ERDA/NASA/1028-77/1
NASA TM-73718

DRIVE TRAIN NORMAL MODES ANALYSIS FOR THE ERDA/NASA 100-KILOWATT WIND TURBINE GENERATOR

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July 1977

Prepared for
ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION
Division of Solar Energy
Federal Wind Energy Program
Under Interagency Agreement E(49-26)-1028

1 Report No	2 Government Accession No	3 Recipient's Catalog No
NASA TM 73718		

4 Title and Subtitle	5 Report Date
DRIVE TRAIN NORMAL MODES ANALYSIS FOR THE ERDA/ NASA 100-KILOWATT WIND TURBINE GENERATOR	July 1977
	6 Performing Organization Code
7 Author(s)	8 Performing Organization Report No
T. L Sullivan, D. R. Miller, and D. A. Spera	E-9266
	10 Work Unit No
9 Performing Organization Name and Address National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135	11 Contract or Grant No
	13 Type of Report and Period Covered Technical Memorandum
12 Sponsoring Agency Name and Address Energy Research and Development Administration Division of Solar Energy Washington, D.C. 20545	14 Sponsoring Agency Code ERDA/NASA/1028-77/1

15 Supplementary Notes

This report was prepared under Interagency Agreement E(49-26)-1028.

16 Abstract

Natural frequencies as a function of power were determined using a finite element model. Two operating conditions were investigated operation with a resistive electrical load and operation synchronized to an electrical utility grid. The influence of certain drive train components on frequencies and mode shapes is shown. An approximate method for obtaining drive train natural frequencies is presented.

17 Key Words (Suggested by Author(s))	18 Distribution Statement
Wind energy; Wind turbine generator, Drive train, Normal modes analysis; Natural frequencies, Mode shapes; Windmill	Unclassified - unlimited STAR Category 44 ERDA Category UC-60

19 Security Classif (of this report)	20 Security Classif (of this page)	21 No of Pages	22 Price*
Unclassified	Unclassified		

* For sale by the National Technical Information Service, Springfield, Virginia 22161

DRIVE TRAIN NORMAL MODES ANALYSIS FOR THE ERDA/NASA

100-KILOWATT WIND TURBINE GENERATOR

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SUMMARY

It is important to know the natural frequencies of wind turbine generator drive trains so that coincidence with driving frequencies can be avoided. To accomplish this for the ERDA/NASA 100 kW Mod-0 wind turbine, the drive train physical characteristics (torsional stiffness and polar mass moments of inertia) were determined. These characteristics were incorporated into a finite element model to determine natural frequencies for generator electrical termination into both a resistive load bank and a utility grid (synchronous operation). An approximate method requiring only hand calculations was also used to estimate drive train natural frequencies. The results from both methods were in close agreement.

Two vibration modes were of interest. In the first or fundamental mode (mode I) all drive train components rotate relative to each other in the same direction. The possibility of reducing the mode I frequency to attenuate higher frequency driving disturbances was investigated. It was found that a significant reduction in frequency could be achieved if the high-speed shaft stiffness was reduced by several orders of magnitude. This might be achieved using an elastomeric material or a torsional steel spring.

In the second mode (mode II) the blades and generator rotate in opposite directions. Mode II natural frequencies were found to be strongly influenced by the stiffness of a Falk coupling. This analysis indicates that replacement of this coupling with a gear coupling will reduce the range of possible resonant operating conditions.

INTRODUCTION

As a part of the national wind energy program directed by the Energy Research and Development Administration (ERDA), NASA's Lewis Research Center is providing project management for wind turbine generating systems ranging from 100 kW (Mod-0, refs. 1 and 2) to more than 1500 kW (Mod-1). An important

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aspect in the design of these systems is proper placement of system natural frequencies with respect to critical exciting frequencies. Proper placement of drive train natural frequencies is especially important because of the inherently low damping in the drive train. Operation at a resonant condition causes excessive loads in the drive train components and reduces the quality of the power generated.

In a previous report (ref. 3) natural frequencies were predicted for the Mod-0 tower, rotor blades and combined system. However, the combined system results did not include the drive train torsional frequencies. Some preliminary estimates of drive train torsional frequencies were made by Lockheed California Company as part of a rotor blade design study.

In order to investigate the possibility of the Mod-0 drive train operating at a resonant condition, this study was made to determine drive train natural frequencies.

This report is organized as follows. First, a physical description of the Mod-0 wind turbine and drive train is given. Then, the drive train stiffnesses and mass moments of inertia are calculated. Using these calculated stiffnesses and inertias, a finite element model of the drive train is developed. Natural frequencies obtained using this model are reported. The role certain drive train components play in drive train vibrations is examined. Finally, the results using an approximate method for determining drive train natural frequencies are compared with those obtained from the finite element model.

DESCRIPTION OF THE MOD-0 WIND TURBINE AND DRIVE TRAIN

Mod-0 Wind Turbine

The Mod-0 wind turbine is an experimental two-bladed, horizontal axis machine with a downwind rotor. The rotor diameter is 125 feet. It is designed to produce 100 kW of power in an 18 mph wind with a rotor speed of 40 rpm. The downwind location of the rotor causes it to see reduced velocity as it passes through the tower wake. This reduced velocity pulse occurs twice per revolution (1.33 Hz) and is the most important driving frequency. Figure 1 shows an overall view of this wind turbine which is located at NASA's Plum Brook Station, near Sandusky, Ohio. A more detailed description of the wind turbine is given in reference 1.

Mod-0 Drive Train

Figure 2 shows an overall view of the drive train. The drive train components are supported by a bedplate connected to the tower by a yaw drive system. Information about the commercially available components of the Mod-0 drive train is given in table I including manufacturer's names and model numbers. Drive train component stiffnesses and mass moments of inertia are tabulated in tables II and III, respectively. Appendix A describes how these values were calculated.

The rotor blades are attached to a hub which contains the mechanism for changing the pitch of the blades. The hub is attached to the low-speed (40-rpm) shaft, which is supported by two bearings mounted on the bedplate. The low-speed shaft is connected to the gearbox through a nonlinear spring (Falk) coupling. The purposes for using this type of coupling were twofold. to protect the gearbox from possible shock loadings coming from the blades and to provide for misalignment between the low-speed shaft and the gearbox input shaft.

In the Mod-0 wind turbine the drive train transmits the torque developed by the rotor to the generator through a 45:1 speed increasing gearbox. The gearbox increases the speed in three stages (triple mesh). A brake disk is located on the high-speed (1800-rpm) output shaft of the gearbox. The high-speed shaft is made in two sections. The upwind section is supported by two bearings and carries a set of 10 belt pulleys on one end. The downwind section of the high-speed shaft connects the pulley shaft to the gearbox through two gear couplings. Power is transmitted from the pulleys on the high-speed shaft to pulleys on the generator shaft by means of 10 V-belts. The Mod-0 wind turbine normally operates with a pulley ratio of 1:1.

FINITE ELEMENT MODELING OF DRIVE TRAIN

A schematic model of the Mod-0 drive train is shown in figure 3. The stiffnesses, k_i, refer to the values tabulated in table II and the mass moments of inertia, M_i, refer to the values tabulated in table III. The speed ratio, n, is 45:1.

The normal modes analysis of undamped systems in the NASTRAN computer code was used to determine drive train natural frequencies and mode shapes. The inverse power method was used for eigenvalue extraction. NASTRAN bar elements were used to represent drive train stiffnesses. Concentrated mass moments of inertia were placed at appropriate nodes to represent drive train masses.

Two concepts were employed to model the speed increase in the gearbox. These were a constant speed model and a link model which are shown schematically in figure 4. Both models gave the same results. For convenience certain stiffnesses and mass moments of inertia have been combined in these models.

