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PLANS - A FINITE ELEMENT PROGRAM FOR NONLINEAR ANALYSIS OF STRUCTURES

VOLUME II - USERS' MANUAL

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A. PIFKO, H. ARMEN, JR., A. LEVY, AND H. LEVINE

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PLANS — A FINITE ELEMENT PROGRAM FOR NONLINEAR ANALYSIS OF STRUCTURES

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by

A. Pifko, H. Armen Jr., A. Levy, and H. Levine

Materials and Structural Mechanics

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Approved by: Richard A. Scheuing
Director of Research

FOREWORD

The PLANS system of programs for the nonlinear analysis of structures was developed within the Grumman Research Department with the cooperation, guidance, and partial support of the NASA Langley Research Center.

The programs are an outgrowth of the work reported in a series of NASA Contractor Reports: CR-803, CR-1649, CR-2351, and CR-2568. The last-named document is the theoretical companion volume to this manual.

The principal developers of PLANS are Drs. A. Pifko, H. Armen Jr., H. Levine, and A. Levy. The successful development of a system as broad in scope and complexity as PLANS requires the efforts of many individuals. The principal developers gratefully acknowledge the contributions of J. S. Miller for his efforts associated with the initial programming. Special thanks also go to Ms. P. Ogilvie, P. Zirk, and E. Yander for their diligence and dedication to the programming effort.

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INTRODUCTION AND INPUT OVERVIEW

INTRODUCTION AND INPUT OVERVIEW

The PLANS system, rather than being one comprehensive computer program, is a collection of finite element programs used for the nonlinear analysis of structures. This collection of programs evolved and is based on the organizational philosophy in which classes of analyses are treated individually based on the physical problem class to be analyzed. On the basis of this concept, each of the independent finite element computer programs of PLANS, with an associated element library, can be individually loaded and used to solve the problem class of interest. A number of programs have been developed for material nonlinear behavior alone and for combined geometric and material nonlinear behavior. Table 1 summarizes the usage, capabilities, and element libraries of the current programs of the PLANS system. These include:

- BEND — for the plastic analysis of built-up structures where bending and membrane effects are significant
- HEX — for problems requiring a three dimensional elastic-plastic analysis
- REVBY — for the plastic analysis of bodies of revolution
- OUT-OF-PLANE MG — for the material and geometric nonlinear analysis of built-up structures

Supplementing these is a SATELLITE program for data debugging and plotting of input geometries.

TABLE 1 OPERATIONAL MODULES OF PLANS – CURRENT STATUS (5/77)

PROGRAM	APPLICATION	ELEMENT LIBRARY	# ELEM	# NODE	TYPE OF LOADING	MATERIAL BEHAVIOR	EQUILIB. CORRECT	SUB-INCREMT.	RESTART	PLOTTING	MESH GENERATOR
BEND	Analysis of 3-D thin-walled built-up structures where bending is significant Plastic analysis only	• Beam (9 cross-sections) • Stringer (CSS-LSS) • Triangle Membrane Family (CST-LST) • Triangle Bending & Membrane	• Variable	• Variable	• Surface • Concentrated • Edge • Thermal (Ex. Beams) • Proportional only • Cyclic	• Isotropic-Orthotropic Kinematic Hardening • Linear & Nonlinear Hardening • Ideally Plastic	Yes	No	Yes	Yes (see Satellite)	None
HEX	Analysis of massive 3-D bodies Plastic analysis only	• 3-D Hexahedron 8 node - 20 node Isoparametric •	• 2500	• 2500	• Surface • Concentrated • Thermal • Cyclic • Proportional only	• Isotropic Kinematic hardening • Linear & Nonlinear Hardening • Isotropic-Orthotropic • Ideally Plastic	Yes	No	Yes	None	Limited to 8 node mes.
REVBY	Axisymmetric analysis of bodies of revolution Plastic analysis only	• Isoparametric Shell of Rev. • Rev. Triangle • Thin Ring	• 600	• 900	• Surface • Concentrated • Line • Cyclic • Proportional only	• Same as HEX	No	No	No	Yes (see Satellite)	None
CPLANE-GM	Analysis of 3-D thin-walled built-up structures where bending is not significant Plastic & Geometric Nonlinearity (Small strain-large deflect.)	• Beam (9 cross-sections) • Stringer • Triangle Membrane Family (CST-LST)	• 600	• 900	• Surface • Concentrated • Edge • Thermal (Ex. Beam.) • Cyclic • Non-Proportional	• As in BEND	Yes	No	Yes	Yes (see Satellite)	None

In addition, a number of spin-off special purpose modules have been developed: FAST for fracture analysis, AXSHEL for the nonlinear analysis of shells of revolution, and BENST for the buckling analysis of large built-up structures. A general purpose nonlinear dynamic analysis module, DYCAST, is in final stages of development. These modules are not included in this document.

The modules that analyze material nonlinearities alone employ the "initial strain" concept within an incremental procedure to account for the effect of plasticity and include the capability for cyclic plastic analysis. The "initial strain" approach does not require that the stiffness matrix be updated at each step in the analysis but rather, the effect of plasticity enters into the analysis as an effective load vector. The cyclic plastic behavior is accounted for by implementing the Prager-Ziegler kinematic hardening theory. Geometric nonlinearities are included in the PLANS system programs by making use of an "updated" or convected coordinate approach, which requires the reformation of the stiffness matrix due to changes in the geometry and stress field during the incremental procedure.

The theoretical foundations upon which the PLANS system programs are based can be found in a companion volume, "PLANS – A Finite Element Program for Nonlinear Analysis of Structures, Volume I – Theoretical Manual," NASA Contractor Report NASA CR-2568.

This present volume describes the input data preparation for the four principal modules of PLANS, BEND, HEX, REVBY, and OUT-OF-PLANE MG as well as the SATELLITE program for input checking. In keeping with the philosophy of the PLANS system, the description of the input for each of the modules is in a self-contained section. Also presented is a section that shows the input data decks for a representative number of sample problems.

The input for all the modules begins with a title card that allows for any 80 character title (specified in columns 1-80). This title serves as a page heading for subsequent computer output. The input data following this card is divided into a number of functional groups, each describing a specific type of input information. These groups are briefly described below and schematically shown in Fig. 1. Each input group must be read in the specific order shown in the figure. In general, each group is delineated with a specific section end card. This is the alphanumeric SEND, left justified on the input card, in columns 1 through 4 or a blank card. The input groups are as follows:

I	Title Card
II	Program Control Parameters and Options
III	Node Specification This section defines an allowable set of node point identification numbers
IV	Element Connectivity Defines each element by specifying its type (i.e., beam, triangle, ..., etc.), identification number and connecting node points
V - VII	Node Point Coordinates Defines the location of each node point in a global cartesian coordinate system
VIII	Node Point Single and Multipoint Constraints Defines boundary conditions and nodal constraint equations
IX	Element Material and Section Properties Section properties include element thickness, area and moment of inertia where applicable.

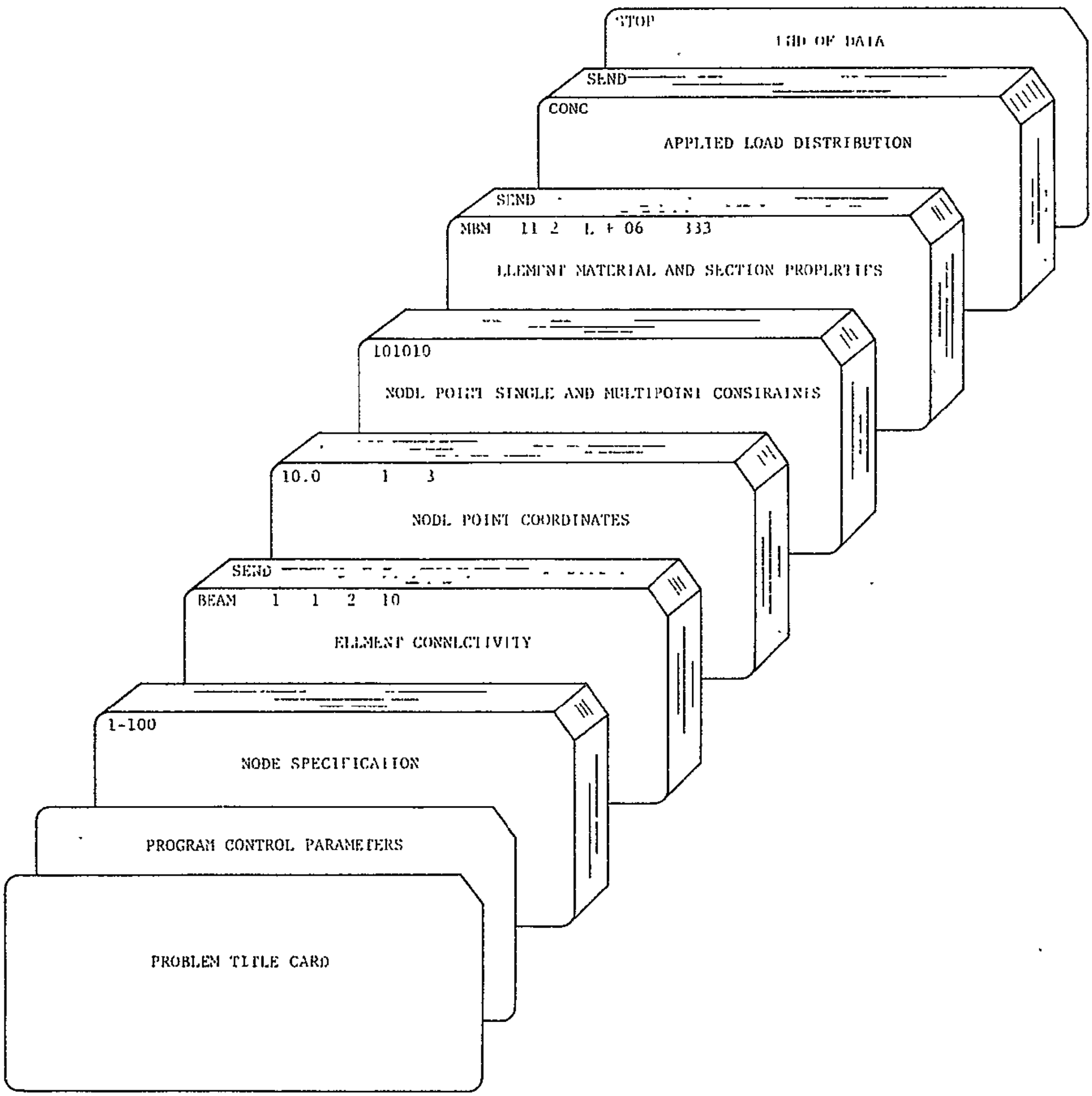


Fig. 1 Order of Groups of Input Data

Material properties include such quantities as Young's modulus, Poisson's ratio, and quantities defining the plastic response such as yield stress and hardening parameters.

X Applied Load

Defines spatial distribution of applied load

The last card in the deck is an alphanumeric STOP or END left justified on an input card in columns 1 through 4. STOP indicates that the job is complete and END indicates another problem deck follows.

Some general rules have been used in designing the input. These rules are listed below:

- Some of the data sections make use of a "keyword" of up to four characters, left justified in their appropriate field to specify an item of data. For example, cards specifying element connectivity for membrane triangles begin with TRIM, and material properties by MAT1. All sections that make use of keywords end with a section end keyword SEND
- Sections that specify data without keywords end with a blank card section delimeter
- The data deck ends with one of two keywords. If END is used, the program reads a new problem data deck. If STOP is used, as will probably be the case for most problems, the job ends
- Two formats for input are used, E15.7 for floating point input (fields of 15) and I5 for integer or fixed point input (fields of 5). The fixed point

(integer) data must be right justified. The floating point data can be written in several forms. For example, 10.0 can be input as:

10.0 any place in the field, or
1.0E+01, 1.0E+1, 1.0E1, where the
field is right justified

There are a number of places in the program where applicable node points or elements must be specified with a set of data. In these cases the nodes or members are specified by entering the appropriate number on the input cards in fields of five. However, for this purpose the user can also utilize a shorthand form of the input. That is, specifying m and -n consecutively is the equivalent of the specification of nodes or elements m, m+1, m+2, ..., n and specifying m, -p, and -n consecutively is the equivalent of the specification of nodes (elements) m, m+p, m+2p, ..., m+kp where m+kp is the highest integer of the form less than or equal to n. For example, the specification of nodes 1 through 100 is written as 1-100 and nodes 1, 3, 5, ..., 99 as 1, -2, -99. This card input appears in fields of 5 (I5 format) with 16 items per card. Any number of continuation cards may be used. A blank I5 field ends the specification.

SECTION 2

INSTRUCTIONS FOR USE OF BEND

A Program for the Elastic-Plastic Analysis of Built-up Structures

INSTRUCTIONS FOR USE OF BEND

BEND is a finite element program to treat the elastic, elastic-plastic or elastic-cyclic plastic response of arbitrary built-up thin walled structures where bending and membrane effects are equally important. The finite element library consists of the following elements:

- Three-node uniform strain triangle
- Six-node linearly varying strain triangle
- Four- and five-node hybrid triangles to be used as transition elements between three- and six-node elements (see Fig. 1a)
- Two-node uniform strain stringer
- Three-node linearly varying strain stringer
- Beam with various cross sections subjected to bending about two planes as well as torsion
- Higher order triangular plate element with bending and membrane capability

The program is capable of treating the elastic and elastic-ideally plastic, linear strain hardening, and nonlinear strain hardening behavior of orthotropic materials. Further, the kinematic hardening theory of plasticity is used (Refs. 1-3) so that provision for cyclic loading conditions involving reversed plastic deformation is included.

The input to the program is categorized in the following sections:

I. Problem Title FORMAT (20A4)

Any 80-character title describing the problem.

II. NPNTC, NPRNT, IRESRT, NUTAP, INPRT FORMAT (5I5)

$0 \leq$ NPNTC ≤ 63:

NPNTC is the sum of the following integers corresponding to the option desired.

If NPNTC = 0 No intermediate printout
= 1 Print the load vector
= 2 Print element stiffness matrix
= 16 Print each element stiffness matrix entry to be stacked with its stacking index
= 32 Print the total stiffness matrix

For example, if it is desired to print the load vector and the total stiffness matrix, NPNTC = 1 + 32 = 33.

NPRNT:

If ≤ 0, perform elastic analysis only

If > 0, perform plastic analysis, printing output every NPRNT increments of load

IRESRT: (See Section XVIII.)

0, Elastic (and/or) plastic run. Do not generate a restart tape

1, Elastic (and/or) plastic (cyclic) run. Build a new restart tape

2, Plastic run with elastic values from previously created restart tape

3, Plastic restart run starting at some specified load level

4, Cyclic restart run

NUTAP:

Applicable only if restarting from a restart tape

0, No new tape written

1, Complete ~~new tape~~ created and additional restart data written

INPRT:

- 0, Write restart tape only at P = PMAX (i.e., at maximum load)
- N, Write restart tape every N increments of load

III. Node Specification (16I5)

This section defines an allowable set of external node point numbers. The maximum node number that can be used is 9999. The program uses this information in two ways. First to set up a table of allowable node points that is used to check all subsequent node point input. Secondly, the program converts each external node number to an internal number consecutively in the order that the node appears on the input card. Consequently the order of the input of external node numbers is completely arbitrary and need not be increasing monotonically. In practice the node numbers should be numbered so as to minimize the bandwidth. Once the input is read the program operates with the internal numbers which are now numbered from 1 through the number of nodes in the model. In this manner the node ordering and therefore the bandwidth of the stiffness matrix can be easily changed and nodes can be inserted or deleted by changing the external node specification.

The input is specified by entering the appropriate number on the input cards in fields of five. However, for this purpose the user can also utilize a shorthand form of the input. That is, specifying m and -n consecutively is the equivalent of the specification of nodes m, m + 1, m + 2, ..., n and specifying m, -p, and -n consecutively is the equivalent of the specifiction of nodes m, m + p, m + 2p, ..., m + kp where m + kp is the highest integer of the form less than or equal to n. For example, the specification of nodes 1 through 100 is written as 1 - 100 and 1, 3, 5, ..., 99 as 1-2-99. This card input appears in fields of 5 (I5 Format) with 16 items per card. Any number of continuation cards may be used. A zero or blank I5 field ends the specification.

IV. Member Connectivity (Node Numbers of Each Member)
FORMAT (A4, 6X, 9I5)

The first alphanumeric field defines the element type:

TRIM - Triangular membrane element (Ref. 3)

BEAM - Beam element

STRG - Stringer element (Ref. 4)

TRIP - Triangular plate bending element (with membrane effects) (Ref. 5)

The first integer field designates the member number. The next integer fields designate the connecting nodes as follows:

TRIM - The nodes for the triangular family of elements are specified around the perimeter beginning with a major (vertex) node, and followed by a minor (midside) node and then alternatively major and minor as shown in Fig. 1. The absence of a minor node must be indicated by a zero or blank field in the proper position.

BEAM - Three node specifications are necessary for the beam element.

Nodes i and j (Fig. 2) which designate the element end points and a third node k, defining the normal to the beam axis about which the section properties are specified. This additional node may be a node of the structural idealization or it may be a "fictitious node" specified just for the purpose of defining the beam section properties. This is shown in Fig. 2.

STRG - Three node specifications are necessary for the stringer element. Nodes i and j, connecting the end points and if desired an additional node designating a midpoint node. This is shown in Fig. 3. A zero or blank for the midpoint node specifies a two-node stringer. The midside node is the third node specified.

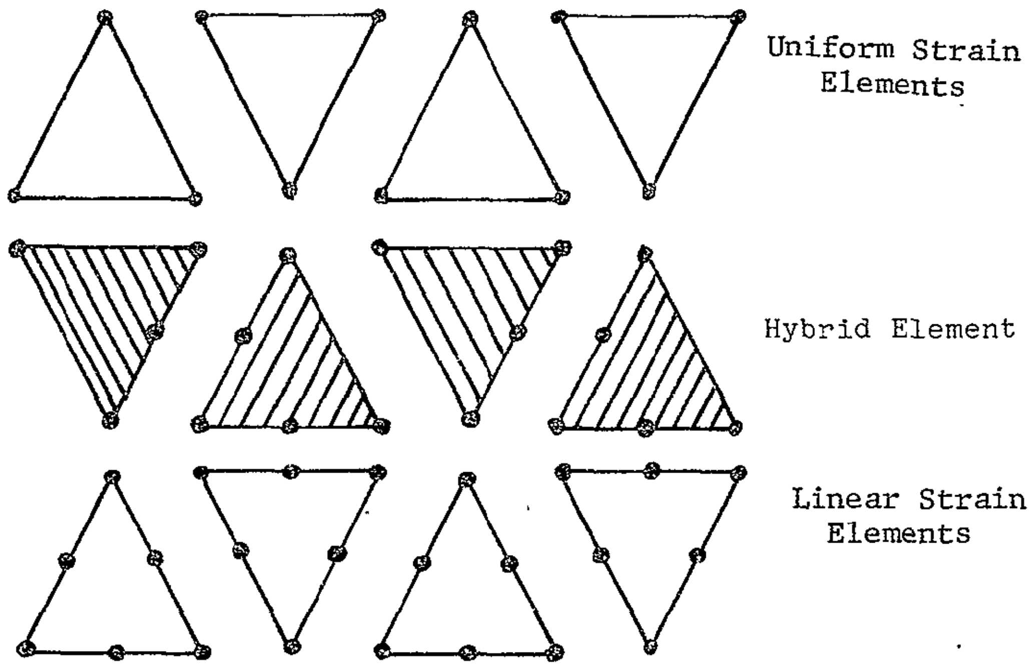


Fig. 1a Triangular Family of Finite Elements Used

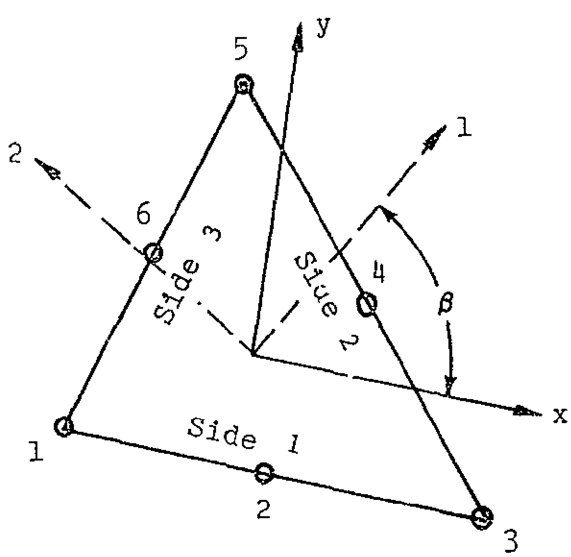


Fig. 1b Elements Topology

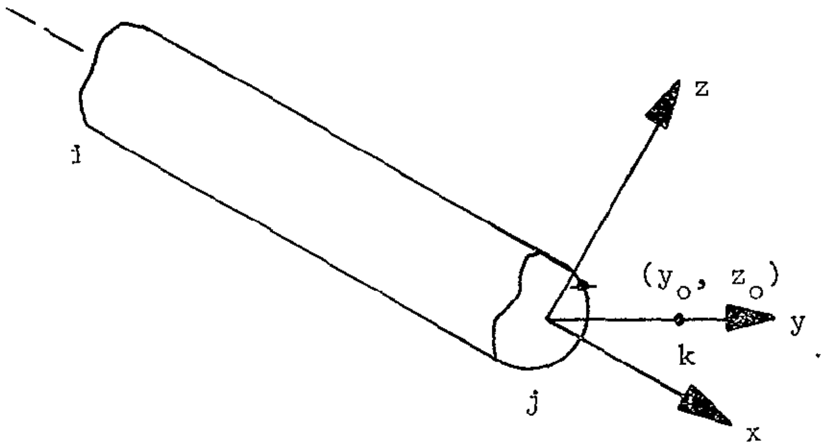


Fig. 2 Beam Element

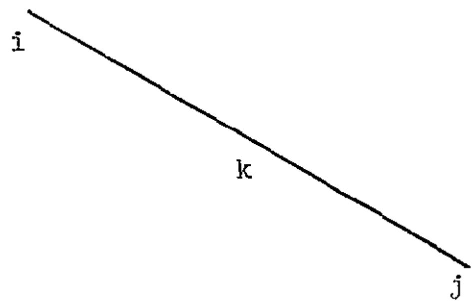


Fig. 3 Stringer Element

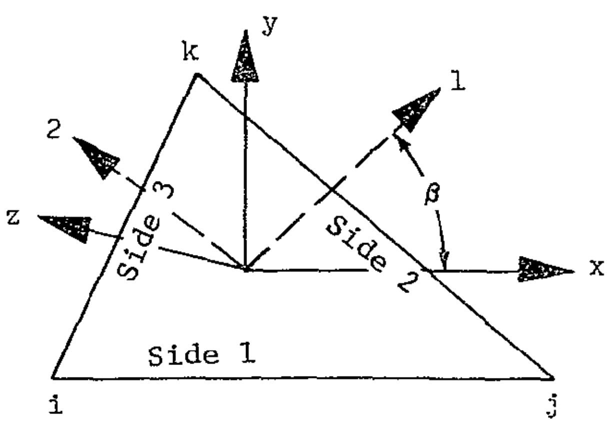


Fig. 4 Triangular Plate Element - Bending and Membrane

TRIP - Three nodes are used to specify the plate element i, j, k as shown in Fig. 4.

If a planar structure is being analyzed using triangular plate and membrane elements, stress and strain output can be calculated with respect to the global coordinate system rather than the element local system by specifying one of the following cards:

GLXY - Structure in the x-y plane

GLXZ - Structure in the x-z plane

GLYZ - Structure in the y-z plane

SEND - Ends the section.

V. X-Coordinates of Nodes FORMAT (E15.7, 13I5)

The x-coordinates of the nodes appearing in the I5 fields are set to the value in the E15.7 field. Any number of continuation cards may be used; their first fifteen columns are ignored. A zero or blank I5 field terminates the card scan for a given x-coordinate. A zero or blank first I5 field (columns 16-20) on a noncontinuation card ends the section. Both shorthand representations of Section III are allowed.

VI. Y-Coordinates of the Nodes. Same as Section V.

VII. Z-Coordinates of the Nodes. Same as Section V.

VIII. <u>Boundary Conditions FORMAT (12I1, 3X, 13I5, /, 15X, 13I5)</u>

The first twelve fields give the boundary conditions specifications in the order: u, v, w, θ_x, θ_y, θ_z, ϵ_x, ϵ_y, ϵ_{xy}, κ_x, κ_y, κ_{xy}; where u, v, w are global displacements in the x, y, z directions, respectively; θ_x, θ_y, θ_z are global rotations with respect to the x, y, z axes; ϵ_x, ϵ_y, ϵ_{xy} are membrane components of the local membrane strain tensor; κ_x, κ_y, κ_{xy} are components of the local curvature tensor. The last six fields for the local membrane strains and curvatures (higher order degrees of freedom) are only used if a plate element is attached to the node. Both tensors are calculated in the local coordinate system (see Fig. 4) of the plate element. This local coordinate system is defined by the first plate element connecting that node that appears in the input stream. For example,

TRIP	3	6	3	2	Node 3 is not listed previously in the element input
TRIP	4	3	6	7	
TRIP	5	7	4	3	

The tensor quantities at node 3 are all defined in the local coordinate system of element 3. See Appendix IV for examples of plate boundary conditions.

The rotational degrees of freedom θ_x, θ_y, θ_z are applicable only for beam and plate elements. If nonzero values are given to rotational or higher order degrees of freedom at a node and beam and/or plate elements are not attached to the node then a singular stiffness matrix will result.

Zero denotes a fixed degree of freedom

One denotes a free degree of freedom

Two will result in the application of a unit generalized displacement, or a corresponding card may be included in Section IX (dependent degrees of freedom) designating the magnitude of the applied generalized displacement.

The 13I5 fields give the applicable nodes for the boundary condition specifications, with both shorthand notations of Section III permitted. Any number of continuation cards may be used for a given specification. However, only the 13I5 fields are used on a continuation card. A zero or blank I5 field terminates the card scan for a given boundary condition specification. Note: If the last field of a card (columns 76-80) is the last specification, an additional blank card (continuation card) must follow. A zero or blank first I5 field (columns 16-20) on a noncontinuation card ends the section. If a node's boundary conditions are not specified in this section, all the degrees of freedom are assumed to be free. To change this default condition, the first card of this section should be set to the desired default (i.e., in the absence of beams and plates, 111000000000) with all nodes used in the problem specified. In the absence of plate elements the default is automatically 111111000000.

IX. Dependent Degrees of Freedom FORMAT (2I5, 2(5X, 2I5, E15.7))

This section designates the input for both single and multipoint constraints as well as applied displacements of the form:

1) $\delta_i = \alpha_1 \delta_{j1} + \alpha_2 \delta_{j2} + \cdots \alpha_n \delta_{jn}$

where δ_i is a dependent degree of freedom,
$\delta_{j1} \cdots \delta_{jn}$ are independent degrees of freedom,
and $\alpha_1, \alpha_2 \cdots \alpha_n$ are coefficients.

2) Rotation of displacements at a node

$$\delta_i = \alpha_1\tilde{\delta}_i + \alpha_2\tilde{\delta}_j + \alpha_3\tilde{\delta}_k$$

$$\delta_j = \beta_1\tilde{\delta}_i + \beta_2\tilde{\delta}_j + \beta_3\tilde{\delta}_k$$

$$\delta_k = \gamma_1\tilde{\delta}_i + \gamma_2\tilde{\delta}_j + \gamma_3\tilde{\delta}_k$$

where the α, β, γ's are the direction cosines of the rotation, δ_i, δ_j, δ_k are the displacements with respect to the original global directions and $\tilde{\delta}_i$, $\tilde{\delta}_j$, $\tilde{\delta}_k$ are the components of displacements at the node with respect to the new coordinate axes. An example of this capability is given in Appendix I.

3) Applied generalized displacement

$$\delta_i = \alpha_1$$

where the coefficient α_1 is the applied generalized displacement.

The first two-fields designate a node number and a degree of freedom (i.e., 1-12). The dependency is defined in the following three fields. The two integer fields designate the node number and degree of freedom number and the coefficient by the floating point field. If there is another dependency for the node, it is designated in a similar fashion in the next three fields. Any number of continuation cards can be used with the first two fields blank. The section is ended by a blank or zero in the third integer field (blank card). Examples of the use of multipoint constraints are in Appendix I.

X. Material and Section Properties

The format for this input is dependent upon the member type. Each type of input begins with a word of up to four characters.

MAT1	Material properties for plane stress, necessary with membrane triangles (TRIM) and bending triangles (TRIP). For use with initially isotropic materials which exhibit perfectly plastic or strain hardening behavior and initially orthotropic materials that exhibit perfectly plastic behavior.
MAT2	Material properties for plane stress, necessary with membrane triangles (TRIM) and bending triangles (TRIP) with initially isotropic or orthotropic behavior that require orthotropic kinematic hardening theory
MBM	Material properties for a beam element
MSTG	Material and section properties for a stringer
THIK	Member thickness for triangular membrane (TRIM) and plate (TRIP) elements
MBET	Angle between local axes and principal directions of orthotropy for TRIM and TRIP elements (see Figs. 1b and 4)
NLRS	Layer and layer output information for TRIP elements
SREC	Beam section properties for a solid rectangular section

SCIR	Beam section properties for a solid circular section
ZSEC	Beam section properties for a Z-section
ISEC	Beam section properties for an I-section
HCIR	Beam section properties for a hollow circular section
HREC	Beam section properties for a hollow rectangular section
LSEC	Beam section properties for an L-section
TSEC	Beam section properties for a T-section
CSEC	Beam section properties for a Channel section
SEND	Ends the section

MAT1 - Plane Stress Material Properties - FORMAT (A4, 1X, 5E15.7, /, 2E15.7, /, 4E15.7, /, 5E15.7, /, (16I5))

The first four cards specify material properties, as follows:

CARD 1: MAT1

EONE = Young's modulus in principal property axis (1)

ETWO = Young's modulus in principal property axis (2)

BETA = No longer used. Set equal to zero.

GONTO = Shear modulus in (1)-(2) principal property plane

VONTO = Poisson's ratio, ν_{12}

Note:

$$\epsilon_1 = \frac{\sigma_1}{E_1} - \frac{\nu_{12}}{E_2}\sigma_2$$

$$\epsilon_2 = \frac{-\nu_{21}}{E_1}\sigma_1 + \frac{\sigma_2}{E_2}$$

$$\gamma_{12} = \frac{\tau_{12}}{G_{12}}$$

CARD 2: TALF-1 = Coefficient of thermal expansion in 1-axis direction, α_1

TALF-2 = Coefficient of thermal expansion in 2-axis direction, α_2

CARD 3: SIGOX = Yield stress in principal 1-direction

SIGOY = Yield stress in principal 2-direction

SIGOZ = Yield stress in principal 3-direction

SIGXY = Shear yield stress in principal 1-2 plane

CARD 4: RMOSN = If RMOSS $\neq 0$; RMOSN = n, the shape parameter used in Ramberg-Osgood representation of stress-strain behavior

If RMOSS = 0; RMOSN = $\bar{\alpha}$ the slope of the linear strain-hardening stress-strain representation, i.e., $\bar{\alpha} = E_T/E$ where E_T is the tangent modulus

RMOSS = If RMOSS $\neq 0$; RMOSS = Ramberg-Osgood parameter $\sigma_{0.7}$ (see Ref. 7)

Note 1: If RMOSN = 0 and RMOSS = 0, the material for the element(s) is assumed to be elastic-ideally plastic

RMOSE = Ramberg-Osgood parameter E (Young's modulus)

YLDST = Yield stress in tension

YLDSC = Yield stress in compression

Input for the yield-stress in tension and compression has been maintained in order to accommodate materials that exhibit initial anisotropic plastic behavior. In this case an initial translation of the yield surface is made consistent with the kinematic hardening theory.[†]

Note: Only initially isotropic materials can be treated when considering linear or nonlinear strain hardening using this property card.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

[†] Isakson, G., Armen, H. Jr., and Pifko, A., "Discrete-Element Methods for the Plastic Analysis of Structures," NASA Contractor's Report NASA CR-803, October 1967.

MAT2 - Plane Stress Material Properties - FORMAT (A4, 1X, 5E15.7, /, 2E15.7, /, 4E15.7, /, 5E15.7, /, E15.7, /, (16I5))

The first five cards specify material properties, as follows:

CARD 1: MAT2

EONE = Young's modulus in principal property axis (1)

ETWO = Young's modulus in principal property axis (2)

BETA = No longer used. Set equal to zero.

GONTO = Shear modulus in (1)-(2) principal property plane

VONTO = Poisson's ratio, ν_{12}

Note:

$$\epsilon_1 = \frac{\sigma_1}{E_1} - \frac{\nu_{12}}{E_2}\sigma_2$$

$$\epsilon_2 = -\frac{\nu_{21}}{E_1}\sigma_1 + \frac{\sigma_2}{E_2}$$

$$\gamma_{12} = \frac{1}{G_{12}}\tau_{12}$$

CARD 2: TALF-1 = Coefficient of thermal expansion in 1-axis direction, α_1

TALF-2 = Coefficient of thermal expansion in 2-axis direction, α_2

CARD 3: SIGOX = Yield stress in principal 1-direction

SIGOY = Yield stress in principal 2-direction

SIGOZ = Yield stress in principal 3-direction

SIGXY = Shear yield stress in principal 1-2 plane

CARD 4: RMONX = If RMOSX ≠ 0; RMONX = n, the shape parameter used in the Ramberg-Osgood representation of the $\sigma_1 - \epsilon_1$ component of the stress-strain behavior.

If RMOSX = 0; RMONX = $\bar{\alpha}_1$ the slope of the linear strain-hardening stress-strain representation in the 1-direction, i.e.,

$\bar{\alpha}_1 = \frac{E_{T1}}{E_1}$ where E_{T1} is the tangent modulus of the $\sigma_1 - \epsilon_1$ curve.

RMOSX If RMOSX ≠ 0; RMOSX = Ramberg-Osgood parameter $\sigma_{0.7}$ in principal 1-direction

RMONY Same as RMONX in principal 2-direction

RMOSY Same as RMOSX in principal 2-direction

RMONXY Same as RMONX for shear component in 1-2 plane; $\tau_{12} - \gamma_{12}$ stress-strain curve

CARD 5: RMOSXY Same as RMOSX for shear component in 1-2 plane; $\tau_{12} - \gamma_{12}$ stress-strain curve

Note: No component of the stress-strain curves may be ideally plastic, i.e., RMONX or RMONY or RMONXY = 0 is not allowed. If either is zero all must be zero and a MAT1 card should be used. The Ramberg-Osgood representation of the stress-strain curve is given by

$$\epsilon = \frac{\sigma}{E} + \frac{3\sigma}{7E}\left(\frac{\sigma}{\sigma_{0.7}}\right)^{n-1}$$

$$\gamma = \frac{\tau}{G} + \frac{3\tau}{7G}\left(\frac{\tau}{\tau_{0.7}}\right)^{n-1}$$

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

MBM - Beam Material Properties - FORMAT (A4, 1X, 5E15.7, /, E15.7, /, (16I5))

CARD 1: MBM

E = Young's modulus

ANU = Poisson's ratio

RMOSS = If not equal to zero, RMOSS equals Ramberg-Osgood parameter, $\sigma_{0.7}$

RMOSN = If RMOSS $\neq$ 0; RMOSN = n, the shape parameter used in Ramberg-Osgood representation of stress-strain behavior. If RMOSS = 0; RMOSN = $\bar{\alpha}$, the slope of the linear strain-hardening stress-strain representation, i.e., $\bar{\alpha} = E_T/E$ where E_T is the tangent modulus

YLDST = Yield stress

CARD 2: TALF = Coefficient of thermal expansion

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. <u>A zero or blank I5 field ends each member listing.</u>

MSTG - Stringer Properties - FORMAT (A4, 1X, 5E15.7, / E15.7 /, (16I5))

CARD 1: MSTG

E = Young's modulus

A = Cross-sectional area

RMOSN = If RMOSS $\neq 0$; RMOSN = n, the shape parameter used in Ramberg-Osgood representation of stress-strain behavior.

If RMOSS = 0; RMOSN = $\bar{\alpha}$, the slope of the linear strain-hardening stress-strain representation, i.e., $\bar{\alpha} = E_T/E$ where E_T is the tangent modulus

RMOSS = If RMOSS $\neq 0$; RMOSS = Ramberg-Osgood parameter $\sigma_{0.7}$

Note 1: If RMOSN =.0 and RMOSS = 0, the material for the element(s) is assumed to be elastic-ideally plastic

YLDST = Yield stress

CARD 2: TALF = Coefficient of thermal expansion

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

THIK - Element Thicknesses - FORMAT (A4, 1X, E15.7, /, (16I5))
(Necessary with membrane elements TRIM) and plate elements (TRIP))

CARD 1: THIK

THICK = Element thickness

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

MBET - Orientation of Axes of Material Anisotropy - FORMAT (A4, 1X, E15.7, /, (16I5))

CARD 1: MBET

BETF = Angle β in degrees between local x-axis and principal 1-axis for material orthotropy. See Figs. 1b and 4. Only applicable for TRIM and TRIP elements

Note: This card is an optional card. The default is BETF = 0.0 for all members. It should be used with initially orthotropic materials or those exhibiting orthotropic strain-hardening characteristics.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

NLRS - Layer and Print Option for TRIP Elements - FORMAT (A4, 1X, /, 2I5)

CARD 1: NLRS

CARD 2: NLRS = Number of layers through the thickness at which stresses are calculated for TRIP elements. NLRS must be an even number ≤ 20

NPLT = Print out option:

= 0; Print stresses, strains, etc. only at top and bottom surfaces

= 1; Print stresses, strains, etc. at each layer through the thickness as well.

Note: This is an optional card. The defaults are NLRS = 10 and NPLT = 0. If one is changed both should be specified. This card controls option for all TRIP elements.

Beam Section Properties - FORMAT (A4, 1X, 5E15.7, /, 3E15.7, /, formats for Cards 3 and 4 (see below), /, (16I5))

The cards specifying beam section properties all start with the following information:

CARD 1: SREC, SCIR, ISEC, ZSEC, HCIR, HREC, LSEC, TSEC, CSEC — Defined at the beginning of the section. The different cross sections are shown in Tables 1 and 2.

A = cross-sectional area

Iyy = moment of inertia about y axis (see Fig. 5)

Izz = moment of inertia about z axis (see Fig. 5)

Iyz = product of inertia

J = torsional rigidity

If the quantities, A, I_{yy}, I_{zz}, I_{yz} are input as zero or blank then these quantities are calculated automatically.

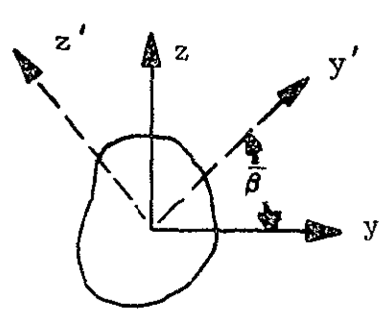


Fig. 5 Definition of Coordinate Axes in Cross Section of Beam Element

CARD 2: YO = Eccentricity of attachment point in the y_o direction

ZO = Eccentricity of attachment point in the z_o direction

$\bar{\beta}$ = angle defining the transformation of the Iy, Iz, Iyz to another reference axis (see Fig. 5)

Additional cards are required according to which section is specified. The notation for each section is shown in Table 1.

SREC - FORMAT (2E15.7)

CARD 3: A = Width

B = Depth

SCIR - FORMAT (E15.7)

CARD 3: R = Radius

ZSEC, ISEC, and CSEC - FORMAT (3E15.7, /, 3E15.7)

CARD 3: A1 = Dimension of upper flange (parallel to y-axis)

A2 = Dimension of web (parallel to z-axis)

A3 = Dimension of lower flange (parallel to y-axis)

CARD 4: T1 = Thickness of upper flange

T2 = Thickness of web

T3 = Thickness of lower flange

HCIR - FORMAT (2E15.7)

CARD 3: R = Outer radius

T = Thickness

HREC, LSEC, and TSEC - FORMAT (4E15.7)

CARD 3: A1 = Width (parallel to y-axis)

A2 = Depth (parallel to z-axis)

T1 = Thickness of upper and/or lower flanges

T2 = Thickness of vertical webs

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

SEND Ends the section.

TABLE 1 SECTION INPUT AND GEOMETRY

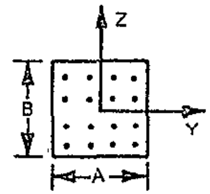
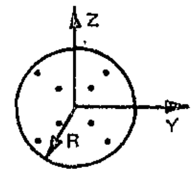
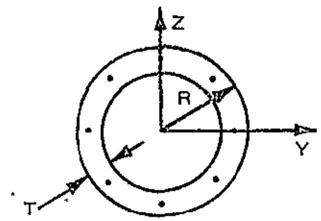
Section	Keyword	Input	
Solid Rectangle	SREC	Area, moments of inertia, I_{yy}, I_{zz}, I_{yz}, J eccentricities, y_o, z_o; transformation angle $\bar{\beta}$. Section width and height, a, b.	
Solid Circular	SCIR	Area, moments of inertia, I_{yy}, I_{zz}, I_{yz}, J eccentricities, y_o, z_o, transformation angle $\bar{\beta}$. Section radius, r.	
Hollow Circular Section	HCIR	Area, moment of inertia, I_{yy}, I_{zz}, I_{yz}, J eccentricities, y_o, z_o, transformation angle $\bar{\beta}$. Outer radius, r, thickness, t.	

