A MODEL FOR SHORE-ATTACHED THERMAL PLUMES IN RIVERS

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ABSTRACT

Side discharge of power plant thermal effluents into natural rivers cause the formation of thermal plumes, the shape and orientation of which depend upon both the discharge conditions and the ambient flow characteristics. When the turbulence level of the ambient flow is high, rapid vertical mixing is achieved between the effluent discharge and the river flow, and the thermal plume becomes shore-attached with the maximum temperatures within the plume occurring along the near shore. In order to determine the isotherm patterns within shore-attached thermal plumes, a solution of the depth-integrated steady state convection-diffusion equation is obtained. The solution is cast in terms of space-cumulative discharge coordinates, thus allowing the inclusion of the effects of cross-channel flow induced by channel curvature in the model. Comparison of the model results with the thermal plume data from power plant sites along the Missouri River indicates good agreement.

INTRODUCTION

In order to meet the increasing electrical energy requirements of the nation, steam-electric power plants of large capacities are being installed or being planned for installation by a number of utilities. However, the low thermal efficiencies of these plants necessitate the rejection of large amounts of waste heat at the generating sites. In situations where it can be successfully demonstrated that discharge of the waste heat into a natural body of water will not cause any harmful effects on the ecology of the receiving aquatic environment, open-cycle cooling using river, lake, or sea water is the most desirable waste-heat-removal method. Closed-cycle cooling systems using cooling towers, cooling ponds, or spray canals involve enormous capital investments for construction; these can be up to 15 times that for an open-cycle system. They may also require a considerable portion of the plant output power for operating the cooling systems, thereby imposing penalties on plant efficiency. Closed-cycle systems also require extensive continuing maintenance. Finally, there is consumptive water loss due to evaporation from these systems. Open-cycle systems, on the other hand, are inexpensive to construct, place minimal demands on the plant output, and evaporative water losses are much less. Thus, lower investment and maintenance costs, improved plant efficiencies, and less water loss all combine to make once-through cooling the most attractive method by far, from an economic point of view, for dissipating power plant waste heat.

The effects of the thermal discharge on a natural stream, and the physical processes involved in heat transport within the flow system are normally analyzed by separating the affected area into two regions: the far-field and the near-field. The determination of far-field temperature effects in a river is important in assessing the cumulative effects of several power plants distributed along the course of the river. On the other hand, the determination of near-field conditions is important in evaluating the possible effects of the thermal discharges on the aquatic biota and drifting organisms. In the vicinity of an outfall, the heated water forms a so-called thermal plume until the combined actions of jet mixing, buoyant and convective spreading, and turbulent mixing eventually lead to complete dilution of the heated water with the ambient flow. The thermal standards of the various regulatory agencies regarding thermal discharges in natural waterways usually specify certain limitations on the size of and temperature rise in thermal plumes. It is therefore important to have reliable methods for predicting configurations of and temperature patterns within thermal plumes.

Several computational methods are available for predicting thermal plume patterns resulting from power plant discharges. Recently Jirka, Abraham and Harleman (4) have published a detailed review of these. Most of these techniques either assume or predict bell-shaped profiles for velocity and temperature along horizontal normals to the plume axis. However, an examination of measured thermal plume patterns near power stations at Omaha, Fort Calhoun, and Brownville (Cooper) on the Missouri River indicates that the temperature profiles are not beel-shaped, but are attached to the shores, with the maximum temperatures in the plumes occurring near or along the shore. None of the currently available thermal plume models predict plume patterns with shore attachment; this deficiency is highlighted by Jirka, Abraham and Harleman (4).

The available thermal plume models fail to properly represent the shore-attached plume patterns because the effects of ambient flow characteristics such as bottom effects, variation of channel shape, non-uniform and varying flow conditions across the channel, and ambient turbulence on the mixing phenomenon are not considered in their formulation. If the turbulence level of the ambient river flow is high, (e.g., the Missouri River), intense mixing between the thermal jet and the ambient flow will be induced, leading to rapid vertical mixing between the two flows. Thereafter, the jet is transported downstream as a coflowing stream, and the mixing is mainly due to turbulent diffusion and transverse convective spreading associated with bend-generated secondary circulation. Weil and Fischer (13) have attempted to relate the effects of stream turbulence to plume behaviour. However, their investigation was restricted to zero-momentum jets discharging at the centerline of the ambient flow. Yeh (14) conducted a series of laboratory experiments to investigate the transverse mixing process for a buoyant effluent in both near and far field regions of an open channel flow. The results showed that the mixing process in the near field region was closely related to the development and decay of buoyancy-generated secondary flow. However in the far field region the transverse mixing was found to be mainly due to turbulent diffusion.

The present study is concerned with the development of a predictive model, based on diffusion concepts, for determining transverse temperature profiles in shore-attached, vertically-mixed thermal plumes in rivers resulting from side discharges of thermal effluents. The details of the analytical development of the model, and its application to field cases are presented in the following sections.

FORMULATION OF DIFFUSION PROBLEM

The formulation of the predictive model for determining the temperature distributions in shore-attached plumes is based on a model of mixing of two rivers outlined by Sayre (9) and the transformation of the two-dimensional depth-integrated steady-state convection-diffusion equation into a simple diffusion equation as presented by Yotsukura and Sayre (17). The solution of the transformed equation is cast in terms of space-cumulative discharge coordinates, (x, p(x,z)), when x and z are the streamwise and cross-channel coordinates, and p(x,z) is the normalized cumulative discharge defined by

$$p = \frac{q_c}{Q_R} = \frac{1}{Q_R} \int_0^z q \, dz \tag{1}$$

where Q_R = total river discharge; q_c = cumulative discharge passing through the river between z = 0 and z at section x; and q = total discharge per unit width. The use of the space-cumulative discharge coordinate system which automatically follows the transverse shifts and meanderings of the flow, was first introduced by Yotsukura and Cobb (15). A procedure for synthesizing the normalized cumulative discharge at cross sections of a river for which the shape is known is outlined in the next section.

The origin of coordinates, $x = 0$, $p = 0$, is located on the discharge bank of the river, just far enough downstream from the outfall so that at the origin initial mixing has taken place, and the temperature rise of the jet is Θ_I. The fraction of the total river discharge occupied by the jet at the origin is given by

$$P = \frac{Q_o \Theta_o}{Q_R \Theta_I} = a \frac{Q_o}{Q_R} \qquad (2)$$

where $a = \Theta_o/\Theta_I$ = initial dilution factor; Θ_o = initial excess temperature of the thermal discharge; and Q_o = volumetric rate of effluent discharge.

