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SUMMARY

Mode I stress intensity coefficients were obtained for an edge-cracked disk (Round Compact Specimen). Results for this plane elastostatic problem, obtained by a boundary collocation analysis are presented for ratios $0.35 < A/D < 1$, where A is the crack length and D is the disk diameter. The results presented are for two complementary types of loading. By superposition of these results the stress intensity factor K_I, for any practical load line location of a pin-loaded round compact specimen can be obtained.

INTRODUCTION

The round compact specimen (edge-cracked disk) is currently being considered by ASTM Committee E-24 on Fracture Testing for incorporation into ASTM Standard Method of Test E 399 on Plane Strain Fracture Toughness of Metallic Materials. In reference 1 values of the mode I stress intensity coefficients Γ_p and Γ_M for round compact specimens are given for the range of ratio of crack length to specimen diameter, A/D, from 0.65 to 0.9 (where crack length A here is the distance from the crack tip to the specimen circumference). The coefficients Γ_p and Γ_M apply to two complementary types of specimen loading; Γ_p to a nominal uniform distribution of stress across the net section, and Γ_M to a nominal bending stress distribution (fig. 1). While these types of loading are impractical in themselves, the two coefficients can be combined to represent any practical case of loading of the specimen by a pair of equal and opposite forces normal to the crack (pin loading). The appropriate value of the stress intensity factor is then obtained from this combination of coefficients. The approach used in reference 1 was to obtain values of Γ_p and Γ_M by the boundary collocation method for a set of ring segment specimens (ref. 2) with successively diminished ratios of internal to external radius to a minimum ratio of 0.07. These results were then extrapolated to estimate values for the limit case of zero internal radius, corresponding to the edge-cracked disk.

To extend and assess the results of reference 1, a more direct approach has now been taken which employs boundary value stress functions and derivatives appropriate to the cracked disk. The present results cover the range of ratio of crack length to specimen diameter, A/D, from 0.35 to 1. These results are given both in tabular form and in the form of interpolation equations obtained by multiple linear and nonlinear regression analysis. It should be appreciated that there is a limitation on the applicability of the present results to practical pin-loaded cracked disk specimens. When the crack tip is close to the load line there will be a significant difference between the actual distribution of loading forces and that assumed in the present model, and the values of the corresponding stress intensity factors will be significantly different. It is estimated that this effect will be negligible when the distance of the crack tip from the load line exceeds 0.15 D.

LIST OF SYMBOLS

A	crack length measured from crack tip to the circumference
a	crack length measured from the crack tip to load line
B	specimen thickness
D	disk diameter
K_I	Mode I stress intensity factor $K_I = K_{IM}$ when $\sigma_p = 0$, and $K_I = K_{Ip}$ when $\sigma_M = 0$
L	load line location
M	resultant moment at nominal neutral axis of specimen
P	applied pin load
$W=(D/2+L)$	distance measured from load line to circumference of specimen
$\Gamma=\dfrac{K_I}{(\sigma_p+\sigma_M)\sqrt{A(1-A/D)}}$	stress intensity coefficient $\Gamma = \Gamma_p$ when $\sigma_M = 0$, and $\Gamma = \Gamma_M$ when $\sigma_p = 0$
$\sigma_M=6M/[B(D-A)^2]$	component of fictitious normal net stress at the crack tip due to M
$\sigma_p=P/[B(D-A)]$	component of fictitious normal net stress due to load P
χ	stress function

APPROACH

As shown in figure 1, the cracked round specimen has a diameter D and is loaded through pins by a couple M_o and opposed tensile forces P normal to the crack with load line offset a distance L from the center of the specimen. Calculations were performed for each of two complementary types of loading: case 1, nominal uniform tension and case 2, nominal bending. For case 1, provided the ratio a/D ($=A/D + L/D - 1/2$) is at least 0.15, and regardless of the load line location L, the computed stress intensity factor K_{Ip} is a function of two variables: nominal uniform stress σ_p and crack length ratio A/D. This case is obtained by adjusting the boundary load condition along A B to produce at the nominal neutral axis position $(D-A)/2$, a resultant moment of zero, and force equal to the applied load P.

For case 2, provided the ratio a/D is at least 0.15, and regardless of the load line location L the computed stress intensity factor K_{IM} is a function of two variables: nominal bending stress at the crack tip σ_M, and crack length ratio A/D. This case is obtained by applying only a courle at boundary A B.

