X-933-77-235

PREPRINT

TMX-71411

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(NASA-TM-X-71411) INTERPOLATION ALGORITHMS AND IMAGE DATA ARTIFACTS (NASA) 20 p HC A02/MF A01 CSCL 12A

N78-10799

Unclas G3/64 52040

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OCTOBER 1977

GSFC

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

X-933-77-235

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ABSTRACT

Interpolation, or resampling coefficients, which are generated from low pass filter Fourier Transforms yield more accurate resampled values than those obtained using cubic spline techniques. This is due to the utilization of six data points rather than four as currently used in cubic spline analysis.

After resampling functions are applied to image data, artifacts which are similar to ringing may become pronounced. These effects are often present in the original data and the interpolation merely enhances them.

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INTERPOLATION ALGORITHMS AND IMAGE DATA ARTIFACTS

I. INTRODUCTION

During the past few years, a great deal of discussion has taken place at various conferences and in various reports regarding the application of digital interpolation techniques to remotely sensed digital images (1-3). The consensus seems to be that resampling using the cubic convolution method leads to the best and most economical results from both a subjective, statistical, and frequency content standpoint. This attitude persists until artifacts such as ringing or other unpleasant visual effects are observed. The tendency at this point is to re-evaluate the resampling algorithm and possibly derive a new one.

Many artifacts, which become pronounced after the application of resampling techniques, are present in the original data and are independent of the interpolation method used. A resampling technique based on band limited filter analysis is described in the next section, and results are compared to those obtained using cubic convolution on a test set of analytic functions.

In Section III, application of this interpolation method is made to a Landsat -1 scene which was used as a test case for the Digital Image Rectification System (DIRS) (4). The scene, Rockwood, Illinois, was widely discussed at the Image 100 User's Conference held at Goddard Space Flight Center during November 1975. Regardless of the resampling techniques used, artifacts which resemble ringing are present in the final image. These features are shown to be present in the original data and are not introduced by the resampling method.

II. AN INTERPOLATION ALGORITHM

A. General Comments

The general criterion for interpolators or resamplers seems to be related to whether or not the resampled values look right and how these new values classify. If values at edges or other locations within the image look strange, the tendency appears to be to change the constants and try again. Sometimes, a different interpolation approach may be formulated.

On occasion one may consider the sin (x)/x as an interpolater, with the usual complaints about the number of side lobes required for N% accuracy and the possibility of introducing spurious features due to ringing. Ringing is caused by the occurrence of sharp edges or discontinuities in the data to be resampled and resembles the interpolating function with respect to the spacing of ringing. It is rare that one will ever encounter this situation in nature especially when the smoothing effects of instrument electronics are considered.

The process of interpolation is fundamentally a low pass filtering operation and should be treated as such, especially when dealing with band limited signals. When a series of weights is convolved with the picture data, the frequency spectrum of the lines is modified by the spectrum of the weights via multiplication. Thus, the ringing effects of a sin (x)/x, the Gibbs phenomenon, can be effectively reduced by apodizing processes (5). Apodizing consists of multiplying the filter weights by a function which is unity and has zero slope at the center (maximum weight) and which approaches zero with zero slope at the final weight.

The following criterion will be used as a basis for choosing an interpolation function (or set of coefficients):

Does the function give correct numeric values for any analytic input function at values between equally spaced points assuming the minimum Nyquist sampling rate? (Note that oversampling will always aid us.) The rationale for this definition is that over a small number of image points along or perpendicular to the flight line, some "best fit" function can be defined such as an N^{th} order polynomial. Admittedly, original data points may be altered slightly, but one can then generate another polynomial based on the differences at the data points, subtract it from the initial polynomial and still remain with an analytic expression.

In view of the previous discussion, the function

$$I_n = \left[1 - (n/N)^2\right]^2 \frac{\sin(\pi n/k)}{(\pi n/k)} \qquad n = 0, \pm 1, \pm 2, \ldots \pm N \qquad (1)$$

is submitted as an interpolation constant generator where

k = the number of interpolated intervals between two original sample points.

N = j · k + 1 where j defines the number of zero crossings of the sin (x)/x side lobes along a positive or negative axis.

This expression is the apodized Fourier Transform of an idealized low pass filter (5).

