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V.R. Murthy

P.S. Barna, Principal Investigator

Technical Report

Prepared for the
National Aeronautics and Space Administration
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Hampton, Virginia

Under
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SUMMARY

The objective of this study is to determine the effect of an attached concentrated mass on the dynamics of helicopter rotor blades. The transmission matrix method is used to determine the natural vibrational characteristics (natural frequencies and mode shapes) of rotor blades. The problems treated are coupled flapwise bending, chordwise bending, and torsional vibration of a twisted nonuniform blade and its special subcase pure torsional vibration. The point transmission matrix for the attached concentrated mass at any spanwise and chordwise locations is derived. The orthogonality relations that exist between the natural modes of rotor blades with an attached concentrated mass are derived. Completely automated computer programs for determination of the natural vibrational characteristics are developed. For computational efficiency and users' convenience the following three separate programs are developed:

1. For determining the natural vibrational characteristics of twisted nonuniform rotor blades undergoing coupled flapwise bending, chordwise bending, and torsional vibration with the provision of an attached point mass at any spanwise and chordwise locations;

2. For determining the natural vibrational chracteristics of twisted nonuniform rotor blades undergoing coupled flapwise bending, chordwise bending, and torsional vibration without the provision of an attached mass; and

3. For determining the natural vibrational characteristics of rotor blades undergoing pure torsional vibrations.

The effect of the following parameters on the collective, cyclic, scissor, and pure torsional modes of a seesaw rotor blade is determined:

1. Effect of collective pitch,

2. Effect of rotation,

3. Effect of magnitude of point mass,

4. Spanwise location of point mass, and

5. Chordwise location of point mass.

The listings of the computer programs developed and a sample output are given.

INTRODUCTION

Helicopters operate in a severe vibrational environment. The vibrations result from mass unbalance, dynamic runout of rotors, torsional vibrations of branched systems, critical shaft conditions, whirl vibrations, etc. The vibrations also result in helicopters due to the fuselage controls and appendages in addition to the complex aerodynamically induced vibrations. Further, the rotor blades are subject to aeroelastic instability problems like divergence and flutter. These problems are becoming more and more important since rotor blades are becoming larger and thinner. It is important to maintain a low level of vibration in helicopters for the comfort of the crew and passengers, to minimize maintenance problems, and to increase the fatigue life of the blades. The determination of natural vibrational characteristics (natural frequencies and associated mode shapes) is an important part of the vibration analyses of helicopters. They are required to eliminate the resonant responses of the blade, and they are also widely used in the series solutions of the response problems. Furthermore, the natural vibration characteristics are of extreme importance in flutter problems and are the basis of nearly all practical flutter analyses.

In this report the natural vibrational characteristics of rotor blades with an attached point mass (at any chordwise and spanwise locations) are determined using the transmission matrix approach. For instance, the attached mass could be a sensor for measuring the angle of attack at any spanwise station. The effect of an attached mass on the natural frequencies is determined. For this purpose the continuous model of a twisted nonuniform blade with coupled flapwise bending, chordwise bending, and torsional degrees of freedom is considered. Completely automated computer programs for determination of the natural vibrational characteristics are developed. For the computational efficiency and users' convenience the following three separate computer programs are developed:

1. For determining the natural vibrational characteristics of twisted nonuniform rotor blades undergoing coupled flapwise bending, chordwise bending, and torsional vibrations with the provision of an attached point mass at any spanwise and chordwise locations;

2. For determining the natural vibrational characteristics of twisted nonuniform rotor blades undergoing coupled flapwise bending, chordwise bending, and torsional vibrations without the provision of an attached mass; and

3. For determining the natural vibrational characteristics of rotor blades undergoing pure torsional vibrations.

Holzer originally employed the transfer matrix method for approximate solutions of differential equations governing the torsional vibrations of rods, and the method is generally known as the Holzer's method (ref. 1). A method quite analogous to the Holzer's method was originated by Myklestad for the treatment of beams (ref. 2). The applicable equations were rearranged and simplified by Thomson to permit a systematic tabular computation and to extend the applicability of the method to more general problems (ref. 3). One of the earliest applications of the transfer matrix method was also the steady-state description of four terminal electrical networks, in which case the method is commonly designated as "four-pole parameters." Molloy (ref. 4) was one of the first to systematically apply four-pole parameters to acoustical, mechanical, and electromechanical vibrations. Pestel and Leckie (ref. 5) have catalogued transfer matrices for uniform elastomechanical elements up to twelfth-order. Rubin (refs. 6, 7) has extended the application through a completely general treatment. Transfer matrices have been applied to a wide variety of engineering problems by a number of researchers, including Targoff (refs. 8, 9), Isakson (refs. 10, 11), Lin (refs. 12, 13), Mercer (ref. 14), Mead (refs. 15, 16), Henderson (refs. 17, 18), McDaniel (refs. 19, 20), and Murthy (refs. 21 to 25) for application to rotor blades, stiffened beams, plates, shells and stiffened rings, etc. Most of the literature except references 21 to 25 deals with the discrete models of the continuous systems.

In the present report, the transmission matrix method is used to obtain the natural vibrational characteristics of the rotor blades with an attached point mass at any spanwise and chordwise locations. The point transmission matrix for the concentrated mass on the rotor blade at any spanwise and chordwise locations is derived. The transmission matrix for the continuous system is obtained by using the procedure described in reference 25. For completeness the derivation of the orthogonality relations that exist between the natural modes which is given in reference 25 is included in the present report with a modification to account for an attached concentrated mass.

NOMENCLATURE

[A]	coefficient matrix of first-order differential equations
B_1, B_2	section constants (defined in reference 26)
b_0	semichord at the root
d	chordwise distance of the attached concentrated mass from the shear center, positive towards leading edge
EI_1	flapwise bending stiffness
EI_2	chordwise bending stiffness
e	distance between mass and elastic axes, positive when mass axis lies ahead of shear center
e_A	distance between area centroid of tensile member and elastic axis
e_0	distance at root between elastic axis and axis about which blade is rotating
GJ	torsional stiffness
k_A	polar radius of gyration of cross-sectional area effective in carrying tensile stresses about elastic axis
k_m	polar radius of gyration of cross-sectional mass about elastic axis $k_m^2 = k_{m_1}^2 + k_{m_2}^2$
k_{m_1}	mass radius of gyration of the cross section about the chord
k_{m_2}	mass radius of gyration of the cross section about an axis perpendicular to the chord passing through the shear center
M	magnitude of the attached concentrated mass
M_x, M_y, M_z	resultant cross-sectional moments about x, y, and z directions respectively, M_x = twisting moment, M_y = flapwise bending moment, M_z = chordwise bending moment

m	mass per unit span
R	span of the rotor (from axis of rotation to the tip of the blade)
T	tension in the blade
[T]	transmission matrix
t	time
V_y, V_z	shear force in y and z directions respectively
v	amplitude of simple harmonic lateral displacement in the plane of rotation, positive in the positive y-direction
w	amplitude of simple harmonic lateral displacement normal to the plane of rotation, positive in the positive z-direction
X_M	spanwise location of the attached concentration mass
x, y, z	right-handed Cartesian coordinate system which rotates with the blade such that the x-axis lies along the undeformed position of the elastic axis, y and z are cross-sectional axes, y-axis is positive towards the leading edge, z-axis is positive vertically upwards
{z}	state vector
β	blade twist prior to deformation
ν	slope of the deflection curve in the plane of the rotation
ϕ	amplitude of simple harmonic torsional deformation, positive leading edge upwards
ψ	slope of the deflection curve normal to the plane of the rotation
Ω	angular velocity of rotation
ω	frequency of vibration

SUPERSCRIPTS

/	differentiation with respect to argument

°	time derivatives
-	nondimensional quantities
T	transpose of the matrix

SUBSCRIPT

0	reference quantities, say at the root

DERIVATION OF TRANSMISSION MATRIX OF CONTINUOUS SYSTEMS

For linear systems, the state vector satisfied a differential equation of the following form

$$\frac{d}{dx}\{z(x)\} = [A(x)]\ \{z(x)\} \tag{1}$$

By definition of the backward transmission matrix

$$\{z(x)\} = [T(x)]\ \{z(0)\} \tag{2}$$

Differentiating this equation with respect to x gives

$$\frac{d}{dx}\{z(x)\} = \frac{d}{dx}[T(x)]\ \{z(0)\} \tag{3}$$

From equation (2) it is obvious that

$$\{z(0)\} = [T(x)]^{-1}\ \{z(x)\} \tag{4}$$

and the inverse of a transmission matrix always exists since the determination of a transmission matrix is unity. Substituting equation (4) into equation (3), the following relation is obtained

$$\frac{d}{dx}\{z(x)\} = \frac{d}{dx}[T(x)]\ [T(x)]^{-1}\{z(x)\} \tag{5}$$

Equating equations (1) and (5) yields

$$[A(x)]\ \{z(x)\} = \frac{d}{dx}\ [T(x)]\ [T(x)]^{-1}\ \{z(x)\}$$

or

$$\left(\frac{d}{dx}\ [T(x)]\ [T(x)]^{-1} - [A(x)]\right) \{z(x)\} = \{0\} \tag{6}$$

since equation (6) must be satisfied for all values of x and for all values of z, it follows that $[A(x)] = \frac{d}{dx}\ [T(x)]\ [T(x)]^{-1}$. Then postmultiplying both sides by $[T(x)]$ gives

$$\frac{d}{dx}\ [T(x)] = [A(x)]\ [T(x)] \tag{7}$$

Therefore, the transmission matrix is given directly by the solution to equation (7). By letting x go to zero in equation (2), the required initial condition becomes

$$[T(0)] = [1]\ , \quad \text{the identity matrix} \tag{8}$$

If equation (7) is solved as a coupled set of first-order differential equations, then equation (8) provides the sufficient number of initial conditions.

BASIC EQUATIONS

The basic differential equations of motion for combined flapwise bending, chordwise bending, and torsion of twisted nonuniform rotor blades are derived in reference 26. Using the transmission matrix formulation of the general case, the natural frequencies of subcases can be determined. These subcases arise if some degree of freedom is decoupled from the combined flapwise bending, chordwise bending, and torsion by virtue of some parameters being zero. Unfortunately, the mock shapes of the subcases cannot be determined from the formulation of the general case, so the subcases must be formulated separately. The differential equations of motion for simple harmonic free vibrations with frequency, ω, are listed below with

$$k_A = B_1 = B_2 = e_A = e_0 = 0$$

Case I: Combined flapwise bending, chordwise bending, and torsion

$$\begin{aligned} -(GJ\phi')' &+ \Omega^2 mxe(-v' \cos\beta + w' \sin\beta) + \Omega^2 me \sin\beta\, v \\ &+ \Omega^2 m(k_{m_2}{}^2 - k_{m_1}{}^2) \cos 2\beta\, \phi - \omega^2 mk_m{}^2 \phi \\ &+ \omega^2 me(v \sin\beta - w \cos\beta) = 0 \end{aligned} \tag{9}$$

$$\begin{aligned} [(EI_1 \cos^2\beta &+ EI_2 \sin^2\beta)w'' + (EI_2 - EI_1)\sin\beta \cos\beta\, v'']'' \\ &- (Tw')' - (\Omega^2 mxe\phi \cos\beta)' \\ &- \omega^2 m(w + e\phi \cos\beta) = 0 \end{aligned} \tag{10}$$

$$\begin{aligned} [(EI_2 - EI_1) \sin\beta \cos\beta\, w'' &+ (EI_1 \sin^2\beta + EI_2 \cos^2\beta)v'']'' \\ &- (Tv')' + (\Omega^2 mxe\phi \sin\beta)' + \Omega^2 me\phi \sin\beta \\ &- \omega^2 m(v - e\phi \sin\beta) - \Omega^2 mv = 0 \end{aligned} \tag{11}$$

$$T' + \Omega^2 mx = 0 \tag{12}$$

Case II: Pure torsion ($e = \beta = 0$)

$$-(GJ\phi')' + \Omega^2 m(k_{m_2}{}^2 - k_{m_1}{}^2)\phi - \omega^2 mk_m{}^2 \phi = 0 \tag{13}$$

For the determination of the transmission matrix, it is required to reduce the governing differential equations of motion to a set of first-order differential equations. The following state vectors are chosen for this purpose.

Case I: Combined flapwise bending, chordwise bending, and torsion.

$$\{z\}^T = \lfloor w,\ v,\ \psi,\ \nu,\ M_x,\ M_z,\ M_y,\ -V_y,\ -V_z \rfloor$$

Case II: Pure torsion

$$\{z\}^T = \lfloor \phi,\ M_x \rfloor$$

The components of the state vector, $\{z\}$, can be chosen in several ways, but they are chosen here such that they represent the physical quantities of deflections, slopes, moments, and shears. This is not absolutely required, but highly preferable for the application of transmission matrices to obtain the natural vibration characteristics. For simplication of the numerical computation the differential equations of motion are non-dimensionalized as shown below

$$\bar{x} = \frac{x}{R}$$

$$\bar{\beta} = \beta$$

$$\bar{e} = \frac{e}{b_0}$$

$$\bar{k}_{m_1}^{2} = \frac{k_{m_1}^{2}}{b_0^{2}}$$

$$\bar{k}_{m_2}^{2} \quad \frac{k_{m_2}^{2}}{b_0^{2}}$$

$$\bar{k}_{m}^{2} \quad \frac{k_{m}^{2}}{b_0^{2}}$$

$$\overline{\Omega}^2 = \frac{\Omega^2 m_0 R^4}{EI_{10}} \quad \text{for Case I}$$

$$\overline{\Omega}^2 = \frac{\Omega^2 m_0 R^4}{GJ_0} \quad \text{for Case II}$$

$$\overline{\omega}^2 = \frac{\overline{\omega}^2 m_0 R^4}{EI_{10}} \quad \text{for Case I}$$

$$\overline{\omega}^2 = \frac{\overline{\omega}^2 m_0 R^4}{GJ_0} \quad \text{for Case II}$$

$$\overline{m} = m/m_0$$

The nondimensional elements of the state vectors are defined as shown below

$$\overline{w} = \frac{w}{b_0}$$

$$\overline{v} = \frac{v}{b_0}$$

$$\overline{\psi} = \frac{\psi R}{b_0}$$

$$\overline{\nu} = \frac{\nu R}{b_0}$$

$$\overline{\phi} = \phi$$

$$M_x = \frac{M_x R^3}{EI_{10} b_0{}^2} \quad \text{for Case I}$$

$$\bar{M}_x = \frac{M_x R^3}{GJ_0 b_0^2} \quad \text{for Case II}$$

$$\bar{M}_y = \frac{M_y R^2}{EI_{10} b_0}$$

$$\bar{M}_z = \frac{M_z R^2}{EI_{10} b_0}$$

$$\bar{V}_y = \frac{V_y R^3}{EI_{10} b_0}$$

$$\bar{V}_z = \frac{V_z R^3}{EI_{10} b_0}$$

The resulting first-order nondimensional equations are given below.

Case I: Combined flapwise bending, chordwise bending, and torsion.

$$\frac{d\bar{w}}{d\bar{x}} = \bar{\psi} \tag{14a}$$

$$\frac{d\bar{v}}{d\bar{x}} = \bar{\nu} \tag{14b}$$

$$\frac{d\bar{\psi}}{d\bar{x}} = -c_2 EI_{10} \bar{M}_z + c_1 EI_{10} \bar{M}_y \tag{14c}$$

$$\frac{d\bar{\nu}}{d\bar{x}} = c_3 EI_{10} \bar{M}_z - c_2 EI_{10} \bar{M}_y \tag{14d}$$

$$\frac{d\bar{\phi}}{d\bar{x}} = \frac{EI_{10} b_0^2}{GJR^2} \bar{M}_x \tag{14e}$$

$$\frac{d\overline{M}_x}{d\overline{x}} = -\overline{\omega}^2\overline{me}\cos\beta\,\overline{w} + (\overline{\omega}^2 + \overline{\Omega}^2)\overline{me}\sin\beta\,\overline{v}$$

$$+ \overline{\Omega}^2\overline{mxe}\cos\beta\,\overline{\psi} - \overline{\Omega}^2\overline{mxe}\sin\beta\,\overline{\nu} \tag{14f}$$

$$+ [\overline{\Omega}^2\overline{m}(\overline{k}_{m_2}^{\,2} - \overline{k}_{m_1}^{\,2})\cos 2\beta - \overline{\omega}^2\overline{mk}_m^{\,2}]\overline{\phi}$$

$$\frac{d\overline{M}_z}{d\overline{x}} = \overline{T\nu} - \overline{\Omega}^2\overline{mex}\sin\beta\,\overline{\phi} - \overline{V}_y \tag{14g}$$

$$\frac{d\overline{M}_y}{d\overline{x}} = \overline{T\psi} + \overline{\Omega}^2\overline{mex}\cos\beta\,\overline{\phi} - \overline{V}_z \tag{14h}$$

$$-\frac{d\overline{V}_y}{d\overline{x}} = (\overline{\omega}^2 + \overline{\Omega}^2)\overline{mv} - (\overline{\omega}^2 + \overline{\Omega}^2)\overline{me}\sin\beta\,\overline{\phi} \tag{14i}$$

$$-\frac{d\overline{V}_z}{d\overline{x}} = \overline{\omega}^2\overline{mw} + \overline{\omega}^2\overline{me}\cos\beta\,\overline{\phi} \tag{14j}$$

Case II: Pure torsion

$$\frac{d\overline{\phi}}{d\overline{x}} = \frac{GJ_0 b_0^{\,2}}{GJR^2}\overline{M}_x \tag{15a}$$

$$\frac{d\overline{M}_x}{d\overline{x}} = [\overline{\Omega}^2\overline{m}(\overline{k}_{m_2}^{\,2} - \overline{k}_{m_1}^{\,2}) - \overline{\omega}^2\overline{mk}_m^{\,2}]\overline{\phi} \tag{15b}$$

where

$$C_1 = \frac{A_{22}}{D}$$

$$C_2 = \frac{A_{12}}{D}$$

$$C_3 = \frac{A_{11}}{D}$$

$$D = A_{11}A_{22} - A_{12}^2$$

$$A_{11} = EI_1 \cos^2 \beta + EI_2 \sin^2 \beta$$

$$A_{12} = (EI_2 - EI_1) \sin \beta \cos \beta$$

$$A_{22} = EI_1 \sin^2 \beta + EI_2 \cos^2 \beta$$

$$\overline{T} = \frac{TR^2}{EI_{10}}$$

Equations (14) and (15) will define the elements of matrix [A] in equation (7) for the evaluation of the transmission matrices of Cases I and II respectively. The required initial conditions are given by equation (8). The nondimensional form of equation (12) is as shown below

$$\frac{d\overline{T}}{d\overline{x}} + \overline{\Omega}^2\overline{mx} = 0 \qquad (16)$$

DERIVATION OF THE POINT TRANSMISSION MATRIX FOR A CONCENTRATED MASS ON A ROTOR BLADE

A concentrated mass is assumed to be attached by a rigid massless bar along the chordline of an airfoil section of the blade as shown in figure 1.

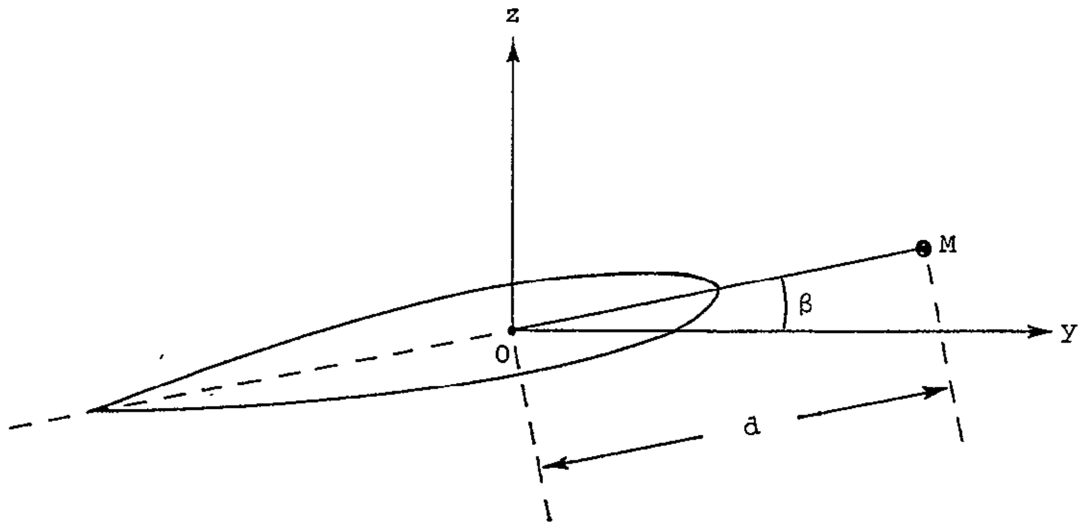


Figure 1. Airfoil section with attached mass.

where

M = attached mass

O = shear center of the cross section

d = distance of the mass from shear center along the chordline

β = twist of the blade

y, z = undeformed coordinate system

Inertial Accelerations

Let x, y, z be undeformed coordinates and x_1, y_1, z_1 be deformed coordinates, and these are related by

$$x_1 = x + u - y \frac{dv}{dx} - z \frac{dw}{dx} \tag{17a}$$

$$y_1 = y + v - z\phi \tag{17b}$$

$$z_1 = z + w + y\phi \tag{17c}$$

Let X, Y, Z be a nonrotating coordinate system and x, y, z be a rotating coordinate system, and $\underset{\sim}{i}$, $\underset{\sim}{j}$, $\underset{\sim}{k}$ be unit vectors along X, Y, Z directions respectively. The position vector $\underset{\sim}{r}$ can be expressed in terms of the rotating coordinates and nonrotating unit vectors as shown below (see figure 2 also).

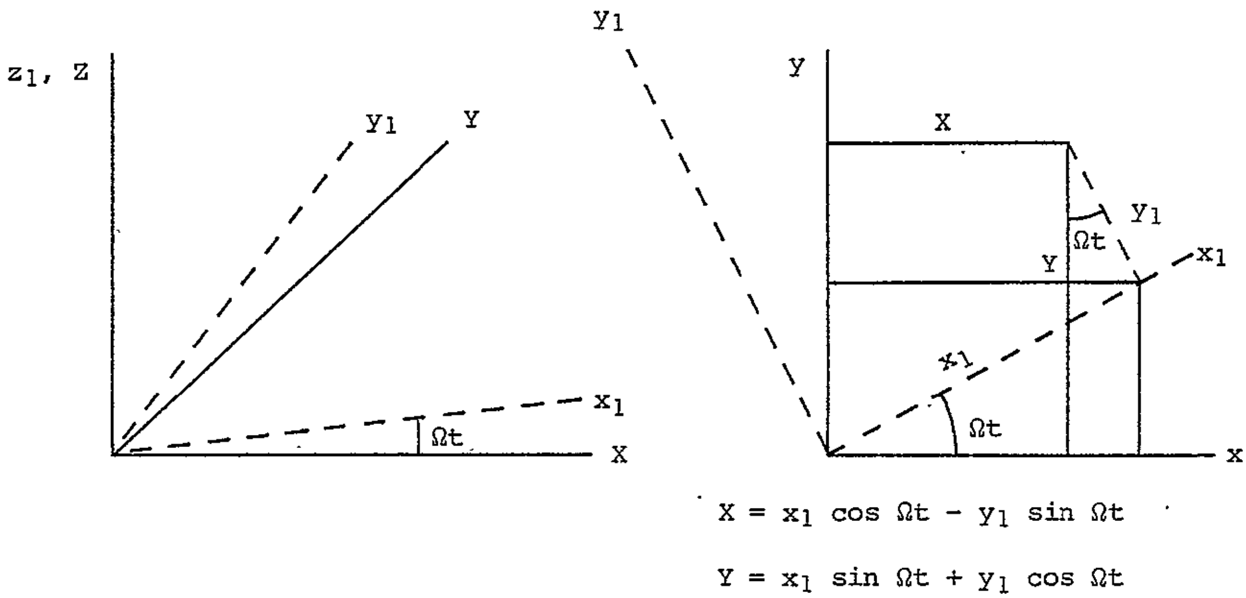


Figure 2. Rotating and nonrotating coordinate systems.

$$\underset{\sim}{r} = \underset{\sim}{i}X + \underset{\sim}{j}Y + \underset{\sim}{k}Z$$

or

$$\underset{\sim}{r} = \underset{\sim}{i}(x_1 \cos \Omega t - y_1 \sin \Omega t) + \underset{\sim}{j}(x_1 \sin \Omega t + y_1 \cos \Omega t) + \underset{\sim}{k}z_1$$

Differentiating this equation with respect to t gives

$$\ddot{\underline{r}} = \underline{i}(\ddot{x}_1 \cos \Omega t - 2\Omega\dot{x}_1 \sin \Omega t - \Omega^2 x_1 \cos \Omega t - \ddot{y}_1 \sin \Omega t$$
$$- 2\Omega\dot{y}_1 \cos \Omega t + \Omega^2 y_1 \sin \Omega t)$$
$$+ \underline{j}(\ddot{x}_1 \sin \Omega t + 2\Omega\dot{x}_1 \cos \Omega t - \Omega^2 x_1 \sin \Omega t + \ddot{y}_1 \cos \Omega t$$
$$- 2\Omega\dot{y}_1 \sin \Omega t - \Omega^2 y_1 \cos \Omega t) + \underline{k}\ddot{z}_1$$

Acceleration components with respect to the rotating coordinate system (x, y, z) are given by substituting $\Omega t = 0$ in the above equation. Denoting these components by a_x, a_y and a_z in x, y, z directions respectively, the following relations can be obtained

$$a_x = \ddot{x}_1 - \Omega^2 x_1 - 2\Omega\dot{y}_1$$

$$a_y = 2\Omega\dot{x}_1 + \ddot{y}_1 - \Omega^2 y_1$$

$$a_z = \ddot{z}_1$$

Substituting equation (17) in these acceleration components yields

$$a_x = \ddot{u} - y\ddot{v}' - z\ddot{w}' - \Omega^2(x + u - yv' - zw') - 2\Omega(\dot{v} - \dot{z}\phi)$$

$$a_y = 2\Omega(\dot{u} - y\dot{v}' - z\dot{w}') + \ddot{v} - z\ddot{\phi} - \Omega^2(y + v - z\phi)$$

$$a_z = \ddot{w} + y\ddot{\phi}$$

By neglecting the small components of usual helicopters the following acceleration components are obtained

$$a_x = -\Omega^2 x \qquad (18a)$$

$$a_y = \ddot{v} - z\ddot{\phi} - \Omega^2(v + y - z\phi) \qquad (18b)$$

$$a_z = \ddot{w} + y\ddot{\phi} \qquad (18c)$$

From geometry of figure 1, it is obvious that

$$y = d \cos \beta \tag{19a}$$

$$z = d \sin \beta \tag{19b}$$

Substituting equation (19) into equation (18), the following acceleration components acting on the concentrated mass are obtained

$$a_x = -\Omega^2 x \tag{20a}$$

$$a_y \doteq \ddot{v} - d \sin \beta\, \ddot{\phi} - \Omega^2 (v + d \cos \beta - d \sin \beta\, \phi) \tag{20b}$$

$$a_z = \ddot{w} + d \cos \beta\, \ddot{\phi} \tag{20c}$$

Force Equilibrium Equations

The free-body diagram of a small element across the concentrated mass is shown in figure 3. For clarity purpose only forces are shown.

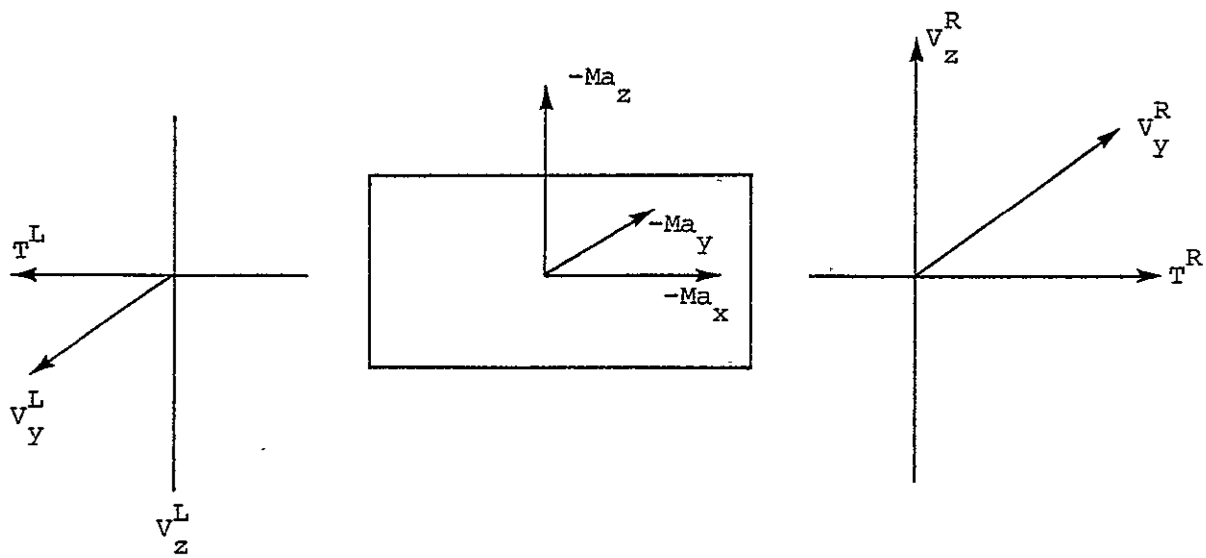


Figure 3. Free-body diagram for forces.

For force equilibrium the following relations must be satisfied

$$T^R = T^L + Ma_x$$

$$V_y^R = V_y^L + Ma_y$$

$$V_y^R = V_z^L + Ma_z$$

Substituting equation (20) in the above equations and assuming simple harmonic free vibrations with frequency, ω , the following equations can be obtained

$$T^R = T^L - M\Omega^2 x \quad (21a)$$

$$V_y^R = V_y^L - (\omega^2 + \Omega^2)Mv + (\omega^2 + \Omega^2)Md \sin \beta \phi \quad (21b)$$

$$V_z^R = V_z^L - \omega^2 Mw - \omega^2 Md \cos \beta \phi \quad (21c)$$

Inertial Moments

From figure 4 the following relations for inertial force vector and position vector can be written

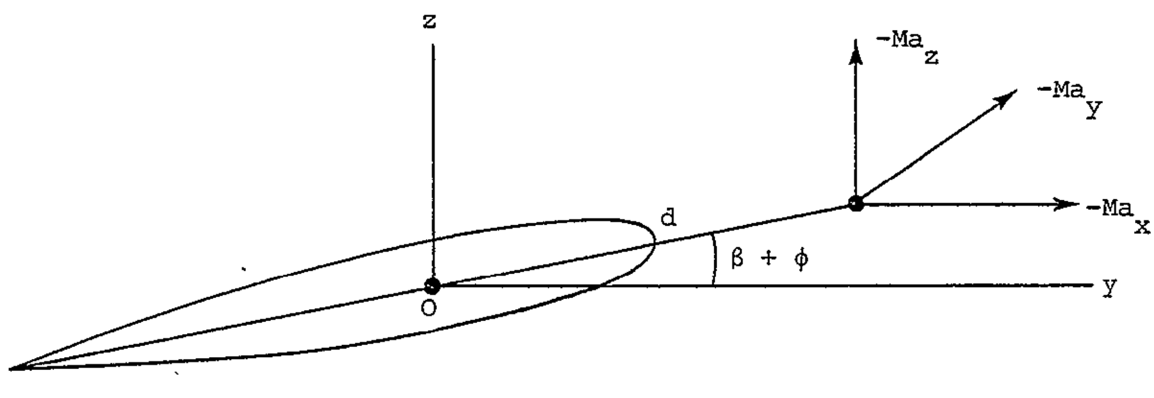


Figure 4. Inertial moments and position vector.

$$\underset{\sim}{F} = -\underset{\sim}{i}Ma_x - \underset{\sim}{j}Ma_y - \underset{\sim}{k}Ma_z$$

$$\underset{\sim}{r} = \underset{\sim}{j}d \cos (\beta + \phi) + \underset{\sim}{k}d \sin (\beta + \phi)$$

or

$$\underset{\sim}{r} = \underset{\sim}{j}d(\cos \beta - \phi \sin \beta) + \underset{\sim}{k}d(\sin \beta + \phi \cos \beta)$$

for small angles of ϕ . The moment vector about the shear center 0 is given by

$$\underset{\sim}{M}_{sc} = \underset{\sim}{r} x \underset{\sim}{F} = \underset{\sim}{i}M_1 + \underset{\sim}{j}M_2 + \underset{\sim}{k}M_3$$

where

$$M_1 = -Md[(\cos \beta - \phi \sin \beta)a_z - (\sin \beta + \phi \cos \beta)a_y]$$

$$M_2 = -Md(\sin \beta + \phi \cos \beta)a_x$$

$$M_3 = Md(\cos \beta - \phi \sin \beta)a_x$$

Substituting equation (20) in the above inertial moment components and neglecting the nonlinear terms the following equations are obtained for simple harmonic free vibrations with frequency, ω .

$$M_1 = \omega^2 Md \cos \beta \; w - (\omega^2 + \Omega^2)Md \sin \beta \; v \cdot (\omega^2 Md^2 - \Omega^2 Md^2 \cos 2\beta)\phi \tag{22a}$$

$$M_2 = \Omega^2 Md \cos \beta \; x \; \phi \tag{22b}$$

$$M_3 = \Omega^2 Md \sin \beta \; x \; \phi \tag{22c}$$

Moment Equilibrium Equations

The free-body diagram of a small element across the concentrated mass is shown in figure 5. For clarity purpose only moments are shown.

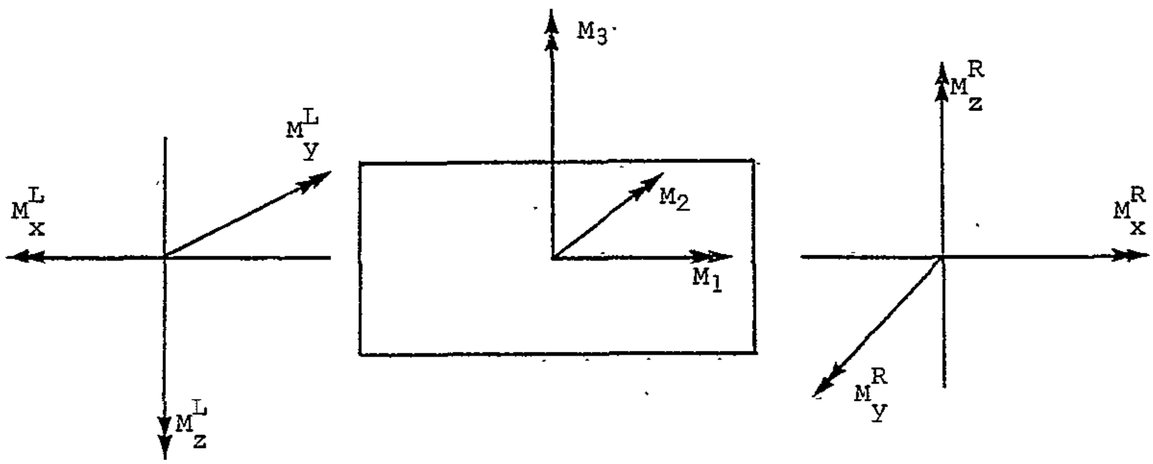


Figure 5. Free-body diagram for moments.

For moment equilibrium the following relations must be satisfied

$$M_x^R = M_x^L - M_1$$

$$M_y^R = M_y^L + M_2$$

$$M_z^R = M_z^L - M_3$$

Substituting equation (22) in the above moment equilibrium equations

$$M_x^R = M_x^L - \omega^2 Md \cos\beta\, w + (\omega^2 + \Omega^2) Md \sin\beta\, v + (-\omega^2 Md^2 + \Omega^2 Md^2 \cos 2\beta)\phi \tag{23a}$$

$$M_y^R = M_y^L + \Omega^2 Md \cos\beta\, x\, \phi \tag{23b}$$

$$M_z^R = M_z^L - \Omega^2 Md \sin \beta x \phi \tag{23c}$$

Point Transmission Matrix

Equations (21) and (23) can be put into a transmission matrix form across the concentrated mass by noting that the deflections and slopes are continuous across the mass. The resulting transmission matrix after nondimensionalization is as shown on the following page [eq. (24)] when

$$\bar{d} = \frac{d}{b_0}$$

$$\bar{M} = \frac{M}{m_0 R}$$

$$\bar{x} = \frac{x}{R} = \text{spanwise location of the mass.}$$

Case II: For this case the point transmission matrix reduces to the following equation:

$$\begin{Bmatrix} \bar{\phi} \\ \bar{M}_x \end{Bmatrix}_R = \begin{bmatrix} 1 & 0 \\ (-\omega^2 + \Omega^2)\overline{Md^2} & 1 \end{bmatrix} \begin{Bmatrix} \bar{\phi} \\ \bar{M}_x \end{Bmatrix}_L \tag{25}$$

NATURAL VIBRATION CHARACTERISTICS

Natural Frequencies

The overall transmission matrix of the blade without the attached mass is obtained by integrating the differential equations given by equation (7) together with the initial conditions given by equation (8). The integration proceeds from $\bar{x} = 0$ to $\bar{x} = 1$. The matrix [A] in equation (7) is the coefficient matrix of the first-order differential equations of motion, and it is obtained from equation (14). The coefficient $\bar{T}$ appearing in

$$
\begin{Bmatrix} \overline{w} \\ \overline{v} \\ \overline{\psi} \\ \overline{\nu} \\ \overline{\phi} \\ \overline{M}_x \\ \overline{M}_z \\ \overline{M}_y \\ -\overline{V}_y \\ -\overline{V}_z \end{Bmatrix}_R
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-\overline{\omega}^2\overline{Md}\cos\beta & (\overline{\omega}^2+\overline{\Omega}^2)\overline{Md}\sin\beta & 0 & 0 & (-\overline{\omega}^2+\overline{\Omega}^2\cos 2\beta)\overline{Md}^2 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\overline{\Omega}^2\overline{Md}\sin\beta\,\overline{x} & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \overline{\Omega}^2\overline{Md}\cos\beta\,\overline{x} & 0 & 0 & 1 & 0 & 0 \\
0 & (\overline{\omega}^2+\overline{\Omega}^2)\overline{M} & 0 & 0 & -(\overline{\omega}^2+\overline{\Omega}^2)\overline{Md}\sin\beta & 0 & 0 & 0 & 1 & 0 \\
\overline{\omega}^2\overline{M} & 0 & 0 & 0 & \overline{\omega}^2\overline{Md}\cos\beta & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{Bmatrix} \overline{w} \\ \overline{v} \\ \overline{\psi} \\ \overline{\nu} \\ \overline{\phi} \\ \overline{M}_x \\ \overline{M}_z \\ \overline{M}_y \\ -\overline{V}_y \\ -\overline{V}_z \end{Bmatrix}
\quad (24)
$$

equation (14) is obtained by solving equation (16) together with the initial condition $\overline{T}(\overline{x} = 1) = 0$. The overall transmission matrix of the blade when a concentrated mass is attached can be obtained as described below.

