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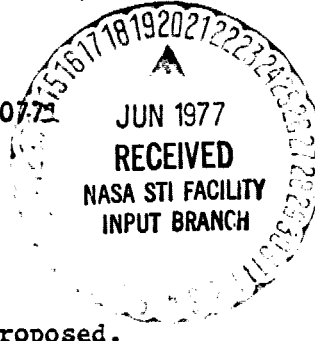
SIMILARITY LAWS OF LUNAR AND TERRESTRIAL VOLCANIC FLOWS\*

S. I. Pai and Y. Hsu

Institute for Physical Science and Technology and Aerospace Engineering  
Department, University of Maryland, College Park Maryland 20742, and

John A. O'Keefe

Goddard Space Flight Center, Greenbelt, Maryland 20771



ABSTRACT

A mathematical model for the volcanic flow in planets is proposed.

This mathematical model, which is one-dimensional, steady duct flow of a mixture of a gas and small solid particles (rock), has been analyzed. We apply this model to the lunar and the terrestrial volcanic flows under geometrically and dynamically similar conditions. Numerical results for the equilibrium two-phase flows of lunar and terrestrial volcanoes under similar conditions are presented. The main results of our theoretical model are (1) the lunar crater is much larger than the corresponding terrestrial crater, (2) the exit velocity from the lunar volcanic flow may be higher than the lunar escape velocity but the exit velocity of terrestrial volcanic flow is much less than that of the lunar case. This result supports the hypothesis that Australian tektites came from the moon, as a stream of a mixture of rock and gas of extremely high speed and (3) the thermal effects on the lunar volcanic flow are much larger than those of the terrestrial case.

\*This research is supported in part by the National Aeronautics and Space Administration under grant No. NSG-5083 to the University of Maryland, Computer time was supported by NASA grant under NSG-398 to the Computer Science Center of the University of Maryland.

N77-26049

Unclas  
34863

G3/91

NASA-TN-X-72610) SIMILARITY LAWS OF LUNAR  
AND TERRESTRIAL VOLCANIC FLOWS (NASA) 36 P  
HC A03/MP A02 CACL 03E

## I. INTRODUCTION

There is a real need for physical and mathematical models of geologic processes, especially in conjunction with planetological studies. The underlying reason is that we cannot observe planetological processes in action. This fact cuts us off from the traditional geologic method of explaining the structures of rocks in terms of processes which we see in operation. It is dangerous to try to explain planetary structures by comparing them with similar terrestrial structures, because we are extrapolating to very different physical conditions. For instance, lunar gravity is 1/6 of terrestrial; atmospheric pressure is  $10^{-12}$  of the earth's; water content of the rock, undetectable; and oxygen fugacity (at magmatic temperatures)  $10^{-6}$  of terrestrial. The extrapolation is so great that it is hard to be sure that a given lunar structure was produced by the same kind of process as an apparently similar terrestrial structure. In particular, in view of the great differences in physical parameters, it is thoroughly unscientific to assume that the nearest analog of a terrestrial structure will be a lunar structure of about the same size.

To some extent, it may be possible to replace the geological principle of uniformitarianism by a planetological principle of homology; we seek to explain planetary structures by comparison with terrestrial structures, after allowance for scaling. There is a famous rule, called the pi-theorem (P.W. Bridgeman, 1922) which states that a physical law can be stated in terms of one or several non-dimensional parameters, each a product of a number of physical variables. If these numbers have the same value in two objects, one on planet A and the other on planet B, both produced by the same process, then it is reasonable to try to explain what is

seen in B in terms of what is known about A. These two cases are in dynamical similarity.

One of the most important problems of planetology is to understand the circular depressions which are the most conspicuous features of the surface of the moon, and the surfaces of most of the planets and satellites which have been examined. These depressions are all called craters; but it is important to realize that the use of one name does not mean that they are all the same kind of object on the moon or the planets, any more than on the earth, where some are due to impact and others to volcanism.

J.F. McCauley (1967) has furnished a useful indication of the truth of this observation, by drawing attention to the craters Maunder (Fig. 1) and Kopff (Fig. 2) in Mare Orientale. The two craters are of roughly the same size (Maunder 55 km, Kopff 40 km) but they are fundamentally different in structure. Kopff has a rim which is polygonal in plan and  $\Delta$ -shaped in cross-section, while the wall of Maunder is circular in plan and much deeper on the inside than the outside; Kopff has a smooth surrounding surface, while that around Maunder is hummocky; Kopff is shallow while Maunder is deep. The point of McCauley's comparison is that both are on Mare Orientale, which has a peculiar, wavy surface, clearly not degraded by any erosional process. The survival of this peculiar surface guarantees that neither crater can have been seriously modified by erosion; both must have been formed in about their observed shape. It seems to follow that the moon has at least two radically different way of forming craters. Since all impact craters tend to be similar, it follows that if Maunder is an impact crater, then Kopff is not. This example is important because it indicates that the moon has large craters of volcanic origin. The explosion craters considered in this paper are probably different from Kopff in

morphology.

Similarly, W.S. Cameron and S.L. Padgett (1974) have drawn attention to regional variations in the frequency of what they call ringdikes, i.e., craters more or less like Kopff. By a similar argument, C.R. Chapman et al. (1971) find that the most subtelescopic craters on the maria are of internal origin, because they are ten times as numerous as the subtelescopic highland craters, whereas telescopic mare craters are one tenth as numerous as telescopic highland craters.

In addition, the lunar origin of tektites, which seems especially probable for Australian and Javanese tektites, their flanges and ring-waves, seems to imply explosive lunar volcanism (O'Keefe, 1976); the argument rests in part on studies of australite ablation, especially those by D.R. Chapman and H.K. Larson (1963). The calculations have been thoroughly checked by independent computations; they seem to rule out any non-lunar origin of tektites. The line of argument is supported by calculations of the size of the crater which would be needed for a terrestrial origin, and of the time needed to produce glass pieces of the necessary size, density and homogeneity, as well as of the probability of hitting compositions as unusual as those of some of the components of the tektite stream fields.

On the other hand, in an elegant morphometric study, R.J. Pike (1974) showed that most lunar craters resemble in shape terrestrial explosion pits, rather than terrestrial volcanoes. The exceptions are the maar-type craters and the tuff rings which are terrestrial, but are more like typical lunar craters than like the majority of terrestrial volcanoes. Pike rejects the the maars and tuff rings, because the maximum size of these objects is about 5 km, while lunar craters range up to 250 to 300 km. Obviously the question

of scale is crucial here.

It thus appears useful to calculate the physics of eruptive volcanism, with the earth and the moon as examples. (Martian eruptive volcanism will then represent an intermediate case.)

In comparing lunar and terrestrial volcanism, we note that water, which is the principal driving gas in a terrestrial volcano, will be less important than hydrogen in a lunar volcano. This is a direct consequence of the low oxygen fugacity in a lunar lava. The equilibrium is governed by the usual relation

$$\frac{(H_2O)}{(H_2) (O_2)^{1/2}} = K \quad (1)$$

where  $(H_2O)$  is the partial pressure of steam, etc., and  $K$  is the equilibrium constant; from the JANAF tables (Stull and Prophet, 1971) we find, at  $1500^\circ\text{C}$   $\log_{10} K = 3.225$  if pressure is in  $\text{N m}^{-2}$  (Newtons per square meter). The fugacity of oxygen in lunar lava at this temperature is known to be about  $10^{-13} \text{ N m}^{-2}$ ; thus is the is the partial pressure of oxygen in contact with the lava. Substituting this in Eq. (1), we find that the ratio of the partial pressures of water and hydrogen is 0.17. Hence in our mathematical model, we consider steam as the gas in terrestrial volcano while hydrogen as the gas in lunar volcano.

## II. MATHEMATICAL MODEL OF VOLCANIC FLOW

To explain the mechanism of volcanic eruption, whether lunar or terrestrial, we may consider the volcano as a two-phase system consisting of the solid particles and a gas. We analyze the one-dimensional flow due to the eruption of such a two-phase system. From the field observation (G. Fielder,

1971) and some preliminary theoretical calculations [T.R. McGetchin and G.W. Ullrich, 1973], the general volcanic flow field is sketched in Fig. 3.

The general flow field may be divided into two regions: the lower part, region I, starts from a great depth in the planet; region I has a height  $H_1$  of the order of 100 km in terrestrial case. At the base, due to chemical reaction, the rock is melting at a temperature of the order of 100 °C. The original pressure is very large [T.R. McGetchin and G.W. Ullrich, 1973]. As a result, a mixture of molten rock and gas flows upward in a duct of almost constant cross-sectional area. In this region, since the pressure is very high, the mixture of the molten rock and the gas behaves like an incompressible fluid. From fluid dynamic point of view, the flow problem is very simple. The flow field should not deviate much from the simple analysis by T.R. McGetchin and G.W. Ullrich [1973]. We shall not discuss it any more.

When the flow is near the surface of the planet, with  $x \geq -H_0$ , where  $x$  is the vertical coordinate, the area of the duct increases and the mixture expands as it flows upward. We call this region as region II, in which the flow field is very complicated. In this paper, we study the essential features in region II of lunar and terrestrial volcanic flows. We are especially interested in the similar flow patterns of a terrestrial volcanic flow with a corresponding lunar volcanic flow.

### III. FUNDAMENTAL EQUATIONS OF VOLCANIC FLOWS AND THE IMPORTANT NON-DIMENSIONAL PARAMETERS.

Since the flow of the mixture comes a long way from the great depth of the planet to the initial section  $x = -H_0$  of region II, it should reach the equilibrium condition such that the initial velocity of the rock and that

of the gas are approximately equal and that the temperatures of the rock and the gas are also equal. It is known as equilibrium flow (S.I. Pai, 1977). In order to get some essential features of the volcanic flow in region II, we assume that the flow in region II remains as equilibrium flow first. We shall investigate the non-equilibrium effects on the flow field in region II in a later paper.

For equilibrium flow of the two-phase system in region II, we consider the following variables:

- velocity of the mixture  $u$
- temperature of the mixture  $T$
- pressure of the mixture  $p$ , and
- density of the mixture  $\rho$

The density of the rock  $\rho_r$  is assumed to be constant and the density of the mixture depends on the density of the gas  $\rho_g$  and the volume fraction of the rock in the mixture  $Z$ .

The values of the variables at the initial section  $x = -H_0$  of cross-sectional area  $A_0$  are given from the calculation of region I as follows:

$$p = p_0, \quad u = u_0, \quad T = T_0, \quad \rho = \rho_0. \quad (2)$$

Since we are interested in similar flows in the lunar and the terrestrial case, the relations between the initial values of these two cases may be obtained by similarity considerations as we shall show in section IV.

