

25-45-13

VI-C-137

IMPORTANT CONSIDERATIONS IN A  
SIMPLE NUMERICAL PLUME MODEL

L.D. Winiarski  
Corvallis Environmental Research Laboratory  
Environmental Protection Agency  
Corvallis, Oregon U.S.A.

ABSTRACT

A simple, inexpensive, numerical plume model based on conservation laws applied to average properties of a puff of plume material can be used to predict the trajectory, width and dilution of plumes, provided that appropriate consideration is given to the average effect of large scale vortex action on both the plume and the surroundings. Consideration of the mechanics of formation of the vortices, as well as momentum balances on a control volume consisting of the total bent over plume provides quantitative limits for these effects. The model predictions show good agreement with a variety of laboratory and field data.

INTRODUCTION

The prediction of the trajectory, dilution and spread of a buoyant plume or jet is an important problem in many diverse fields. The subject might be the discharge of a smoke stack or cooling tower into the atmosphere, the discharge of a submerged jet into a river or ocean or the injection of fuel into a combustion chamber.

IMPORTANT CONSIDERATIONS

It is possible (in principle at least), given enough computer time and core storage, to input this problem in a general purpose, three-dimensional, fluid dynamics computer code where the numerical grid extends to cover the region of the ambient fluid likely to be influenced by the plume. By using a large number of relatively closely spaced grid points one could minimize the uncertainty of some assumptions. Unfortunately this is too expensive.

Resolution and Diffusion

It is not practical to use a grid scale small enough to eliminate the uncertainty. The mathematical description is always uncertain because the distinction between flow defined by a definite velocity vector and flow implied by a diffusion representation is really a mathematical artifice related to the scale of the phenomenae involved relative to the resolution of the calculation system (e.g. the size of the numerical calculation system).

Vortex Motion

Transport by small vortex motion is often accounted for by a diffusion term. However, a significant feature of plume motion is vortex circulations which are of the same scale as the width of the plume. This motion is responsible for most of the entrainment into the plume, therefore it deserves special consideration. To attempt to model the vortex flow in detail would be prohibitively expensive. However, the overall effect of these flows must be included in simple models in a manner that is as consistent as possible with basic conservation principles.

Pressure forces

It is important that model take into account the fact that the plume disturbs the surroundings. The plume experiences a drag due to the fact that it must accelerate not only the mass that it absorbs into the plume but also a portion of the surroundings. The energy for these circulations that are set up in the surroundings must come from the plume. In a simple model it is necessary to make some assumptions as to the magnitude and distribution of the external forces acting on the plume. A check on the overall consistency of these assumptions can be made by considering the integral of these forces on a control volume encompassing the entire bent over plume. The summation of the net vertical component of external forces should equal the initial vertical flux of momentum. The total change of horizontal momentum flux of the initial mass flow should equal the summation of the net horizontal forces on the bent-over plume control volume. These ideas have been important in the development of a successful, inexpensive plume model.

## CALCULATION PROCEDURE

Start by defining an initial mass or puff of material ( $M_0$ ) with density ( $\rho_0$ ) issuing out of a stack of radius ( $b_0$ ) with an initial velocity ( $V_0$ ) in a time instant ( $\Delta t$ ).

$$M = \rho_0 \pi b_0^2 V_0 \Delta t$$

The initial length (H) of the element is:

$$H = V_0 \Delta t$$

Calculate the amount of mass ( $\Delta M$ ) entrained into this puff in the time instant  $\Delta t$ . Calculate the horizontal ( $F_h$ ) and vertical components ( $F_v$ ) of the pressure force acting on the plume. The mass entrained and this pressure force are crucial items to plume prediction. The logic for their calculation will be explained later.

The new mass of the puff is simply the old mass plus the amount entrained during the time instant.

$$M^{t+\Delta t} = M^t + \Delta M$$

The change in horizontal momentum ( $Mu$ ) of the plume equals the horizontal momentum added by the entrained mass plus the impulse added by the horizontal pressure

$$\Delta(Mu) = W\Delta M + F_h \Delta t$$

Note that the horizontal momentum added by the entrained mass comes about by virtue of the fact that this mass comes from the ambient fluid which has a horizontal wind velocity ( $W$ ).

