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# POINT SOURCE DETECTION IN <br> INFRARED ASTRONGMICAL SURVEYS 

## By

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### 1.0 INTRODUCTION.

This is the second report on data processing techniques useful for infrared astronomy data analysis systems. As with the first report (NASA CR-151943), the investigation is restricted to consideration of data from space-based telescope systems operating as survey instruments. In this report the theoretical background for specific point-source detection schemes is completed, and the development of specific algorithms and software for the broad range of requirements is begun.

Section 2 develops the detail detection tests and processing requirements for point-source surveys and evaluates the performance measurement processes. The details of peak detection decisions and correlation detection are covered for the case of general bandlimited white gaussian noise. For non-white noise, a modified correlation test and a matched filter test are presented. A technique for resampling the data which is equivalent to a matched filtering approach is discussed which automatically decorrelates the noise. Implementation of this Karhunen-Loeve filtering is necessarily complicated, but for some kinds of noise an acceptable approach.

Section 3 then reviews a basic processing task to indicate where computation is needed outside of the normal data stream. While the processing used in the primary data reduction task is important, the actual software depends heavily on the specific mission hardware and is best approached anew for each task using the theories of Section 2 of this report and of the previous report, and of several cited authors. For the general signal processing task, a routine for designing digital filters is given based on the theory of Section 2.5. The calibration of detector-filter systems is the most complicated of the tasks off the main processing line; a routine which provides this calibration for blackbody or other input spectra. Finally, the preliminary processing routine for a previous survey
program is presented and briefly discussed to indicate how much processing can be done in a single pass of the data.

The Appendix in Section 4 presents an interesting game which can develop a fuller appreciation and understanding of the complexities of data analysis.

### 2.0 TECHNICAL ASPECTS

This section completes the task begun in the first report of reviewing the theoretical basis for the design of point-source survey data analysis software. The detection techniques for single-channel signal and noise processing are reviewed. The schemes reviewed include peak detection, optimal filtering, correlation, and Karhunen-Loeve filtering. The details of . digital filtering, which is applicable to many aspects of data processing, are reviewed-in the final section.

### 2.1 Detection of Signals in Noise

In most communication systems the errors (false detections and missed signals) are assumed to be of equal importance and with known probabilities. In more general detection problems, however, the a priori probabilities and costs of those errors are difficult to determine. The Neyman-Pearson test was first applied in such a case to radar detection with a peak measurement technique. The criterion can also be applied to more sophisticated detection methods, and in all cases, will give the highest probability of detection at a chosen false-alarm rate. The type of technique used depends on the amount of information available about the expected signal; generally, more-information used will result in a higher detection probability at the chosen false-alarm rate. The likelihood ratio is the test used where the hypothesis is chosen if:

$$
\lambda=\frac{p(s)}{p(n)} \geq \eta
$$

and the counter-hypothesis (no-signal) is chosen otherwise. Here $p(s)$ is the probability density function of the data with a signal present and $p(n)$ is the p.d.f. of the noise alone, and $\eta$ is the decision level chosen to satisfy the false-alarm constraint.

Consider the case of a signal in white noise, such that the signal has a normalized mean value of one. The probability functions are:

$$
\cdot p(s)=\frac{1}{\sqrt{4 \pi}} e^{-(y-1)^{2} / 4} \text { and } p(n)=\frac{1}{\sqrt{4 \pi}} e^{-y^{2} / 4}
$$

Then the likelihood ratio test is:

$$
\lambda(y)=e^{(y / 2)-1} \geq \lambda_{0}
$$

To determine the threshold $\lambda_{0}$, the false-alarm probability is found from:

$$
P(f . a .)=\int_{\gamma}^{\infty} \frac{1}{\sqrt{4 \pi}} e^{-y^{2} / 4} d y
$$

If we want a false-alarm rate of $10 \%$ or less, then $\gamma=1.8$, and we choose the hypothesis if $y \geq 1.8$.

The probability of detection for a single test observątion is:

$$
P(\text { det. })=\int_{\gamma}^{\infty} \frac{1}{\sqrt{4 \pi}} e^{-(y-1)^{2} / 4} d y=0.285
$$

In terms of the likelihood ratio, note that $\lambda(\gamma)=\lambda_{0}=1.9$ and we make a detection whenever $\lambda(y) \geq 1.9$. To improve this rather mediocre performance, several measurements may be tested. With the same false-alarm rate, we choose the decision level differently. If we take $n$ independent samples, the signal-present probability distribution has unity mean and a variance of $\sigma^{2}$, and:

$$
\begin{align*}
\dot{p}_{s}\left(y_{1}, y_{2}, \ldots, y_{n}\right) & =\frac{1}{\sqrt{2 \pi} \sigma} \exp \left[-\frac{\left(y_{j}-1\right)^{2}}{2 \sigma^{2}}\right] \times \ldots \\
& \times \frac{1}{\sqrt{2 \pi} \sigma} \exp \left[-\frac{\left(y_{n}-1\right)^{2}}{2 \sigma^{2}}\right]
\end{align*}
$$

Similarly, the noise-only probability distribution is:

$$
p_{n}(y)=\left(\frac{1}{2 \pi \sigma^{2}}\right)^{n / 2} \exp \left[-\frac{1}{2} \sum_{1}^{n}\left(\frac{y_{i}}{\sigma}\right)^{2}\right.
$$

Taking the logarithm of the likelihood ratio, the decision test is:

$$
\frac{1}{n} \sum_{i=1}^{n} y_{i} \geq \lambda_{0}^{\prime}
$$

where

$$
\lambda_{0}^{\prime}=\frac{1}{2}+\frac{\sigma^{2}}{n} \ln \lambda_{0}
$$

Now the probability of each kind of error is different, and we choose $\lambda_{0}^{\prime}$ by evaluating $P_{1}(f . a)+.P_{2}$ (missed signal), where

$$
P_{\uparrow}(f . a .)=\int_{\lambda_{0}^{\prime}}^{\infty} \ddot{p}_{n}^{\prime}(\bar{y}) d y=\int_{\lambda_{0}^{\prime}}^{\infty}\left(\frac{n}{2 \pi \sigma^{2}}\right)^{1 / 2} e^{-n z^{2} / 2 \sigma^{2}} d z
$$

and

$$
P_{2}(m . s .)=\int_{-\infty}^{\lambda_{0}^{1}} P_{s}(\bar{y}) d y=\int_{-\infty}^{\lambda_{0}^{1}}\left(\frac{n}{2 \pi \sigma^{2}}\right)^{1 / 2} e^{-n(z-1)^{2} / 2 \sigma^{2}} d z
$$

It now becomes clear that improving the performance of simple peak. detection schemes becomes a complicated task even using very little information about the signal. If we use more of the information available, and some of the knowledge about the nature of the noise, a more successful detection scheme can be derived. When multiple detections are made on a single source, the above can be used to evaluate the detection probability.

### 2.2 Correlation Detection

Rather than make a detection test based on only the peaks of the data stream as in the previous example, consider how we might deal with detecting a signal that we know. Let $r_{k}, k=1, \ldots, m$ be the sequential data samples. Assuming additive noise,

$$
r_{k}=\left\{\begin{array}{c}
\left.S_{K}\right\}+n_{k} \\
0
\end{array}\right.
$$

where $S_{R}$ is the $k^{\text {th }}$ value of our expected signal, and $n_{k}$ is the noise sample. We may now derive a likelihood ratio test which uses this information.

First, assume that the noise is bandlimited white noise with power spectral density $S(\omega)=N_{0} / 2$ for $|\omega|<\Omega$ and Zero otherwise: The noise autocorrelation function then is given by:

$$
R(\tau)=\frac{N_{0 \Omega}}{2 \pi} \frac{\sin (\Omega \tau)}{\Omega \tau}
$$

This has its first zero at $\tau=\pi / \Omega$ so that if the received signal is sampled at intervals $\Delta t=\pi / \Omega$ the samples will be uncorrelated, and being gaussian they then will be statistically independent. The probability density functions of the two cases will be:

$$
p_{s}(r)=\left(\frac{1}{2 \pi \sigma_{n}^{2}}\right)^{i m / 2} \exp \left[-\sum_{K=1}^{m} \frac{\left(r_{K}-S_{K}\right)^{2}}{2 \sigma_{n}^{2}}\right]
$$

and

$$
p_{n}(r)=\left(\frac{1}{2 \pi \sigma_{n}^{2}}\right)^{m / 2} \exp \left[-\sum_{k=1}^{m} \frac{\left(r_{k}\right)^{2}}{2 \sigma_{n}^{2}}\right]
$$

and the logarithm of the likelihood ratio test results in the decision test:

$$
\sum_{k=1}^{m} \frac{r_{k} S_{K}}{\sigma_{n}^{2}} \geq \ln \lambda_{0}+\frac{1}{2} \sum_{k=1}^{m} \frac{S_{R}^{2}}{\sigma_{n}^{2}}
$$

Now the left-hand side of 2.2-4 is just the normalized crosscorrelation coefficient of the signal with its expected template. Furthermore, the variance of the noise $\sigma_{n}^{2}$ is just the noise autocovariance function at zero frequency,

$$
\sigma_{n}^{2}=\frac{N_{0} \Omega}{2 \pi}
$$

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Since one of our two signals is zero (noise only), we may define the average signal energy $E$ and the time cross-correlation coefficient $\rho$ by:

$$
E=\frac{1}{2 n} \sum_{k=1}^{m} S_{k}^{2}
$$

and
-

$$
\rho^{\prime}=0
$$

By extending equations 2.2-3 to infinite. bandwidth $\Omega \rightarrow \infty$, the probability density functions for the signal case and the noise-only case can be derived as:

$$
\begin{gather*}
P_{n}(G)=\left[\frac{1}{2 \pi N_{0} E}\right]^{1 / 2} \exp \left[-\frac{(G+E)^{2}}{2 N_{0} E}\right] \\
P_{s}(G)=\left[\frac{1}{2 \pi N_{0} E}\right]^{1 / 2} \exp \left[-\frac{(G-E)^{2}}{2 N_{0} E}\right]
\end{gather*}
$$