Let x_M be the spanwise location of the attached mass. Equation (7) is integrated from $\overline{x} = 0$ to $\overline{x} = \frac{x_M}{R}$ utilizing the initlal conditions given by equation (8). Let $[T_1]$ be the transmission matrix up to the point (x_M) and represent the transmission properties of the system from $\overline{x} = 0$ to $\overline{x} = \frac{x_M}{R}$. The point transmission matrix of the attached mass is computed from equation (24), and let it be denoted by $[T_M]$. Equation (7) can again be integrated from $\overline{x} = \frac{x_M}{R}$ to $\overline{x} = 1$ utilizing the initial conditions given by equation (8). Let $[T_2]$ be the resulting transmission matrix and represent the transmission properties of the system from $\overline{x} = \frac{x_M}{R}$ to $\overline{x} = 1$. Then by using the product rule of the backward transmission matrix, the overall transmission matrix of the system can be obtained from the following equation

$$[T] = [T_2]\ [T_M]\ [T_1]$$

While integrating equation (7) from $\overline{x} = 0$ to $\overline{x} = \frac{x_M}{R}$ to obtain transmission matrix $[T_1]$, coefficient $\overline{T}$ is obtained as usual by integrating equation (16). But this coefficient should be increased by a constant amount $\overline{\Omega^2 M X_M}$ to account for the tension in the blade due to the attached mass. While integrating equation (7) from $\overline{x} = \overline{x}_M$ to $\overline{x} = 1$ this increment in the tension will not be required. This fact is reflected in equation (21a) which is not utilized in obtaining the point transmission matrix given by equation (24). A similar procedure can be used for Case II also when a concentrated mass is attached to the blade. Having obtained the overall transmission matrix of the system either with or without the concentrated mass the frequency determinant can subsequently be obtained as discussed below.

The frequency determinant is dependent on the boundary conditions of the system, and the following three sets of root boundary conditions are assumed for collective, cyclic, and scissor modes of a seesaw rotor blade.

$$M_x = M_y = M_z = V_y = V_z = 0 \quad \text{at} \quad \overline{x} = 1 \quad \text{(tip of the blade)} \tag{26a}$$

Collective Modes

$$w = v = \psi = M_z = 0\ ,\quad M_x = -k_\phi \phi \quad \text{at} \quad \bar{x} = 0 \tag{26b}$$

(root of the blade)

Cyclic Modes

$$w = \nu = v = M_y = 0\ ,\quad M_x = -k_\phi \phi \quad \text{at} \quad \bar{x} = 0 \tag{26c}$$

(root of the blade)

Scissor Modes

$$w = v = \psi = \nu = 0\ ,\quad M_x = -k_\phi \phi \quad \text{at} \quad \bar{x} = 0 \tag{26d}$$

(root of the blade)

By definition of the backward transmission matrix one can write

$$\begin{Bmatrix} \bar{w} \\ \bar{v} \\ \bar{\psi} \\ \bar{\nu} \\ \bar{\phi} \\ \bar{M}_x \\ \bar{M}_z \\ \bar{M}_y \\ -\bar{V}_y \\ -\bar{V}_z \end{Bmatrix}_{\bar{x}=1} = [T_{ij}] \begin{Bmatrix} \bar{w} \\ \bar{v} \\ \bar{\psi} \\ \bar{\nu} \\ \bar{\phi} \\ \bar{M}_x \\ \bar{M}_z \\ \bar{M}_y \\ -\bar{V}_y \\ -\bar{V}_z \end{Bmatrix} \tag{27}$$

where T_{ij} represents the (i, j) th nondimensional element of the transmission matrix from $\bar{x} = 0$ to $\bar{x} = 1$. The frequency determinant corresponding to collective modes is obtained by substituting the tip boundary conditions given by equation (26a) into the output state vector ($\bar{x} = 1$) and the root boundary conditions into the input state vector ($\bar{x} = 0$). This substitution yields five homogeneous equations, and the determinant of the coefficient matrix of these equations must vanish for nontrivial solutions. The resulting frequency determinant is given by the following equation

$$\begin{bmatrix} T_{64} & T_{65} - \bar{k}_\phi T_{66} & T_{68} & T_{69} & T_{610} \\ T_{74} & T_{75} - \bar{k}_\phi T_{76} & T_{78} & T_{79} & T_{710} \\ T_{84} & T_{85} - \bar{k}_\phi T_{86} & T_{88} & T_{89} & T_{810} \\ T_{94} & T_{95} - \bar{k}_\phi T_{96} & T_{98} & T_{99} & T_{910} \\ T_{104} & T_{105} - \bar{k}_\phi T_{106} & T_{108} & T_{109} & T_{1010} \end{bmatrix} = 0$$

where $\bar{k}_\phi$ is nondimensional control system spring rate defined by $\bar{k}_\phi = \frac{k_\phi R^3}{EI_{10} b_0{}^2}$. This represents the resistance to unit torsional deformation due to control systems. By using a similar procedure the frequency determinants corresponding to the cyclic and scissor modes can be obtained, and they are given below

For cyclic modes

$$\begin{bmatrix} T_{63} & T_{65} - \bar{k}_\phi T_{66} & T_{67} & T_{69} & T_{610} \\ T_{73} & T_{75} - \bar{k}_\phi T_{76} & T_{77} & T_{79} & T_{710} \\ T_{83} & T_{85} - \bar{k}_\phi T_{86} & T_{87} & T_{89} & T_{810} \\ T_{93} & T_{95} - \bar{k}_\phi T_{96} & T_{97} & T_{99} & T_{910} \\ T_{103} & T_{105} - \bar{k}_\phi T_{106} & T_{107} & T_{109} & T_{1010} \end{bmatrix} = 0$$

For scissor modes

$$\begin{bmatrix} T_{65} - \bar{k}_\phi T_{66} & T_{67} & T_{68} & T_{69} & T_{610} \\ T_{75} - \bar{k}_\phi T_{76} & T_{77} & T_{78} & T_{79} & T_{710} \\ T_{85} - \bar{k}_\phi T_{86} & T_{87} & T_{88} & T_{89} & T_{810} \\ T_{95} - \bar{k}_\phi T_{96} & T_{97} & T_{98} & T_{99} & T_{919} \\ T_{105} - \bar{k}_\phi T_{106} & T_{107} & T_{108} & T_{109} & T_{1010} \end{bmatrix} = 0$$

For pure torsional vibrations the boundary conditions, the transmission equation, and frequency determinants are given below

Boundary conditions

$$M_x = 0 \quad \text{at} \quad \bar{x} = 1$$

$$M_x = -k_\phi \phi \quad \text{at} \quad \bar{x} = 0$$

Transmission equation

$$\begin{Bmatrix} \bar{\phi} \\ \bar{M}_x \end{Bmatrix}_{\bar{x}=1} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} \bar{\phi} \\ \bar{M}_x \end{Bmatrix}_{\bar{x}=0}$$

Frequency equation

$$T_{21} - \bar{k}_\phi T_{22} = 0$$

The frequency determinant is a function of frequency ω , the solution of which yields the natural frequencies.

Mode Shapes

Having determined the natural frequencies, the associated mode shapes can be obtained subsequently in a straight forward manner. To determine the natural frequencies the overall transmission matrix (i.e., $\bar{x} = 0$ to $\bar{x} = 1$) is required. For determination of the mode shapes the transmission matrix as a function of $\bar{x}$ is required. The transmission matrix as a function of $\bar{x}$ for a blade without the attached mass can be obtained by integrating equation (7) together with the initial conditions. If a numerical scheme is used for this purpose, the transmission matrix is known at various stations along the span. The transmission matrix as a function of $\bar{x}$ for a rotor blade with an attached concentrated mass can be obtained as described below.

Let x_M be the spanwise location of the attached mass. Equation (7) is integrated from $\bar{x} = 0$ to $\bar{x} = x_M$ together with the initial conditions given by equation (8). Let $_{\bar{x}}[T_1]_0$ be the transmission matrix from $\bar{x} = 0$, to $\bar{x}(\bar{x} \leq \overline{x_M})$. The matrix $_{\bar{x}_M}[T_1]_0$ will be the same as matrix $[T_1]$ defined under Natural Frequencies. Let $[T_M]$ be the point transmission matrix of the attached mass. Equation (7) can again be integrated from $\bar{x} = \bar{x}_M$ to $\bar{x} = 1$ utilizing the initial conditions given by equation (8). Let $_{\bar{x}}[T_2]_{\bar{x}_M}(\bar{x}_M^+ \leq \bar{x} \geq 1)$ be the resulting transmission matrix. Then the transmission matrix as a function of $\bar{x}$ for the blade with an attached concentrated mass is given by the following equations

$$[T(\bar{x})] = {}_{\bar{x}}[T_1]_0 \;, \quad 0 \leq \bar{x} \leq \overline{x_M}$$

$$[T(\bar{x})] = {}_{\bar{x}}[T_2]_{\bar{x}_M} \; [T_M] \; {}_{\bar{x}_M}[T_1]_0 \;, \quad \bar{x}_M^+ \leq \bar{x} \leq 1$$

A similar procedure is used to obtain the transmission matrix as a function of $\bar{x}$ for Case II also.

Case I:

Collective Modes

From the definition of the transmission matrix, one can write the following equation

$$
\begin{Bmatrix} \bar{w} \\ \bar{v} \\ \bar{\psi} \\ \bar{\nu} \\ \bar{\phi} \\ \bar{M}_x \\ \bar{M}_z \\ \bar{M}_y \\ -\bar{V}_y \\ -\bar{V}_z \end{Bmatrix}_{\bar{x}=\bar{x}} = [T_y(\bar{x})] \begin{Bmatrix} \bar{w} \\ \bar{v} \\ \bar{\psi} \\ \bar{\nu} \\ \bar{\phi} \\ \bar{M}_x \\ \bar{M}_z \\ \bar{M}_y \\ -\bar{V}_y \\ -\bar{V}_z \end{Bmatrix}_{\bar{x}=0} \tag{28}
$$

By substituting the root boundary conditions given by equation (26b) into the input state vector ($\bar{x} = 0$) of equation (28) and extracting the first, second, and fifth rows and arranging in a matrix form:

$$
\begin{Bmatrix} \bar{w} \\ \bar{v} \\ \bar{\phi} \end{Bmatrix}_{\bar{x}=\bar{x}} = \begin{bmatrix} T_{14}(\bar{x}) & T_{15}(\bar{x}) - \bar{k}_\phi T_{16}(\bar{x}) & T_{18}(\bar{x}) & T_{19}(\bar{x}) & T_{110}(\bar{x}) \\ T_{24}(\bar{x}) & T_{25}(\bar{x}) - \bar{k}_\phi T_{26}(\bar{x}) & T_{28}(\bar{x}) & T_{29}(\bar{x}) & T_{210}(\bar{x}) \\ T_{54}(\bar{x}) & T_{55}(\bar{x}) - \bar{k}_\phi T_{56}(\bar{x}) & T_{58}(\bar{x}) & T_{59}(\bar{x}) & T_{510}(\bar{x}) \end{bmatrix} \begin{Bmatrix} \bar{\nu} \\ \bar{\phi} \\ \bar{M}_y \\ -\bar{V}_y \\ -\bar{V}_z \end{Bmatrix}_{\bar{x}=0} \tag{29}
$$

By substituting the tip boundary conditions given by equation (26a) into the output state vector ($\bar{x} = 1$) and the root boundary conditions given by equation (26b) into the input state vector of equation (27) and extracting first and sixth to ninth rows of the equation while assigning $\bar{w}(\bar{x} = 1) = 1$, the following equation is obtained

$$\begin{bmatrix} T_{14} & T_{15} - \bar{k}_\phi T_{16} & T_{18} & T_{19} & T_{110} \\ T_{64} & T_{65} - \bar{k}_\phi T_{66} & T_{68} & T_{69} & T_{610} \\ T_{74} & T_{75} - \bar{k}_\phi T_{76} & T_{78} & T_{79} & T_{710} \\ T_{84} & T_{85} - \bar{k}_\phi T_{86} & T_{88} & T_{89} & T_{810} \\ T_{94} & T_{95} - \bar{k}_\phi T_{96} & T_{98} & T_{99} & T_{910} \end{bmatrix}_0 \begin{Bmatrix} \nu \\ \phi \\ M_y \\ -V_y \\ -V_z \end{Bmatrix}_{\bar{x}=0} = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{30}$$

Solution to equation (30) can be written as

$$\begin{Bmatrix} \nu \\ \phi \\ M_y \\ -V_y \\ -V_z \end{Bmatrix}_{\bar{x}=0} = \begin{bmatrix} T_{14} & T_{15} - \bar{k}_\phi T_{16} & T_{18} & T_{19} & T_{110} \\ T_{64} & T_{65} - \bar{k}_\phi T_{66} & T_{68} & T_{69} & T_{610} \\ T_{74} & T_{75} - \bar{k}_\phi T_{76} & T_{78} & T_{79} & T_{710} \\ T_{84} & T_{85} - \bar{k}_\phi T_{86} & T_{88} & T_{89} & T_{810} \\ T_{94} & T_{95} - \bar{k}_\phi T_{96} & T_{98} & T_{99} & T_{910} \end{bmatrix}_0^{-1} \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} \alpha_{11} \\ \alpha_{21} \\ \alpha_{31} \\ \alpha_{41} \\ \alpha_{51} \end{Bmatrix} \tag{31}$$

By substituting this equation into equation (29), it is obvious that

$$\begin{Bmatrix} \bar{w} \\ \bar{v} \\ \bar{\phi} \end{Bmatrix}_{\bar{x}=\bar{x}} = \begin{bmatrix} T_{14}(\bar{x}) & T_{15}(\bar{x}) - \bar{k}_\phi T_{16}(\bar{x}) & T_{18}(\bar{x}) & T_{19}(\bar{x}) & T_{110}(\bar{x}) \\ T_{24}(\bar{x}) & T_{25}(\bar{x}) - \bar{k}_\phi T_{26}(\bar{x}) & T_{28}(\bar{x}) & T_{29}(\bar{x}) & T_{210}(\bar{x}) \\ T_{54}(\bar{x}) & T_{55}(\bar{x}) - \bar{k}_\phi T_{56}(\bar{x}) & T_{58}(\bar{x}) & T_{59}(\bar{x}) & T_{510}(\bar{x}) \end{bmatrix} \begin{Bmatrix} \alpha_{11} \\ \alpha_{21} \\ \alpha_{31} \\ \alpha_{41} \\ \alpha_{51} \end{Bmatrix} \tag{32}$$

The mode shapes can now be computed from equation (32). If the differential equations of motion [eqs. (9), (10), and (11)] are decoupled by virtue of some parameters being zero, then the frequency equation will turn out to be the product of frequency equations of the uncoupled systems so that the eigenvalues of the uncoupled systems can be obtained from the coupled formulation. But then the coefficient matrix in equation (30) will be singular and hence cannot be inverted. As a result, the mode shapes of the uncoupled modes cannot be determined from the coupled formulation.

By adopting a similar procedure for cyclic and scissor modes of Case I and for Case II, the following equations can be obtained for the determination of the mode shapes

Cyclic Modes:

$$\begin{Bmatrix} \bar{w} \\ \bar{v} \\ \bar{\phi} \end{Bmatrix}_{\bar{x}=\bar{x}} = \begin{bmatrix} T_{13}(\bar{x}) & T_{15}(\bar{x}) - \bar{k}_\phi T_{16}(\bar{x}) & T_{17}(\bar{x}) & T_{19}(\bar{x}) & T_{110}(\bar{x}) \\ T_{23}(\bar{x}) & T_{25}(\bar{x}) - \bar{k}_\phi T_{26}(\bar{x}) & T_{27}(\bar{x}) & T_{29}(\bar{x}) & T_{210}(\bar{x}) \\ T_{53}(\bar{x}) & T_{55}(\bar{x}) - \bar{k}_\phi T_{56}(\bar{x}) & T_{57}(\bar{x}) & T_{59}(\bar{x}) & T_{510}(\bar{x}) \end{bmatrix} \begin{Bmatrix} \alpha_{12} \\ \alpha_{22} \\ \alpha_{32} \\ \alpha_{42} \\ \alpha_{52} \end{Bmatrix}$$

where

$$\begin{Bmatrix} \alpha_{12} \\ \alpha_{22} \\ \alpha_{32} \\ \alpha_{42} \\ \alpha_{52} \end{Bmatrix} = {}_{1}\begin{bmatrix} T_{13} & T_{15} - \bar{k}_\phi T_{16} & T_{17} & T_{19} & T_{110} \\ T_{63} & T_{65} - \bar{k}_\phi T_{66} & T_{67} & T_{69} & T_{610} \\ T_{73} & T_{75} - \bar{k}_\phi T_{76} & T_{77} & T_{79} & T_{710} \\ T_{83} & T_{85} - \bar{k}_\phi T_{86} & T_{87} & T_{89} & T_{810} \\ T_{93} & T_{95} - \bar{k}_\phi T_{96} & T_{97} & T_{99} & T_{910} \end{bmatrix}^{-1}_{0} \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Scissor Modes:

$$\begin{Bmatrix} \bar{w} \\ \bar{v} \\ \bar{\phi} \end{Bmatrix}_{\bar{x}=\bar{x}} = \begin{bmatrix} T_{15}(\bar{x}) - \bar{k}_\phi T_{16}(\bar{x}) & T_{17}(\bar{x}) & T_{18}(\bar{x}) & T_{19}(\bar{x}) & T_{110}(\bar{x}) \\ T_{25}(\bar{x}) - \bar{k}_\phi T_{26}(\bar{x}) & T_{27}(\bar{x}) & T_{28}(\bar{x}) & T_{29}(\bar{x}) & T_{210}(\bar{x}) \\ T_{55}(\bar{x}) - \bar{k}\, T_{56}(\bar{x}) & T_{57}(\bar{x}) & T_{58}(\bar{x}) & T_{59}(\bar{x}) & T_{510}(\bar{x}) \end{bmatrix} \begin{Bmatrix} \alpha_{13} \\ \alpha_{23} \\ \alpha_{33} \\ \alpha_{43} \\ \alpha_{53} \end{Bmatrix}$$

where

$$\begin{Bmatrix} \alpha_{13} \\ \alpha_{23} \\ \alpha_{33} \\ \alpha_{43} \\ \alpha_{53} \end{Bmatrix} = \begin{bmatrix} T_{15} - \overline{k}_\phi T_{16} & T_{17} & T_{18} & T_{19} & T_{110} \\ T_{65} - \overline{k}_\phi T_{66} & T_{67} & T_{68} & T_{69} & T_{610} \\ T_{75} - \overline{k}_\phi T_{76} & T_{77} & T_{78} & T_{79} & T_{710} \\ T_{85} - \overline{k}_\phi T_{86} & T_{87} & T_{88} & T_{89} & T_{810} \\ T_{95} - \overline{k}_\phi T_{96} & T_{97} & T_{98} & T_{99} & T_{910} \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Case II:

$$\overline{\phi}(\overline{x}) = T_{11}(\overline{x}) - \overline{k}_\phi T_{12}(\overline{x})$$

ORTHOGONALITY OF NATURAL MODES

The governing differential equations of motion given by equations (9), (10), and (11) can be expressed in operator notation as follows

$$L_{11}[\phi] + L_{12}[v] + L_{13}[w] = \omega^2 m k_m{}^2 \phi + \omega^2 me \sin\beta\, v + \omega^2 me \cos\beta\, w \tag{33}$$

$$L_{21}[\phi] + L_{22}[v] + L_{23}[w] = \omega^2 me \cos\beta\, \phi + \omega^2 m w \tag{34}$$

$$L_{31}[\phi] + L_{32}[v] + L_{33}[w] = -\omega^2 me \sin\beta\, \phi + \omega^2 m v \tag{35}$$

where

$$L_{11} \equiv -(GJ)' \frac{d}{dx} - GJ \frac{d^2}{dx^2} + \Omega^2 m (k_{m_2}{}^2 - k_{m_1}{}^2) \cos 2\beta$$

$$L_{12} \equiv -\Omega^2 mxe \sin\beta \frac{d}{dx} + \Omega^2 me \sin\beta$$

$$L_{13} \equiv \Omega^2 mxe \cos\beta \frac{d}{dx}$$

$$L_{21} \equiv -(\Omega^2 mxe \cos\beta)' - \Omega^2 mxe \cos\beta \frac{d}{dx}$$

$$L_{22} \equiv A'' \frac{d^2}{dx^2} + 2A' \frac{d^3}{dx^3} + A \frac{d^4}{dx^4}$$

$$L_{23} \equiv B'' \frac{d^2}{dx^2} + 2B' \frac{d^3}{dx^3} + B \frac{d^4}{dx^4} + \Omega^2 mx \frac{d}{dx} - T \frac{d^2}{dx^2}$$

$$L_{31} \equiv \Omega^2 me \sin\beta + (\Omega^2 mxe \sin\beta)' + \Omega^2 mxe \sin\beta \frac{d}{dx}$$

$$L_{32} \equiv C'' \frac{d}{dx^2} + 2C' \frac{d^3}{dx^3} + C \frac{d^4}{dx^4} + \Omega^2 mx \frac{d}{dx} - T \frac{d^2}{dx^2} - \Omega^2 m$$

$$L_{33} = L_{22}$$

$$A = (EI_2 - EI_1) \sin\beta \cos\beta$$

$$B = EI_1 \cos^2\beta + EI_2 \sin^2\beta$$

$$C = EI_1 \sin^2\beta + EI_2 \cos^2\beta$$

Let ω_r and ω_s be two distinct eigenvalues and (ϕ_r, v_r, w_r) and (ϕ_s, v_s, w_s) be the corresponding eigenfunctions resulting from the solution of the problem described in equations (33), (34), and (35). The eigenvalue problems can be written as

$$L_{11}[\phi_r] + L_{12}[v_r] + L_{13}[w_r] = \omega_r^2 mk_m^2 \phi_r - \omega_r^2 me \sin\beta\, v_r + \omega_r^2 me \cos\beta\, w_r \qquad (36)$$

$$L_{21}[\phi_r] + L_{22}[v_r] + L_{23}[w_r] = \omega_r^2 me \cos\beta\ \phi_r + \omega_r^2 m w_r \qquad (37)$$

$$L_{31}[\phi_r] + L_{32}[v_r] + L_{33}[w_r] = -\omega_r^2 me \sin\beta\ \phi_r + \omega_r^2 m v_r \qquad (38)$$

$$L_{11}[\phi_s] + L_{12}[v_s] + L_{13}[w_s] = \omega_s^2 m k_m^2 \phi_s - \omega_s^2 me \sin\beta\ v_s + \omega_s^2 me \cos\beta\ w_s \qquad (39)$$

$$L_{21}[\phi_s] + L_{22}[v_s] + L_{23}[w_s] = \omega_s^2 me \cos\beta\ \phi_s + \omega_s^2 m w_s \qquad (40)$$

$$L_{31}[\phi_s] + L_{32}[v_s] + L_{33}[w_s] = -\omega_s^2 me \sin\beta\ \phi_s + \omega_s^2 m v_s \qquad (41)$$

Multiplying equation (36) by ϕ_s and equation (39) by ϕ_r, subtracting one result from the other, and integrating both sides of the equation between 0 to R yields

$$\begin{aligned}\int_0^R (\phi_s L_{11}[\phi_r] + \phi_s L_{12}[v_r] + \phi_s L_{13}[w_r] \\ - \phi_r L_{11}[\phi_s] - \phi_r L_{12}[v_s] - \phi_r L_{13}[w_s] dx \\ = (\omega_r^2 - \omega_s^2) \int_0^R m k_m^2 \phi_r \phi_s dx \\ + \int_0^R (-\omega_r^2 me \sin\beta\ v_r \phi_s + \omega_r^2 me \cos\beta\ w_r \phi_s \\ + \omega_s^2 me \sin\beta\ v_s \phi_r - \omega_s^2 me \cos\beta\ w_s \phi_r) dx\end{aligned} \qquad (42)$$

Using a similar procedure on equations (37) and (40) and on equations (38) and (41) the following two equations can be obtained

$$\int_0^R (w_s L_{21}[\phi_r] + w_s L_{22}[v_r] + w_s L_{23}[w_r]$$

$$- w_r L_{21}[\phi_s] - w_r L_{22}[v_s] - w_r L_{23}[w_s])dx$$

$$= (\omega_r^2 - \omega_s^2) \int_0^R m w_r w_s dx \tag{43}$$

$$+ \int_0^R (\omega_r^2 me \cos \beta \; \phi_r w_s - \omega_s^2 me \cos \beta \; \phi_s w_r) dx$$

$$\int_0^R (v_s L_{31}[\phi_r] + v_s L_{32}[v_r] + v_s L_{33}[w_r]$$

$$- v_r L_{31}[\phi_s] - v_r L_{32}[v_s] - v_r L_{33}[w_s])dx$$

$$= (\omega_r^2 - \omega_s^2) \int_0^R m v_r v_s dx \tag{44}$$

$$+ \int_0^R (-\omega_r^2 me \sin \beta \; \phi_r v_s + \omega_s^2 me \sin \beta \; \phi_s v_r) dx$$

Adding equations (42), (43), and (44) yields

$$\int_0^R (\phi_s L_{11}[\phi_r] + \phi_s L_{12}[v_r] + \phi_s L_{13}[w_r]$$

$$- \phi_r L_{11}[\phi_s] - \phi_r L_{12}[v_s] - \phi_r L_{13}[w_s]$$

$$+ w_s L_{21}[\phi_r] + w_s L_{22}[v_r] + w_s L_{23}[w_r]$$

$$- w_r L_{21}[\phi_s] - w_r L_{22}[v_s] - w_r L_{23}[w_s]$$

$$+ v_s L_{31}[\phi_r] + v_s L_{32}[v_r] + v_s L_{33}[w_r]$$

$$- v_r L_{31}[\phi_s] - v_r L_{32}[v_s] - v_r L_{33}[w_s])dx$$

(continued)

$$= (\omega_r^2 - \omega_s^2) \int_0^R m[k_m{}^2\phi_r\phi_s + w_r w_s + v_r v_s$$

(concluded)

$$- e \sin \beta\ (\phi_r v_s + \phi_s v_r) + e \cos \beta\ (\phi_r w_s + \phi_s w_r)]dx$$

Integrating by parts the integrals on the left-hand side of the above equation and simplying the following equation is obtained

$$\begin{aligned}
&(\omega_r{}^2 - \omega_s{}^2) \int_0^R m\{k_m{}^2\phi_r\phi_s + w_r w_s + v_r v_s - e \sin \beta\ (\phi_r v_s + \phi_s v_r) \\
&\quad + e \cos \beta\ (\phi_r w_s + \phi_s w_r)\}dx = \{GJ(\phi_s{}'\phi_r - \phi_r{}'\phi_s)\}_0^R \\
&\quad + \{-v_r(Aw_s{}''' + A'w_s{}'' + Cv_s{}''' + C'v_s{}'' + \Omega^2 mxe \sin \beta\ \phi_s)\}_0^R \\
&\quad + \{-w_r(Bs_s{}''' + B'w_s{}'' + Av_s{}''' + A'v_s{}'' - \Omega^2 mxe \cos \beta\ \phi_s)\}_0^R \\
&\quad + \{v_r{}'(Aw_s{}'' + Cv_s{}'')\}_0^R + \{w_r{}'(Bw_s{}'' + Av_s{}'')\}_0^R \\
&\quad + \{v_s(Aw_r{}''' + A'w_r{}'' + Cv_r{}''' + C'v_r{}'' + \Omega^2 mxe \sin \beta\ \phi_r)\}_0^R \\
&\quad + \{w_s(Bw_r{}''' + B'w_r{}'' + Av_r{}''' + A'v_r{}'' - \Omega^2 mxe \cos \beta\ \phi_r)\}_0^R \\
&\quad + \{-w_s{}'(Bw_r{}'' + Av_r{}'')\}_0^R + \{-v_s{}'(Aw_r{}'' + Cv_r{}'')\}_0^R
\end{aligned} \tag{45}$$

If the boundary conditions of the system are such that the right-hand side of equation (45) is zero, then the differential eigenvalue problem as defined by equations (33), (34), and (35) is said to be a self-adjoint differential eigenvalue problem. The boundary conditions corresponding to fixed, hinge and free ends, etc. make the differential eigenvalue problem of equations (33), (34), and (35) self-adjoint, and the following orthogonality relationship can be identified.

The eigenfunctions (ϕ_r, v_r, w_r) and (ϕ_s, v_s, w_s) corresponding to the distinct eigenvalues ω_r and ω_s respectively are orthogonal in the following fashion

$$\int_0^R m\{k_m^2\phi_r\phi_s + w_r w_s + v_r v_s - e \sin\beta\ (\phi_r v_s + \phi_s v_r) + e \cos\beta\ (\phi_r w_s + \phi_s w_r)\}dx = 0 \tag{46}$$

The vanishing of the right-hand side of equation (45) can be demonstrated, for example, by considering the boundary conditions of the scissor modes. The boundary conditions given by equations (26a) and (26d) can be expressed in terms of the deflections as shown below

$$w = v = w' = v' = 0\ ,\quad GJ\phi' = -k_\phi \phi \quad \text{at} \quad x = 0 \tag{47}$$

$$\left.\begin{aligned} M_x &= GJ\phi' = 0 \\ M_y &= Bw'' + Av'' = 0 \\ M_z &= Aw'' + Cv'' = 0 \\ -V_y &= Aw''' + A'w'' + Cv''' + C'v'' + \Omega^2 mex \sin\beta\ \phi \\ -V_z &= Bw''' + B'w'' + Av''' + A'v'' - \Omega^2 mex \cos\beta\ \phi \end{aligned}\right\} \text{at} \quad x = R \tag{48}$$

By substituting equations (47) and (48) into the right-hand side of equation (45), it is obvious that it vanishes by the fact that the eigenfunctions satisfy all the boundary conditions.

When a concentrated mass M is attached to the blade at the spanwise location $x = x_M$, the orthogonality relation given by equation (46) can be rewritten as

$$\int_0^R [\{m + \delta(x - x_M)M\}\{k_m^2\phi_r\phi_s + w_r w_s + v_r v_s$$
$$- e \sin \beta\ (\phi_r v_s + \phi_s v_r) + e \cos \beta\ (\phi_r w_s + \phi_s w_r)\}]dx = 0$$

or

$$\int_0^R m\{k_m^2\phi_r\phi_s + w_r w_s + v_r v_s - e \sin \beta\ (\phi_r v_s + \phi_s v_r)$$
$$+ e \cos \beta\ (\phi_r w_s + \phi_s w_r)\}dx + M\{d^2\phi_r\phi_s + w_r w_s$$
$$+ v_r v_s - d \sin \beta\ (\phi_r v_s + \phi_s v_r)$$
$$+ d \cos \beta\ (\phi_r w_s + \phi_s w_r)\}_{x=x_M} = 0 \qquad (49)$$

The orthogonality relations given by equations (46) and (49) are valid as long as boundary conditions are self-adjoint and independent of eigenvalues.

NUMERICAL RESULTS AND DISCUSSION

The natural frequencies and associated mode shapes of the Bell Helicopter OH-58A, 206A-1 seesaw rotor blade are determined by using the transmission matrix method. The effects of the following parameters on the natural frequencies corresponding to the collective, cyclic and scissor modes with coupled flapwise bending, chordwise bending, and torsional degrees of freedom are determined:

1. Effect of the collective pitch,
2. Effect of the rotation,
3. Effect of the spanwise location of the concentrated mass,
4. Effect of the chordwise location of the concentration mass, and
5. Effect of the magnitude of the concentrated mass.

As a special subcase the effect of parameter 5 on the pure torsional frequencies of the blade is determined. The properties of the Bell Helicopter OH-58A, 206A-1 are given in table 1. The natural frequencies obtained by varying parameters 1 to 5 are presented in tables 2 to 6 for collective, cyclic and scissor modes. The mode shapes with and without the concentrated mass are given in tables 7 to 9 for collective, cyclic, and scissor modes. The pure torsional frequencies are given in table 10, and the corresponding mode shapes are shown in table 11. The percentage effects of various parameters are given in table 12.

Effect of Collective Pitch

The collective pitch significantly alters the predominantly bending natural frequencies corresponding to the collective and cyclic modes. The effect is more significant on the cyclic mode frequencies compared to the collective mode frequencies. The predominantly torsion and rigid body mode frequencies are not affected by the variation of collective pitch. The scissor mode frequencies are not at all affected by collective pitch. However, the mode shapes including the scissor modes will be altered significantly by the variation of collective pitch. A particular mode is considered as predominant in this report (whether it is a predominantly flapwise bending or chordwise bending or torsion) by comparing the following quantities in a given mode:

1. Maximum torsional deformation in radians,

2. Maximum nondimensional flapwise deflection with respect to the semichord (w/b_0), and

3. Maximum nondimensional chordwise deflection with respect to the semichord (v/b_0).

Effect of Rotational Speed

The rotational speed significantly changes the natural frequencies and mode shapes as expected.

Effect of Magnitude of Attached Mass

The attached point mass significantly affects the predominantly torsional mode frequencies. As the magnitude of the mass is increased, the predominantly bending mode adjacent to a torsional mode will be altered to an additional torsional mode, and in such cases the frequencies and mode shapes are also affected significantly.

Effect of Spanwise Location

In this case predominantly torsional frequencies are also affected. The effect is increased as the mass is moved towards the tip. When a mass of 1.5 lb is attached at midspan, the effect on the predominantly bending frequencies is insignificant, and the first predominantly torsional mode frequency is altered by approximately 5 percent. But when the same mass is moved to the tip, in addition to the significant effect of torsional mode frequency (20%), the bending mode adjacent to the original torsional mode is altered to an additional predominantly torsional mode.

Effect of Chordwise Location

If the mass is attached nearer to elastic axis (quarter chord distance behind the leading edge), the effect on the frequencies including the torsional frequency is insignificant. When the mass is attached right at the leading edge, the effect on the bending frequencies is negligible and the torsional frequency is affected by four to five percent. When the mass is attached at semichord distance away from the leading edge, in addition to the significant effect on torsional frequency (20%), the bending mode adjacent to the original torsional frequency is altered to an additional torsional mode.

Effect of Mass on Pure Torsional Frequencies

The pure torsional frequencies are affected significantly by the addition of a concentrated mass as can be seen from table 10. The effect on the first mode frequency by addition of a 2.0 lb mass at the tip of the blade at a semichord distance away from the leading edge is 28 percent. The percentage effect decreases with the increasing number of mode.

Generalized Masses

The generalized masses due to the addition of a concentrated mass are affected due to the following two reasons

1. Due to the changes in the natural frequencies and mode shapes, and

2. Due to the additional concentrated mass, in which case the new orthogonality relation given by equation (49) should be used instead of the one given by equation (46).

The changes in the generalized masses could be significant. Hence, in the response and stability analyses using the modal analysis, the natural vibration characteristics should be recomputed accounting for the additional mass.

Finally, the description and features of the computer programs developed are given in Appendix A. The listings of the programs and sample output are given in Appendix B.

Some Pertinent Data of the Blade

1. Twist of the blade = -10.6 deg (linear)
2. Collective pitch range = 8 to 22 deg
3. Normal operating rotational speed = 354 RPM
4. Blade chord = 13 in. (uniform)
5. Span of the blade = 211.8 in.
   (Axis of rotation to the tip of the blade)
6. Control system spring rate = 225000 in.-lb/Rad
7. Distance of the blade from the axis of rotation = 18.5 in.

Table 1. Elastic properties of the blade.

No.	Station (in.)	$m \times 10^{-2}$ ($lb\text{-}sec^2/in.^2$)	$EI_1 \times 10^8$ ($lb\text{-}in.^2$)	$EI_2 \times 10^8$ ($lb\text{-}in.^2$)	$GJ \times 10^8$ ($lb\text{-}in.^2$)	e (in.)	$mk_{m_1}^2 \times 10^{-2}$ ($lb\text{-}sec^2$)	$mk_{m_2}^2 \times 10^{-2}$ ($lb\text{-}sec^2$) Through Center of Gravity
1	0.0	0.176	0.77	2.68	0.924	0.0	0.0	0.0
2	1.0	0.176	0.77	2.68	0.924	0.0	0.0	0.0
3	2.1	0.699	5.16	6.50	6.19	0.0	0.0	0.0
4	3.0	0.699	5.16	6.50	6.19	0.0	0.0	0.0
5	3.5	0.404	1.16	1.16	1.39	0.0	0.0	0.0
6	5.5	0.404	1.38	1.42	1.66	0.0	0.0	0.0
7	8.25	0.287	1.38	1.42	1.66	0.0	0.0	0.0
8	8.5	0.443	1.38	1.42	1.66	0.0	0.0	0.0
9	10.25	0.616	4.32	5.18	5.18	0.0	0.0	0.0
10	11.0	0.616	4.32	5.18	5.18	0.0	0.0	0.0
11	12.0	0.443	3.19	2.00	3.83	0.0	0.0	0.0
12	13.5	0.443	3.19	2.00	3.83	0.0	0.0	0.0
13	14.0	0.303	3.10	1.17	3.72	0.0	0.0	0.0
14	16.5	0.217	2.56	1.17	3.07	0.0	0.0	0.0
15	17.5	0.388	2.56	1.17	3.07	0.0	0.0	0.0
16	18.5	0.246	1.108	1.17	1.33	0.0	0.181	1.10
17	20.0	0.254	0.62	4.15	1.0	-0.1	0.172	1.18
18	22.0	0.254	0.58	4.50	0.85	-0.15	0.166	1.23
19	30.0	0.176	0.28	5.00	0.38	-0.20	0.091	1.33

(continued)

Table 1. Elastic properties of the blade (concluded).

No.	Station (in.)	$m \times 10^{-2}$ (lb-sec^2/in.^{2})	$EI_1 \times 10^8$ (lb-in.^{2})	$EI_2 \times 10^8$ (lb-in.^{2})	$GJ \times 10^8$ (lb-in.^{2})	e (in.)	$mk_{m_1}^2 \times 10^{-2}$ (lb-sec^2)	$mk_{m_2}^2 \times 10^{-2}$ (lb-sec^2) Through Center of Gravity
20	35.0	0.148	0.20	5.00	0.25	-0.15	0.057	1.32
21	40.0	0.122	0.14	4.60	0.18	-0.25	0.041	1.26
22	45.0	0.104	0.11	4.25	0.135	-0.16	0.030	1.18
23	50.0	0.085	0.085	3.95	0.108	-0.16	0.023	1.07
24	55.0	0.075	0.065	3.65	0.08	-0.16	0.018	0.983
25	60.0	0.066	0.0582	3.375	0.0698	-0.16	0.015	0.867
26	90.0	0.066	0.0582	2.40	0.0698	-0.40	0.015	0.681
27	102.0	0.211	0.0626	2.38	0.0698	0.39	0.034	0.820
28	110.0	0.211	0.0626	2.38	0.0698	0.39	0.034	0.820
29	120.0	0.066	0.0582	2.34	0.0698	-0.30	0.015	0.681
30	180.0	0.066	0.0582	2.34	0.0698	-0.30	0.015	0.681
31	188.0	0.207	0.0756	2.83	0.0698	0.76	0.024	0.839
32	205.0	0.207	0.0756	2.83	0.0698	0.76	0.024	0.839
33	207.0	0.066	0.0582	2.34	0.0698	-0.30	0.015	0.681
34	209.0	0.066	0.0582	2.34	0.0698	-0.03	0.015	0.681
35	211.8	0.238	0.0582	2.34	0.0698	-0.30	0.015	0.681

Table 2. Effect of collective pitch on natural frequencies, $\Omega = M = 0$.