Let the variable ζ represent the displacement from the origin in the p direction at $x = 0$, as shown in Fig. 1. Then the input transverse temperature distribution at $x = 0$ is given by

$$\left.\begin{aligned} \Theta_I(\zeta,0) &= \frac{\Theta_o}{a}, \quad 0 \leq \zeta \leq P \\ &= 0, \quad P < \zeta \leq 1 \end{aligned}\right\} \qquad (3)$$

Neglecting for the moment reflections from the banks, the normalized transverse temperature distribution function for the case of a continuous vertical line source, extending over the depth of flow and concentrated at $(x = 0, p = \zeta)$ is $f_R(p-\zeta;\ x)$ which is a Gaussian probability distribution function, given by

$$f_R(p_1;x) = \frac{1}{\sigma_p} \left(\frac{1}{\sqrt{2\pi}} \exp \left(-\frac{s^2}{2}\right)\right); \qquad p_1 = p-\zeta \qquad (4)$$

where $s = p_1/\sigma_p$ is the standardized normal variable; and the standard deviation in the p domain is

$$\sigma_p = \sqrt{2D\ x'} \qquad (5)$$

In Eq.(5), D = overall transverse diffusion factor; and $x' = x - x_o$ where x_o = longitudinal coordinate of the virtual source for any subreach determined from

$$D_j(x_{j-1}-x_{o_j}) = D_{j-1}(x_{j-1}-x_{o_{j-1}}) \tag{6}$$

where x_{j-1} = distance from the origin to the upstream end of j-th subreach, and D_j = value of D for the j-th subreach. For the first subreach, $x_{oj} = 0$.

The desired output temperature distribution for an input distribution $\Theta_I(\zeta, 0)$ is given by the convolution integral

$$\Delta T(p,x) = \int_{p=0}^{p=1} f_R(p-\zeta;x)\ \Theta_I(\zeta,0)\ d\zeta \tag{7}$$

Using Eq. (3), Eq. (7) can be written

$$\Delta T(p,x) = \frac{\Theta_o}{a} \int_o^P f_R(p-\zeta;x)\ d\zeta \tag{8}$$

Taking into account now reflections from the banks, the final solution can be obtained from Eq. (8) as

$$\begin{aligned}\Delta T(p,x) = \frac{\Theta_o}{a} \Big(&\Big\{F_R\Big(\frac{p+P}{\sigma_p}\Big) - F_R\Big(\frac{p-P}{\sigma_p}\Big)\Big\} \\ &+ \sum_{n=1}^{\infty}\Big\{F_R\Big(\frac{2n+(p+P)}{\sigma_p}\Big) - F_R\Big(\frac{2n-(p+P)}{\sigma_p}\Big) \\ &\quad + F_R\Big(\frac{2n-(p-P)}{\sigma_p}\Big) - F_R\Big(\frac{2n+(p-P)}{\sigma_p}\Big)\Big\}\Big)\end{aligned} \tag{9}$$

where $F_R(\ )$ is the standardized cumulative normal distribution function corresponding to the probability density function $f_R(\ )$. The terms in the infinite series representing the bank reflections are negligible if $Dx \leq 0.08$.

The value of the initial dilution factor, a, introduced in Eq. (2) is mainly a function of the design and orientation of the discharge outfall structure, the initial jet or plume characteristics, and the ambient flow properties. If the thermal effluent is discharged almost parallel to the river flow, and Θ_o is the temperature excess measured at the discharge canal outlet, the initial dilution factor, a, can have a value of one. For other situations, values of the initial dilution factor between about 2 and 4 can be used (3,7), until better criteria are established.

Maximum Centerline Temperature

The maximum centerline temperature, ΔT_c, for a shore-attached plume occurs along the near shore (p = 0). The solution for ΔT_c, from Eq. (9) is

$$\Delta T_c = \Delta T(0,x) = \frac{\Theta_o}{a}\left(2F_R\left(\frac{P}{\sigma_p}\right) + 2\sum_{n=1}^{\infty}\left\{F_R\left(\frac{2n+P}{\sigma_p}\right) - F_R\left(\frac{2n-P}{\sigma_p}\right)\right\} - 1\right) \qquad (10)$$

The variations of $Q_R \Delta T_c / Q_o \Theta_o$ with Dx for values of P equal to 0.05, 0.10, and 0.20 are shown in Fig. 2. The limiting case, P = 0, corresponds to a source concentrated at the origin. The solution corresponding to Eq. (9) for this limiting case is

$$\Delta T(p,x) = 2\Theta_o \frac{Q_o}{Q_R}\left(f_R(p) + \sum_{n=1}^{\infty}\left\{f_R(2n-p) + f_R(2n+p)\right\}\right) \qquad (11)$$

Substituting for the probability density function $f_R(p)$ by

$$f_R(p) = \frac{1}{\sigma_p\sqrt{2\pi}} \exp\left(-\frac{1}{2}\frac{p^2}{\sigma_p^2}\right), \qquad (12)$$

the solution for maximum temperatures along the shore; (p = 0), corresponding to Eq. (10) for this case is

$$\Delta T_c = \Delta T(0,x) = \Theta_o \frac{Q_o}{Q_R}\sqrt{\frac{2}{\pi}}\,\frac{1}{\sigma_p}\left(1 + 2\sum_{n=1}^{\infty}\exp\left(-2\frac{n^2}{\sigma_p^2}\right)\right), \qquad (13)$$

Eq. (13) is represented by the curve labelled concentrated source in Fig. 2.

Diffusion Factor, D

The diffusion factor, D, in Eq. (5) can be related to the dimensionless tranverse mixing coefficient, $\alpha = E_z/\bar{d}u_*$, by the equation

$$D = \frac{\alpha\,\bar{d}\,u_*}{Q_R^2}\left(\int_0^1 d^2\,u\,dp\right) \qquad (14)$$

if it is assumed that the overall transverse mixing coefficient E_z does not vary across the channel. In Eq. (14) $\bar{d}$ = A/B = average depth of flow section; A = area of river flow section; $u_* = (g\bar{d}S)^{1/2}$ = shear velocity; g = acceleration due to gravity; S = slope of energy gradient; d = local flow depth; and u = q/d = local depth-averaged velocity of flow. Using the Manning formula for the river flow rate,

$$Q_R = \frac{1.49}{n}\,B\,\bar{d}\,(\bar{d})^{2/3}\,(S)^{1/2} \qquad (15)$$

and approximating

$$\int_0^1 d^2\,u\,dp \simeq (\bar{d})^2\,\bar{u} = \bar{d}\,\bar{q}, \qquad (16)$$

where $\bar{q} = \bar{d}\,\bar{u}$ = width-averaged value of q, the relation for the diffusion factor can also be written

$$D = \frac{\alpha}{1.49}\,\frac{n\sqrt{g}}{B^2}\,(\bar{d})^{5/6} \qquad (17)$$

where n = Manning's coefficient. Values of D determined by Eq. (17) are less accurate compared to those by Eq. (14), mainly because of the approximation used in Eq. (16).