We define the non-dimensional quantities (with a bar) as follows;



$$\bar{p} = \frac{p}{p_0}, \quad \bar{T} = \frac{T}{T_0}, \quad \bar{\rho} = \frac{\rho}{\rho_0}, \quad \bar{u} = \frac{u}{u_0}, \quad \bar{x} = \frac{x}{L}, \quad \bar{A} = \frac{A}{A_0}$$

$$\bar{a} = \frac{a}{a_0}, \quad \bar{a}_M = \frac{a_M}{a_{M0}}, \quad (3)$$

where  $L$  is the characteristic length of the flow field of region II which may be taken as  $H_0$ .

The sound speed of the gas  $a$  is given by the following formula:

$$a = \sqrt{\gamma \frac{R_u}{m} T} \quad (4)$$

where  $\gamma$  is the ratio of specific heats  $c_p/c_v$ ,  $R_u$  is the universal gas constant and  $m$  is the molecular weight of the gas.

The non-dimensional equations for one-dimensional steady equilibrium flows of a two phase mixture of solid particles and a gas (S.I. Pai, 1977) are as follows:

(i) Equation of state

$$\bar{p} = \frac{1 - Z_0}{1 - Z} \bar{\rho} \bar{T} \quad (5)$$

where subscript  $0$  refers to values at initial section  $x = -H_0$ .

(ii) Equation of continuity

$$\bar{\rho} \bar{u} \bar{A} = 1 \quad (6)$$

(iii) Equation of motion

$$\frac{d\bar{u}}{d\bar{x}} = \frac{\bar{u}}{\bar{a}_M^2 - M_0^2 \bar{u}^2} \left( -\frac{\bar{a}_M^{-2}}{\bar{A}} \frac{d\bar{A}}{d\bar{x}} + \frac{M_0^2}{Fr_0} + 2c_f M_0^2 \frac{\bar{u}^2}{\bar{D}} \right) \quad (7)$$

where the sound speed of the mixture  $a_M$  is given by the formula:

$$a_M^2 = \frac{\Gamma(1 - k_p)}{(1 - Z)^2} \left( \frac{R_u}{m} \right) T \quad (8)$$

where

$$k_p = \frac{Z\rho_f}{\rho} \quad (9)$$

is the mass concentration of the solid particles in the mixture and  $\Gamma$  is the effective ratio of specific heats of the mixture, i.e.,

$$\Gamma = \gamma \frac{\left(1 + \frac{\eta\delta}{\gamma}\right)}{1 + \eta\delta} \quad (10)$$

where  $\eta = 1 - k_p/k_p$ ,  $\delta = c_s/c_v$ ,  $c_s$  is the specific heat of the solid particles,  $c_v$  is the specific heat of the gas at constant volume and  $c_p$  is the specific heat of the gas at constant pressure.

The initial Mach number  $M_0$  is defined as follows:

$$M_0 = \frac{u_0}{a_0} \quad (11)$$

and the initial Froude number  $Fr_0$  is defined as follows:

$$Fr_0 = \frac{u_0^2}{gL} \quad (12)$$

where  $g$  is the gravitational acceleration of the planet. The friction coefficient of the duct is  $c_f$  and  $\bar{D} = D/L$  where  $D$  is the hydraulic diameter,  $A(x)$  is the cross-sectional area of the duct in region II.

(iv) The energy equation:

$$\begin{aligned}
& [b_1(1-Z)^2 + b_4 \frac{(1-Z)}{\bar{u}\bar{A}} + b_3(1-Z)^2] \bar{a}_M^{-2} + \frac{1}{2} M_o^2 \bar{u}^{-2} + \frac{\bar{M}_o}{Fr_o} \bar{x} - \\
& - \sum_{n=0}^n 2c_f M_o^2 \frac{\bar{u}_{av}}{D_{av}} (\bar{x}_{n+1} - \bar{x}_n) = \quad (13) \\
& = \left[ \frac{k_p}{\Gamma_o} \left( \frac{\delta_o}{\gamma_o - 1} \right) \frac{(1-Z_o)^2}{(1-k_p)} + \frac{k_p}{G_o} \frac{\rho_o}{\rho_{go}} \frac{(1-Z_o)}{\Gamma_o} + \frac{\gamma_o}{(\gamma_o - 1)} \frac{(1-Z_o)^2}{\Gamma_o} \right] + \frac{1}{2} M_o^2 - \frac{M_o^2}{Fr_o}
\end{aligned}$$

where

$$\left. \begin{aligned}
b_1 &= \frac{k_p}{(1-k_p)} \frac{\delta}{(\gamma-1)\Gamma} \\
b_3 &= \left( \frac{\gamma}{\gamma-1} \right) \frac{1}{\Gamma} \\
b_4 &= \frac{k_p}{\Gamma} \frac{1}{C_o} \frac{\rho_o}{\rho_{go}}
\end{aligned} \right\} \quad (14)$$

$$G_o = \frac{\rho_r}{\rho_{go}} \quad (15)$$

is the initial density ratio and  $D_{av}$  is the average hydraulic diameter between two steps and  $u_{av}$  is the average velocity between two steps (Y. Hsu, 1976).

From the non-dimensional equations (5), (6), (7) and (13), we have four important non-dimensional parameters:  $k_p$  (9);  $M_o$  (11);  $Fr_o$  (12) and  $G$  (15), which characterize the volcanic flow. The significance of these parameters is as follows.

The mass concentration  $k_p$  indicates the mass fraction of the solid particles in the mixture, which is important in the determination of the velocity of the mixture.

The initial density ratio  $G$  indicates the relative importance of the gas and the solid particles in the mixture.

The Mach number  $M_0$  is a measure of the compressibility effect of the mixture due to high speed.

The initial Froude number  $Fr_0$  indicates the effect of gravitational force on the flow field.

#### IV. SIMILARITY LAWS OF LUNAR AND TERRESTRIAL VOLCANIC FLOWS

We would like to compare a terrestrial volcanic flow with a similar lunar volcanic flow. For the similitude of these two volcanic flows, we should have both geometrical similitude and dynamic similitude (L.I. Sedov, 1959).

By geometrical similitude, we mean that there is a linear scaling of the region II of the terrestrial and the lunar volcanoes. We should compare these volcanic flow fields with the same non-dimensional initial area  $\bar{A}_0$  and same slope of the non-dimensional variation of the cross-sectional area in region II, i.e., the same  $d\bar{A}/d\bar{x}$ . These two volcanoes are geometrically similar. It should be noted that the ratio of the exit area  $A_g$  to the initial area  $A_0$  for these two volcanoes are not the same because we have to satisfy the conditions of dynamic similarity.

By dynamic similitude, we mean that the most important non-dimensional parameters of the fundamental equations of the volcanic flows for these two cases should be equal. In other words, for dynamic similitude, the non-dimensional parameters:  $M_0$ ,  $G$ ,  $Fr_0$  and  $k_p$  for the similar terrestrial and lunar volcanic flows should be equal. From these conditions, we may determine the corresponding size and initial values of these two similar volcanic flows as follows.

We assume that in the terrestrial volcanic flow, the gas is steam while in the lunar volcanic flow the gas is hydrogen as we have discussed in section I. We have the following values of various characteristic quantities in two geometrically and dynamically similar terrestrial and lunar volcanic flows:

(i) For geometrical similitude, we have in region II

$$\bar{A}_{om} = \bar{A}_{oe} = 1, \quad \left( \frac{d\bar{A}}{d\bar{x}} \right)_m = \left( \frac{d\bar{A}}{d\bar{x}} \right)_e \quad (16)$$

where subscript o refers to the value at the initial section  $x = -H_0$  or  $\bar{x} = -1$ , subscript m refers to the value for lunar volcanic flow while subscript e refers to the value for terrestrial volcanic flow. We compare two similar volcanic flows that satisfy Eq. (16).

(ii) We assume that the initial temperature  $T_0$  is the same in these two similar terrestrial and lunar volcanic flows, i.e.,

$$T_{om} = T_{oe} = 1,000 \text{ } ^\circ\text{C} \quad (17)$$

(iii) In the terrestrial volcanic flow, the principal gas is steam while that in the lunar volcano is hydrogen. Hence at  $T_0 = 1000^\circ\text{C}$ , we have

$$m_e = 18.016; \quad c_{pe} = 0.593 \text{ cal/gm } ^\circ\text{K}; \quad \gamma_e = \frac{c_{pe}}{c_{ve}} = 1.23 \text{ (for steam)} \quad (18a)$$

$$m_m = 2.016; \quad c_{pm} = 3.721 \text{ cal/gm } ^\circ\text{K}; \quad \gamma_m = \frac{c_{pm}}{c_{vm}} = 1.36 \text{ (for hydrogen)}. \quad (18b)$$

The ratio of the initial sound speed in these two cases is

$$\frac{a_{om}}{a_{oe}} = \sqrt{\frac{\gamma_{me}}{\gamma_{em}}} = 3.126 \quad (19)$$

(iv) By dynamical similarity, the initial Mach number of these two similar flows should be the same, i.e.,

$$M_{om} = \frac{u_{om}}{a_{om}} = M_{oe} = \frac{u_{oe}}{a_{oe}} \quad . \quad (20)$$

Hence

$$u_{om} = 3.126 u_{oe} \quad . \quad (21)$$

For dynamical similarity, the initial value of velocity in the lunar case should be 3.126 times that of the terrestrial case.

(v) By dynamical similarity, the density ratio  $G_o$  of these two similar flows should be the same, i.e.,

$$G_{om} = \frac{\rho_r}{\rho_{gom}} = G_{oe} = \frac{\rho_r}{\rho_{goe}} \quad . \quad (22)$$

Hence

$$\rho_{gom} = \rho_{goe} \quad . \quad (23)$$

because we assume that the rock densities  $\rho_r$  in these two cases are the same. Since the initial density of the gas is related to the initial pressure of the mixture by the following formula

$$\rho_{go} = \frac{p_o}{\frac{Ru}{m} T_o} \quad (24)$$

we have the relation of the two initial pressures for those two similar volcanic flows as follows.

$$p_{om} = \frac{m_e}{m_m} p_{oe} \approx 9 p_{oe} \quad . \quad (25)$$

If we assume that the pressures in the reservoirs are the same and because the smaller gravitational forces, the drop in pressure as the flow ascends must be smaller in the lunar case. Hence it is possible that  $p_{om}$  is larger than  $p_{oe}$ .

