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ANALYTIC STUDIES OF LOCAL-SEVERE-STORM OBSERVABLES BY SATELLITES

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16. Abstract

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Abstract (Continued)

descent of tropopause-level, nonrotating air down the central core of a tornado is the only plausible mechanism for attaining 90 m/s swirl speeds. Finally, the need for a stability study to indicate the parametric conditions under which a one-cell vortical storm (tornado cyclone) becomes unstable and passes over into a more intense two-cell structure (rotating thunderstorm with a tornado) is emphasized. Of interest is the degree of interaction with surrounding atmospheric systems, which may explain the longer lifespan of the (more interactive) tornado cyclone relative to the (less interactive) tornado.

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SUMMARY

In the absence of definitive measurement, the nature of the secondary flow in a tornado has long been a subject of controversy. Attention is here concentrated on the exceptionally violent whirlwind, often characterized by a fairly vertical axis of rotation. For a cylindrical polar coordinate system with axis coincident with the axis of rotation, the secondary flow involves the radial and axial velocity components. The thesis advanced is, first, that a violent whirlwind is characterized by swirl speeds relative to the axis of rotation on the order of 90 m/s, with 100 m/s being close to an upper bound. This estimate is based on interpretation of funnel-cloud shape (which also suggests properties of the radial profile of swirl, as well as the maximum magnitude); an error assessment of the funnel-cloud interpretation procedure is developed. Second, computation of ground-level pressure deficits achievable from typical tornado-spawning ambients by idealized thermohydrostatic processes suggests that a two-cell structure is required to sustain such large swirl speeds; that is, moist-adiabatic ascent of low-level air is insufficient, and the compressional heating via dry-adiabatic descent of tropopause-level, nonrotating air down the central core of a tornado is the only plausible mechanism for attaining 90 m/s swirl speeds. Finally, the need for a stability study to indicate the parametric conditions under which a one-cell vortical storm (tornado cyclone) becomes unstable and passes over into a more intense two-cell structure (rotating thunderstorm with a tornado) is emphasized. Of interest is the degree of interaction with surrounding atmospheric systems, which may explain the longer lifespan of the (more interactive) tornado cyclone relative to the (less interactive) tornado.

INTRODUCTION

This report considers a specific topic concerning tornadoes: what observables, if any, can be identified such that the existence of a particularly severe tornado can be detected by reasonable improvements in current geosynchronous weather satellite instrumentation? Questions concerning the stabilization of the satellite to scan a specific suspicious locale, the resolution limits of active and passive sensors, the rapidity of computer processing of the data, and the dissemination of the pertinent information shall not be considered in the ensuing discussion (ref. 1). It is noted that current capability of geosynchronous meteorological satellites is about 750 m resolution by passive (i.e., radiative-energy-receiving only) optical instrumentation, operative in the infrared and visible portions of the electromagnetic spectrum; though normally a thirty-minute interval is used, pictures may be taken as rapidly as every five minutes (ref. 2). Characterization of a "lower tropospheric" state and an "upper tropospheric" state with respect to temperature and moisture is possible in clear areas, or areas under scattered cloud cover; movement of intermediate-sized clouds can be used to infer wind motion, though such results are less accurate over land (ref. 3). Significant improvement in weather satellite instrumentation may be anticipated on the time scale of a decade or more, when active (microwave) sensors may be available. Currently, the top cloud deck is "impenetrable".

Of course, prediction of the onset of a severe tornado is preferable to detection of an already fully developed, highly dangerous whirlwind. However, since a very severe tornado can persist for more than an hour and can leave a damage path several hundred kilometers long (ref. 4), early detection is extremely valuable. Realistically, severe tornado detection remains (along with hailstorm occurrence) the most practical goal of current severe-local-storm (i.e., mesoscale) satellite meteorology. It may be noted that alternative, ground-based instrumentation (radar) can detect only already developed tornadoes occurring within about 100 km (ref. 4), and then only with a large percentage of false alarms (ref. 5). Although instrumentation to monitor intense sferic activity as an indicator of tornado occurrence has received extensive publicity in recent years, the reliability of tornado forecasts based upon such methods [perhaps limited to a fifty-percent false-alarm rate (ref. 6)] has proved rather low in field tests.

Thesis of This Investigation; Directions for Future Research

This literature on tornadoes extends many centuries and is exceedingly voluminous; yet, current knowledge is quite incomplete concerning genesis, mature-stage structure and translation, and decay. Time scale for evolution; spatial scales for subdomains; source(s) of vorticity; magnitude and direction of the axial flow; importance of vertical wind shear (increase in magnitude, and veering in direction, of the horizontal wind with altitude); nature of the flow in the surface inflow (boundary) layer; reduction in eddy transport of momentum in a rapidly rotating flow; and stability of simple vortex structure to transition to a variety of more complicated substructures are among the literally dozens of questions of current controversy. A reader might decide that virtually every point of view on any question is already in the literature; so contribution often resides in establishing which, among many conflicting suggestions, is, in fact, physically pertinent.

Here some exceedingly simple concepts are brought together to identify a satellite observable that may be useful for detecting severe tornadoes.

1. First, a visual observable (the tornado funnel cloud, or condensation boundary) is studied to establish the fact that a severe tornado is characterized by wind speeds of 90 m/sec, with 100 m/sec being close to an upper bound. The investigators first proposed this low an upper bound many years ago, on the basis of examination of funnel-cloud photographs, when swirl speeds approaching sonic were commonly cited (refs. 7-12).

2. Second, characteristic ambients in which Midwestern tornadoes arise are analyzed to establish what pressure deficits are achievable by idealized thermohydrostatic processes (ref. 13-18; ref. 8). It is readily established that moist adiabatic ascent of ground-level air to its upper-tropospheric stability level cannot produce the pressure deficits required for severe tornado swirl speeds; the existence of dry-adiabatic compression of air from near-tropopause heights sinking slowly along the central axis of the tornado is required. [The pressure deficits are readily translated into maximum swirl speed estimates via the cyclostrophic balance, under adoption of appropriate

radial profiles for the azimuthal velocity component, under recognition that, for flows of Mach number of approximately 0.3 or less, the density may be taken as effectively constant for dynamics calculations (ref. 19).] Thus the occurrence of a compressionally heated central downdraft core, an "eye within an eyewall", is established as a necessary and sufficient condition for an intense tornado, in that no alternative plausible physical mechanism appears identifiable to explain tornado swirl speeds.

Nevertheless, the situation is more complicated than sometimes envisioned, in that the funnel-cloud interpretation technique previously discussed indicates a nonmonotonicity in the time-history of the maximum swirl speed during the tornado lifespan; thus, a tornado may experience several transitions between one- and two-cell structure throughout its lifespan, and an "eye" may be a transient phenomenon (ref. 9). Also, since insertion of an eye causes fluid particles to be displaced radially outward, and since outward radial displacement from the axis of rotation implies reduced swirl speeds under conservation of angular momentum, the fact that insertion of an eye implied augmented swirl speeds is not entirely a trivial matter. Thus, further work on two-cell structure is proposed immediately below.

However, it may be worth noting here that appreciable information has been extracted about tornadoes on the basis of cyclostrophic, quasihydrostatic, and axisymmetric modeling; suggestions (ref. 20) to modelers that such approximations should be abandoned for more complicated, more speculative, and more expensive-to-analyze modeling appear unwarranted for many purposes.

3. Third, since the occurrence of an "eye" is a property of a severe tornado, intense further research on this property as a satellite observable is warranted. The sinking air inserted into the central column is upper-tropospheric air, or even more likely, lower-stratospheric air, and thus effectively nonrotating. It is interesting that the onset of a severe tornado has been correlated on some occasions with the temporary cessation of anvil growth (ref. 21) (a phenomenon which seems at least not incompatible with the central downdraft concept); but it is even more interesting that the onset of severe tornadoes has been correlated on several occasions with

rapid descent of a particularly evident cumulonimbus tower, which had penetrated the general level of the cirrus deck (ref. 22) (a phenomenon which seems consistent with the central downdraft concept). Thus, it would seem quite important to delineate the radial dimension and the vertical profile of thermodynamic states that characterize the "eye", in order to determine what may be detectable from a satellite.

There are several properties of the hurricane eye that may be worth brief mention here. First, the thermal anomaly (i.e., the temperature rise above ambient, at fixed altitude) in a hurricane eye may attain 11 K or more in the upper troposphere (ref. 23)*; this thermal differential is well within the discrimination of current satellite instrumentation. Does a similarly large anomaly exist on a broad enough scale to be detectable in the tornado? Second, Carrier (ref. 29) has noted that the insertion of an eye within an eyewall is rapid: it would require a third of an hour in a hurricane (and considerations of plausible mean downdraft speeds from the upper troposphere suggests a roughly comparable, perhaps somewhat abbreviated "eye"-formation time in a tornado). However, inertia-induced oscillations of the eye-eyewall interface during the terminal stages of spin-up may explain the nonmonotonic, quasi-periodic intensity variation of hurricanes, which persists on the scale of eight-to-twelve hours or so (ref. 23). Similar transitory development and removal of a downdraft within the updraft annulus might explain the cyclic decay and reintensification of tornadoes on the scale of five minutes, with the maximum rate of change (increase or decrease) of swirl speed being about 0.45 m/sec^2 and persisting for 90 sec. (ref. 9).

Thus, further research concerning satellite observables should concentrate on properties of a tornado "eye". Also since the long-lived, long-path, extreme-intensity tornado usually develops in connection with the

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*Kuo (ref. 24, p. 597) has remarked: "..... It is rather doubtful that Carrier's mature hurricane model will produce a temperature at the top of the center much above the ambient temperature at that level as observed in real tropical storms." Since Carrier cites dry adiabatic compression of air descending from the tropopause as the source of warming in the eye of a hurricane, and since the same mechanism is being proposed for the tornado, the validity of the criticism warrants discussion. Reference 25, p. 319, rotating thunderstorm (tornado cyclone) (refs. 30-31), a question of extreme interest is to ascertain the conditions such that a one-cell vortex becomes unstable, and the preferred configuration evolves to a two-cell structure in which a tornado with a downdraft core is inserted in the center of the convective storm. It is remarkable that the conditions for which a two-cell structure tends to replace a one-cell structure have never been quantitatively examined (ref. 32). The fact that "vortex breakdown" phenomenon is involved in the sense that there is an abrupt radial spreading of the axial flow (ref. 33) is not at all evident, and the problem should be addressed in more general stability terms. Other stability studies have preoccupied modelers of atmospheric convective activity: one such problem concerns the ability of an internal gravity wave (refs. 34-36) to organize motion in a convectively unstable atmosphere, so that a storm is first initiated (wave-CISK), and another such problem concerns the occasional tendency of a single cyclonic vortex to disintegrate into several smaller cyclonic vortices symmetrically positioned within a cylinder circumscribed

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appears to show the largest temperature increase in the eye relative to ambient, at fixed altitude, occurs at about 20,000 N/m^2 (200 mb, or about 40,000 ft). The thermal anomalies are smaller above and below this altitude. Much above this level, the compression has not acted over sufficient distance to achieve comparable warming, while much below this level, evaporative cooling of water entrained from the eyewall (ref. 26) counteracts warming. The existence of water content in the eye of a hurricane at lower levels is well known (ref. 27, p. 46); in fact, Newton (ref. 28, p. 590) comments: "In a conical area inside the main cloud deck is an eye or clear area in upper levels, where the air is warm and dry. Within the eye, dense broken low clouds are formed, but these do not extend to great heights." Of course, in idealizing the eye as completely dry, one naturally obtains the largest anomaly at sea level. Thus, the source of Kuo's doubt about the ability of Carrier's model to discuss thermal anomaly is unclear.

by the original vortex (suction vortices) (refs. 37-38)[*]. However, from the point of view of severe tornado occurrence, the key stability problem is not **either** of these, but rather the problem suggested here: the conditions that characterize the tendency of an already developed rotating storm without an "eye" to develop an "eye".

Each of the three enumerated points is developed in detail in subsequent sections.