Constant Speed Model

A simple way to model the Mod-0 drive train is to assume that all the components are rotating at the same speed. Actual speed differences are accounted for by multiplying the appropriate stiffnesses and masses by the square of the speed ratio. For example, in the Mod-0 drive train the speed of the low speed shaft was taken as unity and all other speeds referred to it. The gearbox provides a 45:1 speed increase. Hence, all the stiffnesses and masses of the high-speed components (brake disk through the generator) are multiplied by $(45)^2$ or 2025 in the constant speed model.

Link Model

The constant speed model does not provide torque reactions at the gearbox and other places where speed changes occur. These reactions might be important for modal analysis of the complete system. Hence, a second model was devised. Speed changes are accounted for by the ratio of the lengths of two links that connect the low speed end of the drive train to the high speed end. Torque is resolved at the ends of these links.

DRIVE TRAIN NATURAL FREQUENCIES AND MODE SHAPES

The Mod-0 drive train natural frequencies and mode shapes obtained using the constant speed model are reported in this section for both load bank and synchronous operation. The four-mass model shown in figure 4 produces four natural frequencies. Only the first two modes will be discussed further. The third and fourth mode frequencies, which were approximately 33 and 70 Hz, are not of importance in this study because the range of driving frequencies is below about 3 Hz.

Load Bank Operation

During load bank operation the generator field and the electrical load act as a damping constraint without the equivalent of a spring stiffness. Thus, the stiffness, k_7 (fig. 3) is zero for load bank operation.

The natural frequencies predicted for load bank operation as a function of power generated are plotted on figure 5. Mode I is a rigid body mode (frequency = 0 Hz). Mode II traverses a wide range of frequencies because of the nonlinear character of the Falk coupling.

Figure 6 is a plot of measured Mod-0 low-speed shaft torsional moment as a function of power. At a power of 30 to 35 kW, the torsional moment reached a peak indicative of a resonant condition. The frequency of these torsional oscillations was four times the rotational frequency (4P). Note that on figure 5 the predicted frequency crosses 4P at a power of 34 kW. Because an actual resonance and a predicted possible resonance are in such good agreement, it is concluded that the model provides an accurate description of the actual drive train for load bank operation.

Synchronous Operation

During synchronous operation, electromagnetic coupling between the generator and the utility line supplies a constraint to the generator rotor attempting to keep the rotor speed constant. This constraint can be modeled as a nonlinear spring. The stiffness of this spring varies with power level.

The predicted natural frequencies for synchronous operation are compared to those for load bank operation in figure 5. Mode I is seen to be not strongly affected by power level. However, at 100 kW the frequency is close to 1P. This frequency could be excited if the blades are unbalanced.

Mode II frequencies are strongly affected by power in a manner similar to load bank operation. However, the frequencies are higher and the 4P crossing takes place at a power of less than 10 kW.

Mode Shapes

The mode shapes for 0 and 100 kW load bank operation are shown in figure 7. At 0 kW all components from the generator to the gearbox rotate almost as a rigid body with most of the differential rotation taking place in the Falk coupling. Blade rotation is relatively small and is in a direction opposite to that of the generator. As the power increases more differential rotation takes place in the high speed shaft and gearbox and less in the Falk coupling. Again, the blades are nearly stationary.

The mode shapes for synchronous operation are shown in figure 8. In mode I all components are rotating in the same direction with most of the differential rotation taking place in the generator field and in the Falk coupling. As power is increased, more rotation takes place in the field while less occurs in the Falk coupling. For mode II mode shapes during synchronous operation are very similar to those for load bank operation at equivalent power settings.

DRIVE TRAIN COMPONENT INFLUENCES ON NATURAL FREQUENCIES

Certain drive train components exert a strong influence on natural frequencies and mode shapes. In this section the influence of several components is discussed.

Generator

Because of its size and location, the generator spring has a strong influence on drive train natural frequencies. This is especially true at higher power levels when it has the smallest equivalent spring stiffness in the drive train. Therefore, the accuracy of the predicted natural frequencies for synchronous operation is strongly dependent on the accuracy of the calculated generator spring stiffness. The generator spring calculations require knowledge of the line reactance which is dependent on the operating environment of the generator. The generator spring stiffnesses given in this report are based on the best estimate of line reactance at the time of writing. A better estimate of this value may change the calculated generator spring constants and, consequently, the predicted natural frequencies.

Falk Coupling

The Falk coupling strongly influences drive train frequencies and mode shapes. The nonlinear character of this component results in a very wide range of mode II frequencies as power is varied. This greatly increases the chances of operating at a resonant frequency. Replacement of the nonlinear Falk coupling with a stiff linear coupling would reduce the range of mode II frequencies. To determine the amount of reduction possible, drive train frequencies and mode shapes were calculated for a system containing a gear coupling in place of the Falk coupling. It was assumed that the gear coupling had the same stiffness as an equal length of the low speed shaft.

Drive train natural frequencies with a gear coupling are compared to those with a Falk coupling in figure 5. While there is only a very small effect on mode I, mode II natural frequencies are strongly influenced by a gear coupling. The frequencies are increased and the ranges of frequencies are decreased to 0 Hz for load bank operation and to 0.8 Hz for synchronous operation. With a Falk coupling these ranges were 2.5 and 2.9 Hz, respectively.

The mode shapes for the drive train with a gear coupling are shown in figure 9. For load bank operation there is only one mode shape regardless of power because all nonlinear springs have been eliminated. The relative rotation is now more evenly distributed in all components of the drive train. For synchronous operation at 0 kW most of the mode I rotation now takes place in the generator spring. For the remaining mode shapes, a more even distribution of relative rotation takes place in the drive train components than was the case when the Falk coupling was present.

High Speed Shaft

Because of drive train oscillations encountered during synchronous operation of the 100 kW wind turbine and because the mode I frequency was close to 1P at 100 kW, a means of reducing the mode I frequency was thought desirable. One practical means of accomplishing this was to reduce the stiffness of a portion of the high speed shaft. The high speed shaft was made in two parts and the part attached to the gearbox could be easily removed and replaced with a less stiff shaft. The drive train model was used to determine the effect on natural frequency of reducing the high speed shaft stiffness.

Figure 10(a) shows the effect of high speed shaft stiffness on mode I frequencies for 0 and 100 kW power with the Falk coupling and with a gear coupling. A very substantial reduction in high speed shaft stiffness is required before any significant reduction in frequency is obtained. To reduce the frequency at 100 kW to 0.5 P requires a high speed shaft 0.0025 times as stiff as the original shaft. This low stiffness might be obtained with a torsional spring or an elastomeric shaft.

Reducing the high speed shaft stiffness also has an effect on mode II frequencies. This is shown in figure 10(b). Again large changes in stiffness are required to obtain significant changes in frequency. The 0 kW gear coupling case is the most sensitive one to a reduction in high speed shaft stiffness. Since the frequencies for both modes are affected by high speed shaft stiffness, care must be exercised so that a change that places a frequency for one mode in a more desirable location does not have the opposite effect for another mode.

Fluid Coupling

A method for reducing the transmission of drive train oscillations to the generator would be to place a fluid coupling in the high speed shaft. This would not have a significant effect on the drive train stiffness but would add a substantial mass moment of inertia. The mass moment of inertia of a typical fluid coupling of appropriate size for the Mod-0 drive train is estimated to be 15 lb-in-sec^2. Adding this to the 100 kW system reduces the mode II frequency at rated power by 16 percent. The mode I frequency is essentially unaffected.

APPROXIMATION OF DRIVE TRAIN NATURAL FREQUENCIES

Examination of the physical characteristics of the Mod-0 drive train and its vibration mode shapes indicates that it should be possible to approximate the drive train with single degree-of freedom-spring/mass systems. The mass moment of inertia of the blades located at one extremity of the drive train greatly predominates any of the other components. For Mod-0 the blades provide 91 percent of the total effective mass moment of inertia. Therefore, that mass acting against a spring that represents the entire drive train should be a reasonable approximation of mode I. A schematic of this system is shown in figure 11(a).

