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# SINGLE SCATTERING SOLUTION FOR RADIATIVE TRANSFER THROUGH RAYLEIGH AND AEROSOL ATMOSPHERE

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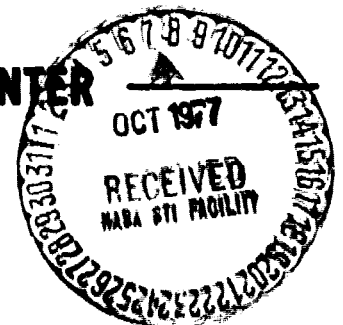
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SINGLE SCATTERING SOLUTION FOR RADIATIVE TRANSFER  
THROUGH RAYLEIGH AND AEROSOL ATMOSPHERE

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**ABSTRACT**

A solution is presented to the radiative transfer of the solar irradiation through a turbid atmosphere, based on the single-scattering approximation, i. e. , an assumption that a photon that underwent scattering either leaves the top of the atmosphere or strikes the surface. The solution depends on a special idealization of the scattering phase function of the aerosols. The equations developed here are subsequently applied to analyze quantitatively (1) the enhancement of the surface irradiation and (2) the enhancement of the scattered radiant emittance as seen from above the atmosphere, caused by the surface reflectance and atmospheric back-scattering. An order-of-magnitude error analysis is presented.

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# SINGLE SCATTERING SOLUTION FOR RADIATIVE TRANSFER THROUGH RAYLEIGH AND AEROSOL ATMOSPHERE

## INTRODUCTION

The problem of radiative transfer through the Earth's atmosphere is highly complex - and an exact solution defies a simple analytical treatment. For a survey of approximation methods see Hunt (1971). In what follows we present a simple approximate solution for solar irradiation based on single-scattering, i. e. , an assumption that a photon once it is scattered, either leaves the atmosphere or strikes the surface. At the surface it can be either absorbed or reflected. The solution can be regarded as the limiting case for an optically thin atmosphere. Furthermore, the simplicity of the solution hinges on an idealization of the aerosol scattering phase function, which is first described.

The equations derived here are applied to analyze the increase in the surface irradiance due to a reflection from a bright surface and subsequent back-scattering, and also to quantitatively describe the inhomogeneities in the atmospheric veil introduced by surface reflection and subsequent upward scattering. These inhomogeneities will be analyzed more fully in a forthcoming paper (Otterman and Fraser, 1978). Also detailed treatment of the atmospheric effects on imaging in Landsat bands is planned (Fraser, Kaufman, Otterman and Podolak, 1978).

## SCATTERING PHASE FUNCTIONS FOR RAYLEIGH AND AEROSOL SCATTERING

The angular dependence of scattering of unpolarized light by the air molecules is described by the well known phase function  $1 + \cos^2 \phi$ , where the angle  $\phi$  is measured from the incident beam. Integration of this expression over a sphere, i. e. , forming the integral  $2\pi \int_0^\pi \sin \phi (1 + \cos^2 \phi) d\phi = 4\pi + \frac{4\pi}{3}$ , indicates easily that in Rayleigh scattering  $\frac{3}{4}$  of the scattering is isotropic and only  $\frac{1}{4}$  predominantly forward or backward.

The phase function of aerosol scattering can vary greatly depending on their electro-magnetic properties, on the size distribution of the particles, and their shape (spherical or more needle like) (Kondratyev, 1969). The phase function is here greatly idealized, and is represented by four parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $m$  as follows:

- i The fraction  $\alpha$  of the scattered photons is limited to a very narrow forward angle.
- ii The fraction  $\beta$  is scattered in direction  $\phi$ , into a solid angle  $2\pi \sin \phi d\phi$ , proportionally to the product of this angle and of  $\cos^m \phi$  ( $m$  is even).
- iii The fraction  $\gamma$  is scattered isotropically.

Since  $\alpha + \beta + \gamma = 1$ , only three of the parameters (including  $m$ ) are independent, and have to be determined appropriately by fitting to a particular phase function.

In this idealized phase function the ratio of forward scattering to back-scattering by the aerosols (i. e., the ratio of scattering into the hemisphere  $0 \leq \phi \leq \pi/2$  to scattering into the hemisphere  $\pi/2 < \phi \leq \pi$ ; in our model also the ratio of scattering into any hemisphere that includes  $\phi = 0$  to the scattering into the opposite hemisphere) is given by  $(1 + \alpha)/(1 - \alpha)$ .

#### DERIVATION OF THE ANALYTICAL EXPRESSIONS

The solar irradiance on a horizontal surface at the top of the atmosphere is given by  $\cos \theta_0$ , where  $\theta_0$  is the solar zenith angle. A plane-parallel atmosphere is assumed; thus the airmass is given by  $1/\cos \theta_0$ . Single scattering is assumed, that is, a photon that is scattered, without undergoing a second encounter impinges on the ground or leaves the atmosphere. The direct irradiance of the surface,  $G_d$ , is

$$G_d = \cos \theta_0 e^{-Q/\cos \theta_0} \quad (1)$$

where  $Q$  is the vertical optical thickness at the wavelength considered.

The components of the optical thickness are Rayleigh scattering,  $R$ , aerosol scattering,  $M$ , and absorption  $B$ :

$$Q = R + M + B \quad (2)$$

According to our idealized phase function, the fraction  $[R + (1 + \alpha)M] / 2Q$  of the photons that are scattered or absorbed from the direct beam, are scattered into a hemisphere below a horizontal plane through the scattering point. Because of the single scattering assumption, this fraction reaches the surface.



