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ESTIMATION OF STIFFENING EFFECT OF SHAFT AND HOUSING MATERIAL OUTSIDE PROJECTED AREA OF A ROLLING ELEMENT BEARING

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16. Abstract

In the analysis of distortions occurring in rolling-element bearings, it is common to neglect the stiffening effect of shafting outside the bearing region. The magnitude of such an effect will be dependent primarily on the bearing width-to-bore ratio, the shaft geometry, and the location of the bearing on the shaft. The work presented here gives an estimate of the stiffening effect for a wide range of these variables. In addition, brief consideration is given to the parallel situation existing at the outer ring housing.

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ESTIMATION OF STIFFENING EFFECT OF SHAFT AND HOUSING MATERIAL OUTSIDE PROJECTED AREA OF A ROLLING ELEMENT BEARING

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SUMMARY

The stiffening effect of the shafting outside the bearing region is frequently neglected in the analysis of deflections occurring in rolling-element bearings. If it is desired to take some account of the effect, while not directly including such shafting in the analysis, there is at present no published data to enable an accurate assessment. The work presented here provides some such information considering a wide range of the parameters on which the stiffening effect is dependent. The data provided encompass six values of bearing width to bore ratio, six values of shaft inner-to-outer diameter ratio, six values of the ratio of shaft length outside the bearing to shaft outer diameter, and two alternative locations of the bearing on the shaft. These values are fully permutated.

A comparable stiffening effect at the outer race housing can also be obtained. Brief consideration is given to this particular aspect. A finite-element computer program was used to generate the results presented.

The results indicate that for some circumstances the stiffening effect can be considerable. It is shown that the material very close to the bearing (or loaded region) is primarily responsible for the increase in stiffness. The extent to which this increase in stiffness is reflected in the change in raceway deflection will depend on the relative stiffnesses of the shaft or housing and corresponding ring.

INTRODUCTION

Occasionally a designer may wish to have an estimate of by how much the diametral clearance (or radial internal clearance) in a rolling-element bearing alters between the

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operating and unmounted conditions. This might be the case, for example, for applications in the aircraft gas-turbine-engine field. Many factors may influence the change in internal clearance. These include the interference fit between rings and their mountings, rotational inertia effects, differential thermal expansion considerations, and the loading applied to the bearing.

The common approach to determining the change in clearance is to estimate deflections at the inner raceway, at the outer raceway, and of the rolling elements separately and to sum the effects. Normally only axisymmetric variations are considered. In dealing with the rings, a width of shaft and housing equal to the bearing axial dimension is taken (ref. 1). Since the shaft and housing will extend outside the projected area of the bearing the stiffness of the portion considered will be less than that actually "seen" by the ring in question. An allowance for the stiffening effect of the material outside the bearing region can easily be made (ref. 1) if appropriate data are at hand. The purpose of this paper is to provide some such data.

The author acknowledges Dr. B. Parsons, Reader in Mechanical Engineering, University of Leeds, with whom in 1974-75, he jointly undertook some private work in the same area as described in the present paper. On that occasion that range of variables covered was considerably less than here, though the method of analysis was identical.

THE ANALYSIS

Inner Ring and Shaft

In dealing with an inner ring and its shaft, the important variables are the ratio of shaft inner to outer diameter d/D, the ratio of bearing width to shaft outer diameter W/D, and the location of the bearing on the shaft. In regard to the last parameter, figure 1 shows the two situations considered. First a bearing of width W is taken to be located centrally upon a shaft, a length L of shafting extending on either side. Second, the bearing is taken to be at the end of a shaft (an end bearing) with a length L of shaft on one side only.

A uniformly hollow shaft is assumed, with the influence of any flanges, pulleys, or changes in section not considered. The extent to which the geometry chosen adequately reflects a particular situation will have to be gaged for each case considered.

In the finite-element computer analysis undertaken, a uniformly distributed load was applied axisymmetrically to the shafting over the outer surface where the bearing was assumed located (fig. 1). The radial deflection in the loaded region of the shaft surface is not uniform, and a mean value δ_m was calculated. If there were no shafting outside the bearing region, the uniform surface deflection δ could be calculated from the

Lamé-Clapeyron formulation for uniformly loaded, hollow cylinders, namely (ref. 2),

$$\delta = \frac{pD}{2E}\left(\frac{D^2 + d^2}{D^2 - d^2} - \mu\right)$$

where p is the uniform compressive stress on the outer diameter, E is Young's modulus, μ is Poisson's ratio, and δ is directed radially inwards.

