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GRAVITY GRADIENT PRELIMINARY INVESTIGATIONS

FINAL REPORT ON EXHIBIT "C"

CONTRACT NAS 9-9200

PART II
LUNAR TIDAL GRAVITY GRADIENTS AND STRESSES

January 30, 1971

Prepared for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
MANNED SPACECRAFT CENTER
Houston, Texas 77058

By

Mark H. Houston
Lloyd G.D. Thompson

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FOREWORD

This report covers a specific section of the Statement of Work, Exhibit "C", of Contract NAS9-9200 concerning tide-produced gravity gradients and crustal stresses on the moon and their possible link with lunar transient events and moonquakes. This is one of four different areas of investigation performed under Exhibit "C" and the results have been prepared as a separate Part II of the Final Report for convenience of future reference to the different subject areas. This Part II and the three reports on the other work areas (Parts I, III and IV) constitute the Final Report due under the Contract.

The results required by the Statement of Work, Exhibit "C", are essentially self-evident from the mathematics, calculations and analyses performed and presented in this report. All items of the Statement of Work are believed to be more than adequately satisfied.

The role of the tide-produced gravity gradients as triggering mechanisms for moonquakes and lunar transient events has long been a subject of interest of the junior author, Lloyd G.D. Thompson and the Contract Technical Monitor, William B. Chapman. The preliminary investigation covered by this report was chiefly performed by Mark H. Houston under the direction and guidance of Thompson and Chapman. In recognition of this, Houston has been selected as the senior author.

PART II

LUNAR TIDAL GRAVITY GRADIENTS AND STRESSES

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Preliminary analysis of the gravity gradients associated with gravity tides on the moon caused by the earth indicates that the relative changes in the gradients are very irregular and large and about 15 times greater than those experienced on earth. Thus gradients, in preference to gravity tides themselves, may well be an important key in correlating tide effects with lunar transient events and moonquakes and also in determining triggering mechanisms for crustal movement and faulting. Preliminary analysis of lunar crustal stresses and strains caused by lunar gravity tides indicates that these factors may be more direct causative agents or triggering mechanisms. In particular, the cubic dilation undergoes relatively large changes and is about 11 times greater on the moon than on earth. Thus it should be correspondingly more important. Development of formulae for the gravity gradient tensor and the stress tensor terms plus computer programs for calculating lunar tidal gravity, gradients, stresses and strains together with some suggested ways in which these terms may play a role in crustal mechanics provides a starting point for further more detailed investigations.

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I. Introduction

Lunar transient events occuring as red patches, bright spots in shadowed areas and other phonomena have been observed on the surface of the moon for over three hundred years. Whether these events are volcanic eruptions, gaseous outflows, static-charged dust clouds or merely optical effects is still unknown.

Several investigators have asserted that transient lunar events may be triggered by tidal stresses within the lunar crust and have examined the correlation between the variation of the vertical gravity tide and the incidence of lunar events. This report developes and presents the formulae for the gravity gradients and tidal stresses associated with the gravity tides on the moon caused by the earth. It further indicates the role of these terms as possible triggering mechanisims for lunar transient events or moonquakes. This work is intended to form the basis for more detailed investigations to establish a correlation between tidal gravity gradients and stresses and lunar events and the role of these terms as triggering agents. The equations and analysis have been considered for the case of gravity tides on the moon caused by the earth only with the solar effects ommitted. Basically, the formulae also apply for the gravity tides on the earth caused by the moon but in this case the ephemeris of the moon must be substituted.

First the gravity gradient tensor is developed and the various terms examined. Assuming particular forms and values for the Love Numbers ℓ and h and the material constants λ and μ, a method for computing the crustal stresses using crustal strains derived from the tidal potential is presented. The tidal stresses and cubic dilation are then examined.

II. Lunar Tidal Gravity Gradients

a) Tide Potential

It can be shown that in selenographic spherical coordinates the gravitational potential, U, on the surface of t[illegible] moon can be expressed as:

$$U = \frac{\gamma M_e}{2} \frac{r^2}{d^3} \left\{ 3 \cos^2 \theta - 1 \right\}$$

where γ = universal gravitational constant

M_e = mass of the earth

d = earth-moon separation

r = radius of the moon

θ = included angle between the sub-terrestial point and the observation location

In astronomical coordinates the included angle, θ, may be related to selenographic coordinates by

$$\cos\theta = \sin\phi \sin\delta + \cos\delta \cos\phi \left\{ \cos\xi \cos\lambda + \sin\xi \sin\lambda \right\}$$

where

ϕ = latitude of the selenographic observation point

δ = latitude of the earth (declination)

λ = longitude of the earth (hour angle)

ξ = longitude of the selenographic observation point

Finally, substituting the expression for θ into the tidal potential and rearranging into Laplace's Separation, the tidal potential may be written

$$U = \frac{3\gamma M_e}{4} \frac{r^2}{d^3} \left\{ \cos^2\phi \cos^2\delta \cos 2(\lambda-\xi) + \sin 2\phi \sin 2\phi \cos(\delta-\xi) + 3\left(\sin^2\phi - \frac{1}{3}\right)\left(\sin^2\delta - \frac{1}{3}\right) \right\}$$

b) Gravity Tides

Harrison (1963) and Sutton et al (1963) have discussed aspects of the tidal variations on the moon.

The lunar gravity gradients are best considered in local coordinates in the lunar surface. If we adopt axes with z positive upward, x positive eastward and y positive northward, then the gravity forces in the x, y, and z direction will be given by,

$$U_x = -\frac{1}{r\cos\phi} \frac{\partial U}{\partial \lambda}$$

$$U_y = -\frac{1}{r} \frac{\partial U}{\partial \phi}$$

$$U_z = -\frac{\partial U}{\partial r}$$

or assuming $\xi \equiv 0$

$$U_x = -\frac{3\gamma M_e}{2}\ \frac{r}{d^3}\left\{-\cos\phi\ \cos^2\delta\ \sin 2\lambda - \sin\phi\ \sin 2\delta\ \sin\lambda\right\}$$

$$U_y = -\frac{3\gamma M_e}{2}\ \frac{r}{d^3}\left\{-\frac{1}{2}\sin 2\phi\ \cos^2\delta\ \cos 2\lambda + \cos 2\phi\ \sin 2\delta\ \cos\lambda + \frac{3}{2}\sin 2\phi\ (\sin^2\delta - \frac{1}{3})\right\}$$

$$U_z = -\frac{3\gamma M_e}{2}\ \frac{r}{d^3}\left\{\cos^2\phi\ \cos^2\delta\ \cos 2\lambda + \sin 2\phi\ \sin 2\delta\ \cos\lambda + 3(\sin^2\phi - \frac{1}{3})(\sin^2\delta - \frac{1}{3})\right\}$$

c) <u>Gravity Gradient Tensor</u>

Similarly, the components of the gravity gradient tensor are,

$$U_{xx} = \frac{1}{r\cos\phi}\ \frac{\partial U_x}{\partial\lambda} - \frac{U_y}{r}\tan\phi + \frac{U_z}{r}$$

$$U_{xy} = \frac{1}{r}\left\{\frac{\partial U_x}{\partial\lambda} + U_x\tan\phi\right\} + \frac{1}{r\cos\phi}\ \frac{\partial U_y}{\partial\lambda}$$

$$U_{xz} = \frac{1}{r\cos\phi}\ \frac{\partial U_z}{\partial\lambda} + \frac{\partial U_x}{\partial r} - \frac{U_x}{r}$$

$$U_{yy} = \frac{1}{r}\ \frac{\partial U_y}{\partial\phi} + \frac{U_z}{r}$$

$$U_{yz} = \frac{\partial U_y}{\partial r} - \frac{U_y}{r} + \frac{1}{r}\ \frac{\partial U_z}{\partial\phi}$$

$$U_{zz} = \frac{U_z}{r}$$

$$U_{yx} = U_{xy}$$

$$U_{zx} = U_{xz}$$

$$U_{zy} = U_{yz} \quad .$$

Substituting, the gradient elements become,

$$U_{xx} = \frac{3\gamma M_e}{2d^3} \left\{ \cos^2\delta \cos 2\lambda + \sin^2\delta - \frac{1}{3} \right\}$$

$$U_{yy} = -\frac{3\gamma M_e}{2d^3} \left\{ \sin^2\phi \cos^2\delta \cos 2\lambda - \sin 2\phi \sin 2\delta \cos\lambda + 3(\cos^2\phi - \tfrac{1}{3})(\sin^2\delta - \tfrac{1}{3}) \right\}$$

$$U_{zz} = -\frac{3\gamma M_e}{2d^3} \left\{ \cos^2\phi \cos^2\delta \cos 2\lambda + \sin 2\phi \sin 2\delta \cos\lambda + 3(\sin^2\phi - \tfrac{1}{3})(\sin^2\delta - \tfrac{1}{3}) \right\}$$

$$U_{yx} = \frac{3\gamma M_e}{2d^3} \left\{ -2 \sin\phi \cos^2\delta \sin 2\lambda + 2 \cos\phi \sin 2\delta \sin\lambda \right\}$$

$$U_{xz} = \frac{3\ M_e}{2d^3} \left\{ \cos\phi \cos^2\delta \sin 2\lambda - \cos^2\phi \cos^2\delta \cos 2\lambda + \sin 2\phi \sin 2\delta (\sin\lambda - \cos\lambda) - \sin\phi \sin 2\delta \sin\lambda - 3(\sin^2\phi - \tfrac{1}{3})(\sin^2\delta - \tfrac{1}{3}) \right\}$$

$$U_{yz} = -\frac{3\gamma M_e}{2d^3} \left\{ -\sin 2\phi \cos^2\delta \cos 2\lambda + 2 \cos 2\phi \sin 2\delta \cos\lambda + 3 \sin 2\phi (\sin^2\delta - \tfrac{1}{3}) \right\}$$

Note that $U_{xx} + U_{yy} + U_{zz} = 0$.

