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A COMPUTERIZED ENGINEERING MODEL FOR EVAPORATIVE WATER COOLING TOWERS

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ABSTRACT

The evaporative cooling tower is often used to reject waste heat from industrial processes, especially power plants and chemical facilities. In this paper, we present a consistent physical model for crossflow and counterflow cooling towers which imposes rigorous heat and mass balances on each increment of the tower under study. Individual towers are characterized by specification of a mass evaporation rate equation.

The solution algorithm allows reduction of test data, interpolation of the reduced data, and comparison of test results to design data. These capabilities can be used to evaluate acceptance tests for new towers, to monitor changes in tower performance as an aid in planning maintenance, and to predict tower performance under changed operating conditions.

INTRODUCTION

The rejection of waste heat is a necessary element in the operation of chemical plants, power plants, and gaseous diffusion plants. These particular processes are of interest because of the enormous quantities of heat involved. For instance, a nuclear-powered electrical generating facility of 1000-megawatts [Mw(e)] capacity will reject about 10^8 Btu/min to the ambient. If the temperature rise in the condenser is limited to 20° F, 0.6 x 10^6 gpm of water must be circulated.

Frequently, this water is cooled and reused in order to reduce water treatment costs and to avoid possible environmental damage. Usually, this recirculated water is cooled using cooling towers in which the bulk of the water is cooled by evaporating a portion into the surrounding atmosphere.

The costs of new cooling towers are substantial. Figures of up to $20/kw of electrical generating capacity are typical, making the cooling tower system for a 1000-Mw(e) nuclear plant worth as much as $20 million. Therefore, the cost and performance of cooling towers and associated peripheral equipment must be included in the trade-offs required in designing the integrated *optimum* process plant.

*Will present paper.

A mathematical model of the cooling tower, manipulated by computer, can provide cooling tower cost and performance data. Inputs and computed results can link this model to the adjoining elements in the system. Also, systematic analysis of data produced by tests of existing cooling towers can be used to identify deterioration and to predict tower performance as operating conditions change. An increase of a few degrees in the temperature of the supply water to the condensers of a power plant can increase the annual energy costs substantially. If such a temperature rise can be attributed to a decrease in tower performance, repairs to offset deterioration and restore cooling capacity may be justified easily, allowing repairs to be planned and funds to be budgeted in an orderly fashion.

COOLING TOWER CODE UTILIZATION - TYPICAL PROBLEMS

The problems usually encountered in analysis of cooling towers for large process plants fall into three general classes which are discussed in some detail in this section.

Analysis of Test Data

Typically a cooling tower yields a set of data at each operating condition tested. Measurements include wet-bulb temperature and barometric pressures of the ambient air, volume flow rate for the fan(s), temperatures of water entering and leaving the tower and flow rate of water onto the tower. A cooling tower is similar to other heat transfer equipment with respect to possessing a performance coefficient which is most often expressed as a function of mass flow rates. The problem is to determine the performance coefficient grouping, $K'aV/C_pL$, and the associated mass flow ratio,

$$\theta \equiv \frac{\text{Water Flow Rate Onto Tower}}{\text{Mass Flow of Dry Air Through Tower}} \equiv \frac{L}{G} \quad . \qquad (1)$$

Data taken at several operating conditions can be reduced with the code and the performance line constructed as indicated in Figure 1.

Comparison of Test Results to Design Point

Design conditions for a tower are quite often represented by a single point on a plot, such as Figure 1. Given the performance line (from recent tests, as above) and the manufacturer's design point, the objective is to calculate quantitative measures of the distance between that line and point. To put the problem in perspective, suppose that the point were the manufacturer's guaranteed performance on a new tower and the line were the result of a preacceptance test on that tower. For the hypothetical case shown, the distance between the two could represent serious economic loss to the purchaser. Adjustment of the purchase price can be more rationally negotiated if the distance between line and point can be quantified. The vertical distance (constant θ) between the point and the line can be quantified as an increase in cold or supply water temperature if the tower is operated at the mass flow ratio, atmospheric conditions and water temperature difference being specified in the design. Another distance of interest is along the constant approach line and quantifies the reduction in water flow rate required to meet the water inlet conditions. A plant operator can convert either of these quantities to an economic loss or gain, as the case may be.

The Tower as Part of a Heat Rejection System

A cooling tower frequently is required to accept a given rate of hot water from a process plant and cool that water to a specified temperature so that it can be returned (after replacing evaporative losses) to the plant for reuse. In such a case, the following parameters will be known: water flow rate, fan volume flow rate, performance line for the tower, wet-bulb temperature and barometric pressure of the ambient air, temperature of makeup water available, temperature difference between water at plant intake and plant discharge, and return water temperature desired. The tower may be capable of exceeding the requirements or its capacity may be inadequate. If excess capacity is available, a portion of the hot water from the plant bypasses the tower and is mixed with the cooled portion of the flow to provide the desired return temperature. The objective of the calculation is to determine the amount of water bypassed, as well as mass flow of air required and rate of water loss due to evaporation. If the tower performance does not meet the plant cooling requirements, the objective is to determine what conditions can be met. As in the previous case, air mass flow rates and evaporative losses are interesting by-products of the calculation.

The potential user of the model presented in this paper should be aware of the limitations of the model. The model is keyed to analysis of existing towers, or to forecast requirements of cooling tower cells of established performance. It is not intended to be able to synthesize a tower design from elemental fill data. Scaling of tower size should be done by adding or subtracting integer numbers of identical tower cells. Although it might be possible to scale height or plan area for identical fill geometry, no experience can be reported here for such calculations. It should also be noted that this model is not intended for use with natural draft towers and that all fans on the forced draft installations being studied must be running.

DERIVATION OF MODEL EQUATIONS

The description of the cooling tower is split into overall (macroscopic) balances of mass and energy and detailed (microscopic) balances of small elements. The microscopic equations which are derived below merely express the mass and energy balances on air and water across the increment and the rate-of-transfer of mass between the water and air. Heat transfer, in all cases, has been assumed to be adequate to maintain the air at the local saturation temperature.

Consider a small increment of tower volume (Figure 2a). Water enters the top and falls through the increment. The water may be thought to be falling as a sheet or film, or as drops of various sizes. In any case, a portion of the water evaporates. Air enters the volume increment from below and exits at the top, carrying the vaporized portion of the water and some droplets with it. The heat required to vaporize the water lost to the air stream is drawn from the unvaporized water, cooling it. This process is repeated through a number of increments, so that the water arriving at the bottom of the tower is cooled appreciably at the expense of some evaporative loss.

