NASA CR-152030

(NASA-CR-152030) SPLINED VERSION OF FLEXSTAB: A CRITICAL ANALYSIS OF ALTERNATE SCHEMES Final Report (Boeing Commercial Airplane Co., Seattle) 47 p HC A03/MF A01 CSCL 01A G3/02
N78-10027
Unclas
52490

Splined Version of FLEXSTAB
A Critical Analysis of Alternate Schemes

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Prepared for
Ames Research Center
under contract NAS2-7729
Task I

National Aeronautics and
Space Administration

August 1977

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA CR-152030		
4. Title and Subtitle: SPLINED VERSION OF FLEXSTAB–A CRITICAL ANALYSIS OF ALTERNATE SCHEMES		5. Report Date: August 1974
		6. Performing Organization Code
7. Author(s): J. A. Weber		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address: Boeing Commercial Airplane Company, P.O. Box 3707, Seattle, Washington 98124		11. Contract or Grant No.: NAS2-7729
		13. Type of Report and Period Covered: Final Report
12. Sponsoring Agency Name and Address: Ames Research Center, National Aeronautics and Space Administration, Moffett Field, California 94035		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

Recommendations are made for improved aerodynamic models and numerical schemes to be considered for inclusion into the FLEXSTAB computer program system. These recommendations are based on a critical analysis of existing and soon-to-be available numerical technology. This work was performed as Task I, of contract NAS2-7729, with the requirement that there be no loss in any capability relative to the NASA-Ames FLEXSTAB version (1.01.00) including the low frequency unsteady aerodynamic capability.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
FLEXSTAB Aeroelasticity Analysis Aerodynamics	Unclassified Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	43	

*For sale by the National Technical Information Service, Springfield, Virginia 22151

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1.0 SUMMARY

The ultimate objective of work identified as Task I of NAS2-7729 is to develop and implement, within the FLEXSTAB computer program system, improved aerodynamic models and methods to replace those currently embodied in the FLEXSTAB aerodynamic influence coefficient (AIC) program.

The work is to be based upon the aerodynamic technology methods and experience which have evolved from the vortex spline scheme developed originally under NASA-Ames contract NAS2-6530 and the subsequent contract work which has gone beyond the original vortex spline to the doublet and source spline concepts, work presently being conducted under NASA-Ames contract NAS2-7729 (*Development of a FLEXSTAB Computer Program*). It is the latter work, in particular, which supplies the impetus, encouragement and strong technological basis to enter into the Task I work leading toward improved aerodynamic methods for the FLEXSTAB computer program system.

The conclusion drawn from this study is that the ultimate objectives of Task I can be achieved with little technical risk and without loss of any capability relative to the NASA-Ames released version 1.01.00. The direction of that development and the ability of the FLEXSTAB computer program system to interact dynamically with advancing technology hinges upon development of a restructured program system. Two options are presented.

Option A assumes that the present FLEXSTAB system will not undergo a program restructuring.

Option B assumes that FLEXSTAB will be restructured. The exact body surface aerodynamic model is recommended to provide an improved modeling capability not presently available.

Option B is the specific recommendation of this report.

2.0 INTRODUCTION

Steady state aerodynamic technology has advanced beyond the original numerical lifting surface technology of *constant pressure panels* introduced by Woodward (refs. 1 through 3); his aerodynamic model for wing-body interference (*interference shell* of constant cross section) has also been superseded. It is the Woodward model which is employed in the current FLEXSTAB program system. New technology, however, has been and is being developed which has the potential to improve the aerodynamic methods within FLEXSTAB. Chief among the characteristics of the new technology are: the capability to treat theoretical models that more closely represent the true configuration surfaces, insensitivity to configuration paneling (and the associated promise of fully automatic paneling), computational simplicity (computation of AIC integral expressions in closed form), and the possibility of producing potential flow information that will provide an adequate foundation for future implementation of drag force prediction involving boundary layer analysis (drag prediction in FLEXSTAB is limited to induced drag).

The work described in this report is an analysis of alternate approaches which presents the relative merits of each of several possible candidate theoretical aerodynamic models and numerical schemes, now or soon to be available. Each of the schemes is evaluated in each of the following specific areas: (1) applicability to the steady flow needs of FLEXSTAB (subsonic and supersonic) and (2) adaptability to (a) the low reduced frequency approximation, (b) Küssner and Wagner functions approximation, and (c) arbitrary reduced frequency analysis. Existing and new technology is defined and reviewed.

The relative merits of each of the various methods are assessed with respect to their features and limitations as well as their practical applicability to the present and the potential growth versions of FLEXSTAB. Particular attention is directed toward the steady flow and low reduced frequency requirements of FLEXSTAB, with a view toward adaptability to arbitrary reduced frequency analysis. Consideration is given to the potential gains in accuracy measured against the computational cost of obtaining a solution.

3.0 SYMBOLS AND ABBREVIATIONS

AIC	aerodynamic influence coefficient
a_∞	speed of sound
℄	centerline
C_p	dimensionless pressure coefficient
D/Dt	total derivative
dx', dy'	differential elements
ESIC	external structural influence coefficient
F	equation of bounding surface
F(S)	singularity strength
ISIC	internal structural influence coefficient
K	kernel function form
k	ratio of specific heats
LD	line doublet
LS	line source
M	Mach number
P, P_∞	static pressure at arbitrary point and at infinity
Q	magnitute of total velocity
t	time
U_∞	magnitude of fluid velocity at infinity
VC	constant vortex
x,y,z	coordinate axes (x, streamwise direction)
∇	gradient operator
ϕ	perturbation velocity potential function
Φ	total velocity potential function
ρ_∞	fluid density at infinity
σ, μ, γ	singularity strength distribution function for source, doublet, and vortex
$\frac{\partial}{\partial t}$	time derivative

4.0 OVERVIEW OF FLEXSTAB

4.1 SOME PRESENT FEATURES*

General Description

FLEXSTAB is a collection of computer programs which have been assembled for aeroelastic analysis of arbitrary airplane-like configurations. The programs can be linked by tape or disk data transfer. These programs can be executed singly or linked for consecutive execution in a single run. Thus, a complete aeroelastic analysis made up of an aerodynamic analysis, structural analysis, calculation of static and dynamic stability derivatives, time history calculation, etc., can be accomplished with minimum user participation leaving the complicated interface calculations to FLEXSTAB (see fig. 1).

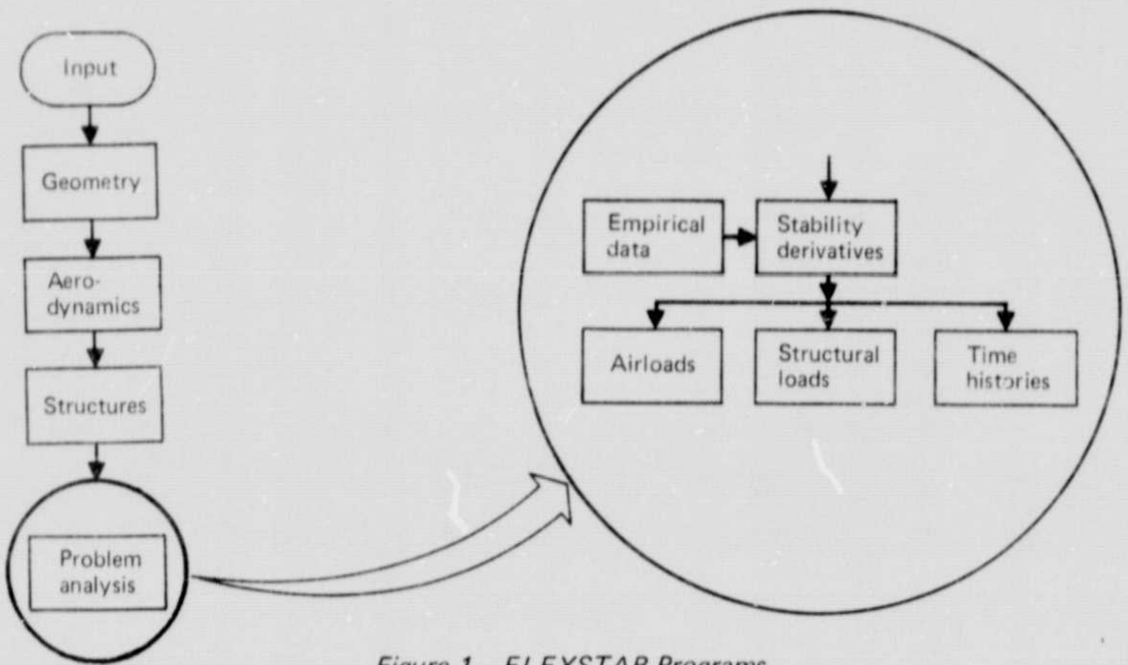


Figure 1.—FLEXSTAB Programs

Boundary Value Problem

The aeroelastic formulation of FLEXSTAB is based upon linearized partial differential flow equations, the associated flow boundary conditions, and a linearized structural model of the configuration (nonlinear effects can be introduced at the problem analysis level, e.g., the introduction of wind tunnel derived-stability derivatives into the time history calculation). A complete description of the aerodynamic boundary value problem

*Tinoco, E. N., and Mercer, J. E.: *FLEXSTAB – A Summary of the Functions and Capabilities of the NASA Flexible Airplane Analysis Computer System*. Boeing document D6-41098, NASA CR-2564, December 1973.

which FLEXSTAB addresses is contained in section 5.0. The aerodynamic and structural models are based upon deformations producing linear pressures, and forces producing linear deformations, respectively. Great simplification results and arbitrary motions can be composed from superpositions of linear solutions.

Numerical Scheme/Aerodynamic Modeling

The numerical models currently employed in FLEXSTAB are *singularity* solutions of the partial differential flow equation. They are of three basic forms within FLEXSTAB: source, doublet, vortex. The source solution is used to represent thickness effects; the doublet and vortex to represent lifting effects. In general, the thickness and lift effects can be separated because of the assumed linearity of the problem. These singularity solutions are distributed over regions of space and are constructed as line segments or panels. The solutions automatically satisfy the differential equations. A complete analysis is accomplished by linearly superimposing combinations of the singularity distributions with the strength of each distribution being determined from the boundary conditions at specific boundary points.