TABLE 1 SECTION INPUT AND GEOMETRY (CONT)

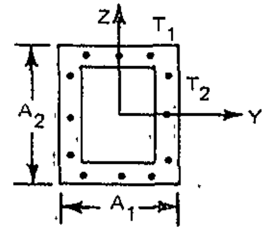
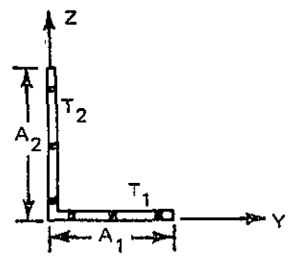
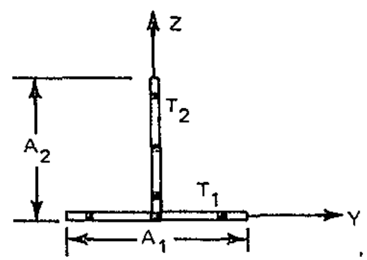
Section	Keyword	Input	
Hollow Rectangular Section	HREC	Area, moment of inertia, I_{yy}, I_{zz}, I_{yz}, J eccentricities, y_o, z_o, transformation angle $\bar{\beta}$. Width, a_1, depth, a_2, thickness of upper and lower flanges, t_1, thickness of vertical webs, t_2	
L-Section	LSEC	Area, moment of inertia, I_{yy}, I_{zz}, I_{yz}, J eccentricities, y_o, z_o, transformation angle $\bar{\beta}$. Dimension of flange and web a_1,a_2 thickness flange, and web, t_1, t_2.	
T-Section	TSEC	Same as for L-Section	

TABLE 1 SECTION INPUT AND GEOMETRY (CONT)

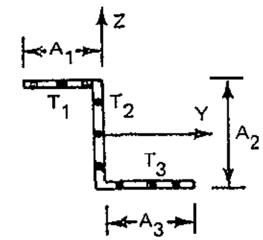
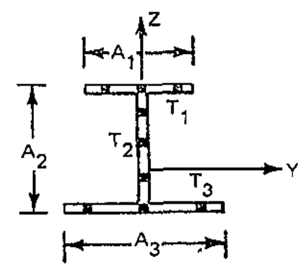
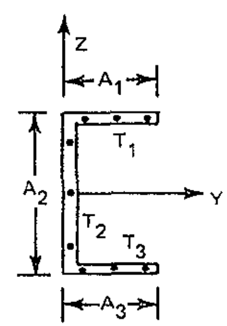
Section	Keyword	Input	
Z-Section	ZSEC	Area, moments of inertia, I_{yy}, I_{zz}, I_{yz}, J eccentricities, y_o, z_o, transformation angle $\bar{\beta}$. Dimension of upper and lower flange, a_1, a_3. Dimension of web, a_2, thickness of upper and lower flange, t_1, t_3, thickness of web, t_2 NOTE: $a_1 = a_3$; $t_1 = t_3$	
I-Section	ISEC	Same as Z-section $a_1 \neq a_3$; $t_1 \neq t_3$	
Channel Section	CSEC	Same as Z-Section $a_1 = a_3$; $t_1 = t_3$	

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS

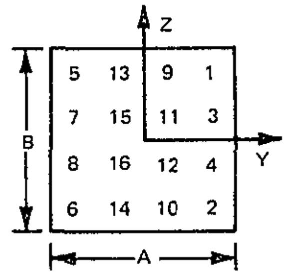
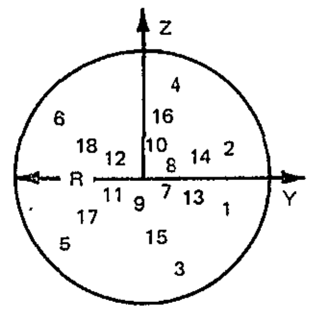
Section	Keyword	Number Stress Points	Location of Stress Points			Location of Stress Points
Solid Rectangle	SREC	16		± y	± z	
			1	0.430568a	0.430568b	
			3	0.430568a	0.169991b	
			9	0.169991a	0.430568b	
			11	0.169991a	0.169991b	Shear Center at Centroid
Solid Circular	SCIR	18		r	± θ°	
			2	0.887a	12.1	
			4	0.887a	60.98	
			6	0.887a	137.05	
			8	0.1127a	12.1	
			10	0.1127a	60.98	
			12	0.1127a	137.05	
			14	0.500a	12.1	
			16	0.500a	60.98	
			18	0.500a	137.05	Shear Center at Centroid

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

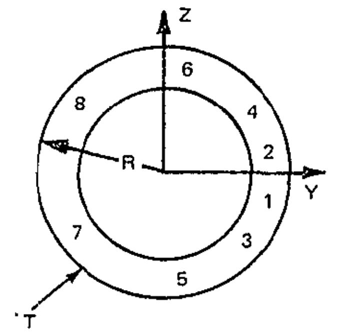
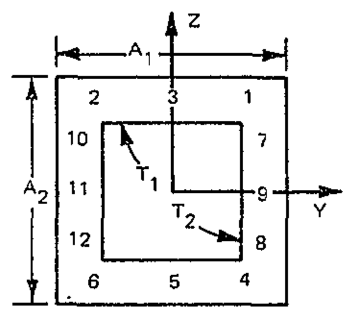
Section	Keyword	Number Stress Points	Location of Stress Points	Location of Stress Points
Hollow Circular Section	HCIR	8	r, θ° 1 a 352.8 2 a 7.2 3 a 323.4 4 a 36.6 5 a 274.6 6 a 85.4 7 a 213.0 8 a 147.0	 Shear Center at Centroid
Hollow Rectangular Section	HREC	12	$\pm y$, $\pm z$ 1 $0.77459\bar{a}_1$ $0.5\bar{a}_2$ 3 0 $0.5\bar{a}_2$ 7 $0.5\bar{a}_1$ $0.77459\bar{a}_2$ 9 $0.5\bar{a}_1$ 0 $\bar{a}_1 = a_1 - t_2$ $\bar{a}_2 = a_2 - t_1$	 Shear Center at Centroid

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

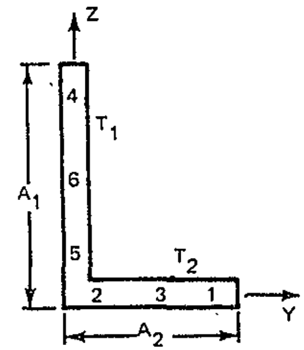
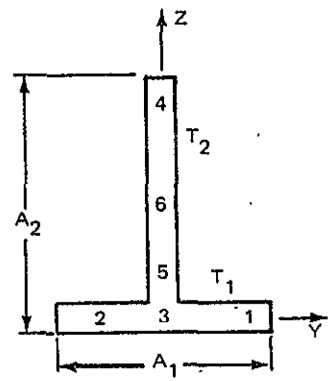
Section	Keyword	Number Stress Points	Location of Stress Points			Location of Stress Points
				y	z	
L-Section	LSEC	6	1	$0.88730\bar{a}_2$	0.0	
			2	$0.11271\bar{a}_2$	0.0	
			3	$0.50000\bar{a}_2$	0.0	
			4	0.0	$0.88730\bar{a}_1$	
			5	0.0	$0.11271\bar{a}_1$	
			6	0.0	$0.50000\bar{a}_1$	Axes at Shear Center
			$\bar{a}_1 = a_1 - \frac{t_2}{2}$, $\bar{a}_2 = a_2 - \frac{t_1}{2}$			
T-Section	TSEC	6	1	$0.77459a_1$	0.0	
			2	$-0.77459a_1$	0.0	
			3	0.0	0.0	
			4	0.0	$0.88730\bar{a}_2 + \frac{t_1}{2}$	
			5	0.0	$0.11271\bar{a}_2 + \frac{t_1}{2}$	
			6	0.0	$0.50000\bar{a}_2 + \frac{t_1}{2}$	Axes at Shear Center
			$\bar{a}_2 = a_2 - t_1$			

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

Section	Keyword	Number Stress Points	Location of Stress Points			Location of Stress Points
				y	z	
Z-Section	ZSEC	9	1	$-0.88730a_1$	$\bar{a}_2$	Shear Center at Centroid
			2	$-0.11271a_1$	$\bar{a}_2$	
			3	$-0.5000a_1$	$\bar{a}_2$	
			4	$0.88730a_1$	$\bar{a}_2$	
			5	$0.11271a_1$	$\bar{a}_2$	
			6	$0.5000a_1$	$\bar{a}_2$	
			7	0.0	$0.77459a_2'$	
			8	0.0	$0.77459a_2'$	
			9	0.0	0.0	
			$\bar{a}_2 = (a_2 - t_1)/2$, $t_1 = t_3$			
			$a_2' = (a_2 - t_1)$, $a_1 = a_3$			

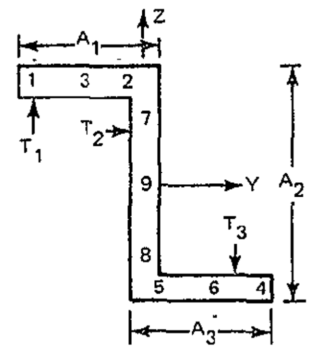


TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

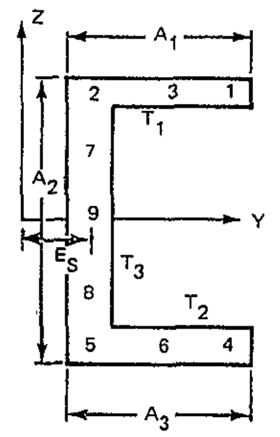
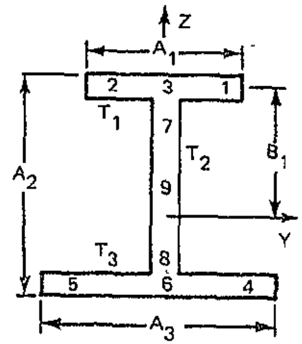
Section	Keyword	Number Stress Points	Location of Stress Points	Location of Stress Points
Channel Section	CSEC	9	y, z 1: $0.88730\bar{a}_1 + e_s$, $0.5\bar{a}_2$ 2: $0.11271\bar{a}_1 + e_s$, $0.5\bar{a}_2$ 3: $0.5000\bar{a}_1 + e_s$, $0.5\bar{a}_2$ 4: $0.88730\bar{a}_1 + e_s$, $-0.5\bar{a}_2$ 5: $0.11271\bar{a}_1 + e_s$, $-0.5\bar{a}_2$ 6: $0.5000\bar{a}_1 + e_s$, $-0.5\bar{a}_2$ 7: e_s, $0.77459\bar{a}_2$ 8: e_s, $-0.77459\bar{a}_2$ 9: e_s, 0.0 e_s - distance from shear center to web center line $e_s = \dfrac{1/2(a_1 - t_2)}{1 + 1/6\left(\dfrac{t_2(a_2 - t_1)}{t_1(a_1 - t_2)}\right)}$, $\bar{a}_2 = a_2 - t_1$ NOTE: $t_1 = t_3$; $a_1 = a_3$	 Axes at Shear Center

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

Section	Keyword	Number Stress Points	Location of Stress Points			Location of Stress Points
I-Section	ISEC	9		y	z	 Axes at Shear Center
			1	$0.774569a_1$	b_1	
			2	$-0.77459a_1$	b_1	
			3	0.0	b_1	
			4	$0.77459a_3$	b_2	
			5	$0.77459a_3$	b_2	
			6	0.0	b_2	
			7	0.0	$0.8873\bar{b}_1 + 0.11271\bar{b}_2$	
			8	0.0	$0.11271\bar{b}_1 + 0.8873\bar{b}_2$	
			9	0.0	$0.5\bar{b}_1 + 0.5\bar{b}_2$	

$$b_1 = \frac{\bar{a}_2\left(\frac{t_3}{t_1}\right)\left(\frac{a_3}{a_1}\right)^3}{\left[1+\left(\frac{t_3}{t_1}\right)\left(\frac{a_3}{a_1}\right)\right]^3}, \quad b_2 = b_1\left(\frac{t_1}{t_3}\right)\left(\frac{a_1}{a_3}\right)^3$$

$$\bar{a}_2 = a_2 - 0.5(t_1 + t_3) \; ; \; \bar{b}_1 = b_1 - 0.5t_1 \; ; \; \bar{b}_2 = b_2 - 0.5t_3$$

XI. Applied Loads

Many different types of loading are currently admissible — concentrated loads at nodes, distributed loads on an edge of a triangular member, distributed surface loads, distributed line loads on beams, and thermal loads. Each loading situation is designated by one of the following four-character key words:

CONC	(Concentrated force or moment at a node)
TRIA	(Distributed edge load, TRIM elements)
SURF	(Distributed surface loads, TRIP element)
EDG1	(Distributed edge load, side 1, TRIP element)
EDG2	(Distributed edge load, side 2, TRIP element)
EDG3	(Distributed edge load, side 3, TRIP element)
BMLO	(Distributed line load, beam element)
TMPU	(Temperatures at upper surface for TRIP element or temperatures for TRIM element)
TMPM	(Temperatures at middle surface for TRIP element)
TMPL	(Temperatures at lower surface for TRIP element)
SEND	(Section end)

Note: The keywords for the section on applied loads are used as section headings. The keyword appearing on an input card designates that the input to follow is associated with a particular type of applied loading. A blank card (as described in succeeding sections) delimits the input and indicates that the next card contains a different keyword. For example, CONC indicates input that follows is for concentrated loads and TRIA; distributed loads on an edge of a membrane triangle. Thus the input is as follows:

CONC

data

for

concentrated

loads

blank card

TRIA

data

for

distributed

loads

blank card

SEND

It should be noted that the keywords can appear in any order and may be specified more than one time in the applied load section.

TRIA - Distributed edge load (TRIM elements) - FORMAT (A4, /, 4I5, 3E15.7, /, 15X, 3E15.7)

Each card gives the load components applied at a member side (in units of force) as follows:

CARD 1: TRIA

CARD 2:	First I5 field:	Number of node (m)
	Second I5 field:	Number of minor node on the side, zero, or blank if there is no minor node

Third I5 field:	Number of other end point node (n)
Fourth I5 field:	NODR - Reference node number
First E15.7 field:	x load component at node m
Second E15.7 field:	y load component at node m
Third E15.7	z load component at node m

CARD 3: First E15.7 field: x load component at node n

Second E15.7 field: y load component at node n

Third E15.7 field: z load component at node n

Note: If NODR = 0 then x, y, z are global directions. If NODR $\neq$ 0 then the x-direction is parallel to the element edge specified by m - n; the y-direction is perpendicular to x in the plane defined by the three points, m, n, NODR; the z-axis is perpendicular to the x-y plane.

The program allows for a linearly varying distributed load from node to node. As many load components as desired may be specified. <u>A blank card (i.e., zero or blank first I5 field) ends the specification of distributed edge loads for TRIM elements</u>.

CONC - Concentrated loads - FORMAT - (A4, /, I5, 3E15.7, /, 5X, 3E15.7)

Each card gives the load components in global directions at a specified node.

CARD 1:	CONC	
CARD 2:	I5 field:	Node number
	First E15.7 field:	Force component F_x
	Second E15.7 field:	Force component F_y
	Third E15.7 field:	Force component F_z
CARD 3:	First E15.7 field	Moment component M_x
	Second E15.7 field	Moment component M_y
	Third E15.7 field	Moment component M_z

A blank card (i.e., zero or blank first I5 field) ends the specification of concentrated loads.

SURF - Distributed Surface Loads (TRIP element) - FORMAT (A4, 1X, 3E15.7, /, 3E15.7, /, 3E15.7, /, (16I5))

CARD 1: SURF

PXI Value of distributed surface load in force/unit area in local element x-direction at node i of element (see Fig. 4).

PYI Value of distributed surface load in force/unit surface area in local y-direction at node i of element

PZI Value of distributed surface load in force/unit surface area in local z-direction at node i of element

CARD 2: PXJ Same as PXI but at node j

PYJ Same as PYI but at node j

PZJ Same as PZI but at node j

CARD 3: PXK Same as PXI but at node k

PYK Same as PYI but at node k

PZK Same as PZI but at node k

Note: A linear variation of the surface loads in the plane of the element is assumed.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

BMLO - Distributed Line Load, Beam Element - FORMAT (A4, /, 4E15.7, /, (16I5))

Card 1: BMLO

CARD 2: PYI Force/unit length in local y-direction (see Fig. 5) at node i

PYJ Force/unit length in local y-direction at node j

PZI Force/unit length in local z-direction at node i

PZJ Force/unit length in local z-direction at node j

Note: A linear variation of the distributed load between nodes is assumed.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing. A blank card ends this section.

EDG1 - Distributed Edge Load, Side 1, TRIP Element - FORMAT (A4, 1X, 3E15.7, /, 3E15.7, /, (16I5))

This card inputs the distributed edge or line loads along side 1 of a TRIP element. Side 1 is the side connecting nodes i and j (see Fig. 4).

CARD 1: EDG1

PXI	Local x-component of force/unit length at i^{th} node
PYI	Local y-component of force/unit length at i^{th} node
PZI	Local z-component of force/unit length at i^{th} node

CARD 2:

PXJ	Same as PXI but at node j
PYJ	Same as PYI but at node j
PZJ	Same as PZI but at node j

Note: A linear variation of the edge loads is assumed between nodes i and j.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

EDG2 - Distributed Edge Load, Side 2, TRIP Element - FORMAT (A4, 1X, 3E15.7, /, 3E15.7 /, (16I5)).

This section inputs the distributed edge or line loads along side 2 of a TRIP element. Side 2 is the side connecting nodes j and k (see Fig. 4). All input is the same as in the EDG1 section with EDG2 replacing EDG1 as the alphameric clue word and "j" replacing "i" and "k" replacing "j" in the description.

EDG3 - Distributed Edge Load, Side 3, TRIP Element - FORMAT (A4, 1X, 3E15.7, /, 3E15.7, /, (16I5))

This section inputs the distributed edge or line loads along side 3 of a TRIP element. Side 3 is the side connecting nodes k and i (see Fig. 4). All input is the same as in the EDG1 section with EDG3 replacing EDG1 as the alphameric clue word and "k" replacing "i" and "i" replacing "k" in the description.

TMPU - Nodal Temperatures At Upper Surface for TRIP Element or Nodal Temperatures for TRIM Element - FORMAT (A4, /, E15.7, 13I5, /, (15X, 13I5))

This section is used to input temperatures at nodes of TRIM elements and at the upper surface (i.e., $z = + h/2$, where h is the plate thickness) of TRIP plate elements. For TRIP elements a parabolic temperature distribution through the thickness is assumed. To input temperatures at the lower surface and middle surface use the TMPL and TMPM cards, respectively.

CARD 1:	TMPU	
CARD 2:	E15.7 field	Temperature at node (TRIM) or temperature at upper surface of node (TRIP)
	I5 Integer fields	Applicable nodes

The temperatures of the nodes appearing in the I5 fields are set to the value in the E15.7 field. Any number of continuation cards may be used; their first fifteen columns are ignored. A zero or blank I5 field terminates the card scan for a given temperature. A zero or blank first I5 field (columns 16-20) on a noncontinuation card ends the TMPU input. Both shorthand representations of Section III are allowed.

Note: For the TRIM element the nodal temperatures are averaged to obtain one element temperature. For the TRIP element a linear in-plane variation of the temperatures from node to node is assumed.

TMPM - Nodal Temperatures at Middle Surface for TRIP Element - FORMAT (A4, /, E15.7, 13I5, /, (15X, 13I5))

This section is used to input temperatures at nodes at the middle surface (z = 0) of TRIP plate elements. For TRIP elements a parabolic temperature distribution through the thickness is assumed. To input temperatures at the upper and lower surface use the TMPU and TMPL cards, respectively.

The input formats and information for the card are identical to the TMPU card with TMPM replacing TMPU.

TMPL - Nodal Temperatures at Lower Surface for TRIP Element - FORMAT (A4, /, E15.7, 13I5, /, (15X, 13I5))

This section is used to input temperatures at nodes at the lower surface (z = - h/2, h is the plate thickness) of TRIP elements. For TRIP elements a parabolic temperature distribution through the thickness is assumed. To input temperatures at the upper and middle surface use the TMPU and TMPM cards, respectively.

The input formats and information for this card are identical to the TMPU card with TMPL replacing TMPU.

A card with SEND in the first four columns ends the section for applied loading.

XII. Members to be Printed - FORMAT (16I5)

Specifies the members whose strains and stresses are to be printed. Both shorthands of Section III are allowed. A blank card or card with only zero entries ends the section.

XIII. Nodes to Be Printed - FORMAT (16I5)

Nodes whose displacements are to be printed as per Section XII.

XIV. Parameters for Plastic Analysis - FORMAT (2E15.7)

PMAX = Maximum load to be applied for this half cycle

PPCT = Load increment as a decimal multiple of yield load.

XV. Parameters for Succeeding Load Cycles - FORMAT (I5, 2E15.7)

NPRNT = If equal to zero, no additional load cycle and end of problem. If greater than zero, print output every NPRNT increments.

PMAX = Maximum load for this load cycle

PPCT = Load increment as percentage of yield load.

Note: PPCT should be equal to one-half the original value since the plastic range is twice the elastic range for initial loading.

XVI. Change Plastic Material Properties for New Load Cycle

The input for this section begins with a four-character word as follows:

MAT1 New material properties for plane stress membrane triangles (TRIM) or plate (TRIP) elements (see Section X for applicability)

MAT2 New material properties for orthotropic strain-hardening materials (see Section X for applicability

MBM New material properties for beam elements

MSTG New material properties for a stringer element

MAT1 - FORMAT (A4, 1X, I5, /, 4E15.7, /, 5E15.7)

CARD 1: MAT1

IMEM One member number from the group that is to be changed

CARD 2: SIGOX, SIGOY, SIGOZ, SIGXY — Defined in Section X

CARD 3: RMOSN, RMOSS, RMOSE, YLDST, YLDSC — Defined in Section X

One set of three cards is required for each MAT1 group to be changed. A MAT1 group may be changed to a MAT2 group after a half load cycle.

MAT2 - FORMAT (A4, 1X, I5, /, 4E15.7, /, 5E15.7, /, E15.7)

CARD 1: MAT2

IMEM One member number from the group that is to be changed

CARD 2: SIGOX, SIGOY, SIGOZ, SIGXY — Defined in Section X

CARD 3: RMONX, RMOSX, RMONY, RMOSY, RMONXY — Defined in Section X

CARD 4: RMOSXY Defined in Section X

One set of four cards is required for each MAT2 group to be changed. A MAT2 group may be changed to a MAT1 group after a half load cycle.

MBM or MSTG - FORMAT (A4, 1X, I5, /, 3E15.7)

CARD 1: MBM
or
MSTG
IMEM (One member number from the group that is to be changed)

CARD 2: RMOSN (Ramberg-Osgood parameters defined in Section X)
RMOSS
YLDST

The section is ended with a section end card, SEND.

Note: If there is no change in material properties, a SEND card must appear in the appropriate place in the input stream.

XVII. Problem End - FORMAT (A4)

Each problem's input must be ended with a card reading "ENDb" where "b" denotes a blank, in columns 1-4. The last problem in a run should end with a card reading "STOP" in column 1-4 instead of "ENDb."

XVIII. Restarting a Problem

The initial restart tape is created on Unit 21. Subsequent restart jobs mount the restart tape as Unit 21 and if desired (NUTAP > 0) copies and continues the restart tape on Unit 22.

CARD 1: Problem Title - FORMAT (20A4)

As in Section I

CARD 2: NPNTC, NPRNT, IRESRT, NUTAP, INPRT - FORMAT (5I5)

As in Section II. Here NPNTC is ignored.

If IRESRT is equal to 2

Restart from an elastic critical load using a previously created restart tape

CARD 3: PMAX, PPCT - FORMAT (2E15.7)

PMAX = Maximum load to be applied for this half cycle

PPCT = Load increment as percentage of yield load

Input is then continued from Section XV

If IRESRT is equal to 3

Continuation from some previously plastic load. The job can be restarted from a previous maximum load or from any intermediate load level.

CARD 3: PMAX, NRSRT - FORMAT (E15.0, I5)

If starting from a previous maximum load NRSRT can be ignored. If starting from some intermediate load level, NRSRT is a unique number obtained as output from the job that generated the restart tape. This output is of the form;

* * * * *

PLASTIC ANALYSIS VALUES FOR RESTART TAPE HAVE BEEN WRITTEN AT P = 1.5000000E+03 FOR PRINT INTERVAL 30

* * * * *

In this case NRSRT is equal to 30 and the load is not necessarily the last maximum value.

Note: For cases when the job ends abnormally (for example when time or lines are exceeded) the job can be restarted with a value for NRSRT obtained from the output. In this case the last value of NRSRT should not be used. This will insure that a complete set of data will be accessed from the restart tape.

If IRESRT is equal to 4

Restart from a previous maximum load level followed by a reversal of a previous load distribution (CYCLE). A reversal of a previous load distribution can be placed on the structure from any previously generated and saved state of stress.

CARD 3: PMAX, NRSRT - FORMAT (E15.0, I5)

If starting from a previous maximum load NRSRT can be ignored. If starting from some intermediate load level, NRSRT is a unique number obtained as output from the job that generated the restart tape. This output has the form shown in the preceding discussion for IRESRT equal to three. The value of PMAX is ignored.

Succeeding cards follow as in Sections XV through XVII.

XIX. Alternate Input Method

All or part of the input may be read from a tape. This tape is mounted as Unit 23 with the appropriate job control cards. The tape is accessed by specifying the keyword, TAPE (FORMAT A4), in the proper place in the card input stream. Input will then be read from the tape until a SEND (A4) is encountered on the tape. This returns the input reader to the card input stream.

Note: All SEND formats used to end sections must be in card form as the SEND encountered on the tape only sends the reader back to the card input stream. Also blank cards which end sections are best not put on the tape but rather on cards as this allows the user to add card input to tape input, i.e., if a blank ends a section and is encountered on the tape format then the section ends and no new data may be added, but if the blank does not occur on tape the section is still open and can be closed with a blank card in the card input stream.

Example:

Input for member connectivity, Section IV, and coordinates have been generated on a tape in the prescribed manner.

Input cards are as follows:

```
TITLE
   0     5
   1 - 100
   TAPE         (reads data from a tape)
   TRIM
     .          (additional elements if desired)
     .
     .
   SEND
   TAPE         (reads X-coordinates from tape)
                (Blank card to end section for X-coordinates)
   TAPE         (reads Y-coordinates from tape)
                (Blank card to end section for Y-coordinates)
   5.0          (card input for Z-coordinates)
     .
     .
     .
                (Blank card ends section for Z-coordinates)
```

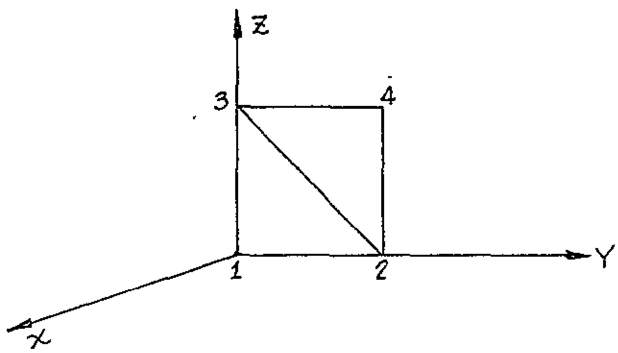
If no additional tape input, remaining card input as usual.

REFERENCES

1. Prager, W., "The Theory of Plasticity: A Survey of Recent Achievements," (James Clayton Lecture) Proc. Inst. Mech. Engrs., Vol. 169, 1955, p. 31.
2. Ziegler, H., "A Modification of Prager's Hardening Rule," Quart. Appl. Math., Vol. 17, No. 1, 1959, p. 55.
3. Armen, H., Jr., Pifko, A., and Levine, H., "Finite Element Analysis of Structures in the Plastic Range," NASA Contractor Report CR-1649, February 1971.
4. De Veubeke, B. F., "Displacement and Equilibrium Models in the Finite Element Method," in Stress Analysis, edited by Zienkiewicz and Hollister, John Wiley and Sons, Ltd., 1965, p. 145.
5. Armen, H., Jr., Levine, H., Pifko, A., and Levy, A., "Nonlinear Analysis of Structures," NASA Contractor Report CR-2351, March 1974; also Grumman Research Department Report RE-454, May 1973.
6. Abramowitz, M. and Stegun, I. A., Eds., Handbook of Mathematical Functions, National Bureau of Standards, Applied Mathematics Series, 1964.
7. Pifko, A., Levine, H. S., and Armen, H. Jr., "PLANS - A Finite Element Program for Nonlinear Analysis of Structures, Volume I - Theoretical Manual," NASA Contractors Report NAS CR-2568, November 1975.

APPENDIX I — MULTIPOINT CONSTRAINTS

EXAMPLE 1



Node 1 and 2 are fixed.

There is an applied displacement at node 3 in the negative y direction. The distance between nodes 3 and 4 remains constant.

Boundary Conditions

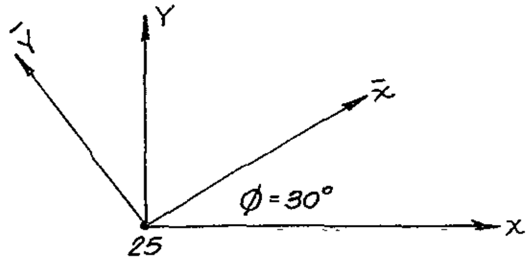
011000000000	1-4	default condition
000000000000	1 2 4	
021000000000	3	
Blank card		

Dependencies

Node	Dof	Node	Dof		
3	2	3	2	-.050	(specified displacement)
4	2	3	2	1.0	restraint condition that
4	3	3	3	1.0	3-4 remain rigid

Note: In the boundary condition cards for node 4, degrees of freedom 2 and 3 must be specified 0 because they are effectively eliminated from the solution (they are dependent degrees of freedom).

EXAMPLE 2 Rotation of Displacements at a Boundary



For this problem we set the displacement $\bar{u}$ equal to zero and $\bar{v}$ free. This corresponds to the normal displacement along $\bar{x}$ equal to zero and the tangential displacement (in $\bar{y}$ direction) free. The global coordinates of the problem are x, y. Thus:

$$u = \bar{u} \cos \varphi - \bar{v} \sin \varphi$$

$$v = \bar{u} \sin \varphi + \bar{v} \cos \varphi$$

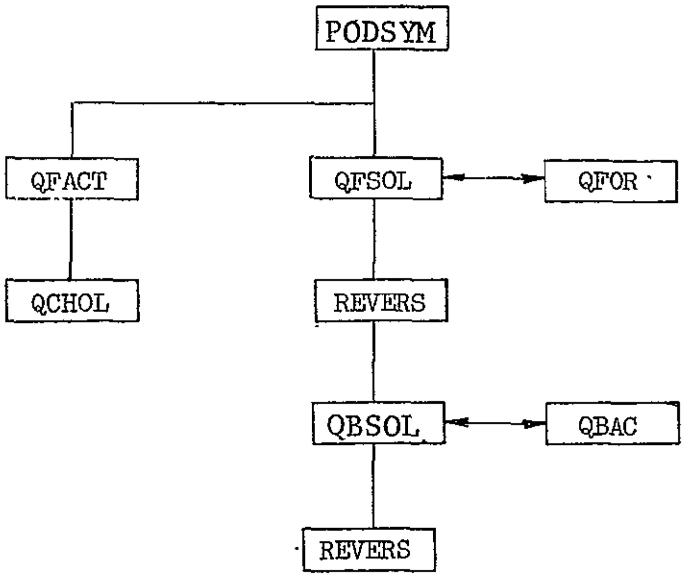
The multipoint constraint cards are:

25	1	25	1	.86602540E00	25	2	-.50E00
25	2	25	1	.50E00	25	2	.8660254E00

Note: On the boundary condition cards, since $\bar{u} = 0.0$ node 25 degree of freedom 1 is specified to be 0 but node 25 degree of freedom 2 is specified 1 or free since it remains an independent degree of freedom (although rotated). If the normal displacement $\bar{u}$ were not fixed (and the transformation performed merely to apply a normal load), it would have a 1 boundary condition specified.

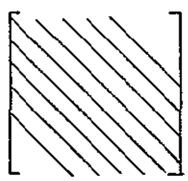
APPENDIX II — SOLUTION PACKAGE

PODSYM - Solution of Symmetric Positive Definite Banded Matrix Equations



Problem

Solve AX = Y, where

A = 

is a banded positive definite symmetric matrix; X is the desired solution vector; and Y is the known right side (load vector). PODSYM is the user interface and supervisory routine. It uses the Cholesky algorithm which factors the total stiffness matrix into LL^T (where L is a lower triangular matrix) and then solves a pair of triangular sets of equations.

The factorization is carried out by subroutine QFACT which supervised the storage and data set allocation and subroutine QCHOL which generates the lower triangular L matrix. QCHOL implements the Cholesky algorithm to factor the positive definite symmetric A matrix as the product of a lower triangular matrix with its transpose:

$$A = LL^T$$

A straightforward argument establishes the possibility of decomposing any positive definite matrix in this fashion. Once L has been obtained, it is not difficult to solve the system of linear equations $AX = Y$ by calculating Z as the solution of the lower triangular system $LZ = Y$, and then X as the solution of the upper triangular system $L^TX = Z$. The forward solution $(LZ = Y)$ is accomplished by subroutines QFSOL and QFOR and the back solution $(L^TX = Z)$ by subroutines QBSOL and QBAC. Before the call to QBSOL, subroutine REVERS is called, which reverses the rows of L so that the last row becomes the first row, etc. This is accomplished in order to sequentially access L^T during the back solution.

The above algorithm is noteworthy for its assured stability and general efficiency. It is possible to carry out an error analysis of the procedure as it is represented on a digital computer; such analysis shows that the computed L matrix satisfies an equation of the form

$$A + E = LL^T$$

with bounds on the elements of E which show that E is small compared to A. The effect of rounding errors made in the subsequent solution of $LZ = Y$ and $L^TS = Z$ may then be taken into account by (implicitly) introducing an additional perturbation F into A, and it is then concluded that the computed solution X_o exactly satisfies the equation

$$(A = E + F)X_o = Y$$

Since $E + F$ can be shown to be small, one would like to infer that X_o is almost an exact solution of the original equations, and unless A is too nearly singular, such a conclusion is indeed warranted. But if A is very ill-conditioned, no such result can be guaranteed, and X_o may be far from the mathematically correct solution; in this event single-precision computation will not suffice for the calculation of an accurate solution, and since the solution will be very sensitive to small errors in A, it is unlikely that even a high-precision computation will yield satisfactory results unless A and Y are known (and supplied) to more than single-precision accuracy. The PODSYM subroutines make a fairly realistic attempt to detect and report pathological conditions of this sort.

The large positive definite matrices that occur in practical work very often contain a large number of zero entries and the program seeks to benefit from the presence of these elements by modifying the standard Cholesky formulae.

$$\ell_{kk} = \left[A_{kk} - \sum_{j=1}^{k-1} \ell_{kj}^2 \right]^{\frac{1}{2}}$$

and

$$\ell_{ik} = \left[A_{ik} - \sum_{j=1}^{k-1} \ell_{ij}\ell_{kj} \right] / \ell_{kk} \quad \text{for} \quad i > k$$

to read instead

$$\ell_{kk} = \left[A_{kk} - \sum_{j=\nu(k)}^{k-1} \ell_{kj}^2 \right]^{\frac{1}{2}}$$

and

$$\ell_{ik} = \left[A_{ik} - \sum_{j=\mu(i,k)}^{k-1} \ell_{ij}\ell_{kj} \right] / \ell_{kk}$$

APPENDIX III — CHANGING THE NUMBER OF NODES AND MEMBERS IN BEND

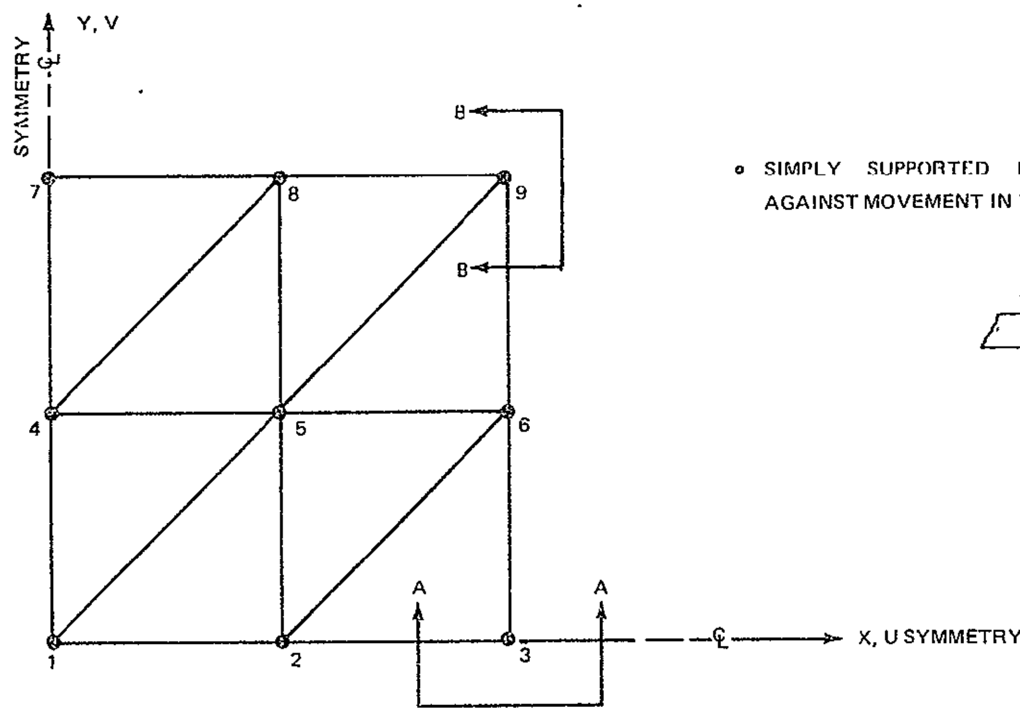
The number of nodes and members allowed for a BEND problem is variable. The following changes must be made in the MAIN program for a successful up-dimensioning or down-dimensioning of nodes and/or members.

- Reset MXMEM equal to the maximum number of members desired
- Reset MXNOD to the maximum number of nodes desired
- Redimension C the main working area so that it is greater than the greater of 10.5 * MXNOD + 6.5 * MXMEM or 24 * MXNOD + 1975
- Set NCORE equal to the dimension of the C array
- For maximum efficiency set MHCON equal to a prime number approximately 1/3 more than MXMEM
- For maximum efficiency set NHCON equal to a prime number approximately 1/3 more than MXNOD. A table of prime numbers appears in Ref. 6, page 870.
- For CDC versions, dimension IRAC (random access array) to MXMEM+1.

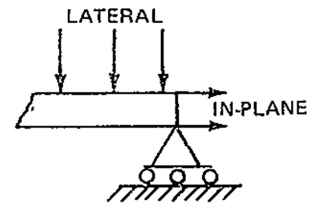
The following limitations on the number of property specifications exist in BEND and can only be changed through redimensioning within the program:

Property or Section	Current Limitation
Number of MAT1 and MAT2 property types	20
Number of MBM property types	20
Number of beam section property types	100
Number of types of edge loads for EDG1, EDG2, EDG3 arrays	100
Number of types of surface loads applied (SURF)	100
Number of multipoint constraint conditions	500
Number of beam element distributed load specification types	100
Number of boundary condition type specifications	100

APPENDIX IV — EXAMPLES OF PLATE BOUNDARY CONDITIONS



o SIMPLY SUPPORTED EDGES NOT RESTRAINED AGAINST MOVEMENT IN THE X-Y PLANE.



SECTION A-A

SECTION B-B

BOUNDARY CONDITION: 1 - FREE; 0 = FIXED

NODE	U	V	W	θ_x	θ_y	θ_z	e_x	e_y	γ_{xy}	κ_{xx}	κ_{yy}	κ_{xy}
1	0	0	1	0	0	0	1	1	0	1	1	0
2	1	0	1	0	1	0	1	1	0	1	1	0
3	1	0	0	0	1	0	1	1	0	0	0	0
4	0	1	1	1	0	0	1	1	0	1	1	0
6	1	1	0	0	1	$0^{(1)}$	1	1	$0^{(2)}$	0	0	1
6	1	1	0	0	1	1	1	1	1	0	0	1
7	0	1	0	1	0	0	1	1	0	0	0	0
8	1	1	0	1	0	$0^{(1)}$	1	1	$0^{(2)}$	0	0	1
8	1	1	0	1	0	1	1	1	1	0	0	1
9	1	1	0	0	0	$0^{(1)}$	1	1	$0^{(2)}$	0	0	1
9	1	1	0	0	0	1	1	1	1	0	0	1

U, V: ALL ZERO IF NO IN-PLANE LOAD

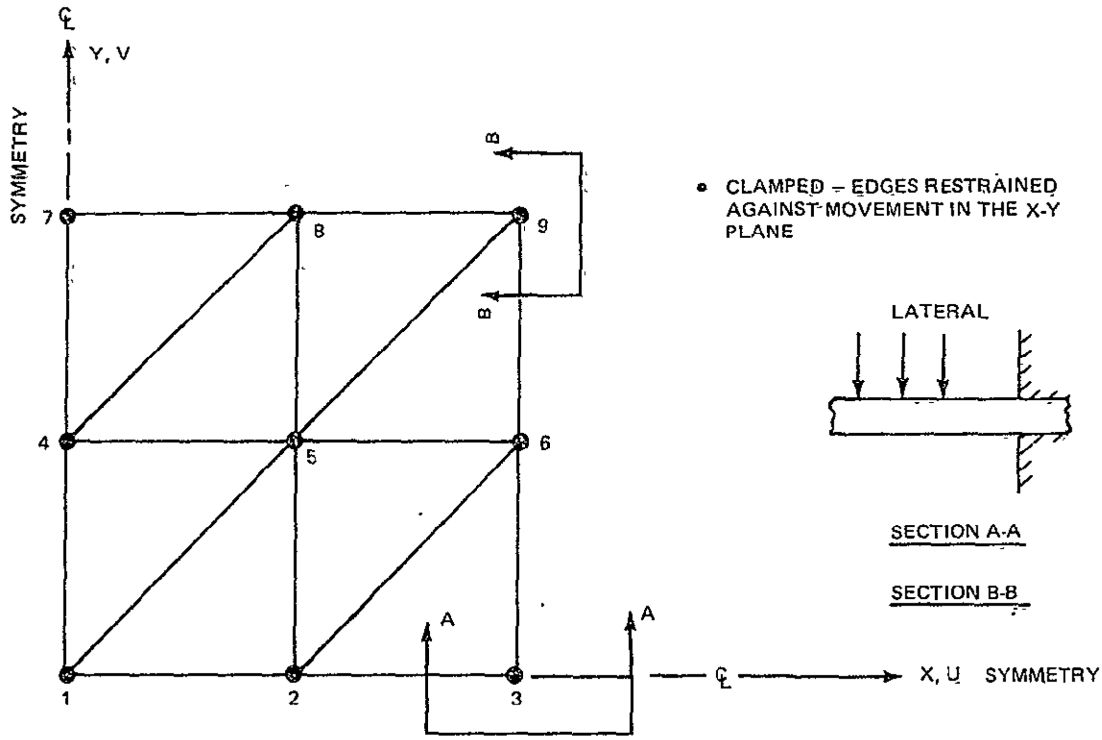
e_x, e_y: ALL ZERO IF NO IN-PLANE LOAD

NOTES:

(1) = 0 FOR ZERO OR UNIFORM DIRECT IN-PLANE LOADS
 = 1 FOR NON-UNIFORM IN-PLANE LOADS

(2) = 0 FOR ZERO IN-PLANE SHEAR LOAD
 = 1 FOR IN-PLANE SHEAR

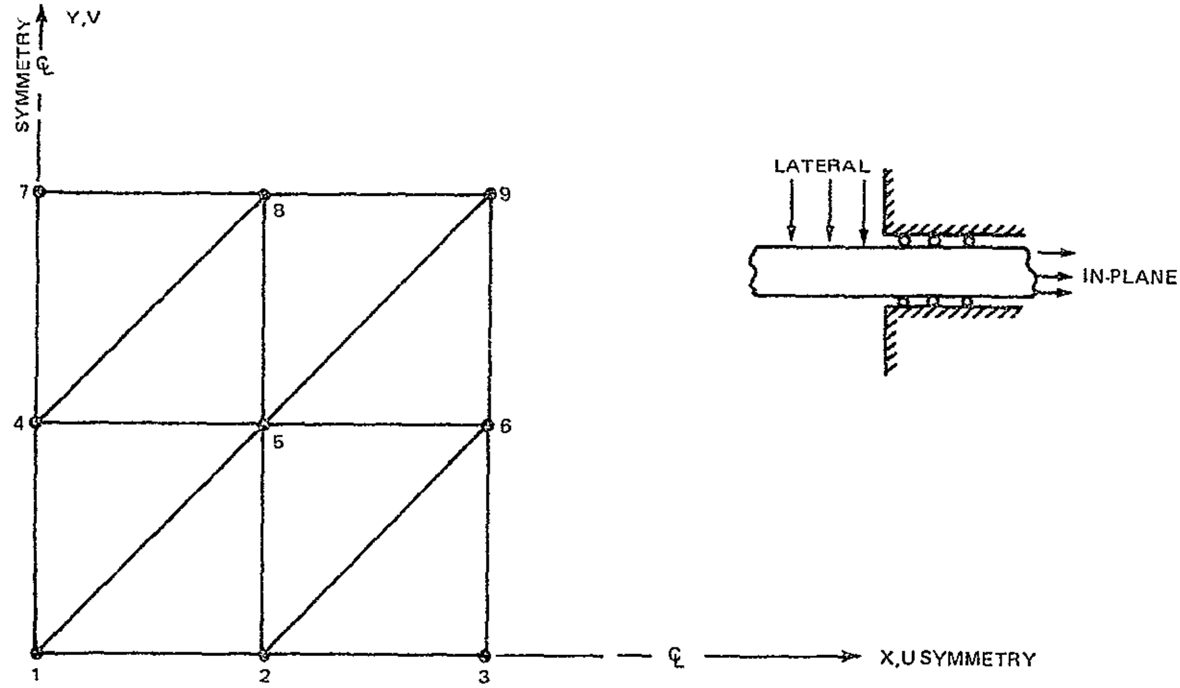
NODE 5 COMPLETELY FREE



BOUNDARY CONDITION: 1 - FREE; 0 = FIXED

NODE	U	V	W	θ_x	θ_y	θ_z	e_x	e_y	γ_{xy}	κ_{xx}	κ_{yy}	κ_{xy}
1	0	0	1	0	0	0	1	1	0	1	1	0
2	1	0	1	0	1	0	1	1	0	1	1	0
3	0	0	0	0	0	0	1	0	0	1	0	0
4	0	1	1	1	0	0	1	1	0	1	1	0
6	0	0	0	0	0	0	1	0	1	1	0	0
7	0	0	0	0	0	0	0	1	0	0	1	0
8	0	0	0	0	0	0	0	1	1	0	1	0
9	0	0	0	0	0	0	0	0	0	0	0	0

NODE 5 COMPLETELY FREE



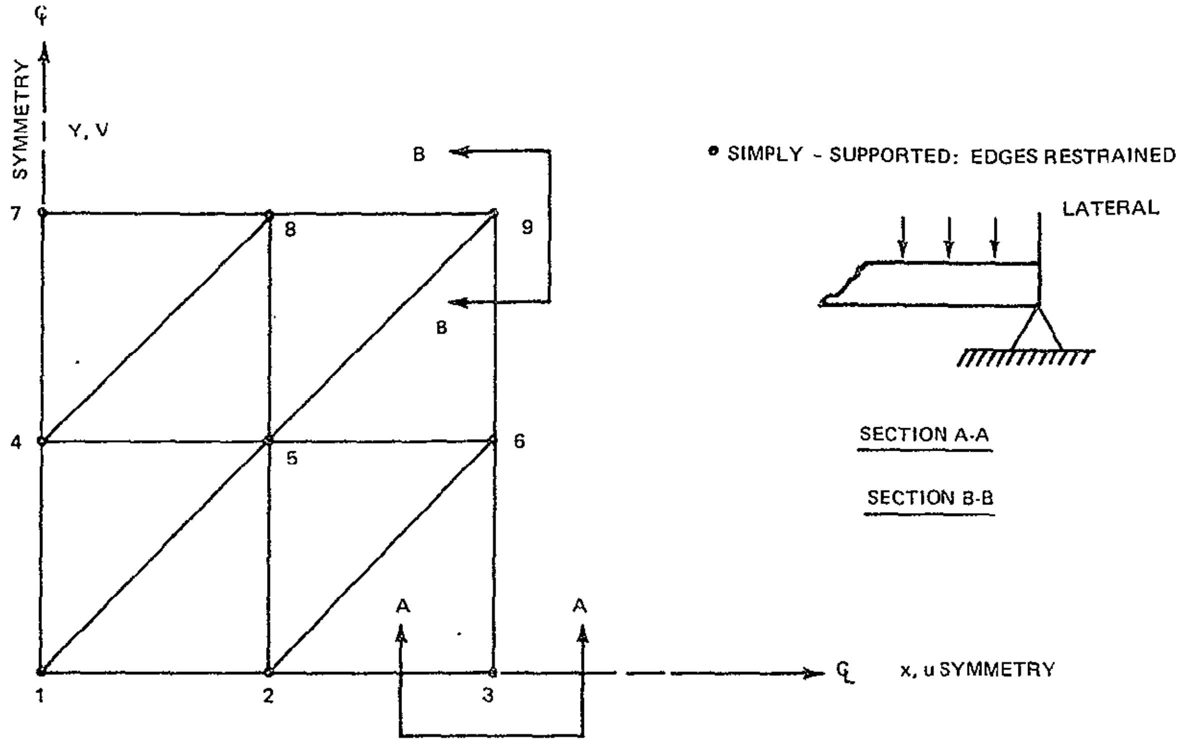
BOUNDARY CONDITION: 1 - FREE; 0 = FIXED

NODE	U	V	W	θ_x	θ_y	θ_z	e_x	e_y	γ_{xy}	κ_{xx}	κ_{yy}	κ_{xy}
1	0	0	1	0	0	0	1	1	0	1	1	0
2	1	0	1	0	1	0	1	1	0	1	1	0
3	1	0	0	0	0	0	1	1	0	1	0	0
4	0	1	1	1	0	0	1	1	0	1	1	0
6	1	1	0	0	0	0(1)	1	1	0(2)	1	0	0
6	1	1	0	0	0	1	1	1	1	1	0	0
7	0	1	0	0	0	0	1	1	0	0	1	0
8	1	1	0	0	0	0(1)	1	1	0(2)	0	1	0
8	1	1	0	0	0	1	1	1	1	0	1	1
9	1	1	0	0	0	0(1)	1	1	0(2)	0	0	0
9	1	1	0	0	0	1	1	1	1	0	0	0

(U, V): ALL ZERO IF NO INPLANE LOAD

(e_x, e_y): ALL ZERO IF NO INPLANE LOAD

NOTES
(1) = 0 FOR ZERO OR UNIFORM DIRECT IN-PLANE LOADS
= 1 FOR NON-UNIFORM IN-PLANE LOADS
(2) = 0 FOR ZERO IN-PLANE SHEAR LOAD
= 1 FOR IN-PLANE SHEAR
NODE 5 COMPLETELY FREE



BOUNDARY CONDITION: 1 - FREE; 0 = FIXED

NODE	U	V	W	θ_x	θ_y	θ_z	e_x	e_y	γ_{xy}	κ_{xx}	κ_{yy}	κ_{xy}
1	0	0	1	0	0	0	1	1	0	1	1	0
2	0	0	1	0	1	0	1	1	0	1	1	0
3	0	0	0	0	1	0	1	$1^{(1)}$	0	0	0	0
3	0	0	0	0	1	0	1	$0^{(2)}$	0	0	0	0
4	0	1	1	1	0	0	1	1	0	1	1	0
6	1	$1^{(1)}$	0	0	1	0	1	$1^{(1)}$	$0^{(1)}$	0	0	1
6	1	$0^{(2)}$	0	0	1	0	1	$0^{(2)}$	$1^{(2)}$	0	0	1
7	0	0	0	1	0	0	$1^{(1)}$	1	0	0	0	0
7	0	0	0	1	0	0	$0^{(2)}$	1	0	0	0	0
8	$1^{(1)}$	0	0	1	0	0	$1^{(1)}$	1	$0^{(1)}$	0	0	1
8	$0^{(2)}$	0	0	1	0	0	$0^{(2)}$	1	$1^{(2)}$	0	0	1
9	0	0	0	0	0	0	$1^{(1)}$	$1^{(1)}$	0	0	0	1
9	0	0	0	0	0	0	$0^{(2)}$	$0^{(2)}$	0	0	0	1

NOTES:

(1) WHEN DISPLACEMENTS ARE NOT RESTRAINED IN THE DIRECTION OF THE EDGE

(2) WHEN DISPLACEMENTS ARE FULLY RESTRAINED

NODE 5 COMPLETELY FREE

SECTION 3

INSTRUCTIONS FOR USE OF HEX

A Program for the Elastic-Plastic Analysis of Three Dimensional Solids

INSTRUCTIONS FOR THE USE OF HEX

HEX is a finite element program to treat the elastic, elastic-plastic or elastic-cyclic plastic response of arbitrary three dimensional solid structures. The program uses a family of isoparametric hexahedra elements (Refs. 1 and 2) consisting of eight-node hexahedra and hexahedra with up to 12 additional midside nodes as shown in Fig. 1. Sample problems from the present program can be found in Ref. 3.