Nature of the Modes	Collective Pitch (Degrees)	Natural Frequencies (Rad/sec), FB = Flapwise Bending, CB = Chordwise Bending, T = Torsion, RB = Rigid Body Mode						
		I Mode	II Mode	III Mode	IV Mode	V Mode	VI Mode	VII Mode
Collective Modes	0	0.0000 RB	8.0043 FB	50.5391 FB	151.1314 FB	163.2120 CB	295.3516 FB	329.0782 T
	8	0.0000 RB	8.0194 FB	50.8441 FB	148.5486 FB	165.3739 CB	295.7274 FB	329.0797 T
	15	0.0000 RB	8.1359 FB	51.0740 FB	145.8596 FB	167.3722 CB	295.1760 FB	329.0792 T
	22	0.0000 RB	8.3805 FB	51.2605 FB	142.7966 FB	169.4166 CB	293.6876 FB	329.0767 T
Cyclic Modes	0	0.0000 RB	24.2381 FB	37.8552 CB	95.3712 FB	191.5022 FB	220.5889 CB	329.0075 T
	8	0.0000 RB	24.0152 FB	36.6160 CB	99.3403 FB	188.9661 FB	222.6689 CB	329.0083 T
	15	0.0000 RB	20.7326 CB	39.0471 FB	105.9727 FB	185.1393 FB	229.0445 CB	329.0115 T
	22	0.0000 RB	17.4641 CB	41.7499 FB	114.0819 FB	181.1205 FB	237.8682 CB	329.0167 T
Scissor Modes	0	7.9954 FB	36.3463 CB	51.4956 FB	153.0226 FB	220.4402 CB	295.7369 FB	329.0796 T
	8	7.9954 FB	36.3463 CB	51.4956 FB	153.0226 FB	220.4402 CB	295.7369 FB	329.0796 T
	15	7.9954 FB	36.3463 CB	51.4956 FB	153.0226 FB	220.4402 CB	295.7369 FB	329.0796 T
	22	7.9954 FB	36.3463 CB	51.4956 FB	153.0226 FB	220.4402 CB	295.7369 FB	329.0796 T

Table 3. Effect of rotational speed on natural frequencies, collective pitch = 15°, M = 0 .

Nature of the Modes	Rotational Speed (RPM)	Natural Frequencies (Rad/sec), FB = Flapwise Bending, CB = Chordwise Bending, T = Torsion, RB = Rigid Body Mode						
		I Mode	II Mode	III Mode	IV Mode	V Mode	VI Mode	VII Mode
Collective Modes	0	0.0000 RB	8.1359 FB	51.0740 FB	145.8596 FB	167.3722 CB	295.1760 FB	329.0792 T
	90	14.0456 FB	57.9631 FB	150.8230 FB	172.0067 FB	302.9627 FB	329.1637 T	455.6245 FB
	180	23.8246 FB	74.6751 FB	160.2750 CB	188.9446 FB	324.5546 FB	329.7059 T	475.8608 FB
	270	33.9126 FB	95.7667 FB	169.7463 CB	217.7807 FB	329.5538 T	357.6205 FB	506.7043 FB
	354	43.2701 FB	117.0175 FB	179.5617 CB	250.7197 FB	330.0171 T	394.2283 FB	541.7232 FB
Cyclic Modes	0	0.0000 RB	20.7326 CB	39.0471 FB	105.9727 FB	185.1393 FB	229.0445 CB	329.0115 T
	90	9.4152 RB	26.5578 CB	43.2346 FB	114.9449 FB	191.1030 FB	232.3227 CB	329.0834 T
	180	18.8336 RB	32.5008 CB	57.6741 FB	138.2622 FB	205.2044 FB	244.0435 FB	329.2929 T
	270	28.2544 RB	35.4246 CB	77.9037 FB	169.5433 FB	219.6862 CB	267.9191 FB	329.6326 T
	354	37.0350 RB	37.3960 CB	98.3988 FB	201.6647 FB	232.0615 CB	299.7101 FB	330.0657 T
Scissor Modes	0	7.9954 FB	36.3463 CB	51.4956 FB	153.0226 FB	220.4402 CB	295.7369 FB	329.0796 T
	90	13.7503 FB	36.8006 CB	58.1819 FB	160.9285 FB	221.7586 CB	303.6304 FB	329.1647 T
	180	23.1564 FB	37.8225 CB	74.7200 FB	182.1923 FB	225.8361 CB	325.4567 FB	329.8134 T
	270	32.1352 FB	39.8275 CB	95.7662 FB	210.8494 FB	233.9084 CB	329.5586 T	359.2020 FB
	354	37.3392 CB	45.0472 FB	117.0532 FB	232.0153 CB	253.7052 FB	330.0189 T	396.6472 FB

Table 4. Effect of magnitude of attached mass on natural frequencies, x_M = 211.8 in., d = 9.7 in., Ω = 354 RPM, collective pitch = 15°.

Nature of the Modes	Magnitude of Attached Mass (lb)	Natural Frequencies (Rad/sec), FB = Flapwise Bending, CB = Chordwise Bending, T = Torsion, RB = Rigid Body Mode						
		I Mode	II Mode	III Mode	IV Mode	V Mode	VI Mode	VII Mode
Collective Modes	0.0	43.2701 FB	117.0175 FB	179.5617 CB	250.7197 FB	330.0171 T	394.2283 RB	541.7232 FB
	0.5	43.2176 FB	117.1580 FB	178.6105 CB	249.0986 FB	296.1035 T	393.7128 FB	540.2327 FB
	1.0	43.1670 FB	117.3013 FB	177.6238 CB	245.1096 FB	275.0553 T	393.4286 FB	539.4606 FB
	1.5	43.1181 FB	117.4469 FB	176.5766 CB	237.0735 T	264.9626 T	393.2712 FB	539.0953 FB
	2.0	43.0709 FB	117.5927 FB	175.4395 CB	227.2062 T	261.3449 T	393.2133 FB	538.9866 FB
Cyclic Modes	0.0	37.0350 RB	37.3960 CB	98.3988 FB	201.6647 FB	232.0615 CB	299.7101 FB	330.0657 T
	0.5	37.0355 RB	37.1011 CB	98.5738 FB	201.5790 FB	230.8393 CB	290.0559 T	304.7066 T
	1.0	36.8128 CB	37.0356 RB	98.7570 FB	201.1643 FB	229.6229 CB	267.6030 T	303.1069 T
	1.5	36.5309 CB	37.0360 RB	98.9514 FB	200.2380 FB	228.2941 CB	250.8627 T	303.3292 FB
	2.0	36.2549 CB	37.0367 RB	99.1519 FB	198.5490 FB	226.5731 CB	239.6249 T	303.8008 FB
Scissor Modes	0.0	37.3392 CB	45.0472 FB	117.0532 FB	232.0153 CB	253.7052 FB	330.0189 T	396.6472 FB
	0.5	37.0930 CB	44.9039 FB	117.1955 FB	230.8191 CB	251.8094 FB	296.3588 T	396.2658 FB
	1.0	36.8515 CB	44.7684 FB	117.3424 FB	229.6282 CB	246.5482 FB	276.6018 T	396.1683 FB
	1.5	36.6091 CB	44.6453 FB	117.4931 FB	228.0837 CB	237.0498 T	268.3211 T	396.2110 FB
	2.0	36.3702 CB	44.5328 FB	117.6440 FB	223.0286 T	230.0157 FB	265.6692 T	396.3542 FB

Table 5. Effect of spanwise location of concentrated mass on natural frequencies, $M = 1.5$ lb, $\Omega = 354$ RPM, collective pitch = 15°, chordwise distance of mass from leading edge = 6.5 in.

Nature of the Modes	Spanwise Location and Chordwise Location From Elastic Axis (in.)	Natural Frequencies (Rad/sec), FB = Flapwise Bending, CB = Chordwise Bending, T = Torsion, RB = Rigid Body Mode						
		I Mode	II Mode	III Mode	IV Mode	V Mode	VI Mode	VII Mode
Collective Modes	$M = 0$	43.2701 FB	117.0175 FB	179.5617 CB	250.7197 FB	330.0171 T	394.2283 FB	541.7232 FB
	$d = 9.7$, $x_M = 211.8$	43.1181 FB	117.4469 FB	176.5766 CB	237.0735 T	264.9626 T	393.2712 FB	539.0953 FB
	$d = 9.7$, $x_M = 158.85$	43.2211 FB	117.4908 FB	179.9896 CB	242.7804 FB	275.9257 T	395.5335 FB	540.7139 FB
	$d = 9.59$, $x_M = 105.9$	43.2764 FB	115.0264 FB	177.2461 CB	249.2111 FB	315.2340 T	392.9685 FB	541.0933 FB
Cyclic Modes	$M = 0$	37.0350 RB	37.9360 CB	98.3988 FB	201.6647 FB	232.0615 CB	299.7101 FB	330.0657 T
	$d = 9.7$, $x_M = 211.8$	36.5309 CB	37.0360 RB	98.9514 FB	200.2380 FB	228.2941 CB	250.8627 T	303.3292 FB
	$d = 9.70$, $x_M = 158.85$	37.0065 CB	37.0313 RB	99.2492 FB	199.8117 FB	230.5302 CB	273.8252 T	298.0209 FB
	$d = 9.59$, $x_M = 105.9$	37.0365 RB	37.3022 CB	97.2172 FB	201.9483 FB	228.1099 CB	295.1667 FB	316.5346 T
Scissor Modes	$M = 0$	37.3392 CB	45.0472 FB	117.0532 FB	232.0153 CB	253.7052 FB	330.0189 T	396.6472 FB
	$d = 9.7$, $x_M = 211.8$	36.6091 CB	44.6453 FB	117.4931 FB	228.0837 CB	237.0498 T	268.3211 T	396.2110 FB
	$d = 9.7$, $x_M = 158.85$	37.0101 CB	44.6684 FB	117.5263 FB	229.6600 FB	248.1115 FB	276.1423 T	397.8221 FB
	$d = 9.59$, $x_M = 105.9$	37.2637 CB	45.0130 FB	115.0627 FB	228.0991 CB	251.8251 FB	315.3651 T	395.4402 FB

Table 6. Effect of chordwise location of concentrated mass on natural frequencies, $M = 1.5$ lb, $\Omega = 354$ RPM, $x_M = 211.8$ in.

Nature of the Modes	Chordwise Locations From Shear Center and Leading Edge (in.)	Natural Frequencies (Rad/sec), FB = Flapwise Bending, CB = Chordwise Bending, T = Torsion, RB = Rigid Body Mode						
		I Mode	II Mode	III Mode	IV Mode	V Mode	VI Mode	VII Mode
Collective Modes	$M = 0$	43.2701 FB	117.0175 FB	179.5617 CB	250.7197 FB	330.0171 T	394.2283 FB	241.7232 FB
	$d = 9.7$ $d_{\ell e} = 6.5$	43.1181 FB	117.4469 FB	176.5766 CB	237.0735 T	264.9626 T	393.2712 FB	539.0953 FB
	$d = 6.45$ $d_{\ell e} = 3.25$	43.1181 FB	117.4893 FB	176.8773 CB	245.2217 FB	289.4397 T	393.0761 FB	537.3240 FB
	$d = 3.2$ $d_{\ell e} = 0.0$	43.1181 FB	117.5110 FB	177.0137 CB	247.0820 FB	315.1198 T	392.4943 FB	534.8055 FB
	$d = 0.05$ $d_{\ell e} = -3.25$	43.1181 FB	117.5159 FB	177.0576 CB	247.4319 FB	328.6644 T	391.4030 FB	531.8813 FB
Cyclic Modes	$M = 0$	37.0350 RB	37.9360 CB	98.3988 FB	201.6647 FB	232.0615 CB	299.7101 FB	330.0657 T
	$d = 9.7$ $d_{\ell e} = 6.5$	36.5309 CB	37.0360 RB	98.9514 FB	200.2380 FB	228.2941 CB	250.8627 T	303.3292 FB
	$d = 6.45$ $d_{\ell e} = 3.25$	36.5351 CB	37.0359 RB	98.9614 FB	201.2544 FB	228.7224 CB	278.7150 T	306.7561 T
	$d = 3.2$ $d_{\ell e} = 0.0$	36.5377 CB	37.0359 RB	98.9669 FB	201.6011 FB	228.8304 CB	293.4813 FB	318.7820 T
	$d = -0.05$ $d_{\ell e} = -3.25$	36.5385 CB	37.0359 RB	98.9684 FB	201.6506 FB	228.8627 CB	296.3261 FB	329.1355 T
Scissor Modes	$M = 0$	37.3392 CB	45.0472 FB	117.0532 FB	232.0153 CB	253.7052 FB	330.0189 T	396.6472 FB
	$d = 9.7$ $d_{\ell e} = 6.5$	36.6091 CB	44.6453 FB	117.4931 FB	228.0837 CB	237.0498 T	268.3211 T	396.2110 FB
	$d = 6.45$ $d_{\ell e} = 3.25$	36.6133 CB	44.6452 FB	117.5322 FB	228.7281 CB	247.1323 FB	290.4035 T	395.8670 FB
	$d = 3.2$ $d_{\ell e} = 0.0$	36.6159 CB	44.6451 FB	117.5522 FB	228.8264 CB	249.7660 FB	315.2585 T	395.0400 FB
	$d = -0.05$ $d_{\ell e} = -3.25$	36.6168 CB	44.6451 FB	117.5567 FB	228.8536 CB	250.2792 FB	328.6690 T	393.6814 FB

Table 7. Collective mode shapes, Ω = 354 RPM, collective pitch = 15°, x_M = 211.8 in., d = 9.7 in., $d_{\ell e}$ = 6.5 in.

Station $\frac{x}{R}$	1. w/b_0 2. v/b_0 3. ϕ (Rad)	I Mode		II Mode		III Mode		IV Mode	
		M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb
0.0	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0000	0.0018	-0.0016	0.0203	-0.0005	-0.0618	-0.0150	-0.2407
0.1	1	0.0078	0.0081	-0.0283	-0.0314	0.0090	0.0078	0.0432	-0.0177
	2	0.0147	0.0147	0.0079	0.0110	-0.2986	-0.3264	0.0302	0.0173
	3	0.0000	0.0017	-0.0015	0.0199	-0.0005	-0.0604	-0.0147	-0.2353
0.2	1	0.0425	0.0437	-0.1501	-0.1653	0.0297	0.0228	0.2128	-0.0867
	2	0.0252	0.0252	0.0295	0.0367	-0.5535	-0.6061	0.0327	0.0395
	3	0.0000	0.0014	-0.0013	0.0167	-0.0003	-0.0506	-0.0122	-0.1965
0.3	1	0.1233	0.1257	-0.4123	-0.4509	0.0243	0.0051	0.4985	-0.2049
	2	0.0255	0.0254	0.0807	0.0941	-0.7467	-0.8195	-0.0002	0.0688
	3	0.0000	0.0005	-0.0006	0.0058	0.0005	-0.0163	-0.0025	-0.0608
0.4	1	0.2324	0.2353	-0.7050	-0.7687	-0.0315	-0.0622	0.6255	-0.2700
	2	0.0205	0.0205	0.1341	0.1534	-0.8356	-0.9233	-0.0140	0.0806
	3	0.0000	-0.0007	0.0000	-0.0090	0.0019	0.0295	0.0120	0.1189
0.5	1	0.3513	0.3543	-0.8845	-0.9671	-0.1182	-0.1525	0.3662	-0.1903
	2	0.0144	0.0146	0.1601	0.1828	-0.7851	-0.8808	0.0253	0.0565
	3	0.0000	-0.0019	0.0002	-0.0242	0.0037	0.0752	0.0286	0.2948

(continued)

Table 7. Collective mode shapes, Ω = 354 RPM, collective pitch = 15°, x_M = 211.8 in., d = 9.7 in., $d_{\ell e}$ = 6.5 in. (continued).

Station $\frac{x}{R}$	1. w/b_0 2. v/b_0 3. ϕ (Rad)	I Mode		II Mode		III Mode		IV Mode	
		M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb
0.6	1	0.4758	0.4786	-0.8411	-0.9304	-0.1972	-0.2250	-0.2654	0.0507
	2	0.0078	0.0084	0.1411	0.1640	-0.5872	-0.6819	0.1052	-0.0030
	3	0.0000	-0.0031	0.0022	-0.0372	0.0051	0.1195	0.0402	0.4647
0.7	1	0.6041	0.6064	-0.5868	-0.6732	-0.2099	-0.2254	-0.8230	0.3103
	2	0.0011	0.0021	0.0843	0.1049	-0.2717	-0.3557	0.1513	-0.0695
	3	0.0000	-0.0043	0.0036	-0.0507	0.0062	0.1619	0.0492	0.6242
0.8	1	0.7347	0.7364	-0.1633	-0.2371	-0.1158	-0.1201	-0.8487	0.4029
	2	-0.0056	-0.0043	0.0009	0.0170	0.1220	0.0585	0.1076	-0.1137
	3	0.0000	-0.0056	0.0039	-0.0650	0.0069	0.2018	0.0530	0.7707
0.9	1	0.8670	0.8679	0.3885	0.3432	0.0901	0.0889	-0.1467	0.1627
	2	-0.0125	-0.0106	-0.0996	-0.0903	0.5559	0.5217	-0.0326	-0.1168
	3	0.0000	-0.0068	0.0038	-0.0796	0.0071	0.2387	0.0518	0.9006
1.0	1	1.0000	1.0000	1.0000	1.0000	0.3546	0.3522	1.0000	-0.3817
	2	-0.0194	-0.0170	-0.2067	-0.2062	1.0000	1.0000	-0.2214	-0.0869
	3	0.0000	-0.0080	0.0039	-0.0928	0.0073	0.2716	0.0532	1.0000

(continued)

Table 7. Collective mode shapes, Ω = 354 RPM, collective pitch = 15°, x_M = 211.8 in., d = 9.7 in., $d_{\ell e}$ = 6.5 in. (continued).

Station $\frac{x}{R}$	1. w/b_0 2. v/b_0 3. ϕ (Rad)	V Mode		VI Mode		VII Mode	
		M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb
0.0	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	-0.2751	-0.2507	0.0747	0.1119	-0.0202	-0.1052
0.1	1	0.0013	0.0321	0.1467	0.1468	0.1742	0.1786
	2	-0.0008	0.0443	-0.0711	-0.0822	-0.2600	-0.2322
	3	-0.2688	-0.2450	0.0731	0.1094	-0.0197	-0.1028
0.2	1	0.0042	0.1575	0.6085	0.6071	0.5465	0.5714
	2	-0.0014	0.0628	-0.1664	-0.1840	-0.4007	-0.3651
	3	-0.2221	-0.2039	0.0608	0.0905	-0.0134	-0.0798
0.3	1	0.0042	0.3659	1.0000	1.0000	0.3808	0.4380
	2	-0.0011	0.0493	-0.2276	-0.2480	-0.3050	-0.2792
	3	-0.0520	-0.0575	0.0201	0.0254	0.0195	0.0194
0.4	1	-0.0001	0.4532	-0.5614	-0.5787	-0.3845	-0.3479
	2	0.0002	0.0407	-0.0946	-0.1155	0.0223	0.0340
	3	0.1748	0.1386	-0.0271	-0.0543	0.0649	0.1527
0.5	1	-0.0062	0.2540	-0.3290	-0.3106	-0.3187	-0.3590
	2	0.0019	0.0586	0.1254	0.1134	0.2155	0.2152
	3	0.3925	0.3320	-0.0644	-0.1221	0.0970	0.2615

(continued)

Table 7. Collective mode shapes, Ω = 354 RPM, collective pitch = 15°, x_M = 211.8 in., d = 9.7 in., $d_{\ell e}$ = 6.5 in. (concluded).

Station $\frac{x}{R}$	1. w/b_0 2. v/b_0 3. ϕ (Rad)	V Mode		VI Mode		VII Mode	
		M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb
0.6	1	-0.0130	-0.2145	-0.6049	-0.6346	0.7251	0.6825
	2	0.0035	0.0928	0.2113	0.2214	0.1618	0.1392
	3	0.5881	0.5108	-0.0887	-0.1728	0.1314	0.3495
0.7	1	0.0038	-0.6065	-0.0474	-0.1228	0.8185	0.8864
	2	0.0011	0.0904	0.1263	0.1583	0.1366	0.0867
	3	0.7547	0.6726	-0.1118	-0.2167	0.1632	0.4060
0.8	1	0.0292	-0.5974	0.5650	0.5245	-0.3048	-0.1986
	2	-0.0031	0.0171	-0.0054	0.0333	0.1667	0.1172
	3	0.8849	0.8110	-0.1284	-0.2480	0.1905	0.4282
0.9	1	-0.0199	-0.1062	0.2970	0.3561	-0.5160	-0.5629
	2	0.0008	-0.1196	-0.0603	-0.0297	0.0145	0.0055
	3	0.9717	0.9219	-0.1321	-0.2587	0.1930	0.3970
1.0	1	-0.2020	0.4709	-0.7969	-0.6947	1.0000	0.8347
	2	0.0178	-0.2805	-0.0405	-0.0187	-0.3401	-0.3019
	3	1.0000	1.0000	-0.1374	-0.2640	0.2047	0.3502

Table 8. Cyclic mode shapes, Ω = 354 RPM, collective pitch = 15°, x_M = 211.8 in., d = 9.7 in., $d_{\ell e}$ = 6.5 in.

Station $\frac{x}{R}$	1. w/b_0 2. v/b_0 3. ϕ (Rad)	I Mode		II Mode		III Mode		IV Mode	
		M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb
0.0	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0000	-0.0031	0.0000	0.0012	-0.0016	0.0097	0.0003	0.1955
0.1	1	0.0996	0.0385	0.0421	0.1003	-0.2188	-0.2379	0.5304	0.5293
	2	-0.0001	0.0221	0.0220	0.0002	0.0124	0.0132	-0.0075	0.0059
	3	0.0000	-0.0031	0.0000	0.0012	-0.0016	0.0095	0.0003	0.1911
0.2	1	0.1992	0.0648	0.0718	0.2005	-0.4324	-0.4699	0.9375	0.9346
	2	-0.0005	0.0783	0.0780	0.0008	0.0419	0.0444	-0.0086	0.0347
	3	0.0000	-0.0026	0.0000	0.0010	-0.0014	0.0079	0.0008	0.1605
0.3	1	0.2990	0.0618	0.0714	0.3004	-0.6168	-0.6703	0.9372	0.9445
	2	-0.0010	0.1557	0.1553	0.0015	0.0732	0.0775	0.0621	0.1354
	3	0.0000	-0.0010	0.0000	0.0004	-0.0009	0.0024	0.0047	0.0560
0.4	1	0.3990	0.0450	0.0563	0.4002	-0.7350	-0.8008	0.4932	0.5353
	2	-0.0017	0.2504	0.2501	0.0024	0.0995	0.1054	0.1896	0.2847
	3	0.0000	0.0012	0.0000	-0.0005	-0.0006	-0.0053	0.0116	-0.0809
0.5	1	0.4990	0.0277	0.0402	0.5000	-0.7465	-0.8193	-0.1578	-0.0819
	2	-0.0024	0.3586	0.3586	0.0035	0.1140	0.1212	0.3003	0.4074
	3	0.0000	0.0034	0.0000	-0.0013	-0.0007	-0.0133	0.0200	-0.2146

(continued)

Table 8. Cyclic mode shapes, Ω = 354 RPM, collective pitch = 15°, x_M = 211.8 in., d = 9.7 in., $d_{\ell e}$ = 6.5 in. (continued).

Station $\frac{x}{R}$	1. w/b_0 2. v/b_0 3. ϕ (Rad)	I Mode		II Mode		III Mode		IV Mode	
		M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb
0.6	1	0.5990	0.0177	0.0252	0.5999	-0.6026	-0.6740	-0.7359	-0.6676
	2	-0.0032	0.4773	0.4774	0.0046	0.1085	0.1160	0.3335	0.4439
	3	0.0000	0.0055	0.0000	-0.0021	0.0005	-0.0200	0.0286	-0.3438
0.7	1	0.6992	-0.0025	0.0119	0.6999	-0.3238	-0.3877	-0.9856	-0.9839
	2	-0.0041	0.6032	0.6034	0.0058	0.0868	0.0942	0.2587	0.3650
	3	0.0000	0.0077	0.0001	-0.0030	0.0012	-0.0270	0.0349	-0.4686
0.8	1	0.7993	-0.0147	0.0004	0.7999	0.0569	0.0060	-0.7515	-0.8514
	2	-0.0050	0.7337	0.7339	0.0071	0.0543	0.0612	0.0757	0.1680
	3	0.0000	0.0098	0.0001	-0.0038	0.0014	-0.0345	0.0372	-0.5886
0.9	1	0.8996	-0.0255	-0.0097	0.9000	0.5117	0.4820	-0.0213	-0.1742
	2	-0.0059	0.8665	0.8666	0.0083	0.0148	0.0205	-0.1904	-0.1288
	3	0.0000	0.0120	0.0001	-0.0047	0.0013	-0.0422	0.0365	-0.7006
1.0	1	1.0000	-0.0353	-0.0193	1.0000	1.0000	1.0000	0.9786	0.8851
	2	-0.0069	1.0000	1.0000	0.0097	-0.0192	-0.0239	-0.4915	-0.4771
	3	0.0000	0.0141	0.0001	-0.0055	0.0014	-0.0493	0.0372	-0.7923

(continued)

Table 8. Cyclic mode shapes, Ω = 354 RPM, collective pitch = 15°, x_M = 211.8 in., d = 9.7 in., $d_{\ell e}$ = 6.5 in. (continued)

Station $\frac{x}{R}$	1. w/b_0 2. v/b_0 3. ϕ (Rad)	V Mode		VI Mode		VII Mode	
		M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb
0.0	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0081	0.1587	-0.0294	-0.2403	-0.2757	-0.2463
0.1	1	0.0139	-0.0887	-0.3716	0.1193	-0.0069	-0.4341
	2	0.1241	0.1303	0.0677	0.0155	0.0009	0.0862
	3	0.0079	0.1551	-0.0287	-0.2348	-0.2695	-0.2408
0.2	1	-0.0425	-0.2197	-0.5957	0.1909	-0.0122	-0.6980
	2	0.4039	0.4207	0.1872	0.0554	0.0025	0.2406
	3	0.0066	0.1294	-0.0247	-0.1957	-0.2227	-0.2007
0.3	1	-0.1911	-0.3535	-0.3282	0.1322	-0.0110	-0.3914
	2	0.7141	0.7281	0.2104	0.1206	0.0030	0.2824
	3	0.0014	0.0381	-0.0133	-0.0574	-0.0524	-0.0596
0.4	1	-0.2369	-0.2995	0.3086	-0.0154	-0.0012	0.3505
	2	0.9448	0.9478	0.1485	0.1864	0.0018	0.2184
	3	-0.0065	-0.0845	0.0009	0.1266	0.1747	0.1270
0.5	1	-0.0539	0.0007	0.6479	-0.1297	0.0053	0.7577
	2	0.9905	0.9914	0.1110	0.2108	0.0011	0.1698
	3	-0.0157	-0.2066	0.0155	0.3073	0.3926	0.3084

(continued)

Table 8. Cyclic mode shapes, Ω = 354 RPM, collective pitch = 15°, x_M = 211.8 in., d = 9.7 in., $d_{\ell e}$ = 6.5 in. (concluded).

Station $\frac{x}{R}$	1. w/b_0 2. v/b_0 3. ϕ (Rad)	V Mode		VI Mode		VII Mode	
		M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb
0.6	1	0.3168	0.4480	0.2107	-0.1180	-0.0052	0.2619
	2	0.8085	0.8257	0.1574	0.1691	0.0029	0.2055
	3	-0.0221	-0.3215	0.0205	0.4803	0.5883	0.4646
0.7	1	0.5984	0.7429	-0.5480	-0.0032	-0.0025	-0.6102
	2	0.4563	0.5061	0.2021	0.0724	0.0021	0.2294
	3	-0.0272	-0.4279	0.0248	0.6405	0.7549	0.6016
0.8	1	0.5273	0.6314	-0.8623	0.1176	0.0145	-0.9897
	2	0.0177	0.1021	0.1438	-0.0526	-0.0013	0.1317
	3	-0.0296	-0.5220	0.0246	0.7844	0.8850	0.7100
0.9	1	0.0036	0.0402	-0.2540	0.1020	-0.0251	-0.3509
	2	-0.4637	-0.3336	-0.0499	-0.1762	0.0012	-0.1172
	3	-0.0293	-0.6021	0.0188	0.9084	0.9717	0.7836
1.0	1	-0.7863	-0.8266	1.0000	-0.1027	-0.1839	1.0000
	2	-0.9385	-0.7731	-0.3179	-0.2839	0.0153	-0.4464
	3	-0.0301	-0.6674	0.0195	1.0000	1.0000	0.8271

Table 9. Scissor mode shapes, Ω = 354 RPM, collective pitch = 15°, x_M = 211.8 in., d = 9.70 in., $d_{\ell e}$ = 6.5 in.

Station $\frac{x}{R}$	1. w/b_0 2. v/b_0 3. ϕ (Rad)	I Mode		II Mode		III Mode		IV Mode	
		M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb
0.0	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0000	-0.0036	0.0000	0.0003	-0.0015	0.0197	0.0088	-0.2398
0.1	1	-0.0002	0.0003	0.0101	0.0101	-0.0278	-0.0307	-0.0065	-0.0134
	2	0.0219	0.0222	0.0148	0.0131	0.0018	0.0024	0.1251	-0.0634
	3	0.0000	-0.0035	0.0000	0.0002	-0.0015	0.0193	0.0086	-0.2344
0.2	1	-0.0133	-0.0106	0.0468	0.0474	-0.1486	-0.1632	-0.0872	-0.0388
	2	0.0778	0.0786	0.0472	0.0412	0.0202	0.0235	0.4076	-0.1999
	3	0.0000	-0.0029	0.0000	0.0002	-0.0013	0.0162	0.0076	-0.1958
0.3	1	-0.0573	-0.0491	0.1229	0.1253	-0.4099	-0.4477	-0.2526	-0.0702
	2	0.1552	0.1559	0.0810	0.0693	0.0698	0.0784	0.7198	-0.3418
	3	0.0000	-0.0011	0.0000	0.0001	-0.0006	0.0056	0.0013	-0.0605
0.4	1	-0.1164	-0.1003	0.2258	0.2295	-0.7020	-0.7646	-0.2895	-0.1143
	2	0.2501	0.2504	0.1198	0.1015	0.1236	0.1377	0.9482	-0.4549
	3	0.0000	0.0014	0.0000	-0.0001	0.0000	0.0056	-0.0076	0.1191
0.5	1	-0.1767	-0.1517	0.3406	0.3450	-0.8812	-0.9626	-0.0692	-0.1464
	2	0.3586	0.3586	0.1666	0.1407	0.1518	0.1700	0.9879	-0.5024
	3	0.0000	0.0038	0.0000	-0.0003	0.0002	-0.0234	-0.0179	0.2959

(continued)

Table 9. Scissor mode shapes, Ω = 354 RPM, collective pitch = 15°, x_M = 211.8 in., d = 9.70 in., $d_{\ell e}$ = 6.5 in. (continued).

Station $\frac{x}{R}$	1. w/b_0 2. v/b_0 3. ϕ (Rad)	I Mode		II Mode		III Mode		IV Mode	
		M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb
0.6	1	-0.2360	-0.2017	0.4636	0.4679	-0.8382	-0.9264	0.3548	-0.1298
	2	0.4774	0.4772	0.2202	0.1859	0.1371	0.1568	0.7988	-0.4618
	3	0.0000	0.0063	0.0000	-0.0005	0.0023	-0.0360	-0.0252	0.4656
0.7	1	-0.2937	-0.2496	0.5925	0.5963	-0.5846	-0.6702	0.6733	-0.0521
	2	0.6034	0.6031	0.2785	0.2356	0.0860	0.1053	0.4437	-0.3363
	3	0.0001	0.0087	0.0000	-0.0007	0.0036	-0.0491	-0.0310	0.6248
0.8	1	-0.3495	-0.2957	0.7238	0.7285	-0.1619	-0.2351	0.5953	0.0566
	2	0.7339	0.7336	0.3399	0.2880	0.0093	0.0265	0.0034	-0.1395
	3	0.0001	0.0112	0.0001	-0.0009	0.0040	-0.0630	-0.0336	0.7703
0.9	1	-0.4041	-0.3402	0.8622	0.8636	0.3892	0.3442	0.0116	0.1111
	2	0.8666	0.8665	0.4029	0.3419	-0.0841	-0.0710	-0.4607	0.1128
	3	0.0001	0.0136	0.0001	-0.0011	0.0038	-0.0771	-0.0333	0.8987
1.0	1	-0.4581	-0.3836	1.0000	1.0000	1.0000	1.0000	-0.8695	0.0570
	2	1.0000	1.0000	0.4662	0.3962	-0.1839	-0.1768	-0.9207	0.3943
	3	0.0001	0.0161	0.0001	-0.0013	0.0039	-0.0899	-0.0342	1.0000

(continued)

Table 9. Scissor mode shapes, Ω = 354 RPM, collective pitch = 15°, x_M = 211.8 in., d = 9.70 in., $d_{\ell e}$ = 6.5 in. (continued).

Station $\frac{x}{R}$	1. w/b_0 2. v/b_0 3. ϕ (Rad)	V Mode		VI Mode		VII Mode	
		M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb
0.0	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	-0.0160	0.2139	-0.2751	-0.2517	0.0745	0.1216
0.1	1	0.0547	0.0232	0.0012	0.0392	0.1396	0.1387
	2	0.0432	-0.0938	-0.0004	0.0360	-0.0284	-0.0320
	3	-0.0156	0.2090	-0.2688	-0.2460	0.0728	0.1188
0.2	1	0.2463	0.1558	0.0038	0.1712	0.5951	0.5920
	2	0.1147	-0.3141	-0.0012	0.0983	-0.1297	-0.1402
	3	-0.0129	0.1746	-0.2221	-0.2046	0.0604	0.0982
0.3	1	0.5506	0.4064	0.0037	0.3713	1.0000	1.0000
	2	0.1607	-0.5698	-0.0014	0.1443	-0.2209	-0.2384
	3	-0.0025	0.0544	-0.0520	-0.0570	0.0195	0.0260
0.4	1	0.6904	0.5110	-0.0004	0.4530	0.5679	0.5848
	2	0.2153	-0.7538	-0.0006	0.1918	-0.1274	-0.1513
	3	0.0130	-0.1038	0.1748	0.1406	-0.0279	-0.0629
0.5	1	0.4264	0.2719	-0.0063	0.2642	-0.3385	-0.3224
	2	0.2873	-0.7704	0.0007	0.2393	0.0624	0.0419
	3	0.0306	-0.2573	0.3925	0.3354	-0.0651	-0.1389

(continued)

Table 9. Scissor mode shapes, Ω = 354 RPM, collective pitch = 15°, x_M = 211.8 in., d = 9.70 in., $d_{\ell e}$ = 6.5 in. (concluded)

Station $\frac{x}{R}$	1. w/b_0 2. v/b_0 3. ϕ (Rad)	V Mode		VI Mode		VII Mode	
		M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb
0.6	1	-0.2356	-0.3005	-0.0130	-0.1799	-0.6291	-0.6602
	2	0.3458	-0.5884	0.0022	0.2615	0.1380	0.1360
	3	0.0430	-0.4077	-0.0892	0.5151	-0.0892	-0.1962
0.7	1	-0.8320	-0.8314	0.0037	-0.5518	-0.0630	-0.1374
	2	0.3151	-0.2804	0.0001	0.2078	0.0641	0.0839
	3	0.0525	-0.5493	-0.1120	0.6773	-0.1120	-0.2451
0.8	1	-0.8808	-0.9040	0.0291	-0.5479	0.5705	0.5310
	2	0.1477	0.0658	-0.0034	0.0545	-0.0359	-0.0056
	3	0.0565	-0.6812	-0.1285	0.8153	-0.1285	-0.2792
0.9	1	-0.1730	-0.2406	-0.0199	-0.1020	0.3072	0.3658
	2	-0.1465	0.3860	0.0012	-0.1799	-0.0423	-0.0134
	3	0.0551	-0.7999	-0.1318	0.9248	-0.1318	-0.2902
1.0	1	0.7545	1.0000	-0.2017	0.5505	-0.5594	-0.7009
	2	-0.4282	0.6675	0.0190	-0.4385	0.0334	0.0614
	3	0.0565	-0.8873	-0.1371	1.0000	-0.1371	-0.2931

Table 10. Effect of magnitude of concentrated mass on torsional frequencies, Ω = 354 RPM, d = 9.70 in., x_M = 211.8 in.

Magnitude of the Mass	Frequencies (Rad/sec)			
	I Mode	II Mode	III Mode	IV Mode
0.0	335.2502	913.6214	1494.7750	2036.0238
0.5	303.0671	845.0031	1388.6847	1909.5994
1.0	277.4304	798.8885	1329.8973	1852.5323
1.5	256.8390	768.4413	1297.7905	1825.8265
2.0	240.0246	747.5134	1278.4021	1810.9621

Table 11. Pure torsional mode shapes, Ω = 354 RPM, d = 9.7 in., x_M = 211.8 in.

Station $\frac{x}{R}$	I Mode		II Mode		III Mode		IV Mode	
	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb	M = 0	M = 2.0 lb
0.0	-0.2758	-0.2423	-0.3990	-0.3941	0.3767	0.3726	-0.4141	-0.4076
0.1	-0.2695	-0.2368	-0.3897	-0.3851	0.3676	0.3637	-0.4035	-0.3975
0.2	-0.2226	-0.1976	-0.2786	-0.2914	0.1870	0.2166	-0.0988	-0.1438
0.3	-0.0513	-0.0593	0.2022	0.1019	-0.5575	-0.4164	0.8986	0.7895
0.4	0.1770	0.1246	0.7467	0.5891	-0.9313	-0.9091	0.5508	0.8170
0.5	0.3959	0.3050	0.9996	0.9207	-0.3831	-0.7277	-0.7391	-0.2955
0.6	0.5922	0.4766	0.8374	0.9937	0.5691	0.0273	-0.7048	-0.9761
0.7	0.7587	0.6361	0.3642	0.8185	1.0000	0.7720	0.4836	-0.3710
0.8	0.8876	0.7795	-0.2406	0.4434	0.5125	0.9863	0.9058	0.7147
0.9	0.9720	0.9031	-0.7554	-0.0403	-0.4430	0.5167	-0.1117	0.8571
1.0	1.0000	1.0000	-0.9481	-0.5096	-0.9081	-0.3537	-0.8792	-0.2628

Table 12. Percentage effects on the natural frequencies.