The transverse mixing coefficient, E_z or α, for a natural river depends upon the characteristics of the river channel and the ambient flow. However, there are as yet no very reliable predictive relationships for evaluating its value. The most reliable method for determining the values of E_z for natural channels is by direct experimentation in the field using tracers. Experiments in undistorted hydraulic models can also be used to determine E_z; vertically distorted hydraulic models do not correctly simulate the transverse mixing processes.

In laboratory flumes several investigators have found values of α ranging from about 0.1 for $B/\bar{d} \simeq 5$ to about 0.2 for $B/\bar{d} \simeq 60$. In reasonably straight uniform reaches of natural channels somewhat larger values of α have been found. This increase is probably due to channel irregularities and weak secondary flows rather than to any scale effect. In sinuous channels, due to bend-induced secondary flow, α values are typically much larger. As shown in Fig. 3, average α values ranging from 0.5 to 2.5 for bends in flumes, and from 0.6 to 10 for bends in the Missouri River, have been found. The functional form of the relationships shown on Fig. 3 was derived by Fischer (2) in a transverse convective dispersion analysis which utilized Rozovskii's (9) radial velocity distribution function for fully-developed secondary flow in the central portion of an idealized curved channel. The gap between the laboratory flume data and the Missouri River data in Fig. 3 indicates that there are still certain factors that are not accounted for in evaluating the transverse mixing coefficient. The equation for the curve passing through the Missouri River data is

$$\alpha = 0.4 \left(\frac{B}{\bar{d}}\right)^2 \left(\frac{\bar{u}}{u_*}\right)^2 \left(\frac{\bar{d}}{R_c}\right)^2 \qquad (18)$$

where $\bar{u} = Q_R/A$ = average river flow velocity; and R_c = radius of curvature of river bend. The value of the numerical coefficient in Eq. (18) may be different for other rivers.

SYNTHESIS OF TRANSVERSE FLOW DISTRIBUTION

The cumulative discharge, q_c, and the normalized cumulative discharge, $p = q_c/Q_R$, in a river section at a given flow discharge can be synthesized from the transverse depth profile of the cross-section, as outlined by Sayre (10). The method is based on the hypothesis that the velocity distribution in natural streams is a function of the transverse depth profile, and uses the relation

$$\frac{q}{\bar{q}} = b_o \left(\frac{d}{\bar{d}}\right)^{b_1} \tag{19}$$

where d = local depth of flow; and b_o and b_1 are coefficients whose values according to the Manning formula (Eq. 19) are $b_o = 1.0$, and $b_1 = 5/3$. Sium (12) investigated the applicability of Eq. (19) for synthesizing the flow distribution in several natural rivers including the Missouri and Mississippi Rivers. That study indicated that the coefficients b_o and b_1 in Eq. (19) varied with the ratio of width to average depth, $B/\bar{d}$. For cross-sections in straight river reaches, approximate values of b_o and b_1 are

$$\begin{aligned} b_o &= 1{\cdot}0,\ b_1 = 5/3;\ 50 \le B/\bar{d} < 70 \\ b_o &= 0{\cdot}92,\ b_1 = 7/4;\quad B/\bar{d} \ge 70 \end{aligned} \tag{20}$$

For cross-sections in bends of sinuous and meandering reaches, approximate values of b_o vary between 0.95 and 0.80, and b_1 between 2.48 and 1.78, as $B/\bar{d}$ increases from 50 to 100. Usually the value of b_o must be adjusted to make p = 1 when z = B.

Fig. 4 shows the variation of measured $q/\bar{q}$ with $d/\bar{d}$ for three cross-sections in the Missouri River, together with the theoretical curve corresponding to $q/\bar{q} = (d/\bar{d})^{5/3}$. A comparison of the measured and synthesized transverse distribution of unit discharge at one of the cross-sections (Mile 410.20) is also shown in Fig. 4. The data shown in Fig. 4 correspond to a Missouri River discharge of 61,300 cfs (1736 cu.m.sec). Synthesized transverse flow distributions at Mile 410.20 corresponding to a river flow rate of 35,000 cfs (991. cu.m/sec) are shown in Fig. 5. To obtain the transverse distribution curve for depth d at the discharge of 35,000 cfs (991 cu.m/sec), the water surface level was dropped by 3.8 ft (1.16 m) from that for 61,300 cfs (1736 cu.m/sec). This drop in water surface elevation, corresponding to reduction in discharge from 61,300 cfs (1736 cu.m/sec) to 35,000 cfs (991 cu.m/sec), was determined from an estimated stage versus discharge curve for the Missouri River at Mile 411. The procedure thus assumes that there is no significant change in the cross-sectional shape as the river discharge and depth are reduced, and also that there is a unique stage-discharge relationship. For alluvial channels, these assumptions are not strictly true. Changes in discharge can cause different amounts of aggradation and/or degradation of the bed in different parts of the cross-section. Also, seasonal shifts in the stage discharge relation may occur that evidently are associated with temperature-related effects on the bed configuration, among other factors. Unfortunately, alluvial channel phenomena are not sufficiently well understood to quatitatively evaluate or significantly improve on these assumptions.

APPLICATIONS OF THE MODEL

The solutions of the thermal plume model, Eqs. (9) and (10), were used to predict plume patterns in the Missouri River downstream from the Fort Calhoun Power Station of the Omaha Public Power Station District near Blair, Nebraska, and the Cooper Nuclear Station of the Nebraska Public Power District near Brownville, Nebraska. Fig. 6 shows the measured (5) and the predicted (7) temperature-rise isotherms for the Fort Calhoun Station corresponding to two Missouri River discharges. A comparison of the measured (6) and predicted values of $\Delta T_c/\Theta_o$ for the Cooper Nuclear Station thermal plumes on four different dates is shown in Fig. 7. Background river flow and plant-discharge data corresponding to these plumes are given in Table 1. The results shown in Figs. 6 and 7 indicate that the model predicts thermal plumes in the Missouri River that are in quite good agreement with the measured plumes.

