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RMS MASSLESS ARM DYNAMICS CAPABILITY IN THE SVDS

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Job Order 86-069

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For

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TECHNICAL REPORT INDEX/ABSTRACT

(See instructions on reverse side.)

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13. ABSTRACT

The equations of motion for the simulation of the Orbiter (Remote Manipulator System) Payload System assuming that the masses and inertias of the arm can be neglected are developed for implementation into the Space Vehicle Dynamics Simulation (SVDS) program. The arm flexibility is incorporated into the equations by the computation of flexibility terms for use in the joint servo model. The approach developed in this report is based on using the Jacobian transformation matrix to transform force and velocity terms between the configuration space and the task space to simplify the form of the equations.

14. SUBJECT TERMS

RMS MASSLESS ARM DYNAMICS CAPABILITY IN THE SVDS

1. INTRODUCTION AND SUMMARY

This report presents the formulation and equations for the dynamics to simulate the Remote Manipulator System (RMS) in the Space Vehicle Dynamics Simulation (SVDS). The equations presented are derived assuming the masses and inertias of the RMS system are small relative to the mass and inertia of the orbiter and payload, and can be neglected.

Section 2 presents a Newtonian formulation of the rigid body equation of motion of the Orbiter-RMS-Payload system. The massless arm assumption and the resolved rate law (ref. 1) are used in these equations. The interaction forces and moments at the tip of the end effector resulting from the output torques at each of the six joints are computed for the payload and the orbiter. These forces are inputs to the equations of motion of the orbiter and for the payload The arm joint rates are computed from the translational and rotational velocities of the two bodies.

Section 3 contains a development of the equations required to compute the arm flexibility matrix, given the flexibility of each member. For this purpose, the arm has been divided into five beams. The first beam contains the longeron and the portion of the arm up to the shoulder pitch joint. The second beam is the upper arm, the third is the lower arm, the fourth is the wrist, and the fifth beam is the hand. From the arm flexibility matrix, a joint flexibility matrix is computed and used to compute six flexibility terms for inclusion in each of the six servos to simulate arm flexibility.

Section 4 presents a set of formulas for the computation of static loads on the arm members of the joints.

Section 5 contains a set of engineering flowcharts presenting the computations required to implement the capabilities described in sections 2-4 into the SVDS program.

Appendix A, The RMS Jacobian Matrix, develops relationships between velocities and forces in a chain of rigid bodies and relates these relationships to the RMS Jacobian matrix.

Appendix B, Calculation of the Arm Flexibility Matrix, presents a set of equations that can be used to compute the flexibility matrix for a chain of beams, given the flexibility matrix for each beam.

Appendix C develops equations for the kinetic energy and the virtual work for a chain of rigid bodies. These equations are used in conjunction with Lagrange's equation to write the equations of motion of a chain of rigid bodies. The masses and inertias of the inner bodies of the chain are assumed zero, and the resulting equations are related to the massless arm equations presented in section 2.

Because of the approximations made in the development of the equations of motion in this report, some comments should be made about the method of verification used, and some general comments about when the validity of these equations will be questionable. The massless arm approach presented herein and subsequently being incorporated into the SVDS program requires extensive checkout to determine its level of validity. The primary checkout procedure used is to compare these results with cases run on the RMS Simulation (RMSS) program. The standard types of checkout techniques for dynamics programs will be used; i.e., simple cases that have closed form solutions, balance of energy and work, and engineering approximation checks.

The limits of validity of the massless arm equations must be determined to a certain extent by comparison with other programs. Some observation can be made, however. The lighter the payload, the worse the approximation, and it is felt that from a rigid body standpoint, about 1000 lb is the minimum payload weight that should be simulated in the rigid body mode. The technique employed here for the incorporation of longeron and arm flexibility allows the simulation of the first six bending modes of the system. As the mass of the payload decreases, the separation between the rigid body bending modes (characterized by end body motions) start to couple with the arm modes (characterized by small motion of the end bodies). How good the approach presented herein is can be determined by comparing the mode shapes and frequencies from a massless arm vibration analysis with a distributed mass arm vibration analysis. From a flexible arm standpoint, payloads of less than 10,000 lb would be suspect at the present time.

2. RIGID BODY EQUATIONS

This section presents the massless arm RMS rigid body equations of motion for implementation into the SVDS program, using a Newtonian approach to the formulation of the equations. Appendix C presents a Lagrangian approach to the development of the equations of motion of a chain of rigid bodies; a Newtonian development for this type of system can be found in ref. 2. The assumption that the arm is massless appears to be reasonable, based on a comparison of the mass of the orbiter and the mass of a prospective payloads relative to the mass of the RMS system itself. The orbiter weighs approximately 180,000 lb, the payloads weigh between 20,000 and 65,000 lb, and the movable portions of the arm weigh approximately 500 lb.

The following relationships can be written from reference 1.

$$\{\tau\} = [J]^T \begin{Bmatrix} M_e \\ F_e \end{Bmatrix} \tag{2-1}$$

$$[J]\{\dot{\theta}\} = \begin{Bmatrix} \omega_R \\ V_R \end{Bmatrix} \tag{2-2}$$

If the arm is assumed to be massless, eq. (2-1) gives the output force and moment acting on the payload caused by the torque motors. An equal and opposite force and moment will act on the orbiter.

Using eqs. (2-1) and (2-2) and the massless arm assumption, the end effector forces and moments can be written

$$\begin{Bmatrix} M_e \\ F_e \end{Bmatrix} = [J]^{T^{-1}} \{\tau\} \tag{2-3}$$

The translational equations for body 1 (orbiter) can be written

$$\left\{\frac{d^2 R_1}{dt^2}\right\} = \frac{1}{m_1}[B_1](\{F_1\} - \{F_e\}) + \{g_1\} \tag{2-4}$$

The rotational equations for body 1 are

$$[\Lambda_1]\{\dot{\omega}_1\} + [\underline{\omega}_1][\Lambda_1]\{\omega_1\} = \{M_1\} - \{M_e\} - [\underline{p}_1]\{F_e\} \tag{2-5}$$

The quaternion rates for body 1 are

$$\begin{bmatrix} \dot{e}_1 \\ \{\dot{e}_2\} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -\{\omega_1\}^T \\ \{\omega_1\} & [\underline{\omega}_1] \end{bmatrix} \begin{bmatrix} e_1 \\ \{e_2\} \end{bmatrix} \tag{2-6}$$