(vi) By dynamical similarity, the initial Froude numbers  $Fr_o$  of these two similar flows should be the same, i.e.,

$$Fr_{om} = \frac{u_{om}^2}{g_m L_m} = Fr_{oe} = \frac{u_{oe}^2}{g_e L_e} \quad (26)$$

Hence for dynamic similarity, the ratio of the two characteristic lengths of these two similar volcanic flows are

$$\frac{L_m}{L_e} = \frac{u_{om}^2}{u_{oe}^2} \frac{g_e}{g_m} \approx 60 \quad (27)$$

We shall take  $L = H_o$ .

(vii) By dynamic similarity, the  $k_p$  should be the same in these two similar volcanic flows. In our number calculations, we calculate the flow field corresponding to various values of  $k_p$  and then compare the terrestrial and lunar volcanic flow with the same  $k_p$ .

For simplicity, in our numerical calculations, we shall assume that

$$\frac{d\bar{A}}{d\bar{x}} = K = \text{constant} \quad (28)$$

We calculate the cases for  $K = 1, 3, \text{ and } 5$  and the corresponding ratio of the exit area  $A_s$  to the initial area  $A_o$  are

$$\begin{aligned} K &= 1, 3, 5 \\ (A_s/A_o)_e &= 2, 4, 6 \\ (A_s/A_o)_m &= 61, 181, 301 \end{aligned}$$

Since we are mainly interested in the flow variables at the exit of the volcano, the ratio  $A_g/A_o$  is more important than the slope  $dA/dx$  which determines the flow inside the duct of the volcano. It is interesting to notice that the exit area, the crater area of the lunar case, is about 30 to 50 times that of the terrestrial crater if the initial areas of the two volcanoes  $A_o$  are the same.

#### V. NUMERICAL SOLUTIONS FOR SIMILAR LUNAR AND TERRESTRIAL EQUILIBRIUM VOLCANIC FLOWS

We shall show later that the viscous effects in region II are negligible (see Fig. 14). Hence in our numerical calculations, we consider the equilibrium inviscid flow only so that the friction terms in Eqs. (7) and (13) are neglected.

The initial conditions in our numerical calculations are:

$$T_o = 1,000^\circ\text{C} , M_o = 1.0 , c_s = 0.3 \text{ cal/gm}^\circ\text{K} , \rho_r = 3.30 \text{ gr/cc}$$

$$p_{oe} = 100 \text{ atmospheres} , L_e = H_{oe} = 1 \text{ km}$$

$$p_{om} = 900 \text{ atmospheres} , L_m = H_{om} = 60 \text{ km}$$

and the values for the gases are given in Eq. (18).

We have calculated two cases.

##### (1) Isothermal case

In this case, we assume that the temperature  $T$  of the mixture remains at its initial temperature  $T_o$ . This is physically possible because when the mixture moves upward, some of the molten rock may be solidified and release sufficient heat so that the resultant temperature of the mixture remains unchanged during the region II. Furthermore, from



our previous experience in ash flow calculations, we find that the isothermal approximation is a good approximation for the terrestrial case (Pai, et al. 1972). We would like to analyze this simpler case first.

(2) Adiabatic case

In this case, we assume that there is no heat transfer between the flow and the wall of the duct, nor viscous dissipation. The temperature of the mixture is not a constant in region II but decreases according to the isentropic process as it ascends.

Detailed numerical results are given in the reference by Y. Hsu (1976). Here we just give some typical results in Figs. 4 to 14 to show the similarities and the essential differences between similar lunar and terrestrial volcanic flows.

In Fig. 4, we compare the actual velocity in the isothermal similar lunar and terrestrial volcanic flows with the mass concentration  $k_p$  as a parameter and the corresponding area ratios  $(A_s/A_o)_e = 4$  and  $(A_s/A_o)_m = 181$ . The lunar crater area is about 45 times that of the similar terrestrial one in this case. The velocity in all cases increases as  $k_p$  decreases. The velocity in the lunar case is much larger than the corresponding value in the terrestrial case. When  $k_p$  is equal to or less than 0.8, the exit velocity of the lunar volcanic flow is larger than the escape velocity on the moon which is 2,372 m/sec.

Fig. 5 shows the effect of the area ratio  $(A_s/A_o)$  on the velocity distribution in the isothermal case. When  $A_s/A_o$  increases, the velocity increases too, but the effect of  $(A_s/A_o)$  on the velocity is smaller than that due to  $k_p$ . In the lunar case, the velocity increases rapidly in the first part of the duct and remains almost constant for the portion of the duct.

Fig. 6 shows the effect of  $k_p$  and  $(A_s/A_o)$  on the pressure

distributions for the isothermal terrestrial volcanic flow. The influence of  $k_p$  on the pressure distribution is small, particularly when  $(A_s/A_o)$  is large.

Fig 7 shows the effect of  $k_p$  and  $(A_s/A_o)$  on the pressure distributions for the isothermal lunar volcanic flow. The influence of  $k_p$  on the pressure distributions is negligible.

Figs.8 to 13 show the results of adiabatic volcanic flows. In Fig. 8, the velocity distributions of the similar lunar and terrestrial volcanic flows with corresponding area ratio  $(A_s/A_o)_e = 4$ , and  $(A_s/A_o)_m = 181$  are given with  $k_p$  as a parameter. In general, these velocity distributions are similar to those of isothermal case (Fig. 4) but the velocity is less than the corresponding value of the isothermal case. The initial rate of increase of velocity of the lunar case is larger than that of the isothermal case. For small value of  $k_p$ , the velocity of the lunar volcanic flow will reach its maximum possible value at a short distance above the initial section. From that point up, the lunar volcanic flow will be a jet of a given width less than the width of the duct moving with the constant velocity of the value of maximum possible value.

Fig. 9 shows the temperature distributions in the terrestrial adiabatic volcanic flow. The temperature drops very fast initially, especially when  $k_p$  is small. In the major portion of the duct, however, the temperature is almost constant but at a smaller value than the initial temperature.

Fig. 10 shows the effects of  $k_p$  and  $(A_s/A_o)$  on the temperature distributions of the adiabatic terrestrial volcanic flow. For large  $k_p$  ( $> 0.8$ ), the isothermal approximation is a good one for terrestrial volcanic flow.

Fig. 11 shows the effects of  $k_p$  and  $(A_s/A_o)$  on the temperature

distributions of the lunar adiabatic volcanic flow. The thermal effect on the lunar volcanic flow is larger than that of terrestrial flow. The isothermal approximation is not good for lunar volcanic flow. The temperature of the lunar adiabatic flow drops very fast initially, especially when  $k_p$  is small. When  $k_p$  is less than 0.5, the condition of maximum possible velocity and zero temperature will be reached at a short distance from the initial section.

Fig. 12 shows the effects of  $k_p$  and  $(A_s/A_o)$  on the pressure distributions for the adiabatic terrestrial volcanic flows while Fig. 13 shows the corresponding pressure distributions of similar lunar adiabatic volcanic flow. The pressure drops initially very fast when  $k_p$  is small but remains almost constant in the major portion of the duct. The pressure in the lunar case is much smaller than the corresponding value in the lunar case even though the initial pressure of the lunar case is larger.

Finally, Fig 14 shows the effect of friction on the velocity distribution on the isothermal terrestrial volcanic flow. The effect of friction is negligible.

## VI. SUMMARY AND CONCLUSIONS

From our theoretical study and numerical results, the following conclusions may be drawn.

(1) The flow fields of similar terrestrial and lunar volcanoes have been calculated by the two-phase flow theory for a mixture of a gas and solid particles, considering both isothermal and adiabatic equilibrium cases.

(2) The important non-dimensional parameters which characterize the volcanic flow are (i) the Mach number (ii) the Froude number, (iii) the

density ratio between the rock and the gas and (iv) the mass concentration of the rock in the volcanic flow.

(3) By dynamical similarity, the above four non-dimensional parameters should be the same. If hydrogen powers lunar volcanoes, this suggests that terrestrial volcanoes with steam as the gas are to be compared (other things being equal) with lunar volcanoes 60 times larger in area. Hence the morphological similarity found by Pike (1974) between terrestrial maars and tuff rings, on the one hand, and the commonest kind of large lunar crater on the other is not necessarily invalid, despite the fact that the lunar craters are 50 times larger than the maars and tuff rings.

(4) In the terrestrial cases, the exit velocity from the volcano is supersonic, but its value is much smaller than the terrestrial escape velocity.

(5) In the lunar cases, the exit velocity from the volcano may be higher than the lunar escape velocity when the mass concentration of the rock is less than  $k_p = 0.8$ .

(6) In the adiabatic case, the lunar volcanic flow may reach its maximum possible velocity before the exit and the volcanic flow will then leave the volcano as a jet stream with velocity greater than the lunar escape velocity. Thus the flow field agrees with the assumed flow pattern of a stream of tektites coming from the moon (D.R. Chapman and H.K. Larson, 1963).

(7) When  $k_p$  is large and  $(A_g/A_o)$  is small, the temperature in the terrestrial cases is quite close to its initial value. When  $k_p$  is small, the initial drop of the temperature in both the terrestrial and the lunar cases is large; in the major portion of the duct, the temperature is almost constant but is less than the initial temperature.

(8) The variation of both the velocity and the pressure in the terrestrial cases is gradual. For the lunar cases, the pressure drops very rapidly to a very low value initially and then remains almost constant for the major portion of the duct. The variation of the pressure with  $k_p$  is always negligibly small.

#### VII. REFERENCES

- Bridgeman, P.W. Dimensional Analysis, Yale University Press, New Haven, Conn. 1922.
- Cameron, W. S. and J.L. Padgett, Possible lunar ring dikes, The Moon, 9, 249-294, 1974.
- Chapman, C.R., J.H. Mosher and G. Simmons, Lunar cratering and erosion from Orbiter 5 photographs, J. Geophys. Res. 75, 1445-1466, 1970.
- Chapman, D.R. and H.K. Larson, The lunar origin of tektites, J. Geophys. Res. 68, 4305-4358, 1963.
- Fielder, G., (Ed.) Geology and Physics of the Moon, Elsevier Pub. Co. New York, 1971).
- Hsu, Y., Similarity Laws of Lunar and Terrestrial Volcanic Flows, Ph.D. thesis, Aerospace Engineering Department, University of Maryland, College Park, Maryland, 1976.
- McCauley, J.F., Geologic results from lunar precursor probes, AIAA paper no 67-862, 1967.
- McGetchin, T.R. and G.W. Ullrich, Deep-seated xenoliths observed terrestrial abundance, distribution and emplacement in some maars and diatremes with influences for the Moon, Mars and Venus, J. Geophys. Res. 78, 1833-1853, 1973.
- O'Keefe, J.A., Tektites and their origin, Elsevier, Amsterdam and New York, 244, 1976.

