University of Cincinnati, Cincinnati, Ohio 45221

THE SEPARATED TURBULENT BOUNDARY LAYER OVER A WAVY WALL

(NASA-CR-155283) THE SEPARATED TURBULENT BOUNDARY LAYER OVER A WAVY WALL Final Report, 1 Jul. 1975 - 30 Sep. 1977 (Cincinnati Univ.) 21 p HC A02/MF A01 CSCL 01A G3/02
N78-12016
Unclas
53612

Final Report,

covering the period

July 1, 1975 - September 30, 1977

NASA Grant No. NSG 1208

Principal Investigators are A. Polak and M.J. Werle, Department of Aerospace Engineering and Applied Mechanics.

The NASA Technical Officer of this grant is Mr. James C. Dunavant, NASA Langley Research Center, Hampton, Virginia 23665.

THE SEPARATED TURBULENT BOUNDARY LAYER OVER A WAVY WALL

NASA Grant No. NSG 1208

Final Report

covering the period

July 1, 1975 - September 30, 1977

by

A. Polak and M.J. Werle

University of Cincinnati, Cincinnati, Ohio

This research was concerned with the development of a prediction method for calculating detailed distributions of surface heating rates and pressures over a corrugated wall, due to flow of a two-dimensional supersonic turbulent boundary layer. Primary attention was given to the turbulent boundary layers which are thick relative to the wave amplitude and at the same time the ratio of the wave amplitude to the wave length is sufficiently high so that a strong interaction develops between the viscous and inviscid regions. This manifests itself in the variation in surface pressure which may be an order of magnitude different from the one calculated by purely inviscid methods. Also, under these flow conditions, separation regions may develop between the peaks of the waves. This means that the nature of the problem is such that classical boundary layer methods are not suitable to effect the solution, simply because the interaction is a first order effect.

To solve this problem effectively, an interacting boundary layer approach was employed. Its essence is that the pressure distribution is not prescribed by the body surface profile, but calculated simultaneously with the viscous flow. A time-like relaxation method using an implicit finite-difference numerical code was adopted. The turbulence was represented by the two layer eddy viscosity model of Cebeci and Smith. An attempt was made to account for the surface curvature effect on the turbulent structure by employing a correction to the eddy viscosity model proposed by Bradshaw. The present predictions were compared with independent experimental and theoretical results. The predictions, in terms of pressure and heating levels, compare reasonably well with the measured data when the curvature correction is included.

The complete details, concerning the formulation of the problem, the numerical method and the discussion of the results for test cases with Mach numbers of 2.5 and 3.5, Reynolds numbers $Re_\infty = 10.8\times10^6$/mm and 32.4×10^6/mm, wall-to-stagnation temperature ratio $T_w/T_o = 0.40$ and 0.81, and ratio of wave amplitude to wave large from 1/96 to 1/12 are presented in a self-contained University of Cincinnati technical report, Report No. AFL 77-11-36 (Reference 1). The results of this research effort were also presented in a paper at the ASME Winter Annual Meeting, Atlanta, Georgia, November 1977. This paper appears as an ASME publication 77-WA/HT -28 (Reference 2). While this work was in progress partial results of this research were presented at the 5th U.S. Air Force/Federal

Republic of Germany Data Exchange Agreement Meeting, Wright Patterson Air Force Base, Ohio, 28-29 April 1976 (Reference 3).

The present investigation lead also to two side efforts, carried out under partial support of the NASA Grant No. NSG 1208. These efforts were aimed at achieving higher accuracy and efficiency in the numerical schemes, applicable to problems of present interest. The first of these studies was concerned with developing a numerical scheme for solving viscous flow problems with regions of predominantly inviscid flow (like the outer region of very thick turbulent boundary layer). This effort resulted in a new second order accurate exponential box scheme. It was shown that this method provides a definite advantage over other numerical schemes. The details of this work were published as a University of Cincinnati technical report, Report No. AFL 77-9-34 (Reference 4) and also resulted in a Master of Science Thesis. A Technical Note summarizing this work was submitted for publication to the AIAA Journal. A copy is attached herein as Appendix A.

The second study aimed at achieving a higher efficiency in calculating laminar and turbulent boundary layers. The fourth order cubic spline method developed recently by Rubin was combined with the Davis Coupled Algorithm for solving simultaneously the boundary layer equations. The details of this work, with application to a set of model flow problems and to the incompressible laminar boundary layer equations will appear as a University of Cincinnati Technical Report (Reference 5) in December 1977. This work appears also as a Master of Science Thesis; the abstract of this Thesis is attached herein as Appendix B.