A computer program for calculating tidal gravity and gravity gradient components on the moon due to the earth is given in Appendix A.

In general, the gradients on the moon vary in a complex manner over the surface of the lunar body and the variations are not as regular as those on earth. The longitude of the earth in the lunar sky changes slowly with lunar libration and the latitude changes with the revolution of the moon in its orbit. Additional variation in the gradients is introduced by the changing earth-moon distance.

The variation in the various gradient components for a lunar month period, the component differences one to another and the differences from place to place on the moon are illustrated in Figures 1, 2, and 3. The tidal contribution to the various gradient terms can be seen in these figures. Figure 1 illustrates the character of the gradients of the vertical component of gravity for a hypothetical case in which the gradients are observed at a planetary latitude of 42°N and the tide-producing body lies at a latitude of 20°S. For comparison, Figure 2 illustrates a similar case in which the latitude of the disturbing body has been changed to 20°N. In addition, Figure 3 shows an example of the calculated gradients U_{xx}, U_{yy} and U_{zz} for the crater Aristarchus.

The vertical gradient component U_{zz} acts in the local vertical and can be seen in Figures 1, 2 and 3 to shift in phase angle and magnitude at different sites on the lunar surface. U_{zx} and U_{zy} are the horizontal rates of change of the vertical component of gravity and can be seen in Figures 1 and 2 to change in phase, amplitude and sign. As can be seen by the equations, U_{xy} is similar in nature to U_{zy}. Figure 3 shows that the horizontal gradients U_{xx} and U_{yy} not only can vary in phase angle and amplitude but also can be positive or negative. At times, all gradient components can have large variations which could be significant in contributing to some mechanism for crustal movement.

On the moon the maximum peak-to-peak variation in the vertical component of the gravity tide is about 1.2 mgal and the corresponding change in the gradient is about 7×10^{-12} gal/cm or 0.007 Eotvos Units.* On earth the maximum peak-to-peak change in vertical gravity tide is about 0.3 mgal and the corresponding change in the gradient is about 5×10^{-13} gal/cm or 0.0005 Eotvos Units.

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* 1 Eotvos Unit = 1 E. U. = 1×10^{-9} gal/cm = 1×10^{-9} cm/sec^2/cm.

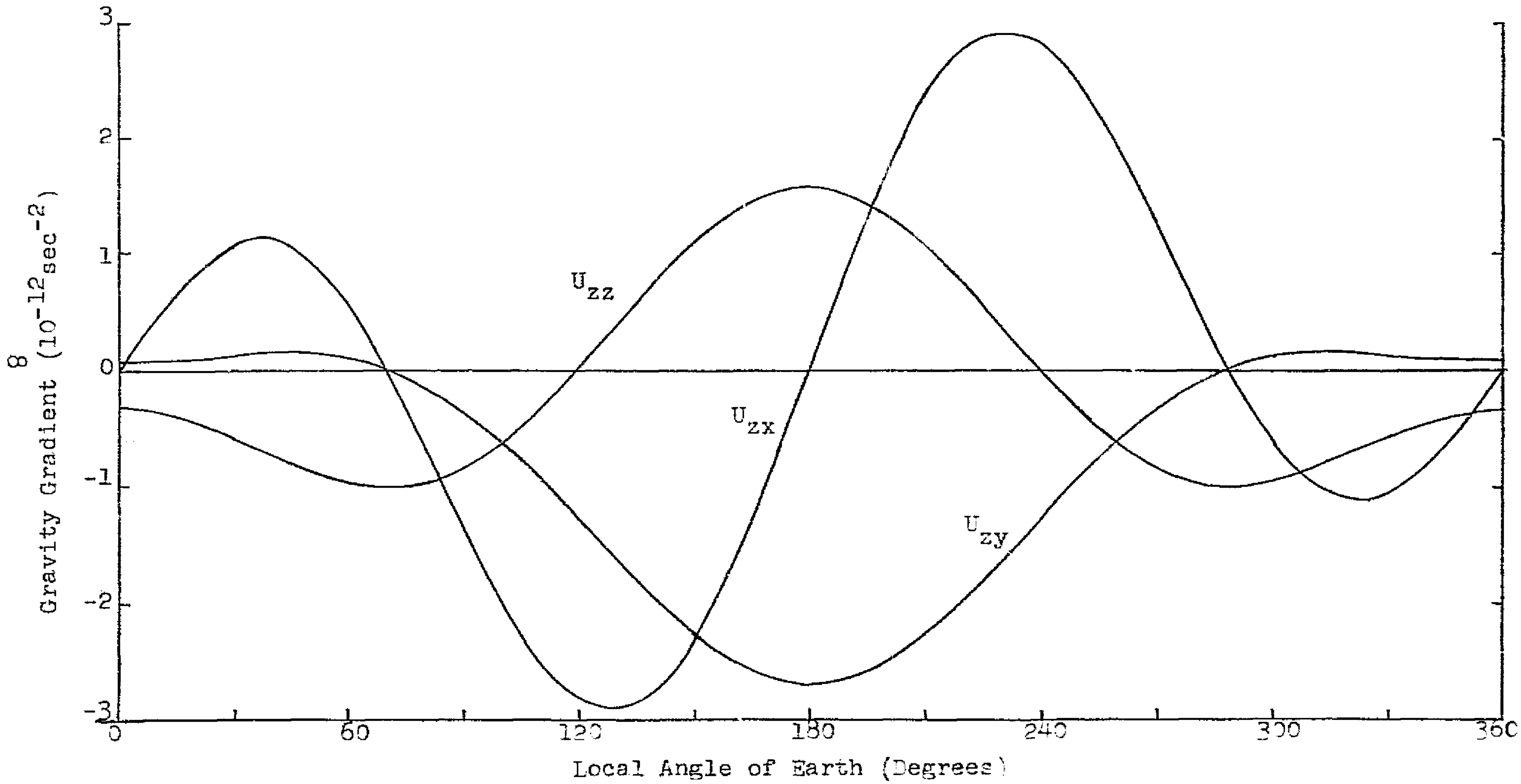

Figure 1. Variation of Gradients of Vertical Component of Lunar Tidal Gravity at 42°N, Earth at 20°S.

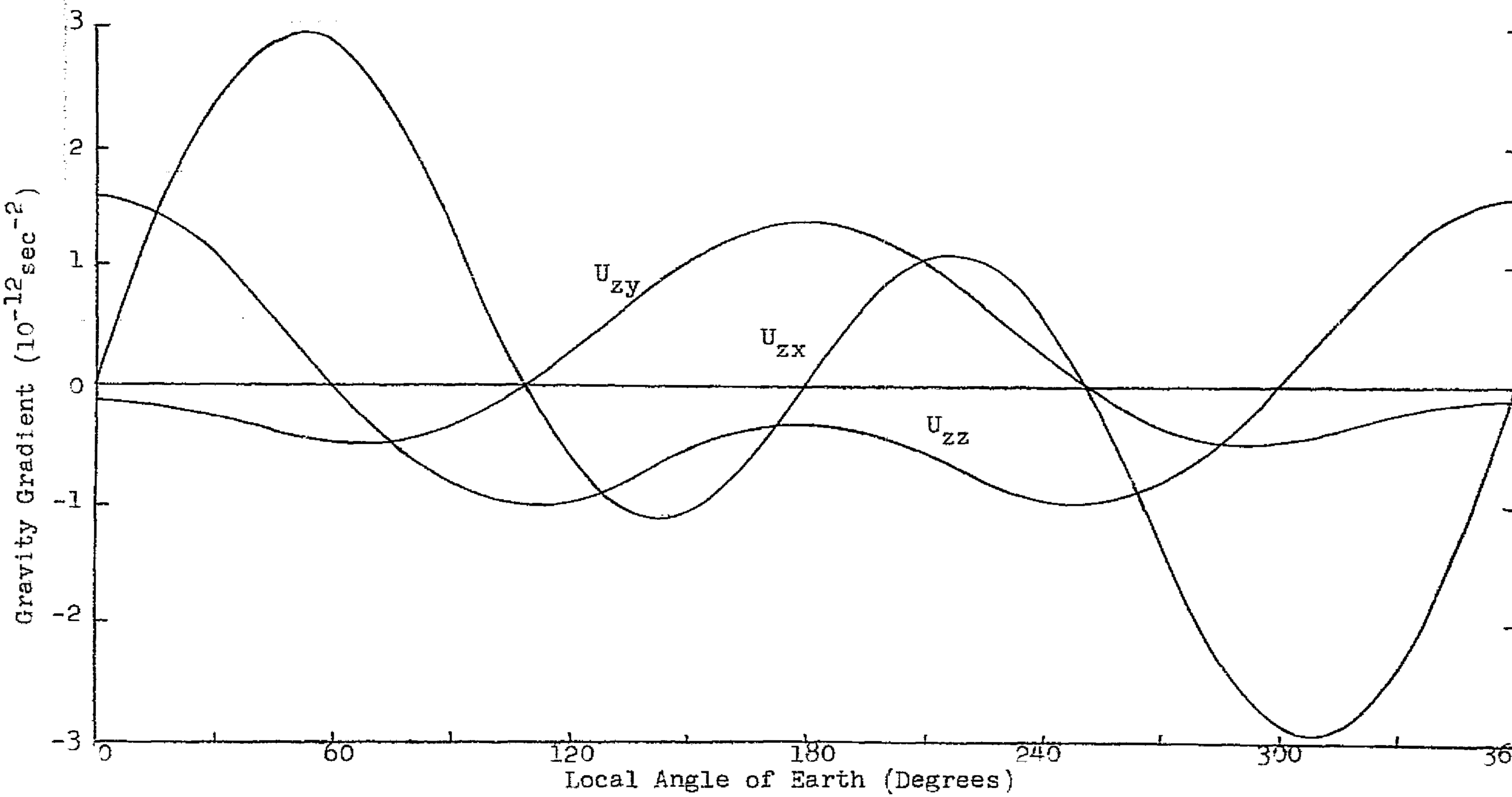

Figure 2. Variation of Gradients of Vertical Component of Lunar Tidal Gravity at 42°N, Earth at 20°N.