Therefore the irradiance  $G_{sd}$  contributed by the scattering from the direct beam to the surface is:

$$G_{sd} = \cos \theta_o \left( 1 - e^{-Q/\cos \theta_o} \right) \frac{R + (1 + \alpha) M}{2Q} \quad (3)$$

The irradiance of the surface,  $G_o$ , when no surface reflection is considered, is:

$$G_o = G_d + G_{sd} = \cos \theta_o \left[ e^{-Q/\cos \theta_o} + \left( 1 - e^{-Q/\cos \theta_o} \right) \frac{R + (1 + \alpha) M}{2Q} \right] \quad (4)$$

A fraction of the irradiance of the surface is reflected, and this fraction in part is back-scattered to the surface. This reflection and back-scattering from the atmosphere form a geometric series effect:

$$G_t = G_o + kG_o + k^2G_o + \dots = \frac{G_o}{1 - k} \quad (5)$$

where  $G_t$  is the total irradiance and  $k$  the coefficient of the reflection back-scattering. This coefficient, assuming reflectance according to the Lambert's law, with reflectivity  $a_o$ , is given by:

$$\begin{aligned} k &= \frac{a_o 2\pi}{\pi} \int_0^{\pi/2} \cos \phi \sin \phi \left( 1 - e^{-Q/\cos \phi} \right) \frac{R + (1 - \alpha) M}{2Q} d\phi \\ &= a_o \frac{R + (1 - \alpha) M}{Q} C_1(Q) \end{aligned} \quad (6)$$

since  $[R + (1 - \alpha) M] / 2Q$  is the fraction of the scattered or absorbed photons that is back-scattered downward, and where

$$\begin{aligned} C_1(Q) &= \int_0^{\pi/2} \cos \phi \sin \phi \left( 1 - e^{-Q/\cos \phi} \right) d\phi \\ &= \frac{1}{2} \left[ 1 - (1 - Q) e^{-Q} + Q^2 \left( +0.5772 + \ln Q + \sum_{n=1}^{\infty} \frac{(-1)^n Q^n}{n n!} \right) \right] \end{aligned} \quad (7)$$

The function  $C_1(Q)$  is tabulated in Appendix A. For  $Q \ll 1$ ,  $C_1(Q) \approx Q$ .

Thus the total irradiance is:<sup>\*</sup>

$$G_t = \cos \theta_o \frac{e^{-Q/\cos \theta_o} + \left(1 - e^{-Q/\cos \theta_o}\right) \frac{R + (1 + \alpha) M}{2Q}}{1 - a_o \frac{R + (1 - \alpha) M}{Q} C_1(Q)} \quad (5.1)$$

which for a Rayleigh atmosphere,  $Q = R$ , can be simplified to

$$G_t = \cos \theta_o \frac{1 + e^{-R/\cos \theta_o}}{2[1 - a_o C_1(R)]} \quad (5.2)$$

The absorption  $E_o$  at the surface is given by

$$E_o = (1 - a_o) G_t \quad (8)$$

and in the atmosphere,  $E_a$

$$\begin{aligned} E_a &= \frac{2 \pi a_o G_t}{\pi} \int_0^{\pi/2} \cos \phi \sin \phi \left(1 - e^{-Q/\cos \phi}\right) d\phi \frac{B}{Q} + \cos \theta_o \left(1 - e^{-Q/\cos \theta_o}\right) \frac{B}{Q} \\ &= \frac{2 G_t a_o B}{Q} C_1(Q) + \cos \theta_o \left(1 - e^{-Q/\cos \theta_o}\right) \frac{B}{Q} \end{aligned} \quad (9)$$

We analyze the regional Earth-atmosphere system hemispheric reflectivity  $A_t$  in terms of the radiant emittance from the top of the atmosphere  $A_t \cos \theta_o$ , which is given by the irradiance  $\cos \theta_o$  less the absorption terms:

$$\begin{aligned} A_t \cos \theta_o &= \cos \theta_o - (E_o + E_a) \\ &= \cos \theta_o \left\{ 1 - \left(1 - e^{-Q/\cos \theta_o}\right) \frac{B}{Q} \right. \\ &\quad \left. - \left[ 1 - a_o + \frac{2 a_o B}{Q} C_1(Q) \right] \frac{e^{-Q/\cos \theta_o} + \left(1 - e^{-Q/\cos \theta_o}\right) \frac{R + (1 + \alpha) M}{2Q}}{1 - a_o \frac{R + (1 - \alpha) M}{Q} C_1(Q)} \right\} \end{aligned} \quad (10)$$

The planetary albedo,  $A_p$ , for the illuminated hemisphere (i. e.  $\theta_o = 0$  is located at the center of the hemisphere) is:

$$A_p = \frac{2\pi \int_0^{\pi/2} \sin \theta_o A_t \cos \theta_o d\theta_o}{2\pi \int_0^{\pi/2} \sin \theta_o \cos \theta_o d\theta_o} =$$

$$1 - \left[ 1 - a_o + \frac{2a_o B}{Q} C_1(Q) \right] \frac{1 - 2C_1(Q) + C_1(Q) \frac{R + (1 + \alpha) M}{Q}}{1 - a_o \frac{R + (1 - \alpha) M}{Q} C_1(Q)} - \frac{2B}{Q} C_1(Q) \quad (11)$$

where the term  $(1 - a_o) \times \dots$  represents the absorption at the surface and  $\frac{2a_o B}{Q} C_1(Q) \times \dots + \frac{2B}{Q} C_1(Q)$  represents the absorption in the atmosphere.

