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## III - Acoustic Far-Field Analysis

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## III - Acoustic Far-Field Analysis

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## LIST OF SYMBOLS

| | |
|---|---|
| $A_n$, $B_n$, $C_n$, ... | Fourier integral coefficients, defined by Equations (36) and (37). |
| A, B, C, D | Characteristic points, defined in Figure (2). |
| a | Speed of sound; m/sec. |
| b | Stream sheet thickness, m. |
| $C_v$ | Specific heat at constant volume; N•m/kg•K. |
| $H_m$, $J_m$, $K_m$ | Duct response functions. |
| H | Absolute total enthalpy per unit mass; N•m/kg. |
| i | $\sqrt{-1}$ |
| i, j | Unit vectors in axial and circumferential directions, respectively. |
| M | Mach number. |
| m | Meridional distance; m. |
| N | Number of grid points along interface of numerical near-field and acoustic far-field solutions. |
| $N_i$ | Number of blades in the $i^{th}$ row. |
| Q, R | Boundary conditions at interface of numerical near-field and acoustic far-field solutions. |
| p | Static pressure, or pressure perturbation; N/m$^2$. |
| r | Radial distance from axis of rotation; m. |
| S | Entropy per unit mass; N•m/kg•K. |
| $S_n$ | Function defined by Equation (49). |
| t | Time; sec. |
| T | Temperature, K. |
| $T_n$ | Function defined by Equation (61). |
| $U_o$ | Reference velocity; m/sec. |
| $\vec{V}$ | Absolute velocity or velocity perturbation vector; m/sec. |
| $V_m$ | Meridional (axial) component of velocity or velocity perturbation; m/sec. |
| $V_\theta$ | Absolute circumferential component of velocity or velocity perturbation; m/sec. |
| x | Axial distance; m. |
| y | Circumferential distance; m. |
| Y | Fundamental wavelength of stage in circumferential direction; m. |
| $\alpha_n$, $\beta_n$ | Acoustic propagation coefficients, defined by Equations (47) and (54); m$^{-1}$. |
| $\delta(\tau)$ | Dirac delta function (0 for $\tau \neq 0$; 1 for $\tau = 0$). |
| $\Delta$ | Difference operator. |
| $\Delta(\tau)$ | Heaviside step function (0 for $\tau < 0$; 1 for $\tau \geq 0$). |

| | |
|---|---|
| $\gamma$ | Ratio of specific heats. |
| $\Omega$ | Non-dimensional frequency, defined by Equation (57). |
| $\omega$ | Frequency; $sec^{-1}$. |
| $\rho$ | Static density or density perturbation, $kg/m^3$. |
| $\theta$ | Circumferential angle; radians. |
| $\zeta$ | Radial component of vorticity; $sec^{-1}$. |
| $\nabla$ | Vector operator. |

Subscripts:

| | |
|---|---|
| a, b, c, o | Evaluated at points A, B, C, O. |
| o | Reference state. |
| $\infty$ | Infinity condition. |
| i | Inlet station. |
| d | Discharge station. |
| m, n | Indices, defined where used. |
| 1 | First blade row. |
| 1 | Irrotational component of velocity vector. |
| 2 | Second blade row. |
| 2 | Solenoidal component of velocity vector. |

Superscripts:

| | |
|---|---|
| $\rightarrow$ | Vector quantity. |
| $-$ | Time to frequency transform. |
| o | Spatial location to spatial harmonic transform. |
| $'$ | Perturbation variable. |

# LIST OF FIGURES

## INTRODUCTION

Under the conditions typically prevalent in highly loaded transonic fan compressor stages, the linearized, small-perturbation approximations to the equations of motion cannot be expected to be descriptive of the flow in the vicinity of the blades. Thus, recourse is made to the numerical solution of the complete nonlinear system of equations, as discussed in Reference (1) in connection with the blade-to-blade program. However, sufficiently far from the blade rows, the amplitude of the flow disturbances will decay to acoustic levels and the linearized, small-perturbation approximations will be descriptive of the far-field. Therefore, an intermediate region in which both analyses are valid should exist at some distance from the blades. The inlet and discharge stations of the blade-to-blade computational domain can serve as the interfaces between the near-field (numerical) and far-field (acoustic) analyses. The present far-field analysis is formulated with respect to an infinite duct model, namely, all outgoing waves should propagate without reflection. It differs, however, from conventional inlet duct analyses in that the signal may begin with an arbitrary transient, associated with the deviation of the assumed initial data in the near-field from the periodic solution which is sought as the asymptotic limit in time. Therefore, the acoustic analysis must recognize that a transient signal will occur during startup and that a simple harmonic time dependence, which is the usual basis of inlet duct acoustics, cannot be assumed. The analysis should allow the transient to radiate outward without reflection, and should be capable of identifying the attainment of a periodic solution by the growth of discrete harmonic components in the solution.

## INTERFACE WITH NEAR-FIELD SOLUTION

The procedures for defining inlet and discharge stations in terms of axial boundaries of the computational domains shown in Figure (1) have been described in References (1) and (2). The present analysis pertains to the flow upstream of the inlet station and downstream of the discharge station, where the passage is assumed to be an annular segment of an infinite cylindrical duct. With respect to the analysis of Reference (1), the streamsheet is assumed to have constant radius, r, and thickness, b. However, the circumferential boundaries of the passage are not necessarily restricted to a single blade-to-blade passage

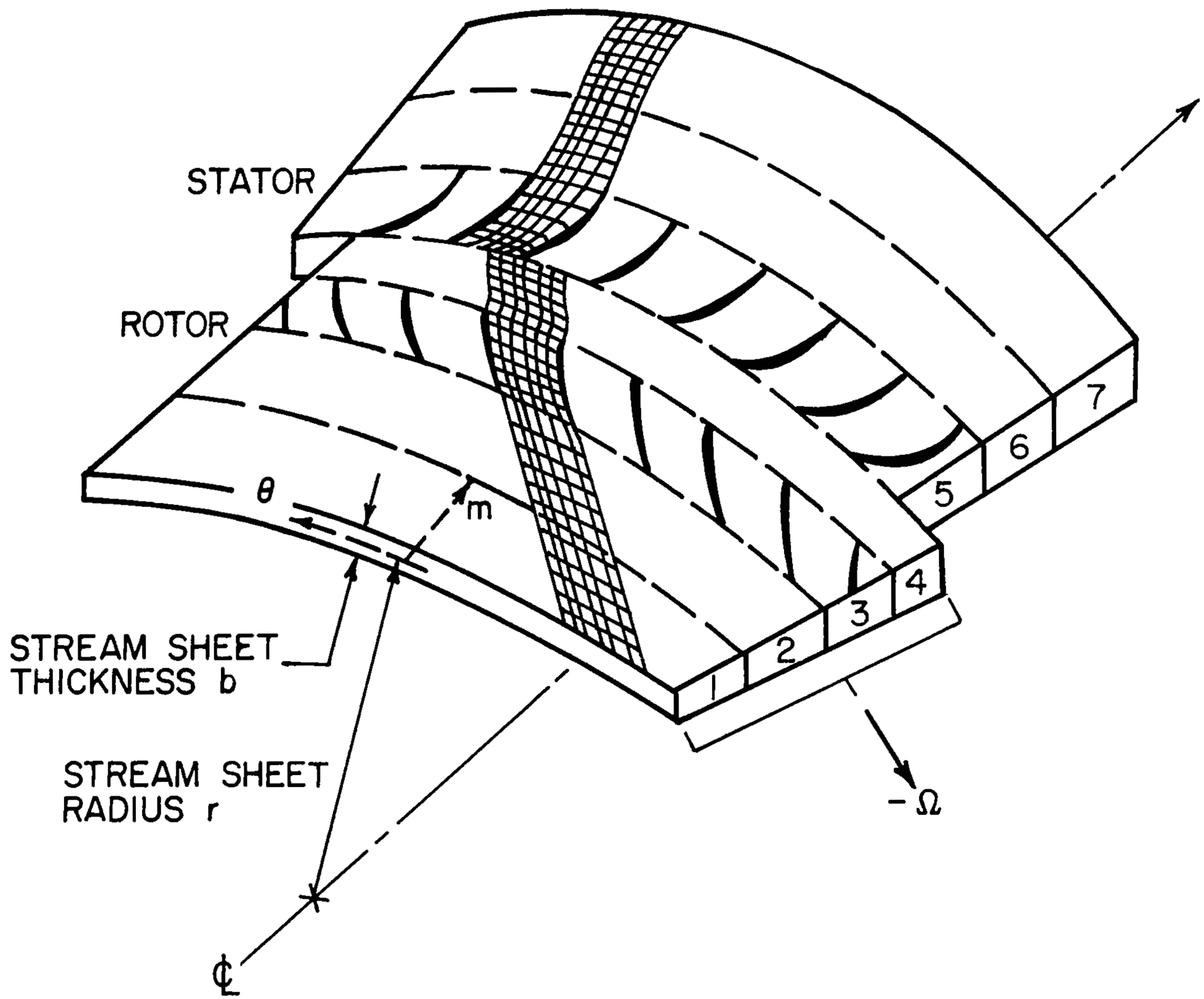


FIGURE 1. BLADE-TO-BLADE COORDINATE SYSTEM AND GRID NETWORK

(as in Reference 1), but should include the angular interval $2\pi/\Delta N$, where $\Delta N = |N_2 - N_1|$. (In the special case $\Delta N=0$, which is usually avoided in practice, use of a single blade-to-blade passage is again permissible.) Use of the present analysis with the computer code described in Reference (2) will, therefore, require additional storage and retrieval of data along the interfaces between the numerical near-field solution and the present acoustic far-field solution, in the same manner as is performed along the interface between domains 4 and 5 of the near-field solution (see Reference 2).

Derivation of the boundary conditions on the near-field (finite difference) solution, at arbitrary stations referred to as the inlet and discharge boundaries of the near field, was presented in Volume 1 (Reference 1). The acoustic analysis is formulated in the absolute frame of reference, and accordingly the boundary point analysis from Reference (1) will be restated here in absolute coordinates.

As discussed in Reference (1), the boundary point solution is obtained from a reference plane method-of-characteristics procedure.* A pair of (approximately) two-dimensional wave motion characteristics and a stream surface characteristic are identified at each grid point on the inlet and discharge boundaries, as shown in Figure (2). The compatibility relations,

$$\frac{dp}{dt} \pm \rho a \frac{dV_m}{dt} = -\gamma p \left(\frac{\partial V_\theta}{r\partial\theta} + \frac{V_m}{rb}\frac{drb}{dm}\right) \pm \rho a V_\theta^2 \left(\frac{1}{r}\frac{dr}{dm}\right) \qquad (1)$$

apply on the wave motion characteristics,

$$\frac{dm}{V_m \pm a} = \frac{rd\theta}{V_\theta} = dt \qquad (2)$$

which are depicted by the lines AO and CO in Figure (2). The equation,

$$S = \text{constant} \qquad (3)$$

applies on the stream path characteristic,

$$\frac{dm}{V_m} = \frac{rd\theta}{V_\theta} = dt \qquad (4)$$

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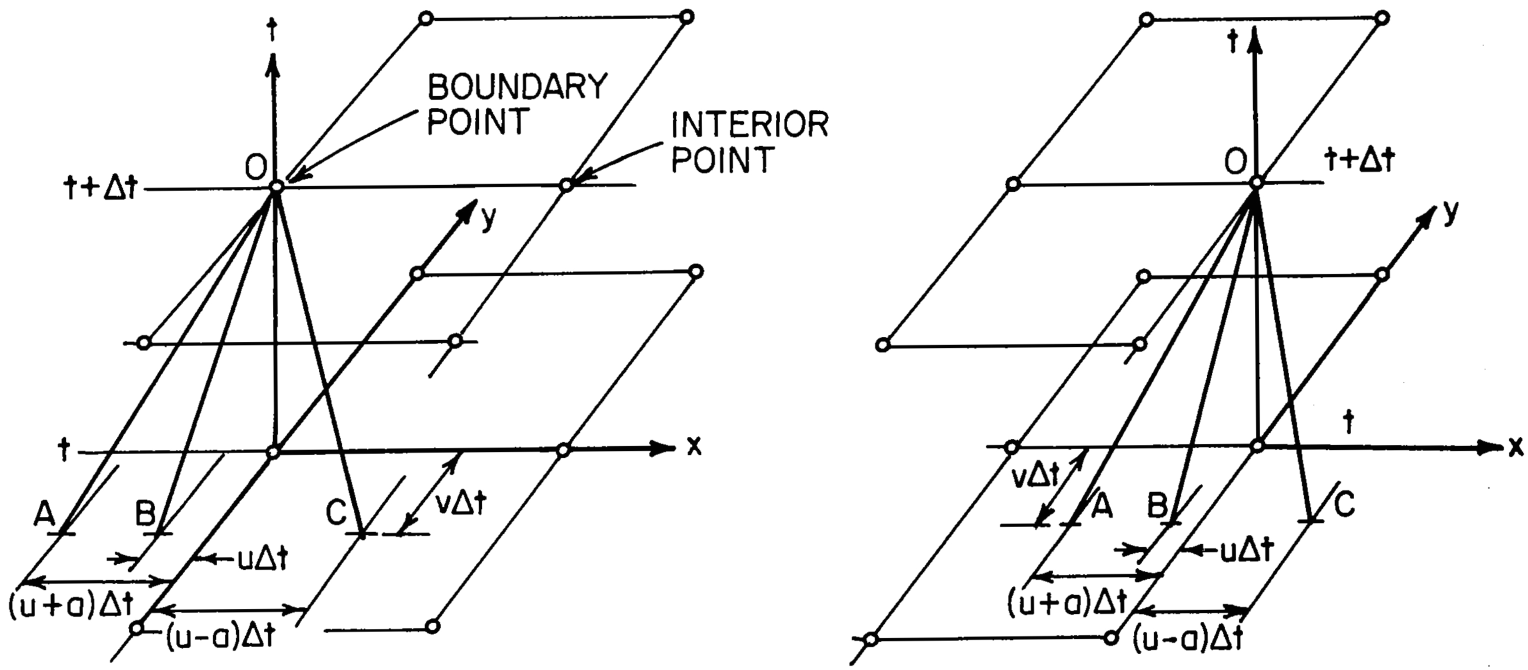
*As pointed out in Reference (1), translation of the reference plane effectively transforms a rotating frame of reference back to an absolute frame. Thus the numerical results are independent of the coordinate system in which the equations are stated.




INLET STATION

DISCHARGE STATION

FIGURE 2. CHARACTERISTIC LINES AND GRID POINTS AT INLET AND DISCHARGE STATIONS

which is depicted by the line BO in Figure (2). In addition, if the flow is isentropic

$$\zeta/\rho = \text{constant}$$

also applies on the stream path characteristic, BO. Finally, the circumferential momentum equation is written as:

$$\frac{\partial V_\theta}{\partial t} = -\frac{\partial H}{r\partial\theta} + T\frac{\partial S}{r\partial\theta} - V_m\,\zeta \qquad (5)$$

The inlet flow is assumed to be isentropic and irrotational; thus:

$$S_i = 0, \quad \text{or} \quad p_i/\rho_i^\gamma = p_{-\infty}/\rho_{-\infty}^\gamma \qquad (6)$$

and

$$\zeta_i = 0 \qquad (7)$$

giving:

$$\left(\frac{\partial V_\theta}{\partial t}\right)_i = -\left(\frac{\partial H}{r\partial\theta}\right)_i \qquad (8)$$

Introducing the small disturbance approximations,

$$p' = p - p_{-\infty} << p_{-\infty} \qquad (9)$$

$$V_m' = V_m - V_{m_{-\infty}} << V_{m_{-\infty}} \qquad (10)$$

$$V_\theta' = V_\theta << V_{m_{-\infty}} \qquad (11)$$

etc.

the compatibility equation on the outgoing wave, CO, at the inlet becomes:

$$\begin{aligned} p_i' - \rho_{-\infty}\, a_{-\infty}\, V_{m_i}' &= p_c' - \rho_{-\infty}\, a_{-\infty}\, V_{m_c}' \\ &- \frac{1}{2}\left(\gamma p_{-\infty}\left(\frac{\partial V_\theta}{r\partial\theta}\right)_i + \gamma\, p_c\left(\frac{\partial V_\theta}{r\partial\theta} + \frac{V_m}{rb}\frac{drb}{dm}\right)_c\right)\Delta t \\ &- \left(\rho_{-\infty}\, a_{-\infty}\, V_{\theta_c}^2\left(\frac{1}{r}\frac{dr}{dm}\right)_c\right)\Delta t \end{aligned} \qquad (12)$$

where the subscript i denotes the inlet value at a new time, $t+\Delta t$, i.e., at point 0 in Figure (2), and subscript c denotes the value at point C. Recall that r and b are assumed constant for $m \leq m_i$. Since $V_\theta$ can be evaluated at all points 0 along the inlet, from integration of Equation (8), and all variables at point C can be obtained by interpolation of known (current) data in the near-field, the right-hand side can be regarded as a known function of $(m_i, \theta, t)$, i.e.:

$$p_i' - \rho_{-\infty} a_{-\infty} V_{m_i}' = \rho_{-\infty} a_{-\infty}^2 Q_i (m_i, \theta, t) \tag{13}$$

$Q_i (m_i, \theta, t)$ is taken as the boundary condition to be placed on the upstream far-field solution. As will be shown in the following section, this condition is sufficient to determine $p_i'$, and thus $p_i$. $V_{m_i}'$ can then be obtained from Equation (13), and $\rho_i$ from Equation (6). As indicated above, $V_{\theta_i}$ is obtained from Equation (8), completing the inlet solution at the new time $t+\Delta t$.

Similar procedures at the discharge boundary lead to the relations:

$$p_d/\rho_d^\gamma = p_b/\rho_b^\gamma \tag{14}$$

$$\left(\frac{\partial V_\theta}{\partial t}\right)_d = -\left(\frac{\partial H}{r\partial \theta} - T\frac{\partial S}{r\partial \theta} + V_m \zeta\right)_d \tag{15}$$

$$p_d' + \rho_\infty a_\infty V_{m_d}' = \rho_\infty a_\infty^2 Q_d (m_d, \theta, t) \tag{16}$$

Equation (16) represents one boundary condition on the downstream far-field. Integration of Equation (15) provides a second boundary condition on the downstream far-field, which is required since it cannot be assumed to be irrotational.

$$V_{\theta_d} = a_\infty R_d (m_d, \theta, t) \tag{17}$$

Due to the finite work input by the rotor, the downstream reference conditions $(\rho_\infty, a_\infty)$ are also not necessarily the same as the upstream reference conditions $(\rho_{-\infty}, a_{-\infty})$. In this case, the far-field analysis provides the value of $p_d'$ and Equations (14), (16) and (17) complete the discharge station solution at the new time $t+\Delta t$.

With these preliminaries in hand, attention is now focussed on the acoustic far-field analysis. In the following discussion the primes on the perturbation variables will be dropped as the acoustic solution is understood to pertain only to perturbation quantities. Furthermore, the subscript $( )_o$ will be used to denote the reference conditions for either boundary, that is, $x \rightarrow \pm \infty$, and all variables without subscript will refer to perturbations with respect to the reference state, for example, $p = p - p_o$ and $\vec{V} = \vec{V} - \vec{V}_o$.

## FAR-FIELD ANALYSIS

The problem of interest is the acoustic radiation from the considered blade rows under steady-state operating conditions. However, development of the numerical solution for the near-field includes generation of a transient flow field associated with the deviation of the assumed initial conditions from a periodic solution, which is sought as an asymptotic limit in time. Therefore, analysis of the far-field must necessarily recognize that a transient signal will occur during startup and that a simple harmonic time dependence, which is the usual basis of engine acoustic analyses, cannot be assumed to be descriptive until the asymptotic limit is approached. The analysis should allow the transient to radiate outward, without reflection, and should be capable of identifying the attainment of a periodic solution by the growth of discrete harmonic components.

Use will be made of a harmonic type analysis to develop an acoustic formulation of the pressure field in which frequency effects are superimposed and the time history obtained by integration. This procedure leads to a convolution integral (i.e., a type of Duhamel integral) in the time domain, which is more directly compatible with the numerical data in the near-field than a result in the frequency domain. (The near-field data consists of values of the pressure, flow velocity, etc., at a discrete series of grid points and at fixed time intervals.)

Since the present effort is addressed toward a cascade formulation, the governing equations are written in a two-dimensional, Cartesian coordinate system. It is noted that the two-dimensional problem could be described by a solution of the wave equation alone, were it not for the fact that the time dependent force distribution on the blades is capable of producing a convected vorticity field (as well as a corresponding entropy field, which, however, is not relevant to the present problem). It will be seen that the convected vorticity field does not contribute to the acoustic pressure field, per se, as distinguished from the irrotational component of the velocity field which is directly coupled with the acoustic pressure. Therefore, the velocity perturbation field downstream of the discharge boundary and upstream of the inlet boundary (see Figure 1) is characterized by the sum of an irrotational ($\nabla$x=0) velocity vector $\vec{V}_1$ and a solenoidal ($\nabla\cdot$=0) velocity vector $\vec{V}_2$, which satisfy the linearized conservation equations. Note that the inlet flow has been assumed to be irrotational, therefore, $\vec{V}_2=0$ in the upstream far-field; however, the analysis will be developed with respect to the more general case pertaining to the downstream far-field.