The heat lost by the water is equated to that gained by the air, giving

$$C_p(m_1 t_1 - m_2 t_2) = G(h_1 - h_2) + 32(m_1 - m_2) \quad .^* \tag{1}$$

m_1 and m_2 are the liquid flow rates entering and leaving the volume, G is mass flow rate of dry air and h_1 and h_2 are enthalpies for saturated air, per unit mass of dry air.

The mass lost by the water is added to the air stream and is expressed as a change in specific humidity (H) of the saturated air:

$$m_1 - m_2 = G(H_1 - H_2) \quad . \tag{3}$$

Equations 1 and 3 express universal physical laws; they apply regardless of the amount of water evaporated. The amount of mass evaporated is expressed in terms of a mass transfer coefficient and driving potential:

$$m_1 - m_2 = \left(\frac{m_1 + m_2}{2}\right) \frac{K'a}{L} \delta V(H'' - H) \quad . \tag{4}$$

H is the specific humidity of the air locally in the volume and H" is the specific humidity of air evaluated at the local water temperature.

Equation 4 is strictly an empirical rate equation. Most often the product K'a would be taken as constant for a tower operating at a particular L/G (water flow to air flow) ratio.

The local mass flow rate $(m_1 + m_2)/2$ is a qualitative measure of variation in shear forces on the drop which should affect the mass transfer rate. Since the coefficient K'a will normally be extracted from experimental data taken over a limited range of air and water flows, this empirical coefficient should not be extrapolated over great variations.

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*The enthalpy of liquid water, $h_L(t)$, is normally calculated using a constant specific heat (with a value of unity) and a zero value at 32°F,

$$h_L(t) = (1)(t - 32) \tag{2}$$

The volume increment is just the total volume of fill divided by the number of increments,

$$\delta V = V/N \quad . \tag{5}$$

The rate equation (4) then takes a familiar form:

$$m_1 - m_2 = \frac{C_p}{N} \left(\frac{m_1 + m_2}{2}\right) \left(\frac{K'aV}{C_p L}\right) (H'' - H) \quad . \tag{4a}$$

These equations (1), (3), and (4a) describe the conservation of mass, energy and the evaporation rate across a small increment of fill as shown in Figure 2a. If the air flow rate G in Eq. (1) is replaced by C, the air flow rate per vertical increment,

$$C = G/N \quad ,$$

and the locations of the variables on the increments are shifted (Figure 2b), the same balance equations are used for an increment of fill in a crossflow tower.

The selection of the humidity potential $H'' - H$ as a driving force for mass transfer rather than the air enthalpy potential $h'' - h$ as a driving force for heat transfer was, to some extent, arbitrary. Aside from personal taste, one advantage and one disadvantage accrue from this choice of rate mechanisms. As an advantage, the Lewis relation (ratio of overall mass transfer to overall heat transfer equals unity) is not required to hold. This compares to the classic model of Baker and Shryock [1] in which the Lewis relation was required to allow an essential algebraic manipulation. As a disadvantage, a difficulty is encountered in integrating Eq. (4a). While the enthalpy of air is nearly constant at fixed dry bulb temperature regardless of the wet bulb temperature, the humidity for fixed dry bulb temperature increases markedly as the wet bulb temperature is increased. Thus, the present model depends on the humidity of the inlet air (i.e, both wet and dry bulb) while the results from the enthalpy-driven models depend only on the wet bulb temperature. Development of an appropriate inlet humidity specification is presently under way.

Solution of Microscopic Balance Equations

The microscopic balance equations (1,3,4a) describe the changes in temperature, enthalpy, humidity and mass flow across an increment of cooling tower volume. The linking of these equations into a model for a crossflow and counterflow tower configuration is described in this section.

A schematic of the increment structure for a crossflow tower is shown in Figure 3. A sweep of the increments starts at the top left, where a hot water temperature and air inlet temperature are known (or assumed). Solving Eqs. (1,3,4a) for the first increment advances the air temperature one increment to the left and the water temperature one increment downward.

By repeating this process for each increment in turn, the water temperature distribution at the bottom and the air temperature distribution into the center of the tower are calculated. The water temperature off the tower is a mass-weighted average of the temperature from each bottom increment. The air temperature is taken as saturated at the average enthalpy.

To solve the equations for one increment, an iterative approach is used. Outlet conditions are assumed, the driving potential is estimated at the center of the increment, and the estimates are corrected. This process is repeated until the correction is arbitrarily small. Details are presented in [2]. Applying the driving potential at the center of the increment is the key to the second-order accuracy of the calculation.

The counterflow tower is modeled as a single vertical column of increments. The scheme is to start at the top and solve each increment iteratively, working to the bottom of the tower. As with the crossflow tower, the driving potential is evaluated at the center of the increment to give second-order accuracy.

Solution of the Macroscopic System

The macroscopic equations are derived from each specific application of the model. For example, reduction of a set of test results to a mass transfer coefficient requires the matching of measured temperatures to values calculated using an assumed mass transfer value. In this problem, the macroscopic equations are of the form,

"Temperature of Cold Water = Measured Value."

With the measured parameters specified in that way, the value of the mass transfer coefficient can be systematically (and automatically) varied to match the data with the calculated values. Another useful form of the macroscopic equations balances a process heat rejection load with the heat rejected by a connected tower.

In both cases (and others), the macroscopic equations are used to set up a multivariable Newton's method, generating corrections for assumed values of the variables linking the macroscopic equations to the tower models. Details are available in [2].

DISCUSSION OF SOME SAMPLE CALCULATIONS

In this section, the results of several typical calculations are examined.

Several sets of test data and a design point for the same tower are given in Table 1. Performance coefficients generated using the model are also tabulated. The data have been reduced using both the crossflow and counterflow algorithms. The values from the crossflow model are higher than the corresponding points from the counterflow routine. This is expected; to do the same cooling job, the lower thermal efficiency of the crossflow layout would require more active volume (V greater), more effective mass transfer (K larger), or more fill (area or *a* greater). The performance coefficients for the crossflow analysis are shown on Figure 4.

The operating profile of air and water temperatures and local evaporation rates is shown in Figure 5 for a counterflow tower operating at the design point listed in Table 1. At the top of the tower, the evaporation rate is highest, but drops rapidly as the water falls through the tower and cools. The water is cooled more rapidly (in terms of temperature reduction per increment) in the upper sections of the tower. The air and water temperature curves approach a limiting value (the wet bulb temperature of the inlet air) near the bottom of the tower.