The new horizontal momentum of the parcel is simply the old horizontal momentum plus the change in horizontal momentum

$$(Mu)^{t+\Delta t} = (Mu)^t + \Delta Mu$$

The new horizontal velocity  $u$ , of the parcel is:

$$u^{t+\Delta t} = \frac{(Mu)^{t+\Delta t}}{M^{t+\Delta t}}$$

The vertical velocity can be solved in a similar manner. First, consider the case of a neutrally buoyant discharge. The change in vertical momentum of the parcel is due to the vertical component of the pressure force provided the undisturbed ambient has no vertical velocity.

$$\Delta(Mv) = F_v \Delta T$$

The new vertical momentum of the parcel is

$$(Mv)^{t+\Delta t} = Mv^t + \Delta Mv$$

The new vertical velocity is

$$v^{t+\Delta t} = \frac{(Mv)^{t+\Delta t}}{M^{t+\Delta t}}$$

The effect of buoyancy is actually a modification of the pressure field and hence  $F_v$ ; however, it can be approximated by a modification of Archimedes principle to compute the acceleration ( $a$ ) due to buoyant force ( $F_b$ ).

$$a = \frac{F_b}{M_{total}} = \frac{\text{displaced fluid weight} - \text{parcel weight}}{\text{total mass moved}}$$

Expressed in terms of the density ( $\rho$ ), the gravitational acceleration ( $g$ ), and the volume ( $vol$ ) of the parcel, this is:

$$a = \frac{(\rho_{amb} - \rho) (vol) g}{(\rho + \rho_{amb}) (vol)}$$

The buoyancy correction to the vertical velocity then is

$$\Delta v = a \Delta t = \frac{(\rho_{amb} - \rho) g \Delta t}{2\rho}$$

$$v^{t+\Delta t} = v^t + \Delta v$$

This approximation for the buoyancy computation which accounts for the displaced ambient mass or virtual mass is not well known; however, evidence for this has been documented (Ref. 3).

The buoyancy calculation depends on updating the density of the parcel at each time step. This can be done from conservation considerations. In the simplest case, the temperature ( $T$ ) is

$$T^{t+\Delta t} = \frac{M^t T + (\Delta M) T_{amb}}{M^{t+\Delta t}}$$

The specific concentration of other parameters (e.g. moisture) can be found in the same way.

Use ambient pressure ( $P$ ) in an equation of state to find the density. For example, the equation for air is:

$$\rho^{t+\Delta t} = \frac{P}{RT^{t+\Delta t}}$$

Refinements to the temperature and density calculations can be made to account for the moisture effects.

The horizontal (X) and vertical (Y) displacements are simply:

$$x^{t+\Delta t} = x^t + \frac{u^t + u^{t+\Delta t}}{2} \Delta t$$

$$y^{t+\Delta t} = y^t + \frac{v^t + v^{t+\Delta t}}{2} \Delta t$$

The total velocity (V) is:

$$V = \sqrt{u^2 + v^2}$$

The length (H) of the parcel along the trajectory changes slightly because the parcel ahead is moving at a velocity less than the parcel behind. This can lead to a change in length:

$$\Delta H = \frac{\partial V}{\partial s} H \Delta t = \frac{\partial V}{\partial t} \frac{\partial t}{\partial s} H \Delta t$$

$$H^{t+\Delta t} = \Delta H + H^t = \frac{H^t (V^t - V^{t-\Delta t})}{\sqrt{(\Delta x)^2 + (\Delta y)^2}} \Delta t + H^t$$

An average radius for the puff can be found from:

$$\rho \pi b^2 H = M$$

$$b = \sqrt{\frac{M}{\rho \pi H}}$$

Time can be updated and the process repeated:

$$t = t + \Delta t$$

Although this integration system is very simple, it is comparable to more elegant integration systems and has been used to illustrate a published inconsistency in the use of such a system (Ref. 2).