In conclusion, it can be stated that the major objective of the work under the present NASA Grant No. NSG 1208 was fulfilled. A method capable of handling strongly interacting flows was successfully adapted to solve the problem of flow of thick turbulent boundary layers over a wavy wall, including multiple separation regions. A supportive effort resulted in developing numerical schemes, which in the future could enhance the accuracy and efficiency of the present state of the interacting numerical algorithm.

The cumulative list of publications resulting during the course of investigation under the support of the NASA Grant No. NSG 1208 is attached under the heading List of References.

LIST OF REFERENCES

1. Polak, A., and Werle, M.J., "Supersonic Separated Turbulent Boundary-Layer Over a Wavy Wall", Report No. AFL 77-11-36, November 1977, Dept. of Aerospace Engineering and Applied Mechanics, University of Cincinnati, Cincinnati, Ohio.

2. Polak, A., and Werle, M.J., "Interacting Turbulent Boundary-Layer Over a Wavy Wall", ASME Paper 77-WA/HT-28. Presented at the ASME Winter Annual Meeting, Atlanta, Georgia, Nov. 27-Dec. 2, 1977.

3. Polak, A., Werle, M.J., and Bertke, S.D., "Numerical Study of Hypersonic Turbulent Interacting Boundary Layers", Proceedings of the 5th U.S. Air Force/Federal Republic of Germany Data Exchange Agreement Meeting, Technical Report AFFDL-TR-77-61, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio 45433.

4. El-Mistikawy, T., and Werle, M.J., "Numerical Methods for Inviscid/Viscous Fluid Flows", Report No. AFL 77-9-34, September 1977, Dept. of Aerospace Engineering and Applied Mechanics, University of Cincinnati, Cincinnati, Ohio. Also a Master of Science Thesis, submitted by Mr. El-Mistikawy to the Department of Aerospace Engineering and Applied Mechanics, June 1977.

5. Vacca, G., Werle, M.J., and Polak, A., "An Application of Fourth Order Spline Collocation Procedure to Boundary Layer Like Problems", Report No. AFL 77-12-37, December 1977, Dept. of Aerospace Engineering and Applied Mechanics, University of Cincinnati, Cincinnati, Ohio. Also Master of Science Thesis, submitted by Mr. G. Vacca to the Department of Aerospace Engineering and Applied Mechanics, December 1977.

APPENDIX A

A NUMERICAL METHOD FOR BOUNDARY LAYERS WITH BLOWING - THE EXPONENTIAL BOX SCHEME†

T.M. El Mistikawy* and M.J. Werle**
Department of Aerospace Engineering and Applied Mechanics
University of Cincinnati
Cincinnati, Ohio

INTRODUCTION

Solutions of boundary layer flows in the presence of massive injection through a porous surface are complicated by the multi-structure nature of the flow field. The flow can be divided into two distinguishable regions (see Refs. 1-4); a basically inviscid turning region near the blowing surface that occupies most of the boundary layer and a thin viscous region in which diffusion effects play a significant role in adjusting the boundary layer flow to the free stream conditions. Similar flow structure and complication occur in a wide class of viscous interaction flow problems (Refs. 5 through 9) where a thin viscous layer actively induces a significant displacement effect to a relatively thick inviscid flow layer. The change in the character of the flow from the viscous to inviscid regions is difficult to accurately capture numerically with a

---

† This work was partially supported by the NASA Langley Research Center under Grant No. NSG 1208 and the Air Force Flight Dynamics Laboratory Contract No. F33615-76-C-3091.

* Graduate Research Assistant.

** Professor, Currently Chief, Gas Dynamics Section, United Technologies Research Center, Hartford, Conn. Member AIAA.

single finite difference representation giving rise to the introduction of various schemes to eliminate numerical "instabilities" ranging from variable grid spacings (Ref. 10) to artificial viscosity concepts (Refs. 11 and 12).

The present study takes a different approach and presents a new numerical scheme that should be useful for solution of a wide class of inviscid/viscous flows. The approach is based on two points; first, that the difficulties encountered by previous schemes can be traced to a misrepresentation of convection effects normal to the principle boundary layer direction (see Ref. 13) and, second, that exponential difference operator concepts (Refs. 14 through 18) can be combined with Keller's box scheme approach (Ref. 19) to produce a stable second order accurate finite difference scheme for such convection-diffusion problems. The technique developed here is demonstrated through application to the solution of the Falkner-Skan equations for self similar boundary layer flow with massive blowing at the surface.