Stress intensity coefficienty Γ_p, Γ_M and Γ are defined as

$$\left.\begin{aligned}
\Gamma_p &= \frac{K_{Ip}}{\sigma_p \sqrt{A(1 - A/D)}} \\
\Gamma_M &= \frac{K_{IM}}{\sigma_M \sqrt{A(1 - A/D)}} \\
\Gamma &= \frac{K_I}{(\sigma_p + \sigma_M) \sqrt{A(1 - A/D)}}
\end{aligned}\right\} \tag{1}$$

For the general case (fig. 1), there are two independent variables L/D and A/D. As an illustration, for a disk with load P and couple M_o at load line location L:

$$\sigma_p = \frac{P}{B(D - A)}$$

and

$$\sigma_M = \frac{3[2M_o + P(2L + A)]}{B(D - A)^2}$$

By superposition we have

$$K_I = K_{IP} + K_{IM} \tag{2}$$

Applying equation (1) and equation (2) we obtain

$$\Gamma = \left(\frac{\sigma_p}{\sigma_p + \sigma_M}\right)\Gamma_P + \left(\frac{\sigma_M}{\sigma_p + \sigma_M}\right)\Gamma_M \tag{3}$$

Replacing σ_p and σ_M by their equivalent definitions we obtain

$$\Gamma = \frac{P(D - A)\Gamma_P + 3[2M_o + P(2L + A)]\Gamma_M}{P(D - A) + 3[2M_o + P(2L + A)]} \tag{4}$$

If at loadline L, a load P is applied through pins and the moment $M_o = 0$, after algebraic manipulation of equation (4) we obtain a more common form

$$\frac{KB\sqrt{D}}{P} = \sqrt{\frac{A/D}{1 - (A/D)}}\left[\Gamma_P + \frac{3(A/D) + (2L/D)}{1 - (A/D)}\Gamma_M\right] \tag{5}$$

RESULTS AND DISCUSSION

As shown in figure 1, the resultant solution of the cracked disk problem is obtained by combining two complementary types of loading.

The first type of loading is based on a nominal constant mid-net section stress $\sigma_p = P/[B(D-A)]$ resulting in the stress intensity coefficient Γ_p as a function of σ_p and A/D. The second solution is based on a nominal pure bending stress at the mid net section where $\sigma_M = 3[P(A+2L)+M_o]/[B(D-A)^2]$ from which the stress intensity coefficient Γ_M as a function of σ_M and A/D is obtained.

Γ_p and Γ_M are obtained using the boundary collocation method with 60 boundary stations and an overdetermined system of equations as detailed in reference 3. Twenty equally spaced boundary stations were taken along the straight portion of the boundary A B and 40 stations equally spaced were taken along the curved boundary B C (fig. A1).

The appropriate stress function boundary conditions for both cases are given in the appendix. The limit values for Γ_p and Γ_M as $A/D \rightarrow 1$ were obtained from reference 4. The values of Γ_p and Γ_M given in table I were obtained for a load line offset $L = 0.35D$. The values given in table II of Γ were obtained directly by boundary collocation with the load line $L = D/4$.

It has been shown (ref. 5) that the pin loaded holes can have a significant effect on the stress intensity factor. It is therefore estimated that the results presented herein will apply when the distance from the crack tip to the load line exceeds 0.15 D.

Table II (eq. (4) with $M_o = 0$) contains a comparison of the present results with those of references 1, 6, and 7, for a pin loaded cracked disk with load line at $L = D/4$. Excellent agreement is obtained. Included in this table are the stress intensity coefficients of the standard rectangular compact specimen (ref. 8). A geometric comparison between the standard rectangular and round compact specimen is given in figure 2. Fitting functions for Γ_p and Γ_M were obtained by linear and nonlinear least squares best fit regression analyses respectively. These functions are:

$$\Gamma_M = 5.611 - 21.246 \frac{A}{D} + 38.149 \left(\frac{A}{D}\right)^2 - 32.169 \left(\frac{A}{D}\right)^3 + 10.320 \left(\frac{A}{D}\right)^4$$

and

$$\Gamma_p = 0.501 + \frac{0.02104[(A/D) - 0.525]}{[(A/D) - 0.525][(A/D) - 0.0798] + 0.0625}$$

These functions are considered to be accurate to less than a percent of the computed solution in the range $0.35 < A/D < 1$. Clearly, the accuracy of application is dependent upon how well the assumed model boundary conditions approximate the real boundary conditions.

