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DEVELOPMENT OF A CURVED PIPE CAPABILITY FOR THE NASTRAN FINITE ELEMENT PROGRAM

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16. Abstract

A curved pipe element capability for the NASTRAN structural analysis program is developed using the NASTRAN dummy element feature. A description is given of the theory involved in the subroutines which describe stiffness, mass, thermal and enforced deformation loads, and force and stress recovery for the curved pipe element. Incorporation of these subroutines into NASTRAN is discussed. Test problems are proposed. Instructions on use of the new element capability are provided.

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SUMMARY

Curved pipes have unique stress distributions and flexibility properties which cannot be easily reproduced using the beam elements of the NASTRAN finite element structural analysis program. The purpose of this research effort was to enhance the application of NASTRAN to the structural analysis of piping systems by introducing a curved pipe element capability. The theory used for the element stiffness matrix was developed from an existing flexibility matrix. The curved pipe capability was implemented using the NASTRAN user dummy element. The dummy element approach is compatible with most of the NASTRAN rigid formats and requires less sophisticated programming than needed for incorporating a new element into NASTRAN. The similarities between the NASTRAN bar element and the curved pipe element made it possible to use the bar element subroutines as the basis for the necessary curved pipe subroutines. An existing program was used to link the subroutines to NASTRAN. Test problems have been prepared and input and output pertaining to the curved pipe element are described along with solution results.

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LIST OF SYMBOLS

A	Cross sectional area of curved pipe element
(a), (b)	Respective ends of curved pipe element
C_A, C_B, C_C, C_D, C_E, C_F	Stress intensification factors
d	Outer diameter of curved pipe element
DL_a	Displacement at end (a) equivalent to applied deformation plus temperature expansion
E	Modulus of elasticity
G.P.A, G.P.B	Adjacent grid points at respective ends (a) and (b)
i, j	Arbitrary points or ends of element
I	Moment of inertia
J	Polar moment of inertia
m	Mass of curved pipe element
R	Radius of curvature of the pipe
$\overline{T}$	Average temperature in pipe wall
T_{AMB}	Ambient temperature
T_i	Temperature on inside of pipe wall
T_m	Temperature at middle of pipe wall
T_o	Temperature at outside of pipe wall
t	Thickness of pipe wall
v	Orientation vector
x, y, z	Coordinate axes for local coordinate system
X, Y, Z	Coordinate axes for basic coordinate system

α	Coefficient of thermal expansion
$\hat{\lambda}_{1,\,2,\,3}$	Unit vectors for the grid point coordinate system
μ	Mass per unit length
ν	Poisson's ratio
$\vec{\omega}$	$(\omega_1^{\,2} + \omega_2^{\,2} + \omega_3^{\,2})^{1/2}$, offset vector, expressed in components ω_1, ω_2, ω_3 in grid point coordinate system
ψ	Incident angle for curved pipe element
ρ	Mass density of pipe material
σ_A	Axial stress at end of curved pipe element
σ_B	Maximum bending stress at end of curved pipe element
σ_H	Hoop stress for curved pipe element
σ_{T_L}	Linear temperature gradient stress for curved pipe element
σ_{T_N}	Nonlinear temperature gradient stress for curved pipe element
τ_F	Shear stress due to lateral forces at end of curved pipe element
τ_T	Shear stress due to torsion at end of curved pipe element
Θ	Rotation at end of element
ΔT_2	Nonlinear temperature gradient parameter

Matrix Symbols

$[B_{ij}]$	6 x 6 matrix relating force vector at i with equivalent force vector at j

$[C_i]$	6 x 6 matrix transforming vectors in the displacement coordinate system at grid point i to the basic coordinate system
$\{DFA\}$	6 x 1 matrix of enforced displacements at end (a) in coordinate system at end (a)
$[E_i]$	6 x 1 matrix relating displacements at the i end of the element to displacements at the adjacent grid point
$\{F_i\}$	6 x 1 matrix defining forces and moments at point i in the appropriate coordinate system
$[f]$	6 x 6 flexibility matrix relating displacements at the center of curvature of the curved pipe element to the corresponding forces
$[I]$	Unit matrix of appropriate size
$[K]$	12 x 12 element stiffness matrix
$[K_{ij}]$	6 x 6 partition of element stiffness matrix
$[K^G]$	12 x 12 global stiffness matrix contribution from curved pipe element
$[K^G_{AA}]$	6 x 6 partition of global stiffness matrix
$[K^o]$	6 x 6 stiffness matrix relating forces at the center of curvature of the curved pipe element to the corresponding displacements
$[M_i]$	6 x 6 mass matrix for end of element
$[M^G]$	12 x 12 global mass matrix from curved pipe element
$[0]$	Null matrix of appropriate size
$\{P\}$	6 x 1 load matrix in appropriate coordinate system
$\{P_D\}$	6 x 1 equivalent force matrix for enforced displacements
$\{S\}$	Matrix of external loads applied to grid points in displacements coordinate system

$[S_i]$	6 x 6 force recovery matrix for end i
$\{S_T\}$	6 x 1 temperature force recovery matrix
$[T_{eb}{}^{(i)}]$	6 x 6 transformation matrix from coordinate system at end (i) of the curved pipe element to the basic coordinate system
$\{u_i\}$	6 x 1 matrix defining displacements at point (i) in the appropriate coordinate system
$\{\ddot{u}_i\}$	6 x 1 matrix defining acceleration components at end i in the appropriate coordinate system
$[\ ]^T$	Transpose of matrix
$[\ ]^{-1}$	Inverse of matrix

INTRODUCTION

Object and Scope

Curved pipes have unique structural properties under flexure, which cannot be easily modelled using standard NASTRAN elements. Flexure causes an ovalization of the cross section, resulting in a significant change in the stress distribution in the curved pipe. Longitudinal bending stresses in the extreme fibers are relieved and high circumferential stresses occur. As a result, the peak stress is in the circumferential direction. This peak stress is conventionally related to that which would exist in a straight pipe subjected to the same moment through a "stress intensification factor." An additional effect of the altered stress redistribution is a decrease in the stiffness of the curved pipe. The ratio of the stiffness of a straight pipe to the stiffness for the curved pipe is denoted the "flexibility factor" and is related to the geometry of the pipe. This behavior could be modelled using NASTRAN bar elements by: (1) using a sufficient number of bars to model the pipe curvature; (2) accounting for stiffness reduction by reducing the moments of inertia; and, (3) using appropriate stress recovery factors to simulate the stress intensification factor. This approach would be cumbersome for a piping system with more than a few bends. The object of this research effort is to simplify the analysis of piping systems using NASTRAN by developing a curved pipe element. This element would minimize the effort of the NASTRAN user and would maintain as many of the standard NASTRAN capabilities as possible. The NASTRAN user dummy element approach was chosen for implementing this new capability because it is the simplest approach to developing a new element and because the present restricted capabilities of the curved pipe element definition make it unsuitable for permanent incorporation into NASTRAN at this point. Also, permanent incorporation should be preceded by a thorough checkout of the proposed element using the dummy element approach. The dummy element can be used for all of the NASTRAN structural rigid formats except the piecewise linear analysis. The element is implemented by first writing FORTRAN IV subroutines for the portions of NASTRAN which are dependent on the element used and then linking these subroutines into NASTRAN. Capabilities which account for offsets of bar ends from grid points, released constraints at bar ends, temperature loads and enforced deformation loads are included for the element. No differential stiffness matrix is generated. A lumped mass approach is used to described inertia properties. Input and output are similar to that for standard NASTRAN elements.

