

77-FM-37

JSC-12960

Euler Angles, Quaternions, and Transformation Matrices

(NASA-TM-74839) SHUTTLE PROGRAM. EULER
ANGLES, QUATERNIONS, AND TRANSFORMATION
MATRICES WORKING RELATIONSHIPS (NASA) 42 p
HC A03/MF A01 CSCI 22A

N77-31234

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Working Relationships

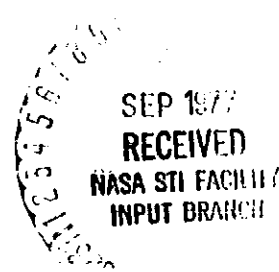
Mission Planning and Analysis Division

July 1977

NASA

National Aeronautics and
Space Administration

Lyndon B. Johnson Space Center
Houston, Texas



77-FM-37

JSC-12960

SHUTTLE PROGRAM


EULER ANGLES, QUATERNIONS, AND TRANSFORMATION MATRICES -

WORKING RELATIONSHIPS


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July 1977

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EULER ANGLES, QUATERNIONS, AND TRANSFORMATION MATRICES -
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By D. M. Henderson
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1.0 INTRODUCTION

Due to the extensive use of the quaternion in the onboard Space Shuttle Computer System, considerable analysis is being performed using relationships between the quaternion, the transformation matrix, and the Euler angles. This Internal Note offers a brief mathematical development of the relationships between the Euler angles and the transformation matrix, the quaternion and the transformation matrix, and the Euler angles and the quaternion. The analysis and equations presented here apply directly to current Space Shuttle problems. Appendix A presents the twelve three-axis Euler transformation matrices as functions of the Euler angles, the equations for the quaternion as a function of the Euler angles, and the Euler angles as a function of the transformation matrix elements.

The equations of Appendix A are a valuable reference in Shuttle analysis work and this Internal Note is the only known document where each of the twelve Euler angle to quaternion relationships are given. Appendix B presents a group of utility subroutines to accomplish the Euler-matrix, quaternion-matrix, and Euler-quaternion relationships of Appendix A.

2.0 DISCUSSION

2.1 Euler Angle Transformation Matrices

The following analysis and utility subroutines are offered to simplify computer programs when working with coordinate transformation matrices and their relationships with the Euler Angles and the Quaternions. The coordinate transformation matrices discussed here are defined using the following figure,

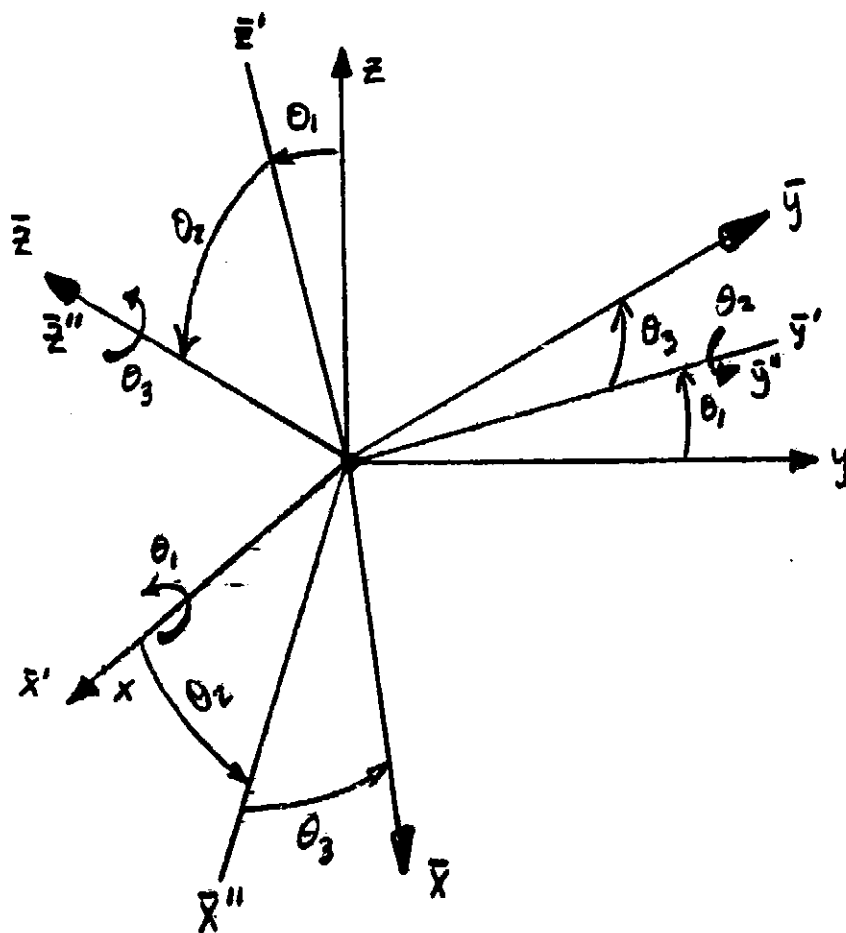


Figure 1.- Coordinate system and Euler angles.

The transformation matrix M, is defined to transform vectors in the \bar{x} - system $(\bar{x}, \bar{y}, \bar{z})$ into the original x-system (x, y, z) and is given by the equation,

$$x = M\bar{x}$$

where (1)

$$x = (x, y, z) \text{ and } \bar{x} = (\bar{x}, \bar{y}, \bar{z}).$$

Using the right-hand rule for positive rotations, the M matrix in (1) above is constructed by the following analysis. The first rotation in Figure 1 above is about the x-axis by the amount θ_1 . The single rotation about the x-axis results in the following transformation,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} \bar{x}' \\ \bar{y}' \\ \bar{z}' \end{pmatrix} \quad (2)$$

or $x = X\bar{x}'$ in matrix form. Rotation about the \bar{y}' -axis by the amount θ_2 yields the intermediate transformation matrix:

$$\begin{pmatrix} \bar{x}' \\ \bar{y}' \\ \bar{z}' \end{pmatrix} = \begin{pmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ 0 & 1 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} \bar{x}'' \\ \bar{y}'' \\ \bar{z}'' \end{pmatrix} \quad (3)$$

or $\bar{x}' = Y\bar{x}''$ in matrix form. Finally rotation about the \bar{z}'' -axis by the amount θ_3 yields the intermediate transformation matrix,

$$\begin{pmatrix} \bar{x}'' \\ \bar{y}'' \\ \bar{z}'' \end{pmatrix} = \begin{pmatrix} \cos\theta_3 & -\sin\theta_3 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} \quad (4)$$

and in matrix form $\bar{x}'' = Z\bar{x}'$. Now using the three equations,

$$\begin{aligned} x &= X\bar{x}' \\ \bar{x}' &= Y\bar{x}'' \\ \bar{x}'' &= Z\bar{x} \end{aligned} \quad (5)$$

by substitution

$$x = (X Y Z) \bar{x}. \quad (6)$$

Then from equation 1,

$$M = (X Y Z) \quad (7)$$

Computation for the M matrix from the indicated matrix multiplication in equation (7) yields,

$$M = \begin{pmatrix} (\cos\theta_2 \cos\theta_3) & (-\cos\theta_2 \sin\theta_3) & (\sin\theta_2) \\ (\cos\theta_1 \sin\theta_3 + \sin\theta_1 \sin\theta_2 \cos\theta_3) & (\cos\theta_1 \cos\theta_3 - \sin\theta_1 \sin\theta_2 \sin\theta_3) & (-\sin\theta_1 \cos\theta_2) \\ (\sin\theta_1 \sin\theta_3 - \cos\theta_1 \sin\theta_2 \cos\theta_3) & (\sin\theta_1 \cos\theta_3 + \cos\theta_1 \sin\theta_2 \sin\theta_3) & (\cos\theta_1 \cos\theta_2) \end{pmatrix} \quad (8)$$

The matrix M in equation (8) is a function of:

- (1) The three Euler angles θ_1 , θ_2 and θ_3 and
- (2) The sequence of rotations used to generate the matrix.