Since the false-alarm rate and the missed sources probabilities are equal when the samples are uncorrelated, the error rate is:

$$
P_{e}=\int_{\dot{\gamma}}^{\infty} \frac{1}{\sqrt{2 \pi}} e^{-z^{2 / 2}} d z
$$

where

$$
\gamma=\left(E / N_{0}\right)^{1 / 2}
$$

and thus we can determine an error rate based on the signal-to-noise ratio, independent of the shape of the signal. Figure 1 shows the error rate as a function of the signal-to-noise power ratio. Note that as long. as the noise samples are uncorrelated, the error rate is also independent of the number of samples in the correlation sum. This apparently unreasonable-result is directly related to the assumption of statistically independent samples. For bandlimited


FIGURE.1. EPROR RATE PERFOPMMACE
FOR cocrelantion detection
white noise there must be $m=\Omega t / \pi=$ constant independent samples in theinterval 0 to F . Oversampling the signal may actually result in degraded performance, as will be discussed in section 2.4. If we choose the value of $\gamma$ in 2.2-8 to achieve a desired error rate, then the probability of detection is

$$
P_{D}=\int_{\gamma-\left(2 E / N_{0}\right)^{1 / 2}}^{(2 \pi)^{-1 / 2}} \exp \left[-z^{2 / 2}\right] d \mathrm{dz} .
$$

which is shown in Figure 2 as a function of signal-to-noise ratio and error rate. .

If we have chosen the normalized signai template properly, our detection test simultaneously makes a best estimate of the signal amplitude. If the signal model is written as a function of a constant amplitude factor $A$, then the maximum likelihood estimate of that amplitude is the solution of:

$$
\sum_{i=1}^{m}\left[r_{i}-s_{i}(A)\right] \frac{\partial s(A)}{\partial A}=0
$$

-or, writing s = A s', we want the solution of:

$$
\sum_{i=1}^{m}\left(r_{i}-\hat{A} S_{i}\right) S_{i}=0
$$

-That solution is

$$
\ddot{\hat{A}-\sum_{i=1}^{m}\left(r_{i} s_{i}\right)} \frac{\sum_{i=1}^{m} s_{i}^{2}}{\text { 2 }}
$$

and now. if $S_{i}$ was normalized such that $\sum S_{i}{ }^{2}=1$; and we re-arrange the terms in 2.2-4, we have the detection test:and ampititude estimate simultaneously:

$$
\hat{A}=\sum_{k=1}^{m} r_{k} S_{R} \geq \sigma_{n}^{2} \ln \lambda_{0}+\frac{1}{2}
$$



- Figure 2.

Now it is clear how the correlation test is a betier detector than a peak test. The correlation test, takes an average of the signals weighted by the expected response as a best esimate of the amplitude. Because it is using $n$ samples of the signal, the improvement in error rate can be as much as $\sqrt{n}$. The uncertainty in the estimate is determined from the noise autocovariance function, as in section 2.3 as:

$$
\sigma_{A}^{2} \cdot \int_{0}^{T} \int_{0}^{t} s(\tau) s(z) R_{n}(z-\tau) d z d \tau
$$

If we have $N$ multiple pulses available from a single source, the decision test 2.2-4 can be modified to:

$$
\sum_{i=1}^{N} \sum_{k=1}^{m} r_{k} s_{k} \geq \sigma_{n}^{2} \ln \lambda_{0}^{1}+\frac{1}{2} \sum_{i=1}^{N} \sum_{k-1}^{m} s_{k}^{2}
$$

The false-alarm rate given by 2.2-8 is not changed. since we are designing our test for a chosen error performance. However, the detection probability improves; the new detection rate is given by 2.2-9 by replacing $E$ with $E^{\prime}$ :

$$
E^{\prime}=\sum_{j=1}^{N} E_{i}
$$

And since all signai energies for a given source are equal, there will be a 3 dB increase in the equivalent performance for each doubling of the number of signals.

As a final note to this discussion, the signal-to-noise ratio used here is the more useful signal power-to-noise power ratio, not the typical peak-to-rms value which has little physical meaning."

### 2.3 Matched Filters and Non-White Noise

The cross-correlation term on the left-hand side of equation 2.2-4 can be replaced by the equivalent matched filter. : If the filter's transfer function is $h\left(t_{j}\right)$, then the output of the filter is

$$
e\left(t_{k}\right)=\sum_{-i=1}^{m} h_{i} r_{k-i}
$$

and by inspection the filter output matches the correlator output if

$$
h_{i}=S_{m-i}
$$

That is, the matched filter is the time-reversed image of the signal expected. It is important to note, however, that because of the time reversal, the matched filter and the correlator output are equal only at time $T$, where the entire signal train (in samples) is within the bounds of the filter or of the correlator.

The matched filter representation is well suited to the case of nonwhite noise. We will show that the optimal detector for non-white noise replaces the left-hand side of 2.2-4 with a filter which is the product of the white-noise matched filter and a pre-whitening filter described in terms of the autocovariance function of the noise. To avoid confusing subscripts, we shall write the filter transformations in terms of time integrals which are the generalized extensions of the summations in section 2.2. The output of the filter at time $T$ is:

$$
e(T)=\int_{0}^{T} h(\tau) \dot{r}(T-\tau) d \tau
$$

The signal and noise components are easily.identified as

$$
S(T)=\int_{0}^{T} h(\tau) S(T-\dot{\tau}) d \tau
$$

$$
N(T)=\int_{0}^{T} h(\tau) n(T-\tau) d \tau
$$

The noise power then can be viritten in terms of the autocovariance function as

$$
\sigma_{n}^{2}=\int_{0}^{T} \int_{0}^{T} h(\tau) h(z) R_{n}(z-\tau) d z d \tau
$$

The optimum signal-to-noise ratio can be found by minimizing the Lagrangian:

$$
L=\int_{0}^{T} \int_{0}^{T} h(\tau) h(z) R_{n}(z-\tau) d z d \tau-\mu \int_{0}^{T} h(\tau) s(T-\tau) d \tau
$$

The resulting variation yields:

$$
\int_{0}^{T} h_{0}(z) R_{n}(\tau-z) d z=s(T-\tau)
$$

The filter which satisfies this relation will maximize the signal-tonoise ratio for a known signal in any noise with autocovariance function $R_{n}(\tau)$. Equation 2.3-6 is, of course, a Fredholm integral equation of the first kind which is solvable only for a restricted group of covariance functions $R_{n}(\tau)$. If, however, we can adequately approximate the integration by replacing the 0 to $T$ limits with $-\infty$ to $+\infty$, then the Fourier transform of 2.3-6 gives immediately

$$
H(s)=\frac{S^{*}(s) e^{-s T}}{S_{n}(s)}
$$

where $s=i w$ and $S_{n}(s)$ is the actual power spectral density function of the noise. This matched filter is then just the white-noise matched filter co nvolved with the actual noise spectrum. This result was derived for the limit $T \rightarrow \pm \infty$, but a detailed derivation shows that it holds wherever the data samples are uncorrelated, which was determined from the zeros of the noise autocovariance function.

As in the previous section, a best estimate of the signal amplitude exists in the presence of non-white noise. That estimate is given by:

$$
\hat{A}=\frac{\int_{0}^{T} h(\tau) r(\tau) d \tau}{\int_{0}^{T} h(\tau) s(\tau) d \tau}
$$

where $h(\tau)$ is the solution of:

$$
s(t)=\int_{0}^{T} R_{n}(t-\tau) h(\tau) d \tau
$$

Comparing this result with 2.3-6, we see that the optimal whitening filter is the best weighting function for the correlation detector and the amplitude estimate in the presence of non-white noise.

### 2.4 Karhunen-Loeve Filtering

The emphasis in the preceding section was an additive white noise. Since this is often invalid, we derived a test based on the noise autocovariance function. For the white noise case we considered a flat bandimited spectrum and found that appropriate uniformly spaced amplitude samples were statistically independent. For colored noise we considered the continuous sampling limit and wrote the detection equations as integral relationships. However, uniformly spaced samples in colored noise are correlated and the sampled case is difficult to evaluate explicitly. There is, however, another method which can be used to generate statisticially independent samples. While these are not amplitude samples, they can be used to construct the same detection and performance tests as previously described. The approach used will be to expand the signal in a series of functions which are orthogonal over the region 0 to $T$.

The functions we seek are a set of $f_{j}(t)$ 's with the normality condition:

$$
\int_{0}^{T} f_{i}(t) f_{j}^{*}(t) d \dot{i}=\left[\left.\begin{array}{ll}
1 & i=j^{\prime} \\
0 & i+j^{\prime}
\end{array} \right\rvert\,\right.
$$

and given these functions, the new samples $r_{k}^{\prime}$ of the data are given by:

$$
r_{k}=\int_{0}^{T} r(t) f_{k}(t) d t
$$

we also need the re-sampled signal template: .

$$
s_{k}=\int_{0}^{T} s(t) f_{k}(t) d t
$$

The eigenfunctions $f_{k}(t)$ are the solutions of the integral equation:

$$
\lambda_{j} f_{j}(t)=\int_{0}^{T} f_{j}(x) R_{n}(t-x) d x
$$

Now we may write the probability density functions for the new sample set, as:

$$
\begin{align*}
& p_{s}=\prod_{k=1}^{N}\left(\frac{1}{2 \pi \lambda_{k}}\right)^{1 / 2} \exp \left[\frac{\left(r_{k}-s_{k}\right)^{2}}{-2 \lambda_{k}}\right] \\
& p_{n}=\prod_{k=1}^{N}\left(\frac{1}{2 \pi \lambda_{k}}\right)^{1 / 2} \exp \left[\frac{r_{k}^{2}}{-2 \lambda_{k}}\right]
\end{align*}
$$

and the detection test becomes:

$$
\sum_{k=1}^{N} \frac{s_{k} r_{k}}{\lambda_{k}} \geq \ln \lambda_{0}+\frac{1}{2} \sum_{k=1}^{N} \frac{s_{r}{ }^{2}}{\lambda_{k}}
$$

which is identical to 2.2-4 except that $\sigma_{n}^{2}$ has been replaced by the eigenvalues $\lambda_{k}$, and the signal samples have been transformed by a weighting function similar to the whitening filter of section 2.3. In this case, however, equations 2.4-2 through 2.4-4 can be written as sums over the time sampled values with no loss of generality, hence, with no degradation in performance caused by correlated samples.