Figure 6 illustrates the convergence of the solution for the incremental (difference) equations to the limiting case of an infinite number of increments. A single set of data has been reduced using the counterflow routine with 6, 10, 20, and 80 increments in the integration routine. When the performance coefficient, K'aV/L, is plotted against the number of increments N, the *kneed* curve results which shows a rapid change followed at large N by asymptotic approach to a constant value. If K'aV/L is plotted against $1/N^2$, the data are covered by a straight line. This linearity indicates that the algorithm is *second-order accurate*; that is, the solution to the discrete balance equations approaches the differential limit (infinite number of increments) as the square of the number of increments. From Figure 6, one can see that the solution of $1/N^2 = 0$ ($N^2 \rightarrow \infty$) is virtually indistinguishable from the solution for N = 80. From an economic viewpoint, the N = 20 solution varies about 0.05% from the N = 80 solution while the computing time varies by about a factor of 4:

$$T_{80}/T_{20} \approx \frac{80}{20} = 4 \quad .$$

On this basis, it is recommended that 20-25 increments be used with the present algorithm for analyzing counterflow cooling towers. For the crossflow algorithm, analysis shows that

$$T_{80}/T_{20} \approx \left(\frac{80}{20}\right)^2 = 16 \quad .$$

Figure 7 is a summary of the calculated results for a particular crossflow cooling tower operated at a particular set of conditions. Detailed profiles of water temperature, air temperature and local evaporation rate for this tower are shown in Figures 8 through 10.

Figure 8 shows the local water temperatures calculated across several horizontal sections in the tower. As expected, the largest portion of the temperature drop occurs in the upper sections of the tower. The top 5% of the fill, for example, accounts for about 12% of the cooling. The bottom 40% of the fill accounts for about 20% of the cooling. This is due to the higher evaporation rates from the hotter water to the larger driving force at the upper left corner of the tower. Below the uppermost sections, the temperature drop across each section is roughly constant from left to right (the temperature curves appear to be roughly parallel). A similar pattern appears in Figure 10 for the local evaporation rate.

In Figure 9, the local saturation temperature for the air is plotted. The air entering nearest the top is exposed to the hottest water and thus gains the most heat. Air in lower horizontal sections is exposed to cooler water and thus gains less energy.

In Figure 10, the local evaporation rates are plotted. The evaporation rates are highest in the upper left corner of the tower where the hottest water meets the coldest air. In the lower portion of the tower, the rates are roughly constant across any horizontal section. This is related to the nearly uniform changes in water temperature seen in Figure 8. The shape of the curve labeled *15* (15% of the fill height below the top of the fill) requires some discussion. The heat transferred from water to air in any section is divided between sensible heat transfer (convection) and latent heat transfer (evaporation). As hotter water is encountered by hotter air, the balance between sensible heat transfer and latent heat transfer shifts. Transfer coefficients change only slightly, but the humidity difference driving the mass transfer (evaporation) increases more than the temperature difference driving the sensible heat transfer. Thus, more heat is transferred by evaporating the water. Since the latent heat of water drops as the temperature increases, the amount of water evaporated to absorb the heat increases. As the air proceeds further to the right in the tower, both driving forces decrease and the total evaporation decreases.

To summarize, the basic physical laws of mass and energy conservation are satisfied to the accuracy of the iteration convergence. The algorithms used for the counterflow and crossflow models are *second-order* accurate and a choice of 20-25 integration increments gives adequate accuracy without inordinate use of expensive computing time. Finally, detailed plots of air temperature, water temperature, and local evaporation rate calculated throughout typical crossflow and counterflow towers are in agreement with physical intuition. The foregoing should aid in establishing confidence in the validity of the physical model and the computer program which evaluates the model.

CONCLUSIONS

In this report, the main features of the mathematical model of the physical transport processes occurring in an induced draft evaporative cooling tower are summarized. Studies of sample problems are presented.

The overall conservation laws for mass and energy are satisfied rigorously, regardless of the number of increments used for integration; this approach allows careful checks on the computer program. The evaporative mass loss is calculated using a modification of the usual rate equation based on the specific humidity difference between water and air as a driving force. Heat transfer rates, if calculated, would be just sufficient to keep the air saturated with moisture. The usual additional assumption that the Lewis relation is unity [1] is not made and, in fact, is redundant. Mass and energy losses by the water are included in the balance equations. Evaporation losses have been calculated (or estimated) in some earlier work [1,3] using the approximate numerical integration of approximate differential equations. The exact (numerical) solution of the incremental balance equations is preferable.

The numerical procedure for solving the model equations is outlined. Computational results indicate that the algorithm is second order in accuracy, allowing as few as 20 increments to be used to integrate the balance equations.

Successful use of these models to optimize the thermal control system for proposed new gaseous diffusion plants by adding or subtracting tower cells of known size and performance has not been detailed here. The success of those applications indicates that the present model has utility in designing optimum heat rejection systems for power stations or chemical process facilities.

REFERENCES

1. Baker, Donald R. and Howard A. Shryock, "A Comprehensive Approach to the Analysis of Cooling Tower Performance," Journal of Heat Transfer, August, 1961, pp. 339-350.

2. Cross, K. E., J. E. Park, J. M. Vance and N. H. Van Wie, Theory and Application of Engineering Models for Cross-Flow and Counterflow Induced Draft Cooling Towers, Union Carbide Corporation, Nuclear Division, Oak Ridge, Tennessee, Report K/CSD-1, May 1976 (available from the National Technical Information Service, U. S. Department of Commerce, 5285 Port Royal Road, Springfield, Virginia 22161, Price: Printed Copy $5.00; Microfiche $2.25).

3. Yadigaroghi, G. and E. J. Pastor, "An Investigation of the Accuracy of the Merkel Equation for Evaporative Cooling Tower Calculations," ASME Paper 74-HT-59, July, 1974.