The radiance at nadir (for an arbitrary  $\theta_o$ ) consists of three elements: the "signal" radiance  $L_{nr}$  from the "target" pixel with reflectivity  $r$ :

$$L_{nr} = \frac{rG_t e^{-Q}}{\pi} \quad (12)$$

the radiance  $L_{nd}$ , scattered from the direct beam

$$L_{nd} = \cos \theta_o \left( 1 - e^{-Q/\cos \theta_o} \right) \left[ \left( (m+1)\beta \cos^m \theta_o + \gamma \right) \frac{M}{4\pi Q} + (\cos^2 \theta_o + 1) \frac{3R}{16\pi Q} \right] \quad (13)$$

and the radiance  $L_{na}$  reflected from the terrain in the general vicinity of the target (reflectivity  $a$ ), and subsequently scattered to nadir:

$$L_{na} = \frac{G_t a}{\pi} \left\{ \left( 1 - e^{-Q} \right) \alpha \frac{M}{Q} + 2\pi \int_0^{\pi/2} \left( 1 - e^{-Q/\cos \phi} \right) \sin \phi \cos \phi \times \right.$$

$$\left. \times \left[ \left( \beta (m+1) \cos^m \phi + \gamma \right) \frac{M}{4\pi Q} + (\cos^2 \phi + 1) \frac{3R}{16\pi Q} \right] d\phi \right\} =$$

$$\begin{aligned}
&= \frac{G_a}{\pi} \left\{ \left[ 2\alpha (1 - e^{-Q}) + \beta (m+1) C_{m+1}(Q) + \gamma C_1(Q) \right] \frac{M}{2Q} + \right. \\
&\quad \left. + \left[ C_3(Q) + C_1(Q) \right] \frac{3R}{8Q} \right\} \quad (14)
\end{aligned}$$

where

$$C_{m+1}(Q) = \int_0^{\pi/2} \left( 1 - e^{-Q/\cos\phi} \right) \sin\phi \cos^{(m+1)}\phi \, d\phi \quad (15)$$

These functions  $C_{m+1}$  are discussed in Appendix A.

The radiance at nadir  $L_n$  is

$$L_n = L_{nr} + L_{nd} + L_{na} \quad (16)$$

## APPLICATIONS

### Enhancement of Irradiance of the Horizontal Surface by the Surface Reflectance and Subsequent Back-Scattering

In what follows we introduce two abbreviations: the fraction  $f$  of the atmospheric optical thickness that effects forward scattering

$$f = \frac{R + (1 + \alpha) M}{2Q} \quad (17)$$

and the fraction  $b$  that effects back-scattering

$$b = \frac{R + (1 - \alpha) M}{2Q} \quad (18)$$

and thus  $f/b$  denotes forward to back-scattering ratio of the atmosphere, and  $1 - (f + b)$  the absorbing fraction.

The surface irradiance due to scattering from the direct beam is

$$G_{sd} = \cos \theta_o \left( 1 - e^{-Q/\cos \theta_o} \right) \frac{R + (1 + \alpha) M}{2Q} = \cos \theta_o \left( 1 - e^{-Q/\cos \theta_o} \right) f \quad (19)$$

The surface irradiance due to surface reflectance and back-scattering is

$$\begin{aligned} G_{rb} &= \cos \theta_o \left[ e^{-Q/\cos \theta_o} + \left( 1 - e^{-Q/\cos \theta_o} \right) f \right] \left[ \frac{1}{1 - 2a_o b C_1(Q)} - 1 \right] \\ &= \frac{\cos \theta_o \left[ e^{-Q/\cos \theta_o} + \left( 1 - e^{-Q/\cos \theta_o} \right) f \right] 2a_o b C_1(Q)}{1 - 2a_o b C_1(Q)} \end{aligned} \quad (20)$$

The enhancement  $S_{rb}$  of the skylight by this effect relative to the scattering from the direct beam therefore is

$$\begin{aligned} S_{rb} &= \frac{G_{rb}}{G_{sd}} = \\ &= \frac{a_o b \left[ e^{-Q/\cos \theta_o} + \left( 1 - e^{-Q/\cos \theta_o} \right) f \right] 2 C_1(Q)}{f \left( 1 - e^{-Q/\cos \theta_o} \right) \left[ 1 - 2a_o b C_1(Q) \right]} = \\ &= a_o b \left[ 1 + \frac{e^{-Q/\cos \theta_o}}{f \left( 1 - e^{-Q/\cos \theta_o} \right)} \right] \frac{2 C_1(Q)}{1 - 2a_o b C_1(Q)} \end{aligned} \quad (21)$$

It can be seen that the enhancement is proportional to  $a_o b$ , but for low  $Q$  and small zenith angles  $\theta_o$  (high solar elevations) it is proportional to  $a_o b/f$ ,

$$\text{because then } \frac{e^{-Q/\cos \theta_o}}{1 - e^{-Q/\cos \theta_o}} \approx \frac{\cos \theta_o}{Q} \gg 1.$$

Under these conditions (since  $C_1(Q) \approx Q$  for low  $Q$ )

$$S_{rb} \approx \frac{2a_0 b}{f} \cos \theta_0 \quad (22)$$

and is thus approximately independent of  $Q$ .

The function  $S_{rb}$ , Eq. 21, is plotted vs.  $a_0$  for  $\theta_0 = 0^\circ$  (see Figure 1)

- i) for  $Q = 0.1$ , and for  $f = b = 1/2$ , i. e., for a Rayleigh atmosphere about  $0.55 \mu\text{m}$
- ii) for  $Q = 0.3$  and for  $f = 3/4$ ,  $b = 1/4$ , which for scattering by aerosols only corresponds to  $\alpha = 0.5$ ; or when Rayleigh optical thickness is  $0.1$  and aerosol optical thickness is  $0.2$ , corresponds to  $\alpha = 0.75$ .
- iii) for  $Q = 0.3$  for  $f = 5/6$ ,  $b = 1/6$ , which for scattering by aerosols only corresponds to  $\alpha = 2/3$ ; or when Rayleigh optical thickness is  $0.1$  and aerosol optical thickness is  $0.2$ , corresponds to  $\alpha = 1.0$ .

