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METHODS OF ANALYZING WIND-TUNNEL DATA FOR DYNAMIC FLIGHT CONDITIONS

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SUMMARY

The effects of power on the stability and the control characteristics of an airplane are discussed and methods of analysis are given for evaluating certain dynamic characteristics of the airplane that are not directly discernible from wind-tunnel tests alone. Data are presented to show how the characteristics of a model tested in a wind tunnel are affected by power.

The response of an airplane to a rolling and a yawing disturbance is discussed, particularly in regard to changes in wing dihedral and fin area. Solutions of the lateral equations of motion are given in a form suitable for direct computations. An approximate formula is developed that permits the rapid estimation of the accelerations produced during pull-up maneuvers involving abrupt elevator deflections.

INTRODUCTION

Some time ago, the NACA undertook an investigation to determine the flying qualities of a low-wing, pursuit monoplane. This airplane (fig. 1), like many of its type, was found to possess several undesirable flying characteristics. Accordingly, a number of modifications to the airplane were recommended. In order to study the effects of the proposed modifications and in order to investigate the reliability of wind-tunnel data for estimating the behavior of the full-scale airplane, a scale model of the airplane, with and without the proposed modifications, was tested in a wind tunnel. Tests were made with and without the propeller operating in order to study the effect of power on the characteristics of the model.

During the course of these wind-tunnel tests, several interesting results were obtained concerning the effects of power on the characteristics exhibited by the model. In the evaluation of the data, certain methods of analysis were employed that were found useful in estimating the dynamic behavior of the airplane, particularly as regards the effects of the proposed modifications. These subjects are treated in the present paper. It is felt that this paper will serve as an aid to other wind-tunnel investigators seeking to evaluate the flight characteristics of an airplane from wind-tunnel results.

The first portion of the paper is concerned with the effect of power on certain of the wind-tunnel characteristics exhibited by a model. Graphs are presented that emphasize the importance of obtaining wind-tunnel data with powered models. The second portion of the paper discusses the lateral motions of the airplane. Concrete applications are included showing how the solutions to the lateral equations of motion were used to estimate the change in dynamic flight characteristics likely to result from the use of the recommended modifications to the airplane. Finally, the longitudinal motion of the airplane is considered. An approximate formula is developed that was found useful in estimating the accelerations and the stick forces likely to develop in an airplane during an abrupt pull-up maneuver.

THE IMPORTANCE OF POWERED MODELS IN WIND-TUNNEL TESTS

Although the influence of the slipstream on the characteristics of an airplane has been appreciated for many years, wind-tunnel tests with propellers operating have not been the usual procedure. The frequency with which undesirable flying characteristics appear in modern airplanes has led to some apprehension concerning the reliability of wind-tunnel data obtained without powered models. The particular testing technique employed in the present investigation will be reserved for detailed discussion in a future paper. It will suffice to say here that the power-on test procedure should be such that data may be obtained for thrust conditions of the model corresponding to those of the full-scale airplane at the lift coefficient and the flight attitude under consideration.

Figure 2 is a plot of the elevator angle required for trim against the indicated air speed at which trim occurs. These curves were evaluated from data secured from (1) wind-tunnel tests with no propeller on the model, (2) actual flight tests with the full-scale airplane, and (3) wind-tunnel tests with a powered model. The shapes of the curves are indices of the static stability (see later discussion of longitudinal motion) displayed by the airplane in flight and by the models in the wind tunnel. The agreement between the flight results and the results obtained with a powered model is good,particularly for the flap-up condition. The larger discrepancy existing for the flap-down condition is partly attributable to the rather large tab deflection (11° nose up) used on the full-scale airplane for this test. The related variation of the elevator stick force required for trim with indicated air speed is not presented here because the model data and the flight data were not directly comparable owing to the absence of a trim tab on the model. In the evaluation of the information obtained from wind-tunnel tests, certain methods of presenting the data have been found to facilitate the analysis. The resulting pitching moment acting on an airplane trimmed for steady equilibrium flight is zero. This condition is true whether or not the airplane is flying yawed. As far as the pilot is concerned, the dihedral effect of the wing manifests itself as a pure rolling moment about an axis lying in the plane of symmetry of the airplane. In wind-tunnel investigations, on the other hand, the rolling moment is usually measured about an axis coincident with the tunnel axis. Consequently, when the model is yawed relative to the tunnel axis, the measured rolling moment about this axis will contain a component of the pitching moment acting on the model unless the model is trimmed for zero pitching moment. In the comparison of the slopes of different rolling-moment curves obtained in yaw tests, it is therefore necessary to know whether the measurements were made with the model trimmed in pitch.

If the system of axes discussed in the subsequent section on lateral motion is employed for the presentation of wind-tunnel data, the necessity for continually checking the trim of the model during yaw tests is avoided. In addition, the data in this form can be used directly in the evaluation of the stability derivatives necessary for the solutions of the equations of motion.

In figure 3 are plotted for various power conditions the rates of change with angle of yaw of the yawing-moment coefficient $dC_n/d\psi$, the rolling-moment coefficient $dC_l/d\psi$, and the lateral-force coefficient $dC_Y/d\psi$. The decrease in dihedral effect $dC_l/d\psi$ with the application of power should be observed. In the condition presented (flap up), the decrease in dihedral effect with the application of power is not great, but, in the flap-down condition and at high lift coefficients, the effect of power may be critical.

An inspection of figure 4 will suffice to illustrate the effect of the slipstream on the characteristics of a control surface placed within its boundaries. The negative yawing moment that exists with neutral rudder when the propeller is operating should be observed. The moment is apparently caused by the rotation of the slipstream. It is, however, recognized that the slipstream rotational characteristics exhibited by the model in this test are probably not identical with the slipstream rotational characteristics that would be obtained in flight with the full-scale airplane, even under similar thrust conditions. The results do indicate, nevertheless, that with the present trend toward greater and greater power, the effect of this slipstream rotation may be a critical factor in tail design.

Figures 2, 3, and 4 have been included to illustrate the effects that power may have on the stability characteristics of an airplane and on the effectiveness of the horizontal and the vertical tail surfaces. These effects are difficult to estimate and consequently recourse must be had to wind-tunnel tests of a model equipped with running propellers. Wind-tunnel tests of a powered model require more care and are more expensive to perform than tests of the conventional type of model. By a judicious choice of tests, however, it is possible to secure the information of greatest value with a minimum of time and expenditure.

THE LATERAL MOTIONS OF THE AIRPLANE

Certain lateral-stability and control characteristics of an airplane can be directly determined from wind-tunnel tests of a model. These characteristics usually include

(1) the directional stability (fig. 3(a)), (2) the dihedral effect (fig. 3(b)), and (3) the static effectiveness of the control surfaces (fig. 4). This information is indispensable to the designer if he is to proportion his airplane properly and is also necessary for the evaluation of the stability derivatives from which the dynamic behavior of the airplane may be predicted.

The dynamic flight characteristics that an airplane will display are not, however, readily obtained. The behavior of an airplane subjected to a lateral disturbance is complicated by the coupling of the ensuing rolling and yawing motions. The coupling of these two motions makes it impossible to estimate the effectiveness of the ailerons and the rudder in flight from static tests of the controls alone. A sound evaluation of these factors can be made only if a knowledge of the motions produced by the controls is available.

The angle of bank produced, for example, in 1 second by the total deflection of the ailerons might be taken as a measure of the effectiveness of the ailerons in producing a sudden roll. If, on the other hand, the airplane is rolled slowly, a greater yawing motion being permitted to develop, the result may be to produce a rolling motion opposed to that generated by the ailerons and of such magnitude that the effect of the ailerons is completely nullified. The airplane may even roll against the ailerons.

In actual flight the pilot might counteract this adverse rolling tendency by coordinating the rudder with the ailerons. In the present analysis, however, only the inherent dynamic characteristics of the airplane are considered. It would be of interest, then, to know the variation of the angle of bank with time after small aileron deflections are applied. For similar reasons, the effectiveness and the sensitivity of the rudder can be most carefully judged if the angle of bank and the angle of yaw produced in a definite time interval by a small rudder deflection are known. Information of this type is particularly valuable in estimating the effect of various combinations of vertical fin area and wing dihedral on the dynamic behavior of an airplane.

Apart from actual flight tests, probably the best method of estimating the dynamic qualities of an airplane is by evaluating the dynamic equations of motion, the data obtained from wind-tunnel tests of a powered model being used.

Assumptions and symbols.- The assumptions generally made in the study of airplane stability are made here. The most important of these assumptions are:

1. The air forces and the moments resulting from displacements of the airplane relative to its steady condition of flight are proportional to the displacements or to their rates of change.

2. The components of moment due to different components of the motion are directly additive. (For example, the rolling moment due to combined rolling and sideslipping may be computed as though the rolling and the sideslipping had occurred separately.)

The axes used in specifying the moments, the angular velocities, and so forth are fixed in the airplane and move relative to the earth and the air. The X axis, passing through the center of gravity of the airplane, is in the plane of symmetry and is so oriented that it points into the relative wind when the airplane is flying steadily in unyawed flight. Also, the axes form a conventional orthogonal system intersecting at the center of gravity. The Z axis points directly downward in the plane of symmetry and the Y axis points along the direction of the right wing. The motions discussed are those of the moving axes relative to the undisturbed air with the exception of the angle of bank, which is measured from the horizontal.