Table 1

Typical Test Results for a Cooling Tower
With Performance Coefficient Calculated Using
Both Crossflow (XF) and Counterflow (CF) Algorithms

	1	2	3	4	Design
Water Flow, 10^6 gal/day	4.22	7.93	6.62	8.89	10.29
Air Flow, 10^6 cfm	0.85	0.85	0.85	0.85	0.5
Water On, °F	125.0	129.8	126.5	135.7	132.3
Water Off, °F	73.3	84.5	80.5	89.9	90.0
Air Wet Bulb, °F	61.8	65.5	62.4	71.1	80.0
Barometer, psia	14.64	14.67	14.62	14.63	14.696
L/G	0.423	0.843	0.687	0.980	1.172
K'aV/L, CF	2.011	1.341	1.440	1.203	1.869
K'aV/L, XF	2.476	1.712	1.789	1.554	2.933

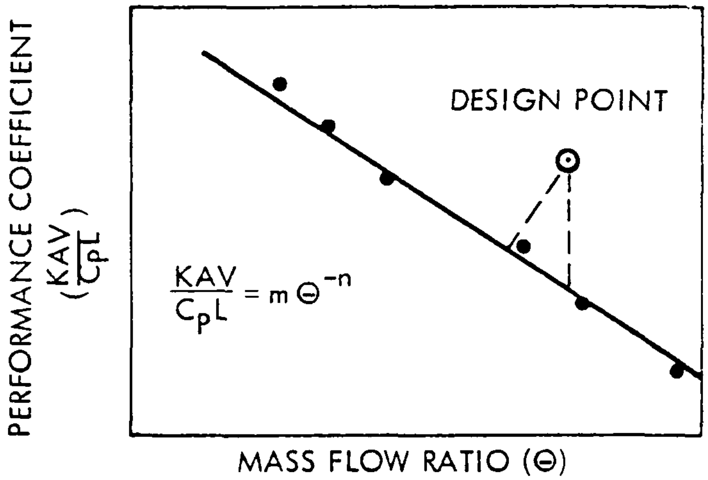

Figure 1. COMPARISON OF DESIGN POINT TO TEST DATA

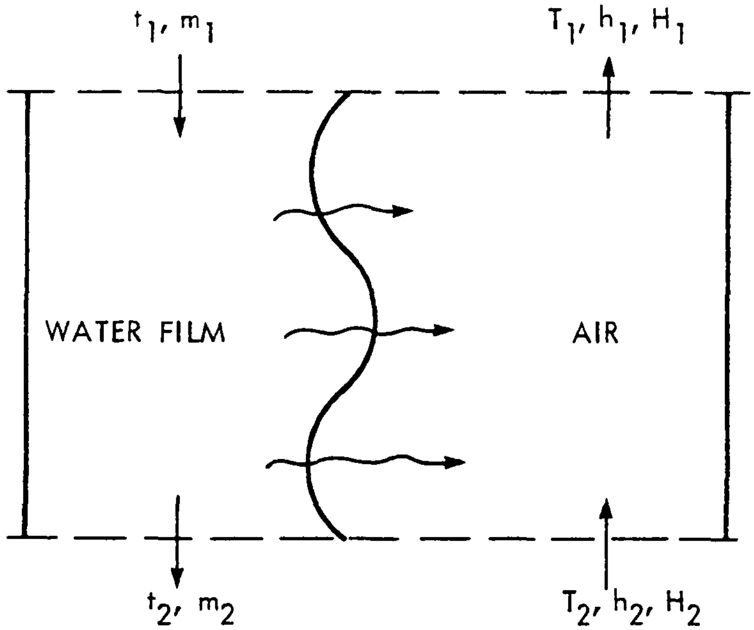

Figure 2a INCREMENT OF THE FILL VOLUME OF A COUNTERFLOW COOLING TOWER

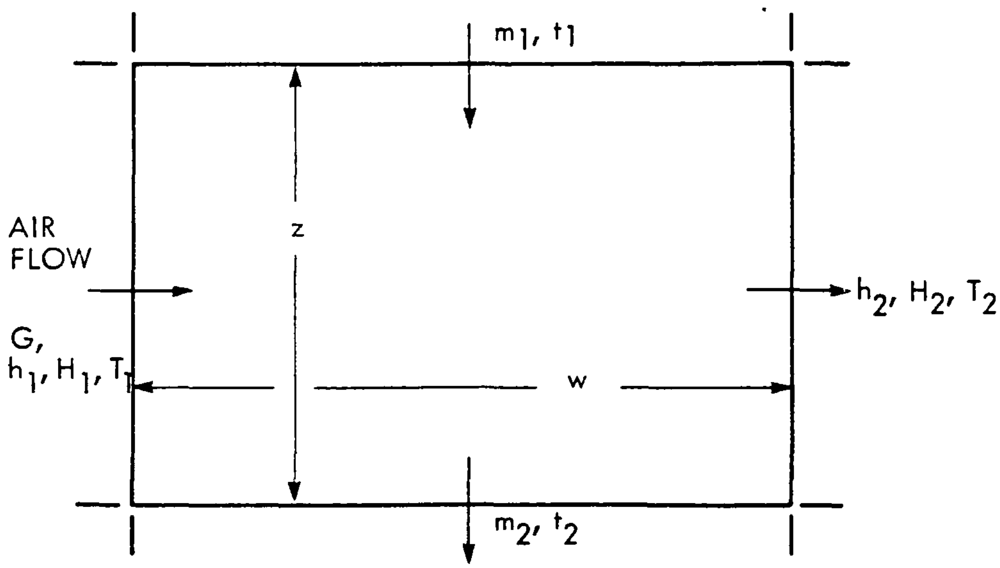

Figure 2b. INCREMENTAL VOLUME OF FILL IN CROSS-FLOW COOLING TOWER.

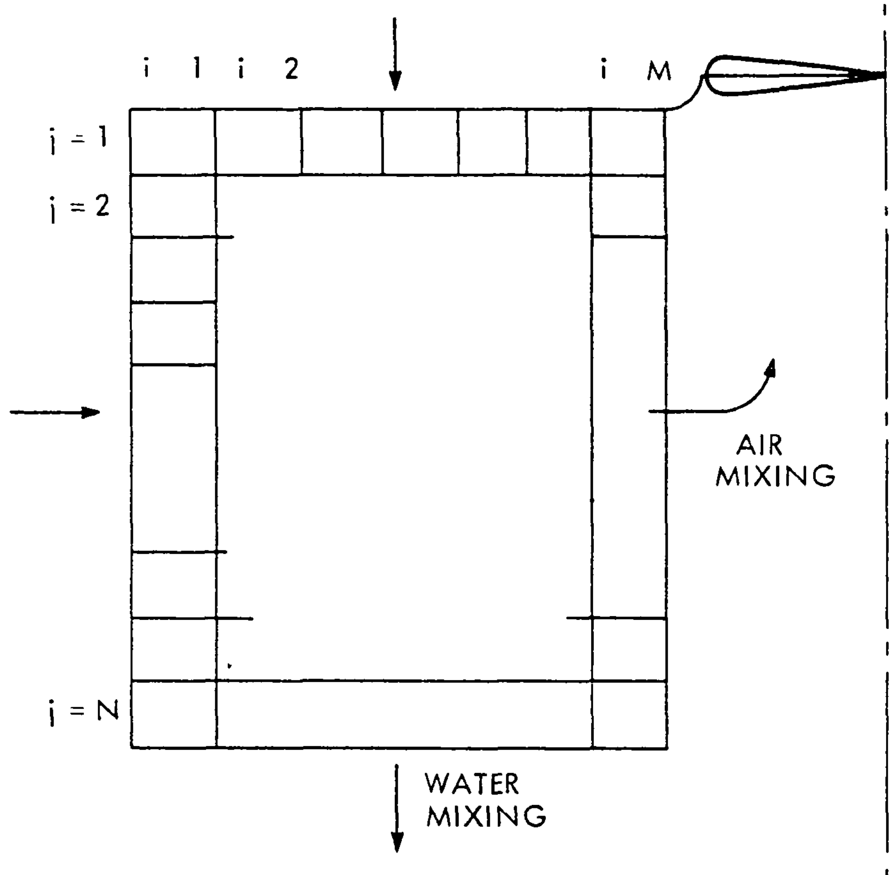

Figure 3 DIVISION OF CROSS-FLOW TOWER INTO INCREMENTAL VOLUMES SHOWING SUBSCRIPTING CONVENTION.