The linearized conservation equations for the perturbation variables can be stated as*

$$\frac{dp}{dt} + \rho_o a_o^2 \nabla\cdot\vec{V} = 0 \tag{18}$$

$$\rho_o \frac{d\vec{V}}{dt} + \nabla p = 0 \tag{19}$$

$$\frac{dp}{dt} - a_o^2 \frac{d\rho}{dt} = \frac{p_o}{C_v} \frac{dS}{dt} \tag{20}$$

where:

$$\vec{V} = \vec{V}_1 + \vec{V}_2 \tag{21}$$

$$\vec{V}_1 = \vec{i}\, V_{m_1} + \vec{j}\, V_{\theta_1} \tag{22}$$

$$\vec{V}_2 = \vec{i}\, V_{m_2} + \vec{j}\, V_{\theta_2} \tag{23}$$

$$\nabla \times \vec{V}_1 = 0 \tag{24}$$

$$\nabla \times \vec{V}_2 = \zeta \neq 0 \tag{25}$$

*These represent linearized versions of Equations (13), (14), (15) and (17) of Reference (1), with r = constant and b = constant.

$$\nabla \cdot \vec{V}_2 = 0 \tag{26}$$

$$U_o = M_o \; a_o \tag{27}$$

and

$$\frac{d}{dt} = \frac{\partial}{\partial t} + U_o \frac{\partial}{\partial m} \tag{28}$$

Since the streamsheet has been assumed to be cylindrical in the far-field, with $b << r$, the conventional Cartesian coordinates $(x,y)$ will be used in place of the meridional coordinates $(m,\theta)$:

$$x = m \tag{29a}$$

$$y = r\theta \tag{29b}$$

Substituting Equation (26) into (18) gives:

$$\frac{dp}{dt} + \rho_o \; a_o^2 \; \nabla \cdot \vec{V}_1 = 0 \tag{30}$$

Substituting Equations (24) and (26) into the curl and divergence of Equation (19), respectively, gives:

$$\frac{d}{dt} \left( \nabla \cdot \vec{V}_1 \right) + \frac{1}{\rho_o} \nabla^2 p = 0 \tag{31}$$

and:

$$\frac{d}{dt} \left( \nabla x \vec{V}_2 \right) = 0 \tag{32}$$

Equations (18) and (19) or (30) and (31) yield the wave equation for the pressure:

$$\frac{1}{a_o^2} \frac{d^2 p}{dt^2} - \nabla^2 p = 0 \tag{33}$$

Therefore, solutions for p are purely radiative. Since Equations (30) and (31) can be also combined to yield the wave equation, solutions for $\vec{V}_1$ are also purely radiative. However, Equation (32) infers that solutions for $\vec{V}_2$ are purely convective. Therefore:

$$\frac{d\vec{V}_2}{dt} = 0 \tag{34}$$

and:

$$\frac{d\vec{V}_1}{dt} + \frac{1}{\rho_o} \nabla p = 0 \qquad (35)$$

Solutions to Equations (33), (34) and (35) can be expressed in terms of a Fourier integral representation:

$$\begin{Bmatrix} p(x,y,t) \\ V_{m_1}(x,y,t) \\ V_{\theta_1}(x,y,t) \end{Bmatrix} = \sum_n e^{-i\alpha_n y} \int_{-\infty}^{\infty} \begin{Bmatrix} \rho_o \, a_o \, A_n(\omega) \\ B_n(\omega) \\ C_n(\omega) \end{Bmatrix} a_o e^{i(\omega t - \beta_n x)} \frac{d\omega}{2\pi} \qquad (36)$$

$$\begin{Bmatrix} V_{m_2}(x,y,t) \\ V_{\theta_2}(x,y,t) \end{Bmatrix} = \sum_n e^{-i\alpha_n y} \int_{-\infty}^{\infty} \begin{Bmatrix} D_n(\omega) \\ E_n(\omega) \end{Bmatrix} a_o e^{i\omega(t - x/U_o)} \frac{d\omega}{2\pi} \qquad (37)$$

Consider first the downstream boundary on which (see Equations 16 and 17):

$$Q_d(y,t) = (p_d + \rho_o \, a_o \, V_{m_d})/(\rho_o \, a_o^2) \qquad (38)$$

$$R_d(y,t) = V_{\theta_d}/a_o \qquad (39)$$

Thus, the boundary data can be expressed as:

$$\begin{Bmatrix} Q(y,t) \\ R(y,t) \end{Bmatrix} = \sum_n e^{-i\alpha_n y} \int_{-\infty}^{\infty} \begin{Bmatrix} \bar{Q}_n^o(\omega) \\ \bar{R}_n^o(\omega) \end{Bmatrix} e^{i\omega t} \frac{d\omega}{2\pi} \qquad (40)$$

where $(\overline{\quad})$ denotes the time to frequency transform, as above, while $(\ )^o$ indicates transformation from spatial location, y, to spatial harmonic, n, which will be discussed later (cf. Equation 55). (The subscript d has been dropped since the ensuing development can apply to either the inlet or discharge boundary.)

Substitution of the above integral forms into Equation (24), (26), (32), (34), (38) and (39) gives:

$$C_n = \frac{\alpha_n}{\beta_n} B_n \tag{41}$$

$$E_n = - \frac{\omega}{U_o \alpha_n} D_n \tag{42}$$

$$B_n = \frac{a_o \beta_n}{\omega - U_o \beta_n} A_n \tag{43}$$

$$A_n + B_n + D_n = \bar{Q}_n^o \tag{44}$$

$$C_n + E_n = \bar{R}_n^o \tag{45}$$

The value of $A_n$, and thus $B_n$, $C_n$, $D_n$ and $E_n$, can thereby be expressed in terms of the transform of the boundary data, viz.:

$$A_n = \frac{\omega - U_o \beta_n}{\omega^2 + (a_o - U_o)\beta_n \omega + U_o a_o \alpha_o^2} [\omega \bar{Q}_n^o + U_o \alpha_n \bar{R}_n^o] \tag{46}$$

The propagation coefficient $\beta_n$ is determined by substituting the integral relation for pressure (Equation 36) into the wave equation (Equation 33):

$$\beta_n = \frac{-\omega M_o \pm (\omega^2 - \alpha_n^2 a_o^2 (1-M_o^2))^{\frac{1}{2}}}{a_o (1-M_o^2)} \tag{47}$$

where the + sign refers to downstream propagation (from the discharge boundary) and the - sign to upstream propagation (from the inlet boundary).

Equations (46) can now be substituted into the corresponding integral relation (Equation 36) to obtain, after some manipulation, the pressure perturbation on the discharge boundary in terms of the specified boundary data.

$$p_d(y,t) = \frac{\rho_o a_o}{(1-M_o^2)} \sum_n \frac{e^{-i\alpha_n y}}{\alpha_n a_o} \int_{-\infty}^{\infty} (\omega - S_n) \left(\frac{\omega}{\alpha_n a_o} \bar{Q}_n^o + M_o \bar{R}_n^o\right) e^{i\omega t} \frac{d\omega}{2\pi} \tag{48}$$

where

$$S_n = (\omega^2 - \alpha_n^2 a_o^2 (1-M_o^2))^{\frac{1}{2}} \tag{4}$$

Consider now the inlet boundary where $\vec{V}_2 = 0$. The boundary condition here is stated as (see Equation 13):

$$Q_i \ (y,t) = (p_i - \rho_o a_o V_{m_i})/(\rho_o a_o^2) \tag{5}$$

Since $\vec{V}_2 = 0$, $D_n = E_n = 0$, and $\bar{R}_n^o$ becomes part of the solution, i.e., it cannot be specified as a boundary condition, consistent with the near field analysis described in the previous section. Thus, $A_n$ is only a function of $Q_i$, viz:

$$A_n = \frac{\omega - U_o \beta_A}{\omega - (a_o + U_o)\beta_n} \bar{Q}_n^o \tag{5}$$

where

$$Q_i(y,t) = \sum_n e^{-i\alpha_n y} \int_{-\infty}^{\infty} \bar{Q}_n^o(\omega) \ e^{i\omega t} \frac{d\omega}{2\pi} \tag{5}$$

Again, with some manipulation, an equation for the pressure perturbation on the inlet boundary is obtained in terms of the specified data:

$$p_i(y,t) = \frac{\rho_o a_o^2}{(1-M_o^2)\ (1+M_o)} \sum_n \frac{e^{-i\alpha_n y}}{\alpha_n^2 a_o^2} \int_{-\infty}^{\infty} (\omega - S_n) \ (\omega + M_o S_n) \ \bar{Q}_n^o \ e^{i\omega t} \frac{d\omega}{2\pi} \tag{5}$$

The selected representation of the solutions, Equations (36) and (37), and boundary conditions, Equations (40) and (52), as Fourier series in the y direction is appropriate for enforcement of the periodicity boundary condition pertaining to spatial variations in this direction. The coefficient $\alpha_n$ is defined accordingly:

$$\alpha_n = \frac{2\pi n}{Y} = \frac{n\Delta N}{r} \tag{54}$$

where Y is the fundamental wave length of the stage (double cascade) configuration (i.e, $Y = 2\pi r/\Delta N$). In addition, the fact that the boundary data is specified at a discrete number of grid points, say N, on the boundaries implies that the Fourier series can only include N terms; n=0, 1, 2...N-1. Since the distance y to each point can be written as mY/N, where m=0, 1, 2...N-1 also, the Fourier series can be expressed in the standard Discrete Fourier Transform (DFT) notation:

$$\begin{Bmatrix} Q(y,t) \\ R(y,t) \end{Bmatrix} = \begin{Bmatrix} Q_m(t) \\ R_m(t) \end{Bmatrix} = \sum_{n=0}^{N-1} \begin{Bmatrix} Q_n^o(t) \\ R_n^o(t) \end{Bmatrix} e^{-\frac{2\pi i nm}{N}} \tag{55}$$

The inverse DFT is then:

$$\begin{Bmatrix} Q_n^o(t) \\ R_n^o(t) \end{Bmatrix} = \frac{1}{N} \sum_{m=0}^{N-1} \begin{Bmatrix} Q_m(t) \\ R_m(t) \end{Bmatrix} e^{2\pi i nm/N} \tag{56}$$

For computational purposes, the following non-dimensional parameters are evident in Equations (48) and (53):

$$\Omega = \omega/\alpha a_o \tag{57}$$

$$\tau = \alpha a_o t \tag{58}$$

where

$$\alpha = 2\pi/Y = \alpha_n/n \tag{59}$$

Then, on the discharge boundary:

$$p_n^o(\tau) = \frac{\rho_o a_o^{\,2}}{(1-M_o^2)n^2} \int_{-\infty}^{\infty} (\Omega-T_n)\ (\Omega\ \bar{Q}_n^o(\Omega) + n\ M_o\ \bar{R}_n^o\ (\Omega)e^{i\Omega\tau})\ \frac{d\Omega}{2\pi} \tag{60}$$

where:

$$T_n = (\Omega^2 - (1-M_o^2)n^2)^{\frac{1}{2}} \tag{61}$$

$$\bar{Q}_n^o(\Omega) = \alpha a_o\ \bar{Q}_n^o(\omega) \tag{62}$$

$$\bar{R}_n^o(\Omega) = \alpha a_o\ \bar{Q}_n^o(\omega) \tag{63}$$

and on the inlet boundary:

$$p_n^o(\tau) = \frac{\rho_o a_o^{\,2}}{(1-M_o^2)\ (1+M_o)n^2} \int_{-\infty}^{\infty} (\Omega-T_n)\ (\Omega+M_o T_n)\ \bar{Q}_n^o(\Omega)e^{i\Omega\tau}\ \frac{d\Omega}{2\pi} \tag{64}$$

Then, on either boundary:

$$p_m(\tau) = \sum_{n=0}^{N-1} e^{-2\pi i n m/N}\ p_n^o(\tau) \tag{65}$$

The integrals of Equations (60) and (64) can be transformed back to the time domain by the following convolution. On the discharge boundary:

$$p_n^o(\tau) = \rho_o a_o^2\ (H_n^o(\tau) * Q_n^o(\tau) + J_n^o(\tau) * R_n^o(\tau)) \tag{66}$$

where * denotes convolution in time. For $n \geq 1$:

$$H_n^o(\tau) = \frac{1}{n^2(1-M_o^2)} \int_{-\infty}^{\infty} \Omega\ (\Omega-T_n)\ e^{i\Omega\tau}\ \frac{d\Omega}{2\pi} \tag{67}$$

$$J_n^o(\tau) = \frac{1}{n(1-M_o^2)} \int_{-\infty}^{\infty} (\Omega - T_n)\, e^{i\Omega\tau}\, \frac{d\Omega}{2\pi} \tag{68}$$

and, for n=0:

$$H_o^o(\tau) = \frac{1}{2}\, \delta(\tau) \tag{69}$$

$$J_o^o(\tau) = 0 \tag{70}$$

$$p_o^o(\tau) = \frac{1}{2}\, \rho_o a_o^2 Q_o^o(\tau) \tag{71}$$

where $\delta$ is the Dirac delta function.

Note that for $\tau \to 0$ with $n \neq 0$:

$$H_n^o(\tau) \to \frac{1}{2}\, \delta(\tau) \tag{72}$$

$$p_n^o(\tau) \to \frac{1}{2}\, \rho_o a_o^2 Q_n^o(\tau) \tag{73}$$

On the inlet boundarcy:

$$p_n^o(\tau) = \rho_o a_o^2 K_n^o(\tau) \ast p_n^o(\tau) \tag{74}$$

where, for $n \geq 1$

$$K_n^o(\tau) = \frac{1}{n^2(1-M_o^2)\ (1+M_o^2)} \int_{-\infty}^{\infty} (\Omega - T_n)\ (\Omega + M_o T_n)\, e^{i\Omega\tau}\, \frac{d\Omega}{2\pi} \tag{75}$$

and, for n=0:

$$K_o^o(\tau) = \frac{1}{2}\,\delta(\tau) \tag{76}$$

$$p_o^o(\tau) = \frac{1}{2}\;\rho_o a_o^2 p_o^o(\tau) \tag{77}$$

while for $t \to 0$ with $n \neq 0$:

$$k_n^o(\tau) = \frac{1}{2}\,\delta(\tau) \tag{78}$$

$$p_n^o(\tau) = \frac{1}{2}\;\rho_o a_o^2 p_n^o(\tau) \tag{79}$$

The integral in Equation (67) is evaluated by dividing the non-dimensional frequency range into three regions, viz: $-\infty$ to $-\Omega_o$, $-\Omega_o$ to $\Omega_o$, and $\Omega_o$ to $\infty$. In the middle region standard DFT techniques are applied, while in the outer regions, $\Omega \to \pm\infty$, the integral is accurately approximated in terms of an exponential integral of the form

$$\int_{\pm\Omega_o}^{\pm\infty} \frac{e^{i\Omega\tau}}{\Omega^{\alpha}}\,d\Omega \tag{80}$$

for which closed form expressions are available. The integral in Equation (68) can be expressed in terms of a Bessel function:

$$\int_{-\infty}^{\infty} e^{i\Omega\tau}\;(\Omega - (\Omega^2-c^2)^{\frac{1}{2}})\;\;\frac{d\Omega}{2\pi} = ic^2\,\frac{J_1(c\tau)}{c\tau}\;\;\Delta(\tau) \tag{81}$$

where c denotes $n(1-M_o^2)^{\frac{1}{2}}$ and $\Delta(\tau)$ is the Heaviside step function. The evaluation of the integral in Equation (75) is carried out in the same fashion as that in Equation (67).

The desired solution for the pressure perturbation on the interface with near-field is finally accomplished by observing that Equation (65) can now be expressed in the form:

$$p_m(t) = \rho_o a_o^2 \sum_{n=0}^{N-1} e^{-2\pi inm/N} \left\{ \begin{array}{l} H_n^o(t) * Q_n^o(t) + J_n^o(t) * R_n^o(t) \\ \\ K_n^o(t) * Q_n^o(t) \end{array} \right\} \quad (82)$$

on the discharge and inlet boundaries respectively. Use of Equation (55) for $P_m(t)$, $Q_m(t)$ and $R_m(t)$ and similar DFT expansions for $H_m$, $J_m$ and $K_m$ then leads to the convolution:

$$p_m(t) = \frac{\rho_o a_o^2}{N} \sum_{n=0}^{N-1} \left\{ \begin{array}{l} H_n(t) * Q_{m-n}(t) + J_n(t) * R_{m-n}(t) \\ \\ K_n(t) * Q_{m-n}(t) \end{array} \right\} \quad (83)$$

on the discharge and inlet boundaries respectively. Thus, a double convolution over both time and distance (in the y direction) is required. The functions* $Q_m(t)$ and $R_m(t)$, therefore, represent the point sources of time-varying strength which are aligned along the considered boundaries, with spatial resolutions consistent with the number of grid points specified.

The functions $H_m(t)$, $J_m(t)$ and $K_m(t)$ are the duct response functions, defined by:

---

*Bear in mind that $Q_m(t)$ denotes either $Q_i\ (y_m,t)$ at the inlet station or $Q_d\ (y_m,t)$ at the discharge station, while $R_m(t)$ denotes $R_d\ (y_m,t)$.

$$
\begin{Bmatrix} H_m(t) \\ J_m(t) \\ K_m(t) \end{Bmatrix} = \sum_{n=0}^{N-1} \begin{Bmatrix} H_n^o(t) \\ J_n^o(t) \\ K_n^o(t) \end{Bmatrix} e^{-2\pi i nm/N}
$$

$$
= \begin{Bmatrix} \frac{1}{2}\,\delta(t) \\ 0 \\ \frac{1}{2}\,\delta(t) \end{Bmatrix} + \sum_{n=1}^{N-1} \begin{Bmatrix} H_n^o(t) \\ J_n^o(t) \\ K_n^o(t) \end{Bmatrix} e^{-2\pi i nm/N} \tag{84}
$$

Finally, it should be noted that only the real part of $p_m(t)$ is of interest. Equations (67), (68) and (75) infer that $H_n^o(\tau)$ and $K_n^o(\tau)$ are purely real functions and $J_n^o(\tau)$ is a pure imaginary function. The boundary data $Q_m(t)$ and $R_m(t)$ are obviously all real numbers. Therefore, it is concluded that

$$
J_m(t) = \sum_n - i\, J_n^o(t)\, \sin \frac{2\pi nm}{N} = \sum_n L_n^o(t) \quad \sin \frac{2\pi nm}{N} \tag{85}
$$

where

$$
L_n^o(\tau) = \frac{1}{n(1-M_o^2)} \int_{-\infty}^{\infty} (\Omega - T_n) e^{i\Omega\tau} \, \frac{d\Omega}{2\pi} \tag{86}
$$

I.e., the solution can be carried out entirely in terms of real functions.

## NUMERICAL EXAMPLE

A group of subroutines including an efficient Fast Fourier Transform routine has been developed, for use in conjunction with the blade-to-blade computer program B2DATL, described in Reference (2), to carry out the indicated convolutions required to solve Equation (83) numerically. Results have been thus far limited to test cases with a simple harmonic input signal. For example, in one of the calculations the discharge station was assumed to be divided into 8 intervals covering a total circumferential distance of 0.1 m. The selected reference (average) Mach number of the discharge flow was 0.8. The flow was assumed to be irrotational so that only one input function Q(y,t) was required, and the second function R(y,t) could be considered as a response function (that is, it was calculated from Q(y,t)). The input function $Q(y,t) = \cos(\Omega\tau - 2\pi n/N)$, with $Q(y,t) = 0$ for $\tau < 0$, was selected for this case, where $\tau = 2\pi N a_o t/Y$, $(n = 0, 1, 2, \ldots, N-1)$ $N = 8$, $Y = 0.1$ m., $a_o = 10^3$ m/sec, and $\Omega = 1$. The input function Q and response function R at n = 0 are plotted in Figure (3). It should be noted that the response function is initially out of phase with the input function because of the assumption that $Q = 0$ for $\tau < 0$. However, the effect of the transient at $\tau = 0$ dies quickly, and after about 1/3 millisecond the response function closely approximates the input function and indicates the desired harmonic solution is being approached asymptotically. The non-dimensional perturbation pressure is plotted in Figure (4). The complete history is shown for the point n = 0, whereas the history of the points n = 1 and 2 is only shown at early times, where a difference in amplitude as well as phase exists. The values of $p_1$ and $p_2$ are not plotted at later times (that is, after about 1/3 millisecond) since the pressure solutions at the various grid points only differ noticeably in the phase angle corresponding to the input function and the differences in amplitude asymptotially decay. However, the complete numerical results for n = 0, 1, 2.....7 are included in Appendix B as the first test case.

## DESCRIPTION OF COMPUTER CODE

### Overview

Program RAFFT (Real Acoustic Far-Field Theory), listed in Appendix A, is designed to compute the real part of the acoustic far-field of a pair of interacting cascades in an infinite duct. This program has not as yet been coupled to program B2DATL which computes the near field of the cascades. It currently consists of two parts. The first part, which is made up of the main program RAFFT, the subroutine GETQR (which calculates the input functions Q and R) and the function routine TRØØT, is at present used solely for the test purposes; it must be replaced by a calling routine in B2DATL which calculates the input

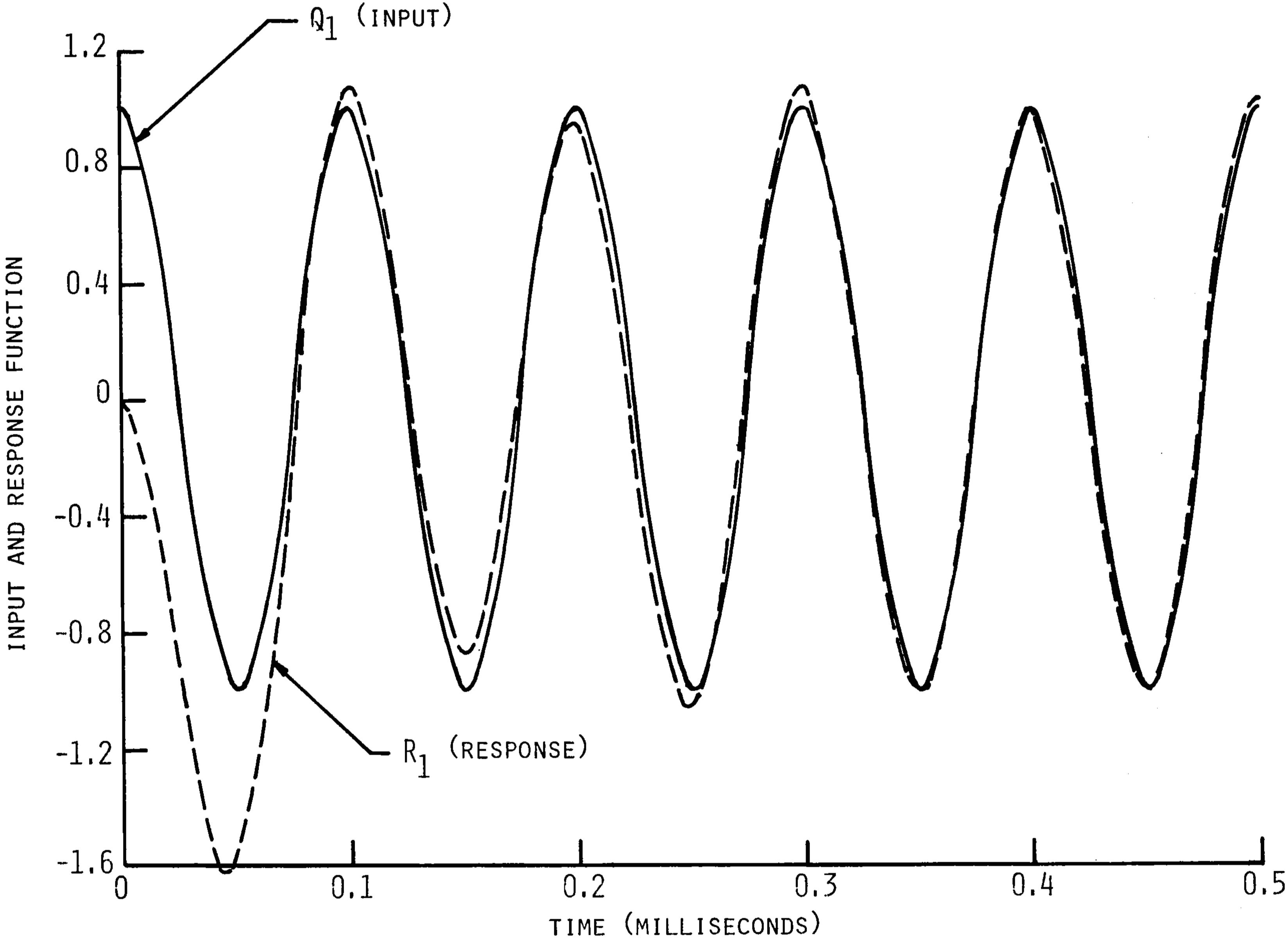



FIGURE 3. INPUT AND RESPONSE FUNCTIONS FOR TEST CASES.

