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PART A

HOWARD UNIVERSITY
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THE DYNAMICS AND OPTIMAL CONTROL OF SPINNING SPACECRAFT WITH MOVABLE TELESCOPING APPENDAGES

PART A

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ABSTRACT

This report considers the dynamics and optimal control of spinning spacecraft with telescoping appendages and is an extension of the research reported in Parts I and II (May 1974 - May 1976). Part I concentrated on the analysis of the motion of a spinning spacecraft during the deployment of telescoping type of varying length appendages and fixed length appendages whose orientation with respect to the main hub can vary. In addition, the use of telescoping appendages to detumble a spacecraft with random spin was also considered. In Part II, the motion and stability analysis of spinning spacecraft with hinged appendages and an application of the linear regulator theory using a quadratic performance index were considered. Also, the time optimal control with a single boom system was considered analytically.

In this report, the problem of optimal control with a minimum time criterion as applied to a single boom system for achieving two axis control is treated in detail. The special case where the initial conditions are such that the system can be driven to the equilibrium state with only a single switching manuever in the bang-bang optimal sequence has been examined analytically. The system responses are presented. Next, our previous application of the linear regulator problem for the optimal control of the telescoping system is extended to consider the effects of measurement and plant noises.

The noise uncertainties are included with an application of the estimator - Kalman filter problem. Different schemes for measuring the components of the angular velocity are considered. Analytical results are obtained for special cases and numerical results are presented for the general case.

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NOMENCLATURE

a	=	offset of the control boom with end mass m_1 from the 2,3 (y,z) plane
A	=	linearized system state matrix
b	=	offset of the control boom with end mass m_1 from the 3,1 (z,x) plane
B	=	linearized system control matrix
c	=	offset of the control boom with end mass m_2 from the 3,1 (z,x) plane
C	=	maximum value of the control U; also, optimal feedback control gain matrix
E	=	expected value operator
d	=	offset of the control boom with end mass m_2 from the 1,2 (x,y) plane
F	=	optimal filter gain matrix
G,H	=	matrices used in defining plant disturbance and measurement processes
I_1, I_2, I_3	=	principal moments of inertia of the main part of the spacecraft
J	=	cost functional for optimal control
ℓ_m	=	maximum value of each control boom length
M	=	mass of main part of the spacecraft
m	=	end mass
m_1, m_2	=	control boom end masses
P	=	covariance of estimation error
Q	=	positive definite symmetric state weighting matrix

R	=	positive definite symmetric control weighting matrix
$S, \hat{S}$	=	covariances of state and estimate, respectively
t	=	time
U	=	control vector
v	=	measurement noise vector
V	=	covariance of measurement noise
w	=	plant noise vector
W	=	covariance of plant noise
x	=	coordinate of the control boom end mass m_2 along the '1' axis (control variable)
X	=	state vector of the system
$\hat{X}$	=	state vector of the estimate
z	=	coordinate of the control boom end mass m_1 along the '3' axis (control variable)
ω_i	=	angular velocities about the 1,2,3 axes, respectively ($i = 1,2,3$)
$\omega_{T_{max}}$	=	maximum expected value of transverse rate
Ω	=	nominal main body spin rate
ε	=	error $(\hat{X} - X)$
α	=	ω_1/Ω, nondimensionalized form of ω_1
β	=	ω_2/Ω, nondimensionalized form of ω_2
γ	=	$\omega_3/\Omega - 1$, variation of the nondimensionalized form of ω_3 from the nominal value
ρ	=	linear damping present in boom driving mechanism
τ	=	Ωt, dimensionless time
τ_s	=	switching time
τ_f	=	final time

ζ	=	z/ℓ_m, dimensionless form of z
ξ	=	x/ℓ_m, dimensionless form of x
θ	=	nutation angle
$\cdot$	=	indicates differentiation with respect to t
$'$	=	indicates differentiation with respect to τ
(0)	=	indicates initial conditions
o	=	indicates original system quantities in the time domain
1,2,3	=	principal axes of main spacecraft

LIST OF ILLUSTRATIONS

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I. INTRODUCTION

This report will describe a continuation of the NASA sponsored research already accomplished during May 1974 - May 1976 (Parts I and II) on the dynamics of spin stabilized spacecraft with movable appendages.[1,2] In Part I, the equations of motion have been developed for the telescoping boom system where all the appendages are extended along the hub principal axes. A control strategy based on an application of Lyapunov's second method was used to recover a randomly tumbling spacecraft and approach a final state of either zero inertial angular velocity or a flat spin about one of the hub principal axes.[1,3] The dynamics of this system during nominal deployment of the booms with a small nutation angle have also been considered both analytically for special cases of a nearly spherical hub, and numerically for the more general case.[1,4]

In Part II, the following topics were treated: the dynamics and an extensive stability analysis of a spacecraft with hinged appendages of fixed length;[2,5] an examination of linear optimal control theory as applied to the deployment maneuver of a telescoping boom system (offset from the hub principal axes) by selecting different integrand function;[2,6] and the time optimal control of a nutating spacecraft using a single offset telescoping boom system.[2]

The topics considered during the present NASA grant are: the problem of optimal control with a minimum time criterion for a single offset boom system; and an application of the linear regulator problem for the optimal control of the offset telescoping system with measurement and plant noises present.

The first phase of the current study will examine the time optimal control of a nutating spacecraft using a single offset telescoping boom system. This is an extension of the work consdiered in Ref.2. It is assumed that the spacecraft consists of a rigid central hub and a movable telescoping boom with an end mass which is linearly offset from the nominal hub spin axis. An advantage of such a telescoping system as used in the control of a spinning spacecraft system is its potential reuse. The use of such a moving (internal) mass device for the detumbling of spacecraft was first proposed and described by Edwards and Kaplan.[7,8] The motion of the control mass was along a linear track fixed in the vehicle where the control variable was taken as the mass acceleration relative to the main part of the spacecraft.

The optimal control of a spin-stabilized spacecraft with one or two movable telescoping booms was the subject of a recent paper.[6] The boom end mass positions were controlled such that a quadratic cost functional involving the weighted components of excess angular velocity plus the control effort itself was minimized when the terminal time was unspecified. For such a system, the computation of the control law involved the solution of the matrix Riccati algebraic equation.[6] It was concluded that for three-axis control at least two offset booms (moving orthogonal to each other) would be required, whereas two-axis (nutation) control could be achieved by using a single offset boom constrained to move parallel to the spin axis.[6]

A very recent investigation by Kunciw and Kaplan[9] utilizes a first-order gradient optimization technique to show how a movable mass control system may be employed to detumble a general asymmetric space station about a principal axis in minimum time. Results indicate that the detumbling time is minimized for larger values of control mass and lengths of the linear track.[9]

The present study extends the work of Ref. 6 and complements that of Ref. 9 by analytically determining the boom (mass) control logic such that the terminal time will be minimized for the case where two-axis control of a symmetric spacecraft is required. The equations of rotational motion are developed and linearized about the desired final state. This problem has been examined analytically for the special case of a single offset boom where it is assumed that the initial conditions are such that the system can be driven to the equilibrium (rest) state with only a single switching maneuver in the bang-bang optimal sequence. For this system it is possible to obtain an analytical solution for the switching and final times in terms of the initial conditions and magnitude of the maximum value of the control force.[10,11] Also the required boom motion can be determined analytically for this linear system. Some typical numerical results based on these solutions are discussed.

The second phase of this year's study in the area of optimal control extends the previous application[2,6] of the deterministic linear regulator problem for the optimal control of an offset telescoping boom system to include the effect of noise uncertainties both in the plant as well as in the measurements.

The differences between the desired state vector components and the actual components with noise included are now incorporated within the control logic with an application of the Kalman filter.[12]

An application of modern control theory to nonrigid spacecraft has been very recently considered in Ref. 13. Here the established procedures of linear quadratic Gaussian optimal estimation and control were developed and interpreted for their application to the problem of attitude control of spacecraft with dynamically significant elastic appendages. The conclusions were that the techniques of modern control theory offer promise for practical applications such as spacecraft attitude control, but that the mathematical theory of modeling needs development and the limitations of spacecraft computer capacity require reduced estimator models.[13]

The present study is an extension of Ref. 6 where the measurement noise and plant noise are now considered in the design of the optimal controller. The equations of rotational motion are developed and linearized about the desired final state. For the purpose of simplicity, the actuator dynamics (the motor-drive mechanism that extends or retracts the booms) will be ignored and the boom mass dynamics will be treated as the control variables. (The assumption was also used in Ref. 13 for a different application.) The measurement noise and plant noise in the physical system are assumed to be white Gaussian processes with zero mean.

For the linear system with quadratic performance indices, it has been shown[12] that the optimal control logic is a Kalman filter used in conjunction with the optimal deterministic controller.

The model of the estimator (filter) is the same as that of the "plant model." The average performance (RMS value) of this controlled system in the presence of noise can be predicted from the covariances of the error and estimate.[12] The general system response for non-zero initial conditions is obtained by simulating the linearized equations with the control and filter gains as obtained from their respective matrix Riccati differential equations.

Both two and three axis optimal control of spinning spacecraft using movable telescoping offset boom systems will be considered. The dynamics of such a system will be studied analytically for special cases and numerically for the general case.

II. TIME OPTIMAL CONTROL WITH SINGLE OFFSET BOOM

For satellites with high attitude accuracy, control jets are often required. However, the maximum torque produced by the jets will be bounded. Also the operation of the thrusters are often limited by the weight and propellant capacity of the thruster system. Instead of jet systems, externally movable appendages can be used for controlling the attitude of the spacecraft.[6] The main advantage of movable appendages is their potential reuse. Optimal control theory can be applied to minimize the time required for returning the state of the (linear) system to its nominal value.

1. Statement of the Time Optimal Control Problem

The equations of motion of a linear, controlled time-invariant system are represented by:

$$X' = AX + BU \tag{2.1}$$

where

X = state vector of the system

A = system (plant) matrix

B = control matrix

U = control vector

Here the problem of determining the control $U(|U| \leq C)$ which forces the system (2.1) from the initial state, $X(0)$, to zero state in minimum time is treated.

An admissible control $U(\tau)$, transferring the system state from $X(0)$ to $X(\tau_f) = 0$, is found from the solution of Eq. (2.1) given by

$$X(\tau) = e^{A\tau} X(0) + \int_0^{\tau} e^{A(\tau-\phi)} BU(\phi)\, d\phi \tag{2.2}$$

For $X(\tau_f) = 0$, Eq. (2.2) reduces to

$$\int_0^{\tau_f} e^{-A\phi} BU(\phi)\, d\phi = -X(0) \tag{2.3}$$

Equation (2.3) will be used to determine the switching and final times of the control.

2. Application to Single Offset Boom System

As an application of the time optimal control theory, the movable single offset boom system as a two-axis nutation damper is shown in Fig. 2.1. The equations of rotational motion are developed and linearized about the desired final state of a spin about the z axis only ($\omega_z=\Omega$).[2] The linearized system equations (Ref. 2, Eqs. (5.4) and (5.5)) for the special case of a symmetrical hub (b=0, without loss of generality) result as:

$$\begin{bmatrix} \alpha' \\ \beta' \end{bmatrix} = \begin{bmatrix} 0 & -e \\ d & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [U] \tag{2.4}$$

where

$$U = n(\zeta'' + \zeta) \tag{2.5}$$

$$d = (\bar{I}_3 - \bar{I} + c_1^2)/(\bar{I} + c_1^2) \tag{2.6}$$

$$e = (\bar{I}_3 - \bar{I})/\bar{I} \tag{2.7}$$

$$n = c_1/(\bar{I} + c_1^2) \tag{2.8}$$

The nondimensionalized quantities are defined as: $\alpha = \omega_1/\Omega$; $\beta = \omega_2/\Omega$; $c_1 = a/\ell_m$; $\zeta = z/\ell_m$; $\bar{I}_i = I_i/\mu\ell_m^2$ $(i = 1,2,3)$ and for the assumed symmetry, $\bar{I}_1 = \bar{I}_2 = \bar{I}$; $\tau = \Omega t$; and τ represents the nondimensionless time. The variational coordinates are α,β whereas ζ represents the control variable and describes the end mass position. From Eq. (2.4) it is seen that the equations for the transverse angular velocity components have the form of a coupled two dimensional harmonic oscillator under the influence of the boom motion as a control force.

The solution for U(t), bringing the system state to rest in minimum τ_f, is known to be $U(\tau) = \pm C$, with the number of switches depending upon the initial state of the system.[10] Considering the initial states that can be driven to rest in a single switch (Fig. 2.2(a)), the control takes the form[11]

$$U(\tau) = K_1 \text{ for } 0 \leq \tau \leq \tau_s;\; U(\tau) = K_2 \text{ for } \tau_s \leq \tau \leq \tau_f \tag{2.9}$$

where

$$|K_1| = |K_2| = C$$

The state transition matrix, basing A on Eq. (2.4), is

$$e^{A\tau} = \begin{bmatrix} \cos \omega_0\tau & -\dfrac{e}{\omega_0} \sin \omega_0\tau \\ \dfrac{d}{\omega_0} \sin \omega_0\tau & \cos \omega_0\tau \end{bmatrix} \tag{2.10}$$

where

$$\omega_0 = \sqrt{de}$$

After substitution of Eqs. (2.9) and (2.10) into Eq. (2.3), one obtains:

$$(1 - \cos \omega_0\tau_s)\, K_1 - (\cos \omega_0\tau_f - \cos \omega_0\tau_s)\, K_2 = \alpha(0)d \tag{2.11}$$

$$(\sin \omega_0\tau_s)\, K_1 + (\sin \omega_0\tau_f - \sin \omega_0\tau_s)\, K_2 = -\beta\,(0)\omega_0 \tag{2.12}$$

The expressions for the switching time, τ_s, and the final time, τ_f, are obtained by solving Eqs. (2.11) and (2.12), with the result:

$$\tau_s = \frac{1}{\omega_0} \left\{ \tan^{-1} \sqrt{(g_2/g_1)^2 - 1} - \tan^{-1} (F/E) \right\} \tag{2.13}$$

$$\tau_f = \frac{1}{\omega_0} \left\{ \tan^{-1} \sqrt{(g_4/g_3)^2 - 1} - \tan^{-1} (F/E) \right\} \tag{2.14}$$

where

$$E = \alpha(0)\, d + K_1; \; F = \beta(0)\omega_0$$

$$\begin{aligned} g_1 &= (2K_2 - K_1)\, K_1 - (E^2 + F^2) \\ g_2 &= 2(K_2 - K_1) \sqrt{E^2+F^2} \\ g_3 &= (2K_2 - K_1)\, K_1 + (E^2 + F^2) \\ g_4 &= 2K_2 \sqrt{E^2+F^2} \end{aligned} \tag{2.15}$$

The control scheme for single switching and the phase plane response of the system for a given initial condition $X(0) = [\alpha(0)\ \beta(0)]^T$ are shown in Figs. 2.2(b) and 2.2(c), respectively.

3. Switching Boundary Determination

The solutions for $\alpha(\tau)$ and $\beta(\tau)$ are obtained from Eq. (2.4), with $U(\tau) = C$, as:

$$\alpha(\tau) = (\alpha(0) + \frac{C}{d}) \cos \omega_0\tau - \frac{\omega_0}{d}\, \beta(0) \sin \omega_0\tau - \frac{C}{d} \tag{2.16}$$

$$\beta(\tau) = \frac{d}{\omega_0}\, (\alpha(0) + \frac{C}{d}) \sin \omega_0\tau + \beta(0) \cos \omega_0\tau \tag{2.17}$$

The equation of the trajectories in the $\alpha(\tau)$, $\beta(\tau)\frac{\omega_0}{d}$ plane can be represented by:

$$(\alpha(\tau) + \frac{C}{d})^2 + (\beta(\tau)\frac{\omega_0}{d})^2 = (\alpha(0) + \frac{C}{d})^2 + (\beta(0)\frac{\omega_0}{d})^2 \tag{2.18}$$

These trajectories are circles with centers at (C/d,0). The switching boundary is composed of semicircles passing through the origin. For any given initial state X(0), the system state moves on the switching boundary (passing through the origin) for $\tau_s \leq \tau \leq \tau_f$ as shown in Figs. 2.2 (a),(c).

In order to reach the origin with a single switching maneuver under the assumption: $\beta(0)=0$, the magnitude of the initial value $\alpha(0)$ is restricted by:

$$|\alpha(0)| \leq |2C/d| \tag{2.19}$$

The control $U(\tau)$ assumes the value $K_1= +C$ and $K_2= -C$ for the given initial state X(0) where $\alpha(0)$ satisfies Ineq. (2.19) and $\beta(0) = 0$. Equations (2.13) and (2.14) are now used to obtain the switching time, τ_s, and final time, τ_f, for this initial condition. The expressions for τ_s and τ_f for this case become:

$$\tau_s = \frac{1}{\omega_0} \tan^{-1}\left[\sqrt{\left(\frac{4CE}{3C^2+E^2}\right)^2 -1}\right] \tag{2.20}$$

$$\tau_f = \frac{1}{\omega_0} \tan^{-1}\left[\sqrt{\left(\frac{2CE}{3C^2-E^2}\right)^2 -1}\right] \tag{2.21}$$

4. System Response - Analytic Results

a. Time Response of $\alpha(\tau)$ and $\beta(\tau)$

Equation (2.4) can be written with Eq. (2.9) in the form:

$$\begin{bmatrix} \alpha' \\ \beta' \end{bmatrix} = \begin{bmatrix} 0 & -e \\ d & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} +C, 0\leq\tau\leq\tau_s \\ -C, \tau_s<\tau\leq\tau_f \end{bmatrix} \tag{2.22}$$

The solutions for $\alpha(\tau)$ and $\beta(\tau)$, obtained using Laplace transform techniques, can be expressed as $(\beta(0) = 0)$:

$$\alpha(\tau) = \alpha(0) \cos \omega_0\tau - \frac{C}{d} \ [(1 - \cos \omega_0\tau) \ a(\tau)$$
$$-2(1 - \cos \omega_0(\tau-\tau_s) \ a(\tau-\tau_s)] \tag{2.23}$$

$$\beta(\tau) = \frac{d}{\omega_0} \alpha(0) \sin \omega_0\tau + \frac{C}{\omega_0} [\sin \omega_0\tau \ a(\tau)$$
$$-2 \sin \omega_0(\tau-\tau_s) \ a(\tau-\tau_s)] \tag{2.24}$$

where:

$a(\tau)$ = unit step function

b. Time Response of the Control Mass Position

The equation of the boom end mass displacement is obtained from Eqs. (2.5) and (2.9) and expressed by

$$n(\zeta''(\tau) + \zeta(\tau)) = -C \text{ for } 0 \leq \tau \leq \tau_s$$
$$= + C \text{ for } \tau_s \leq \tau \leq \tau_f \tag{2.25}$$

and hence the time response of the control mass is given by the following equation $(\tau \geq 0)$:

$$\zeta(\tau) = \frac{C}{n} [\psi_1(\tau) - 2\psi_1(\tau-\tau_s) + \psi_1(\tau-\tau_f)] \tag{2.26}$$

where

$$\psi_1(\tau) = (1-\cos \tau) \ a(\tau)$$

When linear damping (ρ) is assumed to be present in the boom extension mechanism, Eq. (2.25) becomes

$$n(\zeta''(\tau) + 2\rho\zeta'(\tau) + \zeta(\tau)) = U(\tau) \tag{2.27}$$

The solution of Eq. (2.27), with $U(\tau)$ given in Eq. (2.9), can be written as

$$\zeta(\tau) = \frac{C}{n}\left[\psi_2(\tau) - 2\psi_2(\tau-\tau_s) + \psi_2(\tau-\tau_f)\right] \tag{2.28}$$

where

$$\psi_2(\tau) = 1 - \frac{e^{-\rho\tau}}{\sqrt{1-\rho^2}} \sin(\sqrt{1-\rho^2}\,\tau+\Phi)\, a(\tau)$$

$$\Phi = \cos^{-1}(\rho) = \tan^{-1}(\sqrt{1-\rho^2}/\rho)$$

5. Simulation Results

In this section some typical numerical results are presented. The following system parameters are selected (Fig. 2.1):[2]

$I = 1.42 \times 10^7$ kg-m^2(10.5×10^6 slug-ft^2)

$I_3 = 2.03 \times 10^7$ kg-m^2(15.0×10^6 slug-ft^2)

$M = 6.21 \times 10^4$ kg(4258 slug)

$\Omega = 0.314$ rad/sec (3 rpm)

$m = 816$ kg(55.95 slug)

$a = 19.8$ m(65ft), $b = 0$, $\ell_m = z_{max} = 5.4$m(17.72ft)

$\omega_1(0) = 0.0391$ rad/sec, $\omega_2(0) = 0$, $\omega_{T_{max}} = 0.04$ rad/sec.