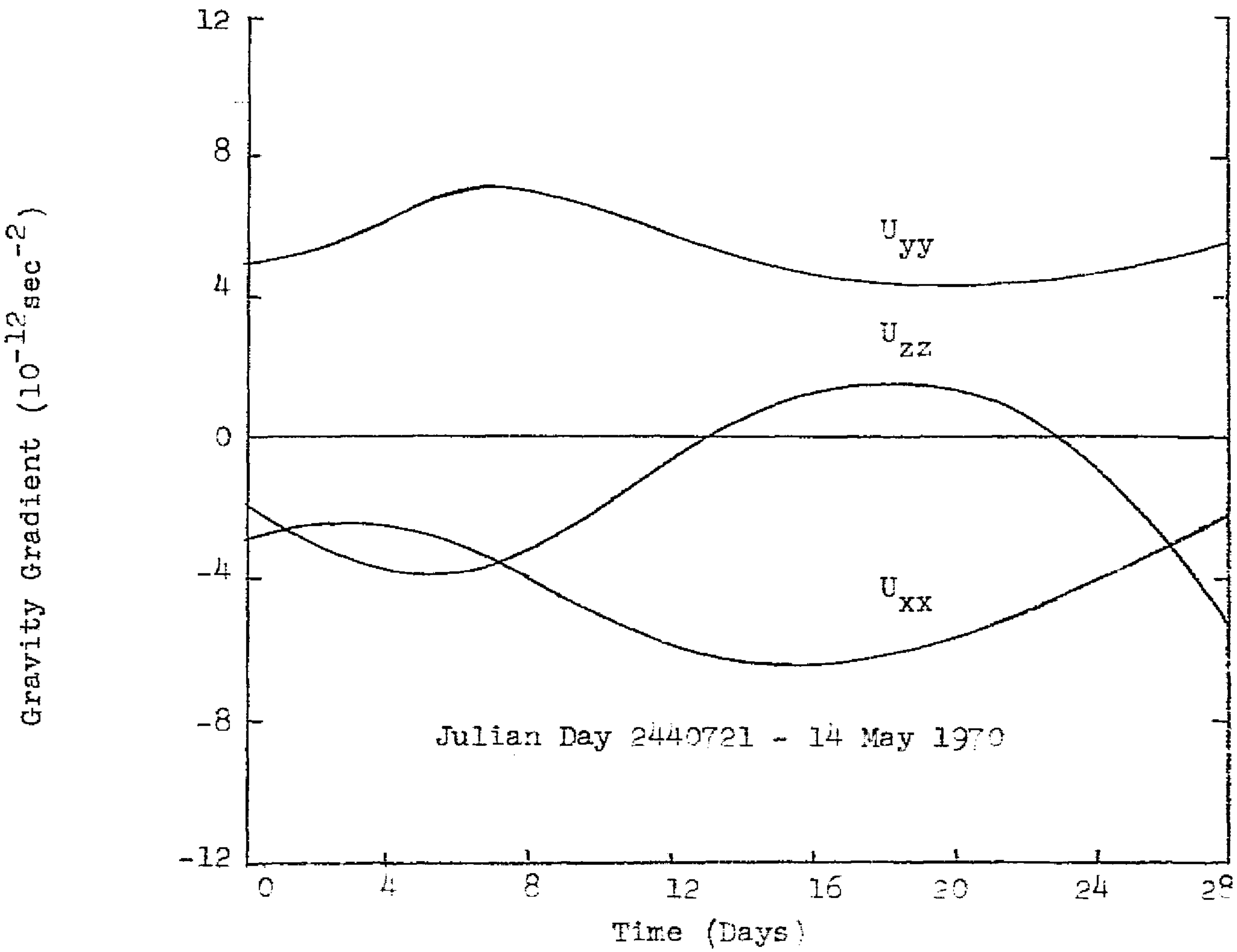

Figure 3. Variation of Principal Gradients of Lunar Tidal Gravity at Crater Aristarchus.

III. Lunar Tidal Stresses

While the variations of the lunar gravity components and the components of the gravity gradient tensor may yield some correlation with the distribution of lunar transient events, further insight into the problem may be obtained by considering tidal stresses induced within the lunar crust to be the triggering mechanism for the lunar events. Hence it is necessary to model the response of a planetary body to a varying external gravity field.

Dix (1968), using Takeuchi's (1950) model of the earth, attempted to show that tidal stresses within the earth may be sufficient to provide a triggering mechanism for the earthquake in Kern County, California on July 21, 1952. In our analysis of stress in the lunar crust we will use a much simpler model for the moon. Our lack of knowledge of the interior structure of the moon precludes any complex model.

a) Displacements

Adopting the previously defined coordinate system, the displacement at the moon's surface can be written,

$$S_x = \frac{\ell}{g \cos\phi} \frac{\partial U}{\partial \lambda}$$

$$S_y = \frac{\ell}{g} \frac{\partial U}{\partial \phi}$$

$$S_z = \frac{h}{g} U$$

where U = gravitational potential

g = acceleration at the moon's surface

h, ℓ = Love Numbers for the vertical and horizontal tides respectively.

Substituting the expression for the potential and assuming g varies only with r gives,

$$S_x = \frac{3\gamma M_e \; \ell \; r^2}{4g \cos\phi \; d^3} \left\{ -\cos^2\phi \cos^2\delta \sin 2\lambda - \sin 2\phi \sin 2\delta \sin\lambda \right\}$$

$$S_y = \frac{3\gamma M_e \; \ell \; r^2}{4gd^3} \left\{ -\sin 2\phi \cos^2\delta \cos 2\lambda + 2 \cos 2\phi \sin 2\delta \cos\lambda + 3 \sin 2\phi \left(\sin^2\delta - \tfrac{1}{3}\right) \right\}$$

$$S_z = \frac{3\gamma M_e \; h \; r^2}{4gd^3} \left\{ \cos^2\phi \cos^2\delta \cos 2\lambda + \sin 2\phi \sin 2\delta \cos\lambda + 3\left(\sin^2\phi - \tfrac{1}{3}\right)\left(\sin^2\delta - \tfrac{1}{3}\right) \right\}$$

b) <u>Strain Tensor</u>

To calculate the strain tensor, we will make the assumption that h, ℓ and g are functions only of the radius r (z direction) and then proceed as before. The elements of the strain tensor are,

$$e_{xx} = \frac{1}{r \cos\phi} \frac{\partial S_x}{\partial \lambda} - \frac{S_y}{r} \tan\phi + \frac{S_z}{r}$$

$$e_{yy} = \frac{1}{r} \frac{\partial S_y}{\partial \phi} + \frac{S_z}{r}$$

$$e_{zz} = \frac{\partial S_z}{\partial r}$$

$$e_{yx} = \frac{1}{r} \frac{\partial S_x}{\partial \phi} + \frac{1}{r} \tan\phi \; S_x + \frac{1}{r \cos\phi} \frac{\partial S_y}{\partial \lambda}$$

$$e_{xz} = \frac{1}{r \cos\phi} \frac{\partial S_z}{\partial x} + \frac{\partial S_z}{\partial r} - \frac{S_x}{r}$$

$$e_{zy} = \frac{\partial S_y}{\partial r} - \frac{S_y}{r} + \frac{1}{r} \frac{\partial S_z}{\partial \phi}$$

$$e_{xy} = e_{yx}$$

$$e_{zx} = e_{xz}$$

$$e_{yz} = e_{zy}$$

Substituting for S_x, S_y, and S_z the tensor components become,

$$e_{xx} = \frac{3\gamma M_e r}{4d^3} \Bigg\{ - \frac{\ell}{\cos^2\phi} \left[\cos^2\phi \cos^2\delta \cos 2\lambda + \sin 2\phi \sin 2\delta \cos\lambda \right]$$

$$- \frac{\ell \sin\phi}{\cos\phi} \Big[- \sin 2\phi \cos^2\delta \cos 2\lambda + 2 \cos 2\phi \sin 2\delta \cos\lambda + 3 \sin 2\phi \, (\sin^2\delta - \tfrac{1}{3}) \Big]$$

$$+ h \Big[\cos^2\phi \cos^2\delta \cos 2\lambda + \sin 2\phi \sin 2\delta \cos\lambda + 3 \, (\sin^2\phi - \tfrac{1}{3})(\sin^2\delta - \tfrac{1}{3}) \Big] \Bigg\}$$

$$e_{yy} = \frac{3\gamma M_e r}{4d^3} \left\{ 2\ell \left[-\sin 2\phi \cos 2\phi \quad (\cos^2\delta \cos 2\lambda + 2 \cos 2\delta \cos\lambda) + 3 \sin 2\phi \quad (\sin^2\delta - \tfrac{1}{3}) \right] \right.$$

$$\left. + h \left[\cos^2\phi \cos^2\delta \cos 2\lambda + \sin 2\phi \sin 2\delta \cos\lambda + 3 (\sin^2\phi - \tfrac{1}{3})(\sin^2\delta - \tfrac{1}{3}) \right] \right\}$$

$$e_{zz} = \frac{3\gamma M_e r}{4\ g\ d^3} \left\{ v \left[h' - h \frac{g'}{g} \right] + 2h \right\}$$

$$\left\{ \cos^2\phi \cos^2\delta \cos 2\lambda + \sin 2\phi \sin 2\delta \cos\lambda + 3 (\sin^2\phi - \tfrac{1}{3})(\sin^2\delta - \tfrac{1}{3}) \right\}$$