A considerable amount of computer programming was required in order to develop the necessary subroutines for the curved pipe element. Maria V. Stephens of the Engineering Analysis Branch, NASA Langley Research Center, provided this service and also provided much needed assistance in dealing with the complicated programming logic involved in the general NASTRAN program. Her assistance is gratefully acknowledged. Also, this research effort was conducted as part of the NASA-ASEE Summer Research Program.

Literature Review

Although several finite element programs include a curved pipe element, details about how the elements function are unavailable. The basis for any finite element is a stiffness or flexibility matrix which relates the forces acting on the element to the resulting displacement. Although several sources (References (1), (2), (3), (4)) have developed and used stiffness matrices for curved beam elements, which have stiffness properties similar to those of curved pipes, relatively little has been done specifically for curved pipes. A flexibility matrix developed in Reference (5) and modified in Reference (6) is used as the basis for this analysis.

ANALYSIS

Development of Subroutine Relationships

In order to introduce a new element capability to NASTRAN via the dummy element approach, five FORTRAN IV subroutines must be developed which: (1) compute the stiffness contributions of the element to the adjacent grid points; (2) compute mass contributions to the adjacent grid points; (3) compute thermal and enforced deformation loads; (4) set up force recovery matrices; and, (5) recover forces and stresses for output. These subroutines must then be linked to the NASTRAN program before the dummy element can be used. The theory involved in these subroutines is discussed in this section.

The development of the subroutine relationships is complicated somewhat by the different coordinate systems involved. NASTRAN allows the user to define displacements at each grid point in a coordinate system of the user's choosing. This set of coordinate systems is then used by NASTRAN in solving for the grid point displacements. However, the properties of an element are generally best defined using a coordinate system peculiar to that specific element. These two coordinate system sets are related to each other in that both coordinate system sets are defined in terms of a single coordinate system, called the "basic" coordinate system. The relationship between the displacement coordinate system and the basic coordinate system is defined on bulk data cards; information needed to define the element coordinate system in terms of the basic system is inputted on bulk data cards, but the actual relationships are developed in the first of the five "dummy" subroutines.

The theory used in developing the stiffness matrix for the curved pipe element was based on the flexibility matrix described by Chen Reference (1). The geometry and notation used by Chen is shown in Figure (1). (The "z" direction is out of the paper). Chen's matrix is written with reference to the center of the arc described by the pipe centerline. It is assumed that end (b) of the bar shown in Figure (1) is fixed and that a rigid connection is made between end (a) and the center point 0, so that the flexibility matrix relates the deflections at point 0 to forces applied at point 0, or, mathematically,

$$\{u_o\}_{6 \times 1} = [f]_{6 \times 6} \{F_o\}_{6 \times 1} \tag{1}$$

where

$\{u_o\} = \{u_{o_y} \; u_{o_y} \; u_{o_z} \; \Theta_{o_x} \; \Theta_{o_y} \; \Theta_{o_z}\}$ is a 6 x 1 column matrix

giving the 3 component deflections and 3 component rotations at point 0 with reference to the coordinate system xyz. The flexibility matrix [f] is a 6 x 6 matrix of influence coefficients. The force vector, $\{F_o\} = \{F_{o_x} \; F_{o_y} \; F_{o_z} \; M_{o_x} \; M_{o_y} \; M_{o_z}\}$ is a 6 x 1 column matrix giving the 3 component forces and the 3 component moments at point 0 with reference to the coordinate system xyz. Equilibrium conditions for the element, the reciprocal theorem, and Equation (1) can be used to get relationships between forces and displacement at the two ends of the element. Specifically, the following relationships are required: (1) a stiffness matrix K_{aa}, relating displacements at end (a) to forces at end (a); (2) a stiffness

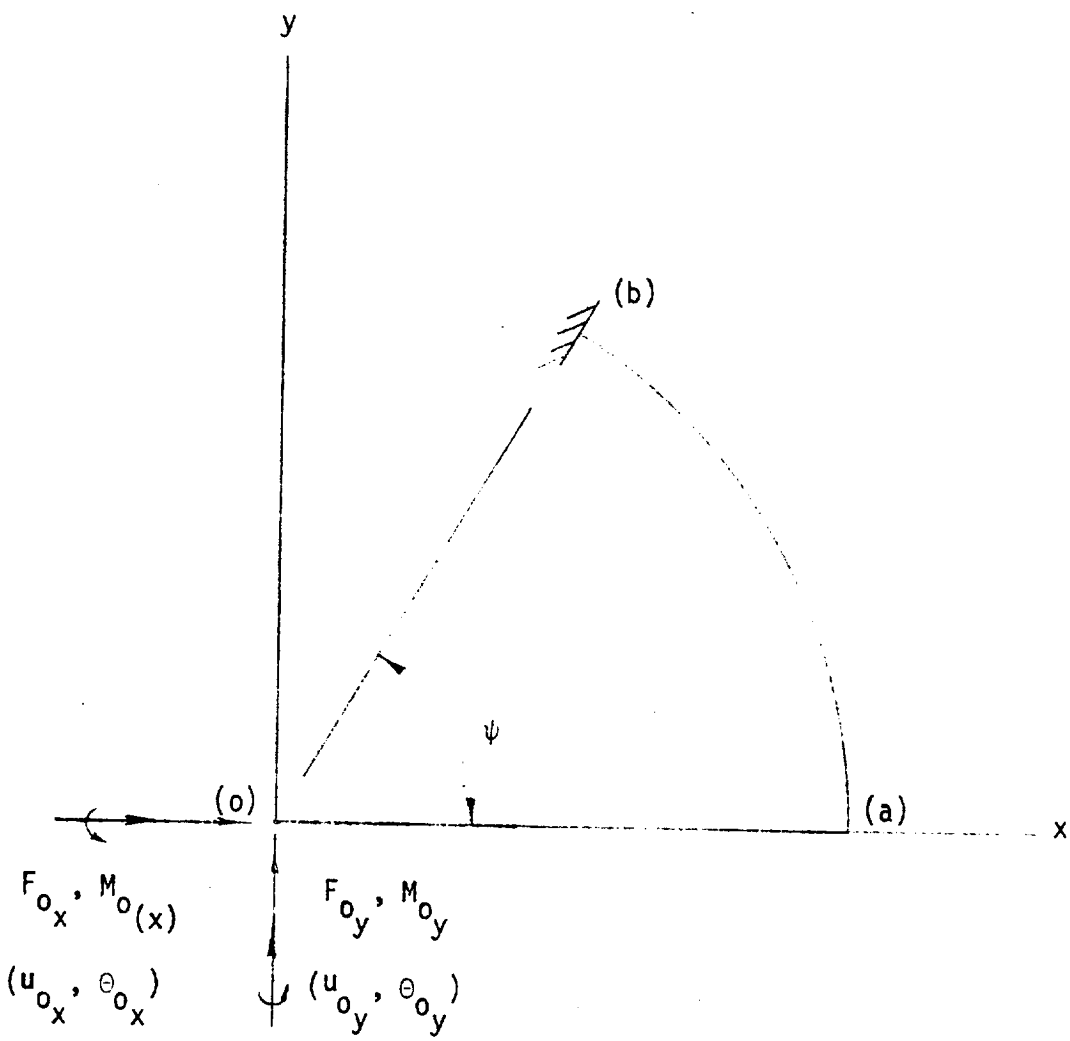

Figure (1). - Geometry and Notation for Chen Flexibility Matrix

matrix, K_{ba}, relating displacements at end (a) to forces at end (b); (3) a stiffness matrix, K_{ab}, relating displacements at end (b) to forces at end (a); and (4) a stiffness matrix, K_{bb}, relating displacements at end (b) to forces at end (b). The notation used for these forces and displacements is given in Figure (2), with the z direction (out of the paper) omitted.