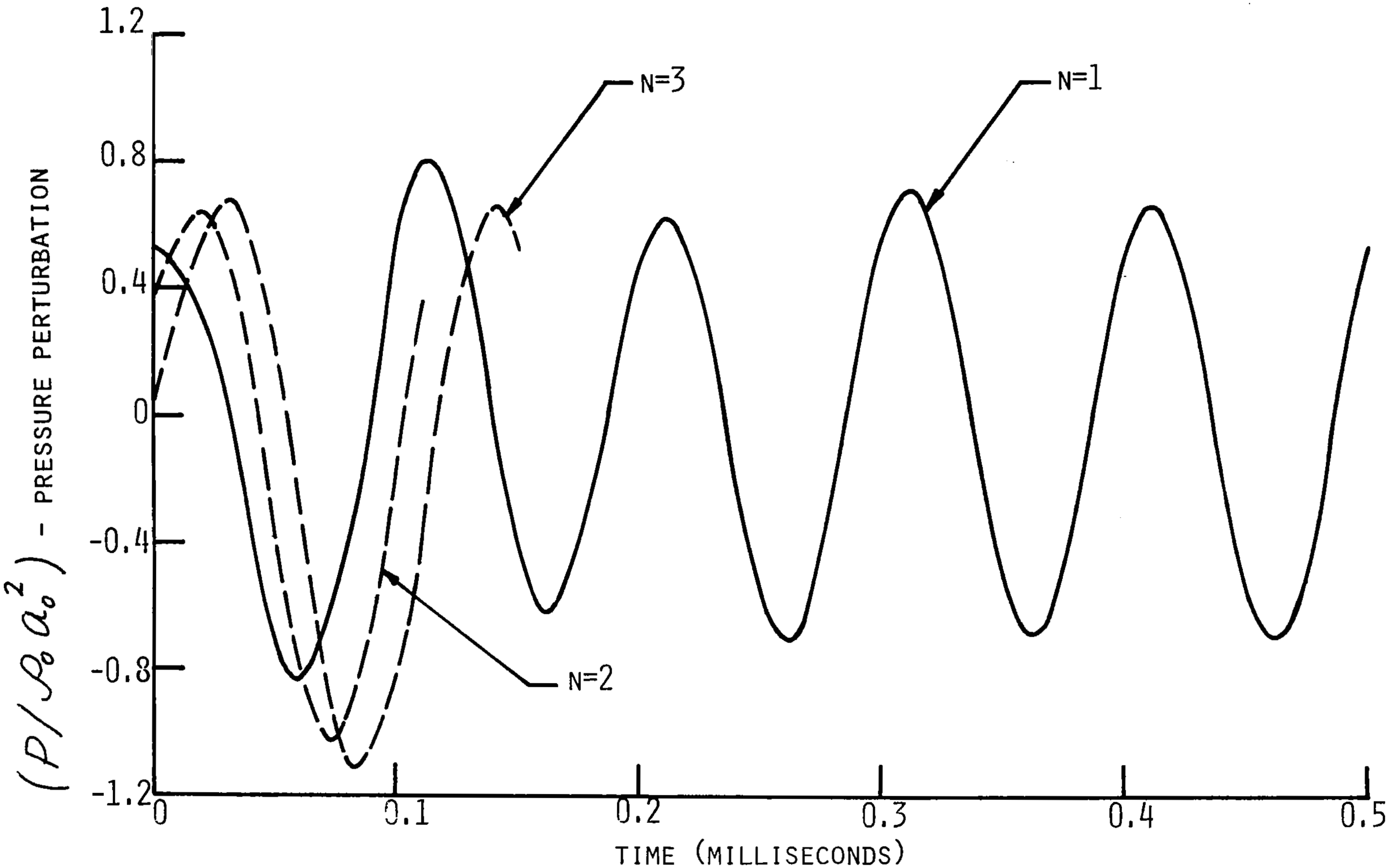



FIGURE 4. PRESSURE PERTURBATIONS AT SEVERAL GRID POINTS

functions Q and R from the boundary data in accord with Equations (38), (39) and (50). The remainder of the program consists of seven subroutines and function routines: AJ1DX, CØNVØL, FFT, FFT2, GREENS, PØLY and TCØNV. Only subroutine CØNVØL is called from the main program; all other subroutines and function routines are called from CØNVØL. Subroutine CØNVØL is called by the statement:

```
CALL CØNVØL (P, Q, R, NY, IT, DTIME, MAXCØN, CMARK, RHØO, C, Y, FIRST,
            NDIMY, NDIMT, PU)
```

which requires all variables as input parameters except P and PU which are the only results. P(IY, IT) is the discharge station perturbation pressure, as a function of discrete spatial location space, IY and time, IT, and PU(IY,IT) is the corresponding inlet pressure.

## Input Data

The input parameters Q(IY,IT), R(IY,IT), NY, IT, DTIME, MAXCØN, CMARK, RHØO, CY, Y, FIRST, NDIMY and NDIMT, required for the evaluation of P(IY,IT) through the statement CALL CØNVØL (...), are defined as follows:

| NAME | DIMENSION | DEFINITION |
|---|---|---|
| Q | (8,1001) | Near-field discharge or inlet boundary condition, $(p' \pm \rho_o a_o V'_m)/\rho_o a_o^2$. See Equations (38) and (50). |
| R | (8,1001) | Circumferential perturbation velocity, $V'_\theta/a_o$. See Equation (39). |
| NY | | Number of grid points in y-direction. |
| IT | | Current index. |
| DTIME | | Time step; seconds. |
| MAXCØN | | Maximum number of points used for convolution in time ($\leq$ 1001, with present dimension of Q and R). |
| CMARK | | Reference Mach number, $M_o$. |
| RHØO | | Reference density, $\rho_o$; kg/m$^3$. |
| C | | Reference speed of sound, $a_o$; m/sec. |

| NAME | DIMENSION | DEFINITION |
|---|---|---|
| Y | | Fundamental wavelength of the cascade ($2\pi r/\Delta N$); m. See Equation (54). |
| ØMEGAO | | Input frequency (non-dimensional), $\Omega$. See Equation (57). |
| FIRST | | A logical parameter which must be set to "FIRST = .TRUE." before subroutine CØNVØL is called for the first time. This avoids repetition of calculations that need to be done only once. Subroutine CØNVØL automatically sets "FIRST=.FALSE." after the initial call. |
| NDIMY | | Dimension of the arrays Q and R in the Y (circumferential) direction, presently 8. (NY $\leq$ NDIMY is necessary.) |
| NDIMT | | Dimension of the arrays Q and R in time, presently 1001. |

Subroutine and Function Sub-Programs

The following seven subroutines and function routines encompass the main portion of the program necessary to determine the acoustic far-field solution:

| NAME | DESCRIPTION |
|---|---|
| CØNVØL | When called the first time, CØNVØL calculates a number of parameters used through the rest of the program, calls subroutine GREENS to calculate the Green's functions $H_m$ and $J_m$. Thereafter, it calls subroutine TCØNV to perform the convolutions, and finally it evaluates Equations (83): $$P(IY,IT)=\frac{RH\emptyset 0\cdot C\cdot C}{NY}\left(\sum_{KY=1}^{NY}[Q(IY,IT)*H(KY,IT)+R(IY,IT)*J(KY,IT)]\right)$$ where * denotes convolution in time. |
| AJ1DX | Evaluates the Bessel function J1(X)/X using a 7th order polynomial approximation. |

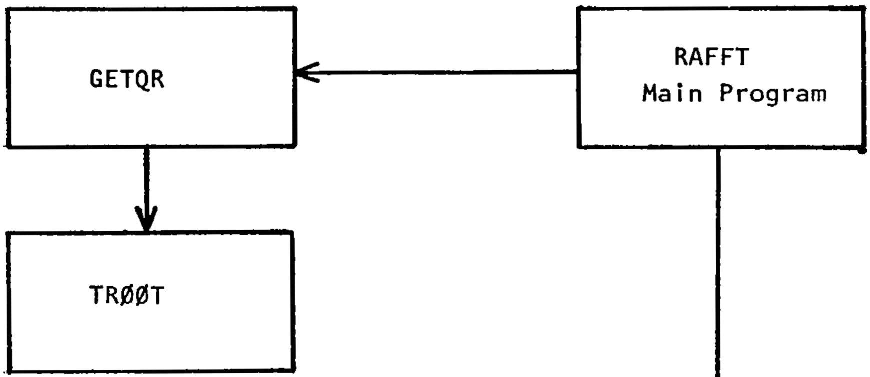
| NAME | DESCRIPTION |
|---|---|
| FFT | Converts a complex array into two real arrays, and then calls FFT2. |
| FFT2 | A fast Fourier transform subroutine written in assembly lanquage*. |
| GREENS | Calculates the Green's functions $H_m$, $J_m$ and $K_m$ in accord with Equation (84) for m = 0, 1, 2....NY-1. In this routine the expression $f(W)=W=\sqrt{W^2-1}$ occurs where $W = \Omega$ (Equation (57)). The sign of the square root is chosen in such a way that f(W) is symmetric in the imaginary part and antisymmetric in the real part, with f(o)=+i. To calculate $H_1(t)$ we evaluate the integral in a finite interval from -WCUT to +WCUT using the fast Fourier transform, where the cut-off frequency is calculated as WCUT=Y/C/DTIME. The contribution from the tails outside the interval (-WCUT,+WCUT) is just 1/2 the delta function, plus terms of second order (i.e., containing the factor 1/WCUT**2). |
| PØLY | Evaluates a polynomial, using as few multiplications as possible. |
| TCØNV | Forms the convolution of two arrays A and B, calculating only one new value $$S_i = \sum_{j=1}^{j_{max}} A(j) * B(i-j)$$ (for one index i=IT) where $j_{max}$=min(i, MAXCØN), and MAXCØN is the maximum number of points used for convolution. |

*Only suitable for use on CDC 6600 computer systems. However, similar FFT routines are generally available for other systems.

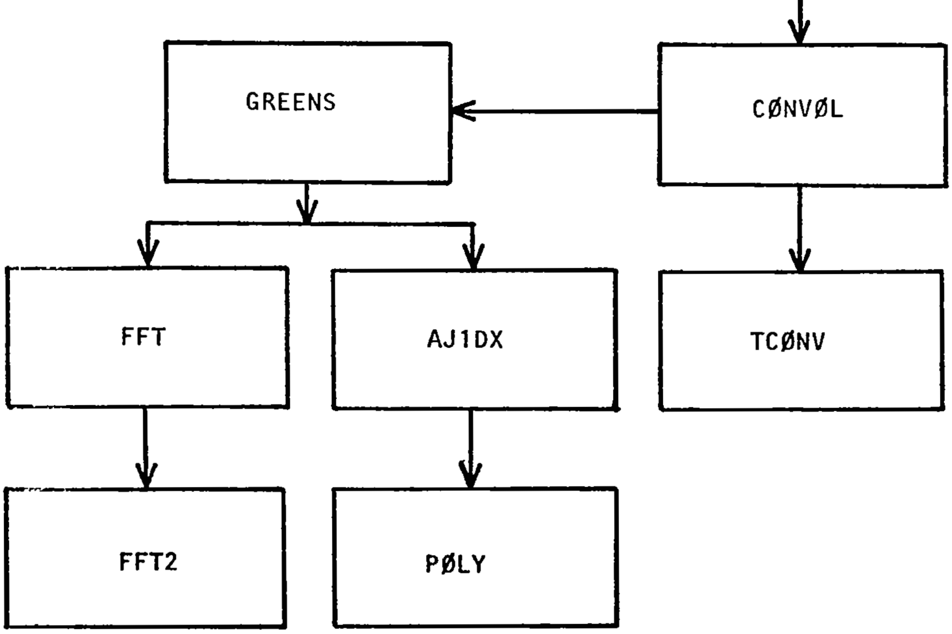
## Flow Chart of the Program

The interconnection between the subprograms are represented in the schematic below:

Part 1 (Generates input functions and prints output. Should be replaced by a calling routine from B2DATL.)




Part 2 (Determines acoustic far-field solution)



## Test Cases

The input function $Q_n(\tau) = \cos(\Omega\tau - 2\pi\frac{n}{N})\cdot\Delta(\tau)$, where $\tau = 2\pi Na_o t/Y$ is non-dimensional time, and N=NY is the number of points in the y-direction, was used for a test example. In this case $R_n(\tau)$ is the response function, rather than input function.

To check $R_n(\tau)$, it was compared with its asymptotic value. Two cases must be distinguished.

1. If $\Omega \geq n\sqrt{1-M^2}$ :

$$R_{n,\,imag}(\tau) = \frac{1}{n(1-M)}[\Omega - T_n(\Omega)] \sin\Omega\tau\,\delta_{n-m} \tag{87}$$

$$R_{n,\,real}(\tau) = \frac{1}{n(1-M)}[\Omega - T_n(\Omega)] \cos\Omega\tau\,\delta_{n-m} \tag{88}$$

$$\text{with}\quad T_n(\Omega) = \sqrt{\Omega^2 - n^2(1-M^2)} \tag{89}$$

2. If $\Omega < n\sqrt{1-M^2}$, substitution of $T_n = -i|T_n|$ yields:

$$R_{n,\,imag}(\tau) = \frac{1}{n(1-M)}[\Omega\sin\Omega\tau + T_n\cos\Omega\tau]\,\delta_{n-m} \tag{90}$$

$$R_{n,\,real}(\tau) = \frac{1}{n(1-M)}[\Omega\cos\Omega\tau - T_n\sin\Omega\tau]\,\delta_{n-m} \tag{91}$$

Two test examples were run, both with m=1, and with $\Omega = 1.0$ in the first case and $\Omega = 0.5$ in the second case. The first test case corresponds to the numerical example discussed in Section IV. Due to the factor $\delta_{n-m}$, $R_n = 0$ except for n=1.

In the first example $\Omega = 1 > n\sqrt{1-M^2} = 0.6$. In the second example $\Omega = 0.5 < n\sqrt{1-M^2} = 0.6$.

As shown in Appendix B, $R_1$ approaches its asymptotic value reasonably well in both cases.

The print-out in Appendix B includes:

- a list of the values of all parameters used
- a comparison of $R(\tau)$ with its asymptotic value
- the input functions Q(IY,IT) and R(IY,IT)
- the Green's functions $H_r(\tau)$ and $J_r(\tau)$ and
- the resulting pressure P(IY,IT).