In Figure 2 the same graphs are repeated for  $\theta^\circ = 60^\circ$ .

Coulson (1968) pointed out that many mineral surfaces have high reflectivity at the longer visible and solar infrared wavelengths, and over such surfaces, surface reflection and subsequent back-scattering from a Rayleigh atmosphere can contribute as much as half of the total skylight particularly at high sun elevations. Indeed, in Figure 1 it can be seen that for a Rayleigh atmosphere one-half of the total skylight ( $S_{rb} = 1.0$ ) is contributed by this effect at  $a_0 = 0.57$ . Such reflectivity can be approached in sandy desert areas at wavelengths  $0.7 - 1.1 \mu\text{m}$  (Otterman and Fraser, 1976), can be encountered over thickly vegetated terrain above  $0.75 \mu\text{m}$  (Colwell, 1974) and can be exceeded over snow at any wavelength. For a real atmosphere, i. e. Rayleigh and aerosol, a reduction of this effect occurs in the ratio  $b/f$ , see Figure 1. For a higher zenith angle the effect is less significant relatively to the scattering from the direct beam, see Figure 2, but remains significant in terms of contribution to the total irradiance, i. e., as compared to the direct irradiance.

### Enhancement of the Atmospheric Veil by the Surface Reflectance and Subsequent Scattering

The enhancement of the atmospheric veil by the surface reflectance and subsequent scattering forms a complimentary question to the problem just discussed. Earlier, we analyzed the fraction of the reflected and scattered radiation that impinges on the ground, here we discuss the fraction that escapes from the

atmosphere. This part of the atmospheric veil appears most strongly above bright terrain, and thus spatial inhomogeneities in this atmospheric veil are introduced even if the atmosphere is horizontally homogeneous. Such inhomogeneities can introduce difficulties in thematic mapping by multispectral radiometry approach and this aspect requires more detailed treatment, in terms of radiances measured from a satellite. Here we analyze the enhancement in the radiant emittance emerging from the top of the atmosphere above a bright terrain.

The radiant emittance due to the scattering from the direct beam is

$$\begin{aligned} W_{sd} &= \cos \theta_o \left( 1 - e^{-Q/\cos \theta_o} \right) \frac{R + (1 - \alpha) M}{2Q} \\ &= \cos \theta_o \left( 1 - e^{-Q/\cos \theta_o} \right) b \end{aligned} \quad (23)$$

and the radiant emittance from surface reflection and subsequent scattering is:

$$\begin{aligned} W_{rf} &= a_o G_t \frac{R + (1 + \alpha) M}{Q} C_1(Q) \\ &= \frac{a_o \cos \theta_o \left[ e^{-Q/\cos \theta_o} + \left( 1 - e^{-Q/\cos \theta_o} \right) f \right] 2f C_1(Q)}{1 - 2a_o b C_1(Q)} \end{aligned} \quad (24)$$

and the enhancement  $S_{rf}$  over bright terrain with reflectivity  $a_o$  as compared to terrain  $a_o = 0$  is

$$\begin{aligned} S_{rf} &= \frac{W_{rf}}{W_{sd}} = a_o f \frac{\left[ e^{-Q/\cos \theta_o} + \left( 1 - e^{-Q/\cos \theta_o} \right) f \right] 2 C_1(Q)}{b \left\{ 1 - 2a_o b C_1(Q) \right\} \left( 1 - e^{-Q/\cos \theta_o} \right)} = \\ &= \frac{a_o f^2}{b} \left[ 1 + \frac{e^{-Q/\cos \theta_o}}{f \left( 1 - e^{-Q/\cos \theta_o} \right)} \right] \frac{2 C_1(Q)}{1 - 2a_o b C_1(Q)} \end{aligned} \quad (25)$$

There is a strong similarity between this expression for  $S_{rf}$  and that for  $S_{rb}$ , Eq. (21), which is not surprising in view of what was said before. For low  $Q$

and small zenith angles (high solar elevations), the enhancement is approximately

$$S_{rf} \approx \frac{2a_0 f}{b} \cos \theta_0 \quad (26)$$

We compute  $S_{rf}$ , Eq. (25), as a function for  $a_0$  for  $\theta^\circ = 0$  (see Figure 3) and, as before in Figure 1:

- i) for  $Q = 0.1$ , and for  $f = b = 1/2$ , i. e., for a Rayleigh atmosphere about  $0.55 \mu\text{m}$
- ii) for  $Q = 0.3$  and for  $f = 3/4$ ,  $b = 1/4$
- iii) for  $Q = 0.3$  and for  $f = 5/6$ ,  $b = 1/6$

and repeat the calculations for  $\theta^\circ = 60^\circ$  (see Figure 4). The case (i) is numerically equal in Figure 3 to that in Figure 1, and in Figure 4 to that in Figure 2, but case (ii) and (iii) are higher in the ratio  $f^2/b^2$  in the corresponding figures. This ratio is 9 for (ii) and 25 for (iii).

It can be seen from Figure 3 that the atmospheric veil can be more than doubled over high reflectivity terrain, since  $S_{rf}$  reaches 1.0 at values of  $a_0$  ranging from 0.14 (case ii) to 0.22 (case iii).

For thematic mapping by multispectral radiometry, the significant question is how to expurge the radiance  $L_{na}$  in Eq. (16), which depends on the surface reflectance from the area around a pixel, and to measure the reflectivity  $r$  of the pixel (or  $re^{-Q}$ ) from  $L_n$ . This will be discussed by Otterman and Fraser (1978).

## DISCUSSION AND CONCLUSIONS

Analytical expressions have been derived for the radiative transfer through the atmosphere for a limiting case of a thin atmosphere. The solution is based on a single encounter (scattering or absorption) approximation to the atmospheric passage of the solar radiation, both for the solar irradiance from the top of the atmosphere downward and for the surface reflection from the bottom of the atmosphere upwards. The simplicity of the solution also depends on the idealization of the scattering phase function.