Body-like elements are composed of a line singularity distribution of sources to represent the thickness, and a line singularity distribution of doublets to represent lifting effects. The source distributions are constants over segments of the line in subsonic flow and linear in supersonic flow; the doublets are quadratic functions.

Wing-like elements are composed of panel singularity distributions of sources (thickness) and vortex elements (lifting effects). The sources are linearly varying in the direction of the free stream; vortex distributions are constant over each panel.

Wing-body interference (lifting) effects on the bodies are carried by cylindrical shells of constant cross sections which shield the body-like elements from all other elements. Vortex panels are distributed over these shells; each panel has a constant strength. On the wing, constant vortex panels carry both the interference and lifting effects.

The leading-edge suction at thin wing leading edges must be computed to provide their contribution to the induced drag (negative contribution). This is done in FLEXSTAB by application of the principle of conserved momentum.

Wing-body intersection is treated in a straight-forward manner. The wing intercepts the body at the interference shell and the intersection line always occurs at panel edges. All carryover interference effects are provided by the vortex panels on the shell and wing. No special treatment is required to provide the appropriate jump in pressure across the wing surface at the intersection because of the constant vortex (or pressure) panel elements.

Nacelles are treated in several ways. In each case they are modeled with line distributions of sources and/or doublets and with an interference shell which may (user option) experience interference effects.

Struts are treated as wing-like surfaces. Struts connect nacelles to either wings or bodies.

Limitations or Restrictions of the Numerical Scheme/Aerodynamic Modeling

The partial differential flow equations and flow boundary conditions, along with the structural model, are linearized. This can be a serious restriction for some flow conditions. The linearized wing assumptions break down for *thick* wings and interference effects would accordingly be incorrect. Panels are required to have their side edges parallel to the direction of the free stream and the vortex panels have an inherent planar wake extending to downstream infinity, the edges of which have discontinuous (infinite) jumps in perturbation downwash velocity. This imposes restrictions on panel size and spacing, with downstream surfaces dictating paneling requirements on upstream surfaces. Boundary point placement can be crucial. A dense control surface paneling may require an overly large number of panels over the remainder of the configuration.

The interference shell represents a mean surface of the body. Accordingly, boundary conditions are not satisfied on the actual body geometry. Results may be adequate for bodies of nearly circular cross sections but can introduce serious error for more general bodies.

It is often risky to place small vortex panels in proximity to large ones. Near a wing-strut intersection this may be unavoidable and numerical errors can be introduced in these regions. This can also occur when paneling the wing in the spanwise direction (stream direction panel spacing is not as critical).

Computation

Very large cases have been run in FLEXSTAB, which is virtually an open ended program. Economic considerations, however, tend to keep the number of aerodynamic singularities to a reasonable level (<500). It is not entirely realistic to quote computation times for the aerodynamic influence coefficient (AIC) as they depend on many factors. Different schemes can only be compared when referenced to exactly the same configuration. Such times are therefore not quoted here.

4.2 FUTURE GROWTH POTENTIAL

Ultimate Objectives

The ultimate objective of Task I is to improve portions of the subsonic and supersonic aerodynamic programs by replacing them with a new FLEXSTAB AIC program consistent with the theoretical aeroelastic formulation currently in FLEXSTAB (but founded upon an advanced generation of aerodynamic building blocks) and to implement these advanced aerodynamic methods into FLEXSTAB. The advanced methods will evolve from adaptations of technological developments conducted under NASA-Ames contracts NAS2-6530 and NAS2-7729. The end product will be a new version of FLEXSTAB having improved aerodynamic features that provide greater accuracy and computational efficiency; and that allow more flexibility in the size, shape, and arrangement of aerodynamic panels. This new version of FLEXSTAB is to have no loss in any capability relative to the initial NASA-Ames released version (1.01.00). In particular, a low frequency type approach to the unsteady aerodynamics is to be retained.

Limitations to FLEXSTAB Growth

As presently constructed, FLEXSTAB is not a system of independent computational modules; it is rather a closely coupled system wherein a change in one portion affects the others. For example, the constant vortex panel AIC's could not be immediately replaced with another numerical model without serious consideration of the impact on the remainder of FLEXSTAB. For this reason, two options are being considered for a new version of FLEXSTAB. The first, designated option A retains the present structure and aerodynamic modeling of FLEXSTAB (line body, mean surface wing, and interference shell); option B involves the restructuring of FLEXSTAB and use of the exact body representation. The two options will be discussed in section 8.0.

5.0 BOUNDARY VALUE PROBLEM DESCRIPTION

5.1 PARTIAL DIFFERENTIAL EQUATION

The partial differential flow equation (ref. 4) is derived from the Eulerian momentum equation, continuity, and a relation for the speed of sound under the assumptions of: perfect fluid, no external force field, no heat conductivity, continuum, inviscid, adiabatic, irrotational, except for bounding surfaces and certain prescribed regions. This equation is linearized assuming certain orders of magnitude for the perturbation velocity components. The resulting partial differential flow equation, with axes fixed to the configuration, is:

$$\phi_{xx}\,(1 - M^2) + \phi_{yy} + \phi_{zz} - \frac{2M}{a_\infty}\,\phi_{xt} - \frac{1}{a_\infty^{\,2}}\,\phi_{tt} = 0 \qquad (1)$$

where:

(x,y, z)	=	coordinate axes (x, streamwise direction)
ϕ	=	perturbation velocity potential function
a_∞	=	speed of sound
t	=	time
M	=	Mach number

Equation (1) is the governing equation for the aerodynamic influence coefficient program within FLEXSTAB as well as for the advanced aerodynamic building blocks. The static pressure at a point in the fluid can be related to the free stream static pressure and dynamic pressure by defining the dimensionless pressure coefficient, C_p,

$$C_p = \frac{P - P_\infty}{\frac{1}{2}\rho_\infty U_\infty^{\,2}} \qquad (2)$$

where:

P, P_∞	=	static pressure at arbitrary point and at infinity
ρ_∞	=	fluid density at infinity
U_∞	=	magnitude of fluid velocity at infinity

An exact expression for C_p, derived under the assumptions listed, written in terms of the velocity potential function is (ref. 5):

$$C_p = \frac{2}{kM^2}\left\{\left[1 - \frac{k-1}{a_\infty^{\,2}}\left(\Phi_t + \frac{Q^2 - U_\infty^{\,2}}{2}\right)\right]^{\frac{k}{k-1}} - 1\right\} \qquad (3)$$

where:

k	=	ratio of specific heats
Φ	=	total velocity potential function
Q	=	magnitude of total velocity

The exact C_p expression for an incompressible fluid is:

$$C_p = 1 - \frac{Q^2}{U_\infty^2} - \frac{2}{U_\infty^2}\Phi_t \tag{4}$$

Each of the expressions for the pressure coefficient ((3), (4)) are independent of the coordinate system used. These equations can be linearized, in which case the resultant expressions may not be independent of the coordinate system. In addition, the linearized expressions will differ depending upon the assumed orders of magnitude of the perturbation velocity components. FLEXSTAB uses a linear expression for C_p.

5.2 BOUNDARY CONDITIONS

If F (x, y, z, t) = 0, is the equation of the bounding (configuration) surface, then the general boundary condition is (ref. 6):

$$\frac{DF}{Dt} = 0 = \frac{\partial F}{\partial t} + \nabla\Phi \cdot \nabla F \tag{5}$$

and is the relation which limits the flow to be tangential to the bounding surface. The boundary condition can be linearized and various expressions obtained depending upon the assumed orders of magnitude of the perturbation velocity components and the coordinate system. FLEXSTAB uses linearized boundary conditions.

5.3 NUMERICAL SCHEMES

Many schemes and techniques have been proposed to solve the aerodynamic boundary value problem. The most promising of the methods for aeroelastic analysis are those which supply sufficient information for the structural representation without unnecessary complexity or over simplification. The methods having the most promise for the FLEXSTAB aeroelastic analysis are *singularity methods*. For completeness, additional methods which have been used for the aerodynamic boundary value problem are listed below as *other methods*.

Singularity Methods

Singularity methods have their foundation in the existence of fundamental solutions of the linear flow equation (1) (ref. 7). Derivatives of fundamental solutions and linear combinations of them also satisfy equation (1). From this knowledge are derived the aerodynamic singularities: source, doublet, and vortex. These singularities are distributed on regions of the bounding surface; their value or strength is determined from the boundary conditions on the bounding surface.

The singularity methods have taken various forms. They have been applied to steady and unsteady and subsonic and supersonic flows (refs. 8 and 9). They have been placed at points, along lines, and over area regions of the bounding surface (refs. 1, 10, and 11). Their strength variations over regions of the bounding surface have been constant, linear, and quadratic (refs. 12 through 14). They have been made to span single and multiple line segments and single and multiple area regions (as well as the entire configuration (refs 14 through 16)). The recommended forms of the singularity solutions under consideration for improving the FLEXSTAB AIC's will be presented in section 7.0.

Other Methods

Among the other methods which have been used for aerodynamic boundary value problems are (refs. 5 and 17 through 22):

Finite difference
Finite element
2-, 3-D characteristics
Strip theory
Slender body theory
Conical flow theory
Newtonian impact theory
Piston theory

None of these other methods are considered adequate for a complete FLEXSTAB aeroelastic analysis.

5.4 AERODYNAMIC MODELING

The following considerations are necessary when modeling an aerodynamic configuration.

Body

It is highly desirable to model the actual body surface, something which FLEXSTAB does not do. The interference shell would no longer be necessary and a more realistic discretization of the body would be obtained in terms of surface boundary conditions and resultant solution (pressure distribution). This modeling has been achieved with success (ref. 23) and has been applied to a wide variety of configurations.

Wing

The wing can be modeled as a thick surface (actual surface discretization) or thin surface (surfaces across which the surface velocities are approximately parallel). In subsonic flow, each of these models has been applied successfully. In supersonic flow the thin wing approximation is considered appropriate, because of the existence of physically thin surfaces and numerical difficulties anticipated when dealing with the exact surfaces in too-close proximity in supersonic flow.