It is convenient to work with Equation (1) in its inverted form,

$$\{F_o\}_{6 \times 1} = [K^o]_{6 \times 6} \{u_o\}_{6 \times 1} \tag{2}$$

where

$$[K^o] = [f]^{-1}.$$

Since (o) - (a) is a rigid connection in Figure (1), forces applied at point (o) could be transferred to end (a) of the element by the equation

$$\{F_a\}_{6 \times 1} = [B_{oa}]_{6 \times 6} \{F_o\} \tag{3}$$

where

$$[B_{oa}] = \left[\begin{array}{ccc|c} & I & & 0 \\ \hline 0 & 0 & 0 & \\ 0 & 0 & R & I \\ 0 & -R & 0 & \end{array}\right]$$

Displacements at point (0) can be written in terms of the displacements at end (a) using the equation

$$\{u_o\}_{6 \times 1} = [B_{oa}]^T_{6 \times 6} \{u_a\} \tag{4}$$

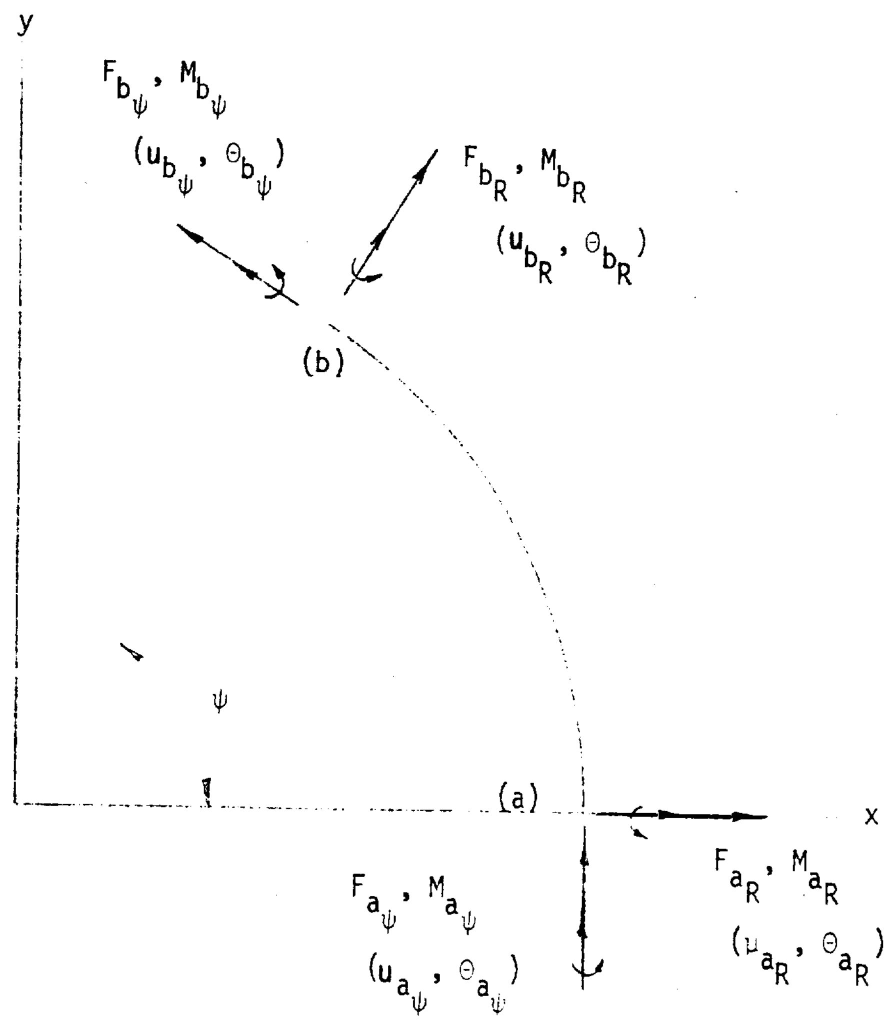

Figure (2). - In Plane Forces and Displacements for the Curved Pipe Element.

Equations (2), (3), and (4) can be combined to get a relationship between forces applied at end (a) and displacements at end (a):

$$\{F_a\}_{6 \times 1} = [K_{aa}]_{6 \times 6} \{u_a\}_{6 \times 1} \tag{5}$$

where

$$[K_{aa}] = [B_{oa}]_{6 \times 6} [K^o]_{6 \times 6} [B_{oa}]^T . \tag{6}$$

Equilibrium requires that forces applied at end (a) be balanced by equal and opposite forces at end (b), in the absence of intermediate loads on the element. Thus, the reaction forces at end (b) associated with forces applied at end (a) can be found by transferring the forces at end (a) to end (b), redefining them in terms of the coordinate system at end (b), and changing the signs. This process can be written in matrix notation as

$$\{F_b\}_{6 \times 1} = - [B_{ab}]_{6 \times 6} \{F_a\}_{6 \times 1} \tag{7}$$

where

$$[B_{ab}] = \begin{bmatrix} \cos\psi & \sin\psi & 0 & 0 & 0 & 0 \\ -\sin\psi & \cos\psi & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -R\sin\psi & \cos\psi & \sin\psi & 0 \\ 0 & 0 & R(1-\cos\psi) & -\sin\psi & \cos\psi & 0 \\ R\sin\psi & R(1-\cos\psi) & 0 & 0 & 0 & 1 \end{bmatrix}$$

Equations (5) and (7) can be combined to yield

$$\{F_b\}_{6 \times 1} = [K_{ba}]_{6 \times 6} \{u_a\}_{6 \times 1} \tag{8}$$

where

$$[K_{ba}]_{6 \times 6} = - [B_{ab}]_{6 \times 6} [K_{aa}]_{6 \times 6} . \tag{9}$$

Forces at both ends of the element due to displacements at end b could be found using the same approach as for displacements at end a. This is not necessary when a relationship between the displacements at end (b) and the forces at end (a) are desired, since the reciprocal theorem requires that

$$\{F_a\}_{6 \times 1} = [K_{ba}]^T_{6 \times 6} \{u_b\}_{6 \times 1} . \tag{10}$$

If end (a) is assumed fixed while end (b) is assumed rigidly attached to point (0), displacements at end (b) can be related to the forces applied at end (b) (using the coordinate system indicated in Figure (2)) by the relation

$$\{F_b\}_{6 \times 1} = [K_{bb}]_{6 \times 6} \{u_b\} \tag{11}$$

where

$$[K_{bb}]_{6 \times 6} = [B_{ob}]_{6 \times 6} [K^o]_{6 \times 6} [B_{ob}]^T_{6 \times 6}$$

and

$$[B_{ob}] = \begin{bmatrix} \cos\psi & \sin\psi & 0 & 0 & 0 & 0 \\ -\sin\psi & \cos\psi & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\psi & \sin\psi & 0 \\ 0 & 0 & R & -\sin\psi & \cos\psi & 0 \\ R\sin\psi & -R\cos\psi & 0 & 0 & 0 & 1 \end{bmatrix} .$$

The relationships between forces and displacements at the ends of the element can now be summarized by combining Equations (5), (8), (10), and (11) to get

$$\left\{\frac{F_a}{F_b}\right\}_{12 \times 1} = [K^E]_{12 \times 12} \left\{\frac{u_a}{u_b}\right\}_{12 \times 1} \tag{12}$$

where

$$[K^E]_{12 \times 12} = \left[\begin{array}{c|c} K_{aa} & K_{ba}^T \\ \hline K_{ba} & K_{bb} \end{array}\right]_{12 \times 12} \tag{13}$$

At times it is desirable to disconnect various degrees of freedom of the ends of the element from the adjacent grid pont. This capability is included in the curved pipe element through the NASTRAN pin flag routine. This routine accounts for disconnected degrees of freedom by operating on the element stiffness matrix, $[K^E]_{12 \times 12}$ to produce the "reduced" stiffness matrix $[K^R]_{12 \times 12}$.