The ratio of δ/δ_m can be thought of as a stiffening factor, and a percentage increase in stiffness Δk, due to the presence of the shafting outside the bearing region, can be defined as follows:

$$\Delta k = \left(\frac{\delta}{\delta_m} - 1\right) \times 100$$

A value of Poisson's ratio of 0.3 was used in all calculations.

Outer Ring and Housing

The generalization of the geometry of the housing in which the outer ring is supported is a more difficult task. Usually there are complicating design features, such as ribbing, which make the specification of a typical arrangement difficult. However, for completeness a limited number of calculations have been performed to determine the stiffening effect for a cylinder loaded axisymmetrically over a portion of its inner surface with a uniform pressure. This geometry is shown in figure 2 and could be taken as representative of some situations.

As figure 2 shows, only the case of a bearing located symmetrically within its housing was examined. The notation for diameters has been reversed from the cases of figure 1, the reference diameter D now being the inner diameter of the housing.

The technique of analysis was identical to that for the shaft situation, the mean radial deflection δ_m at the housing inner surface under the load being determined by a finite-element analysis. The Lamé-Clapeyron formulation for the case of no housing outside the bearing region gives a uniform deflection

$$\delta = \frac{pD}{2E}\left(\frac{d^2 + D^2}{d^2 - D^2} + \mu\right)$$

where p this time is the uniform compressive stress on the inner diameter. Once again, a value of Poisson's ratio of 0.3 was used. A percentage increase in stiffness Δk due to the housing material outside the bearing region can be defined as before.

RESULTS

More attention will be given to the stiffening effect of shaft material outside the bearing region than housing material, because (1) it is more common to have a rotating inner ring with a press fit on the shaft and only a sliding fit at the stationary outer. (The stiffening effect is more likely to be of interest with an interference fit situation.) and (2) the inner ring and shaft geometry is more amenable to generalization and the type of analysis undertaken.

For the inner ring and shaft configurations, the values of the independent variables considered are as follows:

$$d/D = 0,\ 0.1,\ 0.3,\ 0.5,\ 0.7,\ 0.9$$

$$W/D = 0.1,\ 0.25,\ 0.5,\ 0.75,\ 1.0,\ 1.5$$

$$L/D = 1,\ 2,\ 3,\ 4,\ 5,\ 10$$

These values were used for both the central and end bearing situations. A large range has been taken for completeness; for example, the value of W/D of 0.1 represents an extremely narrow bearing, possibly narrower than occurs even in aircraft gas-turbine bearings. Conversely, W/D = 1.5 might only be appropriate for bearings used in tandem arrangements. The values of the percentage increase in stiffness for the shaft loading situations are presented for the central and end bearing cases in tables I and II, respectively. For calculations associated with the outer ring and housing configuration, the range of variables covered is detailed below:

$$d/D = 1.1,\ 1.3,\ 1.5$$

$$W/D = 0.1,\ 0.25,\ 0.5$$

$$L/D = 0.1,\ 0.5,\ 1.0$$

The results are presented in table III.

DISCUSSION

Shaft Stiffening Effect

In examining the results that have been presented, it will be convenient to refer to the typical plots of the data shown in figures 3 and 4. In figure 3 the percentage increase in stiffness is plotted against the length to diameter ratio L/D for various values of W/D and d/D, considering both the central and end bearing arrangements. Figure 4 slows the percentage increase in stiffness plotted this time with the diameter ratio d/D as abscissa. In this figure, one combination of L/D and W/D is taken for each bearing arrangement.

The trends of the results presented in tables I and II are in the main predictable and can be clearly seen in figure 3:

(1) There is a greater stiffening effect for the central rather than the end bearing, other factors remaining constant.

(2) The stiffening effect reduces with increasing W/D, other factors remaining constant.

(3) The stiffening effect reduces with decreasing L/D, other factors remaining constant.

However, it is not quite so clear why, other parameters remaining fixed, the stiffening effect can show a peak as the diameter ratio d/D varies. This is demonstrated in figure 4.

At high values of W/D the stiffening effect is quite small and relatively independent of d/D. However, the percentage increase in stiffness can be considerable at the lower values of W/D, which are more appropriate to aircraft gas-turbine-engine practice. The high values of the increase in stiffness do not reflect, necessarily, effects of comparable magnitude, say, in raceway deflection. The relative stiffnesses of the shaft and inner ring play an important part in what change is obtained in raceway deflection due to the shaft stiffening effect. This is best demonstrated through an example. Consider two hollow cylinders of the same material representing the shaft and inner ring of a rolling-element bearing and having the following dimensions:

	Outer diameter, m	Inner diameter, m
Inner ring	0.117	0.106
Shaft	.106	.071

In practice some interference will probably exist between the two components; however, for the present purposes the value of this is immaterial.