With the basic integration system this simple, attention can be focused on the entrainment and force computation. The problem is to compute these terms in the most physically consistent manner. This is not an easy task. Some insight can be obtained by close observation of an individual puff of material issuing from an orifice.

As the puff emerges from the orifice the ambient fluid is displaced and largely forced to move around to fill the void behind. Relative to the parcel the ambient fluid has an impinging velocity. As the ambient fluid slides around the parcel the shearing action sets up a toroidal vortex which entwines ambient fluid into its center. Initially, the vortex is approximately spherical but can become more like the familiar smoke ring.

Laboratory experiments have shown that the initial growth of the radius with respect to distance is about 1 to 4 (Ref. 4).

This is precisely the rate of growth that would be calculated by assuming that all of the material swept through by a spherical parcel is entrained in the parcel. This implies that the relative impingement velocity and the projected area are the significant controlling parameters. If the frequency of emitted puffs is increased, a continuous plume is developed. However, close inspection of the plume will still show the vestiges of the individual vortices on the edges of the plume. In this case, the vortices can only entrain ambient fluid on the outside edges. The slippage or difference between the plume parcel velocity and that component of the ambient velocity parallel to the plume results in a shearing action which keeps the rolling vortices in motion. The removal of some of the ambient by this shearing entrainment results in an apparent suction or aspiration of the ambient into the plume. This entrainment can be specified by the product of the lateral area of the plume segment, the previously mentioned slippage velocity, and an aspiration or entrainment coefficient ( $\alpha$ ).

$$\text{Aspiration} = \alpha \rho 2b\pi H |V - W \cos\theta|$$

The order of magnitude of this entrainment coefficient is known from experiments of jet discharges in quiescent mediums to be about 0.1 (Ref. 5). In the case of a wind, the component of the wind velocity perpendicular to the plume induces circulation analogous to the circulation of an isolated parcel, except the motion is essentially two-dimensional instead of three-dimensional. That is, two counter rotating vortices are formed which can lead to a splitting (bifurcation) of the plume.

This motion is the two-dimensional analogue of the relative wind impinging on a single moving parcel; therefore, it is logical to assume that the maximum entrainment due to this motion could also be found by multiplying the impinging velocity component by normal area.

$$\text{Impingement} \approx \rho W \sin\theta 2bH$$

These two entrainment calculations, illustrated in Figure 1, represent logical limits for each mechanism but they are not necessarily mutually exclusive.

The maximum horizontal momentum that the plume could extract from the wind both by mass that is actually absorbed into the plume or pressure forces due to deflected mass, can be no greater than the combined momentum of these masses in the ambient.

With this as a postulate for the horizontal momentum, and an assumption for  $F_h$  one can estimate the amount of mass actually entrained.

$$\Delta M = (\text{Aspiration} + \text{Impingement} - \frac{F_h \Delta t}{W})$$

The problem then reduces to estimating  $F_h$ , a precise formulation for  $F_h$  is not known, however, a preliminary function for  $F_h$  is

$$F_h = (\text{Aspiration} + \text{Impingement}) (W-u)$$

The model results using this estimate for  $F_h$  are shown next. Figure 2 is a comparison of model predictions with two data sets for the near field region of a non-buoyant jet. Figure 3 is a comparison of the model predictions with measurements taken in the cooling tower plume.

Figure 4 is a comparison of model-predicted trajectories to a regression fit of Fan's data. Figure 5 shows more completely the predicted average width of the plume and average plume temperature or dilution. More information on the model is available in Reference 6.

#### IMPROVEMENTS TO PLUME MODELS

This distinction between what should be treated as classical turbulent diffusion and what are deterministic motions should be made on the basis of the scale of motion involved, the formulation of the basic law and the degree of resolution implied in the solution procedure.

