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IMPROVED GROUND TRUTH GEOID FOR THE GEOS-3 CALIBRATION AREA

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16. Abstract

The purpose of this investigation is to develop methods and procedures to compute a detailed geoid to be used as geodetic ground truth for the calibration and verification of the GEOS-3 altimeter data. To calibrate and verify the altimeter geoid, it is necessary that an independent absolute geoid, compatible with the tracking station coordinates, be established in the GEOS-3 calibration area. The technique developed is based on "rectifying" the best available detailed geoid so that the rectified geoid will have correct scale, orientation, shape and position with respect to the geocenter.

This rectification procedure was applied to the detailed GSFC gravimetric geoid to make it compatible with a set of geoidal undulations obtained from the tracking station coordinates. The approach involved the development of a mathematical model based on a second degree polynomial, in rectangular Cartesian coordinates, describing the geoid undulations at the control stations. A generalized least squares solution was obtained for the polynomial which describes the variation of the undulation differences between the control stations geoid and the gravimetric geoid.

Three rectified geoids were determined. These geoids correspond to three sets of tracking station data, (1) WFC/C-Band data, (2) GSFC/C-Band data, and (3) OSU-275 data. The absolute accuracy of these rectified geoids is linearly correlated with the uncertainties of the tracking station coordinates and, to a certain extent, with those of the detailed geoid being rectified. Since the accuracy of tracking station coordinates is not better than 2 meters, the problem remains concerning the establishment of a more accurate geoid compatible with future altimetry missions (1 m to 10 cm accuracy).

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IMPROVED GROUND TRUTH GEOID FOR THE GEOS-C CALIBRATION AREA

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1.0 INTRODUCTION

This report covers activities performed by Battelle's Columbus Laboratories (BCL) on behalf of the National Aeronautics and Space Administration, Wallops Flight Center, under Contract No. NAS6-2451. BCL has the responsibility of investigating methods and procedures of geoid rectifications to obtain the geoidal undulations (heights) required as geodetic ground truth for the calibration and verification of the GEOS-C altimetry data.

The objectives of the altimeter experiment, as described in the GEOS-C Mission Plan (reference 13), are to "Demonstrate the utility of satellite altimeters for measuring the geometry of the ocean surface. With sufficient accuracy in the determination of the geocentric position of the spacecraft and with suitable altimetry, the geometry of the ocean surface can be described and mean sea level determinations can be made. This, in turn, will contribute to refinement of the present knowledge of the geoid and to the initial description of the time-varying behavior of the ocean's surface and the larger quasi-steady state departures of the sea surface from the geoid (sea surface slopes, tides, geological effects on the ocean's surface, etc.)"

Because the geoid is an irregular surface that does not exactly conform to any known geometric figure; it is geometrically defined by its physical departures from a chosen regular figure which is usually a reference ellipsoid. In some methods, the departures are determined by linear and angular measurements while,in others, these departures are synthesized from gravity anomalies integrated all over the earth or a combination of both. The latest generation of geoids is deduced from the analysis of the dynamics of satellite orbits or a combination of gravimetry and satellite orbit analysis. Detailed review of geoidal methods and the requirements for a marine geoid compatible with satellite altimetry has been described by Fubara and Mourad, (1972).

One of the goals of the altimeter experiment is to calibrate the altimeter over an ocean area and verify its results. The GEOS-C calibration area is bounded by four ground based tracking stations located at Wallops Flight Center, Bermuda, Merritt Island and at Grand Turk. The purpose of the tracking stations is to determine an accurate orbit for the GEOS-C satellite. Knowing the orbit, the height (h) of the satellite above the ellipsoid becomes known. The height of the subsatellite point on the ocean will be measured directly with the altimeter. After several corrections are made to the altimeter measurement (instrument correction, geometric mean sea level, time varying ocean effects, etc.) the resultant height (H) is referred to the geoid. The difference between the satellite height (h) and the corrected altimeter height measurement (H) will describe the geoidal undulations (separation between the geoid and the reference ellipsoid).

To calibrate and verify the geoidal undulations obtained from the satellite altimeter, it is necessary that an independently determined ground truth geoid which is compatible with the tracking station coordinates, be established in the GEOS-C calibration area. This compatibility is important since the altimetry observations are related to the orbit which, in turn, is computed using the tracking station coordinates. The ground truth geoid implied by the tracking station coordinates is expected to be absolute, correct in scale, shape, orientation and position. Existing geoid models are inconsistent and lack sufficient accuracy for this purpose. The inconsistencies are caused by many factors such as (1) the parameters of the reference ellipsoid, (2) the measuring and reduction techniques, (3) the quantity and quality of data, and (4) the datum origin of the geodetic system.

One of the techniques, geoid rectification, developed and described in this report is aimed at establishing a geodetic ground truth geoid in the GEOS-C calibration area. The technique involves the rectification of the best available detailed gravimetric/satellite geoid to make it compatible with a set of geoidal undulations obtained from the ground tracking station coordinates. The purpose of rectification is to achieve a geoid with true scale, shape, orientation and geocentering. This can, then, be used to verify the satellite altimetry geoid. The availability of three sets of ground tracking station coordinates differing in values from each other dictated that three rectified geoidal models be established.

The absolute accuracy of the resultant geoid is linearly correlated with the uncertainties of the tracking station coordinates and to a certain extent with those of detailed geoids being rectified. Therefore, the success of the rectification depends highly on the ground truth data. Since the accuracy of the tracking stations is not better than 2 meters, the problem remains as to how to get a more accurate geoid compatible with future altimetry missions (1 m to 10 cm).

The primary results, conclusions, and recommendations of this investigation are outlined in Section 2.0 of this report. In Section 3.0 the mathematical formulations of the problem are discussed. Section 4.0 contains the results of the simulation studies. Modification and selection of a mathematical model suitable for the GEOS-C ground truth area is described in Section 5.0. The last Section (6.0) presents the analysis and results of geoid rectification.

2.0 SUMMARY OF RESULTS, CONCLUSIONS AND RECOMMENDATIONS

2.1 Program Summary

The objectives of this investigation have been to develop appropriate geoid rectification technique(s) and provide a detailed geoid, i.e., correct in scale, orientation, and shape that can be used as a geodetic ground truth for the GEOS-C mission with respect to (1) instrumental calibration, (2) ocean geoid determination from satellite altimetry, and (3) computation of quasi-stationary departures from the marine geoid.

The approach used consists of development of a general mathematical model based on a quadratic polynomial in rectangular Cartesian coordinates describing the geoid undulations at the control stations. Before proceeding with the rectification, it was necessary to carry out tests to validate the mathematical model, establish performance criteria, and to correlate the efficiency and accuracy of rectification with the area size, number of control stations and their distribution in the GEOS-C calibration area. The actual data in the calibration area do not have sufficient number of stations to perform these tests. Therefore, it was necessary to rely on simulation data obtained from existing gravimetric and satellite geoids. A generalized least squares solution was obtained for the polynomial which describes the variation of undulation differences between the control stations geoid and the gravimetric geoid.

To determine the coefficients of the polynomial requires a minimum of six and preferably 8-9 control stations. Since the GEOS-C calibration area has only four stations, it was necessary to modify the mathematical model to accommodate this condition without considerable compromise in the accuracy of rectification. The modification involved approximating the general polynomial by one that describes a circle for constant undulation differences. This was possible since the general elliptical shape of the geoid undulation contours can be approximated by arcs of circles without significant loss of accuracy. This required a minimum of four stations. A suitable modified mathematical model was thus selected following several tests using actual tracking station data. This model was then used in implementing the geoid rectification.

2.2 Summary of Results and Conclusions

The major results obtained during this investigation include three rectified geoids for the GEOS-C calibration area. These geoids correspond to three sets of tracking station data: (1) Wallops Flight Center (WFC) C-Band data, (2) Goddard Space Flight Center (GSFC) C-Band data, and (3) Ohio State University (OSU) -275 data. These results indicate that a detailed ground truth geoid can be obtained by rectifying any reasonably reliable and detailed geoid using a set of very sparse ground truth data. Due to the smooth and systematic nature of the difference between the detailed and the ground truth geoids, this difference can be described by a simple function - a polynomial - in a set of two dimensional rectangular Cartesian coordinates of the ground truth stations. The origin of this coordinate system is centered in the area of rectification.

The success of the rectification described in this study depends highly on the quality of the ground truth data. What is expected of the ground truth is an accurate and unique set of absolute undulations at points evenly distributed within and around the area of rectification. However, the inconsistencies among the best available data, which have been used in this study, casts serious doubts about their quality. At the same time, there is insufficient information about each set of data to judge their relative quality. It is, therefore, concluded that establishing an accurate and unique set of control ground truth data is the biggest challenge which must be met in order to satisfy the needs and objectives of future altimetry programs. Following are summaries of specific results and conclustions based on the use of (1) simulated data, and (2) actual data.

Simulation Studies

(1) The mathematical model used, which is a second degree polynomial in a set of two dimensional Cartisean coordinates of the control stations, is quite adequate for rectifying the detailed gravimetric geoids considering the accuracy of the geoids available at present. Higher degree polynomials should give better accuracy in the rectification.

(2) The area of rectification and the number of control stations are highly correlated with the accuracy desired in the rectification. For best results the area must be as small as possible and the number of control stations must be large. With the best accuracy of existing geoids and ground truth data, satisfactory results can be obtained for an area of size similar to that of the GEOS-C calibration area. For the second degree polynomial used in this investigation, there should be a minimum of 6 control stations. For optimum results, eight to nine stations are required.

(3) The distribution of the control stations in this area is very critical. In order to avoid unfavorable distortions in the rectified geoid, the control stations must be uniformly distributed within and around the area of rectification. This requirement emphasizes the need for adding one or two control stations, at sea, in the middle of the calibration area to achieve realistically more accurate rectification.

The data used in these studies are the Marsh-Vincent detailed gravimetric geoid of 1972 for control and the Marsh-Vincent satellite geoid of 1972 to be rectified. The difference between these two geoids is not as smooth and systematic as one would expect for the detailed gravimetric (to be rectified) and the ground truth (control) geoids. Consequently, the performance of this procedure with more realistic data (tracking station coordinates and detailed gravimetric geoid) should be much better than with the simulated data. Therefore, the above conclusions would be valid even for the real data.

Use of Actual Data

(1) Tests with the real data reinforce the conclusions presented in the simulation studies.

(2) The modified second degree polynomial used in the rectification gives adequate accuracy in rectification - well within the accuracy level of the two geoids involved - for the area within and around the area encompassed by the control stations. The error of rectification grows approximately proportional to the square of the distance from this area.

(3) The inconsistencies among the tracking station data indicate that the quality of these data is suspect. Only the OSU-275 data is independent of the detailed gravimetric geoid. The agreement between these two geoids, however, is reasonable except in scale. Their difference at Bermuda is relatively large and has a dominant effect on the rectification, since there is no other station close by. This reinforces the earlier statement about the need to have additional stations, at sea, in the middle of the test area. The geoids corresponding to the C-Band data agree well with the detailed gravimetric geoid, since all these data are based on the same coordinate system and gravity model. However, the number of stations is so few that it is not possible to draw any firm conclusions as to their quality.

(4) The procedure described and applied in this investigation is a viable method for providing a detailed ground truth geoid using only sparse ground truth data for meeting the objectives of the various satellite missions with respect to their calibration and verification needs.

2.3 Recommendations

The following recommendations are based on the results of this investigation:

(1) Determination of an accurate and unique set of absolute geoidal undulations should be made at evenly distributed control stations, within and around the GEOS-C calibration area. This is required because of the inconsistencies among the data available at present. Ramdom variation in the difference between the ground truth and the detailed gravimetric geoids indicates larger uncertainties in the station coordinates than were claimed by the various authors. An investigation should be initiated to identify the best approaches for determining the required station coordinates, properly positioned and oriented with respect to the geocenter, their accuracy, transformation procedures and the combination of various tracking systems data to meet future altimetry objectives.

(2) A program to investigate the feasibility of using geodetic control at sea should be planned and carried out to ensure uniform distribution of control stations for improved accuracy of rectification of the existing detailed gravimetric geoid.

(3) The present geoid rectification of $1^{\circ} \times 1^{\circ}$ details should be extended to 15' X 15' details. This is important since the resolution capability of the altimeter is expected to be better than 15' X 15'.

(4) After the altimeter had been calibrated, portions of the rectified geoid obtained in this investigation should be compared and validated with the altimetry geoid obtained from GEOS-C samples in the calibration area.

3.0 MATHEMATICAL FORMULATION OF THE PROBLEM

As stated in the introductory section, the objective of this investigation is to rectify the given detailed gravimetric geoid so that it is compatible with a set of geoidal heights (undulations) given at some control stations within the altimeter calibration area. These undulations are assumed to be correct in scale and orientation and referred to a geocentric ellipsoid close enough to be a general terrestrial ellipsoid which, by definition, has the following properties

(1) same mass as that of the earth

(2) same volume as that of the geoid

(3) its center coincides with that of the earth

(4) same rotational velocity as the earth

(5) minor axis coincides with the mean axis of rotation of the earth

(6) its surface potential is the same as that of the geoid

(7) the average undulations, referred to this ellipsoid over the whole surface of the earth, is zero.

On the other hand, "detailed gravimetric geoid" implies that the corresponding undulations are absolute except in scale. However, the selected detailed gravimetric geoid, to be rectified in this investigation, is not truly gravimetric. This geoid is computed from a combination of satellite and terrestrial gravity data (Marsh and Vincent, 1974). Apparently the scale for this geoid is introduced through the satellite data which contribute to the determination of the low harmonic component of the undulations. (Even though it is not clear from the report (ibid) that the scale was introduced in this way, personal communication with the author, Marsh, confirms this statement). The higher harmonic component is determined through the terrestrial gravity data. The reference system implied in the detailed gravimetric geoid is GEM6 (Lerch, et al, 1974) with the reference ellipsoid defined by:

semimajor axis (a) = 6378142.0

flattening (f) = 1/298.255

Even though a truly gravimetric geoid is geocentric, the Marsh-Vincent geoid may not be so due to the introduction of the satellite data from the GEM6 system.

Consequently, there would be some differences between the undulations at the control stations and those of the detailed gravimetric geoid. These differences are primarily due to the relative position, orientation and scale between the two coordinate systems, difference in the gravity models used and other systematic errors. Consequently, the difference in the two geoid models is expected to be smooth and systematic.

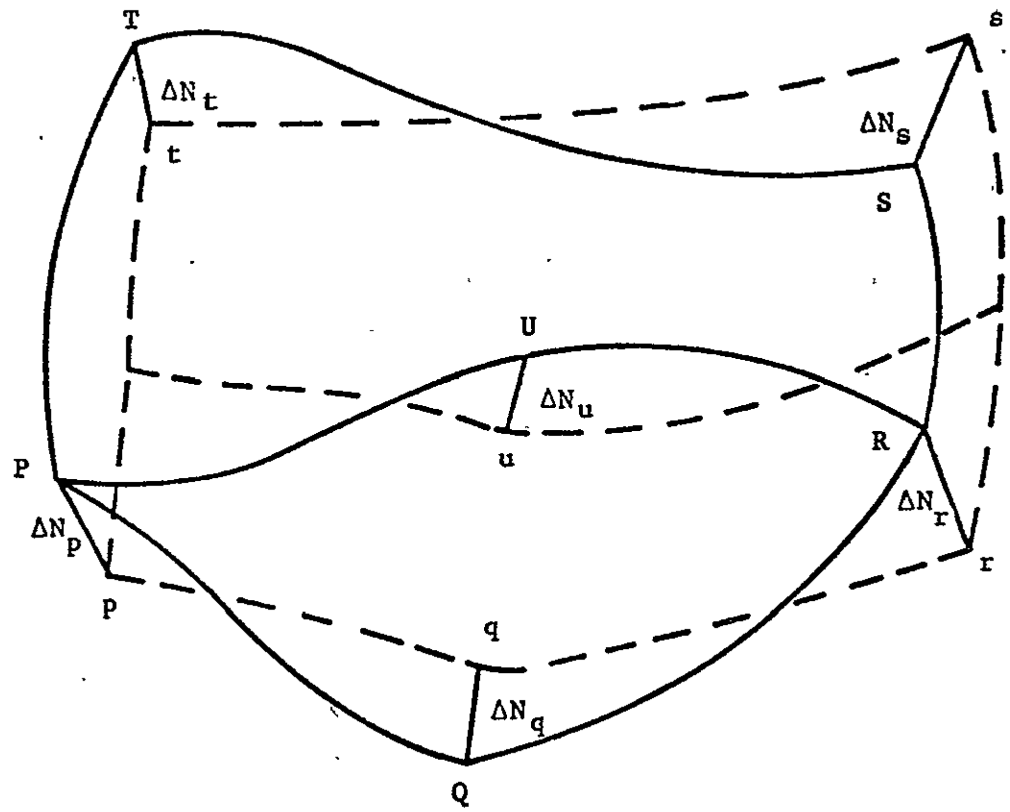
Based on the principle of terrain model leveling by means of control stations in photogrammetry, the geoid in any territory can be rectified to achieve a better scale, shape, orientation and geocentering. The principle is as follows. In Figure 1, P, Q, R, ... U, represents the control stations where an existing geoid PQRSTU is in error and needs rectification. Let the wrong geoid heights at these stations be denoted by $N_{wi} = \left[N_{wp}, N_{wq}, \ldots N_{wu}\right]$. The corresponding accurate geoid heights of the same stations $N_{ci} = \left[N_{cp}, N_{cq}, \ldots N_{cu}\right]$. A rectification is called for if any errors

$$\Delta N_i = N_{ci} - N_{wi} + \Delta h_i \neq 0 \tag{1}$$

where

$$\Delta h_i = \Delta a + a \sin^2\varphi_i \Delta f \tag{2}$$

represents any necessary correction due to changes Δa and Δf in the values of the semi major axis, a, and flattening, f, of the reference ellipsoids involved.