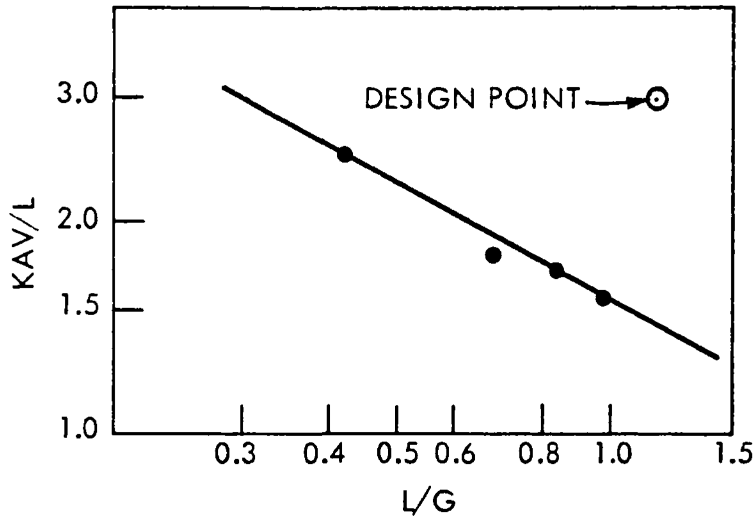

Figure 4 TEST RESULTS FOR CROSS-FLOW COOLING TOWER.