If the cylinders are first taken to be of equal length, it can be shown, using the formulae presented earlier, that the stiffnesses of the shaft and ring (based on their common uniform loading divided by their own deflection at the contacting surface) have the ratio 4.41; that is, the shaft is 4.41 times stiffer than the ring. If now it is taken that the bearing inner ring sits centrally on a shaft having a value of $L/D = 4$ and of $W/D = 0.25$, we see from table I that for $d/D = 0.7$ an increase in stiffness of 106 percent is predicted for the shaft. However, simple calculations using the Lamé-Clapeyron deflection formulae show that the corresponding increase in raceway deflection is only 10 percent. This is because for the case of equal component lengths the shaft is already considerably stiffer than the ring.

As would be expected, the material close to the bearing region influences the stiffening effect most significantly (see fig. 3). Despite some large percentage increase in stiffnesses at $L/D = 1$ (for small values of W/D), lower values of L/D have not been considered. This is primarily because of the attenuation of the effect of raceway deflection as described above. It is worthy of note that for values of L/D greater than 1, the percentage increase in stiffness increases fairly linearly with L/D.

Housing Stiffening Effect

The trends of the stiffening effect, which have been noted already for the shaft, are matched at the housing (see table III). Because of the different range of variables considered for the housing analysis, however, the results serve to emphasize how strongly the material very close to the bearing (or loaded region) influences the stiffening effect. For example, for the following situation,

$$\frac{d}{D} = 1.1 \quad \text{and} \quad \frac{W}{D} = 0.1$$

a length of housing equal to the bearing width on either side effects an increase in stiffness of 146 percent. Extending the housing in each direction to a length 10 times the width only causes a further 46 percent increase in stiffness.

GENERAL COMMENTS

(1) The results presented are to some extent dependent on the finite-element mesh employed to represent the situation in question. A wide range of parameters has been considered, and it would have been inefficient to have investigated optimum element aspect ratios for all the cases dealt with. A brief investigation of some situations in which

extreme values of the independent parameters were combined has indicated that the results would not have been affected by much more than 10 percent for the worst case, and by far less in general if more attention had been paid to the mesh used. Errors are most likely when the shaft length is extremely large compared with the loaded region. The stiffening effect for such situations will be overestimated.

(2) The functioning of the finite-element program used for the analyses was checked against analytical predictions for the case L/D equal to zero. Excellent agreement of the surface deflection values was obtained.

(3) The results presented will be useful in estimating changes in radial internal clearance in a rolling-element bearing where an axisymmetric loading situation is taken to pertain and axial variations are neglected. The data presented may also be useful in plane stress analyses where nonaxisymmetric conditions apply.

CONCLUSIONS

For a wide range of the independent parameters, the stiffening effect of shaft and housing material outside the projected area of a rolling-element bearing has been detailed. Such data will be useful in currently applied techniques to estimate the raceway deflection of aircraft gas-turbine-engine bearings under operating conditions.

The percentage increase in stiffness due to considering material outside the loaded region of a shaft on a housing can be considerable, particularly for bearings having small width to bore ratios. However, the corresponding effect on raceway deflection need not be large since the relative stiffnesses of the shaft or housing and corresponding ring are important in this regard.

It has been shown that the material very close to the bearing has the most substantial stiffening effect. For a shaft with values of length to diameter ratios L/D greater than unity, the percentage increase in stiffness shows an approximately linear variation with L/D for a wide range of other conditions. Also, the stiffening effect has a peak value as the shaft inner to outer diameter ratio is varied over its possible range, other parameters being kept fixed.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, February 15, 1977,
505-04.

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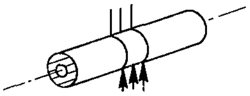
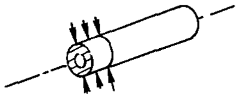
2. Timoshenko, S.: Strength of Materials. Part II: Advanced Theory and Problems. 3rd ed. Van Nostrand Co., Inc., 1956, p. 205.

