NASA TECHNICAL MEMORANDUM

NASA TM - 73,261

(NASA-TM-73261) ACCURACY OF THE KIRCHOFF FORMULA IN DETERMINING ACOUSTIC SHIELDING WITH THE USE OF A FLAT PLATE (NASA) 20 p HC A02/MF A01 CSCL 20A

N77-30906

Unclas

G3/71 42039

NASA TM - 73,261

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Ralph E. Gabrielsen and Joseph E. Davis

Ames Research Center

and

Ames Directorate, USAAMRDL
Ames Research Center
Moffett Field, Calif. 94035

August 1977

ACCURACY OF THE KIRCHOFF FORMULA IN DETERMINING ACOUSTIC SHIELDING WITH THE USE OF A FLAT PLATE

Ralph E. Gabrielsen and Joseph E. Davis*

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INTRODUCTION

In recent years, the desire for reducing aircraft noise has begun to play an increasingly important role in the design of current and proposed commercial and military jet aircraft. The main source of noise on most jet aircraft is the jet engine exhaust. The interaction of this noise with the other surfaces of the aircraft, notably the wings, has a definite effect on the total acoustic signature produced. Numerous studies have suggested that if the jet engines of various aircraft were placed at appropriate locations above the wing instead of below it, the wing would provide a partial shielding of the noise generated by the engines relative to observers on the ground.

As a result of these findings, further experimental and analytical studies are being conducted in order to better understand and eventually predict this shielding in a given situation. The purpose of this study was to analytically predict the shielding effect of an idealized three-dimensional barrier in the presence of an idealized engine noise source by means of the well-known Kirchoff formula. In addition, the results were compared with experimental measurements (ref. 1).

The wing was represented in the analysis by a rectangular flat plate and the engine noise source by a point source. Calculations of the shielding effect were performed for various plate sizes, shapes, locations, and for several point source frequencies ranging from 100 Hz. to 20 kHz. The results are presented in terms of the difference in decibel (dB) level of the shielded versus unshielded situations.

On the following pages the method of analysis is described, the results presented, and conclusions drawn.

METHOD OF ANALYSIS

The acoustic shielding of a point source by a rectangular flat plate was calculated by means of the Kirchoff formula (ref. 2). This formula resulted from assuming that the values for the velocity potential and the

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*Contractor

the normal velocity on a diffractor may be replaced by the free-field values induced by the source on the "illuminated" side, and by zero on the "shadow" side.

Basic Equations and Boundary Conditions

The problem under consideration is described by the wave equation

$$\Delta\phi = \frac{1}{c^2}\,\phi_{tt}$$

with suitable boundary conditions on the surface ∂S of the diffractor, and at infinity, where

ϕ = velocity potential,

c = speed of sound in air,

t = time.

For a periodic acoustic wave, the velocity potential ϕ can be expressed by

$$\phi = ue^{-ikct}$$

where

k = wave constant $2\pi/\lambda$,

λ = wave length of the acoustic waves,

and u is independent of time. Substituting this expression for ϕ into the wave equation gives

$$\nabla^2 u + k^2 u = 0$$

which is the Helmholtz equation. For a given point source u_o, it can be shown by Green's theorem that

$$u(P) = u_o(P) + \frac{1}{4\pi}\int_{\partial S}\left[u(Q)\frac{\partial}{\partial n}\frac{e^{ikr}}{r} - \frac{e^{ikr}}{r}\frac{\partial u(Q)}{\partial n}\right] dS(Q) \qquad (1)$$

providing, of course, that u satisfies the Sommerfeld condition at infinity, where

∂S = surface of the diffractor,

P = arbitrary point in the field,

n = inward normal to ∂S,

$r = |Q-P|$,

Q = arbitrary point on the surface of the diffractor.

The acoustic shielding (ΔSPL) for a given point in space is defined to be the difference in dB level between the shielded and unshielded cases, i.e.,

$$\Delta SPL = 20 \log_{10} \left| \frac{u(P)}{u_o(P)} \right| \tag{2}$$

where the acoustic pressure p is defined as

$$p(P) = -ik\rho_o U(P)$$

and

c = speed of sound in air,

ρ_o = reference density of air.

The Kirchoff Formula

By applying the Kirchoff assumptions discussed above, the Helmholtz-Huygens integral representation yields the formula

$$u(P) \approx u_o(P) + \frac{1}{4\pi} \int_{\partial S_i} \left\{ - u_o(Q) \frac{\partial}{\partial n} \frac{e^{ikr}}{r} + \frac{e^{ikr}}{r} \frac{\partial}{\partial n} \left[u_o(Q) \right] \right\} dS(Q) \tag{3}$$

where ∂S_i denotes the illuminated side of ∂S.

For the problem under consideration, the incident acoustic source is a point so , i.e.,

$$u_o(Q) = \frac{e^{ikr_1}}{r_1}$$

where $r_1 = |Q-R|$, and R is the location of the point source, and the diffractor is a rectangular flat plate of width W and length L. Hence, referring to the coordinate system of figure 1, equation (3) becomes

$$u(P) \approx \frac{e^{ikr_2}}{r_2} + \int_0^W \int_0^L \frac{e^{ik(r_1+r)}}{r_1 r} \left[\frac{Z(R)}{r_1} \left(ik - \frac{1}{r_1} \right) - \frac{Z(P)}{r} \left(ik - \frac{1}{r} \right) \right] dy(Q)\ dx(Q) \tag{4}$$

where

$r_2 = |P-R|$,

$Z(Q) = 0$.

Based upon the literature, it is rather difficult, if not impossible, to determine whether or not the Kirchoff formula would be of practical value for the problem in question. In fact, based upon the literature, it would be possible to present a credible argument either way. However, since its value has been argued for over fifty years, and its actual formulation even modified somewhat in recent years, one is motivated to try it. For this problem, Kirchoff's formula yielded good results based on experimental comparisons.

The calculations of acoustic shielding that are discussed in the remainder of this report were computed using equations (2) and (4). The integral in equation (4) was numerically evaluated using Simpson's rule. The density of the numerical integration mesh was chosen so that the results were less than 1% in error relative to the value of the integral obtained using a much finer mesh.