The body to inertial transformation is computed

$$[B_1] = (\ulcorner I \lrcorner + 2e_1[\underline{e}_2] + 2[\underline{e}_2]^2) \tag{2-7}$$

The rigid body rotational and translational equations of motion for the second body are written

$$\left\{\frac{d^2R_2}{dt^2}\right\} = \frac{1}{m_2}\left([B_2]\{F_2\} + [B_1]\{F_e\}\right) + \{g_2\} \tag{2-8}$$

$$[\Lambda_2]\{\dot{\omega}_2\} + [\underline{\omega}_2][\Lambda_2]\{\omega_2\} = \{M_2\} + [B_{21}]\{M_e\} + [\underline{p}_2][B_{21}]\{F_e\} \tag{2-9}$$

The relative rotational and translational velocity of the tip of the end effector with respect to the orbiter are computed

$$\{V_R\} = [B_1]^T\left(\left\{\frac{dR_2}{dt}\right\} - \left\{\frac{dR_1}{dt}\right\}\right) + [\underline{p}_1]\{\omega_1\} - [B_{12}][\underline{p}_2]\{\omega_2\} \tag{2-10}$$

$$\{\omega_R\} = [B_{12}]\{\omega_2\} - \{\omega_1\} \tag{2-11}$$

Using eq. (2-2), the arm joint rates can be computed as

$$\{\dot{\theta}\} = [J]^{-1}\begin{bmatrix} \{\omega_R\} \\ \{V_R\} \end{bmatrix} \tag{2-12}$$

The following equations are used to compute the secondary body to inertial transformation matrix and the body 1 to body 2 transformation matrix.

$$[B_2] = [B_1][B_{12}] = [B_1][B_{21}]^T \tag{2-13}$$

$$[B_{21}] = [T_{PE}][T_1(\theta_6)][T_3(\theta_5)][T_2(\theta_2 + \theta_3 + \theta_4)][T_3(\theta_1)][T_{LB}] \tag{2-14}$$

Eqs. (2-3) through (2-14) form a complete set of rigid body equations of motion for the Orbiter-RMS-Payload system, assuming that the arm is massless.

Definition of Symbols

$\{\tau\}$ 6×1 matrix of the torques output from the joint servos

$[J]$ 6×6 Jacobian matrix

$\{M_e\}$ 3×1 matrix of the components of the end effector moment vector in the orbiter body system

$\{F_e\}$ 3×1 matrix of the components of the end effector force vector in the orbiter body system

$\{R_1\}$ 3×1 matrix of the components of the orbiter position vector in the inertial system

m_1 Mass of the orbiter

$[B_1]$ 3×3 transformation matrix from the orbiter body system to the inertial system

$\{F_1\}$ 3×1 matrix of the external forces on the orbiter in the orbiter body system

$\{g_1\}$ 3×1 matrix of the components of the gravitational acceleration vector on the orbiter, in the inertial system

$[\Lambda_1]$ 3×3 inertia matrix of the orbiter

$\{\omega_1\}$ 3×1 matrix of the components of the orbiter angular velocity vector in the orbiter body system

$\{M_1\}$ 3×1 matrix of the moments of the external forces about the center of mass (CM) of the orbiter

$\{p_1\}$	3×1 matrix of the position vector from the CM of the orbiter to the tip of the end effector in the orbiter body system
e_1	Scalar part of orbiter quaternion
$\{e_2\}$	3×1 matrix of the vector part of the orbiter quaternion
$\{R_2\}$	Position vector to CM of payload in the inertial system
m_2	Mass of the payload
$[B_2]$	3×3 transformation matrix from the payload body system to the inertial system
$[B_{21}]$	3×3 transformation matrix from the orbiter system to the payload body system
$[B_{12}]$	3×3 transformation matrix from the payload body system to the orbiter body system
$\{p_2\}$	3×1 position vector from the payload CM to the tip of the end effector in the payload body system
$\{V_R\}$	3×1 matrix of the relative translational velocity of the tip of the end effector in the orbiter body system
$\{\omega_R\}$	3×1 matrix of the relative angular velocity of the tip of the end effector in the orbiter body system
$\{\dot{\theta}\}$	6×1 matrix of the joint rates
$[T_{LB}]$	3×3 transformation matrix from the orbiter body system to the longeron system
$[T_{PE}]$	3×3 transformation matrix from the end effector system to the payload body system

3. ARM FLEXIBILITY

This section presents the development of a technique for computing the terms needed to simulated massless arm flexibility, by computing the arm flexibility matrix and then generating the joint flexibility matrix. Joint flexibility terms are generated from the joint flexibility matrix and will be used in the joint servo model to simulate arm flexibility in the massless arm RMS simulation in SVDS. The computation of the arm flexibility matrix is discussed in Appendix B.

Two modes of arm flexibility simulation can be used, passive and active arm flexibility. In the passive arm case, the joints are considered locked and the neutral arm angles are used with current arm angles to compute joints torque from elastic rotations at the joints. In the active case, flexibility terms are generated and used in the joint servo model.

From Appendix B, the flexibility matrix relates end effector elastic displacements and rotations to forces and moments applied at the tip of the end effector.

Using eqs. (2-1) and (2-2), the joint flexibility matrix is developed.

$$[\gamma]\{P_e\} = \{\delta_e\} \tag{3-1}$$

$$[J]^T\{P_e\} = -\{\tau_F\} \tag{3-2}$$

$$[J]\{\delta\theta\} = \{\delta_e\} \tag{3-3}$$

Substituting eqs. (3-2) and (3-3) into eq. (3-1), the following formulas result.

$$-[J]^{-1}[\gamma][J]^{T^{-1}}\{\tau_F\} = \{\delta\theta\} \tag{3-4}$$

$$-[\gamma_J]\{\tau_F\} = \{\delta\theta\} \tag{3-5}$$

where $[\gamma_J]$ is the joint flexibility matrix.

For the passive case, the joint torques resulting from arm flexibility are computed from eq. (3-5) as

$$\{\tau_F\} = -[\gamma_J]^{-1}\{\delta\theta\} \tag{3-6}$$

For the active case, the joint flexibility matrix is used to generate flexibility terms to use in the joint servo model. Presented below is a technique that can be used to generate these terms.

Let $[\gamma_{iJ}^{22}]$ be a 5×5 matrix derived from $[\gamma_J]$ by deleting the ith column and the ith row.

Let $\{\gamma_{iJ}^{21}\}^T$ be a 1×5 matrix derived from $[\gamma_J]$ by taking the ith row and removing the ith element.