Pai, S.I, Two Phase Flows, Chapter V., Vieweg, Wiesbaden, Germany, 1977.

Pai, S.I. T. Hsieh and J.A. O'Keefe, Lunar Ash Flow, Isothermal approximation  
J. Geophys. Res. 77, pp. 3631-3649 (1972).

Pike, R.J. Craters on earth, moon, and Mars: Multivariate analysis classifications  
and mode of origin, Earth and Planetary Sciences Letters, 22,  
245-255, 1974.

Sedov, L.I. Similarity and Dimensional Methods in Mechanics, Academic Press,  
New York, 1959.

Stull, D.R. and H. Prophet, JANAF Thermochemical Tables, Government Printing  
Office, Washington, 1141, 1971.

#### VIII. CAPTIONS AND FIGURES

Fig. 1. The lunar crater Mauser, 55 km in diameter, on Mare Orientale. Note  
the hummocky region behind it, and the very gentle slope of the outside  
wall. The slope of the inner wall has been diminished by slumping. A  
smaller crater, Hohmann, near the bottom of the figure, is analogous  
to Kopff.


Fig. 2. The lunar crater Kopff, 40 km in diameter, in Mare Orientale. Note  
the steep outer wall, and the smooth region around it. Kopff is ob-  
viously a different kind of crater from Mauser, yet it is superposed  
on the characteristic Mare Orientale Terrain, which has not been re-  
worked by cratering. It is thus not likely that Kopff is the worn-  
down remnant of a crater like Mauser.

Fig. 3 Mathematical model for similar lunar and terrestrial volcanic flows.

Fig. 4. Comparison of the actual velocity of isothermal similar lunar and  
terrestrial volcanic flows along the duct with the  $k_p$  as a parameter,

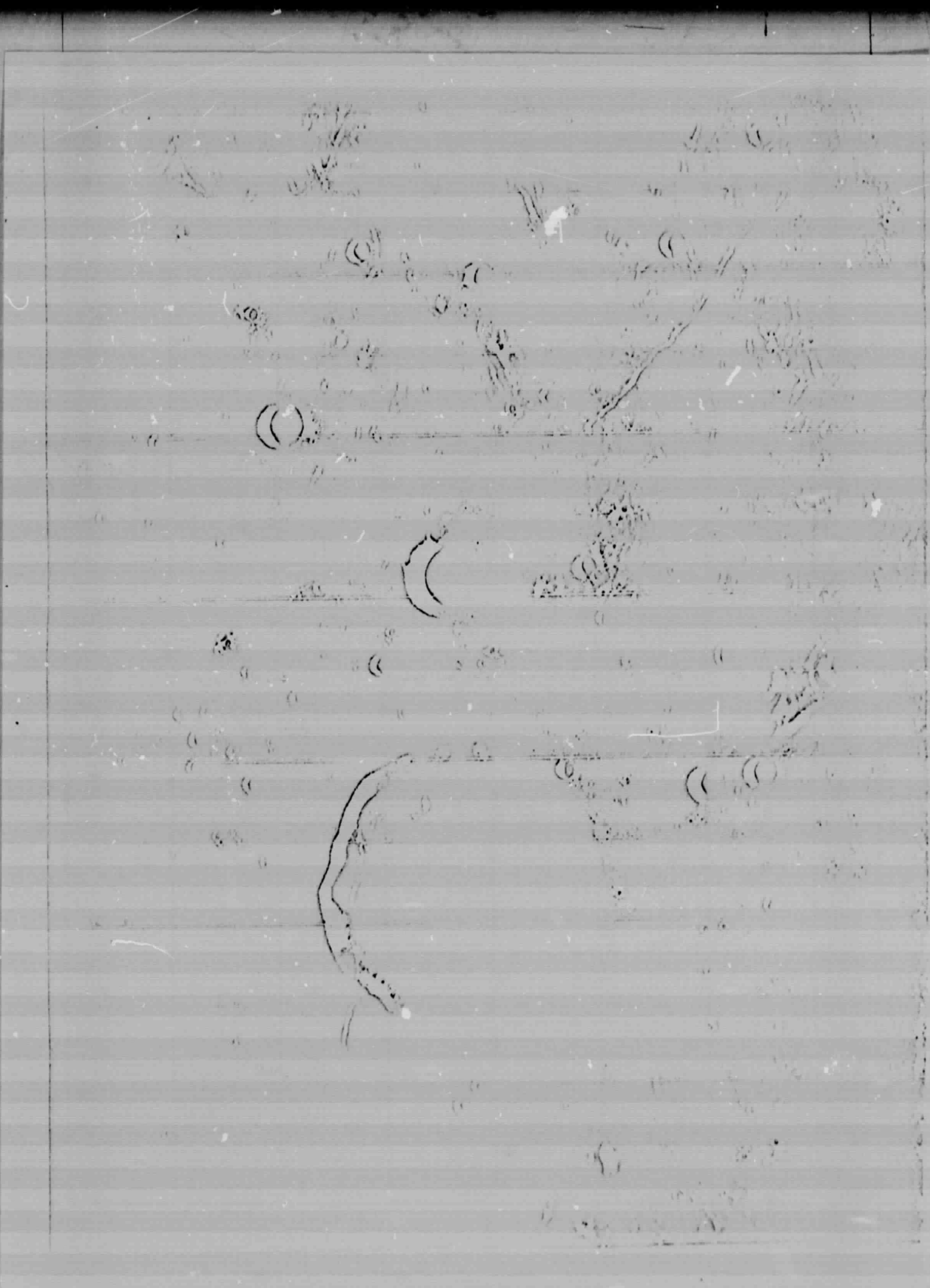
for the corresponding area ratio  $(A_s/A_o)_e = 4$  and  $(A_s/A_o)_m = 181$ .

- Fig. 5. Comparison of the actual velocity distributions of isothermal similar lunar and terrestrial volcanic flows along the duct with the corresponding area ratio as a parameter for a fixed  $k_p = 0.5$ .
- Fig. 6. Effects of  $k_p$  and area ratio to the non-dimensional pressure distributions of isothermal terrestrial volcanic flow along the duct.
- Fig. 7. Effects of  $k_p$  and area ratio on the non-dimensional pressure distributions of isothermal lunar volcanic flow along the duct.
- Fig. 8. Comparison of the actual velocity of adiabatic similar lunar and terrestrial volcanic flows along the duct with the  $k_p$  as a parameter for the corresponding area ratio  $(A_s/A_o)_e = 4$  and  $(A_s/A_o)_m = 181$ .
- Fig. 9. Temperature distribution of the adiabatic terrestrial volcanic flow along the duct with  $k_p$  as a parameter and  $(A_s/A_o)_e = 4$ .
- Fig. 10. Effect of  $k_p$  and area ratio on the temperature distribution of terrestrial volcanic flow along the duct.
- Fig. 11. Effects of  $k_p$  and area ratio on the temperature distribution of lunar volcanic flow along the duct.
- Fig. 12. Effect of  $k_p$  and area ratio on the non-dimensional pressure distribution of adiabatic terrestrial volcanic flow along the duct.
- Fig. 13. Effects of  $k_p$  and area ratio on the non-dimensional pressure distribution of adiabatic lunar volcanic flow along the duct.
- Fig. 14. Effect of friction on isothermal terrestrial volcanic flow.



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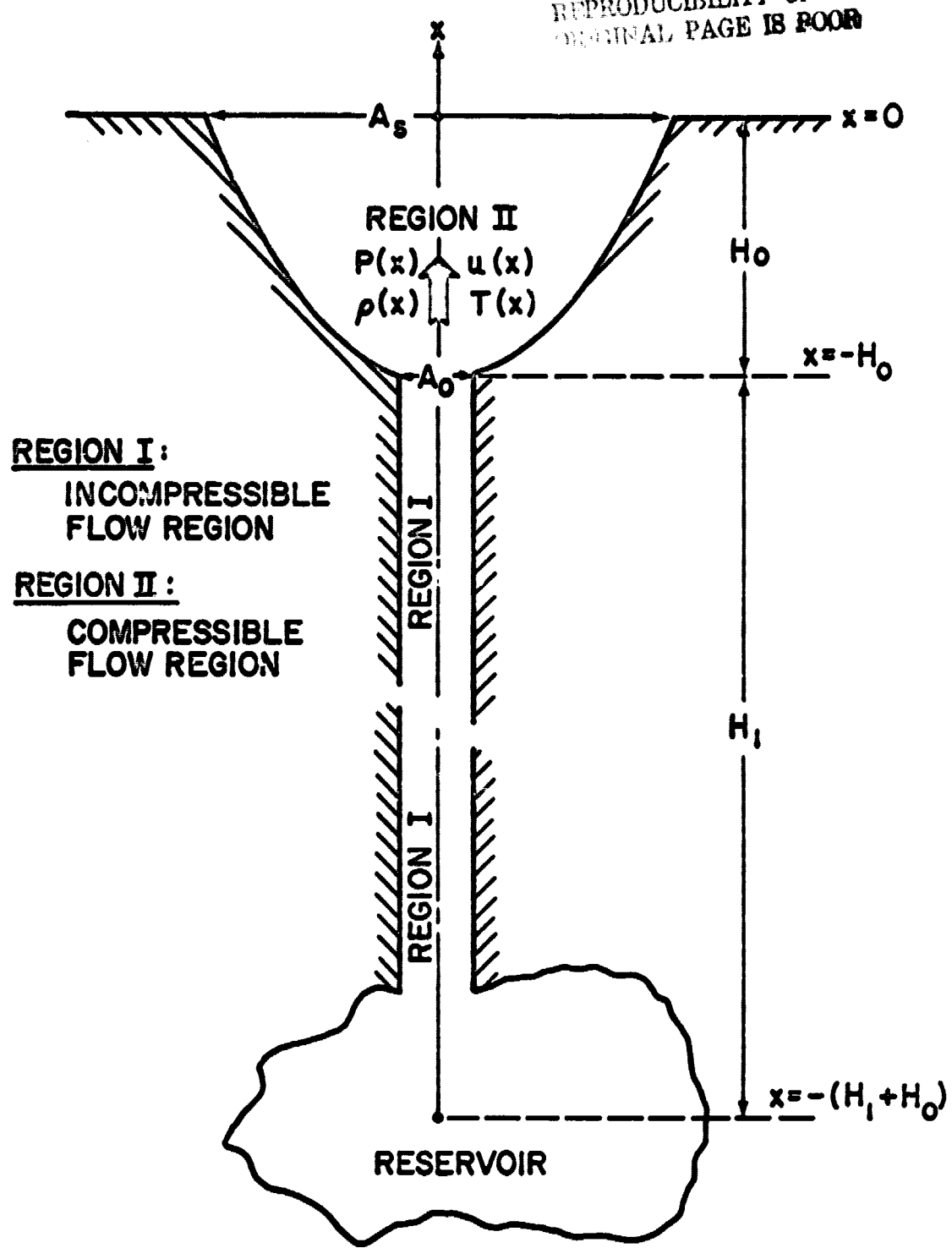