Advances in plume models will come about through a better understanding and more precise description of entrainment and interaction with the pressure field around the plume, which in this simple model is prescribed through the force and buoyancy calculation.

Conservation of mass, momentum and energy over a control volume encompassing the whole plume should be the goal. The degree to which this is achieved is a check on the consistency of the calculation procedure.

Detailed, three-dimensional, finite volume, numerical calculation with a grid spacing fine enough to predict the self generated circulations is presently too expensive for parametric studies of plume behavior. However, a few such calculations could lead to a better understanding of how the force, buoyancy, and entrainment might be specified more precisely in simple, less expensive, calculation systems.

## CONCLUSIONS

A simple, physical calculation scheme based on average mass, and momentum can yield reasonable predictions of plume behavior.

In a steady wind, the high degree of entrainment that occurs relatively close to the orifice can be largely accounted for by deterministic, self induced entrainment mechanisms. Further downstream where the plume horizontal velocity approaches the wind velocity, the diffusion or spread of the plume is controlled by ambient turbulence. However, by then the properties of the plume have already been significantly diluted.

RECOMMENDATION

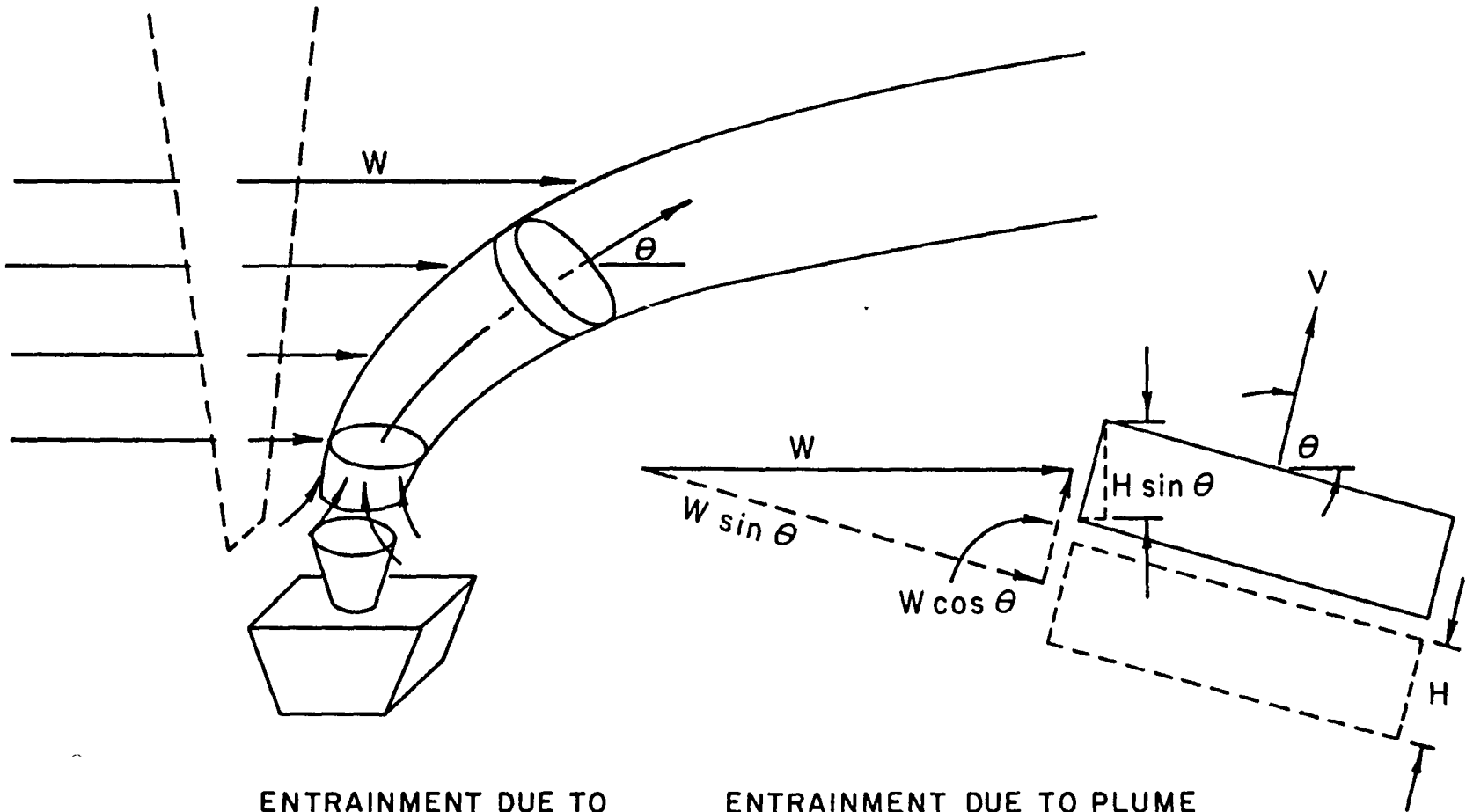
Plume modeling and measurement should make a distinction between entrainment that is due to self-generated vortices and an apparent entrainment or spreading that is a result of fluctuations in wind direction which can cause the plume to oscillate back and forth.