The flexibility term for the ith torque motor can be computed as

$$\gamma_i^e = \gamma_{iiJ} - \{\gamma_{iJ}^{21}\}^T [\gamma_{iJ}^{22}]^{-1} \{\gamma_{iJ}\}^{21} \tag{3-7}$$

4. STATIC LOADS IN THE ARM

In this section, a set of equations to compute the static loads in the arm at each joint are presented. These loads are called static loads because they ignore the inertia of the arm links. Since the dynamics formulated in this report are based on a massless arm this type of loads model is completely consistent with the dynamics. The equations presented in Appendix A of this report are completely applicable, in particular, equation (A-20) can be used. Noting that the end effector forces and moment are computed for the rigid body dynamics these quantities can be used to form the $\{P_n\}$ in eq. (A-20). The following set of equations result.

$$\{M_1\} = [T_{2B}](-\{M_e\} - (\{r_{6B}\} - \{r_{2B}\}) \times \{F_e\})$$

$$\{V_1\} = -[T_{2B}]\{F_e\}$$

$$\{M_2\} = [T_{3B}](\{M_e\} + (\{r_{6B}\} - \{r_{2B}\}) \times \{F_e\})$$

$$\{V_2\} = [T_{3B}]\{F_e\}$$

$$\{M_3\} = [T_{3B}](-\{M_e\} - (\{r_{6B}\} - \{r_{3B}\}) \times \{F_e\})$$

$$\{V_3\} = -[T_{3B}]\{F_e\}$$

$$\{M_4\} = [T_{4B}](\{M_e\} + (\{r_{6B}\} - \{r_{3B}\}) \times \{F_e\})$$

$$\{V_4\} = [T_{4B}]\{F_e\}$$

$$\{M_5\} = [T_{4B}](-\{M_e\} - (\{r_{6B}\} - \{r_{4B}\}) \times \{F_e\})$$

$$\{V_5\} = -[T_{4B}]\{F_e\}$$

$$\{M_6\} = [T_{5B}](\{M_e\} + (\{r_{6B}\} - \{r_{4B}\}) \times \{F_e\})$$

$$\{V_6\} = [T_{5B}]\{F_e\}$$

$$\{M_7\} = [T_{5B}](-\{M_e\} - (\{r_{6B}\} - \{r_{5B}\}) \times \{F_e\})$$

$$\{V_7\} = -[T_{5B}]\{F_e\}$$

$$\{M_8\} = [T_{6B}](\{M_e\} + (\{r_{6B}\} - \{r_{5B}\}) \times \{F_e\})$$

$$\{V_8\} = [T_{6B}]\{F_e\}$$

$$\{M_9\} = -[T_{6B}]\{M_e\}$$

$$\{V_9\} = -[T_{6B}]\{F_e\}$$

where

$\{M_1\},\{V_1\}$ are the moment and force acting on the shoulder in the shoulder yaw coordinate system

$\{M_2\},\{V_2\}$ are the moment and force acting at the shoulder on the upper arm in the upper arm coordinate system

$\{M_3\},\{V_3\}$ are the moment and force acting at the elbow on the upper arm in the upper arm coordinate system

$\{M_4\},\{V_4\}$ are the moment and force acting at the elbow on the lower arm in the lower arm coordinate system

$\{M_5\},\{V_5\}$ are the moment and force acting at the wrist pitch joint on the lower arm in the lower arm coordinate system

$\{M_6\},\{V_6\}$ are the moment and force acting on the wrist at the wrist pitch joint in the wrist system

$\{M_7\},\{V_7\}$ are the moment and force acting on the wrist at the wrist yaw joint in the wrist system

$\{M_8\},\{V_8\}$ are the moment and force acting on the hand at the wrist yaw joint in the hand system

$\{M_9\},\{V_9\}$ are the moment and force acting on the hand at the end effector in the hand system

5. SVDS ROUTINES

This section presents engineering flowcharts of the routines required to interface the massless arm dynamics described in the preceding sections with SVDS. Flowcharts also are presented for the static loads computations in section 4.

Three basic routines are required to interface this capability. RMSFM computes the vehicle interaction forces and moments and the arm joint rates; RMSFLX computes the joint flexibility terms for input to the joint servo model; and RMLOAD computes the static loads on the arm members at the joints.

SUBROUTINE RMSFM

Subroutine RMSFM is called from the Math Model Driver (MMD) each pass of the integration and computes forces and moments on each body resulting from torque motor outputs. The joint angle rates are computed based on the translational and angular velocities of each of the bodies.

START

COMPUTE END EFFECTOR FORCES AND MOMENTS FROM TORQUE MOTOR OUTPUTS

$$\begin{bmatrix} \{M_e\} \\ \{F_e\} \end{bmatrix} = [J]^{T^{-1}} \{\tau\}$$

COMPUTE INTERACTION FORCES AND MOMENTS ON THE ORBITER AND PAYLOAD

$$\{F_{R1}\} = -\{F_e\}$$

$$\{M_{R1}\} = -\{M_e\} - [\underline{p}_1]\{F_e\}$$

$$\{F_{R2}\} = [B_{21}]\{F_e\}$$

$$\{M_{R2}\} = [B_{21}]\{M_e\} + [\underline{p}_2]\{F_{R2}\}$$

COMPUTE RELATIVE VELOCITY AND RELATIVE ROTATIONAL VELOCITY OF END EFFECTOR

$$\{\omega_R\} = [B_{12}]\{\omega_2\} - \{\omega_1\}$$

$$\{V_R\} = [B_1]^T\left(\left\{\frac{dR_2}{dt}\right\} - \left\{\frac{dR_1}{dt}\right\}\right) + [\underline{p}_1]\{\omega_1\} - [B_{12}][\underline{p}_2]\{\omega_2\}$$

COMPUTE JOINT RATES

$$\{\dot{\theta}\} = [J]^{-1}\begin{bmatrix} \{\omega_R\} \\ \{V_R\} \end{bmatrix}$$

RETURN

SUBROUTINE RMSFLX

Subroutine RMSFLX is called from the MMD at some integer multiple of the integration step size, just in front of the torque motor model. This routine computes flexibility terms for input to the torque motor model.