### 2.5 Digital Filtering

The transformations of sections 2.2 through 2.4 can be written as filter transfer functions. Additionally, empirical methods can be used to synthesize a desired transfer function and the equations of those sections can then be used to evaluate the error rate and detection performance. This latter course is often followed when the sampling rate is constrained by some considerations other than those requiring uncorrelated noise samples. Typically the desired filtering is matched to the sample rate and the signal dwell time by the Nyquist theorem and we wish to evaluate the detection performance of such systems. Additionally, it may be desirable to further filter the data to improve the signal-to-noise ratio based on the observed noise spectrum. In this section we will discuss how such a transfer function could be synthesized and then derive the algorithm for converting that analog transfer function to a digital difference equation.

Given an analog impulse response function $H(S)$, the difference equation for the filter function can be derived. Also, given the nominal characteristics desired, the transfer function can be synthesized. Both of these techniques are described below.

The frequency response can generally be described as a series of first-order filters. The transfer function of a low-pass filter is:

$$
H_{i}(S)=\left(\frac{a_{i}}{a_{i}+S}\right) G_{L}
$$

where $a_{i}=2 \pi f_{i}, G_{L}$ is the gain of the filter.
$f_{i}=$ the corner frequency of the filter $(H z)$.


That is, the response of a low-pass filter is flat for $f<f_{i}$, and falls at 6 dB per octave (linearly on a $\log \mathrm{A}-\log \mathrm{f}$ graph).

For a high-pass filter,

$$
H_{i}(S)=\left(\frac{S}{a_{i}+S}\right) G_{H}
$$

which appears as:
$A$

and the slope is the same as before. Higher order forms of these filters have transfer functions which are powers of the above $H(S)^{\prime}$ 's, with the exponent, $n$ equal to the order of the filter. That is, a 3rd order high-pass filter is:

$$
H(S)=\left(\frac{S}{a_{j}+S}\right)^{3} G_{H}
$$

and its response slope increases by a factor of $n$ ( 3 in this example):


Finally, a circuit which can be described by a series of such filters has a transfer function which is a product of the elemental $\mathrm{H}_{\mathrm{j}}(\mathrm{S})$ terms, and a response curve which is a series of line segments with $n( \pm 6 \mathrm{~dB})$ quantum slope changes at each characteristic frequency.

A representative example is demonstrated by the following. The filter consists of a first-order high-pass filter of $f_{i}=4 \mathrm{~Hz}$, and a second-order low-pass filter of $\mathrm{f}_{2}=40 \mathrm{~Hz}$. In addition, the detector acts as a low-pass filter of order 1 at $f_{3}=1 \mathrm{~Hz}$. The overall transfer function is then:

$$
\begin{align*}
& H(S)=\left(\frac{\cdot S}{a_{3}+S}\right)\left(\frac{a_{2}}{a_{2}+S}\right)\left(\frac{a_{1}}{a_{1}+S}\right)^{2} G \\
& \vdots \\
& a_{3}=2 \pi \cdot 4 \mathrm{~Hz} \\
& a_{2}=2 \pi \cdot 1 \mathrm{~Hz} \\
& a_{1}=2 \pi \cdot 40 \mathrm{~Hz} .
\end{align*}
$$

which is pictured as:


The definition of the corner frequency is important here. If we have defined the $f_{i}$ points as the traditional $3 \cdot d B$ or half-power response frequencies, then the values used in 2.5-1 and 2.5-2 for $a_{i}$ must be altered somewhat. Note, for example, that for a single order filter ( $m=1$ ), we have the half-power response at
$H(S) H^{*}(S)=\frac{1}{2}=\frac{a_{i}^{2}}{a_{i}^{2}+S^{2}} \Rightarrow S=a_{i}$
as expected. For an $m^{\text {th }}$ order filter, we find
$H(S) H^{\mu}(S)=\frac{1}{2}=\left(\frac{a_{i}^{2}}{a_{i}^{2}+S^{2}}\right)^{m} \Rightarrow S^{2}=(\sqrt[n]{2}-1) a_{i}^{2}$
and similarly, for a high-pass filter:

$$
S=a_{i} /(\sqrt[n]{2}-1)^{1 / 2}
$$

Given the transfer function $H(S)$, the difference equation can now be determined as follows. First, transform the frequencies. Since we desire the digital equivalent frequency, determine $A_{i}$ by:

$$
A_{i}=\operatorname{TAN}\left(\frac{2 \pi f_{i}^{\top}}{2}\right)=\operatorname{TAN}\left(\frac{a_{i}^{\top}}{2}\right)
$$

where $T$ is the sampling interval, $=1 / S R$ ( $S R$ is the sample rate in $H z)$. Second, transform the $H(S)$ function to an $H(Z)$ function by the substitution:

$$
S \rightarrow \frac{Z-1}{Z+1}
$$

This transformation preserves the square-magnitude response of the system except for a warping of the frequency scale as given by the first relationship (2.5-5). The advantage of this transformation is that aliasing is not introduced by the digital representation, thus avoiding the necessity of "guard" filters (anti-aliasing) which would result in a digital filter of higher order than the original analog transfer function.

The technique theh is to express $H(Z)$ in terms of a polynomial in $Z$, as:

$$
H(Z)=G * \frac{P(Z)}{Q(Z)}
$$

with the order of $Q(Z)$ equal to or greater than $P(Z)$. Most easily, the substitution used is:

$$
\left(A_{i}+S\right) \rightarrow \frac{\left(A_{1}+1\right)\left(Z+U_{i}\right)}{(Z+1)}
$$

where $U_{i}=\frac{\left(A_{i}-1\right)}{\left(A_{i}+1\right)}$ and $P_{i}=A_{i}+1$

The numerator and denominator of $H(Z)$ are then divided by $Z^{n}$, where $n$ is the order of the denominator, resulting in a transfer function which is a ratio of two polynomials of equal order in powers of $Z^{-1}$. Equation 2.5-9 is used for $S$ terms in the numerator. The general form of this $H(Z)$ is:
where:
$A_{i}$ is as defined previously;
$n=$ the number of elemental filters;
$m=$ the number of high-pass filters, and the $A_{i}$ 's are ordered with the low-pass frequencies first.
$P_{i}=A_{i}+1 \quad:$
and
$S_{i}$ are the expanded coefficients of the product function:

$$
\prod_{i=1}^{n}\left(u_{i}+z\right)
$$

such that:

$$
\begin{align*}
S_{1} & =\sum_{i=1}^{n} u_{i} \\
S_{2} & =\sum_{j=2}^{n} U_{j}\left(\sum_{i=1}^{j=1} u_{i}\right) \\
S_{3} & =\sum_{j=3}^{n} U_{R}\left[\sum_{j=2}^{k-1} U_{j}\left(\sum_{i=1}^{j-1} U_{i}\right)\right] \\
S_{r} & =\sum_{m=r}^{n} U_{m} \sum_{\ell=r-1}^{m-1} U_{\ell} \sum_{k=4-2}^{\ell=1}\left[\ldots .\left(\sum_{i=1}^{j-1} A_{i}\right)\right]
\end{align*}
$$

Again from our example 2.5-4, the $H(S)$ transforms to:

$$
H(Z)=\frac{Z^{-4}(Z-1)(Z+1)^{3}}{Z^{-4}\left(Z+U_{1}\right)^{2}\left(Z+U_{2}\right)\left(Z+U_{3}\right)} \frac{A_{1}^{2} A_{2}}{P_{1}^{2} P_{2} P_{3}} G
$$

The Z-transform corresponds exactly with 2.5-12 if we consider the four elements of the filter as having frequencies (in the original form) of 1,$4 ; 40$, and 40 Hz . That is, $U_{7}$ is repeated, but treated as if it were two different terms. Ignoring the constant factor temporarily:

$$
H(Z)=\frac{1-2 Z^{-1}-2 Z^{-3}-z^{-4}}{1+\sum_{i=1}^{4} S_{i}^{\prime} Z^{-1}}
$$

where $S_{1}=2 U_{1}+U_{2}+U_{3}$

$$
\begin{align*}
& s_{2}=u_{1} u_{1}+2 u_{1} u_{2}+2 u_{1} u_{3}+u_{2} u_{3} \\
& s_{3}=u_{1} u_{1} u_{2}+u_{1} u_{1} u_{3}+2 u_{1} u_{2} u_{3} \\
& s_{4}=u_{1}^{2} u_{2} u_{3}
\end{align*}
$$

Now that we have the Z-transform of the $H(S)$ transfer function, the difference equation for the system can be written. Noting that $Z^{-1}$ is the unit delay function, and writing $H(Z)$ as:

$$
H(Z)=\frac{1+\sum_{i=1}^{n} \cdot T_{i} z^{-i}}{1+\sum_{i=1}^{n} S_{i} z^{-i}}=\frac{Y(z)}{X(Z)}
$$

where $Y(Z)$ is the $Z$-transform of the output, and $X(Z)$ is the input transform. Inverting the transform we find:

$$
Y_{j}=x_{j}+\sum_{i=1}^{n} T_{i} x_{j-i}-\sum_{i=1}^{n} S_{i} Y_{j-i}
$$

where $Y_{j}$ is the $j$ th sample of the output series, and $X_{j}$ is the corresponding $j$ th input value. Continuing our example,

$$
Y_{n}=X_{n}-2 X_{n-1}-2 X_{n-3}-X_{n-4}-S_{i} Y_{n-1}-S_{2} Y_{n}-2-S_{3} Y_{n-3}-S_{4} Y_{n-4}
$$

It is interesting to note that because the transformation 2.5-9 is bilinear, the difference equation will always be the same order in $X_{n-i}$ and $y_{n-i}$ terms (except for canceling of some $z^{i}$ terms by the expansion of $\left.(Z+1)^{m}(Z-1)^{l}\right)$.