Leading-Edge Corrections

Leading-edge suction is the term designating the component of induced drag which cannot be obtained by resolution of forces over a thin wing surface in potential flow. It can be obtained by application of the principle of conserved momentum. It arises from the requirement that the flow turn through a finite angle in an infinitesimal distance. The result is an infinite pressure at the (subsonic) edge. In turning, the infinite pressure acting over an infinitesimal area produces a finite contribution to the induced drag, a component which actually exists on a real configuration for which the leading edge has a finite radius.

A thin wing having a sharp leading edge may not have fully attached flow on the edge. The leading edge suction analogy of Polhamus (ref. 24) can be used to obtain the incremental nonlinear lift resulting from the separated vortex flow.

A subsonic leading-edge correction has been formulated (ref. 25) for a thick wing with linearized boundary conditions.

Wing-Body Combination

It is often difficult to model the geometry of a region where a wing and body come together. Thin wing assumptions may break down in these regions and local surface pressures may be in error. A program in which the body can be accurately modeled helps eliminate these problems. In addition, the treatment of the carryover lift must be properly modeled.

Nacelle

It is important to correctly model the inlet and exit flows of a nacelle. In order to accurately predict pressures near a nacelle and the interference effects which it creates, an adequate geometric representation and numerical scheme must be available.

Strut

Struts are usually small in size and may be located relatively close to regions of large numerical influence. It may be difficult to model them accurately.

Wake/Exhaust

The accurate modeling of wakes and jet exhausts may be required in order to represent an important feature of the physical flow as, for example, the wake from a high lift surface. The proximity of the wake or exhaust to control surfaces is important for the calculation of control derivatives, tail sizing, etc. Although the wake surface position is generally unknown, a more accurate placement of the modeled wake might be available by iteration of this nonlinear problem, experimental data, or intuition.

Control Surface

The numerical treatment of control surfaces is important to obtain accurate control surface characteristics. Depending upon the numerical method selected, the control surface modeling may be straightforward or require special model tailoring to avoid numerical instabilities.

6.0 CANDIDATE AERODYNAMIC MODELING SCHEMES

Several aspects of aerodynamic modeling were presented in section 5.4. Each of the items has important bearing on the full boundary value problem solution and, therefore, is given special attention. A discussion is presented here of the specific areas of aerodynamic modeling, the types of models, and their respective features.

Body

Two of the methods to model the body are (1) the linearized boundary condition *interference shell* concept, coupled with separating the body into the slender body lifting and nonlifting problems (introduced by Woodward (refs. 1 through 3) and subsequently used in FLEXSTAB (ref. 26)), and (2) the exact boundary condition surface paneling technique (ref. 23). These methods have not only been used to model fuselage-like bodies but also nacelles. Nacelles are discussed under a separate heading.

Consider first the linearized problem. The actual body may not be slender nor have a circular cross section. However, these approximations may still be made. The approximation is often made that the (noncircular) body has a circular cross section of equivalent area. In addition, the slender body approximation allows splitting the flow problem into the two linearized problems of axial flow over a body of revolution (the nonlifting problem) and a cross flow over the same body (the lifting problem). For slender isolated bodies of revolution in potential flow, the results are good. Woodward (refs. 1 through 3), and Dusto (ref. 26) have both coupled this approximation with a cylindrical interference shell to perform wing-body analysis. The linearized approximation has application to both subsonic and supersonic flow. The results can be quite good for specialized wing-body combinations but a definitive statement regarding a general configuration cannot be made.

Alternatively, in subsonic flow, the actual body surface can be discretized into surface panels to provide a more accurate description of the physical geometry. The governing flow equation and the surface boundary conditions are linear. Paneling the body surface in supersonic flows is still not generally available, although Woodward reports successful results (ref. 27).

The coupling of the linear aeroelastic analysis and the linear aerodynamic problem is through the aerodynamic load. The use of a paneled body surface in the aeroelastic analysis will require additional study because the aerodynamic boundary value problem is linear in the potential function and the perturbation velocities, while the load is a nonlinear function of the velocities if the load expression has not been linearized.

Note: For reference purposes, the following will be useful in section 8.0 (see fig. 2).

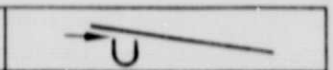
B.1	Line representation	
B.2	Surface representation	

Figure 2.—Notation for Aerodynamic Models of Body

Wing

The wing may be modeled in one of two ways. First, by linearizing the boundary conditions, the wing geometry and boundary conditions may be represented in some mean surface as a decomposition into the nonlifting part (thickness) and lifting part (camber). This is a formulation which lends itself directly to the linear aeroelastic problem. The resulting wing-alone solution for a thin wing shows agreement with more exact methods both in subsonic and supersonic flow, with the possible exception of the subsonic leading edge. For a thick wing, the linearized solution may still be useful although surface pressure may be expected to be in error in some regions. This is generally unimportant for aeroelastic analysis where the aerodynamic load is of primary importance.

Secondly, the actual wing surface may be paneled and exact boundary conditions applied at the wing surface. For thin surfaces, there are two objections to this approach: (1) the solution may be numerically unstable for closely spaced surfaces whose surface perturbation velocities are approximately parallel and (2) the number of panels is doubled, as is the size of the wing influence coefficient matrix. For thick wings, the surface paneling approach may be the only way to achieve satisfactory accuracy for detailed surface pressures. In subsonic flow, this technique has proven successful. It has not yet been successful in supersonic flow, although it has been investigated to a limited extent (ref. 27).

Note: For reference purposes, the following will be useful in section 8.0 (see fig. 3).

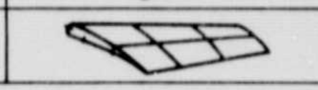
W.1	Mean surface representation	
W.2	Exact surface representation	

Figure 3.—Notation for Aerodynamic Models of Wing

Leading Edge Corrections

If the wing is analyzed as a thick surface, by using surface paneling and exact surface boundary conditions, there is no need to separately calculate the leading-edge suction (or thrust). However, as mentioned in section 5.4, for a thin surface and linearized boundary conditions, it must be calculated separately to obtain an accurate value for the induced drag. For this case, it may also be necessary to introduce a calculation to obtain accurate values of surface pressure near the subsonic leading edge to properly account for the thickness effects (e.g., Riegels' rule, (ref. 25)).

The simplest method to obtain the leading-edge suction is to use the conservation of momentum principle. It requires perturbation velocities in the vicinity of the subsonic leading edge, a straightforward calculation with any of the numerical schemes presented in section 7.0 (for verification of application see ref. 24).

Wing-Body Combination

The isolated body and wing model formulations were discussed in the foregoing under their respective headings. Any combination of those body and wing aerodynamic modeling schemes could be combined for the wing-body problem in a FLEXSTAB type of environment. The combinations which have been successfully demonstrated for subsonic and supersonic flow are shown in table 1 (refs. 1 through 3, 26, and 27).

Table 1.—Wing-Body Aerodynamic Modeling Schemes

Body		Wing		Leading edge corrections	Comment
Line + shell representation	Exact surface	Mean surface	Exact surface		
X		X		X	Subsonic Supersonic
	X	X		X	Subsonic Supersonic
	X		X		Subsonic

Lawrence and Flax (ref. 28) report some of the various formulations which have been used to represent wing-body carry-over effects. These methods are often semi-empirical due to the difficult nature of the problem. (The wing is assumed to be attached to a cylindrical body of constant cross section and infinite length.) The methods are specifically tailored to the flow region, the aspect ratio, and other geometric constraints. Except for the integral representations of the wing-body problem, none of the other formulations presented in the foregoing reference are considered to be candidate aerodynamic modeling schemes in a FLEXSTAB type of environment.

For FLEXSTAB application, the following modeling schemes for the carry-over are being considered. The first is the current interference shell representation of Woodward. In this model, a mean cylindrical shell of constant cross section is used to model the interference effects. The body (a line singularity representation) is considered to be an isolated body in a uniform freestream.

The second scheme is an interference shell of nonconstant cross section.

The third scheme, not properly termed an interference system, is an internal lifting system that is an extension of the wing into the body.

Note: For reference purposes, the following will be useful in section 8.0 (see fig. 4)

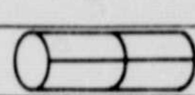
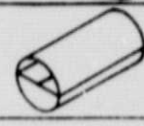
I.1	Interference shell of constant cross section	
I.2	Interference shell of nonconstant cross section	
I.3	Internal lifting surface	

Figure 4.—Notation for Aerodynamic Models for Interaction Effects

Nacelle

The flow characteristics in the region of a nacelle (ref. 29) are probably the most difficult to predict and will be strongly influenced by the aerodynamic modeling scheme selected. A nacelle is generally in close proximity to other surfaces (wing, body, strut, or another nacelle) so that the aerodynamic modeling of the various surfaces cannot be done independently.

There are certain characteristics of nacelles which should be considered, among which are the following:

- Inlet characteristics are important and often must be preserved to guarantee powerplant performance. These include: proper inlet velocity ratios and inlet streamline directions.
- At the exit nozzle it may be necessary to apply the Kutta condition.
- Leading-edge (inlet lip) corrections may be important.
- Exit velocity ratios and streamline directions affect interference pressures.
- Exhaust entrainment and displacement characteristics are particularly important at low speed.
- Momentum changes of the fluid passing through the engine produce forces on the nacelle which cannot be calculated by resolution of surface pressures.
- It may be necessary to properly account for external geometry discontinuities associated with exotic nacelle designs.
- Inlet unstart simulation is very important to the stability and control characteristics (yaw and roll) at high supersonic Mach number (>2).

For FLEXSTAB application, the following modeling schemes are considered candidates for nacelle modeling. The first is a simple flow through nacelle. The nacelle is modeled as an open body; that is, a body with a hole in it. It may have either a constant or nonconstant cross section. No means is provided to control the inlet or exit velocities; they are dictated by the Kutta condition applied at the exit.

A nacelle can also be modeled by representing it as a streamtube whose geometry is specified and which is allowed to deform in an appropriate manner. This is termed a streamtube nacelle.

The third model is a simulation of an engine-on condition. This model has a barrier across the (open) nacelle on which the mass flow is specified.

A schematic of each of these is shown in figure 5, along with notation used in section 8.0.