where $h' = \frac{\partial h}{\partial r}$

$g' = \frac{\partial g}{\partial r}$

$$e_{yx} = \frac{3\gamma M_e r}{4\ g\ d^3} \quad \ell \left\{ 4 \sin\phi \cos^2\delta \sin 2\lambda - 4 \cos\phi \sin 2\delta \sin\lambda + \frac{1}{\cos\ \phi} \sin^2\delta \sin\lambda \right\}$$

$$e_{xz} = \frac{3\gamma M_e r}{4\ g\ d^3} \left\{ h \left[-2 \cos\phi \cos^2\delta \sin 2\lambda - \sin\phi \sin 2\delta \sin\lambda \right] \right.$$

$$+ \left[r \left(h' - h\frac{g'}{g} \right) + 2h \right] \left[\cos^2\phi \cos^2\delta \cos 2\lambda + + \sin 2\phi \sin 2\delta \cos\lambda + 3(\sin^2\phi - \tfrac{1}{3})(\sin^2\delta - \tfrac{1}{3}) \right]$$

$$\left. + \ell \left[\cos\phi \cos^2\delta \sin 2\lambda + 2 \sin\phi \sin 2\delta \sin\lambda \right] \right\}$$

$$e_{zy} = \frac{3\gamma M_e r}{4\, g\, d^3} \left\{ \left[r \left(l' + l\frac{g'}{g} \right) + \ell \right] \left[- \sin 2\phi \cos^2\delta \cos 2\lambda + 2 \cos 2\phi \sin 2\delta \cos\lambda + 3 \sin 2\phi \left(\sin^2\delta - \frac{1}{3} \right) \right] \right.$$

$$\left. + h \left[- \sin 2\phi \cos^2\delta \cos 2\lambda + 2 \cos 2\phi \sin 2\delta \cos\lambda + 3 \sin 2\phi \left(\sin^2\delta - \frac{1}{3} \right) \right] \right\}$$

where $l' = \frac{\partial \ell}{\partial r}$

We can reduce the expressions further by assuming that the density and deformations are homothetic with regard to the center. We assume that

$$\frac{\ell'}{\ell} = \frac{h'}{h} = \frac{g'}{g} = - \frac{2}{r}$$

in which case,

$$e_{zz} = \frac{3\gamma M_e r}{4\, g\, d^3} \quad 2h \left\{ \cos^2\phi \cos^2\delta \cos 2\lambda + \sin 2\phi \sin 2\delta \cos\lambda + 3\left(\sin^2\phi - \frac{1}{3}\right)\left(\sin^2\delta - \frac{1}{3}\right) \right\}$$

$$e_{xz} = \frac{3\gamma M_e r}{4\, g\, d^3} \left\{ 2h \left[- \cos\phi \cos^2\delta \sin 2\lambda - \frac{1}{2} \sin\phi \sin 2\delta \sin\lambda \right. \right.$$

$$+ \cos^2\phi \cos^2\delta \cos 2\lambda + \sin 2\phi \sin 2\delta \cos\lambda$$
$$\left. + 3 \left(\sin^2\phi - \frac{1}{3}\right) \left(\sin^2\delta - \frac{1}{3}\right) \right]$$
$$\left. + \ell \left[\cos\phi \cos^2\delta \sin 2\lambda + 2 \sin\phi \sin 2\delta \sin\lambda \right] \right\}$$

$$e_{zy} = \frac{3\gamma M_e r}{4\, g\, d^3}\left\{ \left[h + \ell \right] \left[-\sin 2\phi \cos^2\delta \cos 2\lambda + 2 \cos 2\phi \sin 2\delta \cos\lambda + 3 \sin 2\phi \left(\sin^2\delta - \tfrac{1}{3}\right) \right] \right\}$$

c) Stress Tensor

Given the above strain tensor e_{ij} and assuming Hooke's Law for a homogeneous isotropic body, the stress tensor can be written in generalized form as,

$$T_{ij} = \lambda\, \delta_{ij}\, e_{ii} + 2\, \mu\, e_{ij}$$

where λ and μ are Lame' constants commonly known as the compressibility and rigidity. Finally, since the general coordinate transformation of a second order tensor is written,

$$T_{\alpha\beta} = \ell_{i\alpha}\, \ell_{j\beta}\, T_{ij}$$

the stress within the lunar crust along any direction, n, whose direction cosines are ℓ_1, ℓ_2, and ℓ_3 are given by

$$T_n = \ell_1^2\, T_{11} + \ell_2^2\, T_{22} + \ell_3^2\, T_{33} + \ell_1\ell_2\, T_{12} + \ell_1\ell_3\, T_{13} + \ell_2\ell_3\, T_{23}$$

IV. Geophysical Mechanisms for Lunar Events

Several possible causative agents for lunar transient events may be considered. Unless some mechanism exists which accumulates the tidal energy over long periods of time, the tidal energy involved is much too small to be a direct causative agent. If faulting or moon quakes are responsible for the observed phenomena, then tidal stresses may serve as a triggering agent. The previous analysis provides a means to calculate tidal stresses in any direction and will thus allow an analysis of the tidal stress along a particularly oriented fault plane. A particular method for calculating the tidal stress along a fault plane is given in Appendix B.

If the lunar crust contains reservoirs of incompressible fluid such as water or molten lava, or contains structures of widely varying compressibility, then variations in the cubic dilation of the lunar crust could be responsible for the transient events. On earth fluid levels in oil and water wells are observed to change with the phase of the solid earth tide. The volume of the liquid remains constant but the volume of the enclosing cavity varies with the cubic dilation of the crust. The tidal cubic dilation can be written

$$\nabla^2 e = e_{xx} + e_{yy} + e_{zz}$$

or $$\nabla^2 e = 2(h-3\ell)\frac{U}{rg} + \frac{2hU}{rg} + \left[\frac{h'}{h} - \frac{g'}{g}\right]\frac{hU}{g}$$

or assuming $\frac{h'}{h} = \frac{g'}{g}$

$$\nabla^2 e = \frac{3\gamma M_e r}{4\, g\, d^3}\left[4h-6\ell\right]\cdot\left[\cos^2\phi\ \cos^2\delta\ \cos 2\lambda + \sin 2\phi\ \sin 2\delta\ \cos\lambda + 3\left(\sin^2\phi - \frac{1}{3}\right)\left(\sin^2\delta - \frac{1}{3}\right)\right]$$

Suppose now we examine the relative magnitude of simple extension and of cubic dilation on the moon as compared to that on the earth. In order to do this, we must select suitable Love Numbers for the moon and the earth. The cubic dilation, D, can be written in terms of the potential and a characteristic number, f.

$$D = f \frac{U}{rg}$$

where $f = r \frac{\partial h}{\partial r} + 6h - 6\ell$

Similarly, crustal extension in the z direction is,

$$\varepsilon = \frac{hU}{g}$$

The two constants for the earth have been experimentally determined to be 0.620 (f_e) and 0.584 (h_e) {Melchior, 1966, p. 300}. Melchior (1966, p. 325+) has shown that for a homogeneous self gravitating sphere,

$$h = \frac{5}{2} \left(1 + \frac{19 \mu}{2g\rho r} \right)^{-1}$$

where μ = rigidity

g = acceleration at the surface

ρ = density

r = radius

Also, $$\ell = \frac{3}{10} h$$

For the moon,

$$= 162,356 \text{ gals}$$

$$\rho = 3.33 \text{ gms/cm}^3$$

$$r = 1.738 \times 10^8 \text{ cm}$$

and if we assume that the rigidity of the moon is equal to that of the earth's crust,

$$\mu = 10^{12} \text{ (cgs)}$$

In that case, $h = .025$, $\ell = .007$ and $f = .055$.

Now taking the ratio of the moon's cubic dilation to that of the earth,

$$\frac{D_m}{D_e} = \frac{f_m}{f_e} \left(\frac{M_e}{M_m}\right)^2 \left(\frac{r_m}{r_e}\right)^3$$

or $$\frac{D_m}{D_e} \approx 11.3$$

The ratio of the moon's radial extension to that of the earth is

$$\frac{S_m}{S_e} = \frac{h_m}{h_e} \left(\frac{M_e}{M_m}\right)^2 \left(\frac{r_m}{r_e}\right)^4$$

$$\text{and} \quad \frac{S_m}{S_e} = 1.5.$$

Thus we see that while the radial extension on the moon is only 1.5 times that of the earth, the amplitude of lunar cubic expansion is eleven times that found on earth and should be correspondingly more important in lunar crustal mechanics.

To further investigate and illustrate this concept, the monthly variation in the vertical and horizontal stresses and cubic dilation on the moon due to the earth have been calculated for the same time at the three craters of Aristarchus, Plato and Alphonsus and are shown in Figures 4, 5 and 6 respectively. The computer program for performing these calculations is given in Appendix C. Figures 4, 5, and 6 show that the variation in the cubic dilation has a very large amplitude while the vertical (T_{zz}) and horizontal (T_{xx}) stresses experience much smaller changes. Further, the behavior of these quantities is significantly different at different locations on the moon. At Aristarchus and Plato (Figures 4 and 5) the cubic dilation and T_{zz} have much the same behavior but T_{xx} is positive at Aristarchus and negative at Plato. At Alphonsus (Figure 6) the amplitude of the cubic dilation is considerably reduced but the magnitude is at a very high positive level. In addition, both T_{xx} and T_{zz} are also positive. This indicates that the stresses and particularly the cubic dilation could well be important causative agents in lunar crustal mechanics and faulting.