The symbols used in the following analysis are defined in the appendix.

Although the axes change their orientation in the airplane with different lift coefficients and probably never coincide with the axes of the principal moments of inertia, the corrections in unstalled flight are small and have been neglected, as have the products of inertia.

Equations of motion.- If the airplane is considered capable of motion in all degrees of lateral freedom, the equations of motion with deflected controls (neglecting the small side forces developed by the deflected controls and by the rolling and the yawing velocities) may be written:

$$\left.\begin{aligned}
\frac{dp}{dt} &= pL_p + rL_r + \beta L_\beta + \delta L_\delta \quad \text{(in rolling)} \\
\frac{dr}{dt} &= pN_p + rN_r + \beta N_\beta + \delta N_\delta \quad \text{(in yawing)} \\
\frac{d\beta}{dt} &= \frac{g\phi}{U_o} - r + \frac{\beta Y_\beta}{U_o} \quad \text{(in sideslipping)}
\end{aligned}\right\} \qquad (1)$$

Also

$$\frac{d\phi}{dt} = p; \quad \frac{d\psi}{dt} = r; \quad \beta = \frac{v}{U_o}$$

In order to solve for any of the variables, it is necessary to integrate this system of linear simultaneous equations. For reasons that will be apparent later, it is convenient to solve the system of equations (1) for separate unit magnitudes of the control disturbance terms, δL_δ and δN_δ; that is, one set of solutions is obtained by letting $\delta L_\delta = 1$ and $\delta N_\delta = 0$, and another set of solutions is obtained by letting $\delta L_\delta = 0$ and $\delta N_\delta = 1$. The unit disturbances are assumed to be instantly applied at zero time and to remain constant thereafter. In order to distinguish the separate solutions, the subscript L will be applied to the solutions obtained when $\delta L_\delta = 1$ and $\delta N_\delta = 0$; whereas the subscript N will be applied to solutions obtained by letting $\delta L_\delta = 0$ and $\delta N_\delta = 1$. Thus, p_L represents the rolling velocity resulting from the application of a pure rolling disturbance of unit magnitude.

If the symbol D is substituted for d/dt and if $\delta L_\delta = 1$ and $\delta N_\delta = 0$, equations (1) may be rewritten in the following form:

$$\left.\begin{aligned} p\,(D-L_p) - rL_r - \beta L_\beta &= 1 \\ p\,(-N_p) + r(D-N_r) - \beta N_\beta &= 0 \\ p\left(-\frac{g}{U_o}\right) + Dr + D\left(D - \frac{Y_\beta}{U_o}\right)\beta &= 0 \end{aligned}\right\} \qquad (2)$$

It can be shown that the solutions of equation (2) are of the form:

$$p_L = p_{L_0} + p_{L_1}e^{\lambda_1 t} + p_{L_2}e^{\lambda_2 t} + p_{L_3}e^{\lambda_3 t} + p_{L_4}e^{\lambda_4 t} \qquad (3)$$

where

p_L resultant rolling velocity due to unit rolling disturbance, that is, $\delta L_\delta = 1$

$p_{L_0} \cdot\cdot p_{L_4}$ constants that depend only on values of stability derivatives and on type of disturbance involved

$\lambda_1 \,..\, \lambda_4$ roots of stability equation $F(D) = 0$,

where

$$F(D) = \begin{vmatrix} D-L_p & -L_r & -L_\beta \\ -N_p & D-N_r & -N_\beta \\ \frac{-g}{U_o} & D & D\left(D-\frac{Y_\beta}{U_o}\right) \end{vmatrix}$$

The constants in equation (3), together with the constants appearing in the expressions for all the remaining components of motion (including the solutions for the unit yawing disturbance $\delta N_\delta = 1$), have been evaluated in terms of the stability derivatives and the roots of the stability equation. The solutions of equation (2) are tabulated in the appendix in a form suitable for computation. The stability equation is also discussed and an alternative form of equation (3) is given for use when the solution of $F(D) = 0$ includes conjugate complex roots of the form $a \pm ib$.

The expressions presented in the appendix were evaluated by applying the operational mathematics of Heaviside to equation (2). For the theory of operational methods the reader is referred to a standard text on the subject, for example, reference 1. Specific applications of the Heaviside treatment to other problems in airplane dynamics may be found in reference 2.

After the complete unit solutions have been obtained, time histories of the motion caused by the unit disturbances can be plotted with very little additional calculation. Because of the linearity of the equations of motion, the unit solutions may be compounded in any arbitrary manner. If, for example, $\delta_a L_{\delta_a}$ represents the rolling acceleration created by the applied aileron rolling moment and $\delta_a N_{\delta_a}$ represents the accompanying yawing acceleration, then, at time t,

$$\phi_t = \delta_a L_{\delta_a} \left(\phi_L\right)_t + \delta_a N_{\delta_a} \left(\phi_N\right)_t \tag{4}$$

where

ϕ_t resultant angle of bank after t seconds

$\left(\phi_L\right)_t$ angle of bank after t seconds due to a unit rolling disturbance

$\left(\phi_N\right)_t$ angle of bank after t seconds due to a unit yawing disturbance

As in the case of the unit disturbance terms, the actual disturbances, $\delta_a L_{\delta_a}$ and $\delta_a N_{\delta_a}$, are assumed to be suddenly applied at zero time and to remain constant thereafter.

Applications.- The practicability of using the solutions to the lateral equations of motion is best illustrated by concrete applications.

The airplane under consideration, as mentioned in the introduction, exhibited certain undesirable flying characteristics in flight tests: The dihedral effect was undesirably low at slow speeds with the flaps down (fig. 3(b) indicates satisfactory dihedral characteristics with the flaps up over the range considered), and it was impossible to raise a wing by use of the rudder alone. In an effort to improve the lateral flying qualities of the airplane, it was proposed to increase the wing dihedral. Because the lateral characteristics of an airplane depend not only on the absolute amount of dihedral but also on the relative amount of weathercock stability present, it was considered necessary to increase the vertical tail area as well as the wing dihedral. The ailerons were unchanged, but the rudder was so modified as to improve its hinge-moment characteristics. These modifications are shown on figure 1.

Wind-tunnel tests were made with a model equipped for power-on tests with and without the proposed modifications. The data from the comparative tests indicated that considerable improvement should result from the incorporation of the modifications on the full-scale airplane. For all flap and power conditions the dihedral effect $dC_l/d\psi$ remained positive, the index of weathercock stability $dC_n/d\psi$ remained negative, and the static characteristics of the rudder appeared satisfactory. Wind-tunnel tests of a model, however, provide no direct information pertaining to the dynamic flight characteristics of the full-scale airplane. Instances have occurred in which the incorporation of similar modifications on a full-scale airplane has affected the control characteristics of the airplane in an adverse manner, particularly at high speeds, in spite of the favorable static characteristics indicated from wind-tunnel tests. Accordingly, it was decided to investigate the response of the airplane, with and without the modifications to the aileron and the rudder controls, by evaluating the solutions to the equations of motion.

Aileron control.- In the particular problem considered here, undesirable aileron control characteristics are likely to be manifested in the form of aileron "heaviness" or stiffness at high speeds. The subsequent analysis, however, is perfectly general and is in no way limited to this high-speed condition. This particular condition is treated, merely as an example, to indicate the general method of procedure. Because the ailerons are identical on both the original and the modified airplanes, aileron heaviness can be physically interpreted as an increase in stick force resulting from the increased aileron deflection necessary to reproduce a given rolling maneuver with the modified airplane. This interpretation of aileron heaviness suggested the following method of analysis:

(a) On the assumption that the original airplane (airplane A) was flying in steady high-speed flight, the angle of bank generated in 5 seconds by a small aileron deflection δ_a was computed.

(b) Then, the aileron deflection $\delta_a + \Delta\delta_a$ necessary to reproduce the identical maneuver with the modified airplane (airplane B) was calculated.

The time for the maneuver (5 sec in this case) is somewhat arbitrary. It should be of sufficient duration, however, to permit full development of the secondary rolling effects introduced by the induced yawing motion.

The magnitude of the increment $\Delta\delta_a$ is a direct measure of the additional stick force required to perform the maneuver (because the ailerons are identical) and may be used as an index of aileron heaviness. If $\Delta\delta_a$ is large and positive, the aileron stick forces on airplane B may be too large to be acceptable and modification of the aileron itself may prove desirable.

The stability derivatives of airplanes A and B were evaluated from the power-on wind-tunnel tests of the two models and from data in references 3 and 4. These derivatives are tabulated in table I, together with other information necessary to evaluate the solutions to the equations of motion.