Fig. 2.3(a) illustrates the variation in the switching time, t_s and final time, t_f, with the normalized value of control effort for a given set of initial conditions: $\omega_1(0) = 0.0391$ rad/sec and $\omega_2(0) = 0$. In order to achieve the desired final state with a single switching maneuver, the magnitude of C must be greater than 0.0275. The final time is rapidly reduced for small increases in the control effort near the minimum value.

For a control effort of C = 0.03 and $\omega_2(0) = 0$, the magnitude of $\omega_1(0)$ must be less than 0.0427 rad/sec, for a single switching maneuver. These conditions are obtained from inequality (2.19). Fig. 2.3(b) demonstrates the manner in which the final time required increases with the larger values of $\omega_1(0)$, whereas the switching time remains essentially constant over a wide range of initial conditions.

The comparison of nutation angle decay for three different control laws is depicted in Fig. 2.4. Initial conditions for all cases were selected such that the initial nutation angle was 5.0 degrees. In the first case, a control law described in Ref. 7 (not based on optimal control theory) was used, which resulted in a final time of 850 seconds to remove the effect of nutation.

Next, the response of the system using a control law based on minimizing a quadratic performance index[2] is presented. The process of determining the weighting matrices for the nondimensionalized form of the state equations is now discussed. (This was not considered in Ref. 2.)

The original equations of the controlled system in the time domain are expressed as:

$$\dot{X}_o = A_o X_o + B_o U_o \tag{2.29}$$

and the cost functional by:

$$J = \int_o^\infty (X_o^T Q_o X_o + U_o^T R_o U_o)\, dt \tag{2.30}$$

The dimensionless equations corresponding to Eqs. (2.29) and (2.30) for the two axis control using a single boom are

$$X' = AX + BU \tag{2.31}$$

$$J = \int_0^{\infty} (X^T Q X + U^T R U) d\tau \qquad (2.32)$$

where the prime indicates differentiation with respect to the dimensionless time, $\tau = \Omega t$. Then, Q and R in Eq. (2.32) can be related to Q_o and R_o in Eq. (2.30) by:

$$Q = \Omega Q_o,\ R = \ell_m^{\,2}\Omega^3 R_o \qquad (2.33)$$

The values of Q_o and R_o are selected as:[6]

$$Q_o = \begin{bmatrix} q_o & 0 \\ 0 & q_o \end{bmatrix},\ R_o = \underset{1x1}{[r_o]} \qquad (2.34)$$

where

$$q_o = 1/\omega_{T\,max}^{2} \qquad (2.35)$$

The matrices, A and B, are readily obtained by comparing Eq. (2.4) with the general form, Eq. (2.31).

The nutation angle decay of the system for the parameters: $\omega_{T\,max}$ = 0.04 rad/sec (maximum expected value of transverse rate) and $[r_o]$ = 0.00372 $m^2\ sec^{-4}$(0.04 $ft^2\ sec^{-4}$) is shown in Fig. 2.4 by the dotted curve. The nutational motion has been effectively removed within 200 secs after control initiation which is about one-fourth of the time obtained using non-optimal control method (Ref. 8). (It should be mentioned that the recent results of Ref. 9 indicate that the gradient technique permits recovery in about one-fourth the time when compared with the non-optimal control law of Refs. 7,8). The response time of the nutation decay can be improved by properly varying the weighting matrices in the performance index.

If a control law is now selected based on the single switching time optimal criteria, a further improvement in response time is obtained. The final time obtained from Eq. (2.21) is 19.47 secs. This analytical result is shown in Fig. 2.4. In Fig. 2.5, a comparison of this analytical result with the numerical integration for the time optimal control system is presented. It is seen that the initial decay of nutation angle (Fig. 2.5 (a)) is extremely rapid approaching a value of 1.1 deg. at the end of the control maneuver (19.47 sec.). With this control mass of 816 kg. (55.95 slug), the analytic solution begins to diverge from the numerical integration results towards the end of the control sequence (Fig. 2.5 (a)) due to the presence of nonlinearities associated with the larger amplitudes of the control mass displacement (Fig. 2.5 (b)). To remove the residual nutation angle here a second switching sequence would be required. Also, it is seen from Fig. 2.5(b) that after approximately 30 secs. without damping in the boom extension mechanism a steady state boom motion would remain with an amplitude of 109.35 ft. (33.33m) with the presence of the boom damping shown, the amplitude of this motion has been completely reduced to zero within 175 secs. Here, it is observed that initially the boom end mass undergoes a very large displacement.

In order to reduce the large displacements of the end mass, the time optimal control of the system with a larger size control mass and a larger main spacecraft mass will be considered.

The values of the masses selected are

$$M = 19.98 \times 10^5 \text{kg} \ (1.37 \times 10^5 \text{ slug})$$
$$m = 2.625 \times 10^4 \text{kg} \ (1800 \text{ slug})$$

such that m/M remains at 0.013 (as in Fig. 2.5 (a)). The other system parameters will remain the same as considered earlier.

Figs. 2.6(a) and (b) show the variation of switching and final times with the control effort and the initial conditions with the new mass parameters. For the same set of initial conditions, in order to achieve the desired final state with a single switching maneuver, $C > 0.042$, and with $C = 0.05$ and $\omega_2(0) = 0$, $\omega_1(0) < 0.0472$ rad/sec. The decay of nutation angle using the control law described in Refs. 2,6 and that resulting from implementation of the time optimal control with a single switch is compared in Fig. 2.7(a). The motion of the boom end mass during and immediately after the nutation decay is illustrated in Fig. 2.7(b). It is seen that a steady state oscillation without damping in the boom extension mechanism would remain with an amplitude of 17.18ft. (5.24m) with the presence of the boom damping shown, the amplitude of this motion has been reduced to 2.1 ft. (0.64m) at 75 secs. For the time optimal results shown, there is negligible difference between the analytic solution and the results of numerical integration.

The time response of the transverse angular rates for the case of Fig. 2.7 is presented in Fig. 2.8. It is observed that the closed form solutions give an excellent correlation with the numerical integration for the large masses, whereas for the smaller masses the closed form solution diverges from the numerical integration results due to the resulting large amplitude boom motion.

Fig. 2.9 represents the dynamic response of the time optimal control system with a control effort of C = 0.1 which is twice of the previous value considered with the presence of the larger control and main spacecraft masses. It is observed that the steady state oscillation of the boom end mass increases from 17.18 ft. (5.24m) to 31.05 ft. (9.46m) for an increase of control effort from C = 0.05 to C = 0.1. The complete control manuever time is reduced to 9.929 sec. from 14.56 sec.

For the general case where the initial conditions do not lie within the single switching region piecewise solutions can be used to obtain the system response analytically, or, as an alternative, the more general gradient technique of Ref. 9 can be employed.