GOVERNING EQUATIONS AND SOLUTION METHOD

The governing equations are taken to be the Falkner-Skan equations with blowing and given here in terms of the Gortler variable η (see Ref. 20) as

$$V_\eta + F = 0 \tag{1a}$$

$$F_{\eta\eta} - VF_\eta + \beta(1-F^2) = 0 \tag{1b}$$

with

$$F(0) = 0 \;; \qquad V(0) = V_w \;; \tag{2a}$$

and $$F \to 1 \text{ as } \eta \to \infty \tag{2b}$$

where V_w is related to the physical injection velocity, F represents the normalized longitudinal velocity component, and β the inviscid pressure gradient parameter.

As in nearly all numerical solution methods, the nonlinear nature of equation (1b) is addressed indirectly. An iterative approach is used to first write this in the form

$$F_{\eta\eta} - (a+b)\, F_\eta + ab\, F = c \tag{3}$$

where a, b are obtained from a previous iteration and are given as

$$a \equiv V/2 + \sqrt{(V/2)^2 + \beta F} \tag{4a}$$

$$b = V/2 - \sqrt{(V/2)^2 + \beta F} \tag{4b}$$

with $$c = -\beta \tag{4c}$$

Whereas classical finite difference techniques seek solutions of equation (3) using Taylor series to generate difference approximations to derivatives over a small grid distance, a different approach is employed here. Following the lead of references 14-18, the coefficients of equation (3) are first approximated over a small grid distance and the resulting equation integrated exactly. With reference to the typical sequential grid points shown in Figure 1, the solution to equation (3) in the interval between points 1 and 2 is given as

$$F_{1,2} = A_1 e^{a_1 \eta} + B_1 e^{b_1 \eta} + f_1 \qquad (5)$$

where $f_1 = c_1/a_1 b_1$ and the coefficients a_1 and b_1 are evaluated at the midpoints of the grid indicated.* Use of equation (5) along with a similar expression for $F_{2,3}$ in the interval between points 2 and 3 and the two continuity conditions that $F_{2+} = F_{2-}$ and $F_{\eta_{2+}} = F_{\eta_{2-}}$ makes it possible to eliminate the arbitrary constants A_i and B_i. The resulting difference equation is

$$\alpha_1 F_1 - (\beta_1 + \beta_2) F_2 + \alpha_2 F_3 = (\alpha_1 - \beta_1) f_1 + (\alpha_2 - \beta_2) f_2 \qquad (6)$$

where

$$\alpha_1 = (a_1 - b_1) e^{b_1 \Delta\eta_1}/\gamma_1 \;; \quad \alpha_2 = (a_2 - b_2) e^{-a_2 \Delta\eta_2}/\gamma_2 \qquad (7a)$$

$$\beta_1 = [a_1 - (1-\gamma_1) b_1]/\gamma_1 \;; \quad \beta_2 = [a_2 (1-\gamma_2) - b_2]/\gamma_2 \qquad (7b)$$

$$\gamma_1 = 1 - e^{-(a_1 - b_1)\Delta\eta_1} \;; \quad \gamma_2 = 1 - e^{-(a_2 - b_2)\Delta\eta_2} \qquad (7c)$$

Equation (6) is a three point difference approximation to equation (3) and, as such, produces a tridiagonal set of algebraic equations that are straightforward to solve using standard techniques. In addition, this difference relation possesses all the favorable properties of the exponential schemes and the "2-point" or "box-scheme" approaches. Equation (6) is found to be always diagonally dominant for $ab < 0$, it is second order accurate in $\Delta\eta_i$ throughout the diffusion region, and most

---

* The use of two points to generate the basic relations for this scheme aligns it with the Box Scheme techniques of Keller, Ref. (19); hence the term "exponential box-scheme" emerges.

importantly, it automatically and smoothly switches to a second order accurate windward difference scheme as the normal convection velocity, V (as represented by the term a+b), becomes large negative or positive.

The companion numerical solution of the continuity equation is obtained substituting equation (5) directly into equation (1a) and integrating to obtain the local relation

$$V_2 = V_1 + A_1(1-e^{a_1 \Delta n_1})/a_1 + B_1(1-e^{b_1 \Delta n_1})/b_1 - f_1 \Delta n_1 \qquad (8)$$

where A_1 and B_1 are obtained in terms of F_1 and F_2 using equation (5).