Variable Parameter	Range of the Parameter	Constant Parameters	Nature of the Mode Shapes	I Mode	II Mode	III Mode	IV Mode	V Mode	VI Mode	VII Mode
Collective Pitch	0, 22 deg.	$\Omega = 0$, $M = 0$	Collective	0.0	4.7	1.4	-5.5	3.8	-0.5	0.0
			Cyclic	0.0	-28.0	10.3	19.6	-5.4	7.8	0.0
			Scissor	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Rotational Speed	0, 354 RPM	$M = 0$	Collective	---	1338.3	251.6	71.9	97.2	33.6	64.6
			Cyclic	---	80.4	152.0	90.3	25.3	30.9	0.3
			Scissor	367.0	23.9	127.3	51.6	15.1	11.6	17.0
Magnitude of the Attached Mass	0, 2.0 lb	x_M = 211.8 in., $d_{\ell e}$ = 6.5 in., Ω = 354 RPM	Collective	-0.5	0.5	-2.3	-9.4	-20.8	-0.3	-0.5
			Cyclic	-2.1	-1.0	0.8	-1.5	-2.4	-20.1	-8.0
			Scissor	-2.6	-1.1	0.5	-3.9	-9.3	-19.5	-0.1
Spanwise Location of the Attached Mass	105.9, 211.8 in.	$d_{\ell e}$ = 6.5 in., M = 1.5 lb, Ω = 354 RPM	Collective	-0.4	2.1	-0.4	-4.9	-16.0	0.1	-0.4
			Cyclic	-1.4	-0.7	1.8	-0.9	0.1	-15.0	-4.2
			Scissor	-1.8	-0.8	2.1	0.0	-5.9	-14.9	0.2
Chordwise Location of the Attached Mass from the Leading Edge	-3.25, 6.5 in.	x_M = 211.8 in., M = 1.5 lb, Ω = 354 RPM	Collective	0.0	-0.1	-0.3	-4.2	-19.4	0.5	1.4
			Cyclic	0.0	0.0	0.0	-0.7	-0.3	-15.3	-7.8
			Scissor	0.0	0.0	0.0	-0.3	-5.3	-18.4	0.6
Magnitude of the Mass	0, 2.0 lb	x_M = 211.8 in., $d_{\ell e}$ = 6.5 in., Ω = 354 RPM	Pure Torsion	-28.4	-18.2	-14.5	-11.1	---	---	---

REFERENCES

1. Holzer, H.: Die Berechnung der Drehschwingungen. Berlin: Springer-verlag, 1921; republished by J. W. Edwards, Publisher Inc., Ann Arbor, MI, 1948.

2. Myklestad, N. O.: New Method of Calculating Natural Modes of Coupled Bending-torsion Vibration of Beams. Trans. ASME, vol. 67, 1945, pp. 61-67.

3. Thomson, W. T.: Matrix Solution of Vibration of Non-uniform Beams. Jour. Appl. Mech., vol. 17, no. 3, 1950, pp. 337-339.

4. Molloy, C. T.: Four Pole Parameters in Vibration Analysis. Colloquium on Mechanical Impedance Methods for Mechanical Vibrations, R. Plunket, ed., Applied Mechanics Division, ASME, 1958, pp. 43-68.

5. Pestel, E. C.; and Leckie, F. A.: Matrix Methods in Elastomechanics. New York: McGraw-Hill Book Company, Inc., 1963.

6. Rubin, S.: Transmission Matrices for Vibration and Their Relation to Admittance and Impedance. Jour. Engr. Industry, Trans. ASME, vol. 86, Series B, no. 1, 1964, pp. 9-21.

7. Rubin, S.: Review of Mechanical Immittance and Transmission Matrix Concepts. Jour. Acous. Soc. America, vol. 41, no. 5, 1967, pp. 1171-1179.

8. Targoff, W. P.: The Associates Matrices of Bending and Coupled Bending-torsion Vibrations. Jour. Aero. Sci., 1947, pp. 579-582.

9. Targoff, W. P.: The Bending Vibrations of a Twisted Rotating Beam. Proc. Third Midwestern Conf. on Solid Mechanics, 1957, pp. 177-194.

10. Isakson, G.; and Eisley, G. J.: Natural Frequencies in Bending of Twisted Rotating Blades. NASA TN D-371, 1960.

11. Isakson, G.; and Eisley, G. J.: Natural Frequencies in Coupled Bending and Torsion of Twisted Rotating and Nonrotating Blades. NASA CR-65, 1964.

12. Lin, Y. K.: Probabilistic Theory of Structural Dynamics. New York: McGraw-Hill Book Company, Inc., 1967.

13. Lin, Y. K.; and McDaniel, T. J.: Dynamics of Beam-type Periodic Structures. Jour. Engr. Industry, vol. 91, 1969, pp. 1133-1141.

14. Mercer, C. A.; and Seavy, C.: Prediction of Natural Frequencies and Normal Modes of Skin-stringer Panel Rows. Jour. Sound and Vibration, vol. 6, 1967, pp. 149-162.

15. Mead, D. J.: Vibration Response and Wave Propagation in Periodic Structures. Jour. Engr. Industry, vol. 93, 1971, pp. 783-792.

16. Mead, D. J.; and Gupta, G. S.: Propagation of Flexural Waves in Infinite, Damped Rib-skin Structures. USAF Report AFML-TR-70-13, 1970.

17. Henderson, J. P.: Vibration Analysis of Curved Skin-stringer Structures Having Tuned Elasomeric Dampers. Ph.D. Thesis, Ohio State University, 1972.

18. Henderson, J. P.; and McDaniel, T. J.: The Analysis of Curved Multispan Structures. Jour. Sound and Vibration, vol. 18, 1970, pp. 203-219.

19. McDaniel, T. J.: Dynamics of Circular Periodic Structures. Jour. Aircraft, vol. 8, 1971, pp. 143-149.

20. McDaniel, T. J.; and Logan, J. D.: Dynamics of Cylindrical Shells with Variable Curvature. Jour. Sound and Vibration, vol. 19, 1971, pp. 39-48.

21. Murthy, V. R.; and Nigam, N. C.: Dynamic Characteristics of Stiffened Rings by Transfer Matrix Approach. Jour. Sound and Vibration, vol. 39, 1975, pp. 237-245.

22. Murthy, V. R.; and McDaniel, T. J.: Solution Bounds to Structural Systems. AIAA Journal, vol. 14, 1976, pp. 111-113.

23. McDaniel, T. J.; and Murthy, V. R.: Solution Bounds to Varying Geometry Beams. Jour. Sound and Vibration, vol. 44, 1976, pp. 431-448.

24. Murthy, V. R.; and Pierce, G. A.: Effect of Phase Angle on Multibladed Rotor Flutter. Jour. Sound and Vibration, vol. 48, 1976.

25. Murthy, V. R.: Dynamic Characteristics of Rotor Blades. Jour. Sound and Vibration, to appear, 1976.

26. Houbolt, J. C.; and Brooks, G. W.: Differential Equations of Motion for Combined Flapwise Bending, Chordwise Bending, and Torsion of Twisted Nonuniform Rotor Blades. NASA Report 1346, 1958.

APPENDIX A

DESCRIPTION OF THE COMPUTER PROGRAMS

Fortran Program I:

Computes the natural frequencies and associated mode shapes of a nonuniform, pretwisted rotor blade with combined flapwise bending, chordwise bending, and torsional degrees of freedom.

I Data Card: IBC, ISTAGE, NS

FORMAT I1, I1, I3

IBC: Variable indicates the nature of modes required

IBC = 1, implies collective modes

IBC = 2, implies cyclic modes

IBC = 3, implies scissor modes

ISTAGE: Program performs two functions

ISTAGE = 1, computes the values of the frequency determinants only

ISTAGE = 2, computes the natural frequencies and mode shapes

NS = Number of stations at which data is provided

II Data Card: SPAN, OMEGA, B, TSR, RPITCH

FORMAT 5 E 14.7

SPAN = Span of the blade (inches), distance between axis of rotation and tip of the blade

OMEGA = Rotational speed of the blade (rotations per minute)

B = Semichord at the root (inches)

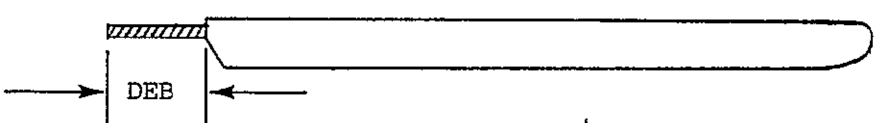
TSR = Control system spring rate (in.-lb/rad)

RPITCH = Collective pitch setting at the root (degrees)

III Data Card: DEB

FORMAT E 14.7

DEB = Distance of the blade from the axis of rotation



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The following data should be provided in the order

STA, MASS, EI1, EI2, GJ, E, BETA, KMIS, KM2S

FORMAT 5E 14.7 on each card

STA = Station locations (inches)

The first station should correspond to the axis of rotation, and the last station should correspond to the tip of the blade. The distance between the stations need not be equal. For accuracy it is preferable to choose the stations such that the variation of structural properties in between can be approximated by a linear relationship.

MASS = Mass per unit span (lb-sec^2/in.^{2})

EI1 = Flapwise bending stiffness (lb-in.^{2})

EI2 = Chordwise bending stiffness (lb-in.^{2})

GJ = Torsional stiffness (lb-in.^{2})

E = Distance between mass and elastic axes (inches) positive when mass axis lies ahead of elastic axis

BETA = Twist of the blade not including the collective pitch (degrees)

KMIS = Mass moment of inertia of the cross-sectional mass about the chord (lb-sec^2)

KM2S = Mass moment of inertia of the cross-sectional mass about an axis perpendicular to the chord passing through the shear center

If the data is provided say at 10 stations, each of the above variables take 2 cards and a total of 18 cards.

If the computer variable ISTAGE = 1 (see the 1st Data Card) then the following card is the last card, otherwise it is last but one card

H1, H2, H3

FORMAT 3 E 14.7

H1 = Starting frequency (rad/sec)

H2 = Frequency increment (rad/sec)

H3 = Ending frequency (rad/sec)

Case 1. ISTAGE = 1.

In this case frequency determinants are only computed for various frequencies. It starts with frequency H1 and increments by H2 and goes up to the value H3. For each of these values it computes the nondimensional frequency determinant and prints. It prints three columns:

I column: Frequency (rad/sec)

II column: Nondimensional frequency $\left(\sqrt{\frac{M_o R^4}{EI_{10}}}\ \text{Frequency}\right)$

III column: Nondimensional frequency determinant value

Case II. ISTAGE = 2.

In this case the natural frequencies and mode shapes are computed. The natural frequencies are computed by frequency scanning technique. Sign changes in the values of frequency determinant are detected starting from value H1 at steps of H2 till the required sign changes are detected or the value H3 is reached. So the required number of frequencies that lie between H1 and H3, whichever is less, are computed. If two frequencies lie closer than increment H2, then there is a chance of missing those frequencies, so H2 has to be chosen such that no two frequencies are closer than H2. This can be estimated by looking at the frequency versus frequency determinant values, which can be obtained when the program is executed under ISTAGE = 1.

The output under ISTAGE = 2 prints the natural frequencies and mode shapes. The natural frequencies are given in (rad/sec), Hertz, and in nondimensional units $\left(\frac{m_o R^4}{EI_{10}}\right)$, and the mode shape deflections are given by the following:

Flapwise deflection w/b_0 (nondimensional)

Chordwise deflection v/b_0 (nondimensional)

Torsional deflection ϕ (radians)

The following card is required when the program is executed under ISTAGE = 2.

NF, ITER, BLANK, DOT, STAR, INC

FORMAT I2, I1, 3A1, I2

NF = Number of frequencies required not exceeding 10

ITER = Number of subdivisions required before interpolation, usually ITER = 1 is sufficient. If ITER = 1, after the frequency is detected within the interval H2, then interval H2 is subdivided and the frequency is detected within the interval of H2/10 before computing the actual frequency by interpolation. If ITER = 2, the interval is reduced to H2/100.

BLANK = one blank space

DOT = ·

STAR = *

INC = increment in the mode number in the normal execution of the program INC = 0. If the first two frequencies are separated by at least 1.0 rad/sec and the third frequency onwards are separated by at least 20 rad/sec, it is not economical (computation wise) to scan the entire frequency range by incrementing at 1.0 rad/sec. In such a case the first two frequencies are detected by scanning with H2 = 1.0 in one execution and the third frequency onwards are computed in a second execution with H2 = 20.0. In the second execution INC = 2 must be fed to keep track of the correct mode number.

Total number of data cards

ISTAGE = 1

Integer greater than $(\frac{9NS}{5}) + 4$

ISTAGE = 2

Integer greater than $(\frac{9NS}{5}) + 5$

Where NS = number of stations

Fortran Program II:

Computes the natural frequencies and associated mode shapes of a nonuniform, pretwisted rotor blade with combined flapwise bending, chordwise bending, and torsional degrees of freedom with an attached concentrated mass at any spanwise and chordwise locations.

I Data Card: IBC, ISTAGE, NS

FORMAT I1, I1, I3

Similar to Program I

II Data Card: SPAN, OMEGA, B, TSR, PMASS

FORMAT 5 E 14.7

SPAN, OMEGA, B, TSR are defined in Program I

PMASS = Magnitude of the attached mass (lb)

III Data Card: CLP, SLP, RPITCH, DEB

RPITCH, DEB are defined in Program I.

CLP = Chordwise location of the mass from shear center (inches), positive forward of shear center

SLP = spanwise location of the mass (inches)

Rest of the data is same as in Program I.

Fortran Program III

Computes the natural frequencies and associated mode shapes of a nonuniform rotor blade with pure torsional degree of freedom with an attached concentrated mass at any spanwise and chordwise locations.

I Data Card: ISTAGE, NS

FORMAT I1, I3

Defined in Program I.

II Data Card: SPAN, B, TSR, OMEGA

FORMAT 4 E 14.7

Defined in Program I.

III Data Card: PMASS, CLP, SLP

FORMAT 3 E 14.7

Defined in Program II.

IV Data Card: onwards

STA, MASS, GJ, KMIS, KM2S

FORMAT 5 E 14.7 on each card

Described in Program I.

After the structural data one or two cards are to be introduced depending on the value of ISTAGE, as discussed under Program I.

APPENDIX B

LISTINGS OF THE COMPUTER PROGRAMS AND SAMPLE OUTPUT

FORTRAN PROGRAM I

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C       ****************************************************************
C       NATURAL VIBRATION CHARACTERISTICS,COUPLED FLAPWISE BENDING,CHORD-
C
C       WISE BENDING AND TORSION,SEE-SAW ROTOR,COLLECTIVE,CYCLIC AND
C
C       SCISSOR MODES.USES FUNCTION DET AND SUBROUTINES INTPOL,PLOT,
C
C       NATFRE AND SHAPES
C       ****************************************************************
C
C       ----------------------------------------------------------------
C       DECLARATION AND COMMON STATEMENTS
C       ----------------------------------------------------------------
C
        REAL MASS,KMS,KM1S,KM2S
        DIMENSION E(101),EI1(101),EI2(101),GJ(101),KMS(101),KM1S(101),
       1KM2S(101),MASS(101),BETA(101),D1(101),D2(101),D3(101),D4(101),
       2D5(101),D6(101),D7(101),D8(101),D9(101),D10(101),D11(101),
       3D12(101),STA(101),PHI(51),W(51),V(51),SL(51),FREQEN(10)
        COMMON/X1/FREQEN,H1,H2,H3,ITER,IJK
        COMMON/X2/J1,PP,FRE,HERTZ,SL,BLANK,DOT,STAR
        COMMON/X3/STA,SPAN
        COMMON/X4/IBC
        COMMON/X5/D1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11,D12,N1,N2
        COMMON/X6/TSR
        COMMON/X7/NS
        COMMON/X8/OMEGAN
        COMMON/X11/IND
C
C       ----------------------------------------------------------------
C       THIS SECTION READS THE DATA OF THE SYSTEM
C       ----------------------------------------------------------------
C
        READ(5,5) IBC,ISTAGE,NS
        READ(5,10) SPAN,OMEGA,B,TSR,RPITCH,DEB
        READ(5,10)(STA(J),J=1,NS),(MASS(J),J=1,NS),(EI1(J),J=1,NS),(EI2(J)
       1,J=1,NS),(GJ(J),J=1,NS),(E(J),J=1,NS),(BETA(J),J=1,NS),(KM1S(J),J=
       21,NS),(KM2S(J),J=1,NS)
        READ(5,10)H1,H2,H3
        IF(ISTAGE.EQ.2) READ(5,15)NF,ITER,BLANK,DOT,STAR,INC
C       ----------------------------------------------------------------
C       THIS SECTION PRINTS THE DATA OF THE SYSTEM
C       ----------------------------------------------------------------
C
        WRITE(6,20)
        WRITE(6,25)
        IF(IBC.EQ.1)WRITE(6,30)
        IF(IBC.EQ.2)WRITE(6,35)
```

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```
      IF(IBC.EQ.3)WRITE(6,40)
      IF(ISTAGE.EQ.2)WRITE(6,45)NF
      WRITE(6,135)H1
      WRITE(6,50)H2
      WRITE(6,150)H3
      IF(ISTAGE.EQ.2)WRITE(6,155)INC
      WRITE(6,55)SPAN
      WRITE(6,60)OMEGA
      WRITE(6,65)B
      WRITE(6,70)TSR
      WRITE(6,190) RPITCH
      WRITE(6,195)DEB
      WRITE(6,90) NS
      WRITE(6,95)
      WRITE(6,100)(STA(J),J=1,NS)
      WRITE(6,105)
      WRITE(6,100)(MASS(J),J=1,NS)
      WRITE(6,110)
      WRITE(6,100)(EI1(J),J=1,NS)
      WRITE(6,115)
      WRITE(6,100)(EI2(J),J=1,NS)
      WRITE(6,120)
      WRITE(6,100)(GJ(J),J=1,NS)
      WRITE(6,125)
      WRITE(6,100)(E(J),J=1,NS)
      WRITE(6,130)
      WRITE(6,100)(BETA(J),J=1,NS)
      WRITE(6,140)
      WRITE(6,100)(KM1S(J),J=1,NS)
      WRITE(6,145)
      WRITE(6,100)(KM2S(J),J=1,NS)
      WRITE(6,25)
C
C     ------------------------------------------------------------------
C     THIS SECTION CALCULATES THE SYSTEM PROPERTIES AT THE REQUIRED
C
C     STATIONS BY INTERPOLATION AND PRINTS THE INTERPOLATED VALUES
C     ------------------------------------------------------------------
C
      CALL INTPOL(MASS)
      CALL INTPOL(EI1)
      CALL INTPOL(EI2)
      CALL INTPOL(GJ)
      CALL INTPOL(E)
      CALL INTPOL(BETA)
      CALL INTPOL(KM1S)
      CALL INTPOL(KM2S)
      WRITE(6,20)
      WRITE(6,25)
```

```
      WRITE(6,175)
      WRITE(6,25)
      WRITE(6,105)
      WRITE(6,100)(MASS(J),J=1,101)
      WRITE(6,110)
      WRITE(6,100)(EI1(J),J=1,101)
      WRITE(6,115)
      WRITE(6,100)(EI2(J),J=1,101)
      WRITE(6,120)
      WRITE(6,100)(GJ(J),J=1,101)
      WRITE(6,125)
      WRITE(6,100)(E(J),J=1,101)
      WRITE(6,130)
      WRITE(6,100)(BETA(J),J=1,101)
      WRITE(6,140)
      WRITE(6,100)(KM1S(J),J=1,101)
      WRITE(6,145)
      WRITE(6,100)(KM2S(J),J=1,101)
      WRITE(6,25)
C
C     ------------------------------------------------------------------
C     THIS SECTION NON-DIMENSIONALIZES THE DATA AND COMPUTES THE
C
C     COEFFICIENTS OF THE FIRST-ORDER DIFFERENTIAL EQUATIONS WHICH ARE
C
C     NOT DEPENDENT ON THE FREQUENCIES
C     ------------------------------------------------------------------
C
      DO 205 J=1,101
      KM1S(J)=KM1S(J)/MASS(J)
      KM2S(J)=KM2S(J)/MASS(J)+E(J)*E(J)
205   KMS(J)=KM1S(J)+KM2S(J)
      NZTS=DEB/SPAN
      NZ=NZTS+1
      DO 206 I=NZ,101
206   BETA(I)=BETA(I)+RPITCH
      PI=4.0*ATAN(1.0)
      OMEGA=OMEGA*PI/30.0
      OMEGAS=OMEGA*OMEGA
      SPANS=SPAN*SPAN
      BS=B*B
      TSR=TSR*SPANS*SPAN/(EI1(1)*BS)
      FACT=SQRT(MASS(1)*SPANS*SPANS/EI1(1))
      OMEGAN=FACT*FACT*OMEGAS
      F1=BS/SPANS
      N1=3
      N2=7
      IF(IBC.EQ.2) N1=4
      IF(IBC.EQ.2) N2=8
```

```
      IF(IBC.EQ.3)N2=4
      DO 210 J=1,101
210   D5(J)=MASS(J)/MASS(1)
      X=0.0
      H5=0.01/24.0
      DO 215 J=1,101
      D2(J)=D5(J)*H5*X
215   X=X+0.01
      D12(101)=0.0
      D12(100)=9.0*D2(101)+19.0*D2(100)-5.0*D2(99)+D2(98)
      DO 220 JJ=2,99
      J=101-JJ
220   D12(J)=D12(J+1)-D2(J+2)+13.0*(D2(J+1)+D2(J))-D2(J-1)
      D12(1)=D12(2)+D2(4)-5.0*D2(3)+19.0*D2(2)+9.0*D2(1)
      X=0.0
      DO 230 J=1,101
      BETA(J)=PI*BETA(J)/180.0
      C=COS(BETA(J))
      S=SIN(BETA(J))
      CS=C*C
      SS=S*S
      A11=EI1(J)*CS+EI2(J)*SS
      A12=(EI2(J)-EI1(J))*C*S
      A22=EI1(J)*SS+EI2(J)*CS
      D=A11*A22-A12*A12
      D1(J)=-EI1(1)*A12/D
      D2(J)=EI1(1)*A22/D
      D3(J)=EI1(1)*A11/D
      D4(J)=EI1(1)*F1/GJ(J)
      D6(J)=D5(J)*E(J)/B
      D7(J)=D6(J)*S
      D6(J)=D6(J)*C
      D8(J)=D6(J)*X*OMEGAN
      D9(J)=D7(J)*X*OMEGAN
      D10(J)=OMEGAN*D5(J)*(KM2S(J)-KM1S(J))*(CS-SS)/BS
      D11(J)=D5(J)*KMS(J)/BS
      D12(J)=D12(J)*OMEGAN
230   X=X+0.01
      H1=H1*FACT
      H2=H2*FACT
      H3=H3*FACT
      IF(ISTAGE.EQ.2) GO TO 240
C
C     ------------------------------------------------------------------
C     THIS SECTION CALCULATES THE FREQUENCY DETERMINANTS OF THE SYSTEM
C     ------------------------------------------------------------------
C
      WRITE(6,20)
      WRITE(6,25)
```

```
      WRITE(6,180)
      WRITE(6,25)
235   P=H1*H1
      IF(H1.GT.H3) GO TO 265
      FR=H1/FACT
      F=DET(P)
      WRITE(6,185)FR,H1,F
      H1=H1+H2
      GOTO 235
C
C     ----------------------------------------------------------------------
C     THIS SECTION CALCULATES THE NATURAL FREQUENCIES AND THE ASSOCIATED
C
C     MODAL FUNCTIONS OF THE SYSTEM.MODAL FUNCTIONS ARE NORMALIZED WITH
C
C     RESPECT TO THE MAXIMUM DEFLECTION OF THE PREDOMINANT MODE
C     ----------------------------------------------------------------------
C
240   CALL NATFRE(NF)
      SL(1)=0.0
      DO 245 J=1,50
245   SL(J+1)=SL(J)+0.02
      IF(IJK.EQ.0) GO TO 261
      DO 260 J=1,IJK
      J1=J+INC
      PP=FREQEN(J)
      P=PP*PP
      IF((J1+IBC).EQ.2.AND.OMEGA.LE.0.0) GO TO 246
      CALL SHAPES(P,W,V,PHI)
      GO TO 248
246   DO 247 I=1,51
      V(I)=SL(I)
      W(I)=0.0
247   PHI(I)=0.0
      GO TO 256
248   IF(IND.EQ.1) GO TO 256
      AMAX=W(1)
      DO 250 I=1,51
      IF(ABS(AMAX).LT.ABS(W(I)))AMAX=W(I)
      IF(ABS(AMAX).LT.ABS(V(I)))AMAX=V(I)
250   IF(ABS(AMAX).LT.ABS(PHI(I)))AMAX=PHI(I)
      DO 255 I=1,51
      W(I)=W(I)/AMAX
      V(I)=V(I)/AMAX
255   PHI(I)=PHI(I)/AMAX
256   CONTINUE
      FRE=PP/FACT
      HERTZ=FRE/(2.0*PI)
      CALL PLOT(W,1)
```

```
      CALL PLOT(V,2)
      CALL PLOT(PHI,3)
260   CONTINUE
261   IF(IJK.LT.NF) WRITE(6,160) IJK
265   IF(ISTAGE.EQ.1) WRITE(6,25)
C
C     ----------------------------------------------------------------
C     FORMATS
C     ----------------------------------------------------------------
C
5     FORMAT(I1,I1,I3)
10    FORMAT(5E14.7)
15    FORMAT(I2,I1,3A1,I2)
20    FORMAT(1H1)
25    FORMAT(//2X,"*****************************************************
     1******************************************************************
     2*****")
30    FORMAT(//35X,"NATURE OF THE MODES",19X,"COLLECTIVE MODES")
35    FORMAT(//35X,"NATURE OF THE MODES",19X,"CYCLIC MODES")
40    FORMAT(//35X,"NATURE OF THE MODES",19X,"SCISSOR MODES")
45    FORMAT(//5X,"NUMBER OF FREQUENCIES REQUIRED
     1                                                 =",I5)
50    FORMAT(//5X,"FREQUENCY INCREMENT(RAD/SEC)
     1                                                 =",E14.7)
55    FORMAT(//5X,"LENGTH OF THE BLADE (INCHES)
     1                                                 =",E14.7)
60    FORMAT(//5X,"ROTATIONAL VELOCITY OF THE BLADE (RPM)
     1                                                 =",E14.7)
65    FORMAT(//5X,"SEMI-CHORD OF THE BLADE (INCHES)
     1                                                 =",E14.7)
70    FORMAT(//5X,"CONTROL SYSTEM SPRING RATE (IN-LB/RAD)
     1                                                 =",E14.7)
90    FORMAT(//5X,"NUMBER OF DATA POINTS
     1                                                 =",I5)
95    FORMAT(// 5X,"STATION LOCATIONS (INCHES)")
100   FORMAT(//7(4X,E14.7))
105   FORMAT(//5X,"MASS PER UNIT LENGTH (LB-SEC**2/IN**2)")
110   FORMAT(//5X,"FLAPWISE BENDING STIFFNESS (LB-IN**2)")
115   FORMAT(//5X,"CHORDWISE BENDING STIFFNESS (LB-IN**2)")
120   FORMAT(//5X,"TORSIONAL STIFFNESS (LB-IN**2)")
125   FORMAT(//5X,"DISTANCE BETWEEN MASS AND ELASTIC AXIS(INCHES)")
130   FORMAT(//5X,"TWIST OF THE BLADE NOT INCLUDING THE COLLECTIVE PITCH
     1(DEGREES)")
135   FORMAT(//5X,"STARTING FREQUENCY (RAD/SEC)
     1                                                 =",E14.7)
140   FORMAT(//5X,"MASS MOMENT OF INERTIA ABOUT THE CHORD (LB-SEC**2)")
145   FORMAT(//5X,"MASS MOMENT OF INERTIA ABOUT AN AXIS PERPENDICULAR TO
     1 THE CHORD THROUGH THE CENTER OF GRAVITY(LB-SEC**2)")
150   FORMAT(//5X,"ENDING FREQUENCY (RAD/SEC)
```

```
     1                                                            =",E14.7)
155   FORMAT(//5X,"INCREMENT IN THE MODE NUMBER
     1                                                            =",I5)
160   FORMAT(//5X,"THE NUMBER OF FREQUENCIES DETECTED WITH IN THE RANGE
     1 ARE ONLY                                                   =",I5)
175   FORMAT(//5X,"THE FOLLOWING ARE THE INTERPOLATED VALUES AT 101 EQUI
     1DISTANT STATIONS")
180   FORMAT(//5X,"THE FOLLOWING COLUMNS ARE 1.FREQUENCY(RAD/SEC) 2.NOND
     1IMENSIONAL FREQUENCY 3.VALUE OF THE FREQUENCY DETERMINANT RESPECTI
     2VELY")
185   FORMAT(10X,E14.7,15X,E14.7,15X,E14.7)
190   FORMAT(//5X,"COLLECTIVE PITCH(DEGREES)
     1                                                            =",E14.7)
195   FORMAT(//5X,"DISTANCE OF THE BLADE FROM THE ROOT (INCHES)
     1                                                            =",E14.7)
      STOP
      END
```

```
      SUBROUTINE NATFRE(N)
C
C     ------------------------------------------------------------------
C     THIS SUBROUTINE SCANS THE FREQUENCY DETERMINANT WITH RESPECT TO
C
C     THE FREQUENCY TILL THE SPECIFIED NUMBER OF SIGN CHANGES ARE
C
C     DETECTED STARTING FROM ZERO FREQUENCY.USES FUNCTION DET
C     ------------------------------------------------------------------
C
      DIMENSION FREQEN(10),JKL(10)
      COMMON/X1/FREQEN,H1,H,H3,ITER,IJK
      IJK=0
      ITRN=0
      DO 5 J=1,N
5     JKL(J)=0
      PP=H1
      P=PP*PP
      F=DET(P)
      IF(ABS(F).GT.0.0001)GO TO 10
      IJK=IJK+1
      JKL(IJK)=1
      FREQEN(IJK)=PP
      PP=PP+H
      P=PP*PP
      F=DET(P)
10    F=SIGN(1.0,F)
15    PP=PP+H
      IF(PP.GT.H3) GO TO 30
      P=PP*PP
      G=DET(P)
      IF(ABS(G).GT.0.0001)GO TO 20
      IJK=IJK+1
      JKL(IJK)=1
      FREQEN(IJK)=PP
      IF(IJK.EQ.N)GO TO 30
      PP=PP+H
      P=PP*PP
      F=DET(P)
      F=SIGN(1.0,F)
      GO TO 15
20    G=SIGN(1.0,G)
      IF(F*G.GT.0.0)GO TO 25
      IJK=IJK+1
      FREQEN(IJK)=PP-H
      IF(IJK.EQ.N)GO TO 30
25    F=G
      GO TO 15
```

```
30    ITRN=ITRN+1
      IF(ITRN.GT.ITER)GO TO 55
      IF(IJK.EQ.0) GO TO 65
      H=H/10.0
      DO 50 J=1,IJK
      IF(JKL(J).EQ.1)GO TO 50
      PP=FREQEN(J)
      P=PP*PP
      F=DET(P)
      F=SIGN(1.0,F)
35    PP=PP+H
      P=PP*PP
      G=DET(P)
      IF(ABS(G).GT.0.0001)GO TO 40
      JKL(J)=1
      FREQEN(J)=PP
      GO TO 50
40    G=SIGN(1.0,G)
      IF(F*G.GT.0.0)GO TO 45
      FREQEN(J)=PP-H
      GO TO 50
45    F=G
      GO TO 35
50    CONTINUE
      GO TO 30
55    DO 60 J=1,IJK
      IF(JKL(J).EQ.1)GO TO 60
      PP=FREQEN(J)
      P=PP*PP
      F=DET(P)
      PP=PP+H
      P=PP*PP
      G=DET(P)
      DIFF=G-F
      FREQEN(J)=PP-G*H/DIFF
60    CONTINUE
65    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PLOT(A,N)
C
C     ------------------------------------------------------------------
C     THIS SUBROUTINE PRINTS THE NATURAL FREQUENCIES AND MODE SHAPES AND
C
C     PLOTS THE MODE SHAPES
C     ------------------------------------------------------------------
C
      REAL LINE
      DIMENSION A(51),SL(51),LINE(51)
      COMMON/X2/J1,PP,FRE,HERTZ,SL,BLANK,DOT,STAR
      COMMON/X11/IND
      WRITE(6,10)
      WRITE(6,20)
      WRITE(6,30)J1,FRE,HERTZ,PP
      WRITE(6,20)
      IF(N.EQ.1)WRITE(6,40)
      IF(N.EQ.2)WRITE(6,50)
      IF(N.EQ.3)WRITE(6,60)
      WRITE(6,70)
      IF(IND.EQ.0) GO TO 75
      WRITE(6,150)
      RETURN
75    DO 80 J=1,6
80    WRITE(6,90)(SL(J),A(J),SL(J+9),A(J+9),SL(J+18),A(J+18),SL(J+27),A(
     1J+27),SL(J+36),A(J+36),SL(J+45),A(J+45))
      WRITE(6,90)(SL(7),A(7),SL(16),A(16),SL(25),A(25),SL(34),A(34),SL(4
     13),A(43))
      WRITE(6,90)(SL(8),A(8),SL(17),A(17),SL(26),A(26),SL(35),A(35),SL(4
     14),A(44))
      WRITE(6,90)(SL(9),A(9),SL(18),A(18),SL(27),A(27),SL(36),A(36),SL(4
     15),A(45))
      WRITE(6,20)
      WRITE(6,10)
      DO 100 J=1,51
100   LINE(J)=DOT
      J=25.0*(A(1)+1.0)+1.5
      LINE(J)=STAR
      WRITE(6,110)(LINE(J),J=1,51)
      DO 120 J=1,51
120   LINE(J)=BLANK
      LINE(26)=DOT
      DO 130 JJ=3,51,2
      J=25.0*(A(JJ)+1.0)+1.5
      LINE(J)=STAR
      WRITE(6,140)(LINE(J),J=1,51)
      LINE(J)=BLANK
130   LINE(26)=DOT
```

```
10    FORMAT(1H1)
20    FORMAT(//2X,"*****************************************************
     1******************************************************************
     2******")
30    FORMAT(//5X," MODE NUMBER =",I2,8X,"FREQ. RAD/SEC =",F10.4,8X,"FRE
     1Q. HERTZ =",F10.4,8X," NON-DIMEN. FREQ. =",F10.4)
40    FORMAT(//50X,"FLAPWISE DEFLECTION/SEMICHORD")
50    FORMAT(//50X,"CHORDWISE DEFLECTION/SEMICHORD")
60    FORMAT(//50X,"TORSIONAL DEFLECTION(RADIANS)")
70    FORMAT(6(4X,"STA X/L",4X,"DEFLN"))
90    FORMAT(//12(2X,F8.4))
110   FORMAT(////////40X,51A1)
140   FORMAT(40X,51A1)
150   FORMAT(//////5X,"THE MATRIX TO BE INVERTED IN THE MODE SHAPES COMP
     1UTATIONS IS SINGULAR AND HENCE COMPUTATIONS ARE ABANDONDED")
      RETURN
      END
```

```
      SUBROUTINE INTPOL(A)
C
C     ------------------------------------------------------------------
C     THIS SUBROUTINE INTERPOLATES THE REQUIRED VALUES.
C     ------------------------------------------------------------------
C
      DIMENSION A(101),STA(101),TABLE(101,1),B(101)
      COMMON /X3/STA,SPAN
      COMMON/X7/NS
      IN=0
      DO 5 J=1,NS
5     IF(ABS(A(J)).LE.0.0) IN=IN+1
      IF(IN.NE.NS)GO TO 15
      DO 10 J=1,101
10    A(J)=0.0
      RETURN
15    A(101)=A(NS)
      NM1=NS-1
      DO 20 I=1,NM1
20    TABLE(I,1)=(A(I+1)-A(I))/(STA(I+1)-STA(I))
      H=SPAN/100.0
      XARG=H
      DO 35 I=2,100
      DO 25J=1,NS
      IF(J.EQ.NS.OR.XARG.LE.STA(J)) GO TO 30
25    CONTINUE
30    MAX=J
      IF(MAX.LE.2) MAX=2
      ISUB=MAX-1
      YEST=TABLE(ISUB,1)
      B(I)=YEST*(XARG-STA(ISUB))+A(ISUB)
35    XARG=XARG+H
      DO  40 J=2,100
40    A(J)= B(J)
      RETURN
      END
```

```
      FUNCTION DET(P)
C
C     ------------------------------------------------------------------
C     THIS FUNCTION CALCULATES THE VALUE OF THE FREQUENCY DETERMINANT
C
C     USES SUBROUTINE TRAMAT
C     ------------------------------------------------------------------
C
      DIMENSION TF(10,10),A(6,6)
      COMMON/X4/IBC
      COMMON/X6/TSR
      COMMON/X9 /TF
      COMMON/X12/DETER
      CALL TRAMAT(P)
      DO 5 J=9,10
      DO 5 I=1,5
5     A(I,J-5)=TF(I+5,J)
      IF(IBC-2)10,20,30
10    DO  15 I=1,5
      A(I,1)=TF(I+5,4)
      A(I,2)=TF(I+5,5)-TF(I+5,6)*TSR
15    A(I,3)=TF(I+5,8)
      GOTO 40
20    DO 25 I=1,5
      A(I,1)=TF(I+5,3)
      A(I,2)=TF(I+5,5)-TF(I+5,6)*TSR
25    A(I,3)=TF(I+5,7)
      GOTO 40
30    DO 35 I=1,5
      A(I,1)=TF(I+5,5)-TF(I+5,6)*TSR
      A(I,2)=TF(I+5,7)
35    A(I,3)=TF(I+5,8)
40    CONTINUE
      CALL SOLUTN(A,1)
      DET=DETER
      RETURN
      END
```

```
      SUBROUTINE TRAMAT(P)
C
C     ------------------------------------------------------------------
C     INTEGRATION OF DIFFERENTIAL EQUATIONS,USES THE FUNCTION RUNGE
C     ------------------------------------------------------------------
C
      DIMENSION D1(101),D2(101),D3(101),D4(101),D5(101),D6(101),D7(101),
     1D8(101),D9(101),D10(101),D11(101),D12(101),V1(10,10),V(10),T(10),
     2TT(51,10,10),TF(10,10)
      COMMON/X5/D1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11,D12,N1,N2
      COMMON/X8/OMEGAN
      COMMON/X9/TF
      COMMON/X10/TT
      DO 10 J=1,10
      DO 5  I=1,10
      TT(1,I,J)=0.0
 5    V1(I,J)=0.0
      TT(1,J,J)=1.0
 10   V1(J,J)=1.0
      M=0
      DO 45 I=3,10
      IF(I.EQ.N1.OR.I.EQ.N2) GO TO 45
      DO 15 J=1,10
 15   V(J)=V1(J,I)
      J=1
      DO 40 L=1,50
 20   K=RUNGE(V,T,J,M)
 25   IF(K.NE.1) GO TO 30
      FACT=P+OMEGAN
      T(1)=V(3)
      T(2)=V(4)
      T(3)=D1(J)*V(7)+D2(J)*V(8)
      T(4)=D3(J)*V(7)+D1(J)*V(8)
      T(5)=D4(J)*V(6)
      T(6)=-P*D6(J)*V(1)+FACT*D7(J)*V(2)+D8(J)*V(3)-D9(J)*V(4)+D10(J)*
     1V(5)-P*D11(J)*V(5)
      T(7)=D12(J)*V(4)+V(9)-D9(J)*V(5)
      T(8)=D12(J)*V(3)+V(10)+D8(J)*V(5)
      T(9)=FACT*D5(J)*V(2)-FACT*D7(J)*V(5)
      T(10)=P*D5(J)*V(1)+P*D6(J)*V(5)
      GO TO 20
 30   DO 35 JJ=1,10
 35   TT(L+1,JJ,I)=V(JJ)
 40   CONTINUE
      DO 44 JJ=1,10
 44   TF(JJ,I)=TT(51,JJ,I)
 45   CONTINUE
      RETURN
```