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[*] The fact that a tornadic vortex is itself composed of smaller-scale transient vortices, sometimes alluded to as secondary, satellite, or suction vortices, has been known for over a century (ref. 39). Such substructure is not regarded further here because it is believed to relate to the scale and intensity of turbulence in a tornado, rather than to the mean velocity and pressure fields of interest here. In fact, the persistence and interaction of large-scale coherent vortical structure, of rather two-dimensional nature, is a topic of intense research in current free turbulent shear flow investigations (ref. 40). Such structure suggests that the mean and instantaneous states of a tornado may be more divergent more often than sometimes conceptualized, but such gustiness (and other properties of the turbulence) are not essential to the time-averaged information sought here.

SYMBOLS

a	radius of maximum swirl in a Rankine vortex, m
c_p	heat capacity at constant pressure of a gas mixture, J/kg · K
c_{p_i}	heat capacity at constant pressure of species i, J/kg · K
g	magnitude of the acceleration of gravity, m/sec^2
H	static energy, $c_pT + gz + LY$, J/kg
h	height of the cloud deck, m
$I(R)$	integral of the centripetal acceleration [see equation (14)]
K	v^2_{max}/gh, dimensionless ratio
L	latent heat of phase transition, J/kg
m_i	molecular weight of species i, kg/mol
$\bar{m}$	parameter in radial profile of swirl [see equation (20)]
$\bar{n}$	parameter in radial profile of swirl [see equation (20)]
P	integral of the reciprocal of density over pressure [see equation (4)]; also, saturation pressure for water vapor in air, N/m^2
p	pressure of a gas mixture, N/m^2
p_i	pressure of species i, N/m^2
R	dimensionless (cylindrical) radial coordinate, r/a
R_i	gas constant for species i, J/kg · K
RH	relative humidity, $P_v/P(T)$
r	cylindrical radial coordinate, m

T	temperature, K
T_d	dew point temperature, K
t	time, sec
U	dimensionless radial velocity component, u/v_{max}
u	cylindrical radial velocity component, m/sec
V	dimensionless swirl (dimensionless azimuthal velocity component), v/v_{max}
v	swirl (azimuthal velocity component), m/sec
W	dimensionless axial velocity component, w/v_{max}
w	axial velocity component, m/sec
Y	mass fraction of species i, ρ_i/ρ
Z	dimensional axial coordinate, z/h
z	axial coordinate, m
θ	azimuthal angle in cylindrical polar coordinates
μ	mixing ratio for water vapor in air, ρ_v/ρ_a
ρ	density of species i, kg/m^3
ρ_i	density of species i, kg/m^3
σ	ratio of molecular weight of water vapor to that of air, m_v/m_a

Subscripts:

a	dry air
ℓ	evaluated on the axis of rotation at the height of the funnel-cloud tip (i.e., its earthwardmost extension)
m	evaluation on the funnel cloud at that altitude at which the peak swirl occurs

m	maximum
v	water vapor
∞	ambient

Mathematical notation:

$\rightarrow$	vector
$\wedge$	unit vector

FUNNEL-CLOUD INTERPRETATION FOR ESTIMATION OF MAXIMUM SWIRL SPEED

It has long been known that the tornado funnel cloud is the earthward extension of the condensation boundary, and, as such, is directly indicative of whirlwind thermodynamics (as opposed to dynamics) (refs. 41, 42). That is, the funnel cloud demarcates the condensation isotherm or isobar (for present purposes, distinction is not critical); although the funnel descends from the ambient cloud deck, the secondary flow pattern is primarily inflow and updraft -- as indicated by the flight of dust and debris.

The ambient cloud deck height is the altitude at which air, being cooled at the existing atmospheric lapse rate as it rises and expands, experiences condensation of its water vapor content. However, static enthalpy may be depleted to supply kinetic energy as well as to create gravitational potential energy, and, in fact, the funnel cloud is the local, transitory displacement of the condensation boundary earthward owing to rapid swirling. The more rapid the swirling, the further earthward the displacement. Although such general concepts are at least eighty years old, the quantitative exploitation of these ideas in terms of an atmospheric observable (ref. 8) has been in the meteorological literature for barely a half-dozen years (ref. 43).

The funnel-photograph-interpretation method is developed here in slightly more general form than in previous presentations because of recent controversies concerning its validity. The basic purpose is to confirm that tornadoes do attain 100 m/sec, but very rarely any higher, maximum swirl speed.

A steady inviscid axisymmetric flow is taken to describe a rapidly swirling flow with vertical axis of symmetry. In conventional notation, for inertial cylindrical polar coordinates with origin on the vertex axis at ground level,

$$\vec{v}(r,z) = u(r,z)\hat{r} + v(r,z)\hat{\theta} + w(r,z)\hat{z} \tag{1}$$

Appreciable deviation from verticality of the axis of symmetry usually indicates an incipient or decaying vortex, rather than the intense whirlwind of interest here.*

The radial and axial components of the momentum conservation equation are

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} \tag{2}$$

and

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \tag{3}$$

If the entropy is constant, then $p = p(\rho)$; above the surface frictional layer, this is felt to be a satisfactory approximation for the investigation of tornado dynamics being undertaken here. It is convenient to define

$$P = \int \frac{dp}{\rho} \tag{4}$$

In the cyclostrophic approximation to equation (2) and the hydrostatic approximation to equation (3), one has

$$\frac{\partial P}{\partial r} = \frac{v^2}{r} \; ; \; \frac{\partial P}{\partial z} = -g \tag{5}$$

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*Intense tornadoes with appreciably slanted axes of rotation can apparently exist, at least for short durations of time, perhaps owing to departure from axisymmetry (refs. 20, 44). However, highly slanted columns of lightened air seem incapable of producing appreciable sustained pressure reduction at the ground by hydrostatic considerations. It is not likely that drawing an analogy with a tilted mercury barometer to explain how a pressure deficit can be transmitted along a slanted column of fluid (as developed in ref. 4) furnishes valid physical insight.

Hence by cross-differentiation

$$\frac{\partial v}{\partial z} = 0 \Longrightarrow v(r,z) \rightarrow v(r) \tag{6}$$[*]

Glaser (ref. 42) suggested that along an isobar dP = 0, so that via

$$v^2(r) = g\ r \left(\frac{\partial z}{\partial r}\right)_p \tag{7}$$

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[*] The discussion below will ultimately arrive at the conclusion that adoption of the cyclostrophic and hydrostatic approximations, as in equations (5) and (6), serves adequately in estimation of the maximum swirl speed in tornadoes, a point noted earlier by Gray (refs. 15,16). First, the peak swirl may be anticipated to occur above the surface (frictional) inflow layer, the one region of the tornado in which angular momentum is not conserved, but rather is dissipated. The swirl monotonically decreases as the ground is approached, at fixed radial distance from the axis of rotation, according to turbulent-boundary-layer analysis for the inflow layer (ref. 45), and also according to the data analysis of the well-documented Dallas, Texas tornado of 2 April 1957 by Hoecker (refs. 46-48). [The inferences drawn by Hoecker require critical examination (ref. 32) because (1) the differential between the radial and azimuthal velocity components of large debris and of fluid particles is not accounted for; and (2) there is but one composite each of the swirl profile and of the secondary flow, drawn from multiple data sources distributed over the half-hour lifetime of the tornado, during which the tornado peak swirl nonmonotonically varied from zero swirl to about 90 m/sec (about 200 mph) (ref. 9).

It is thus no surprise that the velocity and pressure fields given by Hoecker are incompatible with close satisfaction of the cyclostrophic balance, even in portions of the tornado flow field where the cyclostrophic balance should hold. Nevertheless, this inference by Hoecker concerning reduced swirl in the surface layer seems creditable.] Lewellen (ref. 49) has suggested on the basis of the Bodewalt solution that the peak swirl may occur within the surface frictional layer; however, the relevance to intense tornadoes of this laminar solution, in which the fluid away from the ground plane is everywhere in rigid-body rotation, such that there is efflux from the boundary layer at all radial distances from the axis of symmetry, is exceedingly dubious; for intense tornadoes, two-cell structure replaces a rigidly rotating core. Second, even those authors (ref. 20) who infer large axial updrafts near the center of the tornado, and who emphasize departure from hydrostatic equilibrium near the base of an updraft annulus, acknowledge that the maximum Froude number is about 0.35 at altitude of 100 m and decreasing rapidly. At altitude of 300 m, Hoecker infers a maximum Froude number of only 0.02. Since the Froude number is effectively the ratio of inertial acceleration to buoyancy in the conservation of axial momentum, a small Froude number implies the adequacy of the hydrostatic approximation.

one could obtain the swirl profile from the funnel-cloud slope. This process is highly error-prone because differentiation of data is not an accurate procedure. [At or outside the maximum-swirl radius, the curvature of the condensation boundary is convex, from equation (7).] Integration of (5) avoids differentiation of the data and permits identification of a flow observable that leads to rapid estimation of the peak swirl. Attention, thus, is returned to equations (2) and (4).

Although stronger approximations are to be adopted evenutally, for now it is taken that the secondary flow, i.e., the streamline pattern generated by the radial and axial velocity components, is effectively irrotational. Then the vorticity has an axial component only, and

$$v(r,z) \rightarrow v(r), \ \frac{\partial w}{\partial r} = \frac{\partial u}{\partial z} \ . \tag{8}$$

Then equations (2) and (3) admit the integral

$$c_pT + \frac{u^2 + w^2}{2} = -gz + \int \frac{v^2}{r} \, dr + \text{const}, \tag{9}$$

where $P = c_pT$ under the dry adiabatic relationship for an ideal gas, $p \sim \rho^{\gamma}$; the moist adiabatic relationship is intractable and unnecessary for present purposes. Where the flow is irrotational and the potential-vortex

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Thus, for those portions of the tornado in which the peak swirl is anticipated to occur, i.e., for regions above the surface inflow layer, hydrostatics is an excellent approximation. The thickness of the surface frictional layer is on the order of 75 m (ref. 45). Third, although the swirl profile changes modestly with altitude, for those portions of the tornado wind field in which the peak swirl is anticipated to occur, axial variation, either of the magnitude or of the radial position of peak swirl, is small. For example, Hoecker infers that, between altitude of 50 m and of 400 m, the magnitude of the peak swirl in the Dallas tornado was invariant, and the radial position of the peak sloped radially outward from the vertical by but 3°. Thus, one anticipates the usefulness of the hydrostatic and cyclostrophic approximations adopted in writing equations (5) and (6) from the outset.

model holds (i.e., where $vr \sim$ const), equation (9) becomes the isoenergetic relation

$$c_pT + \frac{u^2 + v^2 + w^2}{2} + gz = \text{const.} \tag{10}$$

Obviously, as $r \to 0$ at fixed z, equation (10) fails since v becomes unbounded; it was the failure to recognize the inadequacy of the potential vortex form as $r \to 0$ that unfortunately precluded further progress long ago by Ferrell (ref. 41), that might have significantly accelerated tornado research. Actually, the work of Abdullah (ref. 50) suffered from a similar problem many years later.

The const. in equation (9) is assigned by requiring $T \to T_\infty$, $v \to 0$, $w \to 0$, $u \to u_\infty$ as $r \to \infty$, $z \to 0$:

$$c_p(T-T_\infty) + \frac{u^2-u^2_\infty + w^2}{2} = -gz + \int_\infty^r \frac{[v(r_1)]^2}{r_1}\, dr_1. \tag{11}$$

This is viewed here as the equation for any isotherm, once the flow is known.

In the ambient $r \to \infty$, at $z = h$, the cloud-deck height, the temperature $T = T_d$, the dew-point temperature:

$$c_p(T_d-T_\infty) + \frac{u_d^{\,2} + w_d^{\,2} - u_\infty^{\,2}}{2} = -g\,h, \tag{12}$$

where $u(r \to \infty,\ z = h) \equiv u_d$, $w(r \to \infty,\ z = h) \equiv w_d$. If the dynamic contributions were neglected, the dry-adiabatic lapse, $dT/dz = -g/c_p$, would hold in the ambient from ground level to the lifting condensation level. Clearly, the ambient may be so dry that T_d is very small and h is very large; however, this is exceedingly atypical of convectively unstable ambients that spawn severe tornadoes. The cloud deck height h is usually 675 m, and rarely above 1000 m.*

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*The funnel-cloud method cannot place a theoretical bound on peak swirl speed attainable in a tornado, because it is simply a basis for data interpretation. However, empirically, from processing of much data, it places a bound of about 100 m/sec on peak swirl. Ultimately, in the development which follows, that bound can be traced to the observation that funnel-cloud length does not exceed 675 m (i.e., for a funnel tip touching the ground, the parent cloud has its base at 675 m).