It was seen that for mode II, the blade end of the drive train is essentially grounded. For load bank operation this mode can be approximated by the mass moment of inertia of the generator, pulleys and belts acting against the spring representing the drive train between the generator and the blades. A schematic for this is shown in figure 11(b). For synchronous operation the spring representing the field would be added in parallel to this spring as shown in figure 11(c).

The calculations required to determine the approximate drive train natural frequencies are given in appendix B. In table IV the results are compared to those obtained from the NASTRAN model. For mode I the agreement is excellent. For mode II the maximum difference is about 8 percent.

CONCLUDING REMARKS

The Mod-0 wind turbine drive train has been analyzed. The component elements are documented as to their torsional spring stiffnesses and mass moments of inertia. Using these values the drive train natural vibration frequencies were determined with a finite element model. The major findings of this study are:

1. The validity of the finite element model was verified for load bank operation with experimental data.

2. Natural frequencies obtained using an approximate method were in good agreement with the finite element analysis.

3. The presence of a nonlinear spring coupling in the drive train was found to cause a wide range of mode II frequencies. From the standpoint of reducing the possibility of operating at a resonant condition, it is undesirable to have a nonlinear spring coupling in the drive train of a wind turbine generator.

4. In order to reduce the mode I natural frequency to avoid a potential resonance, it was found necessary to reduce the stiffness of the high-speed shaft by several orders of magnitude. Reducing the high-speed shaft stiffness also reduces mode II frequencies. Therefore, when changing drive train component

stiffnesses, the effect on the frequencies of both modes must be taken into consideration.

5. For mode I natural frequencies the predominating mass moment of inertia was that for the blades, while for mode II it was that of the generator, belts and pulleys.

6. The natural frequencies for synchronous operation are strongly dependent upon the stiffness of the generator spring. A better estimate of Mod-0 line reactance may result in a change in this stiffness.

APPENDIX A

CALCULATION OF DRIVE TRAIN PHYSICAL CHARACTERISTICS

The methods used to calculate the Mod-0 drive train component stiffnesses and rotary mass moments of inertia are presented in this appendix.

Shafts

Besides the shafts in the gearbox, the important drive train shafts are the low-speed shaft, the two part high-speed shaft, and the generator shaft. The dimensions of these shafts as well as their stiffnesses and mass moments of inertia are tabulated in table V. The equations given in appendix C were used to make the required calculations.

Falk Coupling

The stiffness of the Falk coupling was derived from a manufacturer supplied curve of torsional stiffness versus applied torque. Torsional stiffness versus power was obtained assuming a 14 kW loss at 0 kW and a 28 kW loss at 100 kW. Figure 12 is a plot of Falk coupling stiffness versus torque and power.

The mass moment of inertia of the Falk coupling was estimated to be 75 lb-in-sec^2.

Gearbox

The overall gearbox stiffness as well as individual gearbox shaft dimensions, speed ratios and stiffnesses are given in table VI. The overall stiffness was obtained by reducing the combined shaft stiffness by 30 percent. This reduction was the estimated effect of the flexibility of the gear teeth, bearings, shafts in bending, and gearbox case.

Gear dimensions, speed ratios, mass moments of inertia and total mass moment of inertia are given in table VII. The mass moment of inertia of the shafts was estimated to be 242 lb-in-sec^2.

Pulleys and Belts

The pulleys have an outside diameter of 12.5 inches and weigh 132 pounds each. Assuming they can be represented by a solid disk, their mass moment of inertia was calculated using equation (C7).

The derivation of the equation used to calculate the stiffness of the belts is given in appendix C. Calculation of the Mod-0 belt stiffness is included in appendix C as a sample problem.

The mass moment of inertia of the belts was estimated by assuming the belt mass acted at the pulley outside radius. The weight of the belts is 12 pounds.

Generator

The electromagnetic coupling between the generator field and armature that occurs during synchronous operation acts as a spring to ground in the drive train model. For generating systems subjected to low frequency, high intensity pulses like Mod-0, this spring is of great importance. For this study the procedure discussed in Chapter 2 of reference 5 was used to make a torque-angle plot. In the calculations necessary to make this plot, a line reactance of 0.3 was assumed. More accurate determination of this number may result in a change in the calculated torque-angle characteristics of the generator. The torque-angle plot shows the relation between the input torque to the generator and an electrical angle between the generated field voltage and the voltage at the generator terminals. The slope of this curve represents an electrical torsional stiffness.

Because output power is directly proportional to input torque, an electrical torsional stiffness can be determined for a specified value of output power. Once determined, the electrical torsional stiffness can be converted to a mechanical torsional stiffness. Figure 13 shows the generator spring stiffness used in this study as a function of power for operation of the Mod-0 generator under transient conditions.

The mass moment of inertia of the generator as obtained from the manufacturer was 16.9 lb-in-sec^2. This value includes the contribution of the exciter windings and generator shaft.

Rotor Blades and Hub

The rotor blades and hub were assumed to be rigid members of the drive train. Their mass moments of inertia were calculated from summing masses times the square of the distance from the center of rotation.

Brake

The brake disk is 18 inches in diameter and 0.5 inch thick. Equation (C6) was used to calculate its mass moment of inertia.

Fluid Coupling

The mass moment of inertia of the fluid coupling was supplied by the manufacturer. The mass moment of inertia of the oil was neglected.

APPENDIX B

CALCULATION OF APPROXIMATE DRIVE TRAIN NATURAL FREQUENCIES

For calculating mode I natural frequencies, the mass moment of inertia of the blades is considered acting against a spring representing all the drive train components. Its spring constant, k_I, is calculated as follows.

$$k_I = \left(\sum_{i=1}^{3} \frac{1}{k_i} + \frac{1}{n^2} \sum_{i=4}^{7} \frac{1}{k_i} \right)^{-1}$$

where n is the speed ratio and the individual drive train component stiffnesses, k_i, are given in table II. The approximate natural frequency, f_n, is calculated by the equation

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k_I}{M_1}}$$

where M_1 is given in table III.

For calculating mode II natural frequencies, the mass moment of inertia of the generator, pulley and belts is considered acting against a single spring. For load bank operation this spring represents all the drive train components except the field spring. For synchronous operation the load bank spring is added in parallel to the field spring. The spring constant for load bank operation, k_{II_L}, is calculated as follows.

$$k_{II_L} = \left(\sum_{i=1}^{3} \frac{1}{k_i} + \frac{1}{n^2} \sum_{i=4}^{6} \frac{1}{k_i} \right)^{-1}$$

For synchronous operation the spring constant, k_{II_S}, is

$$k_{II_S} = k_{II_L} + n^2 k_7$$

The approximate natural frequency for mode II load bank operation is

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k_{II_L}}{n^2(M_8 + M_9)}}$$

and for mode II synchronous operation is

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k_{II_s}}{n^2(M_8 + M_9)}}$$

A theoretical basis for the mode II approximations can be found in Timoshenko and Young (ref. 5). For two disks of mass moment of inertia I_1 and I_2 connected by a shaft with a torsional spring constant k,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k(I_1 + I_2)}{I_1 I_2}}$$

For the case of $I_1 >> I_2$, this reduces to

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{I_2}}$$

The spring constants used in calculating the approximate natural frequencies are tabulated in table VIII.

APPENDIX C

EQUATIONS USED IN CALCULATING DRIVE TRAIN STIFFNESSES AND MASS MOMENTS OF INERTIA

In this appendix some of the equations required to calculate stiffnesses or mass moments of inertia are derived. Included are equations for determining the torsional stiffnesses of hollow and solid shafts, the stiffness of pulley-belt systems, and the polar mass moment of inertia of shafts and disks.