P, Q, R, ... U = POINTS ON ERRONEOUS GEOID
p, q, r, ... u = POINTS ON CONTROL GEOID

FIGURE 1. ILLUSTRATION OF THE PRINCIPLE OF GEOID RECTIFICATION

3.1 General Mathematical Model

Due to the smooth variation in ΔN_i, a quadratic polynomial in the rectangular coordinates (to be defined later) of the corresponding control station may be adequate to represent this variation in the calibration area. Then, each control station with coordinates (x_i, y_i) will give an equation of the form

$$\Delta N_i + ax_i^2 + bx_i + cx_iy_i + dy_i + ey_i^2 + k = 0 \tag{3}$$

Consequently, there will be n such equations for n control stations.

Before proceeding with the determination of the coefficients a, g, ..., e, k of the polynomial in equation (3), the system to which the coordinates (x_i, y_i) belong must be defined. There are several ways to define this coordinate system. Since the variation of ΔN is very small over an area of size similar to the GEOS-C calibration area, an approximate knowledge of the coordinates of the control stations is sufficient to describe ΔN by the mathematical model given by equation (3). Also, the magnitude of these coordinates must be such that the magnitude of the resulting coefficients of the polynomial would be neither too small nor too large. A system that would meet these criteria is described here.

Let φ_i and λ_i be the geodetic coordinates of the station i and also let the Cartesian coordinates, X^1 and Y^1, of this station be defined as

$$X^1 = \varphi_i \qquad Y^1 = \lambda_i \cdot \text{Cos}\ \varphi_i \tag{4}$$

The coordinates, X_o, and Y_o, of the centroid of all the stations are given by:

$$X_o = \frac{1}{N} \sum_{i=1}^{N} \varphi_i \qquad Y_o = \frac{1}{N} \sum_{i=1}^{N} \lambda_i \cdot \text{Cos}\ \varphi_i \tag{5}$$

where N is the number of stations.

Then, the coordinates x_i, y_i of the i^{th} station referred to the centroid (X_o, Y_o) are given by

$$x_i = X^1 - X_o \qquad y_i = Y^1 - Y_o \tag{6}$$

These coordinates as defined above and the observations N_c and N_w are all that are required for the determination of the coefficients in the polynomial.

3.2 Least Squares Rectification of the Geoidal Heights

Equations (1) and (3) can be combined to give equations of the form

$$N_{ci} - N_{wi} - \Delta h_i + ax_i^2 + bx_i + cx_i y_i + dy_i + ey_i^2 + k = 0 \qquad (7)$$

where

N_{ci} are considered as measured, or known, parameters

a, b, ..., k are unknown parameters and

N_{wi} are the measured data.

If the relative weight matrices associated with the quantities (a, b, ..., e, k), N_{ci} and N_{wi} are P_1, P_2, and P_3, respectively, equation (4) can be written in a general form

$$F_1(X_1^a, X_2^a, L_1^a \; P_1, P_2, P_3) = 0 \qquad (8)$$

where X_1^a, X_2^a and L_1^a are the adjusted or true values of the quantities (a, b, ..., e, k), N_{ci}, and N_{wi} respectively. If the number of equations is greater than or equal to 6, a least squares solution for the parameters can be obtained using the technique described in (Fubara, 1973). Let the observed or a priori values of the quantities X_1, X_2, and L_1 be X_1^o, X_2^o, and L_1^o respectively. Then the solution to the problem begins by linearizing the equations in form

$$\overrightarrow{F_1}\left(\overrightarrow{X_1^o}, \overrightarrow{X_2^o}, \overrightarrow{L_1^o} \;\middle|\; [P_1], [P_2], [P_3]\right) + [A_1][\Delta_1] + [B_1][\Delta_2] + [C_1][V_1] = 0 \qquad (9)$$

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where $[A_1]$, $[B_1]$ and $[C_1]$ are the first partial derivatives of $\vec{F_1}$ with respect to $\vec{X_1}$, $\vec{X_2}$ and $\vec{L_1}$. The true and measured parameter values are then related by

$$\vec{X_1^a} = \vec{X_1^o} + \vec{\Delta_1} \quad (10)$$

$$\vec{X_2^a} = \vec{X_2^o} + \vec{\Delta_2} \quad (11)$$

$$\vec{L_1^a} = \vec{L_1^o} + \vec{V_1} \quad (12)$$

In other words $\vec{\Delta_1}$, $\vec{\Delta_2}$ and $\vec{V_1}$ are corrections to the measurements or a priori values used to estimate the true values of the parameters. It can be shown that these corrections are given by the following equations:

$$\vec{\Delta_1} = -[N]^{-1}[A_1]^*[M_1]^{-1}\vec{W_1} \quad (13)$$

where * indicates matrix transpose and

$$[M_1] = [B_1]\,[P_2]^{-1}\,[B_1]^* + [C_1]\,[P_3]^{-1}\,[C_1]^* \quad (14)$$

$$[N] = [P_1] + [A_1]^*[M_1]^{-1}[A_1] \quad (15)$$

$$\vec{W_1} = \vec{F_1}(\vec{X_1^o}, \vec{X_2^o}, \vec{L_1^o}) \quad (16)$$

and

$$\vec{\Delta_2} = [P_2]^{-1}[B_1]^*[M_1]^{-1}\left[[A_1](\,[A_1]^*[M_1]^{-1}[A_1]\,)^{-1}[A_1]^*[M_1]^{-1} - I\right]\vec{W_1} \quad (17)$$

$$\vec{V_1} = [P_3]^{-1}[C_1]^*\vec{K_1} \quad (18)$$

where

$$\vec{K_1} = -[M_1]^{-1}\,(\,[A_1]\vec{\Delta_1} + \vec{W_1}\,) \quad (19)$$

The present problem is put into this framework by writing equation (3) in the form of equation (8). This means that Δh_i is zero, implying that the reference ellipsoids in both geoids are the same.

$$F_{11} = ax_1^2 + bx_1 + cx_1y_1 + dy_1 + ey_1^2 + k - \Delta N_1 = 0$$

$$F_{12} = ax_2^2 + bx_2 + cx_2y_2 + dy_2 + ey_2^2 + k - \Delta N_2 = 0 \quad (20)$$

$$\bullet$$
$$\bullet$$

$$F_{1n} = ax_n^2 + bx_n + cx_ny_n + dy_n + ey_n^2 + k - \Delta N_n = 0$$

then $\vec{X}_1$, $\vec{X}_2$ and $\vec{L}_1$ are defined as

$$\vec{X}_1^T = (a, b, c, d, e, k) \quad (21)$$

$$\vec{X}_2^T = (N_{c1}, N_{c2}, \ldots, N_{cn}) \quad (22)$$

$$\vec{L}_1^T = (N_{w1}, N_{w2}, \ldots, N_{wn}) \quad (23)$$

where N_{ci} is the absolute geoidal undulation at (x_i, y_i) and N_{wi} is the geoidal undulation from the geoid being rectified. Also

$$\Delta N_i = N_{ci} - N_{wi} \quad (24)$$

With these definitions it follows that

$$[A_1] = \begin{bmatrix} x_1^2 & x_1 & x_1y_1 & y_1 & y_1^2 & 1 \\ x_2^2 & x_2 & x_2y_2 & y_2 & y_2^2 & 1 \\ \bullet & & & & & \\ \bullet & & & & & \\ \bullet & & & & & \\ \bullet & & & & & \\ x_n^2 & x_n & x_ny_n & y_n & y_n^2 & 1 \end{bmatrix} \quad (25)$$

$$[B_1] = -[C_1] = [I_n] \tag{26}$$

where $[I_n]$ is the n x n identity matrix. Furthermore,

$$[P_1] = [0] \tag{27}$$

and, if we assume that the unknown covariances are zero, then

$$[P_2] = \frac{1}{\sigma_s^2} [I_n] \tag{28}$$

$$[P_3] = \frac{1}{\sigma_c^2} [I_n] \tag{29}$$

where σ_s^2 is the variance of the measured absolute geoidal undulations and σ_c^2 is the variance of the undulations from the geoid being rectified.

If the initial estimate of $\overrightarrow{X_1}$ is arbitrarily taken as zero (i.e., $X_1^o = 0$), then

$$\overrightarrow{W_1}^T = [\ \Delta N_1, \Delta N_2, \ldots, \Delta N_n\ ] \tag{30}$$

and

$$\overrightarrow{X_1^{aT}} = \overrightarrow{\Delta_1^T} = [a, b, c, d, e, k] \tag{31}$$

Owing to the simplicity of equations (26)-(29), equations (13)-(19) can be simplified as follows:

$$\overrightarrow{\Delta_1} = - \frac{1}{\sigma_s^2 + \sigma_c^2} [N]^{-1} [A_1]^* \overrightarrow{W_1} \qquad (3.$$

$$[M_1] = [P_2]^{-1} + [P_3]^{-1} = \sigma_c^2 + \sigma_s^2 \tag{33}$$

$$[N] = \frac{1}{\sigma_s^2 + \sigma_c^2} [A_1]^*[A_1] \tag{34}$$

$$\vec{\Delta}_2 = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_c^2} \left[[A_1] \left([A_1]^* [A_1] \right)^{-1} [A_1]^* \vec{W}_1 - \vec{W}_1 \right] \tag{35}$$

$$\vec{V}_1 = - \sigma_c^2 \, \vec{K}_1 \tag{36}$$

$$\vec{K}_1 = - \frac{1}{\sigma_s^2 + \sigma_c^2} \left([A_1] \vec{\Delta}_1 + \vec{W}_1 \right) . \tag{37}$$

From (32) and (34)

$$\vec{\Delta}_1 = - \left([A_1]^* [A_1] \right)^{-1} [A_1]^* \vec{W}_1 \quad . \tag{38}$$

Using (38) in (35),

$$\vec{\Delta}_2 = \frac{\sigma_s^2}{\sigma_c^2 + \sigma_s^2} \left[- [A_1] \vec{\Delta}_1 - \vec{W}_1 \right] \quad . \tag{39}$$

The correction, $\vec{V}$, to $\vec{\Delta N}$ (= N_c - N_w) is $\vec{\Delta}_2 - \vec{V}_1$ which is obtained by subtracting equation (36) with (37) from (39).

$$\vec{V} = \vec{\Delta}_2 - \vec{V}_1 = - [A_1] \vec{\Delta}_1 - \vec{W}_1 \tag{40}$$

Consequently, the desired least squares solution for the coefficients of the polynomial is obtained by evaluating equation (38) and the corrections V for the observations, ΔN, by evaluating equation (40). Once these coefficients are determined the geoidal height at an arbitary point can be rectified by evaluating the corresponding ΔN from equation (3) and adding it to the observation N_w.

4.0 SIMULATION STUDIES

The main thrust of the simulation studies was to:

(1) Validate the mathematical model, described in the previous section, and the related computer programs

(2) establish performance criteria for the actual rectification of the geoid, and

(3) investigate the correlation of the efficiency and accuracy of rectifying the detailed gravimetric geoid with the (i) extent of the area of rectification, (ii) number of control stations, and (iii) distribution of the control stations in a given area.

In order to fully accomplish the desired purpose, there must be sufficient number of stations to perform a least squares fit using the prescribed polynomial and still have enough independent stations to permit a meaningful statistical analysis to determine the "goodness" of the rectification elsewhere in the area. Unfortunately, the number of stations (mostly satellite tracking stations), in the vicinity of the calibration area, for which absolute undulation data are available is not sufficient for this purpose. Consequently, the studies must be performed using some simulated control station data in a realistic manner as possible.

For the purpose of simulation studies, the two geoids used are: (1) Vincent, Strange, and Marsh detailed gravimetric geoid of August, 1972, which is assumed to provide the control data, and (2) Vincent, Strange, and Marsh satellite geoid of August, 1972, which is assumed to be the one to be rectified. Contour maps of the difference between these geoids are available to a scale of approximately 5^{o} to an inch.

Two series of tests were performed. The first explored the effects of varying the number of control stations used for the rectification. It also examined the effects of varying the size of the area involved. The second series examined these same effects in the GEOS-C calibration area.

4.1 First Test Series

For these tests, ten control stations and eleven check stations were selected as shown in Figure 1. The undulations at the control stations were used to obtain a least squares solution for the coefficient, a, b, ..., e, k, of the polynomial experssed in equation (3). These coefficients were then used to compute the undulations (N_c) at the eleven check stations to determine the accuracy of the rectification at other points in the area.

The detailed procedure is as follows:

(1) Choose a subset of 6 to 10 control stations shown in Figure 1 and read their latitudes, longitudes and ΔN's from the contour map.

(2) Compute the corresponding x, y coordinates.

(3) Use these station values in a least squares solution for the coefficiënts in equation (1).

(4) Using the computed values of a, b, c, d, e and k, calculate ΔN for each of the eleven check stations shown in Figure 1.

(5) Read ΔN from the map for each of the check stations and compute the differences, e_i, given by

$$e_i = (\Delta N_i \text{ from map}) - (\Delta N_i \text{ computed})$$

which are the errors in the rectification associated with this procedure.

(6) As a measure of the precision of the rectification, compute the mean error, μ, and the standard deviation, σ :

$$\mu = \frac{1}{11} \sum_{i=11}^{21} e_i \tag{41}$$

$$\sigma = \sqrt{\frac{1}{11} \sum_{i=11}^{21} (e_i^2 - \mu^2)} \tag{42}$$

This procedure was applied to eight different subsets of the ten control stations: four sets of seven stations; one set each of six, eight, nine, and all ten stations. The seven station cases (E,F,G, and H) are shown on Figure 2 and the six, eight, nine and ten station cases (C,J,D, and Q) are shown in Figure 3. These figures show which control stations were used in each test. Also shown are the values of μ and σ for each test.