In accordance with equation (4), the angle of bank assumed by airplane A in t seconds after the application of an aileron deflection δ_a is

$$\phi_t = \delta_a L_{\delta_a} \left(\phi_L\right)_t + \delta_a N_{\delta_a} \left(\phi_N\right)_t$$

In order to evaluate the unit solutions ϕ_L and ϕ_N, it is necessary to obtain the roots of the stability equation $F(D) = 0$. If the appropriate values from table I are substituted into the expression for $F(D)$ given in the appendix,

$$F(D) = D^4 + 20.4555D^3 + 52.7884D^2 + 347.8242D + 5.43760 = 0$$

With the use of the procedure outlined in the appendix for solving quartic equations, the roots of this equation were determined to be:

$$\lambda_1 = -0.01567$$

$$\lambda_2 = -18.6230$$

$$\lambda_3 = -0.908424 + 4.2199i$$

$$\lambda_4 = -0.908424 - 4.2199i$$

Substitution of the appropriate roots and derivatives in the expression for the angle of bank ϕ_L, given in the appendix, yields

$$\begin{aligned}\phi_L = {} & 3.3471 - 3.3497e^{-0.01567t} + 0.002876e^{-18.623t} \\ & + \left(\frac{-0.46778 + 0.1631093i}{1741.79 - 2270.45i}\right)e^{(-0.9084 + 4.2199i)t} \\ & + \left(\frac{-0.46778 - 0.1631093i}{1741.79 + 2270.45i}\right)e^{(-0.9084 - 4.2199i)t}\end{aligned}$$

If the last two terms are combined in accordance with the transformation formula in the appendix, the final expression for ϕ_L can be written:

$$\begin{aligned}\phi_L = {} & 3.3471 - 3.3497e^{-0.01567t} + 0.002876e^{-18.623t} \\ & - 0.00034623e^{-0.9084t} \cos 4.2199\,(t + 0.1354) \qquad (5)\end{aligned}$$

Similarly,

$$\phi_N = 11.5982 - 11.62275e^{-0.01567t} + 0.00038854e^{-18.623t}$$

$$+ 0.043548e^{-0.9084t} \cos 4.2199\ (t + 0.23305) \qquad (6)$$

Practical solutions are most conveniently obtained by graphical addition and subtraction of the component parts of the motion. It can be seen that a subtraction involves the small difference of relatively large quantities. For this reason it is necessary to retain as many absolute figures as possible in the evaluation of the roots of the stability equation and in the evaluation of the individual components of the motion.

Equations (5) and (6) are plotted in figure 5 (a) for airplane A. The corresponding expressions for airplane B are plotted in figure 5 (b).

From figure 5 (a) the unit solutions for ϕ_L and ϕ_N after 5 seconds are

$$\phi_{L_{5\ sec}} = 0.25 \text{ radian}$$

$$\phi_{N_{5\ sec}} = 0.86 \text{ radian}$$

For a 1° aileron deflection from table I,

$$\delta_a L_{\delta_a} = 1.54 \text{ per second}^2$$

and

$$\delta_a N_{\delta_a} = 0$$

Hence, for airplane A,

$$(\phi)_{5\ sec} = (1.54)\ (0.25) = 0.385 \text{ radian} = 22.1^\circ$$

The total aileron deflection necessary to bank airplane B 22.1° in 5 seconds is given by

$$\delta_a = \frac{\phi_t}{L_{\delta_a}\left(\phi_L\right)_t + N_{\delta_a}\left(\phi_N\right)_t}$$

where

$$L_{\delta_a} = \frac{\left(\frac{dC_L}{d\delta_a}\right) qSb}{mk_X{}^2}$$

$$N_{\delta_a} = \frac{\left(\frac{dC_n}{d\delta_a}\right) qSb}{mk_Z{}^2}$$

For airplane B, $L_{\delta_a} = 1.54$ per degree, $N_{\delta_a} = 0$, and $\phi_L = 0.26$ radian; hence,

$$\delta_a = \frac{0.385}{(1.54)\ (0.26)} = 0.96^\circ$$

or

$$\Delta\delta_a = -0.04^\circ$$

In view of the fact that the ailerons on the two airplanes possess the same hinge-moment characteristics, it can be concluded that the aileron stick forces developed during maneuvers will at least be no greater (or heavier) on the modified airplane than on the original airplane.

Rudder control.- The sensitivity of the rudder control, particularly at high speeds, may be judged by the angle of bank and the angle of yaw generated, after a definite time interval, by the yawing moment impressed by the rudder. Here again the time must be of sufficient duration to permit the interaction of secondary rolling and yawing effects.

Airplane A was assumed to be flying level at high speed and to be suddenly subjected to a yawing moment impressed by a given rudder deflection. The angle of bank

ϕ and the angle of yaw ψ, generated after 5 seconds, were calculated. Under similar circumstances, the angle of bank $\phi + \Delta\phi$ and the angle of yaw $\psi + \Delta\psi$ developed by airplane B were similarly calculated. The magnitudes of $\Delta\phi$ and $\Delta\psi$ were taken as a measure of the relative sensitivity of airplanes A and B to their respective rudder controls. The computations follow.

From figure 5 and table I,

	Airplane A	Airplane B
$\left(\phi_L\right)_{5\ sec}$ radian - - - - - - - - -	0.25	0.26
$\left(\phi_N\right)_{5\ sec}$ radian - - - - - - - - -	.86	.98
$\delta_r L_{\delta_r}$, per sec^2 - - - - - - - - -	.308	.410
$\delta_r N_{\delta_r}$, per sec^2 - - - - - - - - -	.549	.652

If a form similar to that of equation (4) is used,

$$\phi_t = \left(\delta_r L_{\delta_r}\right) \left(\phi_L\right)_t \ \left(\delta_r N_{\delta_r}\right) \left(\phi_N\right)_t$$

and the angle of bank of airplane A, 5 seconds after the sudden application of a 1° rudder deflection, is

$$(\phi)_{5\ sec} = (-0.308)\ (0.25) + (0.549)\ (0.86)$$
$$= 0.395 \text{ radian} = 22.6^\circ$$

For airplane B,

$$(\phi)_{5\ sec} = (-0.410)\ (0.26) - (0.652)\ (0.98)$$
$$= 0.532 \text{ radian} = 30.5^\circ$$

and

$$\Delta\phi = 8^\circ$$

The angles of yaw generated by the unit disturbances were obtained in a similar manner: thus,

$$\left(\psi\right)_t = \delta_r L_{\delta_r} \left(\psi_L\right)_t + \delta_r N_{\dot{\delta}_r} \left(\psi_N\right)_t$$

In the maneuver investigated, the unit solutions after 5 seconds were as follows:

	Airplane A	Airplane B
$\left(\psi_L\right)_{5\ sec}$ radian - - - - - - - -	0.04	0.04
$\left(\psi_N\right)_{5\ sec}$ radian - - - - - - - -	.30	.26

For airplane A,

$$(\psi)_{5\ sec} = (-0.308)(0.04) + (0.549)(0.30)$$
$$= 0.153 \text{ radian} = 8.8^\circ$$

and, for airplane B,

$$(\psi)_{5\ sec} = (-0.410)(0.04) + (0.652)\ (0.26)$$
$$= 0.154 \text{ radian} = 8.8^\circ$$

or

$$\Delta\psi = 0^\circ$$

It appears that, for equal rudder deflections, the modified airplane will tend to generate more bank than the original airplane. The increase in the wing dihedral and the modified fin and rudder may result, then, in a rudder control that will be slightly more sensitive at high speeds than the rudder control on the original airplane.

Hinge-moment measurements are, of course, necessary to determine whether the resultant rudder-pedal forces on the modified airplane will be greater or less than those on the original airplane. The hinge-moment characteristics can be compared on the basis of the quantity

$$\left(\frac{S_r c_r}{\frac{1}{2}\rho V^2}\right)\left(\frac{\partial C_h}{\partial \delta_r{}^\circ}\right) = R$$

For airplane A, $R = -0.297$; for airplane B, $R = -0.150$. The resultant pedal forces, on the other hand, depend not only on the hinge-moment characteristics of the rudder but also on its floating characteristics and on the particular deflection required to perform a stipulated maneuver. The resultant hinge moment per degree of rudder deflection may be expressed as follows:

$$\frac{H}{\delta_r{}^\circ} = R\left[1 - \left(\frac{\partial \delta_r}{\partial \psi}\right)_{C_{h_r} = 0} \frac{\psi}{\delta_r}\right]$$

The quantity $\left(\frac{\partial \delta_r}{\partial \psi}\right)_{C_{h_r} = 0}$ is theoretically a constant for any given tail arrangement but actually it is very critical to interference effects at the tail and fluctuates considerably. For small angles of yaw $(\pm 5^\circ)$, $\left(\frac{\partial \delta_r}{\partial \psi}\right)_{C_{h_r} = 0}$ was practically zero for both airplanes A and B; and the quantity R may, therefore, be taken as a measure of the pedal forces for a given rudder deflection.

THE LONGITUDINAL MOTION OF THE AIRPLANE

When the longitudinal stability of an airplane is discussed, the characteristic usually referred to is the "static" longitudinal stability. If static longitudinal stability exists, the dynamic stability characteristics are of minor importance (reference 5).

The usual index for static longitudinal stability is the rate of change of pitching-moment coefficient with lift coefficient dC_m/dC_L or some quantity proportional to it. In flight the most convenient method of evaluating the amount of static stability present is to measure the elevator angle required to trim the airplane at various speeds; the slope, $d\delta_e/dV$ or $d\delta_e/d\alpha$, is an index of the degree of static stability possessed by the airplane. The significance of the ratio $d\delta_e/d\alpha$, methods of evaluating it for power-off and windmilling conditions, and suggested design values are discussed in reference 6. The necessity for power-on wind-tunnel tests for securing the effect of power has already been discussed (fig. 2).