To find the $T_{i}$ coefficients of equations 2.5-17 and 2.5-18, we must expand

$$
(1+Z)^{n-m}(1-Z)^{m}=1+T_{1} Z+\ldots+T_{n} z^{n}
$$

but the gain constant in 2.2-12 of

$$
\prod_{i=1}^{m_{i=1}^{n} A_{i=1}^{n}\left(\frac{G_{i}}{P_{i}}\right)=\operatorname{COEFF} . \ldots} \quad 2 .
$$

must be included as a factor of all the $X_{i}$ terms.

Expanding 2.5-20 can be done using the binomial expansion subroutine attached to expand $(1+Z)^{n-m}$ and $(1-Z)^{m}$, and the polynomial product routine to find the resulting terms. Then if we redefine the subscript of $T$ by one to absorb the 1 on the r.h.s. of 2.5-20, we can include the factor $2.5-21$ easily into the definition of $T_{j}$ :

$$
(l+z)^{n-m}(1-Z)^{m}-T_{1}+T_{2} Z+\ldots+T_{n+1} z^{n}
$$

if we also redefine the $S_{i}$ and set $S_{1}=0$, then the relations 2.2-17 and 2.2-18 can be written:

$$
\cdot
$$

$$
Y_{i}=\sum_{i=1}^{n+1} T_{i} X_{j-i+1}-\sum_{i=1}^{n+1} S_{i} Y_{j-i+1}
$$

$$
=\sum_{i=1}^{n+1}\left(T_{i} x_{j-i+1}-S_{i} Y_{j-i+1}\right)
$$

This redefinition of $S_{i}$ has been included in the attached algorithm (see section 3.1).

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### 3.0 ALGORITHMS AND SOFTWARE

Section 3 of the first repori covered in detail the basic survey point-source processing scheme. However, in order to successfully complete the sky survey and to define the detection test gates, a number of peripheral routines are needed. This section will discuss several of the most important of these routines. Some routines are simple, such as coordinate transformations used for positional matching. Others, although complex and worthy of discussion, are very specifically written for each mission. These routines generally are part of the primary data processing system and are assembled from the formula and algorithms of section 2 of this report and section 3 of the previous paper. Yet, other routines which are a part of the data base merging are decisions and tests for specific types of astronomical sources and depend on the sensor bandpasses and sensitivities and on the spectral characteristics of the sources being searched for. Some routines which are only peripheral to the primary detection scheme are so basic and important that they are worthy of individual discussion here. The complete set of programs designed and tested on the Celestial Mapping Program (CMP) data will be published at a later date when that task is completed.

Two basic programs will be covered here and one front-end detection scheme used on a previous survey program. First, we will discuss an application of the digital filter design scheme of section 2.5. Then the calibration of infrared detectors is discussed and a routine to eva.luate the spectral response of a detector plus filter combination to a variety of stellar spectra.

### 3.1 A Digital Filter Design Aid

The discussion of section 2.5 covered the algorithm for digital. filter design; here, we consider the specific use of the following routines. The program attached does two things. :First, the coefficients of the difference equation

$$
Y_{j}=\sum_{i=1}^{N+1}\left(T_{i} X_{j-i+1}-S_{i} Y_{j-i+1}\right)
$$

where the $X_{i}$ 's are the input data, $Y_{i}$ 's the output data, and $S_{i}$ and $T_{i}$ the filter coefficients determined from the desired frequency response.
'Second, the routine creates a pair of sample response sequences. One is the impulse response function of the filter. If the filter characteristics were chosen to duplicate the response of a detector and its electronics, then this impulse response will model the radiation-hit response. The other response is the system reaction to a square wave. Since the duration of the square wave is equal to the point-source dwell time, the response is approximately the same as a source signature.

PROGRAM TRNSFN（INPUT，OUTPUT）
COMPLEX CASUMPCOEFF，T
c

THE FOLLOWING DIMENSION STATEMENT IS USED TO CREATE ——n A SAMPLE．RESPONSE SEGUENGE FOR A RECTANGULAR INPUT＊＊＊＊ EQUAG IN LENGTH TO THE STAR DWELL TIME，AND THE IMPULSE＊＊＊＊ RESPONSE．SEQUENCE．
$\therefore$ DIMEASION XX（200），XP（200），YY（200），YP（200），TIM（200）
＊＊＊＊
＊＊＊＊
＊＊＊

THE ROUTINE IS DIMENSIUNED FOR TRANSFER FUNCTIONS OF TOTAL ORDER q QR LESS：FOR HIGMER ORDERS，CHANGE THE DIMENSIONS OF ALL THE FOLLOWING TO N＋1．WHERE N IS THE DESIRED ORDER

DIMENSION $C(10), S U M(10), T(10), X(10), Y(10), \angle(10), R S(10), T R(10)$
PRINT 99
99 FORMAT（1HR）
$A 1=1 H C$ \＆$A 2=1 H S$ \＄$A 3=2 H T$ \＆$A 4=1 H F^{\prime}$
N AND M．DESCRIBE THE ORDER DF THE TRANSFER FUNGTION，
$N=$ THE TOTAL ORDER
$M$ z the orden of the highmpass functions
the program will loop for new transfer functions．to end． SET $M=0$ ：（THE LAST DATA CARD CAN BE A B！．ANK TO STOP）

50 cantinue
READ $100, \mathrm{~N}, \mathrm{M}$
100 FGRMAT（215）．
IF（N，EQ：O）GO TO 51
TRANGFURMATIUN
THE DWELL TTME IS DETERMINED FROM THE SCAN RATE SCNRTE ANO
THE DETEGTOR SIZE SIZE.n
SIZE AND SCNRTE ARE VARIABLES USED ONLY FOR GENERATING
THE TEST CASES.
FEAD 10i: SHESIZE SCNRTE

LN=INT( (SH*SIZE/SCNRTE) +0 , S $)$

FRINT 2010NpM, SReLN
 110e4,* DWELL = *,IS,1/)
C THE CORNER FREQUENCIES FOR THE FILTER ELEMENTS CAN BE SPECIFIEO
C AS COMPLEX (REAL + IMAGINARY)
THEY SHOULO BE ORDERED WITH THE M HIGHmPASS ELEMENTS LAST

C EACH ELEMENT HAS A (COMPLEX)GAIN WHICH CANNOT BE ZERQ

102 FORMAT(4F10,6,15)
FRINT 2OS, I,HDR,WOI,NORO

NNENORD.
IF ( (NTAM),GE.N) GOTOT 7

KOI=kDI/(2a**(1; NORD) $\left.m 1_{0}\right)$
GO TO 8
7 KOR=AOR* (2,** (1, NORD)m1, $)$

8 CONTINUE
WAIETAN(WDI\#PI/ SR)
WARETAN(VDR*HI/ SR)
$c$
$c$
$c$
13: CONTINUE
NTENT+1
FRINT 2OR, NT,WDR,WDI,I,WAR,WAL,NT,GAINR,GAINI
THE GAIN CAN BE SET AS ( $1,00,0,00$ )

THE ORDER MUST BE EGUAL TO OR GREATER THAN ONE
THE FREQUENCY CARDS ARE FORMATTED ( $4 F 10.60$ I5) AS FOLLOWS:
FREG(REAL), FREQ(IMAG),GAIN(REAL),GAIN(IMAGI,ORDER
PI=3.141592653579
COEFF: CMPL X (1,0,0,0)
NT: 0
$00: 1=1, N$
EEAD 1OZ,WDR,WOI,GAINR,GAINI,NORD

202 FORMAT(1H, 10X, *WD (*, I2, *) $=*, 2 F 10,6,5 \times, * W A(*, I 2, *)=*, 2 F 10.96,5 X$
1, *GAIN(*,I2,*) = *, 2F10: ${ }^{\circ}$ )
$C(N T)=C M P L X(W A R-1,, W A I) / C M P L_{0} X(W A R+1,, W A I)$
COEFF=COEFF*CMPLX(GADNREGAINI)/CMPLX(WAR + 1, WWAI)
IF (NT:LE: (NMM)) COEFF=COEFFACMPLX(WAR,WAD)
NN二NN-1
IF (NN.GT:O) GO TO 131
IF (NT:EQ.N) GO TO 141

- continue

141 CONTINUE
$C$
$C$
$C$
List the c(i) terms and the value of coeff

203 FORMAT(SHO, /f(C1OX,A1,*(*,ID $=*, 2 F 12,8,1)))$

2 $\operatorname{SUN}(I)=C N P 10(0,0,0,0)$
DO $A I=1, \mathrm{~N}$ $\operatorname{sum}(1)=\operatorname{sum}(1)+C(I)$ IF(InEQ.N) TO TO A DO $3 \mathrm{~J}=2, \mathrm{~N}$
$I F(I+J m 1, L E, N) \operatorname{SUM}(J)=\operatorname{SUM}(J)+\operatorname{SUM}(W+1)+C(I+J m)$


OUTPUT IS NUMBERED SEQUEY ALLY AND THE SAMPLE TIME IS GIVEN IN MILGISECONDS：

DO 11 $I=1,200$
＊＊＊ あ穴内 $\star$ れたれ $\operatorname{TIN}(I)=0,0$ れね出
 $X X(I)=0.00 \quad \$ \quad X P(I)=0,00$
＊＊＊ $Y Y(I)=0.00 \quad \$ \quad Y P(I)=0.00$
＊＊＊${ }^{*}$夫 丸＊＊
$L L=17+L N$
$0012 \mathrm{I}=20 \mathrm{LL}$