Shaft Torsional Stiffness

The twist, θ, per unit length of a hollow circular shaft is

$$\frac{d\theta}{dx} = \frac{T}{GJ}$$

where T is the applied twisting moment and the product GJ is called the torsional rigidity.

$G = E/2(1 + \nu)$ is the shear modulus, where E is Young's modulus and ν is Poisson's ratio. Also, $J = (\pi/32)\left(d_o^4 - d_i^4\right)$ is the polar moment of inertia where d_o is the outside diameter and d_i is the inside diameter.

The total twist for a uniform shaft of length ℓ is

$$\theta = \ell \frac{d\theta}{dx}$$

Thus, the shaft stiffness is

$$k = \frac{T}{\theta} = \frac{GJ}{\ell} = \frac{\pi E}{64(1 + \nu)\ell}\left(d_o^4 - d_i^4\right) \tag{C1}$$

For a solid circular shaft, $d_i = 0$ and

$$k = \frac{\pi E d_o^4}{64(1 + \nu)\ell} \tag{C2}$$

For stiffnesses calculated herein, $E = 30\times10^6$ psi and $\nu = 0.3$, the values for steel.

Belt Stiffness

Consider a belt on two pulleys with one pulley fixed and the second rotating through an angle, θ, due to a torque, T. Then the stiffness of the loaded part of the belt

$$k_\ell = \frac{T}{\theta} = \frac{PR}{\Delta\ell/R} = \frac{PR^2}{\Delta\ell}$$

where $\Delta\ell$ is the change in belt length, R is the pulley radius, and P is the belt load.

$$\Delta\ell = \epsilon\ell = \frac{P}{E_b}\,\ell$$

where ϵ is the belt strain, E_b is the belt elastic modulus and ℓ is the effective length of the belt. Therefore,

$$k_\ell = \frac{R^2 E_b}{\ell}$$

The effective length of the belt is dependent upon the amount of slip that takes place on the pulleys. Here it is assumed that one-fourth of the contact length is free to slip. Then

$$\ell = \ell_o + \frac{\pi R}{4}$$

where ℓ_o is the distance between the pulley centers. Therefore,

$$k_\ell = \frac{R^2 E_b}{\ell_o + \frac{\pi R}{4}}$$

The stiffness of the unloaded part of the belt is dependent upon the belt tension preload and the torque being transmitted. To simplify this analysis it is assumed that the unloaded part of the belt is half as stiff as the loaded part. Hence, the pulley/belt system stiffness

$$k_5 = \frac{1.5\ R^2 E_b}{\ell_o + \frac{\pi R}{4}} \qquad \text{(C3)}$$

For Mod-0 $R = 6.25$ inches, $\ell_o = 30.5$ inches, and $E_b = 19\ 000$ pounds. There are 10 belts. Substituting these values in equation (C3), $k_5 = 3.16\times10^5$ lb-in/rad.

Polar Mass Moment of Inertia

For circular shafts and disks, the polar mass moment of inertia, I_m, can be obtained by considering a hollow circular disk of mass, m, thickness, t, inside radius, r_i, and outside radius, r_o. Then,

$$I_m = \int_{r_i}^{r_o} r^2\, d_m$$

$$d_m = \rho_m\, dV = \rho_m t\, dA = 2\pi r \rho_m t\, dr$$

where ρ_m is mass density, V is volume, and A is area. Hence,

$$I_m = 2\pi\rho_m t \int_{r_i}^{r_o} r^3\, dr$$

Integrating,

$$I_m = \frac{1}{2}\pi\rho_m t\left(r_o^4 - r_i^4\right) \tag{C4}$$

Noting that

$$m = \pi\rho_m t\left(r_o^2 - r_i^2\right)$$

equation (C4) can be written

$$I_m = \frac{1}{2} m\left(r_o^2 + r_i^2\right) \tag{C5}$$

For a solid circular disk, equation (C4) reduces to

$$I_m = \frac{1}{2}\pi\rho_m t r_o^4 \tag{C6}$$

and equation (C6) reduces to

$$I_m = \frac{1}{2} m r_o^2 \tag{C7}$$

For shafts replace the thickness, t, with the shaft length, ℓ.

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TABLE I. - COMMERCIALLY AVAILABLE COMPONENTS IN MOD-0 DRIVE TRAIN

Component	Manufacturer	Identifying number	Remarks
Coupling	Falk	170T10	0.25 in. gap assumed
Gear box	Horsburgh & Scott	170-T (special)	Ratio: 45, Rating 236 hp at 1800 rpm; triple mesh
Generator	General Electric	5SJ4444A23 Y53F1	Synchronous, 125 kVA, 0.8 power factor, 100 kW, 480 V, Y-connected, three-phase, 60 Hz, 1800 rpm
Fluid coupling	American-Standard	500 (size)	

TABLE II. - MOD-0 DRIVE TRAIN COMPONENT STIFFNESSES

Drive train component	Spring designation,	Torsional stiffnesses, 10^8 lb-in/rad
Low-speed shaft	k_1	2.00
Falk coupling	k_2	See fig. 12
Gear box[a]	k_3	1.66
High-speed shaft	k_4	.00668
Belts	k_5	.00316
Generator shaft	k_6	.0419
Generator field	k_7	See fig. 13

[a]See table VI for calculation.

TABLE III. - MOD-0 DRIVE TRAIN ROTARY

MASS MOMENTS OF INERTIA

Drive train component	Mass designation, M_i	Mass moment of inertia, $lb\text{-}in\text{-}sec^2$
Blades	M_1	1 130 400
Hub assembly	M_2	7 250
Low-speed shaft	M_3	140
Gears and shafts[a]	M_4	2 415
Falk coupling	M_5	75
Brake	M_6	3.78
High-speed shaft	M_7	.54
Pulleys and belts	M_8	14.6
Generator rotor	M_9	16.9
Fluid coupling	M_{10}	14.9

[a]See table VII for calculations (contribution of shafts estimated to be 242 $lb\text{-}in\text{-}sec^2$).

TABLE IV. - COMPARISON OF NATURAL FREQUENCIES OBTAINED FROM NASTRAN MODEL AND SINGLE DEGREE OF FREEDOM APPROXIMATE METHOD

Case	Predicted frequency, Hz	
	NASTRAN model	Approximate method
Mode I, synchronized, 0 kW	0.27	0.27
Mode I, synchronized, 100 kW	.63	.63
Mode II, load bank, 0 kW	1.41	1.49
Mode II, load bank, 100 kW	4.02	4.16
Mode II, synchronized, 0 kW	2.18	2.35
Mode II, synchronized, 100 kW	5.16	5.43

TABLE V. - SHAFT DIMENSIONS, STIFFNESSES AND MASS MOMENTS OF INERTIA

Shaft	Shaft dimensions, in.			Stiffness, lb-in/rad	Mass moment of inertia, $lb\text{-}in\text{-}sec^2$
	Outside diameter	Inside diameter	Length		
Low-speed					
Part 1	10.24	3.00	18.0	6.87×10^8	14
Part 2	10.00	3.00	23.4	4.80×10^8	17
Part 3	8.00	3.00	6.7	6.79×10^8	3
Flange					106
High-speed					
Part 1	2.50	0	44.9	9.86×10^5	.20
Part 2	2.93	0	40.5	2.06×10^6	.25
Flange					.09
Generator	2.88	0	18.6	4.19×10^6	.09

TABLE VI. - GEAR BOX SHAFT DIMENSIONS, SPEED RATIOS, AND STIFFNESSES

Shaft dimensions, in.			Shaft number, i	Speed ratio, n_i	Shaft stiffness, k_{G_i}, lb-in/rad
Outside diameter	Inside diameter	Length			
8.00	3.00	16.0	1	1:0	2.84×10^{8}
4.82	0	7.19	2	3.8	8.47×10^{7}
3.75	0	9.00	3	15.0	2.49×10^{7}
2.87	0	13.81	4	45	5.57×10^{6}

$$k_G = \left(\sum_{i=1}^{4} \frac{1}{n_i^2 k_{G_i}} \right)^{-1} = 2.16\times10^{8} \text{ lb-in/rad}$$

$$k_3 = \frac{k_G}{1.3} = 1.66\times10^{8} \text{ lb-in/rad}$$

where 1.3 is a stiffness reduction factor to account for other gear box flexibilities.