Fig. 3

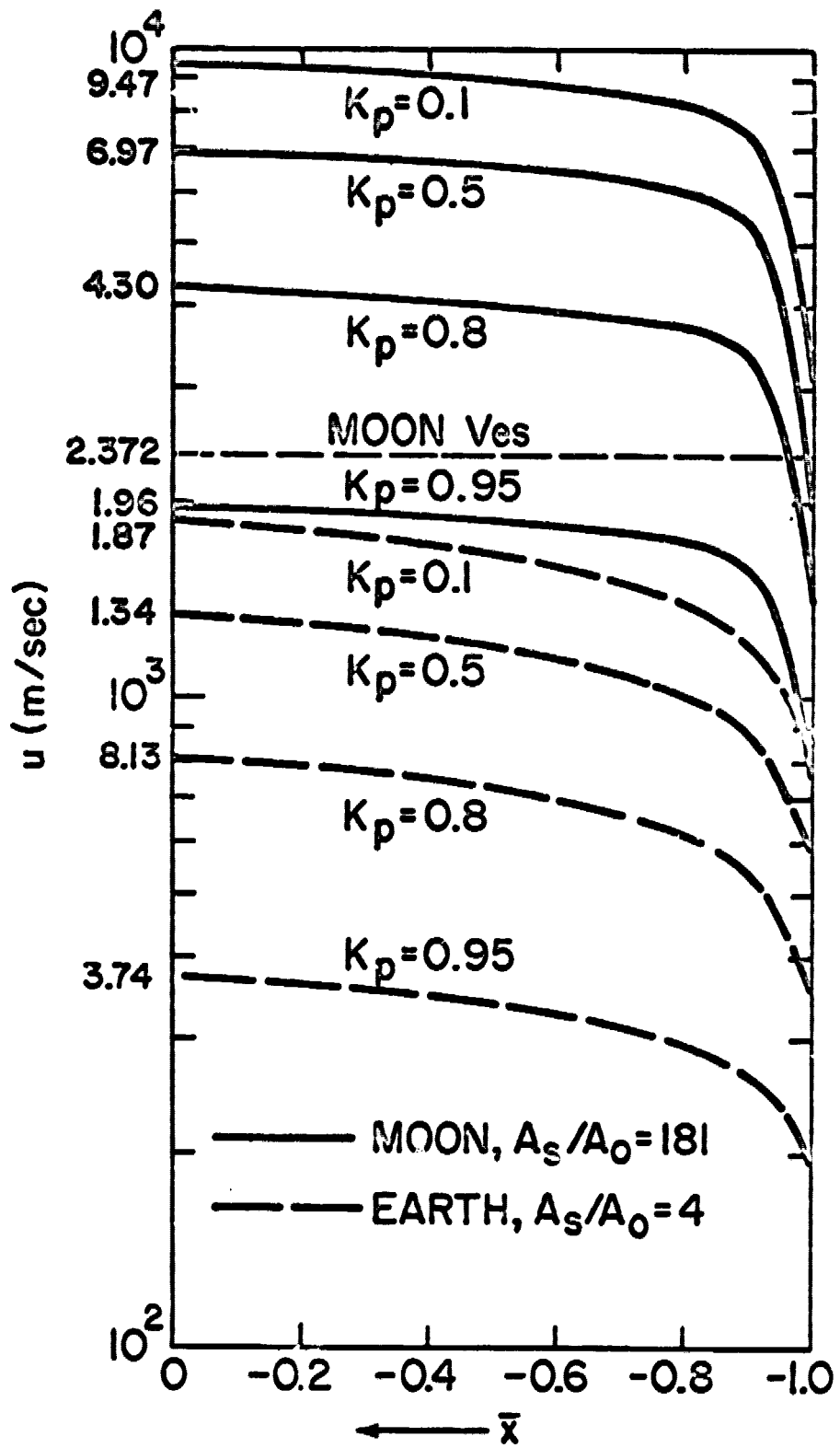


Fig. 4

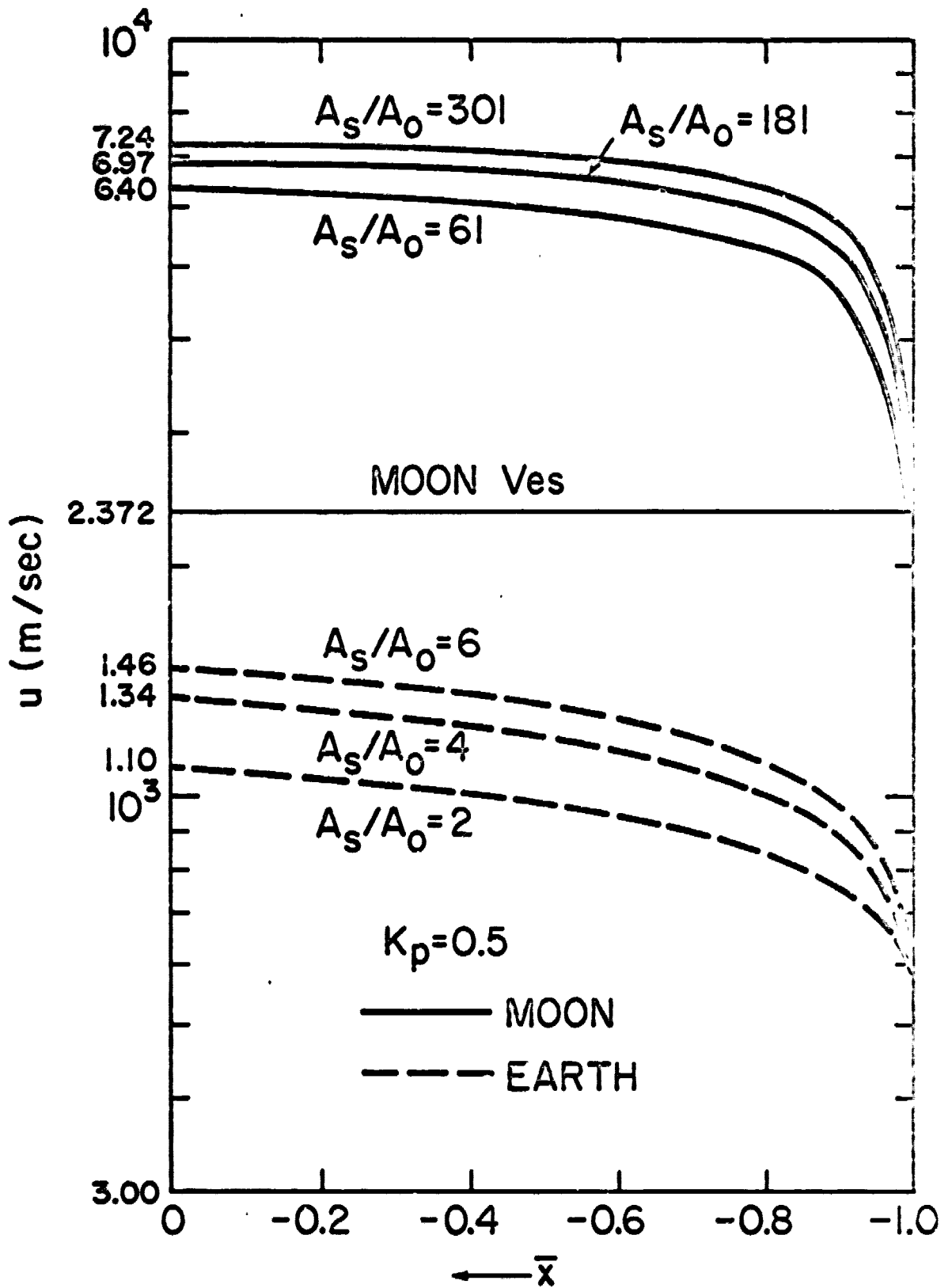


Fig. 5

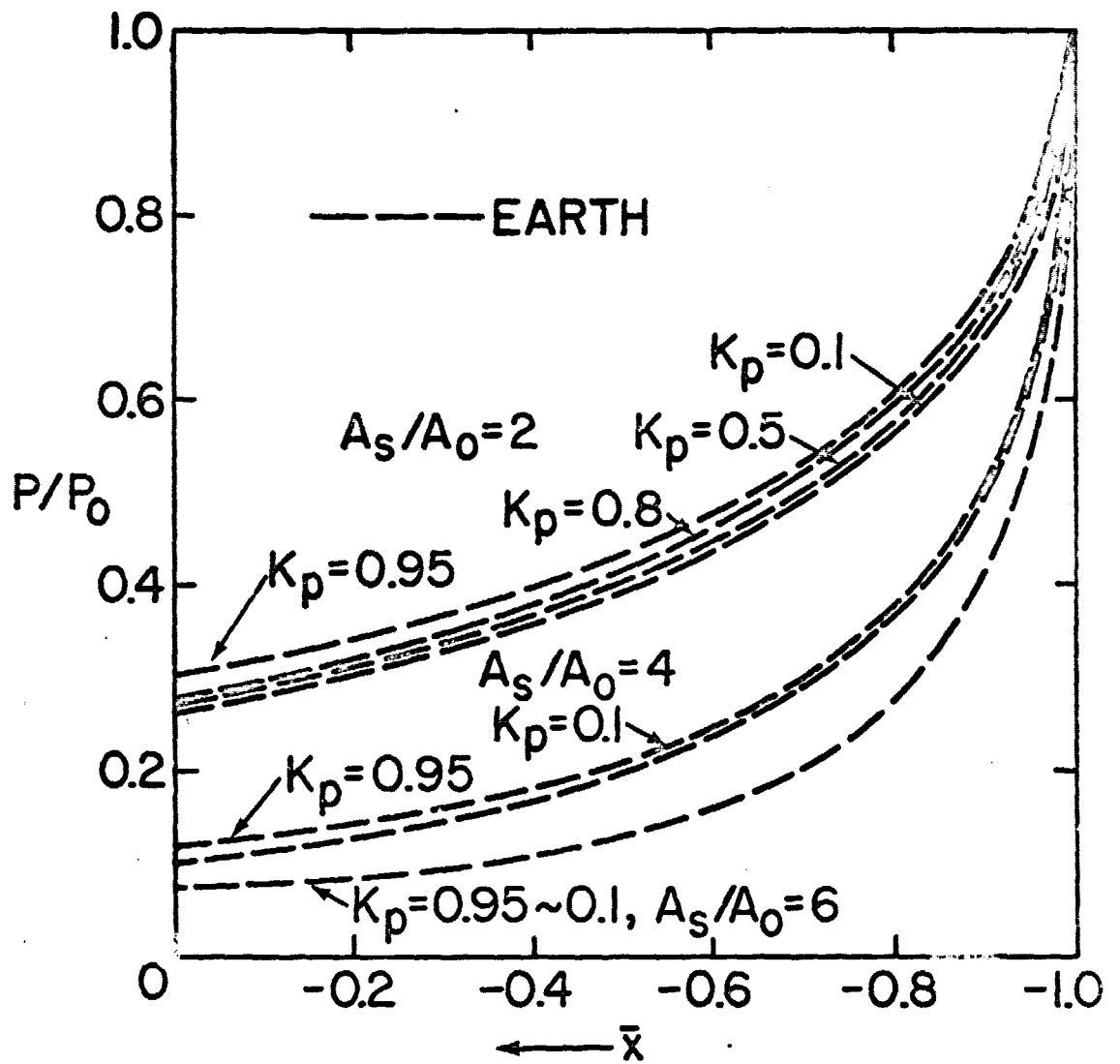