DISCUSSION OF RESULTS

Calculations of the acoustic shielding of a point source by a rectangular flat plate were made for the five different point source and rectangular plate configurations shown in figure 2. These calculations were performed at frequencies ranging from 100 Hz. to 20 kHz. The acoustic shielding ΔSPL was always evaluated at $Z = -5.0$ m along a line (called the traverse line) parallel to the x-axis (see figure 1) at the same y location as that of the point source. The traverse angle at these locations is defined as

$$\theta = \tan^{-1} \frac{X(P) - X(R)}{Z(R) - Z(P)}$$

Some samples of the results are shown in figures 3 through 10. These are representative of the results obtained for all five configurations. Most of the numerical results are compared with experimental measurements by Ahtye and McCulley (ref. 1). The comparisons with the experimental data indicate that the Kirchoff approximation provides good qualitative results for all five combinations at all frequencies and provides good quantitative agreement for all combinations at frequencies above 4 kHz.

In comparing the numerical and experimental results, it is important to realize that the periodic sharp peaks in shielding (locations of maximum negative ΔSPL) are very narrow at the higher frequencies and are difficult to compute or measure accurately. Numerically, the exact point of these peaks can only be found by trial and error which is prohibitively expensive.

Experimentally, these peaks cannot be measured accurately because the microphone that performs the measurement is a finite size much larger than the band width of the peak. Hence, only a spatial average of the pressure can be measured at these peaks.

At periodic locations of minimum shielding (locations of smallest negative ΔSPL) the band widths are not as narrow as for peaks of maximum shielding. Hence, these locations can be defined both numerically and experimentally with sufficient accuracy. Fortunately, it is the values of minimum shielding that are the most important in assessing the acoustic shielding in a given situation.

In general, the acoustic shielding provided by the various rectangular plates was most effective for the higher frequencies and at locations where the point source was hidden geometrically by the plate. The best shielding occurred with the rectangular plate placed closest to the point source and centered in front of it (configuration 1, figs. 3-6). The least shielding occurred with the rectangular plate placed furthest away from the point source and furthest off-center (configuration _, figs. 7-10).

CONCLUSIONS

Based on the good agreement between experimental measurements and the numerical results of the current study, it can be concluded that the Kirchoff approximation provides at least a good qualitative estimation of the acoustic shielding of a point source by a rectangular flat plate for measurements taken in the far field of the flat plate at frequencies ranging from 1 kHz. to 20 kHz. Furthermore, at frequencies greater than 4 kHz, the Kirchoff approximation provides reasonably accurate quantitative predictions of acoustic shielding. It appears to be a valuable tool for predicting far field acoustic shielding and, on that basis, can be of use in preliminary evaluations of noise reducing design considerations for future jet aircraft.

APPENDIX

COMPUTER PROGRAM USED TO PERFORM CALCULATIONS

The predictions of the acoustic shielding of a point source by a rectangular flat plate were performed using the computer program listed in figure 11. The program was written in standard USA Fortran IV and run on a CDC 7600 computer. A description of the user input and the program output is given below. Both the input and output parameters are consistent with the coordinate system of figure 1 and the definitions in the Method of Analysis and Conclusions.

User Input

Card No. 1 - Format(6F10.4)

X1,Y1,Z1,X2,Y2,Z2

where

X1,Y1,Z1 = Coordinates of the point source.

X2,Y2,Z2 = Coordinates of the initial evaluation point along the traverse line where the acoustic shielding is to be computed. The traverse line is always assumed to be parallel to the X-axis. The program automatically computes the velocity potential at a series of points along the traverse line. These points are defined by the parameters DELT and TMAX.

Card No. 2 - Format(I5)

NCASE

where

NCASE = Number of different frequencies at which the calculations are to be performed. Card No. 3 is repeated once for each different frequency.

Card No. 3 - Format(2I5,4F10.4)

N1,N2,F,DX,TMAX,DELT (One card for each frequency)

where

N1 = Number of numerical integration grid points in the X-direction on the rectangular plate.

N2 = Number of numerical integration grid points in the Y-direction on the rectangular plate.

F = Frequency of the point source in Hertz.

DX = Distance between the numerical integration grid points in either the X or Y direction.

TMAX = Maximum traverse angle at which the acoustic shielding is to be evaluated (degrees). The traverse line extends from X2 to the X location whose traverse angle is TMAX.

DELT = Incremental traverse angle between each successive evaluation point along the traverse line. The default value is 0.5 degrees.

Program Output

T2 = Traverse angle at the point at which the acoustic shielding is being evaluated.

V = Magnitude of the complex velocity potential at the traverse angle T2.

TIME = Accumulated compute time in seconds used by the computer program.

DSPL = Acoustic shielding (see Basic Equations and Boundary Conditions) at the traverse angle T2.

VOP = Complex velocity potential (real and imaginary parts) induced by the point source alone at the traverse angle T2.

X,Y,Z = Coordinates of the point source.

Y2,Z2 = The Y and Z coordinates of the traverse line at which the acoustic shielding is computed.

F = Frequency of the acoustic waves.

REFERENCES

1. Ahtye, Warren F.; and McCulley, Geraldine: "Evaluation of Simple Methods for the Prediction of Noise Shielding by Airframe Components," NASA TP-1004, 1977.

2. Baker, B. B.; and Copson, E. T.: "The Mathematical Theory of Huygens' Principle," Oxford University Press, London, 1950.