By examination of equation (7) it is possible to show that there are twelve possible Euler rotational sequences. If the (X Y Z) notation in equation (7) represents a rotation about the X axis, then the Y axis and finally the Z axis, then the following per-

mutations of the rotational order represents the twelve possible Euler angle sets using three rotations,

$$\begin{array}{lll}
 X Y Z & Y X Z & Z X Y \\
 X Z Y & Y Z X & Z Y X \\
 X Y X & Y X Y & Z X Z \\
 X Z X & Y Z Y & Z Y Z
 \end{array} \tag{9}$$

Any repeated axis rotation such as XXY does not represent a three axis rotation but reduces to the two axis rotation XY. Hence the rotations described in (9) above represent all twelve possible sets of Euler angle defining sequences. Conversely then, for a given transformation matrix there are twelve Euler angle sets which can be extracted from the matrix. The Euler matrices corresponding to all possible rotational sequences of (9) above are presented in Appendix A.

The utility subroutines "EULMAT" generates the transformation matrix from a given Euler sequence and the Euler angles. The utility subroutine "MATEUL" extracts the Euler angles from a given Euler rotational sequence. The convention is established that the Euler angles occur in the same sequences as the axis rotations. Using this concept and functional notation, equation (7) could be expressed as

$$M = X Y Z = M(\theta_x, \theta_y, \theta_z) \tag{10}$$

and from (9)

$$M = X Z X = M(\theta_x, \theta_z, \theta'_x) \text{ etc.} \quad (11)$$

This convention is assumed in both subroutines and also used throughout this design note when Euler angles are used. A brief explanation of the use of these two utility subroutines is given in Appendix B.

It is interesting to note that a negative rotation in the single axis rotation matrices of equations (2), (3) and (4) will result in the formation of the transpose of the matrix. However the transpose of M in equation (7) is formed from reversing the order of multiplication and transposing the individual axis rotation equations, i.e.

$$M^T = (X Y Z)^T = (Y Z)^T X^T = Z^T Y^T X^T. \quad (12)$$

Hence the transposes of the matrices of (9) are easily formed by reversing the order of multiplication and transposing each single axis rotation equation. Using the notation in equations (10) and (11) above equation (12) could be written,

$$M^T(\theta_x, \theta_y, \theta_z) = M(-\theta_z, -\theta_y, -\theta_x). \quad (13)$$

It is recommended to avoid confusion that the forward transformation be computed and simply transposed to yield the reverse transformation matrix. All matrices of Appendix A are in the forward form, i.e. $X = M\bar{x}$ and formed from (9).

2.2 Transformation Matrices Using the Hamilton Quaternion

The transformation matrix of equation (1) can be written as a function of the Hamilton Quaternion;

$$\begin{aligned} q_1 &= \cos \omega/2 \\ q_2 &= \cos \alpha \sin \omega/2 \\ q_3 &= \cos \beta \sin \omega/2 \\ q_4 &= \cos \gamma \sin \omega/2, \end{aligned} \tag{14}$$

where ω is the rotation angle about the rotation axis with α , β , and γ direction angles with the x, y and z axes respectively. Notice also that $q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1$, since $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$. The rotation angle, ω , is assumed positive according to the right-hand rule of axis rotation.

The matrix M becomes

$$M = \begin{pmatrix} (q_1^2 + q_2^2 - q_3^2 - q_4^2) & 2(q_2q_3 - q_1q_4) & 2(q_2q_4 + q_1q_3) \\ 2(q_2q_3 + q_1q_4) & (q_1^2 - q_2^2 + q_3^2 - q_4^2) & 2(q_3q_4 - q_1q_2) \\ 2(q_2q_4 - q_1q_3) & 2(q_3q_4 + q_1q_2) & (q_1^2 - q_2^2 - q_3^2 + q_4^2) \end{pmatrix}. \tag{15}$$

For a more detailed discussion of the derivation of equation (15), see Reference 1. Using functional notation, equation (15) can be written,

$$M = M(q_1, q_2, q_3, q_4). \tag{16}$$

Unlike the Euler angle rotational sequences to describe the transformation matrix of equation (1), only two quaternions can be found from equation (15). The two quaternions are:

$$\begin{array}{rcl}
 q_1 & & -q_1 \\
 q_2 & & -q_2 \\
 q_3 & \text{and} & -q_3 \\
 q_4 & & -q_4 .
 \end{array} \tag{17}$$

These two quaternions represent a positive rotation about the rotation axis pointing in one direction and a positive rotation about the same line of rotation pointing in the opposite direction. Both quaternions of (17) satisfy equation (15).

The utility subroutine "QMAT" generates the transformation matrix from a given quaternion. The "QMAT" algorithm generates the matrix as given in equation (15) without duplicating any arithmetic operations. The subroutine "MATQ" extracts the positive quaternion, i.e., $q_1 > 0$, from the transformation matrix and normalizes the results to guarantee an orthogonal matrix. In order to avoid any discontinuity in extracting the quaternion from the transformation matrix, the procedure as described in Reference 2 is used.

Early works by Hamilton (Reference 3) presented the quaternion as having a scalar and a vector part, i.e.,

$$q_1 = S \quad \vec{V} = (q_2, q_3, q_4) \tag{18}$$

and equation (16) could be expressed as,

$$M = M(q_1, q_2, q_3, q_4) = M(S, \vec{V}). \tag{19}$$

For a given quaternion the following relationship is true (from (17) above),

$$M(S, \vec{V}) = M(-S, -\vec{V}). \quad (20)$$

The transpose of the transformation matrix is given by,

$$M^T(S, \vec{V}) = M(-S, \vec{V}) = M(S, -\vec{V}). \quad (21)$$

2.3 Euler Angle and Quaternion Relationships

By examination of equations (10) and (16) the equality,

$$M(X(\theta_1), Y(\theta_2), Z(\theta_3)) = M(\theta_1, \theta_2, \theta_3) = M(q_1, q_2, q_3, q_4) \quad (22)$$

can be written. Based on an equality for each element of the matrix the following nine equations must be true;

$$\begin{aligned} \cos\theta_2 \cos\theta_3 &= q_1^2 + q_2^2 - q_3^2 - q_4^2 \\ -\cos\theta_2 \sin\theta_3 &= 2(q_2q_3 - q_1q_4) \\ \sin\theta_2 &= 2(q_2q_4 + q_1q_3) \\ \cos\theta_1 \sin\theta_3 + \sin\theta_1 \sin\theta_2 \cos\theta_3 &= 2(q_2q_3 + q_1q_4) \\ \cos\theta_1 \cos\theta_3 - \sin\theta_1 \sin\theta_2 \sin\theta_3 &= q_1^2 - q_2^2 + q_3^2 - q_4^2 \\ -\sin\theta_1 \cos\theta_2 &= 2(q_3q_4 - q_1q_2) \\ \sin\theta_1 \sin\theta_3 - \cos\theta_1 \sin\theta_2 \cos\theta_3 &= 2(q_3q_4 - q_1q_3) \\ \sin\theta_1 \cos\theta_3 + \cos\theta_1 \sin\theta_2 \sin\theta_3 &= 2(q_3q_4 + q_1q_2) \\ \cos\theta_1 \cos\theta_2 &= q_1^2 - q_2^2 - q_3^2 + q_4^2. \end{aligned} \quad (23)$$