EXAMPLE

Two examples now follow which will demonstrate the use of table I in conjunction with equation (4) and equation (5).

Example 1

For a pin loaded disk with load P at load line location $L = D/4$, to find the value of $KB\sqrt{W}/P$ where $W = 0.75\ D$ and $a/W = 0.4$. Thus

$a = A - 0.25\ D$, $A/D = 0.55$ and from table I, $\Gamma_p = 0.509$ and $\Gamma_M = 1.064$. From equation (5) since $M_o = 0$, we have $KB\sqrt{W}/P = \sqrt{3/4}\ KB\sqrt{D}/P = 7.618$.

There is a small difference between this value and the value 7.613 given in table II. The values in table II are obtained by direct application of the boundary conditions for this loading and the value 7.618 in the above example is obtained by combination.

Example 2

Given a disk with crack length A, load P, and couple M_o at $L = 0$, to find the resultant stress intensity coefficient Γ. From table I we obtain Γ_p and Γ_M and using equation (4) we have

$$\Gamma = \frac{P(D - A)\Gamma_p + 3(2M_o + PA)\Gamma_M}{PD + 6M_o + 2PA}$$

APPENDIX

The results presented here were obtained by boundary collocation analysis of a homogeneous isotropic specimen under plane elastostatic conditions. This method is described in detail in reference 3. The necessary boundary conditions to be satisfied by the stress function and its normal derivative (fig. A1) are as follows

Along Ⓐ Ⓑ

$$\chi = \rho P \phi \left(\frac{-\sin\alpha \sin\phi}{2\alpha + \sin 2\alpha} + \frac{\cos\alpha \cos\phi}{2\alpha - \cos 2\alpha}\right) + \frac{M_o}{2}\left(\frac{\sin 2\phi - 2\phi \cos 2\alpha}{2\alpha \cos 2\alpha - \sin 2\alpha} - 1\right)$$

$$+ \rho P \cos\phi \left(\frac{-\sin^3\alpha}{2\alpha + \sin 2\alpha} + \frac{\alpha \cos\alpha - \sin\alpha \cos^2\alpha}{2\alpha - \sin 2\alpha}\right)$$

$$- P\rho \sin\phi \left(\frac{\alpha \sin\alpha + \cos\alpha \sin^3\alpha}{2\alpha + \sin 2\alpha} + \frac{\cos^3\alpha}{2\alpha - \sin 2\alpha}\right)$$

and

$$\frac{\partial\chi}{\partial x} = \frac{\partial\chi}{\partial\rho} \cos(\phi + \alpha) - \frac{\partial\chi}{\rho\partial\phi} \sin(\phi + \alpha)$$

Along Ⓑ Ⓒ

$$\chi = P \frac{D}{2} (\cos \Phi + \sin 2\alpha) + Pe - M_o$$

and

$$\frac{\partial\chi}{\partial R} = P \cos \Phi$$

The symbols ρ, ϕ, α, M_o, Φ, and e are defined in figure A1.

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TABLE I. - VALUES OF STRESS INTENSITY COEFFICIENTS

Γ_P AND Γ_M FOR THE EDGE CRACKED DISK

WHEN $a/D > 0.15$

A/D	Γ_P	Γ_M
0.35	0.2587	1.6157
.40	.3857	1.4172
.45	.4538	1.2695
.50	.4901	1.1551
.55	.5090	1.0637
.60	.5184	.9865
.65	.5228	.9255
.70	.5246	.8717
.75	.5251	.8252
.80	.5251	.7845
.85	.5249	.7487
.90	.5242	.7168
.95	.522	.682
[a]1.00	.521	.663

[a]Limit values obtained from ref. 4.