END

```
      FUNCTION RUNGE(Y,F,J,M)
C
C     ------------------------------------------------------------------
C     FOURTH-ORDER RUNGE-KUTTA METHOD
C     ------------------------------------------------------------------
C
      DIMENSION Y(10),F(10),PHI(10),SAVEY(10)
      M=M+1
      GO TO(5,10,20,30,40),M
5     RUNGE=1
      RETURN
10    DO 15 JJ=1,10
      SAVEY(JJ)=Y(JJ)
      PHI(JJ)=F(JJ)
15    Y(JJ)=SAVEY(JJ)+0.01*F(JJ)
      J=J+1
      RUNGE=1
      RETURN
20    DO 25 JJ=1,10
      PHI(JJ)=PHI(JJ)+2.0*F(JJ)
25    Y(JJ)=SAVEY(JJ)+0.01*F(JJ)
      RUNGE=1
      RETURN
30    DO 35 JJ=1,10
      PHI(JJ)=PHI(JJ)+2.0*F(JJ)
35    Y(JJ)=SAVEY(JJ)+0.02*F(JJ)
      J=J+1
      RUNGE=1
      RETURN
40    DO 45 JJ=1,10
45    Y(JJ)=SAVEY(JJ)+(PHI(JJ)+F(JJ))/300.0
      M=0
      RUNGE=0
      RETURN
      END
```

```
      SUBROUTINE SHAPES(P,W,V,PHI)
C
C     -----------------------------------------------------------------
C     THIS SUBROUTINE CALCULATES THE MODE SHAPES.USES THE SUBROUTINE
C
C     TRAMAT
C     -----------------------------------------------------------------
C
      DIMENSION W(51),V(51),PHI(51),A(6,6),TF(10,10),TT(51,10,10),X(5)
      COMMON/X4/IBC
      COMMON/X6/TSR
      COMMON/X9/TF
      COMMON/X10/TT
      COMMON/X11/IND
      COMMON/X13/X
      CALL TRAMAT(P)
      A(1,4)=TF(1,9)
      A(1,5)=TF(1,10)
      A(1,6)=1.0
      DO 10 I=2,5
      DO 5 J=4,5
5     A(I,J)=TF(I+4,J+5)
10    A(I,6)=0.0
      IF(IBC-2)15,25,35
15    A(1,1)=TF(1,4)
      A(1,2)=TF(1,5)-TSR*TF(1,6)
      A(1,3)=TF(1,8)
      DO 20 I=2,5
      A(I,1)=TF(I+4,4)
      A(I,2)=TF(I+4,5)-TSR*TF(I+4,6)
20    A(I,3)=TF(I+4,8)
      GO TO 45
25    A(1,1)=TF(1,3)
      A(1,2)=TF(1,5)-TSR*TF(1,6)
      A(1,3)= TF(1,7)
      DO 30 I=2,5
      A(I,1)=TF(I+4,3)
      A(I,2)=TF(I+4,5)-TSR*TF(I+4,6)
30    A(I,3)=TF(I+4,7)
      GO TO 45
35    A(1,1)=TF(1,5)-TSR*TF(1,6)
      A(1,2)=TF(1,7)
      A(1,3)=TF(1,8)
      DO 40 I=2,5
      A(I,1)=TF(I+4,5)-TSR*TF(I+4,6)
      A(I,2)=TF(I+4,7)
40    A(I,3)=TF(I+4,8)
45    CALL SOLUTN(A,2)
```

```
      IF(IND.EQ.1) GO TO 130
      IF(IBC-2) 100,110,120
  100 DO 105 J=1,51
      W(J)=X(1)*TT(J,1,4)+X(2)*(TT(J,1,5)-TSR*TT(J,1,6))+X(3)*TT(J,1,8)+
     1X(4)*TT(J,1,9)+X(5)*TT(J,1,10)
      V(J)=X(1)*TT(J,2,4)+X(2)*(TT(J,2,5)-TSR*TT(J,2,6))+X(3)*TT(J,2,8)+
     1X(4)*TT(J,2,9)+X(5)*TT(J,2,10)
  105 PHI(J)=X(1)*TT(J,5,4)+X(2)*(TT(J,5,5)-TSR*TT(J,5,6))+X(3)*TT(J,5,8
     1)+X(4)*TT(J,5,9)+X(5)*TT(J,5,10)
      RETURN
  110 DO 115 J=1,51
      W(J)=X(1)*TT(J,1,3)+X(2)*(TT(J,1,5)-TSR*TT(J,1,6))+X(3)*TT(J,1,7)+
     1X(4)*TT(J,1,9)+X(5)*TT(J,1,10)
      V(J)=X(1)*TT(J,2,3)+X(2)*(TT(J,2,5)-TSR*TT(J,2,6))+X(3)*TT(J,2,7)+
     1X(4)*TT(J,2,9)+X(5)*TT(J,2,10)
  115 PHI(J)=X(1)*TT(J,5,3)+X(2)*(TT(J,5,5)-TSR*TT(J,5,6))+X(3)*TT(J,5,7
     1)+X(4)*TT(J,5,9)+X(5)*TT(J,5,10)
      RETURN
  120 DO 125 J=1,51
      W(J)=X(1)*(TT(J,1,5)-TSR*TT(J,1,6))+X(2)*TT(J,1,7)+X(3)*TT(J,1,8)+
     1X(4)*TT(J,1,9)+X(5)*TT(J,1,10)
      V(J)=X(1)*(TT(J,2,5)-TSR*TT(J,2,6))+X(2)*TT(J,2,7)+X(3)*TT(J,2,8)+
     1X(4)*TT(J,2,9)+X(5)*TT(J,2,10)
  125 PHI(J)=X(1)*(TT(J,5,5)-TSR*TT(J,5,6))+X(2)*TT(J,5,7)+X(3)*TT(J,5,8
     1)+X(4)*TT(J,5,9)+X(5)*TT(J,5,10)
  130 RETURN
      END
```

```
      SUBROUTINE SOLUTN(A,NI)
      DIMENSION A(6,6),X(5),IROW(5),JCOL(5),JORD(5)
      COMMON/X11/IND
      COMMON/X12/DETER
      COMMON/X13/X
      IND=0
      N=5
      MAX=N
      IF(NI.EQ.2) MAX=N+1
      DETER=1.0
      DO 80 K=1,N
      KM1=K-1
      PIVOT=0.0
      DO 60 I=1,N
      DO 60 J=1,N
      IF(K.EQ.1)GO TO 55
      DO 50 ISCAN=1,KM1
      DO 50 JSCAN=1,KM1
      IF(I.EQ.IROW(ISCAN)) GO TO 60
      IF(J.EQ.JCOL(JSCAN)) GO TO 60
50    CONTINUE
55    IF(ABS(A(I,J)).LE.ABS(PIVOT)) GO TO 60
      PIVOT=A(I,J)
      IROW(K)=I
      JCOL(K)=J
60    CONTINUE
      IF(ABS(PIVOT).GT.0.1E-20)GO TO 65
      DETER=0.0
      IND=1
      RETURN
65    IROWK=IROW(K)
      JCOLK=JCOL(K)
      DETER=DETER*PIVOT
      DO 70 J=1,MAX
70    A(IROWK,J)=A(IROWK,J)/PIVOT
      A(IROWK,JCOLK)=1.0/PIVOT
      DO 80 I=1,N
      AIJCK=A(I,JCOLK)
      IF(I.EQ.IROWK) GO TO 80
      A(I,JCOLK)=-AIJCK/PIVOT
      DO 75 J=1,MAX
75    IF(J.NE.JCOLK) A(I,J)=A(I,J)-AIJCK*A(IROWK,J)
80    CONTINUE
      DO 85 I=1,N
      IROWI=IROW(I)
      JCOLI=JCOL(I)
      JORD(IROWI)=JCOLI
      IF(NI.EQ.1) GO TO 85
```

```
      X(JCOLI)=A(IROWI,MAX)
 85   CONTINUE
      INTCH=0
      NM1=N-1
      DO 90 I=1,NM1
      IP1=I+1
      DO 90 J=IP1,N
      IF(JORD(J).GE.JORD(I)) GO TO 90
      JTEMP=JORD(J)
      JORD(J)=JORD(I)
      JORD(I)=JTEMP
      INTCH=INTCH+1
 90   CONTINUE
      IF(INTCH/2*2.NE.INTCH)DETER=-DETER
      IF(ABS(DETER).GT.1.0E-30)GO TO 95
      IND=1
 95   RETURN
      END
```


C-2

FORTRAN PROGRAM II

```
C      ****************************************************************
C      NATURAL VIBRATION CHARACTERISTICS,COUPLED FLAPWISE BENDING,CHORD-
C
C      WISE BENDING AND TORSION,SEE-SAW ROTOR,COLLECTIVE,CYCLIC AND
C
C      SCISSOR MODES.USES FUNCTION DET AND SUBROUTINES INTPOL,PLOT,
C
C      NATFRE AND SHAPES
C      ****************************************************************
C
C      ----------------------------------------------------------------
C      DECLARATION AND COMMON STATEMENTS
C      ----------------------------------------------------------------
C
       REAL MASS,KMS,KM1S,KM2S
       DIMENSION E(101),EI1(101),EI2(101),GJ(101),KMS(101),KM1S(101),
      1KM2S(101),MASS(101),BETA(101),D1(101),D2(101),D3(101),D4(101),
      2D5(101),D6(101),D7(101),D8(101),D9(101),D10(101),D11(101),
      3D12(101),STA(101),PHI(51),W(51),V(51),SL(51),FREQEN(10)
       COMMON/X1/FREQEN,H1,H2,H3,ITER,IJK
       COMMON/X2/J1,PP,FRE,HERTZ,SL,BLANK,DOT,STAR
       COMMON/X3/STA,SPAN
       COMMON/X4/IBC
       COMMON/X5/D1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11,D12
       COMMON/X6/TSR
       COMMON/X7/NS
       COMMON/X8/PMASS,CLP,SLP,CBETA,SBETA,CTBETA,N1,N2,JSAVE
       COMMON/X11/IND
       COMMON/X12/OMEGAN
C
C      ----------------------------------------------------------------
C      THIS SECTION READS THE DATA OF THE SYSTEM
C      ----------------------------------------------------------------
C
       READ(5,5) IBC,ISTAGE,NS
       READ(5,10)SPAN,OMEGA,B,TSR,PMASS,CLP,SLP,RPITCH,DEB
       READ(5,10)(STA(J),J=1,NS),(MASS(J),J=1,NS),(EI1(J),J=1,NS),(EI2(J)
      1,J=1,NS),(GJ(J),J=1,NS),(E(J),J=1,NS),(BETA(J),J=1,NS),(KM1S(J),J=
      21,NS),(KM2S(J),J=1,NS)
       READ(5,10)H1,H2,H3
       IF(ISTAGE.EQ.2) READ(5,15)NF,ITER,BLANK,DOT,STAR,INC
C      ----------------------------------------------------------------
C      THIS SECTION PRINTS THE DATA OF THE SYSTEM
C      ----------------------------------------------------------------
C
       WRITE(6,20)
       WRITE(6,25)
       IF(IBC.EQ.1)WRITE(6,30)
```

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```
      IF(IBC.EQ.2)WRITE(6,35)
      IF(IBC.EQ.3)WRITE(6,40)
      IF(ISTAGE.EQ.2)WRITE(6,45)NF
      WRITE(6,135)H1
      WRITE(6,50)H2
      WRITE(6,150)H3
      IF(ISTAGE.EQ.2)WRITE(6,155)INC
      WRITE(6,55)SPAN
      WRITE(6,60)OMEGA
      WRITE(6,65)B
      WRITE(6,70)TSR
      WRITE(6,190) RPITCH
      WRITE(6,195)DEB
      IF(PMASS.LE.0.0)GOTO 200
      WRITE(6,75)PMASS
      WRITE(6,80)CLP
      WRITE(6,85)SLP
200   WRITE(6,90)NS
      WRITE(6,95)
      WRITE(6,100)(STA(J),J=1,NS)
      WRITE(6,105)
      WRITE(6,100)(MASS(J),J=1,NS)
      WRITE(6,110)
      WRITE(6,100)(EI1(J),J=1,NS)
      WRITE(6,115)
      WRITE(6,100)(EI2(J),J=1,NS)
      WRITE(6,120)
      WRITE(6,100)(GJ(J),J=1,NS)
      WRITE(6,125)
      WRITE(6,100)(E(J),J=1,NS)
      WRITE(6,130)
      WRITE(6,100)(BETA(J),J=1,NS)
      WRITE(6,140)
      WRITE(6,100)(KM1S(J),J=1,NS)
      WRITE(6,145)
      WRITE(6,100)(KM2S(J),J=1,NS)
      WRITE(6,25)
C
C     ------------------------------------------------------------------
C     THIS SECTION CALCULATES THE SYSTEM PROPERTIES AT THE REQUIRED
C
C     STATIONS BY INTERPOLATION AND PRINTS THE INTERPOLATED VALUES
C     ------------------------------------------------------------------
C
      CALL INTPOL(MASS)
      CALL INTPOL(EI1)
      CALL INTPOL(EI2)
      CALL INTPOL(GJ)
      CALL INTPOL(E)
```

```
      CALL INTPOL(BETA)
      CALL INTPOL(KM1S)
      CALL INTPOL(KM2S)
      WRITE(6,20)
      WRITE(6,25)
      WRITE(6,175)
      WRITE(6,25)
      WRITE(6,105)
      WRITE(6,100)(MASS(J),J=1,101)
      WRITE(6,110)
      WRITE(6,100)(EI1(J),J=1,101)
      WRITE(6,115)
      WRITE(6,100)(EI2(J),J=1,101)
      WRITE(6,120)
      WRITE(6,100)(GJ(J),J=1,101)
      WRITE(6,125)
      WRITE(6,100)(E(J),J=1,101)
      WRITE(6,130)
      WRITE(6,100)(BETA(J),J=1,101)
      WRITE(6,140)
      WRITE(6,100)(KM1S(J),J=1,101)
      WRITE(6,145)
      WRITE(6,100)(KM2S(J),J=1,101)
      WRITE(6,25)
C
C     ------------------------------------------------------------------
C     THIS SECTION NON-DIMENSIONALIZES THE DATA AND COMPUTES THE
C
C     COEFFICIENTS OF THE FIRST-ORDER DIFFERENTIAL EQUATIONS WHICH ARE
C
C     NOT DEPENDENT ON THE FREQUENCIES
C     ------------------------------------------------------------------
C
      DO 205 J=1,101
      KM1S(J)=KM1S(J)/MASS(J)
      KM2S(J)=KM2S(J)/MASS(J)+E(J)*E(J)
205   KMS(J)=KM1S(J)+KM2S(J)
      NZTS=DEB/SPAN
      NZ=NZTS+1
      DO 206 I=NZ,101
206   BETA(I)=BETA(I)+RPITCH
      PI=4.0*ATAN(1.0)
      OMEGA=OMEGA*PI/30.0
      OMEGAS=OMEGA*OMEGA
      SPANS=SPAN*SPAN
      BS=B*B
      TSR=TSR*SPANS*SPAN/(EI1(1)*BS)
      FACT=SQRT(MASS(1)*SPANS*SPANS/EI1(1))
      OMEGAN=FACT*FACT*OMEGAS
```

```
      IF(PMASS.LE.0.0) SLP=SPAN
      PMASS=PMASS/(SPAN*MASS(1)*386.4)
      CLP=CLP/B
      SLP=SLP/SPAN
      F1=BS/SPANS
      N1=50.0*SLP+0.5
      N2=N1+1
      JSAVE=2*N1+1
      H4=PMASS*SLP
      SBE=BETA(JSAVE)
      CBETA=COS(SBE)
      SBETA=SIN(SBE)
      CTBETA=CBETA*CBETA-SBETA*SBETA
      DO 210 J=1,101
210   D5(J)=MASS(J)/MASS(1)
      X=0.0
      H5=0.01/24.0
      DO 215 J=1,101
      D2(J)=D5(J)*H5*X
215   X=X+0.01
      D12(101)=0.0
      D12(100)=9.0*D2(101)+19.0*D2(100)-5.0*D2(99)+D2(98)
      DO 220 JJ=2,99
      J=101-JJ
220   D12(J)=D12(J+1)-D2(J+2)+13.0*(D2(J+1)+D2(J))-D2(J-1)
      D12(1)=D12(2)+D2(4)-5.0*D2(3)+19.0*D2(2)+9.0*D2(1)
      DO 225 J=1,JSAVE
225   D12(J)=D12(J)+H4
      X=0.0
      DO 230 J=1,101
      BETA(J)=PI*BETA(J)/180.0
      C=COS(BETA(J))
      S=SIN(BETA(J))
      CS=C*C
      SS=S*S
      A11=EI1(J)*CS+EI2(J)*SS
      A12=(EI2(J)-EI1(J))*C*S
      A22=EI1(J)*SS+EI2(J)*CS
      D=A11*A22-A12*A12
      D1(J)=-EI1(1)*A12/D
      D2(J)=EI1(1)*A22/D
      D3(J)=EI1(1)*A11/D
      D4(J)=EI1(1)*F1/GJ(J)
      D6(J)=D5(J)*E(J)/B
      D7(J)=D6(J)*S
      D6(J)=D6(J)*C
      D8(J)=D6(J)*X*OMEGAN
      D9(J)=D7(J)*X*OMEGAN
      D10(J)=OMEGAN*D5(J)*(KM2S(J)-KM1S(J))*(CS-SS)/BS
```

```
      D11(J)=D5(J)*KMS(J)/BS
      D12(J)=D12(J)*OMEGAN
230   X=X+0.01
      H1=H1*FACT
      H2=H2*FACT
      H3=H3*FACT
      IF(ISTAGE.EQ.2) GO TO 240
C
C     -----------------------------------------------------------------
C     THIS SECTION CALCULATES THE FREQUENCY DETERMINANTS OF THE SYSTEM
C     -----------------------------------------------------------------
C
      WRITE(6,20)
      WRITE(6,25)
      WRITE(6,180)
      WRITE(6,25)
235   P=H1*H1
      IF(H1.GT.H3) GO TO 265
      FR=H1/FACT
      F=DET(P)
      WRITE(6,185)FR,H1,F
      H1=H1+H2
      GOTO 235
C
C     -----------------------------------------------------------------
C     THIS SECTION CALCULATES THE NATURAL FREQUENCIES AND THE ASSOCIATED
C
C     MODAL FUNCTIONS OF THE SYSTEM.MODAL FUNCTIONS ARE NORMALIZED WITH
C
C     RESPECT TO THE MAXIMUM DEFLECTION OF THE PREDOMINANT MODE
C     -----------------------------------------------------------------
C
240   CALL NATFRE(NF)
      SL(1)=0.0
      DO 245 J=1,50
245   SL(J+1)=SL(J)+0.02
      IF(IJK.EQ.0) GO TO 261
      DO 260 J=1,IJK
      J1=J+INC
      PP=FREQEN(J)
      P=PP*PP
      IF((J1+IBC).EQ.2.AND.OMEGA.LE.0.0) GO TO 246
      CALL SHAPES(P,W,V,PHI)
      GO TO 248
246   DO 247 I=1,51
      V(I)=SL(I)
      W(I)=0.0
247   PHI(I)=0.0
      GO TO 256
```

```
248   IF(IND.EQ.1) GO TO 256
      AMAX=W(1)
      DO 250 I=1,51
      IF(ABS(AMAX).LT.ABS(W(I)))AMAX=W(I)
      IF(ABS(AMAX).LT.ABS(V(I)))AMAX=V(I)
250   IF(ABS(AMAX).LT.ABS(PHI(I)))AMAX=PHI(I)
      DO 255 I=1,51
      W(I)=W(I)/AMAX
      V(I)=V(I)/AMAX
255   PHI(I)=PHI(I)/AMAX
256   CONTINUE
      FRE=PP/FACT
      HERTZ=FRE/(2.0*PI)
      CALL PLOT(W,1)
      CALL PLOT(V,2)
      CALL PLOT(PHI,3)
260   CONTINUE
261   IF(IJK.LT.NF) WRITE(6,160) IJK
265   IF(ISTAGE.EQ.1) WRITE(6,25)
C
C     ----------------------------------------------------------------
C     FORMATS
C     ----------------------------------------------------------------
C
5     FORMAT(I1,I1,I3)
10    FORMAT(5E14.7)
15    FORMAT(I2,I1,3A1,I2)
20    FORMAT(1H1)
25    FORMAT(//2X,"*****************************************************
     1*************************************************************
     2*****")
30    FORMAT(//35X,"NATURE OF THE MODES",19X,"COLLECTIVE MODES")
35    FORMAT(//35X,"NATURE OF THE MODES",19X,"CYCLIC MODES")
40    FORMAT(//35X,"NATURE OF THE MODES",19X,"SCISSOR MODES")
45    FORMAT(//5X,"NUMBER OF FREQUENCIES REQUIRED
     1                                                   =",I5)
50    FORMAT(//5X,"FREQUENCY INCREMENT(RAD/SEC)
     1                                                   =",E14.7)
55    FORMAT(//5X,"LENGTH OF THE BLADE (INCHES)
     1                                                   =",E14.7)
60    FORMAT(//5X,"ROTATIONAL VELOCITY OF THE BLADE (RPM)
     1                                                   =",E14.7)
65    FORMAT(//5X,"SEMI-CHORD OF THE BLADE (INCHES)
     1                                                   =",E14.7)
70    FORMAT(//5X,"CONTROL SYSTEM SPRING RATE (IN-LB/RAD)
     1                                                   =",E14.7)
75    FORMAT(//5X,"WEIGHT OF THE SENSOR (LB)
     1                                                   =",E14.7)
80    FORMAT(//5X,"CHORDWISE LOCATION OF THE SENSOR (INCHES)
```

```
     1                                                          =",E14.7)
85    FORMAT(//5X,"SPANWISE LOCATION OF THE SENSOR (INCHES)
     1                                                          =",E14.7)
90    FORMAT(//5X,"NUMBER OF DATA POINTS
     1                                                          =",I5)
95    FORMAT(// 5X,"STATION LOCATIONS (INCHES)")
100   FORMAT(//7(4X,E14.7))
105   FORMAT(//5X,"MASS PER UNIT LENGTH (LB-SEC**2/IN**2)")
110   FORMAT(//5X,"FLAPWISE BENDING STIFFNESS (LB-IN**2)")
115   FORMAT(//5X,"CHORDWISE BENDING STIFFNESS (LB-IN**2)")
120   FORMAT(//5X,"TORSIONAL STIFFNESS (LB-IN**2)")
125   FORMAT(//5X,"DISTANCE BETWEEN MASS AND ELASTIC AXIS(INCHES)")
130   FORMAT(//5X,"TWIST OF THE BLADE NOT INCLUDING THE COLLECTIVE PITCH
     1(DEGREES)")
135   FORMAT(//5X,"STARTING FREQUENCY (RAD/SEC)
     1                                                          =",E14.7)
140   FORMAT(//5X,"MASS MOMENT OF INERTIA ABOUT THE CHORD (LB-SEC**2)")
145   FORMAT(//5X,"MASS MOMENT OF INERTIA ABOUT AN AXIS PERPENDICULAR TO
     1THE CHORD THROUGH THE CENTER OF GRAVITY (LB-SEC**2)")
150   FORMAT(//5X,"ENDING FREQUENCY (RAD/SEC)
     1                                                          =",E14.7)
155   FORMAT(//5X,"INCREMENT IN THE MODE NUMBER
     1                                                          =",I5)
160   FORMAT(//5X,"THE NUMBER OF FREQUENCIES DETECTED WITH IN THE RANGE
     1 ARE ONLY                                                 =",I5)
175   FORMAT(//5X"THE FOLLOWING ARE THE INTERPOLATED VALUES AT 101 EQUID
     1ISTANT STATIONS")
180   FORMAT(//5X,"THE FOLLOWING COLUMNS ARE 1.FREQUENCY(RAD/SEC) 2.NOND
     1IMENSIONAL FREQUENCY 3.VALUE OF THE FREQUENCY DETERMINANT RESPECTI
     2VELY")
185   FORMAT(10X,E14.7,15X,E14.7,15X,E14.7)
190   FORMAT(//5X,"COLLECTIVE PITCH(DEGREES)
     1                                                          =",E14.7)
195   FORMAT(//5X,"DISTANCE OF THE BLADE FROM THE ROOT (INCHES)
     1                                                          =",E14.7)
      STOP
      END
```

```
      SUBROUTINE NATFRE(N)
C
C     -----------------------------------------------------------------
C     THIS SUBROUTINE SCANS THE FREQUENCY DETERMINANT WITH RESPECT TO
C
C     THE FREQUENCY TILL THE SPECIFIED NUMBER OF SIGN CHANGES ARE
C
C     DETECTED STARTING FROM ZERO FREQUENCY.USES FUNCTION DET
C     -----------------------------------------------------------------
C
      DIMENSION FREQEN(10),JKL(10)
      COMMON/X1/FREQEN,H1,H,H3,ITER,IJK
      IJK=0
      ITRN=0
      DO 5 J=1,N
5     JKL(J)=0
      PP=H1
      P=PP*PP
      F=DET(P)
      IF(ABS(F).GT.0.0001)GO TO 10
      IJK=IJK+1
      JKL(IJK)=1
      FREQEN(IJK)=PP
      PP=PP+H
      P=PP*PP
      F=DET(P)
10    F=SIGN(1.0,F)
15    PP=PP+H
      IF(PP.GT.H3) GO TO 30
      P=PP*PP
      G=DET(P)
      IF(ABS(G).GT.0.0001)GO TO 20
      IJK=IJK+1
      JKL(IJK)=1
      FREQEN(IJK)=PP
      IF(IJK.EQ.N)GO TO 30
      PP=PP+H
      P=PP*PP
      F=DET(P)
      F=SIGN(1.0,F)
      GO TO 15
20    G=SIGN(1.0,G)
      IF(F*G.GT.0.0)GO TO 25
      IJK=IJK+1
      FREQEN(IJK)=PP-H
      IF(IJK.EQ.N)GO TO 30
25    F=G
      GO TO 15
```

```
30    ITRN=ITRN+1
      IF(ITRN.GT.ITER)GO TO 55
      IF(IJK.EQ.0) GO TO 65
      H=H/10.0
      DO 50 J=1,IJK
      IF(JKL(J).EQ.1)GO TO 50
      PP=FREQEN(J)
      P=PP*PP
      F=DET(P)
      F=SIGN(1.0,F)
35    PP=PP+H
      P=PP*PP
      G=DET(P)
      IF(ABS(G).GT.0.0001)GO TO 40
      JKL(J)=1
      FREQEN(J)=PP
      GO TO 50
40    G=SIGN(1.0,G)
      IF(F*G.GT.0.0)GO TO 45
      FREQEN(J)=PP-H
      GO TO 50
45    F=G
      GO TO 35
50    CONTINUE
      GO TO 30
55    DO 60 J=1,IJK
      IF(JKL(J).EQ.1)GO TO 60
      PP=FREQEN(J)
      P=PP*PP
      F=DET(P)
      PP=PP+H
      P=PP*PP
      G=DET(P)
      DIFF=G-F
      FRECEN(J)=PP-G*H/DIFF
60    CONTINUE
65    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PLOT(A,N)
C
C     ------------------------------------------------------------------
C     THIS SUBROUTINE PRINTS THE NATURAL FREQUENCIES AND MODE SHAPES AND
C
C     PLOTS THE MODE SHAPES
C     -----------------------------------------------   -------------
C
      REAL LINE
      DIMENSION A(51),SL(51),LINE(51)
      COMMON/X2/J1,PP,FRE,HERTZ,SL,BLANK,DOT,STAR
      COMMON/X11/IND
      WRITE(6,10)
      WRITE(6,20)
      WRITE(6,30)J1,FRE,HERTZ,PP
      WRITE(6,20)
      IF(N.EQ.1)WRITE(6,40)
      IF(N.EQ.2)WRITE(6,50)
      IF(N.EQ.3)WRITE(6,60)
      WRITE(6,70)
      IF(IND.EQ.0) GO TO 75
      WRITE(6,150)
      RETURN
75    DO 80 J=1,6
80    WRITE(6,90)(SL(J),A(J),SL(J+9),A(J+9),SL(J+18),A(J+18),SL(J+27),A(
     1J+27),SL(J+36),A(J+36),SL(J+45),A(J+45))
      WRITE(6,90)(SL(7),A(7),SL(16),A(16),SL(25),A(25),SL(34),A(34),SL(4
     13),A(43))
      WRITE(6,90)(SL(8),A(8),SL(17),A(17),SL(26),A(26),SL(35),A(35),SL(4
     14),A(44))
      WRITE(6,90)(SL(9),A(9),SL(18),A(18),SL(27),A(27),SL(36),A(36),SL(4
     15),A(45))
      WRITE(6,20)
      WRITE(6,10)
      DO 100 J=1,51
100   LINE(J)=DOT
      J=25.0*(A(1)+1.0)+1.5
      LINE(J)=STAR
      WRITE(6,110)(LINE(J),J=1,51)
      DO 120 J=1,51
120   LINE(J)=BLANK
      LINE(26)=DOT
      DO 130 JJ=3,51,2
      J=25.0*(A(JJ)+1.0)+1.5
      LINE(J)=STAR
      WRITE(6,140)(LINE(J),J=1,51)
      LINE(J)=BLANK
130   LINE(26)=DOT
```

```
10    FORMAT(1H1)
20    FORMAT(//2X,"*****************************************************
     1******************************************************************
     2******")
30    FORMAT(//5X," MODE NUMBER =",I2,8X,"FREQ. RAD/SEC =",F10.4,8X,"FRE
     1Q. HERTZ =",F10.4,8X," NON-DIMEN. FREQ. =",F10.4)
40    FORMAT(//50X,"FLAPWISE DEFLECTION/SEMICHORD")
50    FORMAT(//50X,"CHORDWISE DEFLECTION/SEMICHORD")
60    FORMAT(//55X,"TORSIONAL DEFLECTION")
70    FORMAT(6(4X,"STA X/L",4X,"DEFLN"))
90    FORMAT(//12(2X,F8.4))
110   FORMAT(////////40X,51A1)
140   FORMAT(40X,51A1)
150   FORMAT(//////5X,"THE MATRIX TO BE INVERTED IN THE MODE SHAPES COMP
     1UTATIONS IS SINGULAR AND HENCE COMPUTATIONS ARE ABANDONDED")
      RETURN
      END
```

```
      SUBROUTINE INTPOL(A)
C
C     ------------------------------------------------------------------
C     THIS SUBROUTINE INTERPOLATES THE REQUIRED VALUES.
C     ------------------------------------------------------------------
C
      DIMENSION A(101),STA(101),TABLE(101,1),B(101)
      COMMON /X3/STA,SPAN
      COMMON/X7/NS
      IN=0
      DO 5 J=1,NS
5     IF(ABS(A(J)).LE.0.0) IN=IN+1
      IF(IN.NE.NS)GO TO 15
      DO 10 J=1,101
10    A(J)=0.0
      RETURN
15    A(101)=A(NS)
      NM1=NS-1
      DO 20 I=1,NM1
20    TABLE(I,1)=(A(I+1)-A(I))/(STA(I+1)-STA(I))
      H=SPAN/100.0
      XARG=H
      DO 35 I=2,100
      DO 25J=1,NS
      IF(J.EQ.NS.OR.XARG.LE.STA(J)) GO TO 30
25    CONTINUE
30    MAX=J
      IF(MAX.LE.2) MAX=2
      ISUB=MAX-1
      YEST=TABLE(ISUB,1)
      B(I)=YEST*(XARG-STA(ISUB))+A(ISUB)
35    XARG=XARG+H
      DO  40 J=2,100
40    A(J)= B(J)
      RETURN
      END
```

```
      FUNCTION DET(P)
C
C     ------------------------------------------------------------------
C     THIS FUNCTION CALCULATES THE VALUE OF THE FREQUENCY DETERMINANT
C
C     USES SUBROUTINE TRAMAT
C     ------------------------------------------------------------------
C
      DIMENSION TF(10,10),A(5,5),IROW(5),JCOL(5),JORD(5)
      COMMON/X4/IBC
      COMMON/X6/TSR
      COMMON/X9 /TF
      CALL TRAMAT(P)
      DO 5 J=9,10
      DO 5 I=1,5
5     A(I,J-5)=TF(I+5,J)
      IF(IBC-2)10,20,30
10    DO  15 I=1,5
      A(I,1)=TF(I+5,4)
      A(I,2)=TF(I+5,5)-TF(I+5,6)*TSR
15    A(I,3)=TF(I+5,8)
      GOTO 40
20    DO 25 I=1,5
      A(I,1)=TF(I+5,3)
      A(I,2)=TF(I+5,5)-TF(I+5,6)*TSR
25    A(I,3)=TF(I+5,7)
      GOTO 40
30    DO 35 I=1,5
      A(I,1)=TF(I+5,5)-TF(I+5,6)*TSR
      A(I,2)=TF(I+5,7)
35    A(I,3)=TF(I+5,8)
40    CONTINUE
      N=5
      DET=1.0
      DO 80 K=1,N
      KM1=K-1
      PIVOT=0.0
      DO 60 I=1,N
      DO 60 J=1,N
      IF(K.EQ.1) GO TO 55
      DO 50 ISCAN=1,KM1
      DO 50 JSCAN=1,KM1
      IF(I.EQ.IROW(ISCAN)) GO TO 60
      IF(J.EQ.JCOL(JSCAN)) GO TO 60
50    CONTINUE
55    IF(ABS(A(I,J)).LE.ABS(PIVOT)) GO TO 60
      PIVOT=A(I,J)
      IROW(K)=I
```

```
      JCOL(K)=J
60    CONTINUE
      IF(ABS(PIVOT).GT.0.1E-20) GO TO 65
      DET=0.0
      RETURN
65    IROWK=IROW(K)
      JCOLK=JCOL(K)
      DET=DET*PIVOT
      DO 70 J=1,N
70    A(IROWK,J)=A(IROWK,J)/PIVOT
      A(IROWK,JCOLK)=1.0/PIVOT
      DO 80 I=1,N
      AIJCK=A(I,JCOLK)
      IF(I.EQ.IROWK) GO TO 80
      A(I,JCOLK)=-AIJCK/PIVOT
      DO 75 J=1,N
75    IF(J.NE.JCOLK) A(I,J)=A(I,J)-AIJCK*A(IROWK,J)
80    CONTINUE
      DO 85 I=1,N
      IROWI=IROW(I)
      JCOLI=JCOL(I)
85    JORD(IROWI)=JCOLI
      INTCH=0
      NM1=N-1
      DO 90 I=1,NM1
      IP1=I+1
      DO 90 J=IP1,N
      IF(JORD(J).GE.JORD(I)) GO TO 90
      JTEMP=JORD(J)
      JORD(J)=JORD(I)
      JORD(I)=JTEMP
      INTCH=INTCH+1
90    CONTINUE
      IF(INTCH/2*2.NE.INTCH)DET=-DET
      RETURN
      END
```

```
      SUBROUTINE TRAMAT(P)
C
C     ------------------------------------------------------------------
C     THIS SUBROUTINE COMPUTES THE TRASMISSION MATRIX THROUGH THE SYSTEM
C
C     USES THE SUBROUTINES INTG AND MATMUL
C     ------------------------------------------------------------------
C
      DIMENSION TF(10,10),TT(51,10,10),PT(10,10),A(10,10),B(10,10),C(10,
     110)
      COMMON/X8/PMASS,CLP,SLP,CBETA,SBETA,CTBETA,N1,N2,JSAVE
      COMMON/X9 /TF
      COMMON/X10/TT
      COMMON/X12/OMEGAN
      DO 10 I=1,10
      DO 5 J=1,10
      TT(1,I,J)=0.0
5     PT(I,J)=0.0
      TT(1,I,I)=1.0
10    PT(I,I)=1.0
      PT(6,1)=-P*PMASS*CLP*CBETA
      PT(10,1)=P*PMASS
      PT(6,2)=(P+OMEGAN)*PMASS*CLP*SBETA
      PT(9,2)=(P+OMEGAN)*PMASS
      PT(6,5)=(-P+OMEGAN*CTBETA)*PMASS*CLP*CLP
      PT(7,5)=-OMEGAN*PMASS*CLP*SLP
      PT(8,5)=-PT(7,5)*CBETA
      PT(7,5)=PT(7,5)*SBETA
      PT(9,5)=-PT(6,2)
      PT(10,5)=-PT(6,1)
      IK=N1
      CALL INTG(P,1,IK,1)
      IF(N1.GE.50) GO TO 15
      IJ=N2
      IL=JSAVE
      CALL INTG(P,IJ,50,IL)
15    DO 20 I=1,10
      DO 20 J=1,10
20    A(I,J)=TT(N2,I,J)
      CALL MATMUL(PT,A,B)
      N=N2+1
      IF(N.GT.51) GO TO 40
      DO 35 I=N,51
      DO 25 J=1,10
      DO 25 K=1,10
25    A(J,K)=TT(I,J,K)
      CALL MATMUL(A,B,C)
      DO 30 J=1,10
```

```
      DO 30 K=1,10
30    TT(I,J,K)=C(J,K)
35    CONTINUE
      GO TO 50
40    DO 45 I=1,10
      DO 45 J=1,10
45    TT(51,I,J)=B(I,J)
50    DO 55 I=1,10
      DO 55 J=1,10
55    TF(I,J)=TT(51,I,J)
      RETURN
      END
```

```
      SUBROUTINE INTG(P,NI,NJ,NK)
C
C     ------------------------------------------------------------------
C     INTEGRATION OF DIFFERENTIAL EQUATIONS,USES THE FUNCTION RUNGE
C     ------------------------------------------------------------------
C
      DIMENSION D1(101),D2(101),D3(101),D4(101),D5(101),D6(101),D7(101),
     1D8(101),D9(101),D10(101),D11(101),D12(101),V1(10,10),V(10),T(10),
     2TT(51,10,10)
      COMMON/X5/D1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11,D12
      COMMON/X12/OMEGAN
      COMMON/X10/TT
      DO 10 J=1,10
      DO 5  I=1,10
5     V1(I,J)=0.0
10    V1(J,J)=1.0
      M=0
      DO 45 I=1,10
      DO 15 J=1,10
15    V(J)=V1(J,I)
      J=NK
      DO 40 L=NI,NJ
20    K=RUNGE(V,T,J,M)
25    IF(K.NE.1) GO TO 30
      FACT=P+OMEGAN
      T(1)=V(3)
      T(2)=V(4)
      T(3)=D1(J)*V(7)+D2(J)*V(8)
      T(4)=D3(J)*V(7)+D1(J)*V(8)
      T(5)=D4(J)*V(6)
      T(6)=-P*D6(J)*V(1)+FACT*D7(J)*V(2)+D8(J)*V(3)-D9(J)*V(4)+D10(J)*
     1V(5)-P*D11(J)*V(5)
      T(7)=D12(J)*V(4)+V(9)-D9(J)*V(5)
      T(8)=D12(J)*V(3)+V(10)+D8(J)*V(5)
      T(9)=FACT*D5(J)*V(2)-FACT*D7(J)*V(5)
      T(10)=P*D5(J)*V(1)+P*D6(J)*V(5)
      GO TO 20
30    DO 35 JJ=1,10
35    TT(L+1,JJ,I)=V(JJ)
40    CONTINUE
45    CONTINUE
      RETURN
      END
```

```
      FUNCTION RUNGE(Y,F,J,M)
C
C     -----------------------------------------------------------------
C     FOURTH-ORDER RUNGE-KUTTA METHOD
C     -----------------------------------------------------------------
C
      DIMENSION Y(10),F(10),PHI(10),SAVEY(10)
      M=M+1
      GO TO(5,10,20,30,40),M
5     RUNGE=1
      RETURN
10    DO 15 JJ=1,10
      SAVEY(JJ)=Y(JJ)
      PHI(JJ)=F(JJ)
15    Y(JJ)=SAVEY(JJ)+0.01*F(JJ)
      J=J+1
      RUNGE=1
      RETURN
20    DO 25 JJ=1,10
      PHI(JJ)=PHI(JJ)+2.0*F(JJ)
25    Y(JJ)=SAVEY(JJ)+0.01*F(JJ)
      RUNGE=1
      RETURN
30    DO 35 JJ=1,10
      PHI(JJ)=PHI(JJ)+2.0*F(JJ)
35    Y(JJ)=SAVEY(JJ)+0.02*F(JJ)
      J=J+1
      RUNGE=1
      RETURN
40    DO 45 JJ=1,10
45    Y(JJ)=SAVEY(JJ)+(PHI(JJ)+F(JJ))/300.0
      M=0
      RUNGE=0
      RETURN
      END
```