TABLE VII. - GEAR DIMENSIONS, SPEED RATIOS, AND MASS MOMENTS OF INERTIA

Gear dimensions, in.		Gear number, i	Speed ratio, n_i	Mass moment of inertia, M_{G_i}, lb-in-sec^2
Pitch diameter	Thickness			
26.9	8.5	1	1.0	336
7.1	9.0	2	3.8	1.79
17.6	5.5	3	3.8	40.1
4.4	6.0	4	15	.17
12.0	3.0	5	15	4.74
4.0	3.4	6	45	.066

$$M_4 = \sum_{i=1}^{6} n_i^2 M_{G_i} = 2173.2 \text{ lb-in-sec}^2.$$

TABLE VIII. - SPRING CONSTANTS USED IN CALCULATION OF APPROXIMATE DRIVE TRAIN NATURAL FREQUENCIES

Case	Power, kW	Spring constant 10^8 lb-in/rad
Mode I, synchronized	0	0.033
	100	.184
Mode II, load bank	0	.056
	100	.435
Mode II, synchronized	0	.139
	100	.741

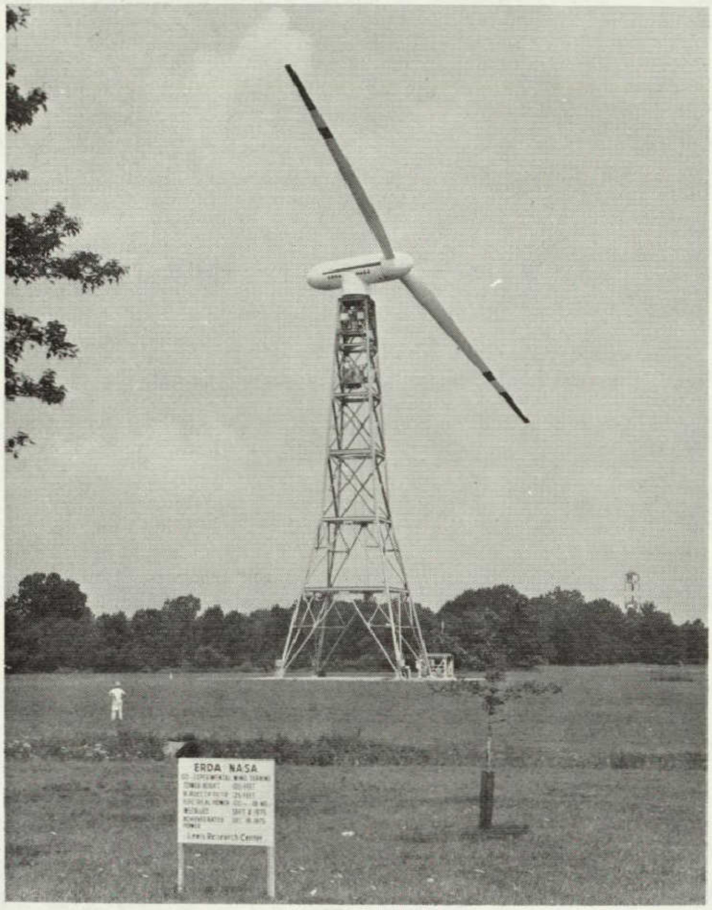

Figure 1. - Mod-0 100 kW wind turbine.

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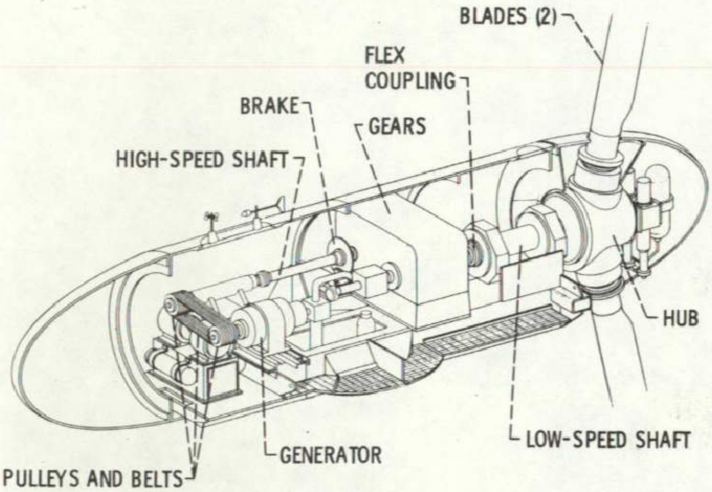

Figure 2. - 100 kw wind turbine generator drive train.

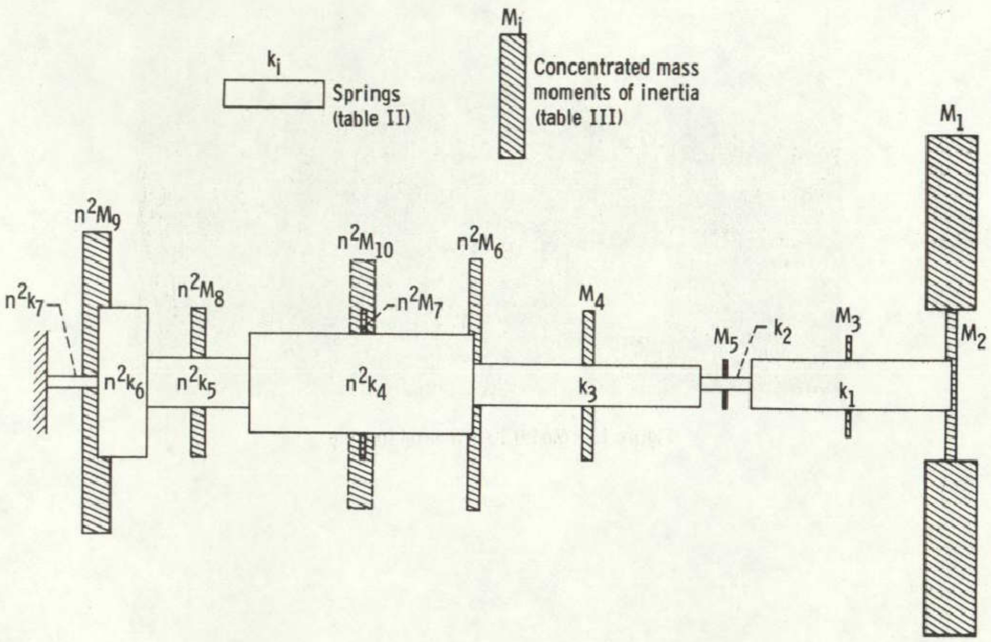

Figure 3. - Schematic of Mod-0 Drive Train (n = 45).

