ± 0.06	286.7 ± 1.4

Table 3

Ocean Tide Parameters from Numerical Models

Source	Tidal Constituent	C_{22} (cm)	ϵ_{22}	C_{42} (cm)	ϵ_{42}
Bogdanov and Magarik (1967)	M_2	4.3	325°	1.7	116°
Pekeris and Accad (1969)	M_2	4.40	340°	1.4	170°
Zahel (1970)	M_2	4.70	345°	1.35	75°
Hendershott (1972 - Model 1)	M_2	5.10	316°	1.2	115°
Hendershott (1972 - Model 2)	M_2	5.4	275°	1.1	75°
Bogdanov and Magarik (1967)	S_2	1.6	310°	0.2	90°

Table 4

Amplitudes and Phases of Ocean Tide Perturbations on the Inclination of 1967-92A

Source	Tidal Constituent	Amplitude (sec)	Phase
Satellite data	M_2	0.031 ± 0.007	319° ± 14°
Goad and Douglas (1976)	M_2	0.0385 ± 0.0060	331° ± 10°
Bogdanov and Magarik (1967)	M_2	0.030	340°
Pekeris and Accad (1969)	M_2	0.031	336°
Zahel (1970)	M_2	0.049	0°.4
Hendershott (1972 - Model 1)	M_2	0.041	322°
Hendershott (1972 - Model 2)	M_2	0.044	280°
Satellite data	S_2	0.283 ± 0.007	286°.7 ± 1°.4
Bogdanov and Magarik (1967)	S_2	0.183	315°

Table 5

Combined Ocean Tide Coefficients

Source	Tidal Constituent	$\hat{C}_{22}$ (cm)
Bogdanov and Magarik (1967)	M_2	3.0
Pekeris and Accad (1969)	M_2	3.1
Zahel (1970)	M_2	4.9
Hendershott (1972 - Model 1)	M_2	4.0
Hendershott (1972 - Model 2)	M_2	4.4
Bogdanov and Magarik (1967)	S_2	1.5