Fig. 6

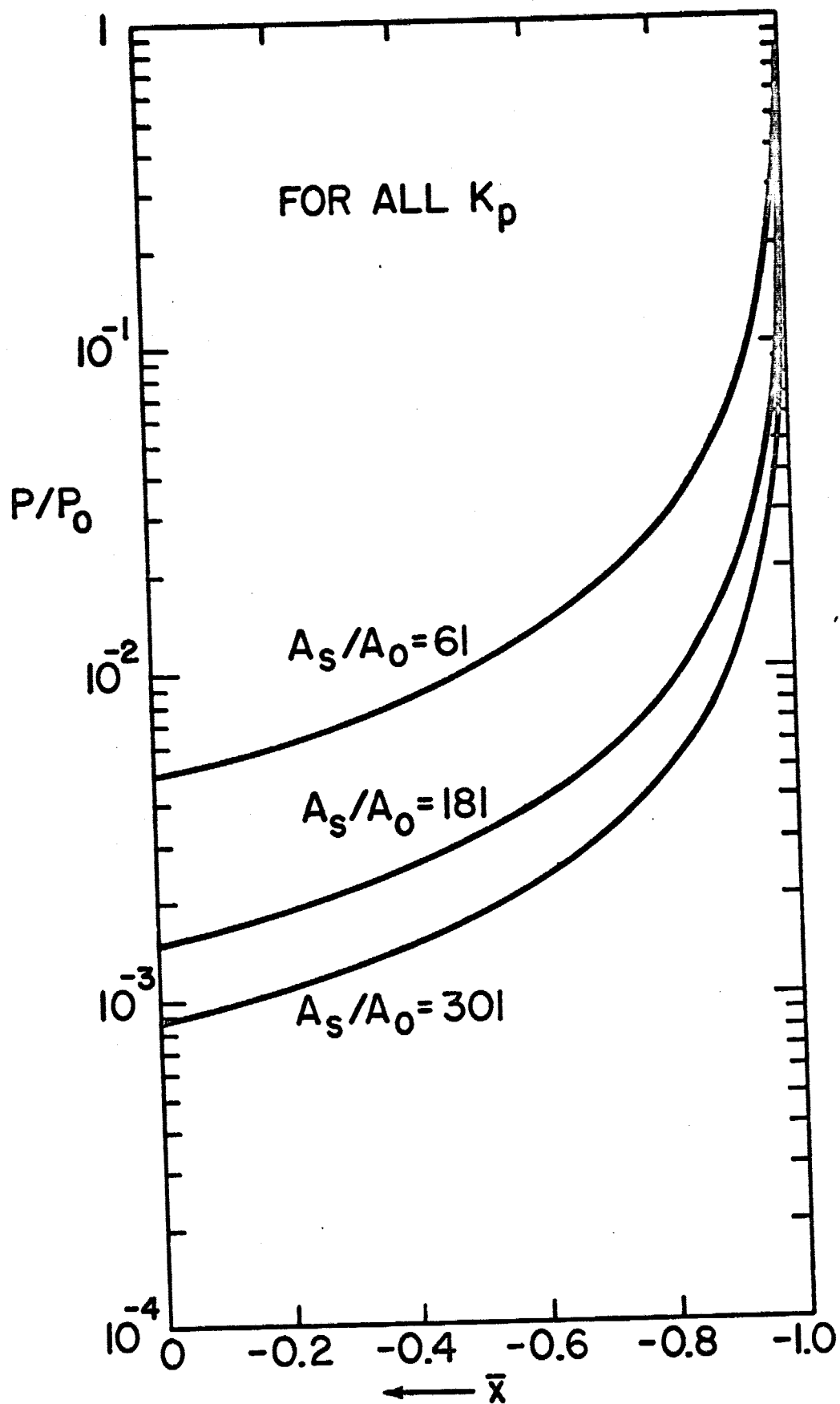


Fig. 7

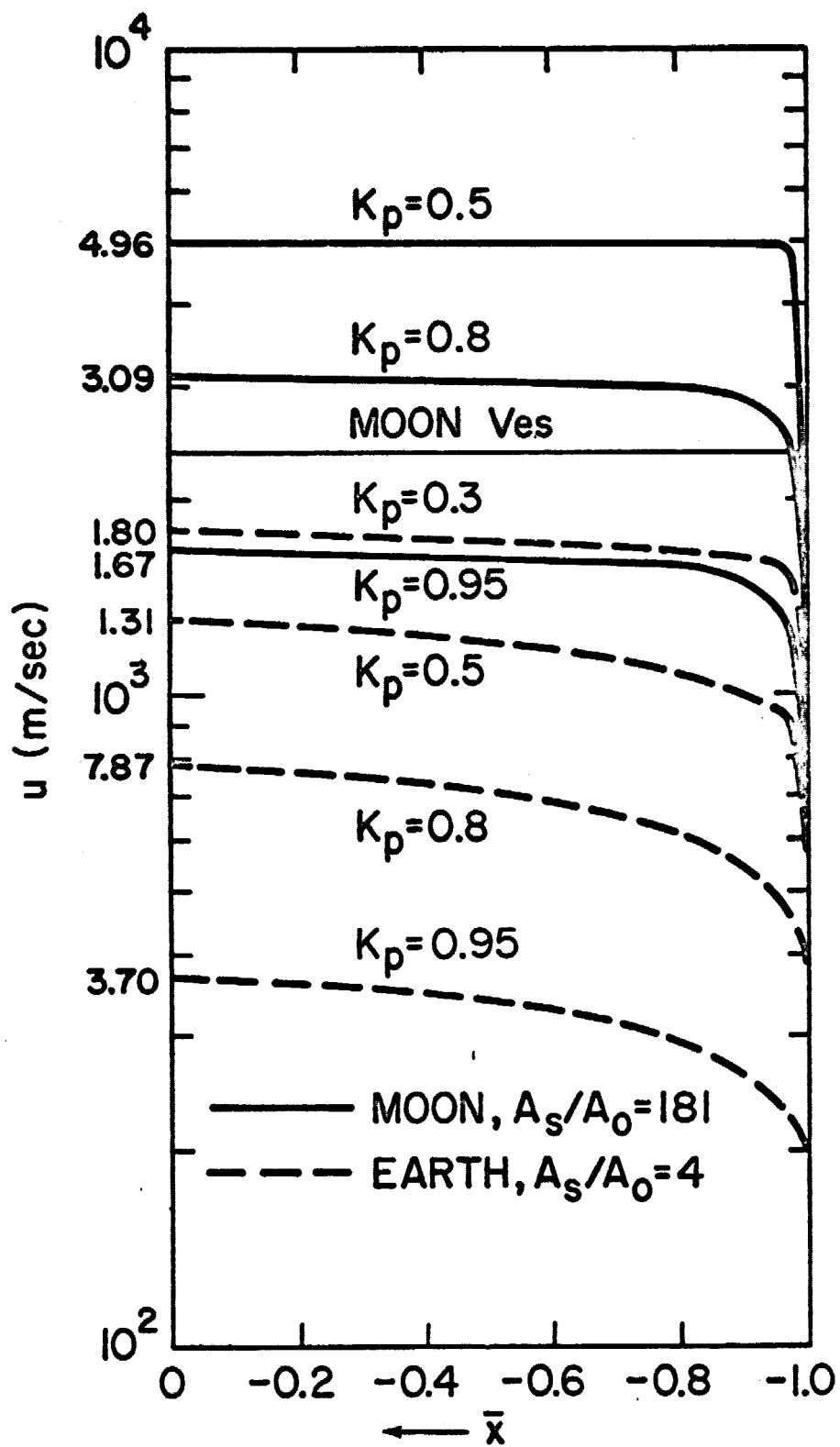


Fig. 8

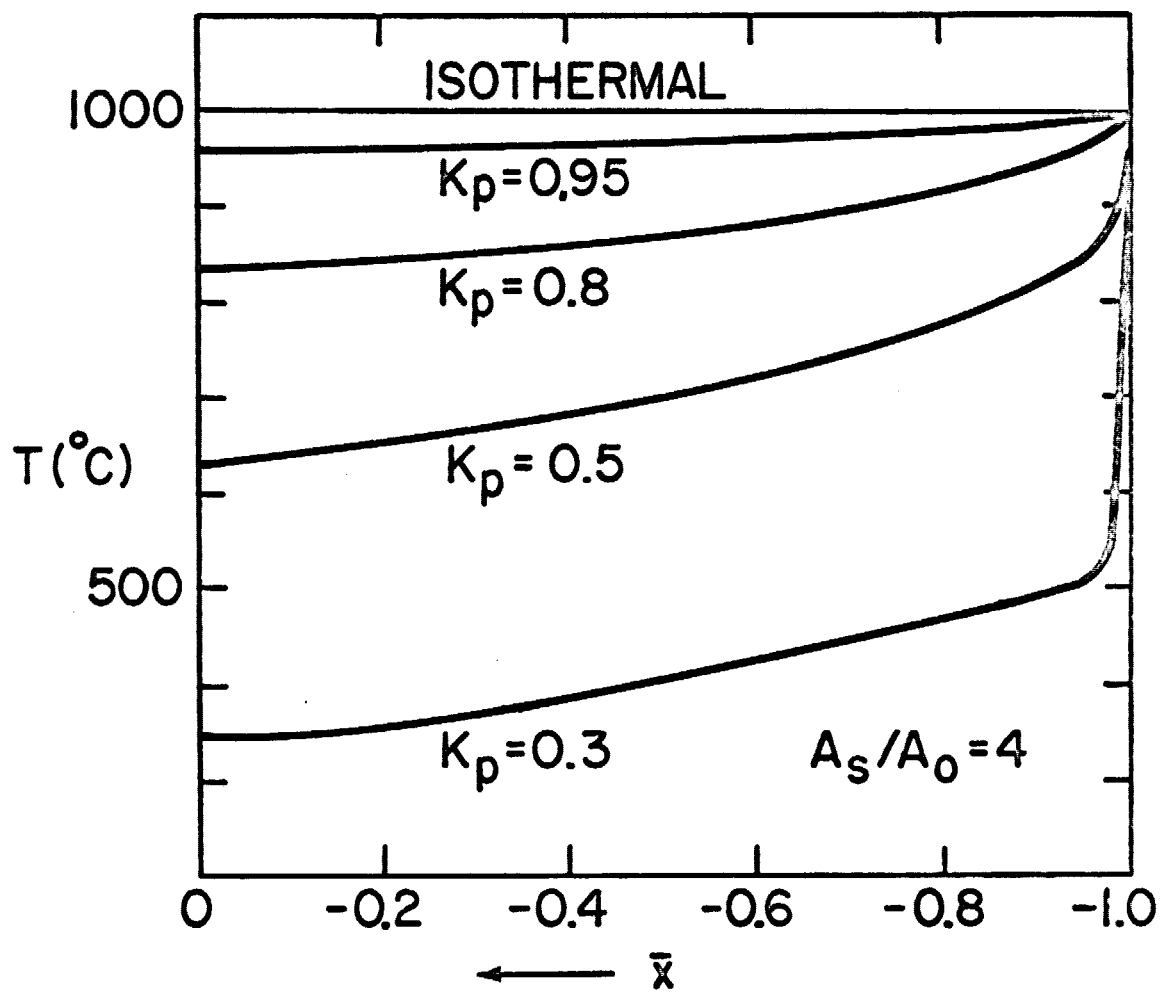