If tunnel data are available from which the floating angle of the elevator may be caluclated for any lift coefficient, the elevator stick force required for trim at any lift coefficient can be calculated from the following formula:

$$P = \left(\frac{d\delta_e}{dx}\right)\left(\frac{\partial C_h}{\partial \delta_e^{\circ}}\,\Delta\delta_e^{\circ}\right)\frac{1}{2}\rho V^2\, S_e c_e \qquad (7)$$

where

- P stick force for trim, pounds
- x linear travel of top of control column, feet
- $\Delta\delta_e^{\circ}$ difference, in degrees, between elevator angle required for trim at lift coefficient under consideration and free-floating angle of elevator

The term $\Delta\delta_e^{\circ}$ may fluctuate considerably with power.

The slope of the curve relating the variation in stick force with forward speed dP/dV depends on the speed at which the airplane is trimmed for zero stick force and, consequently, depends on the initial setting of the trimming tab. In the comparison of the change in the slope of the stick-force curve resulting from modifications to the elevator, care should therefore be taken to orient the elevator trimming-tab settings so that zero stick force always occurs at the same speed. Otherwise, a superficial examination of the curves of stick-force variation with air speed may lead to incorrect conclusions concerning the effectiveness of the tail surfaces or the stability of the airplane.

In addition to providing a means of trimming the airplane in steady flight, the elevator must be capable of changing the airplane flight path. The rate at which this change is accomplished in a quick pull-up can, in a sense, be interpreted as a measure of the effectiveness of the elevator in maneuvers.

It is convenient to take the rate of change of the maximum normal acceleration per unit of elevator deflection as an index of elevator effectiveness in maneuvers. Care must be exercised, however, in interpreting this index. Although it is essential in a pursuit airplane to design an elevator sufficiently powerful to maneuver the airplane to the maximum lift coefficient of the wing, it has been found very undesirable if this condition is fulfilled with a minimum amount of elevator deflection. As discussed in reference 6, satisfactory static stability characteristics require the quantity $d\delta/d\alpha$ to have a value around 0.5. In airplanes that required considerably less stick travel to trim the airplane over the angle-of-attack range inadvertent stalling has frequently occurred in accelerated maneuvers. The optimum value for the rate of change of normal acceleration per unit of elevator deflection is therefore conditioned by the requirements of satisfactory static longitudinal stability.

In the following section a simplified formula is developed that permits the rapid estimation of the normal accelerations developed in abrupt pull-up maneuvers.

<u>Development of a simplified formula for normal accelerations produced in abrupt pull-ups</u>.- To a first order of approximation, the equations of motion in the plane of symmetry involving a disturbance in pitch may be written as follows:

$$\left.\begin{aligned}
\frac{du}{dt} &= uX_u + wX_w + qX_q - g\theta &\quad (a)\\
\frac{dw}{dt} &= uZ_u + wZ_w + q\left(U_o + Z_q\right) - g\theta_o\theta + \delta_e Z_{\delta_e} &\quad (b)\\
\frac{dq}{dt} &= uM_u + wM_w + qM_q + \delta_e M_{\delta_e} &\quad (c)
\end{aligned}\right\}(8)$$

Also

$$\frac{d\theta}{dt} = q$$

where θ_0 is the initial angle the X axis makes with the horizontal axis.

The axes in which equations (8) are expressed are similar to those defined in the section on lateral motion except that the X axis is inclined at an angle θ to the relative wind.

In general, this system of equations must be solved by methods analogous to those used in obtaining the solutions to the lateral equations of motion. The normal acceleration is given by the expression $dw/dt - qU_0$.

These equations have been solved for the airplane, the characteristics of which (evaluated, so far as possible, from wind-tunnel tests on a powered model) are presented in table II. The curve of the normal acceleration resulting from a 1° upward elevator deflection is plotted in figure 5 together with other components of the motion.

The variation of the components of motion shown in figure 6 is typical of an abrupt pull-up maneuver at high speed. The following facts are apparent from this figure:

1. During the time the airplane takes to attain maximum acceleration, the velocity V remains sensibly constant and is equal to U_0, the equilibrium velocity. Accordingly, $u = du = 0$, and equation (8a) can be neglected.

2. In the vicinity of the maximum acceleration, the time rate of change of pitching velocity dq/dt is approximately zero, that is, q is approximately constant.

3. In the vicinity of the maximum acceleration, the acceleration component dw/dt is almost zero and the acceleration thereafter is given almost entirely by the product qU_0.

If Z_{δ_e}, Z_q, and $g\theta_o$ (all of which are small) are neglected and the substitutions $\alpha Z_\alpha = wZ_w$ and $wM_w = \alpha M_\alpha$ are made, equation (8), for an abrupt pull-up maneuver, can be simplified to the following form:

$$\left.\begin{aligned} qU_o + \alpha Z_\alpha &= 0 \\ qM_q + \alpha M_\alpha &= -M_o \end{aligned}\right\}$$

where

$$M_o = \delta_e M_{\delta_e}$$

The solution for qU_o is

$$qU_o = U_o \left(\frac{M_o Z_\alpha}{U_o M_\alpha - Z_\alpha M_q}\right) \tag{9}$$

where

$$M_o = \left(\frac{\partial C_m}{\partial \delta_e}\right) \delta_e \frac{1}{2} \frac{\rho S V^2 c}{m k_Y^2}$$

$$Z_\alpha = -\left(\frac{dC_L}{d\alpha}\right) \frac{1}{2} \frac{\rho S V^2}{m}$$

$$M_\alpha = \left(\frac{dC_m}{d\alpha}\right) \frac{1}{2} \frac{\rho S V^2 c}{m k_Y^2}$$

$$M_q = -F\eta_t \left(\frac{dC_L}{d\alpha}\right)' \frac{1}{2} \frac{\rho S V l^2}{m k_Y^2} \frac{S'}{S}$$

In the expression for M_q, the slipstream factor F normally has a value between 1 and 1.25 for high-speed flight. The tail efficiency factor η_t is always less than 1 and is generally about 0.9. The value of the product $F\eta_t$ is therefore always about unity.

If the airplane is assumed to be flying level before the pull-up maneuver, $mg = C_L \frac{1}{2}\rho S V^2 = C_L \frac{1}{2}\rho S U_o{}^2$. If the substitutions $U_o = V$, $\mu = \dfrac{m}{\frac{1}{2}\rho S l}$, and $F\eta_t = 1$ are made, equation (9) can be reduced to the following simplified form:

$$\frac{qU_o}{\delta_e} = -\frac{g\mu}{C_L}\left[\frac{\partial C_m/\partial\delta_e}{\mu\dfrac{dC_m}{dC_L} - \dfrac{l}{c}\left(\dfrac{dC_L}{d\alpha}\right)'\dfrac{S'}{S}}\right] \qquad (10)$$

where qU_o/δ_e represents the change in normal acceleration per unit of elevator deflection.

Formula (10) gives a value of the normal acceleration produced during an abrupt pull-up maneuver that closely approximates the maximum acceleration which would be obtained by solving the more cumbersome equations of motion. It is the acceleration that will be obtained if the elevator is instantly deflected to its final position and held in that position until the maximum acceleration is reached. The formula gives less accurate results when dC_m/dC_L is small, that is, $-\dfrac{dC_m}{dC_L} < 0.01$. Several calculations made for smaller values of dC_m/dC_L gave results, however, that were in error by less than 7 percent. Greater accuracy is to be expected at small values of C_L on account of the approximations made involving θ. At high values of C_L, these approximations introduce greater errors.

Maneuverability and stability.- The manner in which the normal acceleration produced in a pull-up maneuver is affected by dC_m/dC_L and μ is shown in figure 7. An examination of this figure reveals that, for airplanes with high wing loadings (large μ) and low static stability (small dC_m/dC_L), the normal acceleration produced per degree of elevator is affected by a small change in the static stability. In accelerated maneuvers this effect, as far as the pilot is concerned, will manifest itself in the form of increased stick forces if it is assumed that the pilot wants to produce a given acceleration regardless of the degree of static stability present in the airplane.

The index of static stability dC_m/dC_L for an airplane with the characteristics given in table II is assumed to be -0.022. The curve of the normal acceleration produced in a pull-up from level flight at 448 feet per second is given in figure 6. The maximum change in the normal acceleration attained with an upward elevator deflection of 1^o is about 84.5 feet per second.

From formula (10)

$$\frac{qU_o}{\delta_o{}^o} = -g\,\frac{34.4}{0.10}\left(\frac{-0.021}{(34.4)\,(-0.022) + (-2.01)}\right) = 2.6g \text{ per}$$

degree and qU_o = 83.7 feet per second^2, an error of less than 1 percent.

If the static stability of the airplane is improved by moving the center of gravity forward 0.078c, the expression for the normal acceleration becomes

$$qU_o = -g\,\frac{34.4}{0.10}\left(\frac{-0.021}{(34.4)\,(-0.10) + (-2.07)}\right)\delta_e{}^o$$

$$= (-1.37g)\;\delta_e{}^o$$

If it is desired to load the airplane to its former additional load factor of 2.6g, the elevator movement required with the higher dC_m/dC_L would be

$$\delta_e = \frac{2.6g}{-1.37g} = -1.9^o$$

Accordingly, the pilot would have to exert nearly twice as much stick force to execute the identical pull-up maneuver because of the increased static stability of the airplane. The desirability of modifying the elevator in order to change its hinge-moment characteristics thus partly depends on the relationship between the static stability desired and the magnitude of the stick forces acceptable in accelerated maneuvers.