The program is capable of treating the elastic and the elastic-ideally plastic response of orthotropic materials. In addition, consideration is given to isotropic materials exhibiting elastic-ideally plastic, linear strain hardening, or nonlinear strain hardening behavior. Further, the kinematic hardening theory of plasticity is used (Refs. 4-6) so that provision for cyclic loading conditions involving reversed plastic deformation is included.

The input to the program is categorized in the following sections:

I. Problem Title FORMAT (20A4)

Any 80-character title describing the problem.

II. NPNTC, NPRNT, IRESRT, NUTAP, INPRT FORMAT (5I5)

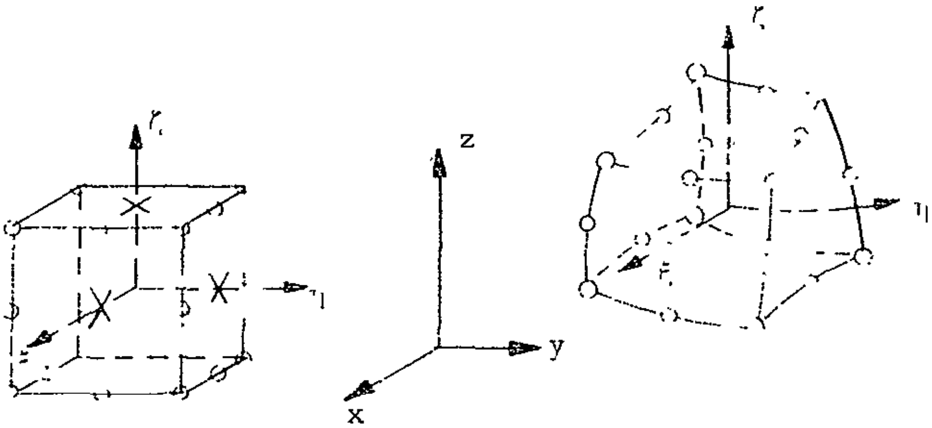
$0 \leq$ NPNTC ≤ 63:

NPNTC is the sum of the following integers corresponding to the option desired.

If NPNTC = 0 No intermediate printout

= 1 Print the load vector

= 2 Print element stiffness matrix



Local Coordinates   Cartesian Coordinates

Fig. 1 Isoparametric Hexahedra

NPNTC = 16 Print each element stiffness matrix entry
to be stacked with its stacking index

= 32 Print the total stiffness matrix

For example, if it is desired to print the load vector and the total stiffness matrix, NPNTC = 1 + 32 = 33.

NPRNT:

If ≤ 0, perform elastic analysis only.

If > 0, perform plastic analysis, printing output every NPRNT increments of load.

IRESRT: (See Section XVIII)

0, Elastic (and/or) plastic run. Do not generate a restart tape

1, Elastic (and/or) plastic (cyclic) run. Build a new restart tape

2, Plastic run with elastic values from previously created restart tape

3, Plastic restart run starting at some specified load level

4, Cyclic restart run

NUTAP:

Applicable only if restarting from a restart tape

0, No new tape written

1, Complete new tape created and additional restart data written

INPRT:

0, Write restart tape only at P = PMAX (i.e., at maximum load)

N, Write restart tape every N increments of load

III. Node Specification (16I5)

This section defines an allowable set of external node point numbers. The maximum node number that can be used is 9999. The program uses this information in two ways. First to set up a table of allowable node points that is used to check all subsequent node point input. Secondly, the program converts each external node number to an internal number consecutively in the order that the node appears on the input card. Consequently the order of the input of external node numbers is completely arbitrary and need not be increasing monotonically. In practice the node numbers should be numbered so as to minimize the bandwidth. Once the input is read the program operates with the internal numbers which are now numbered from 1 through the number of nodes in the model. In this manner the node ordering and therefore the bandwidth of the stiffness matrix can be easily changed and nodes can be inserted or deleted by changing the external node specification.

The input is specified by entering the appropriate number on the input cards in fields of five. However, for this purpose the user can also utilize a shorthand form of the input. That is, specifying m and -n consecutively is the equivalent of the specification of nodes m, m + 1, m + 2, ..., n and specifying m, -p, and -n consecutively is the equivalent of the specifiction of nodes m, m + p, m + 2p, ..., m + kp where m + kp is the highest integer of the form less than or equal to n. For example, the specification of nodes 1 through 100 is written as 1 - 100 and 1, 3, 5, ..., 99 as 1-2-99. This card input appears in fields of 5 (I5 Format) with 16 items per card. Any number of continuation cards may be used. <u>A zero or blank I5 field ends the specification.</u>

IV. <u>Member Connectivity (Node Numbers of Each Member)</u>
FORMAT (A4, 6X, 9I5)

The first alphanumeric field defines the element type as follows

HEX8 - eight node hexahedra element

HX20 - hexahedra element with up to twelve additional midside nodes

MSGN - mesh generator for eight-node hexahedra

SEND - ends the section

The first integer field designates the member number. The next integer field designates the connecting nodes as follows:

HEX8 - The eight corner nodes are specified in the order shown on Fig. 2.

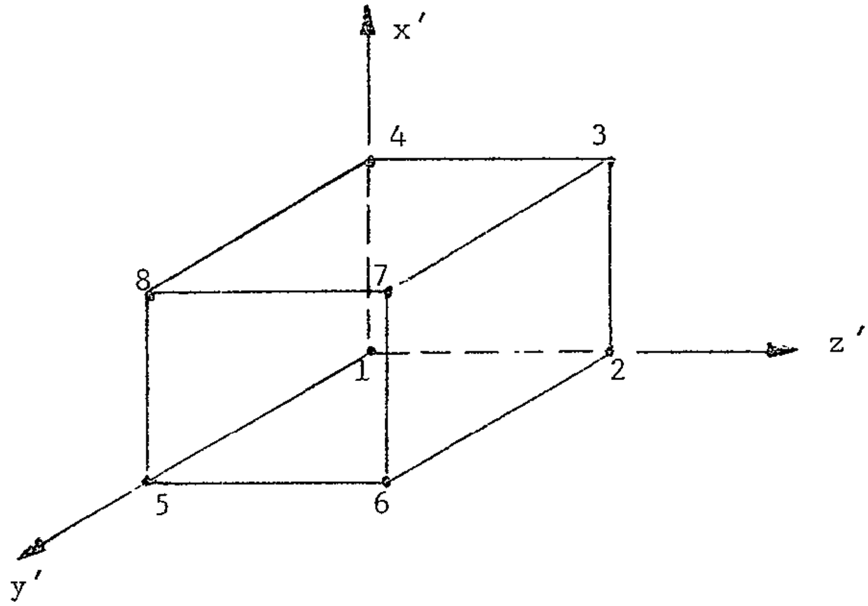


Fig. 2 Order of Corner Nodes

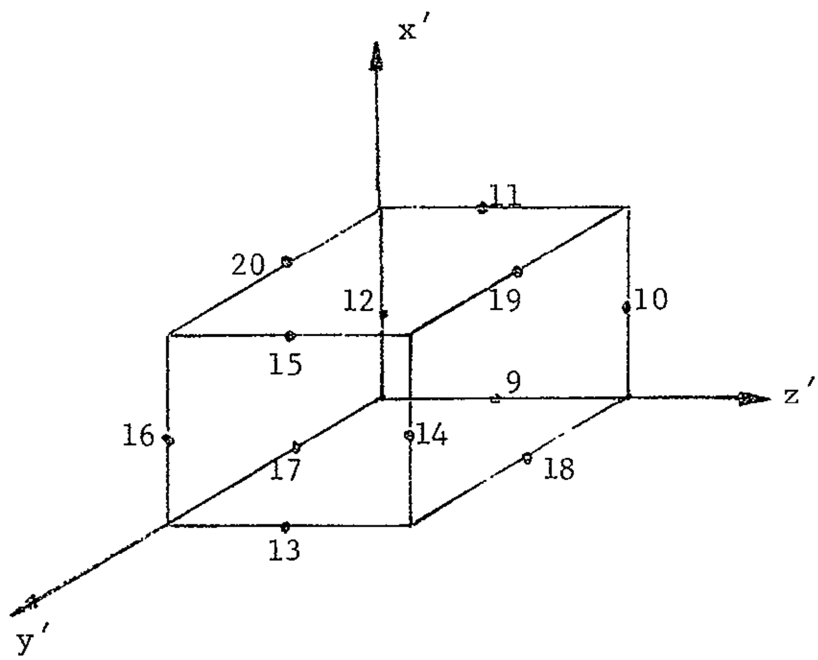


Fig. 3 Order of Midside Nodes

HX20 - The eight corner nodes are specified first in the order shown in Fig. 2. The additional midside nodes are specified on a second card (FORMAT (12I5)) according to the order shown in Fig. 3. An element can be constructed with any combination of midside nodes. If a midside node is not specified, a zero is placed in its corresponding position on the card. If a midside node is specified for an element, a midside node should also be specified for every element coincident with that side.

Note: The hexahedra elements with midside nodes can have curved boundaries. The shape of any side is described by a quadratic polynomial through the three node points. If a side containing a midside node is straight, the coordinates of the midside node can be read in as zero. In this case the nodal coordinates are calculated within the program.

MSGN - The nodal connectivity is automatically generated for eight-node hexahedra according to the convention in Fig. 2. The first card for this input option requires the four-character identifier MSGN. A second card is required with FORMAT (5I5) which specifies the number of element subdivisions in three coordinate directions. The integers in the five fields are the subdivisions in the x_1, x_2, x_3 directions, and the first node number and element number in the mesh. The coordinate system must be right handed. The mesh generator assumes that the nodes are numbered consecutively along the x_1 direction specified in the first I5 field then incremented on the $x_1 - x_3$ face until that face is completely specified. The mesh is then incremented in the x_2 direction and then consecutively numbered on the $x_1 - x_3$ plane. The element numbers are numbered consecutively starting at the specified starting value in the positive x_1 direction. The mesh generating capability still requires that the nodes be defined in Section IV and that the nodal coordinates be explicitly defined in Section VI. An example of the convention for a 4 x 3 x 2 mesh in Cartesian coordinates with the starting node and element equal to one is shown in Fig. 4.

V. X-Coordinates of Nodes FORMAT (E15.7, 13I5)

The global x-coordinates of the nodes appearing in the I5 fields are set to the value in the E15.7 field. Any number of continuation cards may be used; their first fifteen columns are ignored. A zero or blank I5 field terminates the card scan for a given x-coordinate. A zero or blank first I5 field (columns 16-20) on a noncontinuation card ends the section. Both shorthand representations of Section III are allowed.

VI. Y-Coordinates of the Nodes. Same as Section V.

VII. Z-Coordinates of the Nodes. Same as Section V.

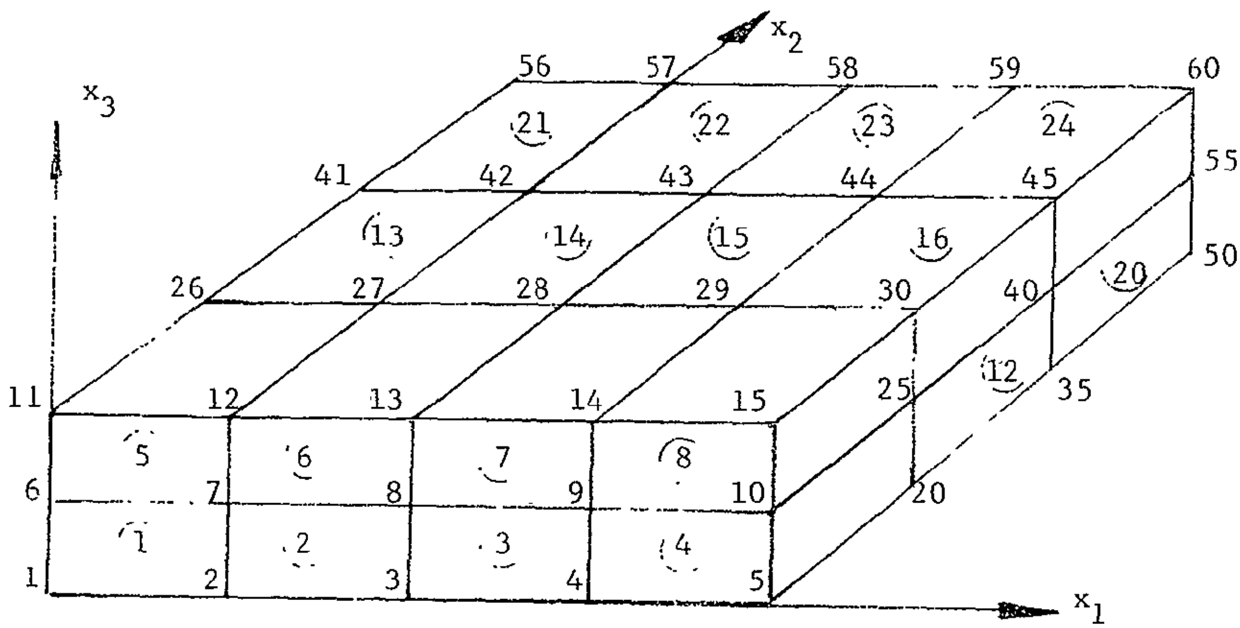


Fig. 4 Node and Member Ordering Convention for Automatic Mesh Generation

VIII. Boundary Conditions FORMAT (I3, 2I1, 5X, 14I5)

The first three fields give the boundary conditions specifications in the order: u, v, w; where u, v, w are global displacements in the global x, y, z directions, respectively.

Zero—denotes a fixed degree of freedom

One denotes a free degree of freedom

Two will result in the application of a unit generalized displacement, or a corresponding card may be included in Section IX (dependent degrees of freedom) designating the magnitude of the applied generalized displacement.

The 14I5 fields give the applicable nodes for the boundary condition specifications, with both shorthand notations of Section III permitted. Any number of continuation cards may be used for a given specification. However, only the 14I5 fields are used on a continuation card. <u>A zero or blank I5 field terminates</u>

the card scan for a given boundary condition specification. Note: if the last field of a card (columns 76-80) is the last specification, an additional blank card (continuation card) must follow. A zero or blank first I5 field (columns 11-15) on a noncontinuation card ends the section. If a node's boundary conditions are not specified in this section, all the degrees of freedom are assumed to be free. To change this default condition, the first card of this section should be set to the desired default with all nodes used in the problem specified. Note: Maximum number of nodes is currently 2500.

IX. Dependent Degrees of Freedom FORMAT (2I5, 2(5X, 2I5, E15.7))

This section designates the input for both single and multipoint constraints as well as applied displacement of the form:

1) $\delta_i = \alpha_1 \delta_1 + \alpha_2 \delta_2 + \cdots \alpha_n \delta_n$

where δ_i is a dependent degree of freedom, $\delta_1 \cdots \delta_n$ are independent degrees of freedom, and $\alpha_1, \alpha_2, \cdots, \alpha_n$ are coefficients.

2) Rotation of displacements at a node

$$\delta_i = \alpha_1 \tilde{\delta}_i + \alpha_2 \tilde{\delta}_j + \alpha_3 \tilde{\delta}_k$$

$$\delta_j = \beta_1 \tilde{\delta}_i + \beta_2 \tilde{\delta}_j + \beta_3 \tilde{\delta}_k$$

$$\delta_k = \gamma_1 \tilde{\delta}_i + \gamma_2 \tilde{\delta}_j + \gamma_3 \tilde{\delta}_k$$

where the α, β, γ's are the direction cosines of the rotation, δ_i, δ_j, δ_k are the displacements with respect to the original global directions and $\tilde{\delta}_i$, $\tilde{\delta}_j$, $\tilde{\delta}_k$ are the components of displacements at the node with respect to the new coordinate axes. An example of this capability is given in Appendix I.

3) Applied generalized displacement

$$\delta_i = \alpha_1$$

where the coefficient α_1 is the applied generalized displacement.

The first two I5 fields designate a node number and a degree of freedom (i.e., 1-3). The dependency is defined in the following three fields. The two integer fields designate the node number and degree of freedom number and the coefficient by the floating point field. If there is another dependency for the node, it is designated in a similar fashion in the next three fields. Any number of continuation cards can be used with the first two fields blank. The section is ended by a blank or zero in the third integer field (blank card). Examples of the use of multipoint constraints are in Appendix I.

X. Material Properties

The input for material properties consists of cards as follows:

CARD 1: FORMAT (A4, 1X, 5E15.7)

MAT1 = four-character identifier
EONE = Young's modulus in principal property axis (1)
ETWO = Young's modulus in principal property axis (2)
ETEE = Young's modulus in principal property axis (3)
VONTO = Poisson's ratio, ν_{12}
VTOTE = Poisson's ratio, ν_{23}

CARD 2: FORMAT (4E15.7)

VTETO = Poisson's ratio, ν_{32}
GONTO = Shear modulus, G_{12}
GTOTE = Shear modulus, G_{23}
GONTE = Shear modulus, G_{13}

CARD 3: FORMAT (3E15.7)

BETA1 =
BETA2 =
BETA3 =
} Angles between 1, 2, 3 material axes and local element x, y, z axes (defined in the note at the end of the section).

CARD 4: FORMAT (3E15.7)

TALF-1 Coefficient of thermal expansion in the principal 1 direction

TALF-2 Coefficient of thermal expansion in the principal 2 direction

TALF-3 Coefficient of thermal expansion in the principal 3 direction

CARD 5: FORMAT (5E15.7)

SIGOX = Yield stress in principal 1 direction

SIGOY = Yield stress in principal 2 direction

SIGOZ = Yield stress in principal 3 direction

SIGOXY = Shear yield stress in principal 1-2 plane

SIGOYZ = Shear yield stress in principal 2-3 plane

CARD 6: FORMAT (5E15.7)

SIGOZX = Shear yield stress in principal 3-1 direction

RMOSN = If RMOSS $\neq 0$; RMOSN = n, the shape parameter used in Ramberg-Osgood representation of stress-strain behavior.

If RMOSS = 0; RMOSN = α the slope of the linear strain-hardening stress-strain representation, i.e., $\alpha = E_T/E$ where E_T is the tangent modulus.

RMOSS = If RMOSS $\neq$ 0; RMOSS = Ramberg-Osgood parameter $\sigma_{0.7}$

Note (1): If RMOSN = 0 and RMOSS = 0 the material for the element(s) is assumed to be elastic-ideally plastic.

YLDST = Yield stress (for isotropic materials)

Note (2): Only initially isotropic materials can be treated when considering linear or nonlinear strain-hardening.

RMOSE = Ramberg-Osgood parameter E (Young's modulus)

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

Note: The transformation between the 1, 2, 3 material axes and the local element x, y, z axes are defined by path dependent angular rotations β_1, β_2, β_3. First a rotation about the 1 axis, β_1, then a rotation about the 2 axis in the transformed system and then a rotation, β_3 about the 3 axis in the transformed system

ACUM - Table describing yield stress versus accumulated plastic strain relationship

FORMAT (A4/A4, 1X, I5, /, A4, 1X, 2E15.7, /, (16I5))

CARD 1:

ACUM - four character identifier

CARD 2:

ACUM - four character identifier

NUMP - number of pairs of points used to represent the yield stress versus accumulated plastic strain

CARD 3:

ACUM - four character identifier

VARSIG - yield stress

SUMEPS - accumulated plastic strain

Card 3 is repeated "NUMP" times where NUMP is the number of pairs in the table. A maximum of 20 pairs is allowed.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

GAUSS or LOBATTO - Designation of stress points within an element

CARD 1: FORMAT (A4, 1X, 3I15)

or { GAUSS = five letter character identifier
LOBAT = five letter character identifier }

NX = number of stress points in local x direction

NY = number of stress points in local y direction

NZ = number of stress points in local z direction

CARD 2: Applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

SEND in columns 1-4 ends the section.

Note: Up to 8 Gauss or 10 Lobatto points are permitted in any one direction and a total of 50 points are permitted in any one element. For local coordinate definitions see Figure 2. For a definition of these integration points see Ref. 7.

XI. Applied Loads

Three different types of loading are currently admissible — concentrated loads at nodes, distributed loads on a surface of a hexahedron member, and thermal loads. Each loading situation is designated by one of the following four-character key words:

CONC	(Concentrated load)
SURF	(Surface distributed load)
TEMP	(Temperatures for thermal loading)

Distributed load -

CARD 1: FORMAT (A4)

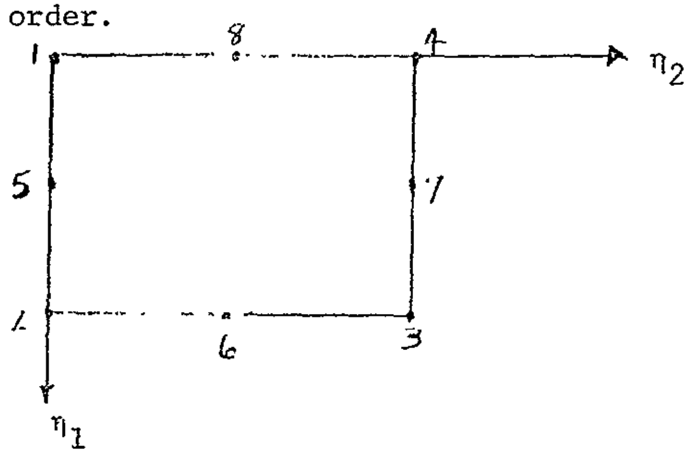
Key word SURF

CARD 2: FORMAT (I5, 5X, 8I5)

First I5 field:	number of nodes on the loaded surface. There is a minimum of four nodes per surface and a maximum of eight in order to accommodate midside nodes.
Second through fifth I5 fields:	These four fields contain the four corner node numbers in counterclockwise or clockwise order.

Sixth through ninth I5 fields: These four fields contain the midside node numbers (if any). If a particular midside node is omitted, a zero or blank is placed in the appropriate field on the input card.

The input order for a given surface is shown below for counter-clockwise order.



The magnitude of the surface tractions at each node specified above follow: There is one card for each node specified.

SUCCEEDING CARDS: FORMAT (I5, 3E15.7)

NUM = Node number specified on CARD 2 (I5 field)

TX = Surface traction in global x-direction

TY = Surface traction in global y-direction

TZ = Surface traction in global z-direction

A blank card ends the input for surface loads.

Note: The keyword SURF appears only once at the beginning of the specification of surface loads. Additional input for surface loads begin with card 2.

Concentrated load -

CARD 1: FORMAT (A4)

Key word CONC

CARD 2: FORMAT (I5, 3E15.7)

NUM = Node number at point of application of load (I5 field)

FX = Load component in the global x-direction (first E15.7 field)

FY = Load component in the global y-direction (second E15.7 field)

FZ = Load component in the global z-direction (third E15.7 field)

As many load components as desired may be specified. A zero or blank first I5 field ends the specification of concentrated loads.

Note: The keyword CONC appears only once at the beginning of the specification of concentrated loads. Additional input for concentrated loads begin with card 2.

Thermal load -

Each card after the first designates the temperature increment from some reference temperature followed by its applicable nodes. The input format is identical to that described for the input of nodal coordinates in Section V.

CARD 1: FORMAT (A4, 1X, E15.7)

Key word TEMP

TREF = Reference temperature in E15.7 field

The succeeding cards are as in Section V for nodal coordinates. That is, the nodal temperature in the first E15.7 field followed by the applicable nodes. A blank card ends the specification of nodal temperatures.

Note: The nodal temperature specified at the corner nodes of an individual hexahedra are interpolated to obtain the temperature used in calculating stresses and strains at the stress points.

SEND in columns 1-4 ends the section for applied loading.

XII. Members to be Printed - FORMAT (16I5)

Specify the members whose strains and stresses are to be printed. Both shorthands of Section III are allowed. A maximum of 2500 members may be specified. Members in excess of 2500 and undefined member numbers are ignored. A blank card or card with only zero entries ends the section.

XIII. Nodes to be Printed - FORMAT (16I5)

Up to 2500 nodes whose displacements are to be printed for the analysis, as per Section XII. A blank card ends the section.

XIV. Parameters for Plastic Analysis - FORMAT (2E15.7)

PMAX = Maximum load to be applied for this half cycle

PPCT = Load increment as percentage of yield load

XV. Parameters for Succeeding Load Cycles - FORMAT (A5, I5, 2E15.7)

CARD 1: FORMAT (A5)

Keyword defining the load type as follows:

CYCLE = Same load distribution as last half cycle

NEW = New load distribution to be specified

If any other word (or a blank card) is used the analysis is complete.

CARD 2: FORMAT (I5, 2E15.7)

NPRNT = If equal to zero, no additional load cycle and end of problem. If greater than zero, print output every NPRNT increments.

PMAX = New maximum load for this load cycle

PPCT = Load increment as percentage of yield load.

Note: When CYCLE is specified, the original load distribution is maintained. In this case PPCT should be set equal to one-half the original value since the elastic range for subsequent cycles is twice the elastic range for initial loading.

XVa. If NEW is used in Section XV a new load must be specified. This is accomplished with input identical to Section XI - Applied Loads. If CYCLE is specified in Section XV this section is omitted.

XVI. Change Plastic Material Properties When Cycle is Specified

CARD 1: FORMAT (A4, 1X, I5)

MAT1 = Key word

IMEM = One member number from the group that is to be changed.

CARD 2: FORMAT (5E15.7

SIGOX
SIGOY
SIGOZ
SIGOXY
SIGOYZ

} Defined in Section X

CARD 3: FORMAT (5E15.7)

SIGOZX
RMOSN
RMOSS
YLDST
RMOSE

} Defined in Section X

The section is ended with a section end card, SEND.

Note: If there is no change in material properties, a SEND card must appear in the appropriate place in the input stream. Succeeding load reversals (CYCLE) or new loads (NEW) are specified with additional input beginning with Section XV. A blank card (or any alphnumeric other than CYCLE or NEW) branches to Section XVII to end the job.

XVII. Problem End - FORMAT (A4)

Each problem's input must be ended with a card reading "ENDb" where "b" denotes a blank, in columns 1-4. The last problem in a run should end with a card reading "STOP" in column 1-4 instead of "ENDb."

XVIII. Restarting a Problem

The initial restart tape is created on Unit 21. Subsequent restart jobs mount the restart tape as Unit 21 and if desired (NUTAP > 0) copies and continues the restart tape on Unit 22.

CARD 1: Problem Title - FORMAT (20A4)

As in Section I

CARD 2: NPNTC, NPRNT, IRESRT, NUTAP, INPRT - FORMAT (5I5)

As in Section II. Here NPNTC is ignored.

If IRESRT is equal to 2

Restart from an elastic critical load using a previously created restart tape

CARD 3: PMAX, PPCT - FORMAT (2E15.7)

PMAX = Maximum load to be applied for this half cycle

PPCT = Load increment as percentage of yield load

Input is then continued from Section XV

If IRESRT is equal to 3

Continuation from some previously plastic load. The job can be restarted from a previous maximum load or from any intermediate load level.

CARD 3: PMAX, NRSRT - FORMAT (E15.0, I5)

If starting from a previous maximum load NRSRT can be ignored. If starting from some intermediate load level, NRSRT is a unique number obtained as output from the job that generated the restart tape. This output is of the form,

* * * * *

PLASTIC ANALYSIS VALUES FOR RESTART TAPE HAVE BEEN
WRITTEN AT P = 1.5000000E+03 FOR PRINT INTERVAL 30

* * * * *

In this case NRSRT is equal to 30 and the load is not necessarily the last maximum value.

Note: For cases when the job ends abnormally (for example when time or lines are exceeded) the job can be restarted with a value for NRSRT obtained from the output. In this case the last value of NRSRT should not be used. This will insure that a complete set of data will be accessed from the restart tape.

If IRESRT is equal to 4

Restart from a previous maximum load level followed by application of a new load (NEW) or reversal of a previous load distribution (CYCLE). A new load or a reversal of a previous load distribution can be placed on the structure from any previously generated and saved state of stress.

CARD 3: PMAX, NRSRT - FORMAT (E15.0, I5)

If starting from a previous maximum load NRSRT can be ignored. If starting from some intermediate load level, NRSTR is a unique number obtained as output from the job that generated the restart tape. This output has the form shown in the preceding discussion for IRESRT equal to three. The value of PMAX is ignored.

Succeeding cards follow as in Sections XV through XVII.

XIX. Alternate Input Method

All or part of the input may be read from a tape. This tape is mounted as Unit 23 with the appropriate job control cards. The tape is accessed by specifying the keyword, TAPE (FORMAT A4), in the proper place in the card input stream. Input will then be read from the tape until a SEND (A4) is encountered on the tape. This returns the input reader to the card input stream.

Example:

Input for member connectivity, Section IV, and coordinates have been generated on a tape in the prescribed manner.

Input cards are as follows:

TITLE

0 5

1 - 100

TAPE (reads data from a tape)

HEX8

.   (additional elements if desired)

.

.

SEND

TAPE   (reads X-coordinates from tape)

(Blank card to end section for X-coordinates)

TAPE   (reads Y-coordinates from tape)

(Blank card to end section for Y-coordinates)

5.0   (card input for Z-coordinates

.

.

.

(Blank card ends section for Z-coordinates)

If no additional tape input, remaining card input as usual.

Note: All SEND formats used to end sections must be in card form as the SEND encountered on the tape only sends the reader back to the card input stream. Also blank cards which end sections are best not put on the tape but rather on cards as this allows the user to add card input to tape input, i.e., if a blank ends a section and is encountered on the tape format then the section ends and no new data may be added, but if the blank does not occur on tape the section is still open and can be closed with a blank card in the card input stream.

REFERENCES

1. Zienkiewicz, O, et al., "Iso-Parametric and Associated Element Families for Two and Three Dimensional Analysis," Finite Element Methods in Stress Analysis, TAPIR, Technical University of Norway, Trondheim, 1969.

2. Levy, A., "A Three Dimensional 'Variable Node' Isoparametric Solid Element," Grumman Research Department Memorandum RM-587, July 1974.

3. Armen, H., Levine, H., Pifko, A., and Levy, A., "Nonlinear Analysis of Structures," NASA Contractors Report CR-2351, March 1974; also Grumman Research Department Report RE-454, May 1973.

4. Prager, W., "The Theory of Plasticity: A Survey of Recent Achievements," (James Clayton Lecture) Proc. Inst. Mech. Engrs., Vol. 169, p. 31, 1955.

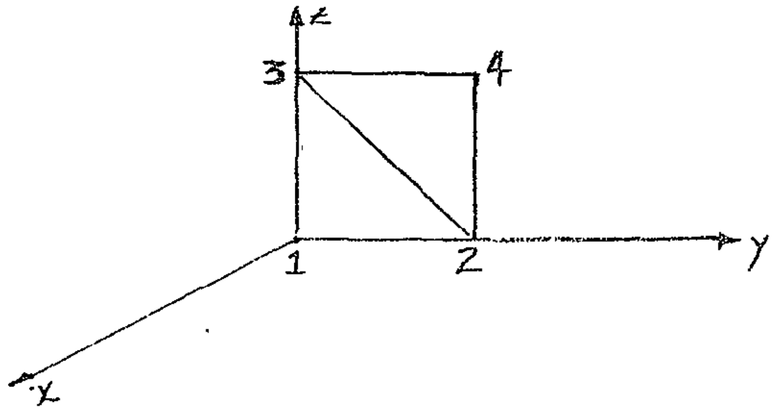
5. Ziegler, H., "A Modification of Prager's Hardening Rule," Quart. Appl. Math., Vol. 17, No. 1, p. 55, 1959.

6. Armen, H., Jr., Pifko, A., and Levine, H., "Finite Element Analysis of Structures in the Plastic Range," NASA Contractor Report CR-1649, February 1971.

7. Abramowitz, M. and Stegun, I. A., Eds., Handbook of Mathematical Functions, National Bureau of Standards, Applied Mathematics Series, 1964.

APPENDIX I — MULTIPOINT CONSTRAINTS

EXAMPLE 1



Node 1 and 2 are fixed.

There is an applied displacement at node 3 in the negative y direction. The distance between nodes 3 and 4 remains constant.

Boundary Conditions

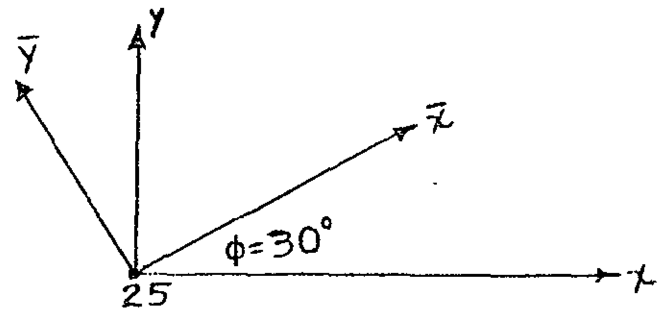
011000	1-4	default condition
001000	1 2 4	
021000 ——	3	
Blank card		

Dependencies

Node	Dof	Node	Dof		
3	2	3	2	-.050	(specified displacement)
4	2	3	2	1.0	restraint condition that
4	3	3	3	1.0	3-4 remain rigid

Note: In the boundary condition cards for node 4, degrees of freedom 2 and 3 must be specified 0 because they are effectively eliminated from the solution (they are dependent degrees of freedom).

EXAMPLE 2 Rotation of Displacements at a Boundary



For this problem we set the displacement $\bar{u}$ equal to zero and $\bar{v}$ free. This corresponds to the normal displacement along $\bar{x}$ equal to zero and the tangential displacement (in $\bar{y}$ direction) free. The global coordinates of the problem are x, y. Thus:

$$u = \bar{u} \cos \varphi - \bar{v} \sin \varphi$$

$$v = \bar{u} \sin \varphi + \bar{v} \cos \varphi$$

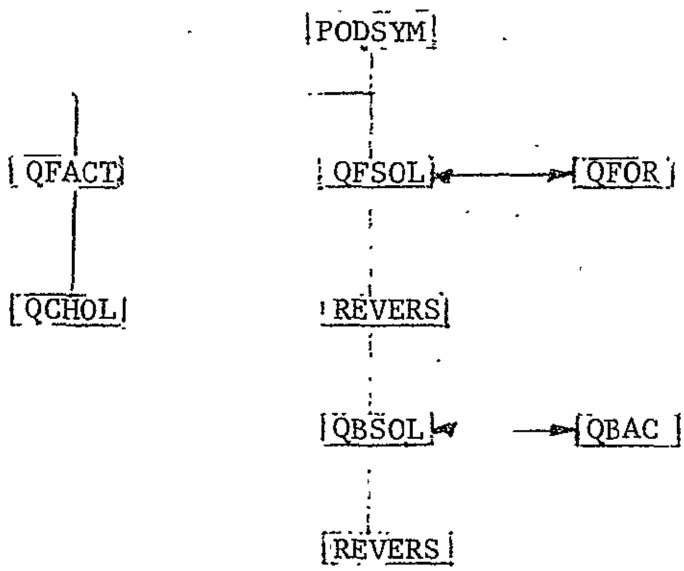
The multipoint constraint cards are:

25	1	25	1	.86602540E00	25	2	-.50E00
25	2	25	1	.50E00	25	2	.8660254E00

Note: On the boundary condition cards, since $\bar{u} = 0.0$ node 25 degree of freedom 1 is specified to be 0 but node 25 degree of freedom 2 is specified 1 or free since it remains an independent degree of freedom (although rotated). If the normal displacement $\bar{u}$ were not fixed (and the transformation performed merely to apply a normal load), it would have a 1 boundary condition specified.

APPENDIX II — SOLUTION PACKAGE

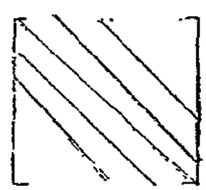
PODSYM - Solution of Symmetric Positive Definite Banded Matrix Equations



Problem

Solve $AX = Y$, where

A =



is a banded positive definite symmetric matrix; X is the desired solution vector; and Y is the known right side (load vector). PODSYM is the user interface and supervisory routine. It uses the Cholesky algorithm which factors the total stiffness matrix into LL^T (where L is a lower triangular matrix) and then solves a pair of triangular sets of equations.

The factorization is carried out by subroutine QFACT which supervised the storage and data set allocation and subroutine QCHOL which generates the lower triangular L matrix. QCHOL implements the Cholesky algorithm to factor the positive definite symmetric A matrix as the product of a lower triangular matrix with its transpose:

$$A = LL^T$$

A straightforward argument establishes the possibility of decomposing any positive definite matrix in this fashion. Once L has been obtained, it is not difficult to solve the system of linear equations $AX = Y$ by calculating Z as the solution of the lower triangular system $LZ = Y$, and then X as the solution of the upper triangular system $L^TX = Z$. The forward solution ($LZ = Y$) is accomplished by subroutines QFSOL and QFOR and the back solution ($L^TX = Z$) by subroutines QBSOL and QBAC. Before the call to QBSOL, subroutine REVERS is called, which reverses the rows of L so that the last row becomes the first row, etc. This is accomplished in order to sequentially access L^T during the back solution.

The above algorithm is noteworthy for its assured stability and general efficiency. It is possible to carry out an error analysis of the procedure as it is represented on a digital computer; such analysis shows that the computed L matrix satisfies an equation of the form

$$A + E = LL^T$$

with bounds on the elements of E which show that E is small compared to A. The effect of rounding errors made in the subsequent solution of $LZ = Y$ and $L^TX = Z$ may then be taken into account by (implicitly) introducing an additional perturbation F into A, and it is then concluded that the computed solution X_o exactly satisfies the equation

$$(A = E + F)X_o = Y$$

Since $E + F$ can be shown to be small, one would like to infer that X_o is almost an exact solution of the original equations, and unless A is too nearly singular, such a conclusion is indeed warranted. But if A is very ill-conditioned, no such result can be guaranteed, and X_o may be far from the mathematically correct solution; in this event single-precision computation will not suffice for the calculation of an accurate solution, and since the solution will be very sensitive to small errors in A, it is unlikely that even a high-precision computation will yield satisfactory results unless A and Y are known (and supplied) to more than single-precision accuracy. The PODSYM subroutines make a fairly realistic attempt to detect and report pathological conditions of this sort.