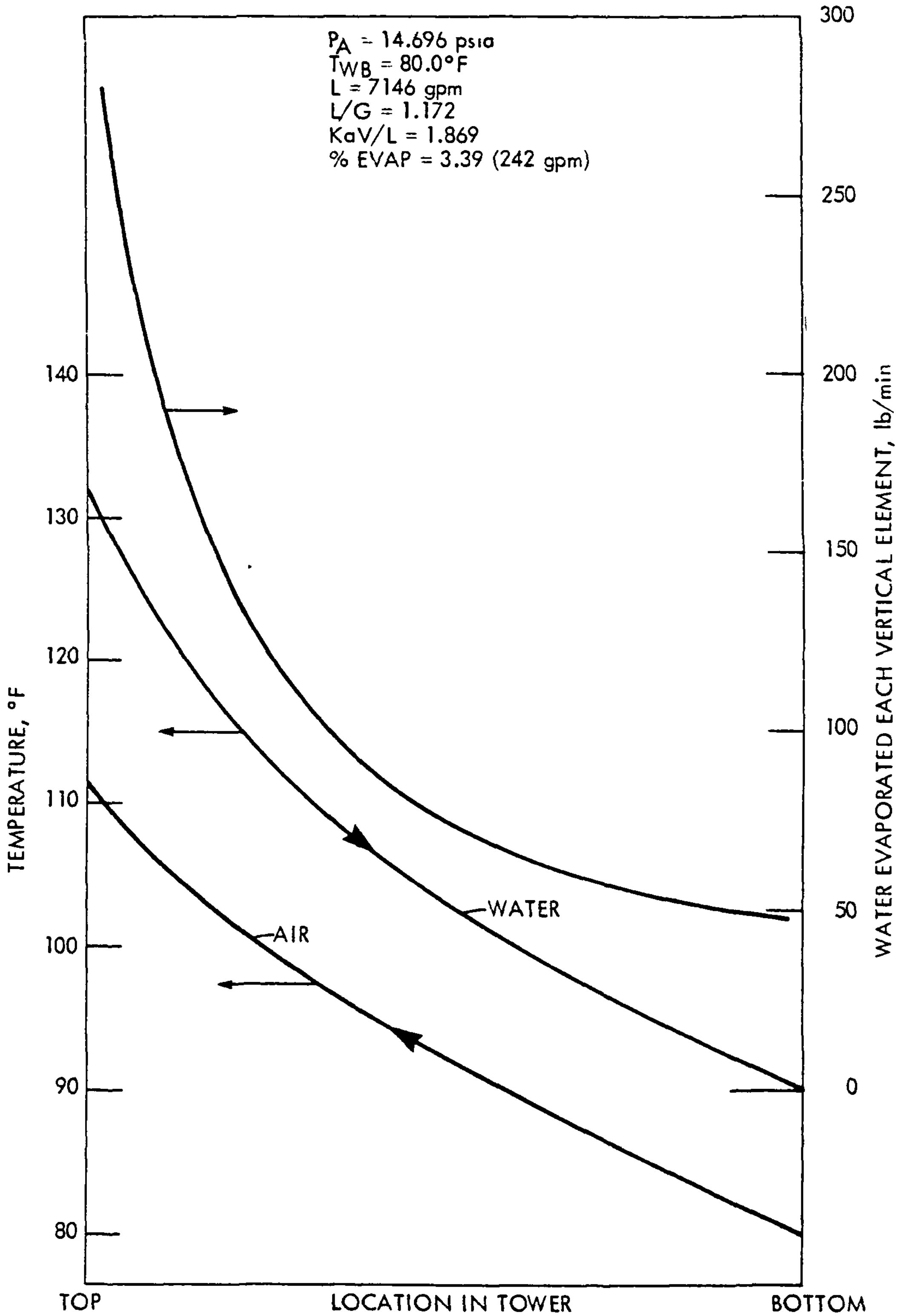

Figure 5. COUNTERFLOW TOWER PROFILES

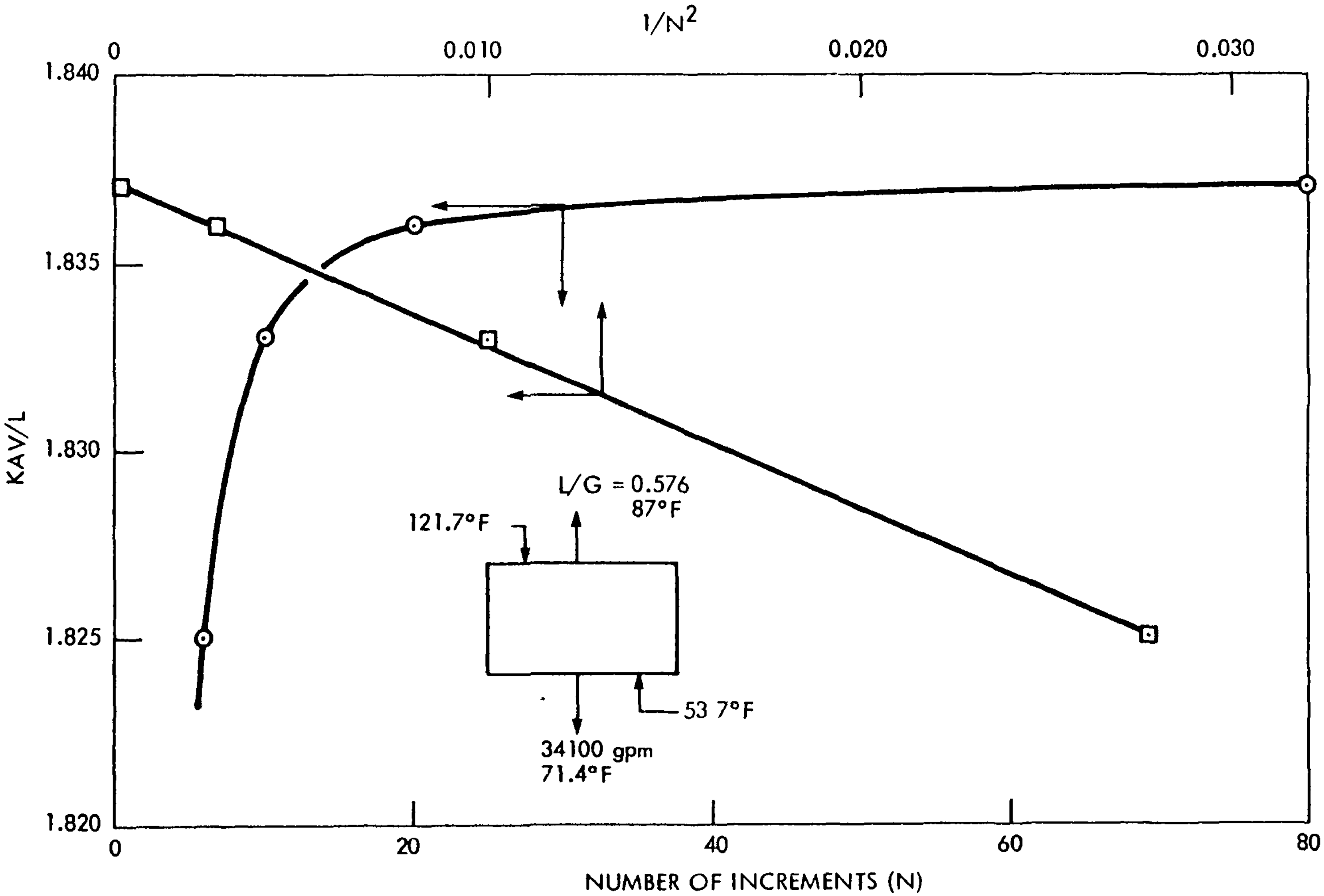

Figure 6. SENSITIVITY OF PERFORMANCE COEFFICIENT TO NUMBER OF INCREMENTS-COUNTERFLOW TOWER

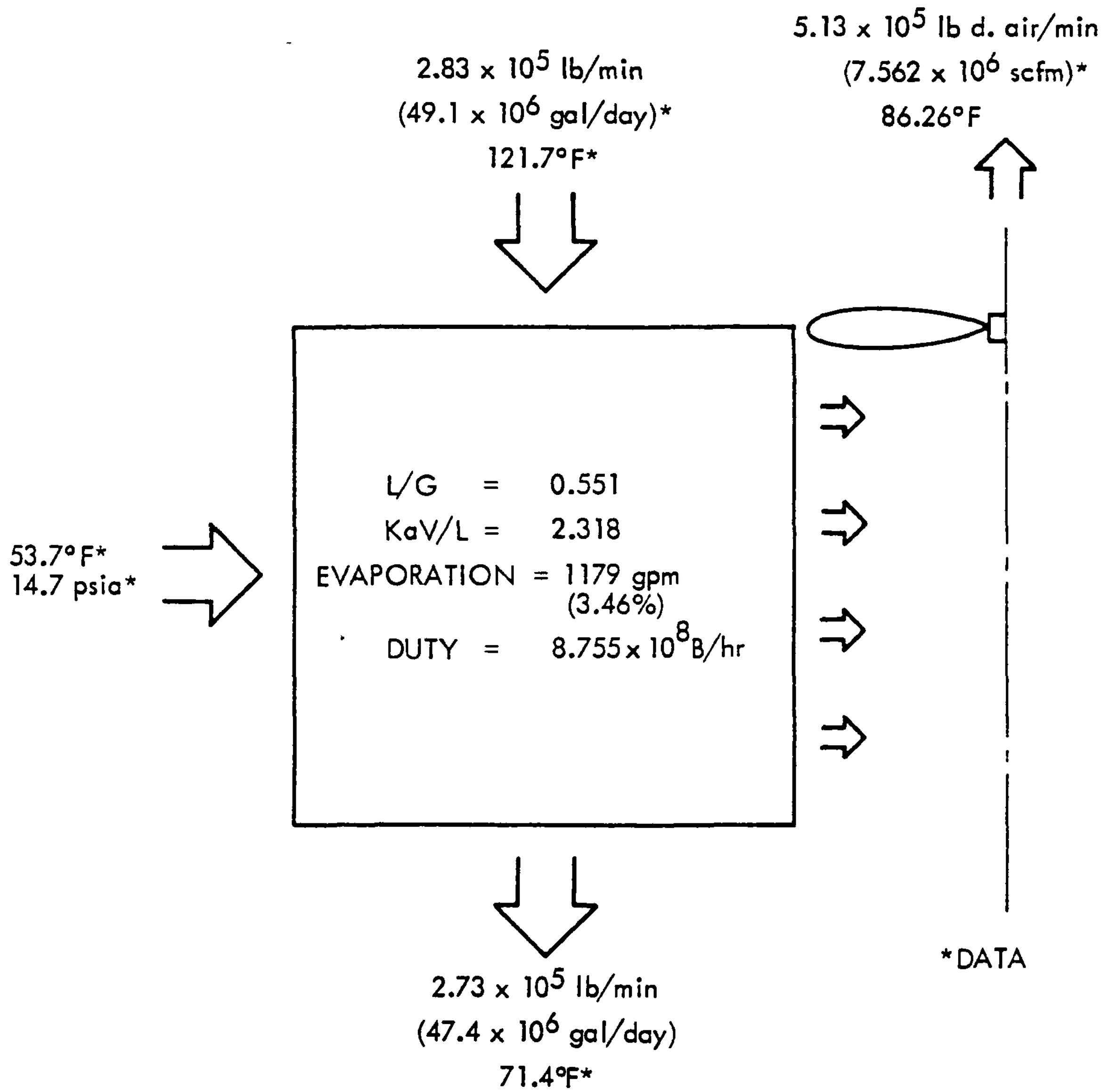