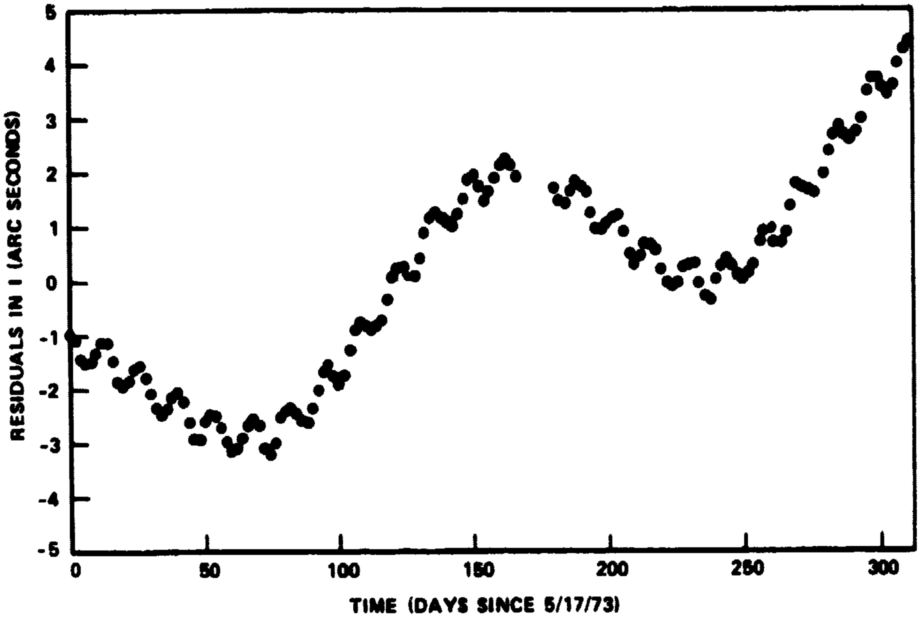

Figure 1. Total Tidal Perturbation of the Inclination of 1967-92A

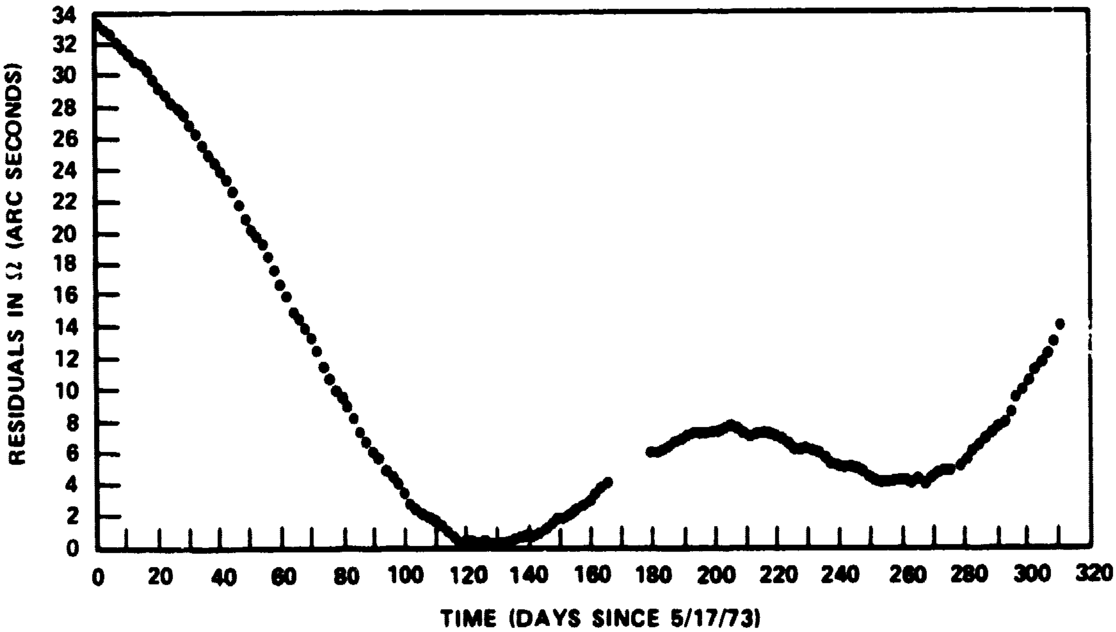

Figure 2. Total Tidal Perturbation of the Longitude of Ascending Node of 1967-92A