Presentation of such a simplified solution has to be accompanied by an assessment of its accuracy, and this is outlined here. The "subsequent encounter"



(i. e., either an absorption-encounter or a scattering-encounter of a photon that first underwent scattering) are analyzed in an order-of-magnitude approximation. Consider that all the first encounters as computed here occur on the average mid-way through the atmosphere. The once scattered photons, (a fraction  $\left(1 - e^{-Q/\cos\theta_0}\right) (R + M)/Q \approx (R + M)/\cos\theta_0$  of the photons impinging on the top of the atmosphere), face, if they are scattered predominantly forward or backward, an absorbing optical thickness of  $B/2 \cos\theta_0$ . The once-scattered photons that undergo subsequent absorption (before possibly impinging on the surface) constitute a fraction  $\sim (R + M) B/2 \cos^2\theta_0$  of the impinging radiation, as compared to a fraction  $\sim B/\cos\theta_0$  that is absorbed before undergoing scattering (and which is accurately computed in our solution). The relative error in computing absorption is thus of the order of  $(R+M)/2 \cos\theta_0$ .

The second source of error lies in that second scattering results in a change of direction. The effective scattering phase function<sup>†</sup> becomes somewhat different from that assumed in the computations. Repeating the argument that the second encounter for the once-scattered photons is approximately proportional to the airmass  $1/2 \cos\theta_0$ , and that the  $\alpha$  fraction of the aerosol scattering represents only a small change in the direction that can be neglected here, we estimate that the fraction of photons that undergoes a change of direction as a result of a second scattering is  $[R + (1 - \alpha)M] / 2 \cos\theta_0$ . In the cases analysed of  $Q = 0.3$  and  $\theta_0 = 60^\circ$ , Figures 2 and 4, this fraction would be between 10 and 15 percent. The net result would be that an appreciable fraction of the scattered photons would be directed at a more nearly random direction relative to the first scattering point. The effective phase function then becomes spherically less asymmetric. The important ratio  $f/b$  becomes somewhat reduced, but the results for a Rayleigh atmosphere would be scarcely affected.

There is one limitation of applying the formulas that has to be stressed. The above error estimates indicate that the errors increase as  $1/\cos\theta_0$ . Beside this indication, that the solution becomes quite inaccurate at the low solar elevations, the assumption that the fraction  $\alpha$  of the aerosol scattering, the " $\alpha$  beam," is scattered forward into a zero-width spherical angle introduces a limitation on the lowest solar elevation for which the formulas can be used. According to our idealized phase function, the " $\alpha$  beam" scattered from the direct beam will always impinge on the surface. In reality, at the very low solar elevations, because of the actual finite angular spread of the " $\alpha$  beam," part of the beam will be directed upwards. The formulas should not be thus used for  $\theta_0 > 70^\circ$ .

While this error analysis indicates the need for caution in applying the formulas derived here, it is claimed that these simple solutions can provide means

of quantitatively assessing the atmospheric scattering effects under low and even medium turbidity conditions, especially for low absorptivity.

The equations are applied to analyze the enhancement of the surface irradiance by surface reflectance and subsequent back-scattering. It is concluded that for high surface reflectance of 0.57, one-half of the scattered irradiance can be contributed by this effect for a Rayleigh atmosphere. For an aerosol and Rayleigh atmosphere, the relative contribution of this effect to the scattered irradiance is reduced in the ratio  $f/b$ , i. e., ratio of the forward scattering optical thickness to the back-scattering optical thickness.

The scattered radiant emittance as seen from above the atmosphere is also enhanced by surface reflectance and subsequent scattering, and for a Rayleigh atmosphere the expressions are numerically the same as in the case of the enhancement of the surface irradiance. For a turbid atmosphere, i. e., Rayleigh + aerosol, the effect is increased proportionally to the  $f/b$  ratio. Thus, over a bright terrain the atmospheric veil is much more pronounced than over a dark terrain. These inhomogeneities constitute an important phenomenon, affecting the multispectral radiometry from space, and deserve detailed studies.

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## FOOTNOTES

\*The total irradiance on the surface when  $a_0 = 1$  for  $\theta_0 = 0$  and for a non-absorbing atmosphere is approximately  $1 + [R + (1 - \alpha)M] / 2$ , and thus is higher than at the top of the atmosphere. This is due to the fact that some photons impinge on the surface more than once. Numerically similar results were obtained by Deirmendjian and Sekera (1954).

†i. e., a function relating the direction probability for photons emerging from the top of the atmosphere or striking the surface, to the direction of the direct beam.

## REFERENCES

Abramowitz, M. and I. A. Stegun, Eds. (1964), Handbook of Mathematical Functions, U. S. Department of Commerce, National Bureau of Standards Applied Mathematics Series 55.

Colwell, J. E. (1974), Vegetation Canopy Reflectance, Remote Sensing of Environment, 3, 175-183.

Coulson, K. L. (1968), Effect of Surface Reflection on the Angular and Spectral Distribution of Skylight, Journal of the Atmospheric Sciences, 25, 759-770.

Deirmendjian, D. and L. Sekera (1954), Global Radiation Resulting from Multiple Scattering in a Rayleigh Atmosphere, Tellus, VI, 382-398.

Fraser, R. S., J. Y. Kaufman, J. Otterman and M. Podolak (1978), Atmospheric Effects on Imaging in the LANDSAT Bands, in preparation, for submission to Applied Optics.

Hunt, G. E. (1971), A Review of Computational Techniques for Analysing the Transfer of Radiation Emerging from a Model Cloudy Atmosphere, J. Quant. Spectrosc. Radiat. Transfer, 11, 655-690.