TABLE II. - COMPARISON OF PRESENT STRESS INTENSITY PARAMETER $K_I B\sqrt{W}/P$ AS A FUNCTION OF a/W WITH THOSE RESULTS OF REFERENCES 1, 6, 7, AND 8 (AS SHOWN IN FIG. 2 FOR L/D = 0.25 AND W = 0.75 D)

a/W	Direct collocation solid disk	Collocation ring segment Ref. 1	Experimental results Ref. 6	Finite elements Ref. 7	Compliance results Ref. 7	Standard rectangular compact specimen Ref. 8
	$K_I B\sqrt{W}/P$					
0.200	4.745	------	------	------	------	4.750
.267	5.472	------	------	------	------	------
.300	------	------	------	------	------	5.844
.333	6.365	------	6.244	------	------	------
.400	7.613	------	7.299	------	------	7.333
.467	9.203	------	8.948	------	------	------
.500	------	------	------	10.185	10.286	9.631
.533	11.378	11.926	11.314	------	------	------
.600	14.532	14.820	14.721	14.738	14.566	13.62
.667	19.368	19.518	19.680	------	------	------
.700	------	------	------	22.653	22.798	21.56
.733	27.411	27.490	26.879	------	------	------
.800	42.773	42.745	------	42.542	------	41.07
.867	79.542	79.523	------	------	------	------
.933	226.	------	------	------	------	------

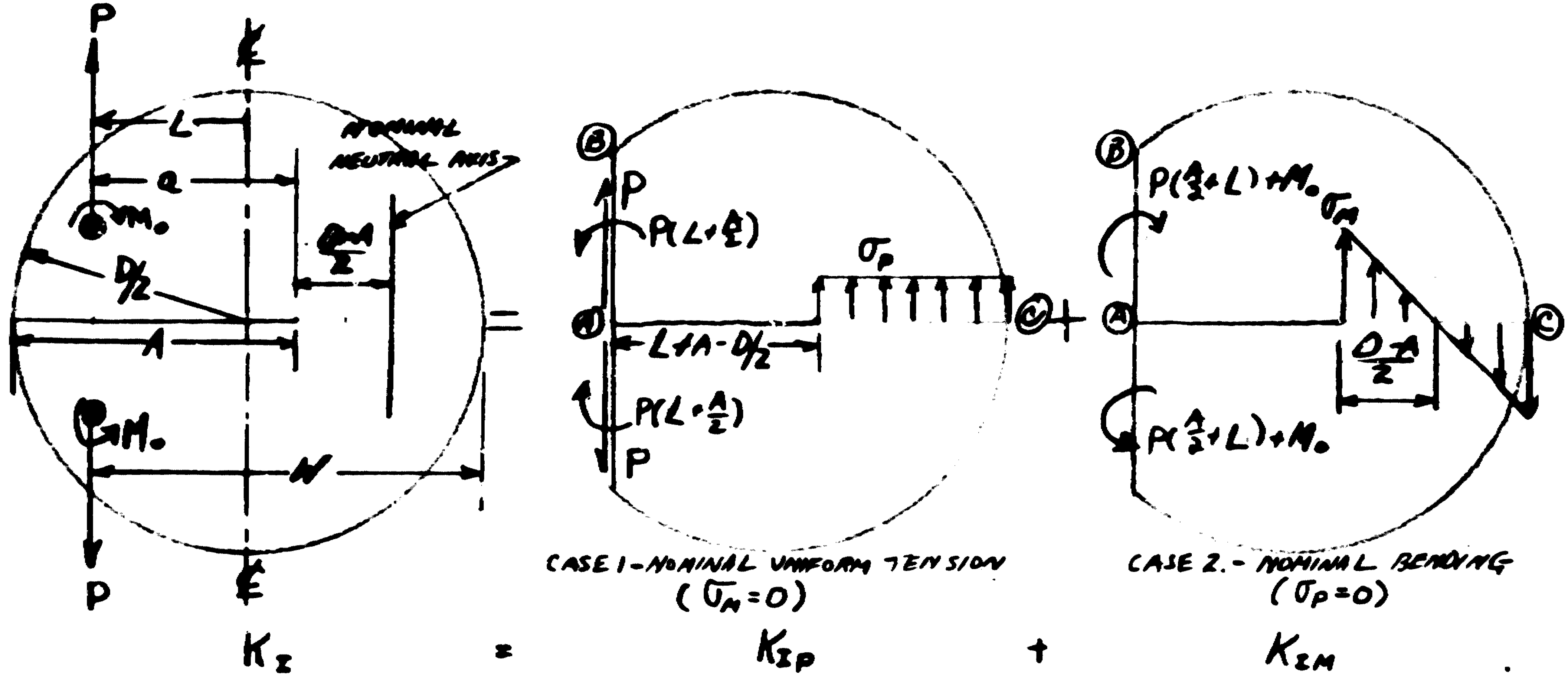

FIGURE 1. APPLICATION OF SUPERPOSITION PRINCIPLE TO SPECIMEN LOADED THROUGH PINS AT A SELECTED DISTANCE L FROM THE CENTERLINE OF THE DISK