The prescription of the momentum and entrainment calculations should be made as consistent as possible with an overall mass momentum, and energy budget of the whole plume.

## REFERENCES

1. Batchelor, G. K. An Introduction to Fluid Dynamics. Cambridge University Press, 1967, paperback reprint 1974.
2. Frick, W. F. and Winiarski, L. D. Comments on The Rise of Moist, Buoyant Plumes. J. of Applied Meteorology, Vol. 14 No. 3, April 1975. p. 421.
3. Hamilton, P. M. The Application of a Pulsed-Light Rangefinder (Lidar) to the Study of Chimney Plumes Phil. Frans. Roy. Soc. Lond. A 265, p. 153. 1969.
4. Turner, J. S. The Flow Into an Expanding Spherical Vortex Journal of Fluid Mechanics, Vol. 18, part 2, p. 195. 1964
5. Turner, J. S. Buoyancy Effects in Fluids Cambridge University Press, 1973.
6. Winiarski, L. D. and Frick, W. F. Cooling Tower Plume Model. EPA-600/3-76-100, Ecological Research Series, U.S. Environmental Protection Agency, Corvallis, Oregon 97330, 1976.





ENTRAINMENT = ENTRAINMENT DUE TO WIND IMPINGEMENT + ENTRAINMENT DUE TO PLUME SEGMENT MOVING WITH VELOCITY RELATIVE TO WIND (ASPIRATION)

$$\rho A_{\text{projected}} W$$

$$\alpha \rho \pi 2bH |\bar{V} - W \cos \theta|$$

VI-C-145

Figure 1. Entrainment mechanisms.