# APPENDIX A

## PROGRAM LISTING

```
      PROGRAM RAFFT(INPUT,OUTPUT,TAPE98)
      DIMENSION P(8,1001),Q(8,1001),R(8,1001),PU(8,1001)
      LOGICAL FIRST
      DIMENSION ASYMPT(8)
      COMPLEX EPOWER,TROOT,E2PI,R1,R7,EW1,EW7
      COMMON/NSTEP/NSTEP
      EPOWER(X)=CEXP(CMPLX(0.,X))
*
* READ AND PRINT PARAMETERS
 10   READ 110,DTIME,NY,NT,MAXCON,CMARK,RHOO,C,Y,WZERO
 110  FORMAT(F10.0,3I10,4F10.0)
      IF(NT.EQ.0) STOP
      PRINT120,DTIME,NY,NT,MAXCON,CMARK,RHOO,C,Y,WZERO
 120  FORMAT(*1PARAMETERS USED*/
     ** DTIME  =*E10.3/
     ** NY     =*I10/
     ** NT     =*I10/
     ** MAXCON =*I10/
     ** CMARK  =*E10.3/
     ** RHOO   =*E10.3/
     ** C      =*E10.3/
     ** Y      =*E10.3/
     ** OMEGAO =*E10.3/ )
*
      NSTEP=5
      NDIMY=8
      NDIMT=1001
      FIRST=.TRUE.
*
 11   CALL SECOND(T0)
      PRINT 114
 114  FORMAT(///* TEST   R1STAR APPROACHES ITS ASYMPTOTIC VALUE*/)
      DO 12 IT=1,NT
* GET NEW VALUES OF Q AND R AT TIME T FOR NY DISCRETE VALUES OF Y
      CALL         GETQR(Q,R,IT,DTIME,NY,CMARK,C,Y,NDIMY,NDIMT,WZERO)
 12   CONTINUE
*
      PRINT 115
 115  FORMAT(*1       TIME     INPUT R(TIME) *)
      DO 15 IT=1,NT,NSTEP
      TIME=(IT-1)*DTIME
 15   PRINT 125,TIME,(R(IY,IT),IY=1,NY)
 125  FORMAT(1X,F10.3,F9.3,7F7.3)
      PRINT 116
 116  FORMAT(*1       TIME     INPUT Q(TIME)*)
      DO 16 IT=1,NT,NSTEP
      TIME=(IT-1)*DTIME
 16   PRINT 125,TIME,(Q(IY,IT),IY=1,NY)
      CALL SECOND(T1)
      TUSED=T1-T0
      PRINT 310,TUSED
*
      DO 20 IT=1,NT
* CALCULATE P(Y,T) FOR DISCRETE VALUES OF Y
* BY CONVOLUTION WITH GREENS FUNCTIONS
      CALL        CONVOL(P,Q,R,NY,IT,DTIME,MAXCON,CMARK,RHOO,C,Y,FIRST,
     *                    NDIMY,NDIMT,PU)
*
* PRINT SELECTED VALUES OF THE RESULTING PRESSURE P
      IF(MOD(IT-1,NSTEP).NE.0) GO TO 20
      IF(IT.EQ.1) PRINT 130
 130  FORMAT(*1       TIME     PRESSURE P(TIME) FOR DISCRETE VALUES OF Y*)
      IF(IT.EQ.1) CALL SECOND(T1)
      TIME=(IT-1)*DTIME
      PRINT 140,TIME, (P(IY,IT),IY=1,NY)
 140  FORMAT(1X,9E10.3)
 20   CONTINUE
      CALL SECOND(T2)
      TUSED=T2-T1
      PRINT 310,TUSED
 310  FORMAT(//* TIME USED =*F7.3* SECONDS*//)
      PRINT 150
 150  FORMAT(*1       TIME     UPSTREAM COMPONENT OF PRESSURE*)
      DO 50 IT=1,NT,NSTEP
      TIME=(IT-1)*DTIME
 50   PRINT 140,TIME,(PU(IY,IT),IY=1,NY)
      END
```

```
      SUBROUTINE GETQR(Q,R,IT,DTIME,NY,CMARK,C,Y,NDIMY,NDIMT,WZERO)

      DIMENSION Q(NDIMY,NDIMT),R(NDIMY,NDIMT)
      REAL J1(1001),J7(1001)
      COMPLEX Q1STAR(1001),Q7STAR(1001),R1STAR,R7STAR,RR(8)
      COMPLEX ASYMPT,TROOT
      COMMON/NSTEP/NSTEP
      PI=3.1415926535898
      TWOPI=2.*PI
      T=(IT-1)*DTIME
      ALPHAC=TWOPI*C/Y
      W0=ALPHAC*WZERO
      WOT=W0*T
      A=SQRT(1.-CMARK**2)
*
      DO 10 IY=1,NY
      Q(IY,IT)=COS(WOT-TWOPI*(IY-1)/NY)
 10   CONTINUE
*
      J1(IT)=(1.+CMARK)*AJ1DX(ALPHAC*A*T)
      Q1STAR(IT)=.5*CEXP(CMPLX(0.,WOT))
      R1STAR=0.
      DO 20 JT=1,IT
      R1STAR=R1STAR+J1(JT)*Q1STAR(IT-JT+1) *(0.,1.)    /PI
 20   CONTINUE
*
      IF(MOD(IT-1,NSTEP).NE.0) GO TO 25
      ASYMPT= (1./(1.-CMARK)) * (WZERO-TROOT(1,CMARK,WZERO))
     *    * CEXP(CMPLX(0.,WOT))
      PRINT 125,IT,R1STAR,ASYMPT
 125  FORMAT(* TIME STEP*I4*    R1STAR =*2F7.3*    ASYMPTOTE =*2F7.3)
 25   CONTINUE
*
      DO 30 IY=1,NY
      R(IY,IT)=REAL(     CEXP(CMPLX(0.,-TWOPI*(IY-1)/NY))*R1STAR   )
 30   CONTINUE
      RETURN
      END
```

```
      COMPLEX FUNCTION TROOT(IY,CMARK,W)
      S=W**2-IY**2*(1.-CMARK**2)
      IF(S.GE.0.) TROOT=SQRT(S)
      IF(S.LT.0.) TROOT=(0.,-1.)*SQRT(-S)
      RETURN
      END
```

```
      SUBROUTINE CONVOL(P,Q,R,NY,IT,DTIME,MAXCON,CMARK,RHO0,C,Y,FIRST,
     *                     NDIMY,NDIMT,PU)
*
* GIVEN THE INPUT FUNCTIONS Q(Y,T) = (P+RHO0*C*U)/(RHO0*C) AND R(Y,T)=V,
* FIND THE PRESSURE P AT X=0 THROUGH CONVOLUTION IN TIME AND SPACE
* P(IY,IT) = RHO0*C*(1/N) * SUM(OVER KY FROM 0 TO N-1) OF
*            ( Q(KY,IT) CONV H(IY-KY,IT) + R(KY,IT) CONV J(IY-KY,IT) )
*
      DIMENSION P(NDIMY,NDIMT),Q(NDIMY,NDIMT),R(NDIMY,NDIMT)
      DIMENSION QCONV(8,8),RCONV(8,8)
      REAL HR(1001,5),JR(1001,3)
      REAL PU(NDIMY,NDIMT),QKCONV(8,8),KR(1001,5)
      LOGICAL FIRST
      COMMON/PI/PI,PIHALF,TWOPI
      DATA PI/3.1415926535898/
*
* IF THIS SUBROUTINE IS CALLED FOR THE FIRST TIME,
* CALCULATE THE GREENS FUNCTIONS HR AND JR
* AND THE NON-DIMENSIONAL TIME STEP DT
      IF(.NOT.FIRST) GO TO 10
      PIHALF=PI/2.
      TWOPI=2.*PI
      RHO0CN=RHO0*C/NY*C
      DT=DTIME*C*TWOPI/Y
      NY2=NY/2
      NY21=NY2+1
      NY22=NY2+2
      NDIMP=NDIMY*NDIMT/2
      CALL SECOND(T0)
      CALL GREENS(HR,JR,DT,MAXCON,NY,CMARK,P,NDIMP,KR)
      CALL SECOND(T1)
      TUSED=T1-T0
      PRINT 310,TUSED
 310  FORMAT(//* TIME USED =*F7.3* SECONDS*//)
      FIRST=.FALSE.
*
* PREPARE A TABLE OF TIME CONVOLUTIONS
* (SAVES COMPUTER TIME BY USING SYMMETRIES)
 10   DO 50 IY=1,NY
      DO 20 KY=1,NY21
      CALL TCONV(Q(IY,1),HR(1,KY),IT,DT,NY,MAXCON,QCONV(IY,KY))
      CALL TCONV(Q(IY,1),KR(1,KY),IT,DT,NY,MAXCON,QKCONV(IY,KY))
 20   CONTINUE
      DO 30 KY1=NY22,NY
      KY=NY+2-KY1
      QKCONV(IY,KY1)=QKCONV(IY,KY)
 30   QCONV(IY,KY1)=QCONV(IY,KY)
*
      RCONV(IY,1)=0.
      RCONV(IY,NY21)=0
      DO 40 KY=2,NY2
      KY1=NY+2-KY
      CALL TCONV(R(IY,1),JR(1,KY-1),IT,DT,NY,MAXCON,RCONV(IY,KY))
 40   RCONV(IY,KY1)=-RCONV(IY,KY)
 50   CONTINUE
*
      DO 70 IY=1,NY
      SUMQ=0.
      SUMR=0.
      SUMK=0.
      DO 60 KY=1,NY
      IKY=IY-KY+1
      IF(IKY.LT.1) IKY=IKY+NY
      SUMQ=SUMQ+QCONV(KY,IKY)
      SUMR=SUMR+RCONV(KY,IKY)
      SUMK=SUMK+QKCONV(KY,IKY)
   60 CONTINUE
      PU(IY,IT)=RHO0CN*SUMK
 70   P(IY,IT)=RHO0CN*(SUMQ+SUMR)
      RETURN
      END
```

```
      SUBROUTINE GREENS(HR,JR,DT,MAXCON,NY,CMARK,P,NDIMP,KR)
*
* GENERATE THE GREENS FUNCTIONS HR(T) AND JR(T)
* FOR T = 0, DT, 2*DT, ..., (MAXCON-1)*DT
* AND FOR R = 0, 1, ..., NY/2 (FOR H),  R = 1, 2, ..., NY/2-1 (FOR J)
*
      REAL HR(1001,5),JR(1001,3),COSJK(8),SINJK(8)
      REAL KR(1001,5)
      COMPLEX S,P(1)
      COMMON/PI/PI,PIHALF,TWOPI
*
* P, WHICH IS NOT DEFINED AT THE BEGINNING, IS USED
* AS DUMMY STORAGE FOR THE FAST FOURIER TRANSFORM
*
* FIND MAXIMUM N WHICH FITS WITHIN SPACE OF P
      N=16384
 10   IF(N.LE.NDIMP) GO TO 20
      N=N/2
      GO TO 10
*
 20   CM=1.-CMARK**2
      A=SQRT(CM)
      WCUT=TWOPI/DT
      DW=WCUT/N
      NY2=NY/2
      NY21=NY2+1
      NYM1=NY-1
      NY2M1=NY2-1
      N4=N/4
* TABLES OF SINES AND COSINES
      DO 25 I=1,NY
      ARG=TWOPI*(I-1)/NY
      SINJK(I)=SIN(ARG)
 25   COSJK(I)=COS(ARG)
*
* CALCULATE H1
 1000 FORMAT(1X,F10.3,F9.3,8F7.3)
 1180 FORMAT(*1       TIME     GREENS FUNCTION HR*)
      PRINT 1180
      DO 30 I=1,N
      W=(I-1)*DW
      SQ=W**2-CM
      IF(SQ.GE.0.) S=SQRT(SQ)
      IF(SQ.LT.0.) S=CMPLX(0.,-SQRT(-SQ))
 30   P(I)=(W-S)*W/CM
*
      CALL FFT(P,N)
*
      DO 40 I=1,N4
* HN IS EVEN, TAKE TWICE REAL PART OF FOURIER TRANSFORM
      P(I)=REAL(P(I))*DW/PI
* DELTA FUNCTION
      IF(I.EQ.1) P(I)=.5/DT
 40   CONTINUE
*
* COSINE SERIES
      DO 70 I=1,MAXCON
      DO 60 J1=1,NY21
      J=J1-1
      SUM=0.
* H0 = 1/2 * DELTA FUNCTION
      IF(I.EQ.1) SUM=.5/DT
      DO 50 K=1,NYM1
      IF(K*I       .GT.N4) GO TO 50
      INDEX=MOD(J*K,NY)+1
      IF(I.GT.1) SUM=SUM+COSJK(INDEX) * K*P(K*I       )
      IF(I.EQ.1) SUM=SUM+COSJK(INDEX) * P(1)
 50   CONTINUE
 60   HR(I,J1)=SUM
      IF(MOD(I,50).EQ.2 .AND. I.NE.2) PRINT 1180
      T=(I-1)*DT
 180  PRINT 1000,T,(HR(I,J),J=1,NY21)
 70   CONTINUE
*
* JN (FOR N=1) IS THE BESSEL FUNCTION M * J1(A*T)/(A*T)
* WITH A = SQRT(1-M**2)
 1190 FORMAT(*1       TIME     GREENS FUNCTION JR*)
      PRINT 1190
      DO 80 I=1,N4
      T=(I-1)*DT
 80   P(I)=CMARK*AJ1DX(A*T)
```

```
*
* SINE SERIES
      DO 190 I=1,MAXCON
      DO 100 J=1,NY2M1
      SUM=0.
      DO 90 K=1,NYM1
      IF(K*I       .GT.N4) GO TO 90
      INDEX=MOD(J*K,NY)+1
      SUM=SUM+SINJK(INDEX) * K*P(K*I       )
 90   CONTINUE
 100  JR(I,J)=SUM
      IF(MOD(I,50).EQ.2 .AND. I.NE.2) PRINT 1190
      T=(I-1)*DT
      PRINT 1000,T,(JR(I,J),J=1,NY2M1)
  190 CONTINUE
*
* CALCULATE K1
 1200 FORMAT(*1       TIME     GREENS FUNCTION KR*)
      PRINT 1200
      DO 130 I=1,N
      W=(I-1)*DW
      SQ=W**2-CM
      IF(SQ.GE.0.) S=SQRT(SQ)
      IF(SQ.LT.0.) S=CMPLX(0.,-SQRT(-SQ))
 130  P(I)=(W-S)*(W+CMARK*S)/(CM*(1.+CMARK))
      CALL FFT(P,N)
      DO 140 I=1,N4
      P(I)=REAL(P(I))*DW/PI
      IF(I.EQ.1) P(I)=.5/DT
 140  CONTINUE
      DO 170 I=1,MAXCON
      DO 160 J1=1,NY21
      J=J1-1
      SUM=0.
      IF(I.EQ.1) SUM=.5/DT
      DO 150 K=1,NYM1
      IF(K*I       .GT.N4) GO TO 150
      INDEX=MOD(J*K,NY)+1
      IF(I.GT.1) SUM=SUM+COSJK(INDEX)*K*P(K*I       )
      IF(I.EQ.1) SUM=SUM+COSJK(INDEX)*P(1)
 150  CONTINUE
 160  KR(I,J1)=SUM
      IF(MOD(I,50).EQ.2 .AND. I.NE.2) PRINT 1200
      T=(I-1)*DT
 200  PRINT 1000,T,(KR(I,J),J=1,NY21)
 170  CONTINUE
*
      RETURN
      END
```

```
      SUBROUTINE TCONV(QR,HJ,IT,DT,NY,MAXCON,RESULT)
*
* CONVOLUTION IN TIME
* OF Q(IY,IT) WITH H(JY,IT)
* OR R(IY,IT) WITH J(JY,IT)
* TO CALCULATE ONE NEW POINT
*
      DIMENSION QR(1),HJ(1)
      RESULT=0.
      MAXT=MAXCON
      IF(IT.LT.MAXCON) MAXT=IT
      J=(IT-1)*NY+1
      DO 10 K=1,MAXT
      RESULT=RESULT+QR(J)*HJ(K)
 10   J=J-NY
      RESULT=RESULT*DT
      RETURN
      END
```

```
      FUNCTION AJ1DX(X)
* BESSEL FUNCTION J1(X)/X
      DIMENSION C1(7),C2(7),C3(7)
      DATA (C1(I),I=1;7)/.5,-.56249985,.21093573,-.03954289,
     *                  .00443319,-.00031761,.00001109/
      DATA (C2(I),I=1,7)/.79788456,.00000156,.01659667,.00017105,
     *                  .00249511,.00113653,.00020033/
      DATA (C3(I),I=1,7)/-2.35619449,.12499612,.00005650,-.00637879,
     *                  .00074348,.00079824,-.00029166/
      AX=ABS(X)
      IF(AX.GT.3.) GO TO 10
      AJ1DX=POLY(C1,(X/3.)**2,7)
      RETURN
 10   F1=POLY(C2,3./AX,7)
      THETA1=AX+POLY(C3,3./AX,7)
      AJ1DX=F1*COS(THETA1)/SQRT(AX)/AX
      RETURN
      END
```

```
      FUNCTION POLY(A,X,N)
* EVALUATE THE POLYNOMIAL A(1)+A(2)*X+...+A(N)*X**(N-1)
      DIMENSION A(1)
      POLY=0.
      XPOWER=1.
      DO 10 I=1,N
      POLY=POLY+A(I)*XPOWER
 10   XPOWER=XPOWER*X
      RETURN
      END
```

```
      SUBROUTINE FFT(XREAL,NSIG)
      REAL XREAL(1)
      IF(NSIG.LT.0) GO TO 10
      N=NSIG
      NSPACE=2
      GO TO 20
 10   N=-NSIG
      NSPACE=-2
 20   CALL FFT2(XREAL,XREAL(2),N,NSPACE)
      RETURN
      END
```

```
         IDENT  FFT2
         ENTRY  FFT2
L100     SX0    B3
         SB4    B0
         SB3    B3-B7
         AX0    1
         SB5    B0
         SB6    X0
         SX1    B5
         EQ     B6,B7,FFT23
L110     SB4    B3-B4
         SB5     B3-B5
         SA2    B1+B4
         SA3    B1+B5
         SA4    B2+B4
         NX7     X2
         SA5    B2+B5
         NX6    X3
         SA7    A3
         SA6    A2
         NX7    X4
         NX6    X5
         SA7    A5
         SA6    A4
         LT     B6,B4,L110
L120     SB4    B4+B7
         SB5    B6+B5
         SA2    B1+B4
         SA3    B1+B5
         SA4    B2+B4
         NX7    X2
         SA5    B2+B5
         NX6    X3
         SA7    A3
         SA6    A2
         NX7    X4
         SX0    B6
         NX6    X5
         SA7    A5
         SA6    A4
L130     AX0    1
         IX1    X1-X0
         PL     X1,L130
         LX0    1
         SB4    B4+B7
         IX1    X1+X0
         SB5    X1
         GE     B5,B4,L110
         LT     B4,B6,L120
FFT23    SA1    A0
FFT2
         SA0   A1
         SA1  A0
         SB1  X1
         SA1  A0+1
         SB2  X1
         SA1  A0+2
         SB3  X1
         SA1  A0+3
         SB4  X1
         SA4    B4
         MX2     1
         SA5    L60
         SA3    B3
         LX2    57
         PX7    X3
         BX6    -X2*X5
          PL    X4,L10
         BX6    X2+X5
         BX4    -X4
L10      LX3    32
         SA6    A5
         NX0    B5,X3
         PX2    X4
         SB7    X4
         DX7    X2*X7
         SA1    B5+S
         SB3    X7
         SB6    X7
         EQ     L40
L20      SA3    CD
         RX4    X2*X1
         RX7    X2*X0
         RX5    X3*X0
         RX6    X3*X1
         RX4    X4-X5
         RX6    X6+X7
         NX5    X4
         RX7     X1-X6
         RX0    X0+X5
         NX1    X7
L30      SB5    B6+B4
         SA2    B1+B4
         SA3    B1+B5
         SA4    B2+B4
         RX6    X2+X3
         SA5    B2+B5
         RX2    X2-X3
         SA6    A2
         RX7    X4+X5
         RX3    X1*X2
         RX4    X4-X5
         SA7    A4
         RX5    X0*X4
         RX2    X0*X2
         RX6    X3-X5
         RX4    X1*X4
         SA6    A3
         RX7    X2+X4
         SB4    B6+B5
         SA7    A5
         LT     B4,B3,L30
         SB5    B4-B3
         BX1    -X1
         SB4    B6-B5
         LT     B5,B4,L30
         SB4    B4+B7
         SA2    SD
         LT     B4,B5,L20
L40      SB4    B0
         SX5    B6
         AX5    1
         SB6    X5
L50      SB5    B6+B4
         SA2    B1+B4
         SA3    B1+B5
         SA4    B2+B4
         RX6    X2+X3
         SA5    B2+B5
         RX7    X2-X3
         SA6    A2
         SA7    A3
         RX6    X4+X5
         SB4    B6+B5
         RX7    X4-X5
         SA6    A4
         SA7    A5
         LT     B4,B3,L50
         EQ     B6,B7,L100
         SA1    A1
         SB4    B7
         RX6    X1*X1
         SA1    A1+1
        FX6    X6+X6
         SA3    ONE
         SA6    CD
L60      BX0    X1
         RX6    X3-X6
         BX7    X0
         NX1    X6
         SA7    SD
         EQ     L30
S        DATA   9.5873799095977346E-5
         DATA   1.9174759731070331E-4
         DATA   3.8349518757139559E-4
         DATA   7.6699031874270453E-4
         DATA   1.5339801862847656E-3
         DATA   3.0679567629659763E-3
         DATA   6.1358846491544754E-3
         DATA   1.2271538285719926E-2
         DATA   2.4541228522912288E-2
         DATA   4.9067674327418014E-2
         DATA   9.8017140329560602E-2
         DATA   1.9509032201612827E-1
         DATA   3.8268343236508977E-1
         DATA   0.7071067811865475
ONE      DATA   1.0
CD
SD
      END
```

# APPENDIX B

## PRINTOUT OF TEST CASES

```
PARAMETERS USED
DTIME  =  .250E-05
NY     =         8
NT     =       201
MAXCON =       101
CMARK  =  .800E+00
RHO0   =  .100E+01
C      =  .100E+04
Y      =  .100E+00
OMEGA0 =  .100E+01


TEST   R1STAR APPROACHES ITS ASYMPTOTIC VALUE

TIME STEP   1    R1STAR =  0.000    .143    ASYMPTOTE =  1.000   0.000
TIME STEP   6    R1STAR =  -.316    .757    ASYMPTOTE =   .707    .707
TIME STEP  11    R1STAR =  -.963    .928    ASYMPTOTE =   .000   1.000
TIME STEP  16    R1STAR = -1.520    .520    ASYMPTOTE =  -.707    .707
TIME STEP  21    R1STAR = -1.596   -.257    ASYMPTOTE = -1.000    .000
TIME STEP  26    R1STAR = -1.065   -.969    ASYMPTOTE =  -.707   -.707
TIME STEP  31    R1STAR =  -.150 -1.209     ASYMPTOTE =  -.000  -1.000
TIME STEP  36    R1STAR =   .701   -.835    ASYMPTOTE =   .707   -.707
TIME STEP  41    R1STAR =  1.068   -.057    ASYMPTOTE =  1.000   -.000
TIME STEP  46    R1STAR =   .801    .687    ASYMPTOTE =   .707    .707
TIME STEP  51    R1STAR =   .101    .983    ASYMPTOTE =   .000   1.000
TIME STEP  56    R1STAR =  -.596    .680    ASYMPTOTE =  -.707    .707
TIME STEP  61    R1STAR =  -.874   -.023    ASYMPTOTE = -1.000    .000
TIME STEP  66    R1STAR =  -.580   -.698    ASYMPTOTE =  -.707   -.707
TIME STEP  71    R1STAR =   .098   -.940    ASYMPTOTE =  -.000  -1.000
TIME STEP  76    R1STAR =   .738   -.603    ASYMPTOTE =   .707   -.707
TIME STEP  81    R1STAR =   .944    .113    ASYMPTOTE =  1.000   -.000
TIME STEP  86    R1STAR =   .577    .782    ASYMPTOTE =   .707    .707
TIME STEP  91    R1STAR =  -.159  1.004     ASYMPTOTE =   .000   1.000
TIME STEP  96    R1STAR =  -.835    .638    ASYMPTOTE =  -.707    .707
TIME STEP 101    R1STAR = -1.052   -.110    ASYMPTOTE = -1.000    .000
TIME STEP 106    R1STAR =  -.672   -.806    ASYMPTOTE =  -.707   -.707
TIME STEP 111    R1STAR =   .094 -1.046     ASYMPTOTE =  -.000  -1.000
TIME STEP 116    R1STAR =   .810   -.687    ASYMPTOTE =   .707   -.707
TIME STEP 121    R1STAR =  1.066    .063    ASYMPTOTE =  1.000   -.000
TIME STEP 126    R1STAR =   .717    .772    ASYMPTOTE =   .707    .707
TIME STEP 131    R1STAR =  -.031  1.029     ASYMPTOTE =   .000   1.000
TIME STEP 136    R1STAR =  -.743    .689    ASYMPTOTE =  -.707    .707
TIME STEP 141    R1STAR = -1.010   -.045    ASYMPTOTE = -1.000    .000
TIME STEP 146    R1STAR =  -.683   -.743    ASYMPTOTE =  -.707   -.707
TIME STEP 151    R1STAR =   .039   -.996    ASYMPTOTE =  -.000  -1.000
TIME STEP 156    R1STAR =   .726   -.660    ASYMPTOTE =   .707   -.707
TIME STEP 161    R1STAR =   .974    .065    ASYMPTOTE =  1.000   -.000
TIME STEP 166    R1STAR =   .637    .751    ASYMPTOTE =   .707    .707
TIME STEP 171    R1STAR =  -.084    .991    ASYMPTOTE =   .000   1.000
TIME STEP 176    R1STAR =  -.762    .644    ASYMPTOTE =  -.707    .707
TIME STEP 181    R1STAR =  -.993   -.088    ASYMPTOTE = -1.000    .000
TIME STEP 186    R1STAR =  -.637   -.775    ASYMPTOTE =  -.707   -.707
TIME STEP 191    R1STAR =   .102 -1.011     ASYMPTOTE =  -.000  -1.000
TIME STEP 196    R1STAR =   .791   -.657    ASYMPTOTE =   .707   -.707
TIME STEP 201    R1STAR =  1.028    .085    ASYMPTOTE =  1.000   -.000
```