Column 1 of Table 1 tabulates this function for $k = 4$ and $j = 3$. If the values are applied to a "picket fence", that is, equally spaced pulses of unit amplitude, a certain amount of ringing will occur as can be seen by summing the appropriate coefficients used in the convolution. This effect is due to using a finite number of interpolation constants to approximate the picket fence at intermediate sample values. Normalizing each interpolation constant to the sum of the values involved in the convolution:

$$C_i = \sum_{n=-j}^{j-1} I_{(nk + i)} \qquad i = 1, 2, \ldots, k-1 \qquad (2)$$

effectively eliminates this effect and values are tabulated in Column 2, Table 1.

Figure 1 illustrates the application of the latter coefficients to a discretely sampled input for a midpoint interpolation. When the coefficients are slid to the right an amount equal to one quarter of original sampling unit, a new value may be calculated. In all cases only 6 data points and the proper choice of 6 of the interpolation coefficients are required. Note that the resampled value is located at the peak of interpolation coefficients.

Table 2 applies the interpolation constants of Table 1, Column 2, to the discrete polynomials ($F(n)$) designated in the headings. The values for n, the theoretical value of $F(n)$, and the interpolated values are shown for each polynomial. The interpolated values are calculated from $F(n)$ with n being an integer. It should be noted that for values midway between the original samples, the interpolated values are nearly exact. This leads one to consider a recursive interpolation technique in which values are calculated for the midpoints, and this new set of data is then used as a basis for interpolating new midpoint values which corresponds to computing values at one fourth of the original spacing. This process is illustrated in the third major heading of Table 2. $F'(n)$ and $F''(n)$ refer to the first and second passes of the interpolation process.

Table 3 illustrates the midpoint evaluation on the cubic polynomial as designated by cubic convolution methods. As a comparison, midpoint convolution for the case where $j = 2$ and $k = 2$ utilizing equation 1 is shown. In both these cases, four data points are used thus agreement with the correct midpoint value is not as good as when six data points are used as in Table 2. Values for integer values of n are shown once.

The Fourier Transforms of the function used ($k = 4$, $j = 3$) are shown in Figure 2 for the unapodized, apodized, and normalized cases. The deviation from the ideal low pass box car is due to the small number of lobes used. Figure 3 shows the transforms of apodized sinc functions for 4, 8 and 16 side lobes on either side of the maximum and the improvement in approaching the box car is noticeable. This is nothing more than a restatement of the fact that interpolated values improve as more data is used.

III. ROCKWOOD ILLINOIS SCENE

Figure 4 is a DIRS (4) processed scene of the Rockwood Illinois area and corresponds to 10 meter resolution. This scene was the basis of much discussion during the Image 100 User's Conference held at Goddard Space Flight Center in the latter part of 1975. The item discussed was the striping effects observed in the lower left and center left regions of the picture. These effects were attributed to the resampling procedure which in this case was based on cubic convolution.

This is not the case. The "striping" is a manifestation of the original data and is due to the way the Landsat Multispectral Scanner (MSS) samples narrow features sufficiently different from the adjacent background. As an example*, consider a cement road 40 meters wide surrounded by water on both sides in a direction which parallels the flight path. The instantaneous field of view of the MSS is about 80 meters square. Due to scanner sampling timing (4), a 22 meter offset occurs between detectors 1 and 6 of a particular band, and earth rotation contributes about a 50 meter offset in opposite direction between detector 6 of swath n, and detector 1 of swath n + 1. Given this situation, it is easy to visualize two adjacent samples from detector 1, swath n acquiring half the road for a net intensity of I, and another sample from swath n, detector 6 acquiring the full road for an intensity of 2I. Thus the modulation of this feature can be 50%, and any interpolation of resampling algorithm should reflect this.

Figure 5 is the original data from the lower half of the scene previously described and consists of 150 pixels and 150 lines. Each pixel and line has been replicated 8 times in order to simulate 10 meter resolution. Additionally each line has been shifted to the nearest fraction of a pixel to allow for the MSS sampling skew. In the regions where Figure 3 shows striping, one observes that the replicated data also shows a striping tendency. The resampling procedure simply smoothes the transitions in these areas and appears as low frequency striping in Figures 4 and 6.

*This analogy is valid if the scanning function is rectangular and sees only one "pixel". In practice the scanning function is a two dimensional smear of the order of 9 pixels by 3 lines. A deconvolution method based on a priori knowledge of the scanner is under investigation by Bendix Corporation.