Kondratyev, K. Ya. (1969), Radiation in the Atmosphere, Academic Press, New York.

Otterman, M. and R. S. Fraser (1976), Earth Atmosphere and Surface Reflectivities in Arid Regions from LANDSAT MSS Data, Remote Sensing of Environment, 5, 247-266.

Otterman, J. and R. S. Fraser (1978), Adjacency Effects on Imaging by Surface Reflection and Atmospheric Scattering, in preparation, for submission to Applied Optics.

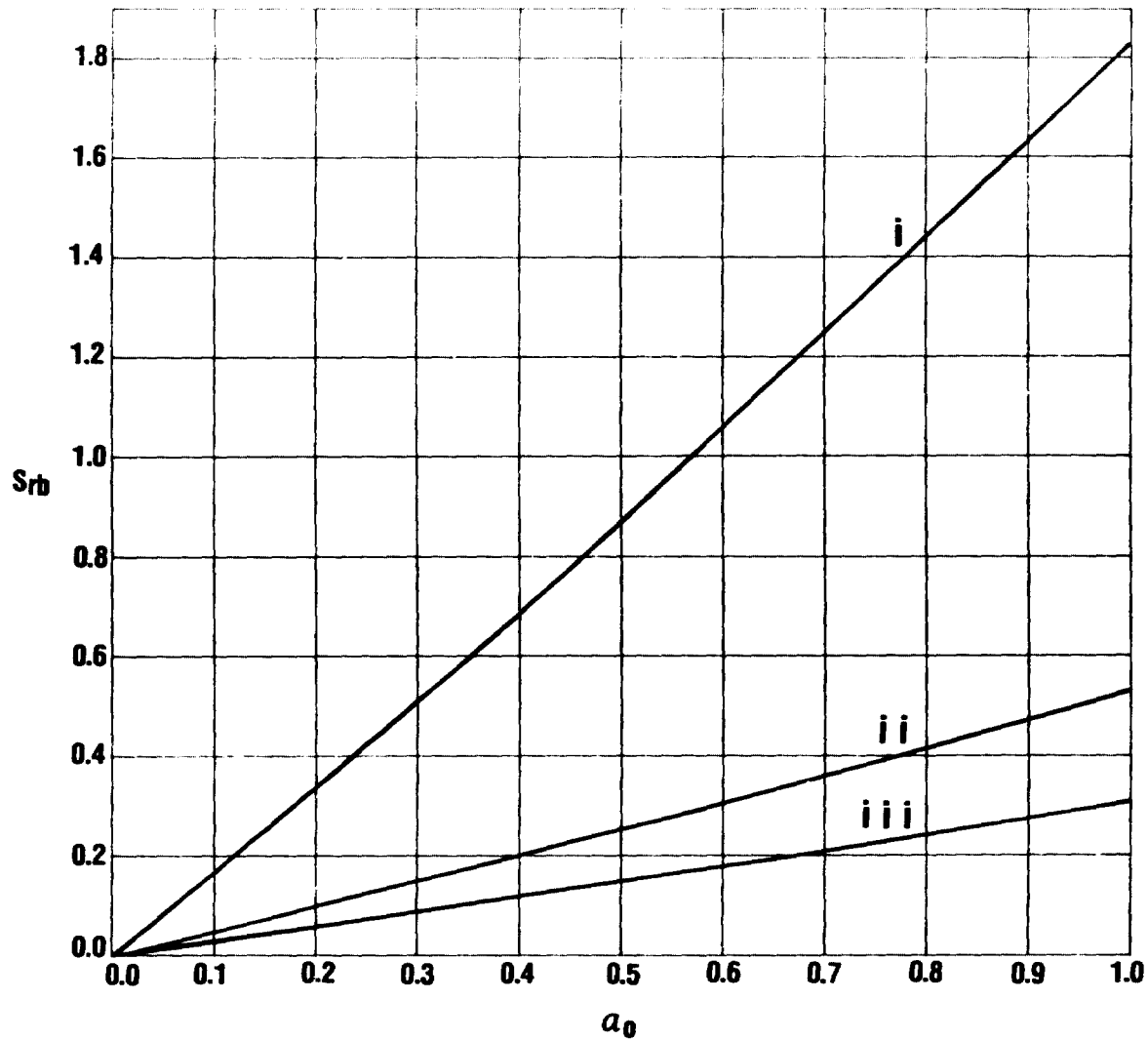


Figure 1. The increase in the surface irradiance by surface reflection and subsequent backscattering,  $S_{rb}$ , for (i)  $Q = 0.1$ ,  $f/b = 1.0$ ; (ii)  $Q = 0.3$ ,  $f/b = 3.0$ ; (iii)  $Q = 0.3$ ,  $f/b = 5.0$ .  $\theta = 0^\circ$