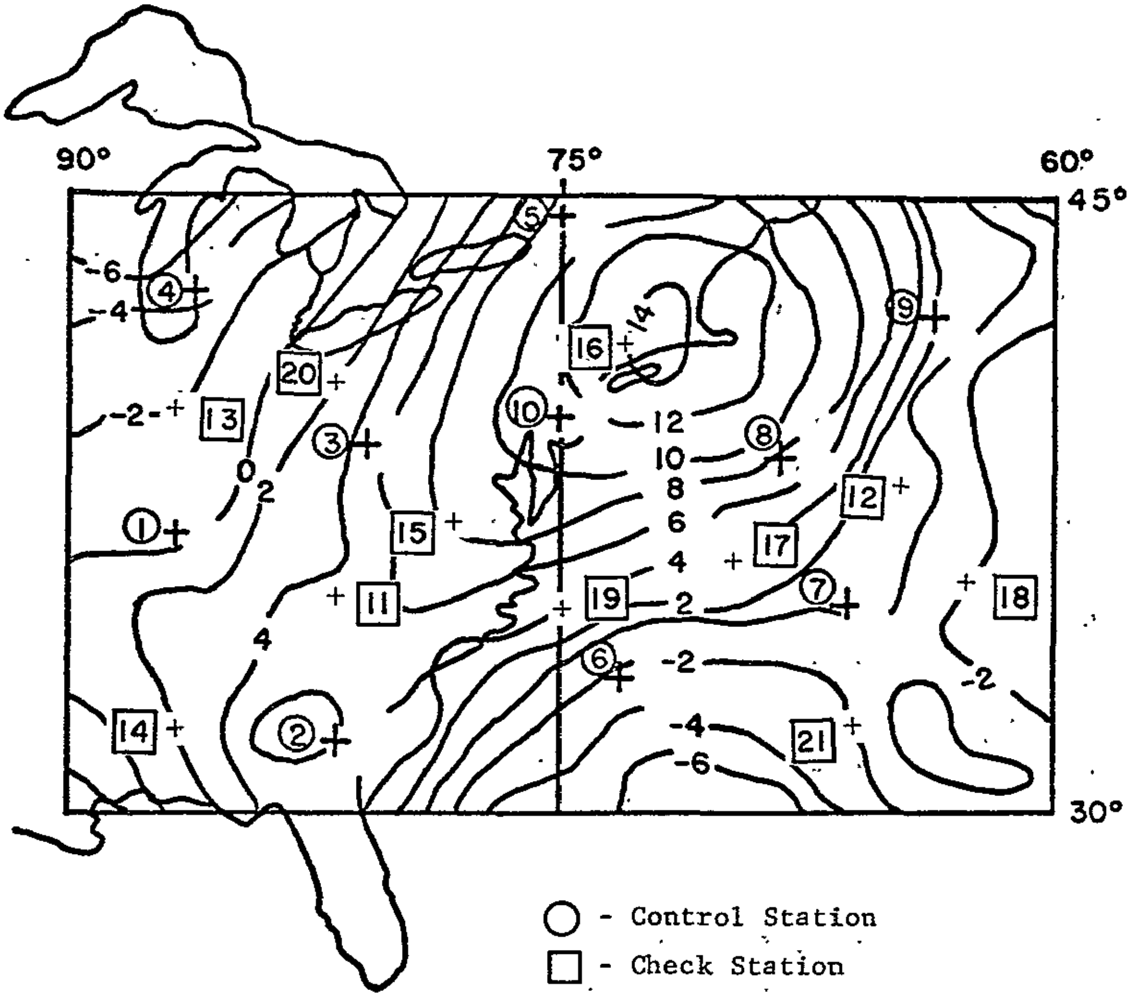
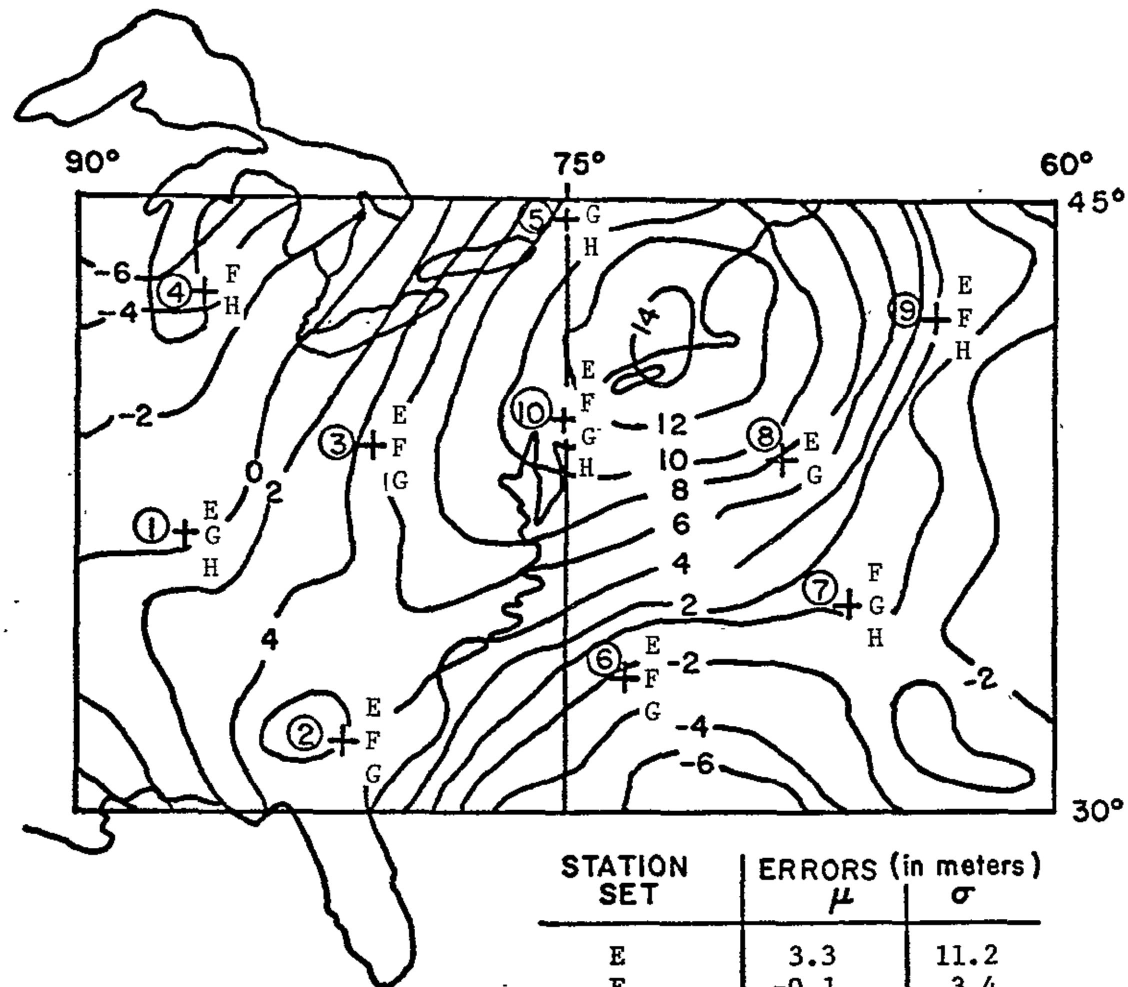
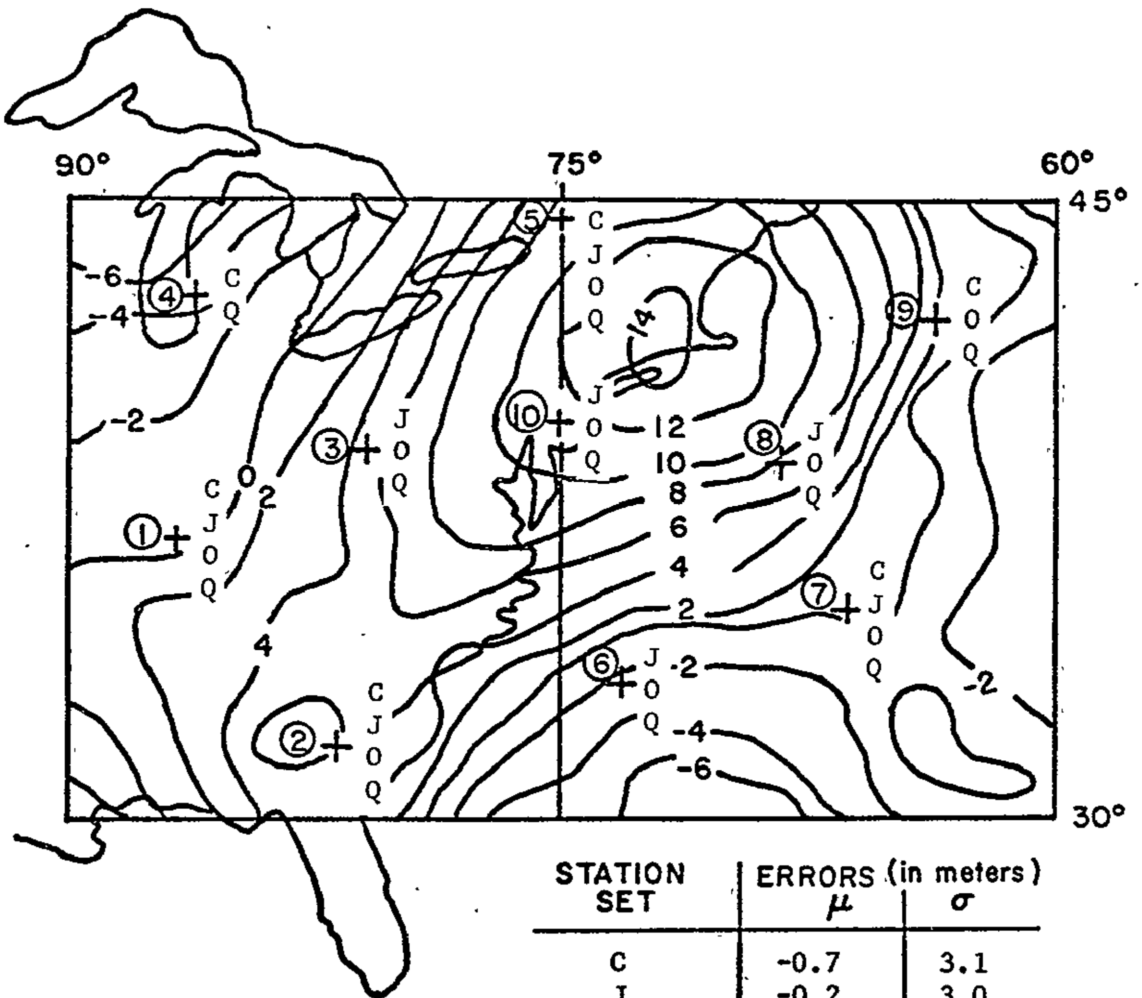


FIGURE 1. TEN CONTROL STATIONS AND ELEVEN CHECK STATIONS



STATION SET	ERRORS (in meters) μ	σ
E	3.3	11.2
F	-0.1	3.4
G	0.2	3.0
H	-0.5	2.7

FIGURE 2. FOUR SETS OF 7 CONTROL STATIONS



STATION SET	ERRORS (in meters) μ	σ
C	-0.7	3.1
J	-0.2	3.0
O	0.3	2.4
Q	0.3	2.4

FIGURE 3. SELECTED SETS OF 6, 8, 9, 10 CONTROL STATIONS

As mentioned earlier, a solution for the coefficients is possible only if the number of the control stations is greater than or equal to six. Six stations will give a unique solution, in which case any erroneous data will certainly distort the rectified geoid model.

Figure 2 indicates that the results of the rectification depend, to some extent, on the particular set of control points even though the number of such points is the same in each set. The reason for any one set of relatively well distributed control stations being better than another is not readily apparent. However, the results are not significantly different except for one set (A) for which there is no apparent reason. Similar results were obtained for sets of 6, 8, 9, and 10 control stations. The results presented on Figures 2 and 3 are summarized in Table 1.

Results in Table 1 show that the rectification improves with the increase in the number of control stations even though the improvement is not very significant for sets of more than 7 or 8 relatively well distributed control stations.

As a brief check on the effect of reducing the size of the test area, a run was made using only the right half of the previously shown test area (Figure 2) considering eight control and ten check stations. Figure 4 shows eight control stations, marked by large dots, and the ten check stations marked by + signs, which were used for this test. The resulting mean error was -0.4 meters and the standard deviation was 2.07 meters. While this is not a substantial improvement over the best results using a set of eight control stations for the entire test area, it is a considerable improvement over the average results of $\overline{|\mu|}$ = 0.8 meters and $\bar{\sigma}$ = 3.8 meters obtained for all (four) sets of eight control stations considered (see Table 1).

TABLE 1. SUMMARY OF RESULTS FROM FIRST TEST SERIES

DATA SET	NO. OF CONTROL STATIONS	CONTROL STATIONS USED 1	2	3	4	5	6	7	8	9	10	RECOVERED ΔN (in meters) μ	σ	MEAN OF THE ABSOLUTE MEAN $\overline{\lvert\mu\rvert}$ (m)	MEAN OF THE STANDARD REVIATION $\overline{\sigma}$ (m)
A	6		X		X		X	X		X	X	-4.4	12.6	2.1	7.3
B	6	X	X	X			X		X	X		3.2	10.7		
C	6	X	X		X	X		X		X		-0.7	3.1		
D	6	X		X		X	X	X	X			0.2	2.8		
E	7	X	X	X			X		X	X	X	3.3	11.2	1.0	5.1
F	7		X	X	X		X	X		X	X	-0.1	3.4		
G	7	X		X		X	X	X	X		X	0.2	3.0		
H	7	X	X		X	X		X		X	X	-0.5	2.7		
I	8	X	X	X		X	X		X	X	X	2.0	6.8	0.8	3.8
J	8	X	X	X		X	X	X	X		X	-0.2	3.0		
K	8	X	X		X	X	X	X		X	X	0.3	2.9		
L	8		X	X	X	X	X	X		X	X	0.6	2.4		
M	9	X	X	X	X	X	X		X	X	X	1.1	4.5	0.5	2.9
N	9		X	X	X	X	X	X	X	X	X	0.3	2.4		
O	9	X	X	X		X	X	X	X	X	X	0.3	2.4		
P	9	X	X	X	X	X		X	X	X	X	0.2	2.4		
Q	10	X	X	X	X	X	X	X	X	X	X	0.3	2.4		

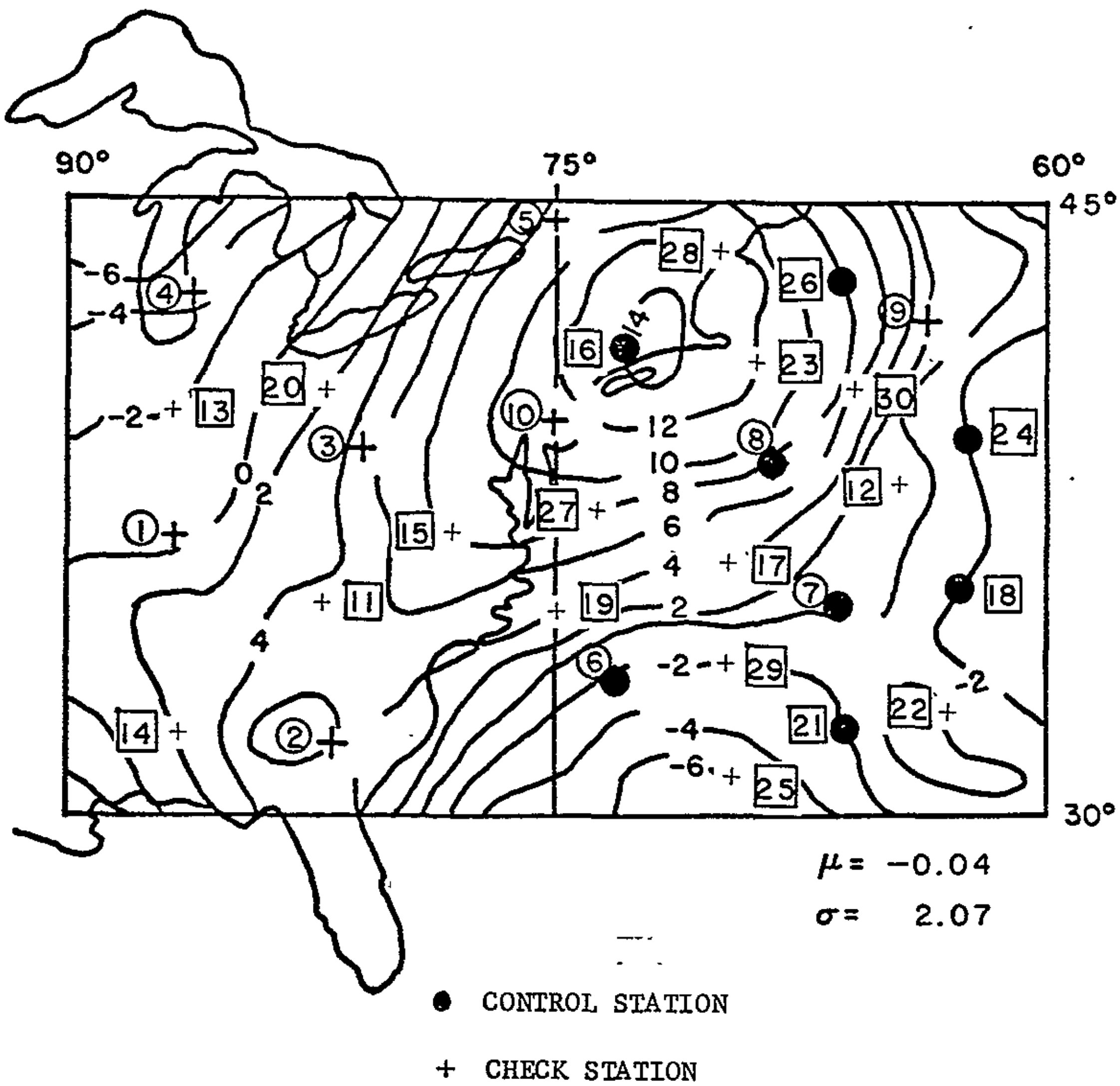


FIGURE 4. EIGHT CONTROL AND TEN CHECK STATIONS IN THE RIGHT HALF OF THE TEST AREA

4.2 Second Test Series

For the second series of tests the area prescribed for GEOS-C calibration was examined. Figure 5 shows the GEOS-C calibration area defined by the triangle whose vertices are Bermuda, Wallops Island and Merritt Island. The large dots indicate the locations of ten check stations selected at random within the primary calibration area. As in the previous tests, the contours on the figure are contours of ΔN, the difference between the undulations of the Vincent, Strange and Marsh, 1972, detailed gravimetric and satellite geoids.