GENERATE GAUSSIAN NQISE AND ADD IT TO THE INPUT DATA：
CALL RANSET（863211）
$A=0$ ，
NOISEEI \＄SNR＝100．
NOISE＝1 SNR＝10． NOISE＝0
IF（NOISEGEQ，（U）GO TO 123
$001221=10,200$
GNOTSE $=0.0$
$00121 \mathrm{~J}=1.12$
121 GNOISE＝GNOISE $+\operatorname{RANF}(A)$
GNOISE＝（GNOTSEm6．0）／SNR
$X X(I)=X X(I)$ \＆GNOISE
122 CONTINUE
123 CONTINUE
$X P(10)=1.00$
IL二N＋1
JFEN＋て
YYMテック999，$\quad \$ Y P M=89999$,
00 14 JこJF． 200
0013 Ini，IL
＊＊＊
$Y Y(J)=Y Y(J)+T R(I) \star X X(J-I+1)-R S(I) \star Y Y(J m I+1)$
$Y P(J)=Y P(J)+T R(I) * X P(J+I+1)=R S(I) * Y P(J \cap I+1)$
13 CONTINUE
IF（YP（J）GGT，YFM）YPM＝YP（J）
IF（YY（J）：GT，YYM）YYM＝YY（J）
14 continue
$0015 \mathrm{~J}=1.200$
$Y Y(J)=Y Y(J) / Y Y M \quad \$ \quad Y P(J)=Y P(J) / Y P M$
15 CONTINUE
FRINT 300．YYM，YPM

FRINT 111，（I，TIM（I），XX（I），YY（I），XP（I），YP（I），I＝1，200）

## SYMBOLIC REFERENCE RAF (R\#:

ENTAY POINTS
45. TRNSFN

### 3.2 Infrared Filter Calibration

The calibration of infrared brightnesses is the single most difficult aspect of a scanning sky survey. For ground-based point-and-integrate systems, it is possible, and in fact common, to make all measurements of the same signal-to-noise ratio by varying the integration. Since the amplitude uncertainty was shown to be a function of the signal-to-noise value in section 2.2, it is clear that uniform photometric accuracy is readily achieved. For survey instruments, the uncertainty of the initial measurements is inversely related to the signal-to-noise value, applying a fundamental limit to the accuracy of the survey measurements which varies both with brightness of the source and its location in the sky.

Further complicating the problem is the fact that the sources have a wide variety of spectrum so that the broad band detectors typical of infrared survey instruments do not have a well-defined intrinsic calibration. It is possible to calibrate the detector voltage in terms of the watts per $\mathrm{cm}^{2}$ it receives. However, if the survey. is measure sources in several colors, or if the calibrations are to be derived from measurements made in a difficult wavelength region, the measurements must be referred to a spectral intensity. The wavelength bandwidth that is needed, however, is dependent on the spectrum of the source being measured. Furthermore, the effective wavelength of that measurement varies with the input spectrum.

The units of the brightness measurement are another problem. The most useful form is the brightness magnitude, defined by

$$
m=-2.5 \log _{10}\left(\frac{B}{B_{0}}\right)
$$

where $B$ is the observed brightness and $B_{0}$ is the reference value. This reference is different for every filter, since it is defined as the response that filter-detector would observe from a particular "standard" star - the archetype is $\alpha$-Lyra, which is defined as a
$10,000^{\circ} \mathrm{K}$ blackbody source with an angular diameter of $1.5697 \mathrm{E}-16$ steradians. The great benefit of this magnitude measurement is that we skirt the question of effective bandwidth. These magnitude measurements still need an effective wavelength, but for blackbody spectra at wavelengths less than 50 micrometers, the effective wavelength varies only very slowly until the source temperature falls below $500^{\circ} \mathrm{K}$. Finally, the magnitude measurements defined by 3.2-1 can be used inversely to find the equivalent blackbody color temperature if measurements are available in two or more bands.

Figures 3, 4, and 5 show the variations in bandwidth, effective wavelength, and magnitude difference for three infrared filters similar to ones commonly used in previous surveys and measurements. The results were derived from the attached filter calibration routine which is self-explanatory.


Figure 4.


Figure 5


Figure 6.
?
PEL, ZH.
ACCOUNT, SPTMEM, T3025.
MAP, CFF.
FTh:
REWITO.PUNCH,
GOPY, PUINCH, OUTPUT.
EXIT
PROGRAM INTGR' (INPUT, OUPPUT, PNNCH, TAPES=INPUT,TAPEG=OUTPUT)
PLANCF: (MCDIFIEC)
$C$
$C m m m m=-m m m m=I N T E G Q T I O N G A N B E$ DOHE WITH OR WITHOUT THE PLANCK FUNCTION,
PROGRAH FUR IKIEGRATING THE PLANCK WLACKHODY KADIATION FUNCTION
OVER AN JNTERVAL CETERMIMEO ALM ATTENUATEO UY FILTERMSENSTTIVITY
TYPE FUNCTTONS.
DATA SHOULO BE IN ORDER OF INCREASING WAVELENGTHE
NFUNCT $\because N O$ : OF PESPCNSE FUMCTION
IKITE $=1$ WRITE KAVELENGTH, IHTENSITY, RADIANCE, PLANCK INTENSITY,
$=0 \quad$ DO MOT URJTE,
NFILX $=1 *$ FLAUCK Fulicidolv TO BE CALCULATEO.

NORFAL = $=0$ TRAFSIAISEION OATA DECK COMEISTS OF TRANS DATA ONL.Y.
$=0$ = $=0$ :RT RCPNALIZE
IPLGT $=1 F L C T$
$=0$ CONCT MLOT
NTRANS = AC: OF SETS OF THANGMISSLON GATA PER RESFONSE FUNCTION,
M $\quad=10$, CF FINCTIORS (1 OR 2).
$N \quad=A O$, CF KAVELENGTH FEF FUINCIIUN (OOW INTEGER),
L $\quad=$ NO. CF TFME. TG GE CALCULAIED (JF THEGE ARE NONE L=I)
EXPER $=$ AARE OF EXFERLHEMT UATA, ETC, FURNAT $A A G$.
TYPE =ICENTIFYING ANAE 〔IF OATAP FORMAT AAG,
CAKD 1. NFUNCT, IKITE, NFLUX, LHAVTB, NOKMAL, IPIOT (6I3)
CAPD Z NTRANS (1I3)
REFEKENCE FLLX IS THE INTEGRAIEW FBUC OF A 10,000 DEGKEE B. B. OF
SIRE 1.5GSTE-16 STR (ALPHA LYKA) , "HE ZLRO MAGHITUDE KEF
ARD IS INTEGRAL. UF E (LANGUA, 10,000$) * H(L A M G D A) A D L A M B D A * O M E G A$
defineo wavelength is the flat hesfonse, effective wavelength
BDWDTH ZERO IS TME FLAT RESFONSE BANDINIUH.
WaVELENGTH IS THE TFUE EFFECTIVE WAVELENGiH FOR B, B, spegtra at
THE GIVEN TENPERATURE
the true effective havelength is the integral of g(mambidart)* R(LAMAOA) *LAMQDACLAMBDA UIVIDED GY THE IATEGRAL OF B(LAMBDA, T) *R(LAVEDA)*DLANBGA
bondth is the true gancidoth at the true effective wavelength for The given temperature
THE TRUE EFFEGTIVE dANDNIDTH IS THE INTEGRAL OF G(mANBDA, T)* p(lafgda)achampda divideo gy b(langida eff,it).

BONDTHDF IS THE PROFER EANDHIDTH FOR THE DEFINED EHFECTIVE WAVELENGTH
 IS THE B, E. EFISSION

COL'MAG IS $-2.5 \times 1$ COGTAT FLUX/KEFERENCE FIUX)
IMT FLUX IS THE IATEGRATED FGUX ON THE DETEGTGR LN GAGMmZASTRA.
Vyeb flux is the integrated flux multiplieg by the fielo of view $=4,5 E-06$ STA AND IS IN $\mathrm{H} * \mathrm{CM}-\mathrm{Z}$
real magfac