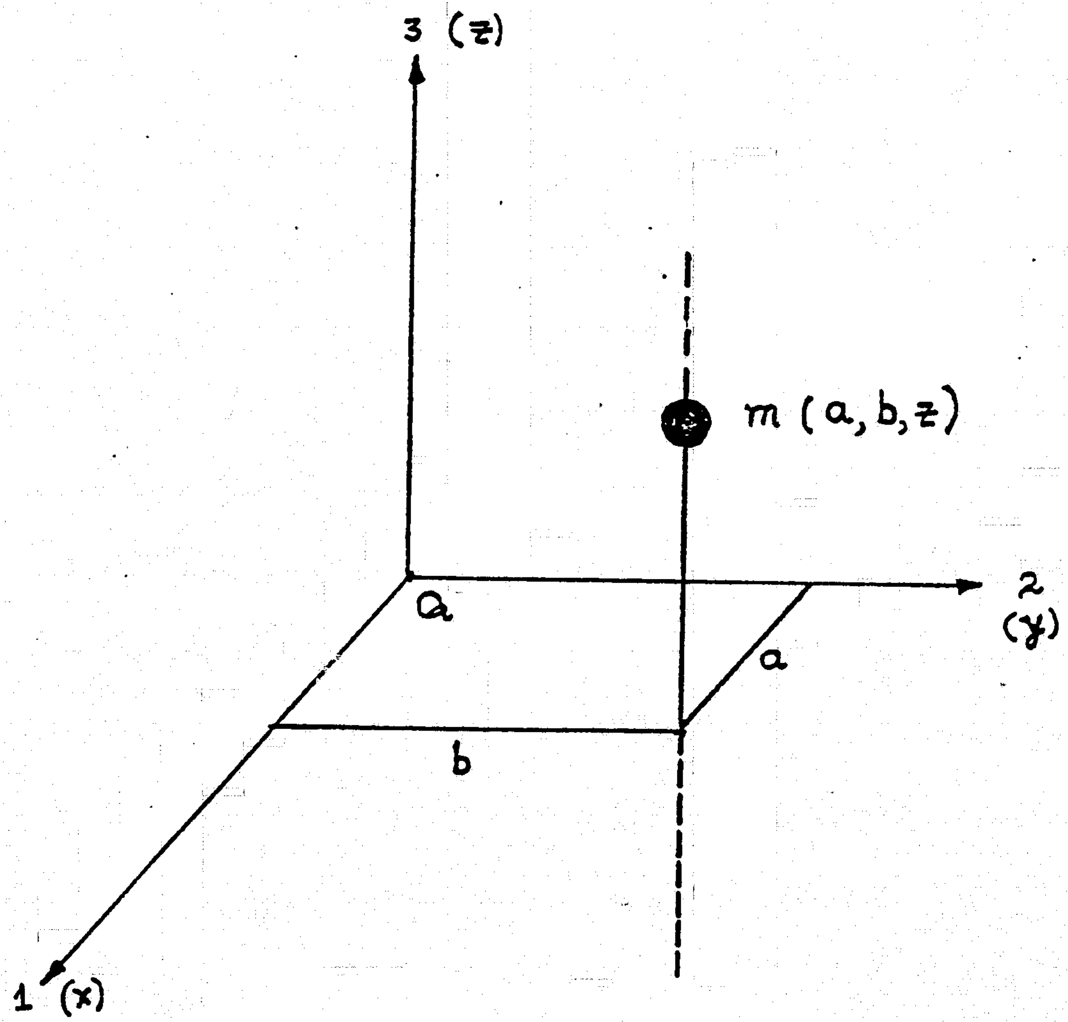

Fig. 2.1. Single Boom Offset System

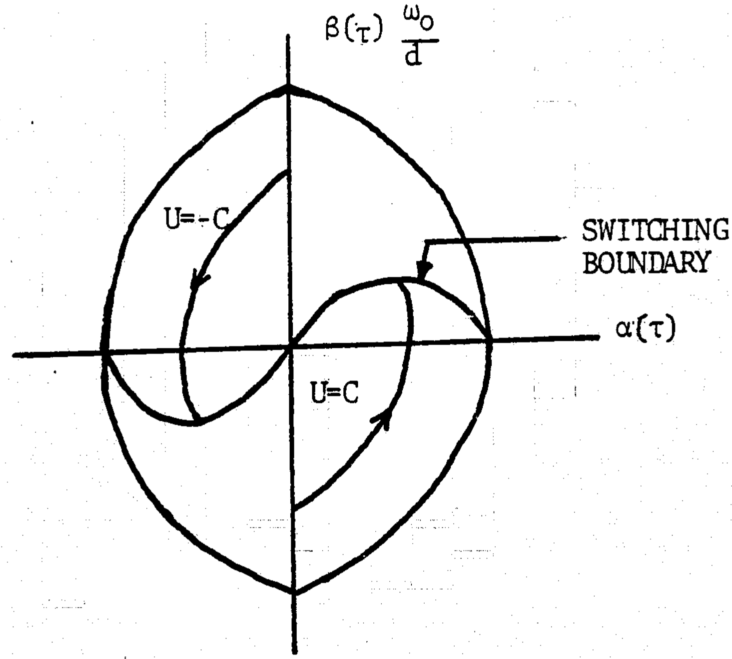

Fig. 2.2(a). Phase Plane Portrait for Single Switching

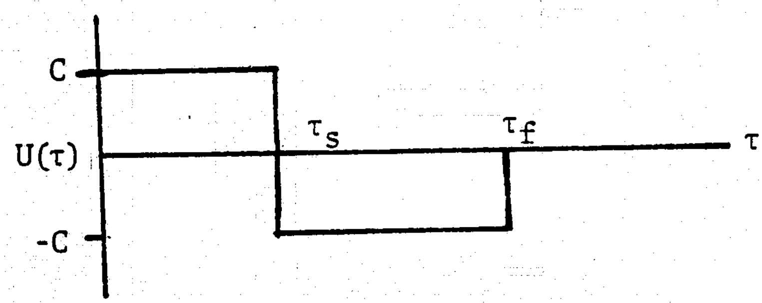

Fig. 2.2(b). Control Scheme for Single Switching

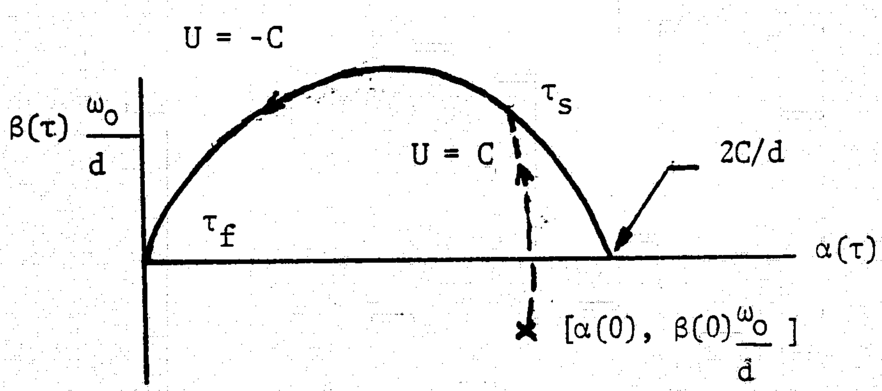

Fig. 2.2(c). Phase Plane of the System for Given Initial Conditions

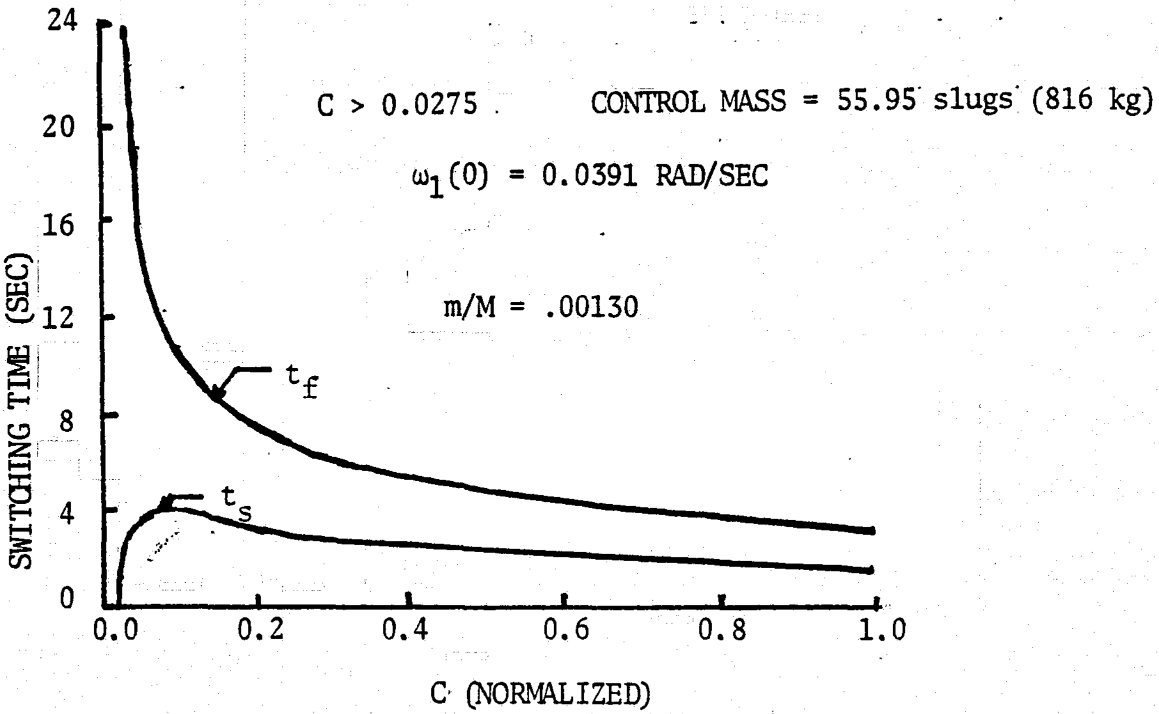

Fig. 2.3(a). Switching Time with Variation of Maximum Control

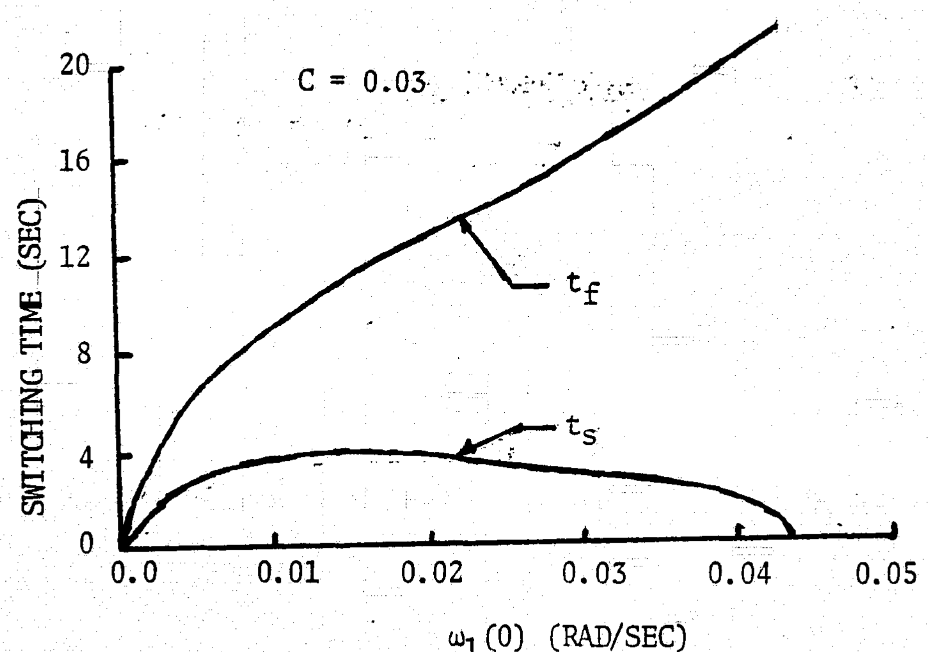

Fig. 2.3(b). Switching Time with Variation of $\omega_1(0)$

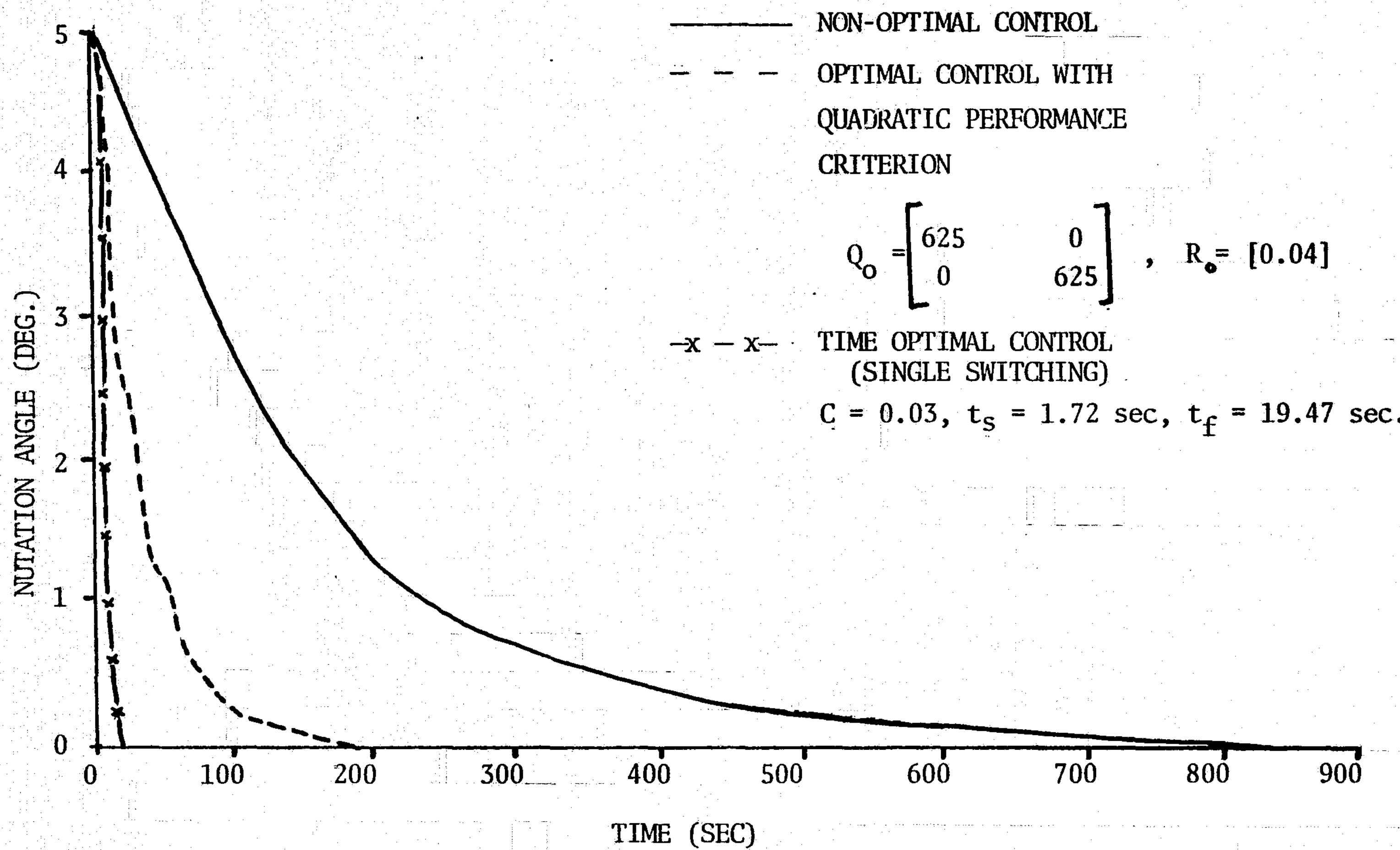

Fig. 2.4. Time Response of Nutation Angle with Different Control Laws

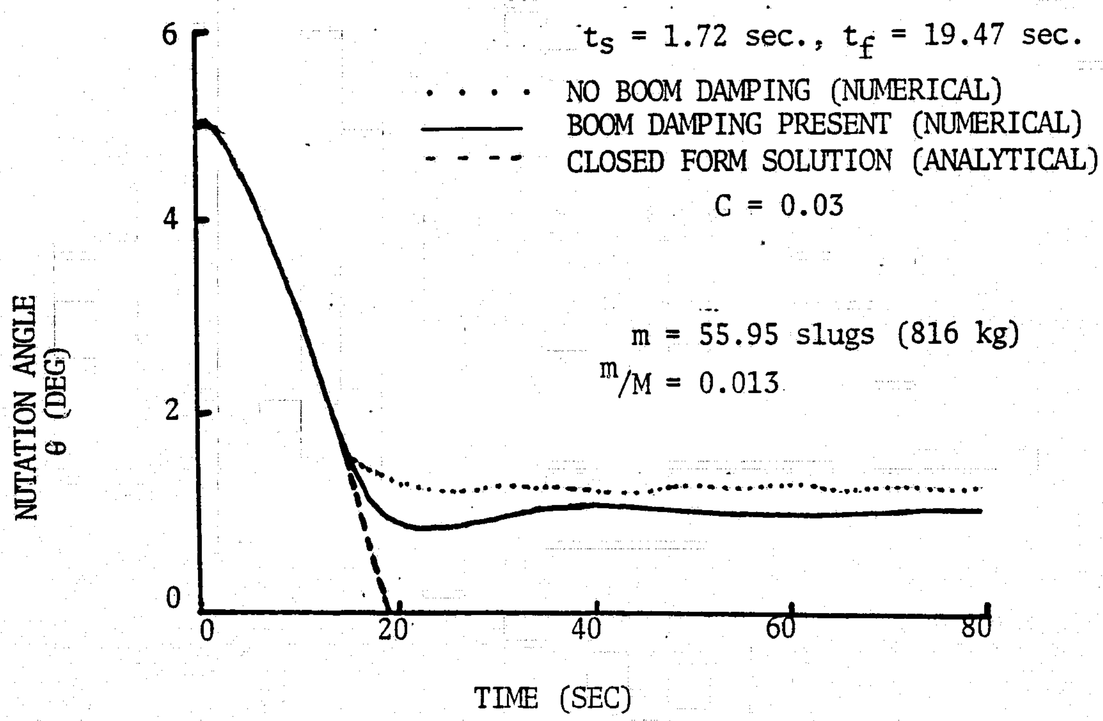

Fig. 2.5(a). Comparison of Analytic and Numerical Results of Nutation Angle Decay

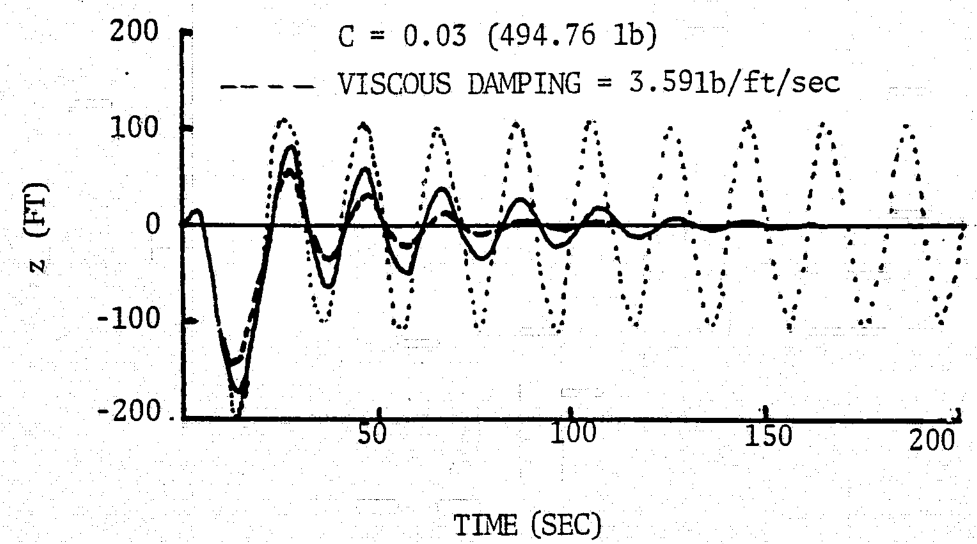

Fig. 2.5(b). Control Mass Displacement

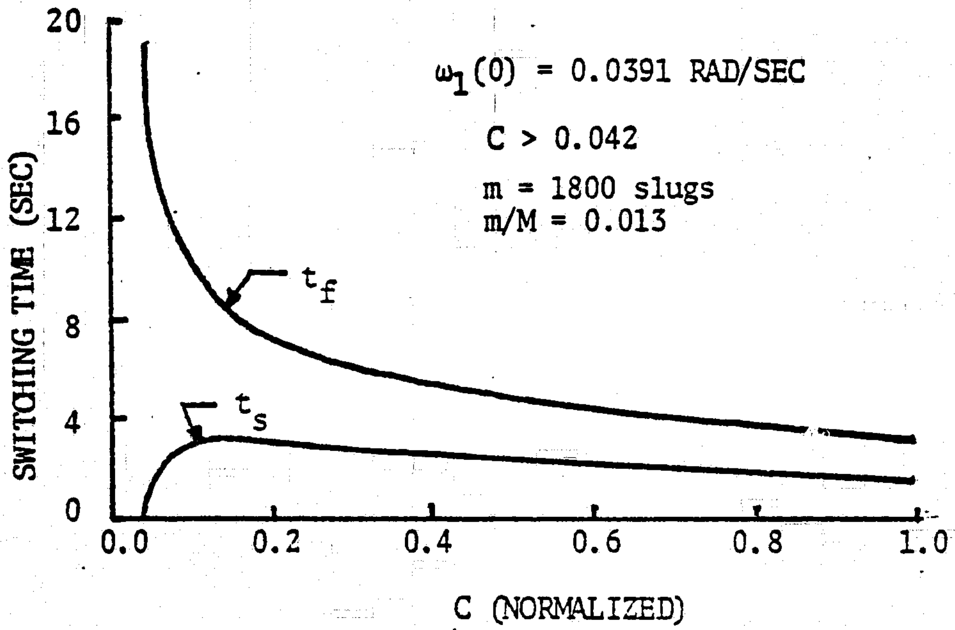

Fig. 2.6(a). Switching Time with Variation of Maximum Control (with Larger Masses)

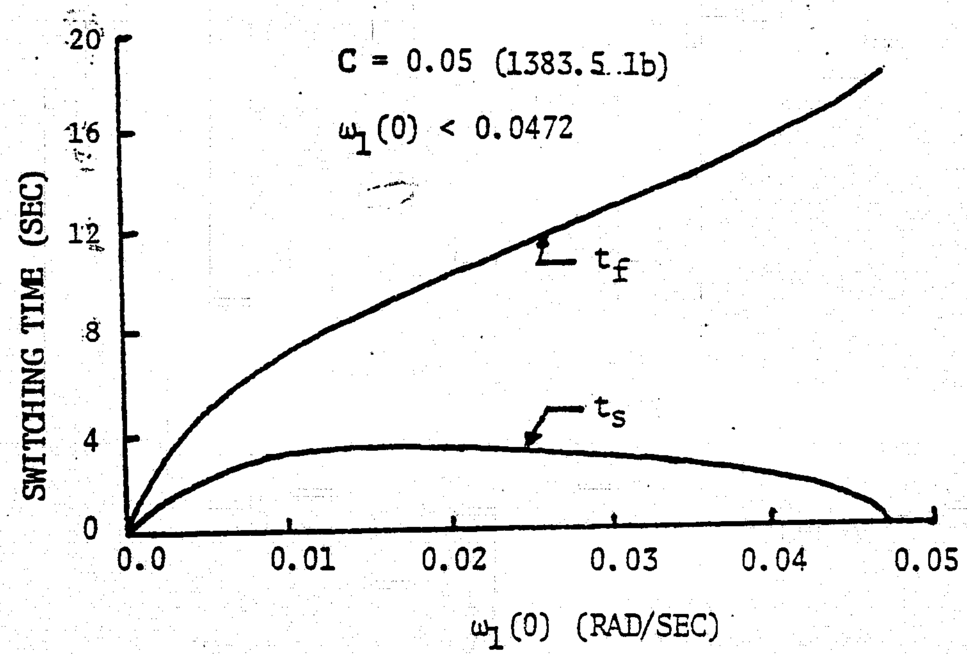

Fig. 2.6(b). Switching Time with Variation of $\omega_1(0)$

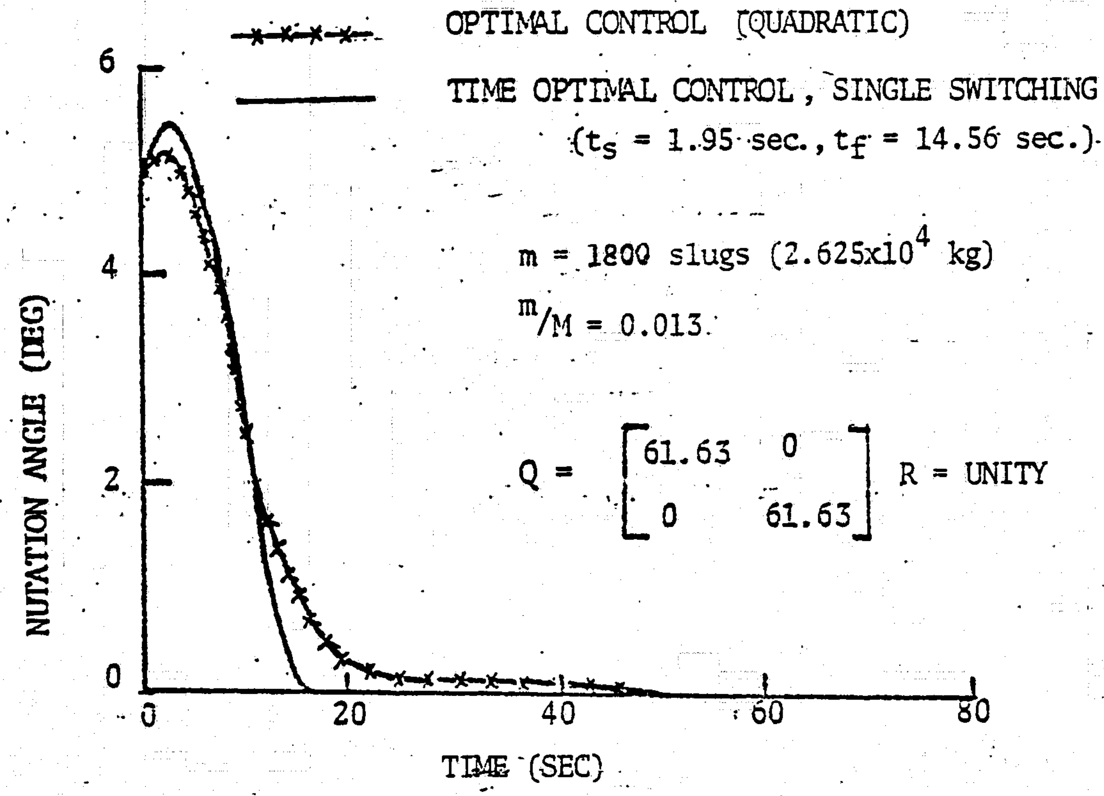

Fig. 2.7(a). Decay of Nutation Angle with Different Optimal Control Laws (with Larger Masses)

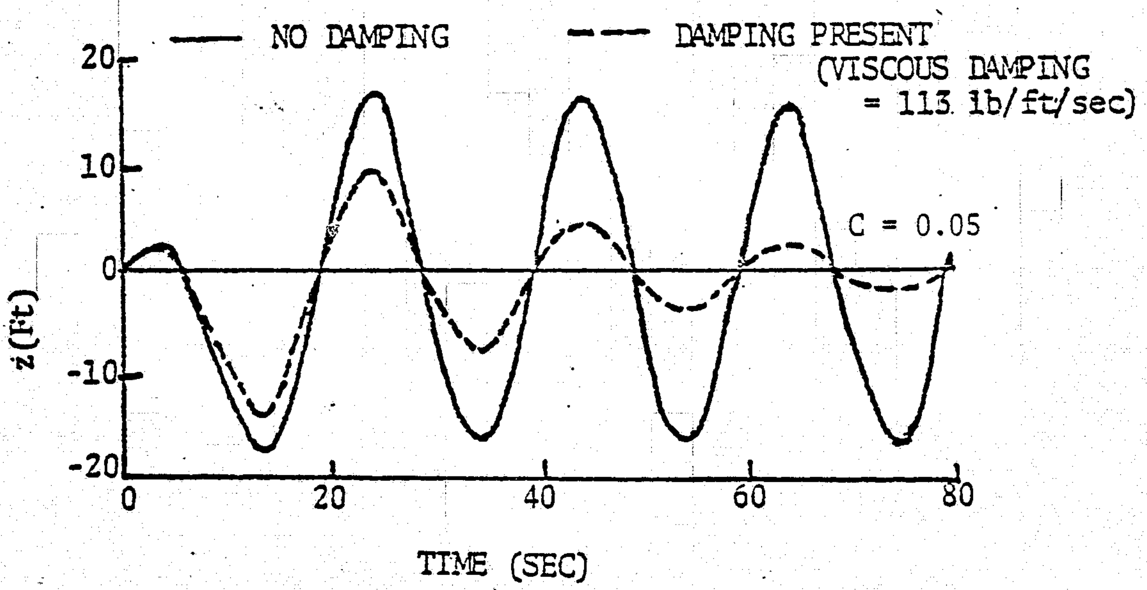

Fig. 2.7(a). Boom End Mass Displacement (with Larger Masses)