It is possible to solve for the values of the quaternion using the trigonometric half angle identities. For this Euler sequence, i.e. $X(\theta_1) Y(\theta_2) Z(\theta_3)$, the following quaternion results;

$$\begin{aligned}
q_1 &= -\sin^{\frac{1}{2}}\theta_1 \sin^{\frac{1}{2}}\theta_2 \sin^{\frac{1}{2}}\theta_3 + \cos^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_3 \\
q_2 &= +\sin^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_3 + \sin^{\frac{1}{2}}\theta_2 \sin^{\frac{1}{2}}\theta_3 \cos^{\frac{1}{2}}\theta_1 \\
q_3 &= -\sin^{\frac{1}{2}}\theta_1 \sin^{\frac{1}{2}}\theta_3 \cos^{\frac{1}{2}}\theta_2 + \sin^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_3 \\
q_4 &= +\sin^{\frac{1}{2}}\theta_1 \sin^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_3 + \sin^{\frac{1}{2}}\theta_3 \cos^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_2
\end{aligned}
\tag{24}$$

Appendix A gives the quaternion as a function of the Euler angles for each of the twelve Euler rotational sequence presented in Section 2.1. The special equations for the quaternion as functions of the Euler angles, like equations (24) above, are sometimes cumbersome to use, especially when multiple Euler sequences are utilized. Less coding, but perhaps more computer operations, are required by using the more general method of first generating the matrix M from the given Euler angle set and then simply extracting the quaternion from the matrix. This method is effected by first a call to "EULMAT" and then a call to "MATQ".

3.0 REFERENCES

1. Working Paper: MDTSCO, TM No. 1.4-MPB-304, E914-8A/B-003,
"Quaternions and Quaternion Transformations," David M.
Henderson, 23 June 1976.
2. Transmittal Memo: MDTSCO, 1.4-MPB-229, "Improving Computer Accuracy
in Extracting Quaternions," David M. Henderson, 9 March 1976.
3. Sir William Rowan Hamilton, LL.D, LL.D. MRIA, D.C.L. CANTAB.,
"Elements of Quaternions" 2 Volumes, Chelsea Publishing Company,
New York, N. Y., 3rd Edition 1969, Library of Congress 68-54711
#8284-0219-1.

APPENDIX A

RELATIONSHIPS FOR THE THREE-AXIS EULER ANGLE ROTATION SEQUENCES

The twelve Euler matrices for each rotation sequence are given based on the single axis rotation equations (2), (3) and (4). The Euler matrices transform vectors from the system that has been rotated into vectors in the stationary system. Also presented here are the equations for the quaternion as a function of the Euler angles and the Euler angles as a function of the matrix elements for each rotation sequence.

$$(1) M = M(X(\theta_1), Y(\theta_2), Z(\theta_3)) = XYZ$$

Axis Rotation Sequence: 1, 2, 3

$$M = \begin{bmatrix} \cos\theta_2 \cos\theta_3 & -\cos\theta_2 \sin\theta_3 & \sin\theta_2 \\ \sin\theta_1 \sin\theta_2 \cos\theta_3 + \cos\theta_1 \sin\theta_3 & -\sin\theta_1 \sin\theta_2 \sin\theta_3 + \cos\theta_1 \cos\theta_3 & -\sin\theta_1 \cos\theta_2 \\ -\cos\theta_1 \sin\theta_2 \cos\theta_3 + \sin\theta_1 \sin\theta_3 & \cos\theta_1 \sin\theta_2 \sin\theta_3 + \sin\theta_1 \cos\theta_3 & \cos\theta_1 \cos\theta_2 \end{bmatrix}$$

$$q_1 = -\sin^{1/2}\theta_1 \sin^{1/2}\theta_2 \sin^{1/2}\theta_3 + \cos^{1/2}\theta_1 \cos^{1/2}\theta_2 \cos^{1/2}\theta_3$$

$$q_2 = \sin^{1/2}\theta_1 \cos^{1/2}\theta_2 \cos^{1/2}\theta_3 + \sin^{1/2}\theta_2 \sin^{1/2}\theta_3 \cos^{1/2}\theta_1$$

$$q_3 = -\sin^{1/2}\theta_1 \sin^{1/2}\theta_3 \cos^{1/2}\theta_2 + \sin^{1/2}\theta_2 \cos^{1/2}\theta_1 \cos^{1/2}\theta_3$$

$$q_4 = \sin^{1/2}\theta_1 \sin^{1/2}\theta_2 \cos^{1/2}\theta_3 + \sin^{1/2}\theta_3 \cos^{1/2}\theta_1 \cos^{1/2}\theta_2$$

$$\theta_1 = \tan^{-1} \left(\frac{-m_{23}}{m_{33}} \right)$$

$$\theta_2 = \tan^{-1} \left(\frac{m_{13}}{\sqrt{1-m_{13}^2}} \right)$$

$$\theta_3 = \tan^{-1} \left(\frac{-m_{12}}{m_{11}} \right)$$

$$(2) M = M(X(\theta_1), Z(\theta_2), Y(\theta_3)) = XZY$$

Axis Rotation Sequence: 1, 3, 2

$$M = \begin{bmatrix} \cos\theta_2 \cos\theta_3 & -\sin\theta_2 & \cos\theta_2 \sin\theta_3 \\ \cos\theta_1 \sin\theta_2 \cos\theta_3 + \sin\theta_1 \sin\theta_3 & \cos\theta_1 \cos\theta_2 & \cos\theta_1 \sin\theta_2 \sin\theta_3 - \sin\theta_1 \cos\theta_3 \\ \sin\theta_1 \sin\theta_2 \cos\theta_3 - \cos\theta_1 \sin\theta_3 & \sin\theta_1 \cos\theta_2 & \sin\theta_1 \sin\theta_2 \sin\theta_3 + \cos\theta_1 \cos\theta_3 \end{bmatrix}$$

$$q_1 = +\sin\frac{1}{2}\theta_1 \sin\frac{1}{2}\theta_2 \sin\frac{1}{2}\theta_3 + \cos\frac{1}{2}\theta_1 \cos\frac{1}{2}\theta_2 \cos\frac{1}{2}\theta_3$$

$$q_2 = +\sin\frac{1}{2}\theta_1 \cos\frac{1}{2}\theta_2 \cos\frac{1}{2}\theta_3 - \sin\frac{1}{2}\theta_2 \sin\frac{1}{2}\theta_3 \cos\frac{1}{2}\theta_1$$