Figure 6 was generated from the original data for Figure 4 using the recursive interpolation discussed in the previous section. Initially, 3 iterations for each line were made (a one eighth pixel interval resampling). The lines were shifted to allow for scanner skew, and then three more recursive interpolations were computed in a "vertical" direction corresponding to a one eighth line resampling. As can be seen, the regions of interest, except for scale, are identical.

IV. CONCLUSIONS

A band limited, convolutional, interpolation technique using few coefficients was developed in Section I. The accuracies obtained when using analytic inputs is very high for new samples calculated at midpoint spacing. The interpolated values in the cases illustrated are closer to the actual calculated values than those obtained using cubic convolution methods.

An additional advantage of this method is the ease with which the rate of cutoff at the cutoff frequency may be varied by varying the number of side lobes. This can be significant if there are major high frequency contributions in the data to be interpolated. The three side lobe coefficients perform adequately when the highest frequency is sampled three or more times. When the sampling rate approaches the Nyquist rate (2 samples per highest frequency) more side lobes are required to improve the filter cutoff (see Figures 2 and 3).

Data artifacts which resemble ringing were shown to be present in the data and not caused by interpolation for a particular Landsat scene discussed in Section II. Recursive convolution, which is a natural offshoot of the bandlimited interpolation, was applied to the data for this scene and results were "eyeball" identical to previous processing.

V. POSSIBLE FUTURE APPLICATION

One of the initial goals of this study was to resample an image using an interpolative nearest neighbor technique. Table lookup procedures would be utilized. Though not accomplished, this could be implemented by deriving interpolation coefficients for each . 1 or . 01 of a pixel, computing the proper nearest neighbor location and applying the looked up coefficients to the data.

No attempts to classify results obtained with the interpolation scheme discussed have been tried, however if "mixels" are avoided, results should be favorable.

VI. ACKNOWLEDGEMENT

The author would like to take this opportunity to thank Peter Van Wie for supplying data and information on the Rockwood Illinois scene, and the Digital Image Rectification System, Dr. Walter Fink whose eloquent arguments led the author to submit this for publication, and Dr. Thomas Lynch for making several suggestions which resulted in the final form of this article.

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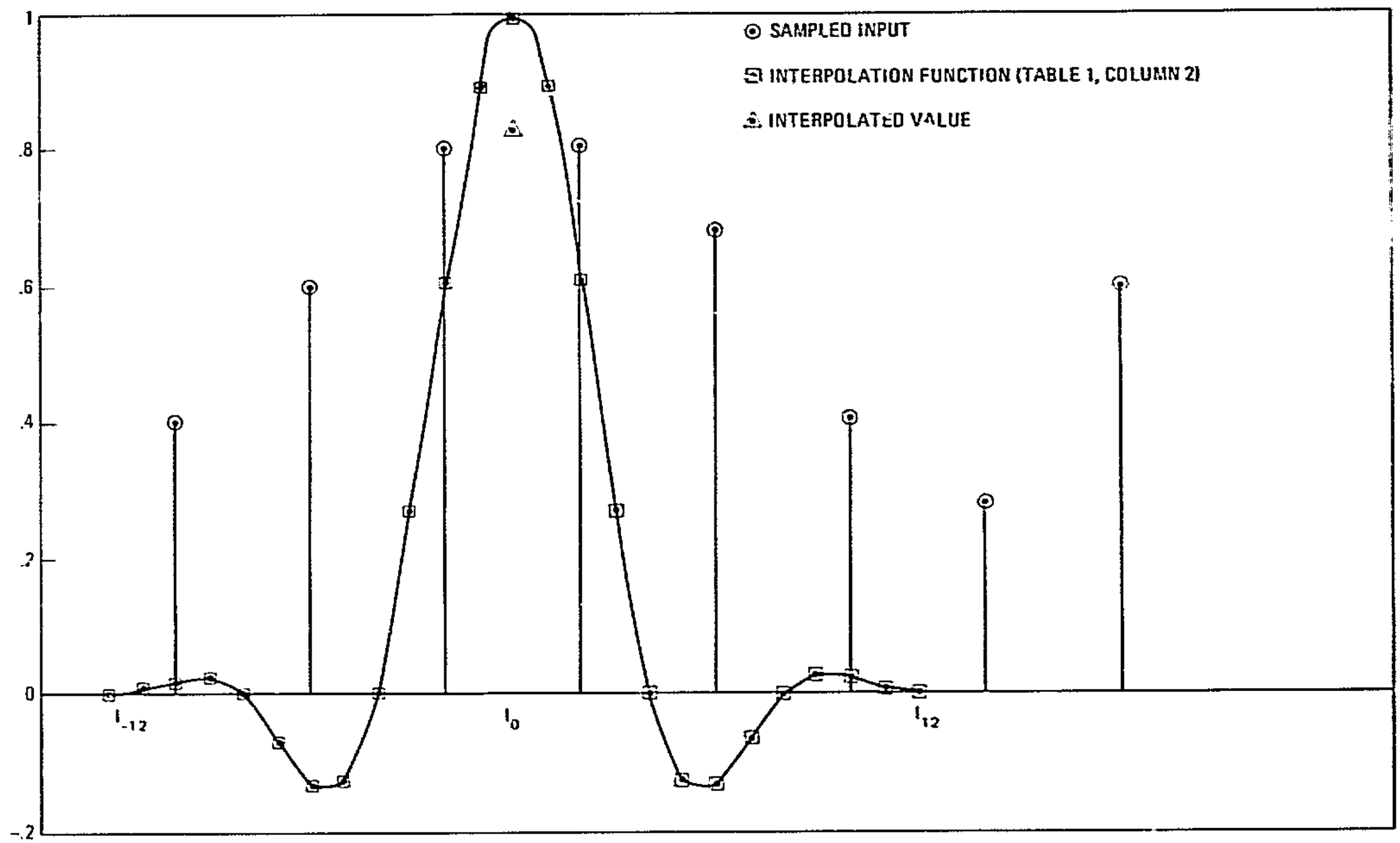