At this point forces at the ends of the element have been related to displacements at the ends of the element, as defined by the coordinate systems shown in Figure (2), by the equation

$$\{F\}_{12 \times 1} = [K^R]_{12 \times 12} \{u\}_{12 \times 1} \,. \qquad (14)$$

where

$$[K^R]_{12 \times 12} = \left[\begin{array}{c c} K^R_{aa} & K^{R^T}_{ba} \\ \hline K^R_{ba} & K^R_{bb} \end{array}\right]$$

The equation solved in NASTRAN is of the form

$$\{S\} = [K^G]\ \{u^G\} \qquad (15)$$

where

$\{S\}$ includes all external loads applied to the grid point, and is therefore equal to the sum of the internal forces applied by the elements to grid point,

$[K^G]$ is the global stiffness matrix, and can be partitioned as

$$\left[K^G\right] = \left[\begin{array}{c|c} K^G_{AA} & K^G_{AB} \\ \hline K^G_{BA} & K^G_{BB} \end{array}\right] \text{, and}$$

$\{u_G\}$ is the complete set of all grid point displacements.

Both matrices $\{S\}$ and $\{u^G\}$ are defined in the displacement coordinate system at each grid point. The global stiffness matrix $[K^G]$ is an assemblage of the stiffness contributions of all the elements. Thus, if the force at the adjacent grid point can be written in terms of the force at the end of the element and the displacement at the end of the element can be written in terms of the displacement at the adjacent grid point, the element's contribution to the global stiffness matrix can be determined using Equation (14). The coordinate systems involved in determining this contribution is shown in Figure (3). Offset vectors ($\vec{\omega}_A = \omega_{A_1} \hat{\lambda}_{A_1} + \omega_{A_2} \hat{\lambda}_{A_2} + \omega_{A_3} \hat{\lambda}_{A_3}$ and $\vec{\omega}_B = \omega_{B_1} \hat{\lambda}_{B_1} + \omega_{B_2} \hat{\lambda}_{B_2} + \omega_{B_3} \hat{\lambda}_{B_3}$) are also shown. The displacements at end (a) of the element can be written in terms of the deflection at grid point A using the relation

$$\{u_a\}_{6 \times 1} = [T_{eb}^{(a)}]^T_{6 \times 6} [C_A][E_A]_{6 \times 6} \{u_A\}_{6 \times 1} \tag{16}$$

where

$$[E_A] = \left[\begin{array}{c|ccc} & 0 & \omega_{A_3} & -\omega_{A_2} \\ I & -\omega_{A_3} & 0 & \omega_{A_1} \\ & \omega_{A_2} & -\omega_{A_1} & 0 \\ \hline 0 & & 1 & \end{array}\right]_{6 \times 6}$$

and determines the displacement at (a) in the displacement coordinate system at grid point (A) given displacements at (A);

$[C_A]$ is an orthogonal transformation matrix which redefines in the basic coordinate system, vectors defined in the displacement coordinate system at A;

$[T_{eb}^{(a)}]$ is a transformation matrix which redefines in the basic coordinate system, vectors defined in the element coordinate system at end (a); since this is an orthogonal matrix, the inverse (which redefines in the element coordinate system at end (a), vectors defined in the basic coordinate system) is equal to the transpose.

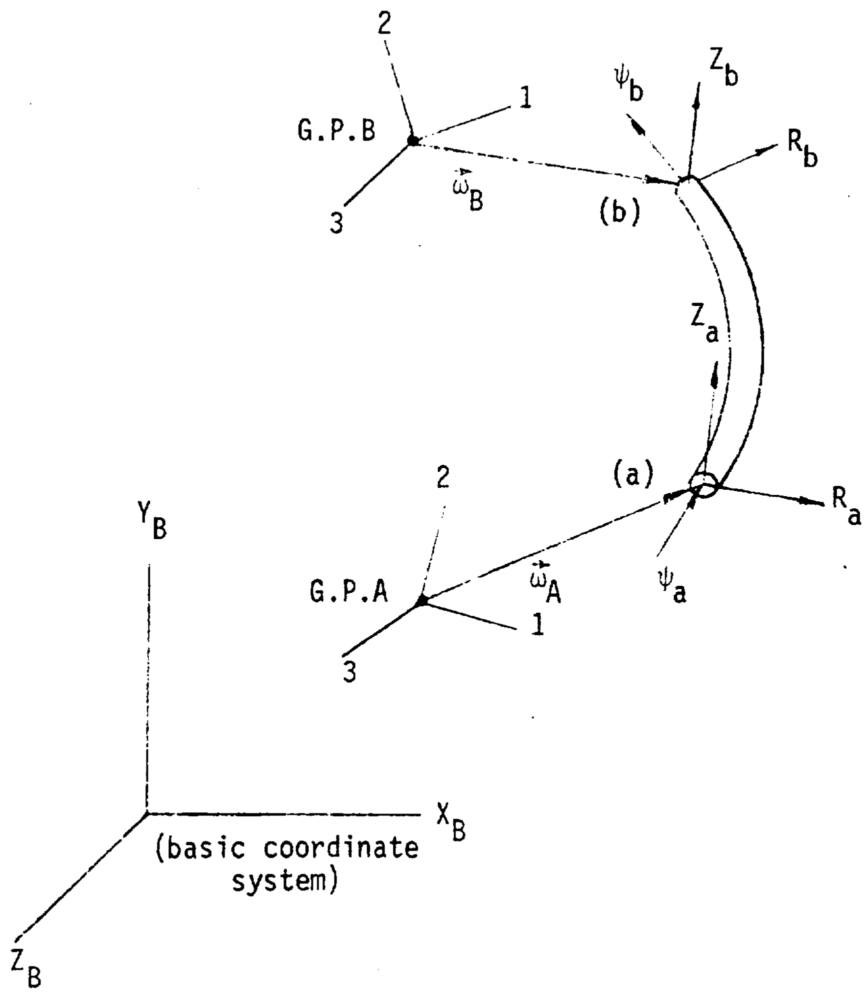

Figure (3). - Coordinate Systems Used in Element Subroutines.

A force vector at grid point A can be written in terms of an equivalent force vector at element end (a) using the relationship

$$\{F_A\} = [E_A]^T\ [C_A]^T\ [T_{eb}^{(a)}]\{F_a\} \tag{17}$$

Combination of Equations (17), (5), and (16) yield

$$\{F_A\}_{6\times 1} = [K_{AA}^G]_{6\times 6}\ \{u_A\}_{6\times 1}$$

where

$$[K_{AA}^G]_{6\times 6} = ([T_{eb}^{(a)}]^T\ [C_A]\ [E_A])^T\ [K_{aa}^R]\ ([T_{eb}^{(a)}]^T\ [C_A][E_A]) \tag{18}$$

Similar efforts produce the matrices

$$[K_{BA}^G]_{6\times 6} = ([T_{eb}^{(b)}]^T\ [C_B]\ [E_B])^T\ [K_{ba}^R]\ ([T_{eb}^{(a)}]^T\ [C_A]\ [E_A]) \tag{19}$$

and

$$[K_{BB}^G]_{6\times 6} = ([T_{eb}^{(b)}]^T\ [C_B]\ [E_B])^T\ [K_{bb}^R]\ ([T_{eb}^{(b)}]\ [C_B]\ [E_B]) \tag{20}$$

where

$$[E_b] = \left[\begin{array}{c|ccc} & 0 & \omega_{B_3} & -\omega_{B_2} \\ I & -\omega_{B_3} & 0 & \omega_{B_1} \\ & \omega_{B_2} & -\omega_{B_1} & 0 \\ \hline 0 & & I & \end{array}\right]_{6\times 6},$$

and $[C_B]$ and $[T_{eb}^{(b)}]$ are transformation matrices at B equivalent to matrices $[C_A]$ and $[T_{eb}^{(a)}]$ at A.