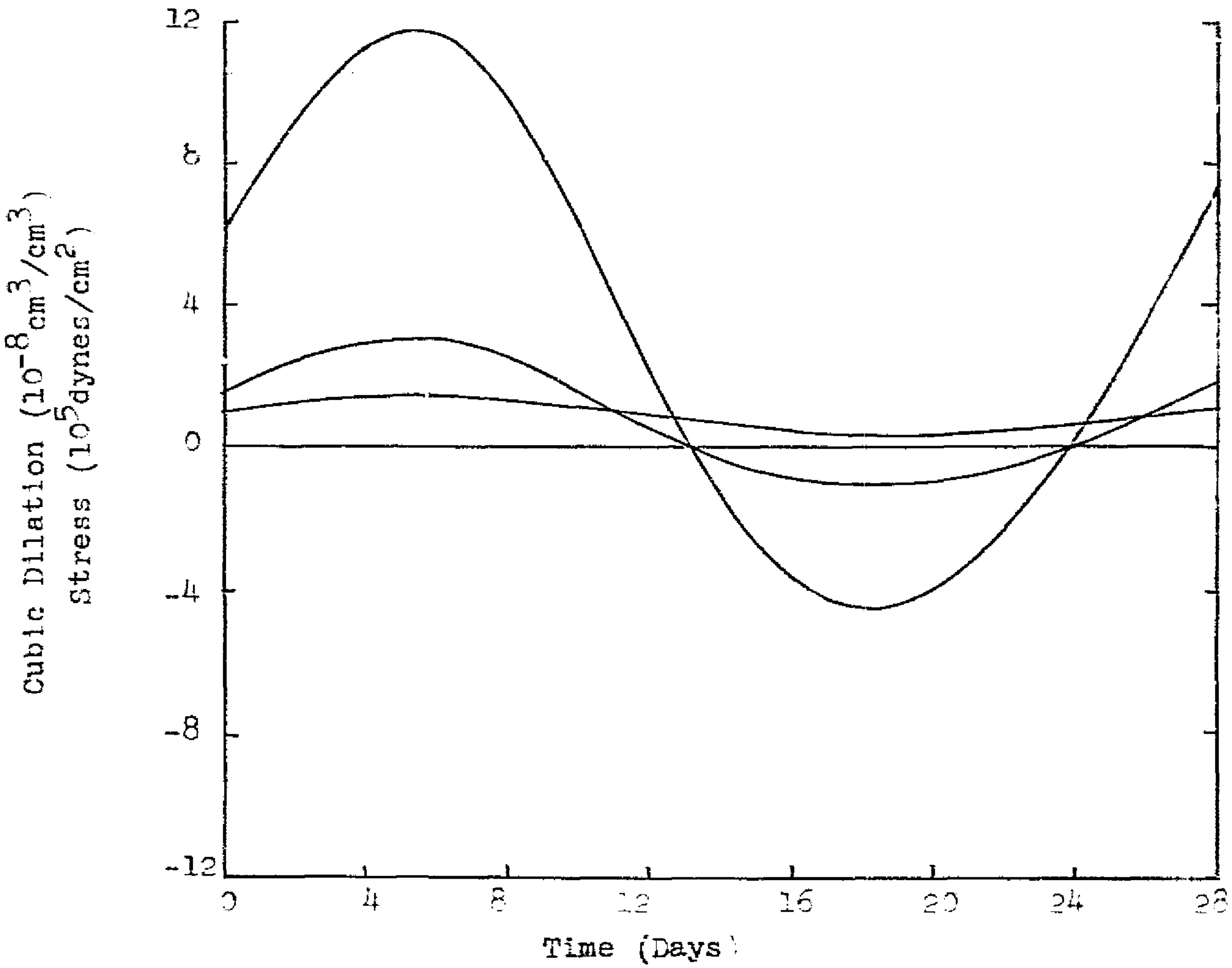

Figure 4. Stress and Cubic Dilation at Crater Aristarchus.