Figure 1. Resampling by Convolution of Interpolation Function with Input Data
Example Shows a Midpoint Computation
Value = .6110 (.8 + .8) = .1323 (.6 + .68) + .02137 (.4 + .4) = .8253

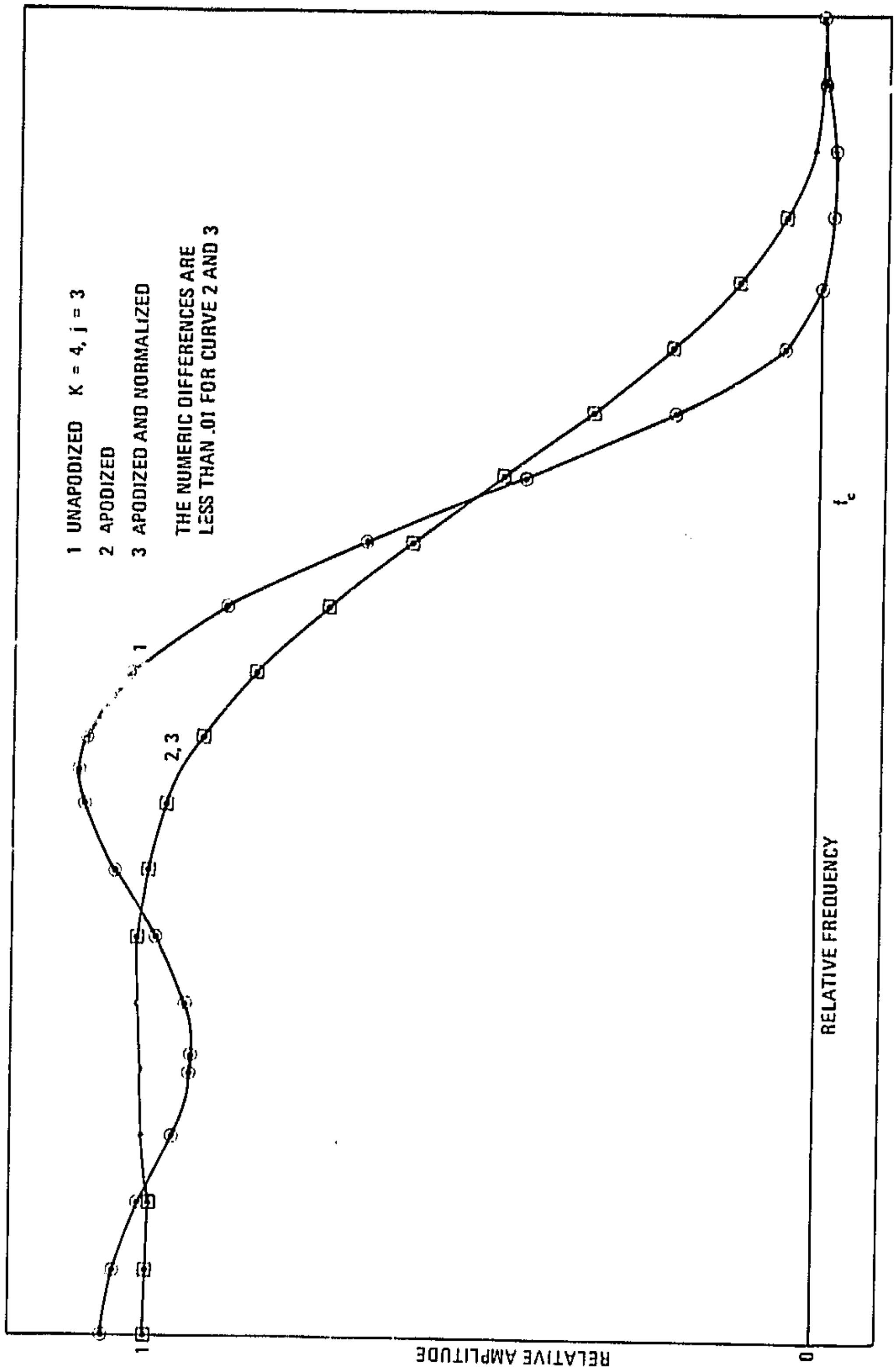

Figure 2. Transfer Function of Interpolation Coefficients

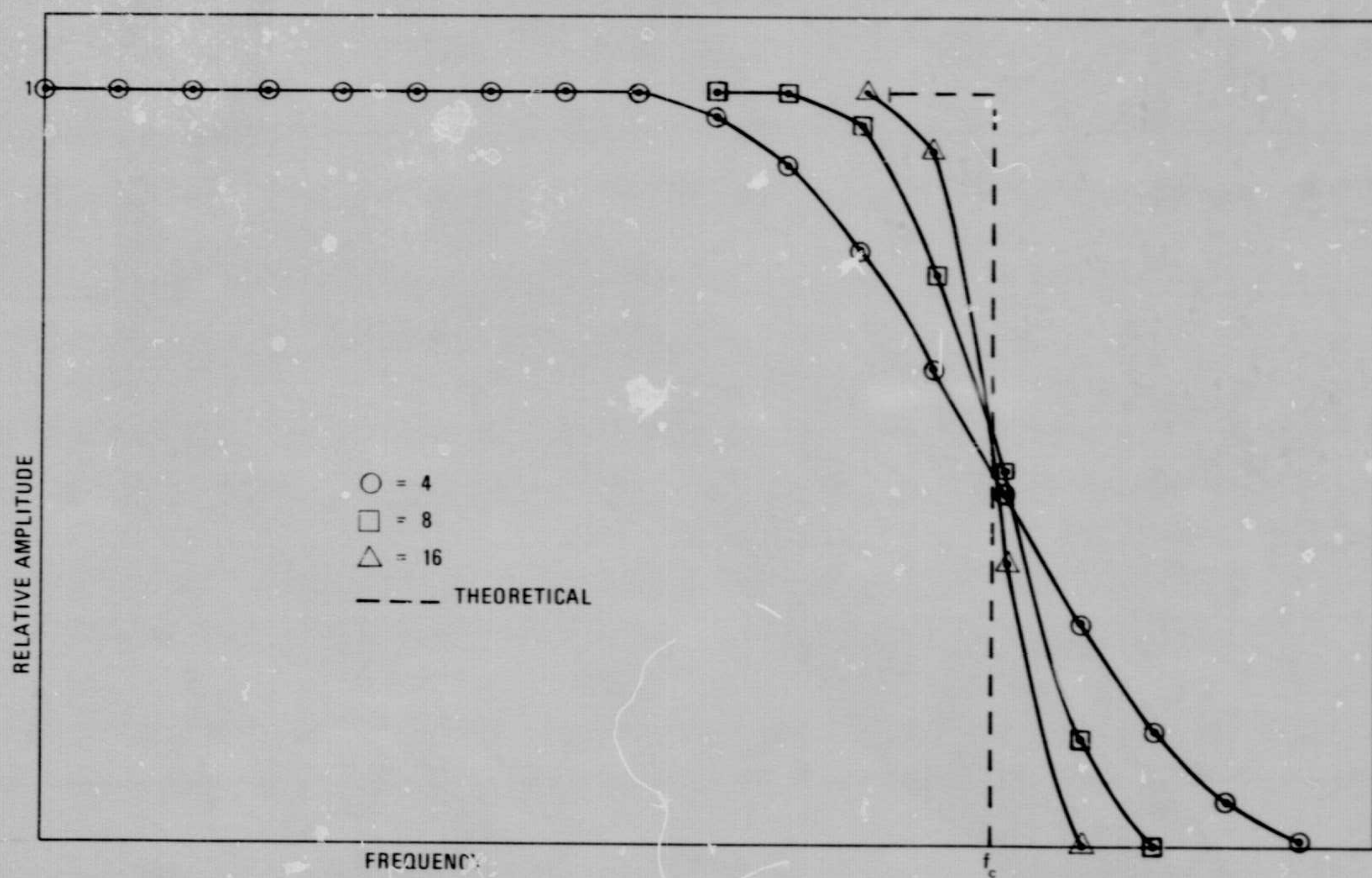

Figure 3. Approximation to an Ideal "Box Car" as a Function of the Number of Side Lobes

Figure 4. DIRS Rectified Scene of Rockwood, Illinois

Figure 5. Original Rockwood, Illinois Data Replicated 8 Times

Figure 6. Rockwood, Illinois Data Interpolated to 10 Meter Resolution by Recursive Convolution

Table 1

Interpolation Function Tabulation

n	1	2
0	1.00000	1.00000
± 1	.88970	.8925
± 2	.60680	.6110
± 3	.26900	.2698
± 4	.00000	.00000
± 5	-.13070	-.1311
± 6	-.13140	-.1323
± 7	-.06485	-.06505
± 8	.00000	.00000