One major capability of the present model is that it permits the inclusion of the effect of the cross-channel flow induced by channel curvature. Because of this, the model can predict expansions and contractions of the thermal plume as the river flow becomes weighted towards one bank or the other. This feature is illustrated in Fig. 8 which shows the predicted thermal plume pattern near the Fort Calhoun Power Station for a Missouri River discharge of 14,000 cfs (396 cu.m/sec). The computations for this case were performed for a 12-mile stretch of the river which was divided into ten subreaches, as shown in the insert map in Fig. 8.

CONCLUSIONS

A computational model, based on a diffusion equation, has been developed for predicting the isotherm pattern for shore-attached thermal plumes in rivers. Comparison of the model results with field measurements indicates reasonably good agreement. Further refinements of the model, especially in defining the initial dilution factor, need to be made based on laboratory investigations. The model can be used to assess the environmental impact of power plant thermal discharges in natural rivers.

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NOTATIONS

The following symbols are used in this paper:

A = area of river flow cross section;
a = initial dilution factor;
B = width of river flow cross section;
b_o = coefficient used in Eq. 19;
b_1 = exponent used in Eq. 19;
D = diffusion factor;

$\underline{d}$ = local depth of flow;
$\bar{d}$ = average depth of river flow cross section;
E_z = overall transverse mixing coefficient;
g = acceleration due to gravity;
n = Manning coefficient;
P = fraction of river flow occupied by plume;
p = normalized cumulative discharge;
Q_o = volumetric discharge rate of thermal effluent;
Q_R = river discharge
$\underline{q}$ = discharge per unit width;
$\bar{q}$ = width-averaged discharge per unit width;
q_c = cumulative discharge from near bank
R_c = radius of curvature of river bend;
S = energy gradient;
$\underline{u}$ = depth-averaged local velocity of flow;
$\bar{u}$ = cross-sectional average velocity of flow;
u_* = shear velocity;
x = longitudinal coordinate;
x_o = position of virtual source;
z = transverse space coordinate;
ΔT = excess temperature within plume;
ΔT_c = centerline or bank excess temperature within plume;
α = dimensionless transverse mixing coefficient;
Θ_o = initial excess temperature;
Θ_I = excess temperature after initial dilution;
σ_p = variance of the normal probability density functions, Eq. 4.

TABLE I.

BACKGROUND DATA CORRESPONDING TO THE THERMAL PLUMES SHOWN IN FIGURES 6 AND 7

Power Station	Date	River Flow cfs (m^3/sec)	Plant Discharge cfs (m^3/sec)	Initial Excess Temperature °F (°C)	Initial Dilution Factor
Ft. Calhoun	5/3/74	33,000 (935)	801 (22.7)	18 (10.0)	2
Ft. Calhoun	12/2/74	18,500 (524)	801 (22.7)	16.2 (9.0)	2
Cooper	5/23/74	48,200 (1365)	725 (20.5)	7.8 (4.3)	1
Cooper	9/27/74	37,100 (1051)	1352 (38.3)	8.6 (4.8)	1
Copper	6/30/75	57,600 (1631)	880 (24.9)	10.1 (5.6)	1
Cooper	9/18/75	66,600 (1886)	1088 (30.8)	10.5 (5.8)	1

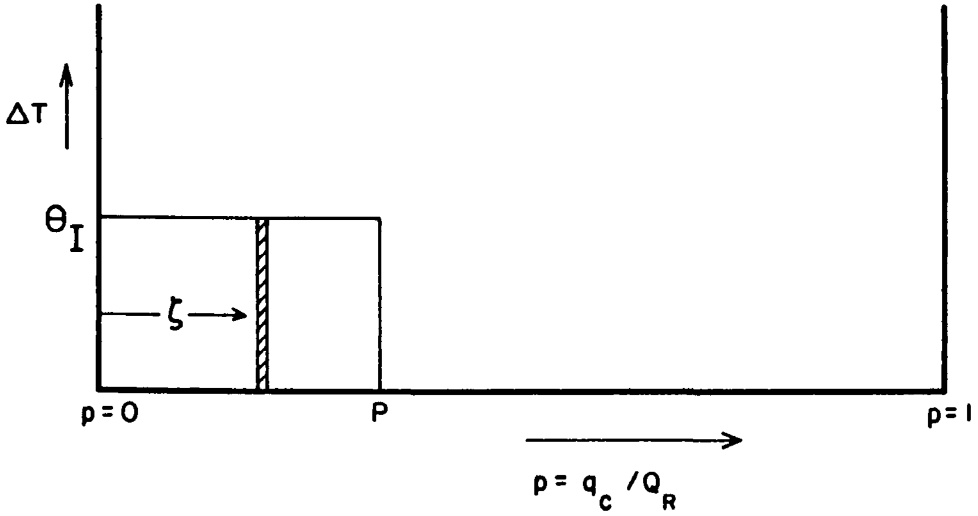

Figure 1. Definition Sketch for the Initial Condition.