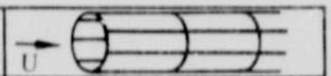
N.1	Flow through nacelle	
N.2	Streamtube nacelle	
N.3	Engine-on nacelle	

Figure 5.—Notation for Aerodynamic Models of Nacelle

Strut

A strut is generally small in size and located close to other surfaces (wing, body, or nacelle). It may be *thin* or *thick* and have either linearized or exact boundary conditions, as was discussed under the *wing* heading. Its physical shape may strongly influence the flow characteristics and its design may be used to control the nearby flow characteristics.

Wake/Exhaust

One wake model and one exhaust model are considered candidates for FLEXSTAB application. The wake model, originating at lifting surface trailing (or leading) edges is specified; that is, its position is assumed to be known. The wake is defined by the geometry of the trailing surface. The planar wake is a subset of this more general model.

The exhaust model is intended to simulate the existence of a plume emanating from a nacelle. A plume shape and entrainment distribution are assumed to be known.

The wake and exhaust models are illustrated in figure 6, along with the notation for these models which is used in section 8.0.

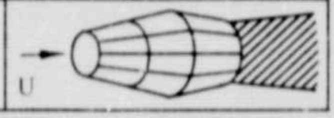
E.1	Specified wake	
E.2	Jet Exhaust	

Figure 6.—Notation for Aerodynamic Models of Wake and Jet Exhaust

Control Surface

The control surfaces can be very adequately modeled with the previously mentioned modeling techniques for the wing and wake. In addition to all of the previous discussions, it is important to have the option to model a complete configuration, as contrasted with modeling schemes (e.g., FLEXSTAB) which assume certain symmetry properties for the configuration and deal only with a half model. Such an option would enable analysis of antisymmetric configurations such as the slewed wing (ref. 30).

7.0 CANDIDATE NUMERICAL SCHEMES

7.1 SELECTION OF CANDIDATE SCHEMES

The numerical schemes considered candidates for FLEXSTAB all fall into the class designated singularity methods. The fundamental solutions of the linearized flow equation (1), are:

Source: $$\phi\,(x,y,z,t) = \iint_{S'} \sigma\,(x',y',t)\; K_\sigma(x',y';\, x,y,z,t)\; dx'dy' \tag{6}$$

Doublet: $$\phi\,(x,y,z,t) = \iint_{S'} \mu\,(x',y',t)\; K_\mu(x',y';\, x,y,z,t)\; dx'dy' \tag{7}$$

Vortex: $$\phi\,(x,y,z,t) = \iint_{S'} \gamma\,(x',y',t)\; K_\gamma(x',y';\, x,y,z,t)\; dx'dy' \tag{8}$$

Each solution ϕ (x, y, z, and t) is expressed as an integral relation involving an unknown singularity strength distribution function (σ, μ, and γ) and a known kernel function form (K_σ, K_μ, and K_γ). The singularity distributions are shown written for the plane $Z' = 0$. The kernel functions depend upon the flow regime (subsonic or supersonic) and whether the flow is steady or nonsteady. However, they are not shown here since the exact forms do not contribute to the discussion, but may be found in references 4 and 31. Directly of interest to this discussion are the forms of the strength distributions: σ, μ, and γ.

Source

Source singularity solutions have probably been used for more varied applications than any other singularity method. This is due first to its mathematical simplicity and second to its ability to simulate a real, physical flow. They have been used to model complete wing-body combinations (ref. 23), nonlifting subsonic flow (ref. 32) and lifting supersonic flow (ref. 33)

A surface distribution of sources has a jump in normal velocity associated with crossing from one side of the surface to the other. Sources have been used to represent nonlifting thick bodies or wings for both steady and nonsteady flows. Supersonic lifting wing problems can also be formulated with sources such that the upper and lower surfaces do not experience one another's influence (Mach box (refs. 33 through 35) and characteristic box (ref. 36)).

In its simplest form, the source strength (σ) is assumed to be constant on a given region of the bounding surface. In general, the strength can be taken arbitrarily, having the parameters which define the variation determined in a specified manner by conditions on the bounding surface.

As a specific example, the source spline scheme is being formulated and developed under Task III of NASA-Ames contract NAS2-7729. It has two characteristics. First, the strength variation is linear in each of two coordinate directions. Second, the surface curvature effects are included within the formulation. The constant strength, flat panel representation (refs. 23 and 37) is available as a special case. The practical application of the source spline is easily recognized and the consistency of the mathematical formulation has been demonstrated (ref. 38). It has application both to subsonic and supersonic flow, both to linearized and exact boundary conditions.

Doublet

A surface distribution of doublets has a jump in tangential velocity associated with crossing from one side of the surface to the other. Such a singularity surface finds application in lifting problems where jumps in velocity potential occur, e.g.; subsonic and supersonic, and for both steady and nonsteady. The doublet is a natural numerical modeling element for the wake region associated with lifting solutions.

The doublet strength (μ) may be taken constant (Doublet Lattice, refs. 39 and 40) or have some nonconstant variation (doublet splines, Task III, NAS2-7729). Although mathematically more simple, the doublet lattice is only a subset of more general distributions, such as the doublet spline.

As a specific example, the doublet spline is being formulated and developed under Task III of NASA-Ames contract NAS2-7729. It has a strength variation which is quadratic in each of two coordinate directions and includes surface curvature effects within the formulation. The vortex lattice type of numerical method is available as a special case. No other reference to the doublet spline is known.

Vortex

Vortex models have an inherent planar (kinematic) wake. They are formulated by superimposing doublet solutions and are useful for lifting problems where the physical location of the wake is relatively unimportant, as is the case for the linearized, thin wing problem. The surface vortex distribution has a jump in tangential velocity associated with crossing from one side of the surface to the other.

The vortex strength (γ) may be constant, as reported by Woodward and used in FLEXSTAB (NAS2-5006), or have a more exotic variation such as the quadratic variation of the vortex spline (NAS2-6530). The vortex singularity has application to lifting problems, e.g.; subsonic and supersonic and for steady and nonsteady.

The vortex spline was introduced under NASA-Ames contract NAS2-6530. Panels are defined over which quadratic strength variations exist in the spanwise direction and linear variations in the chordwise direction. A given spline function, defined by one free parameter governing its value spans a specified grid of neighboring panels such that the following conditions are satisfied for a given vortex spline function:

- The value of the strength and slope (spanwise derivative) of the strength are zero on the edge of the grid except for possible special regions like the tips and the root.

- No discontinuities of value or slope occur within the grid in the spanwise direction.

- Streamwise variations of strength are linear and continuous.

Because of these conditions, although an inherent wake is present, there are no discrete trailing elements which produce infinite perturbation velocities in their vicinity. In addition, perturbation velocities are continuous throughout.

A least squares formulation of the boundary conditions was used to obtain the free parameters characterizing the singularity strength variations. The excellent success of the method has previously been reported (ref. 15). The method has been extended to the nonsteady subsonic flow regime under Task IV of NAS2-7729 with excellent results.

The use of any of the foregoing singularity types are conceptually identical. In general, a singularity type is chosen for its ability to simulate a physical problem (see sec. 6.0, *Aerodynamic Modeling*). Having selected the singularity type, the values of the parameters governing its predetermined type of strength variation are established by imposing the boundary conditions on the boundary surfaces. The process is conceptually independent of the type of singularity variation under consideration (i.e., constant strength, quadratic, etc.).

The singularity methods being considered as candidate schemes include: (1) FLEXSTAB aerodynamics (the reference point), (2) vortex spline (NAS2-6530), (3) source spline (NAS2-7729), (4) doublet spline (NAS2-7729), (5) linearly varying vortex panels of Woodward (refs. 27 and 41) (6) Doublet Lattice (ref. 39), (7) line singularity source (refs. 1 and 10), and (8) line singularity doublets (refs. 1 and 11).

Loading function methods such as that of Rowe (refs. 16 and 42), although singularity methods, are of a type requiring the functions to span great regions of a geometrically restricted configuration and are not considered in this report. These methods can, however, provide valuable comparisons for the methods considered here.

7.2 FEATURES OF CANDIDATE SCHEMES

Some of the features of the candidate schemes will now be discussed. Of interest is the formulation, the application to the boundary value problem, any restrictions imposed, computation features, and the future growth potential for the method. The level to which the scheme has been developed is discussed.

Formulation

The FLEXSTAB steady state problem is formulated using the source and vortex fundamental solutions, equations (6) and (8), with σ and γ constant over *panels*. The source and vortex solutions share common paneling.

The vortex panel has an associated wake which requires quadrilateral or triangular panels having side edges (possibly of zero length) in the direction of the free stream. This is illustrated in figure 7.

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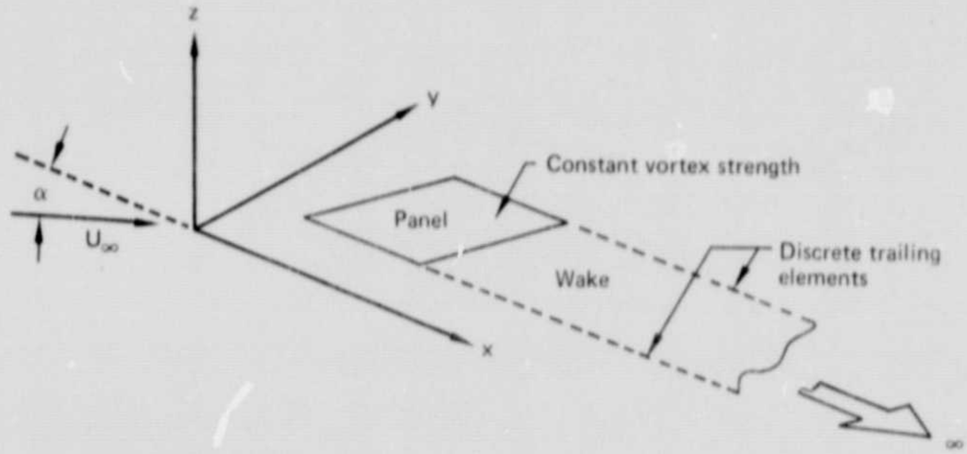

Figure 7.—Constant Vortex Panel Geometry—FLEXSTAB

The panel geometry is defined in a panel axis system such that the panel is in the plane Z = 0. The boundary conditions have been linearized, all being expressed in the plane Z = 0. There is one boundary point per panel.