Figure 7. DATA AND CALCULATED INFORMATION FOR A TYPICAL CROSS-FLOW COOLING TOWER.

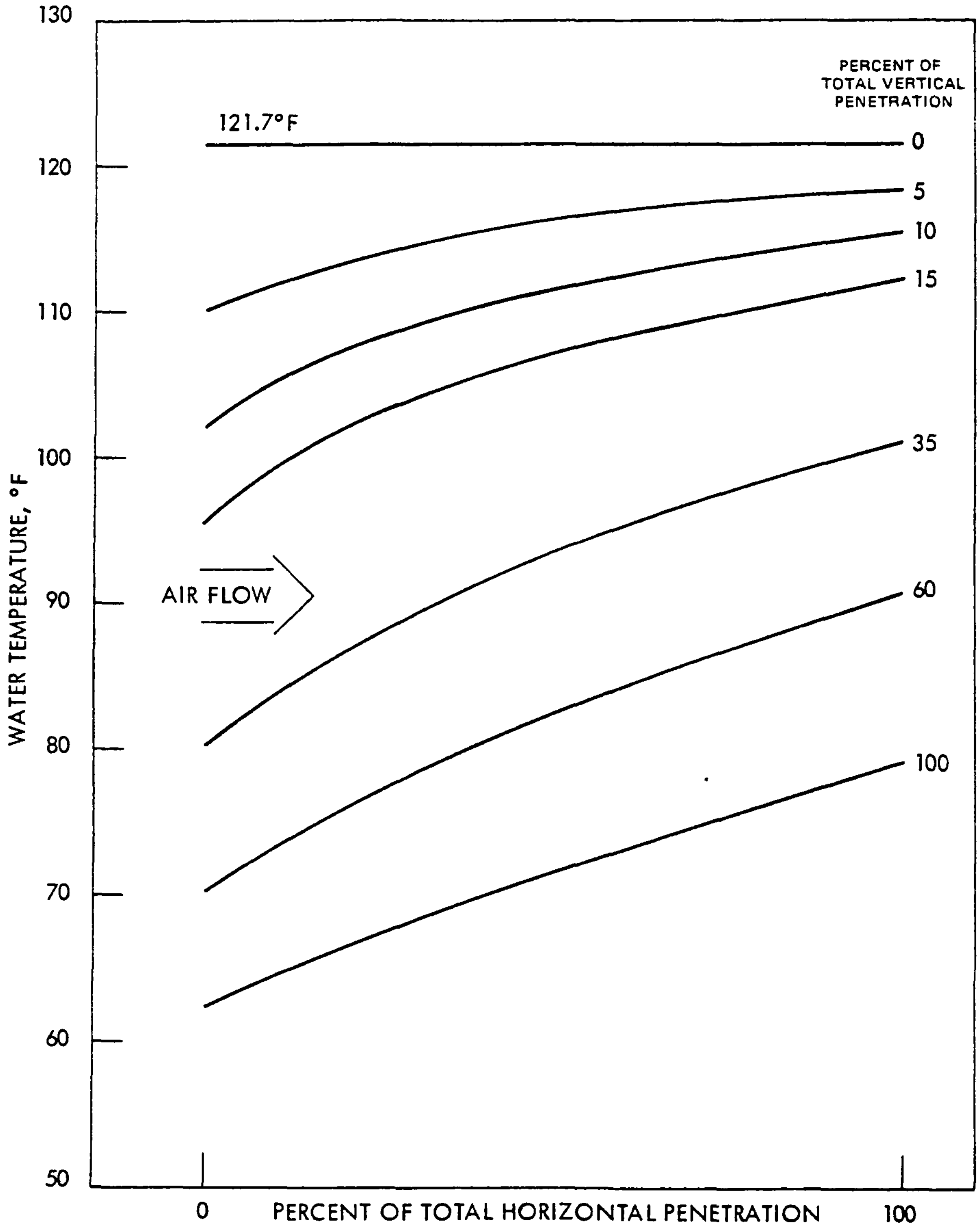

Figure 8. WATER TEMPERATURE DISTRIBUTION IN A CROSS-FLOW COOLING TOWER AT SEVERAL ELEVATIONS.