If one introduces the nondimensionalization (a is the radius at which the swirl $v \to v_{max}$, its maximum value):

$$Z = \frac{z}{h},\ R = \frac{r}{a},\ U = \frac{u}{v_{max}},\ V = \frac{v}{v_{max}},\ W = \frac{w}{v_{max}},\ K = \frac{v_{max}^2}{gh} \qquad (13)$$

then equation (11) for $T=T_d$, the condensation boundary, becomes with of aid of (12),

$$1-Z = K\left\{ I(R) + \left[\frac{U^2 + W^2 - U_d^2 - W_d^2}{2}\right]\right\},\ I(R) = \int_R^\infty \frac{V^2(R_1)}{R_1}\, dR_1 \qquad (14)$$

One can identify K as the ratio of the dominant portion of the kinetic energy to the potential energy associated with ascent to the ambient cloud-deck height. At R=0, U=0 by kinematics, so

$$1-Z_\ell = K\left[I(R=0) + \frac{W_\ell^2 - U_d^2 - W_d^2}{2}\right] \qquad (15)$$

where subscript ℓ denotes evaluation on the axis of symmetry at funnel-cloud height. At R=1, where V=1,

$$1-Z_m = K\left[I(R=1) + \frac{W_m^2 + U_m^2 - U_d^2 - W_d^2}{2}\right] \qquad (16)$$

where subscript m denotes evaluation on the funnel cloud at that height at which the swirl has its maximum. Actually, U_m, U_d are utterly negligible above the surface inflow layer and are henceforth discarded.* With all

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*Much of the fluid above the surface frictional layer, in a mature (quasi-steady) severe tornado, is described, to excellent approximation, by the cyclostrophic balance. Such fluid cannot move radially, to any significant extent; the radial velocity component is nil. Hence, emphasis (ref. 49) on the radial velocity component profile as the basis for characterization of the radial variation of swirl, above the surface inflow layer in a mature tornado, is misguided. What radial profile of the azimuthal velocity component (swirl) holds in the mature tornado is determined during spin-up, i.e., during intensification. If angular momentum is conserved during spin-up, then (above the boundary layer) the potential-vortex profile holds for the swirl in the mature tornado, from the radius of maximum swirl outward from the axis of rotation.

other factors held constant, for $W_\ell > W_d$ ($W_\ell < W_d$), increasing ($W_\ell^2 - W_d^2$) decreases (increases) K, and hence decreases (increases) v_m; neglecting $|W_\ell^2 - W_d^2|$ thus gives a slight overestimate (underestimate) of v_m. Actually, if $z = 0$, i.e., if the funnel cloud just touches the ground, $W_\ell = 0$ by kinematics and no error is incurred if $W_d \doteq 0$.* In any case, it is desirable to render the results independent of the [relatively controversial (ref. 48)] nature of the secondary (radial and axial) flow, so equation (15) is approximated henceforth as (refs. 9,10)

$$1 - Z_\ell = K\, I(0) \tag{17}$$

The term K I(0) is identified as that fraction of the distance earthward from the ambient cloud-deck height to which the funnel cloud descends. Approximating equation (15) as (17) is equivalent to neglecting the convective transport terms in equations (2) and (3). If $Z_\ell = 1$, then the funnel cloud has retreated to the ambient cloud height and $I(0) = 0$, so $V = 0$; whereas if $Z_\ell = 1/4$, the funnel cloud has descended three-quarters the distance to earth from the cloud deck. Also, if $V = 0$ for $0 \lesssim R \lesssim 1$, as would be the case for a nonrotating core, $I(0) = I(1)$, and $Z_m = Z_\ell$; otherwise, the altitude at which the funnel cloud is coincident with the right-circular-cylindrical locus of maximum swirl, Z_m, exceeds the altitude at which the funnel cloud intersects the axis of symmetry, Z_ℓ (ref. 11). In brief, equation (17) becomes

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*For any two-cell vortex, i.e., for any tornado with peak swirl near 100 m/sec or higher, $W_\ell \doteq 0$ on the axis of rotation, and the error incurred by ignoring $(W_\ell^2 - U_d^2 - W_d^2)/2$ is utterly negligible. For a one-cell vortex with peak swirl of about 50 m/sec, typically $W \lesssim 0.4$, $U_d = W_d = 0$, and the error incurred in ignoring $(W_\ell^2 - U_d^2 - W_d^2)/2$ in (15) is an overestimate of peak swirl of less than 4%. Even if one accepts an updraft on the axis of 60m/sec, as inferred in reference 44 off the axis over a very limited range of altitude, the error in peak swirl overestimate incurred by ignoring the cited term is only 30%, since $W_\ell \doteq 1.2$. It is reiterated that a 30% overestimate of peak swirl is virtually the maximum conceivable error that could be incurred owing to adoption of hydrostatics, and could arise only in moderately intense tornadoes (peak swirl about 50 m/sec).

$$v_{max} = \left[\frac{1 - Z_\ell}{I(0)} \, gh \right]^{1/2} \tag{18}$$

Equation (16) without the secondary-flow kinetic energy terms then becomes

$$z_m = h \left[1 - (1 - Z_\ell) \frac{I(1)}{I(0)} \right] \tag{19}$$

For further progress, one must commit oneself to a radial profile for the swirl, and any kinematically reasonable choice is admissible. Furthermore, since it is the square root of the integral of the selected profile that enters, there is no extreme sensitivity of peak swirl to profile choice. Thus, in preference to a swirl profile with continuous derivatives but complicated integrals, simplicity suggests use of the following two-part structure ($\bar{n} > 0$, $\bar{m} > 0$):

$$\frac{v(r)}{v_{max}} = V(R) = \begin{cases} R^{\bar{n}} & 0 \le R \le 1 \\ R^{-\bar{m}} & 1 \le R \le \infty \end{cases} \tag{20}$$

Clearly equation (20) does not obey the kinematic constraint $V \sim R$ as $R \to 0$ unless $\bar{n} = 1$, nor does it give a constant ambient value for the angular momentum (or circulation) unless $\bar{m} = 1$, nor does it give a stable vortex for $R > 1$ according to the Rayleigh criterion (ref. 51) for $\bar{m} > 1$. Here the constraint $1 > \bar{m} > 0$ is accepted, but all $\bar{n} > 0$ are admitted (so the special case of a two-cell vortex, for which $\bar{n} \to \infty$ so the core is nonrotating, can be included within one formalism). Also, $\bar{n} = \bar{m} = 1$ recovers the Rankine vortex. The case $\bar{n} \to \infty$, $\bar{m} = 1/2$ has been empirically fitted to hurricane data (ref. 52). The reader who wishes to pursue the implications of alternatives to equation (20) is reminded that $V(R = 1) = 1$, the absolute maximum by definition, for (19) to hold.

From equations (14) and (20),

$$I(1) = \frac{1}{2\bar{m}}, \quad I(0) = \frac{1}{2\bar{m}} + \frac{1}{2\bar{n}} \tag{21}$$

For the Rankine vortex, i.e., for $\bar{m} = \bar{n} = 1$, under the cyclostrophic approximation with constant density, half the pressure deficit is thus seen to be expended maintaining the rigidly rotating core $(R < 1)$, and half, the outer potential vortex $(R > 1)$. The Rankine vortex is viewed here as a reasonable fit to a one-cell vortex, i.e., a vortex in which the secondary flow in the central region is everywhere characterized by radial inflow and axial updraft. For a two-cell vortex with a nonrotating core, all the pressure deficit is available to maintain the outer swirl, and none is expended on the core flow; furthermore, it will be shown below that the total pressure deficit available to sustain swirl is greater for a two-cell structure than for a one-cell structure for a fixed ambient state, so much greater maximum swirl speeds are associated with the existence of an "eye".

Under equation (21), equations (18) and (19) become, respectively,

$$v_{max} = \left[\frac{2\overline{mn}}{\bar{m}+\bar{n}}\, g(h - z_\ell)\right]^{1/2} = \left[\frac{2\overline{mn}}{\bar{m}+\bar{n}}\, gh(1-Z_\ell)\right]^{1/2}; \tag{22}$$

$$z_m = h\left[1 - \frac{\bar{n}}{\bar{m}+\bar{n}}\,\frac{h-z_\ell}{h}\right] = h\left[1 - \frac{\bar{n}}{\bar{m}+\bar{n}}\,(1 - Z_\ell)\right] \tag{23}$$

Application of equation (22) to circumstances in which alternative methods of maximum swirl speed estimation (such as dust and spray tracking in cinematographic records) are available, suggests (refs. 8-10)

$$\frac{2\overline{mn}}{\bar{m}+\bar{n}} \doteq 1 \tag{24}$$

gives good accuracy. Also, work described below suggests that tornadoes with peak swirls of 100 m/sec or so have two-cell structure, so $\bar{n} \to \infty$. Hence, roughly, $\bar{m} = 1/2$, the same result that holds for hurricanes (ref. 52). For $\bar{n} \to \infty$,

$$z_m = z_\ell,\quad v_{max} = [2\bar{m}g(h-z_\ell)]^{1/2}, \tag{25}$$

while for $\bar{n} = 1$, $\bar{m} = 1$ the maximum swirl speed occurs at the funnel-cloud radius at altitude half way between the funnel tip and the cloud deck:

$$z_m = \frac{h+z_\ell}{2}, \quad v_{max} = [g(h-z_\ell)]^{1/2}. \tag{26}$$

Use of $\bar{m} = 1$ in equation (25) and comparison with (26) shows that, for a potential-vortex outer flow, a nonrotating core permits $2^{1/2}$ times greater swirl speed than a rigidly rotating core. In fact, under the stability constraint $1 \geq \bar{m} \geq 0$, the nonrotating core joined to a potential vortex gives the maximum swirl speed possible for the class of profiles represented by equation (20). Indeed, under the Rayleigh stability criterion, it is difficult to conceive of any swirl profile that could yield a larger maximum swirl speed for a fixed set of parameters.

There is no inherent upper bound for v_{max} in equation (22), even for $1 \geq \bar{m} > 0$; $Z_\ell < 0$ is conceivable in that the funnel cloud could have a finite radius at the ground. To pursue this point, one may examine the funnel cloud shapes from equation (14) under (20), with $U_d = U = W_d = W = 0$. For $\bar{n} \to \infty$,

$$1 - Z = \begin{cases} \dfrac{K}{2\bar{m}} & 0 \leq R \leq 1 \\[2ex] \dfrac{K}{2\bar{m}R^{2\bar{m}}} & 1 \leq R \leq \infty \end{cases} \tag{27}$$

The funnel cloud then appears roughly like the frustum of a cone. For $\bar{n} = 1$,

$$1 - Z = \begin{cases} \dfrac{K}{2}\left(\dfrac{1+\bar{m}}{\bar{m}} - R^2\right) & 0 \leq R \leq 1 \\[2ex] \dfrac{K}{2\bar{m}R^{2\bar{m}}} & 1 \leq R \leq \infty \end{cases} \tag{28}$$

The funnel cloud then appears roughly like a cone. Thus, if dust and debris do not obscure the base of a funnel at finite altitude, the shape of the funnel cloud could conceivably be used as an indicator of the one-cell or two-cell character of the secondary flow; the association of two-cell structure with a nonrotating core, and of one-cell structure with a rotating core, is implicit. Evidently, from equations (27) and (28), for large enough K, Z could be negative at R = 0.

In practice, rarely does (h - z) exceed 675 m; even for $\bar{m} = 1$ equation (25) gives about 107 m/sec, and for $\bar{m} = 0.88$, gives about 100 m/sec. If dust and/or debris obscures the base of the funnel cloud, a conservative estimate of peak swirl can be made on the basis of that portion of the funnel cloud that is visible.

If one adopts a quasisteady approximation, then one can apply equation (18) to data describing the time history of the funnel cloud length. While such data is rarely available, Dergarabedian & Fendell (ref. 9) have indicated the nonmonotonic time history of the maximum swirl speed, the time scale for decay and reintensification, and the maximum rate of change of peak swirl in time for a particular case (ref. 53).