| TIME | INPUT R(TIME) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0. | 0.000 | .101 | .143 | .101 | -.000 | -.101 | -.143 | -.101 |
| .125E-04 | -.316 | .312 | .757 | .759 | .316 | -.312 | -.757 | -.759 |
| .250E-04 | -.963 | -.025 | .928 | 1.337 | .963 | .025 | -.928 | -1.337 |
| .375E-04 | -1.520 | -.707 | .520 | 1.442 | 1.520 | .707 | -.520 | -1.442 |
| .500E-04 | -1.596 | -1.310 | -.257 | .946 | 1.596 | 1.310 | .257 | -.946 |
| .625E-04 | -1.065 | -1.439 | -.969 | .068 | 1.065 | 1.439 | .969 | -.068 |
| .750E-04 | -.150 | -.961 | -1.209 | -.749 | .150 | .961 | 1.209 | .749 |
| .875E-04 | .701 | -.095 | -.835 | -1.066 | -.701 | .095 | .835 | 1.086 |
| .100E-03 | 1.068 | .715 | -.057 | -.796 | -1.068 | -.715 | .057 | .796 |
| .113E-03 | .801 | 1.052 | .687 | -.081 | -.801 | -1.052 | -.687 | .081 |
| .125E-03 | .101 | .766 | .983 | .623 | -.101 | -.766 | -.983 | -.623 |
| .138E-03 | -.596 | .060 | .680 | .902 | .596 | -.060 | -.680 | -.902 |
| .150E-03 | -.874 | -.634 | -.023 | .602 | .874 | .634 | .023 | -.602 |
| .163E-03 | -.580 | -.904 | -.698 | -.084 | .580 | .904 | .698 | .084 |
| .175E-03 | .098 | -.596 | -.940 | -.734 | -.098 | .596 | .940 | .734 |
| .188E-03 | .738 | .095 | -.603 | -.948 | -.738 | -.095 | .603 | .948 |
| .200E-03 | .944 | .748 | .113 | -.588 | -.944 | -.748 | -.113 | .588 |
| .213E-03 | .577 | .962 | .782 | .145 | -.577 | -.962 | -.782 | -.145 |
| .225E-03 | -.159 | .598 | 1.004 | .822 | .159 | -.598 | -1.004 | -.822 |
| .238E-03 | -.835 | -.140 | .638 | 1.041 | .835 | .140 | -.638 | -1.041 |
| .250E-03 | -1.052 | -.822 | -.110 | .666 | 1.052 | .822 | .110 | -.666 |
| .263E-03 | -.672 | -1.046 | -.806 | -.095 | .672 | 1.046 | .806 | .095 |
| .275E-03 | .094 | -.673 | -1.046 | -.806 | -.094 | .673 | 1.046 | .806 |
| .288E-03 | .810 | .086 | -.687 | -1.058 | -.810 | -.086 | .687 | 1.058 |
| .300E-03 | 1.066 | .798 | .063 | -.709 | -1.066 | -.798 | -.063 | .709 |
| .313E-03 | .717 | 1.053 | .772 | .039 | -.717 | -1.053 | -.772 | -.039 |
| .325E-03 | -.031 | .706 | 1.029 | .749 | .031 | -.706 | -1.029 | -.749 |
| .338E-03 | -.743 | -.038 | .689 | 1.013 | .743 | .038 | -.689 | -1.013 |
| .350E-03 | -1.010 | -.746 | -.045 | .683 | 1.010 | .746 | .045 | -.683 |
| .363E-03 | -.683 | -1.008 | -.743 | -.042 | .683 | 1.008 | .743 | .042 |
| .375E-03 | .039 | -.677 | -.996 | -.732 | -.039 | .677 | .996 | .732 |
| .388E-03 | .726 | .047 | -.660 | -.980 | -.726 | -.047 | .660 | .980 |
| .400E-03 | .974 | .735 | .065 | -.643 | -.974 | -.735 | -.065 | .643 |
| .413E-03 | .637 | .981 | .751 | .080 | -.637 | -.981 | -.751 | -.080 |
| .425E-03 | -.084 | .642 | .991 | .760 | .084 | -.642 | -.991 | -.760 |
| .438E-03 | -.762 | -.083 | .644 | .994 | .762 | .083 | -.644 | -.994 |
| .450E-03 | -.993 | -.764 | -.088 | .640 | .993 | .764 | .088 | -.640 |
| .463E-03 | -.637 | -.998 | -.775 | -.097 | .637 | .998 | .775 | .097 |
| .475E-03 | .102 | -.643 | -1.011 | -.787 | -.102 | .643 | 1.011 | .787 |
| .488E-03 | .791 | .095 | -.657 | -1.024 | -.791 | -.095 | .657 | 1.024 |
| .500E-03 | 1.028 | .787 | .085 | -.667 | -1.028 | -.787 | -.085 | .667 |

| TIME | INPUT Q(TIME) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0. | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 | -.000 | .707 |
| .125E-04 | .707 | 1.000 | .707 | .000 | -.707 | -1.000 | -.707 | -.000 |
| .250E-04 | .000 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 |
| .375E-04 | -.707 | .000 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 |
| .500E-04 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 | .000 | -.707 |
| .625E-04 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 | .000 |
| .750E-04 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 |
| .875E-04 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 |
| .100E-03 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 |
| .113E-03 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 |
| .125E-03 | .000 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 |
| .138E-03 | -.707 | .000 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 |
| .150E-03 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 | -.000 | -.707 |
| .163E-03 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 | -.000 |
| .175E-03 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 |
| .188E-03 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 |
| .200E-03 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 |
| .213E-03 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 |
| .225E-03 | .000 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 |
| .238E-03 | -.707 | .000 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 |
| .250E-03 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 | -.000 | -.707 |
| .263E-03 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 | -.000 |
| .275E-03 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 |
| .288E-03 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 |
| .300E-03 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 |
| .313E-03 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 |
| .325E-03 | .000 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 |
| .338E-03 | -.707 | .000 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 |
| .350E-03 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 | -.000 | -.707 |
| .363E-03 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 | -.000 |
| .375E-03 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 |
| .388E-03 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 |
| .400E-03 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 |
| .413E-03 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 |
| .425E-03 | .000 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 |
| .438E-03 | -.707 | .000 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 |
| .450E-03 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 | -.000 | -.707 |
| .463E-03 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 | -.000 |
| .475E-03 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 |
| .488E-03 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 |
| .500E-03 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 |

| TIME | GREENS FUNCTION HR | | | | |
|---|---|---|---|---|---|
| 0.000 | 25.465 | -.000 | -.000 | -.000 | 0.000 |
| .157 | -1.681 | .078 | .284 | .317 | .324 |
| .314 | -2.305 | .183 | .370 | .398 | .404 |
| .471 | -2.630 | .332 | .389 | .395 | .397 |
| .628 | -2.648 | .508 | .341 | .318 | .314 |
| .785 | -2.408 | .678 | .240 | .194 | .185 |
| .942 | -1.999 | .804 | .111 | .060 | .050 |
| 1.100 | -1.525 | .855 | -.016 | -.049 | -.055 |
| 1.257 | -1.081 | .815 | -.113 | -.108 | -.107 |
| 1.414 | -.731 | .688 | -.159 | -.111 | -.104 |
| 1.571 | -.505 | .497 | -.146 | -.069 | -.058 |
| 1.728 | -.395 | .280 | -.077 | -.007 | .003 |
| 1.885 | -.367 | .076 | .030 | .051 | .054 |
| 2.042 | -.379 | -.083 | .153 | .082 | .076 |
| 2.199 | -.392 | -.180 | .267 | .077 | .064 |
| 2.356 | -.384 | -.213 | .351 | .040 | .028 |
| 2.513 | -.348 | -.197 | .390 | -.011 | -.016 |
| 2.670 | -.293 | -.155 | .382 | -.056 | -.047 |
| 2.827 | -.236 | -.110 | .332 | -.078 | -.054 |
| 2.985 | -.191 | -.078 | .255 | -.064 | -.034 |
| 3.142 | -.167 | -.067 | .167 | -.018 | .003 |
| 3.299 | -.162 | -.075 | .085 | .052 | .039 |
| 3.456 | -.170 | -.091 | .020 | .126 | .059 |
| 3.613 | -.179 | -.106 | -.020 | .189 | .053 |
| 3.770 | -.181 | -.111 | -.037 | .228 | .021 |
| 3.927 | -.172 | -.103 | -.036 | .237 | -.023 |
| 4.084 | -.153 | -.087 | -.025 | .219 | -.061 |
| 4.241 | -.132 | -.067 | -.013 | .183 | -.075 |
| 4.398 | -.114 | -.051 | -.006 | .139 | -.051 |
| 4.555 | -.104 | -.044 | -.006 | .097 | .011 |
| 4.712 | -.104 | -.045 | -.014 | .061 | .100 |
| 4.869 | -.110 | -.052 | -.026 | .034 | .197 |
| 5.027 | -.117 | -.060 | -.037 | .015 | .280 |
| 5.184 | -.120 | -.064 | -.043 | .002 | .329 |
| 5.341 | -.116 | -.060 | -.043 | -.007 | .336 |
| 5.498 | -.106 | -.051 | -.036 | -.011 | .303 |
| 5.655 | -.093 | -.040 | -.027 | -.008 | .242 |
| 5.812 | -.083 | -.030 | -.017 | .004 | .170 |
| 5.969 | -.078 | -.025 | -.012 | .024 | .104 |
| 6.126 | -.079 | -.026 | -.012 | .050 | .056 |
| 6.283 | -.085 | -.032 | -.018 | .077 | .031 |
| 6.440 | -.091 | -.038 | -.028 | .099 | .025 |
| 6.597 | -.094 | -.042 | -.037 | .111 | .030 |
| 6.754 | -.092 | -.041 | -.042 | .110 | .037 |
| 6.912 | -.084 | -.034 | -.039 | .095 | .039 |
| 7.069 | -.075 | -.025 | -.027 | .072 | .034 |
| 7.226 | -.067 | -.016 | -.008 | .046 | .022 |
| 7.383 | -.064 | -.011 | .017 | .022 | .008 |
| 7.540 | -.066 | -.013 | .041 | .006 | -.004 |
| 7.697 | -.072 | -.020 | .061 | -.001 | -.010 |
| 7.854 | -.079 | -.029 | .073 | -.000 | -.009 |

| TIME | GREENS FUNCTION HR | | | | |
|---|---|---|---|---|---|
| 8.011 | -.083 | -.036 | .075 | .004 | -.005 |
| 8.168 | -.081 | -.037 | .069 | .009 | .000 |
| 8.325 | -.074 | -.030 | .055 | .010 | .003 |
| 8.482 | -.064 | -.014 | .038 | .006 | .002 |
| 8.639 | -.055 | .006 | .021 | .002 | -.003 |
| 8.796 | -.052 | .027 | .009 | -.005 | -.009 |
| 8.954 | -.057 | .045 | .001 | -.011 | -.014 |
| 9.111 | -.067 | .056 | -.001 | -.013 | -.016 |
| 9.268 | -.079 | .059 | .000 | -.012 | -.015 |
| 9.425 | -.087 | .055 | .003 | -.008 | -.011 |
| 9.582 | -.085 | .046 | .005 | -.005 | -.008 |
| 9.739 | -.071 | .035 | .006 | -.003 | -.005 |
| 9.896 | -.044 | .024 | .004 | -.003 | -.005 |
| 10.053 | -.009 | .015 | -.000 | -.006 | -.008 |
| 10.210 | .025 | .009 | -.005 | -.010 | -.012 |
| 10.367 | .053 | .003 | -.008 | -.014 | -.015 |
| 10.524 | .067 | -.000 | -.010 | -.015 | -.016 |
| 10.681 | .066 | -.003 | -.010 | -.014 | -.015 |
| 10.838 | .052 | -.003 | -.007 | -.010 | -.012 |
| 10.996 | .029 | .000 | -.004 | -.007 | -.008 |
| 11.153 | .005 | .007 | -.002 | -.005 | -.006 |
| 11.310 | -.015 | .017 | -.001 | -.005 | -.006 |
| 11.467 | -.011 | .041 | -.003 | -.020 | -.026 |
| 11.624 | -.021 | .048 | -.007 | -.019 | -.024 |
| 11.781 | -.026 | .052 | -.011 | -.018 | -.020 |
| 11.938 | -.026 | .053 | -.015 | -.016 | -.017 |
| 12.095 | -.024 | .050 | -.015 | -.015 | -.016 |
| 12.252 | -.021 | .045 | -.012 | -.014 | -.016 |
| 12.409 | -.020 | .038 | -.005 | -.014 | -.018 |
| 12.566 | -.021 | .031 | .005 | -.015 | -.021 |
| 12.723 | -.024 | .024 | .016 | -.017 | -.024 |
| 12.881 | -.029 | .019 | .027 | -.019 | -.025 |
| 13.038 | -.034 | .015 | .034 | -.020 | -.025 |
| 13.195 | -.037 | .013 | .038 | -.021 | -.023 |
| 13.352 | -.023 | .013 | .021 | -.020 | -.004 |
| 13.509 | -.027 | .013 | .020 | -.018 | -.003 |
| 13.666 | -.030 | .013 | .017 | -.013 | -.005 |
| 13.823 | -.032 | .013 | .013 | -.007 | -.008 |
| 13.980 | -.033 | .013 | .008 | .001 | -.012 |
| 14.137 | -.032 | .012 | .003 | .008 | -.016 |
| 14.294 | -.031 | .011 | -.001 | .015 | -.019 |
| 14.451 | -.029 | .009 | -.005 | .020 | -.020 |
| 14.608 | -.028 | .008 | -.008 | .022 | -.018 |
| 14.765 | -.027 | .007 | -.009 | .022 | -.012 |
| 14.923 | -.027 | .006 | -.010 | .019 | -.003 |
| 15.080 | -.028 | .006 | -.009 | .014 | .007 |
| 15.237 | -.030 | .006 | -.008 | .007 | .020 |
| 15.394 | -.032 | .007 | -.007 | .001 | .031 |
| 15.551 | -.034 | .007 | -.006 | -.005 | .042 |
| 15.708 | -.035 | .007 | -.006 | -.009 | .049 |

| TIME | GREENS FUNCTION JR | | |
|---|---|---|---|
| 0.000 | -3.698 | -1.520 | -.629 |
| .157 | -3.114 | -1.248 | -.514 |
| .314 | -2.208 | -.851 | -.348 |
| .471 | -1.138 | -.420 | -.171 |
| .628 | -.078 | -.054 | -.024 |
| .785 | .820 | .185 | .072 |
| .942 | 1.448 | .264 | .098 |
| 1.100 | 1.764 | .198 | .073 |
| 1.257 | 1.788 | .050 | .020 |
| 1.414 | 1.590 | -.108 | -.032 |
| 1.571 | 1.267 | -.208 | -.059 |
| 1.728 | .912 | -.203 | -.055 |
| 1.885 | .601 | -.090 | -.024 |
| 2.042 | .375 | .105 | .017 |
| 2.199 | .242 | .329 | .046 |
| 2.356 | .184 | .525 | .050 |
| 2.513 | .172 | .650 | .024 |
| 2.670 | .174 | .681 | -.020 |
| 2.827 | .170 | .625 | -.062 |
| 2.985 | .152 | .510 | -.080 |
| 3.142 | .123 | .373 | -.056 |
| 3.299 | .091 | .249 | .012 |
| 3.456 | .065 | .163 | .114 |
| 3.613 | .051 | .121 | .228 |
| 3.770 | .048 | .113 | .325 |
| 3.927 | .052 | .122 | .383 |
| 4.084 | .055 | .131 | .390 |
| 4.241 | .054 | .128 | .348 |
| 4.398 | .048 | .111 | .273 |
| 4.555 | .037 | .085 | .188 |
| 4.712 | .027 | .059 | .117 |
| 4.869 | .020 | .043 | .075 |
| 5.027 | .019 | .039 | .066 |
| 5.184 | .021 | .046 | .082 |
| 5.341 | .025 | .055 | .108 |
| 5.498 | .027 | .061 | .125 |
| 5.655 | .026 | .058 | .120 |
| 5.812 | .021 | .045 | .089 |
| 5.969 | .014 | .029 | .036 |
| 6.126 | .009 | .016 | -.027 |
| 6.283 | .007 | .011 | -.086 |
| 6.440 | .009 | .017 | -.127 |
| 6.597 | .013 | .031 | -.145 |
| 6.754 | .018 | .045 | -.139 |
| 6.912 | .019 | .051 | -.116 |
| 7.069 | .015 | .042 | -.086 |
| 7.226 | .009 | .016 | -.057 |
| 7.383 | .003 | -.023 | -.037 |
| 7.540 | -.001 | -.066 | -.028 |
| 7.697 | .001 | -.103 | -.027 |
| 7.854 | .008 | -.125 | -.030 |

| TIME | GREENS FUNCTION JR | | |
|---|---|---|---|
| 8.011 | .018 | -.129 | -.033 |
| 8.168 | .024 | -.116 | -.033 |
| 8.325 | .023 | -.093 | -.029 |
| 8.482 | .011 | -.066 | -.023 |
| 8.639 | -.013 | -.044 | -.017 |
| 8.796 | -.045 | -.032 | -.013 |
| 8.954 | -.077 | -.029 | -.012 |
| 9.111 | -.101 | -.032 | -.014 |
| 9.268 | -.113 | -.037 | -.016 |
| 9.425 | -.109 | -.039 | -.016 |
| 9.582 | -.093 | -.037 | -.016 |
| 9.739 | -.069 | -.030 | -.013 |
| 9.896 | -.046 | -.022 | -.010 |
| 10.053 | -.029 | -.016 | -.008 |
| 10.210 | -.023 | -.014 | -.007 |
| 10.367 | -.027 | -.016 | -.007 |
| 10.524 | -.037 | -.020 | -.009 |
| 10.681 | -.046 | -.023 | -.010 |
| 10.838 | -.049 | -.024 | -.011 |
| 10.996 | -.041 | -.020 | -.009 |
| 11.153 | -.024 | -.015 | -.007 |
| 11.310 | .000 | -.009 | -.005 |
| 11.467 | .019 | -.015 | -.010 |
| 11.624 | .037 | -.020 | -.013 |
| 11.781 | .050 | -.023 | -.013 |
| 11.938 | .057 | -.023 | -.011 |
| 12.095 | .056 | -.020 | -.006 |
| 12.252 | .048 | -.012 | -.001 |
| 12.409 | .037 | -.002 | .002 |
| 12.566 | .025 | .011 | .003 |
| 12.723 | .015 | .024 | .001 |
| 12.881 | .008 | .036 | -.004 |
| 13.038 | .006 | .045 | -.010 |
| 13.195 | .007 | .050 | -.015 |
| 13.352 | .004 | .051 | -.010 |
| 13.509 | .004 | .048 | -.003 |
| 13.666 | .005 | .042 | .006 |
| 13.823 | .008 | .034 | .016 |
| 13.980 | .011 | .026 | .025 |
| 14.137 | .013 | .019 | .033 |
| 14.294 | .014 | .014 | .039 |
| 14.451 | .013 | .011 | .042 |
| 14.608 | .011 | .011 | .042 |
| 14.765 | .008 | .012 | .041 |
| 14.923 | .005 | .014 | .039 |
| 15.080 | .003 | .016 | .035 |
| 15.237 | .001 | .017 | .032 |
| 15.394 | .001 | .016 | .028 |
| 15.551 | .001 | .015 | .025 |
| 15.708 | .003 | .012 | .022 |

| TIME | GREENS FUNCTION KR | | | | |
|---|---|---|---|---|---|
| 0.000 | 25.465 | -.000 | -.000 | -.000 | 0.000 |
| .157 | -.187 | .009 | .032 | .035 | .036 |
| .314 | -.256 | .020 | .041 | .044 | .045 |
| .471 | -.292 | .037 | .043 | .044 | .044 |
| .628 | -.294 | .056 | .038 | .035 | .035 |
| .785 | -.268 | .075 | .027 | .022 | .021 |
| .942 | -.222 | .089 | .012 | .007 | .006 |
| 1.100 | -.169 | .095 | -.002 | -.005 | -.006 |
| 1.257 | -.120 | .091 | -.013 | -.012 | -.012 |
| 1.414 | -.081 | .076 | -.018 | -.012 | -.012 |
| 1.571 | -.056 | .055 | -.016 | -.008 | -.006 |
| 1.728 | -.044 | .031 | -.009 | -.001 | .000 |
| 1.885 | -.041 | .008 | .003 | .006 | .006 |
| 2.042 | -.042 | -.009 | .017 | .009 | .008 |
| 2.199 | -.044 | -.020 | .030 | .009 | .007 |
| 2.356 | -.043 | -.024 | .039 | .004 | .003 |
| 2.513 | -.039 | -.022 | .043 | -.001 | -.002 |
| 2.670 | -.033 | -.017 | .042 | -.006 | -.005 |
| 2.827 | -.026 | -.012 | .037 | -.009 | -.006 |
| 2.985 | -.021 | -.009 | .028 | -.007 | -.004 |
| 3.142 | -.019 | -.007 | .019 | -.002 | .000 |
| 3.299 | -.018 | -.008 | .009 | .006 | .004 |
| 3.456 | -.019 | -.010 | .002 | .014 | .007 |
| 3.613 | -.020 | -.012 | -.002 | .021 | .006 |
| 3.770 | -.020 | -.012 | -.004 | .025 | .002 |
| 3.927 | -.019 | -.011 | -.004 | .026 | -.003 |
| 4.084 | -.017 | -.010 | -.003 | .024 | -.007 |
| 4.241 | -.015 | -.007 | -.001 | .020 | -.008 |
| 4.398 | -.013 | -.006 | -.001 | .015 | -.006 |
| 4.555 | -.012 | -.005 | -.001 | .011 | .001 |
| 4.712 | -.012 | -.005 | -.002 | .007 | .011 |
| 4.869 | -.012 | -.006 | -.003 | .004 | .022 |
| 5.027 | -.013 | -.007 | -.004 | .002 | .031 |
| 5.184 | -.013 | -.007 | -.005 | .000 | .037 |
| 5.341 | -.013 | -.007 | -.005 | -.001 | .037 |
| 5.498 | -.012 | -.006 | -.004 | -.001 | .034 |
| 5.655 | -.010 | -.004 | -.003 | -.001 | .027 |
| 5.812 | -.009 | -.003 | -.002 | .000 | .019 |
| 5.969 | -.009 | -.003 | -.001 | .003 | .012 |
| 6.126 | -.009 | -.003 | -.001 | .006 | .006 |
| 6.283 | -.009 | -.004 | -.002 | .009 | .003 |
| 6.440 | -.010 | -.004 | -.003 | .011 | .003 |
| 6.597 | -.010 | -.005 | -.004 | .012 | .003 |
| 6.754 | -.010 | -.005 | -.005 | .012 | .004 |
| 6.912 | -.009 | -.004 | -.004 | .011 | .004 |
| 7.069 | -.008 | -.003 | -.003 | .008 | .004 |
| 7.226 | -.007 | -.002 | -.001 | .005 | .002 |
| 7.383 | -.007 | -.001 | .002 | .002 | .001 |
| 7.540 | -.007 | -.001 | .005 | .001 | -.000 |
| 7.697 | -.008 | -.002 | .007 | -.000 | -.001 |
| 7.854 | -.009 | -.003 | .008 | -.000 | -.001 |