The wake extends to downstream infinity parallel to the X-axis. When a discontinuity of vortex strength or its spanwise derivative is present, as in this formulation, infinite perturbation velocities occur in the vicinity of the discrete trailing elements (see fig. 7).

Mathematically the vortex or pressure panel is a surface across which a discontinuity of tangential perturbation velocity exists. The perturbation velocities are related to the vortex singularity strengths. The perturbation velocities are applied to the boundary condition equations from which the unknown singularity strengths are determined.

The vortex spline scheme is built upon a similar foundation. The paneling is identical to that of figure 7. However, instead of the singularity existing as a distribution over a single panel, each vortex spline, characterized by one free parameter, spans a grid of neighboring panels, generally a grid of 3 or 4 panels in span and 2 panels in the streamwise direction. The spanwise variation is quadratic, the streamwise is linear (see figs. 8 and 9).

A wake region exists in the vortex spline formulation but no discrete trailing elements are present because of the absence of jumps in singularity value or spanwise slope, i.e.; no infinite perturbation velocities occur in the wake.

The linearly varying vortex panels of Woodward (ref. 27) have a variation which is linear chordwise and constant spanwise. Each of the singularity elements is characterized by one free parameter. Unlike the vortex spline, this method does have discrete trailing elements because of the discontinuous (step) spanwise variation and, therefore, infinite perturbation velocities do occur in the wake just as they do for the constant pressure panel method. A similar procedure is reported by Lopez and Shen (ref. 43).

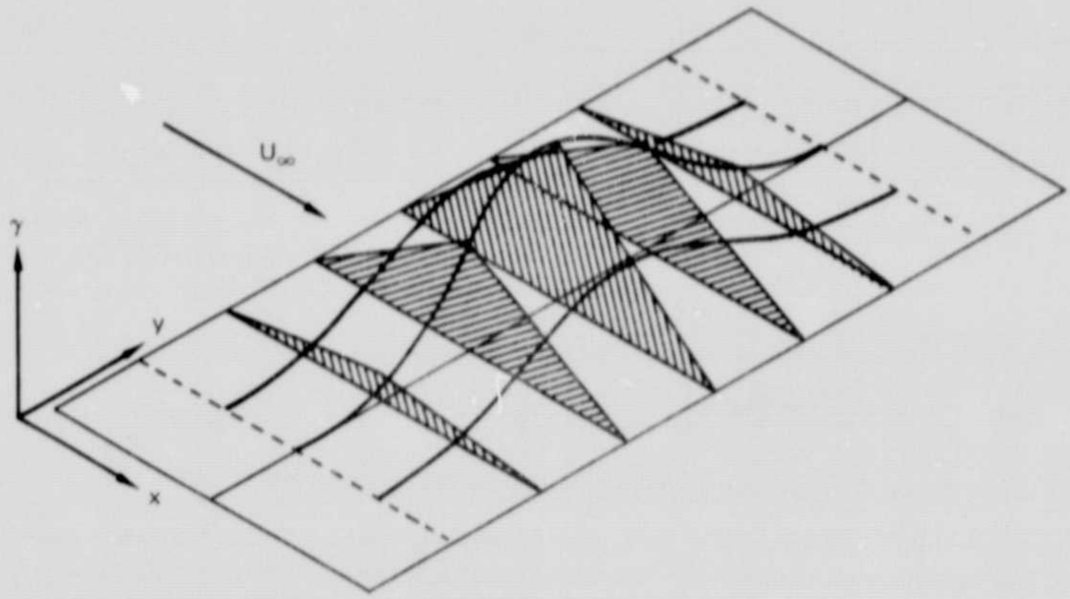

Figure 8.—Surface Spline Distribution of Vorticity

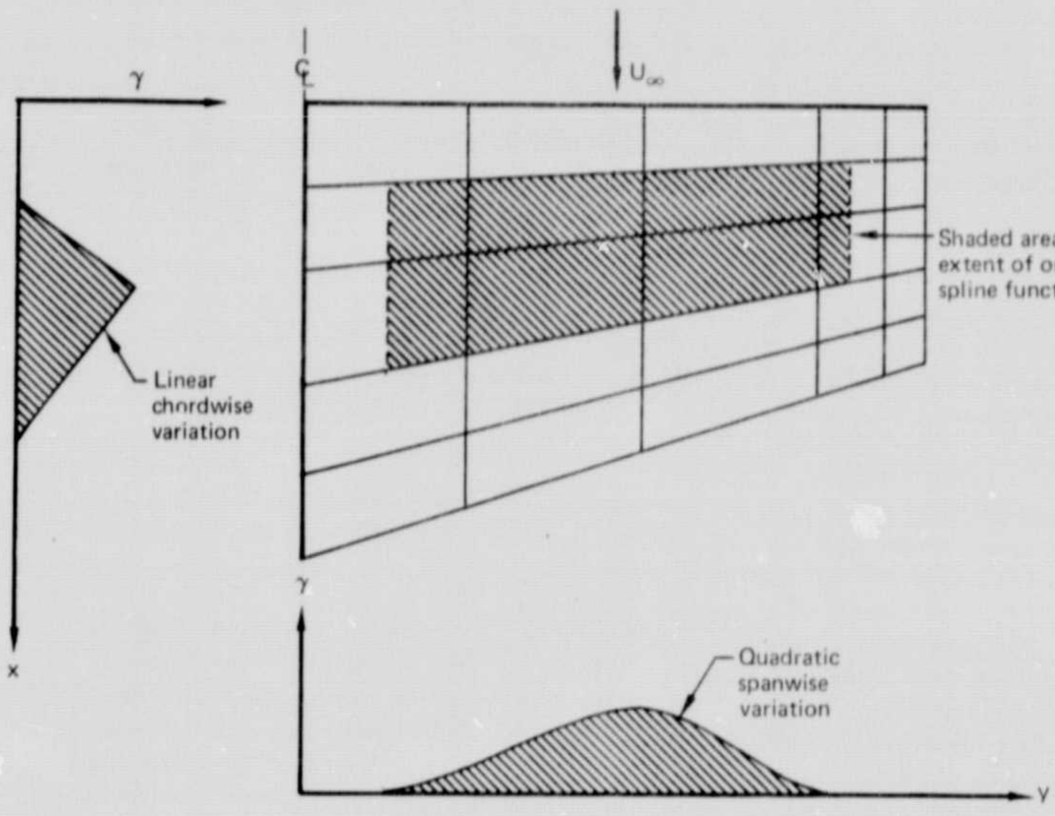

Figure 9.—Spline Functions Distributed on a Wing (Symmetric Flow)

The source spline and doublet spline have no inherent wake. Accordingly, the geometry requirements imposed by the vortex methods are not necessary. The only requirement is that the panels be quadrilateral or triangular. The source and doublet splines are therefore directly amenable to nonlinearized boundary conditions and arbitrary panel configurations, in contrast to the vortex methods.

Mathematically the source and doublet panels are, respectively, surfaces across which either the perturbation normal velocity or the perturbation tangential velocity has a jump discontinuity. Most often the source is used to simulate thickness effects and doublets the lifting effects, however; for thick (nonlinearized) boundary surfaces, both types together can usually be used on the surface and/or in the interior of bodies.

The formulation of the source spline and doublet spline were presented in detail in the proposal submitted to perform this work. (*Task I – Splined Version of FLEXSTAB*, Boeing document D6-41761, April 1974, submitted in response to Task I of RFP letter dated February 6, 1974, NASA-Ames). The source spline formulation has a linear variation in each of two directions. Local surface curvature is also included. The source strengths are not restricted to be continuous in value or slope at panel edges, but continuity can be very nearly enforced by the solution.

For subsonic flow, the three-dimensional doublet spline formulations for analysis and design are not strongly geometry dependent. The doublet distributions are continuous quadratic functions over panels but (from panel to panel) are not necessarily continuous in value and slope at all points of the panel edges. The doublet spline formulation leads to linear equations in the unknown doublet strengths which enables a linear influence coefficient formulation of the entire problem. Surface curvature effects are included. The doublet spline formulation has led to a simplified form for the influence coefficients. Because the doublet distribution is represented by a simple polynomial (quadratic surface function), all integral expressions for the influence coefficients can be integrated in closed form and with considerably fewer terms. This produces less computation time and provides greater numerical reliability.

For supersonic flow, supersonic doublet splines are being developed as a part of the Task III (NAS2-7729) studies. Studies of the supersonic doublet splines were earlier initiated under NASA-Ames contract NAS2-6530. In that study, the doublet characteristic box method was introduced. Briefly, the method has the following features. First, special Mach lines emanating from planform edge breaks are identified. These lines are used to divide the planform into several different regions and each of these regions is divided into a network of panels bounded by Mach lines. Each panel may then fall into a relatively small (<10), restricted class of basic numerical building block elements. A planform is represented by a superposition of these elements. A quadratic doublet singularity distribution, having a specific number of free parameters, is assumed to exist over each panel or building block element. The unknown free parameters are determined by combinations of downwash conditions and vorticity continuity conditions. Application of this method to a cambered triangular region bounded by a supersonic leading edge and one special Mach line has produced results which are indistinguishable from exact theory.

The doublet lattice (fig. 10) is a method applicable only to subsonic flow. A line of acceleration potential doublets lies along a panel quarter chord and has an unknown (constant) singularity strength. One collocation point, located at the panel mid-span and at the three-quarter chord, is used to determine the strength; this boundary point position generally differs from that used for the nonlifting problem. Trailing vortex elements are included to allow for lifting solutions. The doublet lattice method has direct application to the arbitrary frequency nonsteady problem (ref. 39) and has been applied to a wide range of applications (e.g., ref. 44).