$$q_3 = -\sin\frac{1}{2}\theta_1 \sin\frac{1}{2}\theta_2 \cos\frac{1}{2}\theta_3 + \sin\frac{1}{2}\theta_3 \cos\frac{1}{2}\theta_1 \cos\frac{1}{2}\theta_2$$

$$q_4 = +\sin\frac{1}{2}\theta_1 \sin\frac{1}{2}\theta_3 \cos\frac{1}{2}\theta_2 + \sin\frac{1}{2}\theta_2 \cos\frac{1}{2}\theta_1 \cos\frac{1}{2}\theta_3$$

$$\theta_1 = \tan^{-1} \left(\frac{m_{32}}{m_{22}} \right)$$

$$\theta_2 = \tan^{-1} \left(\frac{-m_{12}}{\sqrt{1-m_{12}^2}} \right)$$

$$\theta_3 = \tan^{-1} \left(\frac{m_{13}}{m_{11}} \right)$$

Axis Rotation Sequence: 1, 2, 1

$$M = \begin{bmatrix} \cos\theta_2 & \sin\theta_2\sin\theta_3 & \sin\theta_2\cos\theta_3 \\ \sin\theta_1\sin\theta_2 & \cos\theta_1\cos\theta_3 & -\cos\theta_1\sin\theta_3 \\ -\cos\theta_1\sin\theta_2 & -\sin\theta_1\cos\theta_2\sin\theta_3 & -\sin\theta_1\cos\theta_2\cos\theta_3 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\sin\theta_3 & \cos\theta_1\cos\theta_2\cos\theta_3 \end{bmatrix}$$

$$q_1 = \cos\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 + \theta_3))$$

$$q_2 = \cos\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 + \theta_3))$$

$$q_3 = \sin\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 - \theta_3))$$

$$q_4 = \sin\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 - \theta_3))$$

$$\theta_1 = \tan^{-1} \left(\frac{m_{21}}{-m_{31}} \right)$$

$$\theta_2 = \tan^{-1} \left(\frac{\sqrt{1 - m_{11}^2}}{m_{11}} \right)$$

$$\theta_3 = \tan^{-1} \left(\frac{m_{12}}{m_{13}} \right)$$

$$(4) \quad M = M(X(\theta_1), Z(\theta_2), X(\theta_3)) = XZX$$

Axis Rotation Sequence: 1, 3, 1

$$M = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \cos\theta_3 & \sin\theta_2 \sin\theta_3 \\ \cos\theta_1 \sin\theta_2 & \cos\theta_1 \cos\theta_2 \cos\theta_3 & -\cos\theta_1 \cos\theta_2 \sin\theta_3 \\ \sin\theta_1 \sin\theta_2 & \sin\theta_1 \cos\theta_2 \cos\theta_3 & -\sin\theta_1 \cos\theta_2 \sin\theta_3 \\ & +\cos\theta_1 \sin\theta_3 & +\cos\theta_1 \cos\theta_3 \end{bmatrix}$$

$$q_1 = \cos^{\frac{1}{2}}\theta_2 \cos(\frac{1}{2}(\theta_1 + \theta_3))$$

$$q_2 = \cos^{\frac{1}{2}}\theta_2 \sin(\frac{1}{2}(\theta_1 + \theta_3))$$

$$q_3 = -\sin^{\frac{1}{2}}\theta_2 \sin(\frac{1}{2}(\theta_1 - \theta_3))$$

$$q_4 = \sin^{\frac{1}{2}}\theta_2 \cos(\frac{1}{2}(\theta_1 - \theta_3))$$

$$\theta_1 = \tan^{-1} \left(\frac{m_{31}}{m_{21}} \right)$$

$$\theta_2 = \tan^{-1} \left(\frac{\sqrt{1 - m_{11}^2}}{m_{11}} \right)$$

$$\theta_3 = \tan^{-1} \left(\frac{m_{13}}{-m_{12}} \right)$$

$$(5) M = M(Y(\theta_1), X(\theta_2), Z(\theta_3)) = YXZ$$

Axis Rotation Sequence: 2, 1, 3

$$M = \begin{bmatrix} \sin\theta_1 \sin\theta_2 \sin\theta_3 & \sin\theta_1 \sin\theta_2 \cos\theta_3 & \sin\theta_1 \cos\theta_2 \\ \cos\theta_1 \cos\theta_3 & -\cos\theta_1 \sin\theta_3 & \\ \cos\theta_2 \sin\theta_3 & \cos\theta_2 \cos\theta_3 & -\sin\theta_2 \\ \cos\theta_1 \sin\theta_2 \sin\theta_3 & \cos\theta_1 \sin\theta_2 \cos\theta_3 & \cos\theta_1 \cos\theta_2 \\ -\sin\theta_1 \cos\theta_3 & +\sin\theta_1 \sin\theta_3 & \end{bmatrix}$$

$$q_1 = \sin^{1/2}\theta_1 \sin^{1/2}\theta_2 \sin^{1/2}\theta_3 + \cos^{1/2}\theta_1 \cos^{1/2}\theta_2 \cos^{1/2}\theta_3$$

$$q_2 = \sin^{1/2}\theta_1 \sin^{1/2}\theta_3 \cos^{1/2}\theta_2 + \sin^{1/2}\theta_2 \cos^{1/2}\theta_1 \cos^{1/2}\theta_3$$

$$q_3 = \sin^{1/2}\theta_1 \cos^{1/2}\theta_2 \cos^{1/2}\theta_3 - \sin^{1/2}\theta_2 \sin^{1/2}\theta_3 \cos^{1/2}\theta_1$$

$$q_4 = -\sin^{1/2}\theta_1 \sin^{1/2}\theta_2 \cos^{1/2}\theta_3 + \sin^{1/2}\theta_3 \cos^{1/2}\theta_1 \cos^{1/2}\theta_2$$

$$\theta_1 = \tan^{-1} \left(\frac{m_{31}}{m_{33}} \right)$$

$$\theta_2 = \tan^{-1} \left(\sqrt{\frac{-m_{23}}{1-m_{23}^2}} \right)$$

$$\theta_3 = \tan^{-1} \left(\frac{m_{21}}{m_{22}} \right)$$

$$(6) M = M(Y(\theta_1), Z(\theta_2), X(\theta_3)) = YZX$$

Axis Rotation Sequence: 2, 3, 1

$$M = \begin{bmatrix} \cos\theta_1 \cos\theta_2 & -\cos\theta_1 \sin\theta_2 \cos\theta_3 & \cos\theta_1 \sin\theta_2 \sin\theta_3 \\ \sin\theta_2 & \cos\theta_2 \cos\theta_3 & -\cos\theta_2 \sin\theta_3 \\ -\sin\theta_1 \cos\theta_2 & \sin\theta_1 \sin\theta_2 \cos\theta_3 & -\sin\theta_1 \sin\theta_2 \sin\theta_3 \\ & + \cos\theta_1 \sin\theta_3 & + \cos\theta_1 \cos\theta_3 \end{bmatrix}$$

$$q_1 = -\sin^{1/2}\theta_1 \sin^{1/2}\theta_2 \sin^{1/2}\theta_3 + \cos^{1/2}\theta_1 \cos^{1/2}\theta_2 \cos^{1/2}\theta_3$$