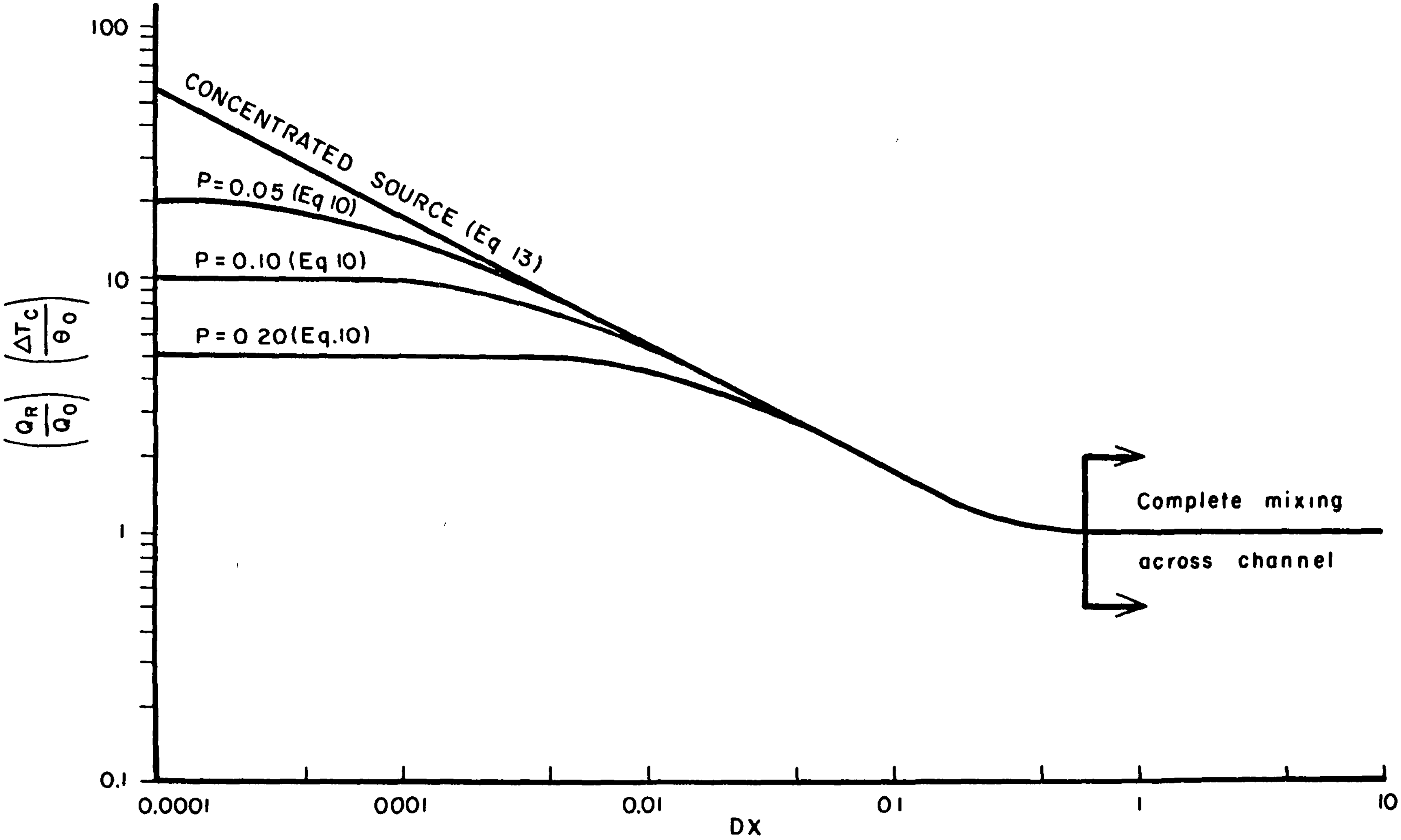

Figure 2. Longitudinal Variation of Maximum Temperature Along the Bank for Various Initial Conditions.

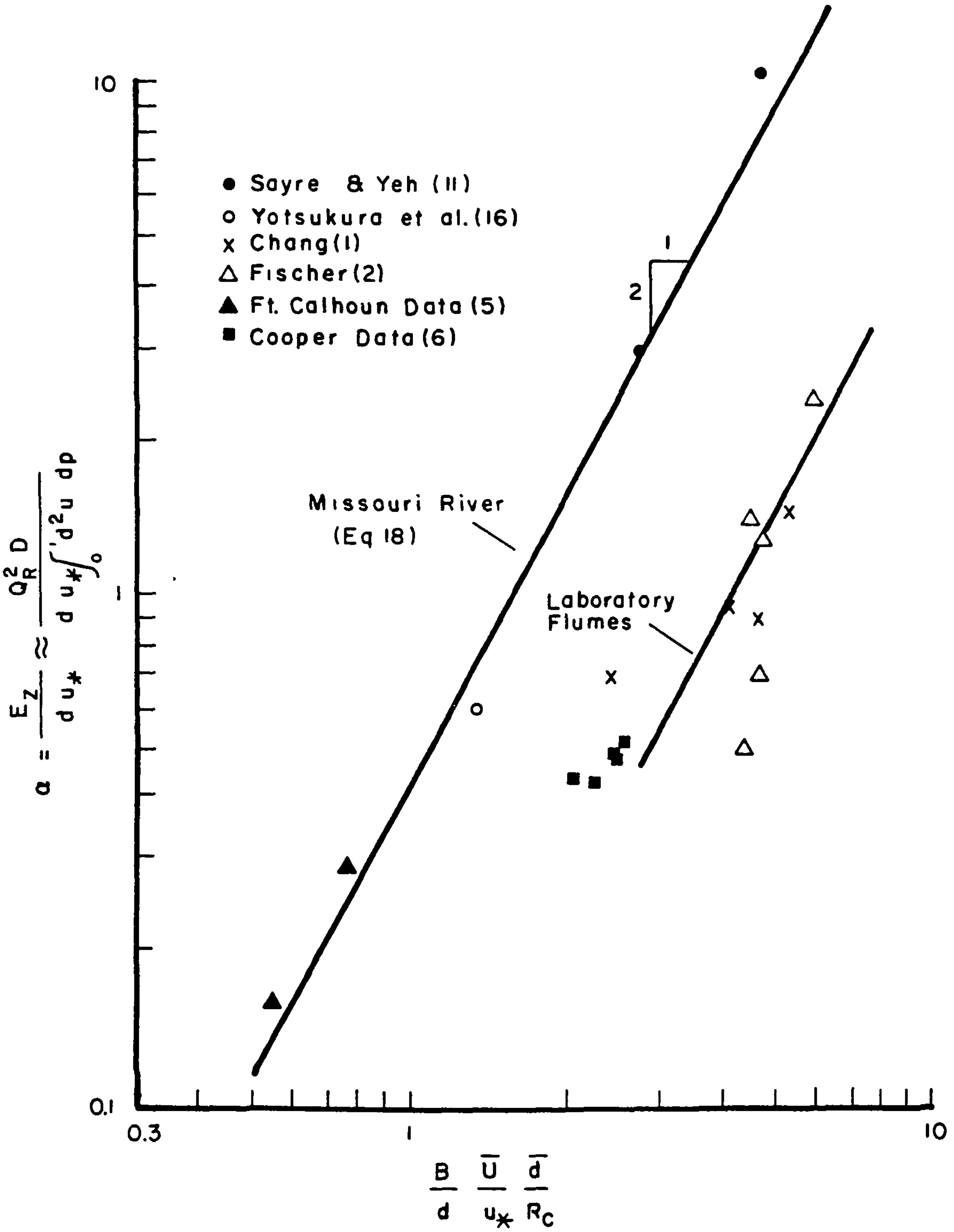

Figure 3. Variation of Transverse Mixing Coefficient in Bends with Bulk Flow and Channel Geometry Parameters.