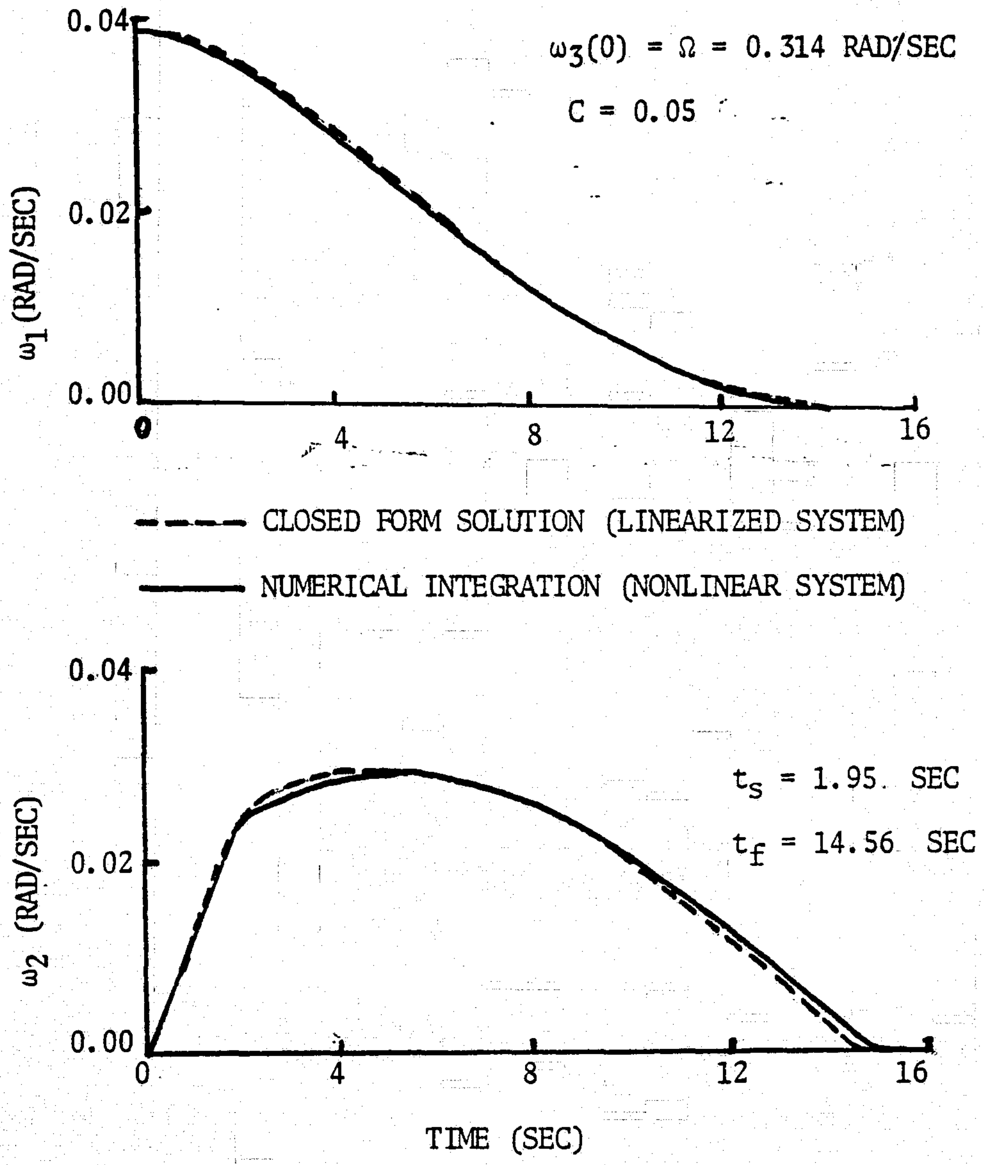

Fig. 2.8. Time Response of Transverse Angular Rates (with Larger Masses)

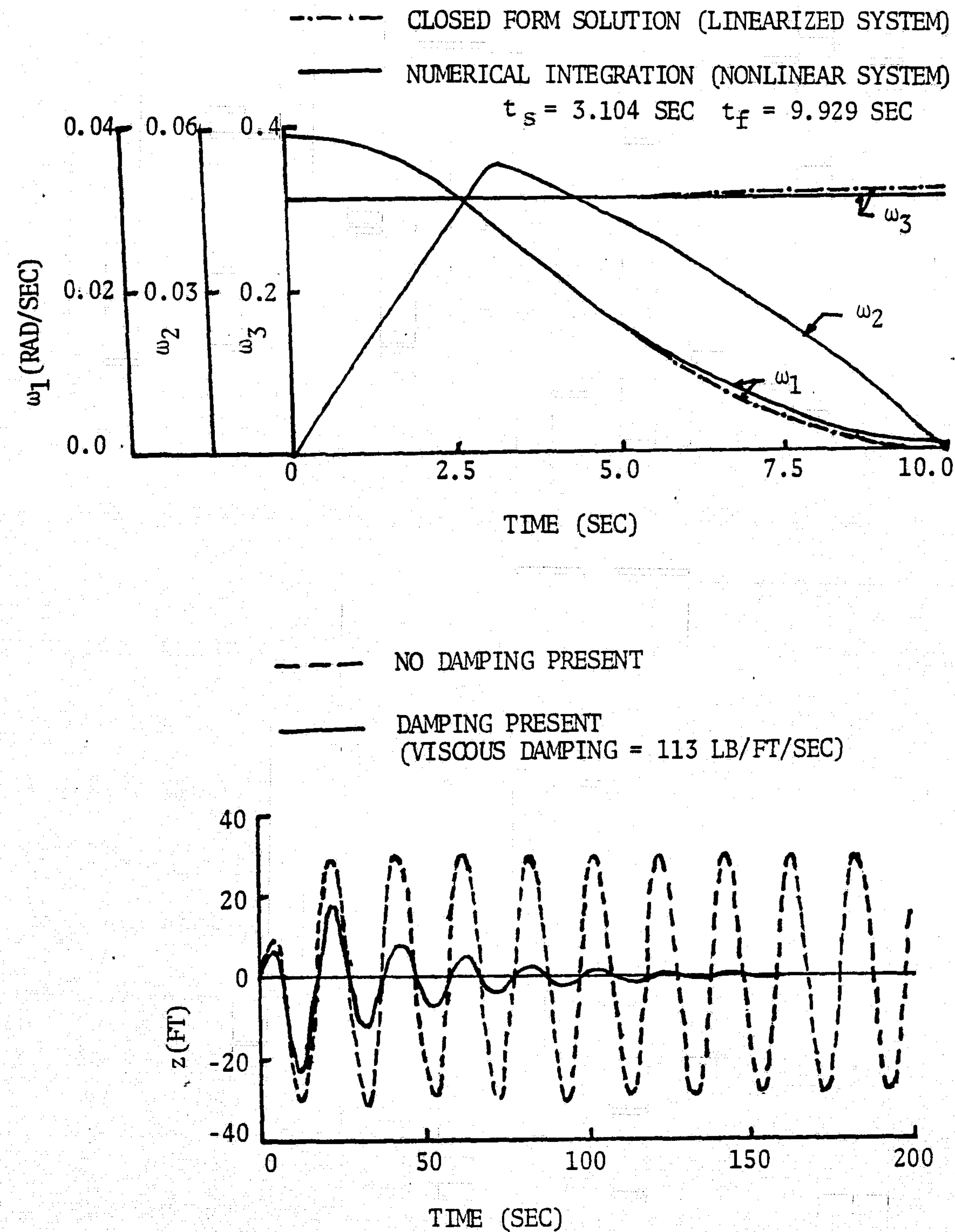

Fig. 2.9. Dynamic Response of System with C = 0.1
(with Larger Masses)

III. OPTIMAL ESTIMATION AND CONTROL

This chapter deals with the application of optimal estimation and control techniques to the attitude control of spinning spacecraft with movable telescoping appendages. The estimation is accomplished using a Kalman filter to obtain estimates of the state (variation of the angular velocity components) of the spacecraft; this estimated state constitutes the input to the controller. In the controller a quadratic performance index is formulated to minimize the components of excess angular velocity plus the control effort; additionally, a linearized model of the overall system is employed.

The attitude control of a spinning spacecraft with the use of one or two movable telescoping appendages is considered (Fig. 3.1). The performance of the system is evaluated by analytical methods for special cases and numerical integration is used for the general case. In this analysis, the dynamics of the driving mechanism are completely ignored, and the control effort needed for the boom movement is assumed to be present instantaneously.

1. Formulation of the Stochastic Optimal Control Problem

This section describes the application of well-known results in optimal linear estimation and control theory to the problem of attitude control of spinning spacecraft. The linear equations of state X and measurement Y are

$$X' = AX + BU + Gw \tag{3.1}$$

$$Y = HX + v \tag{3.2}$$

where the elements of the system state vector, X, represent the variation of angular velocity components of the spacecraft from a nominal state, and the elements of the measurement vector, Y, correspond to the measured components of X with noise present. The vector, w, represents random disturbances that perturb the spacecraft, and the vector, v, represents the measurement noise present in the sensors. The measurement noise (v) and the plant noise (w) are assumed to be white Gaussian processes with zero mean.

The cost functional to be minimized here is taken as a weighted quadratic function of the state vector plus a weighted function of the control:

$$J = E\left\{\lim_{\tau_f \to \infty} \frac{1}{2\tau_f} \int_0^{\tau_f} (X^TQX+U^TRU)d\tau\right\} \tag{3.3}$$

The optimal control vector U minimizing J for control over the internal $0 \leq \tau < \infty$ can be expressed as:[12]

$$U = -C\hat{X} \tag{3.4}$$

where the control gain, C, is related by

$$C = R^{-1}B^TK \tag{3.5}$$

In Eq. (3.5) K is the steady state solution of the matrix Riccati differential equation

$$-K' = KA+A^TK-KBR^{-1}B^TK+Q \tag{3.6}$$

The estimated state, $\hat{X}$, is obtained from

$$\hat{X}' = A\hat{X}+BU+F(Y-H\hat{X}) \tag{3.7}$$

with the filter gain, F, expressed as:

$$F = PH^TV^{-1} \tag{3.8}$$

Again P in Eq. (3.8) results from the steady state solution of the following matrix Riccati differential equation,

$$P' = AP+PA^T-PH^TV^{-1}HP+GWG^T \tag{3.9}$$

The quantities W (covariance of the plant noise) and V (covariance of the measurement noise) are obtained from the following autocorrelations:[12]

$$E\ \{w(\tau)\ w(\tau+\phi)^T\} = W\ \delta(\phi) \tag{3.10}$$

$$E\ \{v(\tau)\ v(\tau+\phi)^T\} = V\ \delta(\phi) \tag{3.11}$$

where $\delta(\phi)$ represents the Dirac delta function.

2. Application to Two Axis Control with a Single Offset Boom

The optimal estimation and control theory stated in Eqs. (3-1) - (3.11) will be applied, at first, to a single offset-boom system providing two axis control and then to a two offset-boom system for three axis control. The present study is an extension of Ref. 6 where the measurement noise and plant noise are now considered in the design of the optimal controller.

The movable telescoping single offset system (m_2=0) is now analyzed (Fig. 3.1). The linearized equations of motion in dimensionless form (Ref. 2, Eqs. (5.4) and (5.5) and Ref. 6) for the special case of a symmetrical hub result as (for b=0):

$$\begin{bmatrix} \alpha' \\ \beta' \end{bmatrix} = \begin{bmatrix} 0 & -e \\ d & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + \begin{bmatrix} 0 \\ n \end{bmatrix} [U] \tag{3.12}$$

where

$$U = \zeta'' + \zeta \tag{3.13}$$

The other quantities in Eq. (3.12) are defined by Eqs. (2.6) - (2.8). Eq. (3.12) can be written in the form: $X' = AX+BU$. The matrices A and B resulting from Eq. (3.12) are

$$A=\begin{bmatrix} 0 & -e \\ d & 0 \end{bmatrix} \quad ; \quad B=\begin{bmatrix} 0 \\ n \end{bmatrix} \tag{3.14}$$

We further assume that the control, U, and the plant noise, w, enter the system as shown in Fig. 3.2. Thus, the equations of motion of the system, with the plant noise present, can be modified to the form shown in Eq. (3.1).

a. Single Measurement System-Analytic Results

(1) Control and Filter Gains Evaluation

The details of the steady state control gain evaluation are available in Refs. 2 and 6. Here only the results are stated. The elements of the K matrix are

$$K_{12} = K_{21} = (r/n^2)\ [d \pm \{d^2+n^2(q/r)\}^{\frac{1}{2}}] \tag{3.15}$$

$$K_{22} = \pm (r/n)\{(q-2eK_{12})/r\}^{\frac{1}{2}} \tag{3.16}$$

$$K_{11} = (K_{22}/e)\{d-n^2(K_{12}/r)\} \tag{3.17}$$

where the sign in front of the radicals is selected such that K is positive definite. The weighting matrices Q and R in the performance index, J, Eq. (3.3), have been selected for the present application as:

$$Q=\begin{bmatrix} q & 0 \\ 0 & q \end{bmatrix} \quad \text{and } R = [r] \tag{3.18}$$

It should be noted that previously for convenience R was selected as the unit matrix. From Eqs. (3.4), (3.5) and (3.13) it is seen that the control has the form

$$U = \zeta''+\zeta = -[C_1\ C_2]\ (X) \tag{3.19}$$

where the control gains are obtained from

$$C_1 = nK_{12}/r; \; C_2 = nK_{22}/r \tag{3.20}$$

Next, the steady state filter gain determination from Eq. (3.9) is considered. For this application of two axis control with a single measurement of α such that $H = [1\ 0]^T$, we assume that the control, U, and the plant noise, w, enter the system together such that B = G. The expansion of the matrix Riccati (filter) equation, Eq. (3.9) with P' = 0 yields

$$2P_{12}e + P_{11}^2/V = 0 \tag{3.21}$$

$$P_{11}d - P_{22}e - P_{11}P_{12}/V = 0 \tag{3.22}$$

$$2P_{12}d - (P_{12}^2/V) + n^2W = 0 \tag{3.23}$$

Eqs. (3.21) - (3.23) can then be solved for the elements of the two dimensional symmetric P matrix as:

$$P_{12} = V\{d \pm (d^2 + n^2(W/V))^{\frac{1}{2}}\} \tag{3.24}$$

$$P_{11} = \pm (-2eVP_{12})^{\frac{1}{2}} \tag{3.25}$$

$$P_{22} = (P_{11}/e)\ (d - (P_{12}/V)) \tag{3.26}$$

Again, the sign in front of the radicals is selected such that P is positive definite. The filter gain, F, obtained from Eq. (3.8), with $H = [1\ 0]^T$, is expressed as

$$F = \frac{1}{V}\begin{bmatrix} P_{11} \\ P_{12} \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \tag{3.27}$$

Thus, the linear model of the plant and the estimator equations become

$$X' = AX - BC\hat{X} + Gw \tag{3.28}$$

$$\hat{X}' = (A - FH - BC)\hat{X} + FHX + Fv \tag{3.29}$$

The general scheme of the stochastic optimal control configuration represented by Eqs. (3.28) and (3.29) is shown in Fig. 3.2. This configuration is taken as the basis for studying the system behavior.

(2) Average Performance of the System

The average performance (RMS value) of the optimally controlled system, Eqs. (3.28) and (3.29), in the presence of plant and measurement noises can be predicted from the covariances of the error and estimate.[12] The state of the controlled system and the state of the estimator are coupled. In terms of $\varepsilon \equiv \hat{X} - X$, Eqs. (3.28) and (3.29) can be written as

$$\varepsilon' = (A-FH)\varepsilon + Fv-Gw \tag{3.30}$$

$$X' = (A-BC)\hat{X} + FH\varepsilon + Fv \tag{3.31}$$

The covariance matrices of ε and $\hat{X}$ (P and $\hat{S}$, respectively), are given by[12]

$$P' = AP + PA^T - FVF^T + GWG^T \tag{3.32}$$

$$\hat{S}' = (A-BC)\,\hat{S} + \hat{S}\,(A-BC)^T + FVF^T \tag{3.33}$$

since $E(\hat{X}\varepsilon^T) = 0$, we have

$$E\,[X(\tau)X(\tau)^T] \equiv S(\tau) = \hat{S}(\tau)+P(\tau) \tag{3.34}$$

These equations allow us to predict the mean square histories of the state variables and their cross-correlations. The mean square values of the control variables and their cross-correlations may be obtained as[12]

$$E\,[U(\tau)\,U(\tau)^T] = C\,\hat{S}\,C^T \tag{3.35}$$

In this section, the steady state analytical solution[14] to the optimal stochastic control problem is given. The analytic solution is obtained from Eqs. (3.32) - (3.35) by using the steady state values of the control and filter gains obtained earlier.

By solving these equations for the special case of a single measurement of $\alpha(\omega_1/\Omega)$ only, the steady state variances of the state and control are obtained as $(\tau \to \infty)$:

$$E\,[\alpha^2(\tau)] = S_{11} = \hat{S}_{11} + P_{11} \tag{3.36}$$

$$E\,[\beta^2(\tau)] = S_{22} = \hat{S}_{22} + P_{22} \tag{3.37}$$

$$E\,[U^2(\tau)] = C_1^{\,2}\,\hat{S}_{11} + 2C_1C_2\hat{S}_{12} + C_2^{\,2}\hat{S}_{22} \tag{3.38}$$

where

$$\hat{S}_{11} = \{VF_1/(d-C_1n)\}\{(C_2n/2e)F_1 - F_2\} \tag{3.39}$$

$$\hat{S}_{12} = (V/2e)F_1^{\,2} \tag{3.40}$$

$$\hat{S}_{22} = (V/C_2n)\{F_2^{\,2} + (d-C_1n)F_1^{\,2}/2e\} \tag{3.41}$$

The elements P_{11} and P_{22} are given in Eqs. (3.25) and (3.26), respectively. This analytical approach can be very useful in preliminary design of sensor measurement schemes.[12]

3. Numerical Results

Some typical numerical results based on the solutions of Sec. 1 and 2 of this chapter will now be discussed. The following system parameters are considered here for numerical integration (Fig. 3.1)[2,6]:

$I = 1.42\text{x}10^7$ kg-m^2 ($10.5\text{x}10^6$ slug-ft^2)

$I_3 = 2.03\text{x}10^7$ kg-m^2 ($15.0\text{x}10^6$ slug-ft^2)

$M = 6.21\text{x}10^4$ kg (4258 slug)

$m = 816$ kg (55.95 slug)

$\Omega = 0.314$ rad/sec. (3 rpm)

$a = c = 19.8$ m (65ft); $b=d=0$

$\omega_1(0) = 0.0391$ rad/sec; $\omega_2(0) = 0$

Four different computer algorithms are required for the numerical solution: 1) the calculation of the control system gains using the matrix Riccati equation for the deterministic system (no-noise)[15], 2) the calculation of the control system gains using the matrix Riccati equation with the filter algorithm[15] i.e. - "filter gains," 3) a means of simulating random noise input to the plant as well as in the measurement device, 4) the simulation of the system dynamics using the control laws from 1) and 2) and a model of the noise from 3).

a. Two Axis Control with Single Offset Boom.

(1) Single Measurement System

When a single boom is offset from the z axis and the hub is symmetrical ($I = I_1 = I_2$), it was shown earlier that the control and filter gains can be obtained analytically. The process of determining the weighting matrices for the nondimensionalized form of the state equations was given in Sec. 5 of the last chapter. Here the details regarding the determination of the covariances of the measurement and plant noises for the nondimensionalized form of the state and measurement equations are considered.

The original state and measurement equations of the controlled system in the time domain are expressed as:

$$X_o = A_o X_o + B_o U_o + G_o \omega_o \quad (3.42)$$

$$Y_o = H_o X_o + v_o \quad (3.43)$$

The dimensionless equations corresponding to Eqs. (3.42) and (3.43) are:

$$X' = AX+BU+G\omega \tag{3.44}$$

$$Y = HX+v \tag{3.45}$$

Then, the covariances of the measurement noise (V) and the plant noise (W) in dimensionless form are related to their original (time domain) values by

$$V = V_o/\Omega^2 \tag{3.46}$$

$$W = W_o/\ell_m^2\Omega^4 \tag{3.47}$$

With the values of $\omega_{T_{max}}$ = 0.04 rad/sec and ℓ_m = 5.4 m (17.72ft), the following numerical values for the parameters result:

$$d = 0.441,\ e = 0.428,\ n = 5.929\text{x}10^{-3}$$

The optimal control law for the values of

$$Q_o = \begin{bmatrix} q_o & 0 \\ 0 & q_o \end{bmatrix},\ q_o = 625\ \text{rad}^{-2}\ \text{sec}^2$$

and R_o = $[\ r_o\ ]$ = 0.00372 m^2 sec^{-4} (0.04 ft^2 sec^{-4}) are obtained as follows. The values of q and r in Eq. (3.18) are related to q_o and γ_o by

$$q = \Omega q_o = 196.25$$

$$r = \ell_m^2\Omega^2 r_o = 0.397$$

Thus, the constant control gain matrix is obtained from Eq. (3.20) as:

$$C = [-3.3188\quad 31.366]$$

The matrix, H, in the measurement equation for a single measurement of ω_1 only becomes

$$H = [\ 1 \quad 0]$$

The covariances of the measurement and plant noises are assumed to have the following values:

$$V_o = 10^{-6}\ rad^2\ sec^{-2}$$

$$W_o = 10^{-6}\ ft^2\ sec^{-4}\ (0.093x10^{-6}m^2sec^{-4})$$

which appear to representative of the current capability of measurement devices. Then, the values of V and W are related to V_o and W_o by

$$V = V_o/\Omega^2 = 0.1014x10^{-4}$$

$$W = W_o/\ell_m^{\ 2}\Omega^4 = 0.3276x10^{-4}$$

The filter gain is obtained from Eq. (3.27) as:

$$F = \begin{bmatrix} 0.0104960 \\ -0.0001287 \end{bmatrix}$$

The dynamic responses of the system are obtained by integrating the linearized plant and estimator equations represented by Eqs. (3.28) and (3.29). The decay of nutation and the motion of the boom end mass is illustrated in Fig. 3.3(a). The nutation angle reaches a value of 0.0142 deg. after 200 secs. from an initial value of 5.0 deg. It is seen that within 100-120 secs. (without damping in the boom extension mechanism) the transient part of the boom end mass motion is removed, leaving a remaining steady state oscillation with an amplitude of 38.5 ft. (11.74m); with the presence of boom damping shown, the amplitude of the boom motion has been reduced to essentially zero within 175 secs.