TABLE I. - INCREASE IN STIFFNESS FOR SHAFT WITH CENTRAL BEARING



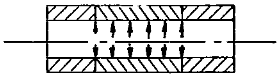
Diameter ratio, d/D	Length to diameter ratio, L/D	Width to diameter ratio, W/D					
		0.1	0.25	0.50	0.75	1.0	1.5
		Percentage increase in stiffness, Δk					
0	1	158	55	23	15	11	9
	2	180	61	25	16	12	9
	3	204	69	29	18	14	10
	4	230	77	32	20	15	11
	5	256	86	35	22	17	12
	10	367	120	47	29	21	16
0.1	1	162	56	24	15	12	9
	2	183	63	26	17	13	10
	3	208	71	29	18	14	11
	4	234	79	33	20	16	12
	5	260	87	36	22	17	13
	10	371	122	48	29	22	16
0.3	1	191	67	27	17	13	9
	2	212	73	30	18	14	10
	3	236	81	33	20	15	11
	4	262	90	36	22	16	12
	5	288	98	40	24	18	13
	10	410	139	55	32	24	17
0.5	1	236	80	30	18	13	10
	2	254	86	33	19	14	11
	3	279	94	36	21	16	11
	4	306	103	40	23	17	12
	5	334	113	44	26	19	13
	10	478	164	64	36	25	18
0.7	1	251	77	27	16	13	10
	2	274	85	30	18	14	10
	3	302	95	35	20	15	11
	4	334	106	39	23	17	12
	5	368	119	44	25	19	13
	10	548	185	68	37	26	18
0.9	1	178	47	18	13	11	9
	2	216	61	23	15	13	10
	3	258	76	28	18	15	11
	4	303	91	32	21	16	12
	5	349	106	37	23	18	13
	10	583	172	53	31	24	18

TABLE II. - INCREASE IN STIFFNESS FOR SHAFT WITH END BEARING

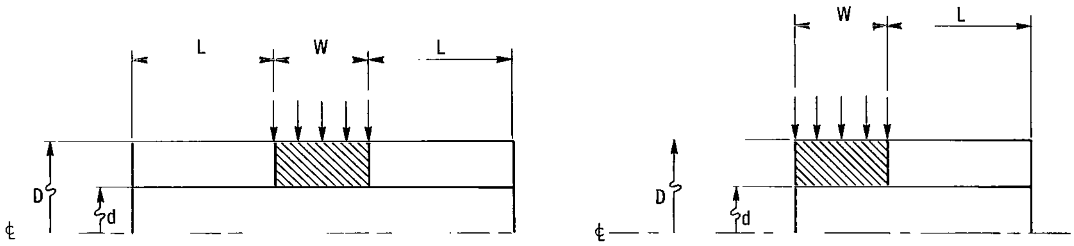


Diameter ratio, d/D	Length to diameter ratio, L/D	Width to diameter ratio, W/D					
		0.1	0.25	0.5	0.75	1.0	1.5
		Percentage increase in stiffness, Δk					
0	1	48	17	11	8	7	6
	2	57	20	12	9	8	7
	3	68	23	13	10	9	7
	4	79	26	14	11	10	8
	5	90	28	16	12	10	8
	10	134	39	20	16	13	11
0.1	1	50	18	11	9	8	6
	2	58	20	12	10	8	7
	3	69	23	13	11	9	7
	4	80	26	15	12	10	8
	5	91	29	16	12	11	9
	10	137	40	21	16	13	11
0.3	1	57	20	12	9	8	6
	2	66	22	13	10	9	7
	3	76	25	14	11	9	8
	4	88	28	15	12	10	8
	5	100	31	17	13	11	9
	10	154	45	22	17	14	11
0.5	1	64	21	12	10	8	7
	2	73	23	13	10	9	7
	3	85	26	14	11	10	8
	4	99	30	16	12	10	8
	5	114	34	17	13	11	9
	10	183	52	24	17	14	11
0.7	1	60	19	12	10	8	7
	2	73	22	13	10	9	7
	3	89	26	15	11	10	8
	4	106	30	16	12	10	8
	5	125	35	18	13	11	9
	10	213	55	25	18	15	11
0.9	1	43	16	11	9	7	6
	2	63	20	13	10	8	7
	3	84	25	15	11	9	7
	4	103	28	16	12	10	8
	5	122	32	18	13	11	9
	10	203	44	24	18	15	11

TABLE III. - INCREASE IN STIFFNESS FOR OUTER RING HOUSING CASE



Diameter ratio, d/D	Length to diameter ratio, L/D	Width to diameter ratio, W/D		
		0.1	0.25	0.5
		Percentage increase in stiffness, Δk		
1.1	0.1	146	37	14
	.5	177	47	17
	1.0	192	52	19
1.3	0.1	177	64	25
	.5	295	93	33
	1.0	318	100	35
1.5	0.1	174	68	30
	.5	347	119	44
	1.0	376	131	50



(a) Central bearing.