RESULTS AND DISCUSSION

The exponential box (EB) scheme described above has been applied and verified for a wide range of test cases (see Ref. 13) with typical results presented here. Typical longitudinal velocity profiles are shown in Figure 2 for blowing at an axisymmetric stagnation point (β = 0.5). The massive blowing case (V_w = 25) is similar to that presented by Liu and Chiu (Ref. 10) except that in that study, compressible heating effects were accounted for due to a cold wall boundary condition. The present case corresponds to the case of an adiabatic wall temperature and was chosen to allow isolated study of the EB scheme for the momentum and continuity equations alone.

In an attempt to demonstrate the multistructure nature of the flow field, Figure 3 shows the individual component contributions to the governing momentum balance equation (1b) in the critical region of the flow ($45 \leq n \leq 55$). Clearly the flow represents an inviscid process below $\eta < 48$ where the normal convection and pressure gradient effects totally balance one another. Only above this region does the diffusion effect play a significant role. With so much of the flow basically inviscid in nature, it should not be surprising to find that diffusion-type numerical schemes (such as the classical three point central difference scheme) have difficulty representing this flow. Such difficulties usually manifest themselves in terms of non-monotonic (jagged) velocity distributions (or local "wiggles"). This point is discussed further in Ref. 13 with the conclusion that this difficulty is largely due to a failure of central difference schemes to adequately model the normal convection effects. It is shown in Ref. 13 that a "windward" representation of the convection terms is necessary to properly represent such flows. The exponential box scheme developed here automatically and smoothly switches from a second order accurate diffusion-type scheme in diffusion regions ($\eta > 48$) to a second order accurate windward scheme in normal convection dominated regions ($\eta < 48$), thus encountering no numerical difficulties or "instabilities" even for a constant grid mesh as per Ref. 10. The accuracy and reliability of the current approach is attested to in Figure 4 where the results of a typical step size study is shown for 3 points in the critical region of the flow field, $45 \leq \eta \leq 55$. Here the EB scheme is seen to maintain second order accuracy (i.e. produce a straightline variation with $\Delta\eta^2$) for a uniform step size $\Delta\eta$ as large as 2 while a central difference scheme (CD) rapidly loses its second order nature. Reference 13 presents a more detailed comparison and shows the current approach to be superior to four other difference methods as well.

REFERENCES

1. Watson, E.G., "The Equation of Similar Profiles in Boundary Layer Theory with Strong Blowing," Proc. Roy. Soc. Ser. A, 294 (1966), pp. 208-234.

2. Kubota, T., Fernandez, F.L., "Boundary Layer Flows with Large Injection and Heat Transfer," AIAA Journal, Vol. 6, No. 1, Jan. 1968, pp. 22-28.

3. Kassoy, D.R., "On Laminar Boundary Layer Blow-Off," SIAM Journal of Applied Math., Vol. 18, No. 1, Jan. 1970, pp. 29-40.

4. Libby, P.A., "Numerical Analysis of Stagnation Point Flows with Massive Blowing," AIAA Journal, Vol. 8, No. 11, Nov. 1970, pp. 2095-2096.

5. Stewartson, K., "Multistructured Boundary Layers on Flat Plates and Related Bodies," appearing in ADVANCES in APPLIED MECHANICS, Vol. 14, Academic Press, San Francisco, 1974.

6. Davis, R.T. and Werle, M.J., "Numerical Methods for Interacting Boundary Layers," Invited Lecture, 1976 Heat Transfer and Fluid Mechanics Institute, Univ. of Calif. at Davis, June 21-23, 1976. See also, Dept. of Aerospace Engineering, Univ. of Cincinnati Rept. No. AFL 76-8-23, August 1976.

7. Burggraf, O.R., "Some Recent Developments in Computation of Viscous Flows," Invited Lecture, Fifth Int. Conf. on Numerical Methods in Fluid Dynamics, appearing in LECTURE NOTES in PHYSICS, Vol. 59, Springer-Verlag, 1976.

8. Sychev, V.V., "On Laminar Separation," Mekhanika Zhidkosti i Gaza, No. 3, 1972, pp. 47-59.

9. Messiter, A.F., "Boundary-Layer Flow Near the Trailing Edge of a Flat Plate," SIAM Journal of Applied Math., Vol. 18, 1970, pp. 241-257.