CONCLUSIONS

This paper is intended to illustrate primarily how power affects the characteristics of a model tested in the wind tunnel and how wind-tunnel data may be used to estimate flying qualities not directly discernible from wind-tunnel tests. The analyses presented in this paper permit the following conclusions to be drawn concerning the methods employed:

(1) In the prediction of the flight characteristics of an airplane operating with power, considerable error may be introduced if wind-tunnel data from tests of a model not equipped with an operating propeller are used.

(2) In the analyses of wind-tunnel rolling-moment data, care should be taken, in the determination of the dihedral effect, to allow for the contribution of the unbalanced pitching moment to the slope of the rolling-moment curve.

(3) The evaluation of the equations of motion permit estimates to be made of the relative effectiveness of the ailerons and of the rudder controls, particularly when changes in wing dihedral are involved as these effects are not readily discernible from static tests of the controls alone. The methods used have been found reasonably accurate, when wind-tunnel data are available, and are not difficult to employ.

(4) The rate of change of normal acceleration per unit of elevator deflection affords a convenient correlation between maneuverability, stability, and elevator-stick forces. The approximate formula developed for calculating the normal accelerations produced during an abrupt pull-up maneuver is simple to evaluate and has been found to yield reasonably accurate results, particularly at high speeds.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., July 22, 1941.

APPENDIX

List of Symbols

U_o	velocity along axis in steady flight
V	velocity along flight path
u	velocity along X axis
v	sideslipping component of velocity
w	velocity along Z axis
p	angular velocity in roll
q	angular velocity in pitch
r	angular velocity in yaw
ϕ	angle of bank
ψ	angle of yaw
α	angle of attack
β	angle of sideslip $(\infty v/U_0)$
θ	angle X axis makes with horizontal
δ	control setting, with appropriate subscript (δ^o indicates that values are in degrees and not in radians)
X, Y, Z	components of force along X, Y, and Z axes, respectively
L	rolling moment about X axis
M	pitching moment about Y axis
N	yawing moment about Z axis
H	hinge moment
b	wing span
c	chord (of wing, unless otherwise subscripted)

S area (of wing, unless otherwise subscripted)

l tail length (distance from center of gravity to tail post)

m mass

μ relative density factor $\frac{m}{1/2\rho S l}$

mk_X^2 moment of inertia about X axis

mk_Y^2 moment of inertia about Y axis

mk_Z^2 moment of inertia about Z axis

t time

D d/dt

ρ density

g gravity (32.2 ft/sec^2)

C_l rolling-moment coefficient $\left(\frac{L}{1/2\rho V^2 Sb}\right)$

C_m pitching-moment coefficient $\left(\frac{M}{1/2\rho V^2 c}\right)$

C_n yawing-moment coefficient $\left(\frac{N}{1/2\rho V^2 Sb}\right)$

C_h hinge-moment coefficient $\left(\frac{H}{1/2\rho V^2 Sc}\right)$

C_Y lateral-force coefficient $\left(\frac{Y}{1/2\rho V^2 S}\right)$

C_L lift coefficient

η_t tail efficiency factor

F empirical slipstream factor to account for contribution of fuselage, wing, and propeller to damping in pitch

Y_β, L_p, L_r, L_δ, L_β, N_p, N_r, N_δ, N_β — lateral-stability derivatives in terms of unit mass or moment of inertia of airplane. For example,

$$Y_\beta = \frac{\partial Y/\partial\beta}{m}$$

$$L_p = \frac{\partial L/\partial p}{mk_X^2}$$

$$N_r = \frac{\partial L/\partial r}{mk_Z^2}$$

X_u, X_w, X_q, Z_u, Z_w, Z_q, Z_δ, Z_α, M_u, M_w, M_q, M_δ, M_α — longitudinal-stability derivatives in terms of unit mass or moment of inertia of airplane. For example,

$$X_u = \frac{\partial X/du}{m}$$

$$Z_\delta = \frac{\partial Z/\partial\delta}{m}$$

$$M_u = \frac{\partial M/du}{mk_Y^2}$$

Subscripts are defined:

- L solutions obtained when $\delta L_\delta = 1$ and $\delta N_\delta = 0$
- N solutions obtained when $\delta L_\delta = 0$ and $\delta N_\delta = 1$
- a aileron
- e elevator
- r rudder
- t time
- o initial value

Primed symbols refer to horizontal tail.

SOLUTIONS OF THE LATERAL EQUATIONS OF MOTION WHEN THE ROLLING DISTURBANCE IS UNITY AND THE YAWING DISTURBANCE IS ZERO ($\delta L_\delta = 1$; $\delta N_\delta = 0$)

The stability equation $F(D) = 0$ is obtained from equation (2) of the text by expanding the third-order determinant formed by the coefficients of the variables. Thus,

$$F(D) = \begin{vmatrix} D-L_p & -L_r & -L_\beta \\ -N_p & D-N_r & -N_\beta \\ -\frac{g}{U_o} & D & D\left(D - \frac{Y_\beta}{U_o}\right) \end{vmatrix}$$

$$= \left(D-L_p\right) \begin{vmatrix} D-N_r & -N_\beta \\ D & D\left(D - \frac{Y_\beta}{U_o}\right) \end{vmatrix}$$

$$+ N_p \begin{vmatrix} -L_r & -L_\beta \\ D & D\left(D - \frac{Y_\beta}{U_o}\right) \end{vmatrix} - \frac{g}{U_o} \begin{vmatrix} -L_r & -L_\beta \\ D-N_r & -N_\beta \end{vmatrix}.$$

$$= D^4 - \left(L_p + N_r + \frac{Y_\beta}{U_o} \right) D^3$$

$$+ \left[\frac{Y_\beta}{U_o} \left(N_r + L_p\right) + N_r L_p - L_r N_p + N_\beta \right] D^2$$

$$+ \left[- \frac{Y_\beta}{U_o} \left(N_r L_p - L_r N_p\right) - L_p N_\beta + L_\beta N_p - \frac{g}{U_o} L_\beta \right] D$$

$$+ \frac{g}{U_o} \left(L_\beta N_r - L_r N_\beta\right) = 0$$

Let λ_1, λ_2, λ_3, and λ_4 be the four roots of this equation $F(D) = 0$ and form the following products:

$$P = \lambda_1 \lambda_2 \lambda_3 \lambda_4 = \frac{g}{U_o} \left(L_\beta N_r - L_r N_\beta\right)$$

$$Q = \lambda_1 \left(\lambda_1 - \lambda_2\right) \left(\lambda_1 - \lambda_3\right) \left(\lambda_1 - \lambda_4\right)$$

$$R = \lambda_2 \left(\lambda_2 - \lambda_1\right) \left(\lambda_2 - \lambda_3\right) \left(\lambda_2 - \lambda_4\right)$$

$$S = \lambda_3 \left(\lambda_3 - \lambda_1\right) \left(\lambda_3 - \lambda_2\right) \left(\lambda_3 - \lambda_4\right)$$

$$T = \lambda_4 \left(\lambda_4 - \lambda_1\right) \left(\lambda_4 - \lambda_2\right) \left(\lambda_4 - \lambda_3\right)$$

The solutions for any of the variables can be expressed in terms of the products of the roots just formed and the stability derivatives. By use of the auxiliary relations $\frac{d\phi}{dt} = p$ and $\frac{d\psi}{dt} = r$ solutions can be written directly for both ϕ and ψ. As it is usually desirable in this work to determine ϕ and ψ rather than p and r, it is convenient to solve for ϕ and ψ directly and then to differentiate each of these solutions, respectively, to obtain p and r if these variables are wanted. The solutions follow:

Angle of Bank ϕ_L

$$\phi_L = \phi_{L_0} + \phi_{L_1} e^{\lambda_1 t} + \phi_{L_2} e^{\lambda_2 t} + \phi_{L_3} e^{\lambda_3 t} + \phi_{L_4} e^{\lambda_4 t}$$

where

$$\phi_{L_0} = \frac{N_\beta + \frac{N_r Y_\beta}{U_0}}{P}$$

$$\phi_{L_1} = \frac{\lambda_1^2 - \lambda_1 \left(N_r + \frac{Y_\beta}{U_0}\right) + \left(N_\beta + \frac{N_r Y_\beta}{U_0}\right)}{Q}$$

$$\phi_{L_2} = \frac{\lambda_2^2 - \lambda_2 \left(N_r + \frac{Y_\beta}{U_0}\right) + \left(N_\beta + \frac{N_r Y_\beta}{U_0}\right)}{R}$$

$$\phi_{L_3} = \frac{\lambda_3^2 - \lambda_3 \left(N_r + \frac{Y_\beta}{U_0}\right) + \left(N_\beta + \frac{N_r Y_\beta}{U_0}\right)}{S}$$

$$\phi_{L_4} = \frac{\lambda_4^2 - \lambda_4 \left(N_r + \frac{Y_\beta}{U_0}\right) + \left(N_\beta + \frac{N_r Y_\beta}{U_0}\right)}{T}$$