The large positive definite matrices that occur in practical work very often contain a large number of zero entries and the program seeks to benefit from the presence of these elements by modifying the standard Cholesky formulae

$$\ell_{kk} = \left[A_{kk} - \sum_{j=1}^{k-1} \ell_{kj}^2 \right]^{\frac{1}{2}}$$

and

$$\ell_{ik} = \left[A_{ik} - \sum_{j=1}^{k-1} \ell_{ij} \ell_{kj} \right] / \ell_{kk} \quad \text{for} \quad i > k$$

to read instead

$$\ell_{kk} = \left[A_{kk} - \sum_{j=\nu(k)}^{k-1} \ell_{kj}^2 \right]^{\frac{1}{2}}$$

and

$$\ell_{ik} = \left[A_{ik} - \sum_{j=\mu(i,k)}^{k-1} \ell_{ij}\ell_{kj} \right] / \ell_{kk}$$

SECTION 4

INSTRUCTIONS FOR THE USE OF REVBY

A Program for the Elastic-Plastic Analysis of Bodies of Revolution

INSTRUCTIONS FOR THE USE OF REVBY

REVBY is a finite element program to treat the elastic, elastic-plastic, or elastic-cyclic plastic response of orthotropic axisymmetric solids of revolution under axisymmetric loadings. Currently, three different types of elements are available for the analysis of thick and thin bodies of revolution:

1. Revolved Triangular Element

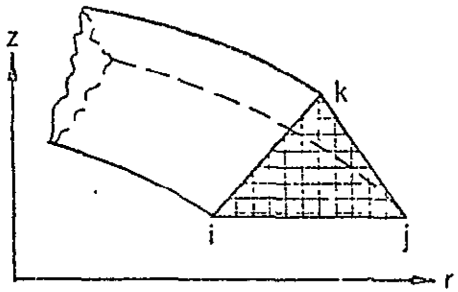
   The revolved triangular element used is similar to the element presented in Ref. 1, restricted to axisymmetric loading, i.e., independent of the circumferential direction, but including orthotropic material properties (see Fig. 1).

2. Thin Shell Element

   The thin shell element used was developed at Grumman and the theoretical derivations are presented in Refs. 2 and 3. Cubic Hermitian polynomials represent the three displacement components. Again, only axisymmetric loading is allowed but orthotropic material properties are included (see Fig. 2).

3. Thin Ring Stiffener Element

   The thin ring stiffener element is similar to the one presented in Ref. 4 with the exception that the ring may be attached with arbitrary eccentricities. No warping of the cross section is allowed and the shear center and centroid must coincide. Five different cross-sectional geometries are available.



Displacement Assumption:

$$u_r = a_1 + a_2 r + a_3 z$$

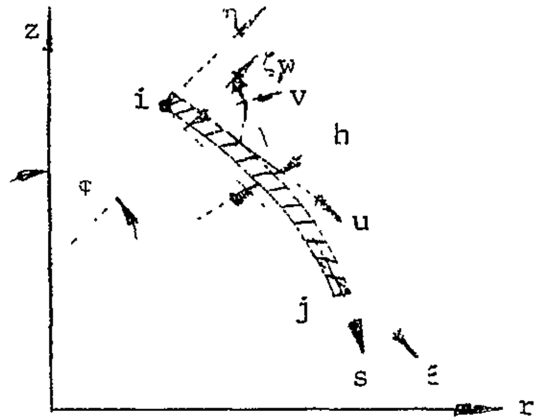
$$u_z = a_4 + a_5 r + a_6 z$$

$$u_\theta = a_7 + a_8 r + a_9 z$$

Initial Strain Distribution:

$$\epsilon_{ij}(r,z) = \text{constant}$$

Fig. 1 Axisymmetric Revolved Triangle



Displacement Assumption:

$$u = a_1 + a_2\xi + a_3\xi^2 + a_4\xi^3$$

$$w = b_1 + b_2\xi + b_3\xi^2 + b_4\xi^3$$

$$v = c_1 + c_2\xi + c_3\xi^2 + c_4\xi^3$$

Initial Strain Distribution:

$$\begin{Bmatrix} \epsilon_s \\ \epsilon_\theta \\ \gamma_{s\theta} \end{Bmatrix} = (1-\xi)\begin{Bmatrix} \epsilon_s^i(\zeta) \\ \epsilon_\theta^i(\zeta) \\ \gamma_{s\theta}^i(\zeta) \end{Bmatrix} + \xi\begin{Bmatrix} \epsilon_s^j(\zeta) \\ \epsilon_\theta^j(\zeta) \\ \gamma_{s\theta}^j(\zeta) \end{Bmatrix}$$

Fig. 2 Axisymmetric Thin Shell Element

The plasticity theory used is presented in Refs. 5, 6, and 7. The input to the program is categorized in the following sections.

I. Problem Title (20A4)

Any 80-character title describing the problem.

II. NPNTC and NPRNT (2I5)

NPNTC: is for selective printing. NPNTC is the sum of the following integers corresponding to the options desired.

If NPNTC =	0,	no intermediate output
=	1,	print total load vector
=	8,	print nonzero element stiffness matrix entries with their stacking indices
=	16,	print the total stiffness matrix

NPRNT:

If $\leq$ 0,	perform elastic analysis only
If > 0,	perform elastic and plastic analysis, printing the results of the plastic analysis every NPRNT load increments

III. Nodes by Partition (16I5)

This section defines an allowable set of external node point numbers. The maximum node number that can be used is 9999. The program uses this information in two ways. First to set up a table of allowable node points that is used to check all subsequent node point input. Secondly, the program converts each external node number to an internal number consecutively in the order that the node appears on the input card. Consequently the order of the input of external node numbers is completely arbitrary and need not be increasing monotonically. In practice the node numbers should be numbered so as to minimize the bandwidth. Once the input is read the program operates with the internal numbers which are now numbered from 1 through the number of nodes in the model. In this manner the node ordering and therefore the bandwidth of the stiffness matrix can be easily changed and nodes can be inserted or deleted by changing the external node specification.

The input is specified by entering the appropriate number on the input cards in fields of five. However, for this purpose the user can also utilize a shorthand form of the input. That is, specifying m and -n consecutively is the equivalent of the specification of nodes m, m + 1, m + 2, ..., n and specifying m, -p, and -n consecutively is the equivalent of the specifiction of nodes m, m + p, m + 2p, ..., m + kp where m + kp is the highest integer of the form less than or equal to n. For example, the specification of nodes 1 through 100 is written as 1 - 100 and 1, 3, 5, ..., 99 as 1-2-99. This card input appears in fields of 5 (I5 Format) with 16 items per card. Any number of continuation cards may be used. A zero or blank I5 field ends the specification.

IV. Node Numbers for Members (A4,6X,4I5)

The first field defines the element type:

TRIR - triangular element designation (Ref. 1)

SHEL - shell element designation (Refs. 2, 3)

RING - ring element designation (Ref. 4)

The first integer field designates a member number and the three following fields give node numbers for the member.

For triangular members, three nodes are specified. The i, j, k nodes are given in counterclockwise order. For shell members, two nodes are specified, i and j, in the positive s direction. For ring members, only one node is specified. SEND in the first field ends the section.

Note: Maximum number of elements (members) allowed is 600 currently.

V. Global Coordinates (E15.7,13I5)

The coordinates are read in a specified order, r first, then z, and last the PHI coordinates. PHI is the angle in radians between the z-axis and the normal to the shell surface, measured positive clockwise (Fig. 2) from the z-axis. The floating point number designates a coordinate value. Following are the applicable node numbers as in Section III.

Only the integer fields are used on continuation cards. A zero or blank node field other than the first field on a noncontinuation card ends the card scan. On a continuation card, the floating field is left blank. A zero or blank first node field on a noncontinuation card ends each section for r, z, and PHI. If there are no shell elements, PHI is set to zero by default and need not be specified.

VI. Boundary Conditions (6I1,4X,14I5)

The first six fields give a boundary condition specification, in the order:

1) u_r, u_z, u_θ, if the node is attached to a triangular member. The last three fields for such a node must be zero.

2) u, w, χ, ϵ_s, v, $\gamma_{s\theta}$, if the node is attached to a shell element. See Fig. 2 for convention. u is the meridional displacement of the shell middle surface, w is the normal displacement, χ is the rotation of a normal to the middle surface, ϵ_s is the meridional membrane strain, v is the circumferential displacement, and $\gamma_{s\theta}$ is the shear strain. At a pole, the condition $u_r = 0$, $\chi = 0$, $v = 0$, $\gamma_{s\theta} = 0$, must be prescribed.

3) u, w, χ if the node is attached to a ring member. The last three fields must be zero.

Zero - denotes a fixed degree of freedom

One - denotes a free degree of freedom

Two - denotes an applied displacement and a corresponding card must be included in Section VII, designating the magnitude of the applied displacement.

The integer fields are the applicable node numbers as in Section III. On continuation cards, only the integer fields are used. If a node's boundary conditions are not specified in this section, all six degrees of freedom are assumed free. To change the default, the first card should be set to the desired default (i.e., for triangles 111000) with all nodes used in the problem then specified.

Blank card ends section.

Note: Maximum number of nodes currently allowed is 900.

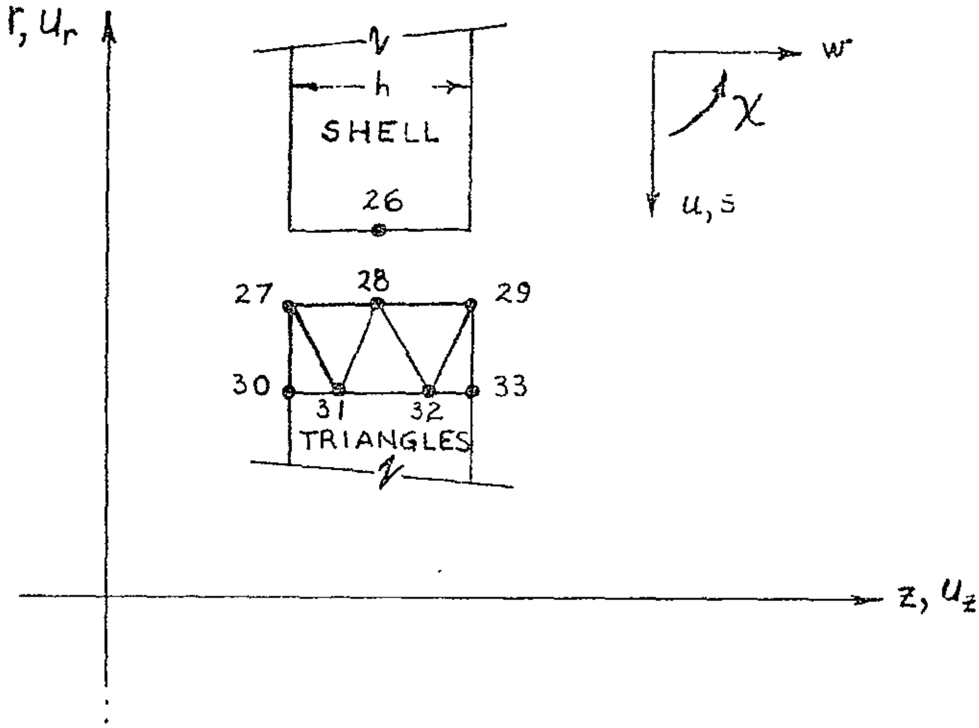
VII. Dependent Degrees of Freedom (2I5,2(5X,2I5,E15.7))

The first two fields designate a node number and its degree of freedom. The dependency is defined in the following three fields. The two integer fields are node number and degree of freedom. The coefficient is contained in the following field. If there is another dependency for the node, it would be designated in the last three fields.

If there is yet another dependency, it would appear on a continuation card with the first two integer fields blank. Both single and multipoint constraints of the form $\delta_i = \alpha_1\delta_{j1} + \alpha_2\delta_{j2} + \cdots + \alpha_n\delta_{jn}$ may be considered as well as applied displacements of the form $\delta_i = \alpha_1$, e.g.,

Blank card ends section.

1. Linking Up a Shell and a Triangle for a Cylinder



From shell theory restrictions

$$u_{r_{27}} = u_{r_{28}} = u_{r_{29}} = w_{26}$$

$$u_{z_{27}} = - u_{26} - h/2\ \chi_{26}$$

$$u_{z_{28}} = - u_{26}$$

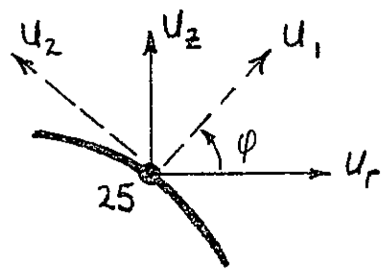
$$u_{z_{29}} = - u_{26} + h/2\ \chi_{26}$$

The input cards, for this case, would appear as (let h = 0.1 inches)

27	1	26	2	1.0 E00			
28	1	26	2	1.0 E00			
29	1	26	2	1.0 E00			
27	2	26	1	-1.0 E00	26	3	- 0.05 E00
28	2	26	1	-1.0 E00			
29	2	26	1	-1.0 E00	26	3	+ 0.05 E00

Note: In the boundary condition cards for nodes 27, 28, and 29, degrees of freedom 1 and 2 must be fixed (specified = 0) because they are effectively eliminated from the solution (they are dependent degrees of freedom).

2. Rotation of Displacements at Boundary



For this problem, we set the displacement normal to the boundary equal to zero. We have:

$$u_1 = u_r \cos \varphi + u_z \sin \varphi$$

$$u_2 = - u_r \sin \varphi + u_z \cos \varphi$$

Let us assume that the node in question is node 25 and $\varphi = 30$ degrees. The multipoint constraint cards are:

25	1	25	1	0.86602540	E00	25	2	0.50 E00
25	2	25	1	- 0.50	E00	25	2	0.86602540 E00

Note: On the boundary condition cards, since $u_1 = 0$, node 25 degree of freedom 1 is specified to be 0 but node 25 degree of freedom 2 is specified 1 (or free) since it remains an independent degree of freedom (although rotated). If the normal displacement u_1 were not fixed (and the transformation performed merely to apply a normal load) it would have a 1 boundary condition specified.

3. Applied Displacement

Suppose for a shell we wish to apply a unit lateral (w) displacement at node 25. We have the multipoint constraint card as

25 2 25 2 1.0 E00

Note: A 2-boundary condition for node 25 degree of freedom 2 is specified on the boundary condition card. The end of this section is designated by a blank or zero in the third integer field (e.g., blank card).

VIII. Material Properties

The format of this input is dependent upon the member type:

A. Triangle Elements (4(5E15.7),3E15 7,/,(16I5))

Card 1	E1	Young's modulus in 1 direction (if $\beta = 0$, θ-direction)
	E2	Young's modulus in 2 direction (if $\beta = 0$, z-direction)
	E3	Young's modulus in 3 direction (if $\beta = 0$, r-direction)
	V12	Poisson's ratio in 1-3 directions
	V23	Poisson's ratio in 2-3 directions
Card 2	V31	Poisson's ratio in 3-1 directions
	G12	Shear modulus in 1-2 plane
	G23	Shear modulus in 2-3 plane
	G13	Shear modulus in 3-1 plane
	β	Orientation of material properties axis with respect to θ-z coordinates (in degrees). This is a rotation about the r-axis and β is positive clockwise.

	SIGOX	Tensile (or compressive) yield stress in θ-direction
	SIGOY	Tensile (or compressive) yield stress in z-direction
	SIGOZ	Tensile (or compressive) yield stress in r-direction
Card 3	SIGXY	Shear yield stress in r-θ plane
	RMOSN	Ramberg-Osgood shape parameter (n), $\epsilon = \sigma/E + (3\sigma/7E)(\sigma/\sigma_{0.7})^{n-1}$. If set = 0.0 material for the group of members is assumed to elastic ideally plastic. If RMOSS = 0.0 and RMOSN $\neq$ 0, RMOSN is ratio of slope of stress-strain curve for linear strain hardening to Young's modulus ($\alpha = E_T/E$ where E_T is the tangent modulus).
	RMOSS	Ramberg-Osgood parameter $\sigma_{0.7}$. If zero and RMOSN not zero have linear strain hardening.
	YLDST	Yield stress in tension (or compression)
Card 4	RMOSE	Ramberg-Osgood parameter E (Young's modulus)
	SIGYZ	Shear yield stress in θ-z plane
	SIGZX	Shear yield stress in z-r plane
Card 5	TALPH (1) TALPH (2) TALPH (3)	3 thermal coefficients of expansion in 1, 2, 3 directions (θ, z, r if $\beta = 0°$)

Note: Orthotropic material properties should not be used for strain hardening problems in the current version of REVBY.

Succeeding cards give applicable members as in Section III. A zero or blank E1 ends the section. If there are no triangular members, no card is required.

B. Shell Elements ((4E15.7,/,5E15.7,/,3E15.7,/,2E15.7)/),(16I5))

1) Material Properties

Card 1	E1	Young's modulus in meridional direction
	E2	Young's modulus in circumferential direction
	G33	Shear modulus in plane of shell median surface
	V12	Poisson's ratio in plane of shell median surface
Card 2	SIGOX	Tensile or compressive yield stress in meridional direction
	SIGOY	Tensile or compressive yield stress in circumferential direction
	SIGOZ	Tensile or compressive yield stress in direction normal to shell surface
	SIGXY	Shear yield stress in plane of shell median surface
	RMOSN	(See Card 3 for triangular members)
Card 3	RMOSS, YLDST, RMOSE	(See Card 4 for triangular members)
Card 4	TALPH (1), TALPH (2)	Two thermal coefficients of expansion in meridional and circumferential direction, respectively

Succeeding cards give applicable members as in Section III. A zero or blank E1 ends the section. If there are no shell elements, no blank card is required.

2) Thickness for Shell Elements (follows the material properties. All nodes must have a thickness specified.)

(E15.7,/(16I5))

First field designates thickness value. Applicable nodes follow as in Section III. Section ends with zero or blank in first field.

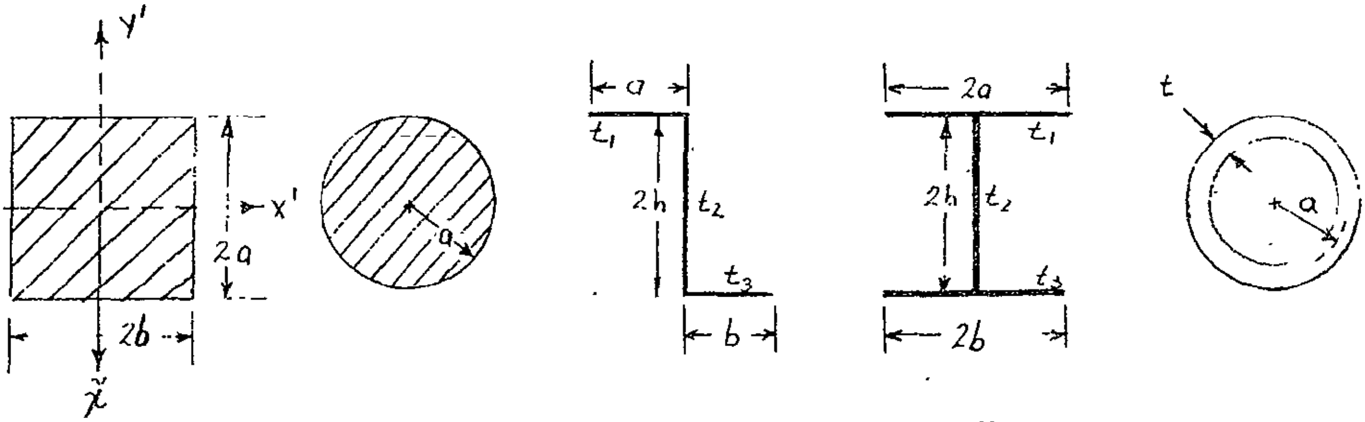
3) NG and NLRS (2I5). For shell elements, one card. NG designates the number of Gauss points used in integration scheme for shell element, and the second field, NLRS, designates number of layers through the thickness at which plastic strains, stresses, etc., are calculated. NG $\leq$ 8, NLRS must be even, and $\leq$ 30.

C. Ring Elements (5E15.7,/,2E15.7,/,A4.6X,3E15.7,/,3E15.7,/, 3E15.7,/,E15.7,(16I5))

Card 1	E	Young's modulus
	A	Cross sectional area of ring
	IX	Moment of inertia of ring about centroidal x'-axis (see diagram below)
	IXY	Product of inertia of ring about centroidal axis
	IY	Moment of inertia of ring about centroidal y'-axis

Card 2	ZO	Distance from shell middle surface to ring centroid (normal to shell middle surface). Plus in negative z-direction.
	ECCEN(Y)	Distance along meridian from ring centroid to point of attachment to shell middle surface. Plus in negative s-direction.
Card 3	SREC	Solid rectangular cross section
	SCIR	Solid circular cross section
	ZSEC	Z-section
	ISEC	I-section
	HCIR	Hollow circular cross section

Remaining three fields designate section dimensions as shown in the diagrams below.



SREC SCIR ZSEC ISEC HCIR

INPUT

	SREC	SCIR	ZSEC	ISEC	HCIR
Card 3	2a	a	a	2a	a
	2b		2h	2h	t
			b	b	
Card 4			t_1	t_1	
			t_2	t_2	
			t_3	t_3	

Note: For ZSEC, $a = b$, $t_1 = t_3$; for ISEC, $2a = 2b$, $t_1 = t_3$.

Card 4 — If section name is ZSEC or ISEC, this card is required for three additional dimensions, otherwise this card is omitted.

Card 5

SIGOX	Yield stress in tension or compression
RMOSN, RMOSS	Same as shell and/or triangle

Card 6 — TALPH (1) Thermal coefficient of expansion

Succeeding cards give applicable members as in Section III. A zero or blank E ends the section. If there are no ring elements, no card is required.

IX. Load Vector Components

This format is also dependent on member type.

A. Triangular Members (I5,3E15.7,/,I10,3E15.7)

The integer field gives node number (i-j pairs); the floating-point fields give components at the respective nodes as follows:

$p_r = 1.0$ corresponds to an applied unit pressure (force/unit area) in the radial direction at node i

$p_z = 1.0$ corresponds to an applied unit pressure (force/unit area) in the z-direction at node i

$p_\theta = 1.0$ corresponds to an applied unit pressure (force/unit area) in the θ-direction at node i

The load between any two adjacent nodes is assumed to vary linearly. Therefore, the load vector is specified by two cards, the first specifying the load at node i and the second, the load at node j. A zero or blank first integer field ends the section. If there are no uniform loads, a blank card is required. If there are no triangular elements, no card is required.

B. Shell Members (3E15.7,/,(16I5))

The three fields designate the meridional component, p_s, normal component, p_z, and the circumferential component, p_θ, of applied stresses at nodes. Applied stresses are assumed to vary linearly from node to node. The applicable nodes follow as in Section III. A blank card ends the section. If there are no uniform loads, a blank card is required. If there are no shell elements, no card is required.

C. Concentrated Loads (4E15.7,/,(16I5))

For triangle component, specify F_r, F_z, and F_θ in r, z, θ directions, and for the shell component specify N_s, Q_s, M_s, and $N_{s\theta}$ in u, w, χ, and v directions, respectively, in lb/in. These are forces per unit length of circumference, except at $r = 0$, where $F_r = F_\theta = 0$ and $N_s = M_s = N_{s\theta} = 0$ and F_z or Q_s represent an actual force (lbs).

Applicable nodes follow as in Section III. A blank or zero card ends the concentrated loads specification section.

X. Nodal Temperatures (A4,E15.7,/(A4,3E15.7,/,(16I5))

If the first field is SEND, there are no temperatures and this section ends. If the first field is blank, there are thermal loads and the floating number field designates the reference temperature value (T_{ref}), which is only for the user's information.

The next cards contain $\Delta T = T - T_{ref}$ at each node (up to three per node depending on the element to which it is attached) in the floating point fields. For a triangular element, one temperature is specified per node. For a shell element, three temperatures are specified, one at the top surface $(+ h/2)$, T_1, one at the median surface $(z = 0)$, T_2, and one at the bottom surface $(- h/2)$, T_3. The program assumes a parabolic distribution of temperature through the thickness (if $T_2 = (T_1 + T_3)/2$, this reduces to a linear variation). For a ring element, three temperatures are specified. A parabolic distribution is assumed through the ring depth. T_1 is the temperature at $\tilde{x} = +a$ or $+h$. T_2 is the temperature at the centroid, and T_3 is the temperature at $\tilde{x} = -a$ or $-h$ (see diagram in Section VIII).

Applicable nodes follow as in Section III.

XI. Members Whose Results are to be Printed for Elastic Solution (16I5) As in Section III

Blank cards end section.

XII. Nodes Whose Results are to be Printed for Elastic Solution (16I5) As in Section III

Blank cards end section.

XIII. Same as Section XI for Plasticity, if applicable.

Blank cards end section.

XIV. Same as Section XII for Plasticity, if applicable.

Blank cards end section.

XV. PMAX and PPCT (2E15.7)

PMAX - Maximum load to be applied for this half cycle

PPCT - Load increment as a percentage of yield load

XVI. NPRNT, PMAX, PPCT (I5,2E15.7)

For succeeding load cycles, one card giving new NPRNT, PMAX, and PPCT. Zero NPRNT signifies no new load cycle and end of problem. For succeeding half-cycles, PPCT equal to one-half the original value should be used since the elastic range is twice the elastic range for initial loading.

XVII. Change in Material Properties (A4)

To change properties of any group of members, read indicator for new material properties (A4).

A. If RING is input, it indicates a change in the RING element material properties. A card with a member number belonging to the group whose properties are to be changed and the new quantities are read in (I5,4E15.7).

	Member Number	Member of group to be changed
	SIGOX	
	RMOSN	New Values
	RMOSS	
	TALPH	

B. NEWM signifies new material properties for either triangle or shell elements. The new values are read in (I5,5E15.7,/, 5E15.7,/,3E15.7)

	Member Number	Member of group to be changed
Card 1	SIGOX	New Values
	SIGOX	
	SIGOZ	
	SIGXY	
	RMOSN	
Card 2	RMOSS	New Values
	YLDST	
	RMOSE	
	SIGYZ	
	SIGZX	
Card 3	TALPH (1)	New Values
	TALPH (2)	
	TALPH (3)	

C. SEND as input indicates no new material properties or end of this section.

XVIII. Each Problem's Input Must Be Ended with a Card Reading "ENDb" Where "b" Denotes a Blank, in Columns 1-4. The Last Problem in a Run Should End with a Card Reading "STOP" in Column 1-4 Instead of "ENDb."

References

1. Wilson, E. L., "Structural Analysis of Axisymmetric Solids," AIAA Journal, Vol. 3, pp. 2269-2274, 1965.

2. Levine, H. S. and Armen, H., Jr., "A Refined Doubly-Curved Axisymmetric Orthotropic Shell Element," Grumman Research Department Memorandum RM-496, February 1971.

3. Levine, H. S., Armen, H., Jr., Winter, R., and Pifko, A., "Nonlinear Behavior of Shells of Revolution Under Cyclic Loading," Grumman Research Department Report RE-426J, April 1972; also presented at the National Symposium on Computerized Structural Analysis and Design, Washington, D.C., March 27-29, 1972; to appear in Journal of Computers and Structures.

4. Cohen, G., "Computer Analysis of Asymmetric Buckling of Ring-Stiffened Orthotropic Shells of Revolution," AIAA Journal, Vol. 6, No. 1, 1968.

5. Isakson, G., Armen, H., Jr., and Pifko, A., "Discrete Elements Methods for the Plastic Analysis of Structures," NASA CR-803, 1967.

6. Armen, H., Jr., Pifko, A., and Levine, H. S., "Finite Element Analysis of Structures in the Plastic Range, NASA CR-1649, 1971.

7. Armen, H., Jr., Levine, H., and Pifko, A., "Plasticity-Theory and Finite Element Applications," Grumman Research Department Report RE-438J, August 1972; also presented at the Second Japanese-U.S. Conference on Matrix Methods, Berkeley, California, August 14-18, 1972; published in Advances in Computational Methods in Structural Mechanics and Design, University of Alabama Press, Huntsville, Alabama.

SECTION 5

INSTRUCTIONS FOR USE OF
OUT-OF-PLANE GM

A Program for the Nonlinear
Analysis of Built-Up Structures

INSTRUCTIONS FOR USE OF OUT-OF-PLANE GM

OUT-OF-PLANE GM is a finite element program for the nonlinear analysis of arbitrary built-up thin walled structures. This program considers the sheet material to be in states of membrane stress. The finite element library consists of the following elements:

- Three-node uniform stress triangle
- Six-node linearly varying stress triangle
- Four- and five-node hybrid triangles to be used as transition elements between three- and six-node elements (see Fig. 1a)
- Two-node uniform stress stringer
- Three-node linearly varying stress stringer
- Beam with various cross sections subjected to bending about two planes as well as torsion

The program is capable of treating the elastic and the elastic-ideally plastic response of orthotropic materials. In addition, consideration is given to isotropic materials exhibiting elastic-ideally plastic, linear strain hardening, or nonlinear strain hardening behavior. Further, the kinematic hardening theory of plasticity is used.

The effects of geometric nonlinear behavior is treated by considering a convected coordinate system that accounts for changes in geometry and by the introduction of an initial stress stiffness matrix. The solution strategy implemented by the program is outlined in a companion volume, "PLANS - A Finite Element Program for Nonlinear Analysis of Structures, Volume I - Theoretical Manual," NASA Contractor Report NASA CR-2568, p. 31.

The input to the program is categorized in the following sections.

I: Problem Title FORMAT (20A4)

Any 80-character title describing the problem.

II. Load and Print Control Parameters FORMAT (6I5,/4E15.7)

CARD 1:

NPNTC is the sum of the following integers corresponding to the option desired.

If NPNTC	= 0	No intermediate printout
	= 1	Print the load vector
	= 2	Print element stiffness matrix
	= 16	Print each element stiffness matrix entry to be stacked with its stacking index
	= 32	Print the total stiffness matrix

For example, if it is desired to print the load vector and the total stiffness matrix, NPNTC = 1 + 32 = 33.

NPRNT $\geq$ 0	Print output every NPRNT increments of load. If NPRNT equals zero the output is printed every load step.
NFORM $\geq$ 0	Reform elastic stiffness matrix to account for changing geometry every NFORM increments of load. The effect of the geometric (initial stress) stiffness matrix is still introduced as an "effective load" in this case.

IREST:	IREST = 0	Do not generate a restart tape
	IREST = 1	Build a restart tape
	IREST = 2	Run continuing from a created restart tape

NUTAP: Applicable only if restarting from a restart tape. When IREST = 2 and NUTAP = 0 no additional restart data is written. If NUTAP · 0 restart tape is continued.

INPRT:
- INPRT = 0 Write restart tape only at P = PMAX (i.e., at maximum load)
- INPRT = N Restart data written every INPRT time steps

NRSRT: Applicable only if restarting from a restart tape. NRSRT is a unique number that is printed in the job that generated the restart tape for <u>each load</u> that data is written on the restart tape. This number defines the starting load for the continuation job (see Section XVII).

CARD 2:

PMAX: Maximum load factor. The load factor multiplies the vector of applied load. If the applied loading is normalized to unity then PMAX represents the magnitude of the largest component of the applied load. If the maximum desired load is input then PMAX is equal to one and each load step represents a percentage of the total load.

DELP: Number of load increments. The load increment ΔP is specified by PMAX/DELP.

PTAN: Load at which full tangent modulus method is used, i.e., the stiffness matrix explicitly includes the initial stress stiffness matrix. If the "tangent modulus" method is to be used for the entire analysis, PTAN = 0.0. In this case the stiffness matrix is reformed in every load step. If the "effective load" method is to be used for the entire analysis, PTAN = PMAX. In this case the stiffness matrix includes only the elastic stiffness matrix and is reformed every NFORM load steps. When 0 < PTAN < PMAX then the program uses the "effective load" method up to PTAN and then implements the "tangent modulus" until the maximum load is reached.

EQMULT: Factor multiplying the equilibrium correction term. EQMULT = 0.0 indicates no equilibrium correction and EQMULT = 1.0 indicates full equilibrium correction.

III. Node Specification (16I5)

This section defines an allowable set of external node point numbers. The maximum node number that can be used is 9999. The program uses this information in two ways. First to set up a table of allowable node points that is used to check all subsequent node point input. Secondly, the program converts each external node number to an internal number consecutively in the order that the node appears on the input card. Consequently the order of the input of external node numbers is completely arbitrary and need not be increasing monotonically. In practice the node numbers should be numbered so as to minimize the bandwidth. Once the input is read the program operates with the internal numbers which are now numbered from 1 through the number of nodes in the model. In this manner the node ordering and therefore the bandwidth of the stiffness matrix can be easily changed and nodes can be inserted or deleted by changing the external node specification.

The input is specified by entering the appropriate number on the input cards in fields of five. However, for this purpose the user can also utilize a shorthand form of the input. That is, specifying m and -n consecutively is the equivalent of the specification of nodes m, m + 1, m + 2, ..., n and specifying m, -p, and -n consecutively is the equivalent of the speeifiction of nodes m, m + p, m + 2p, ..., m + kp where m + kp is the highest integer of the form less than or equal to n. For example, the specification of nodes 1 through 100 is written as 1 - 100 and 1, 3, 5, ..., 99 as 1-2-99. This card input appears in fields of 5 (I5 Format) with 16 items per card. Any number of continuation cards may be used. A zero or blank I5 field ends the specification.

IV. Member Connectivity (Node Number of Each Member)
FORMAT (A4, 6X, 9I5)

The first alphanumeric field defines the element type:

TRIM - Triangular membrane element (Ref. 3)

BEAM - Beam element

STRG - Stringer element

The first integer field designates the member number. The next integer fields designate the connecting nodes as follows:

TRIM - The nodes for the triangular family of elements are specified around the perimeter beginning with a major (vertex) node. Then a minor (midside) node and then alternatively major and minor as shown in Fig. 1. The absence of a minor node must be indicated by a zero or blank field in the proper position.

BEAM - Three node specifications are necessary for the beam element.

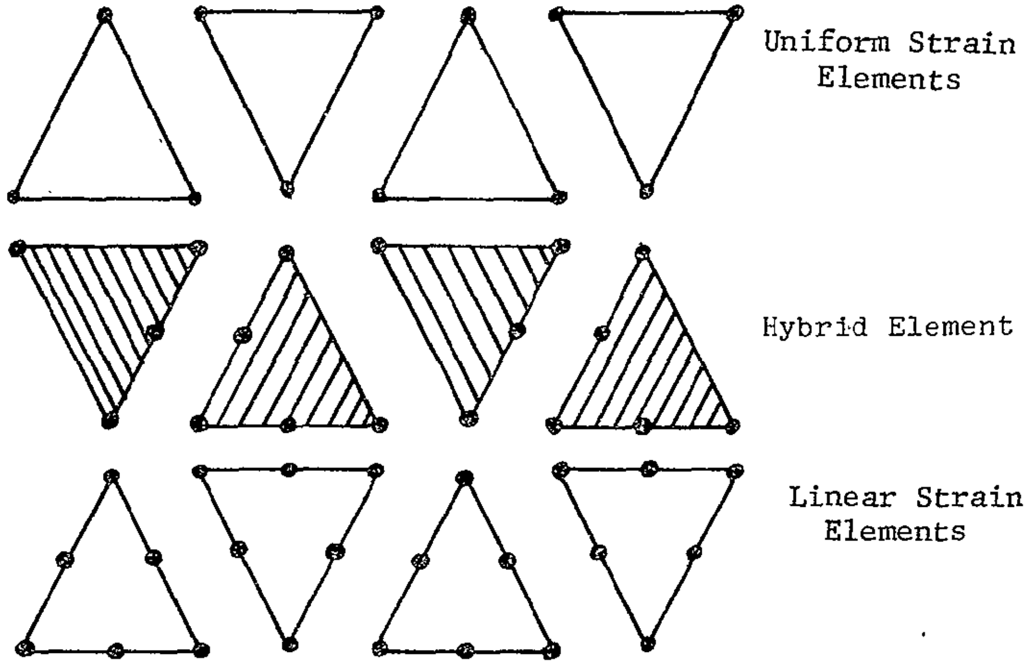


Fig. 1a Triangular Family of Finite Elements Used

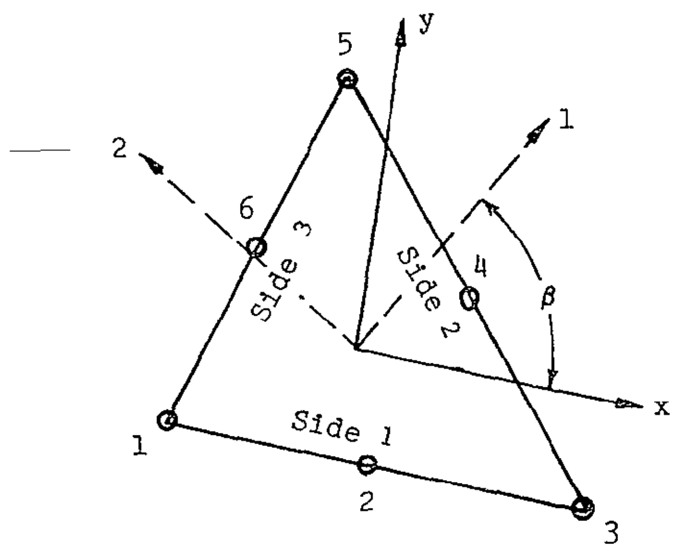


Fig. 1b Elements Topology

Nodes i and j (Fig. 2) which designate the element end points and a third node k, defining the normal to the beam axis about which the section properties are specified. This additional node may be a node of the structural idealization or it may be a "fictitious node" specified just for the purpose of defining the beam section properties. This is shown in Fig. 2.

STRG - Three node specifications are necessary for the stringer element. Nodes i and j, connecting the end points, and if desired an additional node designating a midpoint node. This is shown in Fig. 3. A zero or blank for the midpoint node specifies a two-node stringer. The midside node is the third node specified.

SEND Ends the section

V. X-Coordinates of Nodes FORMAT (E15.7, 13I5)

The x-coordinates of the nodes appearing in the I5 fields are set to the value in the E15.7 field. Any number of continuation cards may be used; their first fifteen columns are ignored. A zero or blank I5—field terminates the card scan for a given x-coordinate. A zero or blank first I5 field (columns 16-20) on a noncontinuation card ends the section. Both shorthand representations of Section III are allowed.

VI. Y-Coordinates of the Nodes. Same as Section V.

VII. Z-Coordinates of the Nodes. Same as Section V.

VIII. Boundary Conditions FORMAT (6I1, 9X, 13I5)

The first six fields give the boundary conditions specifications in the order: u, v, w, θ_x, θ_y, θ_z; where u, v, w are global displacements in the x, y, z directions, respectively, and θ_x, θ_y, θ_z are rotations with respect to the x, y, z axes. The last three fields for θ_x, θ_y, θ_z are only used when using a beam element.

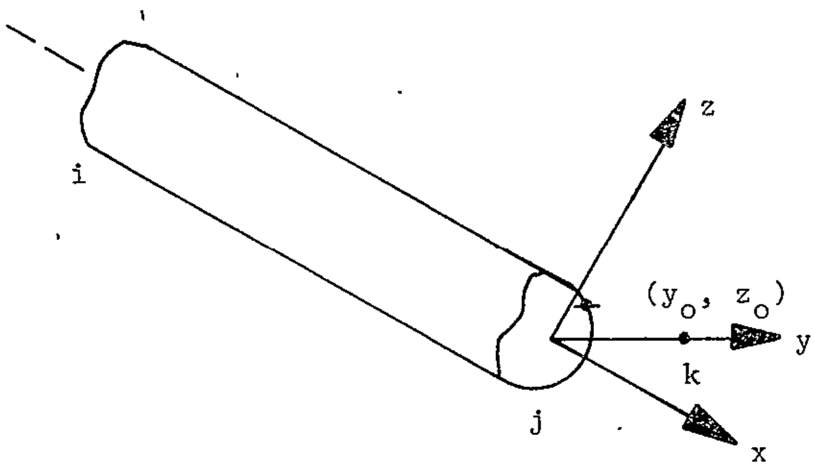


Fig. 2 Beam Element

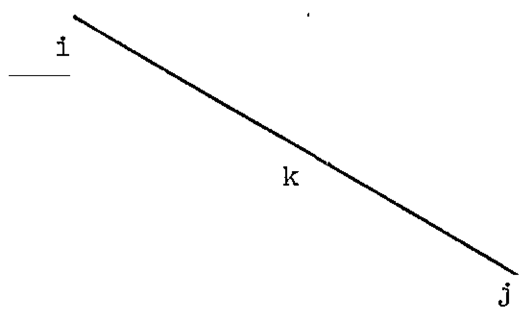


Fig. 3 Stringer Element

Zero denotes a fixed degree of freedom

One denotes a free degree of freedom

Two will result in the application of a unit generalized displacement, or a corresponding card may be included in Section IX (dependent degrees of freedom) designating the magnitude of the applied generalized displacement.

The 13I5 fields give the applicable nodes for the boundary condition specifications, with both shorthand notations of Section III permitted. Any number of continuatuion cards may be used for a given specification. However, only the 13I5 fields are used on a continuation card. A zero or blank I5 field terminates the card scan for a given boundary condition specification. Note: If the last field of a card (columns 76-80) is the last specification, an additional blank card (continuation card) must follow. A zero or blank first I5 field (columns 16-20) on a noncontinuation card ends the section. If a node's boundary conditions are not specified in this section, all the degrees of freedom are assumed to be free. To change this default condition, the first card of this section should be set to the desired default (i.e., in the absence of beams, 111000) with all nodes used in the problem specified. Note: Maximum number of nodes is currently 900.

IX. Dependent Degrees of Freedom FORMAT (2I5, 2(5X, 2I5, E15.7))

This section designates the input for both single and multipoint constraints as well as applied displacement of the form:

1) $\delta_i = \alpha_1\delta_{j1} + \alpha_2\delta_{j2} + \dots \alpha_n\delta_n$

where δ_i is a dependent degree of freedom,

$\delta_{j1} \dots \delta_{jn}$ are independent degrees of freedom

and $\alpha_1, \alpha_2, \dots, \alpha_n$ are coefficients

2) Rotation of displacements at a node

$$\delta_i = \alpha_1\tilde{\delta}_i + \alpha_2\tilde{\delta}_j + \alpha_3\tilde{\delta}_k$$

$$\delta_j = \beta_1\tilde{\delta}_i + \beta_2\tilde{\delta}_j + \beta_3\tilde{\delta}_k$$

$$\delta_k = \gamma_1\tilde{\delta}_i + \gamma_2\tilde{\delta}_j + \gamma_3\tilde{\delta}_k$$

where the α, β, γ's are the direction cosines of the rotation, δ_i, δ_j, δ_k are the displacements with respect to the original global directions, and $\tilde{\delta}_i$, $\tilde{\delta}_j$, $\tilde{\delta}_k$ are the components of displacements at the node with respect to the new coordinate axes. An example of this capability is given in Appendix I.

3) Applied generalized displacement

$$\delta_i = \alpha_1$$

where the coefficient α_1 is the applied generalized displacement.

The first two fields designate a node number and a degree of freedom (i.e., 1-6). The dependency is defined in the following three fields. The two integer fields designate the node number and degree of freedom number and the coefficient by the floating point field. If there is another dependency for the node, it is designated in a similar fashion in the next three fields. Any number of continuation cards can be used with the first two fields blank. The section is ended by a blank or zero in the third integer field (blank card). Examples of the use of multipoint constraints are in Appendix I.

X. Material and Section Properties

The format for this input is dependent upon the member type. Each type of input begins with a word of up to four characters.

MAT1	Material properties for plane stress, necessary with membrane triangles (TRIM)
MBM	Material properties for a beam element
MSTG	Material and section properties for a stringer
THIK	Member thickness for triangular membrane (TRIM) elements
MBET	Angle between local axes and principal directions of orthotropy for TRIM elements
SREC	Beam section properties for a solid rectangular section
SCIR	Beam section properties for a solid circular section
ZSEC	Beam section properties for a Z-section
ISEC	Beam section properties for an I-section
HCIR	Beam section properties for a hollow circular section
HREC	Beam section properties for a hollow rectangular section
LSEC	Beam section properties for an L-section
TSEC	Beam section properties for a T-section
CSEC	Beam section properties for a channel section
SEND	Ends the section

MAT1 - Plane Stress Material Properties - FORMAT (A4, 1X, 5E15.7, /, 2E15.7, /, 4E15.7, /, 5E15.7, /, (16I5))

The first four cards specify material properties, as follows:

CARD 1: MAT1

EONE = Young's modulus in principal property axis (1)

ETWO = Young's modulus in principal property axis (2)

BETA = No longer used. Set equal to zero.