Under a modicum of assumptions, an observer can virtually instantaneously compute the maximum swirl speed from a tornado observable, the funnel cloud length, and the radius of maximum swirl speed from another observable, the funnel-cloud breath, by the above-described method. Alternative approaches are mainly (1) establishing the wind required for postulated modes of structural failure for tornado-inflected damage (ref. 54), or (2) detailed scrutiny of occasional cinematographic records of dust and spray in whirlwinds (ref. 20). Barometric and anemometric data is rare; the intactness and response time of the instrument is often in doubt for the tornado environment; and resolution of data into velocity components relative to the axis of symmetry and into translational speed is virtually unknown.

The assumptions of steady and axisymmetric conditions, while not strictly correct, are believed to be reasonable for the following reasons. The time scale of change in a mature severe tornado is on the order of minutes and thus not pertinent; a tightly wound vortex swirling at 100 m/sec, and translating typically at 17 m/sec, is probably often quite axisymmetric in its inner core. Also, surface heating, evaporative cooling, and mixing ratio variations below the cloud deck could possibly introduce significant nonadiabatic processes. However, large rainfall typically occurs at a distance from a tornado funnel, rather than directly down the core; furthermore, estimation of the cloud deck height on the basis of rapid, undiluted ascent of surface-level air (however it achieves its thermodynamic state) is quite reasonable for the updraft fluid in a severe tornado (ref. 55).

In summary, the knowledge gained for the effort expended appears to provide some useful insight into the physical description of tornadoes, especially in the absence of feasible alternatives for rapid estimation of maximum swirl speed and radius.

THE USE OF THERMODYNAMIC DATA FOR THE TORNADO-SPAWNING AMBIENT TO CHARACTERIZE STRUCTURE

The previous section used interpretation of funnel-cloud photography to establish that tornadoes are capable of maximum swirl speed of 100 m/sec, but not much more. This section seeks to establish, from knowledge of the thermodynamic state of the spawning ambient, what flow configurations must be present in the exceptionally severe tornado. In particular, it is indicated that the ground-level pressure deficits from ambient achieved by having convectively unstable ground-level air rise without dilution to its level of neutral stability is insufficient to sustain such large swirl speeds. Some process other than ascent on the locus of thermodynamic states referred to as a moist adiabat is necessary to explain the necessary pressure deficit. That process appears unavoidably to be dry-adiabatic compression of slowly sinking tropopause-level air, such that an "eye" is inserted within the "eyewall" annulus of rapidly ascending air. The "eyewall" air is rapidly swirling, so it rises without dilution (to a first approximation); it is ambient air, spun up under convectively induced advection, that erupts from the surface inflow layer (ref. 33,56-58). However, the air in the "eye" is lower-stratospheric air, or (less likely) upper-tropospheric air, entrained from the upper "eyewall"; in either case, the air in the "eye" is only slowly rotating.

The methodology to be employed involves well-known combining of the cyclostrophic balance with thermohydrostatics, to translate ground-level pressure differences into swirl estimates. However, some variations are adopted here. The difference in hydrostatically computed weight of three vertical columns of fluid is of interest. The fluid is in general a two-component mixture of dry air and water vapor, both taken as ideal gases, such that three thermodynamic variables are required to specific the state. The three columns are: (1) an ambient column typical of the tornado-spawning environment, with temperature and relative humidity known as a function of pressure by measurement; (2) an "eyewall" column of slightly smaller ground-level pressure, with air rising on the dry adiabat based on local ground-level conditions until the lifting condensation level, and thereafter rising on the moist adiabat; and (3) a moisture-free "eye" column of appreciably smaller ground-level pressure, with dry air sinking on an adiabat. The ambient column characterizes the vertical stratification at the outer edge of the vortex; the "eyewall" characterizes the central convective core of the vortex. For weaker tornadoes only these two columns are present; the sea-level pressure deficit is usually but 1000-2000 N/m^2 (twenty millibars), sometimes far less, and thus inadequate to sustain 100 m/sec swirl speeds. The third, "eye" column represents evolution from a one-cell vortex to a two-cell vortex; such an evolution occurs only in those exceptionally intense tornadoes in which 100 m/sec swirl speeds are sustained by 10^4 N/m^2 (100 mb) ground-level pressure deficits between the axis of rotation and the ambient. Thus, the "eye" represents the occasional insertion of a central downdraft column that radially displaces the updraft to an annulus around the axis of symmetry. Of course, the "eye" need not penetrate from the top of the storm to the ground, though calculations are here made on such a basis. Also, the use of the moist adiabat for the "eyewall" and the dry adiabat for the "eye" are idealizations in that some entrainment is inevitable, the eye is not completely moisture free, and evaporative cooling is being ignored.

The bottom of the storm is taken as a slippery impervious lid (ground plane) that is nonisobaric; the top of the storm is taken as a slippery impervious lid (tropopause) that is isobaric and isothermal. In particular, the tropopause is taken to lie, for all radial positions, at that altitude hydrostatically associated with a particular pair of values for the pressure and temperature in the ambient. The particular pair is that ambient pressure and temperature which match an upper-tropospheric pressure-temperature pair computed for the moist adiabat. This choice of lid yields a slightly different altitude than one computed on the basis of the convectively unstable ambient recovering its sea-level static energy after passing through a midtropospheric minimum.

The calculation involves iteration because the ground-level reference state for the "eyewall" adiabat is not completely known before hand. The temperature and mixing ratio are taken to be those for the sea-level ambient (this procedure probably overestimates the temperature but underestimates the mixing ratio)*, but the pressure at the base of the "eyewall" is not known a priori. A ground-level pressure must be assumed, and confirmed by being recovered. More specifically, the "eyewall" locus of thermodynamic states is established by integrating the adiabat over decreasing pressure until a trial lid is assigned, and thus a trial altitude is ascribed to the tropopause. Then, by use of hydrostatics, one may integrate from the tropopause earthward, and thus determine the self-consistency of the

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*Conceptually, the calculations commence at ground level, in the surface frictional level. Hence, it is more satisfactory to adopt constancy of the static energy, rather than of the potential temperature; constancy of the latter implies isentropic flow. In any case, for a tornado with 50 m/sec peak swirl, the static temperature decrease near the ground from edge to "eyewall" column base is but 1 K, while for a tornado with 100 m/sec peak swirl the decrease is about 4 K. Thus the error in temperature incurred by the isothermal model is at most about one percent. The error in mixing ratio incurred by taking it constant from edge to "eyewall" base is about ten percent or less. Below, when pressure differences are translated into swirl estimate via the cyclostrophic balance, clearly the ground-level pressure differences are being ascribed to hold about 50 m off the ground (where cyclostrophy is a good approximation to the conservation of radial momentum); again, the error incurred by such an inconsistency is small. In practice, data very close to ground level is frequently unavailable, and the datum for altitude (although formally written as ground level) is often above the surface frictional layer.

ground-level pressure computed with the ground-level pressure assumed. Convergence of the iteration is readily achieved.

The reference state for the dry-adiabatic compression in the eye is the tropopause state. The temperature, pressure, and altitude assigned for the compatibility of the ascending adiabat with the ambient are used for the "eye." In this way, just as the "eyewall" gives a readily computed "lower bound" on achievable pressure differential, the "eye" calculation gives a readily computed "upper bound" on achievable pressure differential.

It should be noted that if the "eye"/"eyewall" interface were perfectly vertical to indefinite height, then the additional density reduction in the "eye" would be unavailable to explain rapid swirling of air in the "eyewall" annulus. However, meteorological satellite photography of hurricanes (ref. 23) appears to refute the idea (refs. 59,60) of a perfectly vertical eye/eyewall interface, and to confirm the existence of a finite (even large) outward interfacial slope throughout most of the troposphere [as earlier suggested by Haurwitz (ref. 13) and later re-emphasized by Dergarabedian & Fendell (ref. 8) and by Carrier, Hammond, & George (ref. 61)]. Actually radar and flight data had begun to cast doubt on the vertical eyewall postulate even before the availability of satellite photography (ref. 62). Similar outward slope of the "eye"/"eyewall" interface of a tornado as the updraft approaches its level of neutral stability (the tropopause) is anticipated.

The definition of a precise tropopause is, of course, an idealization. However, it is not clear that some cumulonimbi with undiluted cores "overshoot" the general neutral stability altitude. Davies-Jones and Kessler (ref. 4) lend credence to the concept of "overshooting" owing to excessive upward momentum generated during ascent. Such an effect would counteract part, or all, of the ground-level pressure deficit from ambient achieved by adiabatic ascent. Such an effect is not included here because it is believed to be a highly unlikely phenomena in the subsonic flow fields pertinent to tornadoes. Davies-Jones and Kessler also suggest that non-rotating cumulonibi are more likely to rise higher than rotating cumulonimbi.

This suggestion seems to be the reverse of what is plausible. For any column to rise to its level of neutral stability it must entrain little air during ascent, and organized rotation in a cloud reduces entrainment. The reduction in entrainment at low levels in the atmosphere is of particular significance, so it is the rotation of the cloud in the lower troposphere that is important; since cloud rotation may be appreciably reduced in the upper troposphere, it may be deceiving to examine cloud tops and thereby to infer whether the cumulonimbus possessed significant rotation or not. Different cumulonimbi may rise to different levels of neutral stability for at least two reasons. Less important of the two reasons is the initial thermodynamic constitution of the cloud owing to altitude of column origin. More important is the different amount of entrainment owing to the level of spin-up (or spin-down) of the vortical system. A cumulonimbus generated later during tornado intensification possesses greater rotation, and hence entrains less, than a cumulonimbus generated earlier during intensification. Thus, the later cumulonimbus may appear to "overshoot" the level of neutral stability established by earlier cumulonimbi, but there may be no actual overrunning of neutral stability level entailed. Hence, doubt cast on the present calculational procedure owing to "overshooting" is dismissed.

Further details on the calculational procedure for establishing the ground-level pressure deficits are already set forth in the literature (ref. 23), and are not repeated here, although for convenience the equations are collected in the appendix.

Simplifying the gradient-wind equation into the cyclostrophic balance incurs less than a five-percent overestimate of the maximum swirl speed v_{max} for a tornado with peak swirl of 100 m/sec. For a one-cell tornado without an "eye", use in the cyclostrophic balance of the swirl profile

$$V(r) = \frac{v(r)}{v_{max}} = \begin{cases} R & 0 \leq R \leq 1 \\ \frac{1}{R^{\bar{m}}} & 1 \leq R \leq \infty \end{cases} \qquad (29)$$

appears appropriate. Hence,

$$v_{max} = \left(\frac{2\bar{m}}{\bar{m}+1} \; \frac{p_s - p_e}{\rho_s} \right)^{1/2} \qquad (30)$$

where p_s is the ground-level pressure for the ambient column and p_e, for the "eyewall" column, and the density has been held fixed at the ambient ground level ρ_s.

For a two-cell vortex with an eye, use of the swirl profile

$$V(r) = \frac{v(r)}{v_{max}} = \begin{cases} 0 & 0 \leq R < 1 \\ \frac{1}{R^{\bar{m}}} & 1 \leq R \leq \infty \end{cases} \qquad (31)$$

appears appropriate. Thus, one expects

$$v_{max} = \left(2\bar{m} \; \frac{p_s - p_c}{\rho_s} \right)^{1/2} \qquad (32)$$

where p_c is the ground-level pressure for the "eye". It is recalled that $1 \geq \bar{m} \geq 0$ by stability considerations.

There are many data sources for typical atmospheric stratifications in the vicinity of tornadoes for various regions of the country (e.g., refs. 63,64). Furthermore, many profiles for specific individual events are also available (e.g., ref. 65). Maddox (ref. 66)has criticized "compositing" data from individual cases to form typical tornado-spawning ambients; such procedures do confound data taken "upwind" of an incipient tornado with "downwind" data, and data taken closer in time and/or distance to a tornado event with data taken further remote in time and/or distance. There appears to be merit in Maddox's critique of "compositing", and although composites are analyzed below, the limitations should be kept in mind. However, a greater complaint for present purposes is the frequent incompleteness of tephigram soundings, such that extrapolation of the (static) temperature and/or dew-point temperature profiles to sea-level and/or to the tropopause is necessary.