START

COMPUTE END EFFECTOR MASSLESS ARM FLEXIBILITY MATRIX

$$[\gamma_E] = \sum_{i=1}^{5} \begin{bmatrix} \ulcorner I \lrcorner & [0] \\ [-\tilde{r}_{i5}] & \ulcorner I \lrcorner \end{bmatrix} \begin{bmatrix} [T_{Bi}] & [0] \\ [0] & [T_{Bi}] \end{bmatrix} \begin{bmatrix} [\gamma^i_{11}] & [\gamma^i_{12}] \\ [\gamma^i_{12}]^T & [\gamma^i_{22}] \end{bmatrix} \begin{bmatrix} [T_{iB}] & [0] \\ [0] & [T_{iB}] \end{bmatrix} \begin{bmatrix} \ulcorner I \lrcorner & [\tilde{r}_{i5}] \\ [0] & \ulcorner I \lrcorner \end{bmatrix}$$

where

$$[\gamma^i_{11}] = \begin{bmatrix} \gamma^i_{11} & 0 & 0 \\ 0 & \gamma^i_{22} & 0 \\ 0 & 0 & \gamma^i_{33} \end{bmatrix} \quad [\gamma^i_{12}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \gamma^i_{26} \\ 0 & \gamma^i_{35} & 0 \end{bmatrix} \quad [\gamma^i_{22}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \gamma^i_{55} & 0 \\ 0 & 0 & \gamma^i_{66} \end{bmatrix}$$

COMPUTE JOINT FLEXIBILITY MATRIX

$$[\gamma_J] = [J]^{-1}[\gamma_E][J]^{T^{-1}}$$

COMPUTE FLEXBILITY TERMS FOR TORQUE MOTOR MODEL TO SIMULATE MASSLESS ARM FLEXIBILITY

$$\gamma_i^e = \gamma_{ii_J} - \{\gamma_i^{21}\}^T[\gamma_{iJ}^{22}]^{-1}\{\gamma_{iJ}^{21}\} \quad i=1,\cdots,6$$

where

$[\gamma_{iJ}^{22}]$ is the 5×5 matrix derived from $[\gamma_J]$ by deleting the ith row and column

$\{\gamma_{iJ}^{21}\}^T$ is the 5×1 matrix derived from $[\gamma_J]$ by taking the ith column and deleting the ith element

RETURN

START

COMPUTE END EFFECTOR MASSLESS ARM FLEXIBILITY MATRIX

$$[\gamma_E] = \sum_{i=1}^{S} \begin{bmatrix} \ulcorner I \lrcorner & [0] \\ [-\tilde{r}_{i5}] & \ulcorner I \lrcorner \end{bmatrix} \begin{bmatrix} [T_{Bi}] & [0] \\ [0] & [T_{Bi}] \end{bmatrix} \begin{bmatrix} [\gamma^i_{11}] & [\gamma^i_{12}] \\ [\gamma^i_{12}]^T & [\gamma^i_{22}] \end{bmatrix} \begin{bmatrix} [T_{iB}] & [0] \\ [0] & [T_{iB}] \end{bmatrix} \begin{bmatrix} \ulcorner I \lrcorner & [r_{i5}] \\ [0] & \ulcorner I \lrcorner \end{bmatrix}$$

where

$$[\gamma^i_{11}] = \begin{bmatrix} \gamma^i_{11} & 0 & 0 \\ 0 & \gamma^i_{22} & 0 \\ 0 & 0 & \gamma^i_{33} \end{bmatrix} \quad [\gamma^i_{12}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \gamma^i_{26} \\ 0 & \gamma^i_{35} & 0 \end{bmatrix} \quad [\gamma^i_{22}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \gamma^i_{55} & 0 \\ 0 & 0 & \gamma^i_{60} \end{bmatrix}$$

COMPUTE JOINT FLEXIBILITY MATRIX

$$[\gamma_J] = [J]^{-1} [\gamma_E] [J]^{T^{-1}}$$

COMPUTE FLEXBILITY TERMS FOR TORQUE MOTOR MODEL TO SIMULATE MASSLESS ARM FLEXIBILITY

$$\gamma^e_i = \gamma_{ii_J} - \{\gamma^{21}_i\}^T [\gamma^{22}_{iJ}]^{-1} \{\gamma^{21}_{iJ}\} \qquad i=1,\cdots,6$$

where

$[\gamma^{22}_{iJ}]$ is the 5×5 matrix derived from $[\gamma_J]$ by deleting the ith row and column

$\{\gamma^{21}_{iJ}\}^T$ is the 5×1 matrix derived from $[\gamma_J]$ by taking the ith column and deleting the ith element

RETURN

SUBROUTINE RMLOAD

Subroutine RMLOAD computes the static forces and moments acting on the arm members at the joints and should be called in the output loop at some integer multiple of the integration step size.

START

COMPUTE FORCES AND MOMENTS AT THE JOINTS ON THE ARM MEMBERS

$$\{M_1\} = [T_{2B}](-\{M_e\} - (\{r_{6B}\} - \{r_{2B}\}) \times \{F_e\})$$

$$\{V_1\} = -[T_{2B}]\{F_e\}$$

$$\{M_2\} = [T_{3B}](\{M_e\} + (\{r_{6B}\} - \{r_{2B}\}) \times \{F_e\})$$

$$\{V_2\} = [T_{3B}]\{F_e\}$$

$$\{M_3\} = [T_{3B}](-\{M_e\} - (\{r_{6B}\} - \{r_{3B}\}) \times \{F_e\})$$

$$\{V_3\} = -[T_{3B}]\{F_e\}$$

$$\{M_4\} = [T_{4B}](\{M_e\} + (\{r_{6B}\} - \{r_{3B}\}) \times \{F_e\})$$

$$\{V_4\} = [T_{4B}]\{F_e\}$$

$$\{M_5\} = [T_{4B}](-\{M_e\} - (\{r_{6B}\} - \{r_{4B}\}) \times \{F_e\})$$

$$\{V_5\} = -[T_{4B}]\{F_e\}$$

$$\{M_6\} = [T_{5B}](\{M_e\} + (\{r_{6B}\} - \{r_{4B}\}) \times \{F_e\})$$

$$\{V_6\} = [T_{5B}]\{F_e\}$$

$$\{M_7\} = [T_{5B}](-\{M_e\} - (\{r_{6B}\} - \{r_{5B}\}) \times \{F_e\})$$

$$\{V_7\} = -[T_{5B}]\{F_e\}$$

$$\{M_8\} = [T_{6B}](\{M_e\} + (\{r_{6B}\} - \{r_{5B}\}) \times \{F_e\})$$

$$\{V_8\} = [T_{6B}]\{F_e\}$$

$$\{M_9\} = -[T_{6B}]\{M_e\}$$

$$\{V_9\} = -[T_{6B}]\{F_e\}$$

RETURN

APPENDIX A

THE RMS JACOBIAN MATRIX

The purpose of this appendix is to develop some kinematical relationships that deal with the computation of translational and rotational velocities in a chain of rigid bodies. These same kinematical relationships appear in relating forces and moments in the chain of bodies. As seen in reference 1, for a system of six joint degrees of freedom like the RMS, the so-called Jacobian matrix becomes an invertible transformation relating configuration space velocity and force quantities to task space velocity and force quantities. These same kinematical relationships appear in the computation of the combined flexibility matrix for the system of rigid bodies.