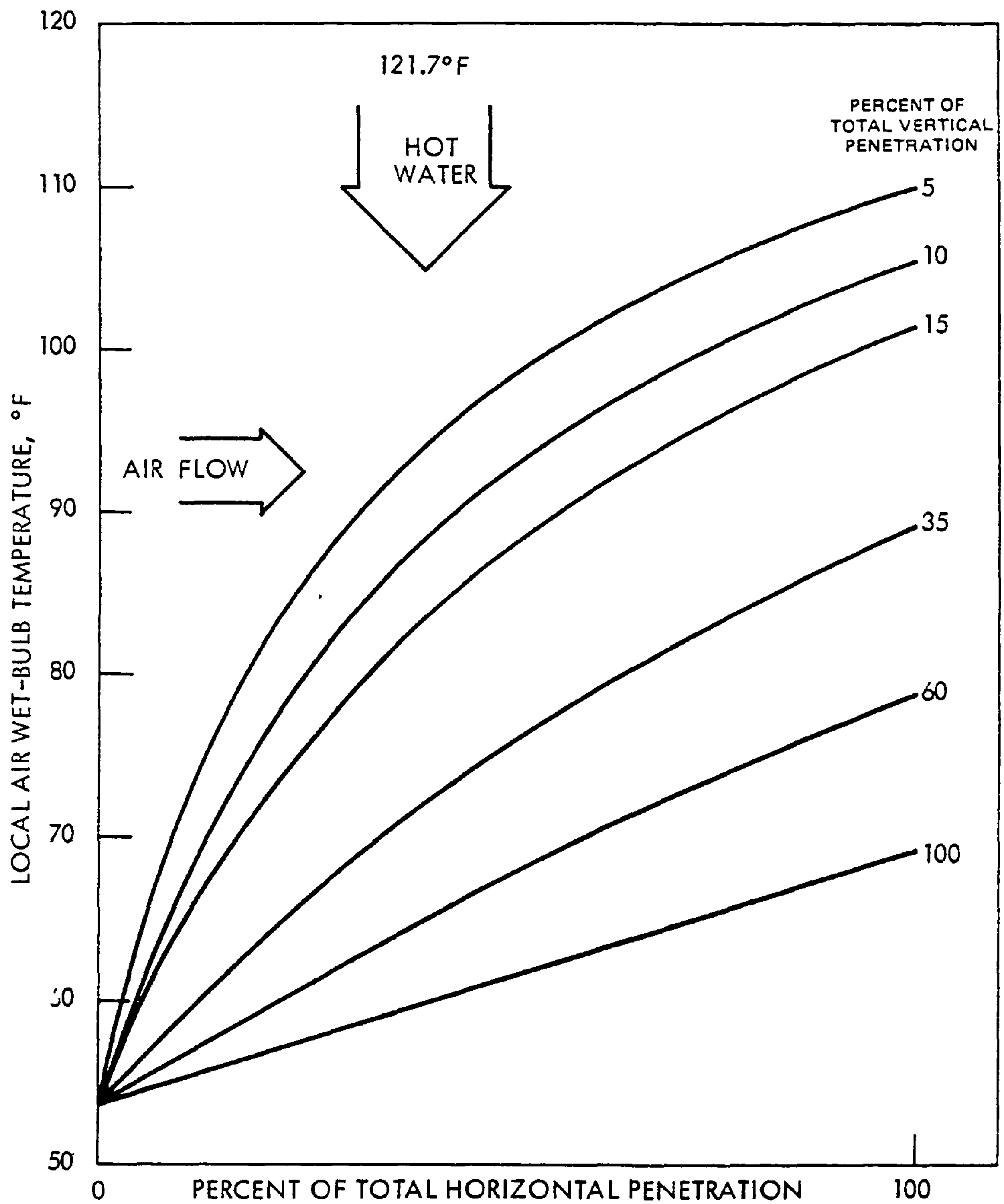

Figure 9. AIR WET-BULB TEMPERATURE DISTRIBUTION IN A CROSS-FLOW COOLING TOWER AT SEVERAL ELEVATIONS.

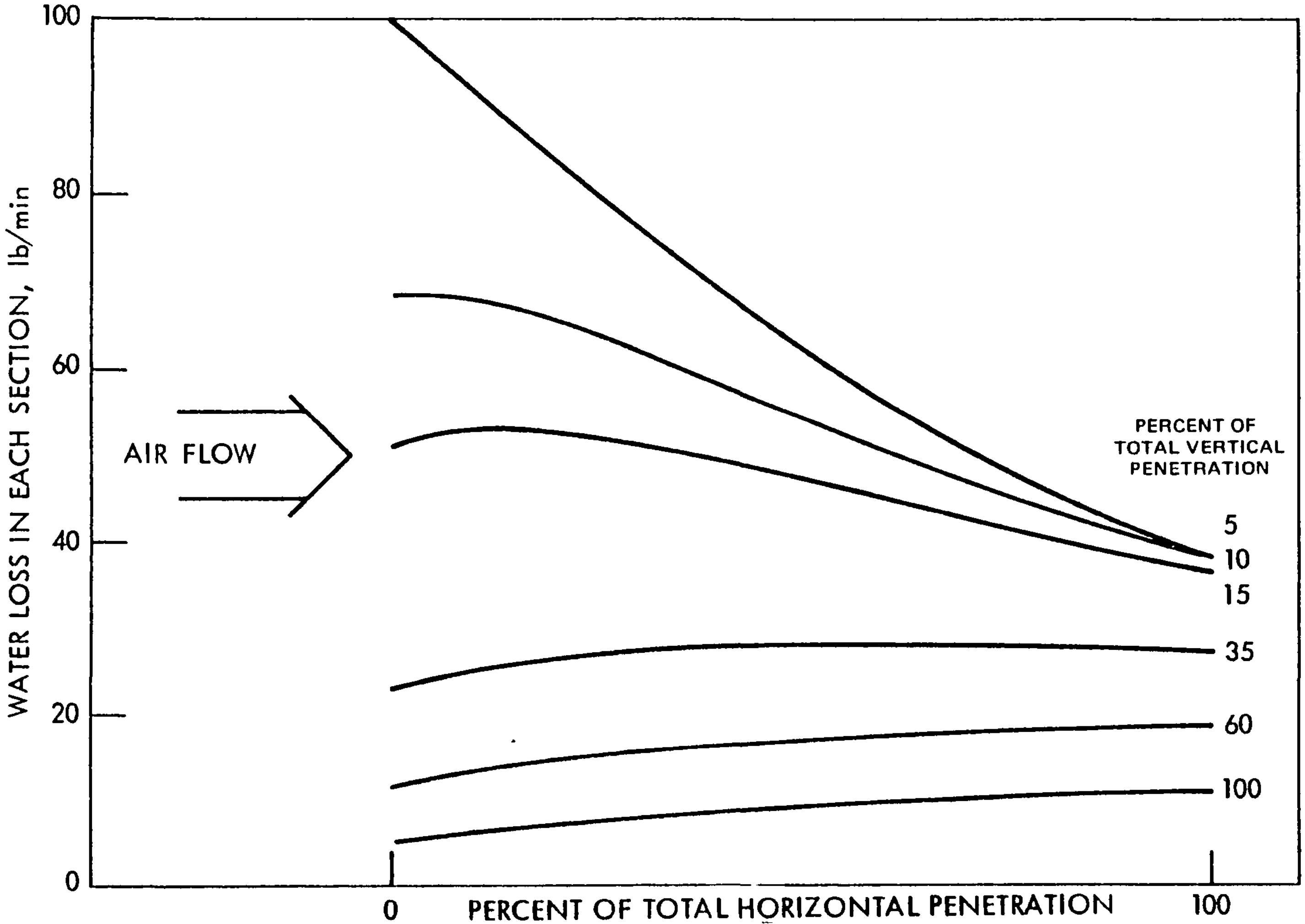

Figure 10 PROFILES OF WATER EVAPORATION RATE AT SEVERAL ELEVATIONS IN A CROSS-FLOW TOWER.