The first set of six control stations selected included Wallops Island, Bermuda, Grand Turk Island, Merritt Island, Eglin Air Force Base and Rosman. These are shown as station set A in Figure 6. The standard deviation, σ, of the recovered ΔN was $\pm$ 3.9 meters. Adding a station at Antigua (station set B) improved the results somewhat as shown by the table in Figure 6. In station set C a control station at sea, in the middle of the main test area, was substituted for Antigua. This, considerably, improved the results.

Based on the logical assumption that better results should be obtained if the control stations were relatively near the test area, the Eglin and Rosman stations were replaced by two coastal stations (9 and 10) and another station (11) at sea was also added in set D (Figure 8). This addition improved, considerably, the results of the rectification (μ = -0.4, σ = 0.61). This last fit involves eight control points and demonstrates that extra control points can significantly improve results by minimizing the effects of points at unfavorable locations. In station set E, Grand Turk Island was eliminated from set D (thus, a set of seven was used); however, the results are improved: μ = 0.14, σ = 0.46.

The configuration in station set F was chosen to represent what might be considered a nearly ideal, but still practical, set of control stations. Eight stations, all in or very close to the primary calibration area and fairly well distributed, were used and the results were quite good (μ = 0.03, σ = 0.41) reinforcing the idea that control stations should be both well distributed and as close as possible to the test area.

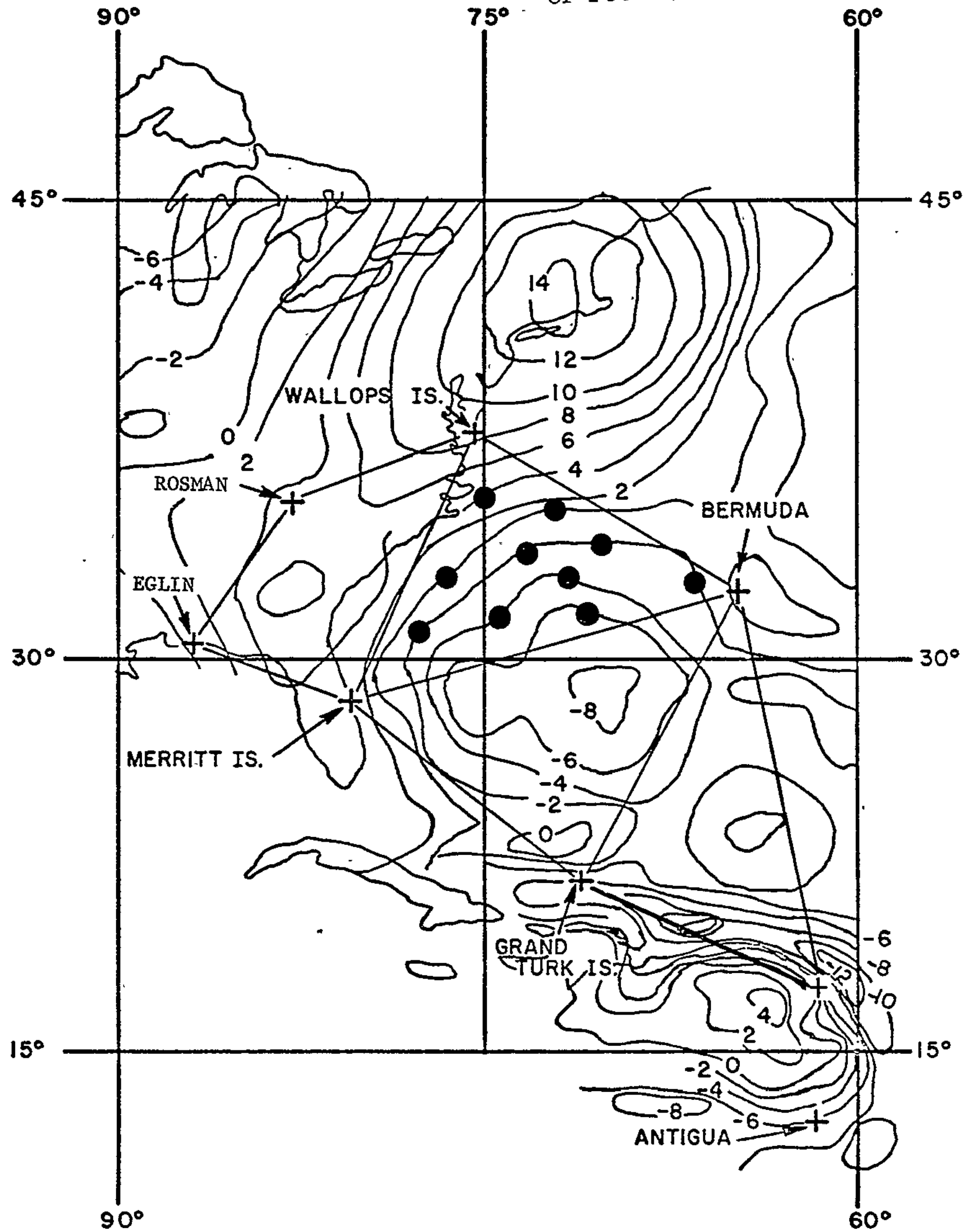
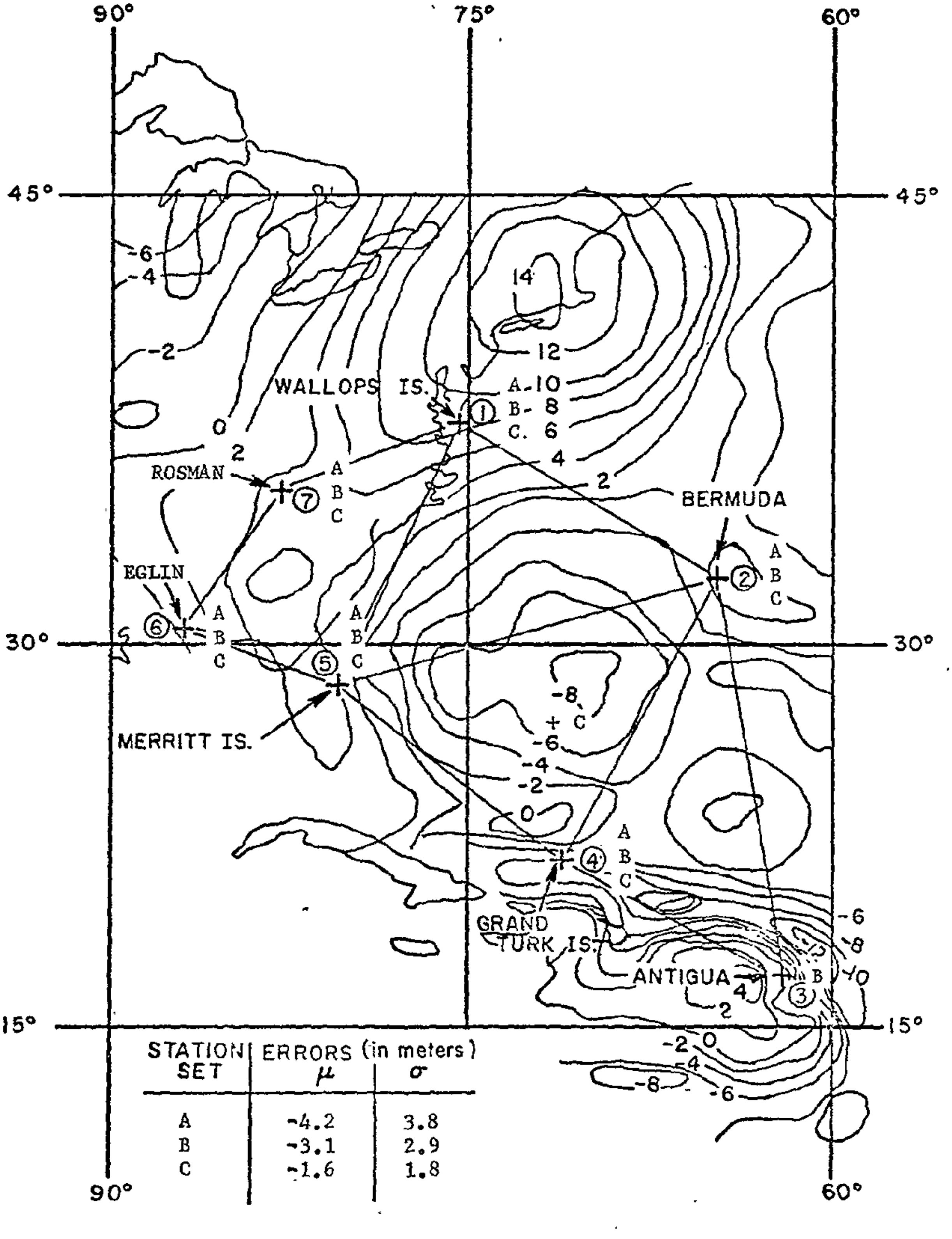
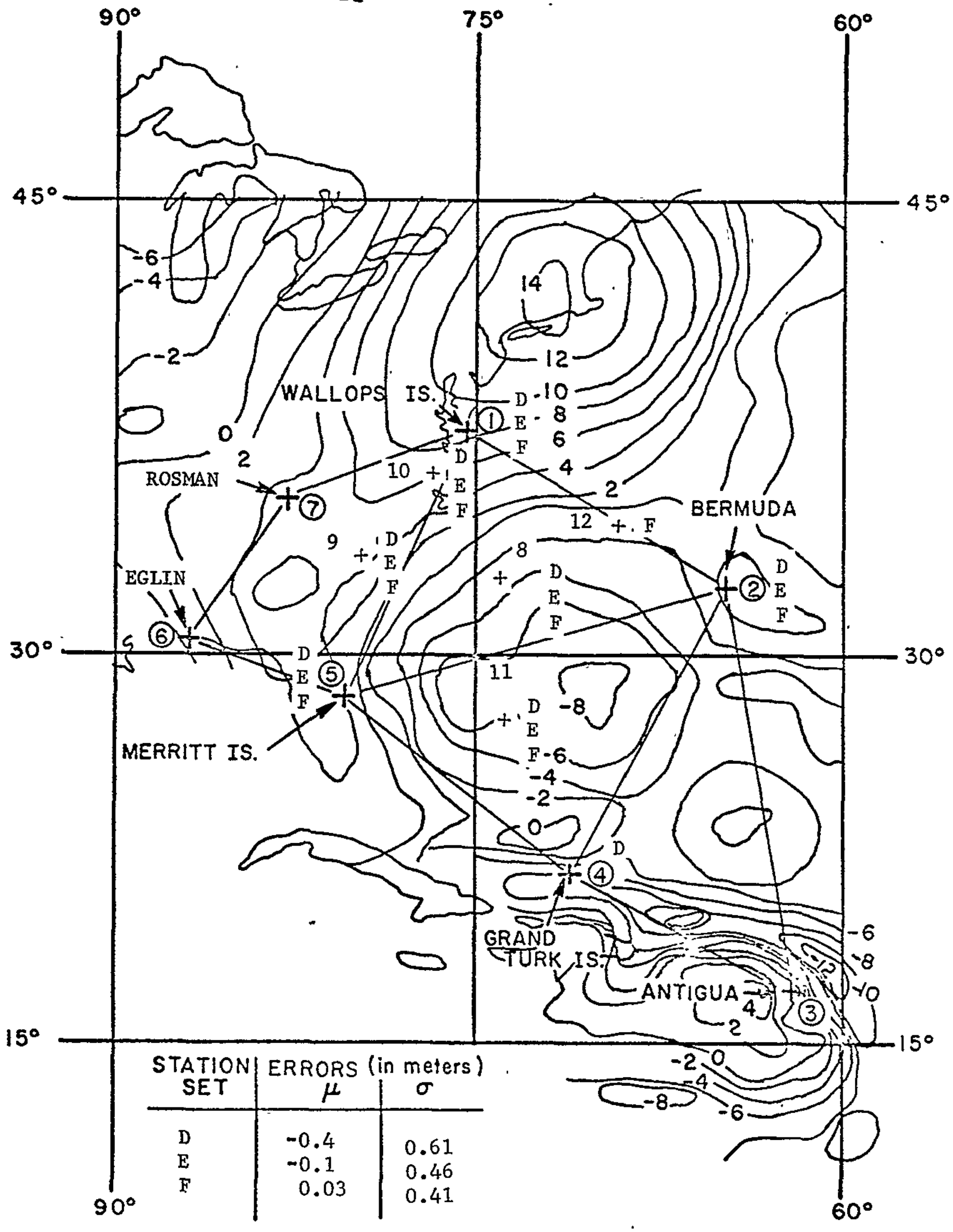


FIGURE 5. TEN CHECK POINTS IN THE GEOS-C PRIMARY TEST AREA



STATION SET	ERRORS (in meters) μ	σ
A	-4.2	3.8
B	-3.1	2.9
C	-1.6	1.8

FIGURE 6. DISTRIBUTION OF THE CONTROL STATIONS FOR SETS A, B, AND C.



STATION SET	ERRORS (in meters) μ	σ
D	-0.4	0.61
E	-0.1	0.46
F	0.03	0.41

FIGURE 7. DISTRIBUTION OF THE CONTROL STATIONS FOR SETS D, E AND F.

4.3 General Conclusions From the Simulation Studies and Comments

Based on the simulation studies conducted so far, the following general conclusions can be drawn.

(1) The mathematical model used is quite representative for the purpose of the proposed rectification considering the accuracy of the geoids used in these studies.

(2) The area in which the rectification is carried out should be as small as possible.

(3) There should be as many control stations as possible, minimum of 6 and preferably 8-9, uniformly distributed within or as close as possible to the area of rectification.

(4) Conclusion (3) emphasizes the importance of adding one or two control stations at sea (in the middle of the test area) to achieve higher accuracy in the rectified geoid.

It must be recognized, however, that the results and conclusions are only as valid as the underlying assumptions made and the data used. For example, the difference between the two geoids used in these simulation studies is due to the higher harmonic (short frequency) geoidal features. The variation of this difference, due to its nature, is very local and is, therefore, very difficult to describe mathematically.

On the other hand, the variation in the difference between the absolute geoid (e.g., computed from the tracking station coordinates) and the detailed gravimetric geoid is expected to be smoother and more systematic. Hence, it can be conjectured that the polynomial representation of this difference in geoids should be more representative. Further tests using more realistic data need to be performed to investigate the correctness of this conjecture. These data must be obtained from satellite tracking station (preferably Geoceiver Stations) coordinates and be derived from dynamic orbit analyses.

5.0 MODIFICATION AND SELECTION OF A SUITABLE MATHEMATICAL MODEL

As mentioned in the last section, the mathematical model used in the simulation studies requires a minimum of 6 and preferably 8-9 control stations. This many tracking stations are not available in the vicinity of the GEOS-C calibration area. Consequently, investigations into possible modifications of the mathematical model are necessary before the actual implementation of the rectification. The modified model must be such that it requires fewer control stations without significant deterioration in the results of rectification. Such modification and the subsequent experiments leading to the selection of the mathematical model to be used in the rectification are presented in this section.

5.1 Modification of the Mathematical Model

The general form of the mathematical model considered to represent the variation of the undulation differences, ΔN, is the second degree polyomial as described in equation (3), which is repeated here for easy reference

$$\Delta N + ax^2 + bx + cxy + dy + ey^2 + k = 0$$

This model will be referred to as Model I in the rest of this report. In this model, the general shape of the contours of ΔN would be elliptical. Since the variation of ΔN in a given area is small, the ellipses can be approximated by circles in which case equation (3) reduces to the form (Model II)

$$\Delta N + A(x^2 + y^2) + Bx + Cy + D = 0 \qquad (43)$$

where A, B, C, and D are the coefficients (constants) of the new polynomial.

A further modification of equation (43) would be to approximate the arcs of circles by straight lines if the curvatures of the arcs are small. In this case, the model reduces to the form (Model III)

$$\Delta N + Bx + Cy + D = 0 \tag{44}$$

Model II requires a minimum of 4 stations while Model III requires a minimum of 3 stations at distinctly different locations. Having formulated the various mathematical models, their relative performance has to be evaluated with realistic data and a selection made for the proposed rectification.

5.2 Tracking Station Data Used for the Investigations

There are 4 sets of (tracking station) coordinates used in these investigations:

(1) Station positions in the Modified Mercury Datum (MMD) as published in NASA Directory (NASA, 1973).