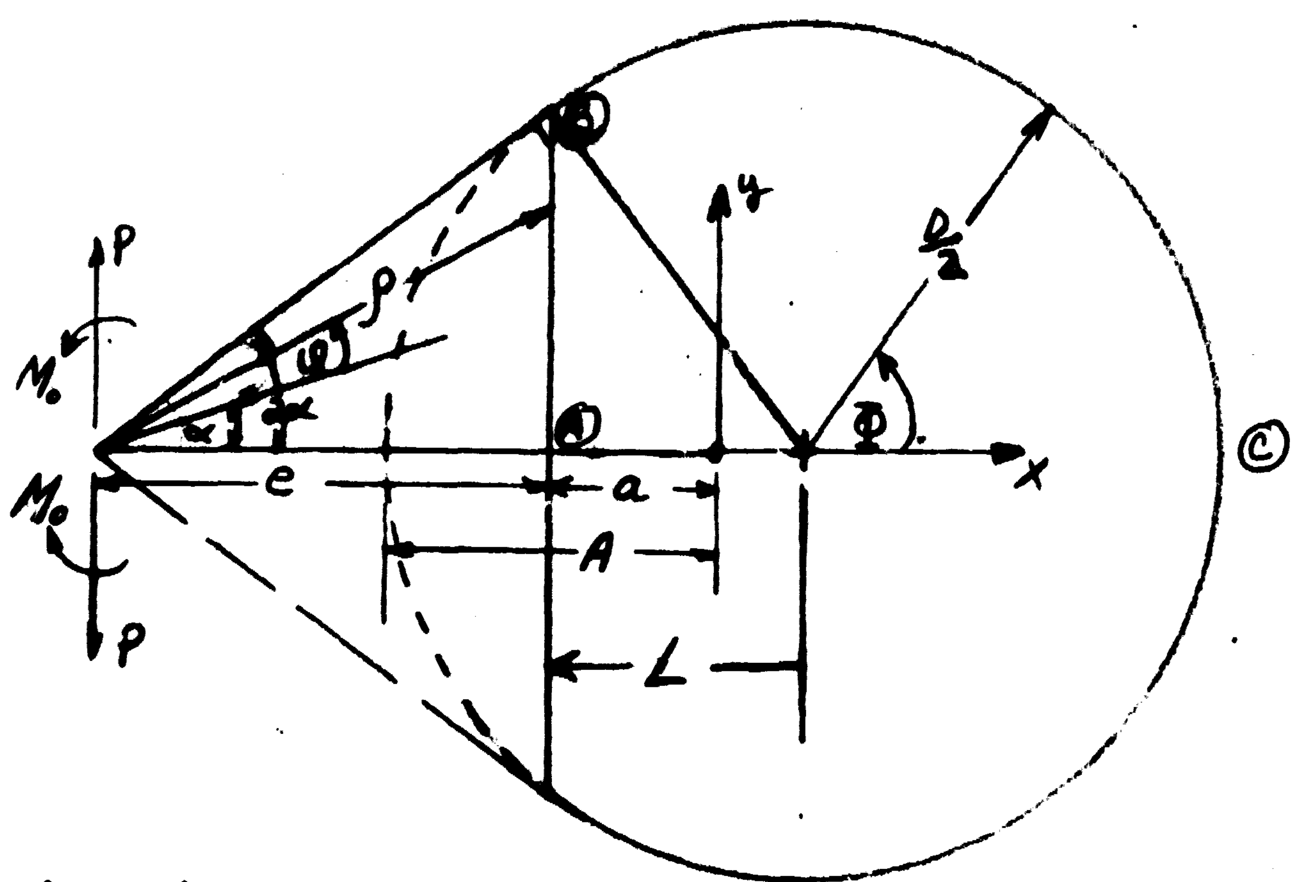

FIGURE A1. ANALYTICAL MODEL OF CRACKED CIRCULAR DISK.
(WHEN PIN LOADED AT LOAD LINE L, $M_o = Pe$)