Angle of Sideslip β_L

$$\beta_L = \beta_{L_0} + \beta_{L_1} e^{\lambda_1 t} + \beta_{L_2} e^{\lambda_2 t} + \beta_{L_3} e^{\lambda_3 t} + \beta_{L_4} e^{\lambda_4 t}$$

where

$$\beta_{L_0} = \frac{g}{U_0} \frac{-N_r}{P}$$

$$\beta_{L_1} = \frac{g}{U_0} \frac{\left(\lambda_1 - N_r\right) - \lambda_1 N_p}{Q}$$

$$\beta_{L_2} = \frac{g}{U_0} \frac{\left(\lambda_2 - N_r\right) - \lambda_2 N_p}{R}$$

$$\beta_{L_3} = \frac{g}{U_0} \frac{\left(\lambda_3 - N_r\right) - \lambda_3 N_p}{S}$$

$$\beta_{L_4} = \frac{g}{U_0} \frac{\left(\lambda_4 - N_r\right) - \lambda_4 N_p}{T}$$

Angle of Yaw ψ_L

$$\psi_L = \psi_{L_0} + \psi_{L_1} e^{\lambda_1 t} + \psi_{L_2} e^{\lambda_2 t} + \psi_{L_3} e^{\lambda_3 t} + \psi_{L_4} e^{\lambda_4 t}$$

where

$$\psi_{L_0} = \frac{N_\beta \frac{g}{U_o}}{P}$$

$$\psi_{L_1} = \frac{N_\beta \frac{g}{U_o} - \lambda_1 \frac{Y_\beta N_p}{U_o}}{Q}$$

$$\psi_{L_2} = \frac{N_\beta \frac{g}{U_o} - \lambda_2 \frac{Y_\beta N_p}{U_o}}{R}$$

$$\psi_{L_3} = \frac{N_\beta \frac{g}{U_o} - \lambda_3 \frac{Y_\beta N_p}{U_o}}{S}$$

$$\psi_{L_4} = \frac{N_\beta \frac{g}{U_o} - \lambda_4 \frac{Y_\beta N_p}{U_o}}{T}$$

Yawing Velocity r_L

$$r_L = \frac{d}{dt} (\psi_L)$$

The yawing velocity is most easily obtained by direct differentiation.

Rolling Velocity p_L

$$p_L = \frac{d}{dt} (\phi_L)$$

The rolling velocity is most easily obtained by direct differentiation.

SOLUTIONS OF THE LATERAL EQUATIONS OF MOTION WHEN THE YAWING DISTURBANCE IS UNITY AND THE ROLLING DISTURBANCE IS ZERO ($\delta N_\delta = 1$; $\delta L_\delta = 0$)

Angle of Bank ϕ_N

$$\phi_N = \phi_{N_0} + \phi_{N_1} e^{\lambda_1 t} + \phi_{N_2} e^{\lambda_2 t} + \phi_{N_3} e^{\lambda_3 t} + \phi_{N_4} e^{\lambda_4 t}$$

where

$$\phi_{N_0} = \frac{-L_\beta - L_r \frac{Y_\beta}{U_0}}{P}$$

$$\phi_{N_1} = \frac{\left(-L_\beta - L_r \frac{Y_\beta}{U_0}\right) + \lambda_1 L_r}{Q}$$

$$\phi_{N_2} = \frac{\left(-L_\beta - L_r \frac{Y_\beta}{U_0}\right) + \lambda_2 L_r}{R}$$

$$\phi_{N_3} = \frac{\left(-L_\beta - L_r \frac{Y_\beta}{U_0}\right) + \lambda_3 L_r}{S}$$

$$\phi_{N_4} = \frac{\left(-L_\beta - L_r \frac{Y_\beta}{U_0}\right) + \lambda_4 L_r}{T}$$

Angle of Sideslip β_N

$$\beta_N = \beta_{N_0} + \beta_{N_1} e^{\lambda_1 t} + \beta_{N_2} e^{\lambda_2 t} + \beta_{N_3} e^{\lambda_3 t} + \beta_{N_4} e^{\lambda_4 t}$$

where

$$\beta_{N_0} = \frac{\frac{gL_r}{U_0}}{P}$$

$$\beta_{N_1} = \frac{\frac{gL_r}{U_0} + \lambda_1 L_p - \lambda_1^2}{Q}$$

$$\beta_{N_2} = \frac{\frac{gL_r}{U_0} + \lambda_2 L_p - \lambda_2^2}{R}$$

$$\beta_{N_3} = \frac{\frac{gL_r}{U_0} + \lambda_3 L_p - \lambda_3^2}{S}$$

$$\beta_{N_4} = \frac{\frac{gL_r}{U_0} + \lambda_4 L_p - \lambda_4^2}{T}$$

Angle of Yaw ψ_N

$$\psi_N = \psi_{N_0} + \psi_{N_1} e^{\lambda_1 t} + \psi_{N_2} e^{\lambda_2 t} + \psi_{N_3} e^{\lambda_3 t} + \psi_{N_4} e^{\lambda_4 t}$$

where

$$\psi_{N_0} = \frac{-L_\beta \frac{g}{U_o}}{P}$$

$$\psi_{N_1} = \frac{-\lambda_1{}^2 \frac{Y_\beta}{U_o} + \lambda_1 L_p \frac{Y_\beta}{U_o} - L_\beta \frac{g}{U_o}}{Q}$$

$$\psi_{N_2} = \frac{-\lambda_2{}^2 \frac{Y_\beta}{U_o} + \lambda_2 L_p \frac{Y_\beta}{U_o} - L_\beta \frac{g}{U_o}}{R}$$

$$\psi_{N_3} = \frac{-\lambda_3{}^2 \frac{Y_\beta}{U_o} + \lambda_3 L_p \frac{Y_\beta}{U_o} - L_\beta \frac{g}{U_o}}{S}$$

$$\psi_{N_4} = \frac{-\lambda_4{}^2 \frac{Y_\beta}{U_o} + \lambda_4 L_p \frac{Y_\beta}{U_o} - L_\beta \frac{g}{U_o}}{T}$$

Yawing Velocity r_N

$$r_N = \frac{d}{dt}(\psi_N)$$

The yawing velocity is most easily obtained by direct differentiation.

Rolling Velocity p_N

$$p_N = \frac{d}{dt}(\phi_N)$$

The rolling velocity is most easily obtained by direct differentiation.

DISCUSSION OF THE EQUATION F(D) = 0

For most airplanes the solution of $F(D) = 0$ will yield both real and imaginary roots. In the solution of the equation, the real roots can first be isolated (Horner's method) and extracted from the equation. The imaginary roots can then be found by solving the resulting quadratic. Because imaginary roots always occur in pairs, two of the roots will be of the form $a \pm ib$, where a and b are constants.

The components of the motion containing the imaginary roots can be combined conveniently into a single term involving only real numbers. In the case of ϕ_L, the solution is

$$\phi_L = \phi_{L_0} + \phi_{L_1} e^{\lambda_1 t} + \phi_{L_2} e^{\lambda_2 t} + \phi_{L_3} e^{\lambda_3 t} + \phi_{L_4} e^{\lambda_4 t}$$

If λ_3 and λ_4 are the conjugate imaginary roots,

$\phi_{L_3} e^{\lambda_3 t} + \phi_{L_4} e^{\lambda_4 t}$ will also be imaginary.

Let $\lambda_3 = a + b$; then ϕ_{L_3} will be imaginary and can be reduced to the form $\frac{E + iF}{G + iH}$. Further,

$$\phi_{L_3} = \frac{E + iF}{G + iH}\,\frac{G - iH}{G - iH} = \frac{EG + FH - i\,(HE - FG)}{G^2 + H^2} = \frac{I + iJ}{K}$$

and

$$\phi_{L_3} e^{\lambda_3 t} + \phi_{L_4} e^{\lambda_4 t} = \frac{2\sqrt{I^2 + J^2}}{K}\, e^{at} \cos b\,(t + \gamma)$$

where

$$\gamma = \frac{1}{b} \tan^{-1}\left(\frac{J}{I}\right)$$

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TABLE I

Airplane	U_o (fps)	S (sq ft)	b (ft)	m (slugs)	mk_X^2 (slug-ft^2)	mk_Z^2 (slug-ft^2)	Y_β (ft/sec^2)
A	448	236	37.3	174	2020	6030	-166
B	448	236	37.3	174	2020	6030	-185

Airplane	L_β (per sec^2)	N_β (per sec^2)	N_p (per sec)	N_r (per sec)	L_p (per sec)	L_r (per sec)
A	-62.7	17.7	-0.076	-1.49	-18.6	0.99
B	-129	31.0	-.088	-1.77	-18.6	1.14

Airplane	$\delta_a L_{\delta_a}$ (per sec^2) (1)	$\delta_a N_{\delta_a}$ (per sec^2) (1)	$\delta_r L_{\delta_r}$ (per sec^2) (2)	$\delta_r N_{\delta_r}$ (per sec^2) (2)
A	1.54	0	-0.308	0.549
B	1.54	0	-.410	.652

[1] $\delta_a{}^\circ = 1^\circ$.
[2] $\delta_r = -1^\circ$.