(2) Coordinates of the C-Band radar stations as determined by Krabill and Klosko - WFC/C-Band - (Krabill, et al, 1974).

(3) Coordinates from the OSU-275 net (Mueller, et al, 1974).

(4) Coordinates of the C-Band radar stations as determined by Marsh, Douglas and Walls - GSFC/C-Band - (Marsh, et al, 1974).

The undulations implied by the tracking station coordinates are referred to the same reference ellipsoid (semi-major axis = 6,378,142.0 m, flattening = 1/298.255) as the one to which the detailed gravimetric geoid is referred. Since the OSU-275 system is not geocentric - coordinates of the origin with respect to the geocenter are X = 17 m, Y = 13 m and Z = 1 m (Mueller, 1974) - it was translated to the geocenter before the undulations were computed.

The undulation differences, ΔN, for the various data sets are presented in Table 2. Graphical presentations of the variation of ΔN are given in Figures 8, 9, 10, and 11.

TABLE 2. DISTRIBUTION OF ΔN FOR STATIONS IN THE VARIOUS DATA SETS

Stn. No.	Name	$\Delta N = N_c - N_o$ (meters)			
		MMD	WFC/C-Band	OSU-275	GSFC/C-Band
1126	Rosman	7.7	--	-10.85	-2.8
3648	Hunter	5.8	--	-11.12	--
3861	Homestead	13.5	--	-8.78	--
4061	Antigua	18.6	5.8	3.25	--
4081	Grand Turk	25.0	1.83	-8.79	2.0
4082	Merritt Island	9.7	-1.2	-9.15	-0.6
4740	Bermuda	21.5	-0.2	-12.86	0.6
4760	Bermuda	--	--	-12.10	--
4840	Wallops Island	4.5	-5.0	-10.0	-2.8
4860	Wallops Island	--	--	-10.0	--
--	Grand Bahama	14.6	--	--	--
--	Eglin AFB	9.5	--	--	--

The variations of ΔN for MMD are quite large (4.5m - 25m) and somewhat random; far from the nature of the ΔNs expected for this type of data. Consequently, these data have not been used in any of the investigations considered here. The magnitude of ΔN for both the C-Band data is considerably smaller than the other two (MMD and OSU-275). This is probably due to the fact that the C-Band data, as well as the detailed gravimetric geoid, use the same gravity model and coordinate system (GEM6). From Figure 9, the WFC/C-Band data indicate that ΔN for Wallops Island and Antiqua are relatively larger and exhibit a slope in the Northwest-Southeast direction with respect to the detailed gravimetric geoid. The GSFC/C-Band data also display a similar pattern except that the slope is smaller.

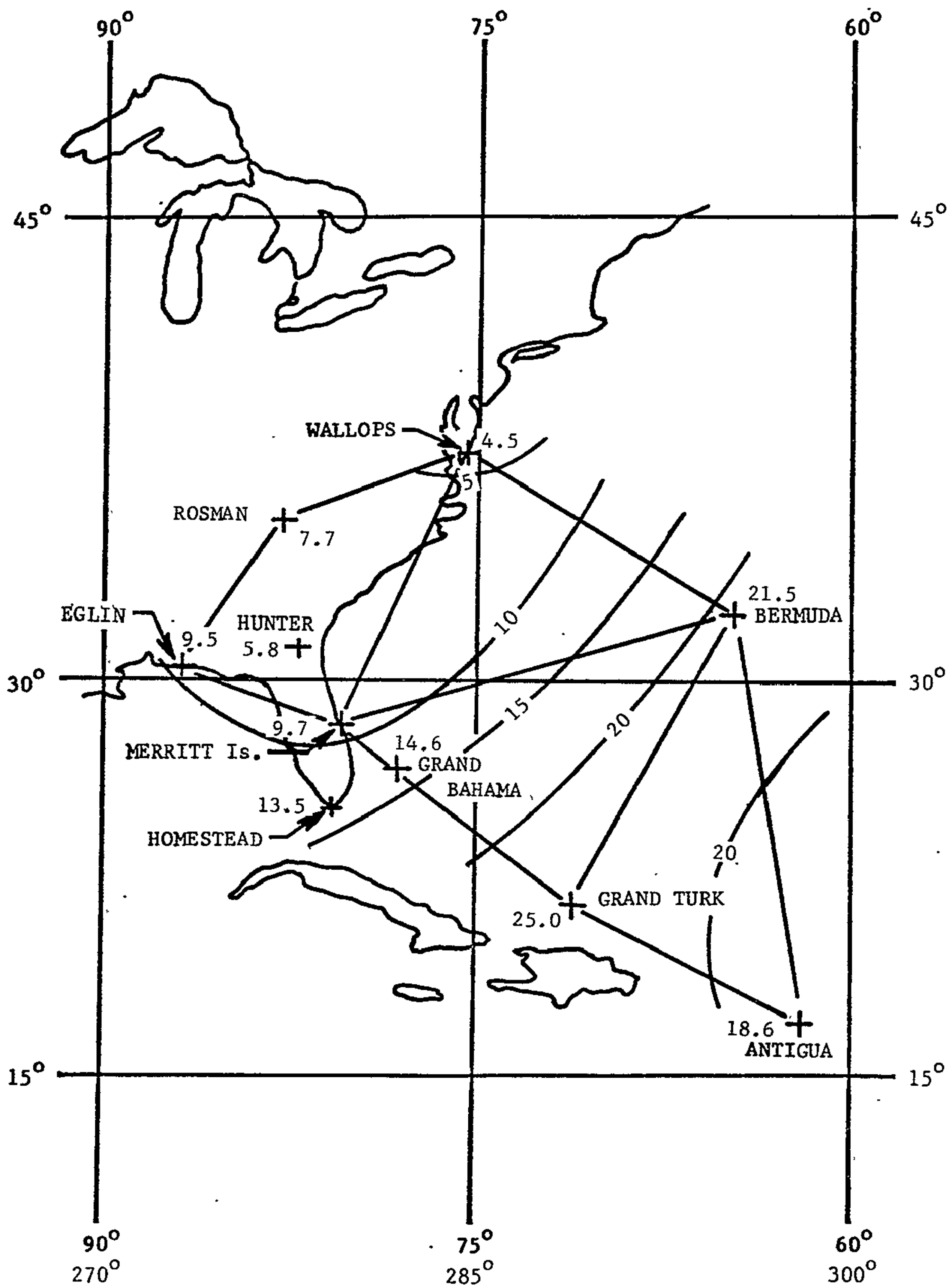


FIGURE 8. THE VARIATION OF ΔN FOR MODIFIED MERCURY DATUM DATA (meters)

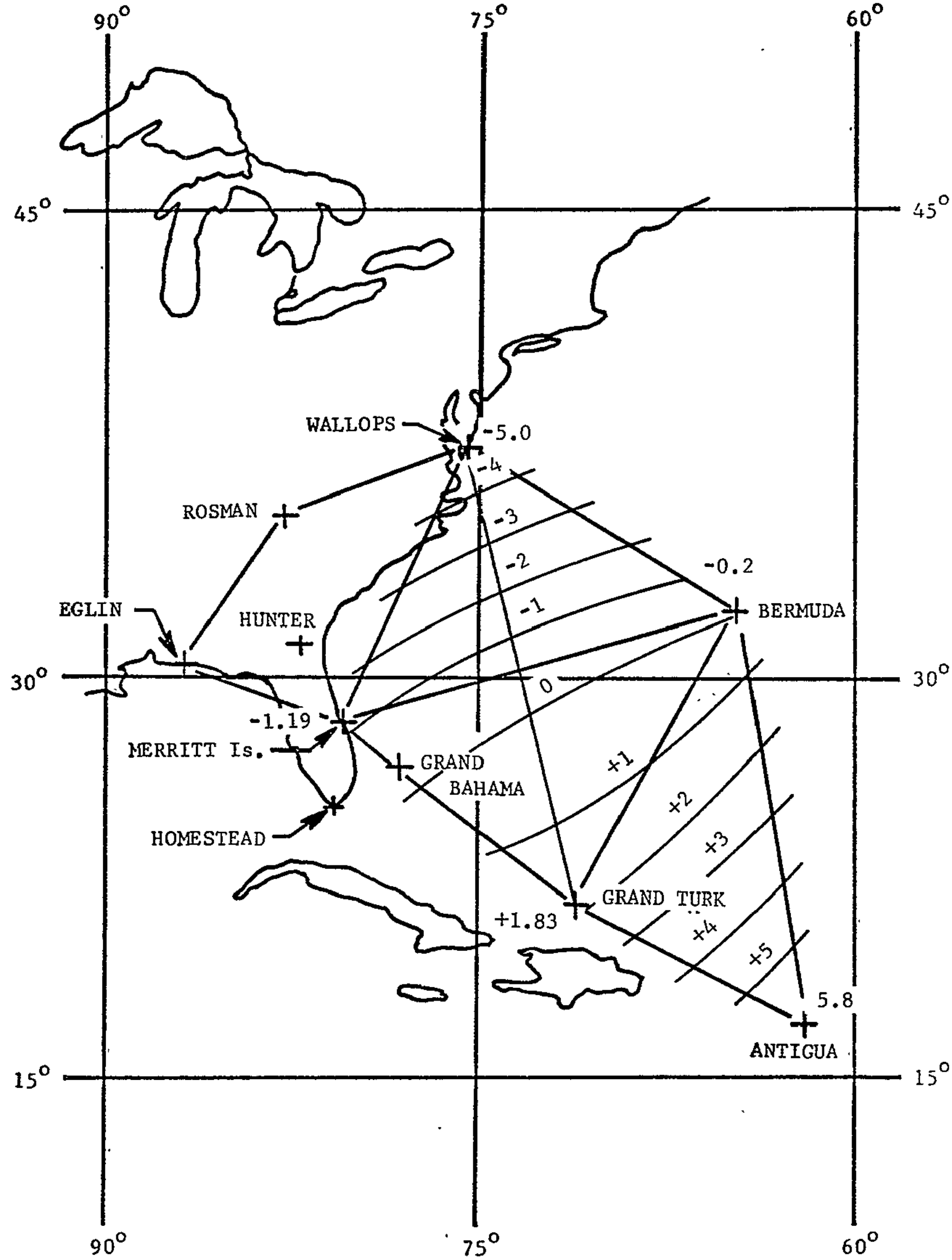


FIGURE 9. THE VARIATION OF ΔN FOR WFC/C-BAND DATA (meters)

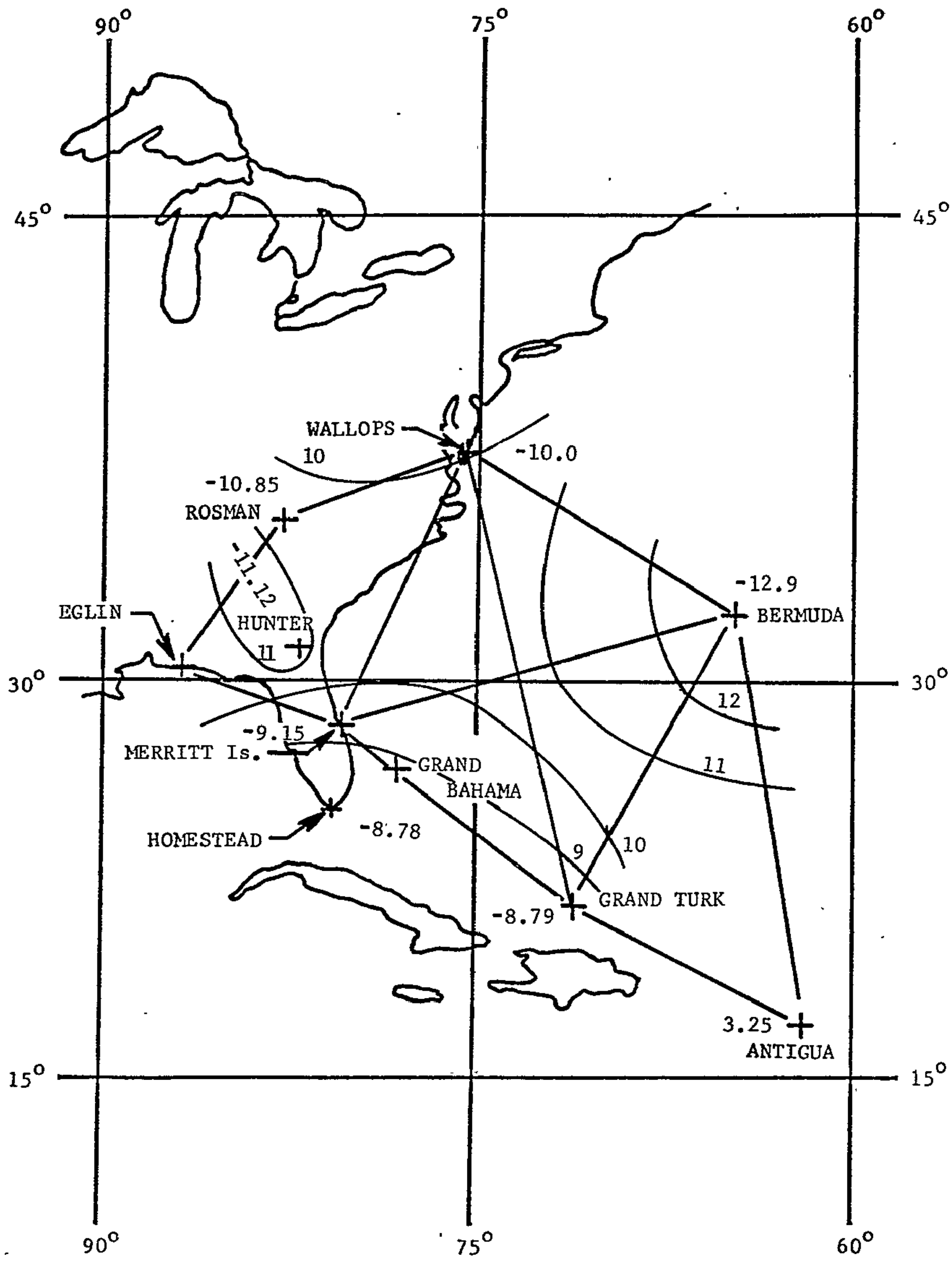


FIGURE 10. THE VARIATION OF ΔN FOR OSU-275 DATA (meters)

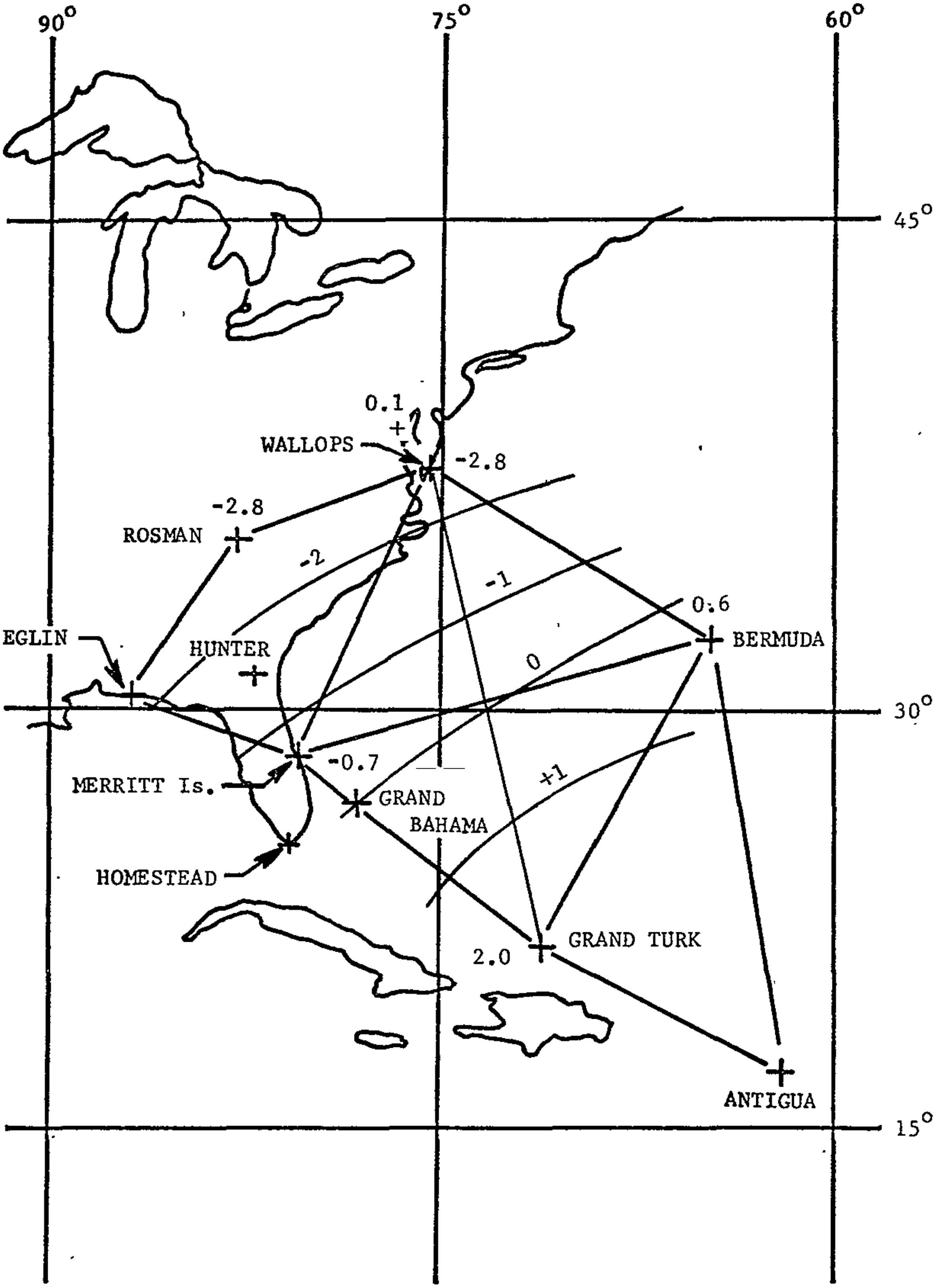


FIGURE 11. THE VARIATION OF ΔN FOR GSFC/C-BAND DATA (meters)

The variations of ΔN in the OSU-275 data are very small with the exception of Antigua. These data also exhibit (Figure 10) a general slope in the Northeast-Southwest direction. The dominant difference between the geoid implied by the OSU-275 data and the detailed gravimetric geoid is the scale. In the case of the OSU-275 data, the scale is introduced using a combination of SECOR, C-Band radar observations, electronic distance measurements and station heights as weighted constraints.