Fig. 9



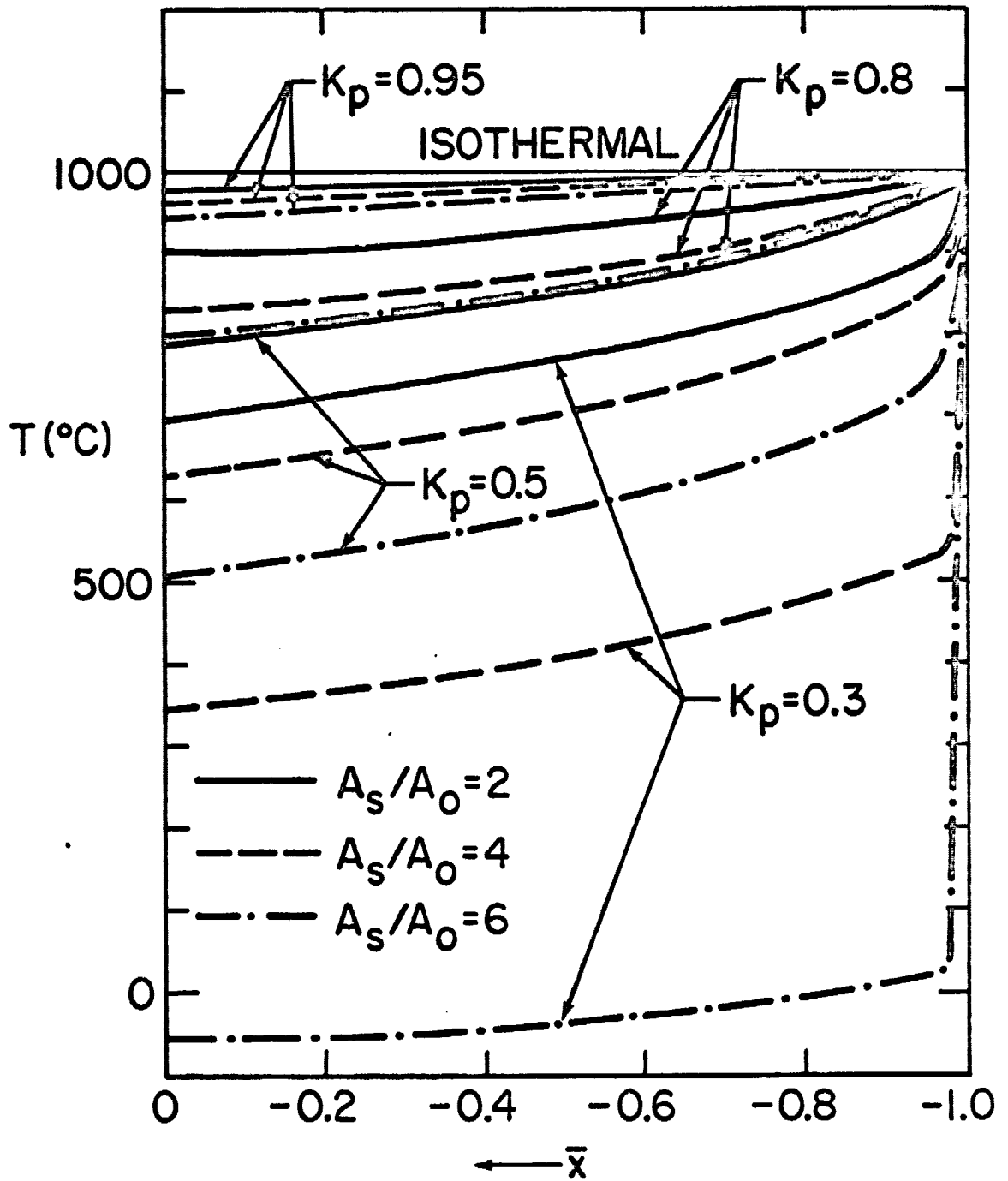


Fig. 10

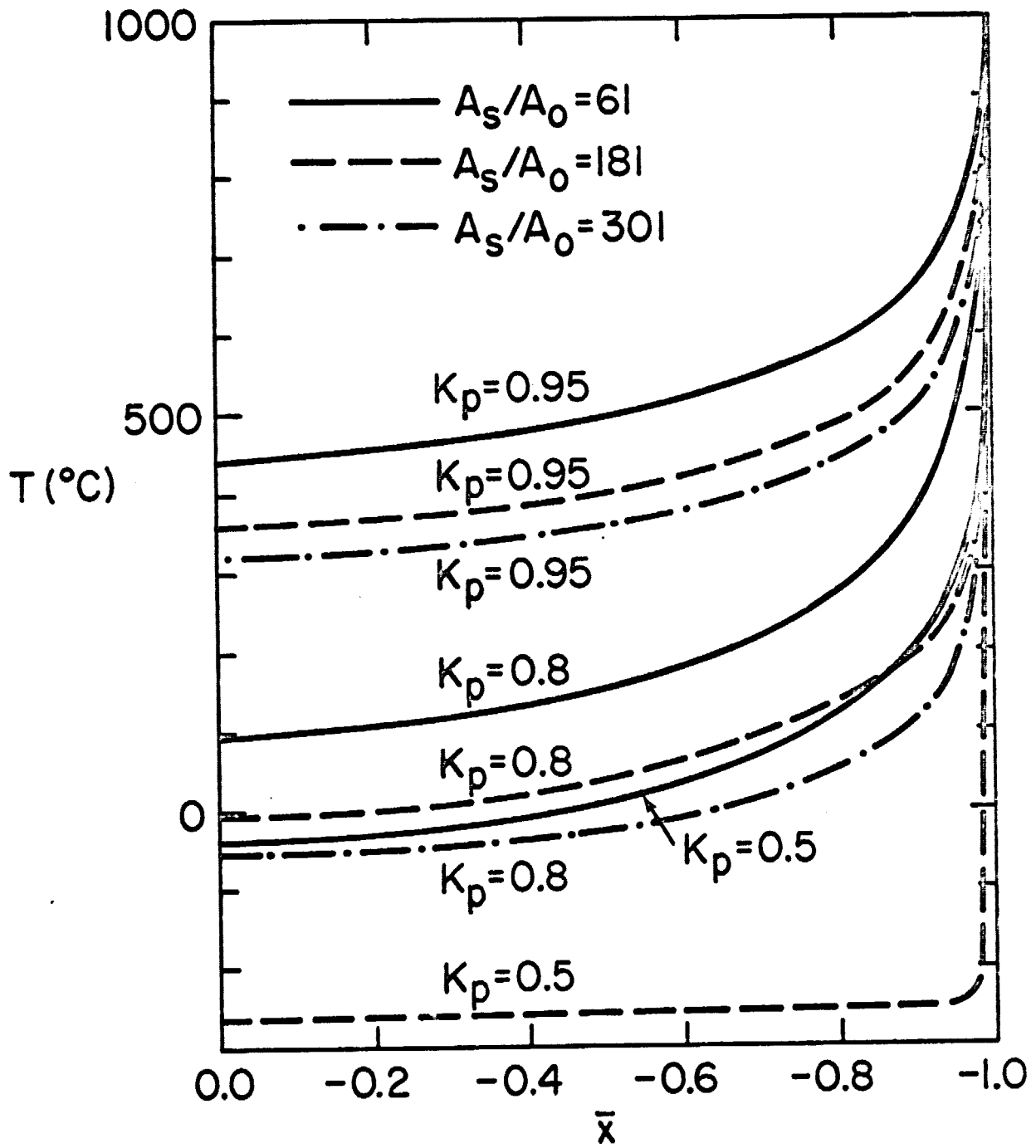


Fig. 11