Finally, application of the reciprocal theorem yields

$$[K^G_{AB}]_{6 \times 6} = [K^G_{BA}]^T_{6 \times 6} \quad . \tag{21}$$

The submatrices given by Equations (18), (19), (20), and (21), provide the contribution of the curved pipe element to the total global stiffness matrix of the structure.

A lumped mass approach was used to define the inertia properties of the curved pipe element. It is assumed that the total mass of the element can be divided equally between the element ends. Second mass moments are not included in the mass matrix. This omission should have a negligible influence for curved pipes with small arc but could be significant for curved pipes with larger arcs. This is because of the lack of compensation for the offset of the center of gravity from a tangent connecting the ends of the element. If it is desired to include the effects of the second mass moments, NASTRAN has a capability for directly inputting these properties. Forces applied to either end of the element can be related to the corresponding acceleration components by the equation

$$\{F_a\}_{6 \times 1} = [M_a]_{6 \times 6} \{\ddot{u}_a\}_{6 \times 1}$$

where

$\{\ddot{u}_a\}$ is a 6 x 1 column matrix giving the linear and angular components of acceleration of end (a) of the element, and

$$[M_a] = \left[\begin{array}{ccc|c} \frac{m}{2} & & & \\ & \frac{m}{2} & & 0 \\ & & \frac{m}{2} & \\ \hline 0 & & & 0 \end{array}\right] = [M_b] \tag{22}$$

with

$$m = R\,\psi\,\{\rho\,\frac{\pi}{4}\,(d^2 - (d - 2t)^2) + \mu\}$$

ρ = mass density of pipe material

μ = non-structural mass per unit length

The NASTRAN solution process requires a relationship between the forces and accelerations at the grid points rather than the ends of the element. A process similar to that used for the stiffness matrix yields

$$\{F_A\}_{6\times 1} = [M^G_{AA}]_{6\times 6}\,\{\ddot{u}_A\}_{6\times 1}$$

where

$$[M^G_{AA}]_{6\times 6} = ([T^{(a)}_{eb}]^T\,[C_A]\,[E_A])^T\,[M_a]\,([T^{(a)}_{eb}]^T\,[C_A]\,[E_A]) \tag{23}$$

Similarly,

$$\{F_B\}_{6\times 1} = [M^G_{BB}]_{6\times 6}\,\{\ddot{u}_A\}_{6\times 1}$$

where

$$[M^G_{BB}]_{6\times 6} = ([T^{(b)}_{eb}]^T\,[C_B]\,[E_B])^T\,[M_b]\,([T^{(b)}_{eb}]^T\,[C_B][E_B]) \tag{24}$$

This formulation assumes no direct relationship between forces at (A) and accelerations at (B) and vice versa. Thus Equations (23) and (24) define the total mass contribution of the element to the structural system.

Stresses due to thermal expansion loads are significant in piping system analysis. Stresses due to misfit may also be of interest. A capability for handling both types of problems has been included in the curved pipe element theory. It is assumed in both cases that the ends of the element move together or apart, whichever the case, without any rotation occurring. The forces required for this compression or extension are calculated and added to the existing external loadings at the grid points. Disconnected degrees of freedom are taken into account in determining these forces. Once the problem has been solved, the forces which would have resulted from the enforced deformation are calculated and subtracted from the forces calculated from the movement of the ends of the element.

The element is assumed to be fixed at end (b) (see Figure (3)) so that the movement associated with the applied deformation or temperature expansion can be interpreted as a displacement of end (a) in the plane of the element. The equivalent grid point loads are

$$\{F_A\}_{6 \times 1} = [E_A]^T \, [C_A]^T \, [T_{eb}^{(a)}] \, [K_{aa}^R] \, \{DL_a\}_{6 \times 1} \tag{25a}$$

and

$$\{F_B\}_{6 \times 1} = [E_B]^T \, [C_B]^T \, [T_{eb}^{(b)}] \, [K_{ba}^R] \, \{DL_a\}_{6 \times 1} \tag{25b}$$

where

$\{DL_a\}$ = the displacement at end (a) in the coordinate system at end (a). This displacement is automatically calculated in the subroutine given the enforced deformation and temperature information. The element temperature is calculated from the relation

$$\bar{T} = \frac{1}{4}(T_o + 2\,T_m + T_i)$$

where

$\bar{T}$ = the temperature used to calculate expansion

T_o = the temperature at the outside of the pipe wall

T_m = the temperature at the middle of the pipe wall

T_i = the temperature at the inside of the pipe wall.

The ambient temperature is inputted with the materials properly card.

The general portion of the NASTRAN program uses the load, stiffness and mass matrices to compute the displacement vector for each grid point. Force and stress recovery procedures are different for each type of element. In the general NASTRAN scheme, matrices for recovery of element forces are set up in one subroutine and actual force recovery takes place in another subroutine. In the first subroutine, the matrices required to determine forces at end (a) due to displacements of end (a) and displacements of end (b) are generated, along with matrices which include the original forces due to temperature expansion. These matrices are defined by the equations

$$[S_a]_{6 \times 6} = [K^R_{aa}]\ [T^{(a)}_{eb}]^T\ [C_A]\ [E_A]\ , \tag{26}$$

$$[S_b]_{6 \times 6} = [K^R_{ab}]\ [T^{(b)}_{eb}]^T\ [C_B]\ [E_B]\ , \tag{27}$$

and

$$\{S_T\}_{6 \times 1} = \alpha\ [K^R_{aa}] \begin{Bmatrix} x_b - x_a \\ y_b - y_a \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} .$$

Once the force recovery matrices have been determined, the forces at end (a) can be calculated by the equation

$$\{P_a\} = [S_a]\{\mu_A\} + [S_b]\{\mu_B\} + \{S_T\} \times (\overline{T} - T_{AMB}) + \{P_D\} \tag{29}$$

where

$\{P_a\} = \{P_{a_R}, P_{a_\Theta}, P_{a_Z}, M_{a_R}, M_{a_\Theta}, M_{a_Z}\}$, force vector at end a,

T_{AMB} = ambient temperature,

$\{P_D\} = [K^R_{aa}]_{6 \times 6} \{DFA\}_{6 \times 1}$

and

$\{DFA\}$ = displacement components at end (a) corresponding to enforced deformation.

The normal force is redefined so that tension is positive:

$$F_\Theta = - P_\Theta .$$

Once the forces at end (a) have been found, Equation (7) can be used to determine the forces at end (b) of the element.