Suppose that we have a chain of rigid bodies with one degree of rotational freedom at each joint. (For more than one degree of freedom at a joint, a null vector can be assumed for the next position vector, thus putting both rotations at the same point). From figure A-1 we can write the following relationship

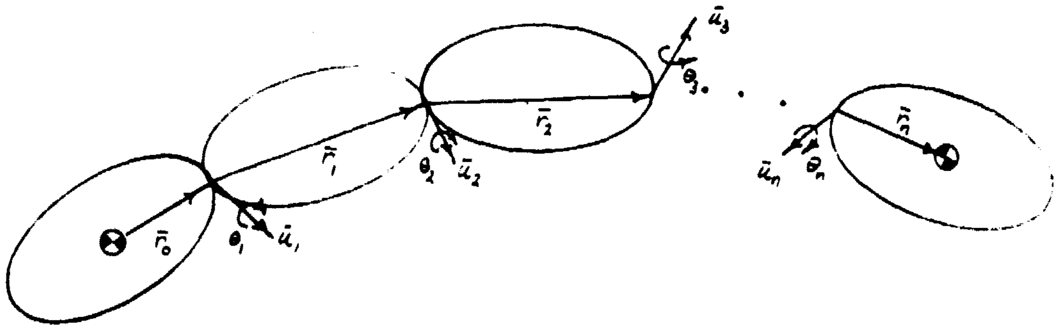

Figure A-1

$$\begin{aligned} \bar{V}_1 &= \bar{V}_0 + \bar{\omega}_0 \times \bar{r}_0 \\ \bar{\omega}_1 &= \bar{\omega}_0 + \dot{\theta}_1 \bar{u}_1 \end{aligned} \tag{A-1}$$

where $\overline{V}_0$ and $\overline{\omega}_0$ are the translational and rotational velocities of body zero (the first body of the chain). A recursive relationship can be written for the velocities of the ith body at joint i in terms of the velocities in body i - 1 at the i - 1 joint.

$$\overline{V}_i = \overline{V}_{i-1} + \overline{\omega}_{i-1} \times \overline{r}_{i-1}$$

$$\overline{\omega}_i = \overline{\omega}_{i-1} + \dot{\theta}_i \overline{u}_i \tag{A-2}$$

Substituting eq. (A-2) into itself, we arrive at a relationship for $\overline{V}_i$ and $\overline{\omega}_i$ in terms of $\overline{V}_0$, $\overline{\omega}_0$, and the joint rates up to the ith body.

$$\overline{V}_i = \overline{V}_0 + \overline{\omega}_0 \times \sum_{j=0}^{i-1} \overline{r}_j + \sum_{j=1}^{i-1} \sum_{k=j}^{i-1} \dot{\theta}_j \overline{u}_j \times \overline{r}_k \tag{A-3}$$

and

$$\overline{\omega}_i = \overline{\omega}_0 + \sum_{j=1}^{i} \overline{u}_j \dot{\theta}_j \tag{A-4}$$

From eqs. (A-3) and (A-4) we can write

$$\begin{bmatrix} \overline{\omega}_i \\ \overline{V}_i \end{bmatrix} = \begin{bmatrix} \ulcorner I \lrcorner & [0] \\ -\sum_{j=0}^{i-1} [\underline{r}_j] & \ulcorner I \lrcorner \end{bmatrix} \begin{bmatrix} \overline{\omega}_0 \\ \overline{V}_0 \end{bmatrix}$$

$$+ \left[\begin{array}{c|c|c|c|c} \overline{u}_1 & \overline{u}_2 & \cdots & \overline{u}_i & \cdots 0 \\ -\sum_{j=1}^{i-1} \overline{r}_j \times \overline{u}_1 & -\sum_{j=2}^{i-1} \overline{r}_j \times \overline{u}_2 & & 0 & 0 \end{array} \right] \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_n \end{bmatrix} \tag{A-5}$$

If we define

$$\overset{6\times 6}{[L_{ij}]} = \overset{6\times 6}{\begin{bmatrix} \ulcorner I \lrcorner & [0] \\ -\sum\limits_{k=i}^{j-1} [\underline{r}_k] & \ulcorner I \lrcorner \end{bmatrix}} \qquad i \neq j \tag{A-6}$$

$$\overset{6\times 6}{[L_{ii}]} = \ulcorner I \lrcorner \tag{A-7}$$

$$\overset{6n\times 6}{[U]} = \begin{bmatrix} \bar{u}_1 & 0 & \cdots \\ 0 & 0 & \cdots \\ 0 & \bar{u}_2 & \cdots \\ 0 & 0 & \bar{u}_n \\ \vdots & & 0 \end{bmatrix} \tag{A-8}$$

$$\{V_i\} = \begin{bmatrix} \bar{\omega}_i \\ \bar{V}_i \end{bmatrix} \quad \text{and} \quad \{\dot{\theta}\} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_n \end{bmatrix} \tag{A-9}$$

Then eq. (A-5) can be written

$$\{V_i\} = [L_{0_i}]\{V_0\} + [[L_{1_i}] \,\vdots\, [L_{2_i}] \cdots \ulcorner I \lrcorner\, [0] \cdots [0]]\,[U]\{\dot{\theta}\} \tag{A-10}$$

If we also define

$$\overset{6\times 6n}{[S_i]} = [[L_{1_i}] \,\vdots\, [L_{2_i}] \cdots \ulcorner I \lrcorner\, [0] \cdots [0]] \tag{A-11}$$

then we can write eq. (A-10) as

$$\{V_i\} = [L_{0_i}]\{V_0\} + [S_i][U]\{\dot{\theta}\} \qquad \text{(A-12)}$$

For the RMS configuration we have seen that the relative velocity between the orbiter and the payload can be written (ref. 1) as

$$\{V_R\} = [J]\{\dot{\theta}\} \qquad \text{(A-13)}$$

Relating eq. (A-13) to eq. (A-12), we see that

$$[J] = [S_6][U] \qquad \text{(A-14)}$$

We have also seen that we can write the output forces and moments from the arm as

$$\{\tau\} = [J]^T \begin{Bmatrix} \bar{M}_e \\ \bar{F}_e \end{Bmatrix} \qquad \text{(A-15)}$$

We will now take a closer look at the force moment kinematic relationships. Consider figure A-2.