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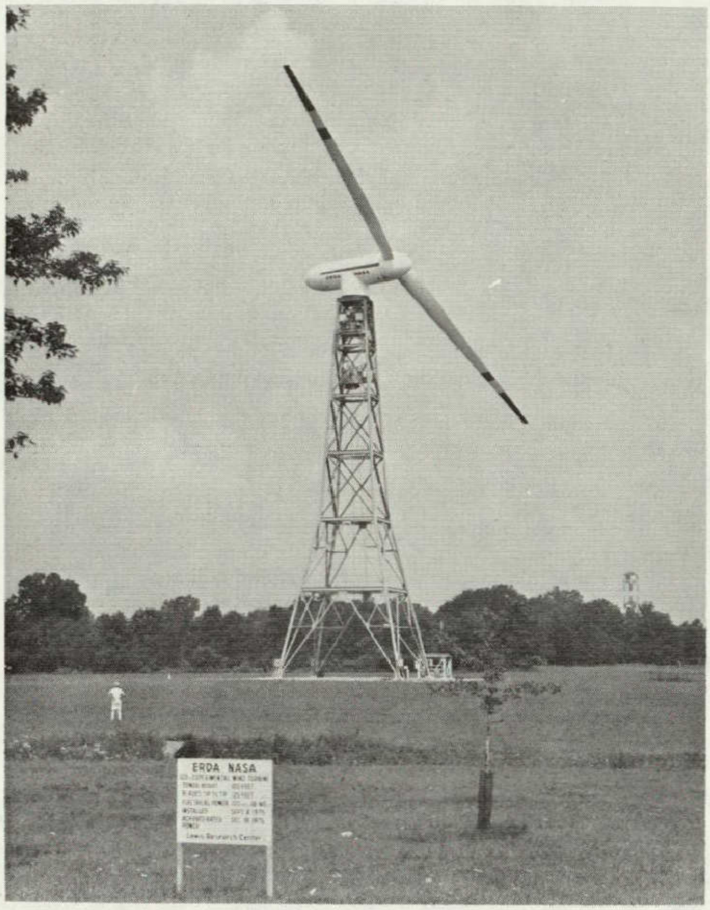

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Figure 1. - Mod-0 100 kW wind turbine.

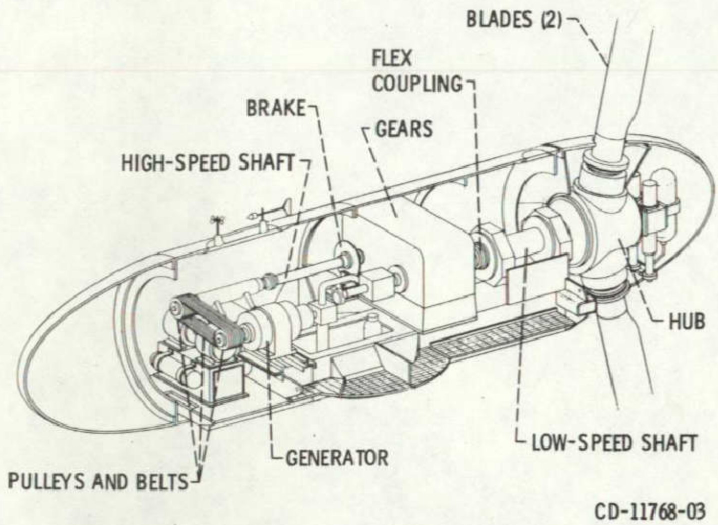


Figure 2. - 100 kw wind turbine generator drive train.

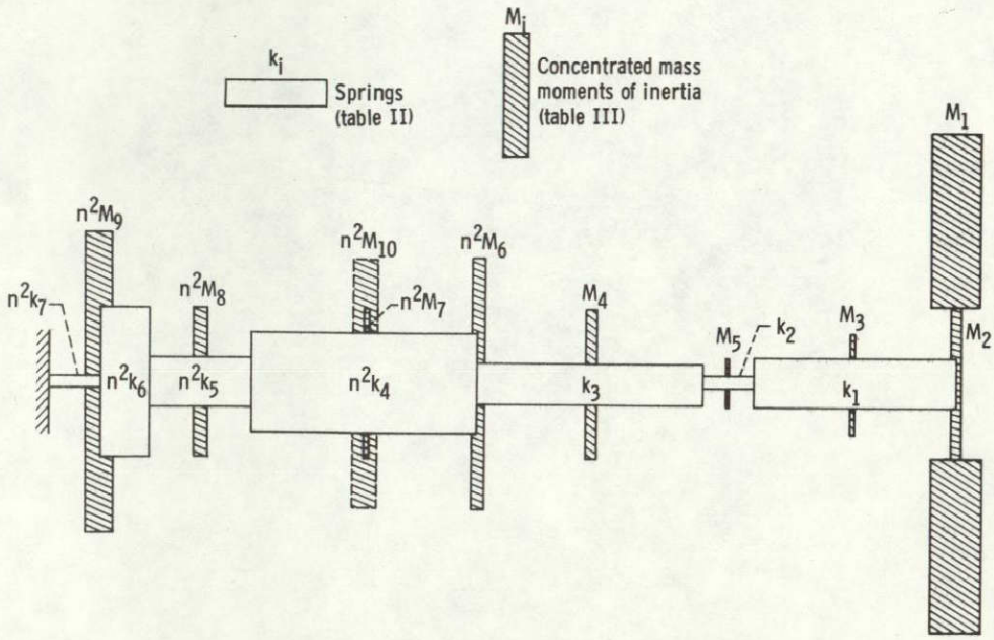


Figure 3. - Schematic of Mod-O Drive Train (n = 45).

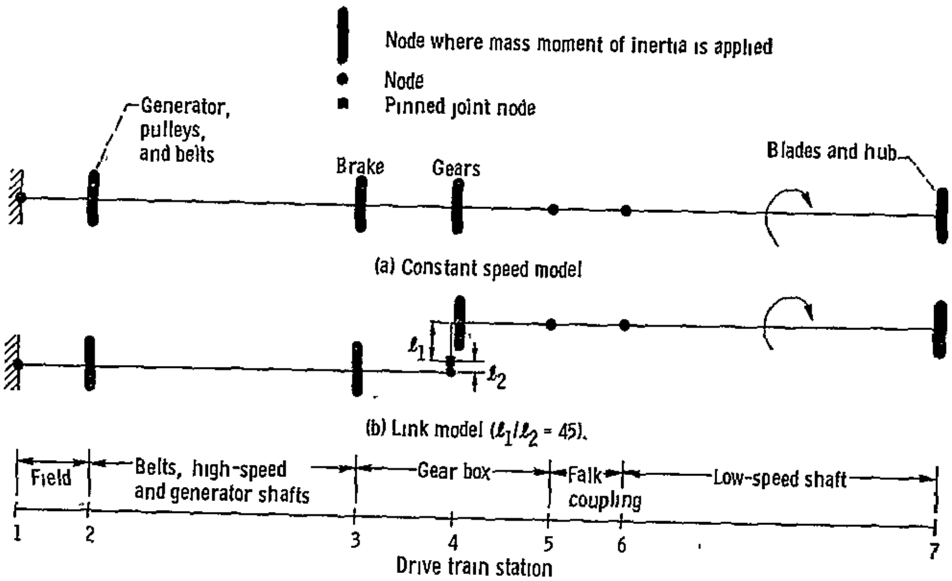


Figure 4 - Finite element model schematics

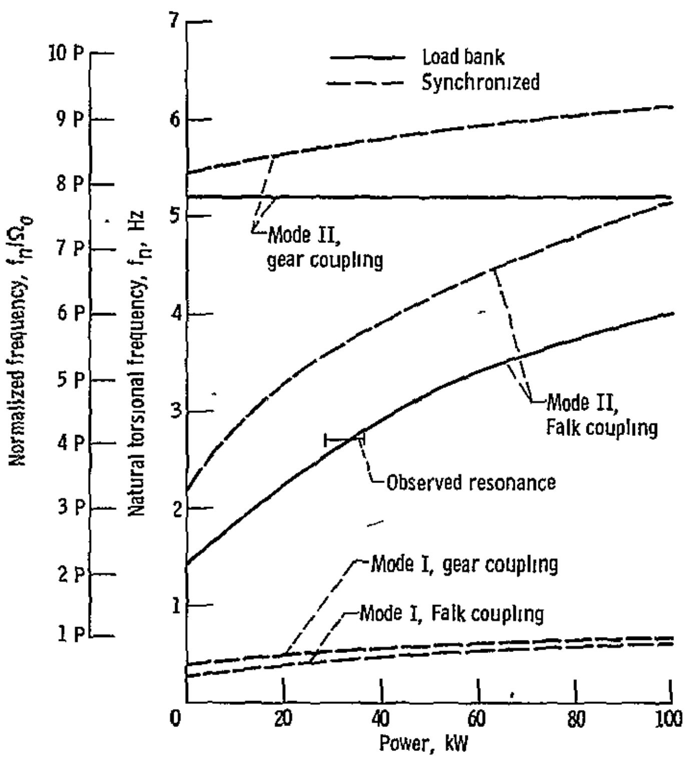


Figure 5 - 100 kW wind turbine drive train natural frequencies
Rotor speed Ω_0 = 0 667 Hz