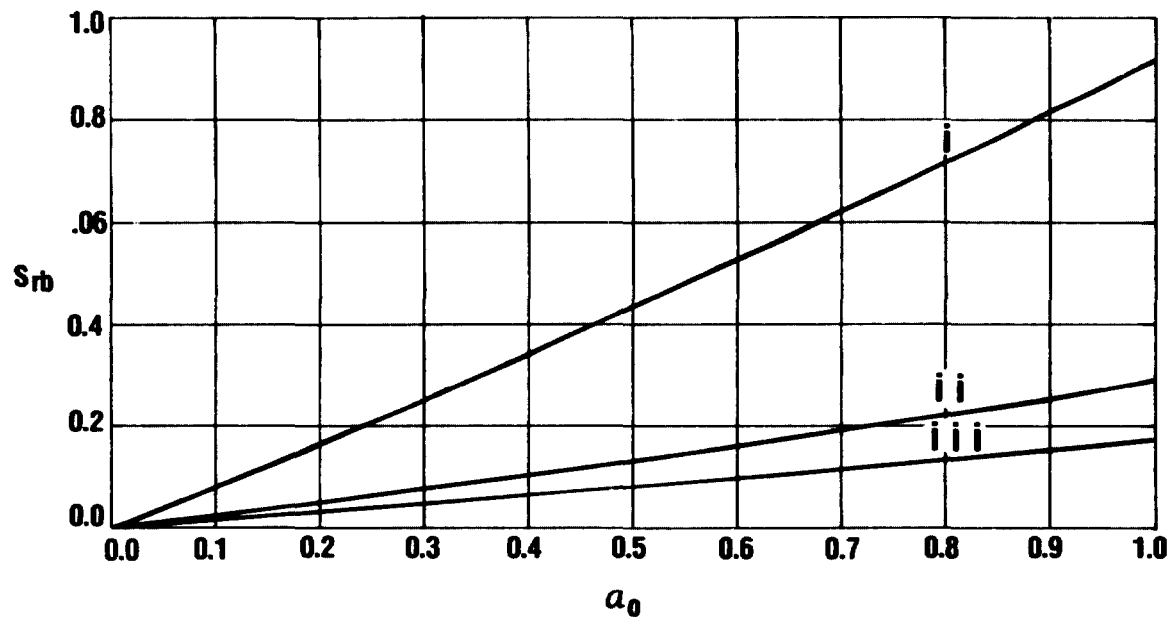


Figure 2. The increase in the surface irradiance by surface reflection and subsequent backscattering,  $S_{rb}$ , for (i)  $Q = 0.1$ ,  $f/b = 1.0$ ; (ii)  $Q = 0.3$ ,  $f/b = 3.0$ ; (iii)  $Q = 0.3$ ,  $f/b = 5.0$ .  $\theta = 60^\circ$

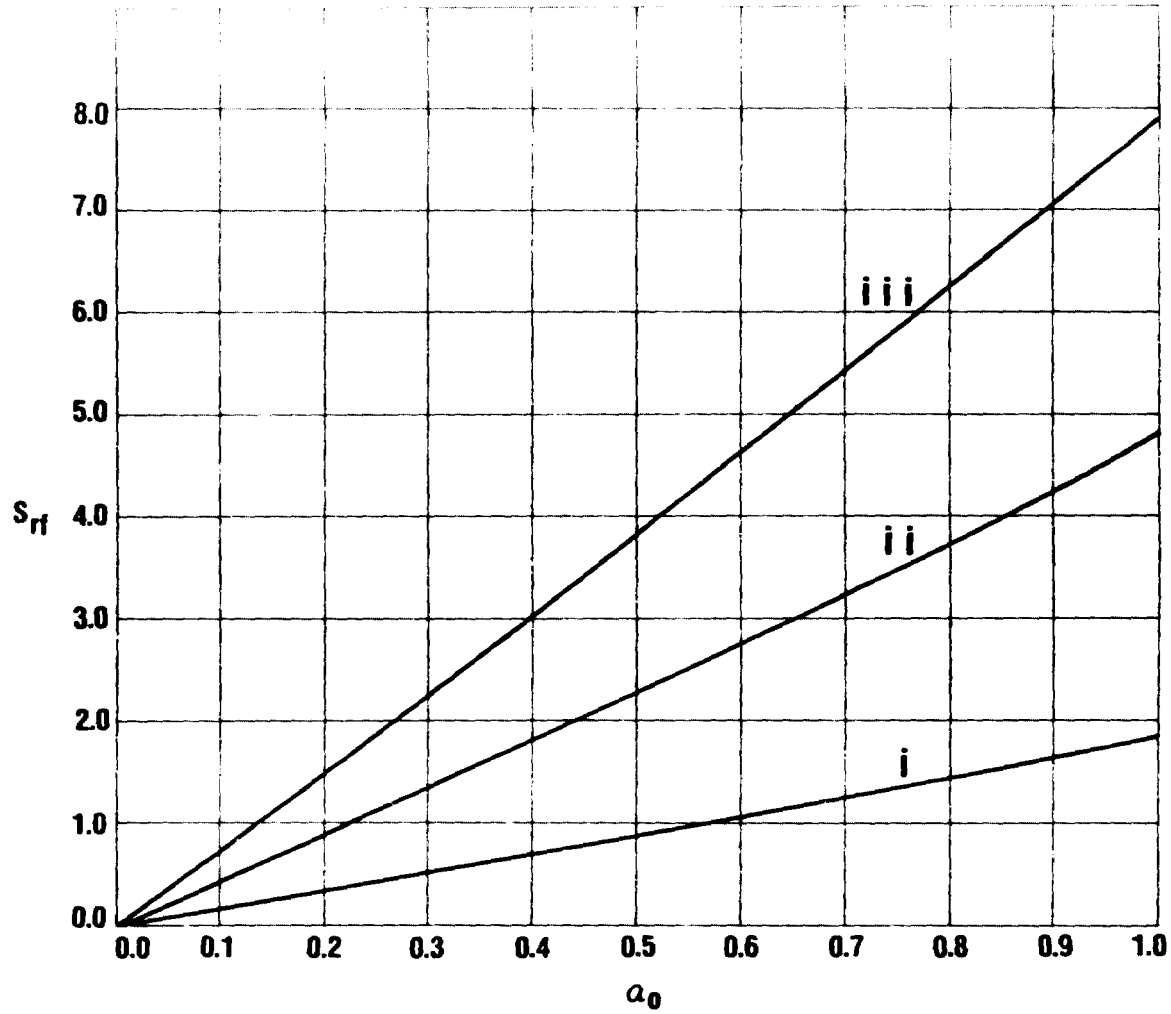


Figure 3. The increase in the atmospheric radiant emittance by surface reflection and subsequent upward scattering,  $S_{rf}$ , for (i)  $Q = 0.1$ ,  $f/b = 1.0$ ; (ii)  $Q = 0.3$ ,  $f/b = 3.0$ ; (iii)  $Q = 0.3$ ,  $f/b = 5.0$ .  $\theta = 0^\circ$