TABLE II

U_o (fps)	S (sq ft)	l (ft)	c (ft)	m (slugs)	$mk_Y{}^2$ (slug-ft^2)	Z_u (per sec)	Z_w (per sec)	Z_q (fps)
448	236	18	6.8	174	4470	-0.144	-3.71	-12.9

X_u (sec)	M_u (per ft-sec) (1)	M_w (per ft-sec)	M_q (per sec)	$\delta_e M_{\delta_e}$ (per sec^2)
-0.044	-0.0005	-0.022	-7.04	-1.84

[1]See reference 7.

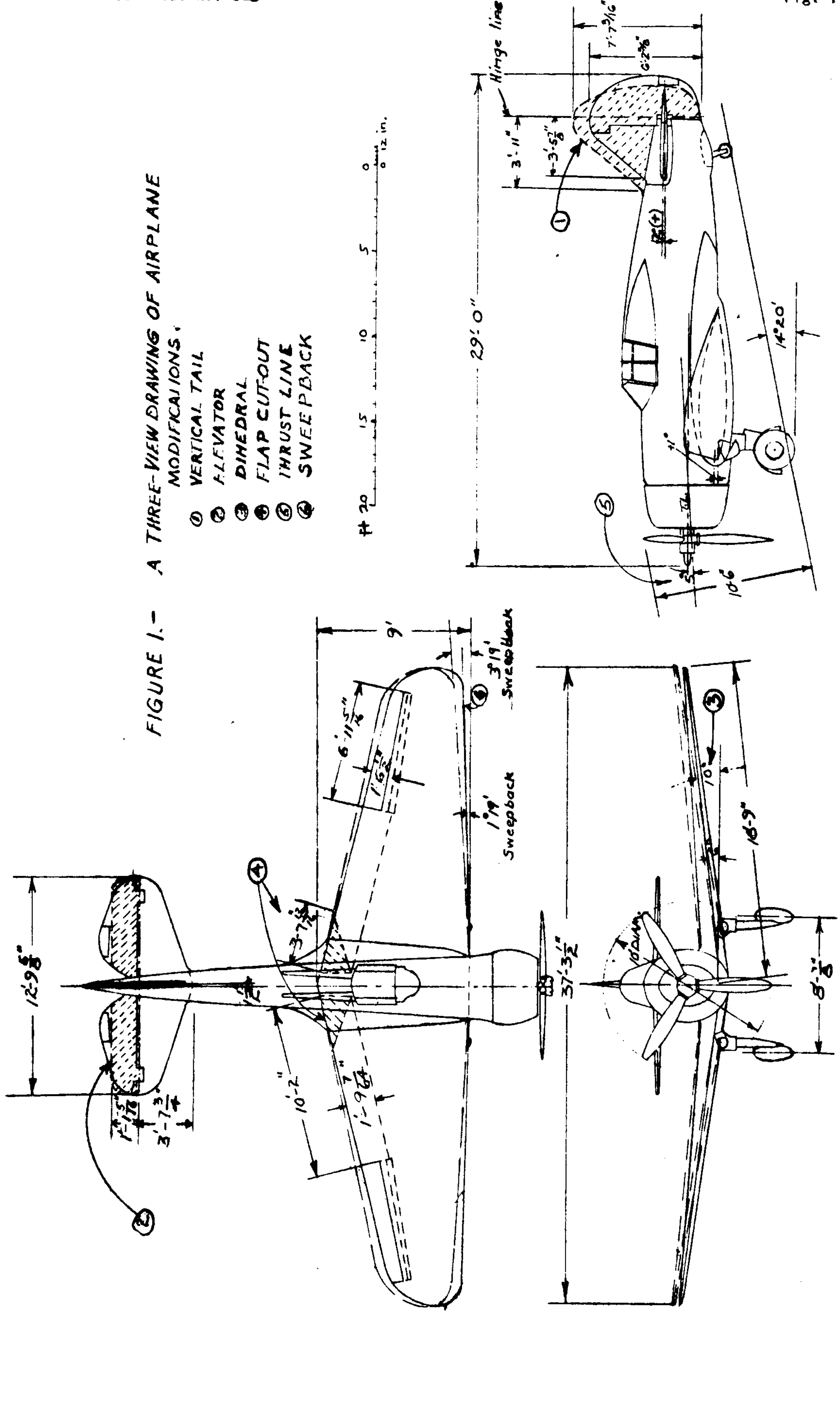


FIGURE 1.- A THREE-VIEW DRAWING OF AIRPLANE

MODIFICATIONS:

1. VERTICAL TAIL
2. ELEVATOR
3. DIHEDRAL
4. FLAP CUT-OUT
5. THRUST LINE
6. SWEEPBACK

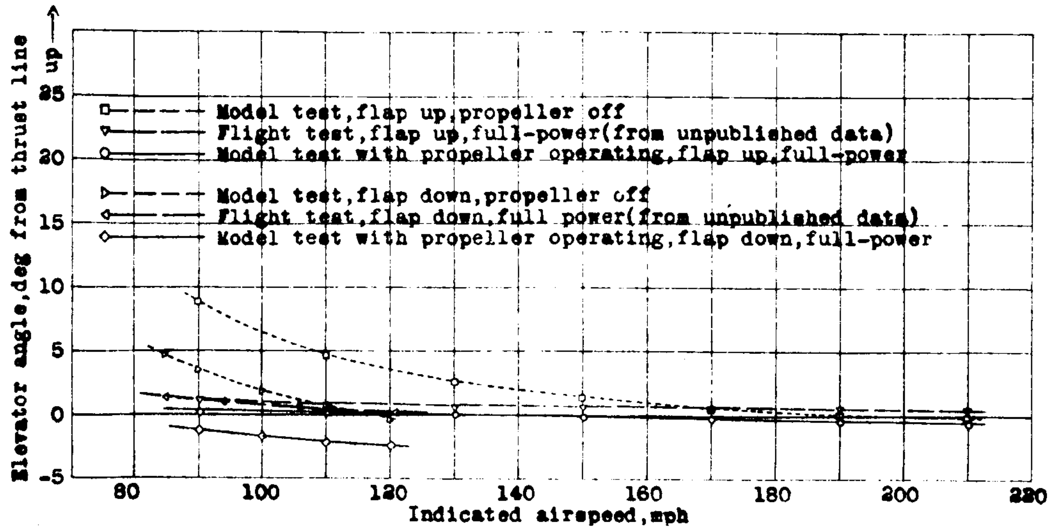
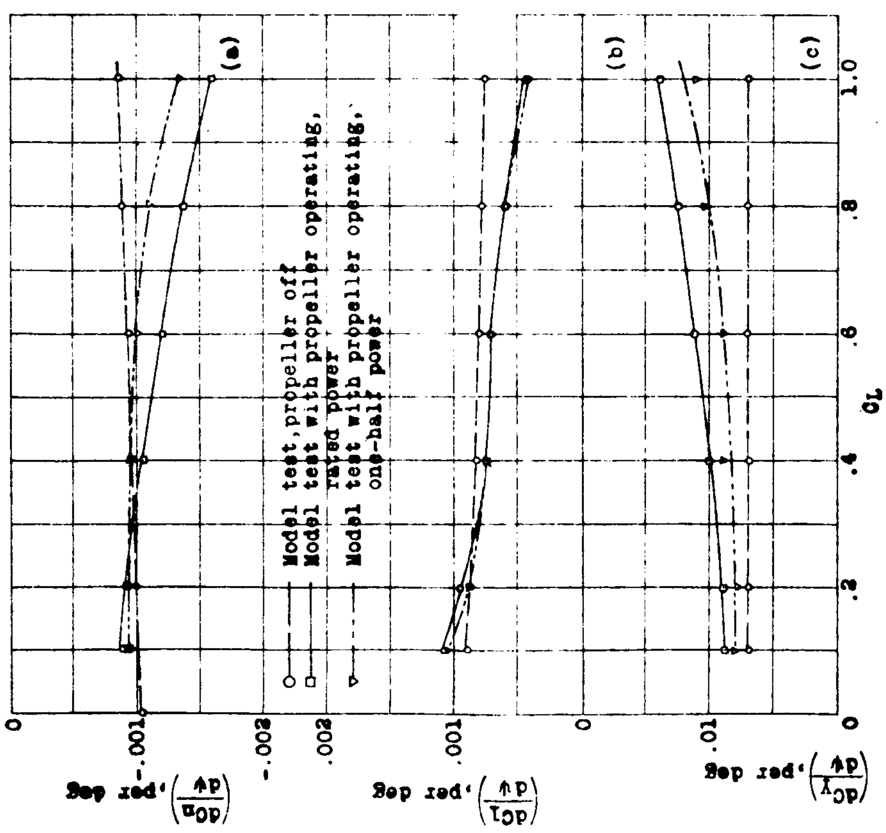


Figure 2.- Variation of elevator angle required for trim with indicated airspeed.



(a) Yawing moment. (b) Rolling moment.
(c) Lateral force.

Figure 3.- Effect of power on the slopes of yawing-moment, rolling-moment, and lateral-force curves. Flap up.