3. Bouwkamp, C. J.: "Diffraction Theory," Reports on Progress in Physics, vol. 17, London, 1954.

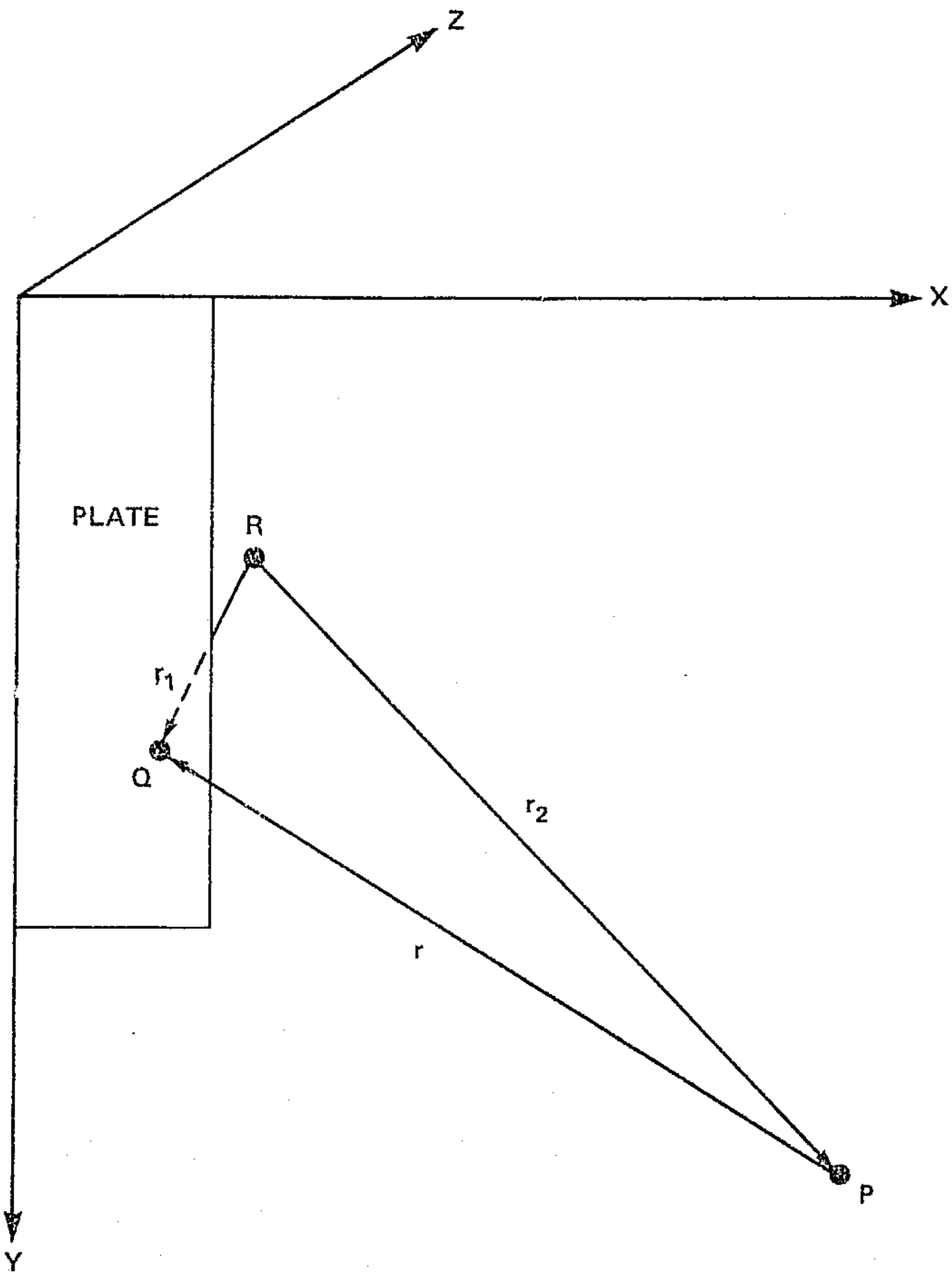

Figure 1.- Coordinate system for point source and rectangular flat plate acoustic field.

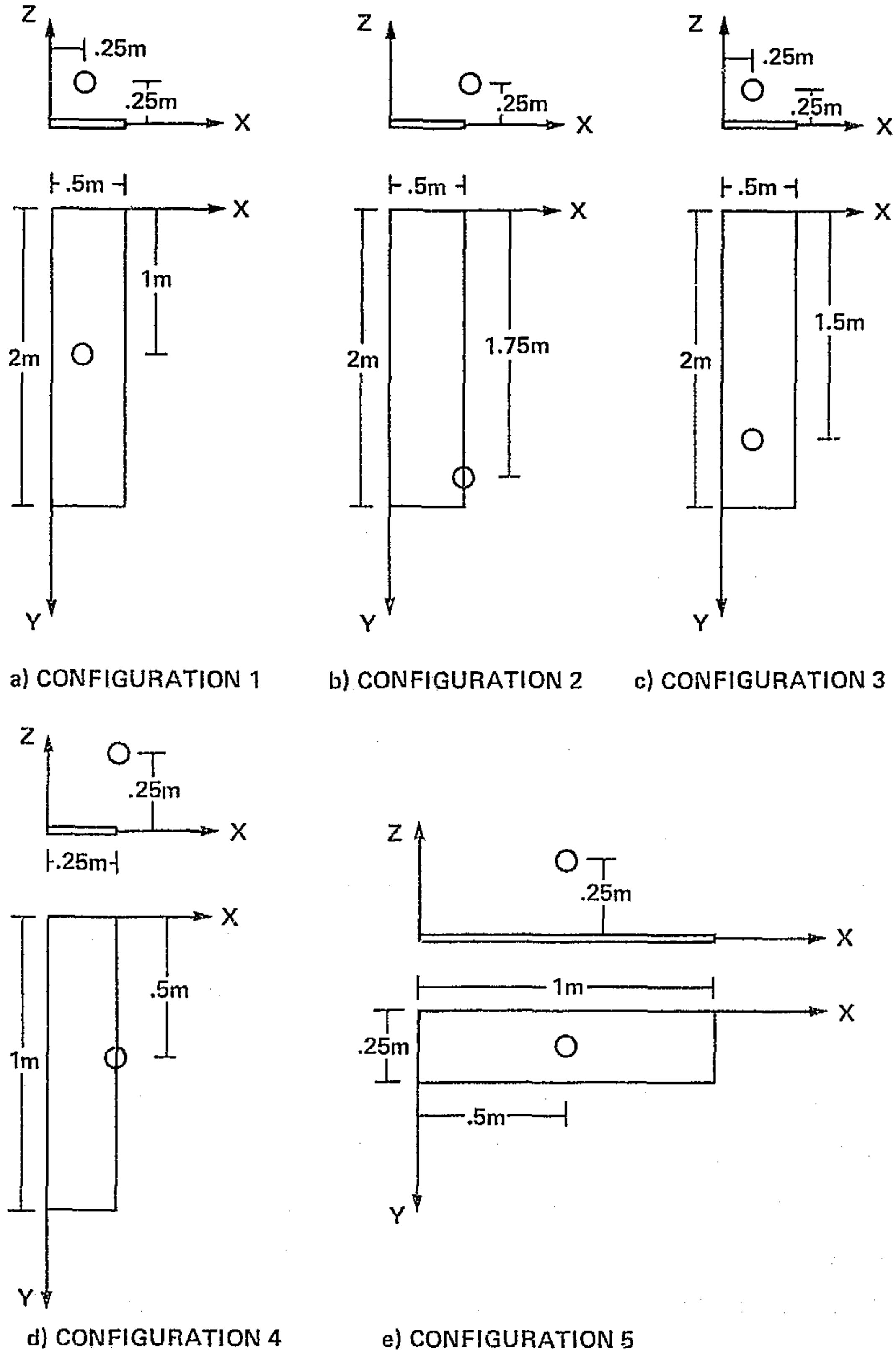

Figure 2.- Configuration for which the acoustic shielding of a point source by a rectangular flat plate were computed.