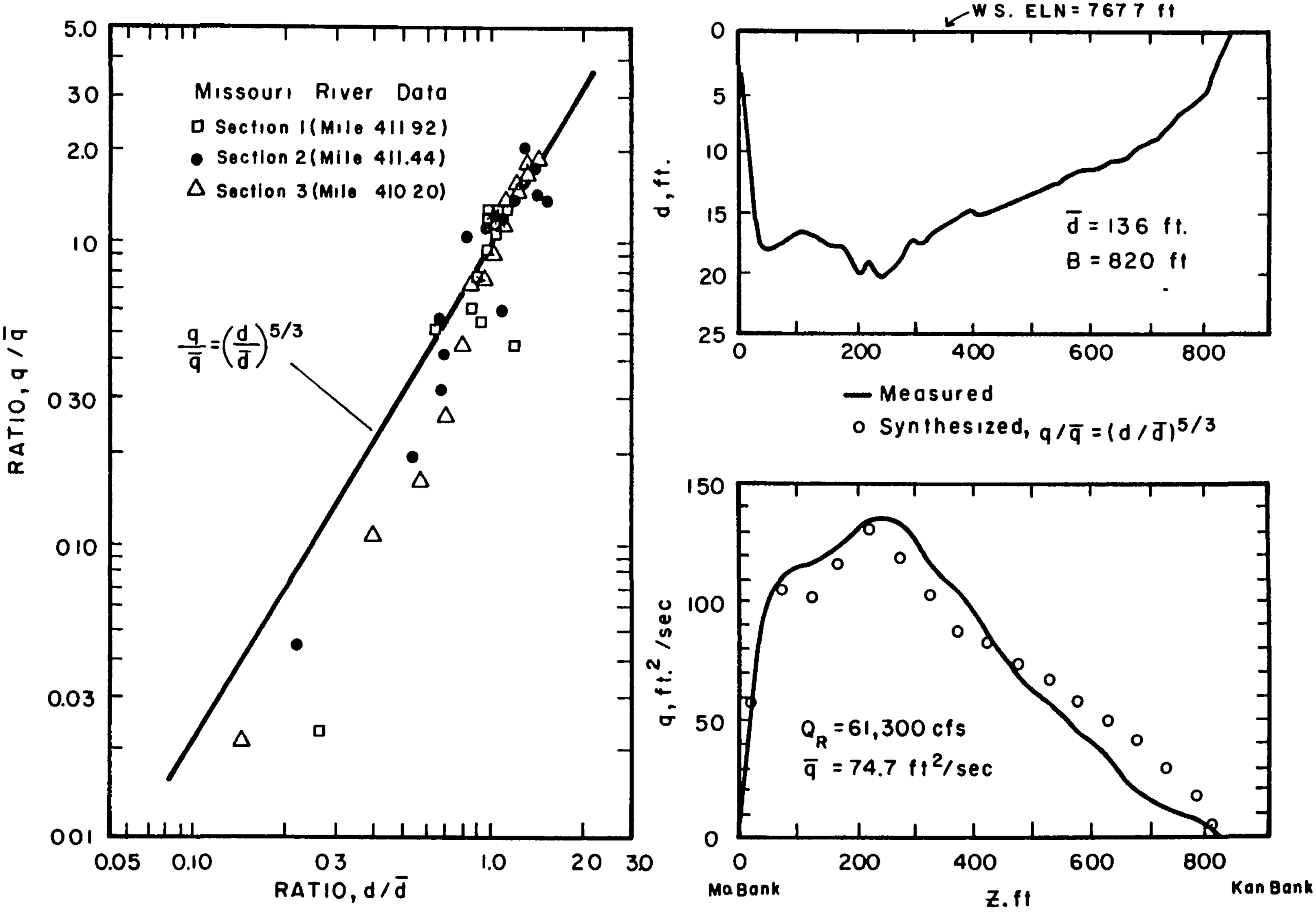

Figure 4. Transverse Distribution of Depth and Unit Discharge Measured at Mile 410.2 in the Missouri River, and the Relative Unit Discharge-Relative Depth Relationship.