1(150), RESP(150), G'LX (150), TYPE(4), EXPER(4)
W」=0
OMEGA $=1,5697 E-16$
PI=3.141592654
$1 \mathrm{COL}=0$
L=O (5, 94) NFUNCT, IRITE,NFLUX, INAVTR, NQRMAL, IPLOT
READ
FORMAT(65.3)
$\mathrm{NJ}=\mathrm{N} . \mathrm{J}+1$
NFLUX=0
$0092[=1,4$
TYPE(1)=GH
$L O L D=L$.
$I C O L=I C O L+1$
$N N=0$
READ (5,99) NTRANS
FURNAT(113)
KEAL (5,100) H.N.L
FURBAT (111,213)
TERF(1)=10000.
INGERT TEWP DEFTNINS CARDS; LEXX, TEMP (2) $=X X, E T C$,
$L=70$
TEMP(2)=100.
00 ? $2 T=3,42$
TENP(I) =TE*P(I-1)+10.
DE 21 I=43.52
IヒKト (1) =TEMF(I-1)+50.
$0022 \mathrm{I}=5 \mathrm{5}, 70$
$\operatorname{THP}(I)=T E N P(1-1)+560$
$N 1=1$ \% $N 2=N$
READ (5,101) (EXPER(K),K=q,4), (WAV(I), RES(I),I=N1,NZ)
HORMAY(4AB/(12FU.4))
$t 1=N+1$
$00^{\circ} 500 I=L 1,150$
$\operatorname{WAV}(I)=0.00$
RES (I) $=0,00$
1 KAV(I) $=0.00$
$R E S P(I)=0,00$
500 CONIIHE
$\mathrm{NC}=\mathrm{N}$
$N A=N A+1$
$1 F(M+1) \cdot 1,1,3$
IF (INAVTR,EEO.1) 50 TO 6OL
REAO (5,112) (TYPE (K),K=1,4), (TRAN(I),I=I,N)
112 FURMAT(4AB/(12F6.4)).
GO TO 600
601 REAU (5,112) (TYPE(K),K=1,4), (MAY(I), TRAN(I),I=1,N)
600 conitinue
DO $2 I=1, N, 1$
RESP(I) $=\mathrm{FE} \mathrm{S}(1) * \operatorname{TRA}(I)$
GU 1011
?
DU $9 \quad I=1, n, 1$
ThAin(1) $=0$.
HESF (I) $=$ PES (I)
CDivTINJE
CUYIINUE
IF (NDFPAL.FG.D) GGTQ 700
UlGNF…
Ni=1

```
22G NSPACE=6
    LINE=0.
    WKITF (o,212) (EXPFR(K),K=1,4),(TYFE(K),K=1,4)
212 FORFAT(fm1,37X,44t/ 4, \,19HTABLE OF INPUP OATA/ 38X,4AG/120X,
```



```
    2,3x,11HFUNCTION(1),3x,11HFUACIIUN(2)//)
    IBG=N1 * INC=N2
    OU 114 L=1RG,INO
    NSPACE=INSPACE+1
    LINE=LINE+1
    It(5-1.SFACE)2(4,206,206
206 HRITE (6,210) NAV(I),RES(I),TRAM(I),WAV(I+35),RES(I+35),TRAN(I+55)
210 fOKMAT(22x,F5,2,7x,F7,4,7x,F7,4,!0X,F5,2,7X,F7,4,7X,F7,4)
    Gu TO 11A
204 NSPACE=1.
    IF(35-1INE)219,214.214
214 WRIIE (G,2!1) WAV(I),RES(I),TRAN(I),WAV(I+35),RES(I+35),TRAN(It35)
211 FORMAT(1HO,CIX,F5,2,7X,F7,4,7X,F7,4,10X,F5,2,7X,FI:4,7X,F7,4)
    G0 10 114
219 NL=I+LINE-1
    IF(N2=N1)250,220,220
114 continut
    G0 TG 250
700 BLGKP=RESP(1)
    D0 30 I=2,N
    IF(GIGRPMKESP(I))32,30,30
32. EIGFP=RESP(I)
zo continue
    DO 33 I=1,N
    RESF(I)=RESP(I)/BIGRF
33 CUNTINUE
    N1=1
720 NSPACE=0
    LINE=O
    WK1HE (6,712) (EXPER(K),K=1,4),(TYPE(K),K=1,4)
712 FORMA!(1H1,46%,4A6/ SOX,19HTABLE OF IAPIIT UATA/ 47X,4AG//10X,
    ILOHMAVELENGTH, 3x,11PGUNCTION(1),3x,11HFUNCTIGN(2), 3x,
    211HFUNCT.(1X2),5x,10HKAVLLFNGTH, 3x,11HFUNCTION(1),3x,
```



```
    410HNORNALIZED//)
    , IbG=N1 & IAL=に2
    ; DO 614 I=IEG,INC
    NSFACY=nspace+1
    LLAE=LINE+1
    If(5-ASPACF)704.706,7.06
```

```
706 JMRITE, (6,710) WAV(I),RES(I)PTKAN(I),FESP(I),NAY(I+35),RES(I+35),
    1THAN(I+35),RESA(T+35)
710 FONM4T(12x,F5,2,3(7x,F7,4),10x,F5,2,3(7x,F7,4))
    GO TO 614
704 NSPACE=1
    IF(35WLINE)719.714,714
714 NRITE (6,711) WAV(I),RES(I),TRAN(I),RESH(I),WAV(I+35),RES(I+35)P
    1TRAN(I+35),RESP(I+35)
711 FORMAT(1HO,11X,F5,2,3(7x,F7,4),10x,F5,2,3(7x,F7,4))
    G0 T0 614
719 N.1=1+LINE-1
    If(N2-N1)}150,720,72
6:4 CONTINUE
750 continue
1111 FORMAT(F10,%,F10,8)
250 IF(iYFLUX,EG.O) GD IO 30%
    WRITE(6,107) (TEHP(I),I=1,L)
    FORMAT(// 55X,IIHTENPERATURE//(9F13,2))
    J=0
    j=J+1
    IF(NFLUX,EQ,0) G0 T0 307
    DU 4 1=1,Ni,1
    CALL SHCTHM(TEMP(J),MAY(I),RESP(I),N(I),NX(I),NXX(I),ZINT)
    GO 10 301
307 00.302, I=1,N
    Wx(1)=%ESF(1)
    4*X.(I)=NX(I)/FI
302 contTmut
IF(IRITE.EO.O) GO TO 300
IYSPACL=5
    L\E=45
    00216 I=1aN
    NSPACE=NSPACE+1
    LDGELINF+1
    If(5-HSPACE)240.242,242
    WRITE (6,120) NAV(I),NX(I),4XX(I),W(I)
```



```
    G0 To 2te
240 NSPACE=
It (45-I.INE) 244, 2440,246
240 MRIIE (6,122) 则V(I),#X(I),wXX(I),W(I)
IZ2 FUPHAT(1H0, С9X,1PE12,5,4X,1PE12,5,5X,1PE12,5,9X,1PEL2,5)
G0 10 216
LINE=1
IF(IKITEqEO.1)GO TO 24马
```

```
    G0 ro 300
2iG cun 1/kN
300 H=HAV(2)=NAV(1)
    \lambda1vT=0.0
    KN=1:C
    DO S I= &,KA,?
    XINT=XINT+(H/3:0)*(hX(I)+4,0*NX(I+1)+NX(I+2)
    conitrue
    IF(NFLUX.ER.1) GOTG 140'
    WRTIE (G,14%) KINT
    HMZERO=XINT
    FURMAT(///1X,11MEFMDKIDIN =1PEI2,5,gH MICRONG)
    BC=BC2=7.
    60 10 144
    NINT=XINT/PI
    XIRF=XINT
    YINT=0.0
    DO7 I=1,KN,2
    YINI=YIMT+(H/3,0)*(h(I)+4,0*W(I+1)+H(I+2))
    cunttnue:
    YINT=YINT/PI.
    EFX=XINTAEIGRF/YINT
    TEFX=AINTABTGFF/ZIMT
    BC:2,5*ALOG10(1/EFX)
    BC2=2.5*ACCf10(1/TEFX)
144 D0 126 I=1,N
126 w*x(I)=~x(I) *MAV(I)
    XXIAT=0:0
    DO 12S I=, KN,2
    XXINT=XXINT+(H/3.0)*(nwx(I)+4.0*WWX(T+1)+NWM(I+2))
128 continue
    XXJNT=XXINT/FI
    EFFT=XXI:\I/XINT
    IF(INFLUX.EG.1) GO TO 14t.
    EFFT=PI*EFFT
    KKJTE (6,130) EFFFT
1.30 FUKNAT(//1X,2SHEFFECTRME WAVELENGIH = 1FE12,5)
lut cunlinve
    If (NFLUX,FQ,O) FAYREF=EFFT
    If(NFLUX,LG,O) GOTC &
    2NON二0.
    CALG SPGTK! (TF:AF(J),EFFT,ZNON,GMAV,ZNON, ZNOH,ZNON)
    Enav=ismav/Fd
    IF(J,NE,j) GQ T0 4451
```



```
    AHREF=XIRP*QMEGA
    PUAC:H 2?2, AHREF
\? FORMAT(5X,*REFERENCE FLUX = *:1PE12,5)
    PUNCH 223,HAVOEF,EXZERC
```



```
    18)
        PUNCH 221
    FORMATCIX,*NO: TENF WAVLNTH BDNOTH ENYHDF ER MAG COL
        IMAG INT.FLUX VIEW FLUKN)
        continue
    ZNON=0.
    CALL SPGTRM (TEHP(U),KAVDEF,ZNON,GNAVOF,ZNON,ZNON,ZNON)
    BWAVDF=BWAVDF/PI
    IF(ByAV,EQ,0,0) GO T0 4441
    DELMAY=XINT/BHAV
    DLGVEF=XINT/RNAVDF
144: cONTIVNE
    MACHAC=#BMAV/EhREF
    ;MAGFAC=ALOGIO(MAGFAC)
    *AGFAC=-2.5*MAGFAC
    BRNAG=-2.5AALCG1O(XIHP*OHEGA/AHREF)
    AfPP=XIRP*4,5L-06
    PUNCH 2OO,TCOL,TEND(I),EFFT,OELWAV,OLWVDF,MAGFAC,BRMAG,XIRF,AIRP
    FURMAT(1X,IS,2X,6FQ,3,2E10.5)
    If (J-k) 6, !-8
    GuntTave.
    IF(mFLUX,EQ.1).g0 T0 E88
    N+|.UX=1
    G0 10 887
    GUNTINUE
    If(HN:MNTRALS)98,97,97
    JF(14J=NFUNGT)95,76,96
    cuntinue.
    step
    END
    SUEROUTINE SPCTRH (TENP,WAV,RESH,W,WX,AXX,WINTG)
    IHSERT DESIHEC EMISSJYITY HERE AS &NS = XXXX
    EMS=1.00
    PI=3.141542654
    IF(wAV,EQ.0:) GO TO 10
    AI=1,43879/(WAV*TEHP/10000.)
    If(AI.GT.8B.) GO TO 10
    Bl=ExP(AI)
    W=(3,741032E-16)/(((NAV/10000%)**5)*(BI-1,))*EMS
    60 10 12
```

$V_{i}=(3.141832 E m 16) * E X P(-A J) /(((N A V / 10000,) \pi * 5)) * F N S$
に $x=$ トF SP*
$W X X=N X / P I$

HETURE
END

### 3.3 A Point-Source Detection Routine

ORIGINAL PAGA IS

- OF POOR QUALITY
The last routine presented here is a program developed for an early sky survey. The purpose in reviewing it here is to illustrate both the breadth of processing which can be done in a single pass of the data and also the complexity of the required software. The routine unpacks and de-commutates the data and checks for errors and gaps. Three background channels are processed, the running noise computed, and the dāta plotted. Within the basic detection loop, the data is tested for signal peaks, correlation peaks, and signal length, and the radiation hits are separated from the data. Estimates of the amplitude and bias level are made, and the position of the position of the signal is found from the time of detection.