```
      SUBROUTINE MATMUL(A,B,C)
C
C     ----------------------------------------------------------------
C     MATRIX MULTIPLICATION
C     ----------------------------------------------------------------
C
      DIMENSION A(10,10),B(10,10),C(10,10)
      DO 5 I=1,10
      DO 5 J=1,10
      C(I,J)=0.0
      DO 5 K=1,10
5     C(I,J)=C(I,J)+A(I,K)*B(K,J)
      RETURN
      END
```

```
      SUBROUTINE SHAPES(P,W,V,PHI)
C
C     ------------------------------------------------------------------
C     THIS SUBROUTINE CALCULATES THE MODE SHAPES.USES THE SUBROUTINE
C
C     TRAMAT
C     ------------------------------------------------------------------
C
      DIMENSION W(51),V(51),PHI(51),A(6,6),TF(10,10),TT(51,10,10),X(5),
     1IROW(5),JCOL(5),JORD(5)
      COMMON/X4/IBC
      COMMON/X6/TSR
      COMMON/X9/TF
      COMMON/X10/TT
      COMMON/X11/IND
      IND=0
      CALL TRAMAT(P)
      A(1,4)=TF(1,9)
      A(1,5)=TF(1,10)
      A(1,6)=1.0
      DO 10 I=2,5
      DO 5 J=4,5
5     A(I,J)=TF(I+4,J+5)
10    A(I,6)=0.0
      IF(IBC-2)15,25,35
15    A(1,1)=TF(1,4)
      A(1,2)=TF(1,5)-TSR*TF(1,6)
      A(1,3)=TF(1,8)
      DO 20 I=2,5
      A(I,1)=TF(I+4,4)
      A(I,2)=TF(I+4,5)-TSR*TF(I+4,6)
20    A(I,3)=TF(I+4,8)
      GO TO 45
25    A(1,1)=TF(1,3)
      A(1,2)=TF(1,5)-TSR*TF(1,6)
      A(1,3)= TF(1,7)
      DO 30 I=2,5
      A(I,1)=TF(I+4,3)
      A(I,2)=TF(I+4,5)-TSR*TF(I+4,6)
30    A(I,3)=TF(I+4,7)
      GO TO 45
35    A(1,1)=TF(1,5)-TSR*TF(1,6)
      A(1,2)=TF(1,7)
      A(1,3)=TF(1,8)
      DO 40 I=2,5
      A(I,1)=TF(I+4,5)-TSR*TF(I+4,6)
      A(I,2)=TF(I+4,7)
40    A(I,3)=TF(I+4,8)
```

```
45    N=5
      MAX=N+1
      DETER=1.0
      DO 80 K=1,N
      KM1=K-1
      PIVOT=0.0
      DO 60 I=1,N
      DO 60 J=1,N
      IF(K.EQ.1)GO TO 55
      DO 50 ISCAN=1,KM1
      DO 50 JSCAN=1,KM1
      IF(I.EQ.IROW(ISCAN)) GO TO 60
      IF(J.EQ.JCOL(JSCAN)) GO TO 60
50    CONTINUE
55    IF(ABS(A(I,J)).LE.ABS(PIVOT)) GO TO 60
      PIVOT=A(I,J)
      IROW(K)=I
      JCOL(K)=J
60    CONTINUE
      IF(ABS(PIVOT).GT.0.1E-20)GO TO 65
      IND=1
      RETURN
65    IROWK=IROW(K)
      JCOLK=JCOL(K)
      DETER=DETER*PIVOT
      DO 70 J=1,MAX
70    A(IROWK,J)=A(IROWK,J)/PIVOT
      A(IROWK,JCOLK)=1.0/PIVOT
      DO 80 I=1,N
      AIJCK=A(I,JCOLK)
      IF(I.EQ.IROWK) GO TO 80
      A(I,JCOLK)=-AIJCK/PIVOT
      DO 75 J=1,MAX
75    IF(J.NE.JCOLK) A(I,J)=A(I,J)-AIJCK*A(IROWK,J)
80    CONTINUE
      DO 85 I=1,N
      IROWI=IROW(I)
      JCOLI=JCOL(I)
      JORD(IROWI)=JCOLI
85    X(JCOLI)=A(IROWI,MAX)
      INTCH=0
      NM1=N-1
      DO 90 I=1,NM1
      IP1=I+1
      DO 90 J=IP1,N
      IF(JORD(J).GE.JORD(I)) GO TO 90
      JTEMP=JORD(J)
      JORD(J)=JORD(I)
      JORD(I)=JTEMP
```

```
      INTCH=INTCH+1
90    CONTINUE
      IF(INTCH/2*2.NE.INTCH)DETER=-DETER
      IF(ABS(DETER).GT.1.0E-30)GO TO 95
      IND=1
      RETURN
95    IF(IBC-2)100,110,120
100   DO 105 J=1,51
      W(J)=X(1)*TT(J,1,4)+X(2)*(TT(J,1,5)-TSR*TT(J,1,6))+X(3)*TT(J,1,8)+
     1X(4)*TT(J,1,9)+X(5)*TT(J,1,10)
      V(J)=X(1)*TT(J,2,4)+X(2)*(TT(J,2,5)-TSR*TT(J,2,6))+X(3)*TT(J,2,8)+
     1X(4)*TT(J,2,9)+X(5)*TT(J,2,10)
105   PHI(J)=X(1)*TT(J,5,4)+X(2)*(TT(J,5,5)-TSR*TT(J,5,6))+X(3)*TT(J,5,8
     1)+X(4)*TT(J,5,9)+X(5)*TT(J,5,10)
      RETURN
110   DO 115 J=1,51
      W(J)=X(1)*TT(J,1,3)+X(2)*(TT(J,1,5)-TSR*TT(J,1,6))+X(3)*TT(J,1,7)+
     1X(4)*TT(J,1,9)+X(5)*TT(J,1,10)
      V(J)=X(1)*TT(J,2,3)+X(2)*(TT(J,2,5)-TSR*TT(J,2,6))+X(3)*TT(J,2,7)+
     1X(4)*TT(J,2,9)+X(5)*TT(J,2,10)
115   PHI(J)=X(1)*TT(J,5,3)+X(2)*(TT(J,5,5)-TSR*TT(J,5,6))+X(3)*TT(J,5,7
     1)+X(4)*TT(J,5,9)+X(5)*TT(J,5,10)
      RETURN
120   DO 125 J=1,51
      W(J)=X(1)*(TT(J,1,5)-TSR*TT(J,1,6))+X(2)*TT(J,1,7)+X(3)*TT(J,1,8)+
     1X(4)*TT(J,1,9)+X(5)*TT(J,1,10)
      V(J)=X(1)*(TT(J,2,5)-TSR*TT(J,2,6))+X(2)*TT(J,2,7)+X(3)*TT(J,2,8)+
     1X(4)*TT(J,2,9)+X(5)*TT(J,2,10)
125   PHI(J)=X(1)*(TT(J,5,5)-TSR*TT(J,5,6))+X(2)*TT(J,5,7)+X(3)*TT(J,5,8
     1)+X(4)*TT(J,5,9)+X(5)*TT(J,5,10)
130   FORMAT(//5X,"THE MATRIX TO BE INVERTED IS SINGULAR IN THE MODE
     1SHAPES COMPUTATIONS AND HENCE ABANDONDED")
      RETURN
      END
```

FORTRAN PROGRAM III

```
C       ******************************************************************
C       NATURAL VIBRATION CHARACTERISTICS,PURE TORSION,USES FUNCTION DET
C
C       AND SUBROUTINES INTPOL,PLOT,NATFRE AND SHAPES
C       ******************************************************************
C
        REAL MASS,KM1S,KM2S,KMS
        DIMENSION GJ(101),KM1S(101),KM2S(101),KMS(101),MASS(101),D1(101),
       1D2(101),D3(101),STA(101),PHI(51),SL(51),FREQEN(10)
        COMMON/X1/FREQEN,H1,H2,H3,ITER,IJK
        COMMON/X2/J1,PP,FRE,HERTZ,SL,BLANK,DOT,STAR
        COMMON/X3/STA,SPAN
        COMMON/X4/D1,D2,D3
        COMMON/X5/TSR
        COMMON/X6/NS
        COMMON/X7/PMASS,CLP,SLP,N1,N2,JSAVE
        COMMON/X11/OMEGAN
C
C       ------------------------------------------------------------------
C       THIS SECTION READS THE DATA OF THE SYSTEM
C       ------------------------------------------------------------------
C
        READ(5,5)ISTAGE,NS
        READ(5,10)SPAN,B,TSR,OMEGA
        READ(5,10)PMASS,CLP,SLP
        READ(5,10)(STA(J),J=1,NS),(MASS(J),J=1,NS),(GJ(J),J=1,NS),
       1(KM1S(J),J=1,NS),(KM2S(J),J=1,NS)
        READ(5,10)H1,H2,H3
        IF(ISTAGE.EQ.2)READ(5,15)NF,ITER,BLANK,DOT,STAR,INC
C
C       ------------------------------------------------------------------
C       THIS SECTION PRINTS THE DATA OF THE SYSTEM
C       ------------------------------------------------------------------
        WRITE(6,20)
        WRITE(6,25)
        IF(ISTAGE.EQ.2)WRITE(6,30)NF
        WRITE(6,35)H1
        WRITE(6,40)H2
        WRITE(6,45)H3
        IF(ISTAGE.EQ.2)WRITE(6,50)INC
        WRITE(6,55)SPAN
        WRITE(6,60)OMEGA
        WRITE(6,65)B
        WRITE(6,70)TSR
        IF(PMASS.LE.0.0)GO TO 200
        WRITE(6,75)PMASS
        WRITE(6,80)CLP
        WRITE(6,85)SLP
```


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```
200   WRITE(6,90)NS
      WRITE(6,95)
      WRITE(6,100)(STA(J),J=1,NS)
      WRITE(6,105)
      WRITE(6,100)(MASS(J),J=1,NS)
      WRITE(6,110)
      WRITE(6,100)(GJ (J),J=1,NS)
      WRITE(6,115)
      WRITE(6,100)(KM1S(J),J=1,NS)
      WRITE(6,120)
      WRITE(6,100)(KM2S(J),J=1,NS)
      WRITE(6,25)
C
C     ------------------------------------------------------------------
C     THIS SECTION CALCULATES THE SYSTEM PROPERTIES AT THE REQUIRED
C
C     STATIONS BY INTERPOLATION AND PRINTS THE INTERPOLATED VALUES
C     ------------------------------------------------------------------
C
      CALL INTPOL(MASS)
      CALL INTPOL(GJ)
      CALL INTPOL(BETA)
      CALL INTPOL(KM1S)
      CALL INTPOL(KM2S)
      WRITE(6,20)
      WRITE(6,25)
      WRITE(6,125)
      WRITE(6,25)
      WRITE(6,105)
      WRITE(6,100)(MASS(J),J=1,101)
      WRITE(6,110)
      WRITE(6,100)(GJ(J),J=1,101)
      WRITE(6,115)
      WRITE(6,100)(KM1S(J),J=1,101)
      WRITE(6,120)
      WRITE(6,100)(KM2S(J),J=1,101)
      WRITE(6,25)
C
C     ------------------------------------------------------------------
C     THIS SECTION COMPUTES THE NON-DIMENSIONAL COEFFICIENTS OF THE
C
C     FIRST-ORDER DIFFERENTIAL EQUATIONS
C     ------------------------------------------------------------------
C
      BS=B*B
      SPANS=SPAN*SPAN
      FI=BS/SPANS
      PI=4.0*ATAN(1.0)
      OMEGA=OMEGA*PI/30.0
```

```
      FACT=MASS(1)*SPANS*SPANS/GJ(1)
      OMEGAN=OMEGA*OMEGA*FACT
      FACT=SQRT(FACT)
      TSR=TSR* SPANS*SPAN/(GJ(1)*BS)
      IF(PMASS.LE.0.0)SLP=SPAN
      PMASS=PMASS/(SPAN*MASS(1)*386.4)
      CLP=CLP/B
      SLP=SLP/SPAN
      N1=50.0*SLP+0.5
      N2=N1+1
      JSAVE=2*N1+1
      DO 205 J=1,101
      KMS(J)=KM1S(J)+KM2S(J)
      D1(J)=GJ(1)*F1/GJ(J)
      D2(J)=OMEGAN*(KM2S(J)-KM1S(J))/(MASS(1)*BS)
205   D3(J)=KMS(J)/(MASS(1)*BS)
      H1=H1*FACT
      H2=H2*FACT
      H3=H3*FACT
      IF(ISTAGE.EQ.2)GO TO 215
C
C     ----------------------------------------------------------------
C     THIS SECTION CALCULATES THE FREQUENCY DETERMINANTS OF THE SYSTEM
C     ----------------------------------------------------------------
C
      WRITE(6,20)
      WRITE(6,25)
      WRITE(6,130)
      WRITE(6,25)
210   P=H1*H1
      IF(H1.GT.H3)GO TO245
      FR=H1/FACT
      F=DET(P)
      WRITE(6,135)FR,H1,F
      H1=H1+H2
      GOTO210
C
C     ----------------------------------------------------------------
C     THIS SECTION CALCULATES THE NATURAL FREQUENCIES AND THE ASSOCIATED
C
C     MODAL FUNCTIONS
C     ----------------------------------------------------------------
C
215   CALL NATFRE(NF)
      SL(1)=0.0
      DO220 J=1,50
220   SL(J+1)=SL(J)+0.02
      DO 240 J=1,IJK
      J1=J+INC
```

```
      PP=FREQEN(J)
      P=PP*PP
      CALL SHAPES(P,PHI)
      AMAX=PHI(1)
      DO 225 I=1,51
225   IF(ABS(AMAX).LT.ABS(PHI(I)))AMAX=PHI(I)
      DO 230 I=1,51
230   PHI(I)=PHI(I)/AMAX
      FRE=PP/FACT
      HERTZ=FRE/(2.0*PI)
235   CALL PLOT(PHI)
240   CONTINUE
      IF(IJK.LT.NF)WRITE(6,140)
      IF(ISTAGE.EQ.1)WRITE(6,25)
245   CONTINUE
C
C     -----------------------------------------------------------------
C     FORMATS
C     -----------------------------------------------------------------
C
5     FORMAT(I1,I3)
10    FORMAT(5E14.7)
15    FORMAT(I2,I1,3A1,I2)
20    FORMAT(1H1)
25    FORMAT(//2X"*****************************************************
     1*****************************************************************
     2*****")
30    FORMAT(//5X,"NUMBER OF FREQUENCIES REQUIRED
     1                                                =",I5)
35    FORMAT(//5X,"STARTING FREQUENCY(RAD/SEC)
     1                                                =",E14.7)
40    FORMAT(//5X,"FREQUENCY INCREMENT(RAD/SEC)
     1                                                =",E14.7)
45    FORMAT(//5X,"ENDING FREQUENCY (RAD/SEC)
     1                                                =",E14.7)
50    FORMAT(//5X,"INCREMENT IN THE MODE NUMBER
     1                                                =",I5)
55    FORMAT(//5X,"LENGTH OF THE BLADE(INCHES)
     1                                                =",E14.7)
60    FORMAT(//5X,"ROTATIONAL VELOCITY OF THE BLADE(RPM)
     1                                                =",E14.7)
65    FORMAT(//5X,"SEMI-CHORD AT THE ROOT(INCHES)
     1                                                =",E14.7)
70    FORMAT(//5X,"CONTROL SYSTEM SPRING RATE(IN-LB/RAD)
     1                                                =",E14.7)
75    FORMAT(//5X,"WEIGHT OF THE SENSOR(LB)
     1                                                =",E14.7)
80    FORMAT(//5X,"CHORDWISE LOCATION OF THE PROBE(INCHES)
     1                                                =",E14.7)
```

```
85    FORMAT(//5X,"SPANWISE  LOCATION OF THE PROBE(INCHES)
     1                                                    =",E14.7)
90    FORMAT(//5X,"NUMBER OF DATA POINTS
     1                                                    =",I5)
95    FORMAT(//5X,"STATION LOCATIONS(INCHES)")
100   FORMAT(//7(4X,E14.7))
105   FORMAT(//5X,"MASS PER UNIT LENGTH(LB-SEC**2)")
110   FORMAT(//5X,"TORSIONAL STIFFNESS(LB-IN**2)")
115   FORMAT(//5X,"MASS MOMENT OF INERTIA ABOUT THE CHORD(LB-SEC**2)")
120   FORMAT(//5X,"MASS MOMENT OF INERTIA ABOUT AN AXIS PERPENDICULAR TO
     1 THE CHORD THROUGH THE CENTER OF GRAVITY(LB-SEC**2)")
125   FORMAT(//5X,"THE FOLLOWING ARE THE INTERPOLATED VALUES AT 101 EQUI
     1DISTANT STATIONS")
130   FORMAT(//5X,"THE FOLLOWING COLUMNS ARE 1.FREQUENCY(RAD/SEC)2.NON-D
     1IMENSIONAL FREQUENCY3.VALUE OF THE FREQUENCY DETERIMINANT RESPECTI
     2VELY")
135   FORMAT(10X,E14.7,15X,E14.7,15X,E14.7)
140   FORMAT(//5X,"THE NUMBER OF FREQUENCIES DETECTED WITHIN THE RANGE A
     1RE ONLY                                             =",I5)
      STOP
      END
```

```
      SUBROUTINE NATFRE(N)
C
C ------------------------------------------------------------------------
C     THIS SUBROUTINE SCANS THE FREQUENCY DETERMINANT WITH RESPECT TO
C
C     THE FREQUENCY TILL THE SPECIFIED NUMBER OF SIGN CHANGES ARE
C
C     DETECTED STARTING FROM ZERO FREQUENCY.USES FUNCTION DET
C ------------------------------------------------------------------------
C
      DIMENSION FREQEN(10),JKL(10)
      COMMON/X1/FREQEN,H1,H,H3,ITER,IJK
      IJK=0
      ITRN=0
      DO 5 J=1,N
5     JKL(J)=0
      PP=H1
      P=PP*PP
      F=DET(P)
      IF(ABS(F).GT.0.0001)GO TO 10
      IJK=IJK+1
      JKL(IJK)=1
      FREQEN(IJK)=PP
      PP=PP+H
      P=PP*PP
      F=DET(P)
10    F=SIGN(1.0,F)
15    PP=PP+H
      P=PP*PP
      G=DET(P)
      IF(ABS(G).GT.0.0001)GO TO 20
      IJK=IJK+1
      JKL(IJK)=1
      FREQEN(IJK)=PP
      IF(IJK.EQ.N)GO TO 30
      IF(PP.GE.H3) GO TO 30
      PP=PP+H
      P=PP*PP
      F=DET(P)
      F=SIGN(1.0,F)
      GO TO 15
20    G=SIGN(1.0,G)
      IF(F*G.GT.0.0)GO TO 25
      IJK=IJK+1
      FREQEN(IJK)=PP-H
      IF(IJK.EQ.N)GO TO 30
      IF(PP.GT.H3) GO TO 30
25    F=G
```

```
      GO TO 15
30    ITRN=ITRN+1
      IF(ITRN.GT.ITER)GO TO 55
      H=H/10.0
      DO 50 J=1,IJK
      IF(JKL(J).EQ.1)GO TO 50
      PP=FREQEN(J)
      P=PP*PP
      F=DET(P)
      F=SIGN(1.0,F)
35    PP=PP+H
      P=PP*PP
      G=DET(P)
      IF(ABS(G).GT.0.0001)GO TO 40
      JKL(J)=1
      FREQEN(J)=PP
      GO TO 50
40    G=SIGN(1.0,G)
      IF(F*G.GT.0.0)GO TO 45
      FREQEN(J)=PP-H
      GO TO 50
45    F=G
      GO TO 35
50    CONTINUE
      GO TO 30
55    DO 60 J=1,IJK
      IF(JKL(J).EQ.1)GO TO 60
      PP=FREQEN(J)
      P=PP*PP
      F=DET(P)
      PP=PP+H
      P=PP*PP
      G=DET(P)
      DIFF=G-F
      FREQEN(J)=PP-G*H/DIFF
60    CONTINUE
      RETURN
      END
      SUBROUTINE PLOT(A)
```

```
C
C     ------------------------------------------------------------------------
C     THIS SUBROUTINE PRINTS THE NATURAL FREQUENCIES AND MODE SHAPES AND
C
C     PLOTS THE MODE SHAPES
C     ------------------------------------------------------------------------
C
      REAL LINE
      DIMENSION A(51),SL(51),LINE(51)
      COMMON/X2/J1,PP,FRE,HERTZ,SL,BLANK,DOT,STAR
      WRITE(6,10)
      WRITE(6,20)
      WRITE(6,30)J1,FRE,HERTZ,PP
      WRITE(6,20)
      WRITE(6,60)
      WRITE(6,70)
75    DO 80 J=1,6
80    WRITE(6,90)(SL(J),A(J),SL(J+9),A(J+9),SL(J+18),A(J+18),SL(J+27),A(
     1J+27),SL(J+36),A(J+36),SL(J+45),A(J+45))
      WRITE(6,90)(SL(7),A(7),SL(16),A(16),SL(25),A(25),SL(34),A(34),SL(4
     13),A(43))
      WRITE(6,90)(SL(8),A(8),SL(17),A(17),SL(26),A(26),SL(35),A(35),SL(4
     14),A(44))
      WRITE(6,90)(SL(9),A(9),SL(18),A(18),SL(27),A(27),SL(36),A(36),SL(4
     15),A(45))
      WRITE(6,20)
      WRITE(6,10)
      DO 100 J=1,51
100   LINE(J)=DOT
      J=25.0*(A(1)+1.0)+1.5
      LINE(J)=STAR
      WRITE(6,110)(LINE(J),J=1,51)
      DO 120 J=1,51
120   LINE(J)=BLANK
      LINE(26)=DOT
      DO 130 JJ=3,51,2
      J=25.0*(A(JJ)+1.0)+1.5
      LINE(J)=STAR
      WRITE(6,140)(LINE(J),J=1,51)
      LINE(J)=BLANK
130   LINE(26)=DOT
10    FORMAT(1H1)
20    FORMAT(//2X,"**************************************************
     1*****************************************************************
     2******")
30    FORMAT(//5X," MODE NUMBER =",I2,8X,"FREQ. RAD/SEC =",F10.4,8X,"FRE
     1Q. HERTZ =",F10.4,8X," NON-DIMEN. FREQ. =",F10.4)
60    FORMAT(//55X,"TORSIONAL DEFLECTION")
```

```
70     FORMAT(6(4X,"STA X/L",4X,"DEFLN"))
90     FORMAT(//12(2X,F8.4))
110    FORMAT(////////40X,51A1)
140    FORMAT(40X,51A1)
       RETURN
       END
       SUBROUTINE INTPOL(A)
```

```
C
C     ------------------------------------------------------------
C     THIS SUBROUTINE INTERPOLATES THE REQUIRED VALUES.
C     ------------------------------------------------------------
C
      DIMENSION A(101),STA(101),TABLE(101,1),B(101)
      COMMON /X3/STA,SPAN
      COMMON/X6/NS
      IN=0
      DO 5 J=1,NS
5     IF(ABS(A(J)).LE.0.0) IN=IN+1
      IF(IN.NE.NS)GO TO 15
      DO 10 J=1,101
10    A(J)=0.0
      RETURN
15    A(101)=A(NS)
      NM1=NS-1
      DO 20 I=1,NM1
20    TABLE(I,1)=(A(I+1)-A(I))/(STA(I+1)-STA(I))
      H=SPAN/100.0
      XARG=H
      DO 35 I=2,100
      DO 25J=1,NS
      IF(J.EQ.NS.OR.XARG.LE.STA(J)) GO TO 30
25    CONTINUE
30    MAX=J
      IF(MAX.LE.2) MAX=2
      ISUB=MAX-1
      YEST=TABLE(ISUB,1)
      B(I)=YEST*(XARG-STA(ISUB))+A(ISUB)
35    XARG=XARG+H
      DO  40 J=2,100
40    A(J)= B(J)
      RETURN
      END
      FUNCTION DET(P)
```

```
C
C     ------------------------------------------------------------------
C     THIS FUNCTION CALCULATES THE VALUE OF THE FREQUENCY DETERMINANT
C
C     USES SUBROUTINE TRAMAT
C     ------------------------------------------------------------------
C
      DIMENSION TF(2,2)
      COMMON/X5/TSR
      COMMON/X9/TF
      CALL TRAMAT(P)
      DET=TF(2,1)-TSR*TF(2,2)
      RETURN
      END
      SUBROUTINE TRAMAT(P)
```

```
C
C     ------------------------------------------------------------------
C     THIS SUBROUTINE COMPUTES THE TRASMISSION MATRIX THROUGH THE SYSTEM
C
C     USES THE SUBROUTINES INTG AND MATMUL
C     ------------------------------------------------------------------
C
      DIMENSION TF(2,2),TT(51,2,2),PT(2,2),A(2,2),B(2,2),C(2,2)
      COMMON/X7/PMASS,CLP,SLP,N1,N2,JSAVE
      COMMON/X9 /TF
      COMMON/X10/TT
      COMMON/X11/OMEGAN
      DO 10 I=1,2
      DO 5 J=1,2
      TT(1,I,J)=0.0
5     PT(I,J)=0.0
      TT(1,I,I)=1.0
10    PT(I,I)=1.0
      PT(2,1)=(-P+OMEGAN)*PMASS*CLP*CLP
      IK=N1
      CALL INTG(P,1,IK,1)
      IF(N1.GE.50) GO TO 15
      IJ=N2
      IL=JSAVE
      CALL INTG(P,IJ,50,IL)
15    DO 20 I=1,2
      DO 20 J=1,2
20    A(I,J)=TT(N2,I,J)
      CALL MATMUL(PT,A,B)
      N=N2+1
      IF(N.GT.51) GO TO 40
      DO 35 I=N,51
      DO 25 J=1,2
      DO 25 K=1,2
25    A(J,K)=TT(I,J,K)
      CALL MATMUL(A,B,C)
      DO 30 J=1,2
      DO 30 K=1,2
30    TT(I,J,K)=C(J,K)
35    CONTINUE
      GO TO 50
40    DO 45 I=1,2
      DO 45 J=1,2
45    TT(51,I,J)=B(I,J)
50    DO 55 I=1,2
      DO 55 J=1,2
55    TF(I,J)=TT(51,I,J)
      RETURN
```

```
      END
      SUBROUTINE INTG(P,NI,NJ,NK)
```

```
C
C      ---------------------------------------------------------------
C      INTEGRATION OF DIFFERENTIAL EQUATIONS,USES THE FUNCTION RUNGE
C      ---------------------------------------------------------------
C
      DIMENSION D1(101),D2(101),D3(101),V1(2,2),V(2),T(2),TT(51,2,2)
      COMMON/X4/D1,D2,D3
      COMMON/X10/TT
      DO 10 J=1,2
      DO 5  I=1,2
5     V1(I,J)=0.0
10    V1(J,J)=1.0
      M=0
      DO 45 I=1,2
      DO 15 J=1,2
15    V(J)=V1(J,I)
      J=NK
      DO 40 L=NI,NJ
20    K=RUNGE(V,T,J,M)
25    IF(K.NE.1) GO TO 30
      T(1)=D1(J)*V(2)
      T(2)=D2(J)*V(1)-P*D3(J)*V(1)
      GO TO 20
30    DO 35 JJ=1,2
35    TT(L+1,JJ,I)=V(JJ)
40    CONTINUE
45    CONTINUE
      RETURN
      END
      FUNCTION RUNGE(Y,F,J,M)
```

```
C
C     ------------------------------------------------------------
C     FOURTH-ORDER RUNGE-KUTTA METHOD
C     ------------------------------------------------------------
C
      DIMENSION Y(2),F(2),PHI(2),SAVEY(2)
      M=M+1
      GO TO(5,10,20,30,40),M
5     RUNGE=1
      RETURN
10    DO 15 JJ=1,2
      SAVEY(JJ)=Y(JJ)
      PHI(JJ)=F(JJ)
15    Y(JJ)=SAVEY(JJ)+0.01*F(JJ)
      J=J+1
      RUNGE=1
      RETURN
20    DO 25 JJ=1,2
      PHI(JJ)=PHI(JJ)+2.0*F(JJ)
25    Y(JJ)=SAVEY(JJ)+0.01*F(JJ)
      RUNGE=1
      RETURN
30    DO 35 JJ=1,2
      PHI(JJ)=PHI(JJ)+2.0*F(JJ)
35    Y(JJ)=SAVEY(JJ)+0.02*F(JJ)
      J=J+1
      RUNGE=1
      RETURN
40    DO 45 JJ=1,2
45    Y(JJ)=SAVEY(JJ)+(PHI(JJ)+F(JJ))/300.0
      M=0
      RUNGE=0
      RETURN
      END
      SUBROUTINE MATMUL(A,B,C)
```

```
C
C     ------------------------------------------------------------------
C     MATRIX MULTIPLICATION
C     ------------------------------------------------------------------
C
      DIMENSION A(2,2),B(2,2),C(2,2)
      DO 5 I=1,2
      DO 5 J=1,2
      C(I,J)=0.0
      DO 5 K=1,2
    5 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      RETURN
      END
      SUBROUTINE SHAPES(P,PHI)
```

```
C
C      ----------------------------------------------------------------
C      THIS SUBROUTINE CALCULATES THE MODE SHAPES.USES THE SUBROUTINE
C
C      TRAMAT
C      ----------------------------------------------------------------
C
       DIMENSION PHI(51),TT(51,2,2)
       COMMON/X5/TSR
       COMMON/X10/TT
       CALL TRAMAT(P)
       DO 5 J=1,51
5      PHI(J)=TT(J,1,1)-TSR*TT(J,1,2)
       RETURN
       END
       END
```

SAMPLE OUTPUT

```
********************************************************************************************************************

NATURE OF THE MODES                                  COLLECTIVE MODES

NUMBER OF FREQUENCIES REQUIRED                                          =  7

STARTING FREQUENCY (RAD/SEC)                                            = 0.

FREQUENCY INCREMENT(RAD/SEC)                                            = .5000000E+01

ENDING FREQUENCY (RAD/SEC)                                              = .8000000E+03

INCREMENT IN THE MODE NUMBER                                            =  0

LENGTH OF THE BLADE (INCHES)                                            = .2118000E+03

ROTATIONAL VELOCITY OF THE BLADE (RPM)                                  = .3540000E+03

SEMI-CHORD OF THE BLADE (INCHES)                                        = .6500000E+01

CONTROL SYSTEM SPRING RATE (IN-LB/RAD)                                  = .2250000E+06

COLLECTIVE PITCH(DEGREES)                                               = .1500000E+02

DISTANCE OF THE BLADE FROM THE ROOT (INCHES)                            = .1850000E+02

NUMBER OF DATA POINTS                                                   =  35

STATION LOCATIONS (INCHES)

0.     .1000000E+01   .2100000E+01   .3000000E+01   .3500000E+01   .5500000E+01   .8250000E+01
```

.8500000E+01	.1025000E+02	.1100000E+02	.1200000E+02	.1350000E+02	.1400000E+02	.1650000E+02
.1750000E+02	.1850000E+02	.2000000E+02	.2200000E+02	.3000000E+02	.3500000E+02	.4000000E+02
.4500000E+02	.5000000E+02	.5500000E+02	.6000000E+02	.9000000E+02	.1020000E+03	.1100000E+03
.1200000E+03	.1800000E+03	.1880000E+03	.2050000E+03	.2070000E+03	.2090000E+03	.2118000E+03

MASS PER UNIT LENGTH (LB-SEC**2/IN**2)

.1760000E-02	.1760000E-02	.6990000E-02	.6990000E-02	.4040000E-02	.4040000E-02	.2870000E-02
.4430000E-02	.6160000E-02	.6160000E-02	.4430000E-02	.4430000E-02	.3030000E-02	.2170000E-02
.3880000E-02	.2460000E-02	.2540000E-02	.2540000E-02	.1760000E-02	.1480000E-02	.1220000E-02
.1040000E-02	.8500000E-03	.7500000E-03	.6500000E-03	.6600000E-03	.2110000E-02	.2110000E-02
.6600000E-03	.6600000E-03	.2070000E-02	.2070000E-02	.6600000E-03	.6600000E-03	.2380000E-02

FLAPWISE BENDING STIFFNESS (LB-IN**2)

.7700000E+08	.7700000E+08	.5160000E+09	.5160000E+09	.1160000E+09	.1380000E+09	.1380000E+09
.1380000E+09	.4320000E+09	.4320000E+09	.3190000E+09	.3190000E+09	.3100000E+09	.2560000E+09
.2560000E+09	.1108000E+09	.6200000E+08	.5400000E+08	.2800000E+08	.2000000E+08	.1400000E+08
.1100000E+08	.8500000E+07	.6500000E+07	.5820000E+07	.5820000E+07	.6260000E+07	.6260000E+07
.5820000E+07	.5820000E+07	.7560000E+07	.7560000E+07	.5820000E+07	.5820000E+07	.5820000E+07

CHORDWISE BENDING STIFFNESS (LB-IN**2)

.2680000E+09	.2680000E+09	.6500000E+09	.6500000E+09	.1160000E+09	.1420000E+09	.1420000E+09
.1420000E+09	.5180000E+09	.5180000E+09	.2000000E+09	.2000000E+09	.1170000E+09	.1170000E+09
.1170000E+09	.1170000E+09	.4150000E+09	.4500000E+09	.5000000E+09	.5000000E+09	.4600000E+09
.4250000E+09	.3950000E+09	.3650000E+09	.3375000E+09	.2400000E+09	.2380000E+09	.2380000E+09
.2340000E+09	.2340000E+09	.2830000E+09	.2830000E+09	.2340000E+09	.2340000E+09	.2340000E+09

TORSIONAL STIFFNESS (LB-IN**2)

.9240000E+08	.9240000E+08	.6190000E+09	.6190000E+09	.1390000E+09	.1660000E+09	.1660000E+09
.1660000E+09	.5180000E+09	.5180000E+09	.3830000E+09	.3830000E+09	.3720000E+09	.3070000E+09
.3070000E+09	.1330000E+09	.1000000E+09	.8500000E+08	.3800000E+08	.2500000E+08	.1800000E+08
.1350000E+08	.1080000E+08	.8000000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07
.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07

DISTANCE BETWEEN MASS AND ELASTIC AXIS(INCHES)

0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.
0.	0.	-.1000000E+00	-.1500000E+00	-.2000000E+00	-.1500000E+00	-.2500000E+00
-.1600000E+00	-.1600000E+00	-.1600000E+00	-.1600000E+00	-.4000000E+00	.3900000E+00	.3900000E+00
-.3000000E+00	-.3000000E+00	.7600000E+00	.7600000E+00	-.3000000E+00	-.3000000E+00	-.3000000E+00

TWIST OF THE BLADE NOT INCLUDING THE COLLECTIVE PITCH(DEGREES)

0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.
0.	0.	-.8200000E-01	-.1920000E+00	-.6310000E+00	-.9050000E+00	-.1179000E+01
-.1453000E+01	-.1727000E+01	-.2002000E+01	-.2276000E+01	-.3921000E+01	-.4579000E+01	-.5018000E+01
-.5566000E+01	-.8856000E+01	-.9295000E+01	-.1022700E+02	-.1033700E+02	-.1044700E+02	-.1060000E+02

MASS MOMENT OF INERTIA ABOUT THE CHORD (LB-SEC**2)

0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.
0.	.1810000E-02	.1720000E-02	.1660000E-02	.9100000E-03	.5700000E-03	.4100000E-03
.3000000E-03	.2300000E-03	.1800000E-03	.1500000E-03	.1500000E-03	.3400000E-03	.3400000E-03
.1500000E-03	.1500000E-03	.2400000E-03	.2400000E-03	.1500000E-03	.1500000E-03	.1500000E-03

MASS MOMENT OF INERTIA ABOUT AN AXIS PERPENDICULAR TO THE CHORD THROUGH THE CENTER OF GRAVITY(LB-SEC**2)

0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.
0.	.1100000E-01	.1180000E-01	.1230000E-01	.1330000E-01	.1320000E-01	.1260000E-01
.1180000E-01	.1070000E-01	.9830000E-02	.8670000E-02	.6810000E-02	.8200000E-02	.8200000E-02
.6810000E-02	.6810000E-02	.8390000E-02	.8390000E-02	.6810000E-02	.6810000E-02	.6810000E-02

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THE FOLLOWING ARE THE INTERPOLATED VALUES AT 101 EQUIDISTANT STATIONS

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MASS PER UNIT LENGTH (LB-SEC**2/IN**2)

.1760000E-02	.6990000E-02	.4040000E-02	.3676662E-02	.4255280E-02	.6160000E-02	.4430000E-02
.2745856E-02	.2929240E-02	.2489973E-02	.2540000E-02	.2413445E-02	.2206940E-02	.2000435E-02
.1793930E-02	.1660880E-02	.1542272E-02	.1427688E-02	.1317552E-02	.1211288E-02	.1135040E-02
.1058792E-02	.9793520E-03	.8988680E-03	.8333600E-03	.7910000E-03	.7487760E-03	.7106520E-03
.6725280E-03	.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03
.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03
.6600000E-03	.7897750E-03	.1045700E-02	.1301625E-02	.1557550E-02	.1813475E-02	.2069400E-02
.2110000E-02	.2110000E-02	.2110000E-02	.2090280E-02	.1783170E-02	.1476060E-02	.1168950E-02
.8618400E-03	.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03
.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03
.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03
.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03	.6600000E-03
.6600000E-03	.6652875E-03	.1038585E-02	.1411882E-02	.1785180E-02	.2070000E-02	.2070000E-02
.2070000E-02	.2070000E-02	.2070000E-02	.2070000E-02	.2070000E-02	.2070000E-02	.1755570E-02
.6600000E-03	.1078943E-02	.2380000E-02				

FLAPWISE BENDING STIFFNESS (LB-IN**2)

.7700000E+08	.5160000E+09	.1240960E+09	.1380000E+09	.1380000E+09	.4320000E+09	.3190000E+09
.2921584E+09	.2560000E+09	.9251627E+08	.5728000E+08	.4978150E+08	.4289800E+08	.3601450E+08
.2913100E+08	.2516800E+08	.2177920E+08	.1879280E+08	.1625120E+08	.1385480E+08	.1258400E+08
.1131320E+08	.1020200E+08	.9143000E+07	.8167200E+07	.7320000E+07	.6490752E+07	.6202704E+07
.5914656E+07	.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07
.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07
.5820000E+07	.5859380E+07	.5937040E+07	.6014700E+07	.6092360E+07	.6170020E+07	.6247680E+07
.6260000E+07	.6260000E+07	.6260000E+07	.6254016E+07	.6160824E+07	.6067632E+07	.5974440E+07
.5881248E+07	.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07
.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07
.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07
.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07	.5820000E+07
.5820000E+07	.5826525E+07	.6287190E+07	.6747855E+07	.7208520E+07	.7560000E+07	.7560000E+07
.7560000E+07	.7560000E+07	.7560000E+07	.7560000E+07	.7560000E+07	.7560000E+07	.7171980E+07

.5820000E+07	.5820000E+07	.5820000E+07

CHORDWISE BENDING STIFFNESS (LB-IN**2)