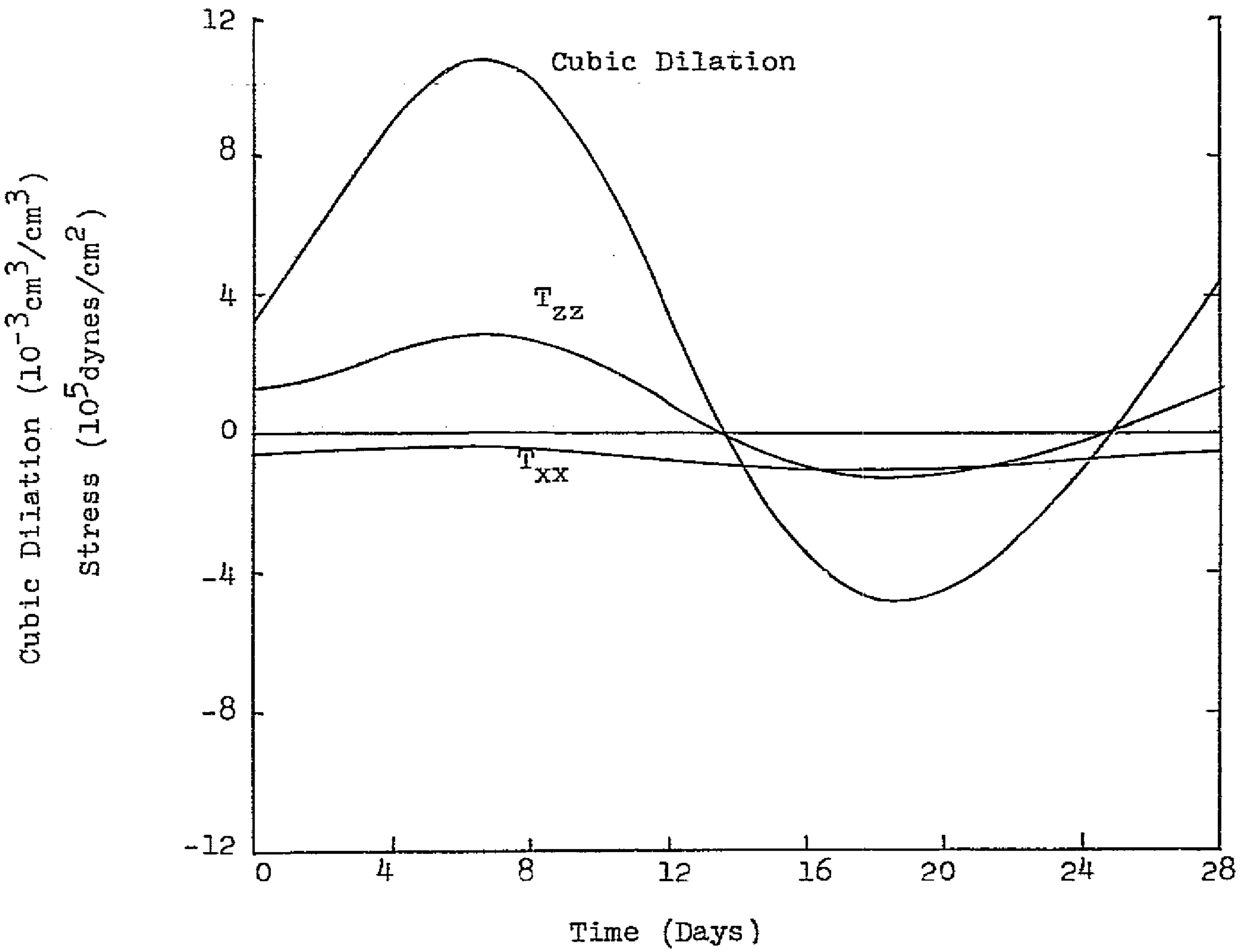

Figure 5. Stress and Cubic Dilation
at Crater Plato

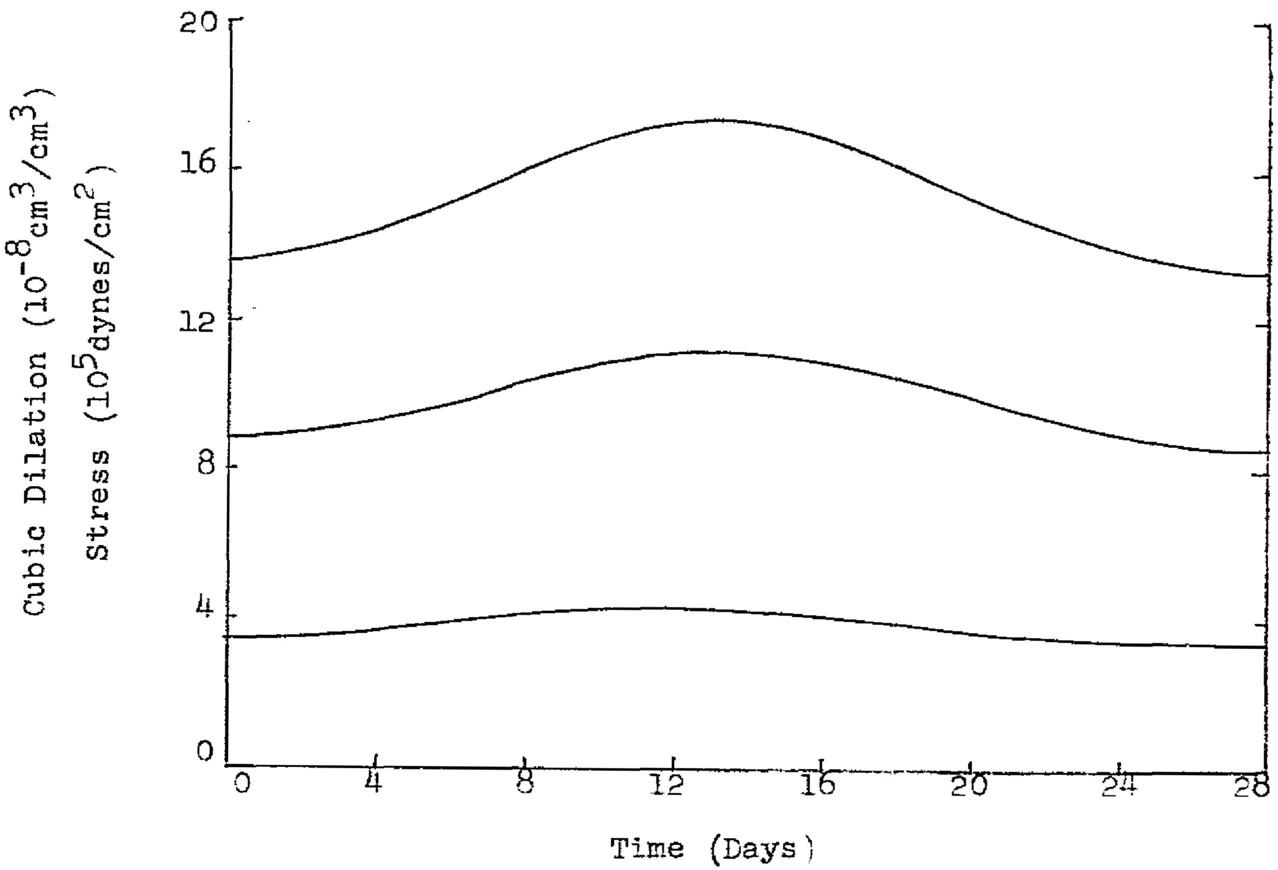

Figure 6. Stress and Cubic Dilation

at Crater Alphonsus

V. Conclusions and Recommendations

From other work, there is an apparent correlation between lunar transient events and/or moonquakes and gravity tides which, by symmetry of equations, applies also to the tidal gravity gradients. Tidal variations in gravity are not likely, in themselves, to be direct causative agents since the tidal energy involved is too small. On the other hand the relative changes in the gradients are significantly irregular and large and are, in fact, 15 times greater on the moon than on the earth. This indicates that gradients (one or more components) could possibly relate more directly to lunar events and the generation of large local differential forces and therefore could be a significant factor or key in crustal mechanics. They may contribute to or have a modifying influence operating in conjunction with other physical mechanisms. More particularly, there may be a mechanism for accumulating tidal effects over a long period of time or there may be large forces created differentially when gradients are considered over large areas or across a fault zone. This preliminary work provides the basis for further and more detailed investigations of these and other approaches to the problem. Such additional studies are highly recommended.

Lunar crustal stresses and cubic dilation caused by gravity tides and gradients could well be important and more direct triggering mechanisms or causative agents in crustal mechanics and faulting. In particular, the lunar cubic dilation exhibits large variations and is eleven times that experienced on earth. Thus it should be correspondingly more important in lunar crustal mechanics. Further investigations of long term stress build-up and the stresses and cubic dilation generated over large adjacent areas and across a fault zone are logical and anticipated continuations of this preliminary work. Such studies are highly recommended, not only in support of lunar investigations but also in the interest of earthquake prediction to which this work also applies.

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APPENDIX A

Computer Program for Calculating Tidal Gravity and Gradients on the Moon due to the Earth.

```
C                 LUNAR TIDES AND GRADIENTS       TIDE1.F4
C                     M HOUSTON 5/14/70
C
C       THIS PROGRAM CALCULATES THE TIDAL ANOMALIES AND TIDAL
C       GRADIENTS ON THE MOON DUE TO THE EARTH.

        IMPLICIT DOUBLE PRECISION (A-H,L,O-Y)
        DIMENSION JYEAR(0/26),JDAY(4,12)
        DATA JYEAR(0)/2415019/,(JDAY(1,K),K=1,12)/
     10,31,60,91,121,152,182,213,244,274,305,335/
        COSQ(THETA) = DCOS(THETA)*DCOS(THETA)
        SINQ(THETA) = DSIN(THETA)*DSIN(THETA)
        COS2(THETA)  = DCOS(2.0D0*THETA)
        SIN2(THETA) = DSIN(2.0D0*THETA)

        EMASS = 5.975D0
        GRAVCN = 6.67D0
        EMDIST = 3.84411D0
        RMOON = (2.73D-1)*(6.37123D8)
        CONST = (1.5D0)*GRAVCN*EMASS/(EMDIST*EMDIST*EMDIST)
        CONST = CONST*(1.0D-11)
C
C       JULIAN CALENDAR 4/2/70
C
        DO 1 J=1,26
1       JYEAR(J) = JYEAR(J-1) + 1461
        DO 2 I=2,4
        DO 2 J=1,12
2       JDAY(I,J) = JDAY(I-1,J) + 365
        JDAY(2,1) = JDAY(2,1) + 1
        JDAY(2,2) = JDAY(2,2) + 1
        TYPE 99
99      FORMAT(/' NUMBER OF CASES (I2)'/)
        NCASE = 0
        ACCEPT 104, ICASE
999     TYPE 100
100     FORMAT(/' DAY,MONTH,YEAR - 3I2'/)
        NCASE = NCASE + 1
        ACCEPT 101,IDAY,IMONTH,IYEAR
101     FORMAT(3I2)
        I1 = IYEAR/4
        I2 = -4*I1+IYEAR+1
        JULDAY = JYEAR(I1) + JDAY(I2,IMONTH) + IDAY
        TYPE 102,JULDAY
102     FORMAT(/' JULIAN CALENDAR DAY =',I15)
        TYPE 103
103     FORMAT(/' NUMBER OF DAYS TIDES AND GRADIENTS DESIRED = '/)
        ACCEPT 104,NDAY
104     FORMAT(I2)
        NDAY = NDAY+JULDAY
```

```
      TYPE 90
90    FORMAT(//' LUNAR OBSERVATION POINT'//' LATITUDE = '/)
      ACCEPT 91,OBLAT
91    FORMAT(D)
      TYPE 92
92    FORMAT(/' LONGITUDE = '/)
      ACCEPT 91,OBLONG

      TYPE 200
200   FORMAT(/' OUTPUT FILE UNIT DESIGNATION (I2) = '/)
      ACCEPT 104,IUNIT
      TYPE 201
201   FORMAT(/' OUTPUT FILE NAME (A5) = '/)
      ACCEPT 202,ZNAME
202   FORMAT(A5)
      CALL OFILE(IUNIT,ZNAME)
      WRITE (IUNIT,190)OBLAT,OBLONG
190   FORMAT(///,1X35HLUNAR OBSERVATION POINT COORDINATES,//
     1 7X8HLATITUDE,/X9HLONGITUDE,//,5X1PD13.6,2X,D13.6)


      RDEG = 1.745329251994329D-2
      OBLAT = OBLAT*RDEG
      OBLONG = OBLONG*RDEG
      JULDAY = JULDAY - 1
69    JULDAY = JULDAY + 1

C
C                    LUNAR EPHEMERIS 4/20/70


      T = DBLE(FLOAT(JULDAY-2415019))/3.6525D4
      PI = 3.141592653589793D0
      AI = (9.21D1)*PI/((1.8D2)*(6.0D1))

      S = 2.7043659D2 + (((2.0D-6)*T+1.98D-3)*T+4.8126789057D5)*T
      P = 3.3432956D2 + (((-1.0D-5)*T-1.032D-2)*T+4.06903403D3)*T
      H = 2.7969668D2 + ((3.0D-4)*T+3.600076892D4)*T
      RMIN = 2.908882086657215D-4
      S = S*RDEG
      P = P*RDEG
      H = H*RDEG


      DRATIO = 1.0D0 + (5.45D-2)*DCOS(S-P) + (2.97D-3)*DCOS((2.0D0)*
     1(S-P)) + (1.002D-2)*DCOS(S-P-2.0D0*(H-P))
     2       + (8.25D-3)*DCOS(2.0D0*(S-H))

      PS = 2.8122083D2 + (((3.0D-6)*T+4.5D-4)*T+1.71902D0)*T
      PS = PS*RDEG
```

```
      S1 = (3.77D2)*DSIN(S-P) + (1.3D1)*DSIN((2.0D0)*(S-P))
    1+ (7.6D1)*DSIN(S-P-(2.0D0)*(H-P)) - (1.1D1)*DSIN(H-PS)
    2+ (4.0D1)*DSIN((2.0D0)*(S-H))

      S1 = S + S1*RMIN
      AN = 2.5918328D2 + (((2.0D-6)*T+2.08D-3)*T-1.93414201D3)*T
      AN = AN*RDEG
      A = DSIN(AI)*DCOS(S1-AN)
      AMU = (1.03D-2)*DSIN((2.0D0)*(S1-AN))*RDEG

      BETA = (3.08D2)*DSIN(S-AN) + (1.7D1)*DSIN(2.0D0*S-P-AN)
    1+ (1.0D1)*DSIN((2.0D0)*(S-H)-S+AN) - (1.7D1)*DSIN(P-AN)

      BETA = BETA*RMIN
      TANB = -DSIN(AI)*DSIN(S1-AN)/DCOS(AI)
      ELAT = (DATAN(TANB)-BETA)
      ELONG = (S1-S+AMU+A*ELAT)

      TANB = -DSIN(AI)*DSIN(S1-AN)/DCOS(AI)
      ELAT = (DATAN(TANB)-BETA)
      ELONG = (S1-S+AMU+A*ELAT)

      E1 = ELAT/RDEG
      E2 = ELONG/RDEG
      WRITE (IUNIT,203)JULDAY
203   FORMAT(/////1X46HEARTH SELENOGRAPHIC COORDINATES FOR JULIAN DAY,
    1 I10)
      WRITE (IUNIT,204)E1,E2,DRATIO
204   FORMAT(//,7X8HLATITUDE,7X9HLONGITUDE,6X8HPARALLAX,
    1 //,5X,1PD13.6,2(2X,D13.6))

C               TIDE AND GRADIENT CALCULATIONS 4/28/70

      DRATIO = DRATIO*DRATIO*DRATIO
      P = OBLAT
      D = ELAT
      L = ELONG - OBLONG

      UZ = COSQ(P)*COSQ(D)*COS2(L)+SIN2(P)*SIN2(D)*DCOS(L)
    1+ ((3.0D0)*SINQ(P)-1.0D0)*(SINQ(D)-1.0D0/3.0D0)
      UZ = -RMOON*DRATIO*UZ*CONST

      UY = -SIN2(P)*COSQ(D)*COS2(L)+COS2(P)*SIN2(D)*DCOS(L)*(2.0D0)
    1+ SIN2(P)*((3.0D0)*SINQ(D)-1.0D0)
      UY = -RMOON*DRATIO*UY*(5.0D-1)*CONST

      UX = DCOS(P)*COSQ(D)*SIN2(L)+DSIN(P)*SIN2(D)*DSIN(L)
      UX = RMOON*DRATIO*UX*CONST
```

```
      UZZ = UZ/RMOON

      UYY = SINQ(P)*COSQ(D)*COS2(L)-SIN2(P)*SIN2(D)*DCOS(L)
     1+ (3.0D0*COSQ(P)-1.0D0)*(SINQ(D)-1.0D0/3.0D0)
      UYY = -DRATIO*CONST*UYY

      UXX = COSQ(D)*COS2(L)+SINQ(D)-1.0D0/3.0D0
      UXX = DRATIO*CONST*UXX

      UYX = -DSIN(P)*COSQ(D)*SIN2(L)+DCOS(P)*SIN2(D)*DSIN(L)
      UYX = (2.0D0)*DRATIO*CONST*UYX

      UXZ = DCOS(P)*COSQ(D)*SIN2(L)-COSQ(P)*COSQ(D)*COS2(L)
     1+ SIN2(P)*SIN2(D)*(DSIN(L)-DCOS(L))-DSIN(P)*SIN2(D)*DSIN(L)
     2- (3.0D0*SINQ(P)-1.0D0)*(SINQ(D)-1.0D0/3.0D0)
      UXZ = DRATIO*CONST*UXZ

      UZY = -SIN2(P)*COSQ(D)*COS2(L)+2.0D0*COS2(P)*SIN2(D)*DCOS(L)
     1+ SIN2(P)*(3.0D0*SINQ(D)-1.0D0)
      UZY = -DRATIO*CONST*UZY

      WRITE (IUNIT,300)UX,UY,UZ
300   FORMAT(///,1X30HTIDAL GRAVITY ANOMALIES (GALS),//,
     1 10X2HUX,13X2HUY,13X2HUZ,/,5X,1PD13.6,2(2X,D13.6))
      WRITE (IUNIT,301)UXX,UYX,UYY,UXZ,UZY,UZZ
301   FORMAT(///,1X29HTIDAL GRADIENT MATRIX (SEC-2),//,
     1 10X3HUXX,/,5X1PD13.6,//,10X3HUXY,12X3HUYY,/,5X
     2 2(D13.6,2X),//,10X3HUXZ,12X3HUYZ,12X3HUZZ,/,5X,3(D13.6,2X))

      IF (JULDAY-NDAY) 69,70,70

70    CONTINUE
      IF (NCASE-ICASE) 999,71,71
71    CONTINUE
      TYPE 302
302   FORMAT(//' FINIS')

      STOP
      END
```