GONTO = Shear modulus in (1)-(2) principal property plane

VONTO = Poisson's ratio, ν_{12}

Note: $\epsilon_1 = \frac{\sigma_1}{E_1} - \frac{\nu_{12}}{E_2} \sigma_2$

$$\epsilon_2 = - \frac{\nu_{21}}{E_1} \sigma_1 + \frac{\sigma_2}{E_2}$$

$$\gamma_{12} = \frac{\tau_{12}}{G_{12}}$$

CARD 2: TALF-1 = Coefficient of thermal expansion in 1-axis direction, α_1

TALF-2 = Coefficient of thermal expansion in 2-axis direction, α_2

CARD 3: SIGOX = Yield stress in principal 1-direction

SIGOY = Yield stress in principal 2-direction

SIGOZ = Yield stress in principal 3-direction

SIGXY = Shear yield stress in principal 1-2 plane

CARD 4: RMOSN = If RMOSS $\neq$ 0; RMOSN = n, the shape parameter used in Ramberg-Osgood representation of stress-strain behavior

If RMOSS = 0; RMOSN = $\bar{\alpha}$ the slope of the linear strain hardening stress-strain representation, i.e., $\bar{\alpha} = E_T/E$ where E_T is the tangent modulus

RMOSS = If RMOSS $\neq$ 0; RMOSS = Ramberg-Osgood parameter $\sigma_{0.7}$

Note 1: If RMOSN = 0 and RMOSS = 0, the material for the element(s) is assumed to be elastic-ideally plastic

RMOSE = Ramberg-Osgood parameter E (Young's modulus)

YLDST = Yield stress in tension

YLDSC = Yield stress in compression

Input for the yield stress in tension and compression has been maintained in order to accommodate materials that exhibit initial anisotropic plastic behavior. In this case an initial translation of the yield surface is made consistent with the kinematic hardening theory.†

Note: Only initially isotropic materials can be treated when considering linear or nonlinear strain hardening using this property card.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. <u>A zero or blank I5 field ends each member listing</u>.

MBET - Orientation of Axes of Material Anisotropy - FORMAT (A4, 1X, E15.7, /, (16I5))

CARD 1: MBET

BETF = Angle β in degrees between local x-axis and principal 1-axis for material orthotropy. See Fig. 4. Only applicable for TRIM elements.

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† Isakson, G., Armen, H. Jr., and Pifko, A., "Discrete-Element Methods for the Plastic Analysis of Structures," NASA Contractor's Report NASA CR-803, October 1967.

Note: This card is an optional card. The default is BETF = 0.0 for all members. It should be used with initially orthotropic materials.

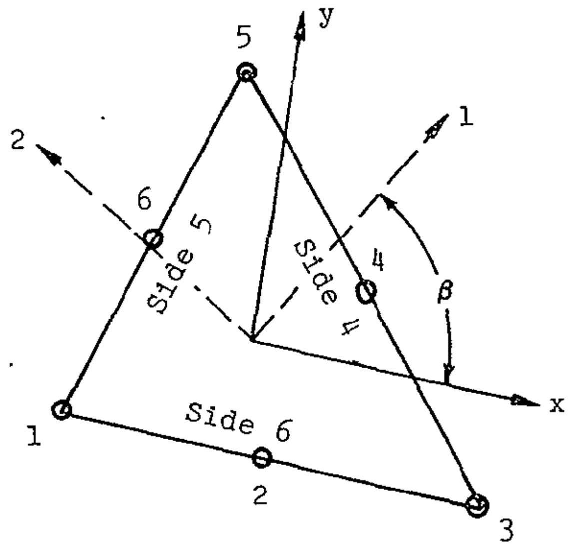


Fig. 4 Orientation of Element Local and Material Axes

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero ar blank I5 field ends each member listing.

MSTG - Stringer Properties - FORMAT (A4, 1X, 5E15.7, /E15.7/, (16I5))

CARD 1: MSTG

E = Young's modulus

A = Cross sectional area

RMOSN = If RMOSS $\neq 0$; RMOSN = n, the shape parameter used in Ramberg-Osgood representation of stress-strain behavior.

If RMOSS = 0; RMOSN = $\bar{\alpha}$, the slope of the linear strain hardening stress-strain representation, i.e., $\bar{\alpha} = E_T/E$ where E_T is the tangent modulus

RMOSS = If RMOSS ≠ 0; RMOSS = Ramberg-Osgood parameter $\sigma_{0.7}$

Note 1: If RMOSN = 0 and RMOSS = 0, the material for the element(s) is assumed to be elastic-ideally plastic

YLDST = Yield stress

CARD 2: TALF = Coefficient of thermal expansion

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

THIK - Element Thicknesses - FORMAT (A4, 1X, E15.7)
Necessary with membrane elements
TRIM

CARD 1: THIK
THICK = Element thickness

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

MBM - Beam Material Properties - FORMAT (A4, 1X, 5E15.7, /, E15.7, /, (16I5))

CARD 1: MBM
E = Young's modulus

ANU = Poisson's ratio

RMOSS = If not equal to zero, RMOSS equals Ramberg-Osgood parameter, $\sigma_{0.7}$

RMOSN = If RMOSS $\neq$ 0; RMOSN = n, the shape parameter used in Ramberg-Osgood representation of stress-strain behavior.

If RMOSS = 0; RMOSN = $\bar{\alpha}$, the slope of the linear strain hardening stress-strain representation, i.e., $\bar{\alpha} = E_T/E$ where E_T is the tangent modulus

YLDST = Yield stress

CARD 2: TALF = Coefficient of thermal expansion

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

Beam Section Properties - FORMAT (A4, 1X, 5E15.7, /, 3E15.7, /, formats for Cards 3 and 4 (see below), /, (16I5))

The cards specifying beam section properties all start with the following information:

CARD 1: SREC, SCIR, ISEC, ZSEC, HCIR, HREC, LSEC, TSEC, CSEC — Defined at the beginning of the section. The different cross sections are shown in Tables 1 and 2.

	A	= cross-sectional area
	Iyy	= moment of inertia about y axis (see Fig. 5)
	Izz	= moment of inertia about z axis (see Fig. 5)
	Iyz	= product of inertia
	J	= torsional rigidity
CARD 2:	YO	= Eccentricity of attachment point in the y_o direction
	ZO	= Eccentricity of attachment point in the z_o direction
	$\bar{\beta}$	= angle defining the transformation of the Iy, Iz, Iyz to another reference axis (see Fig. 5)

Additional cards are required according to which section is specified. The notation for each section is shown in Table 1.

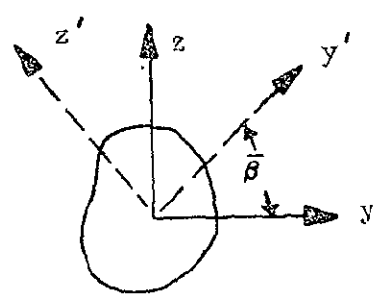


Fig. 5 Definition of Coordinate Axes in Cross Section of Beam Element

SREC - FORMAT (2E15.7)

CARD 3: A = Width

B = Depth

SCIR - FORMAT (E15.7)

CARD 3: R = Radius

ZSEC, ISEC, and CSEC - FORMAT (3E15.7, /, 3E15.7)

CARD 3: A1 = Dimension of upper flange

A2 = Dimension of web

A3 = Dimension of lower flange

CARD 4: T1 = Thickness of upper flange

T2 = Thickness of web

T3 = Thickness of lower flange

HCIR - FORMAT (2E15.7)

CARD 3: R = Outer radius

T = Thickness

HREC, LSEC, and TSEC - FORMAT (4E15.7)

CARD 3: A1 = Width (parallel to y-axis)

A2 = Depth (parallel to z-axis)

T1 = Thickness of upper and/or lower flanges

T2 = Thickness of vertical webs

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

SEND Ends the section.

TABLE 1 SECTION INPUT AND GEOMETRY

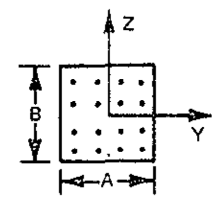
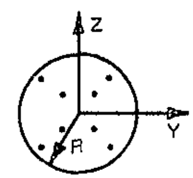
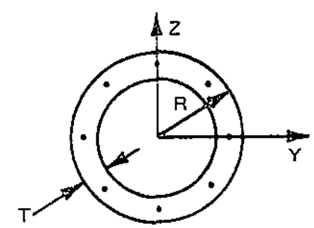
Section	Keyword	Input	
Solid Rectangle	SREC	Area, moments of inertia, I_{yy}, I_{zz}, I_{yz}, J eccentricities, y_o, z_o, transformation angle $\bar{\beta}$. Section width and height, a, b.	
Solid Circular	SCIR	Area, moments of inertia, I_{yy}, I_{zz}, I_{yz}, J eccentricities, y_o, z_o, transformation angle $\bar{\beta}$. Section radius, r.	
Hollow Circular Section	HCIR	Area, moment of inertia, I_{yy}, I_{zz}, I_{yz}, J eccentricities, y_o, z_o, transformation angle $\bar{\beta}$. Outer radius, r, thickness, t.	

TABLE 1 SECTION INPUT AND GEOMETRY (CONT)

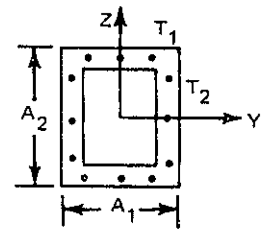
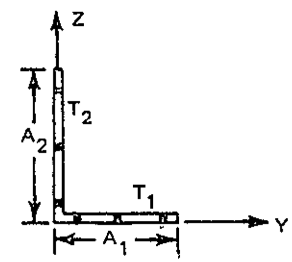
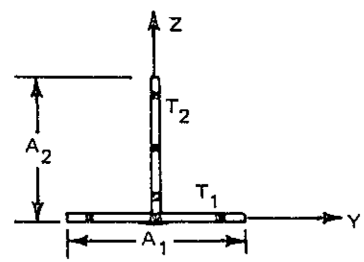
Section	Keyword	Input	
Hollow Rectangular Section	HREC	Area, moment of inertia, I_{yy}, I_{zz}, I_{yz}, J eccentricities, y_o, z_o, transformation angle $\bar{\beta}$. Width, a_1, depth, a_2, thickness of upper and lower flanges, t_1, thickness of vertical webs, t_2	
L-Section	LSEC	Area, moment of inertia, I_{yy}, I_{zz}, I_{yz}, J eccentricities, y_o, z_o, transformation angle $\bar{\beta}$. Dimension of flange and web a_1,a_2 thickness flange, and web, t_1, t_2.	
T-Section	TSEC	Same as for L-Section	

TABLE 1 SECTION INPUT AND GEOMETRY (CONT)

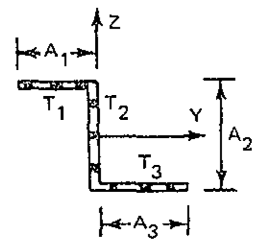
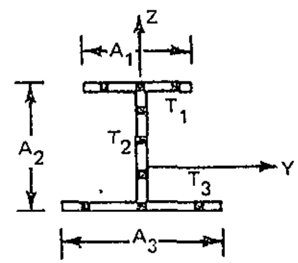
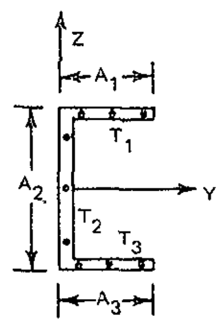
Section	Keyword	Input	
Z-Section	ZSEC	Area, moments of inertia, I_{yy}, I_{zz}, I_{yz}, J eccentricities, y_o, z_o, transformation angle $\bar{\beta}$. Dimension of upper and lower flange, a_1, a_3. Dimension of web, a_2, thickness of upper and lower flange, t_1, t_3, thickness of web, t_2 NOTE: $a_1 = a_3$; $t_1 = t_3$	
I-Section	ISEC	Same as Z-section $a_1 \neq a_3$; $t_1 \neq t_3$	
Channel Section	CSEC	Same as Z-Section $a_1 = a_3$; $t_1 = t_3$	

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS

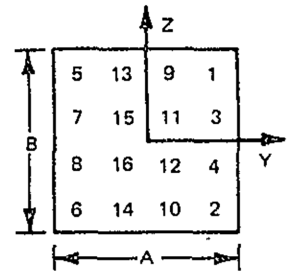
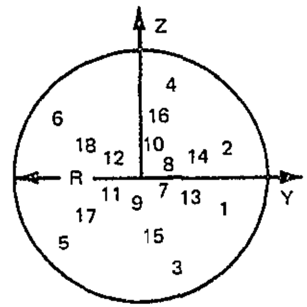
Section	Keyword	Number Stress Points	Location of Stress Points			Location of Stress Points
Solid Rectangle	SREC	16		± y	± z	
			1	0.430568a	0.430568b	
			3	0.430568a	0.169991b	
			9	0.169991a	0.430568b	
			11	0.169991a	0.169991b	
						Shear Center at Centroid
Solid Circular	SCIR	18		r	± θ°	
			2	0.887a	12.1	
			4	0.887a	60.98	
			6	0.887a	137.05	
			8	0.1127a	12.1	
			10	0.1127a	60.98	
			12	0.1127a	137.05	
			14	0.500a	12.1	
			16	0.500a	60.98	
			18	0.500a	137.05	Shear Center at Centroid

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

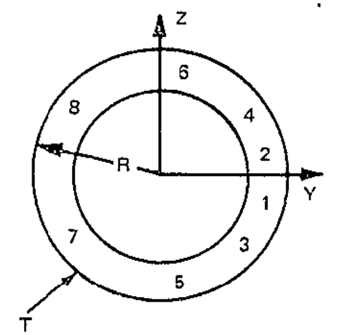
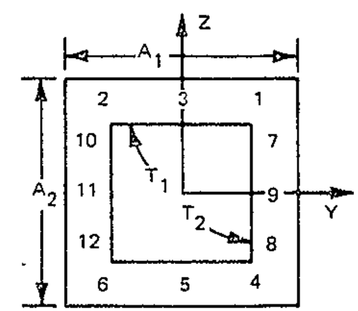
Section	Keyword	Number Stress Points	Location of Stress Points			Location of Stress Points
Hollow Circular Section	HCIR	8		r	θ°	 Shear Center at Centroid
			1	a	352.8	
			2	a	7.2	
			3	a	323.4	
			4	a	36.6	
			5	a	274.6	
			6	a	85.4	
			7	a	213.0	
			8	a	147.0	
Hollow Rectangular Section	HREC	12		± y	± z	 Shear Center at Centroid
			1	$0.77459\bar{a}_1$	$0.5\bar{a}_2$	
			3	0	$0.5\bar{a}_2$	
			7	$0.5\bar{a}_1$	$0.77459\bar{a}_2$	
			9	$0.5\bar{a}_1$	0	
				$\bar{a}_1 = a_1 - t_2$		
				$\bar{a}_2 = a_2 - t_1$		

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

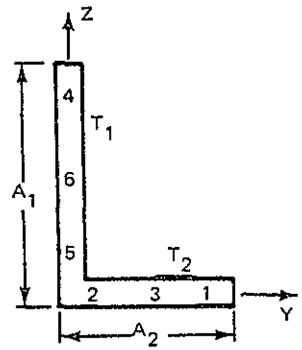
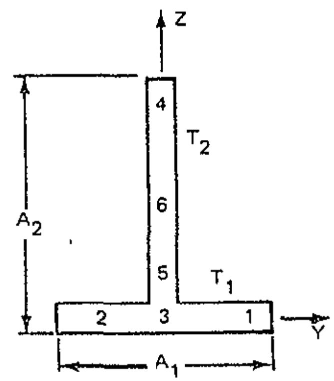
Section	Keyword	Number Stress Points	Location of Stress Points	Location of Stress Points
L-Section	LSEC	6	y z 1 $0.88730\bar{a}_2$ 0.0 2 $0.11271\bar{a}_2$ 0.0 3 $0.50000\bar{a}_2$ 0.0 4 0.0 $0.88730\bar{a}_1$ 5 0.0 $0.11271\bar{a}_1$ 6 0.0 $0.50000\bar{a}_1$ $\bar{a}_1 = a_1 - \frac{t_2}{2}$, $\bar{a}_2 = a_2 - \frac{t_1}{2}$	 Axes at Shear Center
T-Section	TSEC	6	y z 1 $0.77459a_1$ 0.0 2 $-0.77459a_1$ 0.0 3 0.0 0.0 4 0.0 $0.88730\bar{a}_2 + \frac{t_1}{2}$ 5 0.0 $0.11271\bar{a}_2 + \frac{t_1}{2}$ 6 0.0 $0.50000\bar{a}_2 + \frac{t_1}{2}$ $\bar{a}_2 = a_2 - t_1$	 Axes at Shear Center

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

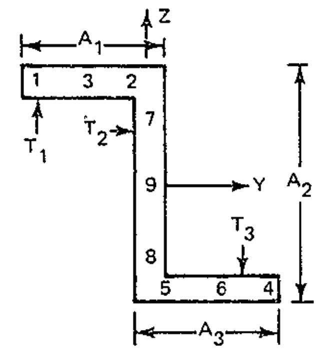
Section	Keyword	Number Stress Points	Location of Stress Points			Location of Stress Points
Z-Section	ZSEC	9		y	z	
			1	$-0.88730a_1$	$\overline{a}_2$	
			2	$-0.11271a_1$	$\overline{a}_2$	
			3	$-0.5000a_1$	$\overline{a}_2$	
			4	$0.88730a_1$	$\overline{a}_2$	
			5	$0.11271a_1$	$\overline{a}_2$	
			6	$0.5000a_1$	$\overline{a}_2$	
			7	0.0	$0.77459a_2'$	Shear Center at Centroid
			8	0.0	$0.77459a_2'$	
			9	0.0	0.0	
			$\overline{a}_2 = (a_2 - t_1)/2$, $t_1 = t_3$			
			$a_2' = (a_2 - t_1)$, $a_1 = a_3$			

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

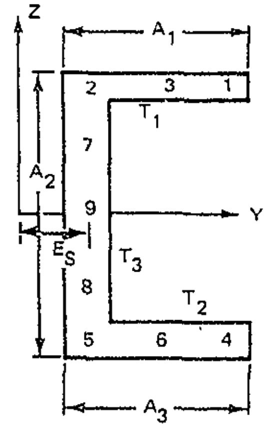
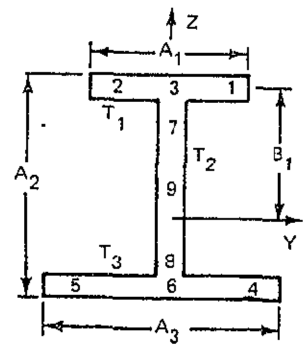
Section	Keyword	Number Stress Points	Location of Stress Points	Location of Stress Points
Channel Section	CSEC	9	y, z 1: $0.88730\bar{a}_1 + e_s$, $0.5\bar{a}_2$ 2: $0.11271\bar{a}_1 + e_s$, $0.5\bar{a}_2$ 3: $0.5000\bar{a}_1 + e_s$, $0.5\bar{a}_2$ 4: $0.88730\bar{a}_1 + e_s$, $-0.5\bar{a}_2$ 5: $0.11271\bar{a}_1 + e_s$, $-0.5\bar{a}_2$ 6: $0.5000\bar{a}_1 + e_s$, $-0.5\bar{a}_2$ 7: e_s, $0.77459\bar{a}_2$ 8: e_s, $-0.77459\bar{a}_2$ 9: e_s, 0.0 e_s - distance from shear center to web center line $e_s = \dfrac{1/2(a_1 - t_2)}{1 + 1/6\left(\dfrac{t_2(a_2 - t_1)}{t_1(a_1 - t_2)}\right)}$, $\bar{a}_2 = a_2 - t_1$ NOTE: $t_1 = t_3$; $a_1 = a_3$	 Axes at Shear Center

TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

Section	Keyword	Number Stress Points	Location of Stress Points			Location of Stress Points
I-Section	ISEC	9		y	z	 Axes at Shear Center
			1	$0.774569a_1$	b_1	
			2	$-0.77459a_1$	b_1	
			3	0.0	b_1	
			4	$0.77459a_3$	b_2	
			5	$0.77459a_3$	b_2	
			6	0.0	b_2	
			7	0.0	$0.8873\bar{b}_1 + 0.11271\bar{b}_2$	
			8	0.0	$0.11271\bar{b}_1 + 0.8873\bar{b}_2$	
			9	0.0	$0.5\bar{b}_1 + 0.5\bar{b}_2$	

$$b_1 = \frac{\bar{a}_2\left(\frac{t_3}{t_1}\right)\left(\frac{a_3}{a_1}\right)^3}{\left(1+\left(\frac{t_3}{t_1}\right)\left(\frac{a_3}{a_1}\right)\right)^3}, \quad b_2 = b_1\left(\frac{t_1}{t_3}\right)\left(\frac{a_1}{a_3}\right)^3$$

$$\bar{a}_2 = a_2 - 0.5(t_1 + t_3) \; ; \; \bar{b}_1 = b_1 - 0.5t_1 \; ; \; \bar{b}_2 = b_2 - 0.5t_3$$

XI. Applied Loads

Two different types of loading are currently admissible — concentrated loads at nodes, and distributed line loads on beams. Each loading situation is designated by one of the following four-character key words:

CONC	(Concentrated force or moment at a node)
BMLO	(Distributed line load, beam element)
SEND	(Section end)

Note: The keywords for the section on applied loads are used as section headings. The keyword appearing on an input card designates that the input to follow is associated with a particular type of applied loading. A blank card (as described in succeeding sections) delimits the input and indicates that the next card contains a different keyword. For example, CONC indicates input that follows is for concentrated loads and TRIA; distributed loads on an edge of a membrane triangle. Thus the input is as follows:

```
CONC
    data
        for
           concentrated
               loads
    blank card
BMLO
SEND
```

It should be noted that the keywords can appear in any order and may be specified more than one time in the applied load section.

CONC - Concentrated loads - FORMAT (A4, /, I5, 3E15.7, /, 5X, 3E15.7)

Each card gives the load components in global directions at a specified node.

CARD 1:	CONC	
CARD 2:	I5 field:	Node number
	First E15.7 field:	Force component F_x
	Second E15.7 field:	Force component F_y
	Third E15.7 field:	Force component F_z
CARD 3:	First E15.7 field	Moment component M_x
	Second E15.7 field	Moment component M_y
	Third E15.7 field	Moment component M_z

A blank card (i.e., zero or blank first I5 field) ends the specification of concentrated loads.

BMLO - Distributed Line Load, Beam Element - FORMAT (A4, /, 4E15.7)

CARD 1:	BMLO	
CARD 2:	PYI	Force/unit length in local y-direction (see Fig. 5) at node i
	PYJ	Force/unit length in local y-direction at node j
	PZI	Force/unit length in local z-direction at node i
	PZJ	Force/unit length in local z-direction at node j

Note: A linear variation of the distributed load between nodes is assumed.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

A card with SEND in the first four columns ends the section for applied loading.

XII. Members to be Printed — FORMAT (16I5)

Specify the members whose strains and stresses are to be printed. Both shorthands of Section III are allowed. A maximum of 600 members may be specified. Members in excess of 600 and undefined member numbers are ignored. A blank card or card with only zero entries ends the section.

XIII. Nodes to Be Printed — FORMAT (16I5)

Up to 900 nodes whose displacements are to be printed for the analysis, as per Section XII.

XIV. Parameters for Succeeding Load Cycles — FORMAT (3I5, /, 3E15.7)

CARD 1:
- NPRNT = If equal to zero, no additional load cycle, end of problem. If greater than zero, print output every NPRNT increments of load
- NFORM = Reform elastic stiffness matrix every NFORM increments of load
- NNEWL = If equal to zero, no new load distribution is to be input. If greater than zero, a new load distribution will be read in

CARD 2:
- PMAX = New maximum load factor
- DELP = Number of load increments for new load cycle
- PTAN = Load at which full tangent modulus is used

XV. New Applied Loads

New load distribution is read in if NNEWL $\neq 0$. Format is identical to Section XI.

XVI. Problem End --FORMAT (A4)

Each problem's input must be ended with a card reading "ENDb" where "b" denotes a blank, in columns 1-4. The last problem in a run should end with a card reading "STOP" in column 1-4 instead of "ENDb."

XVII. Restarting a Problem

The initial restart tape is created on Unit 21. Subsequent restart jobs mount the restart tape as Unit 21 and if desired (NUTAP > 0) copies and continues the restart tape on Unit 22.

CARD 1: Problem Title - FORMAT (2A4)

As in Section I

CARD 2: NPNTC, NPRNT, IFORM, IREST, NUTAP, INPRT, NRSRT

As in Section II. Here NPNTC is ignored and IREST = 2

NRSRT is a unique number obtained as output from the job that generated the restart tape. This output is of the form:

* * * * *

RESTART TAPE HAS BEEN WRITTEN FOR P = 7.100000E+04
NRSRT = 8

* * * * *

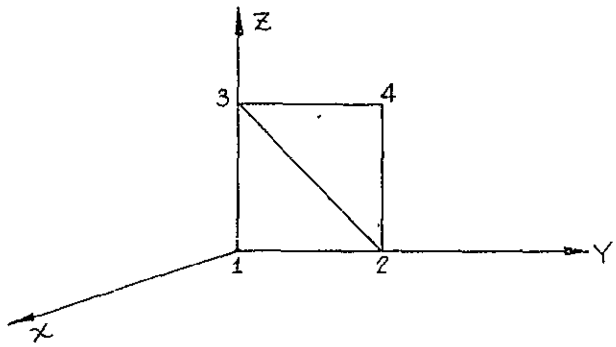
In this case NRSRT is equal to 8 and the job will be restarted at a load factor of 71.0. A restart job can be started at any intermediate load level for which the restart tape has been written.

CARD 3: PMAX, DELP, PTAN, EQMULT

As in Section II

Succeeding cards follow as in Sections XII through XVI.

EXAMPLE 1



Node 1 and 2 are fixed.
There is an applied displacement at node 3 in the negative y-direction.
The distance between nodes 3 and 4 remain constant.

Boundary Conditions

011000	1-4	default condition
000000	1 2 4	
021000	3	
Blank card		

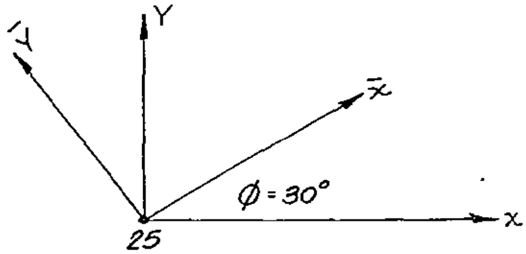
Dependencies

Node	Dof	Node	Dof		
3	2	3	2	0.050	(specified displacement)
4	2	3	2	1.0	restraint condition that
4	3	3	3	1.0	3-4 remain rigid

NOTE: In the boundary condition cards for node 4, degrees of freedom 2 and 3 must be specified 0 because they are effectively eliminated from the solution (they are dependent degrees of freedom).

EXAMPLE 2

Rotation of Displacements at a Boundary



For this problem we set the displacement $\bar{u}$ equal to zero and $\bar{v}$ free. This corresponds to the normal displacement along $\bar{x}$ equal to zero and the tangential displacement (in $\bar{y}$ direction) free. The global coordinates of thepproblem are x, y. Thus:

$$u = \bar{u} \cos \varphi - \bar{v} \sin \varphi$$

$$v = \bar{u} \sin \varphi + \bar{v} \cos \varphi$$

The multipoint constraint cards are:

25	1	25	1	0.86602540E00	25	2	-0.50E00
25	2	25	1	0.50E00	25	2	0.8660254E00

Note: On the boundary condition cards, since $\bar{u}$ = 0.0 node 25 degree of freedom 1 is specified to be 0 but node 25 degree of freedom 2 is specified 1 or free since it remains an independent degree of freedom (although rotated). If the normal displacement $\bar{u}$ were not fixed (and the transformation performed merely to apply a normal load), it would have a 1 boundary condition specified.

SECTION 6

INSTRUCTIONS FOR USE OF SATELLITE

A Preprocessing Program for PLANS

INSTRUCTIONS FOR USE OF SATELLITE

SATELLITE is the preprocessing program associated with the PLANS system programs that checks and plots the undeformed finite element model. Additionally, it calculates the ordering of the external node numbers that leads to the minimum semibandwidth. SATELLITE is currently not operational for the HEX program.

The input for SATELLITE is as follows:

CARD 1: FORMAT (20A4)

Keywords left justified in fields of four. One of the following is required and indicates the program whose data follows.

- PLNE - PLANE program, In-Plane Membrane Program Used for Fracture Analysis
- BEND - BEND program, Elastic-Plastic Analysis of Built-Up Structures
- REVB - REVBY program, Elastic-Plastic Analysis of Bodies of Revolution
- OPGM - OUT-OF-PLANE GM, Nonlinear Analysis of Built-Up Structures
- DYCA - DYCAST program, Nonlinear Dynamic Analysis of Built-Up Structures

Additional optional keywords can be specified in fields of four. Any number of spaces can be skipped on the input card that are multiples of four.

- BAND - Specifies that the optimum semibandwidth is to be calculated. In this case the optimum order of the external node numbers for

Section III data will be printed and punched. These punched data are used with the input deck. If this keyword is not specified the bandwidth will not be optimized.

SCAN - Process and check input data but do not produce any plots

PRIN - Print transformed coordinates to be plotted

The input data deck follows Card 1. Input through the nodal coordinates is necessary. The remainder of the deck may be input although the cards are read and not processed until a STOP or END card is reached. Input for the SATELLITE program continues as follows.

CARD 2: FORMAT (5I5)

This card determines the number and type of plots. Up to five different labeled pictures can be obtained. These are specified with the fixed point numbers right justified in any of the fields of five as follows:

1 - labeled unconnected nodes

2 - labeled nodes, unlabeled members

3 - labeled nodes and members

4 - unlabeled nodes, labeled members

5 - unlabeled nodes and members

CARD 3: FORMAT (4E15.7)

ALPHA - rotation of structure about the global x-axis

BETA - rotation of structure about the global y-axis

GAMMA - rotation of structure about the global z-axis

LENGTH - maximum width of picture in the paper transverse direction. LENGTH is a function of the plotter being used.

If ALPHA, BETA, GAMMA are zero then the picture is projected on the x-y plane with the y direction in the transverse paper direction. The angles lead to a path dependent transformation by rotating the body first about the global x then y and z axes, respectively. The resulting viewing plane is still the original global x-y plane. The viewing plane can be changed with the following card.

CARD 4: FORMAT (6A1)

This card specifies the viewing plane to be used by specifying the six alphanumeric characters XYZ+++. The first two characters specifying the viewing plane with the first indicating the length direction and the second the width of the paper. The last three characters can be the characters + or - and indicate that the normal to the plane is in the positive or negative coordinate direction. For example:

YZX+++ indicates the viewing plane as the global YZ plane with the Z axis in the paper width direction and Y along the paper length

CARD 5: FORMAT (4(A4,1X))

Element types to be omitted. For BEND, OPGM, DYCAST:

BEAM - will omit all beam elements

TRIM - will omit all triangle membrane elements

STGR - will omit all stringer elements

TRIP - will omit all triangular plate elements

For REVBY:

TRIR - will omit all triangle elements

SHEL - will omit all shell elements

RING - will omit all ring elements

If there are no element types to be omitted a blank card must be specified.

CARD 6: FORMAT (16I5)

Nodes to be included in the plot. All the shorthand notations are allowed, i.e., 1 through 100 is input as 1 - 100 and 1, 3, ..., 99 as 1-2-99. A blank card or card with only zero entries ends the section.

If only a part of the structure is to be plotted, this section should include only the nodes that lie in the section. Only these nodes are considered when scaling the plot so that the subsection will be scaled up to the maximum allowable size.

CARD 7: FORMAT (16I5)

Elements to be included in the plot. Input is the same as for Card 6. All nodes associated with the elements to be plotted must be specified with Card 6.

Note: If all the elements and nodes are to be plotted it is sufficient to specify one negative number in the first I5 field whose magnitude is greater than the largest external element/node number in the finite element grid.

CARD 8:   FORMAT (A4)

If this card is blank and

1   END was read at the end of the data deck a new data deck is read starting from Card 1

2   STOP was read at the end of the data deck, the job is complete

If this card is REPT the input will be repeated from Card 2 and the same structure with different plotting parameters will be plotted.

EXAMPLE INPUT

EXAMPLE INPUT

The following pages contain the input data decks for a number of sample problems. The principal intent in providing sample input is to aid the user in better understanding the discussion in the main body of the users' manual. Also provided with each problem is a brief discussion and representative data so that benchmark results are available for comparison. A detailed picture of the sample problem idealizations showing node and element labeling has not been provided. It is suggested that the input plotting program SATELLITE be used to generate computer plots of the input data, if desired.

Many sample problems that demonstrate the capability of the methods used in the PLANS program can be found in Refs. 1-3.

PLANS

BEND MODULE

ELASTIC-PLASTIC ANALYSIS

PREPARED BY

THE RESEARCH DEPARTMENT

OF GRUMMAN AEROSPACE CORPORATION

UNDER PARTIAL FUNDING FROM

THE NASA LANGLEY RESEARCH CENTER

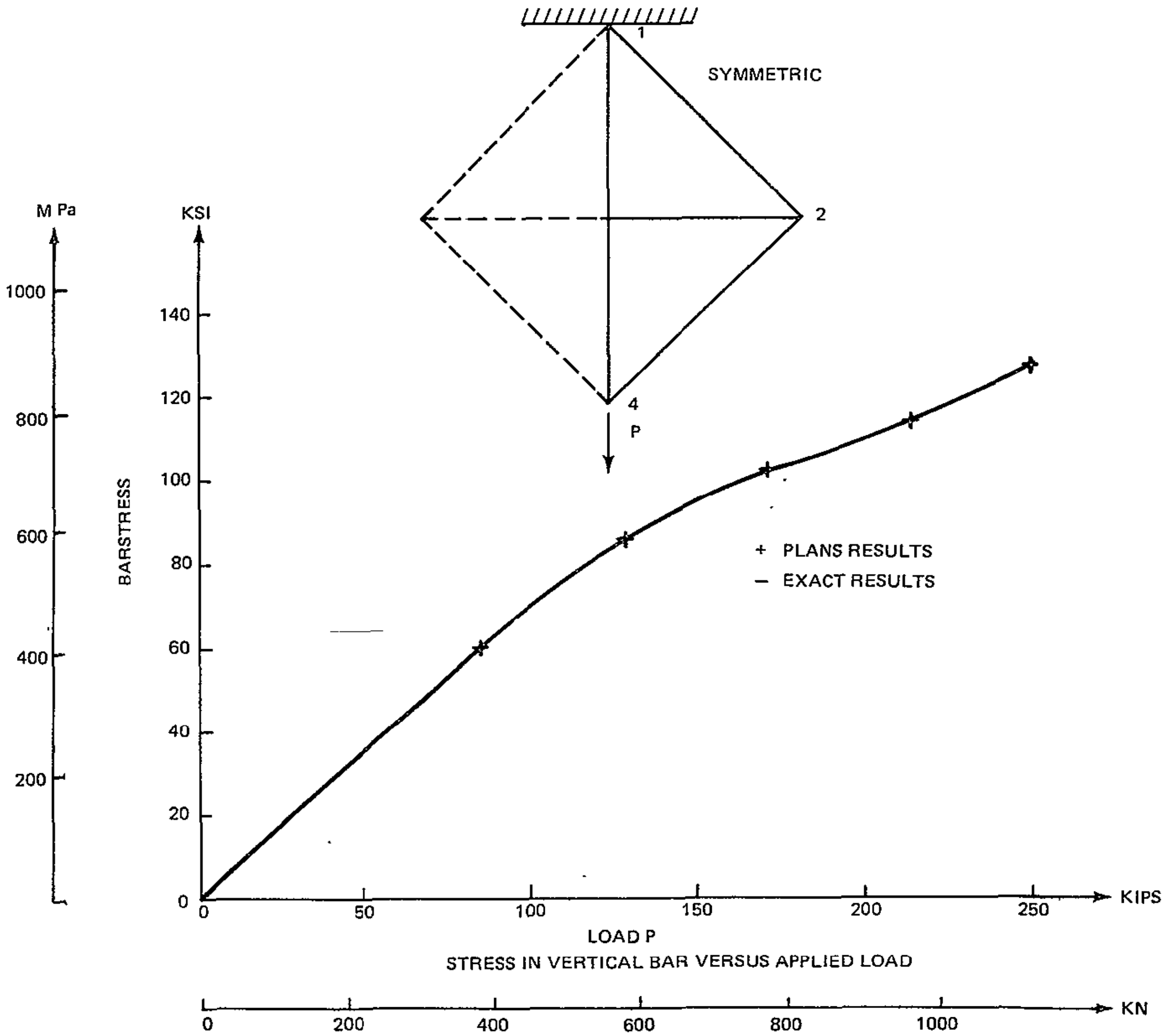
Example Problem No. 1

Program: BEND

Problem Title: Classical Truss - Axial Force Element

Comments: This problem uses the stringer element STGR in a simple 4 element truss. Symmetry conditions were used so that only half the truss was modeled.

The accompanying figure shows the applied load versus the force in the vertical element (element 3). These results were generated using a load step of ΔP = 848.5 lbs or 3774.3 N (which is 0.01 of the initial yield load). Agreement with results from an exact analysis was quite good.



Bend Sample Problem # 1 Classical Truss STRG Axial Force Elements

PROGRAM LISTING OF INPUT DATA CARDS

```
           .........1.........2.........3.........4.........5.........6.........7.........8
           12345678901234567890123456789012345678901234567890123456789012345678901234567890

           BEND SAMPLE PROBLEM #1 CLASSICAL TRUSS  STRG AXIAL FORCE ELEMENTS
               0   50
               1   -4
           STRG          1    1    2
           STRG          2    2    4
           STRG          3    2    3
           STRG          4    1    4
           SEND
                0.0           1
                .5            2    3
                1.0           4
BLANK CARD
                0.0           1    4    3
               .5             2
BLANK CARD
                0.0           1   -4
BLANK CARD
           000000             1    3
           110000             2
           100000             4
BLANK CARD
BLANK CARD
           MSTG      1.0   E&07     1.0   E&00     10.0  E&00     1.0   E&05     6.0   E&04
BLANK CARD
               1   -3
           MSTG      1.0   E&07     0.5   F&00     10.0  E&00     1.0   E&05     6.0   E&04
BLANK CARD
               4
           SEND
           CONC
               4     0.5   E&00     0.0   E&00     0.0   E&00
BLANK CARD
BLANK CARD
           SEND
               1   -4
BLANK CARD
              -4
BLANK CARD
              250000.0         .01
BLANK CARD
           STOP

           .........1.........2.........3.........4.........5.........6.........7.........8
           12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

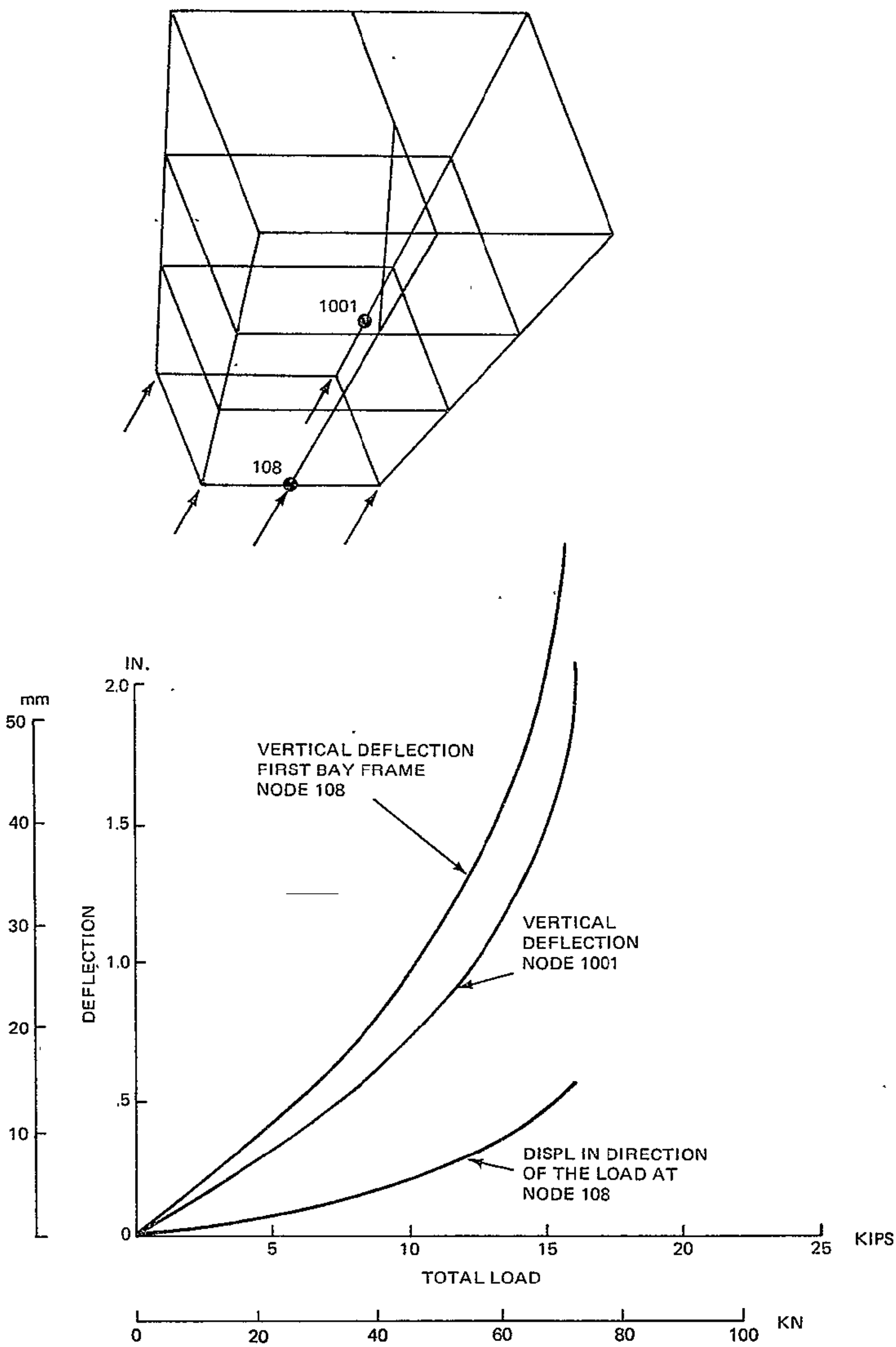
Example Problem No. 2

Program: BEND

Problem Title: NASA Frame with an End Load - BEAM ELEMENTS

Comments: This problem demonstrates the use of beam elements in an open space framework. Fifty seven beam elements of various cross section (L-section, T-section, rectangular section) connecting 40 nodes are used in the model. Multipoint constraints were used to enforce symmetry conditions between the left and right hand side of the structure. Ideally plastic behavior was assumed with a yield stress of 50130 psi or 345.6 MPa.

The figure shows displacement in the vertical and axial directions at the first bulkhead at the centerline (node 108) and at the midpoint between the first two bulkheads in the upper stringer (node 1001). The results indicate a plastic collapse at a load of P = 15000 lbs or 66.7 KN.



Bend Sample Problem # 2 NASA Frame with End Load -

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

BEND SAMPLE PROBLEM =2 NASA FRAME WITH AN END LOAD - BEAM ELEMENTS
    1   20
  101 1001  102  105  201 2000  108 3001 1003  106  202 6000  205  303 4000 2002
  208 3003 1005  206  304 6002  311  403 4002 2004  316 3005  312  408 6004 8000
  411  400 4004  414 8002  422 8004  406
BEAM          1  101  102  108
BEAM          3  106  102  108
BEAM          4  108  106  102
BEAM          5  201  202  208
BEAM          7  206  202  208
BEAM          8  208  206  202
BEAM         10  303  304  316
BEAM         14  312  304  316
BEAM         16  316  312  304
BEAM         18  400  408  406
BEAM         22  422  408  406
BEAM         24  414  422  406
BEAM         25  414  406  422
BEAM         27  406  400  422
BEAM         29  102 2000  106
BEAM         30 2000  202  106
BEAM         31  202 2002  106
BEAM         32 2002  304  106
BEAM         33  304 2004  106
BEAM         34 2004  408  106
BEAM         35 4000  106  102
BEAM         36  206 4000  102
BEAM         37 4002  206  102
BEAM         38  312 4002  102
BEAM         39 4004  312  102
BEAM         40  422 4004  102
BEAM         41 6000  108  311
BEAM         42  208 6000  311
BEAM         43 6002  208  311
BEAM         44  316 6002  311
BEAM         45 6004  316  311
BEAM         46  414 6004  311
BEAM         47 8000  316  311
BEAM         48 8002 8000  311
BEAM         49 8004 8002  311
BEAM         50  406 8004  311
BEAM         53  105  101  108
BEAM         54  105  108  101
BEAM         57  205  201  208

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
....:....1....:....2....:....3....:....4....:....5....:....6....:....7....:....8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

BEAM           58  205  208  201
BEAM           64  311  303  316
BEAM           66  311  316  303
BEAM           68  403  400  406
BEAM           72  411  403  406
BEAM           73  411  414  406
BEAM           75 1001  101  105
BEAM           76  201 1001  105
BEAM           77 1003  201  105
BEAM           78  303 1003  105
BEAM           79 1005  303  105
BEAM           80  403 1005  105
BEAM           81  105 3001  101
BEAM           82 3001  205  101
BEAM           83  205 3003  101
BEAM           84 3003  311  101
BEAM           85  311 3005  101
BEAM           86 3005  411  101
SEND
      56.875E00  101  102  105  106  108
     39.875E00   201  202  205  206  208
      22.625E00  303  304  311  312  316
     0.0    E+00  403  406  408  411  414  422  400
      48.375E00 2000 4000 1001 3001 6000
       31.25E00 2002 4002 1003 3003 6002
     11.3125E00 2004 4004 1005 3005 6004 8002
    16.96875E00 8000
     5.65625E00 8004
BLANK CARD
     0.0    E+00  108  208  316  400  405  414 6000
     0.0    E+00 6004 8000 8002 8004 6002
  12.328125E00  102  106
 -12.328125E00  101  105
  16.031250E00  202  206
 -16.031250E00  201  205
  19.7890625E00  304  312
 -19.7890625E00  303  311
  -24.71875E00  403  411
   24.71875E00  408  422
  14.1796875E00 2000 4000
 17.91015625E00 2002 4002
 22.28390625E00 2004 4004
 -14.1796875E00 1001 3001
-17.91015625E00 1003 3003

....:....1....:....2....:....3....:....4....:....5....:....6....:....7....:....8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
          .........1.........2.........3.........4.........5.........6.........7.........8
          12345678901234567890123456789012345678901234567890123456789012345678901234567890

          -22.25390625E00 1005 3005
BLANK CARD
               C.C    E+00  406
            10.1015625E00   101   102
           -10.1015625E00   105   106   108
            13.1953125E00   201   202
           -13.1953125E00   205   206   208
            16.3203125E00   303   304
           -16.3203125E00   311   312   316
           -8.16015625E00  8002
               20.4375E00   400   403   408
              -20.4375E00   411   414   422
            11.6484375E00 2000 1001
            14.7578125E00 2002 1003
           18.37890625E00 2004 1005
           -11.6484375E00 4000 3001 6000
           -14.7578125E00 4002 3003 6002
          -18.37890625E00 4004 3005 6004
          -12.24023438E00 8000
          -4.08007812 5E00 8004
BLANK CARD
          101010            101   201   303
          101010           6000  6002  6004  8000  8002  8004   108   208   316
          101010            105   205   311
          000000            400   403   408   406   411   414   422
          000000            102   106   202   206   304   312
          000000           1001  1003  1005  3001  3003  3005
BLANK CARD
            102      1      101     1   1.0
            106      1      105     1   1.0
            102      3      101     3   1.0
            106      3      105     3         1.0
            102      5      101     5   1.0
            106      5      105     5   1.0
            202      1      201     1   1.0
            206      1      205     1   1.0
            202      3      201     3   1.0
            206      3      205     3   1.0
            202      5      201     5   1.0
            206      5      205     5   1.0
            304      1      303     1   1.0
            312      1      311     1   1.0
            304      3      303     3   1.0
            312      3      311     3   1.0

          .........1.........2.........3.........4.........5.........6.........7.........8
          12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

  304     5        303     5  1.0
  312     5        311     5  1.0
 1001     1       2000     1       1.0
 1003     1       2002     1       1.0
 1005     1       2004     1       1.0
 3001     1       4000     1       1.0
 3003     1       4002     1       1.0
 3005     1       4004     1       1.0
 1001     2       2000     2      -1.0
 1003     2       2002     2      -1.0
 1005     2       2004     2      -1.0
 3001     2       4000     2      -1.0
 3003     2       4002     2      -1.0
 3005     2       4004     2      -1.0
 1001     3       2000     3       1.0
 1003     3       2002     3       1.0
 1005     3       2004     3       1.0
 3001     3       4000     3       1.0
 3003     3       4002     3       1.0
 3005     3       4004     3       1.0
 1001     4       2000     4      -1.0
 1003     4       2002     4      -1.0
 1005     4       2004     4      -1.0
 3001     4       4000     4      -1.0
 3003     4       4002     4      -1.0
 3005     4       4004     4      -1.0
 1001     5       2000     5       1.0
 1003     5       2002     5       1.0
 1005     5       2004     5       1.0
 3001     5       4000     5       1.0
 3003     5       4002     5       1.0
 3005     5       4004     5       1.0
 1001     6       2000     6      -1.0
 1003     6       2002     6      -1.0
 1005     6       2004     6      -1.0
 3001     6       4000     6      -1.0
 3003     6       4002     6      -1.0
 3005     6       4004     6      -1.0
```