Figures 1-6 present computations based on an upper-air sounding [commencing at 94 000 N/m^2 (940 mb)] at Cordell, Oklahoma at 2300 GMT on 10 June 1967 (ref. 65, p. 29, fig. 12). A low-level warm moist layer of northerly Gulf air (2500 m deep), with temperature distribution approximately dry-adiabatic and with uniform mixing ratio, lay beneath a shallow stable layer around 71 000 N/m^2 (710 mb). The existence of a deep dry layer of westerly air above a moist layer marks this ambient as convectively unstable. In time the low-level inversion disappeared and the moist layer deepened. About 12 600 sec (three-and-one-half hours) after the sounding, a tornado was sighted 48 000 m (26 n mi) north-northeast of Cordell, but no tornado came closer than 30 000 m (16 n mi). The ambient presented by Thresher has a temperature of 303 K and relative humidity of 0.46 at 48 000 N/m^2 (940 mb) (assigned to be 0 m); the storm lid, as calculated by the above-described procedure for slightly smoothed data, is 12 750 m where the temperature is 208 K and the pressure 16 100 N/m^2 (161 mb). The moist adiabat yields about 4250 N/m^2 (42.5 mb) pressure deficit from ambient (hence, a maximum swirl of 63 m/sec for $\overline{m} = 1$, 51 m/sec for $\overline{m} = 0.5$, for a rigidly rotating core). The dry adiabat yields a pressure deficit of 9900 N/m^2 (99 mb) from ambient (hence, a maximum swirl of 136 m/sec for $\overline{m} = 1$, and 96 m/sec for the more plausible value of $\overline{m} = 0.5$). Clearly, the existence or nonexistence of the "eye" is the difference between the moderately destructive, 50 m/sec swirl and the devastating, 100 m/sec swirl.

Miller (ref. 63) presents a composite Midwest pretornado air mass sounding, characterized by a temperature inversion between 80 000 N/m^2 and 85 000 N/m^2 (between 800 mb and 850 mb). The relative humidity rises from 0.6 at 98 000 N/m^2 (980 mb) to 0.9 at 82 000 N/m^2 (820 mb), then falls abruptly to zero at 80 000 N/m^2 (800 mb). Moist-adiabatic ascent yields almost an 1100 N/m^2 (11 mb) deficit from ambient; this deficit implies a maximum swirl speed of 30 m/sec mph for $\overline{m} = 1$, and 25 m/sec mph for $\overline{m} = 0.5$. Insertion of a dry "eye" gives a 5400 N/m^2 (54 mb) pressure deficit from ambient; this deficit implies a maximum swirl of 97 mph for $\overline{m} = 1$ and of 69 mph for $\overline{m} = 0.5$.

Darkow (ref. 64) averaged 45 tornado proximity soundings taken within 6300 sec (105 minutes) and 80 000 m of tornadoes of the Central Plains or Gulf Coast. Darkow's data yields but a 200 N/m^2 (2 mb) deficit by moist adiabatic ascent (so the maximum swirl is 14 m/sec for $\overline{m} = 1$, and 11 m/sec for $\overline{m} = 0.5$), and 3800 N/m^2 (38 mb) deficit by dry-adiabatic descent (so the maximum swirl is 84 m/sec for $\overline{m} = 1$, and 59 m/sec for $\overline{m} = 0.5$). In short, some ambients which can barely sustain swirling under a one-cell structure, can give a formidable tornado under two-cell structure.

Why does the importance of an "eye" to achieving very large swirl speeds in a tornado remain a far from universally accepted insight? First, suggestions by Abdullah (ref. 50), on the basis of nearly uniformly valid potential vortex theory, that near-sonic swirl speeds could arise in tornadoes, led some researchers (refs. 67-70) to seek a role for electrohydrodynamic and magnetohydrodynamic forces in tornado generation, and for Joule heating as a source of thermal instability. Although the gross overestimate of tornado swirl speeds, and of the possible role of electromagnetic effects in the dynamic and energy balances of a whirlwind, involved in such suggestions has long been known (ref. 71), speculations concerning alternative physical mechanisms to explain exceptional tornado intensity persist (refs. 72-76).

Second, the early solutions displaying two-cell structure in a tall thin vortex (refs. 77-80) invariably associated the central downdraft with cold temperatures (ref. 48). These solutions are incompatible with the concept that the existence of an "eye" implies extreme intensity; the solution are meteorologically inconsistent because all permitted axially distributed radial inflow to the mature vortex, whereas all the influx to the intense vortex is confined to the surface frictional layer. Sullivan's solution (ref. 78), in fact, finds the two-cell vortex has a smaller core, lower maximum swirl speed, and higher central pressure than its one-cell counterpart, for the same ambient convergence and circulation. Only Gutman (ref. 77), among these early analytic modelers, notes that, physically, vortices with downdraft cores are particularly intense; however, his solution does not recognize that the central downdraft emanates from descent of high-level, nonrotating air down the core. Gutman's Rankine-vortex-like swirl has a magnitude independent of the ambient stratification.

Third, observational data is inconclusive. Hoecker (refs. 46,47) inferred a Rankine-type vortex even for a central downdraft, possibly owing to the previously-alluded-to compositing of data distributed over the half-hour lifespan of the tornado. More recently, Sinclair (ref. 81) claimed to measure a cold, rigidly rotating core with downdraft at the central axis of a dustdevil. His argument for the stability of such a vortex is not convincing.

Fourth, there may be an unstated concern that strong frictional coupling between the "eyewall" and the "eye" precludes the reversal of axial flow direction over a narrow scale, implicit in two-cell structure in the core of a tall thin vortex. On a linear and laminar basis, "sidewall" (Stewartson-type) boundary layers are thicker than "endwall" (Ekman-type) boundary layers in rotating flow (ref. 51). Also, though little is known about eddy diffusion in a tornado (ref. 48, p. 158), it is noted that horizontal diffusion coefficients exceed vertical diffusion coefficients by two orders of magnitude in other atmospheric-flow contexts (ref. 82). Nevertheless, linear laminar theory, and turbulence models for other atmospheric contexts, have little relevance for a tornado. In ref. 45 it is suggested that, for the surface inflow layer under the intense portion or a severe mature tornado, the role of friction is confined to a small near-ground sublayer (about 6 m thick) of the total inflow layer (about 65 m thick). When the flow in the surface inflow layer "erupts" vertically to form the "eyewall" annulus, it is through this thin sublayer that frictional coupling between the "eye" and "eyewall" occurs. Thus, there is spatially limited diffusive interaction between "eye" and "eyewall" in a tornado, and no basis exists to cast doubt on the requirement for two-cell structure in severe tornadoes.

Fifth, radar returns from a tornado cyclone sometimes indicate an echo-free central vault from ground level to 10 km; from 10 km to the tropopause, there are invariably strong radar returns. Such strong returns are interpreted to indicate the existence of large densities and large numbers of of large droplets (ref. 83). One may venture to infer that a dry region exists at lower levels in the core, but not at higher altitudes. This structure is incompatible with two-cell concepts, in which a compressionally heated "eye" sinks earthward from tropopause levels. Of course, precisely what a radar return means in physical terms is not always completely transparent. In any case, there is no reason why the echo-free vault of a tornado cyclone is to be associated in a conclusive way with the core of a smaller scale tornado. Thus, while the anomalous radar returns are interesting, they are not readily interpreted in a convincing way.

The weight of any of these objections to the thesis that the intense tornado is characterized by an "eye" of virtually rotationless, tropopause-level air, compressionally heated during descent at the center of the vortex, does not appear to be significant; however, in toto, they perhaps explain why the concept does not seem to be universally accepted at present.

THE DETECTION OF TORNADOES BY GEOSYNCHRONOUS SATELLITES

Anticipation of Tornado Onset

As already noted earlier, practicality limits the current goal of geosynchronous satellites designed for severe-mesoscale-storm meteorology to detection of the existing, exceptionally intense tornado, as opposed to anticipation of its onset. One reason for such limitation on satellite utility is that the known atmospheric criteria suggestive of an incipient severe storm do not permit distinction whether the impending weather involves a hailstorm, tornado cyclone, thunderstorm, superoutbreak of tornadoes, or whatever (refs. 60, 84). Nevertheless some brief discussion of the matter is now undertaken.

<u>The static energy</u>. - The main observable from a satellite indicative of a strom-prone atmosphere is believed to be vertical profile of the equivalent potential temperature, or equivalently, the static energy. In conventional notation, this quantity H is the sum of the heat content (c_pT); the gravitational potential energy (gz); and the heat potential owing to moisture content (LY) [where L is the latent heat of phase transition, and Y is the mass fraction of water vapor, so $Y \doteq \mu$, the mixing ratio]. The kinetic

energy ($q^2/2$) [where q is the magnitude of the wind speed relative to the earth] is, even for a tornado, uniformly bounded to but a roughly one-percent correction to H as defined, and thus is a normally negligible contribution. Unlike the tropics, the midlatitudes are usually characterized by a static energy profile that increases monotonically with altitude (ref. 62). This is indicative of a stable atmosphere, even with account account for moisture. In storm-prone regions, however, locally and temporarily, warm and/or moist air underlies cool and/or dry air, such that a midtropospheric minimum of static energy arises; in the upper troposphere, the low-level values are recovered. The larger the discrepancy between surface and midtropospheric values, the more is the likelihood that cumulonimbus activity may arise in the convectively unstable ambient.

For the sake of gross characterization, at ground level $(H/c_p) \doteq 350$ K in a storm-prone Midwestern ambient, with $T \doteq 300$ K, $(gz/c_p) \doteq 0$ K, $(LY/c_p) \doteq 50$ K. A midtropospheric minimum of crudely 325 K might be reached roughly around 65 000 N/m^2 (650 mb), or almost 3000 m, where $T \doteq 280$ K, $(gz/c_p) \doteq 30$ K, and $(LY/c_p) \doteq 15$ K. The ground-level value of (H/c_p) might be recovered around 15 000 N/m^2 (150 mb), or almost 13 700 m, where $T \doteq 210$ K, $(gz/c_p) \doteq 140$ K, and $(LY/c_p) \doteq 0$ K. In Colorado thunderstorms the interacting air masses may have surface dew point of only 283 K but different temperature, while Oklahoma dryline storms (Shaefer 1973) arise in air masses of comparable temperature but appreciably different relative humidity. Thus, the relative contributions of the three factors may vary, and the static energy is a useful quantity precisely because it conveys the convective instability common to different storm-prone ambients (refs. 64,85-87). A geosynchronous satellite may be particularly well suited for measuring the vertical profile of static energy on a refined spatial scale horizontally, to help quantify further the correlation between storm intensity and ambient stratification of static energy. (Of course, one actually measures a triplet of thermodynamic state variables, such as temperature, pressure, and relative humidity, and then computes the static energy.)

The required measurements are by no means trivial for existing satellite instrumentation. Significant vertical resolution is required.

Also, incrementation of moisture by 1.4 kg water vapor per 1000 kg air is a significant increase characteristic of tornado occurrence; this incrementation corresponds to an increase of only about 2 K in the dew point, and so modest a change would challenge satellite instrumentation.

Darkow & Livingston (ref. 88) have recently emphasized that, in a thunderstorm, an equivalence of 30 K difference typically exists between the surface values of the static energy of the evaporatively cooled downdraft of the cold sector and the convectively unstable air of the warm sector. Thus the warm and cold sectors of a developing thunderstorm may be distinguished by a ten-percent change in surface static energy, which may be a more discernible difference than the typical one-percent change in surface pressure across the advancing pseudocold front (gust front). Thus, surface maps of static energy may serve as discriminants of where thunderstorms are developing, perhaps even better than vertical profiles of static energy indicate where tornadoes may develop.

<u>The vertical wind shear</u>.- The problem with use of the vertical profiles of static energy as an indicator of where thunderstorms may arise is that they are an incomplete reflection of the physical processes involved. Much as there remains to be learned about thunderstorm genesis, it seems clear that the presence of appreciable vertical wind shear plays an instigating role (ref. 15,89). This is somewhat anomalous because for tropical cyclones it is the absence of vertical wind shear that precludes (so-called) ventilation (dispersal of lightened air in various directions at various altitudes,such that large horizontal pressure gradients are not sustained).