APPENDIX B

Method for Calculation of Tidal Stresses Along a Fault Plane.

CALCULATION OF TIDAL STRESS ALONG A FAULT PLANE

The following is a method for resolving the tidal stresses along a fault plane whose normal is denoted by the direction cosines ℓ_1, ℓ_2, and ℓ_3 (x_1, x_2, x_3). The formulation transforms the stress tensor e_{ij} into a new coordinate system which contains e_{ij}'. The stress normal to the plane e_{33} and the azimuthal dependence of the stress within the plane can then be easily computed.

1) Normal to desire plane of calculation has direction cosines ℓ_1, ℓ_2, ℓ_3.

2) Tilt new coordinate system so that x_3' $(\hat{z})$ has directional cosines of ℓ_1, ℓ_2, ℓ_3, i.e.

$$A_{13} = \ell_1$$

$$A_{23} = \ell_2$$

$$A_{33} = \ell_3$$

3) Rotate new axes around z' axis until the angle between

x' and z axis is $\pi/2$ or

$$A_{31} = 0$$

Now $$A_{31}^2 + A_{32}^2 + A_{33}^2 = 1$$

$$A_{32} = \sqrt{1-A_{33}^2}$$

$$A_{13} = A_{21} A_{32} - A_{22} A_{31}$$

$$A_{21} = \frac{A_{13}}{A_{32}} = \frac{A_{13}}{\sqrt{1 - A_{33}^2}}$$

$$A_{11}^2 + A_{21}^2 + A_{31}^2 = 1$$

$$A_{11}^2 = 1 - A_{21}^2$$

$$= \frac{1 - A_{33}^2 - A_{13}^2}{1 - A_{33}^2}$$

$$A_{11} = \sqrt{\frac{1 - A_{33}^2 - A_{13}^2}{1 - A_{33}^2}}$$

$$A_{22} = A_{11} A_{33} - A_{13} A_{31}$$

$$A_{22} = A_{33} \sqrt{\frac{1 - A_{33}^2 - A_{13}^2}{1 - A_{33}^2}}$$

$$A_{12}^2 + A_{22}^2 + A_{32}^2 = 1$$

$$A_{12} = \sqrt{1 - A_{22}^2 - A_{32}^2}$$

$$A_{12}^2 = 1 - A_{33}^2 \left\{ \frac{1 - A_{33}^2 - A_{13}^2}{1 - A_{33}^2} \right\} - A_{13}^2$$

$$A_{12}^2 = \frac{1 - A_{33}^2 - A_{33}^2 - A_{33}^4 - A_{13}^2}{1 \ A_{32}^2}$$

$$A_{12} = \sqrt{\frac{1 - A_{13}^2 - A_{33}^2 \ (1 + A_{33}^2)}{1 - A_{33}^2}}$$

Now we have the complete transformation matrix A_{ij}

$$X'_j = A_{ij} X_i$$

$$X_i = A_{ij} X'_j$$

For the transformation of a second order tensor e_{ij}, the transformation is,

$$e_{ij}' = A_{mi} A_{nj} e_{mn}$$

The tension in any direction (with respect to the new coordinates) with direction cosines α_1, α_2, α_3 will be given by

$$T = \alpha_1^2 \; e_{11} + \alpha_2^2 \; e_{zz} + \alpha_3^2 \; e_{zz} + \alpha_1\alpha_2 \; e_{12} + \alpha_1\alpha_3 \; e_{13} + \alpha_2\alpha_3 \; e_{23}$$

at the surface of the moon because of limiting conditions $e_{13} = e_{23} = 0$.

Along the normal to the defined tilt plane which is X_3' in our new coordinate system, the stress will be,

$$T = e_{33} \; .$$

Finally, stress in the tangential plane of the surface of the moon as a function of azimuth will be,

$$\alpha_3 = 0 \quad , \quad \alpha_2^2 = 1-\alpha_1^2$$

$$T = \alpha_1^2 \; e_{11} + (1-\alpha_1^2) \; e_{22} + \alpha_1 \sqrt{1-\alpha_1^2} \; e_{12}$$