$$q_2 = +\sin^{1/2}\theta_1 \sin^{1/2}\theta_2 \cos^{1/2}\theta_3 + \sin^{1/2}\theta_3 \cos^{1/2}\theta_1 \cos^{1/2}\theta_2$$

$$q_3 = +\sin^{1/2}\theta_1 \cos^{1/2}\theta_2 \cos^{1/2}\theta_3 + \sin^{1/2}\theta_2 \sin^{1/2}\theta_3 \cos^{1/2}\theta_1$$

$$q_4 = -\sin^{1/2}\theta_1 \sin^{1/2}\theta_3 \cos^{1/2}\theta_2 + \sin^{1/2}\theta_2 \cos^{1/2}\theta_1 \cos^{1/2}\theta_3$$

$$\theta_1 = \tan^{-1} \left(\frac{-m_{31}}{m_{11}} \right)$$

$$\theta_2 = \tan^{-1} \left(\frac{m_{21}}{\sqrt{1-m_{21}^2}} \right)$$

$$\theta_3 = \tan^{-1} \left(\frac{-m_{23}}{m_{22}} \right)$$

$$(7) M = M(Y(\theta_1), X(\theta_2), Y(\theta_3)) = YXY$$

Axis Rotation Sequence: 2, 1, 2

$$M = \begin{bmatrix} -\sin\theta_1 \cos\theta_2 \sin\theta_3 & \sin\theta_1 \sin\theta_2 & \sin\theta_1 \cos\theta_2 \cos\theta_3 \\ +\cos\theta_1 \cos\theta_3 & & +\cos\theta_1 \sin\theta_3 \\ \sin\theta_2 \sin\theta_3 & \cos\theta_2 & -\sin\theta_2 \cos\theta_3 \\ -\cos\theta_1 \cos\theta_2 \sin\theta_3 & \cos\theta_1 \sin\theta_2 & \cos\theta_1 \cos\theta_2 \cos\theta_3 \\ -\sin\theta_1 \cos\theta_3 & & -\sin\theta_1 \sin\theta_3 \end{bmatrix}$$

$$q_1 = +\cos\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 + \theta_3))$$

$$q_2 = +\sin\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 - \theta_3))$$

$$q_3 = +\cos\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 + \theta_3))$$

$$q_4 = -\sin\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 - \theta_3))$$

$$\theta_1 = \tan^{-1} \left(\frac{m_{12}}{m_{32}} \right)$$

$$\theta_2 = \tan^{-1} \left(\frac{\sqrt{1-m_{22}^2}}{m_{22}} \right)$$

$$\theta_3 = \tan^{-1} \left(\frac{m_{21}}{-m_{23}} \right)$$

$$(8) \quad M = M(Y(\theta_1), Z(\theta_2), Y(\theta_3)) = YZY$$

Axis Rotation Sequence: 2, 3, 2

$$M = \begin{bmatrix} \cos\theta_1 \cos\theta_2 \cos\theta_3 & -\cos\theta_1 \sin\theta_2 & \cos\theta_1 \cos\theta_2 \sin\theta_3 \\ -\sin\theta_1 \sin\theta_3 & & +\sin\theta_1 \cos\theta_3 \\ \sin\theta_2 \cos\theta_3 & \cos\theta_2 & \sin\theta_2 \sin\theta_3 \\ -\sin\theta_1 \cos\theta_2 \cos\theta_3 & \sin\theta_1 \sin\theta_2 & -\sin\theta_1 \cos\theta_2 \sin\theta_3 \\ -\cos\theta_1 \sin\theta_3 & & +\cos\theta_1 \cos\theta_3 \end{bmatrix}$$

$$q_1 = +\cos\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 + \theta_3))$$

$$q_2 = +\sin\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 - \theta_3))$$

$$q_3 = +\cos\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 + \theta_3))$$

$$q_4 = +\sin\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 - \theta_3))$$

$$\theta_1 = \tan^{-1} \left(\frac{m_{32}}{-m_{12}} \right)$$

$$\theta_2 = \tan^{-1} \left(\frac{\sqrt{1-m_{22}^2}}{m_{22}} \right)$$

$$\theta_3 = \tan^{-1} \left(\frac{m_{23}}{m_{21}} \right)$$

$$(9) M = M(Z(\theta_1), X(\theta_2), Y(\theta_3)) = ZXY$$

Axis Rotation Sequence: 3, 1, 2

$$M = \begin{bmatrix} -\sin\theta_1 \sin\theta_2 \sin\theta_3 & -\sin\theta_1 \cos\theta_2 & \sin\theta_1 \sin\theta_2 \cos\theta_3 \\ \cos\theta_1 \cos\theta_3 & & +\cos\theta_1 \sin\theta_3 \\ \cos\theta_1 \sin\theta_2 \sin\theta_3 & \cos\theta_1 \cos\theta_2 & -\cos\theta_1 \sin\theta_2 \cos\theta_3 \\ +\sin\theta_1 \cos\theta_3 & & +\sin\theta_1 \sin\theta_3 \\ -\cos\theta_2 \sin\theta_3 & \sin\theta_2 & \cos\theta_2 \cos\theta_3 \end{bmatrix}$$

$$q_1 = -\sin\frac{1}{2}\theta_1 \sin\frac{1}{2}\theta_2 \sin\frac{1}{2}\theta_3 + \cos\frac{1}{2}\theta_1 \cos\frac{1}{2}\theta_2 \cos\frac{1}{2}\theta_3$$

$$q_2 = -\sin\frac{1}{2}\theta_1 \sin\frac{1}{2}\theta_3 \cos\frac{1}{2}\theta_2 + \sin\frac{1}{2}\theta_2 \cos\frac{1}{2}\theta_1 \cos\frac{1}{2}\theta_3$$

$$q_3 = +\sin\frac{1}{2}\theta_1 \sin\frac{1}{2}\theta_2 \cos\frac{1}{2}\theta_3 + \sin\frac{1}{2}\theta_3 \cos\frac{1}{2}\theta_1 \cos\frac{1}{2}\theta_2$$

$$q_4 = +\sin\frac{1}{2}\theta_1 \cos\frac{1}{2}\theta_2 \cos\frac{1}{2}\theta_3 + \sin\frac{1}{2}\theta_2 \sin\frac{1}{2}\theta_3 \cos\frac{1}{2}\theta_1$$

$$\theta_1 = \tan^{-1} \left(\frac{-m_{12}}{m_{22}} \right)$$

$$\theta_2 = \tan^{-1} \left(\frac{m_{32}}{\sqrt{1-m_{32}^2}} \right)$$

$$\theta_3 = \tan^{-1} \left(\frac{-m_{31}}{m_{33}} \right)$$

$$(10) M = M(Z(\theta_1), Y(\theta_2), X(\theta_3)) = ZYX$$

Axis Rotation Sequence: 3, 2, 1

$$M = \begin{bmatrix} \cos\theta_1 \cos\theta_2 & \cos\theta_1 \sin\theta_2 \sin\theta_3 & \cos\theta_1 \sin\theta_2 \cos\theta_3 \\ \sin\theta_1 \cos\theta_2 & \sin\theta_1 \sin\theta_2 \sin\theta_3 & \sin\theta_1 \sin\theta_2 \cos\theta_3 \\ -\sin\theta_2 & \cos\theta_2 \sin\theta_3 & \cos\theta_2 \cos\theta_3 \end{bmatrix}$$