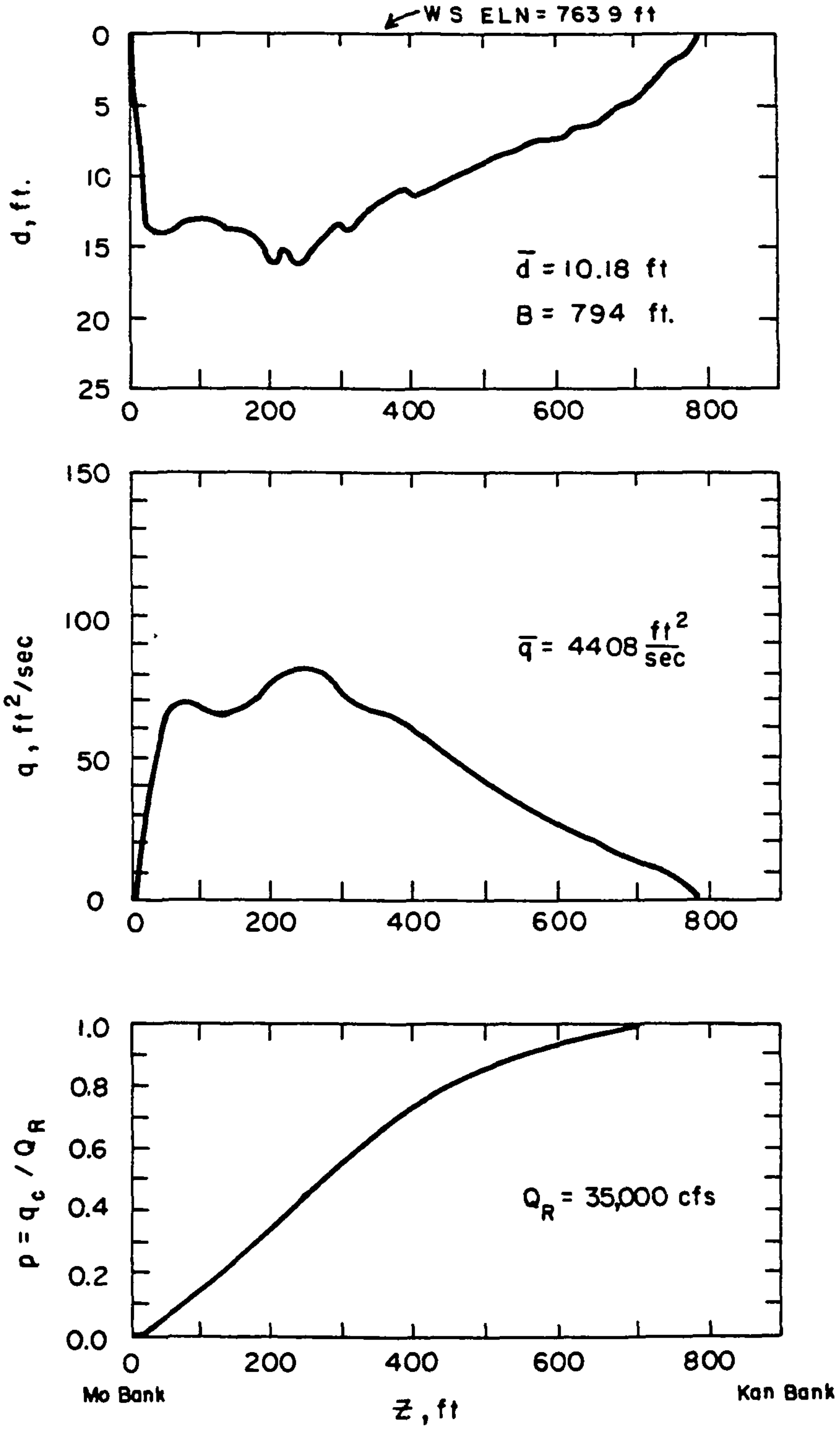

Figure 5. Synthesized Distribution of Depth, Unit Discharge, and Cumulative Discharge at Mile 410.2 in the Missouri River for a Flow Rate of 35,000 CFS.

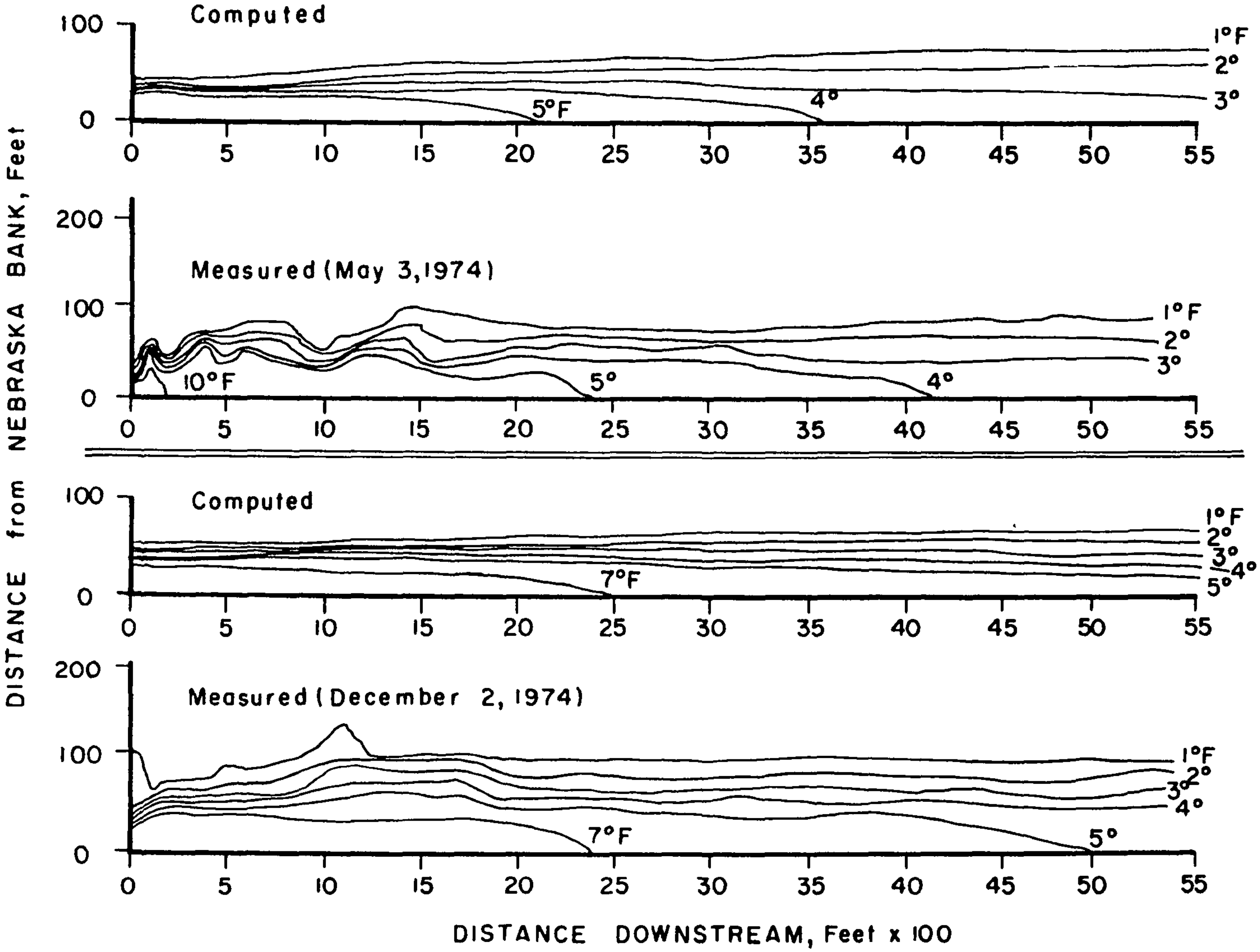

Figure 6. Comparison of Measured and Predicted Temperature-Rise Isotherms for the Missouri River Thermal Plumes Near the Fort Calhoun Power Station, Ft. Calhoun, Nebraska.