The stresses to be calculated for the curved pipe element are:

For the element:

Hoop stress: $$\sigma_H = \frac{C_E\, P_r\, d}{2t} \;; \tag{30}$$

Linear temperature gradient stress:

$$\sigma_{T_L} = \frac{C_F\, E\, \alpha\, (T_o - T_i)}{2\,(1 - \nu)} \;;$$

Nonlinear temperature gradient stress:

$$\sigma_{T_N} = \frac{E\, \alpha\, \Delta T_2}{(1 - \nu)} \;;$$

At each end:

Axial stress:

$$\sigma_A = \frac{C_A\, F_\Theta}{A} \;;$$

Torsional shear stress:

$$\tau_T = \frac{C_B\, M_\Theta\, d}{J} \;;$$

Bending stress:

$$\sigma_B = \frac{C_D\, d\, \sqrt{(M_R)^2 + (M_Z)^2}}{J} \;;$$

Lateral shear stress:

$$\tau_F = \frac{C_c\, \sqrt{(P_R)^2 + (P_Z)^2}}{A} \;,$$

where

$C_{A,\ B,\ C,\ D,\ E,\ F}$ = stress intensity factors input by the user,

E = modulus of elasticity,

α = coefficient of thermal expansion,

d = outer diameter,

t = wall thickness,

ν = Poisson's ratio,

A = cross sectional area,

J = polar moment of inertia.

$$\Delta T_2 = \max \begin{cases} |T_o - \bar{T}| - \frac{1}{2}\ |T_o - T_i| \\ |T_i - \bar{T}| - \frac{1}{2}\ |T_o - T_i| \end{cases}$$

Incorporation of Dummy Element Into NASTRAN

The subroutines developed for a curved pipe become an operable reality in NASTRAN only after they have been coded in FORTRAN IV and only after several links in NASTRAN have been changed to allow for the use of dummy elements. A programmer familiar with NASTRAN data files and tables could eventually produce the necessary FORTRAN IV coding from scratch; however, a significant amount of time can be saved if the subroutines are patterned after an existing set of NASTRAN subroutines. This approach also increases efficiency since it allows the programmer to take advantage of existing NASTRAN subroutines and accounts for NASTRAN idiosyncracies, such as the preference for recalculation of matrices rather than storage of the matrices. FORTRAN IV coding used for the curved pipe dummy element closely parallels the coding used in NASTRAN for the bar element. NASTRAN subroutines GMMATS (GMMATD for double precision) and TRANSS (TRANSD for double precision) were particularly convenient in making the necessary changes to bar element subroutines. The former, described in Articles 3.4.32 and 3.4.33 of Reference (8), is a general matrix multiplication and transpose subroutine, while the latter, described in Articles 3.4.37 and 3.4.38 of the same reference, is a subroutine which maps a vector from a local coordinate system to the basic coordinate system.

Communication between the subroutines and the functional module or other subroutines is simplified due to the similarity between the curved pipe element subroutines and the NASTRAN bar element subroutines. For all of the subroutines except the one which recovers element forces and stresses, it is necessary to retrieve information inputted on GRID, CDUM3, PDUM3, DEFORM, and MAT1 bulk data cards. Information on the DEFORM and MAT1 cards are stored and retrieved for the dummy element exactly as for the bar element. Information from the applicable GRID, CDUM3, and PDUM3 cards are stored in an orderly fashion in an array called the Element Connection and Property Table (ECPT). The format of the ECPT for all NASTRAN elements including dummy elements is described in Article 2.3.8.3 (and preceding articles) of Reference (8). The format of the ECPT for the dummy element depends on entries on the bulk data ADUMi card which defines the attributes of the dummy element. Generally, the format consists of: element identification number; all information on the element connection card after the property identification number; the material identification number; all information given on the element property card after the material identification number; the coordinate system identification number and basic coordinates for the grid points associated with the element; and the element temperature. Declarations in the bar subroutines were altered for the dummy element subroutines only to account for differences in the ECPT and in appropriate internal parameters. NASTRAN bookkeeping parameters were not changed.

The routine which recovers element forces and stresses (SDUM32) receives, as partial input, the output for the subroutine which sets up the element force recovery matrices. Most of the communication required between this subroutine and the functional module SDR2 (Stress Data Recovery - Phase 2), is done with data block COMMON/SDR2X7/ described in Article 4.46.9 of Reference (8). The first 100 locations of this block are reserved for input parameters, the second 100 locations are for stress output parameters, and the third 100 locations are for force output parameters. The actual output format for dummy element forces and stresses is built into NASTRAN, rather than being set by the user. This is undesirable for the curved pipe element since the NASTRAN formats allow only nine forces and nine stresses per element as output, whereas a format allowing twelve forces and fourteen stresses is desirable in order to account for conditions at both ends of the element. The possibility of using WRITE statements in the force and stress recovery subroutine was considered and rejected because the dummy element output would then be located at random in the output of a NASTRAN job and would be difficult to interpret. The dummy formats could be changed using ALTER statements but this approach appears to be quite difficult. One alternative would be to output stresses instead of forces in the force output format since the pipe system analyst is more interested in stress output than force output. A second alternative would be to output forces and stresses at only one end of the element, since forces, and, therefore, stresses, can then be calculated for the opposite end. This is the approach presently being used.

Once the dummy element subroutines have been completed, a "linkedit" run is necessary to load them into the NASTRAN executable. The "linkedit" involves changes within NASTRAN itself. In this case the necessary "linkedit" programming was provided by the NASTRAN Systems Management Office, NASA-Langley Research Center. It was found that the "linkedit" used for one level of NASTRAN is not always completely acceptable for a different level.

The most efficient approach to checking out a dummy element is to create "driver" programs which simulate NASTRAN with respect to each of the element subroutines so that trial input can be used to check each subroutine. All subroutines would be checked in this manner before the "linkedit" occurred. However, time limitations have prevented development of these "driver" programs for this effort, so present attempts at checking involve actual NASTRAN runs on a single element test problem.

Once the dummy element subroutines have been debugged, application of NASTRAN with the dummy curved pipe element to the analysis of piping systems with curved sections can be compared to solutions from other sources. A NASTRAN formulation of Example #1 of Reference (9) has been prepared for comparison. Appendix II gives detailed information about the test problem and the test problem from Reference (9).

CONCLUDING REMARKS

At this point significant advancement has been made in the development of a curved pipe element. The new element will simplify curved pipe analysis primarily because a single element is used rather than multiple elements, the geometry definition is simplified, and input/output are tailored specifically to pipe analysis. For generality, the flexibility factor and stress intensification factors will be inputted by the user. The NASTRAN user dummy element has proven to be a relatively simple means of instituting a new element, although format restrictions have caused problems. Use of subroutines from an existing NASTRAN element simplify communications with the NASTRAN functional modules but could produce errors in programming in sections where the two elements differ. The NASTRAN Systems Management Office provided the necessary linkedit routine and also provided assistance in removing inherent NASTRAN errors. Apparently the linkedit may require modification for different NASTRAN levels. Once the curved pipe element has been completely debugged, a straight pipe element should be considered so that piping analyses could be performed completely with appropriate NASTRAN input and output.

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APPENDIX I

UTILIZATION OF CURVED PIPE ELEMENT IN PIPING SYSTEM ANALYSIS

The NASTRAN user can use the dummy curve pipe element almost as if it were a standard NASTRAN element. Some variations have been made from "standard" NASTRAN bulk data formats so that input will be as efficient as possible. Terms which may be dependent on the radius of curvature are restricted to the connection card, since the radius of curvature is likely to vary for different elements. Pin flags and offsets, conditions not expected to occur often in piping systems, were moved from connection cards to property cards to make room for the radius of curvature dependent terms. One ADUM3 card with non-varying entries must be included to describe the nature of the dummy element used. The required ADUM3, CDUM3, and PDUM3 cards are defined on the following pages. A sketch of the curved pipe element forces and geometry is given in Figure (4).

Output for the element consists of forces and stresses at end (a) of the curved pipe (see Figure (4)). The forces given are, in order, the radial shear force (outward is positive), the axial force (tension positive), the out of plane shear force, and the moments in the element R, ψ, and Z directions at end (a). Stresses given for end (a) are: hoop stress, linear temperature gradient stress, nonlinear temperature gradient stress, axial stress (tension is positive), shearing stress due to torsion, maximum bending stress, and shearing stress due to lateral loads.