10. Liu, T.M., Chiu, H.H., "Fast and Stable Numerical Method for Boundary Layer Flow with Massive Blowing," AIAA Journal, 1976, pp. 114-116.

11. Roache, P.J., "Computational Fluid Dynamics," Hermosa Publishers, Albuquerque, NM, 1976.

12. Briley, W.R., McDonald, H., "Implicit Numerical Method for the Multidimensional Compressible Navier-Stokes equations," Rept. M911363-6, United Aircraft Res. Labs., Nov. 1973.

13. El Mistikawy, T.M. and Werle, M.J., "Numerical Methods for Inviscid/Viscous Fluid Flows," University of Cincinnati, Dept. of Aerospace Engineering Rept. No. AFL 77-9-34, Sept. 1977.

14. Allen, D.N. de G. and Southwell, R.V., "Relaxation Methods Applied to Determine the Motion in Two Dimensions of a Viscous Fluid Past a Fixed Cylinder," Quart. Jour. Mech. and Applied Math., Vol. 8, 1955, pp. 129-145.

15. Dennis, S.C.R., "Finite Difference Associated with Second-Order Differential Equations," Quart. Jour. Mech. and Applied Math., Vol. 23, 1960, pp. 487-507; see also "The Numerical Solution of the Vorticity Transport Equation," appearing in Lecture Notes in Physics, Vol. 19, pp. 120-129, 1973, Springer-Verlag.

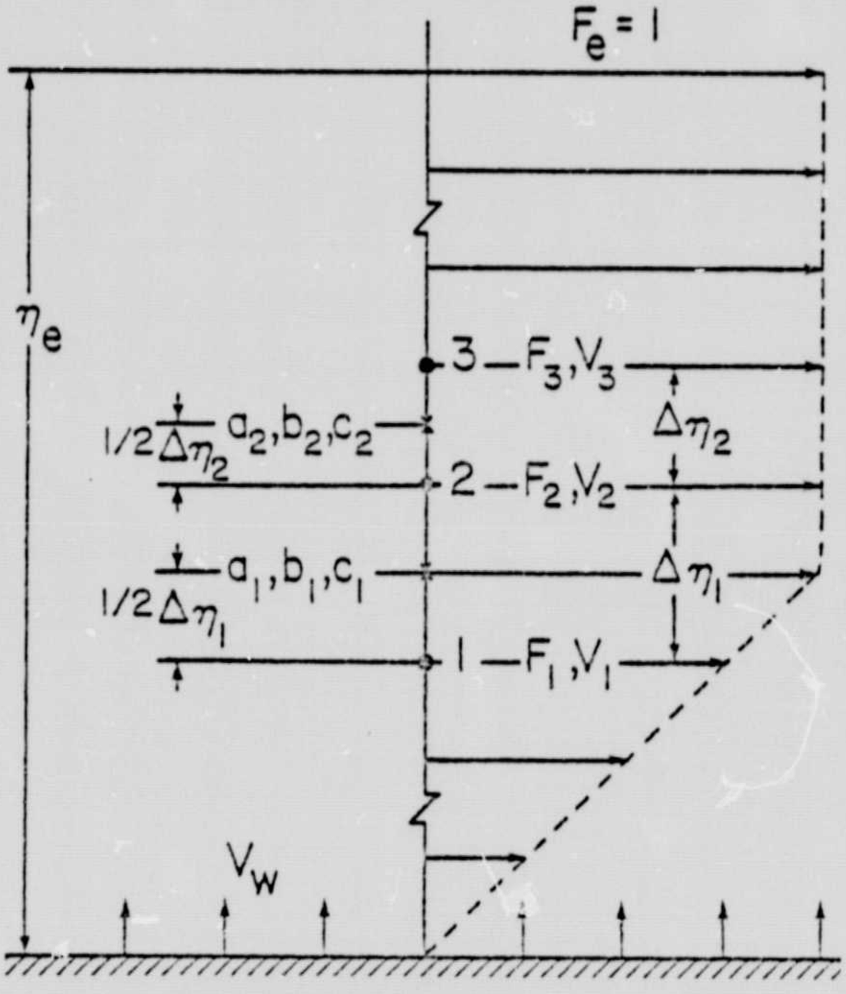
16. Pruess, Steven A., "Solving Linear Boundary Value Problems by Approximating the Coefficients," Math. of Comp., Vol. 27, 1973, pp. 551-561.

17. Rose, Milton E., "Weak-Element Approximations to Elliptic Differential Equations," Numer. Math., Vol. 24, 1975, pp. 185-204.