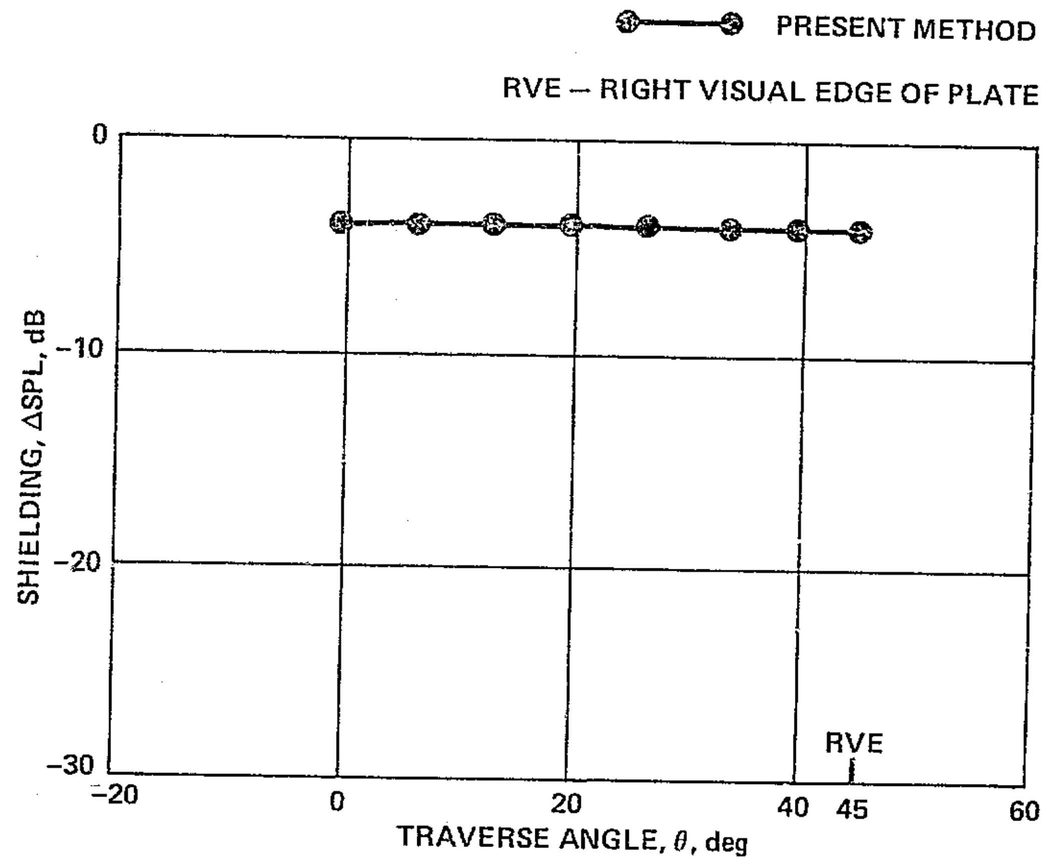

Figure 3.- Acoustic shielding for configuration 1 at a frequency of 0.1 kHz along a traverse line at $Z = -5.0$ m.

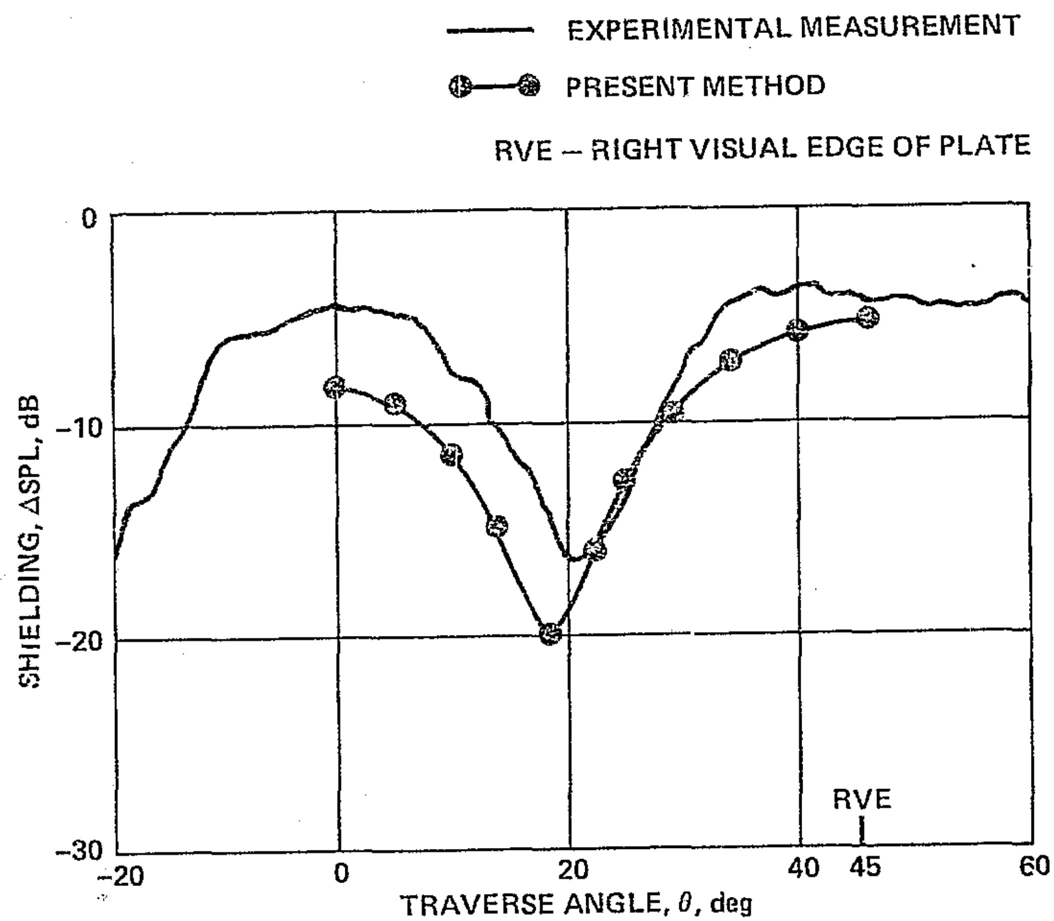

Figure 4.- Acoustic shielding for configuration 1 at a frequency of 1 kHz along a traverse line at Z = -5.0 m.