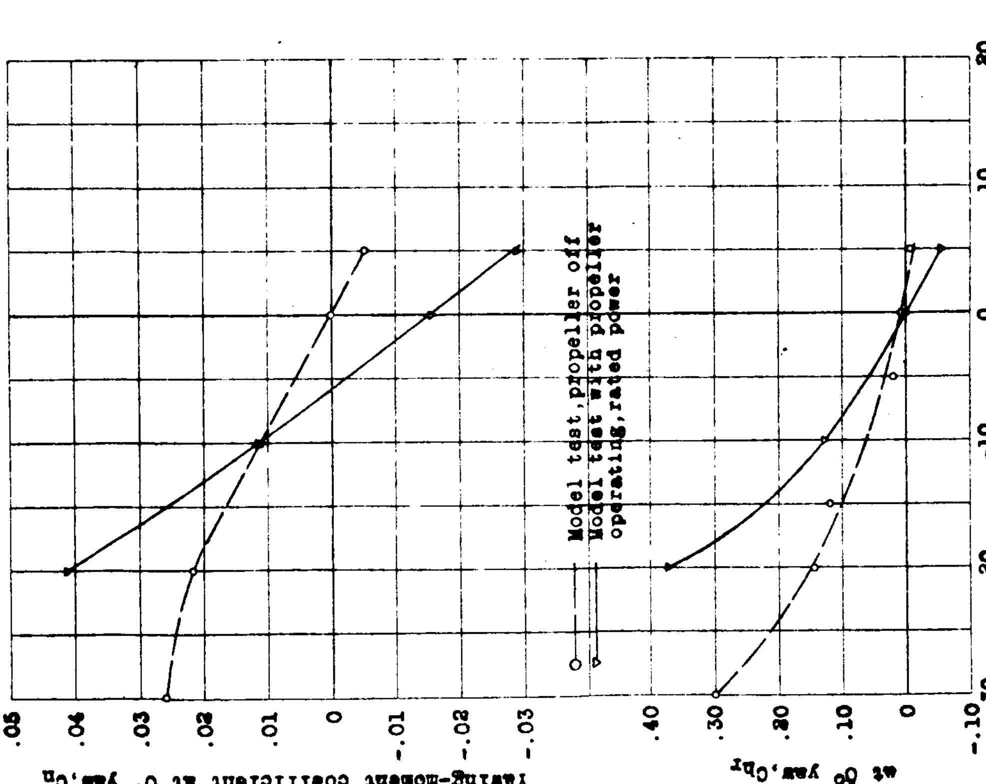
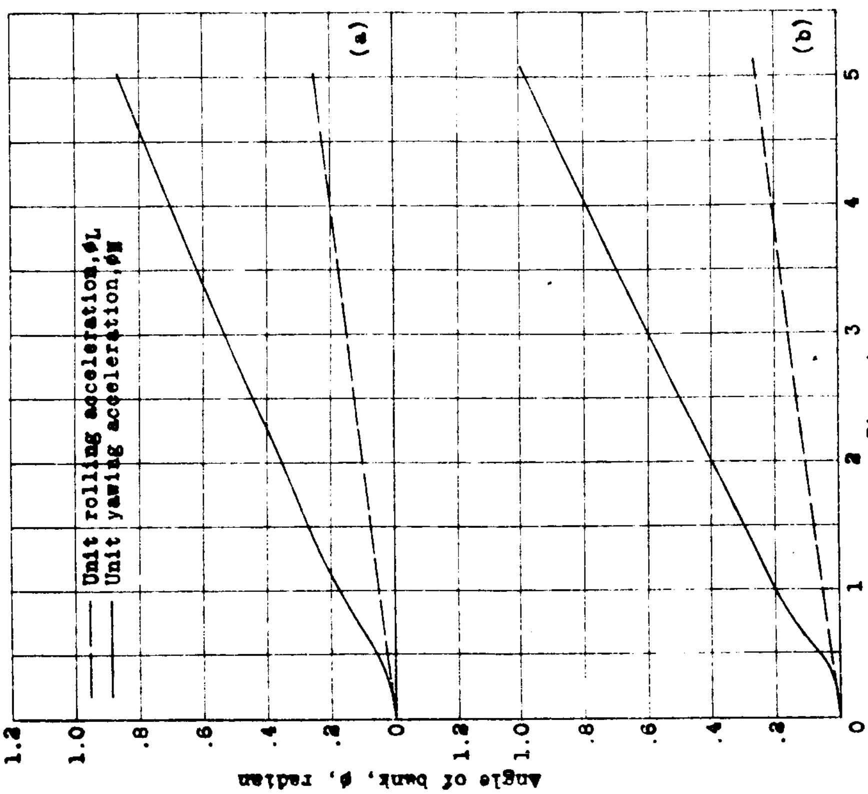


Figure 4.- Effect of power on rudder characteristics. The yawing-moment coefficients have been corrected for fin offset. C_L = 0.9.



(a) Airplane A. (b) Airplane B.

Figure 5.- Angles of bank generated by unit rolling and yawing accelerations applied at t = 0. U_0, 448 feet per second.

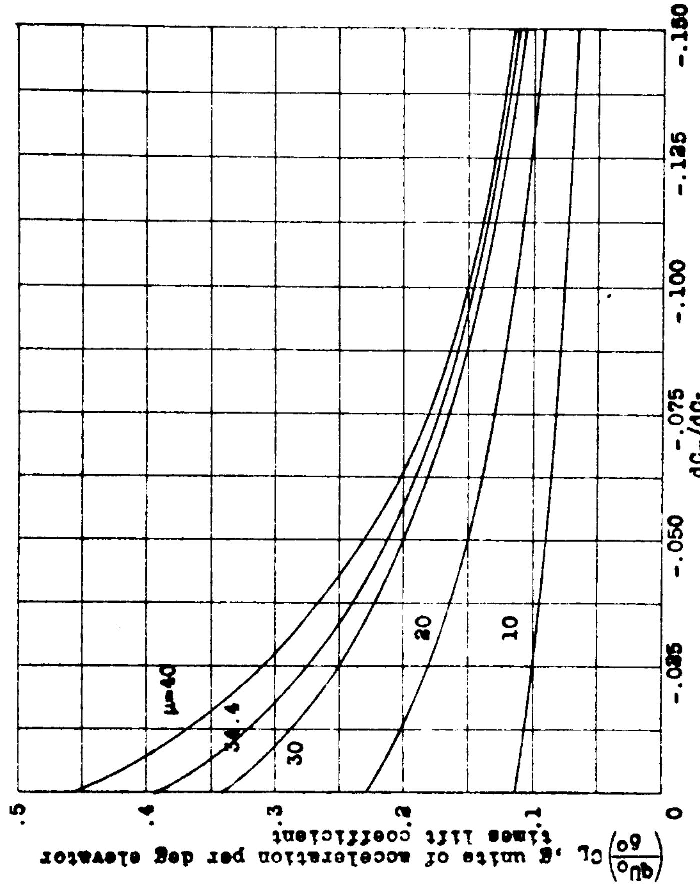


Figure 7.- Variation of normal acceleration per unit of elevation deflection with dC_m/dC_L. Abrupt pull-up maneuver.

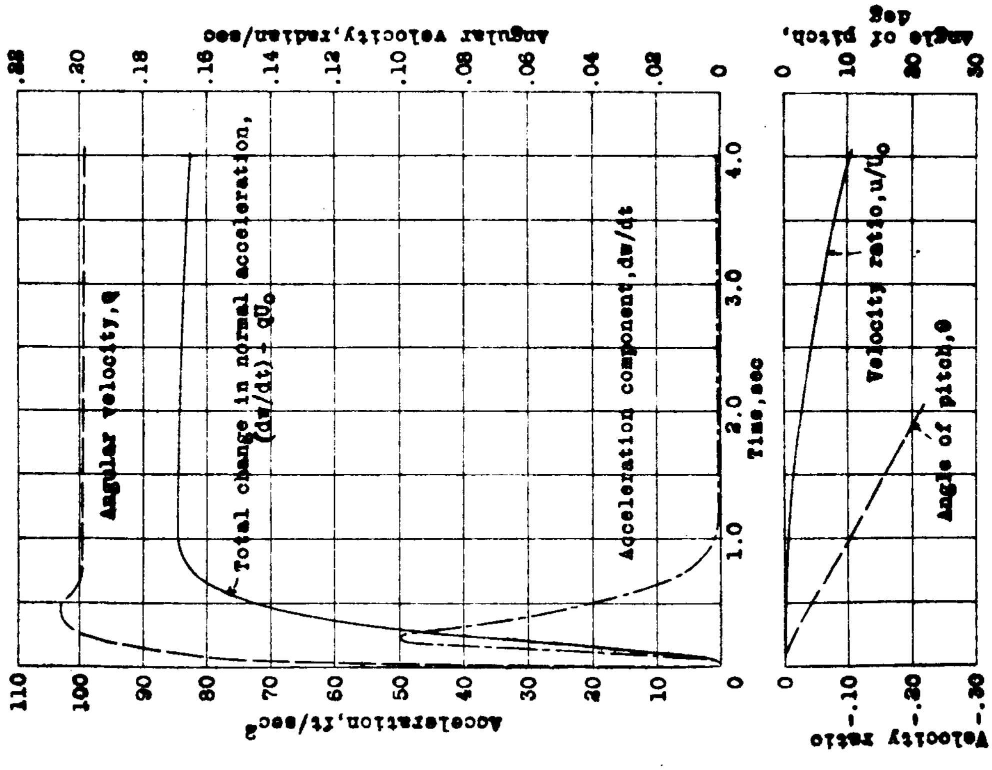


Figure 6.- Abrupt pull-up at high speed. δ_e, -1°.