| TIME | GREENS FUNCTION KR | | | | |
|---|---|---|---|---|---|
| 8.011 | -.009 | -.004 | .008 | .000 | -.001 |
| 8.168 | -.009 | -.004 | .008 | .001 | .000 |
| 8.325 | -.008 | -.003 | .006 | .001 | .000 |
| 8.482 | -.007 | -.002 | .004 | .001 | .000 |
| 8.639 | -.006 | .001 | .002 | .000 | -.000 |
| 8.796 | -.006 | .003 | .001 | -.001 | -.001 |
| 8.954 | -.006 | .005 | .000 | -.001 | -.002 |
| 9.111 | -.007 | .006 | -.000 | -.001 | -.002 |
| 9.268 | -.009 | .007 | .000 | -.001 | -.002 |
| 9.425 | -.010 | .006 | .000 | -.001 | -.001 |
| 9.582 | -.009 | .005 | .001 | -.001 | -.001 |
| 9.739 | -.008 | .004 | .001 | -.000 | -.001 |
| 9.896 | -.005 | .003 | .000 | -.000 | -.001 |
| 10.053 | -.001 | .002 | -.000 | -.001 | -.001 |
| 10.210 | .003 | .001 | -.001 | -.001 | -.001 |
| 10.367 | .006 | .000 | -.001 | -.002 | -.002 |
| 10.524 | .007 | -.000 | -.001 | -.002 | -.002 |
| 10.681 | .007 | -.000 | -.001 | -.002 | -.002 |
| 10.838 | .006 | -.000 | -.001 | -.001 | -.001 |
| 10.996 | .003 | .000 | -.000 | -.001 | -.001 |
| 11.153 | .001 | .001 | -.000 | -.001 | -.001 |
| 11.310 | -.002 | .002 | -.000 | -.001 | -.001 |
| 11.467 | -.001 | .005 | -.000 | -.002 | -.003 |
| 11.624 | -.002 | .005 | -.001 | -.002 | -.003 |
| 11.781 | -.003 | .006 | -.001 | -.002 | -.002 |
| 11.938 | -.003 | .006 | -.002 | -.002 | -.002 |
| 12.095 | -.003 | .006 | -.002 | -.002 | -.002 |
| 12.252 | -.002 | .005 | -.001 | -.002 | -.002 |
| 12.409 | -.002 | .004 | -.001 | -.002 | -.002 |
| 12.566 | -.002 | .003 | .001 | -.002 | -.002 |
| 12.723 | -.003 | .003 | .002 | -.002 | -.003 |
| 12.881 | -.003 | .002 | .003 | -.002 | -.003 |
| 13.038 | -.004 | .002 | .004 | -.002 | -.003 |
| 13.195 | -.004 | .001 | .004 | -.002 | -.003 |
| 13.352 | -.003 | .001 | .002 | -.002 | -.000 |
| 13.509 | -.003 | .001 | .002 | -.002 | -.000 |
| 13.666 | -.003 | .001 | .002 | -.001 | -.001 |
| 13.823 | -.004 | .001 | .001 | -.001 | -.001 |
| 13.980 | -.004 | .001 | .001 | .000 | -.001 |
| 14.137 | -.004 | .001 | .000 | .001 | -.002 |
| 14.294 | -.003 | .001 | -.000 | .002 | -.002 |
| 14.451 | -.003 | .001 | -.001 | .002 | -.002 |
| 14.608 | -.003 | .001 | -.001 | .002 | -.002 |
| 14.765 | -.003 | .001 | -.001 | .002 | -.001 |
| 14.923 | -.003 | .001 | -.001 | .002 | -.000 |
| 15.080 | -.003 | .001 | -.001 | .002 | .001 |
| 15.237 | -.003 | .001 | -.001 | .001 | .002 |
| 15.394 | -.004 | .001 | -.001 | .000 | .003 |
| 15.551 | -.004 | .001 | -.001 | -.001 | .005 |
| 15.708 | -.004 | .001 | -.001 | -.001 | .005 |

TIME PRESSURE P(TIME) FOR DISCRETE VALUES OF Y

| TIME | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0. | .526E+06 | .372E+06 | -.760E-09 | -.372E+06 | -.526E+06 | -.372E+06 | -.822E-08 | .372E+06 |
| .125E-04 | .427E+06 | .570E+06 | .379E+06 | -.339E+05 | -.427E+06 | -.570E+06 | -.379E+06 | .339E+05 |
| .250E-04 | .221E+06 | .603E+06 | .631E+06 | .290E+06 | -.221E+06 | -.603E+06 | -.631E+06 | -.290E+06 |
| .375E-04 | -.254E+06 | .254E+06 | .624E+06 | .628E+06 | .264E+06 | -.254E+06 | -.624E+06 | -.628E+06 |
| .500E-04 | -.692E+06 | -.321E+06 | .238E+06 | .658E+06 | .692E+06 | .321E+06 | -.238E+06 | -.658E+06 |
| .625E-04 | -.823E+06 | -.843E+06 | -.368E+06 | .322E+06 | .823E+06 | .843E+06 | .368E+06 | -.322E+06 |
| .750E-04 | -.530E+06 | -.102E+07 | -.909E+06 | -.268E+06 | .530E+06 | .102E+07 | .909E+06 | .268E+06 |
| .875E-04 | .298E+05 | -.752E+06 | -.109E+07 | -.794E+06 | -.298E+05 | .752E+06 | .109E+07 | .794E+06 |
| .100E-03 | .576E+06 | -.176E+06 | -.825E+06 | -.991E+06 | -.576E+06 | .176E+06 | .825E+06 | .991E+06 |
| .113E-03 | .802E+06 | .397E+06 | -.241E+06 | -.737E+06 | -.802E+06 | -.397E+06 | .241E+06 | .737E+06 |
| .125E-03 | .609E+06 | .679E+06 | .351E+06 | -.162E+06 | -.609E+06 | -.679E+06 | -.351E+06 | .162E+06 |
| .138E-03 | .109E+06 | .537E+06 | .651E+06 | .383E+06 | -.109E+06 | -.537E+06 | -.651E+06 | -.383E+06 |
| .150E-03 | -.393E+06 | .971E+05 | .530E+06 | .653E+06 | .393E+06 | -.971E+05 | -.530E+06 | -.653E+06 |
| .163E-03 | -.620E+06 | -.365E+06 | .105E+06 | .513E+06 | .620E+06 | .365E+06 | -.105E+06 | -.513E+06 |
| .175E-03 | -.446E+06 | -.559E+06 | -.344E+06 | .721E+05 | .446E+06 | .559E+06 | .344E+06 | -.721E+05 |
| .188E-03 | .719E+04 | -.373E+06 | -.534E+06 | -.383E+06 | -.719E+04 | .373E+06 | .534E+06 | .383E+06 |
| .200E-03 | .460E+06 | .797E+05 | -.347E+06 | -.570E+06 | -.460E+06 | -.797E+05 | .347E+06 | .570E+06 |
| .213E-03 | .628E+06 | .515E+06 | .100E+06 | -.373E+06 | -.628E+06 | -.515E+06 | -.100E+06 | .373E+06 |
| .225E-03 | .404E+06 | .659E+06 | .527E+06 | .868E+05 | -.404E+06 | -.659E+06 | -.527E+06 | -.868E+05 |
| .238E-03 | -.847E+05 | .408E+06 | .661E+06 | .527E+06 | .847E+05 | -.408E+06 | -.661E+06 | -.527E+06 |
| .250E-03 | -.551E+06 | -.105E+06 | .402E+06 | .673E+06 | .551E+06 | .105E+06 | -.402E+06 | -.673E+06 |
| .263E-03 | -.706E+06 | -.581E+06 | -.115E+06 | .418E+06 | .706E+06 | .581E+06 | .115E+06 | -.418E+06 |
| .275E-03 | -.457E+06 | -.742E+06 | -.593E+06 | -.963E+05 | .457E+06 | .742E+06 | .593E+06 | .963E+05 |
| .288E-03 | .576E+05 | -.493E+06 | -.755E+06 | -.575E+06 | -.576E+05 | .493E+06 | .755E+06 | .575E+06 |
| .300E-03 | .542E+06 | .268E+05 | -.504E+06 | -.740E+06 | -.542E+06 | -.268E+05 | .504E+06 | .740E+06 |
| .313E-03 | .718E+06 | .521E+06 | .188E+05 | -.494E+06 | -.718E+06 | -.521E+06 | -.188E+05 | .494E+06 |
| .325E-03 | .485E+06 | .708E+06 | .517E+06 | .228E+05 | -.485E+06 | -.708E+06 | -.517E+06 | -.228E+05 |
| .338E-03 | -.205E+05 | .486E+06 | .708E+06 | .515E+06 | .205E+05 | -.486E+06 | -.708E+06 | -.515E+06 |
| .350E-03 | -.504E+06 | -.990E+04 | .490E+06 | .702E+06 | .504E+06 | .990E+04 | -.490E+06 | -.702E+06 |
| .363E-03 | -.685E+06 | -.487E+06 | -.449E+04 | .481E+06 | .685E+06 | .487E+06 | .449E+04 | -.481E+06 |
| .375E-03 | -.462E+06 | -.667E+06 | -.481E+06 | -.134E+05 | .462E+06 | .667E+06 | .481E+06 | .134E+05 |
| .388E-03 | .296E+05 | -.447E+06 | -.662E+06 | -.489E+06 | -.296E+05 | .447E+06 | .662E+06 | .489E+06 |
| .400E-03 | .499E+06 | .398E+05 | -.443E+06 | -.666E+06 | -.499E+06 | -.398E+05 | .443E+06 | .666E+06 |
| .413E-03 | .670E+06 | .503E+06 | .415E+05 | -.444E+06 | -.670E+06 | -.503E+06 | -.415E+05 | .444E+06 |
| .425E-03 | .441E+06 | .667E+06 | .502E+06 | .435E+05 | -.441E+06 | -.667E+06 | -.502E+06 | -.435E+05 |
| .438E-03 | -.525E+05 | .433E+06 | .664E+06 | .507E+06 | .525E+05 | -.433E+06 | -.664E+06 | -.507E+06 |
| .450E-03 | -.519E+06 | -.637E+05 | .429E+06 | .670E+06 | .519E+06 | .637E+05 | -.429E+06 | -.670E+06 |
| .463E-03 | -.683E+06 | -.531E+06 | -.678E+05 | .435E+06 | .683E+06 | .531E+06 | .678E+05 | -.435E+06 |
| .475E-03 | -.445E+06 | -.692E+06 | -.534E+06 | -.632E+05 | .445E+06 | .692E+06 | .534E+06 | .632E+05 |
| .488E-03 | .572E+05 | -.451E+06 | -.695E+06 | -.532E+06 | -.572E+05 | .451E+06 | .695E+06 | .532E+06 |
| .500E-03 | .531E+06 | .562E+05 | -.451E+06 | -.694E+06 | -.531E+06 | -.562E+05 | .451E+06 | .694E+06 |

| TIME | UPSTREAM COMPONENT OF PRESSURE | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0. | .500E+06 | .354E+06 | -.858E-09 | -.354E+06 | -.500E+06 | -.354E+06 | -.809E-08 | .354E+06 |
| .125E-04 | .327E+06 | .475E+06 | .345E+06 | .123E+05 | -.327E+06 | -.475E+06 | -.345E+06 | -.123E+05 |
| .250E-04 | -.119E+05 | .327E+06 | .474E+06 | .343E+06 | .119E+05 | -.327E+06 | -.474E+06 | -.343E+06 |
| .375E-04 | -.349E+06 | -.167E+05 | .326E+06 | .477E+06 | .349E+06 | .167E+05 | -.326E+06 | -.477E+06 |
| .500E-04 | -.480E+06 | -.352E+06 | -.180E+05 | .327E+06 | .480E+06 | .352E+06 | .180E+05 | -.327E+06 |
| .625E-04 | -.333E+06 | -.486E+06 | -.354E+06 | -.150E+05 | .333E+06 | .486E+06 | .354E+06 | .150E+05 |
| .750E-04 | .112E+05 | -.336E+06 | -.487E+06 | -.352E+06 | -.112E+05 | .336E+06 | .487E+06 | .352E+06 |
| .875E-04 | .347E+06 | .650E+04 | -.338E+06 | -.485E+06 | -.347E+06 | -.650E+04 | .338E+06 | .485E+06 |
| .100E-03 | .482E+06 | .344E+06 | .547E+04 | -.337E+06 | -.482E+06 | -.344E+06 | -.547E+04 | .337E+06 |
| .113E-03 | .334E+06 | .479E+06 | .343E+06 | .691E+04 | -.334E+06 | -.479E+06 | -.343E+06 | -.691E+04 |
| .125E-03 | -.808E+04 | .333E+06 | .478E+06 | .344E+06 | .808E+04 | -.333E+06 | -.478E+06 | -.344E+06 |
| .138E-03 | -.345E+06 | -.918E+04 | .332E+06 | .479E+06 | .345E+06 | .918E+04 | -.332E+06 | -.479E+06 |
| .150E-03 | -.479E+06 | -.345E+06 | -.909E+04 | .332E+06 | .479E+06 | .345E+06 | .909E+04 | -.332E+06 |
| .163E-03 | -.332E+06 | -.478E+06 | -.345E+06 | -.922E+04 | .332E+06 | .478E+06 | .345E+06 | .922E+04 |
| .175E-03 | .100E+05 | -.331E+06 | -.478E+06 | -.345E+06 | -.100E+05 | .331E+06 | .478E+06 | .345E+06 |
| .188E-03 | .346E+06 | .107E+05 | -.331E+06 | -.478E+06 | -.346E+06 | -.107E+05 | .331E+06 | .478E+06 |
| .200E-03 | .479E+06 | .347E+06 | .109E+05 | -.331E+06 | -.479E+06 | -.347E+06 | -.109E+05 | .331E+06 |
| .213E-03 | .332E+06 | .480E+06 | .347E+06 | .108E+05 | -.332E+06 | -.480E+06 | -.347E+06 | -.108E+05 |
| .225E-03 | -.107E+05 | .332E+06 | .480E+06 | .347E+06 | .107E+05 | -.332E+06 | -.480E+06 | -.347E+06 |
| .238E-03 | -.347E+06 | -.110E+05 | .332E+06 | .480E+06 | .347E+06 | .110E+05 | -.332E+06 | -.480E+06 |
| .250E-03 | -.480E+06 | -.348E+06 | -.112E+05 | .332E+06 | .480E+06 | .348E+06 | .112E+05 | -.332E+06 |
| .263E-03 | -.332E+06 | -.480E+06 | -.348E+06 | -.112E+05 | .332E+06 | .480E+06 | .348E+06 | .112E+05 |
| .275E-03 | .112E+05 | -.332E+06 | -.480E+06 | -.348E+06 | -.112E+05 | .332E+06 | .480E+06 | .348E+06 |
| .288E-03 | .348E+06 | .112E+05 | -.332E+06 | -.480E+06 | -.348E+06 | -.112E+05 | .332E+06 | .480E+06 |
| .300E-03 | .480E+06 | .348E+06 | .112E+05 | -.332E+06 | -.480E+06 | -.348E+06 | -.112E+05 | .332E+06 |
| .313E-03 | .332E+06 | .480E+06 | .348E+06 | .112E+05 | -.332E+06 | -.480E+06 | -.348E+06 | -.112E+05 |
| .325E-03 | -.112E+05 | .332E+06 | .480E+06 | .348E+06 | .112E+05 | -.332E+06 | -.480E+06 | -.348E+06 |
| .338E-03 | -.348E+06 | -.112E+05 | .332E+06 | .480E+06 | .348E+06 | .112E+05 | -.332E+06 | -.480E+06 |
| .350E-03 | -.480E+06 | -.348E+06 | -.112E+05 | .332E+06 | .480E+06 | .348E+06 | .112E+05 | -.332E+06 |
| .363E-03 | -.332E+06 | -.480E+06 | -.348E+06 | -.112E+05 | .332E+06 | .480E+06 | .348E+06 | .112E+05 |
| .375E-03 | .112E+05 | -.332E+06 | -.480E+06 | -.348E+06 | -.112E+05 | .332E+06 | .480E+06 | .348E+06 |
| .388E-03 | .348E+06 | .112E+05 | -.332E+06 | -.480E+06 | -.348E+06 | -.112E+05 | .332E+06 | .480E+06 |
| .400E-03 | .480E+06 | .348E+06 | .112E+05 | -.332E+06 | -.480E+06 | -.348E+06 | -.112E+05 | .332E+06 |
| .413E-03 | .332E+06 | .480E+06 | .348E+06 | .112E+05 | -.332E+06 | -.480E+06 | -.348E+06 | -.112E+05 |
| .425E-03 | -.112E+05 | .332E+06 | .480E+06 | .348E+06 | .112E+05 | -.332E+06 | -.480E+06 | -.348E+06 |
| .438E-03 | -.348E+06 | -.112E+05 | .332E+06 | .480E+06 | .348E+06 | .112E+05 | -.332E+06 | -.480E+06 |
| .450E-03 | -.480E+06 | -.348E+06 | -.112E+05 | .332E+06 | .480E+06 | .348E+06 | .112E+05 | -.332E+06 |
| .463E-03 | -.332E+06 | -.480E+06 | -.348E+06 | -.112E+05 | .332E+06 | .480E+06 | .348E+06 | .112E+05 |
| .475E-03 | .112E+05 | -.332E+06 | -.480E+06 | -.348E+06 | -.112E+05 | .332E+06 | .480E+06 | .348E+06 |
| .488E-03 | .348E+06 | .112E+05 | -.332E+06 | -.480E+06 | -.348E+06 | -.112E+05 | .332E+06 | .480E+06 |
| .500E-03 | .480E+06 | .348E+06 | .112E+05 | -.332E+06 | -.480E+06 | -.348E+06 | -.112E+05 | .332E+06 |

```
PARAMETERS USED
DTIME   =  .250E-05
NY      =         8
NT      =       201
MAXCON  =       101
CMARK   =  .800E+00
RHOO    =  .100E+01
C       =  .100E+04
Y       =  .100E+00
OMEGAO  =  .500E+00


TEST   R1STAR APPROACHES ITS ASYMPTOTIC VALUE

TIME STEP   1    R1STAR =  0.000    .143   ASYMPTOTE =  2.500  1.658
TIME STEP   6    R1STAR =  -.166    .827   ASYMPTOTE =  1.675  2.489
TIME STEP  11    R1STAR =  -.575   1.353   ASYMPTOTE =   .595  2.940
TIME STEP  16    R1STAR = -1.143   1.608   ASYMPTOTE =  -.575  2.944
TIME STEP  21    R1STAR = -1.750   1.526   ASYMPTOTE = -1.658  2.500
TIME STEP  26    R1STAR = -2.262   1.102   ASYMPTOTE = -2.489  1.675
TIME STEP  31    R1STAR = -2.554    .397   ASYMPTOTE = -2.940   .595
TIME STEP  36    R1STAR = -2.537   -.475   ASYMPTOTE = -2.944  -.575
TIME STEP  41    R1STAR = -2.171  -1.369   ASYMPTOTE = -2.500 -1.656
TIME STEP  46    R1STAR = -1.479  -2.124   ASYMPTOTE = -1.675 -2.489
TIME STEP  51    R1STAR =  -.543  -2.602   ASYMPTOTE =  -.595 -2.940
TIME STEP  56    R1STAR =   .508  -2.703   ASYMPTOTE =   .575 -2.944
TIME STEP  61    R1STAR =  1.518  -2.390   ASYMPTOTE =  1.658 -2.500
TIME STEP  66    R1STAR =  2.329  -1.695   ASYMPTOTE =  2.489 -1.675
TIME STEP  71    R1STAR =  2.807   -.714   ASYMPTOTE =  2.940  -.595
TIME STEP  76    R1STAR =  2.869    .406   ASYMPTOTE =  2.944   .575
TIME STEP  81    R1STAR =  2.493   1.492   ASYMPTOTE =  2.500  1.658
TIME STEP  86    R1STAR =  1.727   2.369   ASYMPTOTE =  1.675  2.489
TIME STEP  91    R1STAR =   .682   2.894   ASYMPTOTE =   .595  2.940
TIME STEP  96    R1STAR =  -.484   2.977   ASYMPTOTE =  -.575  2.944
TIME STEP 101    R1STAR = -1.592   2.597   ASYMPTOTE = -1.658  2.500
TIME STEP 106    R1STAR = -2.468   1.807   ASYMPTOTE = -2.489  1.675
TIME STEP 111    R1STAR = -2.972    .725   ASYMPTOTE = -2.940   .595
TIME STEP 116    R1STAR = -3.021   -.483   ASYMPTOTE = -2.944  -.575
TIME STEP 121    R1STAR = -2.603  -1.626   ASYMPTOTE = -2.500 -1.658
TIME STEP 126    R1STAR = -1.778  -2.527   ASYMPTOTE = -1.675 -2.489
TIME STEP 131    R1STAR =  -.672  -3.040   ASYMPTOTE =  -.595 -2.940
TIME STEP 136    R1STAR =   .546  -3.083   ASYMPTOTE =   .575 -2.944
TIME STEP 141    R1STAR =  1.686  -2.647   ASYMPTOTE =  1.658 -2.500
TIME STEP 146    R1STAR =  2.570  -1.797   ASYMPTOTE =  2.489 -1.675
TIME STEP 151    R1STAR =  3.061   -.664   ASYMPTOTE =  2.940  -.595
TIME STEP 156    R1STAR =  3.079    .576   ASYMPTOTE =  2.944   .575
TIME STEP 161    R1STAR =  2.621   1.730   ASYMPTOTE =  2.500  1.658
TIME STEP 166    R1STAR =  1.757   2.617   ASYMPTOTE =  1.675  2.489
TIME STEP 171    R1STAR =   .620   3.101   ASYMPTOTE =   .595  2.940
TIME STEP 176    R1STAR =  -.615   3.105   ASYMPTOTE =  -.575  2.944
TIME STEP 181    R1STAR = -1.756   2.628   ASYMPTOTE = -1.658  2.500
TIME STEP 186    R1STAR = -2.626   1.746   ASYMPTOTE = -2.489  1.675
TIME STEP 191    R1STAR = -3.092    .593   ASYMPTOTE = -2.940   .595
TIME STEP 196    R1STAR = -3.082   -.649   ASYMPTOTE = -2.944  -.575
TIME STEP 201    R1STAR = -2.597  -1.790   ASYMPTOTE = -2.500 -1.658
```