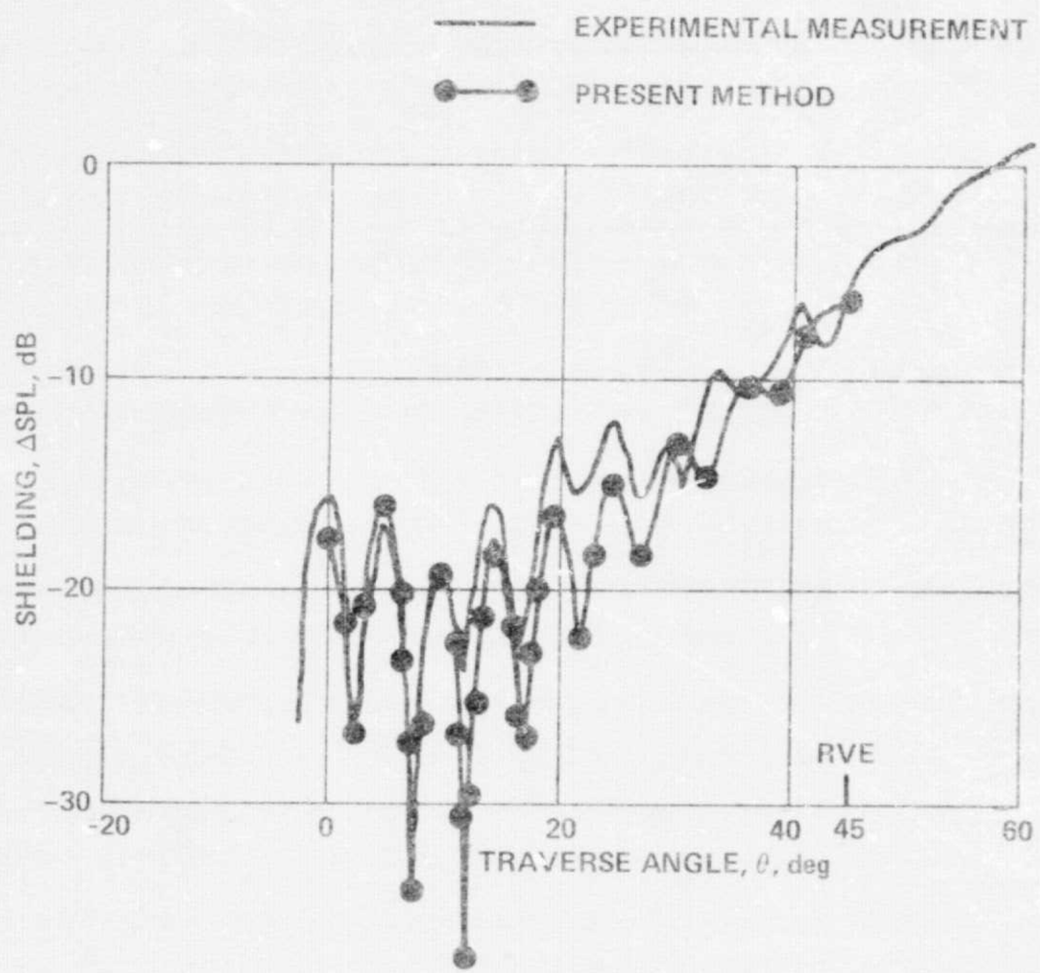

Figure 5.- Acoustic shielding for configuration 1 at a frequency of 8 kHz along a traverse line at $Z = -5.0$ m.

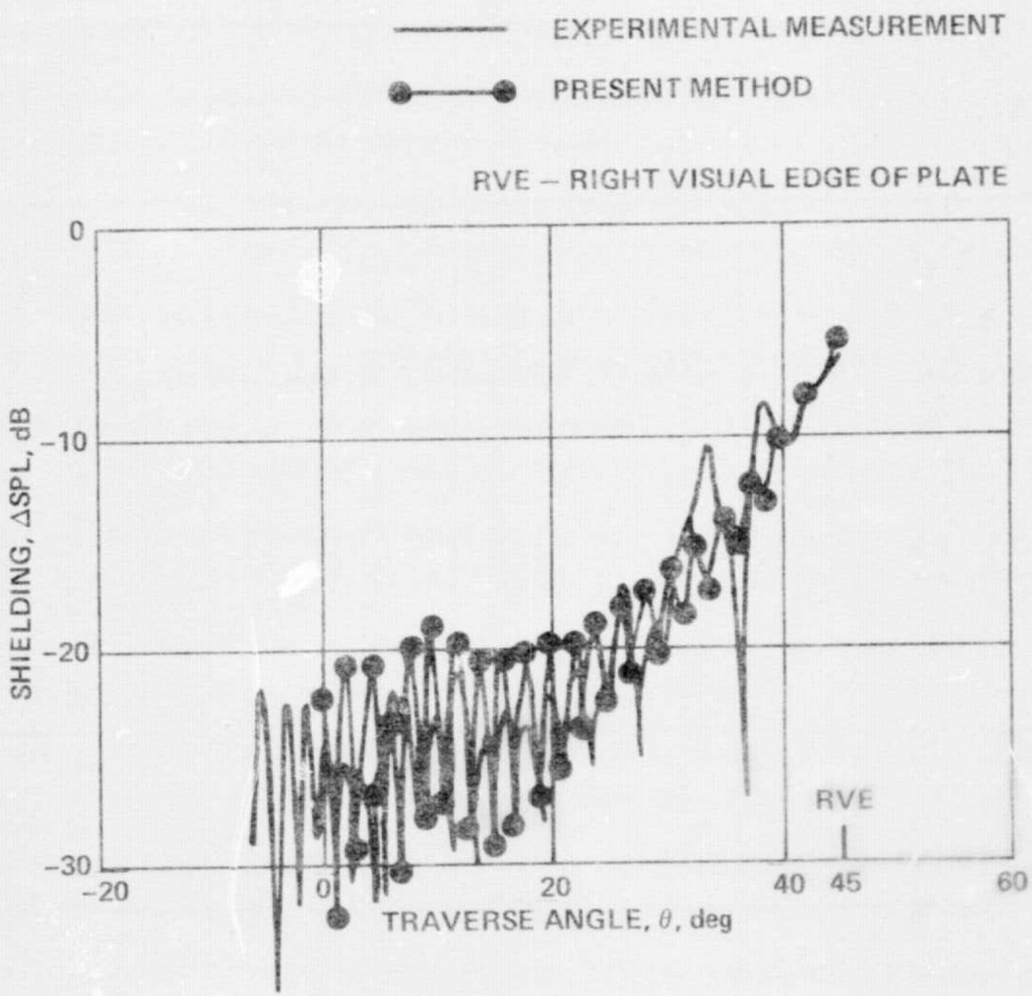

Figure 6.- Acoustic shielding for configuration 1 at a frequency of 20 kHz along a traverse line at $Z = -5.0$ m.