The conventional explanation for the need for vertical wind shear in most midlatitude thunderstorms notes first that the steering level for nonrotating thunderstorms (which move roughly at 10-20 m/sec) is crudely 4000 m. That is, at a height of around 4000 m, an observer moving with the storm would observe no relative ambient wind past the storm. The southeasterly low-level influx and updraft of warm moist air is thus canted upshear in the absence of westerly surface winds, before being bent downshear into the anvil by the 25 m/sec southwesterly winds typical of midtropospheric levels (ref. 62). Thus, there is fall-out of precipitation into cool dry air on the rear flank; re-evaporation of moisture causes the further-cooled rear-flank air to sink to the surface, where it spreads into (1) the gust front that initiates condensation by lifting the convectively unstable southeasterly influx and (2) some upshear flow at the surface in the cold dry sector. Numerical simulation confirms that vertical wind shear reduces the intensity, but prolongs the lifetime, of a squall line (refs. 90-92).

The Oklahoma network of dense surface instrumentation has detected some exceptionally heavy precipitation in connection with the passage of several cold fronts, either because the squall line was immediately adjacent to the front (refs. 93,94) (instead of being displaced 100 000 m or so to the east, as normally schematized), or because the upgliding of warm air at the front initiated exceptional precipitation (in addition to the more traditional prefrontal squall-line activity far to the east)(ref. 95). These events are noted because the absence of an evaporatively cooled downdraft on the rear flank is the only missing item that prevents identifying these frontal events as thunderstorms. Gravity currents (ref. 93) and evaporatively cooled downdrafts downwind of the front (ref. 95) have been cited as sustainers of these storms, which have given evidence of nearby gravity wave activity (ref. 95). It would be useful to contrast the duration of these frontal storms without vertical wind shear to the duration of typical thunderstorms with vertical wind shear.

However, for present purposes it is sufficient to note that a geosynchronous satellite would have to take account of the vertical wind shear, as well as the vertical profile of static energy, to characterize a storm-prone region. But even tracking these two items may be insufficient, because thunderstorm ingredients may be present for an extended time interval before a triggering mechanism (such as an internal gravity wave, to be discussed next) initiates and/or intensifies squall-line development.

Internal gravity waves as storm initiators.- A useful concept, due mainly to Charney (refs. 96,97), to characterize a wide variety of tropical meteorological phenomena of different scale, and to indicate how cumulus- and synoptic-scale motions may interact to mutual enhancement, is CISK (conditional instability of the second kind). The concept is that low-level convergence of heat and moisture may feed deep cumulus (and cumulonimbus) activity; this results in lightening of columns of air relative to their (possibly distance) neighbors, and thus in horizontal pressure gradients. The pressure gradients maintain the convergence, and even amplify it in a kind of positive-feedback cycle. Charney originally had the surface frictional layer (Ekman-type boundary layer) in mind as the low-level convergence mechanism, but Rosmond (ref. 98), and Lindzen (ref. 99) suggested that an alternative inviscid internal-gravity-wave mechanism, arising because of the continuous decrease of density with height in the lidless atmosphere (refs. 100,101), may serve as the convergence mechanism.

Whatever merits the CISK concept has for the tropics, there have been midlatitude observations of (1) banded structure to precipitation patterns in connection with the fall phenomenon of the coastal front in New England and also on the Pacific coast (ref. 36); (2) simultaneous existence of three active parallel prefrontal squall lines, themselves parallel to the cold front, in a jumbo outbreak of tornadoes (ref. 30); and (3) identification of a wave (of wavelength 300 000 - 500 000 m, of period 7200 - 14 400 sec, and of propagation speed 35-45 m/sec) as a spawner or intensifier of Midwestern thunderstorm cells in regions of sufficient moisture of sufficient depth (ref. 102). Thus, the quarter-century-old concept that a (somehow ducted) gravity wave may organize lower-troposphere confluence and divergence, the latent-heat release from the resulting cumulus activity strengthening the wave in a positive-feedback mechanism, has been the subject of a flurry of thunderstorm research activity from American to Japan (ref. 34) in the last half-dozen years.

In addition to the observational studies, there have been a plethora of linear two-dimensional studies of eigenvalue problems (e.g., refs. 35, 103) designed to establish the compatibility of these waves with known two-layer atmospheric dynamics and rather crude parameterizations of cumulus heating. These works, which variously include Oseen-linearized advections, friction, and vertical stratification, in a sense virtually assume the answer they seek by the cumulus parameterizations adopted. Invariably, the diabatic heating is related to the upflux at the subcloud-layer and cloud-layer interface. Internal gravity waves have also been observed to arise in nonlinear numerical simulations of midlatitude storms (refs. 104,105), with amplitudes of 500 m (refs. 90,91),

The waves apparently lead to downflow and outflow under troughs, inflow and updraft under ridges; thus the troughs are precipitation-free and the ridges, precipitation-prone. In particular, the maximum updraft occurs midway between a trough and ridge, but the maximum vertical displacement occurs under a ridge (ref. 35).

An evaluation.- Even if a geosynchronous satellite could measure and process all the requisite data with respect to static energy, vertical wind shear, and internal gravity waves, and possibly some other quantities for which discussion will be omitted for brevity, the problem of anticipating tornadoes, as distinct from thunderstorms, would not be resolved. It is reiterated that conditions that lead to a severe thunderstorm without tornadic activity are not distinguishable from conditions that lead to a multiple tornado outbreak, under current understanding. Thus, the general use of geosynchronous satellites to predict tornado occurrence is probably further away than the use of such satellites to detect existing tornadoes.

Attention would seem more profitably concentrated on rotating thunderstorms [which are particularly prone to generation of a family of long-life, long-path, high-intensity tornadoes (refs. 83,106)], and on properties of the "eye" that distinguishes the perilous tornado from the less destructive whirlwind.

PROPERTIES OF THE "EYE" IN A VERY INTENSE TORNADO: NEEDED RESEARCH

Previous discussion has established the existence of an "eye" as a requirement for thermohydrodynamic explanation of the 100 m/sec, and slightly greater, swirl speeds achieved in exceptionally violent tornadoes. In fact, for some ambients, it appears that the idealization of a completely dry "eye" is a fairly good approximation, in that virtually all the pressure reduction achieved by descent of dry, compressionally heated air from the tropopause to ground level is required to achieve the accepted swirl speeds. The fact that precipitation in severe thunderstorms often occurs away from the tornado funnel (ref. 107, p. 74; ref. 108; ref. 109, p. 384) suggests that evaporative cooling is sometimes a minor consideration for whirlwind thermohydrodynamics.

Elucidation of the "eye"/"eyewall" slope (the radial extent of the "eye" as a function of height and time), as well as elucidation of further details of the dynamics and thermodynamics within the "eye" itself, warrants further consideration. Because of decrease of density with altitude, it is clear that the "eye"/"eyewall" interface slopes radially outward with increasing height above the ground; aircraft flights directly over severe-tornado-engendering thunderstorms would be a useful source of information on cloud-top observables. Any periodicity in "eye" definition and/or properties, as a function of flow parameters, should be identified. The radial dimension of the "eye" is particularly crucial to the question of geosynchronous satellite detection of exceptionally severe tornadoes, because thermal anomaly, sense and magnitude of the vertical velocity component, and the radial profile of the swirl are plausible discriminants between one-cell and two-cell structure, if only the scale of instrument resolution were refined enough. Only the very largest severe whirlwinds could possibly be detected under current scales of resolution; many severe whirlwinds of smaller horizontal scale would not be detected unless resolution is improved by a factor of two-to-five times current specifications. The likely return for any extensive commitment of resources to improved resolution should be defined as carefully as possibly.

If anticipation of tornado occurrence by meteorological satellite designed for severe mesoscale storms appears improbable, and if detection of existing very severe tornadoes is theoretically possible provided instrumentation were refined enough to discern the "eye", should not modeling concentrate exclusively on elucidating the scale and properties of the "eye"? Not exclusively, for two reasons: (1) the required improvement in resolution may be unattainable, because of practical considerations, for a large number of severe whirlwinds of limited radial extent, so some other tell-tale sign must be identified; and (2) it is very difficult to make much progress on any one subdivisional feature of a local severe storm, because interfacial compatibility with respect to mass, momentum, and energy among subdivions provides strong constraints. Thus, it is important for credibility of any model of a tornado cyclone containing a tornado that the treatments of the (1) low-level inflow layer; (2) the updraft core; (3) the tropopause-level outflow layer; (4) the (possible) central "eye" with downdraft of tropopause-level dry air; (5) the bulk of the swirling air outside the core between the inflow and outflow levels; and (6) the interaction (if significant) with surrounding portions of the mesoscale circulation pattern, all be rendered mutually compatible.

For these reasons, modeling may prove deficient if it concentrates entirely on the outflow-layer cloud dynamics, as the feature of the storm that satellite instrumentation does currently discern, or entirely on the "eye", as a feature of the storm that satellite instrumentation may possibly discern. Thus, even while the focus is on upper-level observables for meteorological satellites, a more complete storm model is required.

CONDITIONS FOR THE TRANSITION FROM ONE-CELL TO TWO-CELL STRUCTURE: NEEDED RESEARCH

The transition from (1) organized convective activity with rotation (tornado cyclone) to (2) a more intense mode, in which a compressionally heated core of tropopause-level dry nonrotating is inserted within an updraft annulus (supercell with a tornado), is viewed here as a stability problem. Under what circumstances does a moderate one-cell "eyeless" convective vortex turn into a devastating two-cell convective vortex with an "eye"? The nonmonotonic time-history of tornado intensity, previously discussed, suggests the reverse transition also occurs; in fact, an oscillatory mode of existence alternating between one- and two-cell behavior may arise. Perhaps the sequential spawning of a family of tornadoes by one supercell, often with a time interval between successive whirlwinds, is also related to such transitions.

There are three steps to treating this stability problem in terms of sensible but simple models of the atmospheric phenomena. First, one must set up a simple model of the tornado cyclone with low-level cyclonic influx, high-level outflux, and ascent up a central core. The primary velocity component is a Rankine-type swirl, and the thermohydrodynamics of condensation-heat-driven flow must be included. Second, one must perturb, about the basic state, this idealization of a tornado cyclone, to formulate a linear homogeneous eigenvalue problem to determine under what conditions small disturbances to the one-cell vortex tend to amplify in time, and, presumably, to lead to a two-cell vortex. If the proper physics is included in the basic state, under hopefully some (but hopefully not all) circumstances, the small disturbance will grow in time. Third, one must ask about the mechanisms and the time scale on which the two-cell structure decays. The smaller horizontal scale associated with a tornado (two-cell structure, with an eye), as compared with the large horizontal scale associated with the tornado cyclone (one-cell structure, without an eye), may be expected to lead to a more intense but shorter-lived phenomenon. Each of these three steps is now examined in somewhat greater detail.

The basic state (the tornado cyclone).- The storm is viewed as open, i.e., there is convective (but not diffusive) transport freely across lateral boundaries, as required by the interior dynamics. At the outset, a mature axisymmetric one-cell storm warrants consideration. The swirling is strong enough to require retention of at least some nonlinear inertial terms.

It may turn out that nonaxisymmetric effects are critical to "eye" generation. The radial scale of the tornado cyclone requires definition, but it is believed to be large enough that the rotation of the earth is the source of storm rotation; formulation in noninertial coordinates, with retention of the Coriolis force, is believed necessary. The cyclonic sense of rotation of tornado cyclones, and virtually all well-documented tornadoes (ref. 4, p. 553), suggests such a source for storm circulation. The time scale for spin-up (a few hours) under typical rates of convergence (about 10^{-3}/sec) in mesoscale lows also seems compatible with such a source of rotation (ref. 4, p. 582; refs. 12,110). The vertical wind shear, the increase in magnitude of the zonal wind with increasing altitude from latitudinal gradients in temperature, is another source of departure from axisymmetry that may warrant retention; so is storm translation.

The contribution to the heat budget by change of phase of water substance poses formulation challenges. The convective diffusion of water vapor is complemented by (1) depletion under saturated conditions according to the locus of thermodynamic states defined by a moist adiabat, and by (2) accretion owing to re-evaporation of condensed moisture. Even the moist-adiabat treatment of condensation is not trivial, since ascent from different altitudes in the stratified ambient might warrant inclusion.

An even more formidable difficulty is that, in the absence of diffusion, a steady swirling flow may imply a unique steady state characterized by a cyclostrophic equilibrium. The centrifugal force and the radial pressure gradient balance in the bulk of the high-swirl region. In order to obtain a mass throughput (inflow at low levels, upflow near the axis, and outflow at high levels), one may have to include axial diffusion (to introduce the "tea-cup" effect of the surface inflow layer) and/or unsteadiness (to permit the axially distributed radial influx associated with spin-up). An adequate parameterization of turbulent diffusion in a rotating mesoscale storm is still an unresolved matter, and the variation of the ambient thermodynamic state in time may warrant retention in an unsteady formulation. The introduction of frictional forces and/or a relatively slow time scale of evolution in the basic state renders more difficult the ensuing perturbation problem.