The random noise present in the sensor measuring scheme, v, which is simulated by generating random numbers with a zero mean Gaussian distribution, and the actual output of the sensor ($Y_1 = \omega_1 + v$) are illustrated in Fig. 3.3(b). Also indicated are the time response of the transverse angular velocity components (ω_1, ω) and their estimated values ($\hat{\omega}_1, \hat{\omega}_2$). It is observed that the estimate of the state: $(\hat{\omega}_1 \; \hat{\omega}_2)^T$ has a very good correlation with the actual state: $(\omega_1 \; \omega_2)^T$ for the given initial conditions. Within the plotting accuracy this difference can not be detected.

Next, the case of a larger control mass and a larger main spacecraft mass, as considered in the earlier chapter, is taken as an example for studying the dynamic response of the system. The values of the masses selected are:

$$M = 19.98 \times 10^5 \text{ kg } (1.37 \times 10^5 \text{ slug})$$

$$m = 2.625 \times 10^4 \text{ kg } (1800 \text{ slug})$$

such that m/M remains at 0.013 (as in Fig. 3.3(a)).

Figs. 3.4(a) and (b) show the time histories of the control and filter gains for the single offset boom case with the single measurement of angular velocity ω_1 only. In all these cases, the responses indicate that the transients are brief, and steady state values of K and F are rapidly reached. For this special case, the steady state control and filter gains are obtained analytically by solving the respective algebraic matrix Riccati equation. The plant noise and measurement noise are assumed to have covariance values of $W = V = 10^{-6}$ (dimensionless).

The weighting matrices of state, Q, and control, R, are indicated in the figure. Also the steady state average performance of the system is obtained analytically[12] from the covariance of the error and estimate. The average value of the nutation angle from the covariance analysis of Sec. 2. a(2) is obtained as 0.02 deg.

The dynamic responses of the system for constant (steady state) control and filter gains are shown in Fig. 3.5 for an initial nutation angle of 5.0 degrees. The spacecraft considered is spinning at an angular velocity of $\Omega = 0.314$ rad/sec. about the 'z' axis. For the selected weighting matrices, Q and R, as shown, the decay of nutation angle is represented in Fig. 3.5(a) where the maximum amplitude of boom length is taken as 17.72 ft. (5.4m). The motion of the boom end mass during nutation decay is illustrated. It is seen that, after approximately 25 secs. without damping in the boom extension mechanism, a steady state oscillation would remain with an amplitude of 13.69 ft. (4.17 m.); with the presence of the boom damping shown, the amplitude of the motion has been essentially reduced to zero within 100 secs. The random noise present in the sensor, actual output of the sensor and the transient response of the angular velocity components and their estimates are shown in Fig. 3.5(b).

The effect of the intensity of the sensor and plant noise is considered next for the single measurement (ω_1) scheme.

When the covariance is changed from the earlier value of $W=V=10^{-6}$ to a new value of 10^{-4}, the transient response of the system remains essentially unchanged except the steady state (RMS) value of the nutation angle is increased from 0.02 degree to 0.2 degree.

(2) Two Measurement System

The transient response of the filter gains (F) when both measurements ω_1 and ω_2 are made is shown in Fig. 3.6. The resulting dynamic response of the system for the larger control mass produced by the constant control and filter gains coincides with the earlier case of single measurement of ω_1 only. Also the system transient response for the deterministic case[6] (no noise present) is compared with the stochastic system transient response and it is found that both have essentially the same response for the system parameters and initial conditions considered here. It can be seen that the stochastic system has a steady state (RMS) nutation angle of 0.02 degree which is not characteristic of the deterministic case.[6] (This is consistent with the analytic result described earlier.)

b. Three Axis Control Using Two Offset Booms

For the general case of three-axis control numerical methods are used to solve the matrix Riccati equation.[15] The dynamics of the controlled system are obtained by numerical integration of Eqs. (3.28) and (3.29) as developed in Refs. 2 and 6.

The matrices A and B in Eqs. (3.28) and (3.29) for this application of two boom system for three axis control, using the larger control masses, with the selected system parameters are obtained as:

$$A=\begin{bmatrix} 0.0018015 & -0.6667980 & 0.0 \\ 0.6667980 & -0.0018015 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$B^T=\begin{bmatrix} 0.0082732 & 0.637957 & 0.0 \\ 0.0 & 0.0 & 0.384811 \end{bmatrix}$$

The reference length ℓ_m has been assumed to be 100 ft. (30.48m) and the control signals and plant noises are assumed to enter together such that B = G. In the measuring scheme all the variations of angular velocity components of the spacecraft from a nominal state are measured.

The optimal control constant gain, C, obtained by solving the matrix Riccati equation (Eq. (3.6)) numerically can be expressed as[2,6]:

$$C = \begin{bmatrix} -2.2242 & 3.8489 & 0.0 \\ 0.0 & 0.0 & 3.1401 \end{bmatrix}$$

Similarly, the optimal filter constant gain, F, obtained from the steady state solutions of Eq. (3.9) can be written in the form:

$$F = \begin{bmatrix} 0.3607 & -0.1047 & 0.0 \\ -0.1047 & 0.5045 & 0.0 \\ 0.0 & 0.0 & 0.3848 \end{bmatrix}$$

An example of the application of a two boom system for three axis control is shown in Fig. 3.7. where initial perturbations are assumed to be present in all three angular rates. The decay of nutation angle is showns in Fig. 3.7(a).

It is seen that the nutation is reduced to approximately zero within 30 secs. Also, shown in the figure are the time responses of the z - boom motion when there is no damping present in the boom driving mechanism and when it is present. With the value shown in the figure for boom damping, the boom motion can be reduced to zero within 100 secs.

The response of x - boom motion as shown in Fig. 3.7(a) increases linearly with time without any limitation. This is due to the fact that control of the spin angular velocity magnitude for the linearized system depends on the x - boom motion as seen from the numerical values of the matrices indicated earlier. Due to the stochastic disturbances, the angular velocity component about the '3' axis fluctuates from its nominal value, which forces the x - boom as shown in Fig. 3.7(a). As this is not desirable, a spring force is now assumed to be present in the x - boom mechanism as shown which brings the x - boom within a maximum amplitude of 23.5 ft. (7.16m). The damping present in the boom for the selected system parameters is very weak and, hence, additional damping in the x - boom driving mechanism is needed to bring the x - boom motion to zero value in a reasonable time period.

The time responses of the angular velocity components ω_1, ω_2 and ω_3 and their respective estimates $\hat{\omega}_1, \hat{\omega}_2$, and $\hat{\omega}_3$ are illustrated in Fig. 3.7(b). It is seen that the difference between the estimates of the components of angular velocity and the actual values of these components is negligible within the plotting accuracy.

The spin angular velocity reaches the desired value of 0.314 rad/sec. from an initial value of 0.35 rad/sec. within 20 secs. and fluctuates with a small amplitude (not observable on the figure) which is responsible for the x - boom motion dissucssed earlier.

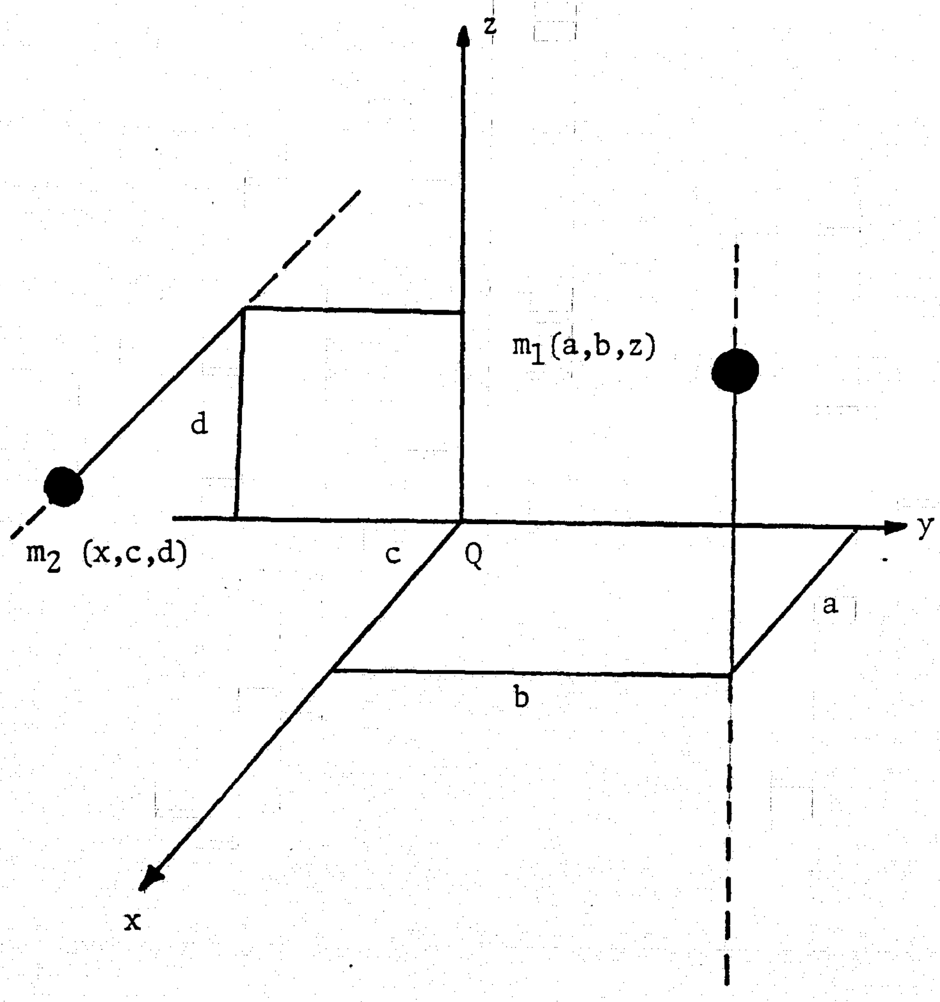

Fig. 3.1. Two Boom Orientation System

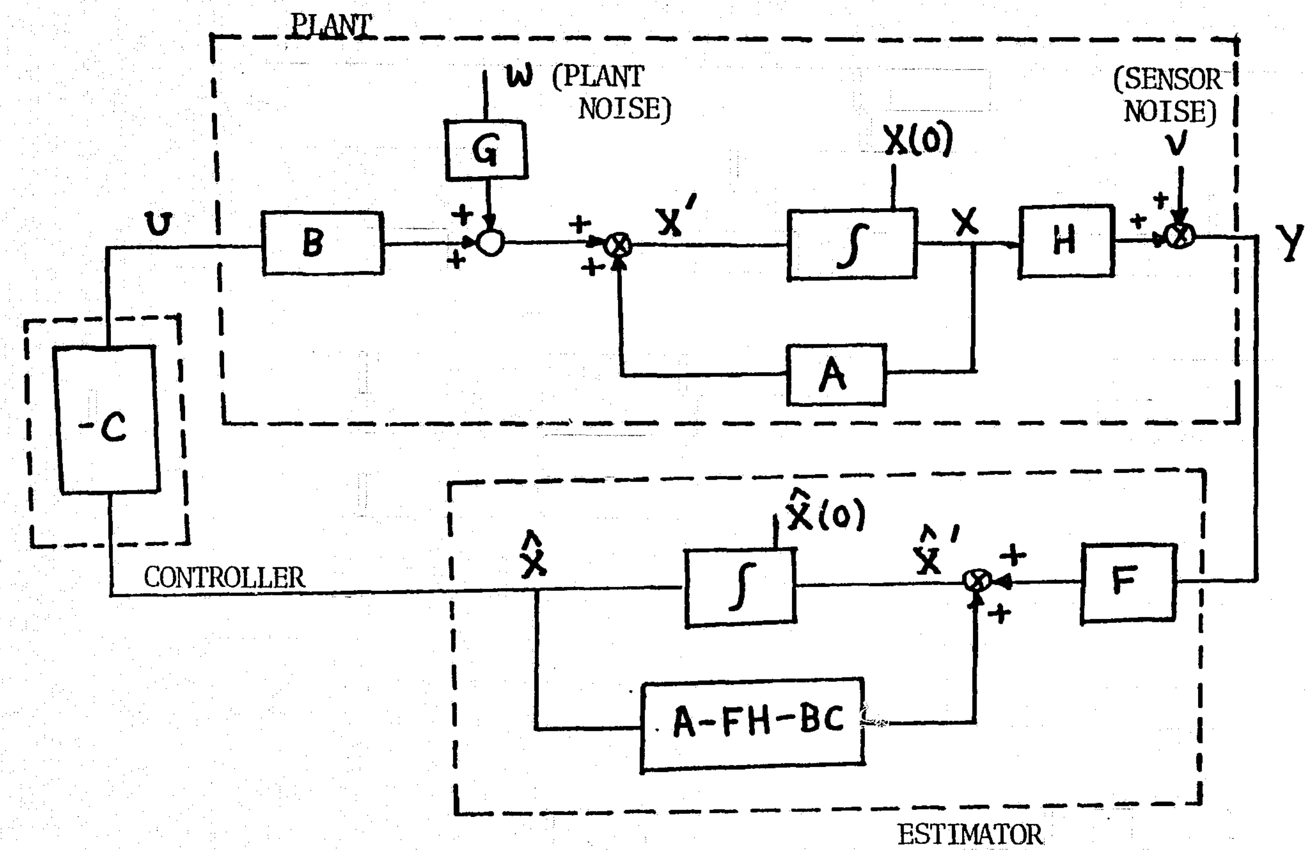

Fig. 3.2. Stochastic Optimal Control Configuration

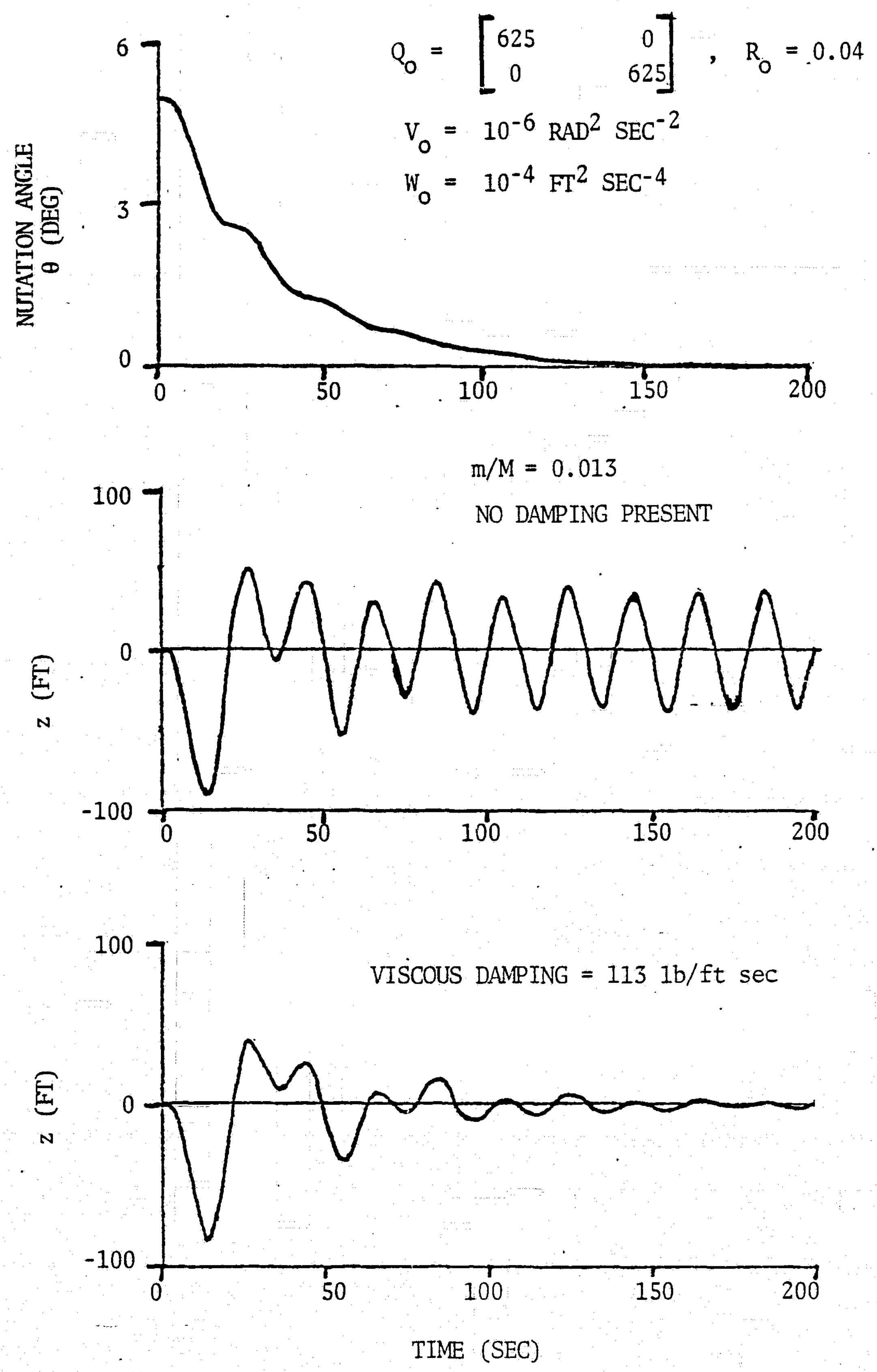

Fig. 3.3(a) Time Response of Nutation Angle (θ) and z - Boom End Mass Displacement for Single Boom Case with Single Measurement ω_1