BLANK CARD

```
MdM              11.2E06        .3333E00                                     50130.E00
```

BLANK CARD

```
    1    3    4   75    7    8   10   14   16   18   22   24   25   27   29  -50
   53   54   57   58   64   66   68   72   73   75  -86    5
SREC               2.0E00         .1667E00       .5667E00        0.0E00         .458

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
..........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

              0.0EC0              0.0E00
              2.0E00              1.0E00
        1        3        4        5        7        8       10       14       16       18       22       24       68       72       73       53
       54       57       58       64       66
SREC                    6.0E00             0.5E00             18.0E00              0.0E00             1.791
              0.0E00              0.0E00
              6.0EC0              1.0E00
       25       27
TSEC          .4474E00      .0448881      .0729367        0.0              7.0293E-03
         0.0          .90625
         1.C             2.09              .2775              .09375
       41      -46
TSEC          .4474E00      .0448881      .0729367        0.0              7.0293E-03

         1.0             2.09              .2775              .09375
       47      -50
LSEC          .1211E00 .01099               .01099          -.007087          1.57675E-04
         .9375          .9375
         1.C             1.0            .0625             .0625
       29      -40      75      -86
SEND
CONC
   101        -.13
BLANK CARD
   102        -.13
BLANK CARD
   105        -.13
BLANK CARD
   106        -.13
BLANK CARD
   108        -.48
BLANK CARD
BLANK CARD
SEND
       29      -50
BLANK CARD
-9000
BLANK CARD
          50000.0         .01
BLANK CARD

STOP

..........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

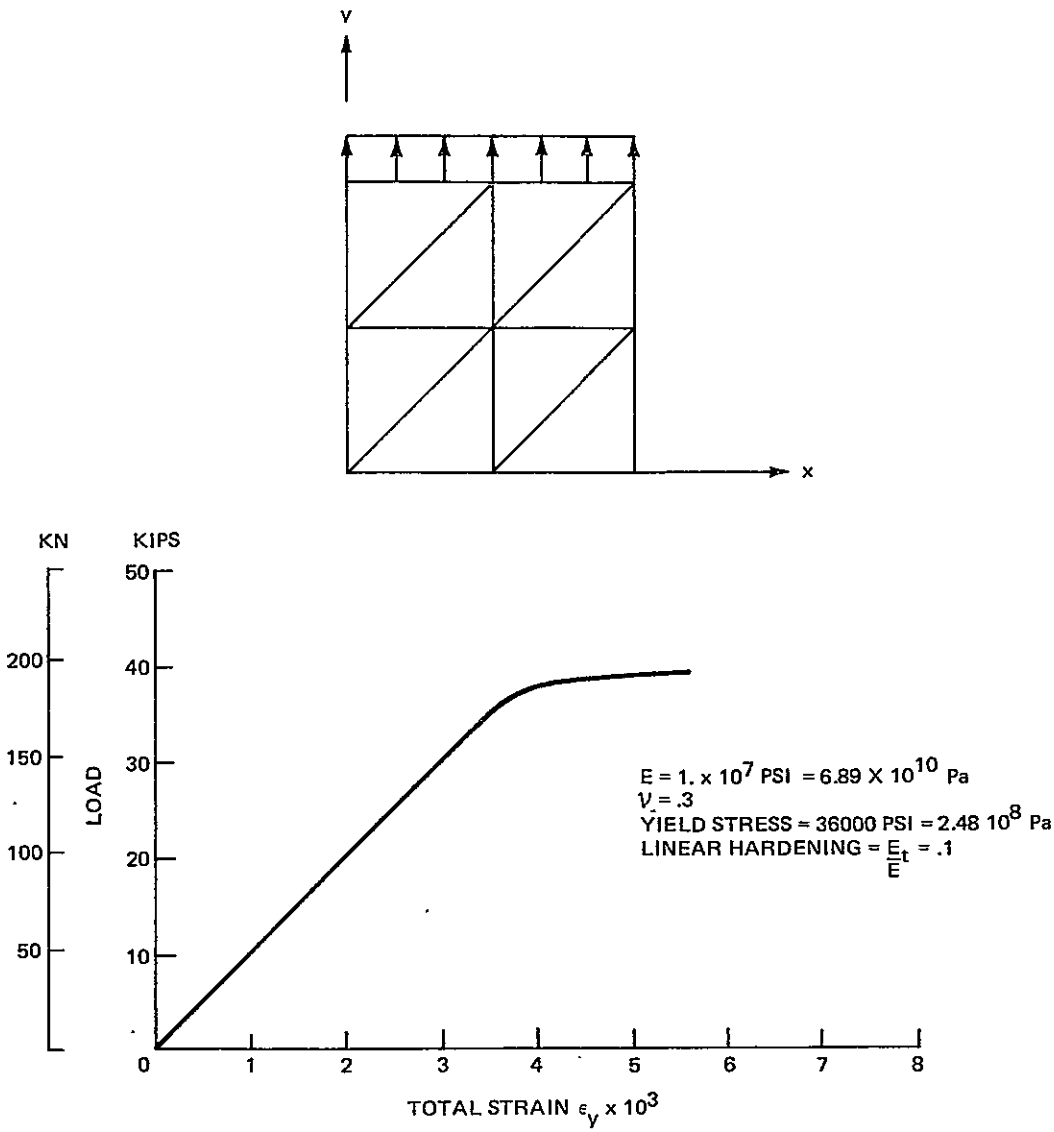
Example Problem No. 3

Program: BEND

Problem Title: 8 Element Square Plate

Comments: This problem represents a simple statically determinate case that uses all combinations of the triangular membrane element, TRIM. These range from 3-node constant strain triangles, the 6-node linear strain triangle, and the two interface cases with 4 and 5 nodes. A distributed edge load (using the TRIA feature) is applied on one edge.

The figure shows load versus longitudinal strain for a linear strain hardening material.



LOAD VERSUS TOTAL STRAIN ϵ_y

BEND SAMPLE PROBLEM #3 EIGHT ELEMENT SQUARE MEMBRANE

FRCGRAM LISTING OF INPUT CATA CARDS

```
••••••••••1•••••••••2•••••••••3•••••••••4•••••••••5•••••••••6•••••••••7•••••••••8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

BEND   SAMPLE PROPLEM =3 8 ELEMENT SQUARE PLATE  TRIM ELEMENTS
    1       1
  101  -105   202  -205   301   303   305   404   405   501   503   505
TRIM          301   301     0   303     0   503     0
TRIM          401   503     0   501     0   301     0
TRIM          201   303     0   301     0   101   202
TRIM          402   505     0   503     0   303   404
TRIM          202   305     0   303   203   103   204
TRIM          302   303     0   305   405   505   404
TRIM          101   101   102   103   203   303   202
TRIM          102   103   104   105   205   305   204
SEND
  0.0        E 00   101   301   501
  5.0        E 00   103   303   503
 10.0        E 00   105   305   505
BLANK CARD
  0.0        E 0C   101   103   105
     5.CCCCE 00     301   303   305
     1C.000E 0C     501   503   505
BLANK CARD
  0.0        E 00   101   103   105
     C.COCOF 00     301   303   305
     C.CCCOE 0C     501   503   505
BLANK CARD
CC000C              101
100000              102  -105
11000C              202  -205   303   305   404   405   503   505
C1000C              301   501
BLANK CARD
BLANK CARD
MATI   1.0         E 07  1.0         E 07           0.0       2.0785   E 06      .3
     1.0    E-06     1.0    E-06
  3.6       E 04   3.6       E 04   3.6       E 04  2.0785    E 04
  .1                   0.0          1.0        E 07    3.6    E 04   3.6      E 04
 101   1C2  201   202   301   302   401   402
THIK     1.0
 101   102  201   202   301   302   401   402
SEND
TRIA
 501         503   301      0.0            -1.0
                    0.0          -1.0
 503         505   303      C.0            -1.0
                    0.0          -1.0
BLANK CARD

••••••••••1•••••••••2•••••••••3•••••••••4•••••••••5•••••••••6•••••••••7•••••••••8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PFCGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

              SEND
              -1000
BLANK CARD
              -1000
BLANK CARD
                  5CCC0.0             .01
BLANK CARD
              STOP
```

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

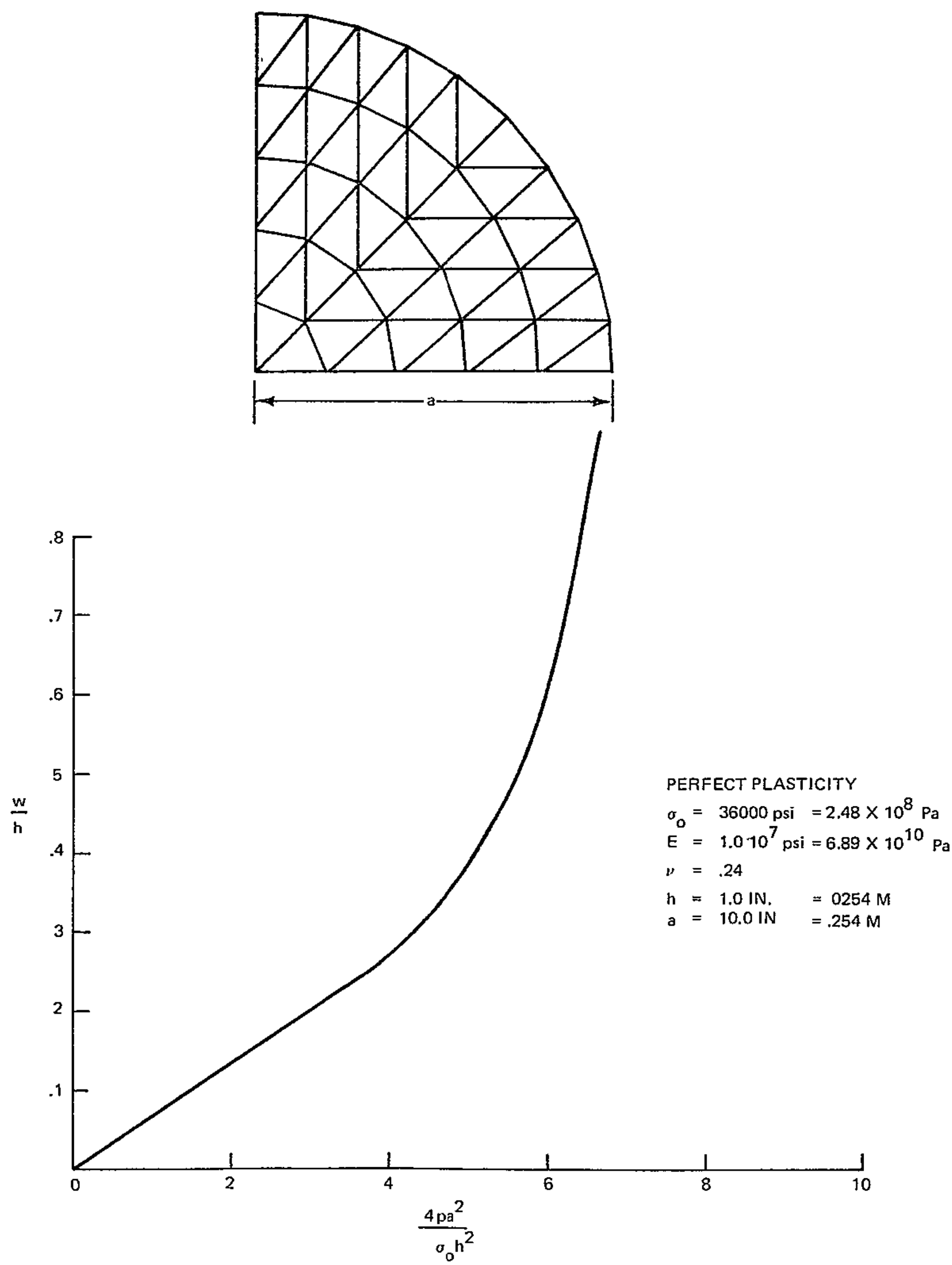
Example Problem No. 4

Program: BEND

Problem Title: Collapse of a Uniformly Loaded Circular Plate

Comments: This problem demonstrates the use of the plate element TRIP. One quadrant of a uniformly loaded, simply supported circular plate was modeled consisting of 50 elements, 36 nodes, and 151 degrees of freedom. The multipoint constraint feature was used to transform the edge rotations to their normal and tangential components in order to apply the simple support boundary conditions ($\theta_t = 0$). To be noted also is that the order of the element input was set so that the local directions of the curvatures at each node (defined by the first element containing that node) line up with the plate boundaries and lines of symmetry. This was done to enforce boundary/symmetry conditions for the higher order degrees of freedom (see Appendix IV, Section 2).

A complete discussion of the plastic analysis of this plate as well as a number of other plate configurations is given in Ref. 2. The figure shown here is for the load versus central deflection up to the collapse load.



LOAD VERSUS CENTRAL DEFLECTION

Bend Sample Problem #4 Uniformly Loaded Simply Supported Circular Plate

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

BEND SAMPLE PROBLEM =4 UNIFORMLY LOADED CIRCULAR PLATE
    1    10         1         1        50
    1   -36
TRIP                1         2         1         3
TRIP                2         1         4         3
TRIP                3         5         2         6
TRIP                4         2         3         6
TRIP                5         7         6         3
TRIP                6         8         7         3
TRIP                7         3         4         8
TRIP                8         4         9         8
TRIP                9        10         5        11
TRIP               10         5         6        11
TRIP               11        12        11         6
TRIP               12         6         7        12
TRIP               13        13        12         7
TRIP               14        14        13         7
TRIP               15         7         8        14
TRIP               16        15        14         8
TRIP               17         8         9        15
TRIP               18         9        16        15
TRIP               19        17        10        18
TRIP               20        10        11        18
TRIP               21        19        18        11
TRIP               22        11        12        19
TRIP               23        20        19        12
TRIP               24        12        13        20
TRIP               25        21        20        13
TRIP               26        22        21        13
TRIP               27        13        14        22
TRIP               28        23        22        14
TRIP               29        14        15        23
TRIP               30        24        23        15
TRIP               31        15        16        24
TRIP               32        16        25        24
TRIP               33        26        17        27
TRIP               34        17        18        27
TRIP               35        28        27        18
TRIP               36        18        19        28
TRIP               37        29        28        19
TRIP               38        19        20        29
TRIP               39        30        29        20
TRIP               40        20        21        30
TRIP               41        31        30        21

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

TRIP         42   32   31   21
TRIP         43   21   22   32
TRIP         44   33   32   22
TRIP         45   22   23   33
TRIP         46   34   33   23
TRIP         47   23   24   34
TRIP         50   25   36   35
TRIP         48   35   34   24
TRIP         49   24   25   35
SEND
 0.0                1    2    5   10   17   26
 1.41422            3    6   11   18   27
 2.0                4
 2.82844            7   12   19   28
 3.74162            8
 4.0                9
 4.24266           13   20   29
 5.29146           14
 5.83092           15
 6.0               16
 5.65688           21   30
 6.78232           22
 7.48328           23
 7.874             24
 8.0               25
 7.0711            31
 8.24621           32
 9.05538           33
 9.59166           34
 9.89949           35
10.0               36
```

BLANK CARD

```
 0.0                1    4    9   16   25   36
 2.0                2
 1.41422            3    8   15   24   35
 4.0                5
 3.74162            6
 2.82844            7   14   23   34
 6.0               10
 5.83092           11
 5.29146           12
 4.24266           13   22   33
 8.0               17
 7.874             18

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

  7.48328             19
  6.78232             20
  5.65688             21    32
10.0                  26
  9.89949             27
  9.59166             28
  9.05538             29
  8.24621             30
  7.0711              31
BLANK CARD
  0.0                  1   -36
BLANK CARD
001110000111           1   -36
001000000110           1
001100000110           2     5    10    17
001010000110           4     9    16    25
000010000001          27   -35
000010000000          26    36
BLANK CARD
     26    4          26     5   -1.0
     26    5          26     5    0.0
     27    4          27     5    -.98994904
     27    5          27     5     .14142199
     28    4          28     5    -.95916595
     28    5          28     5     .28284397
     29    4          29     5    -.90553799
     29    5          29     5     .42426596
     30    4          30     5    -.82462101
     30    5          30     5     .56568804
     31    4          31     5    -.70711002
     31    5          31     5     .70711002
     32    4          32     5    -.56568804
     32    5          32     5     .82462101
     33    4          33     5    -.42426596
     33    5          33     5     .90553799
     34    4          34     5    -.28284397
     34    5          34     5     .95916595
     35    4          35     5    -.14142199
     35    5          35     5     .98994904
     36    4          36     5    0.0
     36    5          36     5     1.0
BLANK CARD
MAT1       1.0    E+07      1.0   E+07      0.0             4.032258E+06     .24
     0.0               0.0

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
            .........1.........2.........3.........4.........5.........6.........7.........8
            12345678901234567890123456789012345678901234567890123456789012345678901234567890

                 36000.0        36000.0        36000.0        20784.6
                 0.0            0.0            1.0     E+07     36000.0                  36000.0
                1  -50
            THICK     1.0
                1  -50
            NLRS
               10     1
            SEND
            SURF      0.0            0.0                   -1.0
                 0.0            0.0                 -1.0
                 0.0            0.0                 -1.0
                1  -50
            SEND
                1    2
BLANK CARD
                1  -36
BLANK CARD
                 630.0          .01

BLANK CARD

            STOP




            .........1.........2.........3.........4.........5.........6.........7.........8
            12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

Example Problem No. 5

Program: BEND

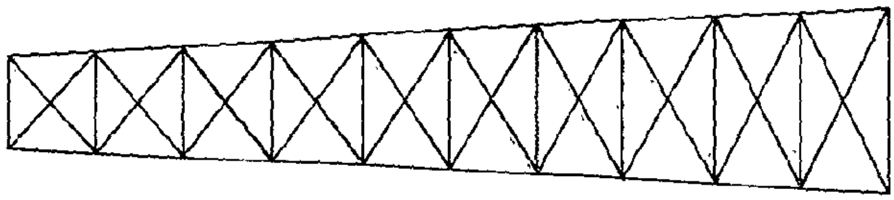
Problem Title: Thermoplastic Analysis of a Circular Disk

Comments: This problem demonstrates the thermal stress capability in conjunction with the triangular membrane element. To do this, an elastic plastic analysis of an annular disk subjected to a steady state axisymmetric radial temperature distribution was performed. Since the response is axisymmetric, only a slice of the disk was modeled with the multipoint constraint feature being used to enforce tangential and normal boundary conditions on one edge. Forty constant strain triangular elements connecting 32 nodes were used in the idealization. The steady state temperature distribution is applied at nodes according to the relation

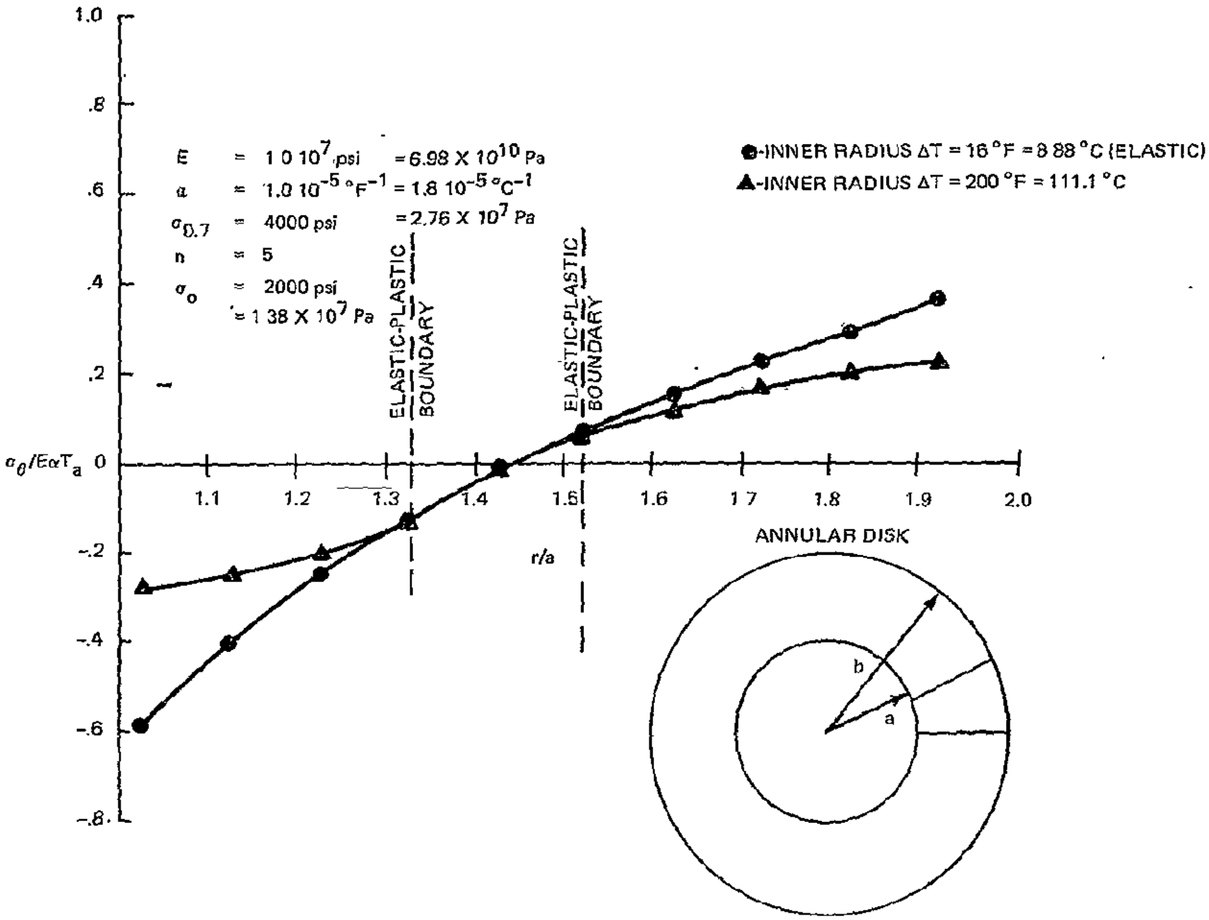
$$T = T_a - (T_b - T_a)\ \ell n(r/a)/\ell n(b/a)$$

where T_a and T_b are the inner and outer radius temperature, respectively.

Results shown in the figure are for dimensionless circumferential stress versus the radial coordinate. Comparison between these results and Ref. 4 are good.



FINITE ELEMENT MODEL OF A SECTOR OF THE DISK



DISTRIBUTION OF CIRCUMFERENTIAL STRESS

Bend Sample Problem #5 Thermoplastic Analysis of Circular Disk

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

BEND  SAMPLE PROBLEM #5 THERMOPLASTIC ANALYSIS OF CIRCULAR DISK
    0    5    1
    1   -3    7   -9   13  -15   19  -21   25  -27   31  -33   37  -39   43  -45
   49  -51   55  -57   61   63
TRIM         01   01    0   03    0   02
TRIM         02   03    0   09    0   02
TRIM         03   09    0   07    0   02
TRIM         04   07    0   01    0   02
TRIM         05   07    0   09    0   08
TRIM         06   09    0   15    0   08
TRIM         07   15    0   13    0   08
TRIM         08   13    0   07    0   08
TRIM         09   13    0   15    0   14
TRIM         10   15    0   21    0   14
TRIM         11   21    0   19    0   14
TRIM         12   19    0   13    0   14
TRIM         13   19    0   21    0   20
TRIM         14   21    0   27    0   20
TRIM         15   27    0   25    0   20
TRIM         16   25    0   19    0   20
TRIM         17   25    0   27    0   26
TRIM         18   27    0   33    0   26
TRIM         19   33    0   31    0   26
TRIM         20   31    0   25    0   26
TRIM         21   31    0   33    0   32
TRIM         22   33    0   39    0   32
TRIM         23   39    0   37    0   32
TRIM         24   37    0   31    0   32
TRIM         25   37    0   39    0   38
TRIM         26   39    0   45    0   38
TRIM         27   45    0   43    0   38
TRIM         28   43    0   37    0   38
TRIM         29   43    0   45    0   44
TRIM         30   45    0   51    0   44
TRIM         31   51    0   49    0   44
TRIM         32   49    0   43    0   44
TRIM         33   49    0   51    0   50
TRIM         34   51    0   57    0   50
TRIM         35   57    0   55    0   50
TRIM         36   55    0   49    0   50
TRIM         37   55    0   57    0   56
TRIM         38   57    0   63    0   56
TRIM         39   63    0   61    0   56
TRIM         40   61    0   55    0   56

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

SEND
     1.0     E 00     3     6
     1.1     E 00     9    12
     1.2     E 00    15    18
     1.3     E 00    21    24
     1.4     E 00    27    30
     1.5     E 00    33    36
     1.6     E 00    39    42
     1.7     E 00    45    48
     1.8     E 00    51    54
     1.9     E 00    57    60
     2.0     E 00    63    66
   .99452189E 00     1     4
  1.0939741 E 00     7    10
  1.1934262 E 00    13    16
  1.2928784 E 00    19    22
  1.3923306 E 00    25    28
  1.4917828 E 00    31    34
  1.5912350 E 00    37    40
  1.6814254 E 00    43    46
  1.7901394 E 00    49    52
  1.8895916 E 00    55    58
  1.9890437 E 00    61    64
       1.05  E 00     2     5
       1.15  E 00     8    11
       1.25  E 00    14    17
        1.35 E 00    20    23
       1.45  E 00    26    29
       1.55  E 00    32    35
       1.65  E 00    38    41
        1.75 E 00    44    47
       1.85  E 00    50    53
        1.95 E 00    56    59
BLANK CARD
     0.0     E 00     3    -3    -66
   .0523359 E 00      2     5
   .0575695 E 00      8    11
   .0628031 E 00     14    17
   .0680367 E 00     20    23
   .0732703 E 00     26    29
   .0785039 E 00     32    35
   .0837375 E 00     38    41
   .0889711 E 00     44    47
   .0942047 E 00     50    53

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
••••••••••1•••••••••2•••••••••3•••••••••4•••••••••5•••••••••6•••••••••7•••••••••8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

             .0994383 E 00     56    59
             .1046712 E 00     62    65
             .1045284 E 00      1     4
             .1149813 E 00      7    10
             .1254341 E 00     13    16
             .1358870 E 00     19    22
             .1463398 E 00     25    28
             .1567926 E 00     31    34
             .1672455 E 00     37    40
             .1776983 E 00     43    46
             .1881512 E 00     49    52
             .1986040 E 00     55    58
             .2090569 E 00     61    64
BLANK CARD
                 0.0            1   -63
BLANK CARD
         110000000000           1    -3     7    -9    13   -15    19   -21    25   -27    31   -33    37
                              -39    43   -45    49   -51    55   -57    61    63
         100000000000           1    -6   -61     3    -6   -63
BLANK CARD
                1     1         1     1 .9945218              1     2 -.1045284
                1     2         1     1 .1045284              1     2  .9945218
                7     1         7     1 .9945218              7     2 -.1045284
                7     2         7     1 .1045284              7     2  .9945218
               13     1        13     1 .9945218             13     2 -.1045284
               13     2        13     1 .1045284             13     2  .9945218
               19     1        19     1 .9945218             19     2 -.1045284
               19     2        19     1 .1045284             19     2  .9945218
               25     1        25     1 .9945218             25     2 -.1045284
               25     2        25     1 .1045284             25     2  .9945218
               31     1        31     1 .9945218             31     2 -.1045284
               31     2        31     1 .1045284             31     2  .9945218
               37     1        37     1 .9945218             37     2 -.1045284
               37     2        37     1 .1045284             37     2  .9945218
               43     1        43     1 .9945218             43     2 -.1045284
               43     2        43     1 .1045284             43     2  .9945218
               49     1        49     1 .9945218             49     2 -.1045284
               49     2        49     1 .1045284             49     2  .9945218
               55     1        55     1 .9945218             55     2 -.1045284
               55     2        55     1 .1045284             55     2  .9945218
               61     1        61     1 .9945218             61     2 -.1045284
               61     2        61     1 .1045284             61     2  .9945218
BLANK CARD
         MAT1       1.0      E 07      1.0     E 07       0.0     E 00   3.846153 E 06       .30000E 00

••••••••••1•••••••••2•••••••••3•••••••••4•••••••••5•••••••••6•••••••••7•••••••••8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

     1.0   E-05     1.0   E-05
     2.0   E 03     2.0   E 03     2.0   F 03   1.1547002E 03
    5.0    E CC     4.0   F 03     1.0   E 07     2.0   E 03      2.0   E 03
    1  -40
THIK      .1
    1  -40
SEND
TMPU
    1.0             1    3
  .8624964          7    9
  .7369655         13   15
  .6214883         19   21
  .5145732         25   27
  .4150375         31   33
  .3219281         37   39
  .2344652         43   45
  .1520030         49   51
  .07400054        55   57
  .0               61   63
.92961067           2
.79836614           8
.6780719           14
.56704059          20
.46394710          26
.36773178          32
.27753398          38
.19264508          44
.11247473          50
.036525879         56
```

BLANK CARD

```
SEND
    1  -40
```

BLANK CARD

```
    1  -63
```

BLANK CARD

```
     200.0          .05
```

BLANK CARD

```
STOP
```

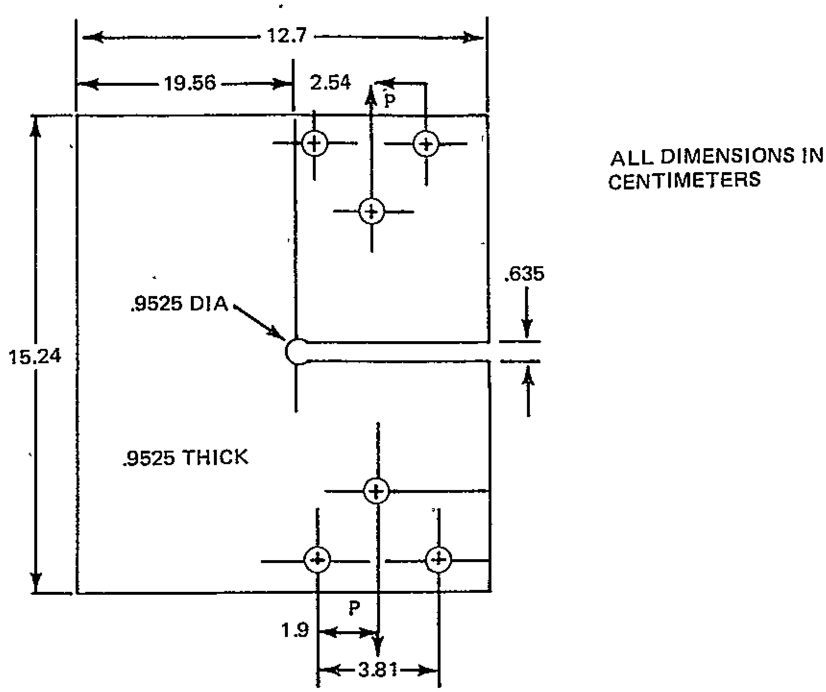
```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

Example Problem No. 6

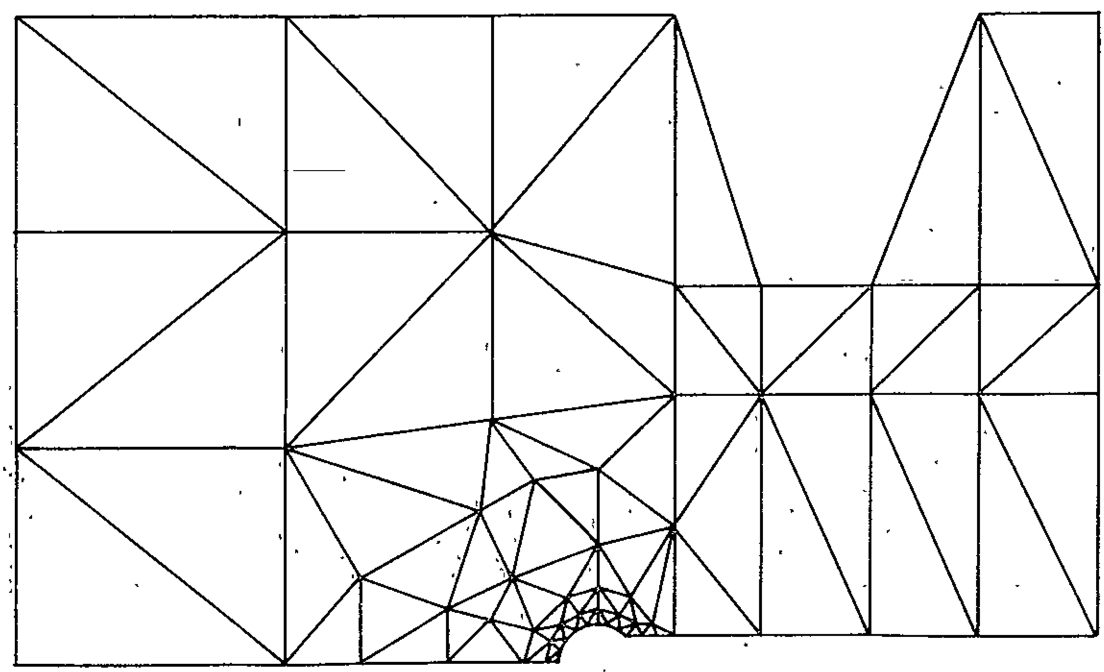
Program: BEND

Problem Title: Elastic Plastic Analysis of an SAE Keyhole Specimen

Comments: The accompanying figures show an SAE keyhole specimen, the finite element idealization of half of the plate, and results for the strain at the notch versus load. The idealization uses 92 constant strain elements connecting 64 nodes that leads to 120 degrees of freedom.

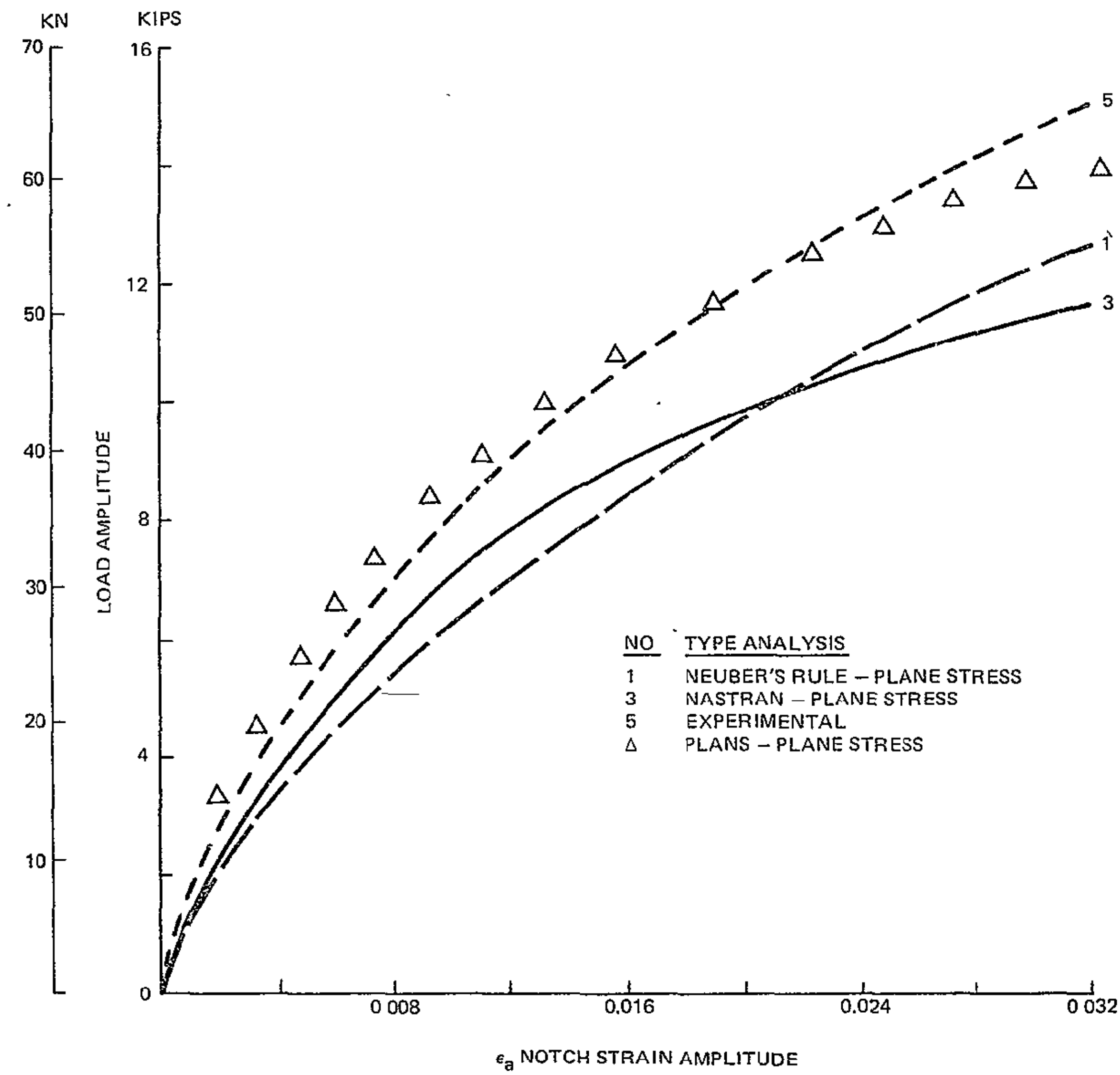


SAE KEYHOLE SPECIMEN



FINITE ELEMENT IDEALIZATION

Bend Sample Problem #6 Load versus Notch Strain for SAE Keyhole Specimen



Bend Sample Problem #6 (Continued)

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

BEND SAMPLE PROBLEM # 6  SAE KEYHOLE SPECIMEN
    1    0    1    1   50
    1  -64
GLXY
TRIM           1    1         2         10
TRIM           2    2         3         10
TRIM           3    3        12         10
TRIM           4   10        12         11
TRIM           5   11        12         19
TRIM           6   20        19         12
TRIM           7   13        20         12
TRIM           8   21        20         13
TRIM           9   22        21         13
TRIM          10    3        13         12
TRIM          11    3         4         13
TRIM          12    4         5         13
TRIM          13    5        14         13
TRIM          14   14        22         13
TRIM          15   15        22         14
TRIM          16    6        15         14
TRIM          17    5         6         14
TRIM          18   25         6          5
TRIM          19   25        27          6
TRIM          20   26        27         25
TRIM          21   27        28          6
TRIM          22   28         7          6
TRIM          23    6        16         15
TRIM          24    6         7         16
TRIM          25    7        17         16
TRIM          26   16        17         23
TRIM          27   17        18         23
TRIM          28   18        24         23
TRIM          29    8        18         17
TRIM          30    8         9         18
TRIM          31    8        30          9
TRIM          32    8        29         30
TRIM          33   28        29          7
TRIM          34    7        29          8
TRIM          35    7         8         17
TRIM          36   25         5         64
TRIM          37   64         5          4
TRIM          38   63        64          4
TRIM          39   62        63          4
TRIM          40   62         4          3

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

TRIM         41   61        62         3
TRIM         42    2        61         3
TRIM         43    2        59        61
TRIM         44   59        60        61
TRIM         45   60        62        61
TRIM         46   60        57        62
TRIM         47   57        63        62
TRIM         48   57        58        63
TRIM         49   58        64        63
TRIM         50   58        25        64
TRIM         51   59        55        60
TRIM         52   55        56        60
TRIM         53   56        57        60
TRIM         54   56        49        57
TRIM         55   55        48        56
TRIM         56   48        49        56
TRIM         57   49        50        57
TRIM         58   50        58        57
TRIM         59   50        51        58
TRIM         60   51        52        58
TRIM         61   52        25        58
TRIM         62   52        53        25
TRIM         63   53        26        25
TRIM         64   48        40        41
TRIM         65   48        41        49
TRIM         66   41        42        49
TRIM         67   49        42        50
TRIM         68   42        43        50
TRIM         69   43        51        50
TRIM         70   43        44        51
TRIM         71   44        45        51
TRIM         72   45        52        51
TRIM         73   45        46        52
TRIM         74   46        53        52
TRIM         75   47        53        46
TRIM         76   47        54        53
TRIM         77   54        26        53
TRIM         78   40        31        32
TRIM         79   40        32        41
TRIM         80   32        33        41
TRIM         81   41        33        42
TRIM         82   33        34        42
TRIM         83   42        34        43
TRIM         84   34        35        43

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

TRIM         85   35        44        43
TRIM         86   35        36        44
TRIM         87   36        37        44
TRIM         88   37        45        44
TRIM         89   37        38        45
TRIM         90   38        46        45
TRIM         91   38        39        46
TRIM         92   39        47        46
SEND
     0.0000E 00    1
     1.2500E 00    2
     1.2500E 00    3
     2.2000E 00    4
     3.0500E 00    5
     3.4500E 00    6
     3.9500E 00    7
     4.4500E 00    8
     5.0000E 00    9
     0.0000E 00   10
     0.0000E 00   11
     1.2500E 00   12
     2.2000E 00   13
     3.0500E 00   14
     3.4500E 00   15
     3.9500E 00   16
     4.4500E 00   17
     5.0000E 00   18
     0.0000E 00   19
     1.2500E 00   20
     2.2000E 00   21
     3.0500E 00   22
     4.4500E 00   23
     5.0000E 00   24
     3.0500E 00   25
     3.0500E 00   26
     3.4500E 00   27
     3.9500E 00   28
     4.4500E 00   29
     5.0000E 00   30
     2.5125E 00   31
     2.5217E 00   32
     2.5483E 00   33
     2.5900E 00   34
     2.6421E 00   35

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

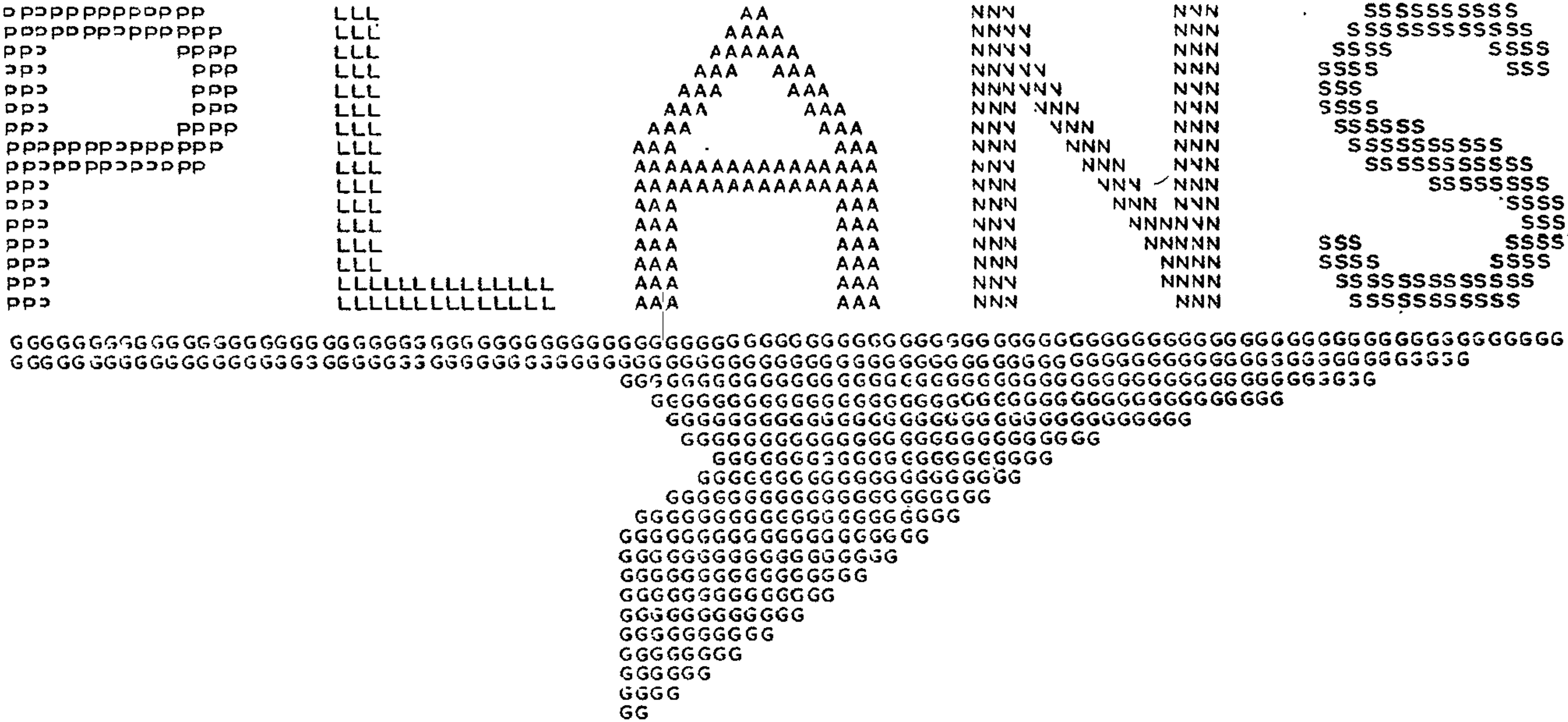
```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