$$q_1 = +\sin^{\frac{1}{2}}\theta_1 \sin^{\frac{1}{2}}\theta_2 \sin^{\frac{1}{2}}\theta_3 + \cos^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_3$$

$$q_2 = -\sin^{\frac{1}{2}}\theta_1 \sin^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_3 + \sin^{\frac{1}{2}}\theta_3 \cos^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_2$$

$$q_3 = +\sin^{\frac{1}{2}}\theta_1 \sin^{\frac{1}{2}}\theta_3 \cos^{\frac{1}{2}}\theta_2 + \sin^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_3$$

$$q_4 = +\sin^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_3 - \sin^{\frac{1}{2}}\theta_2 \sin^{\frac{1}{2}}\theta_3 \cos^{\frac{1}{2}}\theta_1$$

$$\theta_1 = \tan^{-1} \left(\frac{m_{21}}{m_{11}} \right)$$

$$\theta_2 = \tan^{-1} \left(\frac{-m_{31}}{\sqrt{1-m_{31}^2}} \right)$$

$$\theta_3 = \tan^{-1} \left(\frac{m_{32}}{m_{33}} \right)$$

$$(11) M = M(Z(\theta_1), X(\theta_2), Z(\theta_3)) = ZXZ$$

Axis Rotation Sequence: 3, 1, 3

$$M = \begin{bmatrix} -\sin\theta_1 \cos\theta_2 \sin\theta_3 & -\sin\theta_1 \cos\theta_2 \cos\theta_3 & \sin\theta_1 \sin\theta_2 \\ +\cos\theta_1 \cos\theta_3 & -\cos\theta_1 \cos\theta_3 & -\cos\theta_1 \sin\theta_2 \\ \cos\theta_1 \cos\theta_2 \sin\theta_3 & \cos\theta_1 \cos\theta_2 \cos\theta_3 & \cos\theta_1 \sin\theta_2 \\ +\sin\theta_1 \cos\theta_3 & -\sin\theta_1 \sin\theta_3 & \cos\theta_2 \\ \sin\theta_2 \sin\theta_3 & \sin\theta_2 \cos\theta_3 & \cos\theta_2 \end{bmatrix}$$

$$q_1 = +\cos\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 + \theta_3))$$

$$q_2 = +\sin\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 - \theta_3))$$

$$q_3 = +\sin\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 - \theta_3))$$

$$q_4 = +\cos\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 + \theta_3))$$

$$\theta_1 = \tan^{-1} \left(\frac{m_{13}}{-m_{23}} \right)$$

$$\theta_2 = \tan^{-1} \left(\frac{\sqrt{1-m_{33}^2}}{m_{33}} \right)$$

$$\theta_3 = \tan^{-1} \left(\frac{m_{31}}{m_{32}} \right)$$

$$12) M = M(Z(\theta_1), Y(\theta_2), Z(\theta_3)) = ZYZ$$

Axis Rotation Sequence: 3, 2, 3

$$M = \begin{bmatrix} \cos\theta_1 \cos\theta_2 \cos\theta_3 & -\cos\theta_1 \cos\theta_2 \sin\theta_3 & \cos\theta_1 \sin\theta_2 \\ -\sin\theta_1 \sin\theta_3 & -\sin\theta_1 \cos\theta_3 & \\ \sin\theta_1 \cos\theta_2 \cos\theta_3 & -\sin\theta_1 \cos\theta_2 \sin\theta_3 & \sin\theta_1 \sin\theta_2 \\ +\cos\theta_1 \sin\theta_3 & +\cos\theta_1 \cos\theta_3 & \\ -\sin\theta_2 \cos\theta_3 & \sin\theta_2 \sin\theta_3 & \cos\theta_2 \end{bmatrix}$$

$$q_1 = +\cos\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 + \theta_3))$$

$$q_2 = -\sin\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 - \theta_3))$$

$$q_3 = +\sin\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 - \theta_3))$$

$$q_4 = +\cos\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 + \theta_3))$$

$$\theta_1 = \tan^{-1} \left(\frac{m_{23}}{m_{13}} \right)$$

$$\theta_2 = \tan^{-1} \left(\frac{\sqrt{1-m_{33}^2}}{m_{33}} \right)$$

$$\theta_3 = \tan^{-1} \left(\frac{m_{32}}{-m_{31}} \right)$$

APPENDIX B
COMPUTER SUBROUTINES FOR THE RELATIONSHIPS

The following subroutines with a brief description of their use are presented in this appendix.

- (1) "EULMAT" - Generates the transformation matrix from a given set of Euler angles and an axis rotation sequence.
- (2) "MATEUL" - Extracts the Euler angles from the given transformation matrix and an axis rotation sequence.
- (3) "QMAT" - Generates the transformation matrix from a given quaternion.
- (4) "MATQ" - Extracts the quaternion from a given transformation matrix.
- (5) "YPRQ" - Generates the quaternion directly from the yaw-pitch-roll Euler angles.
- (6) "POSNOR" - Computes the positive-normalized quaternion from the given quaternion.

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NAME: EULMAT

PURPOSE: Generates a 3 x 3 transformation matrix from a given sequence and Euler angle set.

INPUT: ISEQ - Rotation Sequence (Integer Array (3); i.e., 1, 2, 3)
EUL - Euler Angles in radians, in "ISEQ" Order; ARRAY (3)

OUTPUT: A - The 3 x 3 transformation matrix

ALGORITHM REFERENCE: Appendix A; Euler Sequences (1) thru (12).

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EULER ANGLES TO THE TRANSFORMATION MATRIX

FOR IS FULMAT,EULMAT
FOR SCE3-02/19/77-06:24:23 (,0)

SUBROUTINE LULMAT ENTRY POINT 002237

STORAGE USED: CODE(1) 002230; DATA(0) 002124; BLANK COMMON(2)

EXTERNAL REFERENCES (BLOCK, NAME)

0023 SIN
0024 COS
0025 MEXP35

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0001	0001	1001	0001	00013	1000	0001	00010	
0001	0001	142	0001	00014	1000	0001	00014	
0001	0001	1620	0001	00014	1000	0001	00014	
0001	0001	30L	0001	00013	P	0001	00013	
0001	0001	INJ01	0001	00014	J	0001	00014	
0000	R	002047	SINA	0001	00053	TEMP	0000	00053