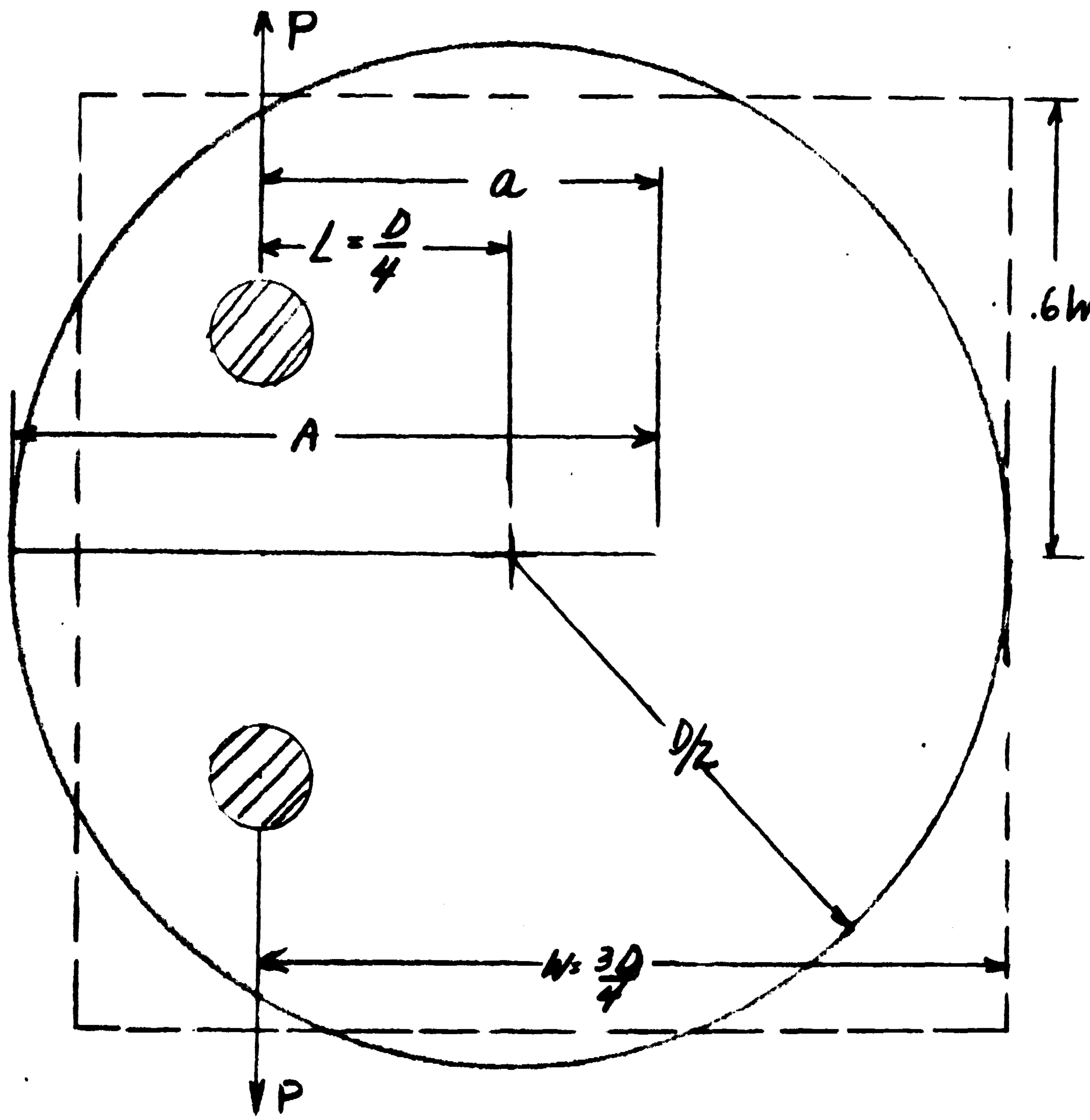

FIGURE 2. GEOMETRIC COMPARISON OF THE ROUND COMPACT SPECIMEN WITH THE STANDARD RECTANGULAR COMPACT SPECIMEN FOR LOAD LINE LOCATION $L = D/4$