          2.7000E 00    36
          2.7579E 00    37
          2.8102E 00    38
          2.8375E 00    39
          2.4500E 00    40
          2.4750E 00    41
          2.5250E 00    42
          2.6000F 00    43
          2.7000E 00    44
          2.8000E 00    45
          2.8750E 00    46
          2.9000E 00    47
          2.3500E 00    48
          2.4000E 00    49
          2.5500E 00    50
          2.7000E 00    51
          2.8500E 00    52
          2.9500E 00    53
          2.9750E 00    54
          2.0000E 00    55
          2.2000E 00    56
          2.3000E 00    57
          2.7000E 00    58
          1.6000E 00    59
          2.0000E 00    60
          1.6000E 00    61
          2.1500F 00    62
          2.4000E 00    63
          2.7000E 00    64
BLANK CARD
          0.0000E 00     1
          0.0000E 00     2
          1.0000E 00     3
          1.1250E 00     4
          1.2500E 00     5
          1.2500E 00     6
          1.2500F 00     7
          1.2500F 00     8
          1.2500E 00     9
          1.0000E 00    10
          2.0000E 00    11
          2.0000E 00    12
          2.0000E 00    13
          1.7500E 00    14

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

    1.7500E 00    15
    1.7500E 00    16
    1.7500E 00    17
    1.7500E 00    18
    3.0000E 00    19
    3.0000E 00    20
    3.0000E 00    21
    3.0000E 00    22
    3.0000E 00    23
    3.0000E 00    24
    0.6250E 00    25
    0.1250E 00    26
    0.1250E 00    27
    0.1250E 00    28
    0.1250E 00    29
    0.1250E 00    30
    0.0000E 00    31
    0.0579E 00    32
    0.1102E 00    33
    0.1517E 00    34
    0.1783E 00    35
    0.1875E 00    36
    0.1783E 00    37
    0.1517E 00    38
    0.1250E 00    39
    0.0000E 00    40
    0.1000E 00    41
    0.1750E 00    42
    0.2250E 00    43
    0.2500E 00    44
    0.2250E 00    45
    0.1750E 00    46
    0.1250E 00    47
    0.0000E 00    48
    0.1500E 00    49
    0.3000E 00    50
    0.3500E 00    51
    0.3000E 00    52
    0.1750E 00    53
    0.1250E 00    54
    0.0000E 00    55
    0.2000E 00    56
    0.4000E 00    57
    0.5500E 00    58

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
          ..........1.........2.........3.........4.........5.........6.........7.........8
          12345678901234567890123456789012345678901234567890123456789012345678901234567890

              0.0000E 00    59
              0.2500E 00    60
              0.4000E 00    61
              0.7000E 00    62
              0.8500E 00    63
              0.9000E 00    64
BLANK CARD
              0.0000E 00     1   -64
BLANK CARD
          110000000000       1   -64
          000000000000       1
          100000000000       2    31    40    48    55    59
BLANK CARD
BLANK CARD
          MAT1     29.50000E 06     29.500E 06     0.0000E 00   1.134620E 07    0.3000E 00
              0.0000E 00     0.0000E 00
              35.000E 03     35.000E 03     35.000E 03    2.02070E 04
             5.26320E 00    38.5890E 03    29.5000E 06     35.000E 03    35.0000E 03
             1   -92
          THIK      0.3750E 00
             1   -92
          SEND
          CONC
             15     0.0000E 00     -1.000E 00     0.0000E 00
                    0.0000E 00      0.000E 00      0.000E 00
             16     0.0000E 00     -1.000E 00      0.000E 00
                     0.000E 00      0.000E 00      0.000E 00
BLANK CARD
          SEND
             1   -92
BLANK CARD
             1   -64
BLANK CARD
             -7.5000E 03     0.0100E 00
BLANK CARD
          STOP

          ..........1.........2.........3.........4.........5.........6.........7.........8
          12345678901234567890123456789012345678901234567890123456789012345678901234567890
```



REVBY MODULE

ELASTIC-PLASTIC ANALYSIS

PREPARED BY

THE RESEARCH DEPARTMENT

OF GRUMMAN AEROSPACE CORPORATION

UNDER PARTIAL FUNDING FROM

THE NASA LANGLEY RESEARCH CENTER

Example Problem No. 7

Program: REVBY

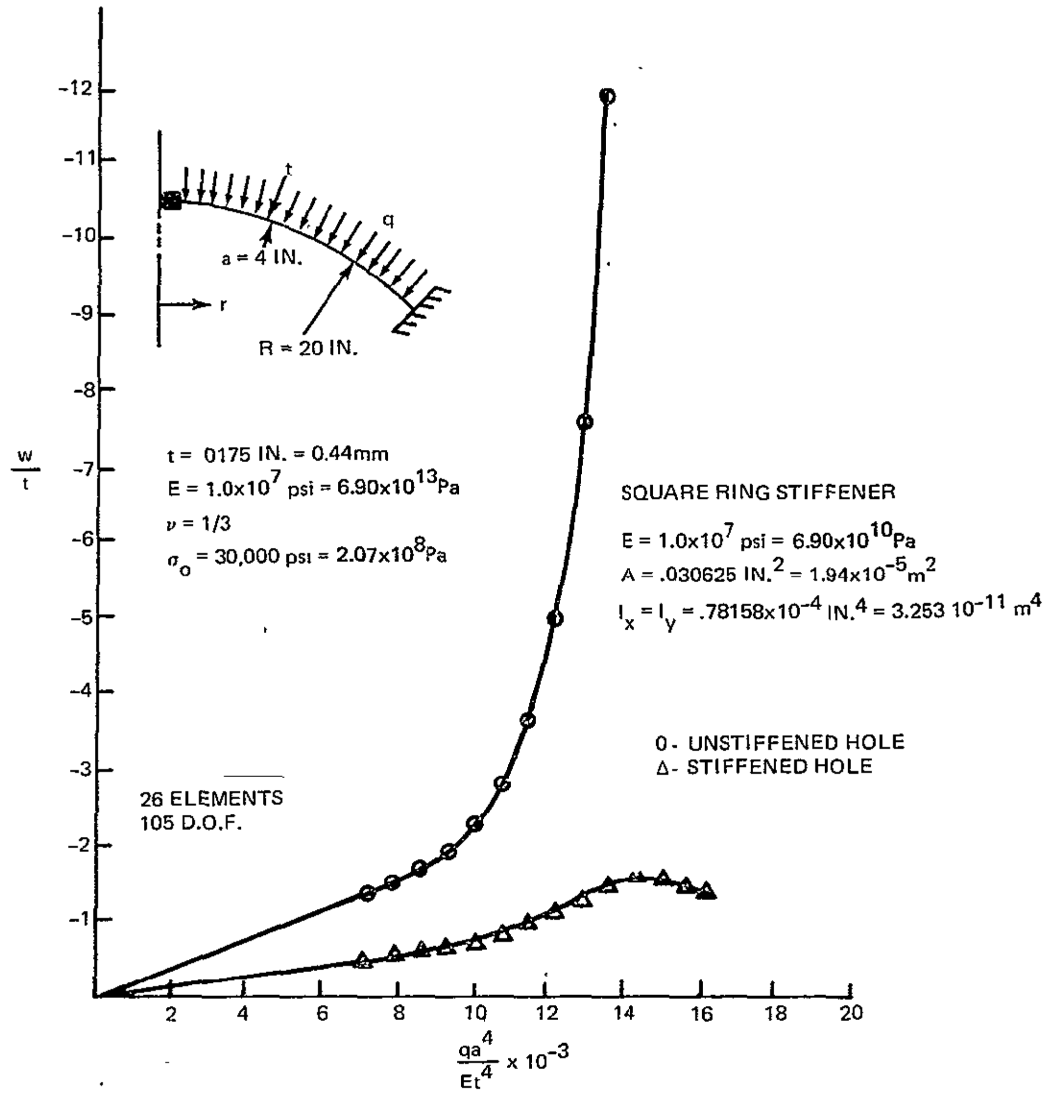
Problem Title: Plastic Analysis of a Ring Stiffened Spherical Shell

Comments: This problem demonstrates the use of the shell and ring element. The structure consists of a uniformly loaded spherical shell with a stiffened circular hole at the shell apex. The problem is idealized using 26 axisymmetric shell elements and one ring element connecting 27 nodes.

The figure shows the normal displacement versus applied pressure at the ring shell interface and also results for an unstiffened shell. Sudden collapse occurs for the stiffened shell at $qa^4/Et^4 = 15000$.

Additional results for this problem are in Ref. 3.

LOAD-DEFLECTION CURVES FOR RING-STIFFENED SPHERICAL SHELL UNDER EXTERNAL PRESSURE



(a) NORMAL DISPLACEMENT AT THE HOLE BOUNDARY

REVBY Sample Problem # 1 Ring Stiffened Spherical Shell

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

REVBY SAMPLE  PROBLEM =1 RING STIFFENED SPHERICAL SHELL
    1    10
    1   -27
SHEL              1       1       2
SHEL              2       2       3
SHEL              3       3       4
SHEL              4       4       5
SHEL              5       5       6
SHEL              6       6       7
SHEL              7       7       8
SHEL              8       8       9
SHEL              9       9      10
SHEL             10      10      11
SHEL             11      11      12
SHEL             12      12      13
SHEL             13      13      14
SHEL             14      14      15
SHEL             15      15      16
SHEL             16      16      17
SHEL             17      17      18
SHEL             18      18      19
SHEL             19      19      20
SHEL             20      20      21
SHEL             21      21      22
SHEL             22      22      23
SHEL             23      23      24
SHEL             24      24      25
SHEL             25      25      26
SHEL             26      26      27
RING             27       1
SEND
   3.9999998E-01        1
   4.8724484E-01        2
   5.7448047E-01        3
   6.6170520E-01        4
   7.4891728E-01        5
   8.6517763E-01        6
   9.8140872E-01        7
   1.0976057E 00        8
   1.2834435E 00        9
   1.4691706E 00       10
   1.6547689E 00       11
   1.8402252E 00       12
   2.0255213E 00       13

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

     2.2106419E 00    14
     2.3955717E 00    15
     2.5802937E 00    16
     2.7647915E 00    17
     2.9490499E 00    18
     3.1330528E 00    19
     3.3167849E 00    20
     3.4314718E 00    21
     3.5460434E 00    22
     3.6604939E 00    23
     3.7462511E 00    24
     3.8319378E 00    25
     3.9175501E 00    26
     4.0030890E 00    27
BLANK CARD
    -4.0003993E-03     1
    -5.9360676E-03     2
    -8.2523972E-03     3
    -1.0949340E-02     4
    -1.4026847E-02     5
    -1.8722069E-02     6
    -2.4093587E-02     7
    -3.0141219E-02     8
    -4.1223209E-02     9
    -5.4034557E-02    10
    -6.8574131E-02    11
    -8.4840655E-02    12
    -1.0283279E-01    13
    -1.2254894E-01    14
    -1.4398742E-01    15
    -1.6714633E-01    16
    -1.9202363E-01    17
    -2.1861726E-01    18
    -2.4692488E-01    19
    -2.7694398E-01    20
    -2.9657394E-01    21
    -3.1687075E-01    22
    -3.3783376E-01    23
    -3.5399282E-01    24
    -3.7052590E-01    25
    -3.8743269E-01    26
    -4.0471286E-01    27
BLANK CARD
     2.0001329E-02     1
```


```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

   2.4364647E-02     2
   2.8727978E-02     3
   3.3091299E-02     4
   3.7454616E-02     5
   4.3272387E-02     6
   4.9090147E-02     7
   5.4907918E-02     8
   6.4216316E-02     9
   7.3524714E-02    10
   8.2833171E-02    11
   9.2141569E-02    12
   1.0145003E-01    13
   1.1075842E-01    14
   1.2006682E-01    15
   1.2937528E-01    16
   1.3868368E-01    17
   1.4799213E-01    18
   1.5730053E-01    19
   1.6660893E-01    20
   1.7242670E-01    21
   1.7824447E-01    22
   1.8406224E-01    23
   1.8842554E-01    24
   1.9278890E-01    25
   1.9715220E-01    26
   2.0151556E-01    27
BLANK CARD
111100          1   -26
000100         27
BLANK CARD
BLANK CARD
   1.0000000E 07  1.0000000E 07  0.3750000E 07  0.3333333E 00
   0.3000000E 05  0.3000000E 05  0.3000000E 05  0.1732000E 05  0.0000000E 00
   0.0000000E 00  0.3000000E 05  1.0000000E 07
   1.0000000E-05  1.0000000E-05
    1   -26
BLANK CARD
   1.7499998E-02
    1   -27
BLANK CARD
    6    10
         1.0E 07      3.0625E-02      7.8153E-05 0.00000000E 00      7.8158E-05
         0.0E00        8.75E-02
SPEC .             .175E00         .175E00

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

           4.0E03           0.0E00           0.0E00
  1.0000000E-05
   27
BLANK CARD
  0.0000000E 00 -1.0000000E 00  0.0000000E 00
    1   -27
BLANK CARD
BLANK CARD
SEND
  -27
BLANK CARD
  -27
BLANK CARD
   26    27
BLANK CARD
  -27
BLANK CARD
      60.0            .01
BLANK CARD
STOP
```



```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

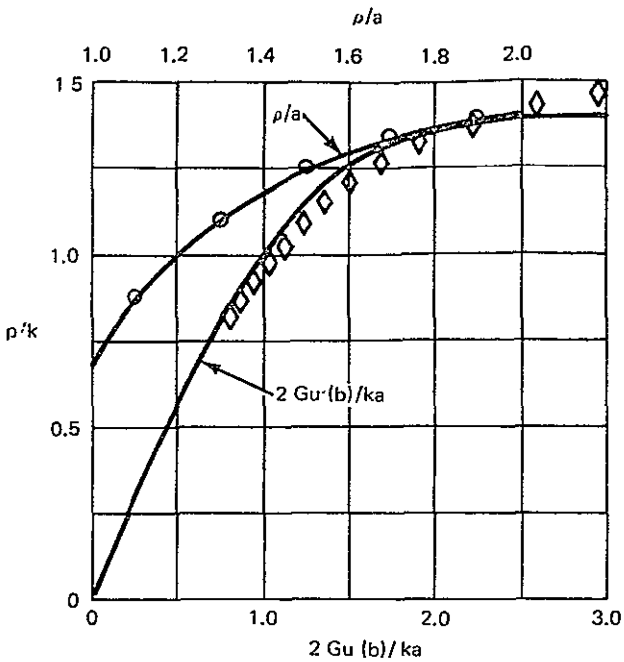
Example Problem No. 8

Program: REVBY

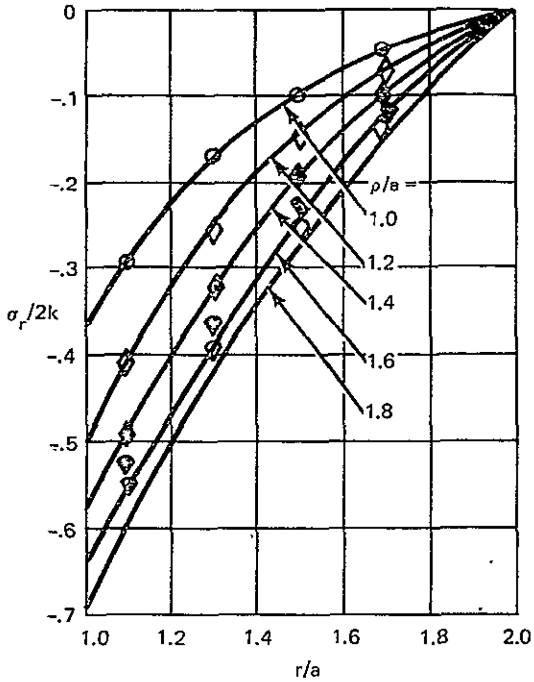
Problem Title: Thick Tube Under Internal Pressure

Comments: This problem demonstrates the use of the axisymmetric revolved triangular element, TRIR. Five bays of four triangles each (20) are used to idealize thick tube under internal pressure.

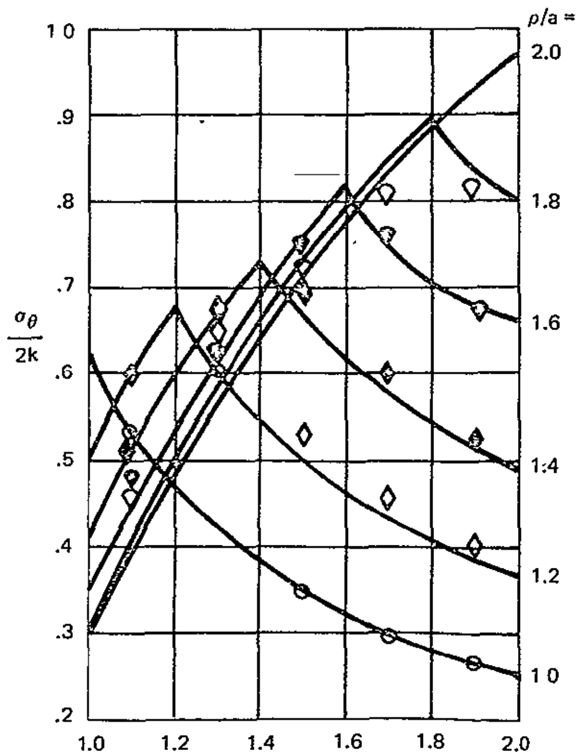
A series of results are shown in the figure and compared with a solution in Ref. 5. Additional discussion of this problem is in Ref. 3.



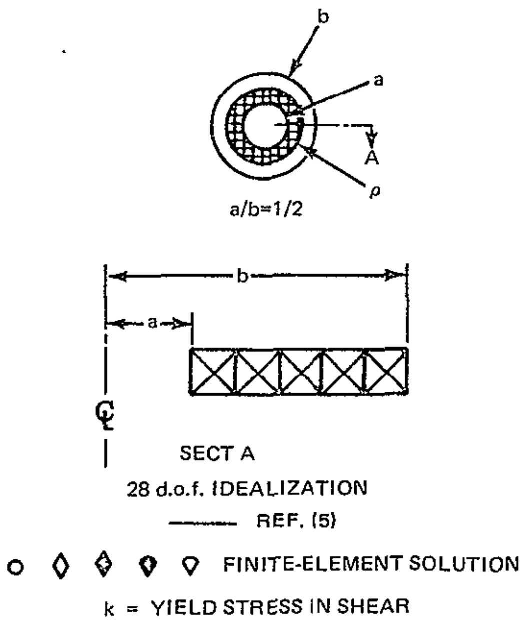
PRESSURE p VERSUS RADIAL DISPLACEMENT u (b) AND RADIUS OF ELASTIC-PLASTIC BOUNDARY, ρ



DISTRIBUTION OF RADIAL STRESS



DISTRIBUTION OF CIRCUMFERENTIAL STRESS



REVBY Sample Problem # 2 Thick Cylinder Under Internal Pressure

PROGRAM LISTING OF INPUT DATA CARDS

```
         1         2         3         4         5         6         7         8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

HEAVY SAMPLE PROBLEM #2 THICK CYLINDER UNDER INTERNAL PRESSURE
    1    1
    1   -6  101 -105    7  -12
TRIR          1    1  101    7
TRIR          2  101    8    7
TRIR          3  101    2    8
TRIR          4    1    2  101
TRIR          5    2  102    8
TRIR          6  102    9    8
TRIR          7    3    9  102
TRIR          8    2    3  102
TRIR          9    3  103    9
TRIR         10  103   10    9
TRIR         11  103    4   10
TRIR         12    3    4  103
TRIR         13    4  104   10
TRIR         14  104   11   10
TRIR         15  104    5   11
TRIR         16    4    5  104
TRIR         17    5  105   11
TRIR         18  105   12   11
TRIR         19  105    6   12
TRIR         20    5    6  105
SE[illegible]
  5.0000000E-01    1    7
  6.0000000E-01    2    8
  7.0000000E-01    3    9
  8.0000000E-01    4   10
  9.0000000E-01    5   11
  1.0000000E 00    6   12
  5.5000000E-01  101
  6.5000000E-01  102
  7.5000000E-01  103
  8.5000000E-01  104
  9.5000000E-01  105
BLANK CARD
  [illegible]E 03    1   -6
  5.0000000E-02  101 -105
  1.0000000E-01    7  -12
BLANK CARD
BLANK CARD
100100        1   -6
110100      101 -105    7  -12
BLANK CARD
BLANK CARD

         1         2         3         4         5         6         7         8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

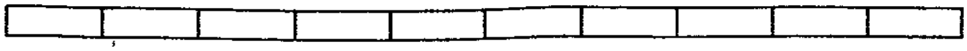
```
                                  1         2         3         4         5         6         7         8
                         12345678901234567890123456789012345678901234567890123456789012345678901234567890

                            1.0000000E 07  1.0000000E 07          1.0E07  3.0000000E-01  3.0000000E-01
                            3.0000000E-01  3.8491540E 06  3.8491540E 06  3.8491540E 06         0.0E00
                            3.6000000E 04  3.6000000E 04  3.6000000E 04  2.0784000E 04  0.0000000E 00
                            4.0000000E 04  3.6000000E 04  1.0000000E 07  2.0784000E 04  2.0784000E 04
                                   0.1E-04        0.1E-04        0.1E-04
                              1   -20
BLANK CARD
                              7   1.0000000E 00          0.0E00          0.0E00
                                    1   1.0000000E 00          0.0E00          0.0E00
BLANK CARD
BLANK CARD
                         SEND
                           -20
BLANK CARD
                              1   -12   101  -105
BLANK CARD
                           -20
BLANK CARD
                              1   -12   101  -105
BLANK CARD
                            0.2000000E 05  1.0000000E-02
                              5         -20.E03          .005E00
                         SEND
BLANK CARD
                         STOP

                                  1         2         3         4         5         6         7         8
                         12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

Example Problem No. 9

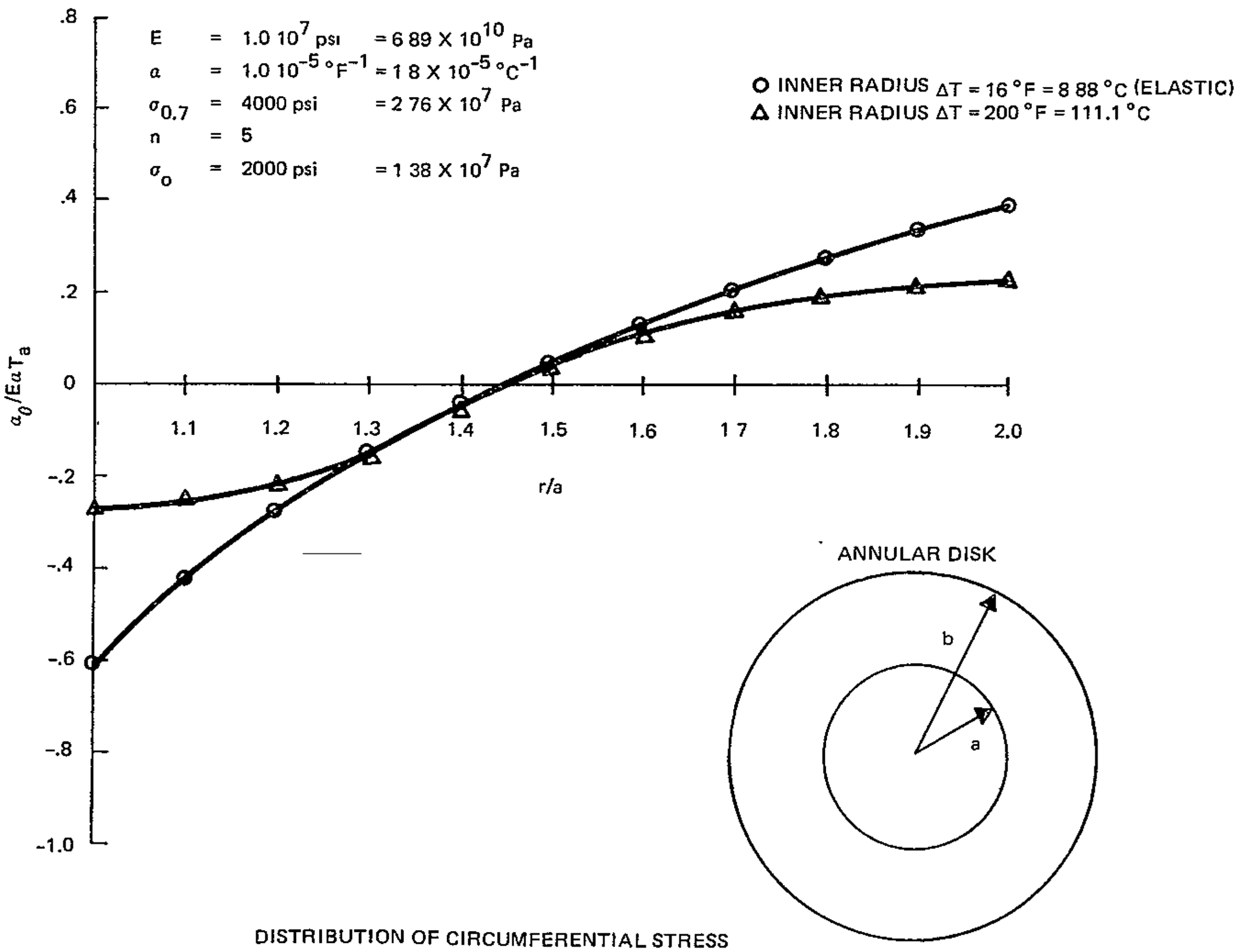
Program: REVBY

Problem Title: Thermoplastic Analysis of a Circular Disk

Comments: This problem is the same as sample problem 5 using the BEND program. Here 10 shell elements are used connecting 11 nodes leading to 22 degrees of freedom. Comparison made with the results of Ref. 4 indicate excellent agreement.



FINITE ELEMENT IDEALIZATION USING SHELL ELEMENTS



DISTRIBUTION OF CIRCUMFERENTIAL STRESS

REVBY Sample Problem #3 Thermoplastic Analysis of a Circular Disk Shell Elements

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

REVBY SAMPLE PROBLEM # 3  THERMOPLASTIC ANALYSIS OF CIRCULAR DISK - II
    0    10
    1   -11
SHEL            1    1    2
SHEL            2    2    3
SHEL            3    3    4
SHEL            4    4    5
SHEL            5    5    6
SHEL            6    6    7
SHEL            7    7    8
SHEL            8    8    9
SHEL            9    9   10
SHEL           10   10   11
SEND
  1.0000000E 00      1
  1.1000000E 00      2
  1.2000000E 00      3
  1.3000000E 00      4
  1.4000000E 00      5
  1.5000000E 00      6
  1.6000000E 00      7
  1.7000000E 00      8
  1.8000000E 00      9
  1.9000000E 00     10
  2.0000000E 00     11
```

BLANK CARD

```
  0.0000000E 00      1   -11
```

BLANK CARD

```
  0.0000000E 00      1   -11
```

BLANK CARD

```
100100          1   -11
```

BLANK CARD

BLANK CARD

```
 10.0000000E 06 10.0000000E 06  0.3840153E 07  0.3000000E 00
  2.0000000E 03  2.0000000E 03  2.0000000E 03  1.1547005E 03  0.5000000E 01
  4.0000000E 03  2.0000000E 03 10.0000000E 06
  1.0000000E-05  1.0000000E-05
    1   -10
```

BLANK CARD

```
  0.1000000E 00
    1   -11
```

BLANK CARD

```
    6    10
```

BLANK CARD

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

                0.0000000E 00
                1.0000000E 00   1.0000000E 00   1.0000000E 00
             1
                8.6249648E-01   8.6249648E-01   8.6249648E-01
             2
                7.3696559E-01   7.3696559E-01   7.3696559E-01
             3
                5.1457317E-01   5.1457317E-01   5.1457317E-01
             5
                4.1503750E-01   4.1503750E-01   4.1503750E-01
             6
                3.2192809E-01   3.2192809E-01   3.2192809E-01
             7
                2.3445525E-01   2.3445525E-01   2.3445525E-01
             8
                1.5200309E-01   1.5200309E-01   1.5200309E-01
             9
                7.4000581E-02   7.4000581E-02   7.4000581E-02
            10
                0.0000000E 00   0.0000000E 00   0.0000000E 00
            11
                6.2148838E-01   6.2148838E-01   6.2148838E-01
             4
SEND
           -11
BLANK CARD
           -12
BLANK CARD
           -11
BLANK CARD
           -12
BLANK CARD
              200.0          .050
BLANK CARD
STOP

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

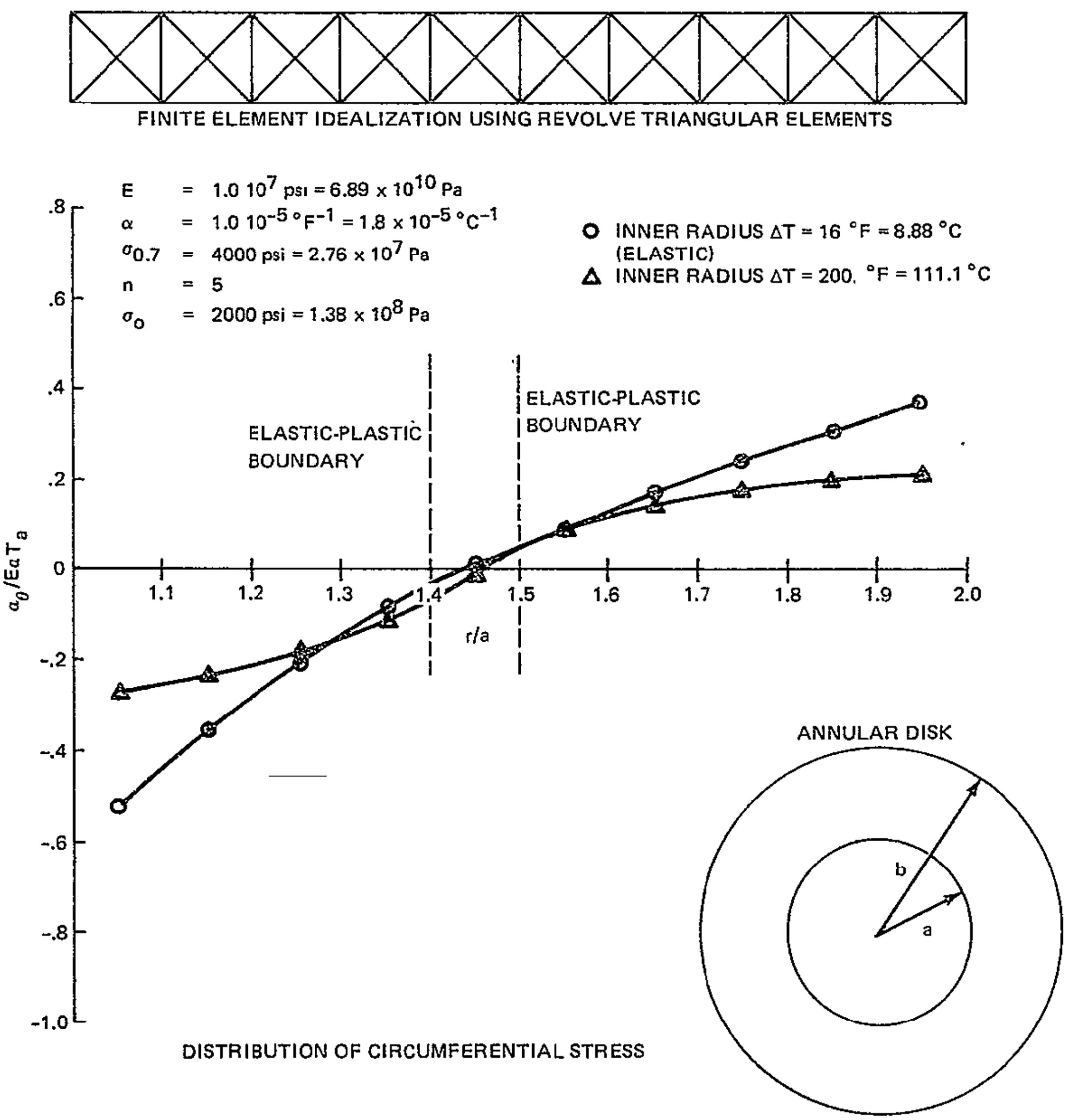
```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

REVBY  SAMPLE PROBLEM =4  THERMOPLASTIC ANALYSIS OF CIRCULAR DISK - I
     0   10
     1  -32
TRIR             1       1      23      12
TRIR             2       1       2      23
TRIR             3       2      13      23
TRIR             4      13      12      23
TRIR             5       2      24      13
TRIR             6       2       3      24
TRIR             7       3      14      24
TRIR             8      14      13      24
TRIR             9       3      25      14
TRIR            10       3       4      25
TRIR            11       4      15      25
TRIR            12      15      14      25
TRIR            13       4      26      15
TRIR            14       4       5      26
TRIR            15       5      16      26
TRIR            16      16      15      26
TRIR            17       5      27      16
TRIR            18       5       6      27
TRIR            19       6      17      27
TRIR            20      17      16      27
TRIR            21       6      28      17
TRIR            22       6       7      28
TRIR            23       7      18      28
TRIR            24      18      17      28
TRIR            25       7      29      18
TRIR            26       7       8      29
TRIR            27       8      19      29
TRIR            28      19      18      29
TRIR            29       8      30      19
TRIR            30       8       9      30
TRIR            31       9      20      30
TRIR            32      20      19      30
TRIR            33       9      31      20
TRIR            34       9      10      31
TRIR            35      10      21      31
TRIR            36      21      20      31
TRIR            37      10      32      21
TRIR            38      10      11      32
TRIR            39      11      22      32
TRIR            40      22      21      32
SEND

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

Example Problem No. 10

Program: REVBY

Problem Title: Thermoplastic Analysis of a Circular Disk

Comments: Same as sample problem 5 (BEND) and 8 (REVBY). Use here is made of 40 revolved triangular element connceting 32 nodes leading to 53 degrees of freedom. Results shown in the figure are for average stresses at the intersection of the diagonals of each bay. Again comparison with the results from Ref. 4 are good.



REVBY Sample Problem # 4 Thermoplastic Analysis of a Circular Disk

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

                 1.0E00     1    12
                 1.1E00     2    13
                 1.2E00     3    14
                 1.3E00     4    15
                 1.4E00     5    16
                 1.5E00     6    17
                 1.6E00     7    18
                 1.7E00     8    19
                 1.8E00     9    20
                 1.9E00    10    21
                 2.0E00    11    22
                1.05E00    23
                1.15E00    24
                1.25E00    25
                1.35E00    26
                1.45E00    27
                1.55E00    28
                1.65E00    29
                1.75E00    30
                1.85E00    31
                1.95E00    32
BLANK CARD
                00.0E00     1   -11
                0.10E00    12   -22
                0.05E00    23   -32
BLANK CARD
                 0.0E00     1   -32
BLANK CARD
100000              1   -11
110000             12   -32
BLANK CARD
BLANK CARD
  1.0000000E 07  1.0000000E 07  1.0000000E 07  0.3000000E 00  0.3000000E 00
  0.3000000E 00  3.8461530E 06  3.8461530E 06  3.8461530E 06  0.0000000E 00
  2.0000000E 03  2.0000000E 03  2.0000000E 03  1.1547004E 03  5.0000000E 00
  4.0000000E 03  2.0000000E 03  1.0000000E 07  1.1547004E 03  1.1547004E 03
  0.1000000E-04  0.1000000E-04  0.1000000E-04
    1   -40
BLANK CARD
BLANK CARD
BLANK CARD
               0.0000000E 00
               1.0000000E 00          0.0E00          0.0E00
    1    12

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

         9.2961067E-01             0.0E00              0.0E00
   23
         8.6249648E-01             0.0E00              0.0E00
   13     2
         7.9836614E-01             0.0E00              0.0E00
   24
         7.3696559E-01             0.0E00              0.0E00
   14     3
         6.7807190E-01             0.0E00              0.0E00
   25
         6.2148838E-01             0.0E00              0.0E00
   15     4
         5.6704059E-01             0.0E00              0.0E00
   26
         5.1457317E-01             0.0E00              0.0E00
   16     5
         4.6394710E-01             0.0E00              0.0E00
   27
         4.1503750E-01             0.0E00              0.0E00
   17     6
         3.6773178E-01             0.0E00              0.0E00
   28
         3.2192809E-01             0.0E00              0.0E00
    7     18
         2.7753398E-01             0.0E00              0.0E00
   29
         2.3446525E-01             0.0E00              0.0E00
    8     19
         1.9264508E-01             0.0E00              0.0E00
   30
         1.5200309E-01             0.0E00              0.0E00
    9     20
         1.1247473E-01             0.0E00              0.0E00
   31
         7.4000581E-02             0.0E00              0.0E00
   10     21
         3.6525879E-02             0.0E00              0.0E00
   32
                    0.0E00         0.0E00              0.0E00
   11     22
$END
  -40
  -32

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

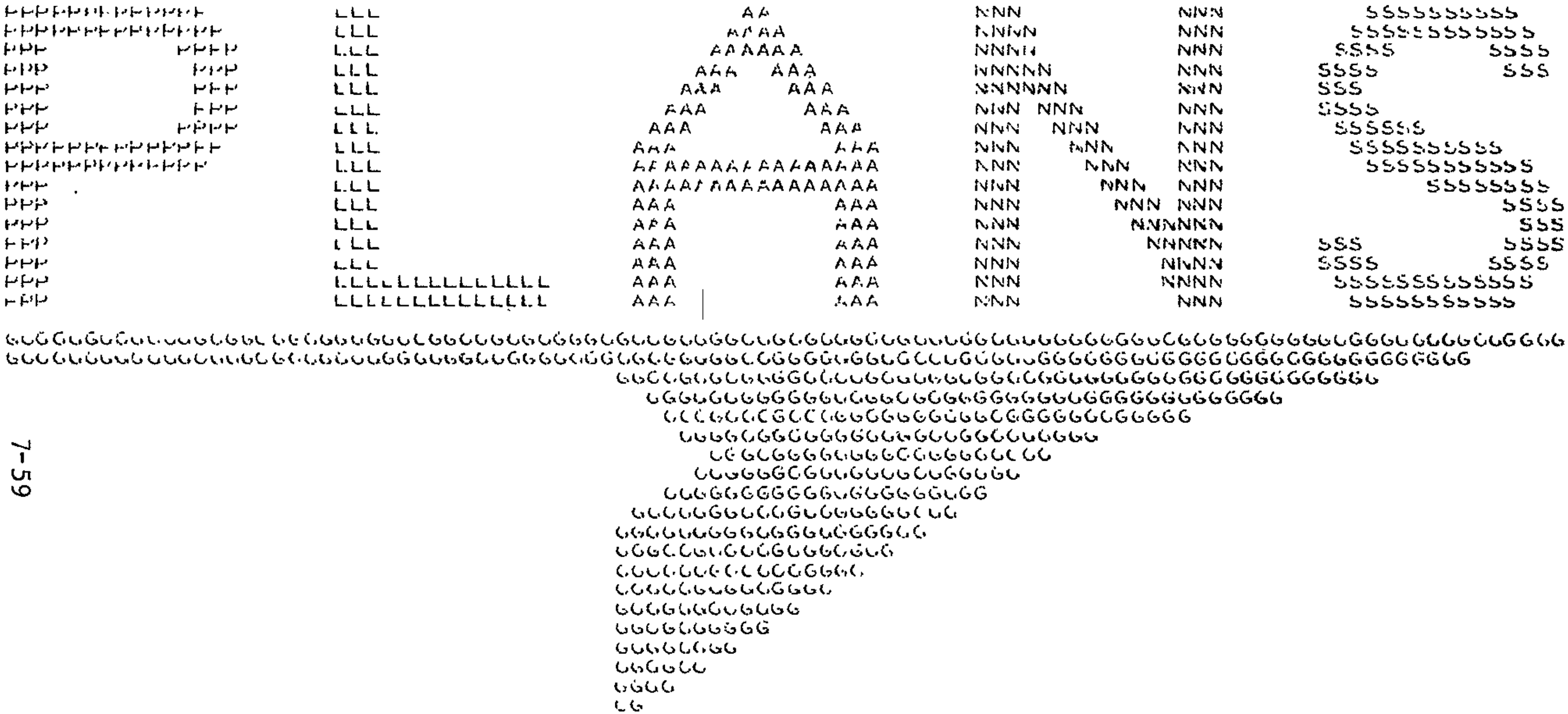
BLANK CARD

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

BLANK CARD
```
  -40
```
BLANK CARD
```
  -32
```
BLANK CARD
```
  2.0000000E 02   0.0100000E 00
```
BLANK CARD
```
STOP
```

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```



HEX MODULE

ELASTIC-PLASTIC ANALYSIS

PREPARED BY

THE RESEARCH DEPARTMENT

OF GRUMMAN AEROSPACE CORPORATION

UNDER PARTIAL FUNDING FROM

THE NASA LANGLEY RESEARCH CENTER

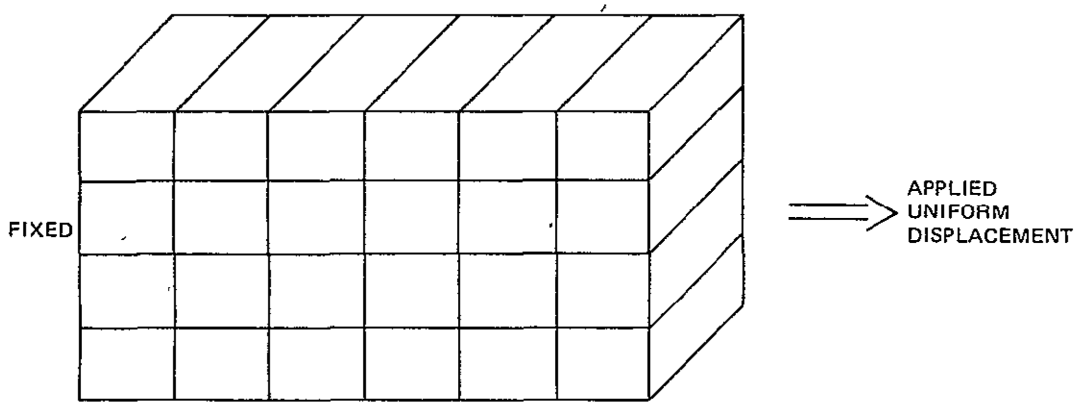
Example Problem No. 11

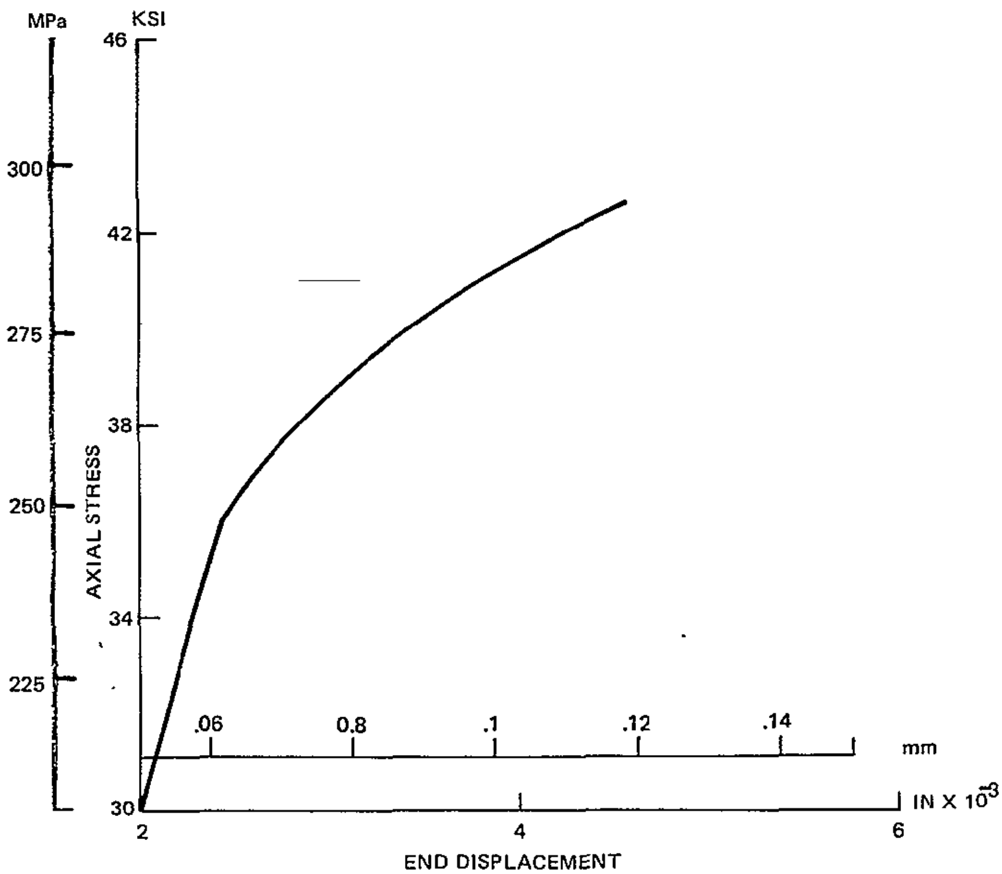
Program: HEX

Problem Title: Bar Subjected to an End Displacement

Comments: This is a simple statically determinate problem to demonstrate the use of the HEX element. The automatic mesh generation feature is used (MSGN) to specify a 1 x 6 x 4 mesh of 24 elements. An applied edge displacement is imposed on one end.

The figure shows the axial stress versus applied end displacement.





HEX SAMPLE PROBLEM # 1 BAR SUBJECTED TO AN END DISPLACEMENT

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

HEX SAMPLE PROBLEM # 1 LONG BAR SUBJECTED TO AN APPLIED END DISPLACEMENT
    0    5    1    1    5
    1  -70
MSGN
    1    6    4    1    1
SEND
  0.0000000E 00    1    3    5    7    9    9   11   13   15   17   19   21   23
  0.0000000E 00   25   27   29   31   33   35   37   39   41   43   45   47   49
  0.0000000E 00   51   53   55   57   59   61   63   65   67   69
  2.5000000E-01    2    4    6    8   10   12   14   16   18   20   22   24   26
  2.5000000E-01   28   30   32   34   36   38   40   42   44   46   48   50   52
  2.5000000E-01   54   56   58   60   62   64   66   68   70
BLANK CARD
  0.0000000E 00    1  -10
  5.0000000E-01   11  -20
  8.7500000E-01   21  -30
  1.2500000E 00   31  -40
  1.5000000E 00   41  -50
  1.7500000E 00   51  -60
  2.0000000E 00   61  -70
BLANK CARD
 -5.0000000E-01    1    2   11   12   21   22   31   32   41   42   51   52   61
 -5.0000000E-01   62
 -2.5000000E-01    3    4   13   14   23   24   33   34   43   44   53   54   63
 -2.5000000E-01   64
  0.0000000E 00    5    6   15   16   25   26   35   36   45   46   55   56   65
  0.0000000E 00   66
  2.5000000E-01    7    8   17   18   27   28   37   38   47   48   57   58   67
  2.5000000E-01   68
  5.0000000E-01    9   10   19   20   29   30   39   40   49   50   59   60   69
  5.0000000E-01   70
BLANK CARD
  001          1    3    7    9
  101          2    4    8   10
  000          5
  100          6
  121         61  -70
BLANK CARD
   61    2         70    2      1.0
   62    2         70    2      1.0
   63    2         70    2      1.0
   64    2         70    2      1.0
   65    2         70    2      1.0
   66    2         70    2      1.0

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

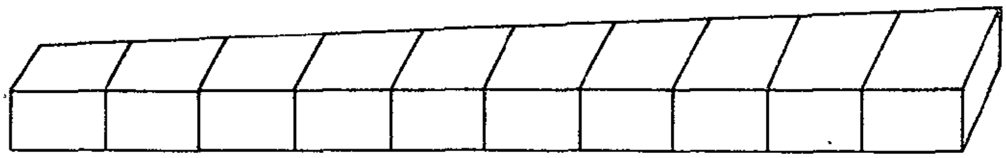
```
             .........1.........2.........3.........4.........5.........6.........7.........8
             12345678901234567890123456789012345678901234567890123456789012345678901234567890