In all the data sets considered, the magnitude of the ΔN for Antigua is larger than for the other stations. This may be due to its location being on a large geoidal slope and the resulting inaccuracy of the undulations as computed by Marsh-Vincent using mean anomalies of $1^{\circ} \times 1^{\circ}$ blocks which are not sensitive enough to reflect the local geoidal features. Wallops Island also exhibits similar, but relatively smaller, undulations for which no explanation can be found at this point. However, the number of stations in the C-Band data is too small to make any firm conclusions as to how good these data are.

The indications are that all data, except MMD, considered here are realistic. The slope exhibited by the WFC/C-Band data is excessive considering the fact that both geoids involved are oriented using dynamic mode of data analyses. On the other hand, the GSFC data are quite close to an ideal case. However, the number of stations available in both C-Band data sets is too limited for any investigation into the selection of the mathematical model. Even though the variations of ΔN in the OSU-275 data are not very systematic or smooth, they are well within the accuracy level of the station coordinates. Furthermore, the number of stations in these data are sufficient to permit detailed investigations into the model selection. Consequently, the OSU-275 data are used in the test leading to the selection of the mathematical model for the proposed rectification of the detailed gravimetric geoid. However, it is proposed to rectify the gravimetric geoid with respect to all three - WFC/C-Band, OSU-275, GSFC/C-Band - sets of data.

5.3 Comparison of the Mathematical Models

As stated earlier, the OSU-275 data have been used in a series of tests to establish criteria on which the performance of each model can be compared for the selection of a model for the proposed rectification of geoids. It is also proposed to test the selected model with the C-Band data and to compare the results with those of OSU-275 data.

Of the 10 stations given in the OSU-275 data, there are two stations each at Wallops Island and Bermuda. These stations are not far enough apart to play an independent role in determining the coefficients in the polynomials assumed. Therefore, the following 8 stations are used in the proposed tests.

Rosman (1126)
Hunter (3648)
Homestead (3861)
Antigua (4061)
Grand Turk (4081)
Merritt Island (4082)
Bermuda (4740)
Wallops Island (4840)

The performance of each model depends on how well each describes the behaviour of the point values of ΔN given at the tracking stations and also on how well the ΔNs at other stations are predicted. Let the undulation differences computed by using the coefficients determined in the least squares fit of the polynomial to the given data be ΔN^1. Then, $e = (\Delta N - \Delta N^1)$ would be the error in representing ΔN by ΔN^1. The root mean square value (σ) of e for the stations included in the least squares adjustment can be a measure of the fit of the assumed mathematical model to the given data. On the other hand, σ for the stations that are not involved in the adjustment would be a measure of the quality of rectification.

The following tests have been performed in order to select an appropriate mathematical model which optimizes the errors of rectification and the required number of tracking stations.

(1) All eight stations are included in the determination of the coefficients of the polynomials by least squares adjustment.

(2) In order to compute the error of rectification, ΔN^1 has to be evaluated at a station not included in the adjustment. For this purpose, Hunter which is located within the area bounded by the tracking stations, is left out of the adjustment while the other seven are left in. In this test, e for Hunter would be the error of rectification.

(3) Since the ΔN at Antigua is suspect and also this station is remote from the area of calibration, Test (2) is repeated with Antigua also left out.

(4) Rosman and Antigua stations are left out due to their remoteness from the calibration area. Adjustment is performed with the data at Hunter, Homestead, Grand Turk, Bermuda and Wallops Island while the error of rectification is checked at Merritt Island. This test is carried out with Models II and III only, since the problem is under determined with Model I (5 stations and 6 unknowns).

(5) In this test, only those stations (Hunter, Merritt Island, Grand Turk, Bermuda and Wallops Island) which are within or very near the calibration area are used in the adjustment. Only the error of fit is computed in this test which is performed only with Model II.

The results of these tests are presented in Table 3. Tests (1)-(3) are intended to contribute directly to the selection of the appropriate mathematrical model while Tests (4) and (5) will show the improvement in the performance of the scheduled model when the area bounded by the tracking stations is narrowed down to the calibration area.

Examination of the results, in Table 3, indicate:

(1) There is no significant difference in the results for Models I and II but the results deteriorate significantly for Model III.

(2) The selection of the station at Hunter for rectification appears poor. For instance, if the check station had been selected near a control station which had a poor fit (maximum error of fit was 2.8, 4.7, 5.8 m for Models I, II, and III respectively) the results would have been just the opposite. However, the errors of rectification are within the accuracy level of tracking station coordinates (2m). Therefore, no meaningful conclusions can be drawn from the results of rectification at Hunter.

TABLE 3. COMPARISON OF THE MATHEMATICAL MODELS

	Rms fit (σ m)					ERROR OF RECOVERY (m)		
	Test 1	Test 2	Test 3	Test 4	Test 5	Test 2 (Hunter)	Test 3 (Hunter)	Test 4 (M.I.)
Model I	1.5	1.7	0.0	--	--	-1.5	1.3	--
Model II	1.8	2.2	0.2	0.4	0.4	-0.6	1.2	-0.7
Model III	3.0	3.4	0.6	0.6	--	0.2	0.8	-1.0

(3) Model II fits and rectifies satisfactorily on or very near the area of calibration.

However, lack of sufficient number of tracking stations uniformly distributed within the calibration area limits further testing in support of these conclusions.

Under the circumstances, in view of the satisfactory performance of Model II with the available data, this model is selected for the proposed rectification of the detailed gravimetric geoid.

Performance of Model II with C-Band Data

The following tracking stations with C-Band data have been used, with Model II.

WFC	GSFC
Grand Turk	Grand Turk
Merritt Island	Merritt Island
Bermuda	Bermuda
Wallops Island	Wallops Island
Antigua	Rosman

Polynomial fits have been done on two sets of stations for WFC data

(1) All stations

(2) All stations except Antigua.

In the first fit, σ was 0.4 m. The second fit is, of course, perfect since the degrees of freedom is reduced to zero. The error of rectification (in the 2^{nd} fit) at Antigua is 2.6m. This error is large and may be due to

i. the inaccuracy of ΔN at this station,

ii. this station being remote from the area of calibration, or

iii. the extrapolation of the polynomial.

On the other hand, five stations fit with GSFC data was excellent ($\sigma = 0.0$). Even though the coordinates for Greenbelt, Maryland were provided in the GSFC data, they were not used in this investigation as the station height was derived from the detailed gravimetric geoid (Marsh, et al, 1974). Unfortunately, due to lack of additional stations within the calibration area, it is not possible to make any further studies on how well the rectification can be done with the selected model and the C-Band data.

6.0 GEOID RECTIFICATION AND ANALYSIS

The objective of this investigation is to provide a detailed geoid, correct in scale, orientation, and shape, that can satisfy the ground truth needs relative to the GEOS-C mission with respect to (1) instrument calibration, (2) ocean geoid determination from satellite altimetry, and (3) computation of quasi-stationary departures from the ocean geoid. None of the existing geoids conform to these requirements (Decker, 1972). It has been shown that repeated Doppler and C-Band radar observations at a number of stations, adjusted in a dynamic mode, can result in the geoidal undulations at the stations being accurate to about 2 meters (Anderle, 1971a, 1971b, Schwarz, 1972, Krabill, et al, 1974). Any available detailed geoid can be rectified to be compatible with Doppler tracking station coordinates. The degree of compatibility depends on the accuracy of the tracking station coordinates. Consequently, the best available detailed geoid, which is the Marsh-Vincent geoid of March, 1974, is used for the rectification.

There are no Doppler station coordinates available at the present time for all stations in the calibration area. The three sets of available data - WFC/C-Band, OSU-275, GSFC/C-Band, - described in the previous section, are considered to be close to being the data from Doppler coordinates. However, all three sets are significantly different from each other. Since it is not possible to determine which is the best set, all three are used to rectify the detailed geoid.

Rectification is done in an almost square area of 26 degrees in latitude (from 19^{o}N to 45^{o}N) and 24 degrees in longitude (274^{o}E to 298^{o}E). This area includes the entire calibration area.

The input data from the detailed gravimetric geoid consists of a set of undulations (computed) at the centers of each 1^{o} X 1^{o} block in the area of rectification. These data are in no sense mean values for these blocks, but are point values, except that they do not contain the harmonic components higher than that represented by 1° X 1° mean anomalies. The reason for this is that the minimum size of anomaly blocks used in computing these undulations is 1° X 1° (Marsh, et al, 1974). Figure 12 shows the variations of the 1° X 1° undulations within the selected area in the form of contours. The numerical values of this 1° X 1° grid of undulation are presented in Table 4.

The undulation differences, ΔN, computed at the centers of the 1° X 1° blocks, using the polynomial in equation (3) for the three data sets are presented in Tables 5, 6, and 7. Graphical representations of these differences appear on Figures 13, 14, and 15. These are called the "difference geoids" since these are differences between the detailed gravimetric geoid and the geoids implied by the tracking station coordinates. It would be instructive to compare these difference geoids to those shown in Figures 9, 10, and 11, where the contours are hand-sketched. In the case of both C-Band data sets, the differences are small. However, in the case of the OSU-275 data the differences are significant. The reason for this is evident from a close examination of Figure 10. Even though the magnitudes of ΔN at Rosman and Hunter are relatively large, their effects in the determination of the polynomial are minimized by the close proximity of the other stations (Merritt Island, Homestead, and Wallops Island). On the other hand, due to the remoteness of Bermuda, the large ΔN at Bermuda has a dominant influence in the determination of the polynomial. However, the difference between the two different geoids (Figures 10 and 14) for the OSU-275 data are within the accuracy level of the tracking station coordinates. It must be emphasized that the preceeding statement is true only for the general area bounded by the tracking stations considered here. A general point of interest concerning the shape of the contours as shown on Figures 13, 14, and 15 is the shape of the contours of ΔN. The deviation of these curves from being circular is due to the convergence of the meridians on the earth. In other words, it is due to the contour map being a square projection of the area from a spheroidal earth.

The rectified geoids - the sum of the difference geoid and the detailed gravimetric geoid - corresponding to the three sets of tracking station data, are presented in Tables 8, 9, and 10. Their corresponding contour maps appear on Figures 16, 17, and 18. Comparing these geoids with the detailed gravimetric geoid, it is noted that the distortions in the difference geoid corresponding to the OSU-275 data have significant effects in the rectified geoid. While the general shape of the contours of the undulations (N) remains the same in most part of the area of rectification, there is significant distortion in the Northwestern quarter of the area. However, this area falls outside the main GEOS-C calibration area.

Having obtained the rectified geoids, how does one make sure of the quality of the resultant rectification? The accuracy expressed by σ (equation 43) of the polynomial fit to the control station data can be considered a measure of the quality of rectification at or very near the control stations. However, a low value for σ does not guarantee good rectification in the center of the calibration area where there is no control station available. Therefore, it would be instructional to compare the two geoids - the absolute geoid as given by the tracking station coordinates and the rectified geoid - along profiles running through the area.

The difference, ΔN, in undulations are defined only at the control stations. The difference geoid between stations is defined by linear interpolation technique and sketching contours of ΔN. This procedure provides the basis for comparing the absolute geoid with the rectified one along any given profile within the calibration area.

Four profiles have been chosen to verify the quality of rectification, Wallops - Bermuda, Wallops - Merritt Island, Wallops - Grand Turk, and Merritt Island - Bermuda. These profiles, for each of the data sets used, are presented in Figures 19, 20, and 21 where the profiles on the original geoid are shown by "+ - + - +" and those on the rectified geoid are shown by "o - o - o".

For the WFC/C-Band data, the agreement between the geoids along the selected profiles is very good. The largest deviation of about 0.5 m occurs near Grand Turk. Comparison of these geoids for the OSU-275 data (Figure 20) is relatively poor. For the most part, the deviation along the profiles is about 0.5 m. Figure 21 indicates that the agreement between the two geoids is excellent for the GSFC/C-Band data, with a maximum deviation of about 0.2 m.

Some of the general conclusions that could be drawn from the above results and analysis are as follows:

(1) The procedure and the mathematical model used in the rectification are quite valid. However, a more general model will improve the results provided there are a sufficient number of tracking stations in the area.

(2) Results of the OSU-275 data emphasize the importance of having the tracking stations uniformly distributed in the area and sufficiently close enough to permit reasonably accurate interpolation between them.

(3) Significant differences among the tracking station data indicate the inaccuracy and inconsistency among these data. What is required is a unique set of tracking station data which will result in a set of absolute geoid undulations. One approach would be to obtain a unique set of data referred to a common datum which is determined by subjecting all the available data to some form of a regression analysis, for example, Least Squares Adjustment. However, such a procedure would be meaningful only if there is sufficient number of tracking stations (at least 10) distributed evenly around the earth in each of the data sets. This requirement is not even partially met with the available data.

It is, therefore, recommended that sufficient Doppler data at all the tracking stations in the vicinity of the calibration area, be collected during the first phase of the GEOS-C data collection and that a new set of coordinates be determined for these stations from a dynamic mode of data analysis. These data could then be used to rectify the detailed gravimetric geoid to give a unique and detailed geoid for the altimetry calibration purposes.