Inspection of a sample portion of the preliminary detection list reveals some of the basic problems which the following merging routines will need to deal with. The most complex problem is that the correlation coefficient does not track well with some of the other measurements of a good signal. For example, source number 22 has a good correlation coefficient, but the estimated amplitude is less than half the peak height, and the amplitude estimate peak is significantly shifted in time from the data peak and also from the peak correlation coefficient. The correlation coefficient is below a reasonable error gate, but the same is true for signal number 10 , where except for a slightly low $\rho$, the signal is very good. This pattern persisted throughout the data, and a careful study revealed that the poor $\rho$ values were the result of an uncertainty in the detector bias. Since the sensor used bidirectional logarithmic amplifiers, the uncertainty in bias led to a possible error in de-compressing the amplifier functions which would tend to warp the signal shape significantly and degrade the value of the correlation coefficient, and also warping the noise spectrum.

The software presented here was not designed to minimize its use of computer resources which would probably have resulted in a separation of the multiple functions of this routine. Furthermore,
using a maximum sensitivily test which allowed a $10 \%$ error rate, the resulting data was not significantly compressed. Of course, making multiple measurements of the signal quality and not immediately testing on them. This allowed manual inspection of the data quality and careful adjustment of the tests which followed providing a sound study basis for a larger detection scheme.





``` \(320=, 29,2621212,212,211,100,213,21-, 215,215,217,210,3,301,352,505\),
```



```
G
OATA（FROGIOTT，I＝1，3）／10HEELZMANN \(2,13 H 121\) SLG，1JHNAL TRACES／
\(r\)
```



``` \(123,21,25,27,28,29,30,31,32,15,54,33,36,37,35,39,-3,+1,42,43,+-1+6\),
```



```
0
REX（ \(-1,4\)（YY（I），\(I=1,-3)\)
44
45 FONATTIX，ITO，E2כ．141
PRINT \(\div 6\)
45 FU－MA（1HI）
1PLOT \(=3\)
SYMHT \(=0.0 \mathrm{E}\)
DX＝0．2R0
FSTAZ \(=-23.70\)
TJUEZでい。
TJOS＝200．
SCNTHME171． 2 SCNEEN＝110．1
\(O Y=4.0\)
TAMX＝0．
SUMYY＝？。
SUTY＝0．
\(0021 \mathrm{I}=1, \mathrm{~L} 0\)
YYTIEYYTIT7DO．
SUMY \(=\) SIMMY \(+Y Y(I)\)
```



```
21．\(N Y=I\)
```



```
SCNEND＝SCNTIN＋SCNLEN．
XURX \(=\) SMIL \(=N / D X\)
\(X M A X=x^{M} A X \neq 1.398\)
NSYMS＝IMT（XMAXT2．）
OFFSET＝5． 5
YMIX \(=12.1\)
ICOLOR＝1
\(\angle F A C T=1 . \mathrm{B}\)
ZLONG \(=7 *(X M A X+10\).
IFTIPLCT．EO．0）60 TO i
GALL FLTIDZ（DPOGID，ZLCNG，YMAX，ZFAGT）
CALL NEサDEM（3）
7
：
CCNTINUE
DO i \(\mathrm{J}=1, \mathrm{E} 5\)
SYSKTJI＝SYSN（J）FSYSNTJ
QMS（J）\(=0\) ．
मHAXJ（J）＝RTAXJ（J）＝－5！
```


$\operatorname{KRJ}(J)=K A J(J)=0$
$0 \mathrm{~F}=\mathrm{x} 5=$.
PRFCNT $(J)=0$
Commit J ) $=0$
KPRIME $(J)=$ ?

YNン1(J)=
Y!
KF:AK $(3)=$ -
KSTAPTJ)
MAFKÉP(J)=, TOUS.

$X \operatorname{AR}(J)=3 \cdot 1$
PI=3.1415920535
SANDO $=359$.
$F R: Q A=1.0$
$F R=Q B=-9$

RETA $=2 . * P I * F E E G 8$
TATA=TतNTAEPHAC(2.*SENOF)
TAMB=TAN(BETA/ (2. $\because=S A M P P))$
$0 \cdot A=1 .+$ TATM
OMA=1. TAMA
OF $3=1 .+T \mathrm{~T}$ T?
OME=1. T2NB
00 9 I $=1,3$
Q ICOLCNT (T) = O

$E R P=1.80$
NSTAP=:
NP.JCT $=$ C

$R A D=\varepsilon .7337233^{*} \mathrm{PI} / 12$.
COT=CP=COSTEEC?
SIDECP=SIN(DECP)
n= TN: TRU=:
START=-1
REJECT= C
$\mathrm{NU}=5$
OBJECT=0
PHREC= 019
IFITLOT. $=1.2150104$
CALL AXIS (2.0,0.0,7HAIIMUFH, $-7, X M A X, 0.0, F S T A Z, ~ D X, 1 J .0)$

GALL AYIS (1.0,3.0,12HSIGNHL SCALE,12,5.0, Э0.0,-2.*JY,3Y,2J.3)
CALL FLOT(2.U,U.T,-3)
$X S Y M=0$.
TO 11 J=1, WSYMS
$Y S Y M=0$.
XSYM $=\times 5 \mathrm{MF}$ ?
$0011 \mathrm{~K}=1,18$
YSYITVYSYM+0. 5
11 CALL SYM?OL (XSYM, YSYM, SYMHT, $3,0,0,-1$ )
YL-ASI=-2.5~UY
YMOST $=9.5 * D Y$
-Z CONTTNUE


```
    SD\eta=1. 
```



```
    OC|EE.K=1,:9
    DTA(J,< =0, 人(J,K)
    ILソL=*ソ(J.N)-IOFST(J)
    IF(ILVI.I:, -5-\) ILVL=-E.C
    TF(ILVL.GT.E5!) ILVL=5=C
    IF(ILVL.LI. i) MLVL=561-ILML
    IF(ILVL.OE.:) MLVL=ILVL+1
    IF(ILJL,1,!) CNA(J,V)=-3ET(ivL,JL)
    IF(ILVL,G&,:) n\S(J,K)=RET(ILVL)
    IF (ICCLJE.NE.1) GO TO EJ
    C
    E0 CONTIWUE
    IFIICCLOR.NE.IT GO 1O 1S
    DO 33 k=1,39
    RSU(J,R)=FST(J,K+S?)
    BI\triangleS(J,K)=9IAS(J,K+33)
    ANन (J,K) =nTr\J,K+3#)
    SUMX=SUMXY=SUMXX=G.
    0% 31 <K=K,39
    SU:HX=SUMX+\capTA (3,KK)
    SUNTXX=CU:XX+0TA(J,KR)**?
    z1 SUMXY=SU4XY +DTA(J,KK)FYY(KX-X K+1)
        00 SC KL=1,K
            SU:TX=SUMOX SHX (J,KL)
            SUSRX=SUAXYFDNX(J,KL)*=2
    32. SUMXY=SUYYY +DNX(J,KL)*YY(4T-K+KL)
    XDENONELT.FSUYTX-SURXFSUMX
    RNUH=L2. *SUNXY SUMXFSU*Y
```



```
    DSO(J,<+?S)=RNUN*RNUM/ (XCEHOH* YDENOM)
    GOTO
    35 RSO(J,k+20)=0.
    37 IFTYOEMOY:=6.0.) GO TO 33
    AMF (J,K+ZO)=PNUM/YOENOM
    0, 的 :3
    38 A:H= (3,K+70)=0.
    39 IFTVT:+0:=0.10. GO TG +2
    BTAS(J,K+ZЭ)=(SULAYY*SUMX-SUHY*SUMXY)/YDENOH
    60 10 33
    L2. 3TAS (J,K+3\geqslant)=0.
    33- CONITKUE
    15 CONTINUF
    IF(3EGIN\) GO TD 7
    C
    C
    C
    ए0 14 <=1,7प
    XN(J)=OTA (J,K)
```




KSi4F $(J)=\mathrm{K}$
OTAMAX（J）＝DTA（J，K）
R＂：AK（J）
3G：TIP（J）$=$ TA $+($ KSTAF（J）$+J / 7 E) / 350.$.
Ir（KPRミME（j），EG．j）GO TO 9？

IF（ZNC（J）．EC．．．）GO TO 125

1 re TO 192
195 CO：TINE


NP JCT＝？！P．JTT＋1

 2रमगXJTJ，पTMAX（J）

30 T0 103
C
ITA STUSC＝HAS BEEAFOUNG WHTLK EXCEEDS THE REFERENEELEVEL BY THFEE SIGMA，CALCULATE THE EEGINING TIME AVD THE TIME CF PEAK， ANJ OUTPUT THE STAR DATA TOTHE RECOROTAGFILE．

CoUnt $(J)=\operatorname{CCUNT}(J)+1$

WRITE（2，2）FHREC，J，KSTAR（J）；KPEAK（J），KPRIME（J），JTAMAX（J），BGNTIM（J）
 2MAXJ（J），？IVAX（J）

KAJ $(\mathrm{J})=\mathrm{KP} \cdot \mathrm{J}(\mathrm{J})=0$

$\operatorname{KSTAR}(\mathrm{J})=0$
KFRIT：$=(J)=3$
$K P=A K(J)=0$
DTयसम大（J）$=$－2．0．