(b) End bearing.

Figure 1. - Shaft loading configurations.

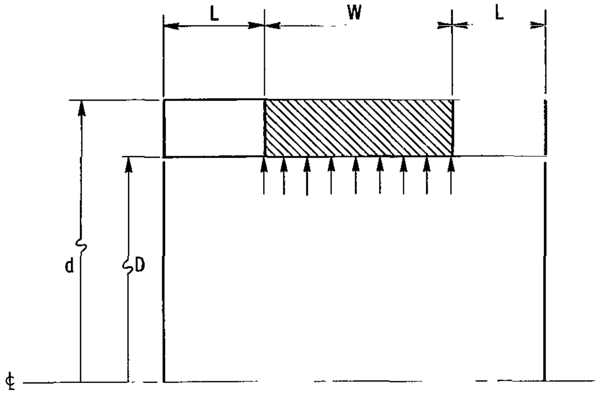
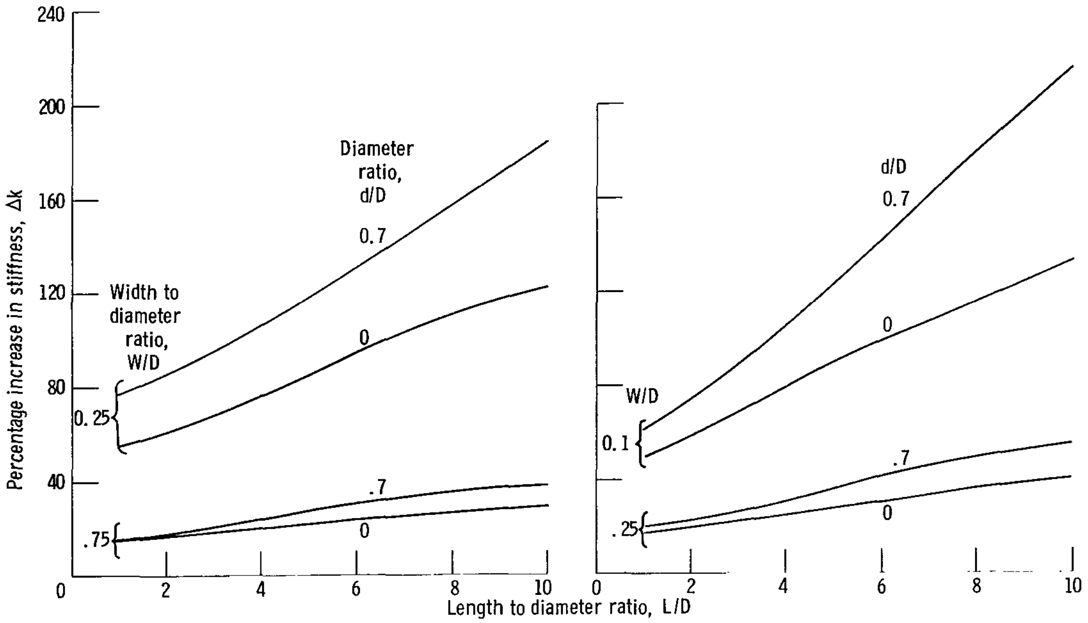


Figure 2. - Outer ring housing loading configuration.



(a) Central bearing.

(b) End bearing.

Figure 3. - Typical variations of percentage increase in stiffness.

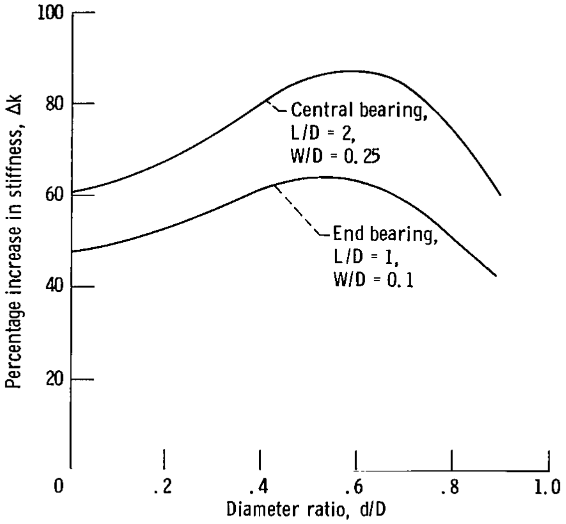


Figure 4. - Typical variation of percentage increase in stiffness with diameter ratio.

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