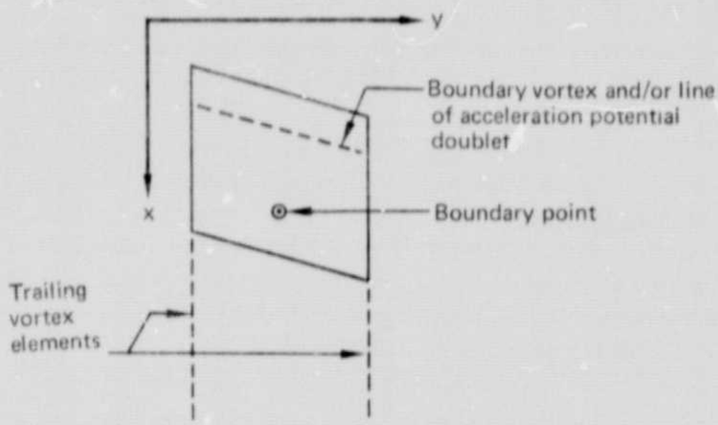

Figure 10.—Doublet Lattice Formulation

The line source and line doublet methods distribute sources and doublets along an axial line, the variation of which may be constant, linear, quadratic, etc., in a coordinate system representative of the line. Boundary points are located at selected points along the line at the real (or imagined) surface. Figure 11 illustrates the line singularity formulation.

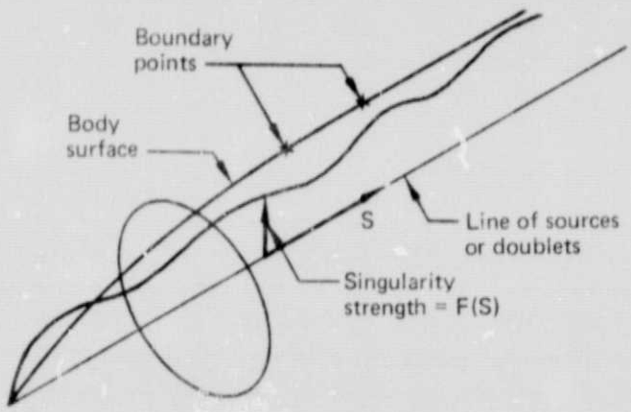

Figure 11.—Line Singularity Formulation

Boundary Value Problem Application

The vortex methods are generally used with linearized boundary conditions because of the inherent wake. The source and doublet methods are used with linearized boundary conditions and also with exact boundary conditions in subsonic flow. For supersonic flow, Woodward (ref. 27) has been successful in using sources on a body surface with exact boundary conditions. The work of Task III (NAS2-7729) has investigated source and doublet spline surface paneling in supersonic flow. The line singularity methods employ linearized boundary conditions.

The vortex methods are more restricted in paneling requirements than the source and doublet. Of the two candidate vortex methods, FLEXSTAB's constant pressure panels have the most severe paneling restrictions, having not only the wake imposed restrictions on downstream paneling but also restrictions on the relative size of neighboring panels and the location of the boundary points. The vortex spline does not suffer as markedly from any of these paneling restrictions. However, it may be more difficult to set up the geometry of a general configuration for the vortex spline. The restrictions seem to be of minor concern for the source and doublet methods. Cases have been run using the doublet spline for which panel corner points were generated in a random manner with no loss in computational accuracy compared to cases run with more conventional paneling.

The use of doublets to simulate a lifting configuration would require a paneled wake geometry across which a jump in potential (doublet strength) can occur. This is an advantage when the geometry of the wake is important as in high lift configuration applications. Only a few panels may be necessary to represent the entire wake, e.g.; in two-dimensional flow, one semi-infinite doublet panel can represent the entire inherent planar wake produced by the vortex method.

A note of caution is in order concerning the vortex spline method. If the geometric panel grid is planar, the vortex spline produces no discrete trailing elements. If the geometry has curvature, the curvature must be continuous in value and spanwise slope to avoid discrete trailing elements produced by the geometric discontinuity. This not only imposes additional geometry paneling requirements, but also generates additional complexity within the influence coefficient kernel functions (see eq. (8)). Whereas the other candidate schemes require only single boundary points per panel, the vortex spline may use the method of least squares which requires four boundary points per panel in supersonic flow and two in subsonic flow. This increases the size of the influence coefficient matrices and, accordingly, the computation of either a matrix solution or a matrix inversion, one of which is necessary for the analysis problem. Since the constant pressure panel method of FLEXSTAB has the discrete trailing elements in general, whether they are produced by discontinuities in singularity strength or geometry, and it, therefore, can be applied to configurations of arbitrary spanwise geometry when proper account is taken of the paneling requirements.

Any of the singularity schemes presented may be applied to the solution of the appropriate nonsteady boundary value problem. The governing differential equation and boundary condition are given by the linear equations (1) and (2). A linear relationship can be found relating the zero-order potential solution (steady-state flow) and the first-order in time potential through a nonhomogeneous linear boundary value problem.

Computation Features

A quantitative comparison of the computation features of the candidate methods is difficult. Certainly the efficiency of the coded algorithms determines the length of the computation time for any of the methods. In addition, the form of the integration (i.e., analytical and/or numerical quadrature of the integral expressions), even if efficiently coded, may not prove to be the fastest manner of computation possible. Table 2 has been assembled for the purpose of illustrating the items which affect the computational speed and efficiency along with other features of the various schemes. Note that some items, such as surface curvature, may increase a local computation cost but may decrease the total (global) cost because less panels are required.

Table 2.—Comparison Features of Candidate Numerical Schemes

	Constant vortex panel FLEXSTAB	Vortex spline NAS2-6530	Source spline NAS2-7729	Doublet spline NAS2-7729	New woodward — linear-in-chord	Doublet lattice	Line singularities
Presently developed	No	No	Yes	Yes	No	No	No
Curv. incl. AIC's avail.	Yes	Yes planar	Sub yes / Sup no	Sub yes / Sup no	Yes	Yes	Yes
Present form of integrations	Analytic log $\tan^{-1}$	Analytic and numerical	Analytic log $\tan^{-1}$	Analytic log $\tan^{-1}$	Analytic	Analytic	Analytic Sinh^{-1} Cosh^{-1}
Inherent wake	Yes	Yes	No	No	Yes	No	No
No. control pts./panel subsonic	1	2 least squares	[a] 1	[a] 1	1	1	1 per segment
supersonic	1	4 least squares	[a] 1	[a] 1	1	Not applicable	1 per segment
Sing. st. var. chordwise	Constant	Linear	Linear	Quadratic	Linear	Constant	Up to quadratic
spanwise	Constant	Quadratic	Linear	Quadratic	Constant		
Rel. comp. time per AIC (no curvature	Moderate	Most	Least	Moderate	Moderate	Least	Least
Rel. no. of panels required	Most	Moderate to least	Least	Least	Moderate	Moderate	Line segments
Paneled wake req'd. for lifting solution	No	No	Yes	Yes	No	Yes	Not applicable
Paneling restrictions	Major, particularly spanwise	Planar: minor nonplanar ???	Very vew	Very few	Major spanwise	Major spanwise and chordwise	Not applicable
Boundary conditions	Linearized	Linearized	Nonlinear or linear	Nonlinear or linear	Nonlinear (subsonic only) or linear	Linear	Linear
Applicable to general configurations	Yes, mean surface only	May require maj. work mean surface	Yes	Yes	Yes, mean surface only for supersonic	Yes, mean surface only	Bodies of circular cross section

[a]Internal panels of a network

Current Level of Development (August 1974)

The FLEXSTAB constant vortex panel aerodynamic influence coefficient program is fully operational. This includes routines to perform the low frequency nonsteady analysis for nonplanar configurations. The vortex spline has been formulated for the steady flow, planar configuration case with some work in subsonic nonsteady flow. In subsonic flow, both the source and doublet splines are formulated for steady nonplanar flow. In steady supersonic flow, the doublet and source spline are under development. The doublet lattice method (subsonic flow only) is well developed and documented for steady and nonsteady flow. The line singularity method is similarly available for both subsonic and supersonic flow.

Future Growth Potential

The constant pressure panel has been successfully used for the subsonic and supersonic flow regimes, both steady and low frequency nonsteady. For steady flow, a linear chordwise variation has been introduced (ref. 27). The vortex spline is considered less general than the constant pressure panel due to the configuration restrictions previously discussed.

The source and doublet spline methods are proving to be extremely powerful and have application to a wide range of problems. To illustrate the level at which the source and doublet splines are performing, it was the numerical building block used in the work for NASA-Langley contract NAS1-12185 and its follow-on contract NAS1-13833. The objective was to conduct an analytical study, develop a method of solution, and develop a computer program to predict the subsonic aerodynamic loads on a delta-like wing over which a leading-edge vortex exists. This is a very complex nonlinear mixed analysis and design problem requiring an iteration procedure for solution. The source and doublet spline successfully handle this problem, whereas the vortex panel methods have little chance for success. The source spline with curvature has recently been used to calculate the subsonic potential flow over a sphere. A constant source panel method (ref. 32) required 1342 elements while the source spline required only 162 elements to achieve the same accuracy.

Figure 12 is a qualitative illustration of where the various panel singularity methods fall on a line of computational accuracy. Note that some items on the line are not attainable as, for example, exact solutions are difficult to obtain (one of the only known exact three-dimensional solutions is for incompressible flow over a circular wing).