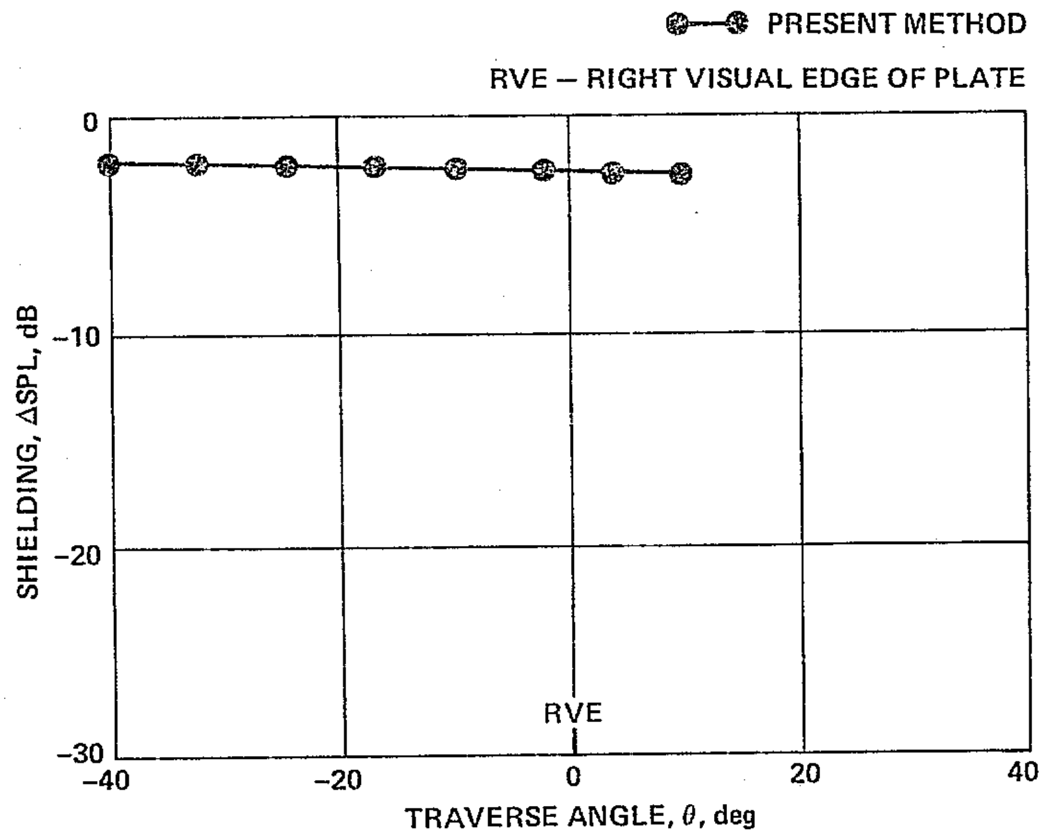

Figure 7.- Acoustic shielding for configuration 2 at a frequency of 0.1 kHz along a traverse line at $Z = -5.0$ m.

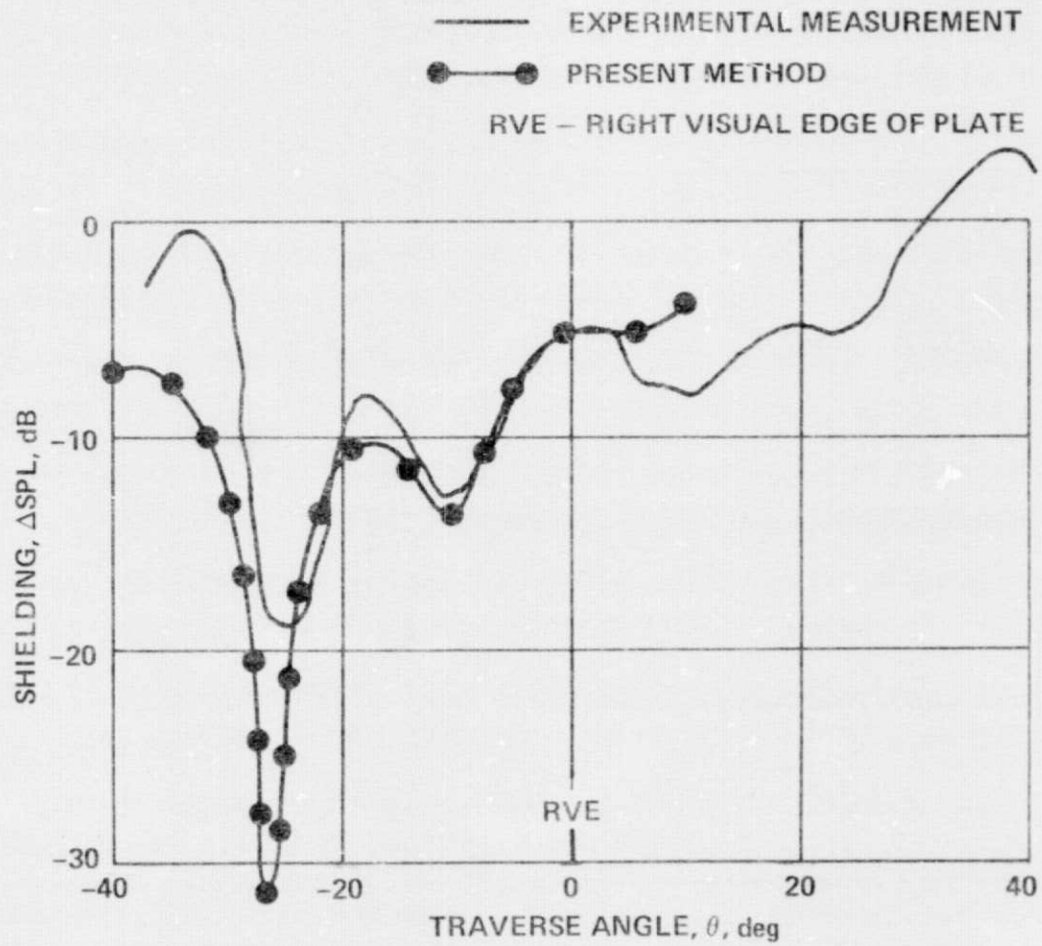

Figure 8.- Acoustic shielding for configuration 2 at a frequency of 2 kHz along a traverse line at $Z = -5.0$ m.