                67    2          70    2     1.0
                68    2          70    2     1.0
                69    2          70    2     1.0
                70    2          70    2  .003114
BLANK CARD
             MAT1     3.0     E 07     3.0    E 07     3.0    E 07     .33333E 00     .33333E 00
                 .333333E 00     1.125 E 07     1.125 E 07     1.125 E 07
                  0.0    E 00     0.0    E 00     0.0    E 00
                  12.    E-06     12.    E-06     12.    E-06
                  3.6    E 04     3.6    F 04     3.6    E 04     2.0784E 04     2.0784E 04
                 2.0784 E 04     10.    E 00    4.0     E 04     3.6    E 04     3.0    E 07
                 1   -24
             SEND
             SEND
              -100
BLANK CARD
              -100
BLANK CARD
                  1.5            .01
BLANK CARD
             STOP

             .........1.........2.........3.........4.........5.........6.........7.........8
             12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

Example Problem No. 12

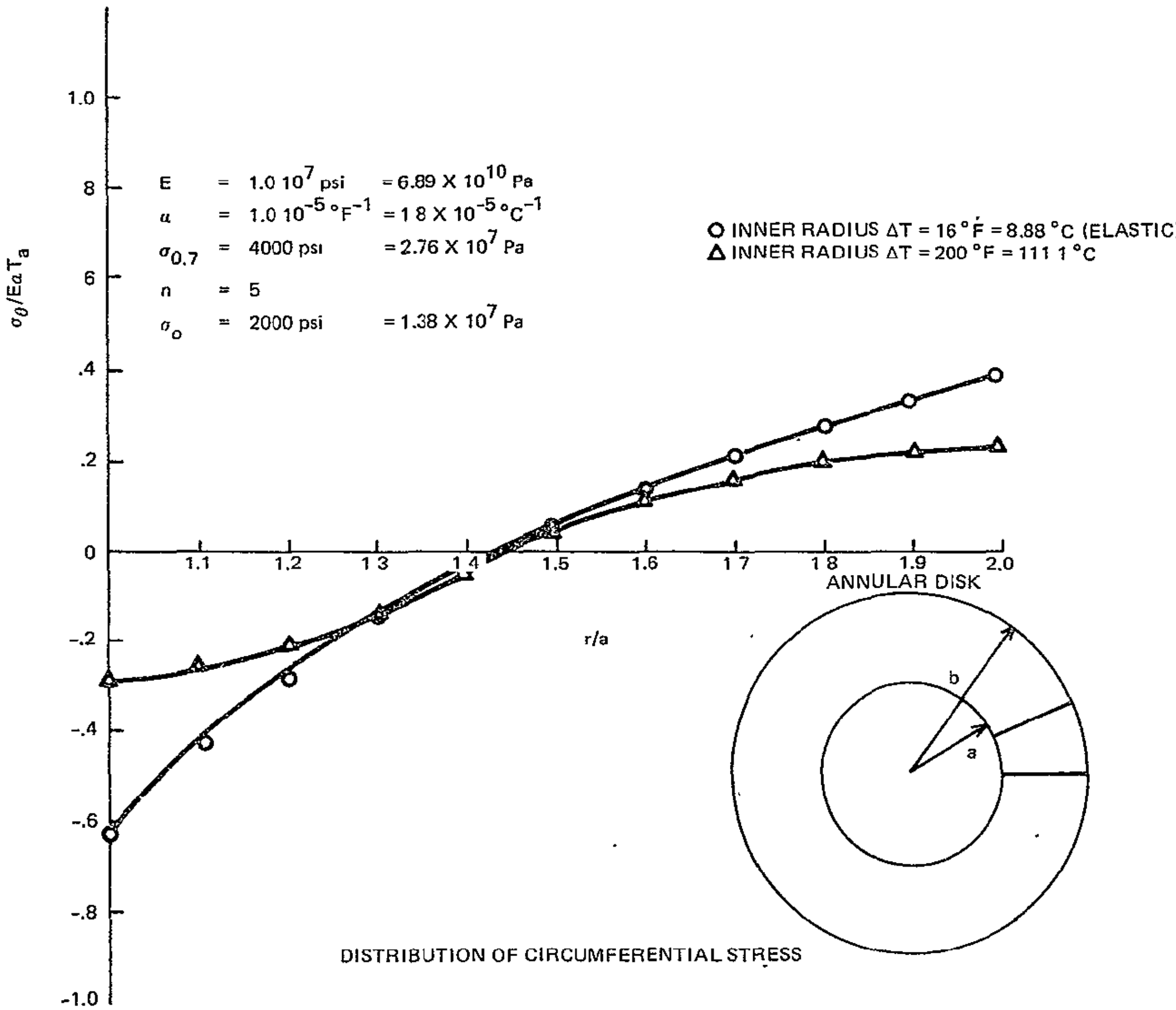
Program: HEX

Problem Title: Thermoplastic Analysis of a Circular Disk

Comments: This problem is the same as problems 5 (BEND), 8, and 9 (REVBY). Here the idealization of a slice of the disk is used with 10 hexahedra elements with midside nodes in the radial direction. Thirty Lobatto stress points are used in each direction, 10 in the radial, 3 in the circumference, and 1 in the thickness direction.



FINITE ELEMENT MODEL OF A SECTOR OF THE DISK



DISTRIBUTION OF CIRCUMFERENTIAL STRESS

HEX Sample Problem # 2 Thermoplastic Disk

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

HEX SAMPLE PROBLEM #2 THERMOPLASTIC ANALYSIS OF ANNULAR DISK
    1   20    1    1   50
    1   -6  101 -104    7  -12  105 -108   13  -18  109 -112   19  -24  113 -116
   25  -30  117 -120   31  -36  121 -124   37  -42  125 -128   43  -48  129 -132
   49  -54  133 -136   55  -60  137 -140   61  -66
HX20          1    1    3    9    7    4    6   12   10
    2  103    8  101    5  104   11  102
HX20          2    7    9   15   13   10   12   18   16
    8  107   14  105   11  108   17  106
HX20          3   13   15   21   19   16   18   24   22
   14  111   20  109   17  112   23  110
HX20          4   19   21   27   25   22   24   30   28
   20  115   26  113   23  116   29  114
HX20          5   25   27   33   31   28   30   36   34
   26  119   32  117   29  120   35  118
HX20          6   31   33   39   37   34   36   42   40
   32  123   38  121   35  124   41  122
HX20          7   37   39   45   43   40   42   48   46
   38  127   44  125   41  128   47  126
HX20          8   43   45   51   49   46   48   54   52
   44  131   50  129   47  132   53  130
HX20          9   49   51   57   55   52   54   60   58
   50  135   56  133   53  136   59  134
HX20         10   55   57   63   61   58   60   66   64
   56  139   62  137   59  140   65  138
SEND
      1.0      E+00    3    6
      1.1      E+00    9   12
      1.2      E+00   15   18
      1.3      E+00   21   24
      1.4      E+00   27   30
      1.5      E+00   33   36
      1.6      E+00   39   42
      1.7      E+00   45   48
      1.8      E+00   51   54
      1.9      E+00   57   60
      2.0      E+00   63   66
  .99452169E+00    1    4
 1.0439741 E+00    7   10
 1.1934262 E+00   13   16
 1.2926784 E+00   19   22
 1.3923306 E+00   25   28
 1.4917828 E+00   31   34
 1.5912350 E+00   37   40

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

  1.0814254 E+00    43    46
  1.7961394 E+00    49    52
  1.8895910 E+00    55    58
  1.9690437 E+00    61    64
   .9986295 E+00     2     5
  1.0984924 E+00     8    11
  1.1983554 E+00    14    17
  1.2982183 E+00    20    23
  1.3980813 E+00    26    29
  1.4979443 E+00    32    35
  1.5978072 E+00    38    41
  1.6976702 E+00    44    47
  1.7975331 E+00    50    53
  1.8973961 E+00    56    59
  1.9972590 E+00    62    65
        0.0         101  -140
BLANK CARD
       0.0  E+00     3    -3   -66
   .0523359 E+00     2     5
   .0575095 E+00     8    11
   .0628031 E+00    14    17
   .0680367 E+00    20    23
   .0732703 E+00    26    29
   .0785039 E+00    32    35
   .0837375 E+00    38    41
   .0889711 E+00    44    47
   .0942047 E+00    50    53
   .0994383 E+00    56    59
   .1046712 E+00    62    65
   .1045284 E+00     1     4
   .1149813 E+00     7    10
   .1254341 E+00    13    16
   .1358870 E+00    19    22
   .1463398 E+00    25    28
   .1567926 E+00    31    34
   .1672455 E+00    37    40
   .1776983 E+00    43    46
   .1881512 E+00    49    52
   .1986040 E+00    55    58
   .2090569 E+00    61    64
       0.0          101  -140
BLANK CARD
  0.0           E+00     3    -6   -63      2    -6   -62      1    -6   -61
        0.1     E+00     4    -6   -64      5    -6   -65      6    -6   -66

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

       0.0            101  -140
BLANK CARD
   100             3    -6   -63     1     -6   -61
   100           101    -4  -137   103     -4  -139
   110             2    -6   -62
   101             4    -6   -64     6     -6   -66
   101           102    -4  -136   104     -4  -140
BLANK CARD
     1     1           1     1  .9945218              1     2 -.1045284
     1     2           1     1  .1045284              1     2  .9945218
     4     1           4     1  .9945218              4     2 -.1045284
     4     2           4     1  .1045284              4     2  .9945218
     7     1           7     1  .9945218              7     2 -.1045284
     7     2           7     1  .1045284              7     2  .9945218
    10     1          10     1  .9945218             10     2 -.1045284
    10     2          10     1  .1045284             10     2  .9945218
    13     1          13     1  .9945218             13     2 -.1045284
    13     2          13     1  .1045284             13     2  .9945218
    16     1          16     1  .9945218             16     2 -.1045284
    16     2          16     1  .1045284             16     2  .9945218
    19     1          19     1  .9945218             19     2 -.1045284
    19     2          19     1  .1045284             19     2  .9945218
    22     1          22     1  .9945218             22     2 -.1045284
    22     2          22     1  .1045284             22     2  .9945218
    25     1          25     1  .9945218             25     2 -.1045284
    25     2          25     1  .1045284             25     2  .9945218
    28     1          28     1  .9945218             28     2 -.1045284
    28     2          28     1  .1045284             28     2  .9945218
    31     1          31     1  .9945218             31     2 -.1045284
    31     2          31     1  .1045284             31     2  .9945218
    34     1          34     1  .9945218             34     2 -.1045284
    34     2          34     1  .1045284             34     2  .9945218
    37     1          37     1  .9945218             37     2 -.1045284
    37     2          37     1  .1045284             37     2  .9945218
    40     1          40     1  .9945218             40     2 -.1045284
    40     2          40     1  .1045284             40     2  .9945218
    43     1          43     1  .9945218             43     2 -.1045284
    43     2          43     1  .1045284             43     2  .9945218
    46     1          46     1  .9945218             46     2 -.1045284
    46     2          46     1  .1045284             46     2  .9945218
    49     1          49     1  .9945218             49     2 -.1045284
    49     2          49     1  .1045284             49     2  .9945218
    52     1          52     1  .9945218             52     2 -.1045284
    52     2          52     1  .1045284             52     2  .9945218

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

55	1	55	1	.9945218	55	2	-.1045284
55	2	55	1	.1045284	55	2	.9945218
58	1	58	1	.9945218	58	2	-.1045284
58	2	58	1	.1045284	58	2	.9945218
61	1	61	1	.9945218	61	2	-.1045284
61	2	61	1	.1045284	61	2	.9945218
64	1	64	1	.9945218	64	2	-.1045284
64	2	64	1	.1045284	64	2	.9945218
101	1	101	1	.9945218	101	2	-.1045284
101	2	101	1	.1045284	101	2	.9945218
105	1	105	1	.9945218	105	2	-.1045284
105	2	105	1	.1045284	105	2	.9945218
109	1	109	1	.9945218	109	2	-.1045284
109	2	109	1	.1045284	109	2	.9945218
113	1	113	1	.9945218	113	2	-.1045284
113	2	113	1	.1045284	113	2	.9945218
117	1	117	1	.9945218	117	2	-.1045284
117	2	117	1	.1045284	117	2	.9945218
121	1	121	1	.9945218	121	2	-.1045284
121	2	121	1	.1045284	121	2	.9945218
125	1	125	1	.9945218	125	2	-.1045284
125	2	125	1	.1045284	125	2	.9945218
129	1	129	1	.9945218	129	2	-.1045284
129	2	129	1	.1045284	129	2	.9945218
133	1	133	1	.9945218	133	2	-.1045284
133	2	133	1	.1045284	133	2	.9945218
137	1	137	1	.9945218	137	2	-.1045284
137	2	137	1	.1045284	137	2	.9945218
102	1	102	1	.9945218	102	2	-.1045284
102	2	102	1	.1045284	102	2	.9945218
106	1	106	1	.9945218	106	2	-.1045284
106	2	106	1	.1045284	106	2	.9945218
110	1	110	1	.9945218	110	2	-.1045284
110	2	110	1	.1045284	110	2	.9945218
114	1	114	1	.9945218	114	2	-.1045284
114	2	114	1	.1045284	114	2	.9945218
118	1	118	1	.9945218	118	2	-.1045284
118	2	118	1	.1045284	118	2	.9945218
122	1	122	1	.9945218	122	2	-.1045284
122	2	122	1	.1045284	122	2	.9945218
126	1	126	1	.9945218	126	2	-.1045284
126	2	126	1	.1045284	126	2	.9945218
130	1	130	1	.9945218	130	2	-.1045284
130	2	130	1	.1045284	130	2	.9945218

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

   134     1          134     1  .9945218              134     2 -.1045284
   134     2          134     1  .1045284              134     2  .9945218
   138     1          138     1  .9945218              138     2 -.1045284
   138     2          138     1  .1045284              138     2  .9945218
BLANK CARD
 MAT1      1.0     E+07   1.0       E+07   1.0     E+07     .3             .3
   .3              3.846153 E+06  3.846153 E+06  3.846153 E+06
   1.              2.0            1.0
   .1       E-04   .1       E-04   .1      E-04
  2.0       E+03  2.0       E+03  2.0      E+03  1.1547004E+03 1.1547004 E+03
  1.1547004E+03   5.0       E+00  4.0      E+03  2.0       E+03 1.0       E+07
    1   -10
 LOBAT             10               1              3
    1   -10
 SEND
 TEMP      6.0
    1.0            1   -6
     .9296167    101 -104
  .8624964         7  -12
     .7983061    105 -108
  .7369655        13  -18
     .6700719    109 -112
  .6214865        19  -24
     .5670406    113 -116
  .5145732        25  -30
     .4639471    117 -120
  .4150375        31  -36
     .3677318    121 -124
  .3219281        37  -42
     .2775340    125 -128
  .2344052        43  -48
     .1926451    129 -132
  .1520030        49  -54
     .1124747    133 -136
  .0740054        55  -60
     .03652568   137 -140
  .0              61  -66
BLANK CARD
 SEND
    1   -10
BLANK CARD
    1   -60   101 -140
BLANK CARD
    200.0          .05

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

BLANK CARD

STOP


```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

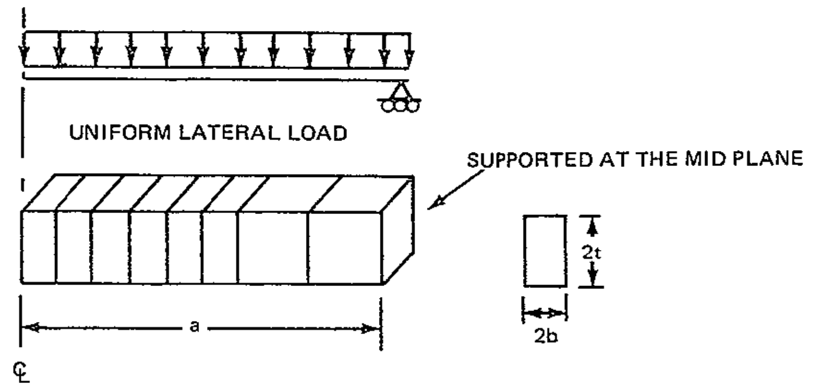
Example Problem No. 13

Program: HEX

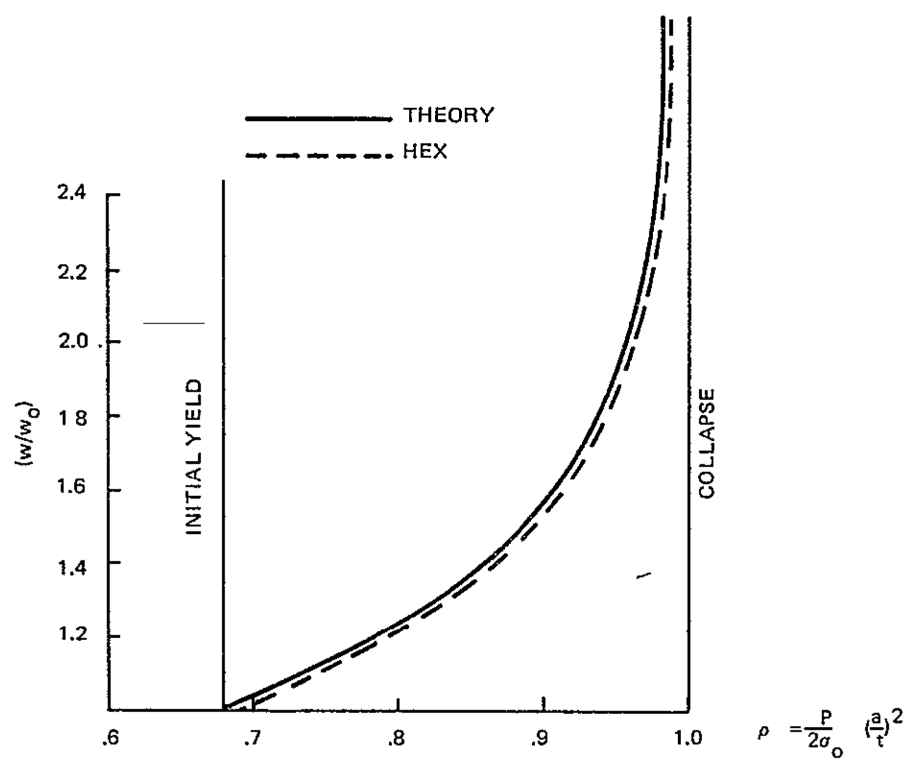
Problem Title: Collapse of a Simply Supported Uniformly Loaded Beam

Comments: Ten 20 node hexahedra elements are used to model half of a simply supported beam. A 1 x 1 x 8 array of Lobatto points are used (8 through the thickness) to determine stresses within each element. Lobatto points were chosen in order to have stress points at the surface. Eight points were taken through the thickness in order to accurately define an elastic plastic boundary through the thickness.

The figure shows the central deflection versus load for an ideally plastic material. Results are in good agreement with Ref. 5.



FINITE ELEMENT MODEL OF HALF OF THE BEAM



LOAD VS. NON-DIMENSIONAL CENTER DEFLECTION (W_O CORRESPONDS TO DISPLACEMENT AT INITIAL YIELD) OF A SIMPLY-SUPPORTED BEAM. HEX RESULTS CORRESPOND TO AN1X1X8 ARRAY OF LOBATTO POINTS.

HEX Sample Problem #3 Collapse of a Uniformly Loaded Beam

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

HEX SAMPLE PROBLEM # 3 COLLAPSE OF SIMPLY SUPPORTED UNIFORMLY LOADED BEAM
    1   20
    1   -6  101  102    7  -16  103  104   17  -26  105  106   27  -36  107  108
   37  -46  109  110   47  -56  111  112   57  -66  113  114   67  -76  115  116
   77  -86  117  118
HX20          1    1    3   13   11    4    6   16   14                     333
    2    8   12    7    5   10   15    9  101  102  104  103
HX20          2   11   13   23   21   14   16   26   24                     333
   12   18   22   17   15   20   25   19  103  104  106  105
HX20          3   21   23   33   31   24   26   36   34                     333
   22   28   32   27   25   30   35   29  105  106  108  107
HX20          4   31   33   43   41   34   36   46   44                     333
   32   38   42   37   35   40   45   39  107  108  110  109
HX20          5   41   43   53   51   44   46   56   54                     333
   42   48   52   47   45   50   55   49  109  110  112  111
HX20          6   51   53   63   61   54   56   66   64                     333
   52   58   62   57   55   60   65   59  111  112  114  113
HX20          7   61   63   73   71   64   66   76   74                     333
   62   68   72   67   65   70   75   69  113  114  116  115
HX20          8   71   73   83   81   74   76   86   84                     333
   72   78   82   77   75   80   85   79  115  116  118  117
SEND
  0.0                1   -6
  0.05               7  -10
  0.1               11  -16
  0.15              17  -20
  0.2               21  -26
  0.25              27  -30
  0.3               31  -36
  0.35              37  -40
  0.4               41  -46
  0.45              47  -50
  0.5               51  -56
  0.55              57  -60
  0.6               61  -66
  0.7               67  -70
  0.8               71  -76
  0.9               77  -80
  1.0               81  -86
  0.0              101  102
  0.1              103  104
  0.2              105  106
  0.3              107  108
  0.4              109  110

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

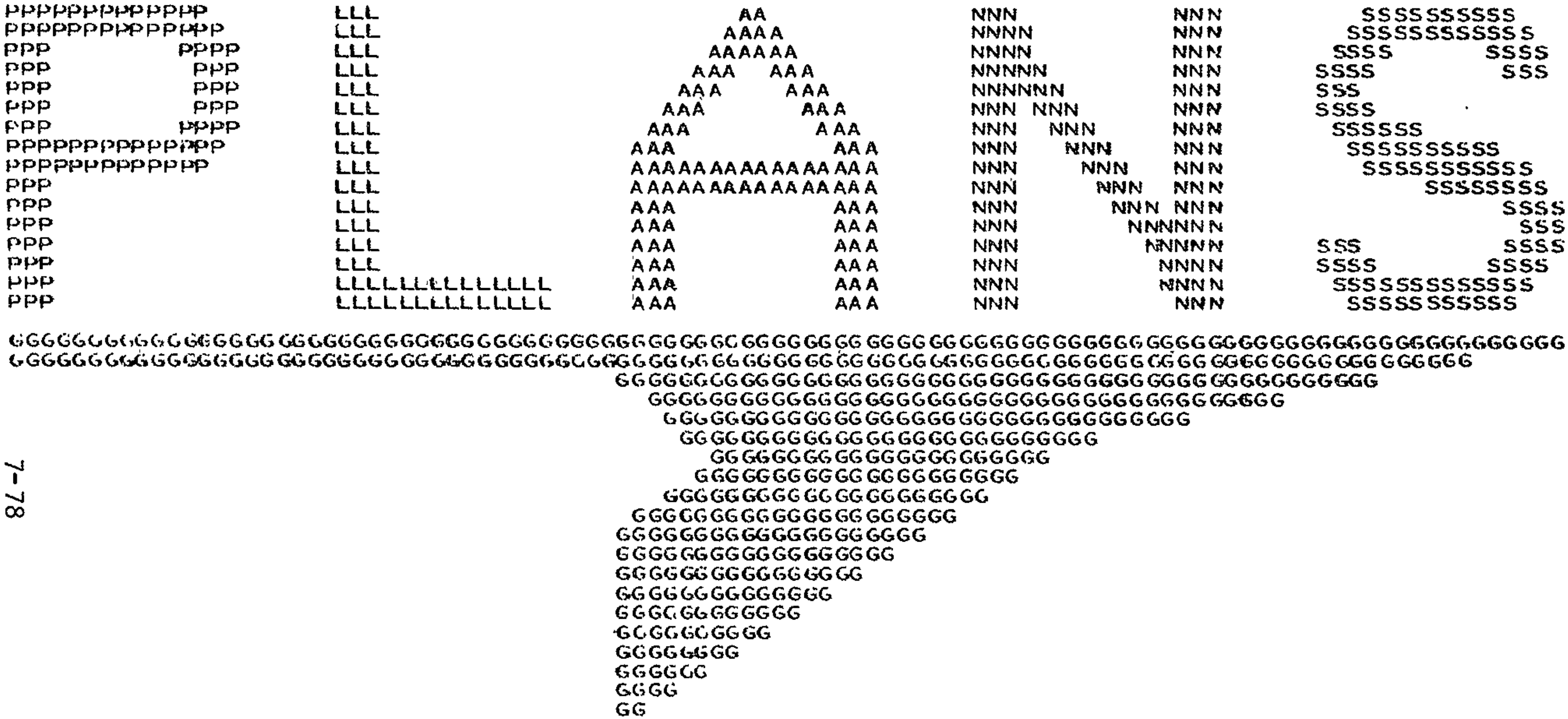
```
            ..........1.........2.........3.........4.........5.........6.........7.........8
            12345678901234567890123456789012345678901234567890123456789012345678901234567890

              0.5            111  112
              0.6            113  114
              0.8            115  116
              1.0           .117  118
BLANK CARD
              0.0              1  -86
              0.1              4    5    6    9   10   14   15   16   19   20   24   25   26
                              29   30   34   35   36   39   40   44   45   46   49   50   54
                              55   56   59   60   64   65   66   69   70   74   75   76   79
                              80   84   85   86
              0.05           101 -118
BLANK CARD
              0.0              1  -86
              0.05             2    5   12   15   22   25   32   35   42   45   52   55   62
                              65   72   75   82   85
              0.1              3    6    8   10   13   16   18   20   23   26   28   30   33
                              36   38   40   43   46   48   50   53   56   58   60   63   66
                              68   70   73   76   78   80   83   86
              0.0            101   -2 -117
              0.1            102   -2 -118
BLANK CARD
              011          1   -6
              110         82   85
              001        101  102
              101        103 -118
BLANK CARD
BLANK CARD
            MAT1         3.0E+07        3.0E+07        3.0E+07  0.3            0.3
                     0.3       1.15385F+07    1.15385E+07    1.15385E+07
BLANK CARD
BLANK CARD
                   30000.0     30000.0      30000.0        17320.0        17320.0
                   17320.0                                 30000.0
                1   -8
            LOBAT              1              1              8
                1   -8
            SEND
            SURF
                8         3    13   16    6    8  104   10  102
                3                                 -1.0
                6                                 -1.0
                8                                 -1.0
               10                                 -1.0
               13                                 -1.0

            ..........1.........2.........3.........4.........5.........6.........7.........8
            12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

   16                               -1.0
  102                               -1.0
  104                               -1.0
    8        13   23   26   16   16  106   20  104
   13                               -1.0
   16                               -1.0
   18                               -1.0
   20                               -1.0
   23                               -1.0
   26                               -1.0
  104                               -1.0
  106                               -1.0
    8        23   33   36   26   28  108   30  106
   23                               -1.0
   26                               -1.0
   28                               -1.0
   30                               -1.0
   33                               -1.0
   36                               -1.0
  106                               -1.0
  108                               -1.0
    8        33   43   46   36   38  110   40  108
   33                               -1.0
   36                               -1.0
   38                               -1.0
   40                               -1.0
   43                               -1.0
   46                               -1.0
  108                               -1.0
  110                               -1.0
    8        43   53   56   46   48  112   50  110
   43                               -1.0
   46                               -1.0
   48                               -1.0
   50                               -1.0
   53                               -1.0
   56                               -1.0
  110                               -1.0
  112                               -1.0
    8        53   63   66   56   58  114   60  112
   53                               -1.0
   56                               -1.0
   58                               -1.0
   60                               -1.0

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

      63                                  -1.0
      66                                  -1.0
     112                                  -1.0
     114                                  -1.0
       8          63   73   76   66   68   116   70  114
      63                                  -1.0
      66                                  -1.0
      68                                  -1.0
      70                                  -1.0
      73                                  -1.0
      76                                  -1.0
     114                                  -1.0
     116                                  -1.0
       8          73   83   86   76   78   118   80  116
      73                                  -1.0
      76                                  -1.0
      78                                  -1.0
      80                                  -1.0
      83                                  -1.0
      86                                  -1.0
     116                                  -1.0
     118                                  -1.0
BLANK CARD
$END
       1    -6    8
BLANK CARD
       1   -86  101 -118
BLANK CARD
          180.0      0.01

BLANK CARD

STOP

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```



GEOMNL MODULE

ELASTIC-PLASTIC ANALYSIS

PREPARED BY

THE RESEARCH DEPARTMENT

OF GRUMMAN AEROSPACE CORPORATIC

UNDER PARTIAL FUNDING FROM

THE NASA LANGLEY RESEARCH CENTE

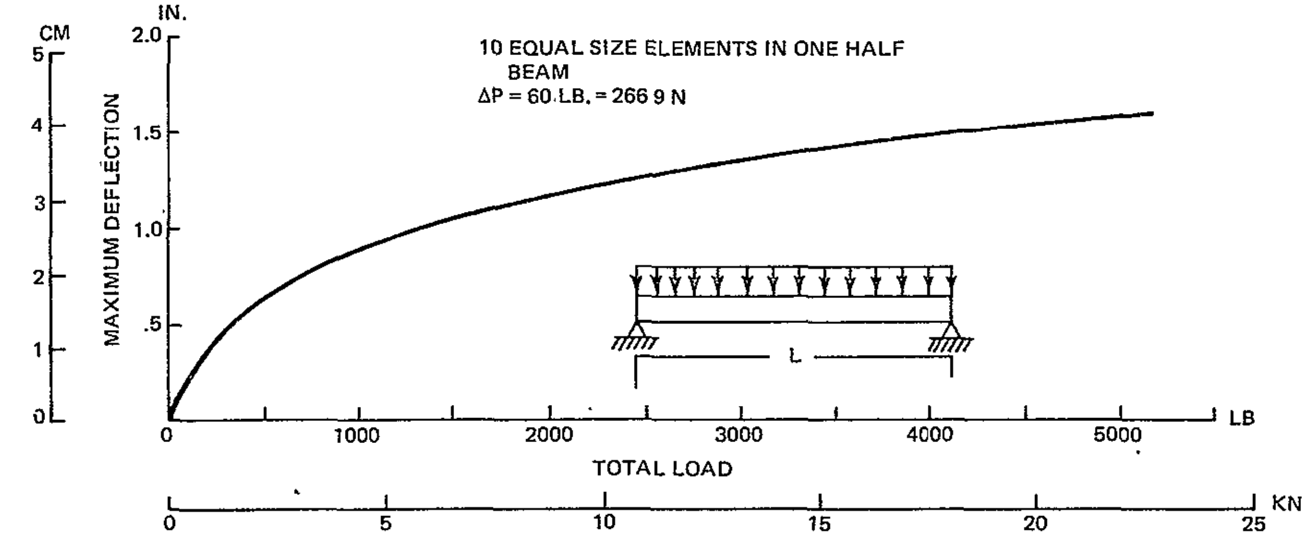
Example Problem No. 14

Program: OPLANE-GM

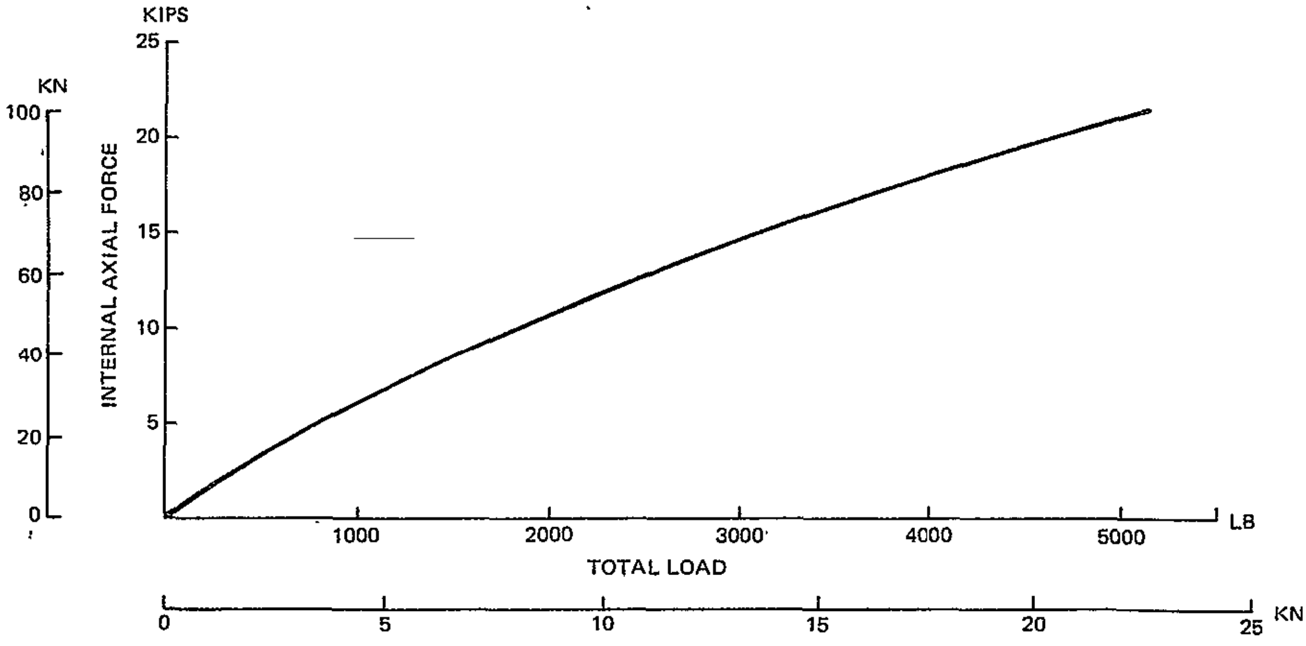
Problem Title: Uniformly Loaded Restrained Beam

Comments: Ten beam elements are used to model half a simply supported restrained beam. A full tangent modulus method is used (PTAN = 0.0).

The figure shows the central deflection and internal axial force versus the total load.



LOAD DEFLECTION BEHAVIOR



VARIATION OF INTERNAL AXIAL FORCE WITH LOAD

OPLANE-MG Sample Problem # 1 Uniformly Loaded Restrained Beam

PROGRAM LISTING OF INPUT DATA CARDS

```
.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890

UPLANEMG SAMPLE PROBLEM = 1 UNIFORMLY LOADED RESTRAINED BEAM
        1    5         1         0      1        1    10
         1CC.0                  1CC.C                  0.0                 1.0
        1  -12
BEAM                   1         1      2        12
BEAM                   2         2      3        12
BEAM                   3         3      4        12
BEAM                   4         4      5        12
BEAM                   5         5      6        12
BEAM                   6         6      7        12
BEAM                   7         7      8        12
BEAM                   8         8      9        12
BEAM                   9         9     10        12
BEAM                  10        10     11        12
SEND
          C.C      E&CC          1     12
          3.C      E&CC          2
          6.C      E&CC          3
          9.C      E&C0          4
         12.C      E&CC          5
         15.C      E&CC          6
         18.C      E&CC          7
         21.C      E&CG          8
         24.C      E&CC          9
         27.C      E&CC         1C
         30.C      E&C0         11
BLANK CARD
          C.C      E&CC          1    -12
BLANK CARD
          C.C      E&CC          1    -11
         1C.C      E&CC         12
BLANK CARD
CGOOGC                          12
C000 1C                          1
1C101C                           2    -1C
C010CC                          11
BLANK CARD
BLANK CARD
MBM          1.0       E&C7      .25                                          3.0 E&09
         C.G                C.C
        1  -10
,SR EC       1.2                 .144            .1              0.0              0.0
             0.0                 0.C             0.0
             1.0                 1.2

.........1.........2.........3.........4.........5.........6.........7.........8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

PROGRAM LISTING OF INPUT DATA CARDS

```
         1         2         3         4         5         6         7         8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
SEND    1   -10
BMLO
    -1.0                -1.0
        1   -10
BLANK CARD
SEND
        1    -5
BLANK CARD
        1   -11
BLANK CARD
BLANK CARD
END
         1         2         3         4         5         6         7         8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

REFERENCES

1. Isakson, G., Armen, H. Jr., and Pifko, A., "Discrete-Element Methods for the Plastic Analysis of Structures," NASA Contractor Report CR-803, October 1967.

2. Armen, H. Jr., Pifko, A., and Levine, H., "Finite Element Analysis of Structures in the Plastic Range," NASA Contractor Report CR-1649, February 1971.

3. Armen, H. Jr., Levine, H., Pifko, A., and Levy, A., "Nonlinear Analysis of Structures," NASA Contractor Report CR-2351, March 1974.

4. Dastidar, D. and Ghosh, P., "Stresses and Strains in the Plastic Range in an Annular Disk due to Steady State Radial Temperature Variation," International Journal of Mechanical Sciences, Vol. 14, pp. 501-510, 1972.

5. Prager, W. and Hodge, P. Jr., Theory of Perfectly Plastic Solids, J. Wiley and Sons, Inc., 1951.