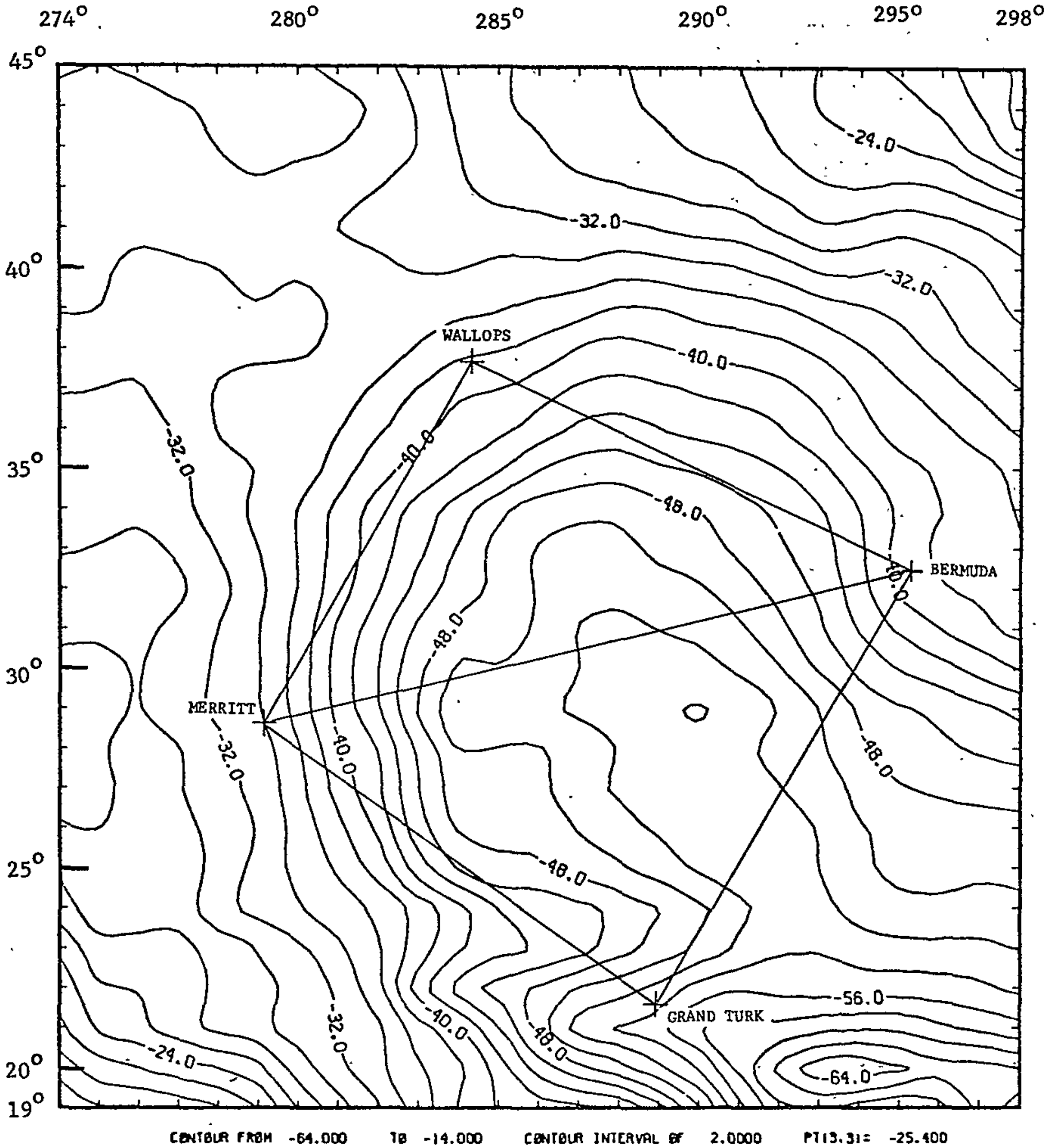


FIGURE 12. DETAILED GRAVIMETRIC GEOID (meters)

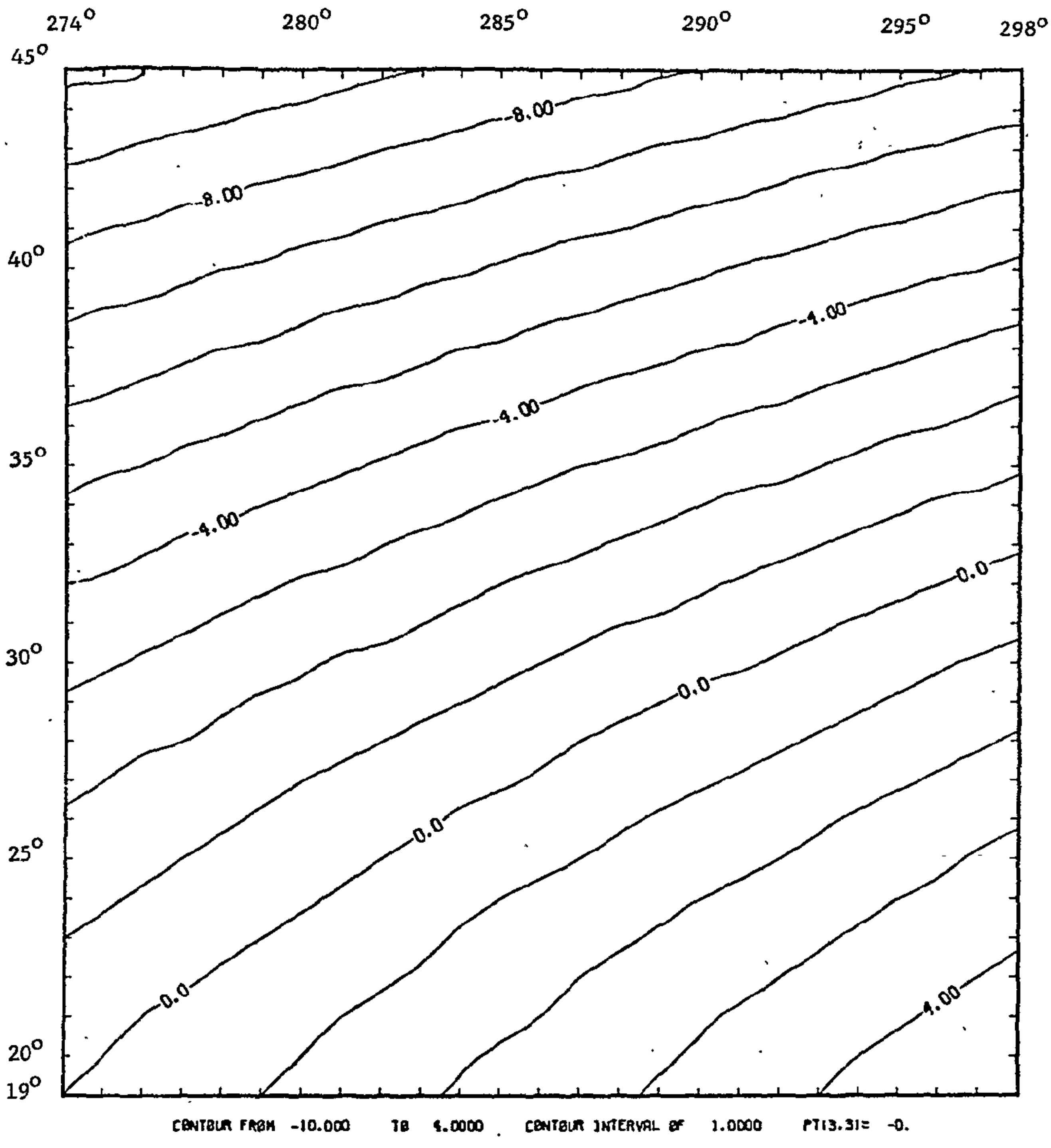


FIGURE 13. DIFFERENCE GEOID FOR WFC/C-BAND DATA (meters)

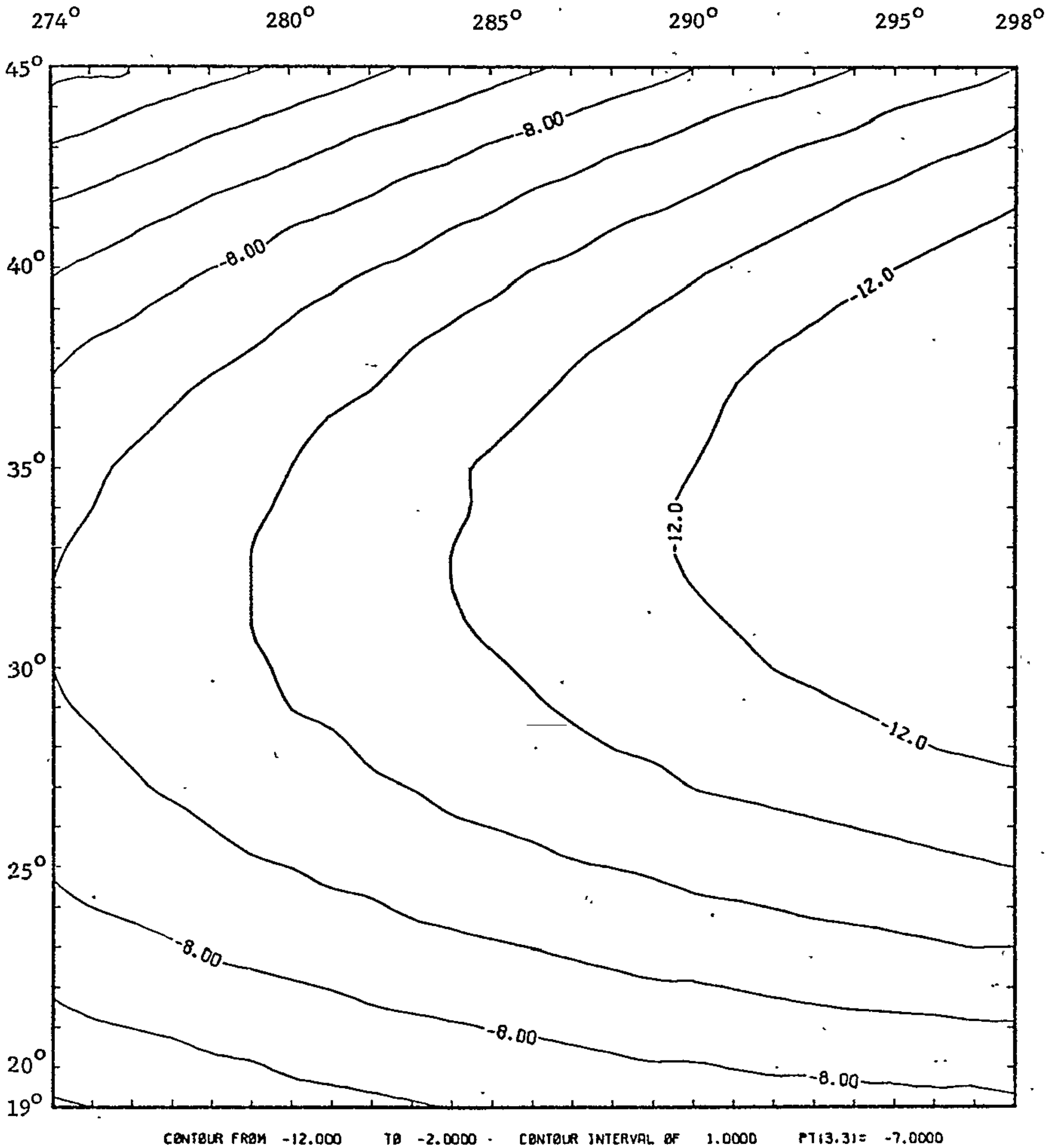


FIGURE 14. DIFFERENCE GEOID FOR OSU-275 DATA (meters)

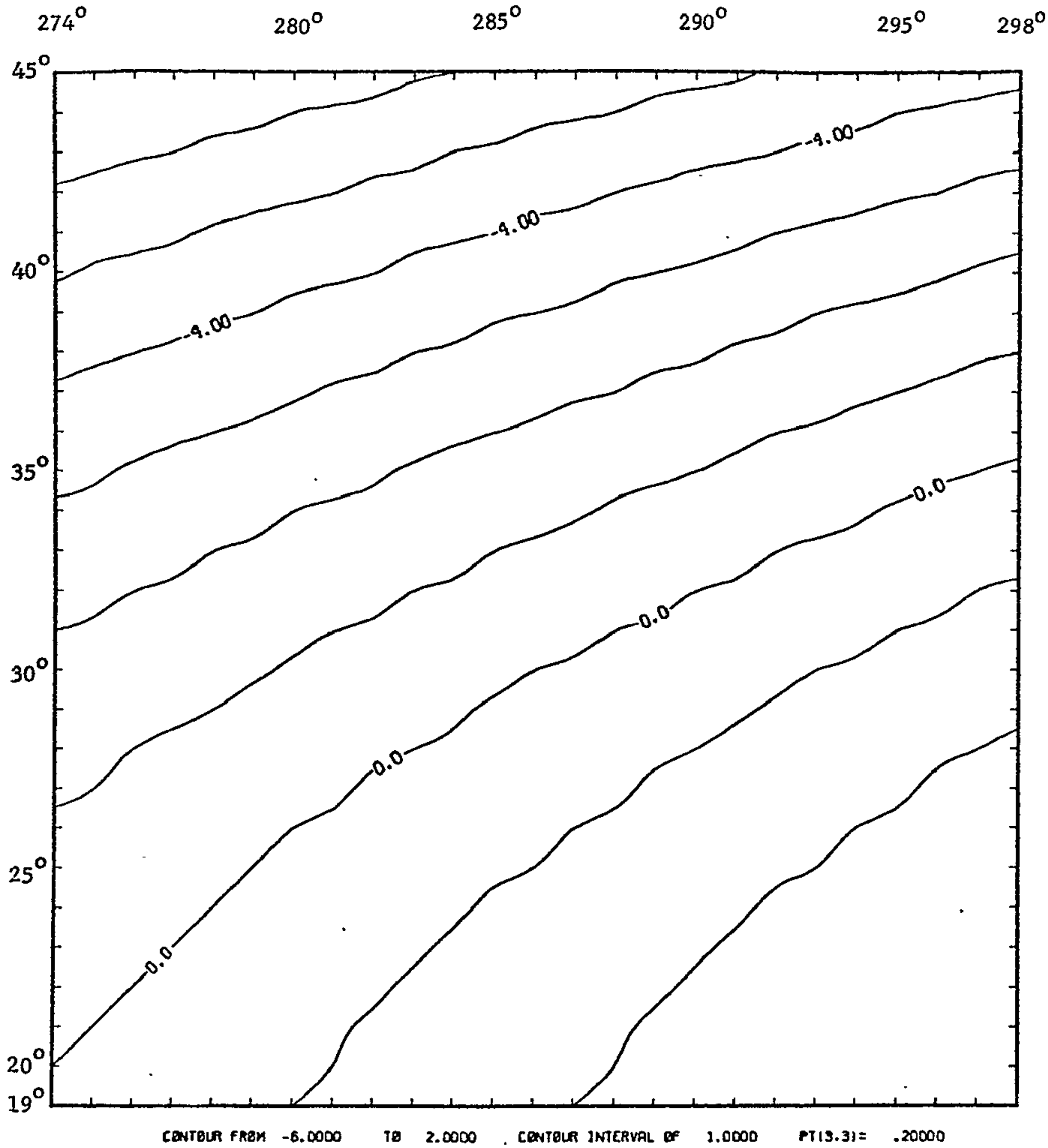


FIGURE 15. DIFFERENCE GEOID FOR GSFC/C-BAND DATA (meters)

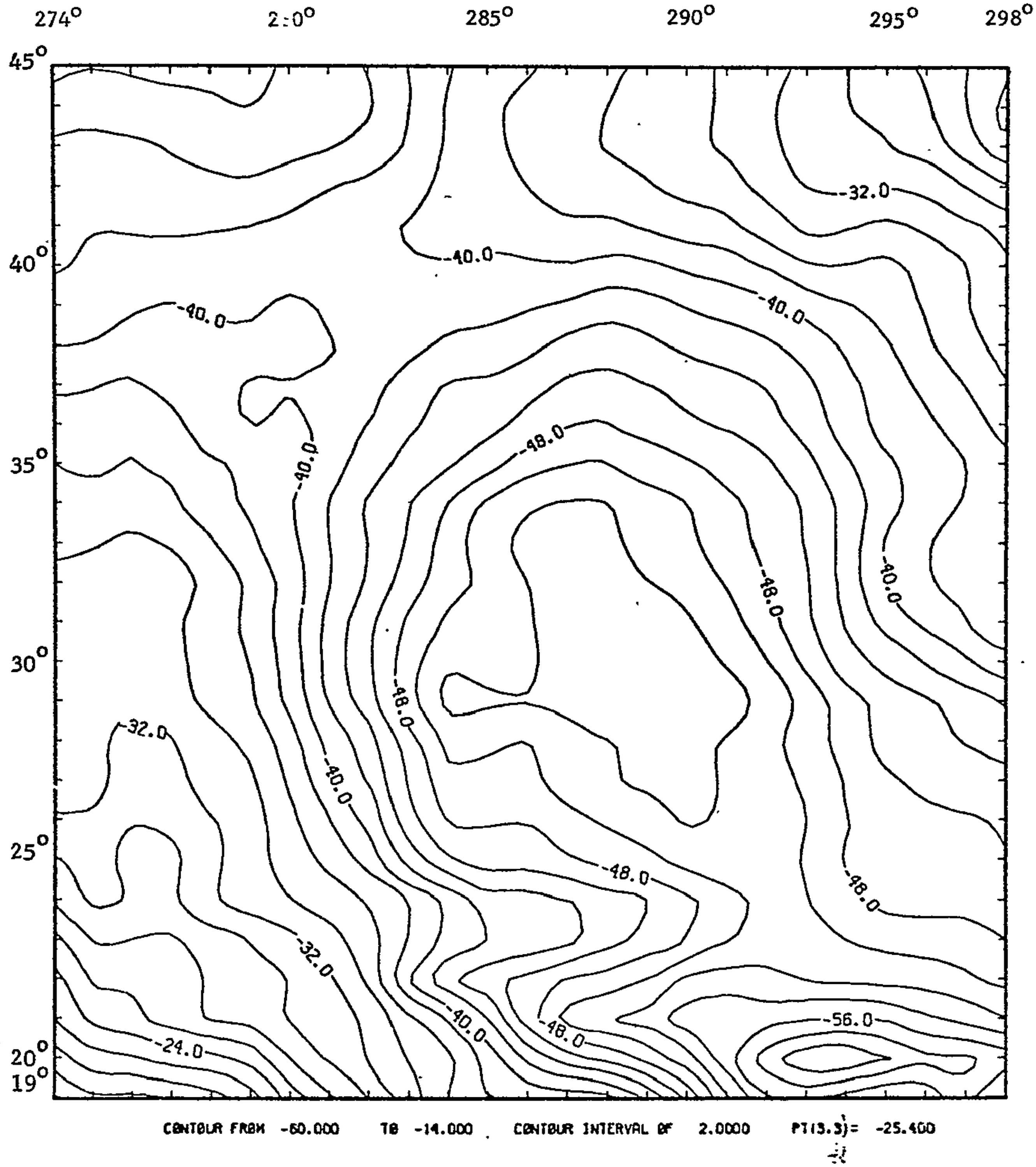


FIGURE 16. RECTIFIED GEOID FOR WFC/C-BAND DATA (meters)