In view of the difficulties just enumerated in representing the tornado cyclone for perturbation purposes, the following simplifications seem worthy of consideration. First, attention may be concentrated entirely on the core of the storm [above the surface inflow layer but below the tropopause (a slippery but impervious isobaric isothermal lid), between the axis of rotation and (roughly) the radius of maximum swirl]; it is here that the transition from one-cell to two-cell structure has the most evident effect. Then, a steady axisymmetric inviscid model, free of Coriolis effects, seems reasonable. Angular momentum is conserved along streamlines. If re-evaporation of condensate is ignored,then the static energy is also conserved along streamlines; in fact, if the surface inflow layer, which "feeds" the core throughput from below, is taken to be characterized by roughly uniform static enthalpy, then the core flow is almost isentropic and the same moist-adiabatic locus of thermodynamic states nearly characterizes all streamlines. While the flow is dynamically incompressible, the hydrostatic stratification warrants retention. Hopefully, confement of attention to the core, and simplification of the swirling updraft occurring there, suffices to describe the tornado cyclone for the stability study that ensues.

The perturbation problem for instability of a one-cell vortex.- The perturbation problem might best be formulated under the following philosophy. All dependent variables are expressed as the sum of a known basic state (steady and axisymmetric) plus a time-dependent nonaxisymmetric three-dimensional perturbation. This representation is substituted into the exact boundary/initial-value problem, including Coriolis acceleration. By definition of the basic state, all terms independent of perturbational quantities sum to zero; that is, the basic state is an exact solution mathematically, albeit one not necessarily observed physically, because it may be unstable to small disturbances. Terms of quadratic (or higher) order in perturbational quantities are discarded as of negligibly small magnitude, relative to terms linear in perturbational quantities (which are retained).

In the simplest case, for cylindrical polar coordinates, with radial coordinate r, azimuthal angle θ, and axial coordinate z,

$$d(r,\theta,z,t) \doteq d_0(r,z) + d_1(r,z) \exp(i\tilde{n}\theta + \sigma t) \qquad (33)$$

The normal-mode representation in angle θ and time t for the perturbational quantity associated with dependent variable d is readily possibly only if the basic state d_0 is steady and axisymmetric.

In any case, it is only after the linear eigenvalue problem has been formally derived that the variable coefficients associated with the basic state may be approximated for tractability, if it is demonstrable that such modification does not change the character of the results.*

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*Of course, in practice, only an approximation to $d_0(r,z)$ is probably available in convenient form anyway, so further simplification for purposes of the eigenvalue problem is a quite reasonable procedure.

Nevertheless, the variable-coefficient problem may require numerical treatment. One seeks the parametric conditions for the basic flow for which σ has a positive real part for any $\tilde{n}$, for this implies that the one-cell storm is unstable.

Decay of a tornado.- A tornado cyclone (a one-cell vortex) may persist for six hours, while a tornado (a two-cell vortex) typically decays in less than an hour, often well less than an hour (ref. 107). The reason for this more rapid decay of the more intense system may be a "return flow" on a smaller radial scale that is self-destructive. The larger-scale one-cell storm is viewed as an open system that readily interacts with the nearby atmosphere. The surroundings furnish the low-level input to the rotating thunderstorm, and transport away the mass, momentum, and energy of the diverging upper-tropospheric outflow. Any sensible heat in the outflow is passed to adjacent systems in the upper troposphere.

In contrast, the smaller-scale two-cell storm may be a closed system that does not have appreciable convective transport (only diffusive transport) across its lateral boundaries. The diverging upper-level outflow would then sink necessarily as it moves radially outward. Compressionally heating of the descending air, and the attendant lightening under hydrostatic considerations, in the flow outside the updraft annulus, would counteract the compressional heating of the descending air, and the attendant lightening under hydrostatic considerations, in the "eye". The central pressure deficit achieved by descent of stratospheric air would be removed by the descent of throughput air from the upper-tropospheric outflow. Without a large radial pressure gradient to maintain rotation, the two-cell configuration would break up. If so, vortex decay, like vortex spin-up, is a far more complicated process than a simple counterbalance of convectively induced advection and radial diffusion.

Since descent of stratospheric air is entrailed in the "eye", the storm lid for a closed system cannot be placed at the tropopause, but rather is placed at a higher altitude.

The diverging upper-level outflow may be so decelerated in swirl owing to (1) the role of friction in dissipating angular momentum during inflow, and (2) the decrease in angular speed with distance from the axis of rotation during outflow, that anticyclonic rotation relative to the earth arises. A similar phenomenon has long been understood for hurricanes (ref. 25).

CONCLUDING REMARKS

Inferences have been drawn from funnel-cloud interpretation that the severe tornado attains peak swirl speed, relative to the axis of rotation, of about 100 m/sec. Inferences have also been drawn, from processing of air in a tornado-spawning ambient through idealized sequences of thermohydrostatic states, that low-level pressure deficits consistent with such swirls can be attained only by having a two-cell structure for the tornado (an "eye" within an "eyewall"). Thus, for a passive geosynchronous meteorological satellite (with sufficient resolution of its instrument), detection of an existing severe tornado might be achieved in the future by detection of a compressionally heated central nonrotating sinking of upper-tropospheric, or lower-stratospheric, air, within a rapidly rotating and rapidly rising updraft annulus.

For elucidation of properties of an existing "eye", and for progress toward anticipating the onset of an "eye" (tornadic stage) in a rotating thunderstorm, a challenging but important stability problem is proposed for investigation. The problem concerns identification of conditions such that one-cell structure becomes unstable, and presumably evolves to two-cell structure. With this problem, tornado research begins to move from emphasis on the low-level swirling flow in the tornado, to emphasis on midtropospheric and upper-tropospheric properties of the dynamic energetics of a whirlwind. Such direction is appropriate when elucidating the observables in tornadic severe local storm available to proposed geosynchronous mesoscale-meteorology satellites.

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APPENDIX

FORMULAE FOR THE TEPHIGRAM METHOD

Equations of state

$$p_a = \rho_a R_a T \tag{A1}$$

$$\sigma p_v = \rho_v R_a T \tag{A2}$$

$$p = p_a + p_v \tag{A3}$$

$$\rho = \rho_a + \rho_v \tag{A4}$$

Hydrostatics

$$\frac{\partial p}{\partial z} = -\rho g \tag{A5}$$

Moisture

$$RH = \frac{p_v}{P(T)} \quad \text{, where} \quad P(T) \text{ is empirically derived}^{*} \tag{A6}$$

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[*] An impirical curve-fit to the saturation vapor pressure (in mb) as a function of temperature (in K) is given by (ref. 23):

$$P(T) = 6.1078 \exp\left[a\,(T - 273.16)/(T - b)\right] \tag{A7}$$

where

$$\left.\begin{aligned} a &= 21.8745585 \\ b &= 7.66 \end{aligned}\right\} \quad \text{over ice}$$

$$\left.\begin{aligned} a &= 17.2693882 \\ b &= 35.86 \end{aligned}\right\} \quad \text{over water}$$

It is noted that 0.01 mb are equivalent to 1 N/m^2.

Dry adiabat

$$\frac{dT}{dp} = \frac{1}{\rho c_{p_a}} \tag{A8}$$

Moist adiabat**

$$\frac{dT}{dp} = \frac{1/\rho + (L\sigma/p^2x^2)\ P}{c_p + (L\sigma/px^2)\ (dP/dT)},\ x = 1 - [(1 - \sigma)\ P/p] \tag{A9}$$

Cyclostrophic balance (approximation to conservation of radial momentum)

$$\frac{\partial p}{\partial r} = \rho v^2/r \tag{A11}$$

** L is treated as constant in this derivation. In fact, L (in cal/gm) is a slowly varying function of T (in K):

$$L = \begin{cases} 594.9 - 0.51\ (T - 273) & 273 \le T \le 313 \\ 594.9 + 79.7 & T < 273 \end{cases} \tag{A10}$$

Actually, at least a few degrees supercooling normally occurs before freezing takes place in the atmosphere, and some authors adopt models with freezing deferred to about 253 K. It is noted that 2.39×10^{-4} cal/gm are equivalent to 1 J/kg.

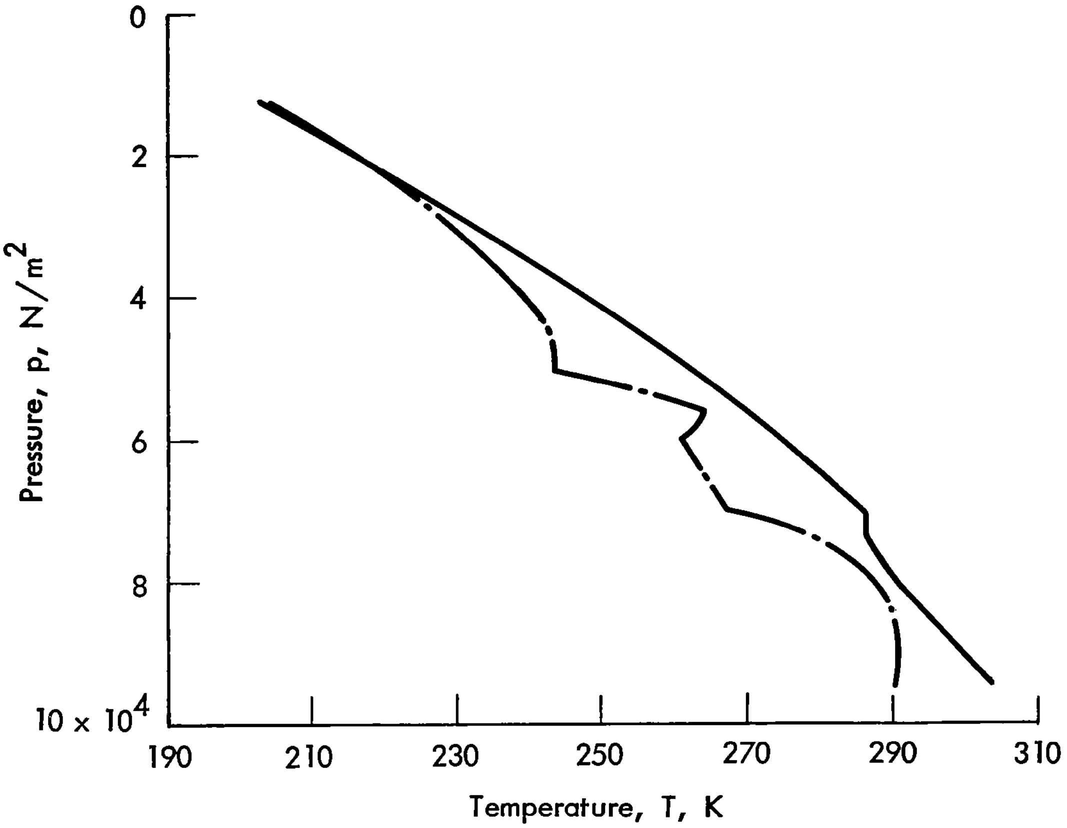

Figure 1. - Slightly smoothed data for the dew-point temperature T_d (dot-dash curve) and the static temperature T (solid curve) as a function of pressure p, for Cordell, Oklahoma, 10 June 1967, 2300 GMT.