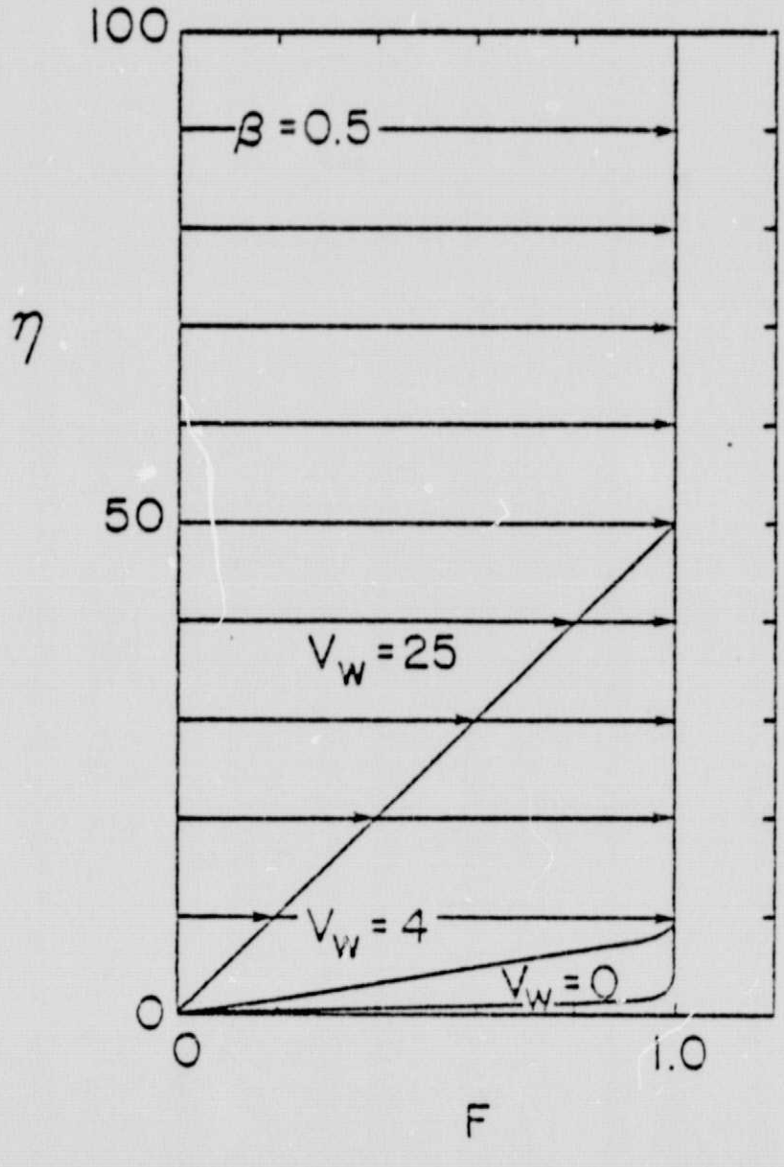
18. Roscoe, D.F., "New Methods for the Derivation of Stable Difference Representations for Differential Equations," J. Inst. Maths. Applics., Vol. 16, 1975, pp. 291-301.