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SUBROUTINE EULMAT(ISEC,EUL,A)
DIMENSION ISEC(3),EUL(3),A(3,3)
DIMENSION X(3,3,3),R(3,3)
DO 100 K=1,3
DO 10 I=1,3
DO 5 J=1,3
X(I,J,K)=0
IF(I.EQ.J) X(I,J,K)=1.0
CONTINUE
CONTINUE
IF(ISEC(K).LE.0) GO TO 100
SINA=SIN(FUL(K))
COSA=COS(FUL(K))
IF(ISEC(K).EQ.2) GO TO 20
IF(ISEC(K).EQ.3) GO TO 30
X(1,1,K)=COSA
X(2,2,K)=-SINA
X(3,2,K)=SINA
X(3,1,K)=COSA
GO TO 100
20. X(1,1,K)=COSA
X(1,2,K)=-SINA
X(3,1,K)=-SINA
X(3,2,K)=COSA
GO TO 100
30. X(1,1,K)=COSA
X(1,2,K)=-SINA
X(3,1,K)=SINA
X(3,2,K)=COSA

```

EULER ANGLES TO THE TRANSFORMATION MATRIX

(CONTINUED)

00153	1*	100	CONTINUE
00155	2*	100	DO 400 L=1,2
00160	3*		M=3-L
00161	4*	200	DO 300 I=1,3
00164	5*	200	DO 300 J=1,3
00167	6*		TEMP=0
00170	7*	200	DO 250 K=1,2
00173	8*		IF(L.EQ.1) HOLD=X(K,J,3)
00175	9*		IF(L.EQ.2) HOLD=X(K,J,1)
00177	0*		IF(ABS(HOLD).LT.1.E-10) GO TO 250
00201	4*		IF(ABS(X(I,K,M)).LT.1.E-10) GO TO 250
00203	4*		TEMP=TEMP+X(I,K,M)*HOLD
00204	4*	250	CONTINUE
00206	4*		IF(L.EQ.1) A(I,J)=TEMP
00210	4*		IF(L.EQ.2) A(I,J)=TEMP
00213	4*	300	CONTINUE
00215	4*	400	CONTINUE
00217	4*		RETURN
00220	4*		END

END OF COMPILATION:

NO DIAGNOSTICS.

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NAME: MATEUL

PURPOSE: Extracts the Euler angles from the given transformation matrix and the required Euler rotational sequence.

INPUT: ISEQ - Rotation sequence, (Integer Array (3), i.e., 1,2,3.)
A - The 3 x 3 transformation

OUTPUT: EUL - The Euler angles, in "ISEQ" order; ARRAY(3).

ALGORITHM REFERENCE: Appendix A; Euler angles as a function of the matrix elements, sequences (1) thru (12).

TRANSFORMATION MATRIX TO THE EULER ANGLES

T.

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JFOR, IS MATEUL, MATEUL
FOR S2E3-02/19/77-06:24:25 (L0)

SUBROUTINE MATEUL ENTRY POINT 070335

STORAGE USED: CODE(1) 000353; DATA(5) 00052; BLANK COMMON(2)

EXTERNAL REFERENCES (BLOCK, NAME)

003	SORT
004	ATAN2
005	NEPR3

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

001	000054	10L	001	000056	15L	001	000057
001	000110	30L	001	000220	40L	001	000227
001	000251	50L	001	000297	60L	001	000307
005	000304	BSIGN	005	000305	CSIGN	005	R 000301
005	000305	FNUM	005	000306	I	005	I 000300
005	000306	J	005	000307	JJ	005	I 000303

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SUBROUTINE MATEUL(ISE0,A,EUL)
DIMENSION A(3,3),EUL(3)
DIMENSION ISE0(3)
J=ISE0(1)
K=ISE0(2)
L=ISE0(3)
IECK=1
IF(IJ.EQ.1) IECK=4.95
BSIGN=1.0
CSIGN=1.0
IF(IJ.EQ.1) GO TO 10
IF(IJ.EQ.2) GO TO 20
IF(IJ.NE.1) GO TO 5
BSIGN=-1.0
IF(IJ.EQ.1) L=2
GO TO 3
CSIGN=-1.0
IF(IECK.NE.0) L=1
GO TO 3
IF(IJ.NE.2) GO TO 15
BSIGN=-1.0
IF(IECK.NE.0) L=3
GO TO 3
IF(IJ.NE.3) GO TO 25
BSIGN=-1.0
    
```

TRANSFORMATION MATRIX TO THE EULER ANGLES
(CONTINUED)

00150	29*	IF (IEOK.NE.0) L=1
00152	30*	GO TO 30
00153	31*	25 CSIGN=-1.0
00154	32*	IF (IEOK.NE.0) L=3
00156	33*	30 JO L=1,3
00161	34*	FNSGN=1.0
00162	35*	FDSGN=1.0
00163	36*	IF (N.EQ.2) GO TO 70
00165	37*	IF (N.EQ.1) GO TO 50
00167	38*	IF (IEOK.NE.0) GO TO 40
00171	39*	FNSGN=BSIGN
00172	40*	JJ=1
00173	41*	GO TO 45
00174	42*	40 JJ=L
00175	43*	IF (BSIGN.GT.0.0) FDSGNE=-1.0
00177	44*	45 FNUM=FNSGN*A(I,J)
00200	45*	FDEN=FDSGN*A(I,JJ)
00201	46*	GO TO 90
00202	47*	50 IF (IEOK.NE.0) GO TO 55
00204	48*	FNSGN=BSIGN
00205	49*	II=K
00206	50*	JJ=K
00207	51*	GO TO 60
00210	52*	55 FDSGN=BSIGN
00211	53*	II=L
00212	54*	JJ=I
00213	55*	60 FNUM=FNSGN*A(I,J,K)
00214	56*	FDEN=FDSGN*A(I,JJ)
00215	57*	GO TO 90
00216	58*	70 IF (IEOK.NE.0) GO TO 80
00220	59*	FNUM=CSIGN*A(I,K)
00221	60*	FDEN=SQRT(1.0-A(I,K)**2)
00222	61*	GO TO 90
00223	62*	80 FNUM=SQRT(1.0-A(I,I)**2)
00224	63*	FDEN=A(I,I)
00225	64*	90 CUL(N)=ATAN2(FNUM,FDEN)
00226	65*	100 CONTINUE
00231	66*	RETURN
	67*	END

END OF COMPILATION:

NO DIAGNOSTICS.

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NAME: QMAT

PURPOSE: Generates the transformation matrix from the given quaternion.

INPUT: Q - The quaternion; ARRAY(4).

OUTPUT: A - The 3 x 3 transformation matrix

ALGORITHM REFERENCE: Equation (15) from Section 2.2.

QUATERNION TO THE TRANSFORMATION MATRIX

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FOR IS QMAT, QMAT
FOR SPEC-02/19/77-06:24:19 (,0)

SUBROUTINE QMAT ENTRY POINT 000077

STORAGE USED: CODE(11) 000103; DATA(0) 000000; BLANK COMMON(2)

EXTERNAL REFERENCES (BLOCK, NAME)

0003 NERR31

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0000 00007 INJPA 0000 P 00000 P2 0000 P 000001
0000 R 00004 P5 0000 R 00000 TEMP

```
0101 1* SUBROUTINE QMAT(C,A)
0103 2* DIMENSION Q(4),A(3,3)
0104 3* P2=Q(2)+Q(2)
0105 4* P3=Q(3)+Q(3)
0106 5* Q4=Q(4)+Q(4)
0107 6* P5=P2*Q(2)
0108 7* P6=P4+Q(4)
0109 8* TEMP=1.0-P3*Q(3)
0110 9* A(1,1)=TEMP-P6
0111 10* A(2,2)=1.0-P5-P6
0112 11* A(3,3)=TEMP-P5
0113 12* P5=P2*Q(3)
0114 13* P5=P4*Q(1)
0115 14* A(1,2)=P5-P6
0116 15* A(2,1)=P5+P6
0117 16* P5=P2*Q(4)
0118 17* P6=P3*Q(1)
0119 18* A(1,3)=P5+P6
0120 19* A(3,1)=P5-P6
0121 20* P5=P3*Q(4)
0122 21* P5=P2*Q(1)
0123 22* A(2,3)=P5-P6
0124 23* A(3,2)=P5+P6
0125 24* RETURN
0132 25* END
```

END OF COMPILATION. NO DIAGNOSTICS.