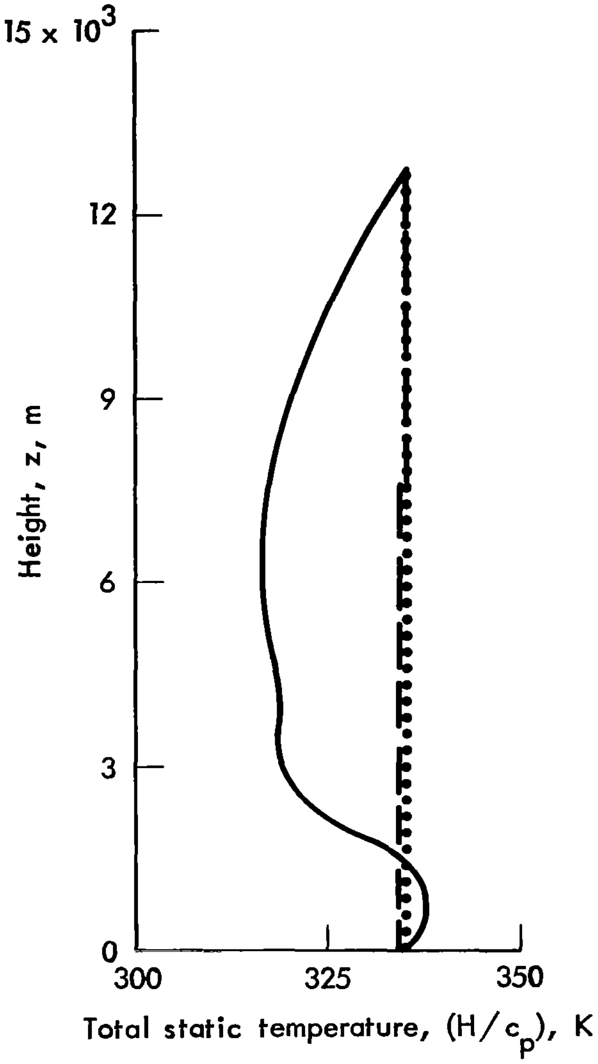

Figure 2.- The ambient (solid curve), moist-adiabatic (dashed curve), and dry-adiabatic (dotted curve) profiles with height z of (H/c_p), a temperature based on the static energy, with z = 0 assigned to 94 000 N/m^2.

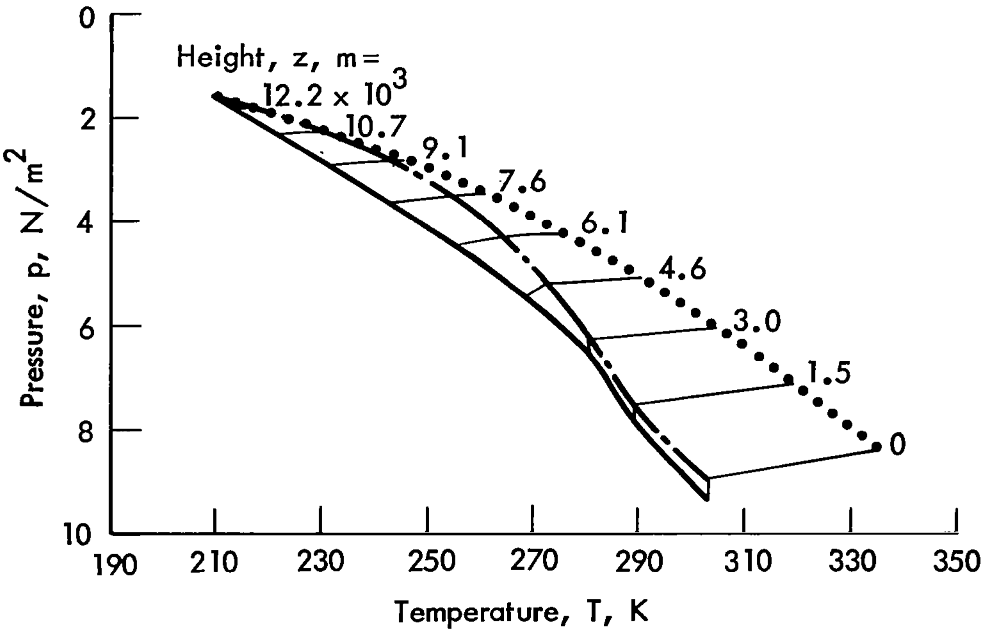

Figure 3. - Pressure p vs. temperature T for the ambient (solid curve), moist-adiabatic (dashed curve), and dry-adiabatic (dotted curve) columns, based on the data of Fig. 1. Constant-altitude contours are noted. Free convection on a moist adiabat is inhibited by the midtropospheric inversion, which dissipated prior to nearby tornado occurrence.

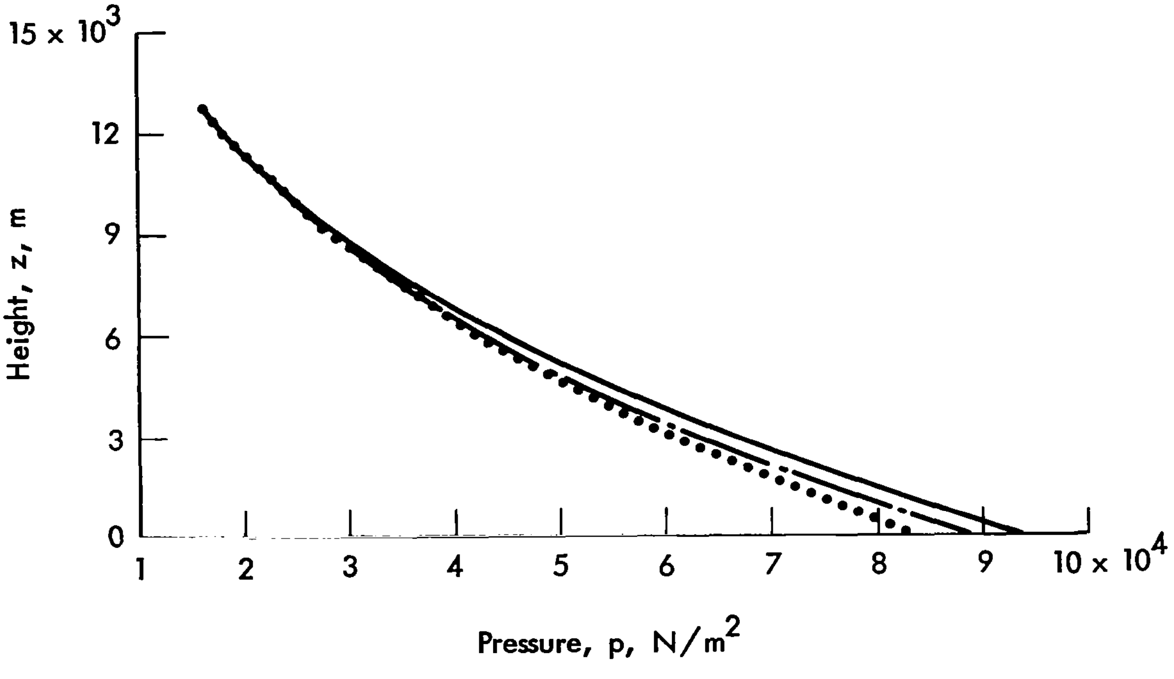

Figure 4. - The appreciably greater pressure deficit from ambient (solid curve) achieved by a dry adiabat (dotted curve), relative to that achieved by a moist adiabat (dashed curve), in the lower troposphere, as found for the data of Fig. 1.

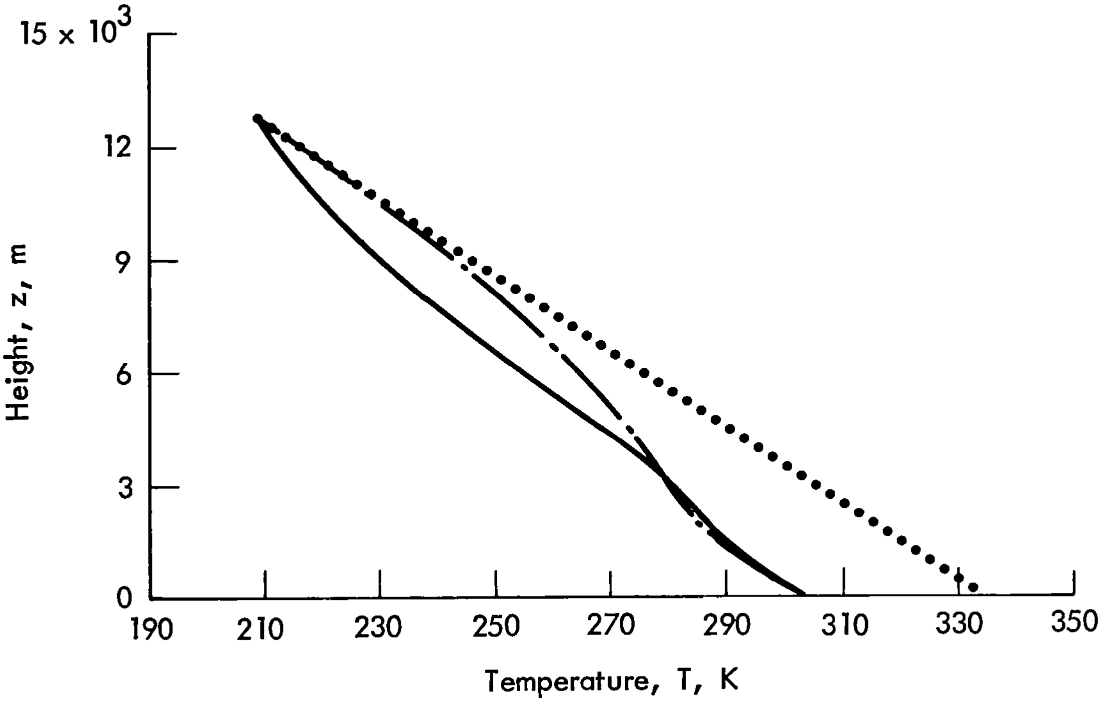

Figure 5. - Recasting static temperature T vs. altitude z results given in Fig. 3. Idealization of a moisture-free "eye" descending from tropopause to z = 0 produces the large temperatures on the dry adiabat (dotted curve), relative to those on the moist adiabat (dashed curve) and on the ambient (solid curve).

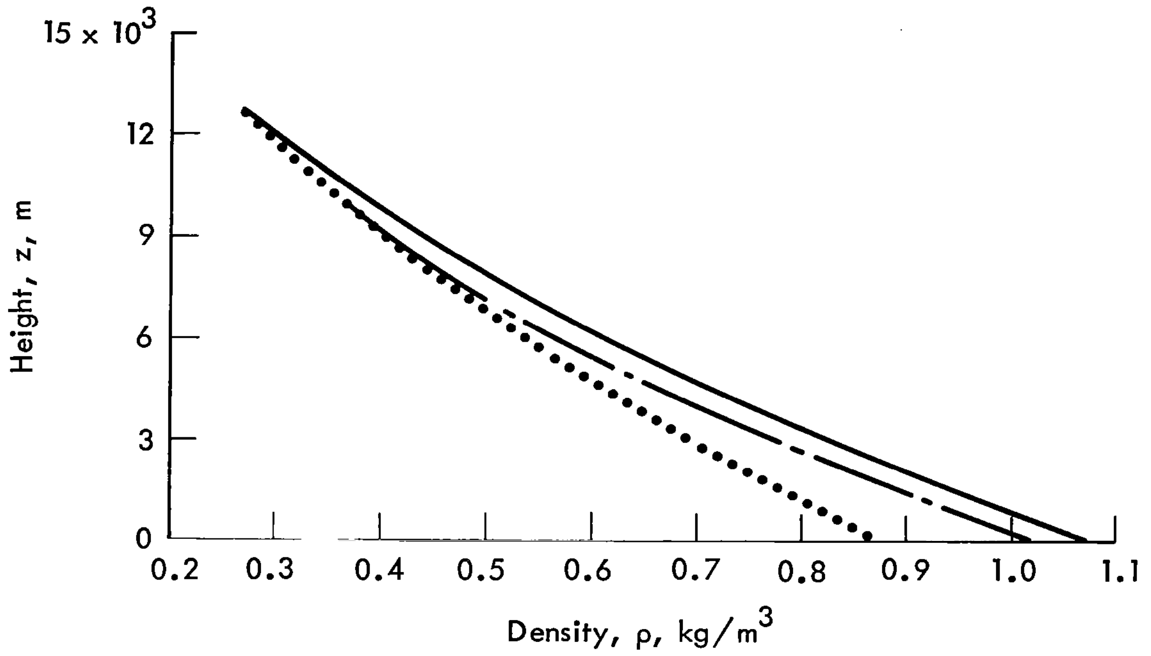

Figure 6. - Density ρ vs. altitude z for the ambient (solid curve), moist-adiabatic (dashed curve), and dry-adiabatic (dotted curve) columns, based on the data of Fig. 1. Density variation at ground level is less than twenty percent, despite the idealization adopted.

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