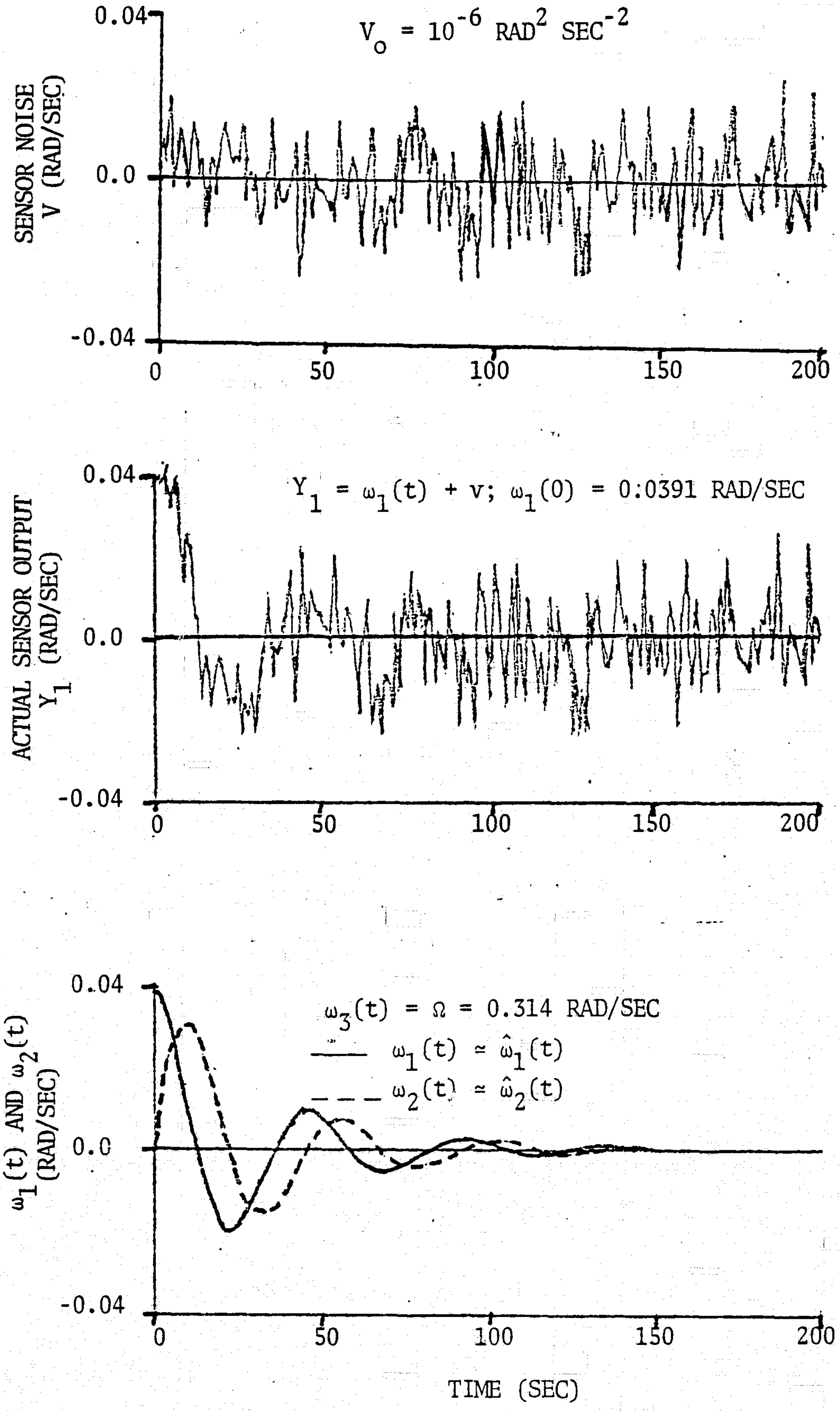

Fig. 3.3(b). Time Response of Sensor Noise (v), Actual Sensor Output (Y_1) and Transverse Angular Velocity Components (ω_1 and ω_2) for Single Boom Case with Single Measurement ω_1

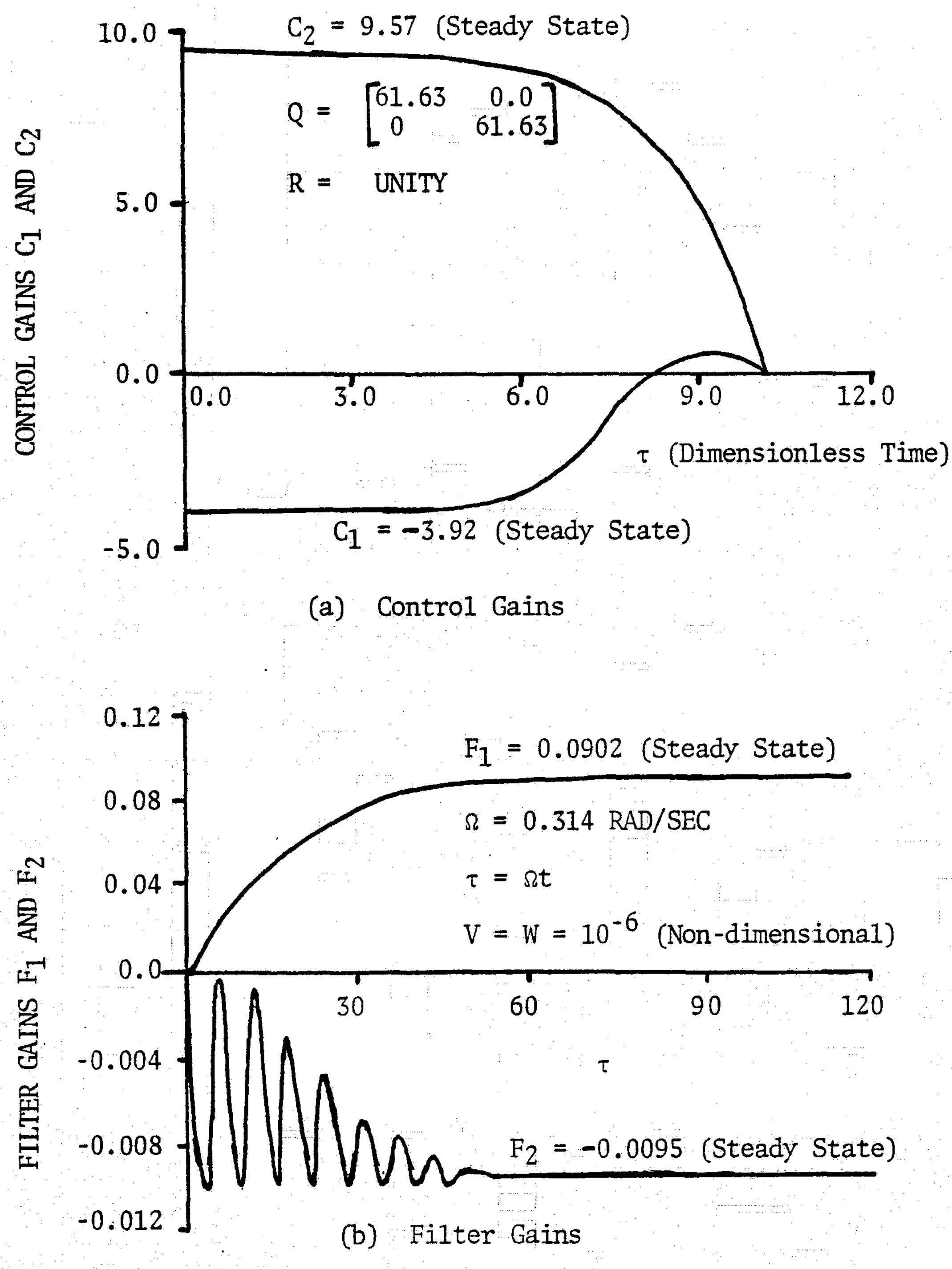

Fig. 3.4. Control and Filter Gains for Single Boom Case with Single Measurement ω_1 (with Larger Masses)

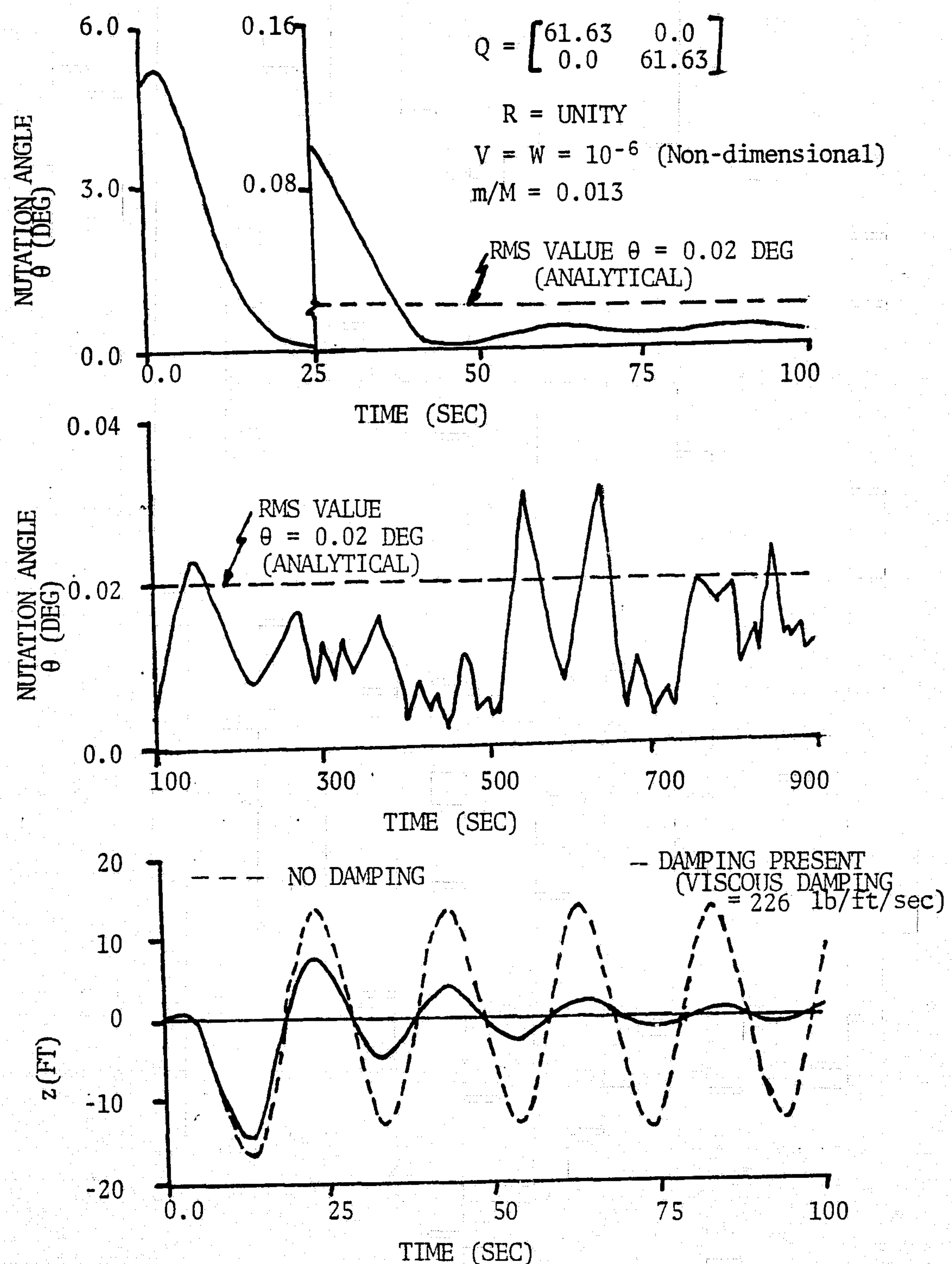

Fig. 3.5(a). Time Responses of Nutation Angle (θ) and z- Boom End Mass Displacement for Single Boom Case with Single Measurement ω_1 (with Larger Masses)

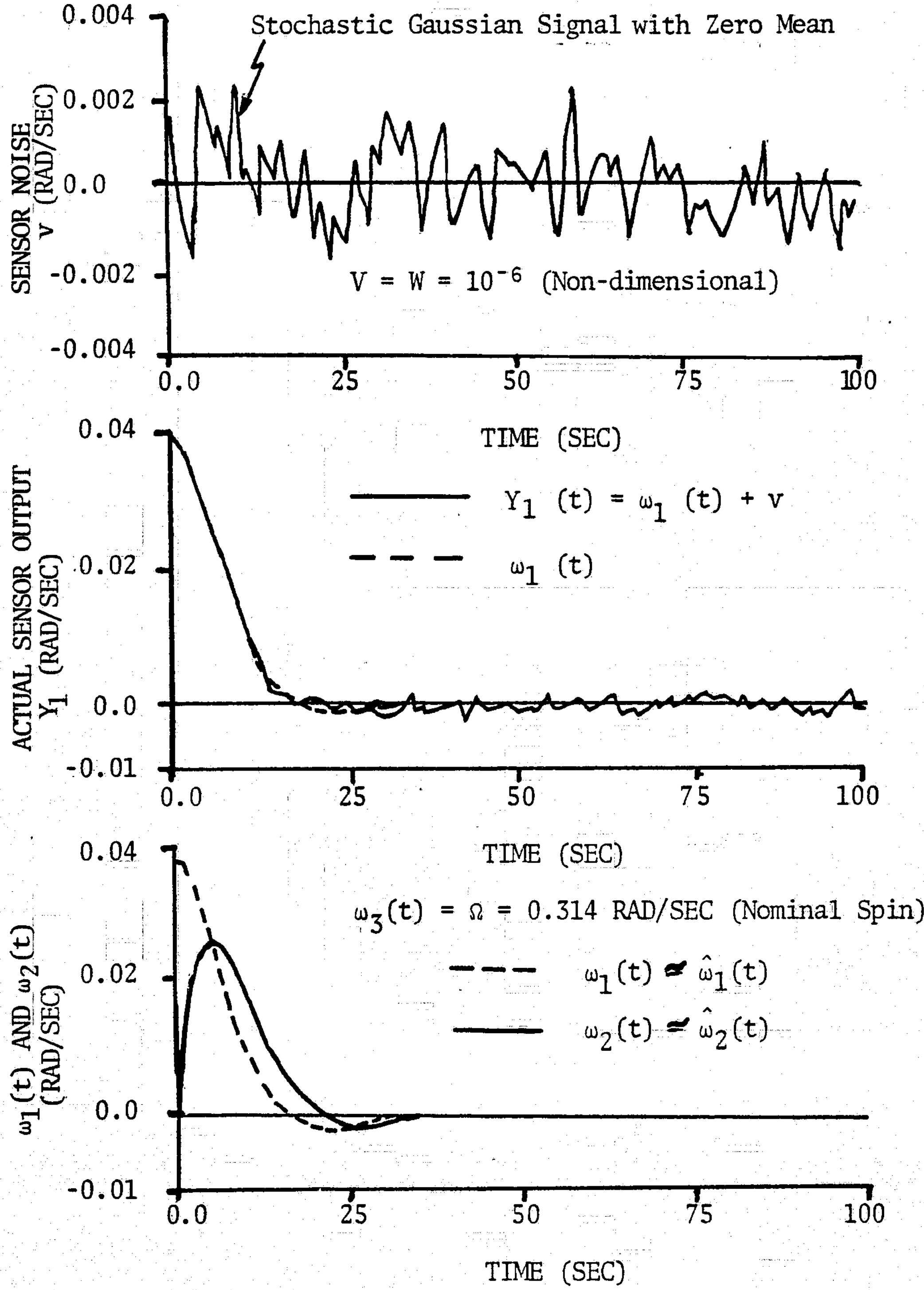

Fig. 3.5(b). Time Responses of Sensor Noise (v), Actual Sensor Output (Y_1) and Transverse Angular Velocity Components (ω_1 and ω_2) for Single Boom Case with Single Measurement ω_1 (with Larger Mass)

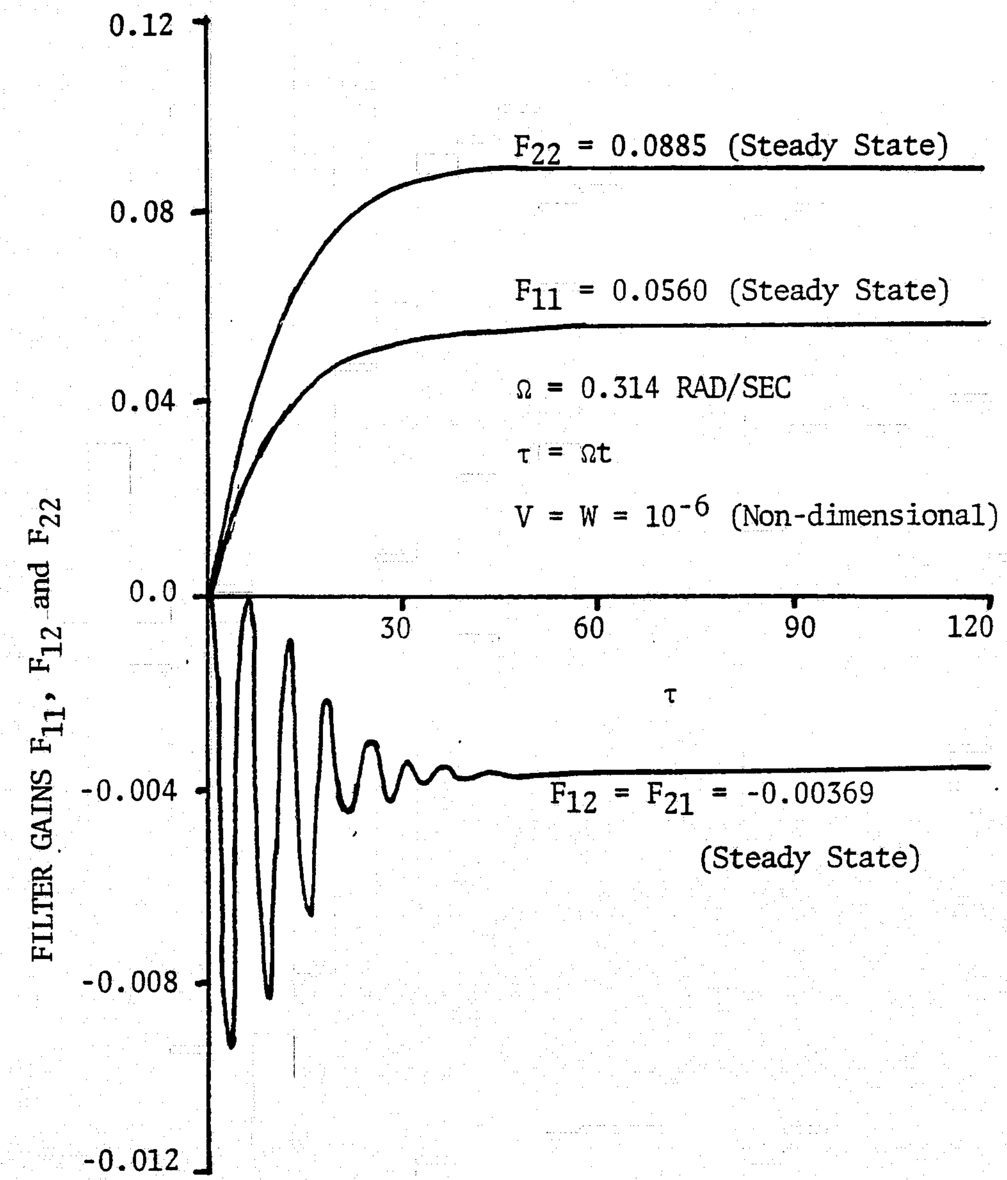

Fig. 3.6. Filter Gains for Single Boom Case with Both Measurements ω_1 and ω_2 (with Larger Masses)