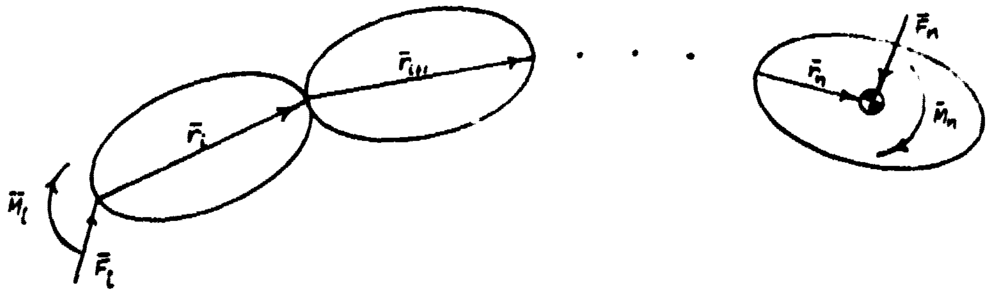

Figure A-2

If we consider this chain to be in static equilibrium, then we can write

$$\bar{F}_i = \bar{F}_n \tag{A-16}$$

and

$$\bar{M}_i = \bar{M}_n + \sum_{j=i}^{n} \bar{r}_i \times \bar{F}_n \tag{A-17}$$

We can write eqs. (A-16) and (A-17) as

$$\begin{bmatrix} \bar{M}_i \\ \\ \bar{F}_i \end{bmatrix} \quad \begin{bmatrix} \lceil I \rfloor + \sum_{j=i}^{n} [\underline{r}_j] \\ \\ 0 \qquad \lceil I \rfloor \end{bmatrix} \begin{bmatrix} \bar{M}_n \\ \\ \bar{F}_n \end{bmatrix} \tag{A-18}$$

Defining

$$\{P_i\} = \begin{bmatrix} \bar{M}_i \\ \\ \bar{F}_i \end{bmatrix} \tag{A-19}$$

and using definitions from above, we can write eq. (A-18) as

$$\{P_i\} = [L_{in+1}]^T \{P_n\} \tag{A-20}$$

We have not discussed the fact that the various vectors given are in different coordinate systems that change relative to each other as the bodies move and the angles change. We will now write these transformations between the coordinate systems fixed in each of the moving bodies. For simplicity without loss in generality, it will be assumed that all of the body fixed coordinate systems are aligned when all the angles are zero.

We will now adopt the following notation: $[B_{ij}]$ denotes the transformation from the coordinate system fixed in body j to the coordinates system fixed in body i. We will also note that

$$[B_{i\ i-1}] = (\lceil I \rfloor - \sin \theta_i\ [\underline{u}_i] + (i - \cos \theta_i)[\underline{u}_i]^2) \tag{A-21}$$

and that

$$[B_{ij}] = [B_{i\ i-1}][B_{i-1\ i-2}]\cdots[B_{j+2\ j+1}][B_{j+1\ j}] \quad i \geq j \qquad \text{(A-22)}$$

We will assume that the matrices defined in the first part of this appendix are written such that everything is transformed to the 0 system unless otherwise specified.

APPENDIX B

CALCULATION OF THE ARM FLEXIBILITY MATRIX

The purpose of this appendix is to present a set of formulas for computing the arm flexibility matrix or the flexibility matrix for a chain of bodies. By flexibility matrix is meant the matrix that relates the forces applied to the end of a cantilevered chain of bodies to the resulting static-elastic displacement. For a given cantilevered body (fig. B-1), we can write

$$\overset{6\times 6}{[\gamma]} \overset{6\times 1}{\begin{bmatrix} \{M\} \\ \{F\} \end{bmatrix}} = \overset{6\times 1}{\begin{bmatrix} \{\delta\psi\} \\ -\{\delta\} \end{bmatrix}} \tag{B-1}$$

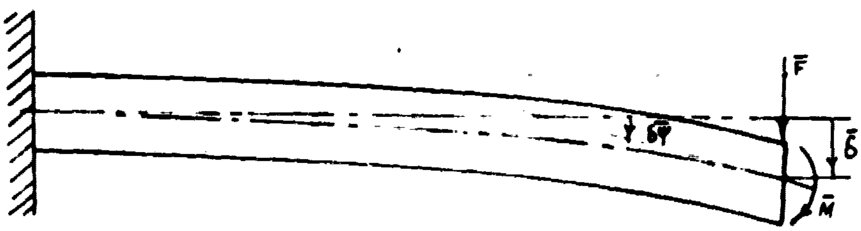

Figure B-1

Suppose the body depicted in figure B-1 is the ith body in a chain; if we apply a load to the end of the chain of bodies, we can compute the load at the ith joint in terms of the end load. Assuming $\bar{r}_n = \bar{0}$, from eq. (A-20) we have the load at the ith hinge as

$$\{P_i\} = [L_{in}]^T\{P_n\} \tag{B-2}$$

The elastic deflection at the ith joint can be written as

$$\{\delta_i\} = [\gamma_i]\{P_i\} = [\gamma_i][L_{in}]^T\{P_n\} \tag{B-3}$$

The resulting deflection at the end of the beam from an elastic deflection at the ith joint assuming small angular deflection is

$$\{\delta_n^i\} = [L_{in}]\{\delta_i^n\} = [L_{in}][\gamma_i][L_{in}]^T \{P_n\} \tag{B-4}$$

The total deflection at the end of the beam can now be written as the sum of the deflections expressed in eq. (B-4).

$$\{\delta_n\} = \sum_{i=1}^{n} \{\delta_n^i\} = \sum_{i=1}^{n} [L_{in}][\gamma_i][L_{in}]^T\{P_n\} = [\gamma]\{P_n\} \tag{B-5}$$