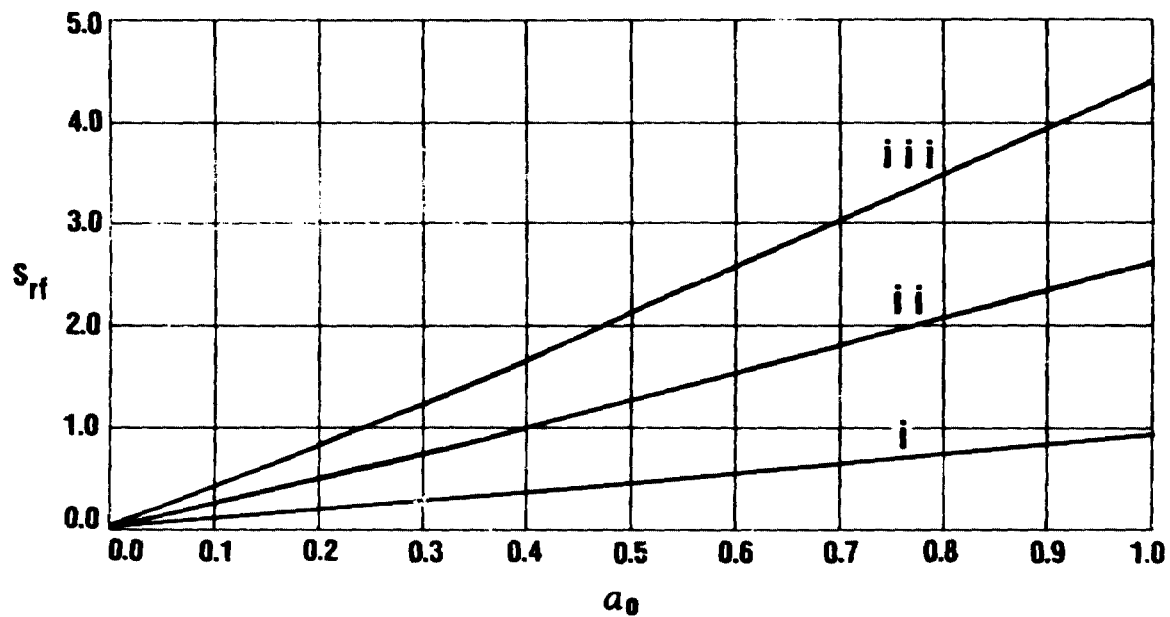


Figure 4. The increase in the atmospheric radiant emittance by surface reflection and subsequent upward scattering,  $S_{rf}$ , for (i)  $Q = 0.1$ ,  $f/b = 1.0$ ; (ii)  $Q = 0.3$ ,  $f/b = 3.0$ ; (iii)  $Q = 0.3$ ,  $f/b = 5.0$ .  $\theta = 60^\circ$



## APPENDIX A

The functions  $C_m(Q)$  are integrable for  $0 < Q < 1$ ,  $m > 0$  as follows:

$$\begin{aligned}
 C_m(Q) &= \int_0^{\pi/2} \sin \psi \cos^m \psi (1 - e^{-Q/\cos \psi}) d\psi \\
 &= \int_0^{\pi/2} \sin \psi \cos^m \psi d\psi - \int_0^{\pi/2} \sin \psi \cos^m \psi e^{-Q/\cos \psi} d\psi \\
 &= \frac{1}{m+1} - \int_1^{\infty} \frac{e^{-Qx}}{x^{m+2}} dx
 \end{aligned} \tag{A1}$$

where in the second integral a substitution was made:  $\cos \psi = \frac{1}{x}$  and therefore  $\sin \psi d\psi = \frac{dx}{x^2}$ . The second integral is an exponential integral  $E$  of order  $m+2$ :

$$\int_1^{\infty} \frac{e^{-Qx}}{x^{m+2}} dx = E_{m+2}(Q) \tag{A2}$$

which can be expressed in terms of the exponential integral of order 1 by the following recursion formula:

$$E_{m+1}(Q) = \frac{1}{m} [e^{-Q} - QE_m(Q)]. \tag{A3}$$

In the particular case for computing  $C_1(Q)$ :

$$E_3(Q) = \frac{1}{2} (1-Q) e^{-Q} + \frac{Q^2}{2} E_1(Q) \tag{A4}$$

$E_1(Q)$  can be represented in terms of a power series in  $Q$ :

$$E_1(Q) = -0.5772 - \ln Q - \sum_1^{\infty} \frac{(-1)^n Q^n}{n n!} \tag{A5}$$

(Eq. 5.1.10, p. 229, in Abramowitz and Stegun, 1964), and thus:

$$C_1(Q) = \frac{1}{2} [1 - (1-Q)e^{-Q} - Q^2 E_1(Q)] . \quad (A6)$$

$$= \frac{1}{2} \left[ 1 - (1-Q)e^{-Q} - Q^2 \left( -0.5772 - \ln Q - \sum_1^{\infty} \frac{(-1)^n Q^n}{nn!} \right) \right]$$

Brief tabulation of this function is given below.

Table A-1

<u>Q</u>	<u>C<sub>1</sub>(Q)</u>
0.00	.00000
0.05	.04508
0.10	.08371
0.15	.11772
0.20	.14805
0.25	.17532
0.30	.19996
0.35	.22233
0.40	.24271
0.45	.26134
0.50	.27839
0.55	.29405
0.60	.30845
0.65	.32171