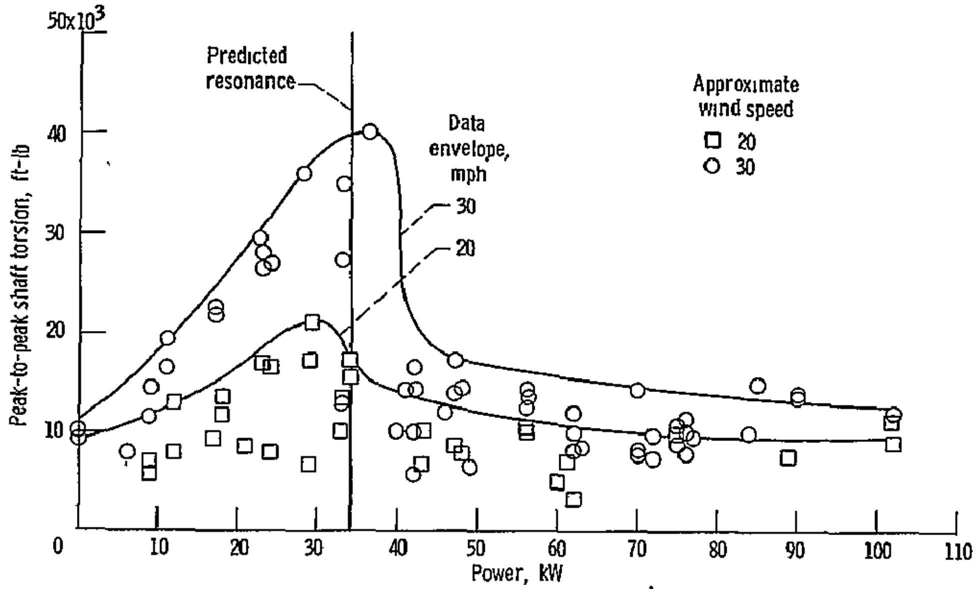


Figure 6 - Low-speed shaft torsional bending moment for load bank operation

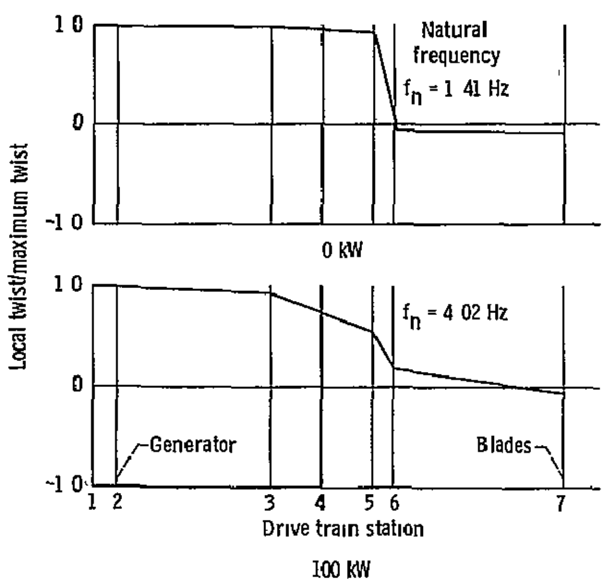


Figure 7 - Wind turbine drive train vibration mode shapes for load bank operation

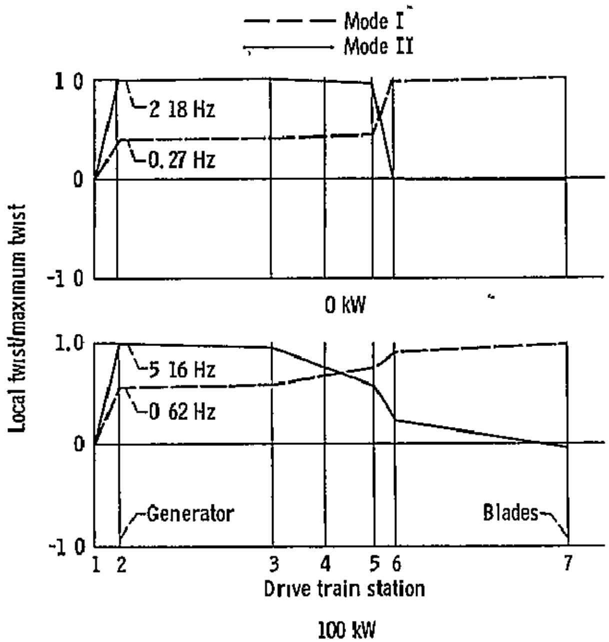


Figure 8 - Wind turbine drive train vibration mode shapes for synchronous operation