IF（ $(. N C T$. MARKER（J））．AND．（K，EQ．39）$) K A J(J)=K A J(J)-39$

91 CONTINUE
GOTO 0
fog COHTINJE
© GACKGRCUMO DATA GHANNELS，IN MV（J，（）
C J＝25，4ラ，5L．．．K＝1，1039
C
SUH万K $=$ ．
SUSOBK＝0．
IqK $=(J-7) 719$
D0 15 $\mathrm{K}=1,39$
POTNT $=\mathrm{VN}(J, \mathrm{~K}) / .2048$
VAL＝POTNT＊PKFAC（IBK）
TIHE＝TRFTK＋J／7U．1／35U．
IF（J．E日．2E）WRITE（8）VAL，TIME
IFTJ．ET：ज5 WRITE（G）VALGTIME



1AT T $=*, F 1$. 4 , * SECONOS*)

 मराTE(:1:14)

 2 HAVELENGTH*, 1
HR-TEた, ICT
100? FO:MAT(1HO, EX,*OミJECTS DETECTED ON EACH CHANNEL*)

1, (CHAN(I), COUNT (I) , $I=4, \dot{E}, 63$ )

GRTTE (E, ICGE)
IOU6 FONMAT(1HETSX,*SIGNALS REJECTED ON EACH CRANNELFT
WRTTE(E,100E) (CHAM(I), RPPCHT (I), $I=8,25),(C H A N(I), ~ K R P C N T(I), I=27,4$ 14), (CHदN(I), RPPCNT(I), $1=4,6,63)$




C


## 

TFTN $+5+5+1 i$

गत्र Emmit

```
IF(MF:=0.? \()^{\prime}\) GOT0-82
```



$N T=?$
तुण
402 COHTINUS
=N:FILE 4
DE:TSO $\because$
IFTIF[T. .TET. GU 0 O
rcclor= 3 GLCP+1
1-TLCELE $\cdot$ ET. $315010:$
$X M-W=X " n x+10$.
TO $3=1, I 5=$ RTE
8 BACKSFAC= 1
CALL FOT (XTEN:T, T,-3)

万OTO:
5 COMIINUE
ZNTFIL-
IF (IFLOT, $\Xi 0.0$ ) GO TO 7T?
CATL ENDOL


VERIT OETMILS UTOTCSTS UF PFUEL-N

I VAL AFZAY SAME OPEZPAND NOT SUBSORIDTED, FIRST ELEIENT MILL OE USEO.

MPCLIC REFERENSE MAF $(P=3)$
UTS $\quad$ EFLLHE REFERENCES
MIST:




### 4.0 APPENDIX: A SIGNAL PROCESSING GAME

The aim of this game is to develop skills in signal processing. The input data for this game are the recorded data $U_{R}(t)$. It is assumed that the non-uniform scanning velocity has been corrected for already. The time coordinate is given in discrete numbers $t=0,1,2 \ldots 63$. We may consider $U_{T}(t)$ as being about one quarter of a single horizontal scan ( $\beta=$ constants).

The rules of the games are as follows. The "investigator" gets the • sheet "Recorded Data $U_{R}(t)$ " and the sheet "Problem \#1." After solving this problem he will give the solution to the "monitor" and to the "game constructor." Now he may start on problem \#2, and so on. But it is important that the investigator does not get the next problem sheet before he has finished the previous problem. The reason is that the formulation of the later problems contains parts. of the answers to the earlier problems. This has to do with the basic structure of this simulation game: for performing any meaningful signal processing operation one must have some knowledge about the original signal and/or the noise. For example, in problem \#l the investigator is told that the noise is additive and non-negative. In the later problems, the investigator will be supplied with even more a priori information. Naturally, this should enable him to extract the signals better and better. But the methods for doing this increase in complexity.

On the very last pages, following the problems, the design of the "recorded data" is explained, and the true original signal is. unveiled. Obviously those pages should not be given to the investigator before he has solved all the problems.

$$
\cdots \text { RECORDED DATA } U_{R}(t)
$$

$t$ is the discrete time variable running from $t=0$ to $t=63^{\circ}$

| $t$ | $U_{R}(t)$ | $t$ | $U_{R}(t)$ | $t$ | $U_{R}(t)$ | $t$ | $U_{R}(t)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 55 | 16 | 110 | 32 | 39 | 48 | 383 |
| 1 | 25 | 17 | 184 | 33 | 56 | 49 | 10 |
| 2 | 85 | 18 | 29 | 34 | 05 | 50 | 69 |
| 3 | 61 | 19 | 51 | 35 | 15 | 51 | 58 |
| 4 | 20 | 20 | 42 | 36 | 95 | 52 | 52 |
| 5 | 95 | 21 | 78 | 37 | 09 | 53 | 66 |
| 6 | 07 | 22 | 09 | 38 | 81 | 54 | 79 |
| 7 | 00 | 23 | 15 | 39 | 21 | 55 | 134 |
| 8 | 62 | 24 | 13 | 40 | 81 | 56 | 94 |
| 9 | 79 | 25 | 50 | 41 | 399 | 57 | 102 |
| 10 | 148 | 26 | 99 | 42 | 312 | 58 | 108 |
| 11 | 105 | 27 | 54 | 43 | 348 | 59 | 94 |
| 12 | 125 | 28 | 99 | 44 | 303 | 60 | 56 |
| 13 | 125 | 29 | 35 | 45 | 383 | 61 | 67 |
| 14 | 173 | 30 | 98 | 46 | 317 | 62 | 51 |
| 15 | 181 | 31 | 02 | 47 | 317 | 63 | 63 |

## Problem \#1

Given are the recorded data $U_{R}(t)$ with $t=G, 1 \ldots 63$. Wanted are the original data $U_{0}(t)$, which represent the "one-dimensional equivalent object radiation" $S_{E}(\alpha)$. We assume that the known influences of the telescope $\left[M\left(x^{\prime}, y^{\prime}\right) ; R\left(x^{\prime}, y^{\prime}\right)\right]$ and of the electrical system $[G(t)]$ have been compensated already or are negiigible. But the recorded signal $U_{R}(t)$ is corrupted by additive noise $N(t)$ :

$$
\cdot U_{R}(t)=U_{0}(t)+N(t)
$$

The only features known about the original signal $U_{0}(t)$ and about the noise $N(t)$ are that they are non-negative:

$$
U_{0}(t) \geq 0 ; \quad N(t) \geq 0
$$

Furthermore, the noise $N(t)$ is stationary, which means that the noise properties are not "drifting." In other words, short-term average features of the noise remain the same from the beginning to the end of the observation.

Try to utilize the given a priori information for computing a new signal $U_{j}(t)$ from $U_{R}(t)$, which somehow is better than $U_{R}(t)$ as an approximated representation of $U_{0}(t)$. Plot $U_{7}(t)$ as a continuous curve, and also $U_{R}(t)$ for comparison.

Probiem \#2
Given are the facts:

$$
U_{0}(t) \geq 0 ; \quad N(t) \geq 0 ; \quad \bar{N}=50
$$

By $\bar{N}$ we mean the linear average of the noise. This $\bar{N}$ can be visualized as the dark current of the photoreceiver as measured with an instrument which rejects high frequencies.

Based on these facts, try to compute a better signal $U_{2}(t)$ from $U_{R}(t)$. Plot both $U_{2}(t)$ and $U_{R}(t)$.

## Problem \#3

Given ane the same facts as in the previous problem. In addition, it is known that the noise is approximately "white."

$$
\begin{aligned}
& N(t)=\bar{N}+n(t) ; \quad \tilde{n}(v)=\int n(t) e^{-2 \pi i v t} d v ; \\
& v=m / 64 ; \quad m=-32,-31, \ldots-1,0,+1, \ldots+30,+31 ;
\end{aligned}
$$

$|\tilde{n}(v)|^{2}$ constant. The amount of this "constant" is not known. Try to deduce it from the recorded data $U_{R}(t)$. You might have to make an intelligent guess.

## Problej \# \#

Given are the same facts as in the previous problems, including the . "constlant" which describes the noise power level.

$$
|\tilde{n}(v)|^{2} \approx \frac{16}{3} * 10^{4} \text { in }-\frac{1}{2} \leq v \leq+\frac{1}{2}
$$

Now that $|\tilde{n}(\nu)|^{2}$ is known and $U_{R}()$ is computable, can you apply the Wiener-filter theory, at least in a guessed approximation? Try it and compute $U_{4}(t)$. $P \operatorname{lot} U_{4}(t)$ and $U_{R}(t)$. Hing: represent $\left|\tilde{U}_{0}(\nu)\right|^{2}$ by a gaussian function of suitable peak power and width. Signal processing specialists always try it with a gaussian function if they don't know a better way.

$$
\left|\tilde{u}_{0}(v)\right|^{2} \approx p_{4} e^{-\pi\left(v / v_{4}\right)^{2}} .
$$

## Problem \#5

Try the same approach as in the previous problem, but with a guessed $\operatorname{sinc}^{2}$-shaped $\left|U_{0}(\nu)\right|^{2}$

$$
\left|U_{0}(v)\right|^{2} \approx P_{5} \operatorname{sinc}^{2}\left(v / v_{5}\right) ; \quad \sin c z=\frac{\sin \pi z}{\pi z}
$$

Plot the result $U_{5}(t)$ and also $U_{R}(t)$ for comparison.

## Problem \#6

Try the same approach as in the previous problem, but a somewhat different guess for $\left|\tilde{U}_{0}(v)\right|^{2}$

$$
\left|\tilde{U}_{0}(v)\right|^{2}=P_{6} \operatorname{sinc}^{2}\left(v / v_{6}\right)+\left(P_{0}-P_{6}\right) \delta_{0}
$$

Herein $\delta_{0}$ means a function which is equal to 1 for $v^{\prime} \leq 0$ and equal to 0 for $v \neq 0$. Plot $U_{6}(t)$ and $U_{R}(t)$.

## Problem \#7

Based on all of the accumulated experience, try your own signal processing approach or simply guess what $U_{0}(t)$ might. have been. Call it. $U_{7}(t)$. Plot $U_{7}(t)$ and $U_{R}(t)$.