| TIME | INPUT R(TIME) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0. | 0.000 | .101 | .143 | .101 | -.000 | -.101 | -.143 | -.101 |
| .125E-04 | -.166 | .468 | .827 | .702 | .166 | -.468 | -.827 | -.702 |
| .250E-04 | -.575 | .550 | 1.353 | 1.363 | .575 | -.550 | -1.353 | -1.363 |
| .375E-04 | -1.143 | .329 | 1.608 | 1.945 | 1.143 | -.329 | -1.608 | -1.945 |
| .500E-04 | -1.750 | -.159 | 1.526 | 2.316 | 1.750 | .159 | -1.526 | -2.316 |
| .625E-04 | -2.262 | -.820 | 1.102 | 2.379 | 2.262 | .820 | -1.102 | -2.379 |
| .750E-04 | -2.554 | -1.525 | .397 | 2.087 | 2.554 | 1.525 | -.397 | -2.087 |
| .875E-04 | -2.537 | -2.130 | -.475 | 1.458 | 2.537 | 2.130 | .475 | -1.458 |
| .100E-03 | -2.171 | -2.503 | -1.369 | .567 | 2.171 | 2.503 | 1.369 | -.567 |
| .113E-03 | -1.479 | -2.548 | -2.124 | -.456 | 1.479 | 2.548 | 2.124 | .456 |
| .125E-03 | -.543 | -2.224 | -2.602 | -1.456 | .543 | 2.224 | 2.602 | 1.456 |
| .138E-03 | .508 | -1.552 | -2.703 | -2.271 | -.508 | 1.552 | 2.703 | 2.271 |
| .150E-03 | 1.518 | -.617 | -2.390 | -2.764 | -1.518 | .617 | 2.390 | 2.764 |
| .163E-03 | 2.329 | .448 | -1.695 | -2.845 | -2.329 | -.448 | 1.695 | 2.845 |
| .175E-03 | 2.807 | 1.480 | -.714 | -2.490 | -2.807 | -1.480 | .714 | 2.490 |
| .188E-03 | 2.869 | 2.316 | .406 | -1.741 | -2.869 | -2.316 | -.406 | 1.741 |
| .200E-03 | 2.493 | 2.818 | 1.492 | -.708 | -2.493 | -2.818 | -1.492 | .708 |
| .213E-03 | 1.727 | 2.896 | 2.369 | .454 | -1.727 | -2.896 | -2.369 | -.454 |
| .225E-03 | .682 | 2.529 | 2.894 | 1.564 | -.682 | -2.529 | -2.894 | -1.564 |
| .238E-03 | -.484 | 1.763 | 2.977 | 2.448 | .484 | -1.763 | -2.977 | -2.448 |
| .250E-03 | -1.592 | .711 | 2.597 | 2.962 | 1.592 | -.711 | -2.597 | -2.962 |
| .263E-03 | -2.468 | -.468 | 1.807 | 3.023 | 2.468 | .468 | -1.807 | -3.023 |
| .275E-03 | -2.972 | -1.589 | .725 | 2.614 | 2.972 | 1.589 | -.725 | -2.614 |
| .288E-03 | -3.021 | -2.478 | -.483 | 1.795 | 3.021 | 2.478 | .483 | -1.795 |
| .300E-03 | -2.603 | -2.991 | -1.626 | .690 | 2.603 | 2.991 | 1.626 | -.690 |
| .313E-03 | -1.778 | -3.044 | -2.527 | -.529 | 1.778 | 3.044 | 2.527 | .529 |
| .325E-03 | -.672 | -2.624 | -3.040 | -1.675 | .672 | 2.624 | 3.040 | 1.675 |
| .338E-03 | .546 | -1.794 | -3.083 | -2.566 | -.546 | 1.794 | 3.083 | 2.566 |
| .350E-03 | 1.686 | -.680 | -2.647 | -3.064 | -1.686 | .680 | 2.647 | 3.064 |
| .363E-03 | 2.570 | .547 | -1.797 | -3.088 | -2.570 | -.547 | 1.797 | 3.088 |
| .375E-03 | 3.061 | 1.695 | -.664 | -2.634 | -3.061 | -1.695 | .664 | 2.634 |
| .388E-03 | 3.079 | 2.584 | .576 | -1.770 | -3.079 | -2.584 | -.576 | 1.770 |
| .400E-03 | 2.621 | 3.076 | 1.730 | -.630 | -2.621 | -3.076 | -1.730 | .630 |
| .413E-03 | 1.757 | 3.093 | 2.617 | .608 | -1.757 | -3.093 | -2.617 | -.608 |
| .425E-03 | .620 | 2.631 | 3.101 | 1.755 | -.620 | -2.631 | -3.101 | -1.755 |
| .438E-03 | -.615 | 1.761 | 3.105 | 2.630 | .615 | -1.761 | -3.105 | -2.630 |
| .450E-03 | -1.756 | .617 | 2.628 | 3.100 | 1.756 | -.617 | -2.628 | -3.100 |
| .463E-03 | -2.626 | -.623 | 1.746 | 3.091 | 2.626 | .623 | -1.746 | -3.091 |
| .475E-03 | -3.092 | -1.767 | .593 | 2.606 | 3.092 | 1.767 | -.593 | -2.606 |
| .468E-03 | -3.082 | -2.638 | -.649 | 1.720 | 3.082 | 2.638 | .649 | -1.720 |
| .500E-03 | -2.597 | -3.102 | -1.790 | .570 | 2.597 | 3.102 | 1.790 | -.570 |

| TIME | INPUT Q(TIME) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0. | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 | -.000 | .707 |
| .125E-04 | .924 | .924 | .383 | -.383 | -.924 | -.924 | -.383 | .383 |
| .250E-04 | .707 | 1.000 | .707 | .000 | -.707 | -1.000 | -.707 | -.000 |
| .375E-04 | .383 | .924 | .924 | .383 | -.383 | -.924 | -.924 | -.383 |
| .500E-04 | .000 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 |
| .625E-04 | -.383 | .383 | .924 | .924 | .383 | -.383 | -.924 | -.924 |
| .750E-04 | -.707 | .000 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 |
| .875E-04 | -.924 | -.383 | .383 | .924 | .924 | .383 | -.383 | -.924 |
| .100E-03 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 | .000 | -.707 |
| .113E-03 | -.924 | -.924 | -.383 | .383 | .924 | .924 | .383 | -.383 |
| .125E-03 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 | .000 |
| .138E-03 | -.383 | -.924 | -.924 | -.383 | .383 | .924 | .924 | .383 |
| .150E-03 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 |
| .163E-03 | .383 | -.383 | -.924 | -.924 | -.383 | .383 | .924 | .924 |
| .175E-03 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 |
| .188E-03 | .924 | .383 | -.383 | -.924 | -.924 | -.383 | .383 | .924 |
| .200E-03 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 |
| .213E-03 | .924 | .924 | .383 | -.383 | -.924 | -.924 | -.383 | .383 |
| .225E-03 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 |
| .238E-03 | .383 | .924 | .924 | .383 | -.383 | -.924 | -.924 | -.383 |
| .250E-03 | .000 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 |
| .263E-03 | -.383 | .383 | .924 | .924 | .383 | -.383 | -.924 | -.924 |
| .275E-03 | -.707 | .000 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 |
| .288E-03 | -.924 | -.383 | .383 | .924 | .924 | .383 | -.383 | -.924 |
| .300E-03 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 | -.000 | -.707 |
| .313E-03 | -.924 | -.924 | -.383 | .383 | .924 | .924 | .383 | -.383 |
| .325E-03 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 | -.000 |
| .338E-03 | -.383 | -.924 | -.924 | -.383 | .383 | .924 | .924 | .383 |
| .350E-03 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 |
| .363E-03 | .383 | -.383 | -.924 | -.924 | -.383 | .383 | .924 | .924 |
| .375E-03 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 | 1.000 |
| .388E-03 | .924 | .383 | -.383 | -.924 | -.924 | -.383 | .383 | .924 |
| .400E-03 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 | .707 |
| .413E-03 | .924 | .924 | .383 | -.383 | -.924 | -.924 | -.383 | .383 |
| .425E-03 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 | .000 |
| .438E-03 | .383 | .924 | .924 | .383 | -.383 | -.924 | -.924 | -.383 |
| .450E-03 | .000 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 | -.707 |
| .463E-03 | -.383 | .383 | .924 | .924 | .383 | -.383 | -.924 | -.924 |
| .475E-03 | -.707 | .000 | .707 | 1.000 | .707 | -.000 | -.707 | -1.000 |
| .488E-03 | -.924 | -.383 | .383 | .924 | .924 | .383 | -.383 | -.924 |
| .500E-03 | -1.000 | -.707 | .000 | .707 | 1.000 | .707 | -.000 | -.707 |

| TIME | GREENS FUNCTION HR | | | | |
|---|---|---|---|---|---|
| 0.000 | 25.465 | -.000 | -.000 | -.000 | 0.000 |
| .157 | -1.681 | .078 | .284 | .317 | .324 |
| .314 | -2.305 | .183 | .370 | .398 | .404 |
| .471 | -2.630 | .332 | .389 | .395 | .397 |
| .628 | -2.648 | .508 | .341 | .318 | .314 |
| .785 | -2.408 | .678 | .240 | .194 | .185 |
| .942 | -1.999 | .804 | .111 | .060 | .050 |
| 1.100 | -1.525 | .855 | -.016 | -.049 | -.055 |
| 1.257 | -1.081 | .815 | -.113 | -.108 | -.107 |
| 1.414 | -.731 | .688 | -.159 | -.111 | -.104 |
| 1.571 | -.505 | .497 | -.146 | -.069 | -.058 |
| 1.728 | -.395 | .280 | -.077 | -.007 | .003 |
| 1.885 | -.367 | .076 | .030 | .051 | .054 |
| 2.042 | -.379 | -.083 | .153 | .082 | .076 |
| 2.199 | -.392 | -.180 | .267 | .077 | .064 |
| 2.356 | -.384 | -.213 | .351 | .040 | .028 |
| 2.513 | -.348 | -.197 | .390 | -.011 | -.016 |
| 2.670 | -.293 | -.155 | .382 | -.056 | -.047 |
| 2.827 | -.236 | -.110 | .332 | -.078 | -.054 |
| 2.985 | -.191 | -.078 | .255 | -.064 | -.034 |
| 3.142 | -.167 | -.067 | .167 | -.018 | .003 |
| 3.299 | -.162 | -.075 | .085 | .052 | .039 |
| 3.456 | -.170 | -.091 | .020 | .126 | .059 |
| 3.613 | -.179 | -.106 | -.020 | .189 | .053 |
| 3.770 | -.181 | -.111 | -.037 | .228 | .021 |
| 3.927 | -.172 | -.103 | -.036 | .237 | -.023 |
| 4.084 | -.153 | -.087 | -.025 | .219 | -.061 |
| 4.241 | -.132 | -.067 | -.013 | .183 | -.075 |
| 4.398 | -.114 | -.051 | -.006 | .139 | -.051 |
| 4.555 | -.104 | -.044 | -.006 | .097 | .011 |
| 4.712 | -.104 | -.045 | -.014 | .061 | .100 |
| 4.869 | -.110 | -.052 | -.026 | .034 | .197 |
| 5.027 | -.117 | -.060 | -.037 | .015 | .280 |
| 5.184 | -.120 | -.064 | -.043 | .002 | .329 |
| 5.341 | -.116 | -.060 | -.043 | -.007 | .336 |
| 5.498 | -.106 | -.051 | -.036 | -.011 | .303 |
| 5.655 | -.093 | -.040 | -.027 | -.008 | .242 |
| 5.812 | -.083 | -.030 | -.017 | .004 | .170 |
| 5.969 | -.078 | -.025 | -.012 | .024 | .104 |
| 6.126 | -.079 | -.026 | -.012 | .050 | .056 |
| 6.283 | -.085 | -.032 | -.018 | .077 | .031 |
| 6.440 | -.091 | -.038 | -.028 | .099 | .025 |
| 6.597 | -.094 | -.042 | -.037 | .111 | .030 |
| 6.754 | -.092 | -.041 | -.042 | .110 | .037 |
| 6.912 | -.084 | -.034 | -.039 | .095 | .039 |
| 7.069 | -.075 | -.025 | -.027 | .072 | .034 |
| 7.226 | -.067 | -.016 | -.008 | .046 | .022 |
| 7.383 | -.064 | -.011 | .017 | .022 | .008 |
| 7.540 | -.066 | -.013 | .041 | .006 | -.004 |
| 7.697 | -.072 | -.020 | .061 | -.001 | -.010 |
| 7.854 | -.079 | -.029 | .073 | -.000 | -.009 |
| TIME | GREENS FUNCTION HR | | | | |
| 8.011 | -.083 | -.036 | .075 | .004 | -.005 |
| 8.168 | -.081 | -.037 | .069 | .009 | .000 |
| 8.325 | -.074 | -.030 | .055 | .010 | .003 |
| 8.482 | -.064 | -.014 | .038 | .008 | .002 |
| 8.639 | -.055 | .006 | .021 | .002 | -.003 |
| 8.796 | -.052 | .027 | .009 | -.005 | -.009 |
| 8.954 | -.057 | .045 | .001 | -.011 | -.014 |
| 9.111 | -.067 | .056 | -.001 | -.013 | -.016 |
| 9.268 | -.079 | .059 | .000 | -.012 | -.015 |
| 9.425 | -.087 | .055 | .003 | -.008 | -.011 |
| 9.582 | -.085 | .046 | .005 | -.005 | -.008 |
| 9.739 | -.071 | .035 | .006 | -.003 | -.005 |
| 9.896 | -.044 | .024 | .004 | -.003 | -.005 |
| 10.053 | -.009 | .015 | -.000 | -.006 | -.008 |
| 10.210 | .025 | .009 | -.005 | -.010 | -.012 |
| 10.367 | .053 | .003 | -.008 | -.014 | -.015 |
| 10.524 | .067 | -.000 | -.010 | -.015 | -.016 |
| 10.681 | .066 | -.003 | -.010 | -.014 | -.015 |
| 10.838 | .052 | -.003 | -.007 | -.010 | -.012 |
| 10.996 | .029 | .000 | -.004 | -.007 | -.008 |
| 11.153 | .005 | .007 | -.002 | -.005 | -.006 |
| 11.310 | -.015 | .017 | -.001 | -.005 | -.006 |
| 11.467 | -.011 | .041 | -.003 | -.020 | -.026 |
| 11.624 | -.021 | .048 | -.007 | -.019 | -.024 |
| 11.781 | -.026 | .052 | -.011 | -.018 | -.020 |
| 11.938 | -.026 | .053 | -.015 | -.016 | -.017 |
| 12.095 | -.024 | .050 | -.015 | -.015 | -.016 |
| 12.252 | -.021 | .045 | -.012 | -.014 | -.016 |
| 12.409 | -.020 | .038 | -.005 | -.014 | -.018 |
| 12.566 | -.021 | .031 | .005 | -.015 | -.021 |
| 12.723 | -.024 | .024 | .016 | -.017 | -.024 |
| 12.881 | -.029 | .019 | .027 | -.019 | -.025 |
| 13.038 | -.034 | .015 | .034 | -.020 | -.025 |
| 13.195 | -.037 | .013 | .038 | -.021 | -.023 |
| 13.352 | -.025 | .013 | .021 | -.020 | -.004 |
| 13.509 | -.027 | .013 | .020 | -.018 | -.003 |
| 13.666 | -.030 | .013 | .017 | -.013 | -.005 |
| 13.823 | -.032 | .013 | .013 | -.007 | -.008 |
| 13.980 | -.033 | .013 | .008 | .001 | -.012 |
| 14.137 | -.032 | .012 | .003 | .008 | -.016 |
| 14.294 | -.031 | .011 | -.001 | .015 | -.019 |
| 14.451 | -.029 | .009 | -.005 | .020 | -.020 |
| 14.608 | -.028 | .008 | -.008 | .022 | -.018 |
| 14.765 | -.027 | .007 | -.009 | .022 | -.012 |
| 14.923 | -.027 | .006 | -.010 | .019 | -.003 |
| 15.080 | -.028 | .006 | -.009 | .014 | .007 |
| 15.237 | -.030 | .006 | -.008 | .007 | .020 |
| 15.394 | -.032 | .007 | -.007 | .001 | .031 |
| 15.551 | -.034 | .007 | -.006 | -.005 | .042 |
| 15.708 | -.035 | .007 | -.006 | -.009 | .049 |

| TIME | GREENS FUNCTION JR | | |
|---|---|---|---|
| 0.000 | -3.698 | -1.520 | -.629 |
| .157 | -3.114 | -1.248 | -.514 |
| .314 | -2.208 | -.851 | -.348 |
| .471 | -1.138 | -.420 | -.171 |
| .628 | -.078 | -.054 | -.024 |
| .785 | .820 | .185 | .072 |
| .942 | 1.448 | .264 | .098 |
| 1.100 | 1.764 | .198 | .073 |
| 1.257 | 1.788 | .050 | .020 |
| 1.414 | 1.590 | -.108 | -.032 |
| 1.571 | 1.267 | -.208 | -.059 |
| 1.728 | .912 | -.203 | -.055 |
| 1.885 | .601 | -.090 | -.024 |
| 2.042 | .375 | .105 | .017 |
| 2.199 | .242 | .329 | .046 |
| 2.356 | .184 | .525 | .050 |
| 2.513 | .172 | .650 | .024 |
| 2.670 | .174 | .681 | -.020 |
| 2.827 | .170 | .625 | -.062 |
| 2.985 | .152 | .510 | -.080 |
| 3.142 | .123 | .373 | -.056 |
| 3.299 | .091 | .249 | .012 |
| 3.456 | .065 | .163 | .114 |
| 3.613 | .051 | .121 | .228 |
| 3.770 | .048 | .113 | .325 |
| 3.927 | .052 | .122 | .383 |
| 4.084 | .055 | .131 | .390 |
| 4.241 | .054 | .128 | .348 |
| 4.398 | .048 | .111 | .273 |
| 4.555 | .037 | .085 | .188 |
| 4.712 | .027 | .059 | .117 |
| 4.869 | .020 | .043 | .075 |
| 5.027 | .019 | .039 | .066 |
| 5.184 | .021 | .046 | .082 |
| 5.341 | .025 | .055 | .108 |
| 5.498 | .027 | .061 | .125 |
| 5.655 | .026 | .058 | .120 |
| 5.812 | .021 | .045 | .089 |
| 5.969 | .014 | .029 | .036 |
| 6.126 | .009 | .016 | -.027 |
| 6.283 | .007 | .011 | -.086 |
| 6.440 | .009 | .017 | -.127 |
| 6.597 | .013 | .031 | -.145 |
| 6.754 | .018 | .045 | -.139 |
| 6.912 | .019 | .051 | -.116 |
| 7.069 | .015 | .042 | -.086 |
| 7.226 | .009 | .016 | -.057 |
| 7.383 | .003 | -.023 | -.037 |
| 7.540 | -.001 | -.066 | -.028 |
| 7.697 | .001 | -.103 | -.027 |
| 7.854 | .008 | -.125 | -.030 |

| TIME | GREENS FUNCTION JR | | |
|---|---|---|---|
| 8.011 | .018 | -.129 | -.033 |
| 8.168 | .024 | -.116 | -.033 |
| 8.325 | .023 | -.093 | -.029 |
| 8.482 | .011 | -.066 | -.023 |
| 8.639 | -.013 | -.044 | -.017 |
| 8.796 | -.045 | -.032 | -.013 |
| 8.954 | -.077 | -.029 | -.012 |
| 9.111 | -.101 | -.032 | -.014 |
| 9.268 | -.113 | -.037 | -.016 |
| 9.425 | -.109 | -.039 | -.016 |
| 9.582 | -.093 | -.037 | -.016 |
| 9.739 | -.069 | -.030 | -.013 |
| 9.896 | -.046 | -.022 | -.010 |
| 10.053 | -.029 | -.016 | -.008 |
| 10.210 | -.023 | -.014 | -.007 |
| 10.367 | -.027 | -.016 | -.007 |
| 10.524 | -.037 | -.020 | -.009 |
| 10.681 | -.046 | -.023 | -.010 |
| 10.838 | -.049 | -.024 | -.011 |
| 10.996 | -.041 | -.020 | -.009 |
| 11.153 | -.024 | -.015 | -.007 |
| 11.310 | .000 | -.009 | -.005 |
| 11.467 | .019 | -.015 | -.010 |
| 11.624 | .037 | -.020 | -.013 |
| 11.781 | .050 | -.023 | -.013 |
| 11.938 | .057 | -.023 | -.011 |
| 12.095 | .056 | -.020 | -.006 |
| 12.252 | .048 | -.012 | -.001 |
| 12.409 | .037 | -.002 | .002 |
| 12.566 | .025 | .011 | .003 |
| 12.723 | .015 | .024 | .001 |
| 12.881 | .008 | .036 | -.004 |
| 13.038 | .006 | .045 | -.010 |
| 13.195 | .007 | .050 | -.015 |
| 13.352 | .004 | .051 | -.010 |
| 13.509 | .004 | .048 | -.003 |
| 13.666 | .005 | .042 | .006 |
| 13.823 | .008 | .034 | .016 |
| 13.980 | .011 | .026 | .025 |
| 14.137 | .013 | .019 | .033 |
| 14.294 | .014 | .014 | .039 |
| 14.451 | .013 | .011 | .042 |
| 14.608 | .011 | .011 | .042 |
| 14.765 | .008 | .012 | .041 |
| 14.923 | .005 | .014 | .039 |
| 15.080 | .003 | .016 | .035 |
| 15.237 | .001 | .017 | .032 |
| 15.394 | .001 | .016 | .028 |
| 15.551 | .001 | .015 | .025 |
| 15.708 | .003 | .012 | .022 |