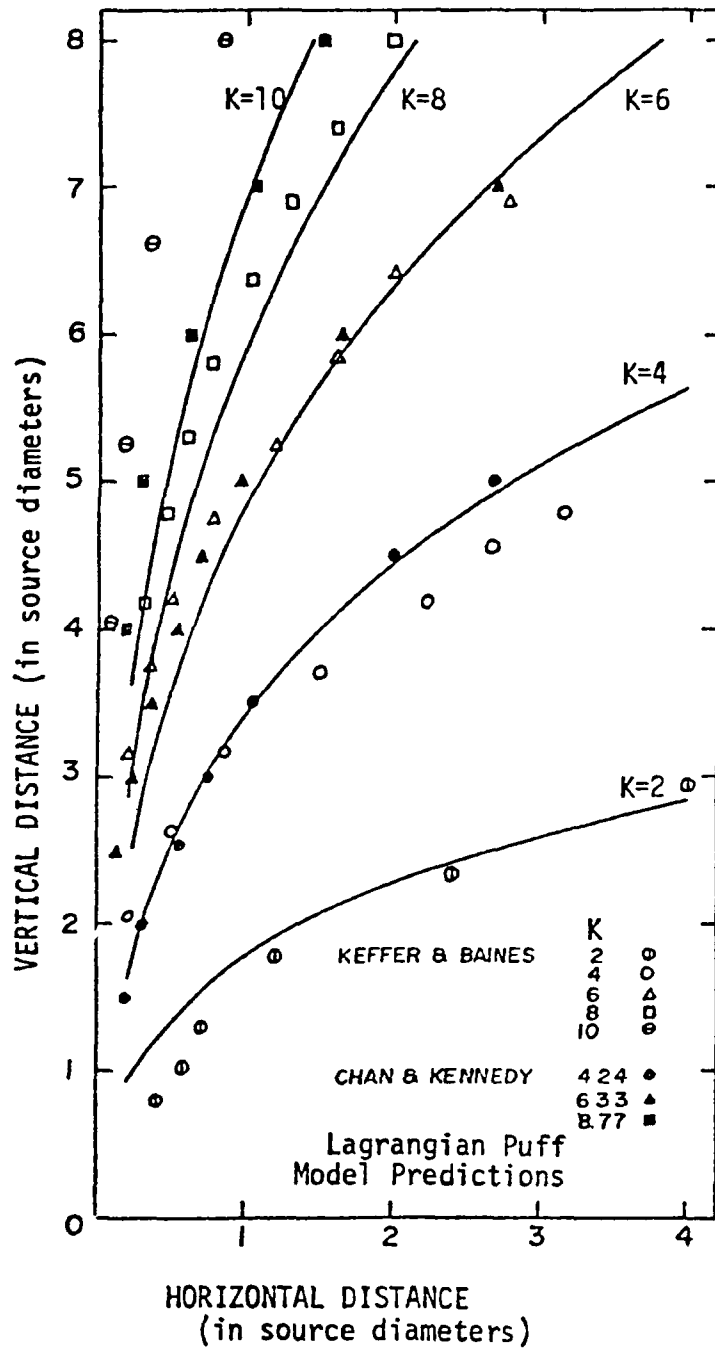


Figure 2. Lagrangian Puff Model predictions compared with jet data.

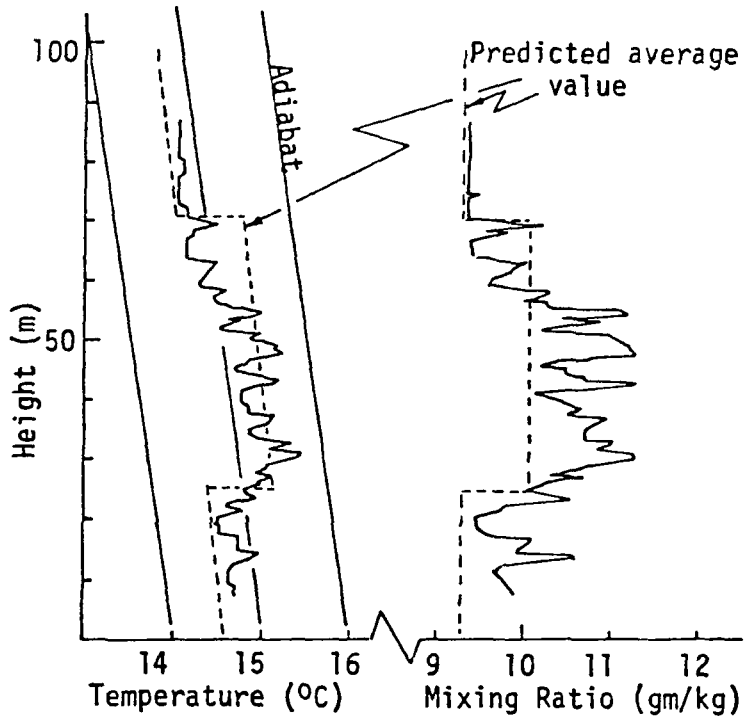


Figure 3. Single cell cooling tower plume profile. Average of Runs 1 and 2.