The dummy element output formats provide headings for forces only as F1-F9 and headings for stresses only as S1-S9. Table (1) describes the force and stress output. Note that the outputted stresses have been pre-multiplied by stress intensification factors CA through CF, which were input by the user.

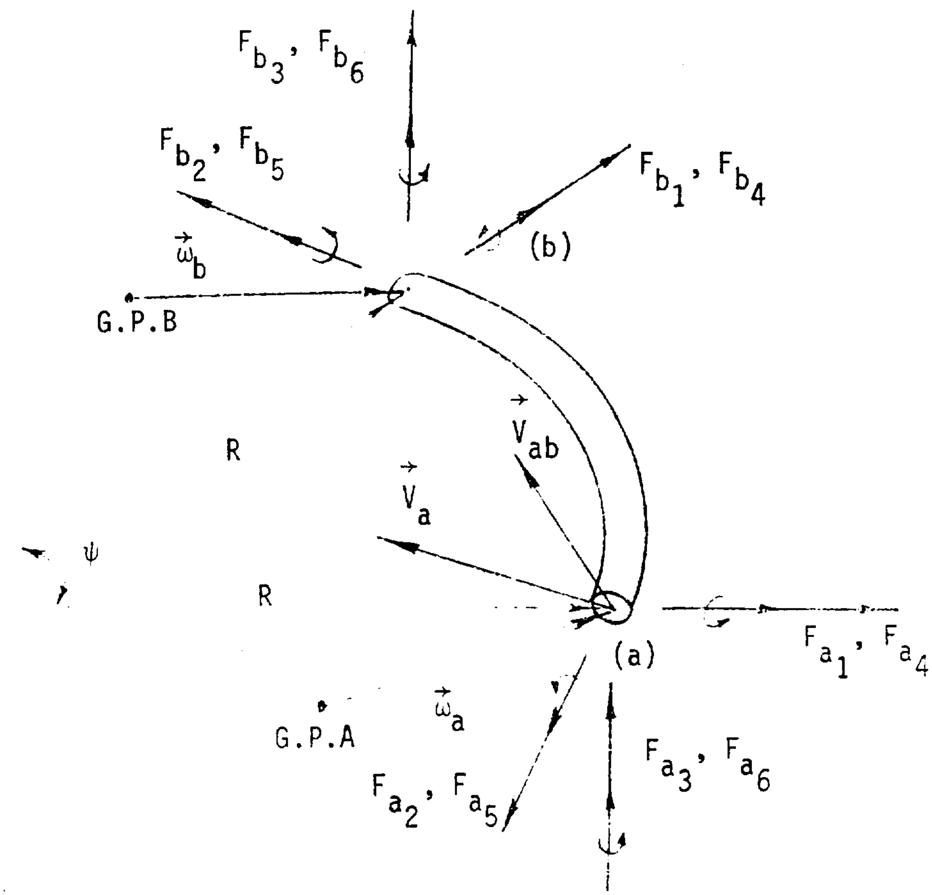

Figure (4). - Element Forces and Geometry.

Input Data Card ADUM3 Dummy Element Attributes

Description: Defines attributes of curved pipe element

Format:

1	2	3	4	5	6	7	8	9	10
ADUM3	NG	NC	NP	ND					
ADUM3	2	12	14	6					

Field:

NG Number of grid points connected by DUM3 element (2)

NC Number of additional entries on CDUM3 card (12)

NP Number of additional entries on PDUM3 card (14)

ND Number of displacement components at each grid point used in generation of differential stiffness matrix (6)

Remarks:

1. No differential stiffness matrix is actually generated.
2. Entries on ADUM3 card for curved pipe elements will always be as shown above.
3. Only one ADUM3 card should be present in the bulk data deck.

Input Data Card CDUM3 Curved Pipe Element Connection

Description: Defines a curved pipe element using the dummy element capability

Format and Example:

1	2	3	4	5	6	7	8	9	10
CDUM3	EID	PID	GA	GB	X1	X2	X3	R	abc
CDUM3	66	21	170	180			1.	4.5	DC3
+bc	PI	F	CA	CB	CC	CD	CE	CF	
+C3	10.	3.2							

Field:

EID Unique element identification number (Integer > 0)

PID Identification number of a PDUM3 property card (Integer > 0)

GA, GB Grid point identification numbers of connection points (Integer > 0; GA $\neq$ GB)

X1, X2, X3 Components of vector $\vec{v}$ at end (a), measured at end (a), parallel to the components of the displacement coordinate system for GA, to determine (with the vector from end (a) to end (b)) the orientation of the element coordinate system. The vector $\vec{v}$ lies in the plane of curvature of the element and is directed from end (a) to the inside of the curve and to the inside of the vector from end (a) to end (b). (Real, $X1^2 + X2^2 + X3^2 > 0$)

R Radius of curvature of the element (Real)

PI Internal pressure (Real)

F Flexibility factor (Real)

CA, CB, CC, CD, CE, CF Stress recovery coefficients for direct stresses, torsional stresses, shear stresses, bending stresses, hoop stresses, linear temperature gradient stresses, non-linear temperature gradient stresses (Real; default = 1)

Remarks:

1. Element identification numbers must be unique with respect to all other element identification numbers.

2. See the sketch for a description of the curved pipe geometry.

Input Data Card PDUM3 Curved Pipe Property

Description: Defines the properties of a curved pipe element referenced by a CDUM3 card

Format and Example:

1	2	3	4	5	6	7	8	9	10
PDUM3	PID	MID	DO	TH	NSM	TO	TM	TI	abc
PDUM3	21	10	3.5	0.216		100.	100.	100.	DP3

+bc	PA	PB	Z1A	Z2A	Z3A	Z1B	Z2B	Z3B	
+P3									

Field:

PID Property identification number (Integer > 0)

MID Material identification number (Integer > 0)

DO Outer diameter (Real)

TH Wall thickness (Real)

NSM Non-structural mass per unit length (Real)

TO, TM, TI Outside, middle, and inside wall temperature (Real)

PA, PB Pin flags for pipe ends (a) and (b), respectively, that are used to insure that the pipe cannot resist a force or moment corresponding to the pin flag at that respective end of the pipe. (Up to 5 of the unique digits 1-6 anywhere in the field with no imbedded blanks; integer > 0) (These degrees of freedom codes refer to the element forces and not global forces).

Z1A, Z2A, Z3A, Z1B, Z2B, Z3B Components of offset vectors $\vec{\omega}_a$ and $\vec{\omega}_b$, respectively, in displacement coordinate systems at grid points at ends (a) and (b), respectively (Real or blank).

Remarks:

1. PDUM3 cards may only reference MAT1 material cards since the curved pipe element is restricted to structural applications.

2. Note that separate property cards will be necessary when pin flags and/or offset are used.

3. See the sketch for a description of the curved pipe geometry.

4. If there are no pin flags or offsets, the continuation card may be omitted.

TABLE (1)

Curved Pipe Dummy Elements

Element Printout Explanations

Force Output

Label	Explanation
	[All forces at end (a)]
EL ID	Element identification number
F1	In plane shear force component (outward is positive)
F2	Axial force (tension is positive)
F3	Out of plane shear force component
F4	Bending moment about radial axis
F5	Torsional moment
F6	Bending moment about axis normal to plane

Stress Output

	[All stresses at end (a)]
EL ID	Element identification number
S1	Hoop stress
S2	Linear temperature gradient stress
S3	Nonlinear temperature gradient stress
S4	Axial stress (tension is positive)
S5	Shear stress due to torsion
S6	Maximum bending stress
S7	Maximum shear stress due to lateral loads

APPENDIX II

TEST PROBLEMS

The one element piping system shown in Figure (5) was used for initial tests of the dummy curved pipe element on NASTRAN. This problem was also solved by hand as a check on the NASTRAN run. Although the deflections and element forces found using the dummy element proved to be correct, the single point constraint forces were in error. The error can be traced to the transformation matrix involved in determining the global stiffness matrix from the element stiffness matrix. At this point the specific nature of the error is unknown.