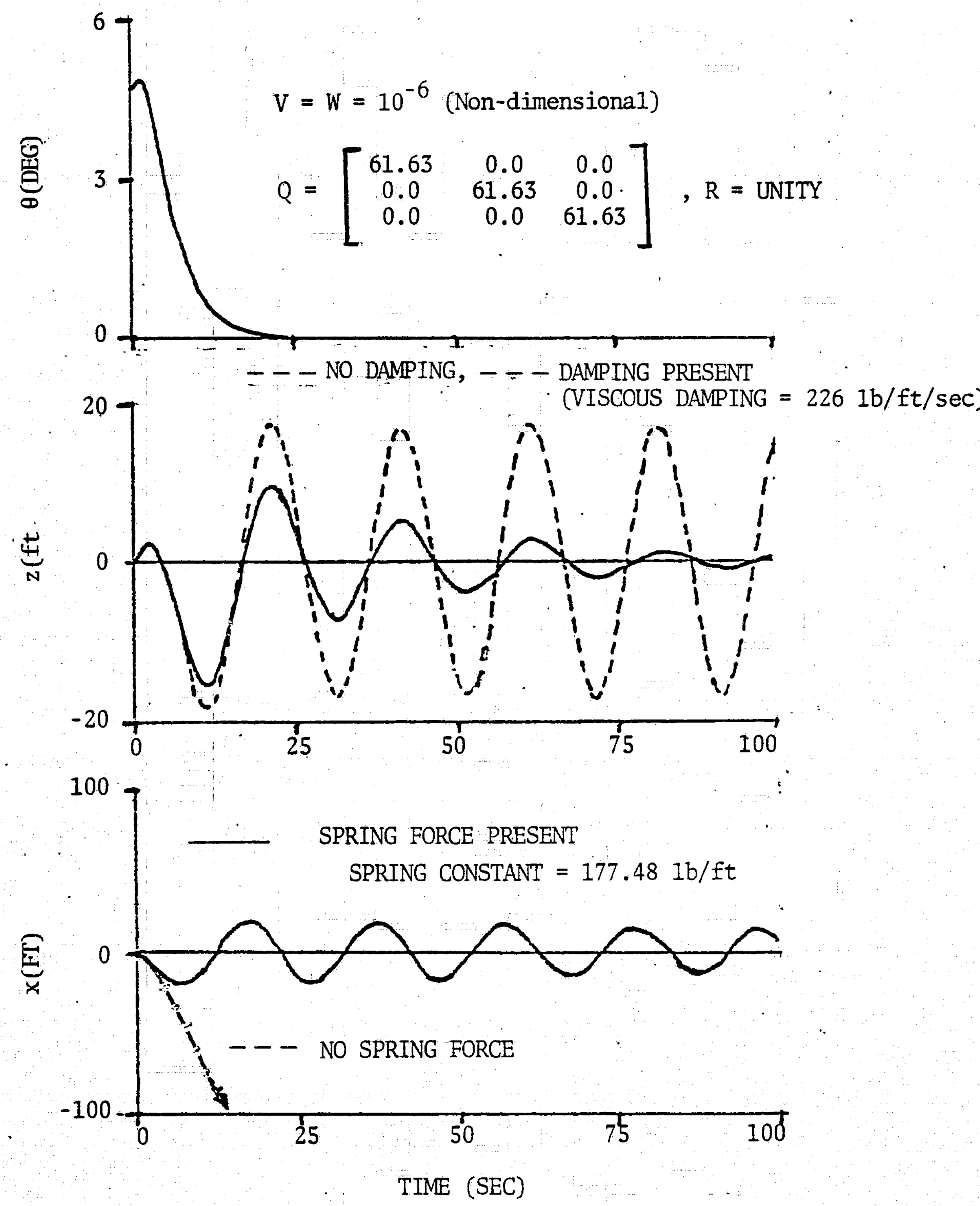

Fig. 3.7(a). Time Response of Nutation Angle (θ), z - Boom and x - Boom End Mass Displacements for Two Booms Case with Three Measurements (with Larger Masses)

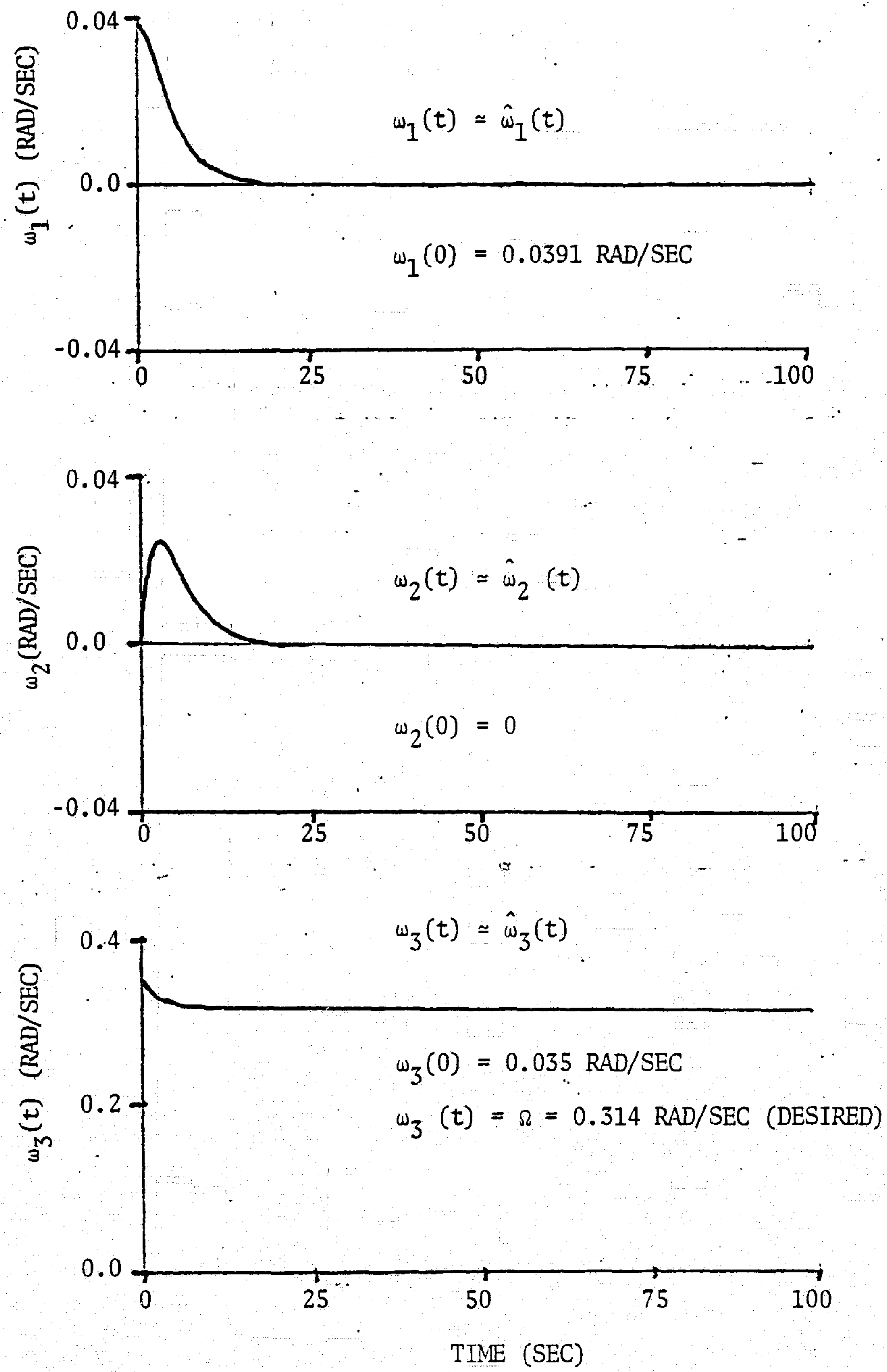

Fig. 3.7(b). Time Response of Angular Velocity Components for Two Booms Case with Three Measurements (with Larger Masses)

IV. CONCLUDING COMMENTS

As a result of the analysis and numerical results the following conclusions regarding the time optimal and stochastic optimal control using telescoping booms can be made:

A. TIME OPTIMAL CONTROL

1. The time optimal control problem is solved analytically for a single offset boom where the initial conditions are such that the system can be driven to the equilibrium state with a single switching in the bang-bang optimal sequence.

2. The large amplitudes of the single offset control boom displacement can be reduced to a reasonable practical value by increasing the size of the control mass. This also reduces the nutation angle steady state amplitude to a negligible value.

3. For the general case where the initial conditions do not lie within the single switching region piecewise solutions can be used to obtain the system response.

B. STOCHASTIC OPTIMAL CONTROL

1. In the area of stochastic optimal control, the average performance of the controlled system in the presence of plant and measurement noises for the case of single boom offset system with the measurement of one of the transverse angular velocity components can be predicted from the covariances of the error and estimate.

2. The dynamics of spinning spacecraft using one or two movable telescoping offset boom systems for the general case with stochastic inputs are obtained using numerical integration. It is seen that the difference between the estimates of the components of the angular velocity and the actual values of these components is negligble for the covariances, initial conditions, and system parameters considered here.

3. In the case of three axis control using two orthogonal booms, the motion of the boom in a direction orthogonal to the spin axis increases linearly with time. This is due to the fact that control of the spin angular velocity magnitude for the linearized system depends on this boom motion. Due to the stochastic disturbances, the spin magnitude fluctuates from its nominal value, which forces the (x) boom. In order to remove this undesirable motion, additional damping in this boom driving mechanism and a spring force acting on the boom mass is needed. In the deterministic case, the spin magnitude reaches the desired value and remains constant; the motion of the (x) boom reaches a steay state (constant) amplitude.

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COMPUTER PROGRAMS

A. TIME OPTIMAL CONTROL

```
!LISTING
; C       TIME OPT. CONTROL-NUM. INT.- NL EQN
;         EXTERNAL BSOC1,BSOC2
;         DIMENSION PARM(5),Y(7),DY(7),WORK(8,7),SIZE(7)
;         REAL I1,I2,I3,M8,M1,M2,LM
;         COMMON Y
;         COMMON I1,I2,I3,M8,M1,M2,LM
;         COMMON N,M
;         COMMON AA,BB,CC,DD
;         COMMON C(3,3)
;         EQUIVALENCE (Y(1),W1N),(Y(2),W2N),(Y(3),W3N),(Y(4),DZN)
;         EQUIVALENCE (Y(5),DXN),(Y(6),ZN),(Y(7),XN)
;         CALL INOUT(2,5)
;         CALL OPEN (1, 'SELLAPPAN' ,3,IER)
;         IF(IER.NE.1) STOP UNABLE TO OPEN FILE
;         READ(2,91) TMAX,STEP,TOL
;         READ(2,91) SIZE
;      91 FORMAT(8F10.0)
;         PARM(1)=0.0
;         PARM(2)=TMAX
;         PARM(3)=STEP
; C       INITIAL VALUES
;         READ(2,911) I1,I2,I3,M8
;         READ(2,911) M1,M2,LM
;         READ(2,911) AA,BB,CC,DD
;     911 FORMAT(4F20.4)
;         W1N=0.0391/0.314
;         W2N=0.0
;         W3N=(0.314-0.314)/0.314
;         DZN=0.0
;         DXN=0.0
;         ZN=0.0
;         XN=0.0
;         N=7
;         M=3
;         WRITE(5,92) TMAX,STEP,TOL
;         WRITE(5,95) I1,I2,I3
;         WRITE(5,96) M1,M2,LM
;         WRITE(5,97) AA,BB,CC,DD
;         WRITE(5,98) SIZE
;         WRITE(5,99)
;      92 FORMAT('1TMAX=',F8.4,10X,'STEP=',F8.4,10X,'TOL=',F8.6)
;      95 FORMAT('0I1=',F15.2,2X,'I2=',F15.2,2X,'I3=',F15.2)
;      96 FORMAT('0M1=',F15.2,2X,'M2=',F15.2,2X,'LM=',F15.2)
;      97 FORMAT('0AA=',F8.4,5X,'BB=',F8.4,5X,'CC=',F8.4,5X,'DD=',F8.4)
;      98 FORMAT('0SIZE',7F8.4)
;      99 FORMAT('1',T6,'T',T17,'W1',T30,'W2',T43,'W3',T55,'DZ',
;        2T69,'DX',T81,'Z',T94,'X',T108,'THETA',T115,'IHLF',/)
;         CALL RKSCL(N,SIZE,DY,TOL,PARM)
;         CALL RKGS(PARM,Y,DY,N,IHLF,BSOC1,BSOC2,WORK)
;         WRITE(5,100) IHLF
;     100 FORMAT('0IHLF=',I3)
;         CALL EXIT
;         END
```

```
PROGRAM IS RELOCATABLE
                              .TITL    .MAIN
!FORT/A/B/E/P/S FORT.LS/L
!LISTING
;       SUBROUTINE BSOC1(T,Y,DY)
;       DIMENSION Y(7),DY(7),C(3,3)
;       REAL I1,I2,I3,MB,M1,M2,LM
;       COMMON W1N,W2N,W3N,DZN,DXN,ZN,XN
;       COMMON I1,I2,I3,MB,M1,M2,LM
;       COMMON N,M
;       COMMON AA,BB,CC,DD
;       COMMON C
;       TS=0.54
;       TF=6.12
;       IF(T.GT.TS) GO TO 40
;       CO=+0.03
;       GO TO 30
;    40 IF(T.GT.TF) GO TO 50
;       CO=-0.03
;       GO TO 30
;    50 CO=0.0
;    30 CONTINUE
; C     CAL.OF.COEFF.FOR LHS OF MATRIX EQN
;       U=M1*(MB+M2)/(MB+M1+M2)
;       ULM=U*LM*LM
; C     NONDIMENSIONALIZATION
;       AI1=I1/ULM
;       AI2=I2/ULM
;       AI3=I3/ULM
;       C1=AA/LM
;       C2=BB/LM
;       C3=CC/LM
;       C4=DD/LM
;       U1=M1*(MB+M2)/((MB+M1+M2)*U)
;       U2=M2*(MB+M1)/((MB+M1+M2)*U)
;       U3=M1*M2/((MB+M1+M2)*U)
;       C(1,1)=AI1+U1*(C2*C2+ZN*ZN)+U2*(C3*C3+C4*C4)
;      *-2.0*U3*(C2*C3+C4*ZN)
;       C(1,2)=-U1*(C1*C2)-U2*C3*XN+U3*(C1*C3+C2*XN)
;       C(1,3)=-U1*(C1*ZN)-U2*C4*XN+U3*(C1*C4+ZN*XN)
;       C(2,1)=-U1*(C1*C2)-U2*C3*XN+U3*(C1*C3+C2*XN)
;       C(2,2)=AI2+U1*(C1*C1+ZN*ZN)+U2*(C4*C4+XN*XN)
;      *-2.0*U3*(C1*XN+C4*ZN)
;       C(2,3)=-U1*(C2*ZN)-U2*C3*C4+U3*(C2*C4+C3*ZN)
;       C(3,1)=-U1*(C1*ZN)-U2*C4*XN+U3*(C1*C4+ZN*XN)
;       C(3,2)=-U1*(C2*ZN)-U2*C3*C4+U3*(C2*C4+C3*ZN)
;       C(3,3)=AI3+U1*(C1*C1+C2*C2)+U2*(C3*C3+XN*XN)
;      *-2.0*U3*(C2*C3+C1*XN)
; C     CAL. OF RHS OF MATRIX EQN
;       CN=C1/(AI1+C1*C1)
;       DF=0.0
;       AZN=(CO/CN)-DF*DZN-ZN
;       AXN=0.0
;       SN=1.0+W3N
;       A11=(AI3-AI2)*W2N*SN
;       A121=-C1*ZN*W1N*W2N+(C2*C2-ZN*ZN)*W2N*SN+C1*C2*W1N*SN
;       A122=2.0*ZN*DZN*W1N+C2*ZN*(SN*SN-W2N*W2N)+C2*AZN
;       A123=-C4*XN*W1N*W2N+(C3*C3-C4*C4)*W2N*SN+C3*XN*W1N*SN
;       A124=-2.0*C3*DXN*W2N-2.0*C4*DXN*SN+C3*C4*(SN*SN-W2N*W2N)
;       A12=U1*(A121+A122)+U2*(A123+A124)
;       A15=-(C1*C4+ZN*XN)*W1N*W2N+2.0*(C2*C3-C4*ZN)*W2N*SN
```

```
;       A16=(C1*C3+C2*XN)*W1N*SN+2.0*C4*DZN*W1N-2.0*C2*DXN*W2N
;       A17=-2.0*DXN*ZN*SN+(C2*C4+C3*ZN)*(SN*SN-W2N*W2N)+C3*AZN
;       A13=U3*(A15+A16+A17)
;       DY(1)=-(A11+A12-A13)
;       B11=(AI1-AI3)*SN*W1N
;       B121=-C2*ZN*W1N*W2N+C1*C2*W2N*SN+(C1*C1-ZN*ZN)*SN*W1N
;       B122=-2.0*ZN*DZN*W2N+C1*ZN*(SN*SN-W1N*W1N)+C1*AZN
;       B123=-C3*C4*W1N*W2N+C3*XN*W2N*SN+(-C4*C4+XN*XN)*SN*W1N
;       B124=-2.0*XN*DXN*W2N+C4*XN*(SN*SN-W1N*W1N)-C4*AXN
;       B12=U1*(B121+B122)+U2*(B123+B124)
;       B15=-(C2*C4+C3*ZN)*W1N*W2N+(C1*C3+C2*XN)*W2N*SN
;       B16=2.0*(C1*XN-C4*ZN)*SN*W1N-2.0*(C1*DXN+C4*DZN)*W2N
;       B17=(C1*C4+XN*ZN)*(SN*SN-W1N*W1N)+XN*AZN-ZN*AXN
;       B13=U3*(B15+B16+B17)
;       DY(2)=-(B11-B12+B13)
;       C11=(AI2-AI1)*W1N*W2N
;       C121=(C2*C2-C1*C1)*W1N*W2N-C1*ZN*W2N*SN+C2*ZN*SN*W1N
;       C122=2.0*C1*DZN*W1N+2.0*C2*DZN*W2N+C1*C2*(W1N*W1N-W2N*W2N)
;       C123=(C3*C3-XN*XN)*W1N*W2N-C4*XN*W2N*SN+C3*C4*SN*W1N
;       C124=-2.0*XN*DXN*SN+C3*XN*(W1N*W1N-W2N*W2N)+C3*AXN
;       C12=U1*(C121+C122)+U2*(C123+C124)
;       C15=2.0*(C2*C3-C1*XN)*W1N*W2N-(C1*C4+XN*ZN)*W2N*SN
;       C16=(C2*C4+C3*ZN)*SN*W1N+2.0*XN*DZN*W1N+2.0*C3*DXN*W2N
;       C17=-2.0*C1*DXN*SN+(C1*C3+C2*XN)*(W1N*W1N-W2N*W2N)+C2*AXN
;       C13=U3*(C15+C16+C17)
;       DY(3)=-(C11-C12+C13)
;       DY(4)=AZN
;       DY(5)=AXN
;       DY(6)=DZN
;       DY(7)=DXN
;       CALL SIMQ(C,DY,M,KS)
;       IF(KS) 3,2,3
;     2 RETURN
;     3 WRITE(5,4)
;     4 FORMAT(// ' SINGULAR EQUATIONS')
;       RETURN
;       END

PROGRAM IS RELOCATABLE
                         .TITL   BSOC1
!FORT/A/B/E/P/S FORT.LS/L
!LISTING
;       SUBROUTINE BSOC2(T,Y,DY,IHLF,NDUM,P)
;         LOGICAL RKNXT
;       DIMENSION Y(7),DY(7),DUMMY(7)
;       DIMENSION X(7)
;       REAL I1,I2,I3,MB,M1,M2,LM
;       COMMON W1N,W2N,W3N,DZN,DXN,ZN,XN
;       COMMON I1,I2,I3,MB,M1,M2,LM
;       COMMON N,M
;       COMMON AA,BB,CC,DD
;       DEG=57.2957795
;       CALL BSOC1(T,Y,DUMMY)
;       WS=0.314
;       X(1)=Y(1)*WS
;       X(2)=Y(2)*WS
;       X(3)=(1.0+Y(3))*WS
;       X(4)=Y(4)*LM*WS
;       X(5)=Y(5)*LM*WS
;       X(6)=Y(6)*LM
;       X(7)=Y(7)*LM
;       H1=I1*X(1)
```

```
;         H2=I2*X(2)
;         H3=I3*X(3)
;         THETA=ATAN2(SQRT(H1*H1+H2*H2),H3)*DEG
;         TA=T/WS
;         TP=TA+0.00005
;         IF(.NOT.RKNXT(IHLF)) GO TO 8
;         WRITE(5,1) TP,X,THETA,IHLF
;       1 FORMAT(1X,F9.4,7E13.7,F9.4,I10)
;         WRITE BINARY(1) TA,THETA,X(7),X(6),X(3),X(2),X(1)
;       8 CONTINUE
;         RETURN
;         END

PROGRAM IS RELOCATABLE
                        .TITL   BSOC2
!RLDR/M TMP/S 001 002 003 DP0:SSP.LB FORT.LB
!DELETE/V SELLAPPAN
DELETED SELLAPPAN
!CREATE SELLAPPAN
!EXEC
```