NAME:

MATQ

PURPOSE:

Extracts the positive quaternion from the given transformation matrix.

INPUT:

A - The 3 x 3 transformation matrix

OUTPUT:

Q - The positive quaternion; ARRAY(4).

ALGORITHM REFERENCE:

See Reference 2.

TP
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TRANSFORMATION MATRIX TO THE QUATERNION

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FOR SI MATQ,MATQ
FOR SRE3-02/19/77-06:24:21 (C)

SUBROUTINE MATQ ENTRY POINT 00003

STORAGE USED: CODE(1) 000220; DATATO) 00050; BLANK COMMON(2)

EXTERNAL REFERENCES (BLOCK, NAME)

0002 SORT
0004 NERK29

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0001	000473	10L	0001	000052	1076	0001	000157
0001	000111	3EL	0001	000101	40L	0001	000116
0001	000018	INJPL	0001	000000	0	0000	R 000000

00101 1*
 00102 2*
 00103 3*
 00104 4*
 00105 5*
 00106 6*
 00107 7*
 00108 8*
 00109 9*
 00110 10*
 00111 11*
 00112 12*
 00113 13*
 00114 14*
 00115 15*
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 00138 38*
 00139 39*
 00140 40*
 00141 41*
 00142 42*
 00143 43*
 00144 44*
 00145 45*
 00146 46*
 00147 47*
 00148 48*
 00149 49*
 00150 50*
 00151 51*
 00152 52*
 00153 53*
 00154 54*
 00155 55*
 00156 56*
 00157 57*

```

SUBROUTINE MATQ(A,Q)
DIMENSION A(3,3),Q(4,16)
I=1
BIG=0.0
DO 40 J=1,4
  Q(I,J)=0.0
  IF(IJ.EQ.2) GO TO 10
  IF(IJ.EQ.3) GO TO 20
  IF(IJ.EQ.4) GO TO 30
  Q(I,J)=1.0
  TEMPE=A(1,1)+A(2,2)+A(3,3)+1.0
  T(IJ)=0.0
  GO TO 35
10 TEMPE=A(1,1)-A(2,2)-A(3,3)+1.0
  T(IJ)=A(3,2)-A(2,3)
  GO TO 35
20 TEMPE=-A(1,1)+A(2,2)-A(3,3)+1.0
  T(IJ)=A(1,3)-A(3,1)
  GO TO 35
30 TEMPE=-A(1,1)-A(2,2)+A(3,3)+1.0
  T(IJ)=A(2,1)-A(1,2)
35 IF(TEMP.LT.BIG) GO TO 40
  BIG=TEMP
  I=J
40 CONTINUE
  IF(IJ.EQ.1) GO TO 60
  I(1)=0.5*SQRT(IPI)
  IF(IJ.EQ.1) Q(11)=APR(C.25*T(11)/O(11))
  TEMPE=0.25/O(11)
  DO 50 J=2,4
    Q(IJ)=TEMP*T(IJ)
50 CONTINUE
60 RETURN
END

```

END OF COMPILATION: NO DIAGNOSTICS.

NAME: YPRQ

PURPOSE: Generates the quaternion directly from the yaw-pitch-roll Euler angles, i.e., a 3, 2, 1 Euler sequence.

INPUT: YPR - The yaw-pitch-roll Euler angles; ARRAY (3).

OUTPUT: QO - The positive quaternion, ARRAY (4).

ALGORITHM REFERENCE: Appendix A, the quaternion equations for Euler sequence (10), a 3, 2, 1 sequence.

NOTE: This subroutine calls "POSNOR" to take the normal of the positive quaternion.

YPRQ-PITCH-ROLL EULER ANGLES TO THE QUATERNION

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FOR SOE3-02/19/77-06:24:03 (,0)

SUBROUTINE YPRO ENTRY POINT DTG114

STORAGE USED: CODE(1) 000101; DATA(0) 000020; BLANK COMMON(2) 00

EXTERNAL REFERENCES (BLOCK, NAME)

0003 POSNR
0004 COS
0005 SIN
0006 NERR3

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0000 R 000011 CP 0000 R 000011 CR 0000 R 000007 C
0000 P 000004 HY 0000 000016 INOPS 0000 R 000000 C
0000 R 000012 SY

```
00101 1* SUBROUTINE YPRQ(YPR,Q0)
00103 2* DIMENSION YPR(3),Q(4),LO(4)
00104 3* HY=.50*YPR(1)
00105 4* HP=.50*YPR(2)
00106 5* HRE=.50*YPR(3)
00107 6* CY=COS(HY)
00108 7* CP=COS(HP)
00109 8* CR=COS(HR)
00110 9* SY=SIN(HY)
00111 10* SP=SIN(HP)
00112 11* SR=SIN(HR)
00113 12* Q(1)=CY*CP*CR+SY*SP*SR
00114 13* Q(2)=CY*CP*SR-SY*SP*CR
00115 14* Q(3)=CY*SP*CR+SY*CP*SR
00116 15* Q(4)=-CY*SP*SR+SY*CP*CR
00117 16* CALL POSNR(Q,Q0)
00118 17* RETURN
00119 18* END
```

END OF COMPILATION: NO DIAGNOSTICS.

NAME: POSNOR

PURPOSE: To output the positive and normalized quaternion
from the given quaternion.

INPUT: Q - The quaternion; ARRAY (4).

OUTPUT: QO - The positive-normalized quaternion;
ARRAY (4).

ALGORITHM REFERENCE:

1. If the sign of Q(1) is negative:

Set $QO(I) = -Q(I)$ for $I = 1, 2, 3, 4$.

2. Set $QO(I) = QO(I)/TEMP$

where $TEMP = \sqrt{QO_1^2 + QO_2^2 + QO_3^2 + QO_4^2}$

SELECTS THE POSITIVE QUATERNION AND NORMALIZES

FOR IS POSNOR, POSNOR
FOR SDE3-02/1977-06:24:14 (, 0)

SUBROUTINE POSNOR ENTRY POINT 00055

STORAGE USED: CODE(1) 00067; DATA(1) 00017; BLANK COMMON(2) 0

EXTERNAL REFERENCES (BLOCK, NAME)

0003 SQR
0004 NLPDS

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0001 00016 1110 0001 00036 1210 0000 1 00000
0000 R 00000 TEMP

```
00101 1* SUBROUTINE POSNOR(0,0)
00102 2* DIMENSION Q(4),Q0(4)
00103 3* TEMP=1.0
00104 4* IF(Q(1).LT.0.0) TEMP=-1.0
00105 5* SUM=0.0
00106 6* DO 10 I=1,4
00107 7* Q0(I)=TEMP*Q(I)
00108 8* SUM=SUM+Q0(I)*Q0(I)
00109 9*
00110 10* CONTINUE
00111 11* TEMP=1.0/SQRT(SUM)
00112 12* DO 100 I=1,4
00113 13* Q0(I)=TEMP*Q0(I)
00114 14* CONTINUE
00115 15* RETURN
00116 END
```

END OF COMPILATION: NO DIAGNOSTICS.

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