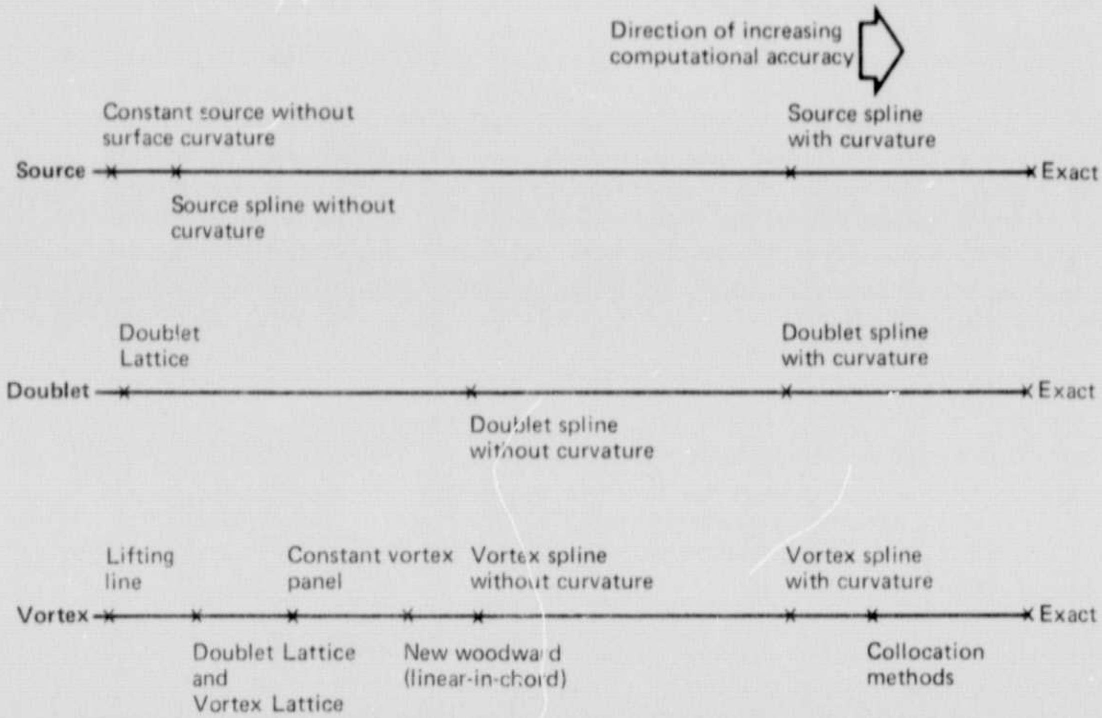

Figure 12.—Panel Singularity Methods

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8.0 AERODYNAMIC MODELING/NUMERICAL SCHEME COMBINATIONS

It is the purpose of this section to make recommendations from among the candidate schemes of sections 6.0 and 7.0 and to assess the impact of the recommended combinations on the ultimate objectives of Task I. An estimate is made of the resources required to achieve those objectives.

8.1 RECOMMENDATIONS

Before recommendations can be made, it is necessary to consider the particular version of FLEXSTAB to which they relate. This was briefly introduced in section 4.2. The present FLEXSTAB system is an intimate collection of programs. A revision of any magnitude generally impacts the entire program. This is because the original program was formulated without any particular regard to a modularized structure. The condition was precipitated by taking off-the-shelf programs (primarily the AIC programs) and forcing conformity at another level of the FLEXSTAB system, for example, the aerodynamic programs and internal structural influence coefficient and external structural influence coefficient programs (ISIC and ESIC) were interfaced within the stability derivatives and static stability programs. This has produced a close-coupled system of programs for which the general interface problem was never fully addressed. A change within the aerodynamics program is sure to affect portions of the SD&SS program. This formulation of FLEXSTAB is here labeled a Level 1 version for later discussion.

A Level 2 version of FLEXSTAB is one which has had careful attention paid to a *modular structure* type of formulation. In simplistic form, one routine could be unplugged and another substituted in its place. A Level 2 version of FLEXSTAB would not have the inherent weaknesses of the Level 1 version. It would be able to dynamically participate in the changing environment of advancing technology with the flexibility to specialize it for specific applications. The interfaces at which fundamental data flow takes place is of primary importance for Level 2 work. This data flow should be clearly identified and the particular format specified. Level 2 provides the best chance that advancing technology can be received into the FLEXSTAB environment.

A general flight vehicle configuration is composed of many items: wing surface, body, control surface, nacelle (engine), strut, wake and/or exhaust region. It is considered advantageous to have various degrees of component modeling. This enables flexibility of application for the users and a gradation of accuracy for the various levels of analysis common to the aeroelastic design cycle, from preliminary design through final configuration analysis.

For purposes of discussion, a notation has been developed to allow easy reference to the various candidate aerodynamic (table 3) and numerical schemes (table 4) presented in the preceding two sections. Only certain combinations of these aerodynamic models and numerical schemes are possible. Table 5 illustrates them with an X. Within table 5 is shown the present FLEXSTAB combination for wing-body analysis (B.1:LS, LD; W.1:

SS (constant), VC; I.1: VC; N.1: VC). The body is represented by line singularity distributions of line sources and line doublets. The wing is a mean surface wing of constant source and vortex panel, and the interference shell is composed of constant vortex panels, as is the flow through nacelle.

Table 3.—Aerodynamic Modeling Schemes

• Body	
B.1	Line representation for isolated body (+ I.1, and I.2)
B.2	Surface panel representation (+ I.3)
• Wing	
W.1	Mean surface
W.2	Exact surface
• Interference	
I.1	Interference shell of constant CS
I.2	Interference shell of nonconstant CS
I.3	Internal lifting surface
• Nacelle	
N.1	Flow through (either W.1, W.2, I.1, or I.2)
N.2	Streamline nacelle (appropriate B.1)
N.3	Engine on (W.1, W.2, 1.1, or 1.2)
• Exhaust and wake	
E.1	Specified wake
E.2	Jet exhaust wake

Table 4.—Numerical Schemes

• Line	
LS	Line source
LD	Line doublet
• Source Panel	
SS	Source spline
• Doublet panel	
DS	Doublet spline
DL	Doublet Lattice
• Vortex panel	
VC	Constant
VS	Vortex spline
VL	Linear-in-chord

Table 5.—Possible Combinations of Numerical Schemes and Aerodynamic Models

		Line		Source	Doublet		Vortex		
		LS	LD	SS	DS	DL	VC	VS	VL
Body	B.1	X	X						
	B.2			X	X	X			
Wing	W.1			X	X	X	X	X	X
	W.2			X	X				
Interference	I.1				X	X	X	X	X
	I.2				X	X			
	I.3				X	X	Planar only		
Nacelle	N.1			X	X	Linearized boundary condition only			
	N.2	X	X						
	N.3			X	X	X	X	X	X
Wake/exhaust	E.1				X	X	Planar only		
	E.2			X	X	Constant cross section			
Wing-body	W.1			X	X	X	X	X	X
	+								
	B.2			X	X	X			
	+								
	I.3				X	X	Planar only		
	W.1			X	X	X	X	X	X
	+								
	B.1	X	X						
	I.1			X	X	X	X	X	X
	or								
	I.2				X	X	X	X	X

Two options are now presented which are aimed at the two particular version levels previously discussed for the FLEXSTAB system. Within each option, recommendations are presented to achieve the ultimate objectives of Task I. Within the recommendations there is not yet a clear choice of one numerical scheme over another. There are four reasons for this.

1. Not all candidate schemes (e.g., supersonic doublet spline) are sufficiently developed to warrant an unqualified acceptance or rejection. Such schemes are being included with the idea that they will be more fully evaluated at a later time.

2. Different levels of modeling combinations and levels of accuracy should be available to a user to more adequately match his immediate need and the knowledge he has of his configuration. This may be possible under option B.

3. Verification of the recommended combinations is discussed in section 8.3. The studies conducted at the time of verification will not necessarily eliminate a particular modeling combination, but instead will serve to establish the important comparison characteristics of the several modeling combinations.

4. The impact of the modeling combinations on the low frequency capability is discussed in section 8.2 in a qualitative manner. Such discussion can only be quantitatively established at the time of verification (see point 3).

Because the formulations of the doublet splines and source splines include higher order singularity strength variations and surface curvature and because the other schemes are subsets of these, they are to be considered the preferred schemes.

Option A

Option A assumes that only a Level 1 version of the FLEXSTAB system is available, that is, the present FLEXSTAB will not undergo a restructuring procedure. Accordingly, the entire aerodynamic modeling of FLEXSTAB is retained, i.e.; line body, mean surface wing, interference shell, etc. The use of an exact body aerodynamic model in the Level 1 version is considered unwise. The numerical methods recommended under option A are the new technology methods emerging from the work of Task III (NAS2-7729) along with the vortex spline (NAS2-6530). These numerical methods would be applied to the wings and shells.

Several key points can be noted regarding the numerical methods. Source splines are used to simulate thickness effects for the (linearized boundary condition) wing. Vortex splines or doublet splines may be used for lifting effects of the wings and shells, although the choice of doublet splines allows the flexibility to use the specified nonplanar wake model (E.1). The linear-in-chord vortex method is not recommended because it is not a significant improvement over the constant pressure panel method, and is a subset of the vortex spline. The recommended combinations for option A are shown in table 6.

Table 6.—Option A: Recommendation of Combinations for Wing-Body Analysis

	Line		Source	Doublet		Vortex	
	LS	LD	SS	DS	DL	VC	VS
B.1	X	X					
W.1			X	X	X	X	X
I.1				X		X	X
I.2				X			

Option B

Option B assumes that FLEXSTAB will be restructured to the status of a Level 2 system of programs. The exact body surface aerodynamic model is recommended (B.2) to provide an improved modeling capability not presently available. The selection of the exact body model all but eliminates the vortex numerical methods for the exact body representation because of their inherent wake, but does not necessarily eliminate them for other modeling applications (e.g., wing representation).

The aerodynamic model recommended for the wing is the mean surface representation presented with option A and used within the present FLEXSTAB system. This model employs linearized boundary conditions and includes thickness and lifting effects. This analysis is generally adequate for sections up to 15% thick. For aeroelastic analysis, the load is important. Surface pressures may also be obtained.

Apart from the direct application for aeroelastic analysis, there is another point which should be considered. This is the calculation of lateral-directional derivatives (ref. 45). When the wing dihedral is very near zero and the linear representation is used, it is difficult to compute certain coupled stability derivatives. For nonzero wing dihedral, the method shows no such difficulty and those particular stability derivatives are of second-order.

The exact surface representation of the wing (W.2) would exhibit no difficulty for near zero dihedral, unlike the linearized one. However, it has not been firmly established that the linearized model could not be properly formulated to remove this difficulty. This should be investigated.

In addition to its direct application to aeroelastic analysis, the mean surface representation would result in smaller matrix sizes and therefore faster computation times.

The numerical methods recommended for the mean surface wing are the source splines for thickness and vortex or doublet splines for the lifting effects. The use of doublet splines allows the flexibility to use the specified nonplanar wake model (E.1) and is thus preferred. The linear-in-chord vortex method is not recommended because it is not a significant improvement over the constant pressure panel method, and is a subset of the vortex spline. The combinations for option B are shown in table 7.