.2680000E+09	.6500000E+09	.1255680E+09	.1420000E+09	.1420000E+09	.5180000E+09	.2000000E+09
.1170000E+09	.1170000E+09	.2286507E+09	.4356500E+09	.4581125E+09	.4713500E+09	.4845875E+09
.4978250E+09	.5000000E+09	.5000000E+09	.4919520E+09	.4750080E+09	.4583060E+09	.4434800E+09
.4286540E+09	.4154240E+09	.4027160E+09	.3900080E+09	.3773000E+09	.3646260E+09	.3529770E+09
.3413280E+09	.3328785E+09	.3259950E+09	.3191115E+09	.3122280E+09	.3053445E+09	.2984610E+09
.2915775E+09	.2846940E+09	.2778105E+09	.2709270E+09	.2640435E+09	.2571600E+09	.2502765E+09
.2433930E+09	.2398210E+09	.2394680E+09	.2391150E+09	.2387620E+09	.2384090E+09	.2380560E+09
.2380000E+09	.2380000E+09	.2380000E+09	.2379456E+09	.2370984E+09	.2362512E+09	.2354040E+09
.2345568E+09	.2340000E+09	.2340000E+09	.2340000E+09	.2340000E+09	.2340000E+09	.2340000E+09
.2340000E+09	.2340000E+09	.2340000E+09	.2340000E+09	.2340000E+09	.2340000E+09	.2340000E+09
.2340000E+09	.2340000E+09	.2340000E+09	.2340000E+09	.2340000E+09	.2340000E+09	.2340000E+09
.2340000E+09	.2340000E+09	.2340000E+09	.2340000E+09	.2340000E+09	.2340000E+09	.2340000E+09
.2340000E+09	.2341937E+09	.2471565E+09	.2601292E+09	.2731020E+09	.2830000E+09	.2830000E+09
.2830000E+09	.2830000E+09	.2830000E+09	.2830000E+09	.2830000E+09	.2830000E+09	.2720730E+09
.2340000E+09	.2340000E+09	.2340000E+09				

TORSIONAL STIFFNESS (LB-IN**2)

.9240000E+08	.6190000E+09	.1489360E+09	.1660000E+09	.1660000E+09	.5180000E+09	.3830000E+09
.3505240E+09	.3070000E+09	.1206360E+09	.9115000E+08	.7737425E+08	.6493100E+08	.5248775E+08
.4004450E+08	.3339800E+08	.2789120E+08	.2359160E+08	.2062640E+08	.1778220E+08	.1587600E+08
.1396980E+08	.1263816E+08	.1149444E+08	.1033408E+08	.9148000E+07	.7986128E+07	.7554056E+07
.7121984E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07
.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07
.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07
.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07
.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07
.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07
.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07
.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07
.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07
.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07	.6980000E+07
.6980000E+07	.6980000E+07	.6980000E+07				

DISTANCE BETWEEN MASS AND ELASTIC AXIS(INCHES)

0.	0.	0.	0.	0.	0.	0.
0.	0.	-.3746667E-01	-.1295000E+00	-.1581125E+00	-.1713500E+00	-.1845875E+00
-.1978250E+00	-.1823000E+00	-.1611200E+00	-.1701200E+00	-.2124800E+00	-.2456440E+00	-.2075200E+00

-.1693960E+00	-.1600000E+00	-.1600000E+00	-.1600000E+00	-.1600000E+00	-.1600000E+00	-.1600000E+00
-.1600000E+00	-.1713760E+00	-.1883200F+00	-.2052640E+00	-.2222080E+00	-.2391520E+00	-.2560960E+00
-.2730400E+00	-.2899840F+00	-.3069280F+00	-.3238720E+00	-.3408160E+00	-.3577600E+00	-.3747040E+00
-.3916480E+00	-.3292950E+00	-.1898600F+00	-.5042500E-01	.8901000E-01	.2284450E+00	.3678800E+00
.3900000E+00	.3900000E+00	.3900000E+00	.3806160E+00	.2344740E+00	.8833200E-01	-.5781000E-01
-.2039520E+00	-.3000000E+00	-.3000000E+00	-.3000000E+00	-.3000000F+00	-.3000000E+00	-.3000000E+00
-.3000000E+00	-.3000000F+00	-.3000000F+00	-.3000000F+00	-.3000000F+00	-.3000000E+00	-.3000000E+00
-.3000000E+00	-.3000000E+00	-.3000000E+00	-.3000000E+00	-.3000000E+00	-.3000000E+00	-.3000000E+00
-.3000000E+00	-.3000000E+00	-.3000000F+00	-.3000000E+00	-.3000000F+00	-.3000000F+00	-.3000000E+00
-.3000000F+00	-.2960250E+00	-.1539000E-01	.2652450F+00	.5458800F+00	.7600000E+00	.7600000E+00
.7600000E+00	.7600000E+00	.7600000F+00	.7600000E+00	.7600000E+00	.7600000E+00	.5236200E+00
-.3000000E+00	-.3000000F+00	-.3000000F+00				

TWIST OF THE BLADE NOT INCLUDING THE COLLECTIVE PITCH(DEGREES)

0.	0.	0.	0.	0.	0.	0.
0.	0.	-.3072267E-01	-.1469000E+00	-.2632277F+00	-.3794530E+00	-.4956782E+00
-.6119035E+00	-.7279960E+00	-.8440624E+00	-.9601288E+00	-.1076195E+01	-.1192262E+01	-.1308328E+01
-.1424394E+01	-.1540461E+01	-.1656527E+01	-.1772760E+01	-.1889250E+01	-.2005726E+01	-.2121793E+01
-.2237859E+01	-.2353973E+01	-.2470110E+01	-.2586247E+01	-.2702384E+01	-.2818521E+01	-.2934658E+01
-.3050795E+01	-.3166932E+01	-.3283069E+01	-.3399206E+01	-.3515343E+01	-.3631480E+01	-.3747617E+01
-.3863754E+01	-.3979891E+01	-.4096028E+01	-.4212165E+01	-.4328302F+01	-.4444439E+01	-.4560576E+01
-.4676787E+01	-.4793012F+01	-.4909238F+01	-.5025453E+01	-.5141519E+01	-.5257586E+01	-.5373652E+01
-.5489718E+01	-.5605809E+01	-.5721946F+01	-.5838083F+01	-.5954220E+01	-.6070357E+01	-.6186494E+01
-.6302631E+01	-.6418768F+01	-.6534905E+01	-.6651042E+01	-.6767179F+01	-.6883316F+01	-.6999453E+01
-.7115590E+01	-.7231727E+01	-.7347864F+01	-.7464001F+01	-.7580138E+01	-.7696275E+01	-.7812412E+01
-.7928549E+01	-.8044686E+01	-.8160823E+01	-.8276960E+01	-.8393097E+01	-.8509234E+01	-.8625371E+01
-.8741508F+01	-.8857646E+01	-.8973871E+01	-.9090097E+01	-.9206322F+01	-.9322521E+01	-.9439638E+01
-.9554754E+01	-.9670870E+01	-.9786986E+01	-.9903103E+01	-.1001922E+02	-.1013534E+02	-.1025153E+02
-.1036802F+02	-.1048427E+02	-.1060000E+02				

MASS MOMENT OF INERTIA ABOUT THE CHORD (LB-SEC**2)

0.	0.	0.	0.	0.	0.	0.
0.	0.	.1776280F-02	.1684600F-02	.1538313E-02	.1339750E-02	.1141188E-02
.9426250E-03	.7896400E-03	.6456160E-03	.5378080E-03	.4700320E-03	.4046760E-03	.3580800E-03
.3114840E-03	.2776560E-03	.2480040F-03	.2216800E-03	.2005000E-03	.1795920E-03	.1668840E-03
.1541760E-03	.1500000E-03	.1500000E-03	.1500000E-03	.1500000F-03	.1500000E-03	.1500000E-03
.1500000E-03	.1500000E-03	.1500000E-03	.1500000E-03	.1500000F-03	.1500000E-03	.1500000E-03
.1500000E-03	.1670050E-03	.2005400F-03	.2340750E-03	.2676100F-03	.3011450E-03	.3346800E-03
.3400000F-03	.3400000E-03	.3400000F-03	.3374160F-03	.2971740E-03	.2569320E-03	.2166900E-03
.1764480E-03	.1500000F-03	.1500000F-03	.1500000F-03	.1500000F-03	.1500000E-03	.1500000E-03
.1500000E-03	.1500000E-03	.1500000F-03	.1500000F-03	.1500000F-03	.1500000E-03	.1500000E-03
.1500000E-03	.1500000E-03	.1500000F-03	.1500000E-03	.1500000E-03	.1500000E-03	.1500000E-03
.1500000E-03	.1500000F-03	.1500000F-03	.1500000F-03	.1500000E-03	.1500000E-03	.1500000E-03

.1500000E-03	.1503375E-03	.1741650E-03	.1979925E-03	.2218200E-03	.2400000E-03	.2400000E-03
.2400000E-03	.2400000E-03	.2400000E-03	.2400000E-03	.2400000E-03	.2400000E-03	.2199300E-03
.1500000E-03	.1500000E-03	.1500000E-03				

MASS MOMENT OF INERTIA ABOUT AN AXIS PERPENDICULAR TO THE CHORD THROUGH THE CENTER OF GRAVITY(LB-SEC**2)

0.	0.	0.	0.	0.	0.	0.
0.	0.	.1129973E-01	.1209500E-01	.1246225E-01	.1272700E-01	.1299175E-01
.1325650E-01	.1326460E-01	.1322224E-01	.1307928E-01	.1282512E-01	.1256128E-01	.1222240E-01
.1188352E-01	.1144888E-01	.1098292E-01	.1055523E-01	.1018670E-01	.9814224E-02	.9322848E-02
.8831472E-02	.8581836E-02	.8450520E-02	.8319204E-02	.8187888E-02	.8056572E-02	.7925256E-02
.7793940E-02	.7662624E-02	.7531308E-02	.7399992E-02	.7268676E-02	.7137360E-02	.7006044E-02
.6874728E-02	.6934405E-02	.7179740E-02	.7425075E-02	.7670410E-02	.7915745E-02	.8161080E-02
.8200000E-02	.8200000E-02	.8200000E-02	.8181096E-02	.7886694E-02	.7592292E-02	.7297890E-02
.7003488E-02	.6810000E-02	.6810000E-02	.6810000E-02	.6810000E-02	.6810000E-02	.6810000E-02
.6810000E-02	.6810000E-02	.6810000E-02	.6810000E-02	.6810000E-02	.6810000E-02	.6810000E-02
.6810000E-02	.6810000E-02	.6810000E-02	.6810000E-02	.6810000E-02	.6810000E-02	.6810000E-02
.6810000E-02	.6810000E-02	.6810000E-02	.6810000E-02	.6810000E-02	.6810000E-02	.6810000E-02
.6810000E-02	.6815925E-02	.7234230E-02	.7652535E-02	.8070840E-02	.8390000E-02	.8390000E-02
.8390000E-02	.8390000E-02	.8390000E-02	.8390000E-02	.8390000E-02	.8390000E-02	.8037660E-02
.6810000E-02	.6810000E-02	.6810000E-02				

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MODE NUMBER = 1 FREQ. RAD/SEC = 43.2701 FREQ. HERTZ = 6.8867 NON-DIMEN. FREQ. = 9.2801

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FLAPWISE DEFLECTION

STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	0.0000	.1800	.0319	.3600	.1871	.5400	.4005	.7200	.6300	.9000	.8670
.0200	.0004	.2000	.0425	.3800	.2095	.5600	.4255	.7400	.6561	.9200	.8935
.0400	.0015	.2200	.0551	.4000	.2324	.5800	.4506	.7600	.6822	.9400	.9201
.0600	.0032	.2400	.0696	.4200	.2556	.6000	.4758	.7800	.7084	.9600	.9468
.0800	.0052	.2600	.0859	.4400	.2792	.6200	.5013	.8000	.7347	.9800	.9734
.1000	.0078	.2800	.1039	.4600	.3030	.6400	.5268	.8200	.7610	1.0000	1.0000
.1200	.0115	.3000	.1233	.4800	.3270	.6600	.5524	.8400	.7875		
.1400	.0166	.3200	.1438	.5000	.3513	.6800	.5782	.8600	.8139		
.1600	.0233	.3400	.1651	.5200	.3758	.7000	.6041	.8800	.8404		

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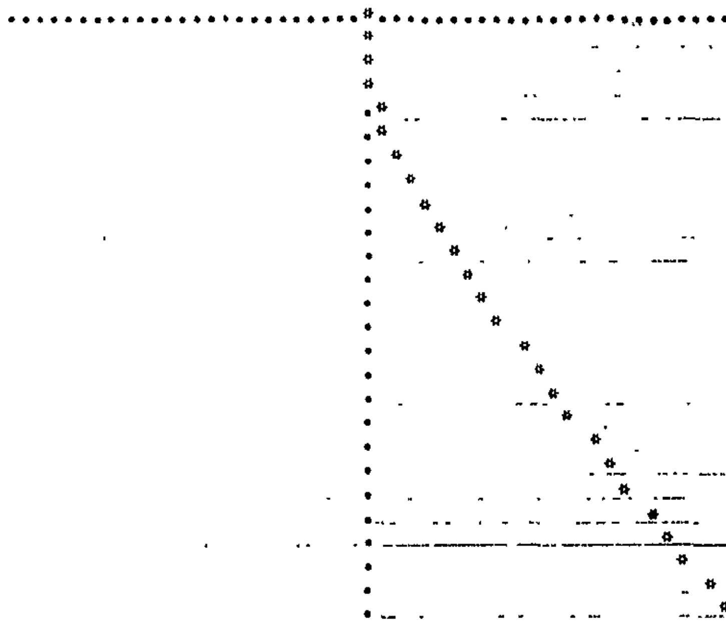

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MODE NUMBER = 1 FREQ. RAD/SEC = 43.2701 FREQ. HERTZ = 6.8867 NON-DIMEN. FREQ. = 9.2801

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CHORDWISE DEFLECTION

STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	0.0000	.1800	.0239	.3600	.0228	.5400	.0118	.7200	-.0002	.9000	-.0125
.0200	.0029	.2000	.0252	.3800	.0217	.5600	.0105	.7400	-.0016	.9200	-.0139
.0400	.0058	.2200	.0260	.4000	.0205	.5800	.0092	.7600	-.0029	.9400	-.0153
.0600	.0087	.2400	.0264	.4200	.0193	.6000	.0078	.7800	-.0043	.9600	-.0167
.0800	.0117	.2600	.0264	.4400	.0181	.6200	.0065	.8000	-.0056	.9800	-.0180
.1000	.0147	.2800	.0261	.4600	.0169	.6400	.0052	.8200	-.0070	1.0000	-.0194
.1200	.0175	.3000	.0255	.4800	.0156	.6600	.0038	.8400	-.0084		
.1400	.0201	.3200	.0247	.5000	.0144	.6800	.0025	.8600	-.0098		
.1600	.0222	.3400	.0238	.5200	.0131	.7000	.0011	.8800	-.0111		

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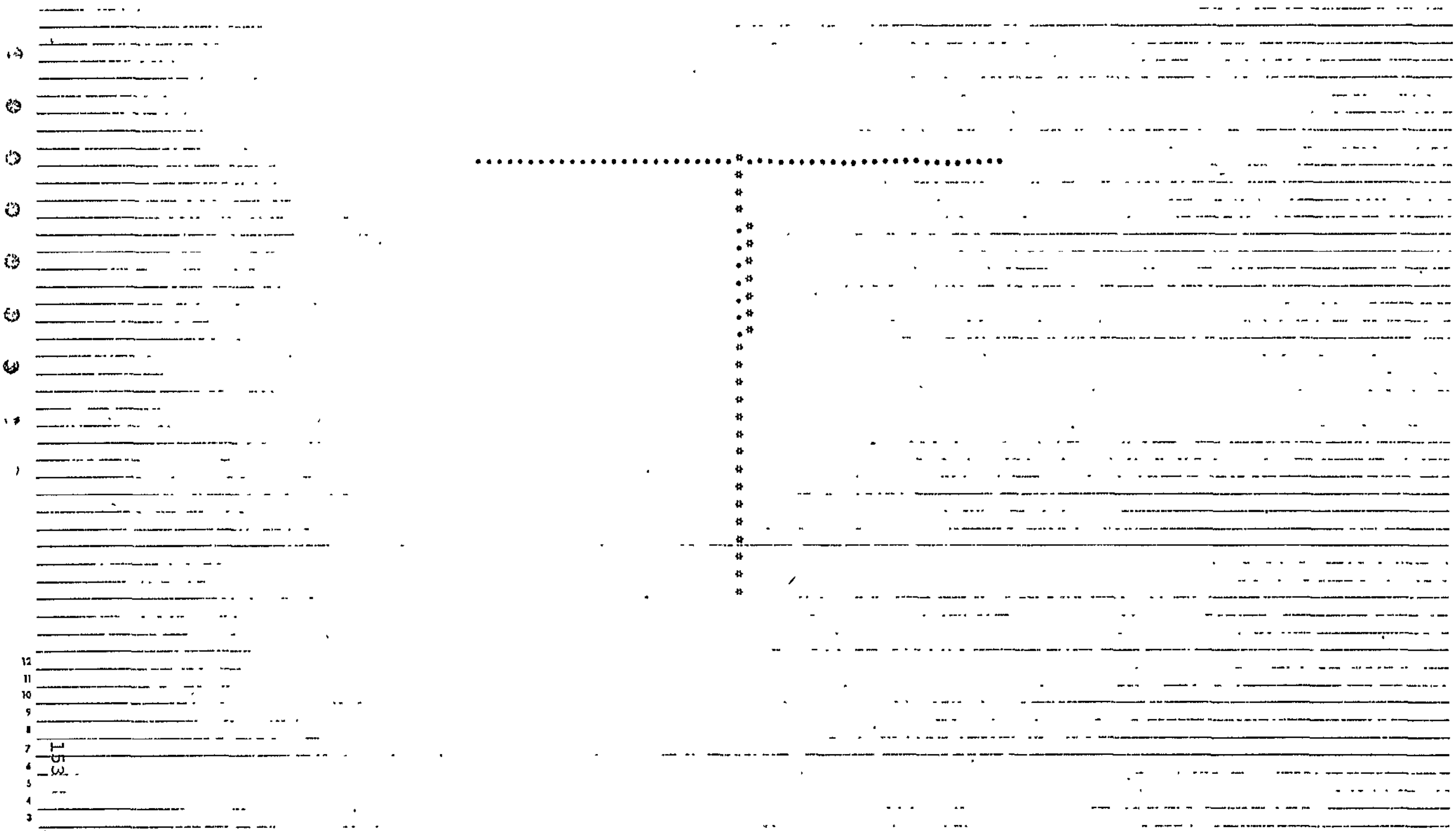


MODE NUMBER = 1 FREQ. RAD/SEC = 43.2701 FREQ. HERTZ = 6.8867 NON-DIMEN. FREQ. = 9.2801

TORSIONAL DEFLECTION

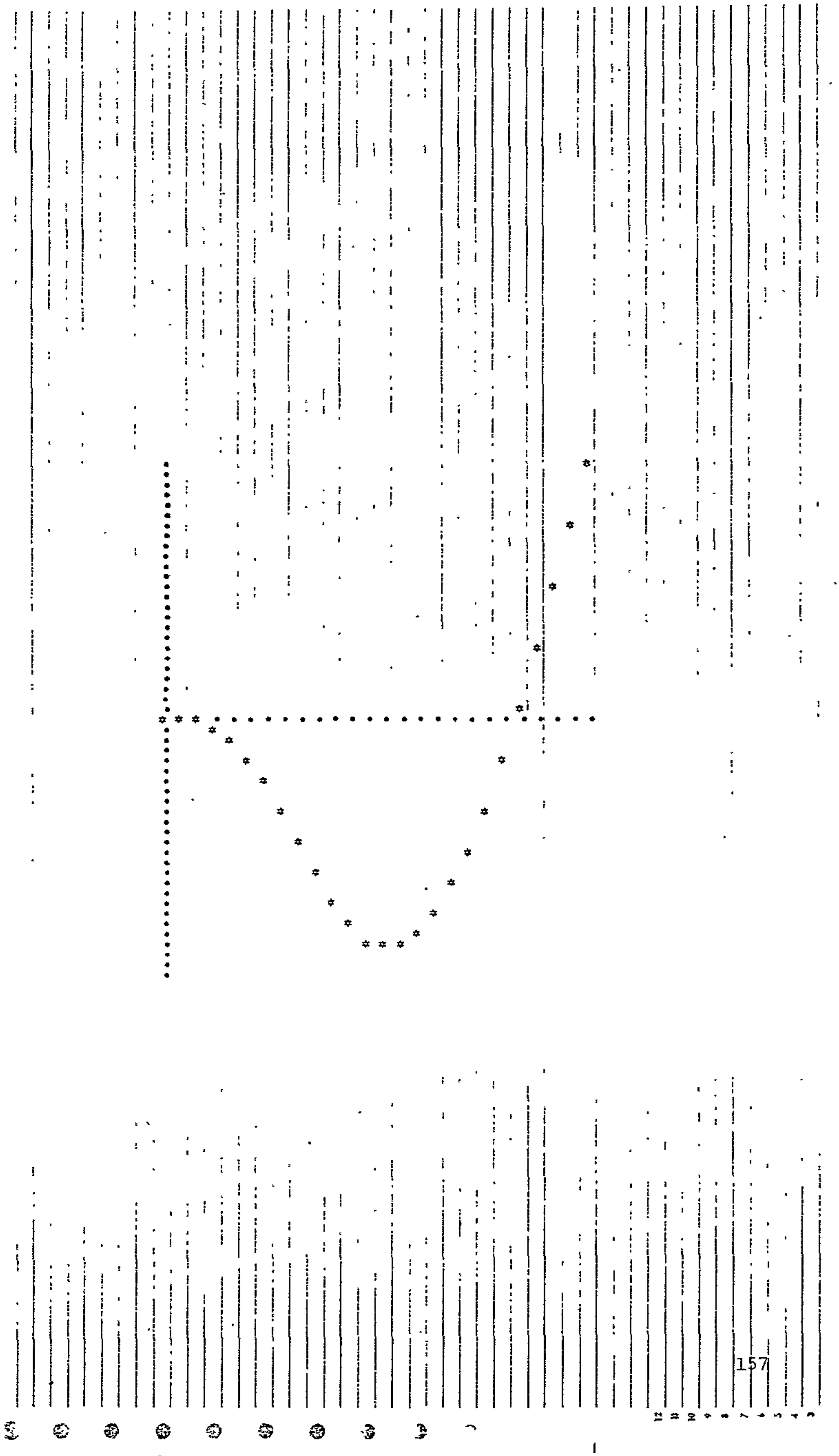
STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	-.0000	.1800	-.0000	.3600	-.0000	.5400	-.0000	.7200	-.0000	.9000	-.0000
.0200	-.0000	.2000	-.0000	.3800	-.0000	.5600	-.0000	.7400	-.0000	.9200	-.0000
.0400	-.0000	.2200	-.0000	.4000	-.0000	.5800	-.0000	.7600	-.0000	.9400	-.0000
.0600	-.0000	.2400	-.0000	.4200	-.0000	.6000	-.0000	.7800	-.0000	.9600	-.0000
.0800	-.0000	.2600	-.0000	.4400	-.0000	.6200	-.0000	.8000	-.0000	.9800	-.0000
.1000	-.0000	.2800	-.0000	.4600	-.0000	.6400	-.0000	.8200	-.0000	1.0000	-.0000
.1200	-.0000	.3000	-.0000	.4800	-.0000	.6600	-.0000	.8400	-.0000		
.1400	-.0000	.3200	-.0000	.5000	-.0000	.6800	-.0000	.8600	-.0000		
.1600	-.0000	.3400	-.0000	.5200	-.0000	.7000	-.0000	.8800	-.0000		

[illegible]

MODE NUMBER = 2 FREQ. RAD/SEC = 117.0175 FREQ. HERTZ = 18.6239 NON-DIMEN. FREQ. = 25.0965

FLAPWISE DEFLECTION

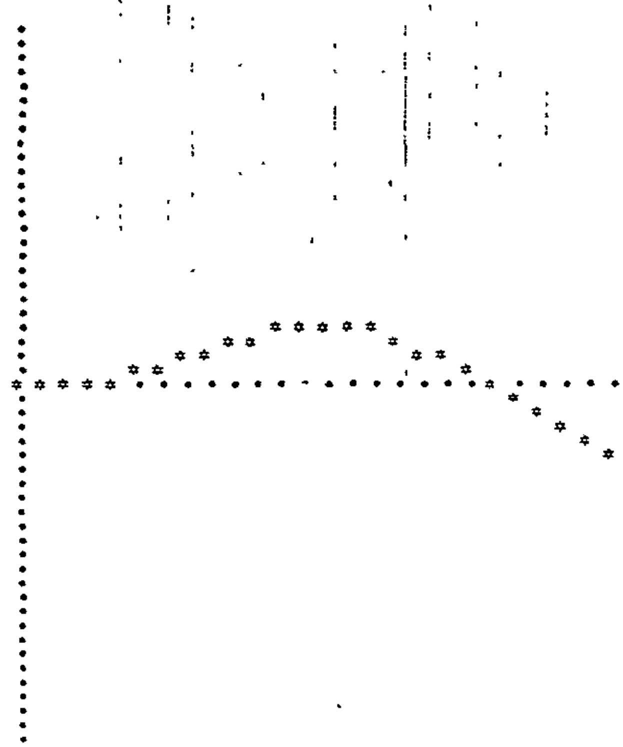
STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	0.0000	.1800	-.1136	.3600	-.5939	.5400	-.8957	.7200	-.5145	.9000	.3885
.0200	-.0014	.2000	-.1501	.3800	-.6511	.5600	-.8867	.7400	-.4357	.9200	.5085
.0400	-.0053	.2200	-.1930	.4000	-.7050	.5800	-.8683	.7600	-.3507	.9400	.6303
.0600	-.0116	.2400	-.2415	.4200	-.7544	.6000	-.8411	.7800	-.2598	.9600	.7531
.0800	-.0190	.2600	-.2950	.4400	-.7983	.6200	-.8055	.8000	-.1633	.9800	.8765
.1000	-.0283	.2800	-.3524	.4600	-.8353	.6400	-.7620	.8200	-.0617	1.0000	1.0000
.1200	-.0416	.3000	-.4123	.4800	-.8644	.6600	-.7108	.8400	.0449		
.1400	-.0597	.3200	-.4734	.5000	-.8845	.6800	-.6524	.8600	.1559		
.1600	-.0836	.3400	-.5343	.5200	-.8950	.7000	-.5868	.8800	.2707		



MODE NUMBER = 2 FREQ. RAD/SEC = 117.0175 FREQ. HERTZ = 18.6239 NON-DIMEN. FREQ. = 25.0965

CHORDWISE DEFLECTION

STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	0.0000	.1800	.0225	.3600	.1146	.5400	.1579	.7200	.0695	.9000	-.0996
.0200	.0018	.2000	.0295	.3800	.1248	.5600	.1540	.7400	.0536	.9200	-.1208
.0400	.0036	.2200	.0378	.4000	.1341	.5800	.1484	.7600	.0369	.9400	-.1422
.0600	.0053	.2400	.0474	.4200	.1423	.6000	.1411	.7800	.0193	.9600	-.1636
.0800	.0067	.2600	.0579	.4400	.1492	.6200	.1324	.8000	.0009	.9800	-.1852
.1000	.0079	.2800	.0691	.4600	.1546	.6400	.1222	.8200	-.0181	1.0000	-.2067
.1200	.0098	.3000	.0807	.4800	.1583	.6600	.1108	.8400	-.0377		
.1400	.0127	.3200	.0923	.5000	.1601	.6800	.0981	.8600	-.0579		
.1600	.0169	.3400	.1037	.5200	.1599	.7000	.0843	.8800	-.0786		



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MODE NUMBER = 2   FREQ. RAD/SEC = 117.0175   FREQ. HERTZ = 18.6239   NON-DIMEN. FREQ. = 25.0965

****

TORSIONAL DEFLECTION

STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	-.0016	.1800	-.0014	.3600	-.0002	.5400	.0009	.7200	.0037	.9000	.0038
.0200	-.0015	.2000	-.0013	.3800	-.0001	.5600	.0014	.7400	.0038	.9200	.0038
.0400	-.0015	.2200	-.0012	.4000	.0000	.5800	.0018	.7600	.0039	.9400	.0039
.0600	-.0015	.2400	-.0011	.4200	.0001	.6000	.0022	.7800	.0039	.9600	.0039
.0800	-.0015	.2600	-.0009	.4400	.0001	.6200	.0026	.8000	.0039	.9800	.0039
.1000	-.0015	.2800	-.0008	.4600	.0001	.6400	.0029	.8200	.0039	1.0000	.0039
.1200	-.0015	.3000	-.0006	.4800	.0001	.6600	.0032	.8400	.0039		
.1400	-.0015	.3200	-.0004	.5000	.0002	.6800	.0034	.8600	.0038		
.1600	-.0014	.3400	-.0003	.5200	.0005	.7000	.0036	.8800	.0038		

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MODE NUMBER = 3   FREQ. RAD/SEC = 179.5617   FREQ. HERTZ = 28.5781   NON-DIMEN. FREQ. = 38.5103

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FLAPWISE DEFLECTION

STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	0.0000	.1800	.0261	.3600	-.0041	.5400	-.1540	.7200	-.2004	.9000	.0901
.0200	.0003	.2000	.0297	.3800	-.0171	.5600	-.1703	.7400	-.1864	.9200	.1407
.0400	.0014	.2200	.0322	.4000	-.0315	.5800	-.1849	.7600	-.1676	.9400	.1931
.0600	.0032	.2400	.0333	.4200	-.0472	.6000	-.1972	.7800	-.1441	.9600	.2465
.0800	.0056	.2600	.0325	.4400	-.0640	.6200	-.2067	.8000	-.1158	.9800	.3005
.1000	.0090	.2800	.0296	.4600	-.0815	.6400	-.2131	.8200	-.0829	1.0000	.3546
.1200	.0130	.3000	.0243	.4800	-.0997	.6600	-.2160	.8400	-.0454		
.1400	.0174	.3200	.0168	.5000	-.1182	.6800	-.2151	.8600	-.0037		
.1600	.0218	.3400	.0073	.5200	-.1364	.7000	-.2099	.8800	.0417		

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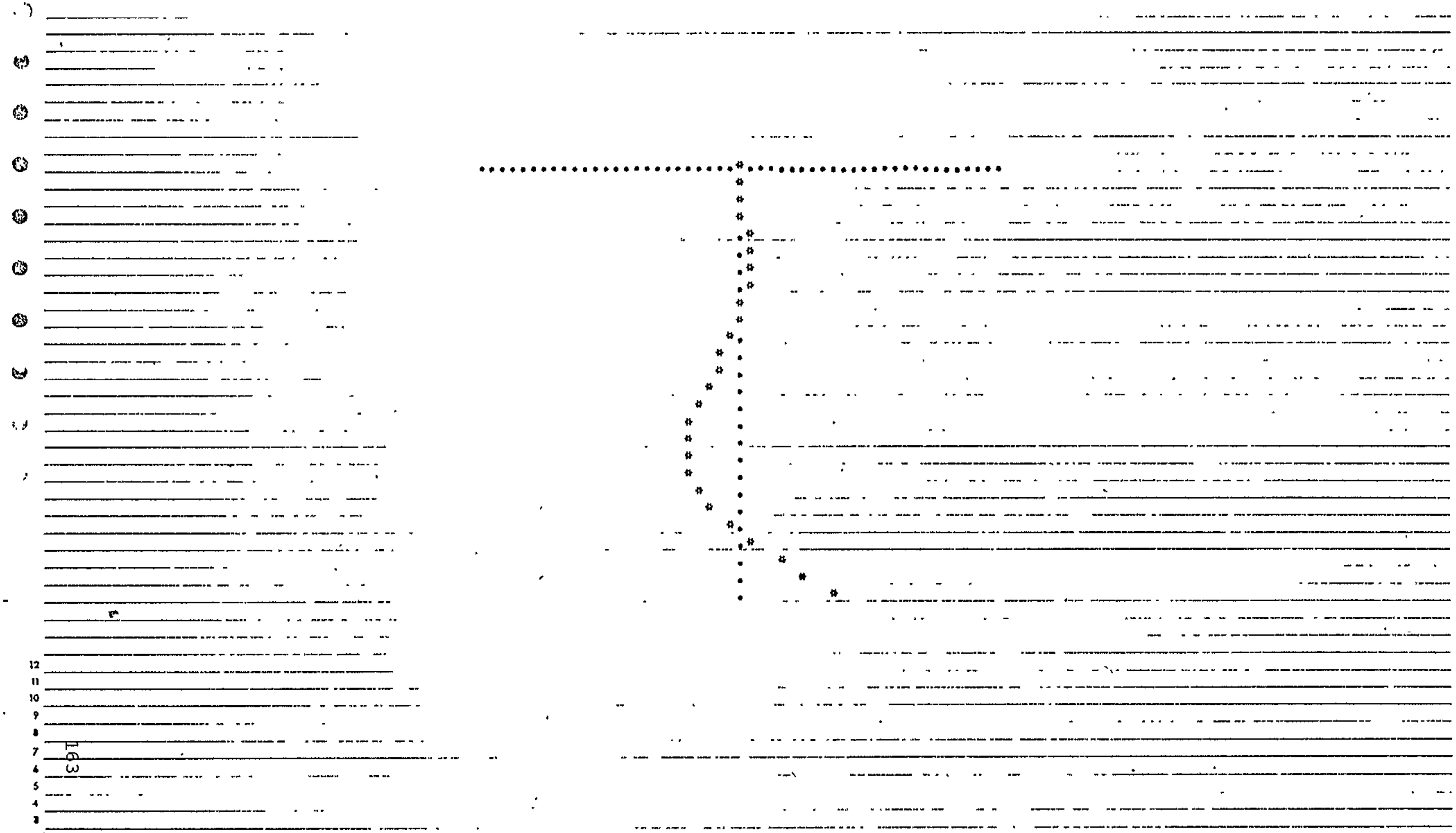
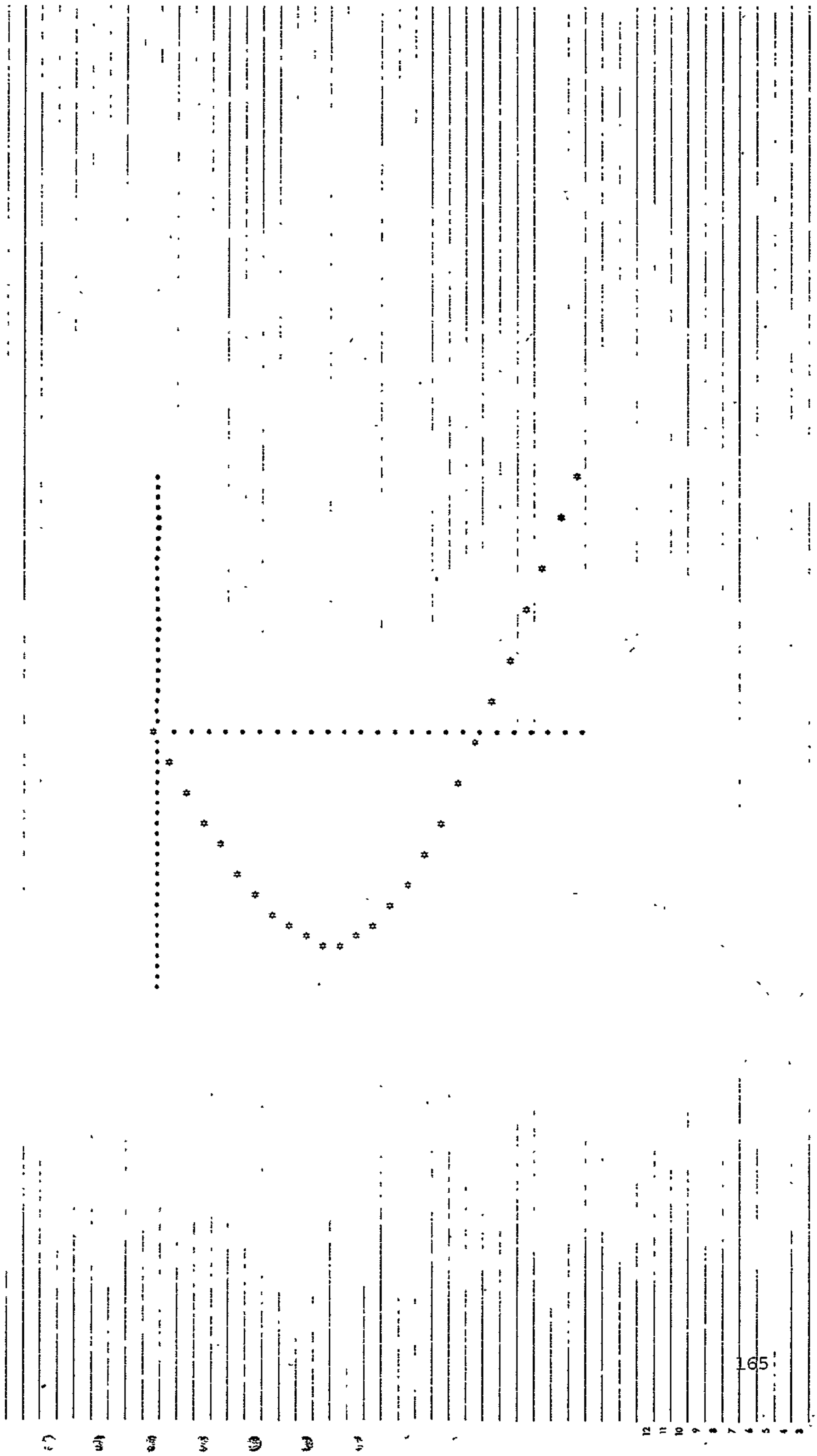


MODE NUMBER = 3 FREQ. RAD/SEC = 179.5617 FREQ. HERTZ = 28.5781 NON-DIMEN. FREQ. = 38.5103

CHORDWISE DEFLECTION

STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	0.0000	.1800	-.5060	.3600	-.8149	.5400	-.7225	.7200	-.1979	.9000	.5559
.0200	-.0623	.2000	-.5535	.3800	-.8279	.5600	-.6826	.7400	-.1214	.9200	.6445
.0400	-.1239	.2200	-.5985	.4000	-.8356	.5800	-.6374	.7600	-.0423	.9400	.7333
.0600	-.1844	.2400	-.6408	.4200	-.8377	.6000	-.5872	.7800	.0389	.9600	.8221
.0800	-.2430	.2600	-.6798	.4400	-.8338	.6200	-.5322	.8000	.1220	.9800	.9111
.1000	-.2986	.2800	-.7152	.4600	-.8238	.6400	-.4729	.8200	.2067	1.0000	1.0000
.1200	-.3526	.3000	-.7467	.4800	-.8075	.6600	-.4095	.8400	.2927		
.1400	-.4053	.3200	-.7740	.5000	-.7851	.6800	-.3423	.8600	.3798		
.1600	-.4565	.3400	-.7968	.5200	-.7567	.7000	-.2717	.8800	.4676		



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MODE NUMBER = 3 FREQ. RAD/SEC = 179.5617 FREQ. HERTZ = 28.5781 NON-DIMEN. FREQ. = 38.5103