19. Keller, H.B., "Numerical Methods for Two-Point Boundary Value Problems," Blaisdel Publishing Co., 1968.

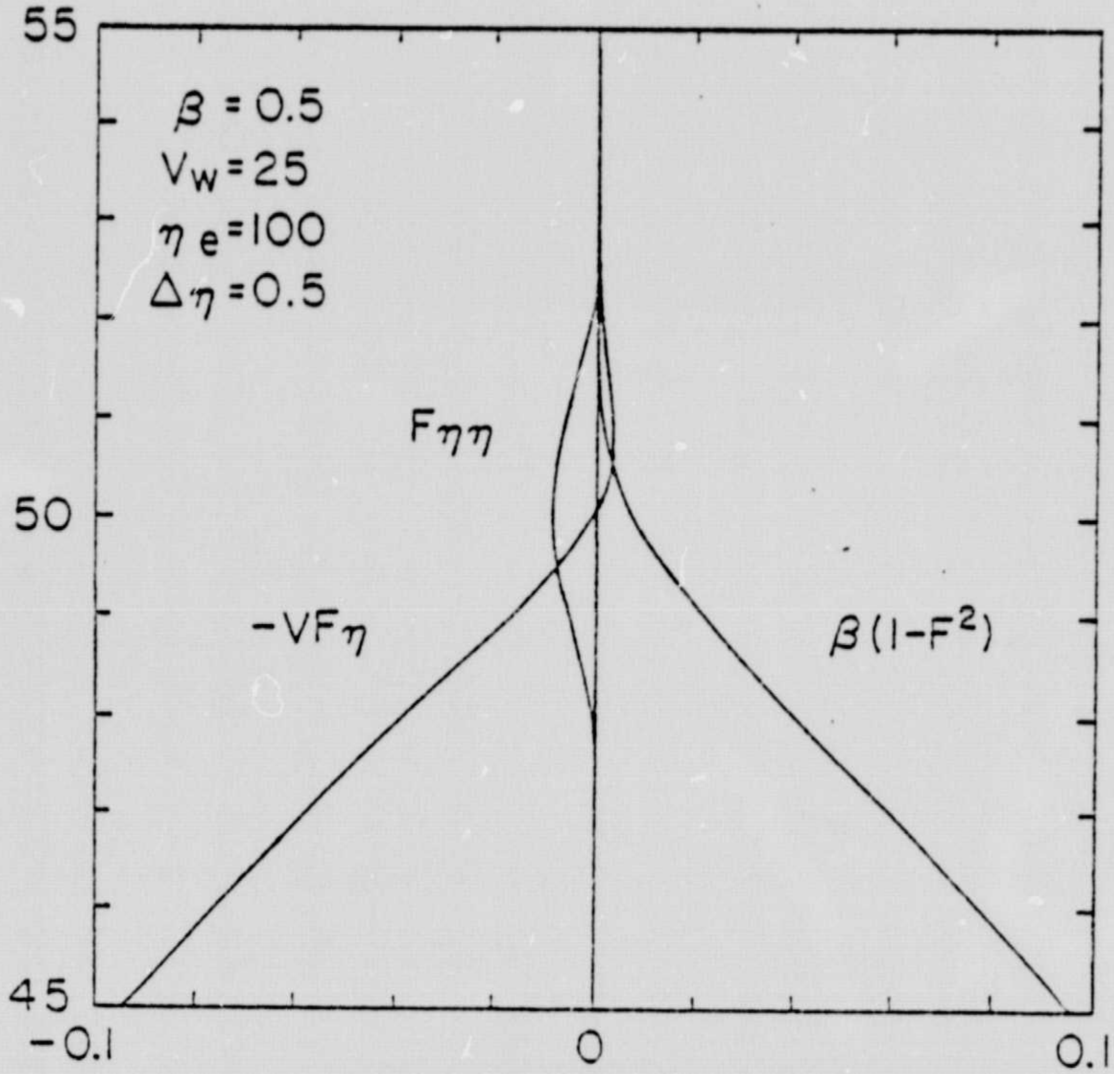
20. Rosenhead, L., (editor), LAMINAR BOUNDARY LAYERS, Oxford at the Carendon Press, 1963, Great Britain.