| TIME | GREENS FUNCTION KR | | | | |
|---|---|---|---|---|---|
| 0.000 | 25.465 | -.000 | -.000 | -.000 | 0.000 |
| .157 | -.187 | .009 | .032 | .035 | .036 |
| .314 | -.256 | .020 | .041 | .044 | .045 |
| .471 | -.292 | .037 | .043 | .044 | .044 |
| .628 | -.294 | .056 | .038 | .035 | .035 |
| .785 | -.268 | .075 | .027 | .022 | .021 |
| .942 | -.222 | .089 | .012 | .007 | .006 |
| 1.100 | -.169 | .095 | -.002 | -.005 | -.006 |
| 1.257 | -.120 | .091 | -.013 | -.012 | -.012 |
| 1.414 | -.081 | .076 | -.018 | -.012 | -.012 |
| 1.571 | -.056 | .055 | -.016 | -.008 | -.006 |
| 1.728 | -.044 | .031 | -.009 | -.001 | .000 |
| 1.885 | -.041 | .008 | .003 | .006 | .006 |
| 2.042 | -.042 | -.009 | .017 | .009 | .008 |
| 2.199 | -.044 | -.020 | .030 | .009 | .007 |
| 2.356 | -.043 | -.024 | .039 | .004 | .003 |
| 2.513 | -.039 | -.022 | .043 | -.001 | -.002 |
| 2.670 | -.033 | -.017 | .042 | -.006 | -.005 |
| 2.827 | -.026 | -.012 | .037 | -.009 | -.006 |
| 2.985 | -.021 | -.009 | .028 | -.007 | -.004 |
| 3.142 | -.019 | -.007 | .019 | -.002 | .000 |
| 3.299 | -.018 | -.008 | .009 | .006 | .004 |
| 3.456 | -.019 | -.010 | .002 | .014 | .007 |
| 3.613 | -.020 | -.012 | -.002 | .021 | .006 |
| 3.770 | -.020 | -.012 | -.004 | .025 | .002 |
| 3.927 | -.019 | -.011 | -.004 | .026 | -.003 |
| 4.084 | -.017 | -.010 | -.003 | .024 | -.007 |
| 4.241 | -.015 | -.007 | -.001 | .020 | -.008 |
| 4.398 | -.013 | -.006 | -.001 | .015 | -.006 |
| 4.555 | -.012 | -.005 | -.001 | .011 | .001 |
| 4.712 | -.012 | -.005 | -.002 | .007 | .011 |
| 4.869 | -.012 | -.006 | -.003 | .004 | .022 |
| 5.027 | -.013 | -.007 | -.004 | .002 | .031 |
| 5.184 | -.013 | -.007 | -.005 | .000 | .037 |
| 5.341 | -.013 | -.007 | -.005 | -.001 | .037 |
| 5.498 | -.012 | -.006 | -.004 | -.001 | .034 |
| 5.655 | -.010 | -.004 | -.003 | -.001 | .027 |
| 5.812 | -.009 | -.003 | -.002 | .000 | .019 |
| 5.969 | -.009 | -.003 | -.001 | .003 | .012 |
| 6.126 | -.009 | -.003 | -.001 | .006 | .006 |
| 6.283 | -.009 | -.004 | -.002 | .009 | .003 |
| 6.440 | -.010 | -.004 | -.003 | .011 | .003 |
| 6.597 | -.010 | -.005 | -.004 | .012 | .003 |
| 6.754 | -.010 | -.005 | -.005 | .012 | .004 |
| 6.912 | -.009 | -.004 | -.004 | .011 | .004 |
| 7.069 | -.008 | -.003 | -.003 | .008 | .004 |
| 7.226 | -.007 | -.002 | -.001 | .005 | .002 |
| 7.383 | -.007 | -.001 | .002 | .002 | .001 |
| 7.540 | -.007 | -.001 | .005 | .001 | -.000 |
| 7.697 | -.008 | -.002 | .007 | -.000 | -.001 |
| 7.854 | -.009 | -.003 | .008 | -.000 | -.001 |

| TIME | GREENS FUNCTION KR | | | | |
|---|---|---|---|---|---|
| 8.011 | -.009 | -.004 | .008 | .000 | -.001 |
| 8.168 | -.009 | -.004 | .008 | .001 | .000 |
| 8.325 | -.008 | -.003 | .006 | .001 | .000 |
| 8.482 | -.007 | -.002 | .004 | .001 | .000 |
| 8.639 | -.006 | .001 | .002 | .000 | -.000 |
| 8.796 | -.006 | .003 | .001 | -.001 | -.001 |
| 8.954 | -.006 | .005 | .000 | -.001 | -.002 |
| 9.111 | -.007 | .006 | -.000 | -.001 | -.002 |
| 9.268 | -.009 | .007 | .000 | -.001 | -.002 |
| 9.425 | -.010 | .006 | .000 | -.001 | -.001 |
| 9.582 | -.009 | .005 | .001 | -.001 | -.001 |
| 9.739 | -.008 | .004 | .001 | -.000 | -.001 |
| 9.896 | -.005 | .003 | .000 | -.000 | -.001 |
| 10.053 | -.001 | .002 | -.000 | -.001 | -.001 |
| 10.210 | .003 | .001 | -.001 | -.001 | -.001 |
| 10.367 | .006 | .000 | -.001 | -.002 | -.002 |
| 10.524 | .007 | -.000 | -.001 | -.002 | -.002 |
| 10.681 | .007 | -.000 | -.001 | -.002 | -.002 |
| 10.838 | .006 | -.000 | -.001 | -.001 | -.001 |
| 10.996 | .003 | .000 | -.000 | -.001 | -.001 |
| 11.153 | .001 | .001 | -.000 | -.001 | -.001 |
| 11.310 | -.002 | .002 | -.000 | -.001 | -.001 |
| 11.467 | -.001 | .005 | -.000 | -.002 | -.003 |
| 11.624 | -.002 | .005 | -.001 | -.002 | -.003 |
| 11.781 | -.003 | .006 | -.001 | -.002 | -.002 |
| 11.938 | -.003 | .006 | -.002 | -.002 | -.002 |
| 12.095 | -.003 | .006 | -.002 | -.002 | -.002 |
| 12.252 | -.002 | .005 | -.001 | -.002 | -.002 |
| 12.409 | -.002 | .004 | -.001 | -.002 | -.002 |
| 12.566 | -.002 | .003 | .001 | -.002 | -.002 |
| 12.723 | -.003 | .003 | .002 | -.002 | -.003 |
| 12.881 | -.003 | .002 | .003 | -.002 | -.003 |
| 13.038 | -.004 | .002 | .004 | -.002 | -.003 |
| 13.195 | -.004 | .001 | .004 | -.002 | -.003 |
| 13.352 | -.003 | .001 | .002 | -.002 | -.000 |
| 13.509 | -.003 | .001 | .002 | -.002 | -.000 |
| 13.666 | -.003 | .001 | .002 | -.001 | -.001 |
| 13.823 | -.004 | .001 | .001 | -.001 | -.001 |
| 13.980 | -.004 | .001 | .001 | .000 | -.001 |
| 14.137 | -.004 | .001 | .000 | .001 | -.002 |
| 14.294 | -.003 | .001 | -.000 | .002 | -.002 |
| 14.451 | -.003 | .001 | -.001 | .002 | -.002 |
| 14.608 | -.003 | .001 | -.001 | .002 | -.002 |
| 14.765 | -.003 | .001 | -.001 | .002 | -.001 |
| 14.923 | -.003 | .001 | -.001 | .002 | -.000 |
| 15.080 | -.003 | .001 | -.001 | .002 | .001 |
| 15.237 | -.003 | .001 | -.001 | .001 | .002 |
| 15.394 | -.004 | .001 | -.001 | .000 | .003 |
| 15.551 | -.004 | .001 | -.001 | -.001 | .005 |
| 15.708 | -.004 | .001 | -.001 | -.001 | .005 |

TIME    PRESSURE P(TIME) FOR DISCRETE VALUES OF Y

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0. | .526E+06 | .372E+06 | -.760E-09 | -.372E+06 | -.526E+06 | -.372E+06 | -.822E-08 | .372E+06 |
| .125E-04 | .542E+06 | .528E+06 | .205E+06 | -.238E+06 | -.542E+06 | -.528E+06 | -.205E+06 | .238E+06 |
| .250E-04 | .620E+06 | .741E+06 | .428E+06 | -.135E+06 | -.620E+06 | -.741E+06 | -.428E+06 | .135E+06 |
| .375E-04 | .440E+06 | .766E+06 | .644E+06 | .144E+06 | -.440E+06 | -.766E+06 | -.644E+06 | -.144E+06 |
| .500E-04 | .926E+05 | .599E+06 | .755E+06 | .468E+06 | -.926E+05 | -.599E+06 | -.755E+06 | -.468E+06 |
| .625E-04 | -.377E+06 | .235E+06 | .709E+06 | .767E+06 | .377E+06 | -.235E+06 | -.709E+06 | -.767E+06 |
| .750E-04 | -.839E+06 | -.256E+06 | .476E+06 | .930E+06 | .839E+06 | .256E+06 | -.476E+06 | -.930E+06 |
| .875E-04 | -.121E+07 | -.798E+06 | .798E+05 | .911E+06 | .121E+07 | .798E+06 | -.798E+05 | -.911E+06 |
| .100E-03 | -.138E+07 | -.128E+07 | -.427E+06 | .673E+06 | .138E+07 | .128E+07 | .427E+06 | -.673E+06 |
| .113E-03 | -.131E+07 | -.160E+07 | -.955E+06 | .252E+06 | .131E+07 | .160E+07 | .955E+06 | -.252E+06 |
| .125E-03 | -.983E+06 | -.169E+07 | -.141E+07 | -.301E+06 | .983E+06 | .169E+07 | .141E+07 | .301E+06 |
| .138E-03 | -.447E+06 | -.151E+07 | -.169E+07 | -.880E+06 | .447E+06 | .151E+07 | .169E+07 | .880E+06 |
| .150E-03 | .222E+06 | -.107E+07 | -.174E+07 | -.139E+07 | -.222E+06 | .107E+07 | .174E+07 | .139E+07 |
| .163E-03 | .904E+06 | -.433E+06 | -.152E+07 | -.171E+07 | -.904E+06 | .433E+06 | .152E+07 | .171E+07 |
| .175E-03 | .149E+07 | .312E+06 | -.104E+07 | -.179E+07 | -.149E+07 | -.312E+06 | .104E+07 | .179E+07 |
| .188E-03 | .186E+07 | .104E+07 | -.384E+06 | -.158E+07 | -.186E+07 | -.104E+07 | .384E+06 | .158E+07 |
| .200E-03 | .194E+07 | .163E+07 | .369E+06 | -.111E+07 | -.194E+07 | -.163E+07 | -.369E+06 | .111E+07 |
| .213E-03 | .172E+07 | .199E+07 | .109E+07 | -.439E+06 | -.172E+07 | -.199E+07 | -.109E+07 | .439E+06 |
| .225E-03 | .120E+07 | .203E+07 | .167E+07 | .336E+06 | -.120E+07 | -.203E+07 | -.167E+07 | -.336E+06 |
| .238E-03 | .470E+06 | .175E+07 | .201E+07 | .109E+07 | -.470E+06 | -.175E+07 | -.201E+07 | -.109E+07 |
| .250E-03 | -.358E+06 | .118E+07 | .203E+07 | .169E+07 | .358E+06 | -.118E+07 | -.203E+07 | -.169E+07 |
| .263E-03 | -.114E+07 | .419E+06 | .173E+07 | .203E+07 | .114E+07 | -.419E+06 | -.173E+07 | -.203E+07 |
| .275E-03 | -.176E+07 | -.425E+06 | .116E+07 | .206E+07 | .176E+07 | .425E+06 | -.116E+07 | -.206E+07 |
| .288E-03 | -.211E+07 | -.122E+07 | .391E+06 | .177E+07 | .211E+07 | .122E+07 | -.391E+06 | -.177E+07 |
| .300E-03 | -.213E+07 | -.183E+07 | -.453E+06 | .119E+07 | .213E+07 | .183E+07 | .453E+06 | -.119E+07 |
| .313E-03 | -.182E+07 | -.216E+07 | -.124E+07 | .412E+06 | .182E+07 | .216E+07 | .124E+07 | -.412E+06 |
| .325E-03 | -.122E+07 | -.217E+07 | -.184E+07 | -.441E+06 | .122E+07 | .217E+07 | .184E+07 | .441E+06 |
| .338E-03 | -.419E+06 | -.183E+07 | -.217E+07 | -.124E+07 | .419E+06 | .183E+07 | .217E+07 | .124E+07 |
| .350E-03 | .456E+06 | -.121E+07 | -.216E+07 | -.185E+07 | -.456E+06 | .121E+07 | .216E+07 | .185E+07 |
| .363E-03 | .127E+07 | -.392E+06 | -.182E+07 | -.218E+07 | -.127E+07 | .392E+06 | .182E+07 | .218E+07 |
| .375E-03 | .189E+07 | .490E+06 | -.119E+07 | -.218E+07 | -.189E+07 | -.490E+06 | .119E+07 | .218E+07 |
| .388E-03 | .222E+07 | .130E+07 | -.378E+06 | -.184E+07 | -.222E+07 | -.130E+07 | .378E+06 | .184E+07 |
| .400E-03 | .221E+07 | .191E+07 | .501E+06 | -.121E+07 | -.221E+07 | -.191E+07 | -.501E+06 | .121E+07 |
| .413E-03 | .185E+07 | .223E+07 | .131E+07 | -.383E+06 | -.185E+07 | -.223E+07 | -.131E+07 | .383E+06 |
| .425E-03 | .120E+07 | .221E+07 | .192E+07 | .503E+06 | -.120E+07 | -.221E+07 | -.192E+07 | -.503E+06 |
| .438E-03 | .370E+06 | .184E+07 | .223E+07 | .131E+07 | -.370E+06 | -.184E+07 | -.223E+07 | -.131E+07 |
| .450E-03 | -.524E+06 | .119E+07 | .220E+07 | .193E+07 | .524E+06 | -.119E+07 | -.220E+07 | -.193E+07 |
| .463E-03 | -.134E+07 | .347E+06 | .183E+07 | .224E+07 | .134E+07 | -.347E+06 | -.183E+07 | -.224E+07 |
| .475E-03 | -.195E+07 | -.546E+06 | .118E+07 | .221E+07 | .195E+07 | .546E+06 | -.118E+07 | -.221E+07 |
| .488E-03 | -.226E+07 | -.136E+07 | .340E+06 | .184E+07 | .226E+07 | .136E+07 | -.340E+06 | -.184E+07 |
| .500E-03 | -.222E+07 | -.196E+07 | -.550E+06 | .118E+07 | .222E+07 | .196E+07 | .550E+06 | -.118E+07 |

| TIME | UPSTREAM COMPONENT OF PRESSURE | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0. | .500E+06 | .354E+06 | -.858E-09 | -.354E+06 | -.500E+06 | -.354E+06 | -.809E-08 | .354E+06 |
| .125E-04 | .434E+06 | .439E+06 | .187E+06 | -.175E+06 | -.434E+06 | -.439E+06 | -.187E+06 | .175E+06 |
| .250E-04 | .330E+06 | .472E+06 | .338E+06 | .595E+04 | -.330E+06 | -.472E+06 | -.338E+06 | -.595E+04 |
| .375E-04 | .169E+06 | .429E+06 | .438E+06 | .190E+06 | -.169E+06 | -.429E+06 | -.438E+06 | -.190E+06 |
| .500E-04 | -.145E+05 | .321E+06 | .469E+06 | .342E+06 | .145E+05 | -.321E+06 | -.469E+06 | -.342E+06 |
| .625E-04 | -.199E+06 | .161E+06 | .427E+06 | .442E+06 | .199E+06 | -.161E+06 | -.427E+05 | -.442E+06 |
| .750E-04 | -.351E+06 | -.240E+05 | .317E+06 | .473E+06 | .351E+06 | .240E+05 | -.317E+05 | -.473E+06 |
| .875E-04 | -.451E+06 | -.207E+06 | .158E+06 | .431E+06 | .451E+05 | .207E+06 | -.158E+05 | -.431E+06 |
| .100E-03 | -.480E+06 | -.359E+06 | -.273E+05 | .320E+06 | .480E+06 | .359E+06 | .273E+05 | -.320E+06 |
| .113E-03 | -.437E+06 | -.457E+06 | -.210E+06 | .161E+06 | .437E+06 | .457E+06 | .210E+05 | -.161E+06 |
| .125E-03 | -.324E+06 | -.465E+06 | -.361E+06 | .258E+05 | .324E+06 | .485E+06 | .361E+06 | .258E+05 |
| .138E-03 | -.163E+06 | -.439E+06 | -.458E+06 | -.209E+06 | .163E+06 | .439E+06 | .458E+06 | .209E+06 |
| .150E-03 | .251E+05 | -.325E+06 | -.485E+06 | -.361E+06 | -.251E+05 | .325E+06 | .485E+06 | .361E+06 |
| .163E-03 | .209E+06 | -.162E+06 | -.439E+06 | -.458E+06 | -.209E+06 | .162E+06 | .439E+06 | .458E+06 |
| .175E-03 | .362E+06 | .260E+05 | -.325E+06 | -.486E+06 | -.362E+06 | -.260E+05 | .325E+06 | .486E+06 |
| .188E-03 | .459E+06 | .210E+06 | -.162E+06 | -.439E+05 | -.459E+06 | -.210E+06 | .162E+06 | .439E+06 |
| .200E-03 | .487E+06 | .363E+06 | .265E+05 | -.326E+06 | -.487E+06 | -.363E+06 | -.265E+05 | .326E+06 |
| .213E-03 | .440E+06 | .461E+06 | .211E+06 | -.162E+06 | -.440E+06 | -.461E+06 | -.211E+05 | .162E+06 |
| .225E-03 | .326E+06 | .488E+06 | .364E+06 | .264E+05 | -.326E+06 | -.488E+06 | -.364E+06 | -.264E+05 |
| .238E-03 | .162E+06 | .440E+06 | .461E+06 | .211E+06 | -.162E+06 | -.440E+06 | -.461E+06 | -.211E+06 |
| .250E-03 | -.272E+05 | .325E+06 | .487E+06 | .364E+06 | .272E+05 | -.325E+06 | -.487E+06 | -.364E+06 |
| .263E-03 | -.212E+06 | .161E+06 | .440E+06 | .461E+06 | .212E+06 | -.161E+06 | -.440E+06 | -.461E+06 |
| .275E-03 | -.364E+06 | -.272E+05 | .325E+06 | .487E+06 | .364E+06 | .272E+05 | -.325E+06 | -.487E+06 |
| .288E-03 | -.461E+06 | -.212E+06 | .161E+06 | .440E+06 | .461E+06 | .212E+06 | -.161E+05 | -.440E+06 |
| .300E-03 | -.487E+06 | -.364E+05 | -.272E+05 | .325E+06 | .487E+06 | .364E+06 | .272E+05 | -.325E+06 |
| .313E-03 | -.440E+05 | -.461E+06 | -.212E+06 | .161E+06 | .440E+06 | .461E+06 | .212E+06 | -.161E+06 |
| .325E-03 | -.325E+06 | -.487E+06 | -.364E+06 | -.272E+05 | .325E+06 | .487E+06 | .364E+06 | .272E+05 |
| .338E-03 | -.161E+06 | -.440E+06 | -.461E+06 | -.212E+06 | .161E+06 | .440E+06 | .461E+06 | .212E+06 |
| .350E-03 | .272E+05 | -.325E+06 | -.487E+06 | -.364E+06 | -.272E+05 | .325E+06 | .487E+06 | .364E+06 |
| .363E-03 | .212E+06 | -.161E+06 | -.440E+06 | -.461E+06 | -.212E+06 | .161E+06 | .440E+06 | .461E+06 |
| .375E-03 | .364E+06 | .272E+05 | -.325E+06 | -.487E+06 | -.364E+06 | -.272E+05 | .325E+06 | .487E+06 |
| .388E-03 | .461E+06 | .212E+06 | -.161E+06 | -.440E+06 | -.461E+06 | -.212E+06 | .161E+06 | .440E+06 |
| .400E-03 | .487E+06 | .364E+06 | .272E+05 | -.325E+06 | -.487E+05 | -.364E+06 | -.272E+05 | .325E+06 |
| .413E-03 | .440E+06 | .461E+06 | .212E+06 | -.161E+06 | -.440E+06 | -.461E+06 | -.212E+06 | .161E+06 |
| .425E-03 | .325E+06 | .487E+06 | .364E+06 | .272E+05 | -.325E+06 | -.487E+06 | -.364E+06 | -.272E+05 |
| .438E-03 | .161E+06 | .440E+06 | .461E+06 | .212E+06 | -.161E+06 | -.440E+06 | -.461E+06 | -.212E+06 |
| .450E-03 | -.272E+05 | .325E+06 | .487E+06 | .364E+06 | .272E+05 | -.325E+06 | -.487E+06 | -.364E+06 |
| .463E-03 | -.212E+06 | .161E+06 | .440E+06 | .461E+06 | .212E+06 | -.161E+06 | -.440E+05 | -.461E+06 |
| .475E-03 | -.364E+06 | -.272E+05 | .325E+06 | .487E+06 | .364E+06 | .272E+05 | -.325E+06 | -.487E+06 |
| .488E-03 | -.461E+06 | -.212E+06 | .161E+06 | .440E+06 | .461E+06 | .212E+06 | -.161E+06 | -.440E+06 |
| .500E-03 | -.487E+06 | -.364E+06 | -.272E+05 | .325E+06 | .487E+06 | .364E+06 | .272E+05 | -.325E+06 |

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16. Abstract

A small perturbation type analysis has been developed for the acoustic far field in an infinite duct extending upstream and downstream of an axial turbomachinery stage. The analysis is designed to interface with the numerical solution of the near field of the blade rows described in NASA CR-2900 and thereby to provide the necessary closure condition to complete the statement of infinite duct boundary conditions for the subject problem. The present analysis differs from conventional inlet duct analyses in that a simple harmonic time dependence was not assumed, since a transient signal is generated by the numerical near-field solution and periodicity is attained only asymptotically. A description of the computer code developed to carry out the necessary convolutions numerically is included, as well as the results of a sample application using an impulsively initiated harmonic signal.


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