A second, more involved piping system problem was also coded for NASTRAN. This problem, shown in Figure 6, was taken from Reference (9) and provides a means of comparison of the NASTRAN solution with an available independent solution. Details of this problem are available in Reference (9).

R = 4.5"

Outer diameter = 3.5"

Modulus of elasticity = $.229 \times 10^8$ psi

Moment of inertia = 3.017 in^4

Shear Modulus = 1.15×10^7 psi

$\psi = 90°$

Flexibility factor = 2

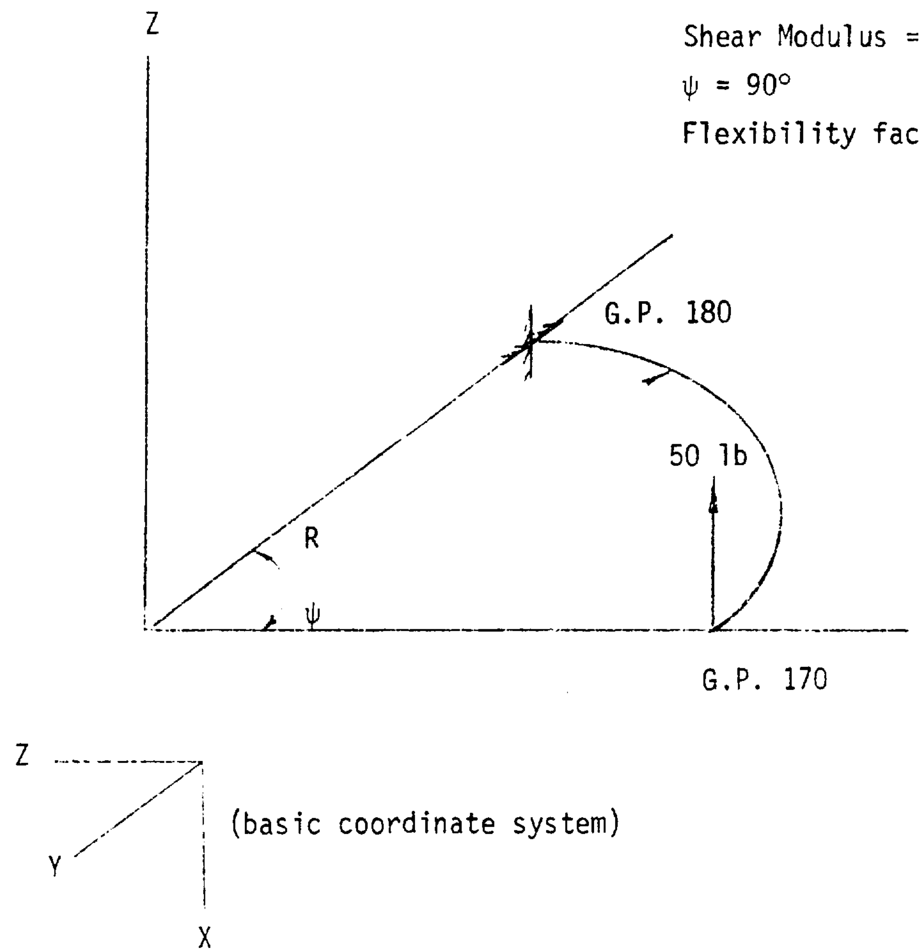

Figure (5). - Test Problem - Single Curved Element.

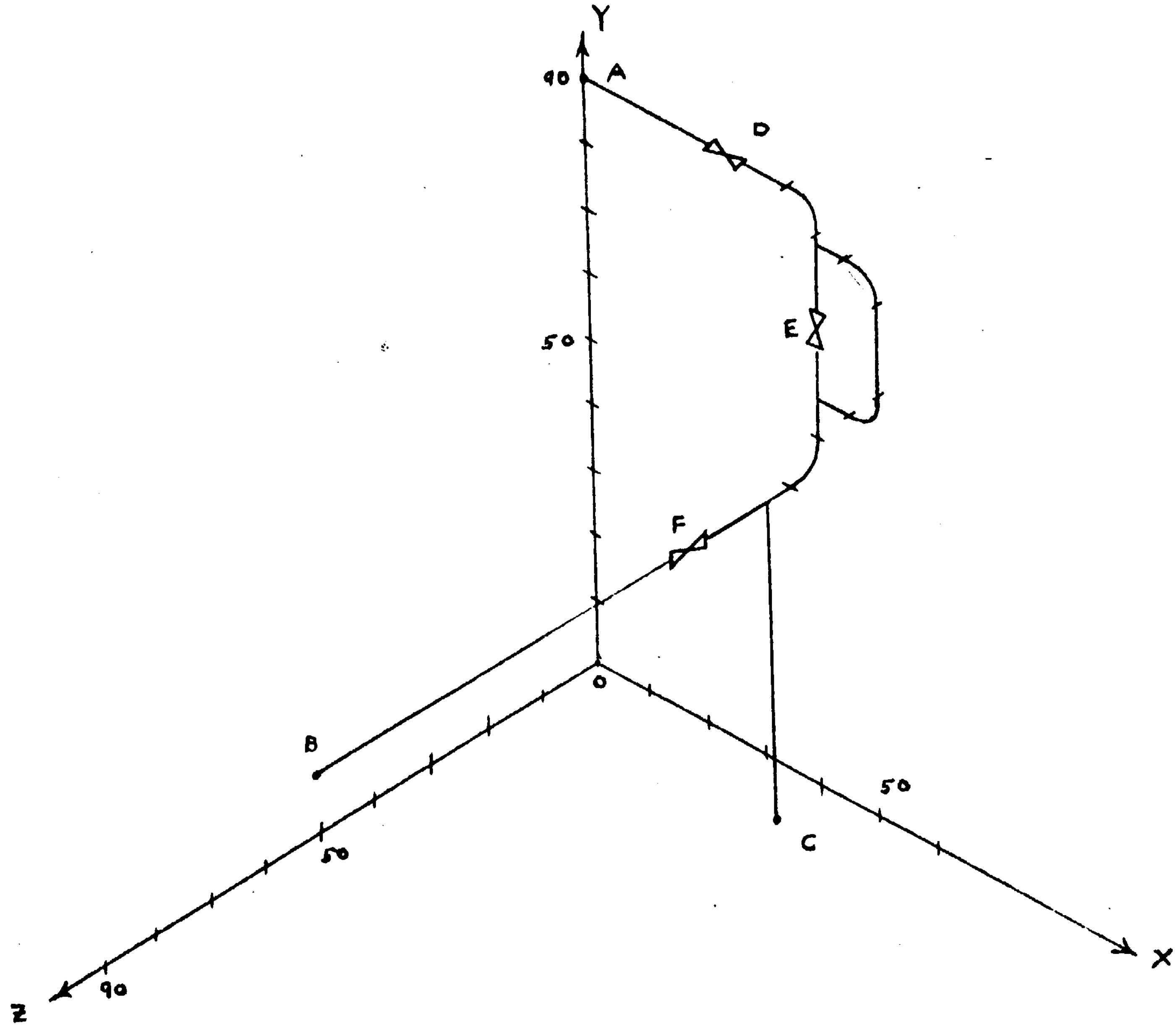

Figure (6). - ANSYS Example #1.