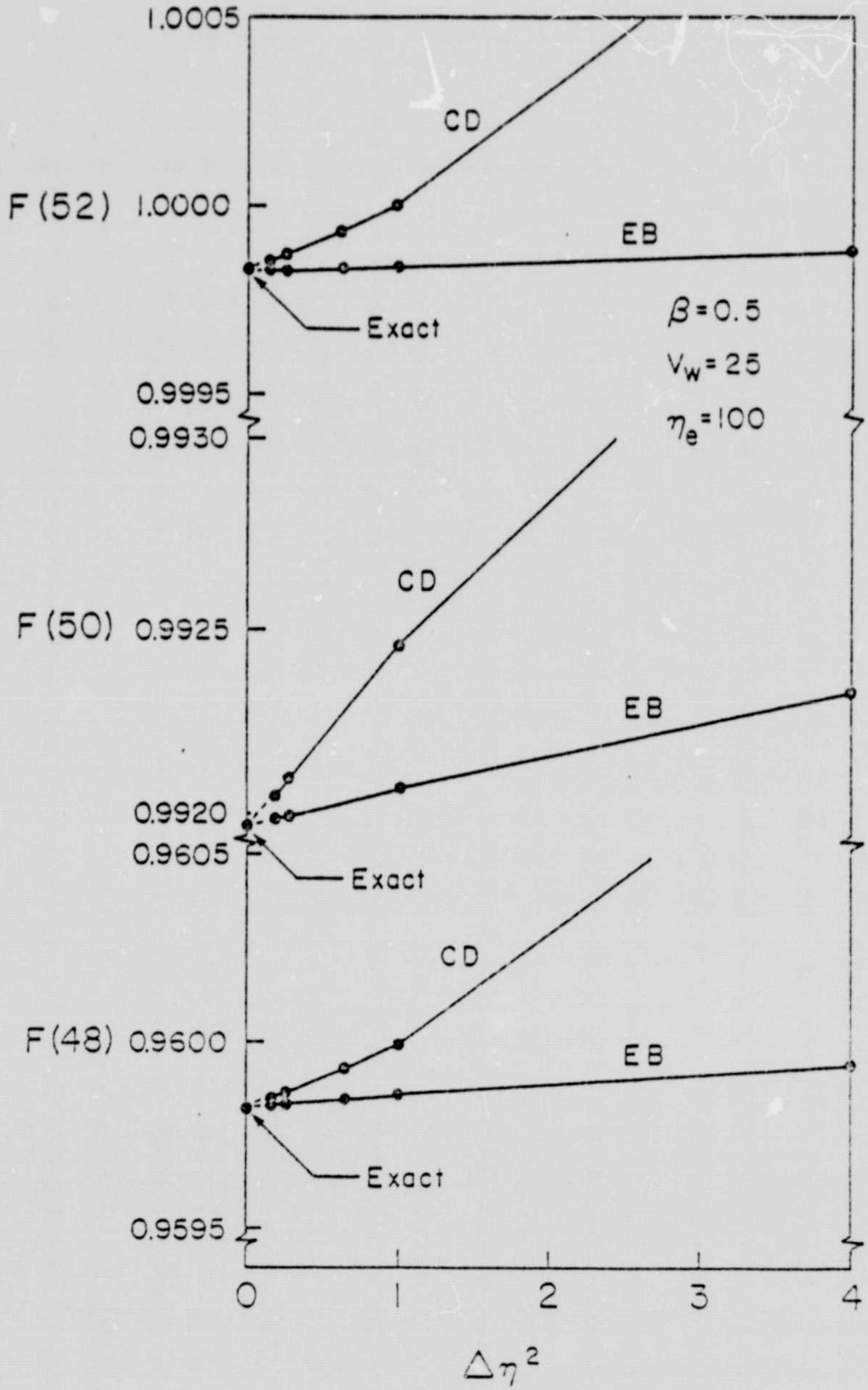


$F_e = 1$
η_e
3 — F_3, V_3
$\Delta\eta_2$
1/2 $\Delta\eta_2$
a_2, b_2, c_2
2 — F_2, V_2
a_1, b_1, c_1
$\Delta\eta_1$
1/2 $\Delta\eta_1$
1 — F_1, V_1
V_w



100
50
0
η
β = 0.5
V_W = 25
V_W = 4
V_W = 0
0
1.0
F


55
50
45
-0.1
0
0.1
β = 0.5
V_W = 25
η e = 100
Δη = 0.5
$F_{\eta\eta}$
$-VF_\eta$
$\beta(1-F^2)$


1.0005
CD
F(52) 1.0000
EB
Exact
β=0.5
V_w=25
η_e=100
0.9995
0.9930
CD
F(50) 0.9925
EB
0.9920
0.9605
Exact
CD
F(48) 0.9600
EB
Exact
0.9595
0
1
2
3
4
$\Delta\eta^2$

APPENDIX B

AN APPLICATION OF FOURTH ORDER SPLINE COLLOCATION PROCEDURES TO BOUNDARY LAYER LIKE FLOWS

A thesis submitted to the

Department of Aerospace Engineering and Applied Mechanics

College of Engineering

Division of Graduate Studies

UNIVERSITY OF CINCINNATI

in partial fulfillment of the
requirements for the degree of

Master of Science

1977

by

GAETANO VACCA

B.Sc. in Mech.E., Universita' Degli Studi di Bari 1974

ABSTRACT

A study and application of the fourth-order spline collocation procedure, numerical solution of boundary laver like differential equations, is presented. A simple inversion algorithm (represting a fourth-order version of the Davis Coupled Scheme) for the simultaneous solution of the resulting difference equations is given. Particular attention is focused on the boundary condition representation for the spline second-derivative approximations. Solutions using the spline procedure, as well as the three-point finite difference method, are presented for several model problems in order to assess and improve the spline numerical scheme. Application of the resulting algorithm to the incompressible laminar self similar boundary layer equations is presented.