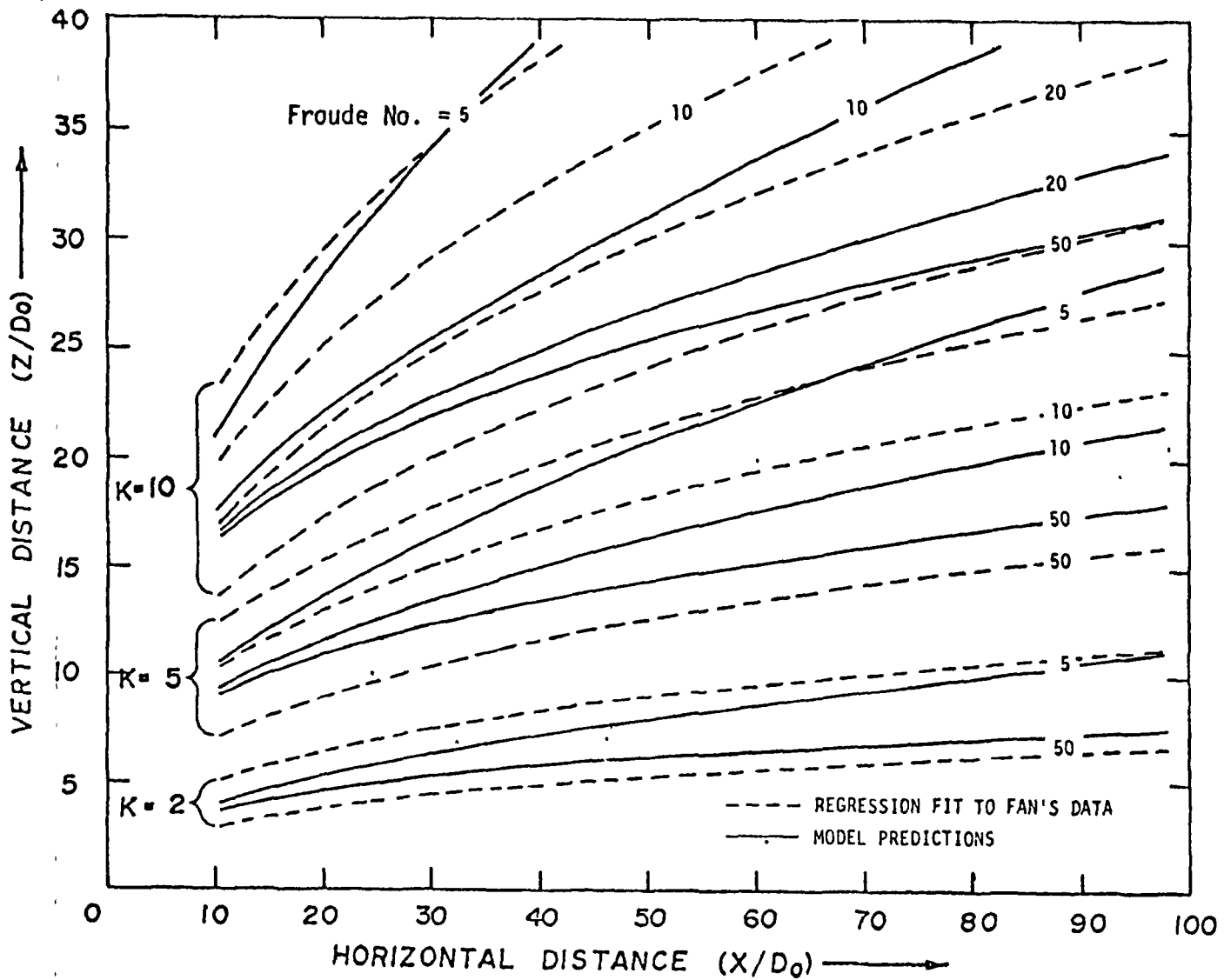


Figure 4. Comparison between model trajectory predictions and regression fit trajectories based on Fan.