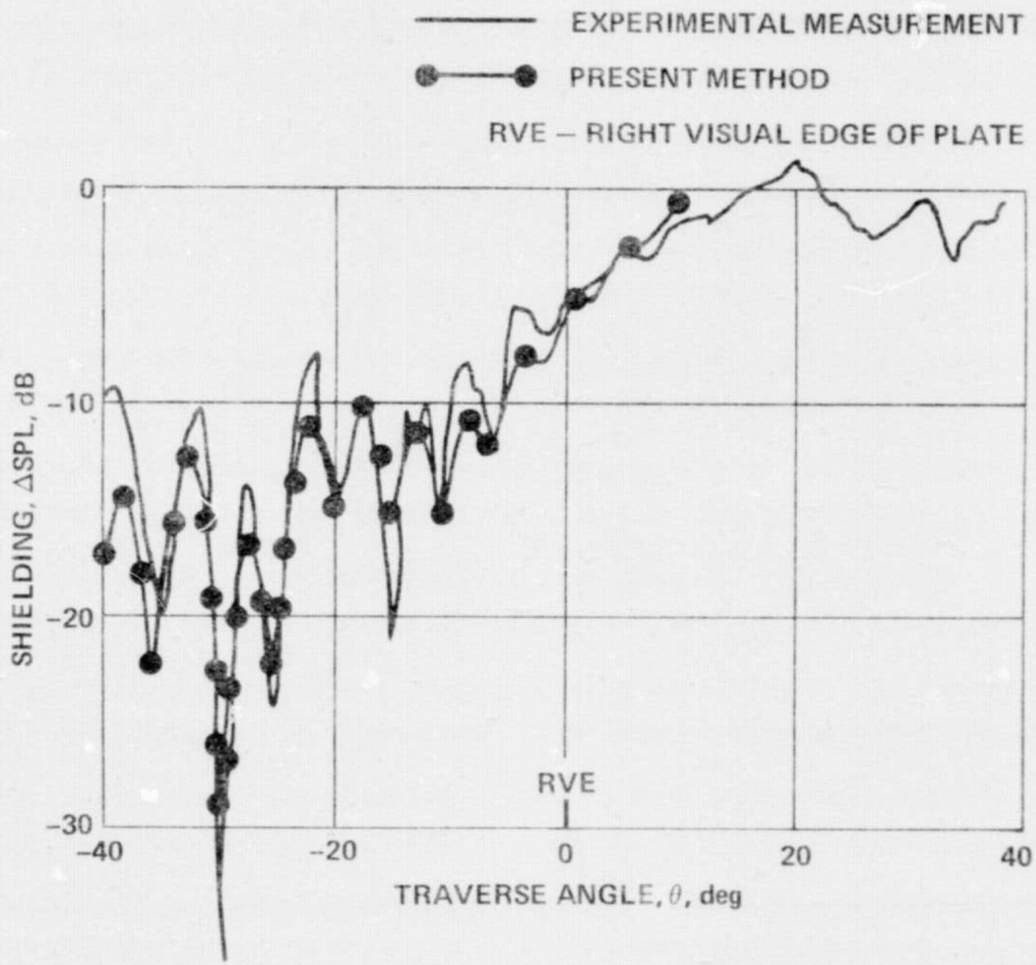

Figure 9.- Acoustic shielding for configuration 2 at a frequency of 8 kHz along a traverse line at $Z = -5.0$ m.