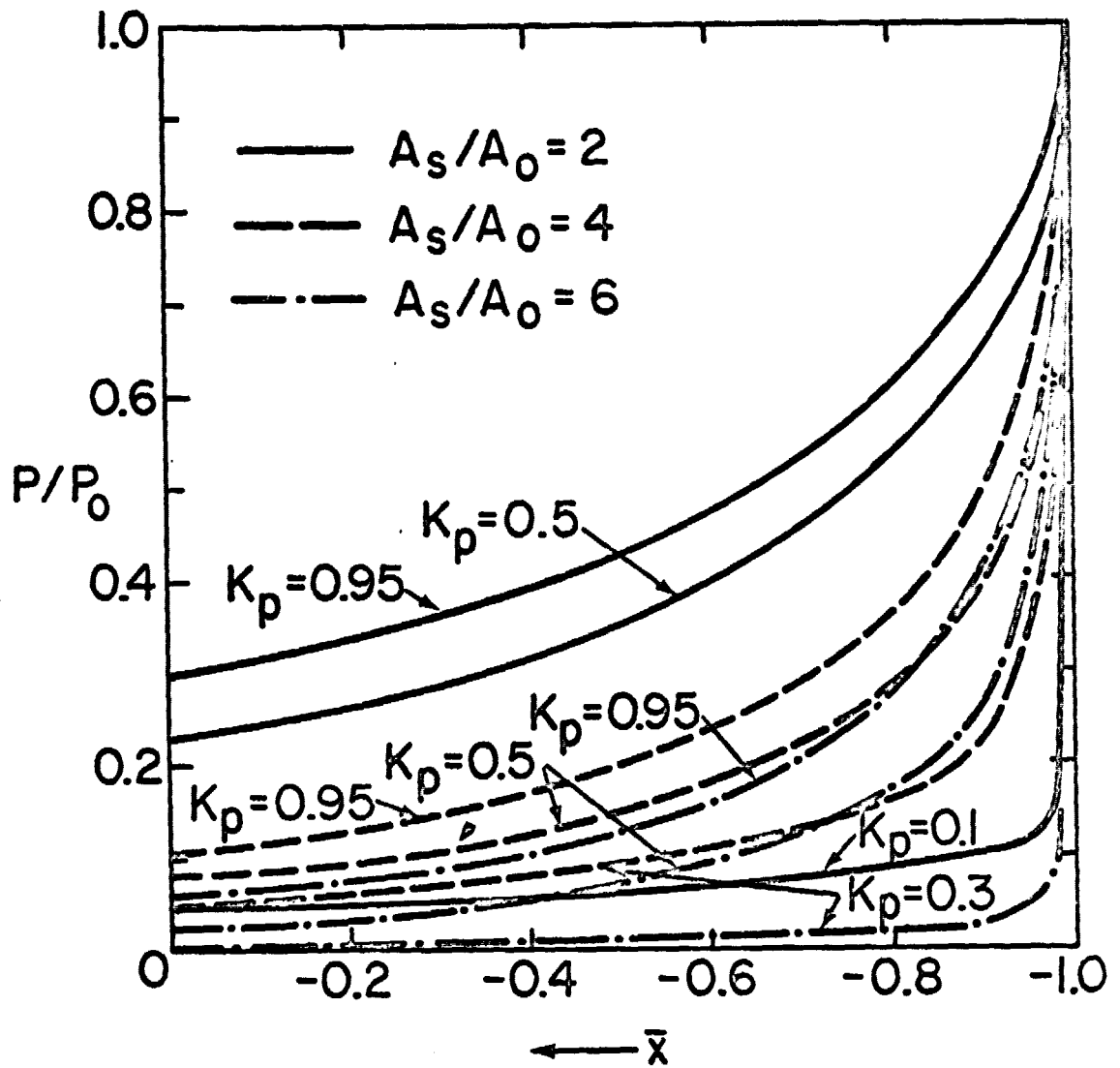


Fig. 12

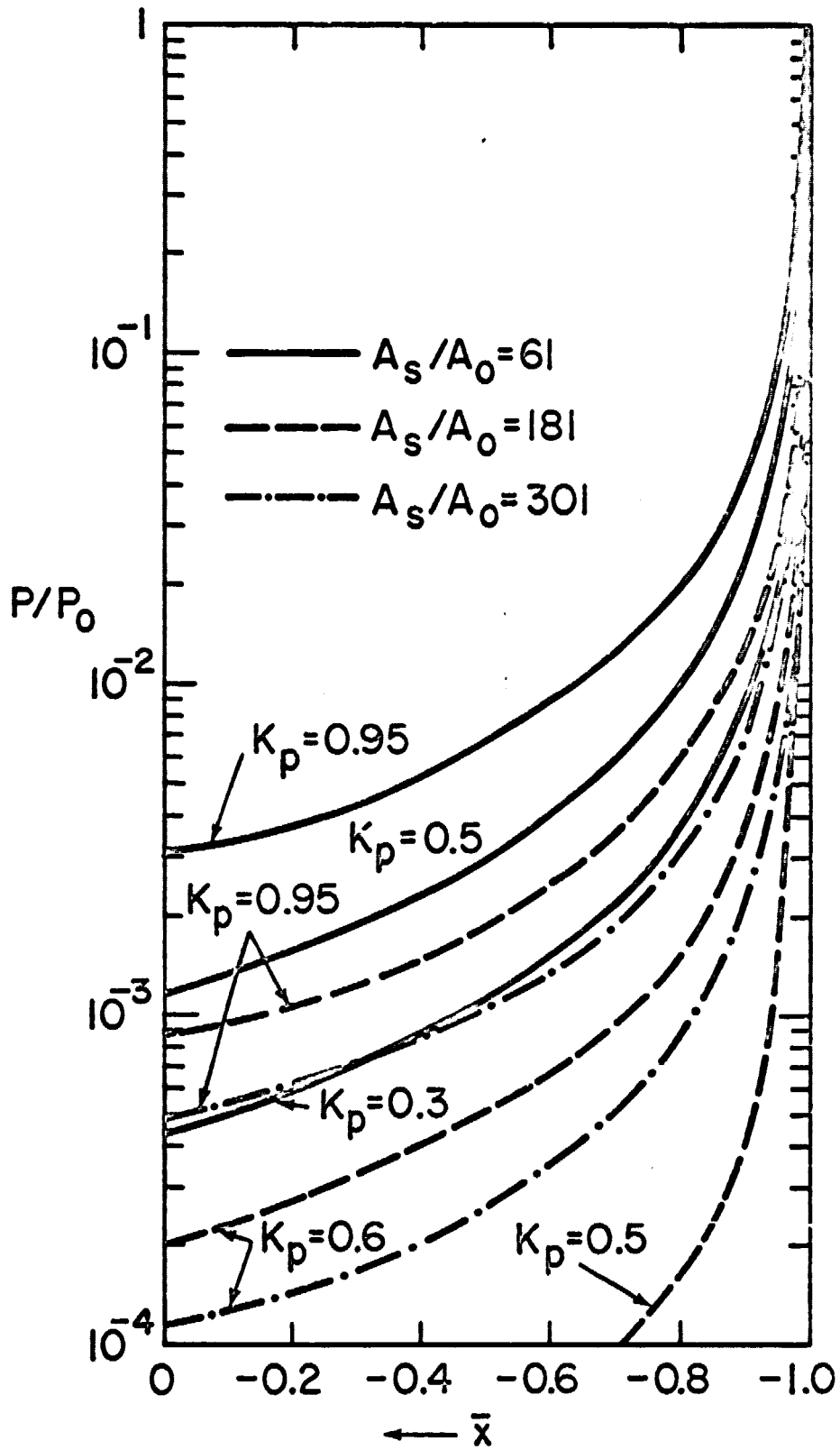


Fig. 13

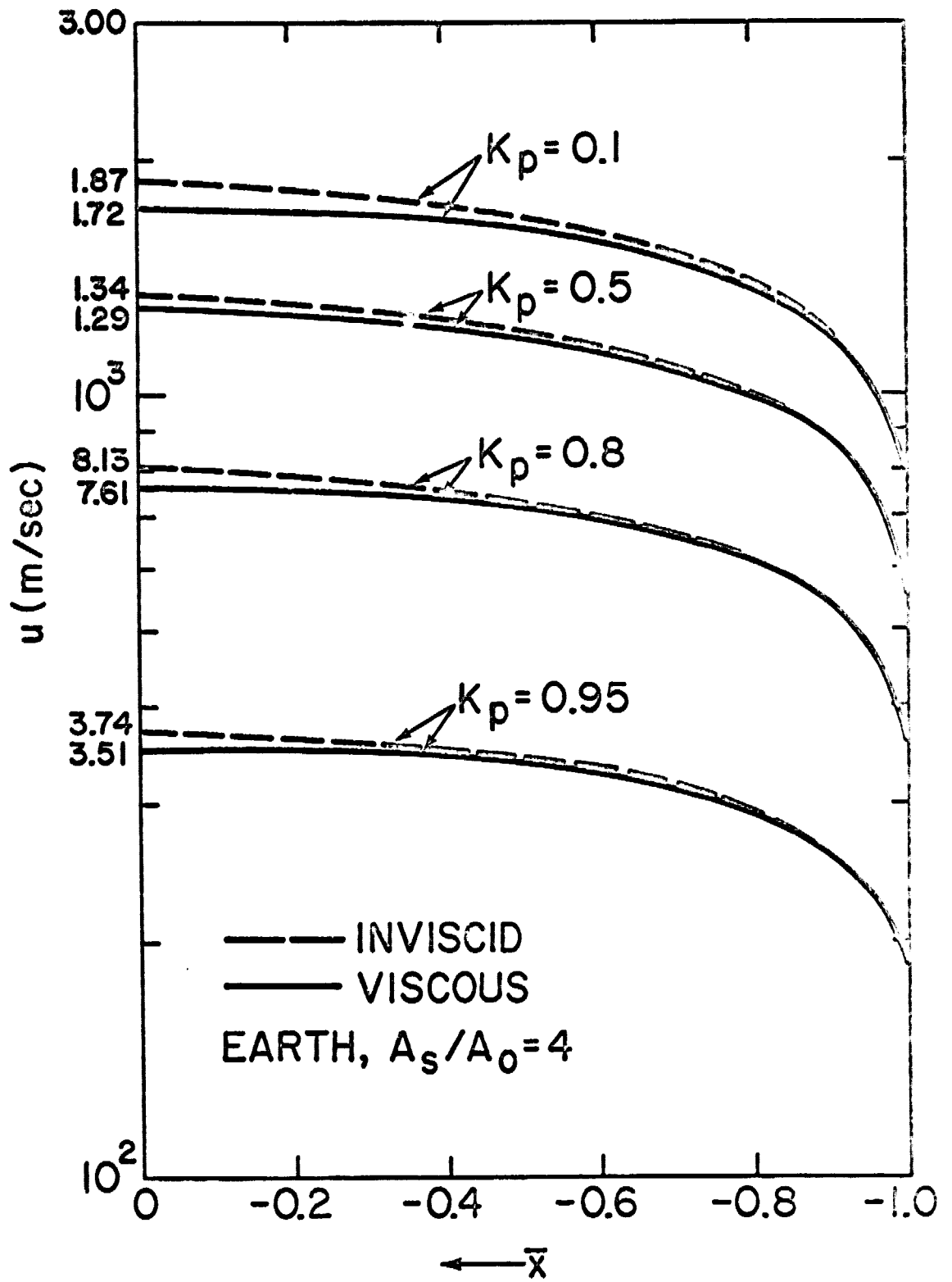


Fig. 14