B. KALMAN FILTER PROGRAM

```
!LISTING
; C       PROGRAM II
; C       RICCATI EQUATION PROGRAM (RICATI)
;         DIMENSION A(10,10),B(10,10),C(10,10),R(10,10),Q(10,10),
;        * K(10,10),D(10,10),X(10),F(10,10),E(10,10),
;        *  G(10,10),H(10,10),S(10,10)
;         DIMENSION V(3,3),LL(3),MM(3)
;         REAL K
;         INTEGER OPTION,BLANK
;         COMMON/KALMAN/ICC,IFF,BLANK
;         DATA ICC,IFF,BLANK/'C ','F ',' '/
;         CALL INOUT(2,5)
;  1000 FORMAT(1H1,5X,37HOPTIMAL CONTROL/KALMAN FILTER PROGRAM/)
;  1001 FORMAT(3I2)
;  1003 FORMAT (1H0,5X,13H THE A MATRIX/)
;  1004 FORMAT (1H0,5X,13H THE B MATRIX/)
;  1005 FORMAT (1H0,5X,13H THE C MATRIX/)
;  1006 FORMAT(8F10.3)
;  1040 FORMAT(4F20.0)
;  1007 FORMAT (A1,9X,2F10.3,I3)
;  1008 FORMAT(1H0,45(1H*))
;  1009 FORMAT (1H0,5X,21H*** FILTER OPTION ***/)
;  1010 FORMAT (1H0,5X,22H*** CONTROL OPTION ***/)
;  1011 FORMAT(6(1PE20.8))
;  1012 FORMAT(1H0,5X,13H THE R MATRIX/)
;  1013 FORMAT(1H0,5X,13H THE Q MATRIX/)
;  1020 FORMAT(1H0,5X,13H THE V MATRIX/)
;  1014 FORMAT (1H0,5X,19H INITIAL CONDITIONS/)
;  1015 FORMAT(1H0,5X,8H TIME = ,1PE20.8/6X,5HGAINS)
;  1016 FORMAT(1H0,5X,21HSTEADY STATE SOLUTION//
;        * 6X,6H GAINS/)
;   100 READ(2,1001) N,M,L
;         WRITE(5,1000)
;         WRITE(5,1001) N,M,L
;         WRITE(5,1008)
;         WRITE(5,1003)
;         DO 110 I=1,N
;         READ(2,1006) (A(I,J),J=1,N)
;   110 WRITE(5,1011) (A(I,J),J=1,N)
;         WRITE(5,1004)
;         DO 120 I=1,M
;         READ(2,1006) (B(J,I),J=1,N)
;   120 WRITE(5,1011) (B(J,I),J=1,N)
;         WRITE(5,1005)
;         DO 130 I=1,L
;         READ (2,1006) (C(I,J),J=1,N)
;   130 WRITE(5,1011) (C(I,J),J=1,N)
;   150 READ(2,1007,END=999) OPTION,T1,T2,NPT
;         WRITE(5,1008)
;         IF(OPTION.EQ.BLANK) GO TO 100
;         IF(OPTION.EQ.ICC) GO TO 300
;         WRITE(5,1009)
;         NR=L
;         NQ=M
```

```
;       DO 230 I=1,N
;       DO 210 J=1,L
;   210 E(I,J)=C(J,I)
;       DO 220 J=1,M
;   220 D(I,J)=B(I,J)
;       DO 230 J=1,N
;   230 F(I,J)=A(I,J)
;       GO TO 400
;   300 WRITE(5,1010)
;       NR=M
;       NQ=N
;       DO 330 I=1,N
;       DO 310 J=1,M
;   310 E(I,J)=B(I,J)
;       DO 330 J=1,N
;   330 F(I,J)=A(J,I)
;   400 WRITE(5,1012)
;       DO 410 I=1,NR
;       READ(2,1040) (R(I,J),J=1,NR)
;   410 WRITE(5,1011) (R(I,J),J=1,NR)
;       WRITE(5,1013)
;       DO 420 I=1,NQ
;       READ(2,1040) (Q(I,J),J=1,NQ)
;   420 WRITE(5,1011) (Q(I,J),J=1,NQ)
;       DO 430 I=1,NR
;       DO 430 J=1,NR
;   430 V(I,J)=R(I,J)
;       CALL MINV(V,NR,U,LL,MM)
;       WRITE(5,1020)
;       DO 425 I=1,NR
;   425 WRITE(5,1011) (V(I,J),J=1,NR)
;       DO 440 I=1,NR
;       DO 440 J=1,N
;       R(I,J)=0.0
;       DO 440 II=1,NR
;   440 R(I,J)=R(I,J)+V(I,II)*E(J,II)
;       IF(OPTION.EQ.ICC) GO TO 500
;       DO 450 I=1,N
;       DO 450 J=1,NQ
;       G(I,J)=0.0
;       DO 450 II=1,NQ
;   450 G(I,J)=G(I,J)+D(I,II)*Q(II,J)
;       DO 460 I=1,N
;       DO 460 J=1,N
;       Q(I,J)=0.0
;       DO 460 II=1,NQ
;   460 Q(I,J)=Q(I,J)+G(I,II)*D(J,II)
;   500 IF(NPT.GT.0) GO TO 530
;       DO 520 I=1,N
;       DO 510 J=1,N
;   510 G(I,J)=0.0
;   520 G(I,I)=1.0
;       EPS=0.1
;       GO TO 570
;   530 WRITE(5,1014)
;       DO 540 I=1,N
;       READ(2,1006) (G(I,J),J=1,N)
;   540 WRITE(5,1011) (G(I,J),J=1,N)
;       WRITE(5,1008)
;       TIME=ABS(T2-T1)
;       PTS=200.0*TIME
;       XX=NPT
;       XX=PTS/XX
```

```
;       ID=XX
;       DI=ID
;       YY=ABS(XX-DI)
;       IF(YY.GT.0.05) ID=ID+1
;       IT=PTS
;       EPS=0.005
;       TIME=T1
;       IF(OPTION.EQ.ICC) TIME=T2
;       WRITE(5,1015) TIME
;       DO 560 I=1,NR
;       DO 550 J=1,N
;       K(I,J)=0.0
;       DO 550 II=1,N
;   550 K(I,J)=K(I,J)+R(I,II)*G(II,J)
;   560 WRITE(5,1011) (K(I,J),J=1,N)
;   570 LC=0
;       ICH=ID
;   575 DO 580 I=1,NR
;       DO 580 J=1,N
;       K(I,J)=0.0
;       DO 580 II=1,N
;   580 K(I,J)=K(I,J)+R(I,II)*G(II,J)
;       DO 590 I=1,N
;       DO 590 J=1,N
;       H(I,J)=0.0
;       DO 590 II=1,NR
;   590 H(I,J)=H(I,J)+E(I,II)*K(II,J)
;       DO 610 I=1,N
;       DO 610 J=1,N
;       D(I,J)=Q(I,J)
;       DO 600 II=1,N
;   600 D(I,J)=D(I,J)+F(I,II)*G(II,J)+G(I,II)*F(J,II)-G(I,II)*H(II,J)
;   610 S(I,J)=G(I,J)+D(I,J)*EPS
;       IF(NPT.LE.0) GO TO 640
;       LC=LC+1
;       IF(LC.LT.ICH) GO TO 625
;       ICH=ICH+ID
;       ALC=LC
;       T=ALC*EPS
;       TIME=T1+T
;       IF(OPTION.EQ.ICC) TIME=T2-T
;       WRITE(5,1015) TIME
;       DO 620 I=1,NR
;   620 WRITE(5,1011) (K(I,J),J=1,N)
;       IF(LC.GE.IT) GO TO 150
;   625 DO 630 I=1,N
;       DO 630 J=1,N
;   630 G(I,J)=S(I,J)
;       GO TO 575
;   640 SUM=0.0
;       DO 650 I=1,N
;       DO 650 J=1,N
;       SUM=SUM+ABS(D(I,J))
;   650 G(I,J)=S(I,J)
;       IF(SUM.GT.0.01) GO TO 575
;       WRITE(5,1016)
;       DO 660 I=1,NR
;   660 WRITE(5,1011) (K(I,J),J=1,N)
;       GO TO 150
;   999 CONTINUE
;       CALL EXIT
;       END
```

```
PROGRAM IS RELOCATABLE
                          .TITL    .MAIN
!RLDR/M TMP/S 001 DP0:SSP.LB FORT.LB
!EXEC

    OPTIMAL CONTROL/KALMAN FILTER PROGRAM

 3 2 3

 *******************************************

     THE A MATRIX

   1.80150000E -3    -6.66798000E -1     0.00000000E  0
   6.66798000E -1    -1.80150000E -3     0.00000000E  0
   0.00000000E  0     0.00000000E  0     0.00000000E  0

     THE B MATRIX

   8.27320300E -3     6.37957000E -1     0.00000000E  0
   0.00000000E  0     0.00000000E  0     3.84811000E -1

     THE C MATRIX

   1.00000000E  0     0.00000000E  0     0.00000000E  0
   0.00000000E  0     1.00000000E  0     0.00000000E  0
   0.00000000E  0     0.00000000E  0     1.00000000E  0

 *******************************************

    *** FILTER OPTION ***

     THE R MATRIX

   1.00000000E -6     0.00000000E  0     0.00000000E  0
   0.00000000E  0     1.00000000E -6     0.00000000E  0
   0.00000000E  0     0.00000000E  0     1.00000000E -6

     THE Q MATRIX

   1.00000000E -6     0.00000000E  0
   0.00000000E  0     1.00000000E -6

     THE V MATRIX

   9.99999600E  5     0.00000000E  0     0.00000000E  0
   0.00000000E  0     9.99999600E  5     0.00000000E  0
   0.00000000E  0     0.00000000E  0     9.99999600E  5

     INITIAL CONDITIONS

   0.00000000E  0     0.00000000E  0     0.00000000E  0
   0.00000000E  0     0.00000000E  0     0.00000000E  0
   0.00000000E  0     0.00000000E  0     0.00000000E  0

 *******************************************
```

C. THREE AXIS OPTIMAL ATTITUDE CONTROL

```
!LISTING
; C      STOCHASTIC OPTIMAL ATTITUDE CONTROL
; C      THREE AXIS CONTROL-TWO BOOM SYSTEM
;        EXTERNAL SOAC1,SOAC2
;        DIMENSION PARM(5),Y(10),DY(10),WORK(8,10),SIZE(10)
;        REAL I1,I2,I3,LM
;        COMMON Y
;        COMMON I1,I2,I3,LM
;        COMMON N
;        COMMON W1,W2,VN1,VN2,VN3
;        COMMON IXW1,SW1,AMW1
;        COMMON IXW2,SW2,AMW2
;        COMMON IXV1,SV1,AMV1
;        COMMON IXV2,SV2,AMV2
;        COMMON IXV3,SV3,AMV3
;        EQUIVALENCE (Y(1),W1N),(Y(2),W2N),(Y(3),W3N)
;        EQUIVALENCE (Y(4),EW1N),(Y(5),EW2N),(Y(6),EW3N)
;        EQUIVALENCE (Y(7),DZN),(Y(8),DXN),(Y(9),ZN),(Y(10),XN)
;        CALL INOUT(2,5)
;        CALL OPEN (1, 'SELLAPPAN' ,3,IER)
;        IF(IER.NE.1) STOP UNABLE TO OPEN FILE
;        READ(2,91) TMAX,STEP,TOL
;        READ(2,91) SIZE
;        READ(2,90) I1,I2,I3,LM
;     90 FORMAT(4F20.0)
;     91 FORMAT(10F8.0)
;        PARM(1)=0.0
;        PARM(2)=TMAX
;        PARM(3)=STEP
;        N=10
;        W1N=0.0391/0.314
;        W2N=0.0
;        W3N=(0.35-0.314)/0.314
;        EW1N=0.0391/0.314
;        EW2N=0.0
;        EW3N=(0.35-0.314)/0.314
;        DZN=0.0
;        ZN=0.0
;        DXN=0.0
;        XN=0.0
;        IXW1=1245
;        SW1=0.001
;        AMW1=0.0
;        IXW2=1245
;        SW2=0.001
;        AMW2=0.0
;        IXV1=1245
;        SV1=0.001
;        AMV1=0.0
;        IXV2=1245
;        SV2=0.001
;        AMV2=0.0
;        IXV3=1245
;        SV3=0.001
```

```
;          AMV3=0.0
;          WRITE(5,92) TMAX,STEP,TOL
;          WRITE(5,95) I1,I2,I3
;          WRITE(5,98) SIZE
;       92 FORMAT('1TMAX=',F8.4,10X,'STEP=',F8.4,10X,'TOL=',F8.6)
;       95 FORMAT('0I1=',F15.2,2X,'I2=',F15.2,2X,'I3=',F15.2)
;       98 FORMAT('0SIZE',10F8.4)
;          CALL RKSCL(N,SIZE,DY,TOL,PARM)
;          CALL RKGS(PARM,Y,DY,N,IHLF,SOAC1,SOAC2,WORK)
;          CALL EXIT
;          END


PROGRAM IS RELOCATABLE
                              .TITL    .MAIN
!FORT/A/B/E/P/S FORT.LS/L
!LISTING
;          SUBROUTINE SOAC1(T,Y,DY)
;          DIMENSION Y(10),DY(10)
;          REAL I1,I2,I3,LM
;          COMMON W1N,W2N,W3N,EW1N,EW2N,EW3N,DZN,DXN,ZN,XN
;          COMMON I1,I2,I3,LM
;          COMMON N
;          COMMON W1,W2,VN1,VN2,VN3
;          COMMON IXW1,SW1,AMW1
;          COMMON IXW2,SW2,AMW2
;          COMMON IXV1,SV1,AMV1
;          COMMON IXV2,SV2,AMV2
;          COMMON IXV3,SV3,AMV3
;          CALL GAUSS(IXW1,SW1,AMW1,W1)
;          CALL GAUSS(IXW2,SW2,AMW2,W2)
;          CALL GAUSS(IXV1,SV1,AMV1,VN1)
;          CALL GAUSS(IXV2,SV2,AMV2,VN2)
;          CALL GAUSS(IXV3,SV3,AMV3,VN3)
;          A11=0.0018015
;          A12=0.666798
;          B11=0.0082733
;          B21=0.637957
;          B32=0.384811
;          C11=-2.2242
;          C12=3.8489
;          C23=3.14
;          F11=0.3607
;          F12=-0.1047
;          F13=0.0
;          F22=0.5045
;          F23=0.0
;          F33=0.3848
;          CD=0.2
;          AZN=-(C11*EW1N+C12*EW2N)-CD*DZN-ZN
;          F1=0.02598
;          AXN=-(C23*EW3N+F1*DXN)-XN
;          DY(1)=A11*W1N-A12*W2N-B11*C11*EW1N-B11*C12*EW2N+B11*W1
;          DY(2)=A12*W1N-A11*W2N-B21*C11*EW1N-B21*C12*EW2N+B21*W2
;          DY(3)=-B32*C23*EW3N+B32*W2
;          DY(4)=(A11-F11-B11*C11)*EW1N-(A12+F12+B11*C12)*EW2N-F13*EW3N
;        *+F11*(W1N+VN1)+F12*(W2N+VN2)+F13*(W3N+VN3)
;          DY(5)=(A12-F12-B21*C11)*EW1N-(A11+F22+B21*C12)*EW2N-F23*EW3N
;        *+F12*(W1N+VN1)+F22*(W2N+VN2)+F23*(W3N+VN3)
;          DY(6)=-F13*EW1N-F23*EW2N-(F33+B32*C23)*EW3N+F13*(W1N+VN1)
;        *+F23*(W2N+VN2)+F33*(W3N+VN3)
;          DY(7)=AZN
;          DY(8)=AXN
```

```
;       DY(9)=DZN
;       DY(10)=DXN
;       RETURN
;       END

PROGRAM IS RELOCATABLE
                                .TITL    SOAC1
!FORT/A/B/E/P/S FORT.LS/L
!LISTING
;       SUBROUTINE SOAC2(T,Y,DY,IHLF,NDUM,P)
;         LOGICAL RKNXT
;       DIMENSION Y(10),DY(10),DUMMY(10)
;       DIMENSION X(10)
;       REAL I1,I2,I3,LM
;       COMMON W1N,W2N,W3N,EW1N,EW2N,EW3N,DZN,DXN,ZN,XN
;       COMMON I1,I2,I3,LM
;       COMMON N
;       COMMON W1,W2,VN1,VN2,VN3
;       COMMON IXW1,SW1,AMW1
;       COMMON IXW2,SW2,AMW2
;       COMMON IXV1,SV1,AMV1
;       COMMON IXV2,SV2,AMV2
;       COMMON IXV3,SV3,AMV3
;       DEG=57.2957795
;       CALL SOAC1(T,Y,DUMMY)
;       WS=0.314
;       X(1)=Y(1)*WS
;       X(2)=Y(2)*WS
;       X(3)=(Y(3)+1.0)*WS
;       X(4)=Y(4)*WS
;       X(5)=Y(5)*WS
;       X(6)=(Y(6)+1.0)*WS
;       X(7)=Y(7)*LM*WS
;       X(8)=Y(8)*LM*WS
;       X(9)=Y(9)*LM
;       X(10)=Y(10)*LM
;       H1=I1*X(1)
;       H2=I2*X(2)
;       H3=I3*X(3)
;       THETA=ATAN2(SQRT(H1*H1+H2*H2),H3)*DEG
;       TA=T/WS
;       TP=TA+0.00005
;       IF(.NOT.RKNXT(IHLF)) GO TO 8
;       WRITE(5,1) TP,X(1),X(4),X(2),X(5),X(3),X(6),X(9),X(10),THETA,IHLF
;     1 FORMAT(1X,F9.4,8F13.7,F9.4,I2)
;       WRITE BINARY(1) TA,THETA,X(10),X(9),X(6),X(3),X(5),X(2),X(4),X(1)
;     8 CONTINUE
;       RETURN
;       END

PROGRAM IS RELOCATABLE
                                .TITL    SOAC2
!RLDR/M TMP/S 001 002 003 DP0:SSP.LB FORT.LB
!DELETE/V SELLAPPAN
DELETED SELLAPPAN
!CREATE SELLAPPAN
!EXEC
```