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TORSIONAL DEFLECTION

STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	-.0005	.1800	-.0004	.3600	.0013	.5400	.0043	.7200	.0064	.9000	.0071
.0200	-.0005	.2000	-.0003	.3800	.0016	.5600	.0045	.7400	.0066	.9200	.0072
.0400	-.0005	.2200	-.0003	.4000	.0019	.5800	.0048	.7600	.0067	.9400	.0072
.0600	-.0005	.2400	-.0001	.4200	.0023	.6000	.0051	.7800	.0068	.9600	.0073
.0800	-.0005	.2600	.0000	.4400	.0026	.6200	.0054	.8000	.0069	.9800	.0073
.1000	-.0005	.2800	.0003	.4600	.0030	.6400	.0056	.8200	.0069	1.0000	.0073
.1200	-.0005	.3000	.0005	.4800	.0033	.6600	.0058	.8400	.0070		
.1400	-.0005	.3200	.0007	.5000	.0037	.6800	.0060	.8600	.0070		
.1600	-.0005	.3400	.0010	.5200	.0040	.7000	.0062	.8800	.0070		

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MODE NUMBER = 4 FREQ. RAD/SEC = 250.7197 FREQ. HERTZ = 39.9033 NON-DIMEN. FREQ. = 53.7714

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FLAPWISE DEFLECTION

STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	0.0000	.1800	.1643	.3600	.6109	.5400	.1395	.7200	-.8818	.9000	-.1467
.0200	.0021	.2000	.2128	.3800	.6253	.5600	.0088	.7400	-.9157	.9200	.0642
.0400	.0083	.2200	.2673	.4000	.6255	.5800	-.1277	.7600	-.9225	.9400	.2888
.0600	.0180	.2400	.3257	.4200	.6100	.6000	-.2654	.7800	-.9006	.9600	.5223
.0800	.0293	.2600	.3857	.4400	.5773	.6200	-.3999	.8000	-.8487	.9800	.7603
.1000	.0432	.2800	.4443	.4600	.5259	.6400	-.5269	.8200	-.7662	1.0000	1.0000
.1200	.0628	.3000	.4985	.4800	.4554	.6600	-.6422	.8400	-.6532		
.1400	.0891	.3200	.5457	.5000	.3662	.6800	-.7421	.8600	-.5100		
.1600	.1229	.3400	.5838	.5200	.2599	.7000	-.8230	.8800	-.3395		

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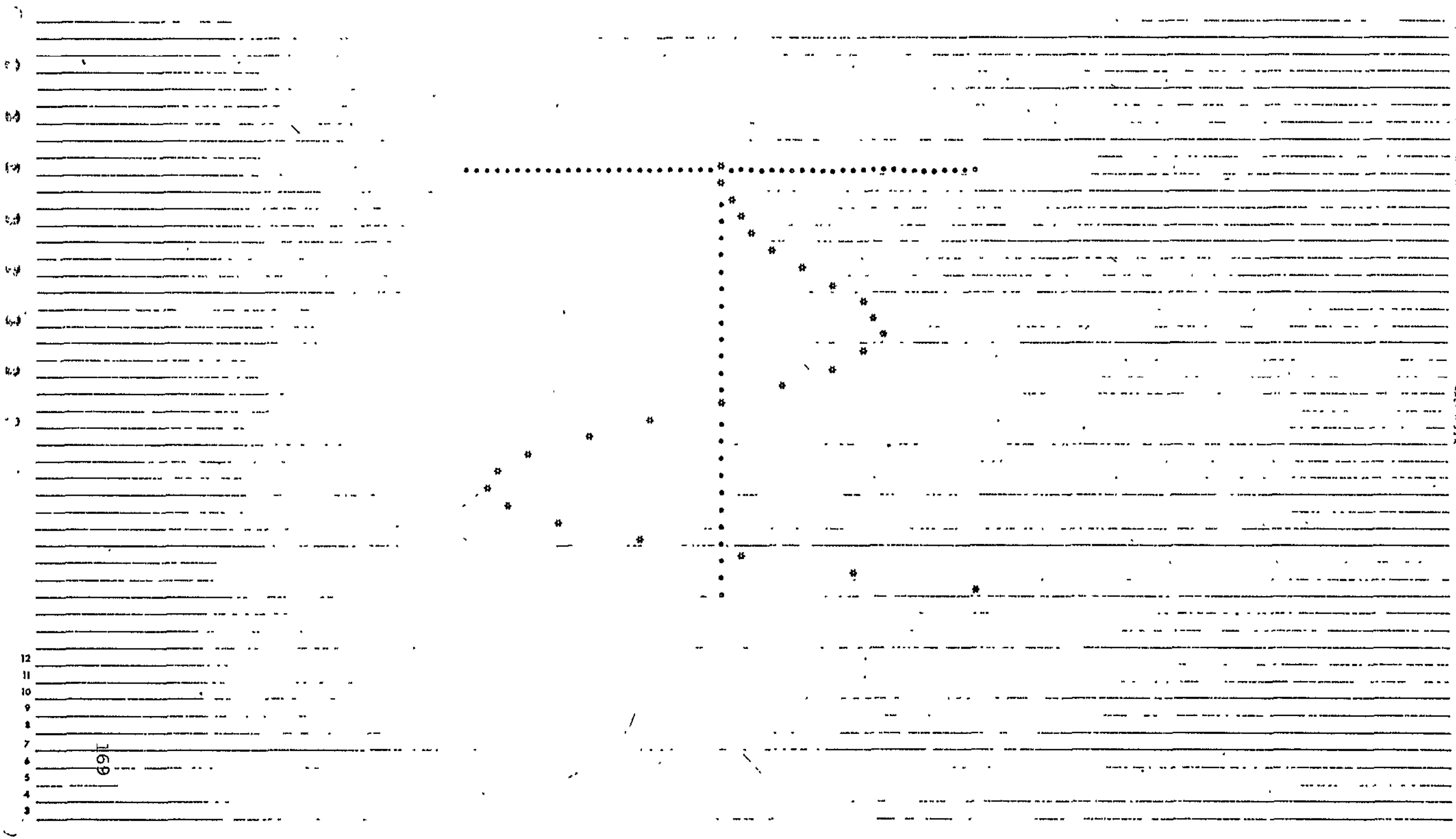
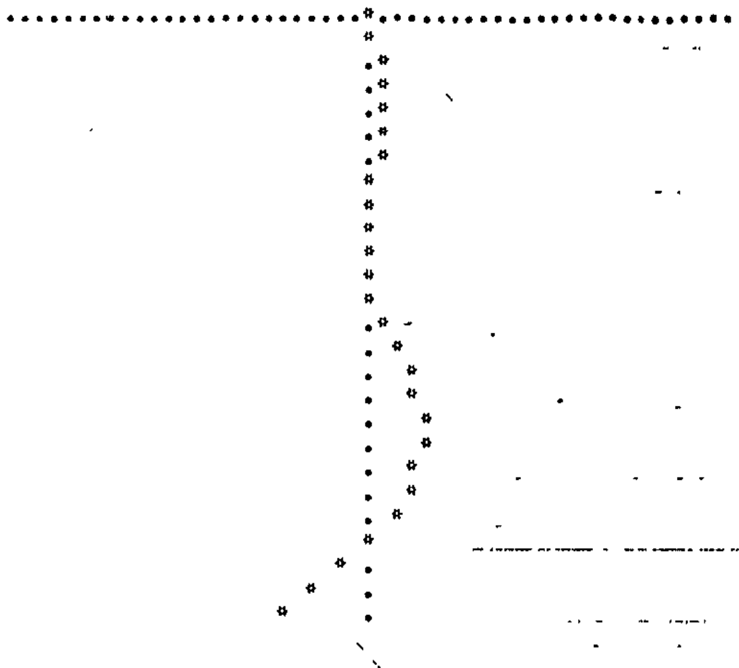


MODE NUMBER = 4 FREQ. RAD/SEC = 250.7197 FREQ. HERTZ = 39.9033 NON-DIMEN. FREQ. = 53.7714

CHORDWISE DEFLECTION

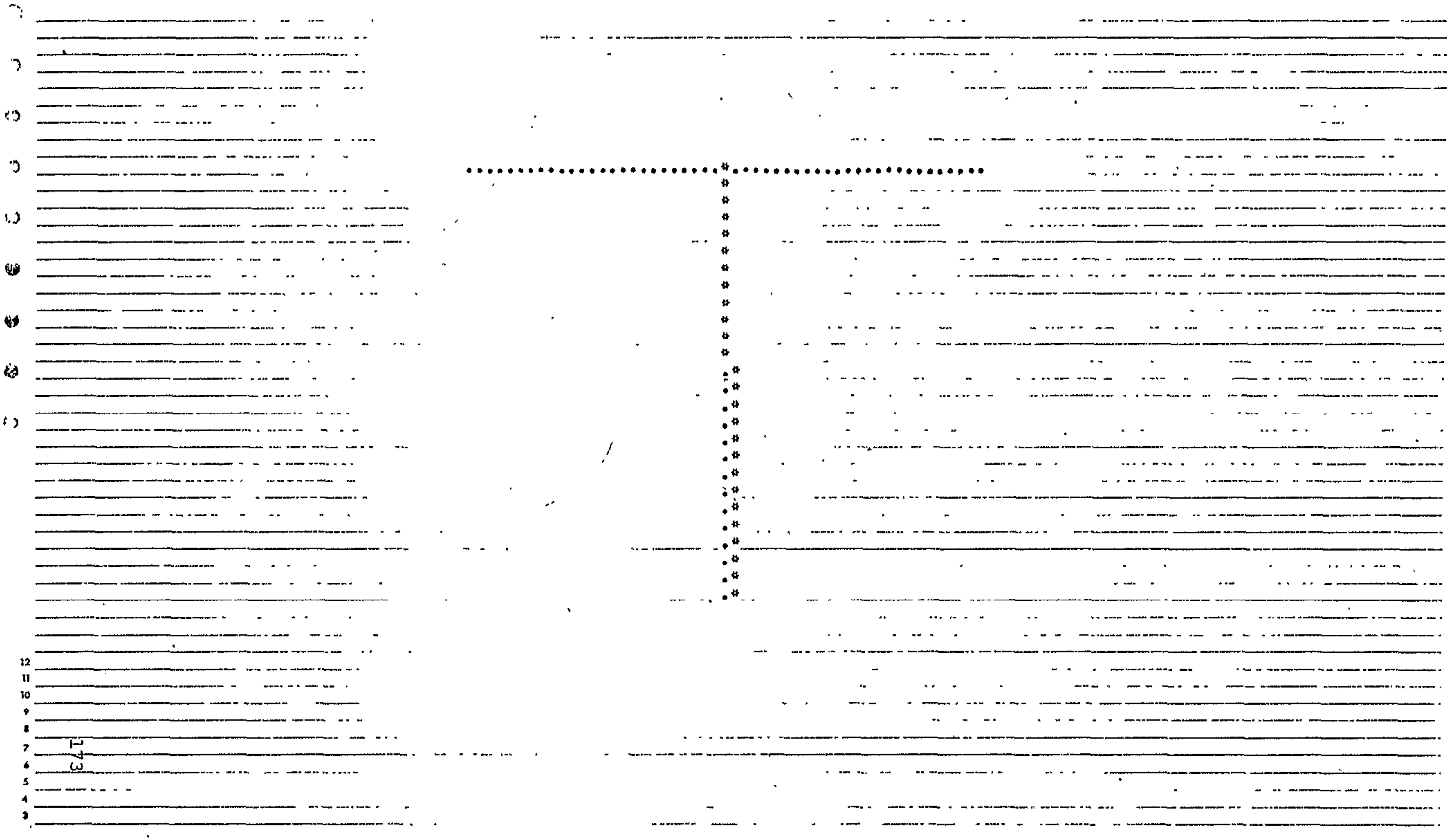
STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	0.0000	.1800	.0362	.3600	-.0137	.5400	.0560	.7200	.1507	.9000	-.0326
.0200	.0063	.2000	.0327	.3800	-.0149	.5600	.0727	.7400	.1461	.9200	-.0686
.0400	.0125	.2200	.0276	.4000	-.0140	.5800	.0893	.7600	.1374	.9400	-.1059
.0600	.0184	.2400	.0212	.4200	-.0110	.6000	.1052	.7800	.1246	.9600	-.1441
.0800	.0244	.2600	.0140	.4400	-.0056	.6200	.1195	.8000	.1076	.9800	-.1826
.1000	.0302	.2800	.0067	.4600	.0023	.6400	.1318	.8200	.0865	1.0000	-.22[illegible]4
.1200	.0345	.3000	-.0002	.4800	.0127	.6600	.1415	.8400	.0616		
.1400	.0371	.3200	-.0061	.5000	.0253	.6800	.1482	.8600	.0331		
.1600	.0376	.3400	-.0107	.5200	.0400	.7000	.1513	.8800	.0015		



MODE NUMBER = 4   FREQ. RAD/SEC = 250.7197   FREQ. HERTZ = 39.9033   NON-DIMEN. FREQ. = 53.7714

TORSIONAL DEFLECTION

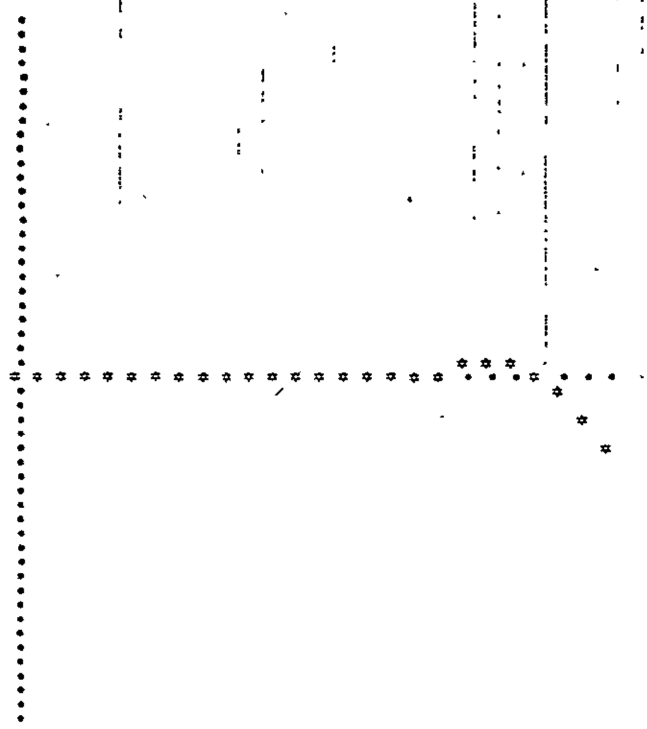
STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	-.0150	.1800	-.0131	.3600	.0059	.5400	.0336	.7200	.0505	.9000	.0518
.0200	-.0150	.2000	-.0122	.3800	.0089	.5600	.0358	.7400	.0515	.9200	.0522
.0400	-.0149	.2200	-.0110	.4000	.0120	.5800	.0381	.7600	.0523	.9400	.0528
.0600	-.0149	.2400	-.0095	.4200	.0153	.6000	.0402	.7800	.0528	.9600	.0532
.0800	-.0148	.2600	-.0075	.4400	.0187	.6200	.0423	.8000	.0530	.9800	.0532
.1000	-.0147	.2800	-.0051	.4600	.0222	.6400	.0443	.8200	.0530	1.0000	.0530
.1200	-.0145	.3000	-.0025	.4800	.0256	.6600	.0461	.8400	.0528		
.1400	-.0142	.3200	.0002	.5000	.0286	.6800	.0478	.8600	.0524		
.1600	-.0138	.3400	.0030	.5200	.0312	.7000	.0492	.8800	.0519		



MODE NUMBER = 5   FREQ. RAD/SEC = 330.0171   FREQ. HERTZ = 52.5239   NON-DIMEN. FREQ. = 70.7782

FLAPWISE DEFLECTION

STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	0.0000	.1800	.0037	.3600	.0018	.5400	-.0103	.7200	.0097	.9000	-.0199
.0200	.0001	.2000	.0042	.3800	.0009	.5600	-.0120	.7400	.0158	.9200	-.0481
.0400	.0003	.2200	.0046	.4000	-.0001	.5800	-.0130	.7600	.0215	.9400	-.0820
.0600	.0006	.2400	.0049	.4200	-.0010	.6000	-.0130	.7800	.0263	.9600	-.1202
.0800	.0009	.2600	.0049	.4400	-.0020	.6200	-.0119	.8000	.0292	.9800	-.1607
.1000	.0013	.2800	.0046	.4600	-.0031	.6400	-.0096	.8200	.0294	1.0000	-.2020
.1200	.0018	.3000	.0042	.4800	-.0045	.6600	-.0061	.8400	.0258		
.1400	.0024	.3200	.0035	.5000	-.0062	.6800	-.0016	.8600	.0170		
.1600	.0031	.3400	.0027	.5200	-.0082	.7000	.0038	.8800	.0019		



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MODE NUMBER = 5 FREQ, RAD/SEC = 330.0171 FREQ, HERTZ = 52.5239 NON-DIMEN, FREQ. = 70.7782

****

CHORDWISE DEFLECTION

STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	0.0000	.1800	-.0013	.3600	-.0004	.5400	.0028	.7200	.0002	.9000	.0008
.0200	-.0002	.2000	-.0014	.3800	-.0001	.5600	.0032	.7400	-.0008	.9200	.0035
.0400	-.0003	.2200	-.0015	.4000	.0002	.5800	.0035	.7600	-.0017	.9400	.0067
.0600	-.0005	.2400	-.0015	.4200	.0005	.6000	.0035	.7800	-.0025	.9600	.0102
.0800	-.0007	.2600	-.0014	.4400	.0008	.6200	.0034	.8000	-.0031	.9800	.0140
.1000	-.0008	.2800	-.0013	.4600	.0011	.6400	.0031	.8200	-.0034	1.0000	.0178
.1200	-.0009	.3000	-.0011	.4800	.0015	.6600	.0026	.8400	-.0032		
.1400	-.0010	.3200	-.0009	.5000	.0019	.6800	.0019	.8600	-.0025		
.1600	-.0012	.3400	-.0006	.5200	.0024	.7000	.0011	.8800	-.0012		

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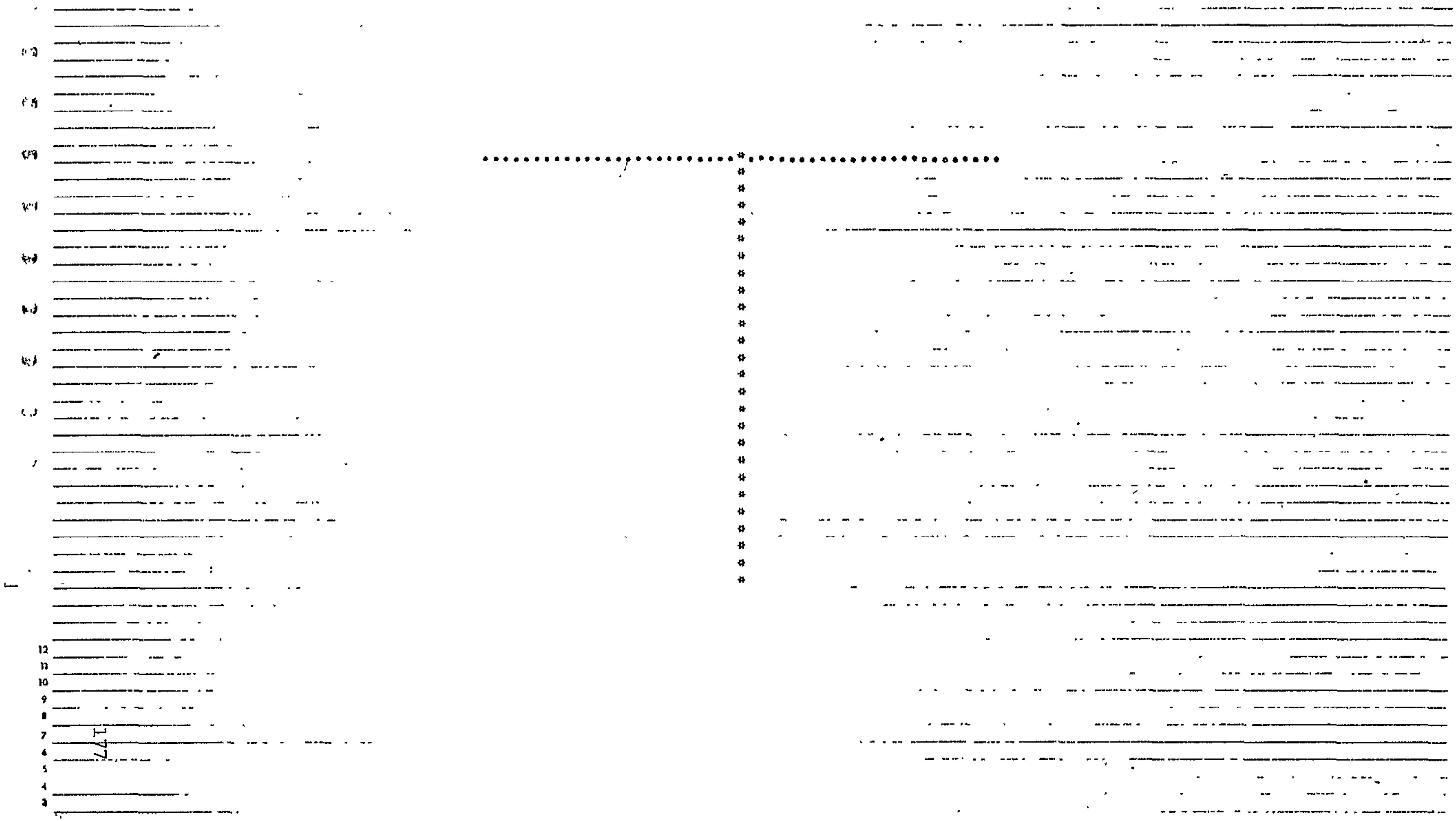

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MODE NUMBER = 5 FREQ. RAD/SEC = 330.0171 FREQ. HERTZ = 52.5239 NON-DIMEN. FREQ. = 70.7782

****

TORSIONAL DEFLECTION

STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	-.2751	.1800	-.2389	.3600	.0845	.5400	.4738	.7200	.7839	.9000	.9717
.0200	-.2740	.2000	-.2221	.3800	.1298	.5600	.5130	.7400	.8116	.9200	.9823
.0400	-.2724	.2200	-.2004	.4000	.1748	.5800	.5511	.7600	.8376	.9400	.9903
.0600	-.2717	.2400	-.1734	.4200	.2195	.6000	.5881	.7800	.8621	.9600	.9958
.0800	-.2709	.2600	-.1390	.4400	.2637	.6200	.6240	.8000	.8849	.9800	.9989
.1000	-.2688	.2800	-.0973	.4600	.3073	.6400	.6587	.8200	.9060	1.0000	1.0000
.1200	-.2653	.3000	-.0520	.4800	.3503	.6600	.6921	.8400	.9253		
.1400	-.2599	.3200	-.0065	.5000	.3925	.6800	.7241	.8600	.9428		
.1600	-.2513	.3400	.0391	.5200	.4337	.7000	.7547	.8800	.9584		

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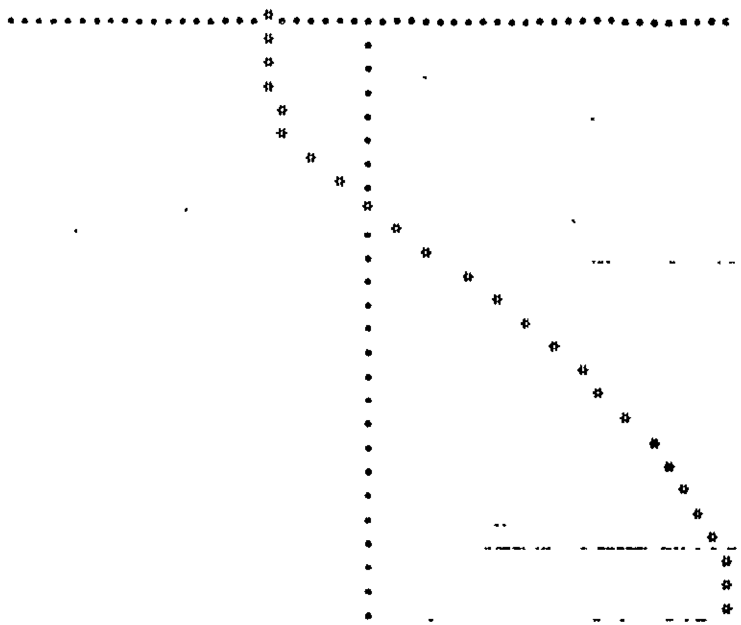

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MODE NUMBER = 6   FREQ. RAD/SEC = 394.2283   FREQ. HERTZ = 62.7434   NON-DIMEN. FREQ. = 84.5494

****

FLAPWISE DEFLECTION

STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	0.0000	.1800	.4926	.3600	.8379	.5400	-.5590	.7200	.1042	.9000	.2970
.0200	.0074	.2000	.6085	.3800	.7135	.5600	-.6150	.7400	.2499	.9200	.1169
.0400	.0285	.2200	.7243	.4000	.5614	.5800	-.6297	.7600	.3809	.9400	-.0915
.0600	.0615	.2400	.8310	.4200	.3885	.6000	-.6049	.7800	.4885	.9600	-.3188
.0800	.0999	.2600	.9188	.4400	.2030	.6200	-.5440	.8000	.5650	.9800	-.5559
.1000	.1467	.2800	.9774	.4600	.0144	.6400	-.4516	.8200	.6032	1.0000	-.7969
.1200	.2093	.3000	1.0000	.4800	-.1667	.6600	-.3334	.8400	.5975		
.1400	.2886	.3200	.9839	.5000	-.3290	.6800	-.1961	.8600	.5433		
.1600	.3841	.3400	.9292	.5200	-.4622	.7000	-.0474	.8800	.4412		

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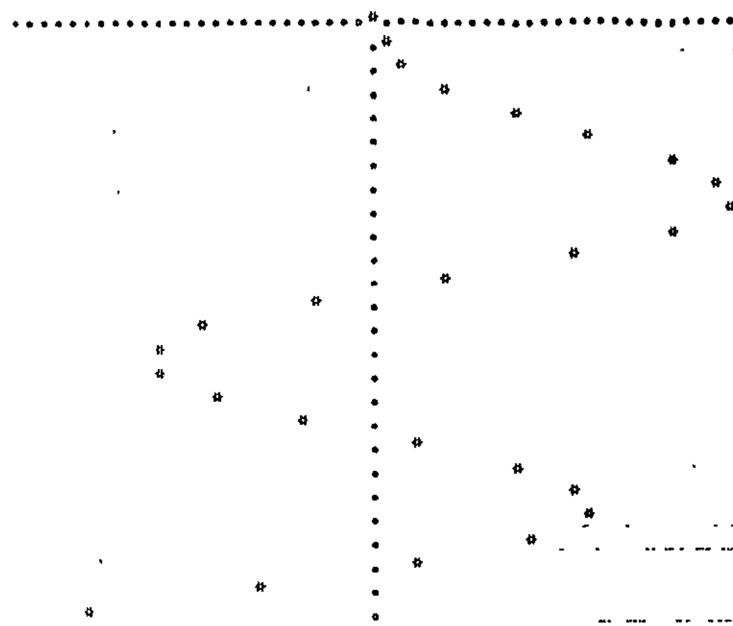

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MODE NUMBER = 6   FREQ, RAD/SEC = 394.2283   FREQ, HERTZ = 62.7434   NON-DIMEN, FREQ. = 84.5494

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CHORDWISE DEFLECTION

STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	0.0000	.1800	-.1432	.3600	-.1695	.5400	.1842	.7200	.0988	.9000	-.0603
.0200	-.0160	.2000	-.1664	.3800	-.1348	.5600	.2014	.7400	.0708	.9200	-.0596
.0400	-.0317	.2200	-.1888	.4000	-.0946	.5800	.2103	.7600	.0434	.9400	-.0565
.0600	-.0469	.2400	-.2082	.4200	-.0505	.6000	.2113	.7800	.0176	.9600	-.0518
.0800	-.0601	.2600	-.2224	.4400	-.0042	.6200	.2051	.8000	-.0054	.9800	-.0463
.1000	-.0711	.2800	-.2294	.4600	.0419	.6400	.1924	.8200	-.0250	1.0000	-.0405
.1200	-.0846	.3000	-.2276	.4800	.0859	.6600	.1743	.8400	-.0404		
.1400	-.1013	.3200	-.2167	.5000	.1254	.6800	.1518	.8600	-.0513		
.1600	-.1211	.3400	-.1971	.5200	.1586	.7000	.1263	.8800	-.0577		

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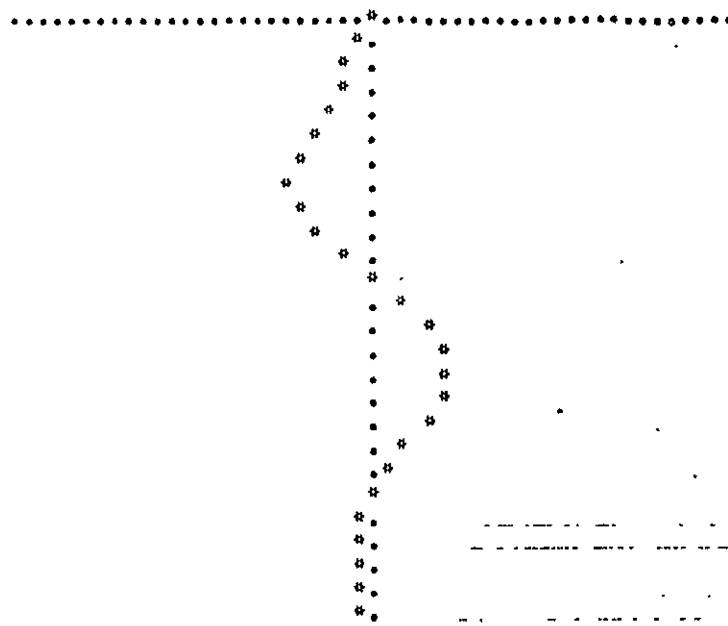
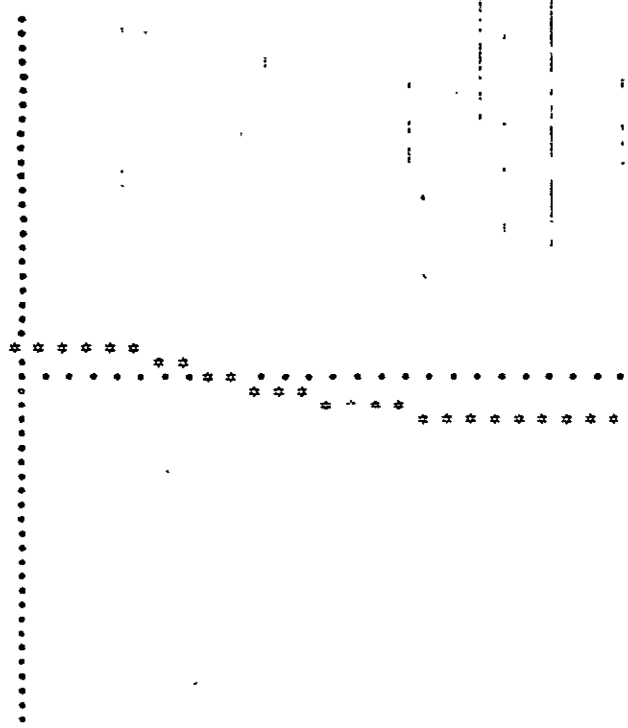


MODE NUMBER = 6   FREQ. RAD/SEC = 394.2283   FREQ. HERTZ = 62.7434   NON-DIMEN. FREQ. = 84.5494

TORSIONAL DEFLECTION

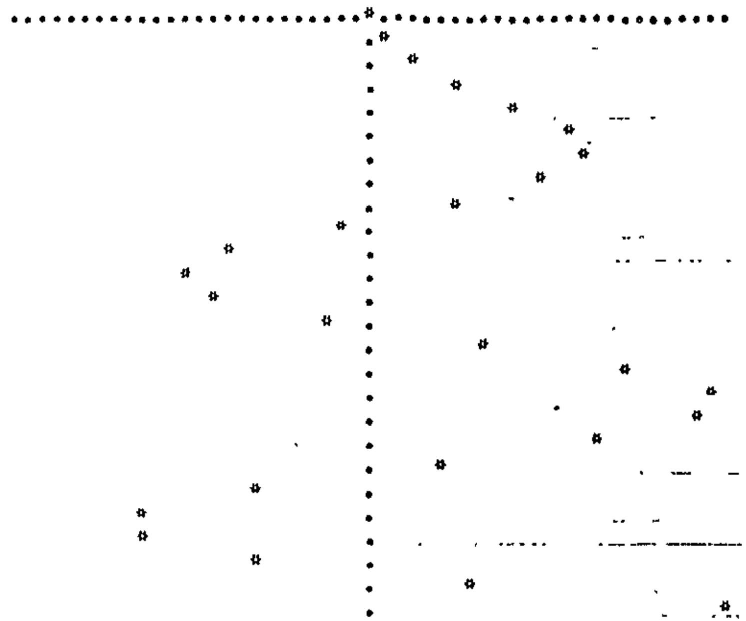
STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	.0747	.1800	.0651	.3600	-.0095	.5400	-.0752	.7200	-.1159	.9000	-.1321
.0200	.0745	.2000	.0608	.3800	-.0185	.5600	-.0797	.7400	-.1198	.9200	-.1338
.0400	.0740	.2200	.0553	.4000	-.0271	.5800	-.0842	.7600	-.1232	.9400	-.1358
.0600	.0738	.2400	.0487	.4200	-.0352	.6000	-.0887	.7800	-.1261	.9600	-.1373
.0800	.0736	.2600	.0404	.4400	-.0429	.6200	-.0934	.8000	-.1284	.9800	-.1376
.1000	.0731	.2800	.0305	.4600	-.0503	.6400	-.0981	.8200	-.1301	1.0000	-.1374
.1200	.0721	.3000	.0201	.4800	-.0576	.6600	-.1028	.8400	-.1311		
.1400	.0706	.3200	.0098	.5000	-.0644	.6800	-.1073	.8600	-.1314		
.1600	.0683	.3400	-.0001	.5200	-.0703	.7000	-.1118	.8800	-.1314		



MODE NUMBER = 7 FREQ. RAD/SEC = 541.7232 FREQ. HERTZ = 86.2179 NON-DIMEN. FREQ. = 116.1824

FLAPWISE DEFLECTION

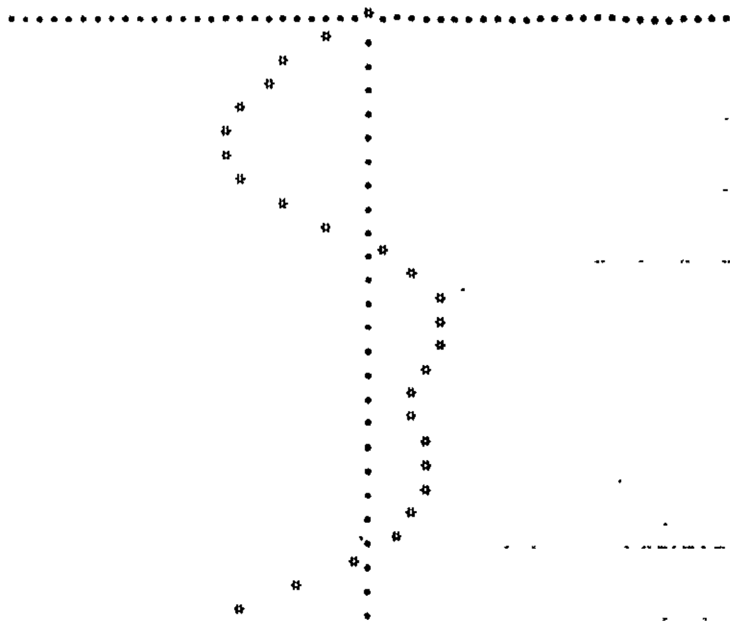
STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	0.0000	.1800	.4808	.3600	-.0919	.5400	.0829	.7200	.6455	.9000	-.5160
.0200	.0090	.2000	.5465	.3800	-.2496	.5600	.3126	.7400	.4275	.9200	-.3057
.0400	.0341	.2200	.5885	.4000	-.3845	.5800	.5332	.7600	.1822	.9400	-.0286
.0600	.0731	.2400	.5981	.4200	-.4831	.6000	.7251	.7800	-.0695	.9600	.2945
.0800	.1189	.2600	.5681	.4400	-.5331	.6200	.8715	.8000	-.3048	.9800	.6424
.1000	.1742	.2800	.4945	.4600	-.5245	.6400	.9596	.8200	-.5005	1.0000	1.0000
.1200	.2420	.3000	.3808	.4800	-.4522	.6600	.9811	.8400	-.6343		
.1400	.3197	.3200	.2369	.5000	-.3187	.6800	.9332	.8600	-.6859		
.1600	.4019	.3400	.0749	.5200	-.1345	.7000	.8185	.8800	-.6453		



MODE NUMBER = 7 FREQ. RAD/SEC = 541.7232 FREQ. HERTZ = 86.2179 NON-DIMEN. FREQ. = 116.1824

CHORDWISE DEFLECTION

STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	0.0000	.1800	-.3832	.3600	-.1142	.5400	.2092	.7200	.1444	.9000	.0145
.0200	-.0612	.2000	-.4007	.3800	-.0446	.5600	.1953	.7400	.1536	.9200	-.0470
.0400	-.1200	.2200	-.4081	.4000	.0223	.5800	.1785	.7600	.1621	.9400	-.1155
.0600	-.1749	.2400	-.4033	.4200	.0832	.6000	.1618	.7800	.1672	.9600	-.1885
.0800	-.2227	.2600	-.3846	.4400	.1350	.6200	.1475	.8000	.1667	.9800	-.2639
.1000	-.2600	.2800	-.3515	.4600	.1752	.6400	.1373	.8200	.1581	1.0000	-.3401
.1200	-.2949	.3000	-.3050	.4800	.2020	.6600	.1321	.8400	.1394		
.1400	-.3280	.3200	-.2478	.5000	.2155	.6800	.1320	.8600	.1091		
.1600	-.3581	.3400	-.1831	.5200	.2170	.7000	.1366	.8800	.0671		



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MODE NUMBER = 7   FREQ. RAD/SEC = 541.7232   FREQ. HERTZ = 86.2179   NON-DIMEN. FREQ. = 116.1824

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TORSIONAL DEFLECTION

STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN	STA X/L	DEFLN
0.0000	-.0202	.1800	-.0161	.3600	.0475	.5400	.1119	.7200	.1697	.9000	.1930
.0200	-.0201	.2000	-.0134	.3800	.0565	.5600	.1188	.7400	.1759	.9200	.1962
.0400	-.0200	.2200	-.0097	.4000	.0649	.5800	.1252	.7600	.1816	.9400	.2007
.0600	-.0199	.2400	-.0047	.4200	.0725	.6000	.1314	.7800	.1866	.9600	.2043
.0800	-.0199	.2600	.0019	.4400	.0791	.6200	.1376	.8000	.1905	.9800	.2051
.1000	-.0197	.2800	.0103	.4600	.0848	.6400	.1438	.8200	.1932	1.0000	.2047
.1200	-.0194	.3000	.0195	.4800	.0904	.6600	.1502	.8400	.1944		
.1400	-.0188	.3200	.0288	.5000	.0970	.6800	.1567	.8600	.1939		
.1600	-.0178	.3400	.0383	.5200	.1044	.7000	.1632	.8800	.1926		

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