So we see that

$$[\gamma] = \sum_{i=1}^{n} [L_{in}][\gamma_i][L_{in}]^T \tag{B-6}$$

We will assume that the $[\gamma_i]$ matrices in the equations above are sparse matrices. Looking at figure B-2 we will assume that the $[\gamma_i]$ matrices have the following form

$$[\gamma_i] = \begin{bmatrix} \gamma_{11}^i & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_{22}^i & 0 & 0 & 0 & \gamma_{26}^i \\ 0 & 0 & \gamma_{33}^i & 0 & \gamma_{35}^i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_{53}^i & 0 & \gamma_{55}^i & 0 \\ 0 & \gamma_{62}^i & 0 & 0 & 0 & \gamma_{66}^i \end{bmatrix} \tag{B-7}$$

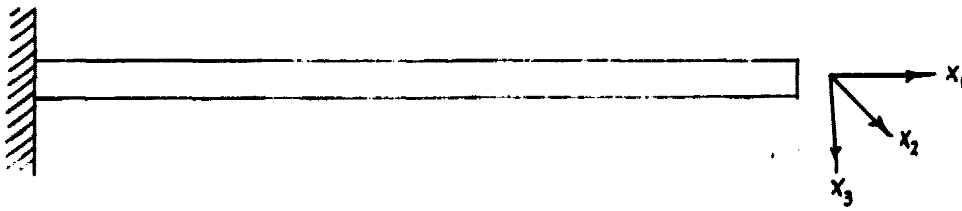

Figure B-2

It is noted that $[\gamma_i]$ is symmetric so there are only seven independent non-zero values in $[\gamma_i]$. All the elements of eq. (B-7) are positive numbers except for γ^i_{26} and γ^i_{62}, which are negative.

APPENDIX C

LAGRANGIAN APPROACH TO THE RIGID BODY EQUATIONS OF MOTION FOR A CHAIN OF RIGID BODIES

The Rigid Body equations of motion for a chain of bodies will be developed in this appendix, using the relationships developed in Appendix A. After writing these equations in rather general terms, we will assume that we have six joints and that all of the inner bodies are massless so that the approximations made in assuming that the arm is massless can be noted.

We will now look at the kinetic energy of the ith body in a chain of rigid bodies (fig. C-1).

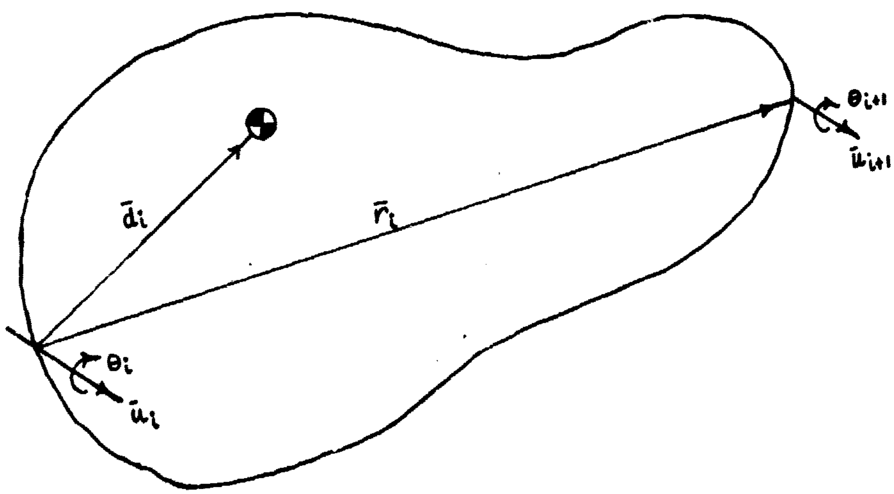

Figure C-1

$$T_i = \frac{1}{2} m_i \overline{V}_i^{CM} \lceil I \rfloor \overline{V}_i^{CM} + \overline{\omega}_i \cdot \overline{\Lambda}_i \overline{\omega}_i \qquad \text{(C-1)}$$

The kinetic energy of the ith body written in terms of the velocities at the ith joint is

$$\overline{V}_i = \overline{V}_i^{CM} - \overline{\omega}_i \times \overline{d}_i \tag{C-2}$$

or

$$\overline{V}_i^{CM} = \overline{V}_i + \overline{\omega}_i \times \overline{d}_i = \overline{V}_i - \overline{d}_i \times \overline{\omega}_i \tag{C-3}$$

Substituting eq. (C-3) into eq. (C-1)

$$T_i = \frac{1}{2} \begin{bmatrix} \overline{\omega}_i \\ \overline{V}_i \end{bmatrix}^T \left[\begin{array}{c:c} [\Lambda_i] - m_i [\underline{d}_i]^2 & -m_i [\underline{d}_i] \\ m_i [\underline{d}_i] & m_i \ulcorner I \lrcorner \end{array}\right] \begin{bmatrix} \overline{\omega}_i \\ \overline{V}_i \end{bmatrix} = \frac{1}{2} \{V_i\}^T [A_i] \{V_i\} \tag{C-4}$$

From eq. (C-4) we can write the kinetic energy of the entire chain of rigid body as

$$T = \frac{1}{2} \sum_{i=0}^{n} \{V_i\}^T [A_i] \{V_i\} \tag{C-5}$$

Using eq. (A-12) in eq. (C-5) we can write

$$T = \frac{1}{2} \sum_{i=0}^{n} \left(\{V_0\}^T [L_{0_i}]^T + \{\dot{\theta}\}^T [U]^T [S_i]^T \right) [A_i] \left([L_{0_i}] \{V_0\} + [S_i][U]\{\dot{\theta}\} \right)$$

$$= \frac{1}{2} \begin{bmatrix} \{V_0\} \\ \{\dot{\theta}\} \end{bmatrix}^T \begin{bmatrix} \sum_{i=0}^{n} [L_{0_i}]^T [A_i][L_{0_i}] & \sum_{i=0}^{n} [L_{0_i}]^T [A_i][S_i][U] \\ \sum_{i=0}^{n} [U]^T [S_i]^T [A_i][L_{0_i}] & \sum_{i=0}^{n} [U]^T [S_i]^T [A_1][S_1][U] \end{bmatrix} \begin{bmatrix} \{V_0\} \\ \{\dot{\theta}\} \end{bmatrix} \tag{C-6}$$

From eq. (C-6) we can write

$$T = \frac{1}{2}\{\dot{q}\}^T[A]\{\dot{q}\} \tag{C-7}$$

where

$$\{\dot{q}\} = \begin{bmatrix} \{V_0\} \\ \{\dot{\theta}\} \end{bmatrix} \tag{C-8}$$

and in general [A] is a function of the q_i's.