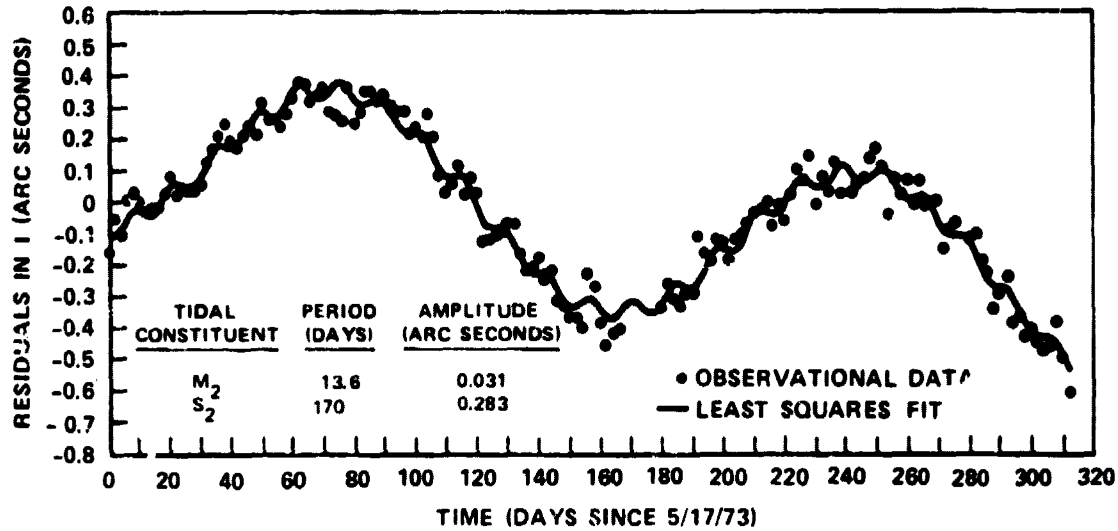

Figure 3. Perturbation of the Inclination of 1967-92A
Caused by Ocean Tides

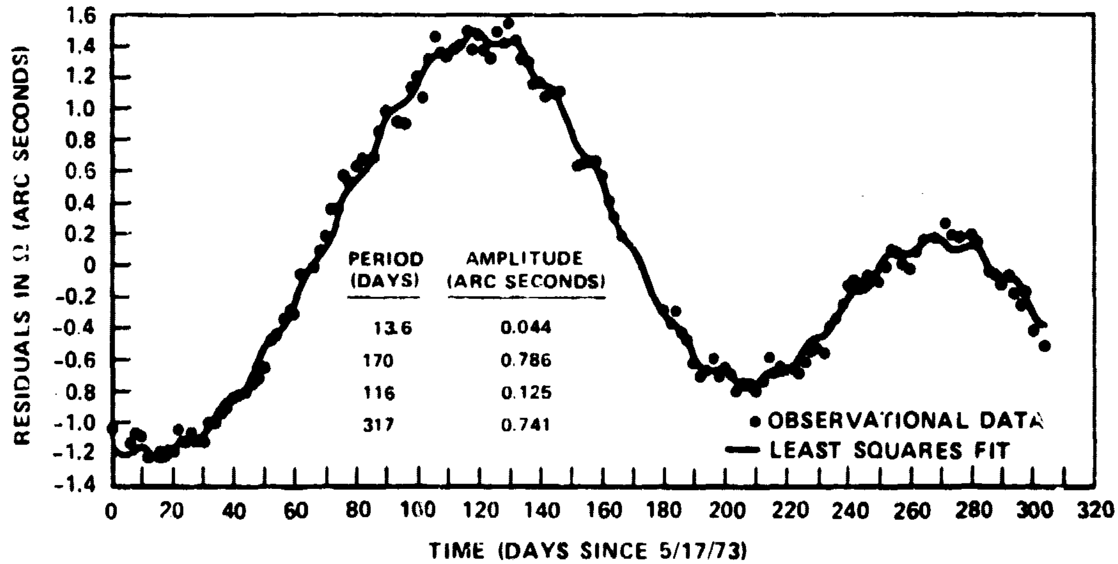

Figure 4. Perturbation of the Longitude of Ascending Node of 1967-92A
Caused by Ocean Tides

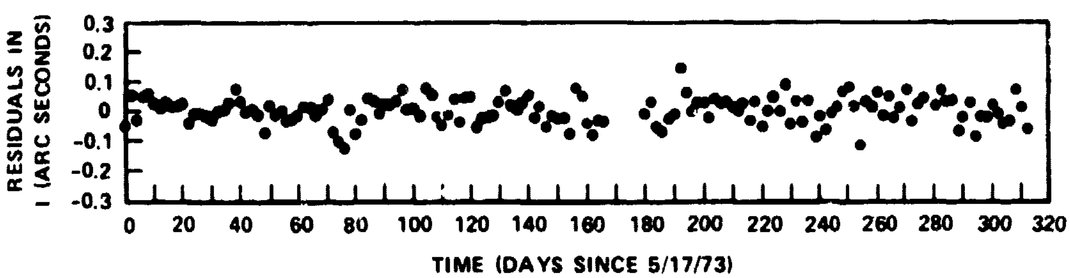

Figure 5. 1967-92A Inclination Residuals After Removal of Tidal Effects

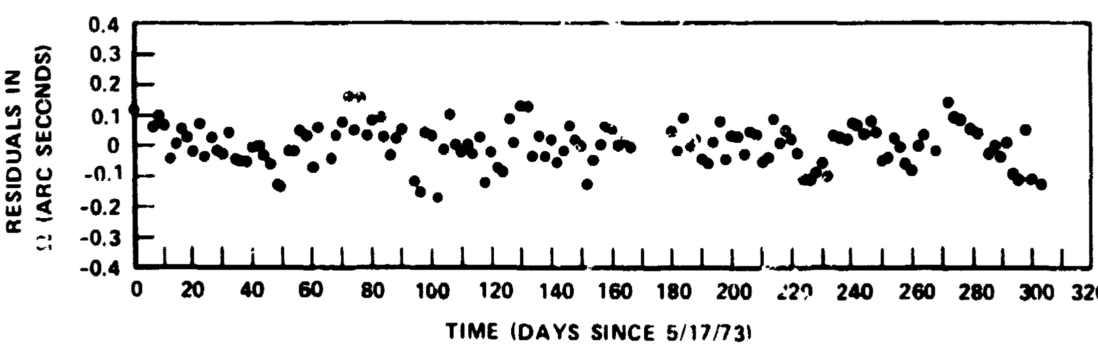

Figure 6. 1967-92A Longitude of Ascending Node Residuals After Removal of Tidal Effects