± 9	.02712	.02721
± 10	.02122	.02137
± 11	.00660	.00662
± 12	.00000	.00000

$$\text{Col. 1} = \left[1 - (n/N)^2\right]^2 * \frac{\sin(\pi n/4)}{\pi n/4} = I_n$$

$$\text{Col. 2} = I_n/(I_1 + I_5 + I_9 + I_{-3} + I_{-7} + I_{-11})$$

for n = 1, 5, 9, -3, -7, -11 etc.

Denominator = C_1

Table 2

Sample Computations Using Table 1 Coefficients

$F(n) = (n-25)^2$			$F(n) = n^3 - 8000$			Recursive Evaluation of $F(n) = n^3 - 8000$			
n	F (n)	Calc F (n)	n	F (n)	Calc F (n)	n	F (n)	F' (n)	F'' (n)
22	9	9	18	-2168	-2168	18	-2168		
23	4	4	19	-1141	-1141	18.5	-1668.4	-1670	
24	1	1	20	0	0	19	-1141		
24.25	.5625	.5743	20.25	303.8	286.3	19.5	-585.1	-586.5	
24.5	.25	.2271	20.5	615.1	613.7	20	0		
24.75	.0625	.0469	20.75	934.2	951.4	20.25	303.8		302.7
25	0	0	21	1261	1261	20.5	615.1	613.7	
26	1	1	22	2648	2648	20.75	934.2		933.2
27	4	4	23	4167	4167	21	1261		
			23.25	4568	4545	21.5	1938.4	1937	
			23.5	4977.9	4976	22	2648		
			23.75	5396	5419	22.5	3390.6	3389	
43	324	324	24	5824	5824	23	4167		
44	361	361	25	7625	7625	23.25	4568		4567
45	400	400	26	9576	9576	23.5	4977.9	4976	
45.25	410.06	409.5				23.75	5396		5396
45.5	420.25	420.2				24	5824		
45.75	430.56	431.1				24.5	6706	6704	
46	441	441				25	7625		
47	484	484							
48	529	529							

Note: If coefficients are applied to 1, -1, 1, -1, . . ., 1, -1 in recursive manner sin (x) will be calculated to better than 3% at values of $\pi / 2^n$

Table 3

Comparison of Interpolation
Using Cubic Convolution and Interpolation Function

$$F(n) = n^3 - 8000$$

n	F (n)	Cubic Conv. F (n)	Eq. 1 Convolution
18	-2168		
19	-1141		
20	0		
20.5	615.1	600	609
21	1261		
22	2648		
23	4167		
23.5	4977.9	4960	4969
24	5824		
25	7625		