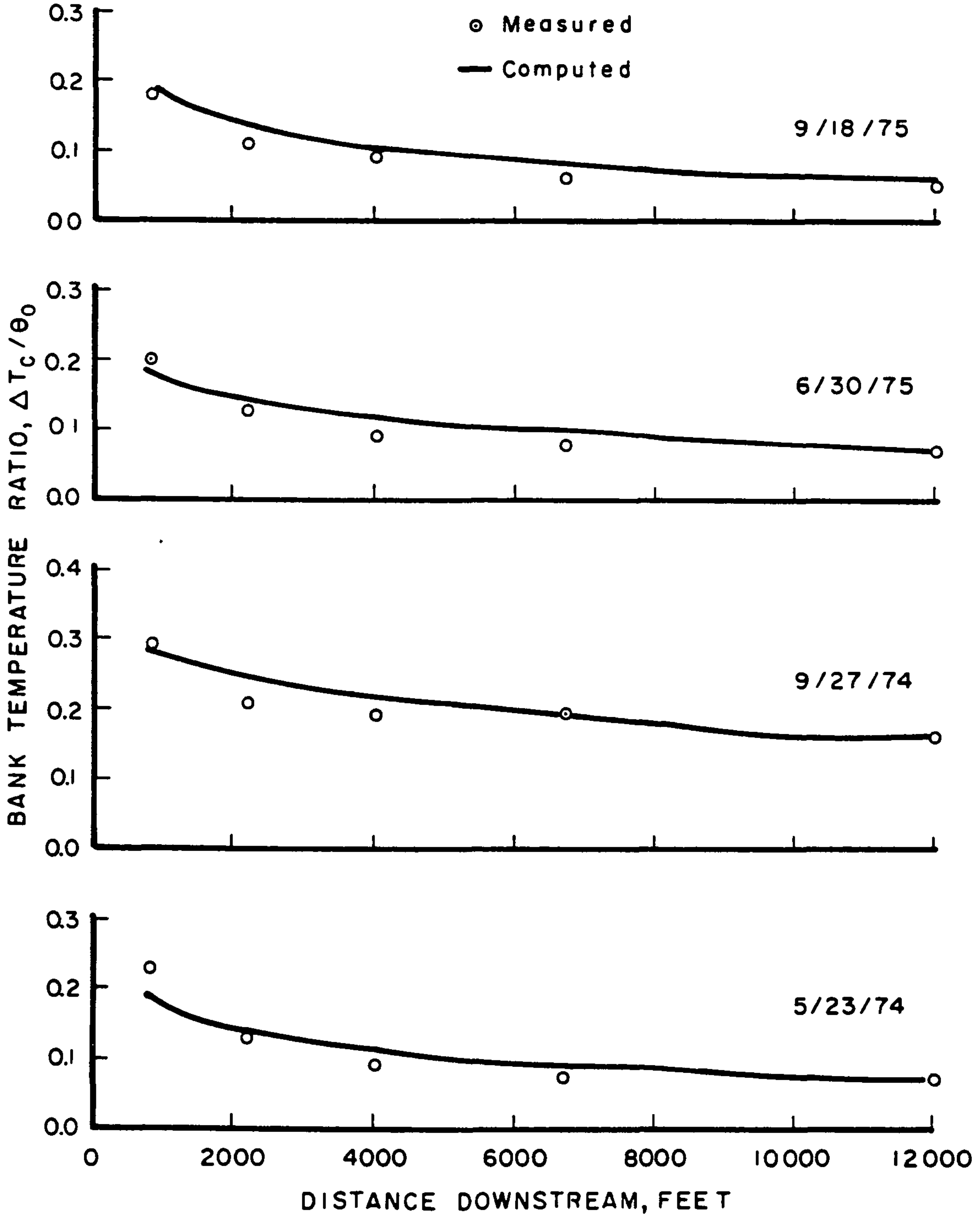

Figure 7. Comparison of Measured and Predicted Maximum Temperatures Along the Bank for the Missouri River Thermal Plumes Near the Cooper Nuclear Station, Brownville, Nebraska.

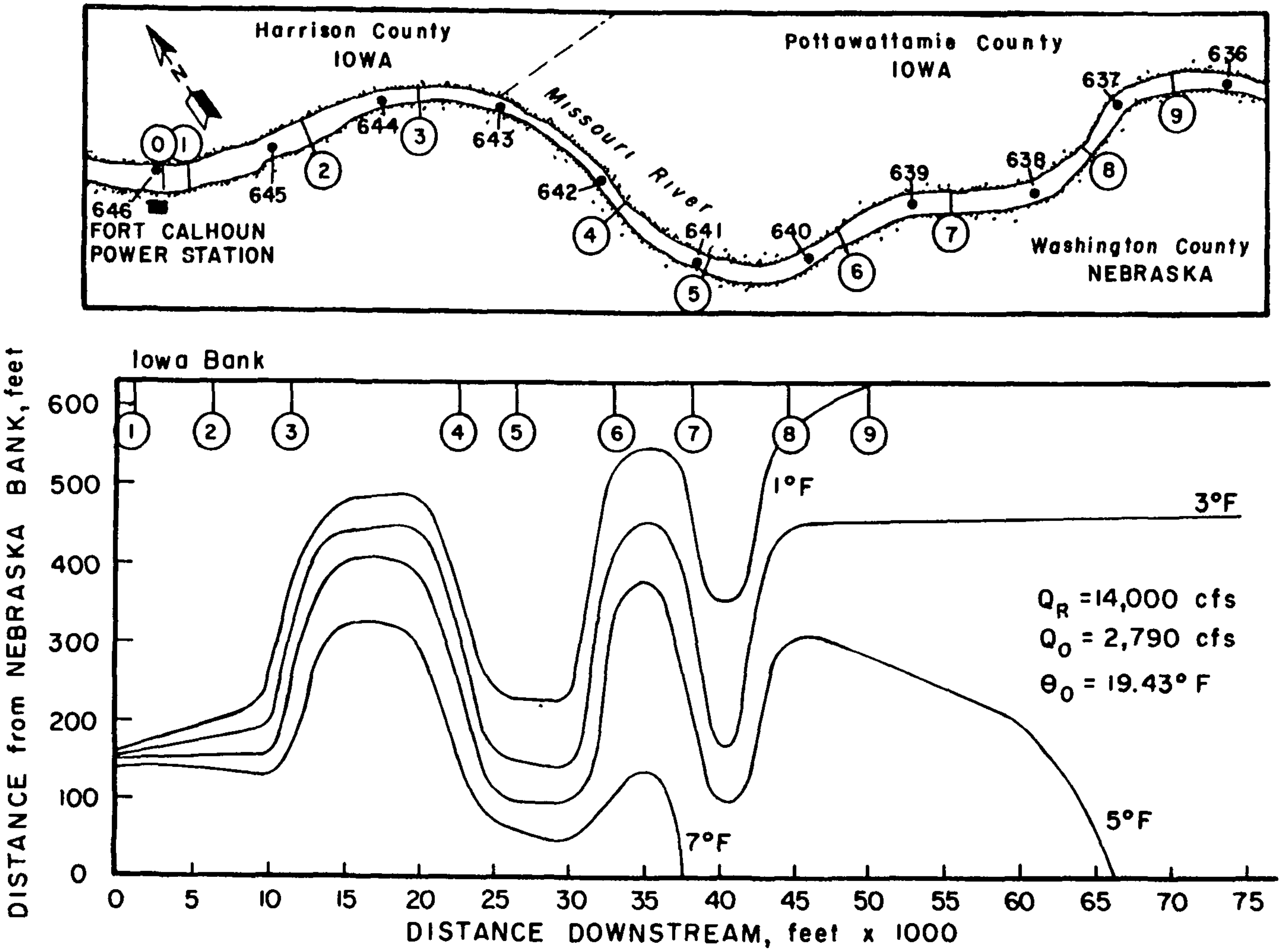

Figure 8. Predicted Temperature-Rise Isotherm Pattern for Low Flow Conditions in the Missouri River Near the Fort Calhoun Power Station, Ft. Calhoun, Nebraska.