APPENDIX C

Computer Program for Calculating Lunar Tidal Stresses and Strains due to the Earth.

```
C                LUNAR STRESS AND STRAIN FROM EARTH TIDES
C                     M HOUSTON 5/14/70
C
C        THIS PROGRAM CALCULATES THE TIDAL STRESSES AND STRAINS
C         ON THE MOON DUE TO THE EARTH.

         IMPLICIT DOUBLE PRECISION (A-H,L,O-Y)
         DIMENSION JYEAR(0/26),JDAY(4,12),E(3,3),Q(3,3)
         DATA JYEAR(0)/2415019/,(JDAY(1,K),K=1,12)/
     10,31,60,91,121,152,182,213,244,274,305,335/
         COSQ(THETA) = DCOS(THETA)*DCOS(THETA)
         SINQ(THETA) = DSIN(THETA)*DSIN(THETA)
         COS2(THETA)  = DCOS(2.0D0*THETA)
         SIN2(THETA) = DSIN(2.0D0*THETA)

         GMOON = 1.67D2
         EMDIST = 3.84411D10
         RMOON = (2.73D-1)*(6.37123D8)

         TYPE 400
400      FORMAT(/' LUNAR LOVE NUMBERS, H AND L '/)
         ACCEPT 91,HLOVE
         ACCEPT 91,LLOVE
         TYPE 401
401      FORMAT(/' LUNAR COMPRESSIBILITY AND RIGIDITY, LAMBDA MU'/)
         ACCEPT 91,LAMBDA
         ACCEPT 91,LMU

         TYPE 402
402      FORMAT(/' DIRECTION COSINES FOR STRESS DIRECTION '/)
         ACCEPT 91,L1
         ACCEPT 91,L2
         ACCEPT 91,L3
         CONST = (3.0D0)*RMOON*(6.67D-8)*(5.983D27)
         CONST = CONST/((4.0D0)*EMDIST*EMDIST*EMDIST*GMOON)
C
C        JULIAN CALENDAR 4/2/70
C
         DO 1 J=1,26
1        JYEAR(J) = JYEAR(J-1) + 1461
         DO 2 I=2,4
         DO 2 J=1,12
2        JDAY(I,J) = JDAY(I-1,J) + 365
         JDAY(2,1) = JDAY(2,1) + 1
         JDAY(2,2) = JDAY(2,2) + 1
         TYPE 99
99       FORMAT(/' NUMBER OF CASES (I2)'/)
```

```
      NCASE = 0
      ACCEPT 104, ICASF
999   TYPE 100
100   FORMAT(/' DAY,MONTH,YEAR - 3I2'/)
      NCASE = NCASE + 1
      ACCEPT 101,IDAY,IMONTH,IYEAR
101   FORMAT(3I2)
      I1 = IYEAR/4
      I2 = -4*I1+IYEAR+1
      JULDAY = JYEAR(I1) + JDAY(I2,IMONTH) + IDAY
      TYPE 102,JULDAY
102   FORMAT(/' JULIAN CALENDAR DAY =',I15)
      TYPE 103

103   FORMAT(/' NUMBER OF DAYS -1 FOR WHICH RESULTS DESIRED = '/)
      ACCEPT 104,NDAY
104   FORMAT(I2)
      NDAY = NDAY+JULDAY

      TYPE 90
90    FORMAT(//' LUNAR OBSERVATION POINT'//' LATITUDE = '/)
      ACCEPT 91,OBLAT
91    FORMAT(D)
      TYPE 92
92    FORMAT(/' LONGITUDE = '/)
      ACCEPT 91,OBLONG

      TYPE 200
200   FORMAT(/' OUTPUT FILE UNIT DESIGNATION (I2) = '/)
      ACCEPT 104,IUNIT
      TYPE 201
201   FORMAT(/' OUTPUT FILE NAME (A5) = '/)
      ACCEPT 202,ZNAME
202   FORMAT(A5)
      CALL OFILE(IUNIT,ZNAME)
      WRITE (IUNIT,190)OBLAT,OBLONG
190   FORMAT(///,1X35HLUNAR OBSERVATION POINT COORDINATES,//
     1 7X8HLATITUDE,/X9HLONGITUDE,//,5X1PD13.6,2X,D13.6)

      RDEG = 1.745329251994329D-2
      OBLAT = OBLAT*RDEG
      OBLONG = OBLONG*RDEG
      JULDAY = JULDAY - 1
69    JULDAY = JULDAY + 1
```

```
C
C                        LUNAR EPHEMERIS 4/20/70

      T = DBLE(FLOAT(JULDAY-2415019))/3.6525D4
      PI = 3.14159265358979 3D0
      AI = (9.21D1)*PI/((1.8D2)*(6.0D1))

      S = 2.7043659D2 + (((2.0D-6)*T+1.98D-3)*T+4.8126789057D5)*T
      P = 3.3432956D2 + (((-1.0D-5)*T-1.032D-2)*T+4.06903403D3)*T
      H = 2.7969668D2 + ((3.0D-4)*T+3.600076892D4)*T
      RMIN = 2.9088820866572 15D-4
      S = S*RDEG
      P = P*RDEG
      H = H*RDEG


      DRATIO = 1.0D0 + (5.45D-2)*DCOS(S-P) + (2.97D-3)*DCOS((2.0D0)*
     1(S-P)) + (1.002D-2)*DCOS(S-P-2.0D0*(H-P))
     2       + (8.25D-3)*DCOS(2.0D0*(S-H))

      PS = 2.8122083D2 + (((3.0D-6)*T+4.5D-4)*T+1.71902D0)*T
      PS = PS*RDEG

      S1 = (3.77D2)*DSIN(S-P) + (1.3D1)*DSIN((2.0D0)*(S-P))
     1+ (7.6D1)*DSIN(S-P-(2.0D0)*(H-P)) - (1.1D1)*DSIN(H-PS)
     2+ (4.0D1)*DSIN((2.0D0)*(S-H))

      S1 = S + S1*RMIN


      AN = 2.5918328D2 + (((2.0D-6)*T+2.08D-3)*T-1.93414201D3)*T
      AN = AN*RDEG
      A = DSIN(AI)*DCOS(S1-AN)
      AMU = (1.03D-2)*DSIN((2.0D0)*(S1-AN))*RDEG


      BETA = (3.08D2)*DSIN(S-AN) + (1.7D1)*DSIN(2.0D0*S-P-AN)
     1+ (1.0D1)*DSIN((2.0D0)*(S-H)-S+AN) - (1.7D1)*DSIN(P-AN)

      BETA = BETA*RMIN
      TANB = -DSIN(AI)*DSIN(S1-AN)/DCOS(AI)
      ELAT = (DATAN(TANB)-BETA)
      ELONG = (S1-S+AMU+A*ELAT)

      TANB = -DSIN(AI)*DSIN(S1-AN)/DCOS(AI)
      ELAT = (DATAN(TANB)-BETA)
      ELONG = (S1-S+AMU+A*ELAT)
```

```
      E1 = ELAT/RDEG
      E2 = ELONG/RDEG
      WRITE (IUNIT,203)JULDAY
203   FORMAT(/////1X46HEARTH SELENOGRAPHIC COORDINATES FOR JULIAN DAY,
     I10)
      WRITE (IUNIT,204)E1,E2,DRATIO
204   FORMAT(//,7X8HLATITUDE,7X9HLONGITUDE,6X8HPARALLAX,
     1//,5X,1PD13.6,2(2X,D13.6))

C                STRESS AND STRAIN CALCULATIONS 7/21/70

      DRATIO = DRATIO*DRATIO*DRATIO
      P = OBLAT
      D = ELAT
      L = ELONG - OBLONG

      E(3,3) = COSQ(P)*COSQ(D)*COS2(L)+SIN2(P)*SIN2(D)*DCOS(L)
     1+ ((3.0D0)*SINQ(P)-1.0D0)*(SINQ(D)-1.0D0/3.0D0)
      CUBIC = (4.0D0*HLOVE-6.0D0*LLOVE)*CONST*E(3,3)*DRATIO
      E(3,3) = (2.0D0)*HLOVE*CONST*E(3,3)*DRATIO

      E(2,2) = 2.0D0*LLOVE*(-SIN2(P)*COS2(P)*(COSQ(D)*COS2(L)
     1 +2.0D0*COS2(D)*DCOS(L))+3.0D0*SIN2(P)*SINQ(D)-SIN2(P))
     2 +HLOVE*(COSQ(P)*COSQ(D)*COS2(L)+SIN2(P)*SIN2(D)*DCOS(L)
     3 +(3.0D0*SINQ(P)-1.0D0)*(SINQ(D)-1.0D0/3.0D0))
      E(2,2) = CONST*E(2,2)*DRATIO

      E(1,1) = -(LLOVE/COSQ(P))*(COSQ(P)*COSQ(D)*COS2(L)
     1 +SIN2(P)*SIN2(D)*DCOS(L))-(LLOVE*DSIN(P)/DCOS(P))*
     2 (SIN2(P)*COSQ(D)*COS2(L)+2.0D0*COS2(P)*SIN2(D)*DCOS(L)
     3 +3.0D0*SIN2(P)*SINQ(D)-SIN2(P)) + HLOVE*(COSQ(P)*COSQ(D)*COS2(L)
     4 +SIN2(P)*SIN2(D)*DCOS(L)+(3.0D0*SINQ(P)-1.0D0)*(SINQ(D)
     5 -1.0D0/3.0D0))
      E(1,1) = CONST*E(1,1)*DRATIO

      E(1,2) = 4.0D0*(DSIN(P)*COSQ(D)*SIN2(L)-DCOS(P)*SIN2(D)*DSIN(L))
     1 +SIN2(D)*DSIN(L)/DCOS(P)
      E(1,2) = LLOVE*CONST*E(1,2)*DRATIO

      E(1,3) = 2.0D0*HLOVE*(-DCOS(P)*COSQ(D)*SIN2(L)
     1 -5.0D-1*DSIN(P)*SIN2(D)*DSIN(L)
     2 +COSQ(P)*COSQ(D)*COS2(L)+SIN2(P)*SIN2(D)*DCOS(L)
     3 +(3.0D0*SINQ(P)-1.0D0)*(SINQ(D)-1.0D0/3.0D0))
     4 + LLOVE*(DCOS(P)*COSQ(D)*SIN2(L)
     5 + 2.0D0*DSIN(P)*SIN2(D)*DSIN(L))
      E(1,3) = CONST*E(1,3)*DRATIO

      E(2,3) = (HLOVE+LLOVE)*(-SIN2(P)*COSQ(D)*COS2(L)
```

```
     1 +2.0D0*COS2(P)*SIN2(D)*DCOS(L)
     2 +3.0D0*SIN2(P)*SINQ(D)-SIN2(P))
       E(2,3) = CONST*E(2,3)*DRATIO

       WRITE (IUNIT,403)E(1,1),E(1,2),E(2,2),E(1,3),E(2,3),E(3,3)
403    FORMAT(///,1X19HTIDAL STRAIN TENSOR ,//,
     1 10X3HEXX,/,5X1PD13.6,//,10X3HEXY,12X3HEYY,/,5X
     2 2(D13.6,2X),//,10X3HEXZ,12X3HEYZ,12X3HEZZ,/,5X,3(D13.6,2X))

C         TIDAL STRESS CALCULATION

       DO 450 I=1,3
450    Q(I,I) = LAMBDA*E(I,I) + 2.0D0*LMU*E(I,I)
       Q(1,2) = 2.0D0*LMU*E(1,2)
       Q(2,3) = 2.0D0*LMU*E(2,3)
       Q(1,3) = 2.0D0*LMU*E(1,3)

       WRITE (IUNIT,404)Q(1,1),Q(1,2),Q(2,2),Q(1,3),Q(2,3),Q(3,3)
404    FORMAT(///,1X19HTIDAL STRESS TENSOR ,//,
     1 10X3HTXX,/,5X1PD13.6,//,10X3HTXY,12X3HTYY,/,5X
     2 2(D13.6,2X),//,10X3HTXZ,12X3HTYZ,12X3HTZZ,/,5X3(D13.6,2X))

       WRITE (IUNIT,405)CUBIC
405    FORMAT(//,' CUBIC DILATION = ',1PD13.6)

       TD = L1*L1*Q(1,1)+L2*L2*Q(2,2)+L3*L3*Q(3,3)+L1*L2*Q(1,2)

C        NOTE THAT BECAUSE OF LIMITING CONDITIONS, NEAR THE SURFACE
C        OF THE MOON, T(1,3) = T(2,3) = 0


       WRITE (IUNIT,406)TD,L1,L2,L3
406    FORMAT(//,1X6HSTRESS ,9X17HDIRECTION COSINES ,/2(1PD13.6,2X),
     1 2(/15X,D13.6))

       IF (JULDAY-NDAY) 69,70,70

70     CONTINUE
       END FILE IUNIT
       IF (NCASE-ICASE) 999,71,71
71     CONTINUE
       TYPE 302
302    FORMAT(//' FINIS')


       STOP
       END
```