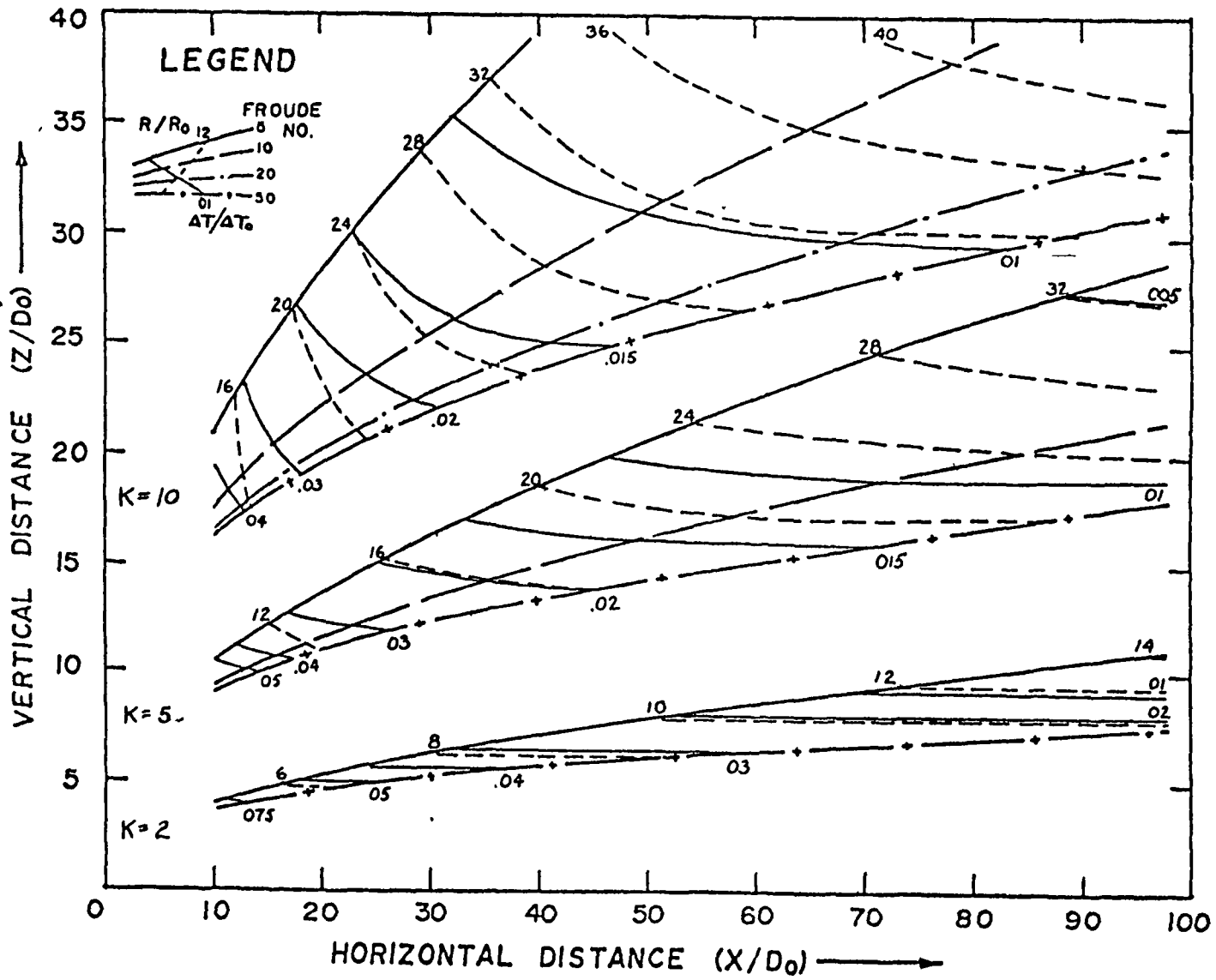


Figure 5. Model predictions of buoyant temperature plume in water.