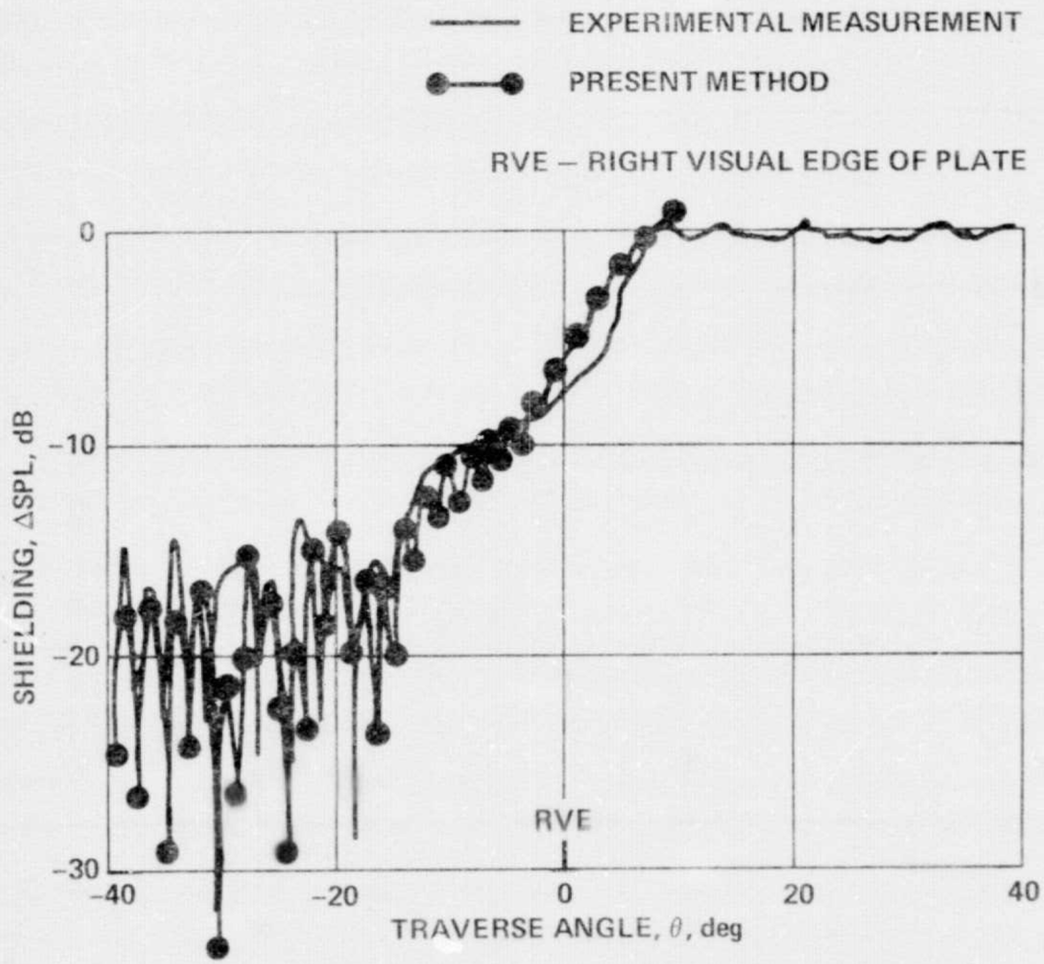

Figure 10.- Acoustic shielding for configuration 2 at a frequency of 20 kHz along a traverse line at $Z = -5.0$ m.

```
      PROGRAM KIRCH(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      COMPLEX VOP,C2,C4,VOS,C5,DVOS,C7,RLE,DRLE,VSUMX,VSUMY,V
      CALL SECOND(TIME)
      WRITE(6,101) TIME
      PI=3.14159265359
      PI2=2.0*PI
      PI4=2.0*PI2
      RAD=57.29577951
      READ(5,900) X1,Y1,Z1,X2,Y2,Z2
  900 FORMAT(6F10.0)
      READ(5,901) NCASE
      X20=X2
      DO 6 ICASE=1,NCASE
      X2=X20
      READ(5,901) N1,N2,F,DX,TMAX,DELT
  901 FORMAT(2I5,4F10.0)
      IF(DELT.LT.1.E-06) DELT=.5
      C0=347.412
      DSR=DX*DX/9.
      FK=PI2*F/C0
      WRITE (6,102)
  102 FORMAT(1H1,41HKIRCHOFF APPROXIMATION-HALF PLANE BARRIER//)
      WRITE(6,103) X1,Y1,Z1,Y2,Z2,F
  103 FORMAT(1H0, 3HX= ,F6.3,2X,3HY= ,F6.3,2X,3HZ= ,F6.3,2X,4HY2= ,F6.3,
     12X,4HZ2= , F7.3,2X,3HF= ,E12.5/)
      WRITE(6,104)
  104 FORMAT(1H0,4X,5HTHETA,14X,1HV,12X,4HTIME,11X,4H SPL,12X,3HVCP/)
   10 CONTINUE
      R=SQRT((Z2-Z1)**2+(X2-X1)**2+(Y2-Y1)**2)
      C1=FK*R
      C2=CMPLX(0.,C1)
      VOP=CEXP(C2)/R
      VOPR=REAL(VOP)
      VOPI=AIMAG(VOP)
      VOPMAG=SQRT(VOPR**2+VOPI**2)
      VSUMX=CMPLX(0.0,0.0)
      VRLD=CMPLX(0.0,0.0)
      VSDR=CMPLX(0.0,0.0)
      N1P1=N1+1
      N2P1=N2+1
      Z12=Z1*Z1
      Z22=Z2*Z2
      AX=1.
      DO 2 I=1,N1P1
      XI=(I-1.)*DX
      XI2=XI*XI
      AY=1.
      VSUMY=CMPLX(0.0,0.0)
```


Figure 11.- Listing of computer program used to determine the acoustic shielding of a point source by a rectangular flat plate.

```
      DO 1 J=1,N2P1
      YJ=(J-1.)*DX
      YJ2=YJ*YJ
      RS=SQRT((X1-XI)**2+(Y1-YJ)**2+Z12)
      RL=SQRT((X2-XI)**2+(Y2-YJ)**2+Z22)
      C3=FK*RS
      C4=CMPLX(0.,C3)
      RLI=1./RL
      RSI=1./RS
      VOS=CEXP(C4)/RS
      C5=CMPLX(0.,FK)
      DVOS=VOS*(C5-RSI)*(Z1*RSI)
      C6=FK*RL
      C7=CMPLX(0.,C6)
      RLE=CEXP(C7)*RLI
      DRLE=RLE*(C5-RLI)*(Z2*RLI)
      VSUMY=VSUMY+(RLE*DVOS-VOS*DRLE)*AY
      IF(AY.LT.3.0) GO TO 3
      AY=2.
      IF(J.EQ.N2) AY=1.
      GO TO 1
3     AY=4.
    1 CONTINUE
      VSUMX=VSUMX+VSUMY*AX
      IF(AX.LT.3.) GO TO 5
      AX=2.
      IF(I.EQ.N1) AX=1.
      GO TO 2
5     AX=4.
2     CONTINUE
      V=(VSUMX*DSK)/PI4+VOP
      VR=REAL(V)
      VI=AIMAG(V)
      VMAG=SQRT(VR**2+VI**2)
      T2=ATAN((X2-X1)/(Z1-Z2))*RAD
      DV=VMAG/VOPMAG
      DSPL=20.*ALOG10(DV)
      CALL SECOND(TIME)
      WRITE(6,101) T2,VMAG,TIME,DSPL,VOP
  101 FORMAT(8E14.4)
      THET=T2+DELT
      X2=(Z1-Z2)*TAN(THET/RAD)+X1
      IF(T2.LT.TMAX) GO TO 10
6     CONTINUE
      STOP
      END
```


Figure 11.- Concluded.

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No
TM-73,261		
4. Title and Subtitle ACCURACY OF THE KIRCHOFF FORMULA IN DETERMINING ACOUSTIC SHIELDING WITH THE USE OF A FLAT PLATE		5. Report Date 6. Performing Organization Code
7. Author(s) Ralph E. Gabrielsen and Joseph E. Davis*		8. Performing Organization Report No. A-7111
9. Performing Organization Name and Address Ames Research Center, NASA and Ames Directorate, USAAMRDL Ames Research Center, Moffett Field, Calif. 94035		10. Work Unit No. 505-10-41 11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration, Washington, D.C. 20546 and U.S. Army Air Mobility R&D Laboratory, Moffett Field, Calif. 94035		13. Type of Report and Period Covered Technical Memorandum 14. Sponsoring Agency Code
15. Supplementary Notes *Contractor		

16. Abstract

The Kirchoff formula is used to determine the acoustic shielding of a point source by a rectangular plate for frequencies ranging from 100 Hz. to 20 kHz. Comparisons are made with experimental measurement. The agreement is excellent at frequencies above 4 kHz. and reasonable at frequencies below 4 kHz.

17. Key Words (Suggested by Author(s))	18. Distribution Statement		
Acoustics	Unlimited - Unclassified STAR Category - 71		
19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	20	$3.25

*For sale by the National Technical Information Service, Springfield, Virginia 22161