Table 7.—Option B: Recommendation of Combinations for Wing-Body Analysis

	Line		Source	Doublet		Vortex	
	LS	LD	SS	DS	DL	VC	VS
B.2			X				
W.1			X	X	X	X	X
I.3				X	X	Planar only	

Nacelle, Wake, and Exhaust Combinations

Both options A and B can employ the following combinations within their analysis. A linearized boundary condition formulation is suggested for option A; either exact or linearized boundary conditions are suggested for option B.

Three alternate nacelle aerodynamic models were presented in section 6.0: flow through, streamtube (entrainment), and engine-on. For models N.1 and N.3, the source spline is recommended for thickness effects and the vortex spline (linearized boundary conditions) for lifting effects. The use of the doublet spline allows the flexibility to use the specified, nonplanar wake model (E.1). The nacelle model (N.2) employs a line source and thereby is able to simulate entrainment effects (as an entrainment model it is more properly termed a jet exhaust model). These alternate models allow a range of capability and modeling sophistication not presently available in FLEXSTAB. Nacelles no longer need to be circular nor to be of constant cross section.

The wake aerodynamic model (E.1) is specified by its geometric description, the planar wake being a subset of the nonplanar. Because of the absence of an inherent planar trailing wake, the doublet methods are recommended for the numerical scheme.

The jet exhaust model (E.2) employs either the doublet splines and/or source splines for its numerical description. The plume shape can be specified by the user or set by a predetermined *default* option.

The combinations just described are shown schematically in table 8.

Table 8.—Recommendation of Combination for Nacelle Wake and Jet Exhaust Analysis

	Line		Source	Doublet		Vortex	
	LS	LD	SS	DS	DL	VC	VS
N.1			X	X	X	X	X
N.2	X						
N.3			X	X	X	X	X
E.1				X			
E.2			X	X			

8.2 DISCUSSION OF TASK I OBJECTIVES

Each of the combinations recommended in section 8.1 (especially option B), adds markedly to the present aerodynamics capability of the FLEXSTAB system. Because the constant pressure panel has not been eliminated from consideration it is clear that no capability, relative to the NASA-Ames released version 1.01.00, has been lost. The combinations offer additional variety for modeling and accuracy levels not yet available in the FLEXSTAB system.

The present nonsteady capability within FLEXSTAB is limited to the low-frequency approximation. None of the above combinations is expected to cause a loss of this capability. In fact, the studies of Task IV (NAS2-7729) with the vortex spline have demonstrated conclusively its applicability to the full nonsteady problem. The doublet methods will no doubt also have application to the more general arbitrary frequency and Küssner-Wagner formulations of the nonsteady problem (ref. 46). This of course will need to be verified outside of the present study for the FLEXSTAB environment.

It is possible to formulate the aerodynamics program in a manner to guarantee that it can be interfaced with other program types (e.g., structures). This will require much work to insure that appropriate interface data is made available for the wide range of potential uses. The present FLEXSTAB is not formulated with the *functional module* concept; the aerodynamics, structures, geometry, etc., are intimately related. This point will determine the future course of the present study and the ultimate fulfillment of the Task I objectives. It is outside the scope of the present study to offer recommendations as to how FLEXSTAB should be structured. Unless a major reorganization is performed to formulate the functional module concept for FLEXSTAB (Level 2 version), little chance is given that FLEXSTAB will interact dynamically with advancing technology.

Retention of the Level 1 version of the program will dictate that any improved aerodynamic capability will have to be converted into an equivalent constant pressure panel formulation at the local aerodynamic/structures interface. The only payoff from an improved aerodynamic program in a Level 1 program will be the output data from the aerodynamic program, with little improvement to be observed downstream of that program. It is virtually impossible to incorporate a second order lateral/directional capability into the Level 1 program.

The option A payoff is a minimum impact on the FLEXSTAB system and low technical risk. Because of the minimum impact, a minimum time would be required to develop a working program, assuming the existence of the low frequency formulation and presently nonexistent routines such as the supersonic doublet numerical scheme. Numerically, there would be an expected increase in accuracy, elimination of control point sensitivity, elimination of paneling sensitivity, and a reduced sensitivity to tail arrangement. In addition, a working version of the FLEXSTAB system, which incorporates the improved numerical techniques, would be available quickly and with a minimum of effort. The limitations of option A include the retention of the crude aerodynamic model (interference shell, etc.) which brings into question the advisability of incorporating improved numerical methods and the relative impossibility of a systemic program capable of accepting advancing aerodynamic technology.

The significant payoff from option B is the ability to model the actual configuration, a point considered extremely important for quality results from a potential flow program. Associated with this development is a restructuring of the FLEXSTAB system, the effect of which is to impact heavily the time required to obtain a new version of FLEXSTAB. The corresponding technical risk is higher, not in terms of technical feasibility but in terms of development time because unforeseen difficulties often cause schedule slides. Accordingly, the investigation and development of the restructuring procedure and development of individual modules must be done with due regard to careful, parallel development and with special attention to seemingly infinitesimal detail. The anticipated numerical improvements hold the promise of an aeroelastic capability of a quality never before achieved.

8.3 VERIFICATION PROCEDURES

Verification is intended to provide quantitative grounds for comparison of the combinations and to define their respective regions of application. Verification will not generally establish superiority of one combination over another in all characteristics (e.g., accuracy, efficiency of computation, absence of geometric constraints, applicability to general configurations, etc.). Instead, verification supplies confirmation that the modeling combination (aerodynamic model and numerical scheme) can supply the key features required to model the physical flow problem of interest to the user. Verification is not a part of this contract but would be one of the items of work to help fulfill the Task I ultimate objectives.

Standards of Comparison

Three standards of comparison for a combination are: exact analytic theory, other existing numerical methods, and experimental data. The nature of experimental data often makes it a good qualitative comparison for a potential flow solution. Exact solutions of the boundary value problems would be the ideal standard of comparison but these are not generally available.

Of the three standards, comparison of the combinations with other existing numerical methods offer the most flexible and valid verification for those combinations. Such methods do exist and their validity is widely recognized. These include the methods reported in references 15, 16, 23, 26, and 27.

Variables of Comparison

Many quantities can be compared. The pressure distribution on a configuration, however, is the primary aerodynamic quantity of interest. Other quantities such as total lift and moment, sectional lift and moment, etc., are integrated quantities and may mask difficulties inherent in a particular scheme. These should not be considered primary quantities. Data should also be compiled demonstrating the economics of using a particular combination. This includes computation time to achieve a specified level of accuracy.

Items to be Verified

The following items should be addressed as part of the verification procedures for the various combinations.

- What are the basic spline formulations required in regions such as wing-body intersection, wing planform breaks, and wing tips?
- Do the combinations have application to the low frequency nonsteady flow problem?
- Does the combination exhibit numerical stability?
- What is the sensitivity to panel size, arrangement, and control point placement?
- Does the solution converge for increasing panel density?
- Are the combinations adequate to model the physical flow?

Follow-On Items of Work

The follow-on items of work associated with options A and B are:

Option A

1. Make a comprehensive evaluation and verification against the present FLEXSTAB and against more refined aerodynamic models (e.g., ref. 23).

2. Address the FLEXSTAB interface problem and the conversion to a constant pressure panel equivalent.

3. Make decision to proceed or scrap.

Option B

1. Validate the aerodynamic models and numerical methods recommended.

2. Address the FLEXSTAB interface (must be done in parallel with any FLEXSTAB restructuring plans).

3. Make decision to proceed or scrap.

8.4 RESOURCES REQUIRED

The estimated resources (August 1974 figures) required to achieve the individual items of work toward the fulfillment of the ultimate objectives of Task I are presented below. The remaining work is broken into phases (2 through 4) and expressed individually for each option (A and B). Note that for option B, no estimate is made of the resources required to develop a Level 2 version of FLEXSTAB. All resource estimates are contained in table 9.

Option A

The assumptions made to generate the figures contained in table 9 for option A are:

- Source splines and doublet splines will be the fundamental numerical building blocks.

- The subroutines will be available for steady-state doublet and source splines, subsonic and supersonic.

- The low frequency doublet building blocks will be available.

- A converter will be developed to obtain constant pressure panel equivalents.

- The present FLEXSTAB aerodynamic modeling is retained.

The manpower level is 4.58 manyears.

Table 9.—Resources Required to Fulfill the Ultimate Objectives of Task I

August 1974 figures

	Phase		Manmonths BCAC[a]		Manmonths BCS[b]		Computer res. CU's		Flow time months		
Option A	2	Verify	8	30.5	4	24.5	200	434	8	8	25
		Interface	2.5		2.5		–		5		
	3	Modeling	4		0.5		–		4	4	
		Converter	3		0.5		–		3		
	4	Program	2		10		20		10	13	
		Implement	2		2		4		2		
		Verify	4		1		200		4		
		Document	5		4		10		5		
Option B	2	Verify	10	34.5	6	30	200	454	10	10	34
		Interface	2.5		2.5		–		5		
	3	Modeling	6		0.5		–		6	6	
		Interface	2		1		–		2		
	4	Program	3		12		40		12	18	
		Implement	1		1		4		1		
		Verify	4		1		200		4		
		Document	6		6		10		6		

[a]BCAC = Boeing Commercial Airplane Company

[b]BCS = Boeing Computer Services, Inc.

Option B

The assumptions made to generate the figures contained in table 9 for option B are:

- Option B costs are independent of a parallel, Level 2 development.
- Source and doublet splines will be the fundamental numerical building blocks.
- The subroutines will be available for steady-state doublet and source splines, subsonic and supersonic.
- The low frequency doublet building blocks will be available.
- The exact surface representation will be the aerodynamic model for the body.

The manpower level is 5.38 manyears.

9.0 CONCLUSIONS

The ultimate objectives of Task I can be achieved with little technical risk and without loss of any capability relative to the NASA-Ames released version 1.01.00. The direction of that development and the ability of the FLEXSTAB computer program system to interact dynamically with advancing technology hinges upon development of a restructured program system. Option B is recommended if the restructuring is done; otherwise, option A is recommended, the payoff versus the cost of which is questionable. Option B is the specific recommendation of this report.

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August 31, 1974

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