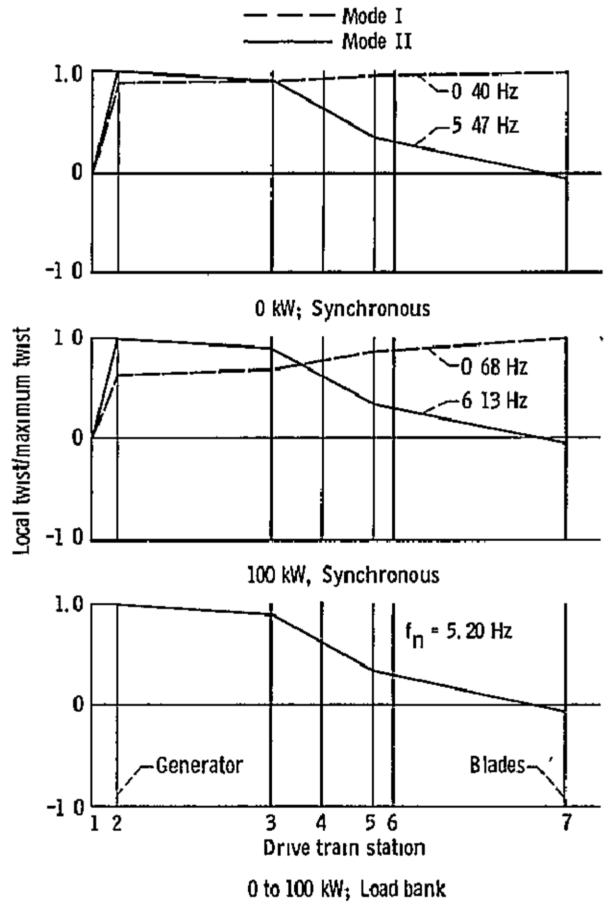
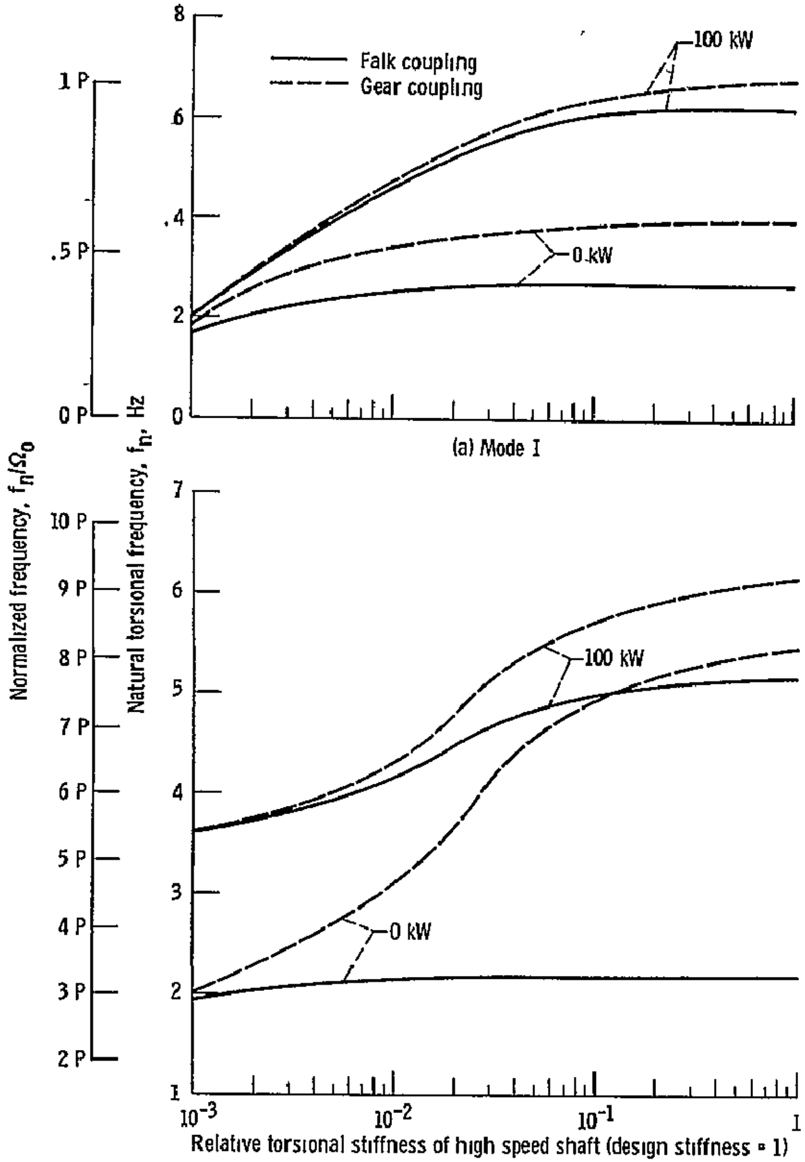


Figure 9 - Wind turbine drive train vibration mode shapes for gear coupling replacing Falk coupling



(b) Mode II.

Figure 10. - Effect of high-speed shaft stiffness on drive train natural frequency. Rotor speed Ω_0 = 0.667 Hz

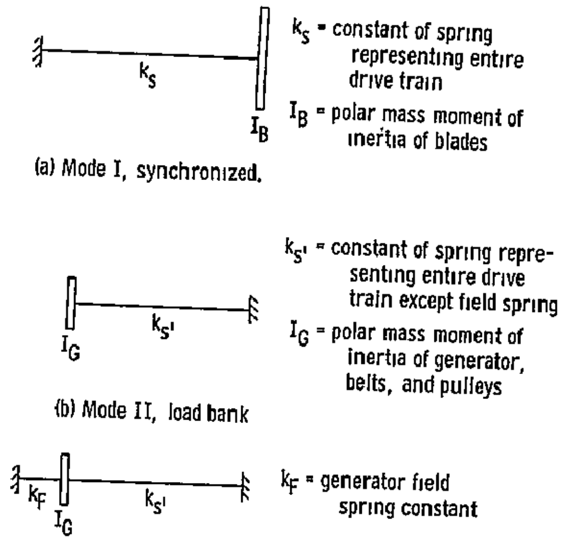


Figure 11 - Schematics of spring/mass systems approximating drive train

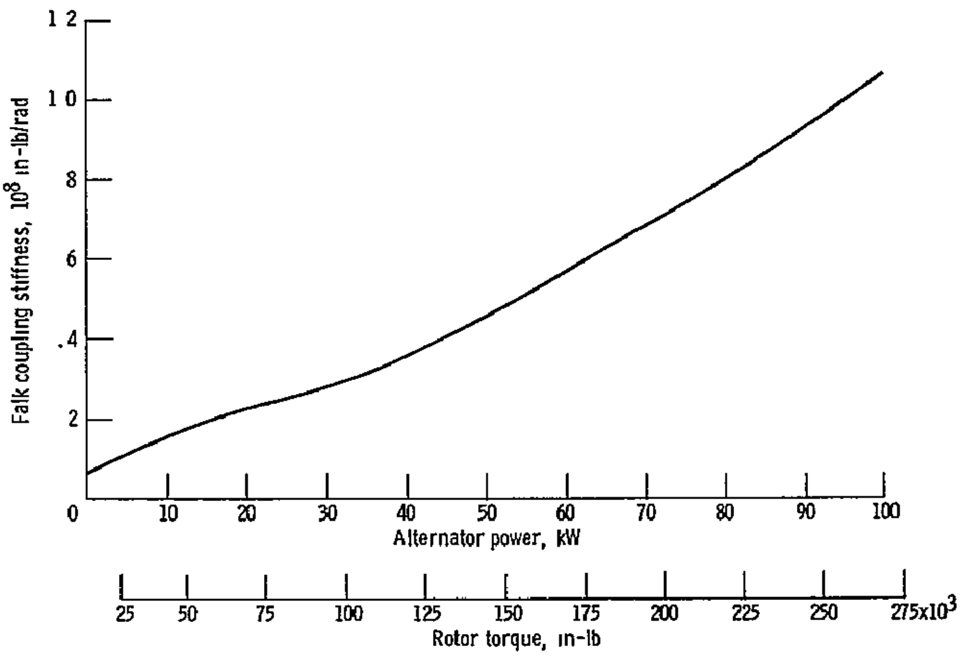


Figure 12. - Mod-0 Falk coupling stiffness versus power output

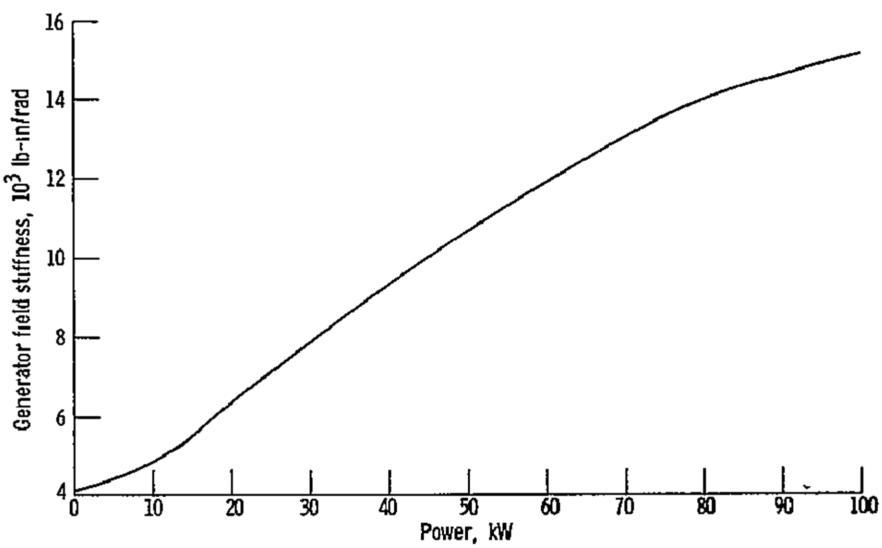


Figure 13 - Mod-0 generator field stiffness versus power for transient condition

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