This completes writing expressions for the kinetic energy of the system. As we are dealing with rigid bodies, we will assume that the strain energy of the system is zero. To complete the ingredients for Lagrange equations, we must formulate the virtual work of the system to define the generalized forces, the virtual work due to the torque motors at the joints can be written

$$\delta W_J = \{\delta\theta\}^T\{\tau\} = \delta t\{\dot{\theta}\}\{\tau\} \tag{C-9}$$

The virtual work of the external forces and moments applied to the ith body can be written as

$$\delta W_e = \sum_{i=0}^{n} \{\delta R_i\}^T \begin{Bmatrix} \bar{M}_i \\ \bar{F}_i \end{Bmatrix} = \delta t \sum_{i=0}^{n} \{V_i\}^T \begin{Bmatrix} \bar{M}_i \\ \bar{F}_i \end{Bmatrix} \tag{C-10}$$

Using eq. (A-12) in eq. (C-10) we have

$$\delta W = \delta W_J + \delta W_C = \{\delta\theta\}^T\{\tau\} + \delta t \sum_{i=0}^{n} \left(\{V_0\}^T [L_{0_i}]^T + \{\dot{\theta}\}^T [U]^T [S_i]^T \right) \begin{Bmatrix} \bar{M}_i \\ \bar{F}_i \end{Bmatrix}$$

$$= \{\delta R_0\}^T \sum_{i=0}^{n} [L_{0_i}]^T \begin{Bmatrix} \bar{M}_i \\ \bar{F}_i \end{Bmatrix} + \{\delta\theta\}^T \left(\{\tau\} + \sum_{i=0}^{n} [U]^T [S_i]^T \begin{Bmatrix} \bar{M}_i \\ \bar{F}_i \end{Bmatrix} \right) \tag{C-11}$$

or

$$\delta W = \{\delta q\}^T \begin{bmatrix} \sum_{i=0}^{n} [L_{0_i}]^T \begin{Bmatrix} \overline{M}_i \\ \overline{F}_i \end{Bmatrix} \\ \text{- - - - - - - - - - - - -} \\ \{\tau\} + \sum_{i=0}^{n} [U]^T [S_i]^T \begin{Bmatrix} \overline{M}_i \\ \overline{F}_i \end{Bmatrix} \end{bmatrix} \tag{C-12}$$

and since

$$\delta W = \{\delta q\}^T \{Q\} \tag{C-13}$$

where {Q} are the generalized forces, we have completed the development of the terms needed.

Substituting eq. (C-5) into Lagrange's equations we have

$$\frac{d}{dt}([A]\{\dot{q}\}) - \frac{\partial}{\partial\{q\}}(\{\dot{q}\}^T[A]\{\dot{q}\}) = \{Q\} \tag{C-14}$$

or

$$[A]\{\ddot{q}\} + \left(\sum_{j=7}^{6+n} q_j \left[\frac{\partial A}{\partial q_j}\right]\right)\{\dot{q}\} - \begin{bmatrix} \{0\}^T \\ \vdots \\ \{0\}^T \\ \{\dot{q}\}^T \left[\frac{\partial A}{\partial q_7}\right] \\ \vdots \\ \{\dot{q}\}^T \left[\frac{\partial A}{\partial q_{6+n}}\right] \end{bmatrix} \{\dot{q}\} = \{Q\} \tag{C-15}$$

or

$$[A]\{\ddot{q}\} + [B]\{\dot{q}\} = \{Q\} \tag{C-16}$$

where

$$[B] = \sum_{j=7}^{6+n} q_j \left[\frac{\partial A}{\partial q_j}\right] - \begin{bmatrix} \{0\}^T \\ \vdots \\ \{0\}^T \\ \{\dot{q}\}^T \left[\frac{\partial A}{\partial q_7}\right] \\ \vdots \\ \{\dot{q}\}^T \left[\frac{\partial A}{\partial q_{6+n}}\right] \end{bmatrix} \tag{C-17}$$

Let us now go back to eq. (C-5) and assume that

$$[A_i] = 0 \qquad i = 1,\cdots,5 \qquad n = 6$$

then

$$T = \frac{1}{2}\{V_0\}^T[A_0]\{V_0\} + \frac{1}{2}\{V_6\}^T[A_6]\{V_6\} \tag{C-18}$$

Also

$$T = \frac{1}{2}\begin{bmatrix}\{V_0\} \\ \\ \{\dot{\theta}\}\end{bmatrix}^T \begin{bmatrix} [A_0] + [L_{0_6}]^T[A_6][L_{0_6}] & [L_{0_6}][A_6][S_6][U] \\ \\ [U]^T[S_6]^T[A_6][L_{0_6}] & [U]^T[S_6]^T[A_6][S_6][U] \end{bmatrix} \begin{bmatrix}\{\overline{V}_0\} \\ \\ \{\dot{\theta}\}\end{bmatrix} \tag{C-19}$$

Now from eq. (A-14) we see that

$$T = \frac{1}{2}\begin{bmatrix}\{V_0\}^T \\ \\ \{\dot{\theta}\}\end{bmatrix} \begin{bmatrix} [A_0] + [L_{0_6}]^T[A_6][L_{0_6}] & [L_{0_6}][A_6][J] \\ \\ [J]^T[A_6][L_{0_6}] & [J]^T[A_6][J] \end{bmatrix} \begin{bmatrix}\{V_0\} \\ \\ \{\dot{\theta}\}\end{bmatrix}$$

As we have seen for the arm, [J] is an invertible transformation (except for singularity points). So eq. (C-18) is a valid form of kinetic energy with a valid set of 12 generalized coordinates. If we assume that the external forces and moments on the interior bodies are zero, then we need only to determine the form to the contributions to the generalized forces from the torque motors.

$$\delta W_J = \{\delta\theta\}^T\{\tau\} = \delta t\{\dot{\theta}\}^T\{\tau\} \tag{C-20}$$

also

$$[J]\{\dot{\theta}\} = \{V_R\} = \{V_6\} - [L_{on}]\{V_0\} \tag{C-21}$$

or

$$\{\dot{\theta}\} = [J]^{-1}\{V_6\} - [J]^{-1}[L_{on}]\{V_0\} \tag{C-22}$$

$$\begin{aligned} \delta W_J &= \delta t(\{V_6\}^T[J]^{-1^T} - \{V_0\}^T[L_{on}]^T[J]^{-1^T})\{\tau\} \\ &= \{\delta R_6\}^T[J]^{-1^T}\{\tau\} - \{\delta R_0\}^T[L_{on}]^T[J]^{-1^T}\{\tau\} \end{aligned} \tag{C-23}$$

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