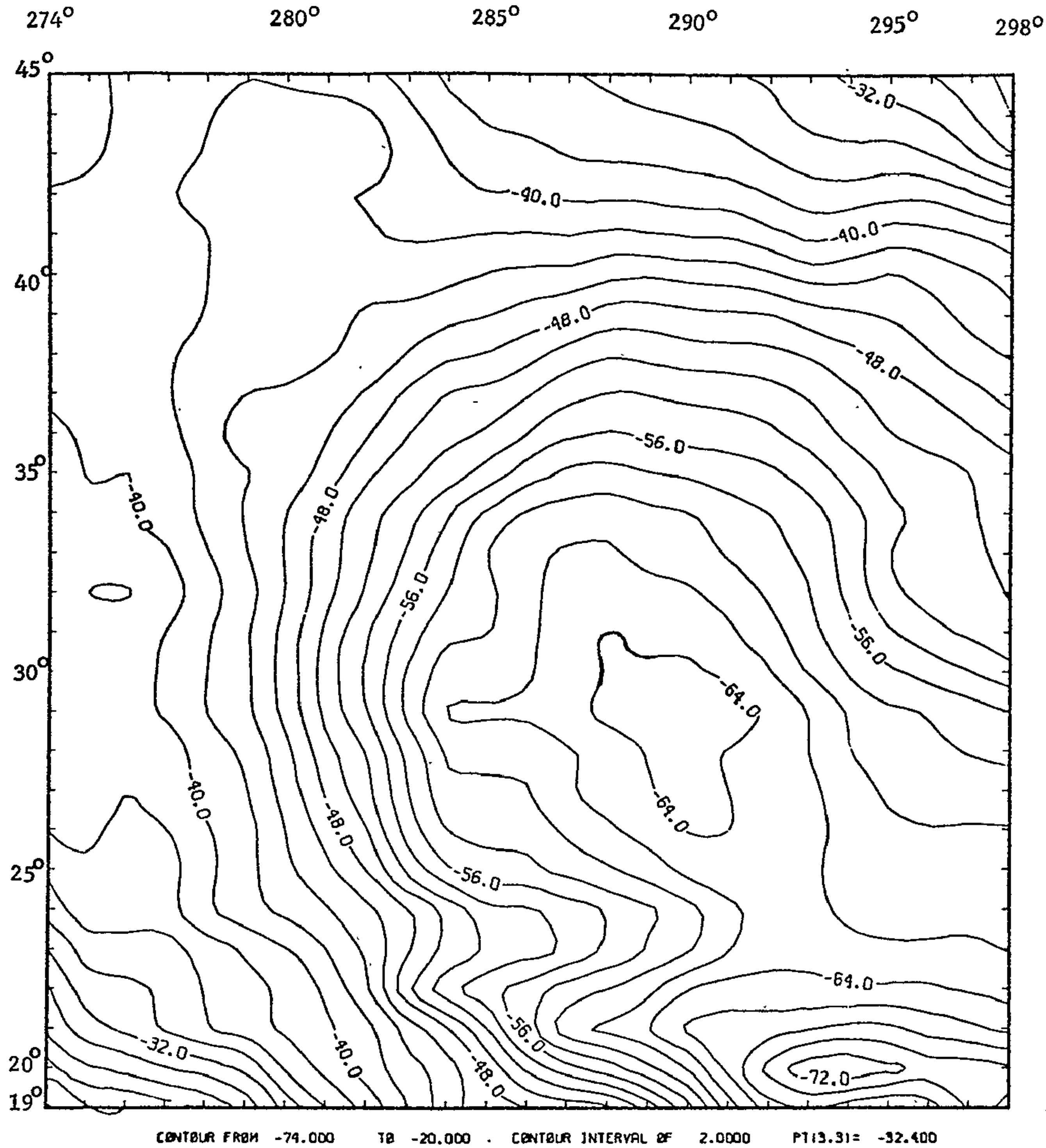


FIGURE 17. RECTIFIED GEOID FOR OSU-275 DATA (meters)

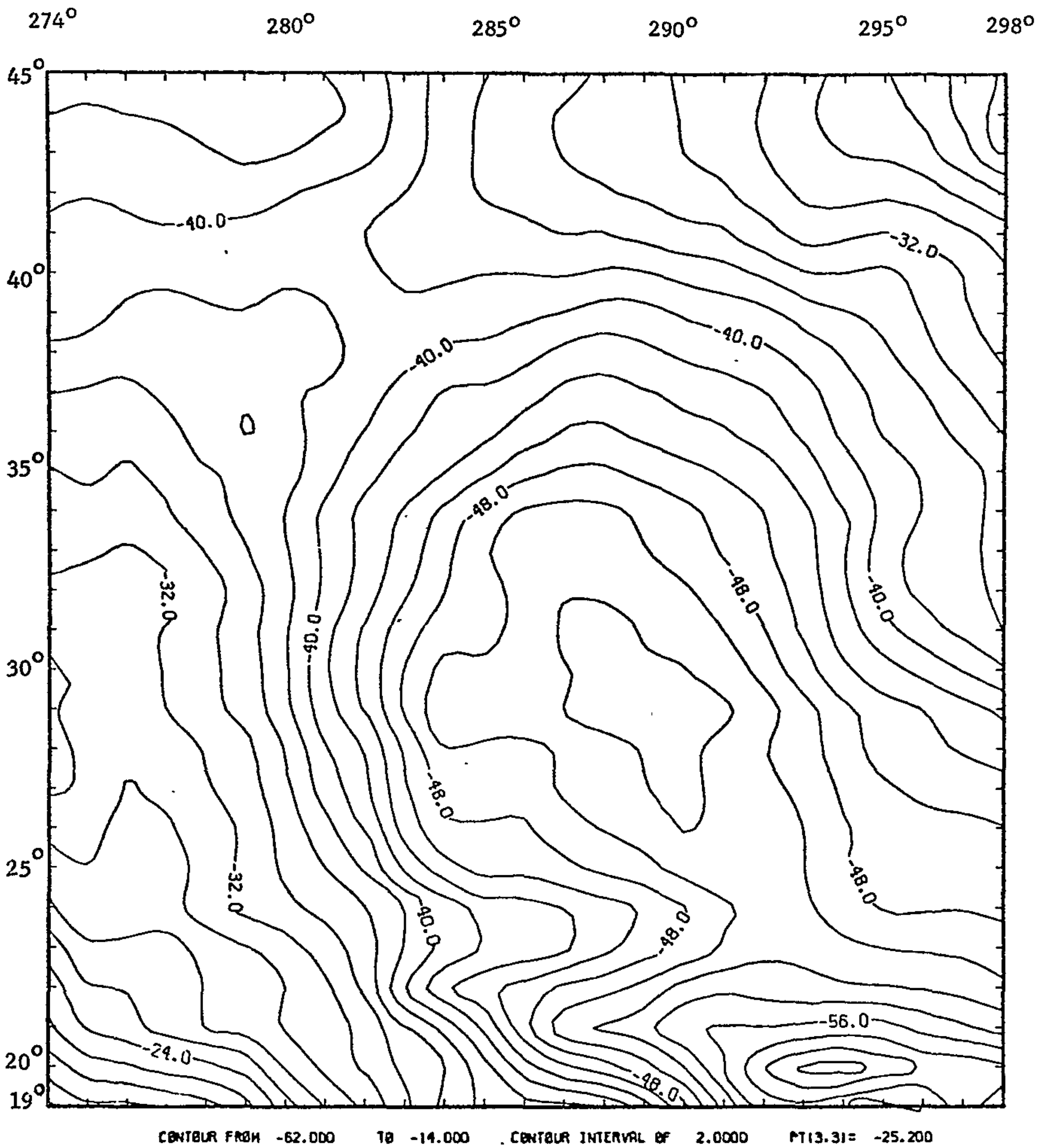


FIGURE 18. RECTIFIED GEOID FOR GSFC/C-BAND DATA (meters)

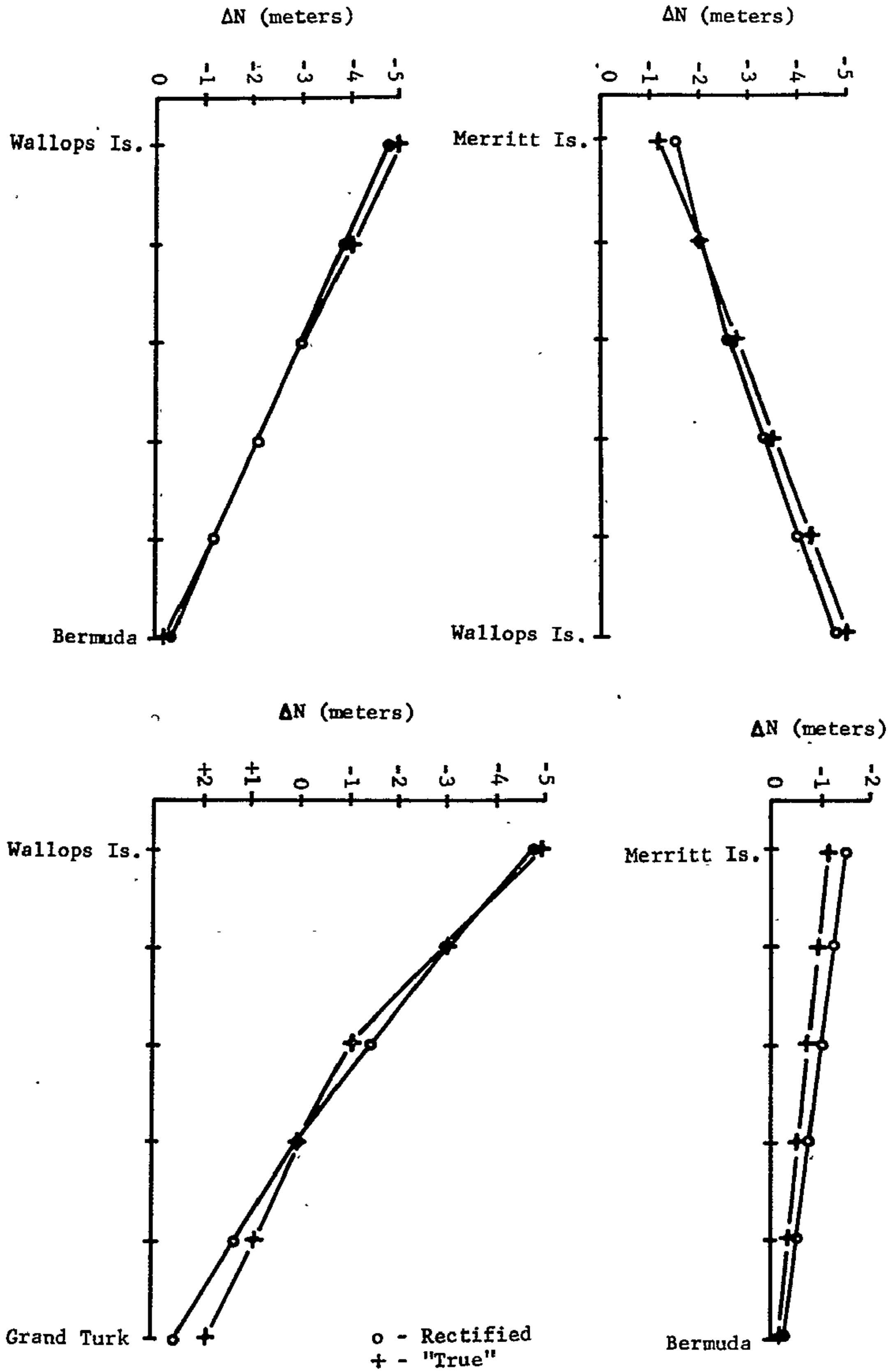


FIGURE 19. COMPARISON OF THE RECTIFIED AND "TRUE" DIFFERENCE GEOIDS FOR WFC/C-BAND DATA

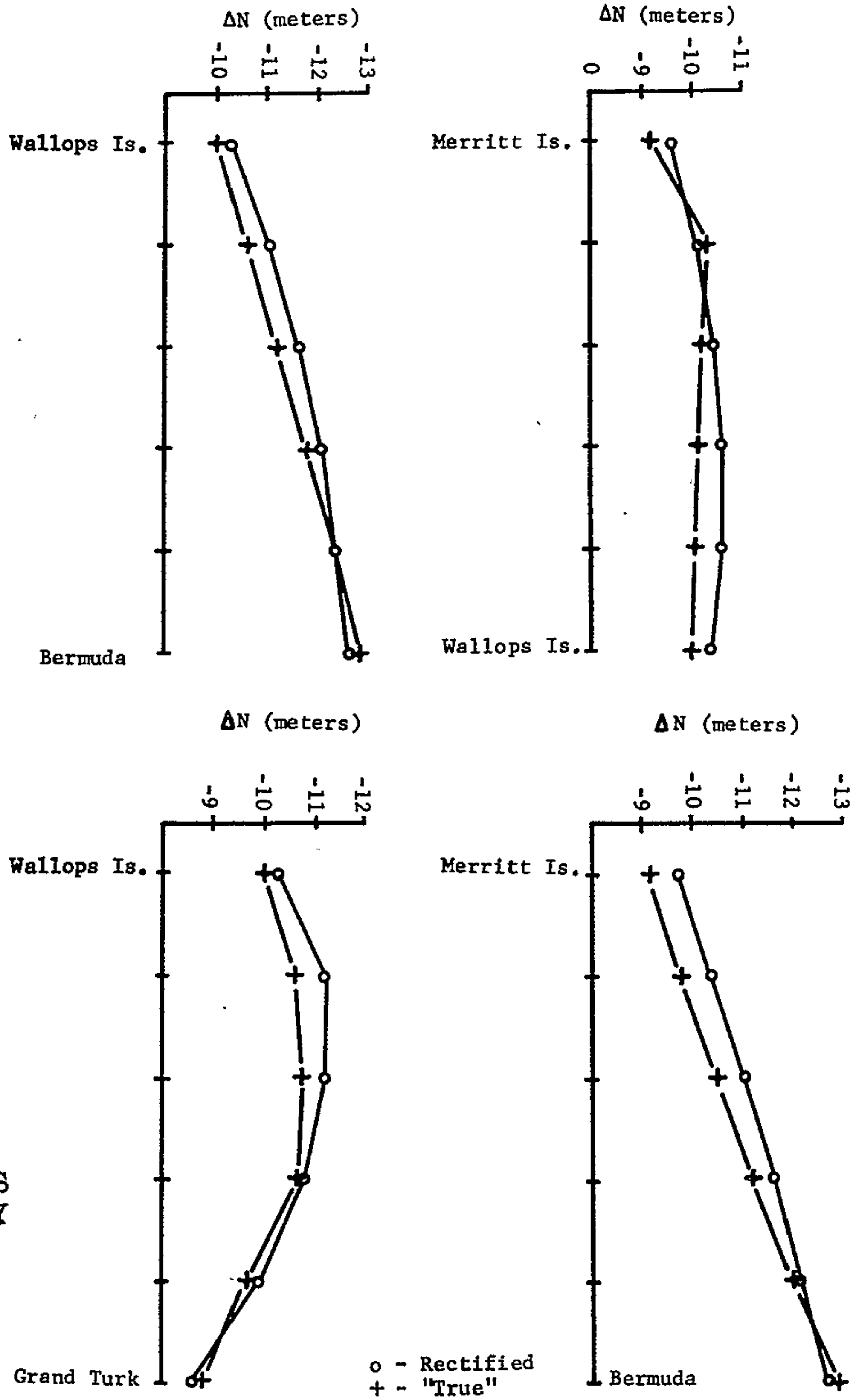


FIGURE 20. COMPARISON OF THE RECTIFIED AND "TRUE" DIFFERENCE GEOIDS FOR OSU-275 DATA

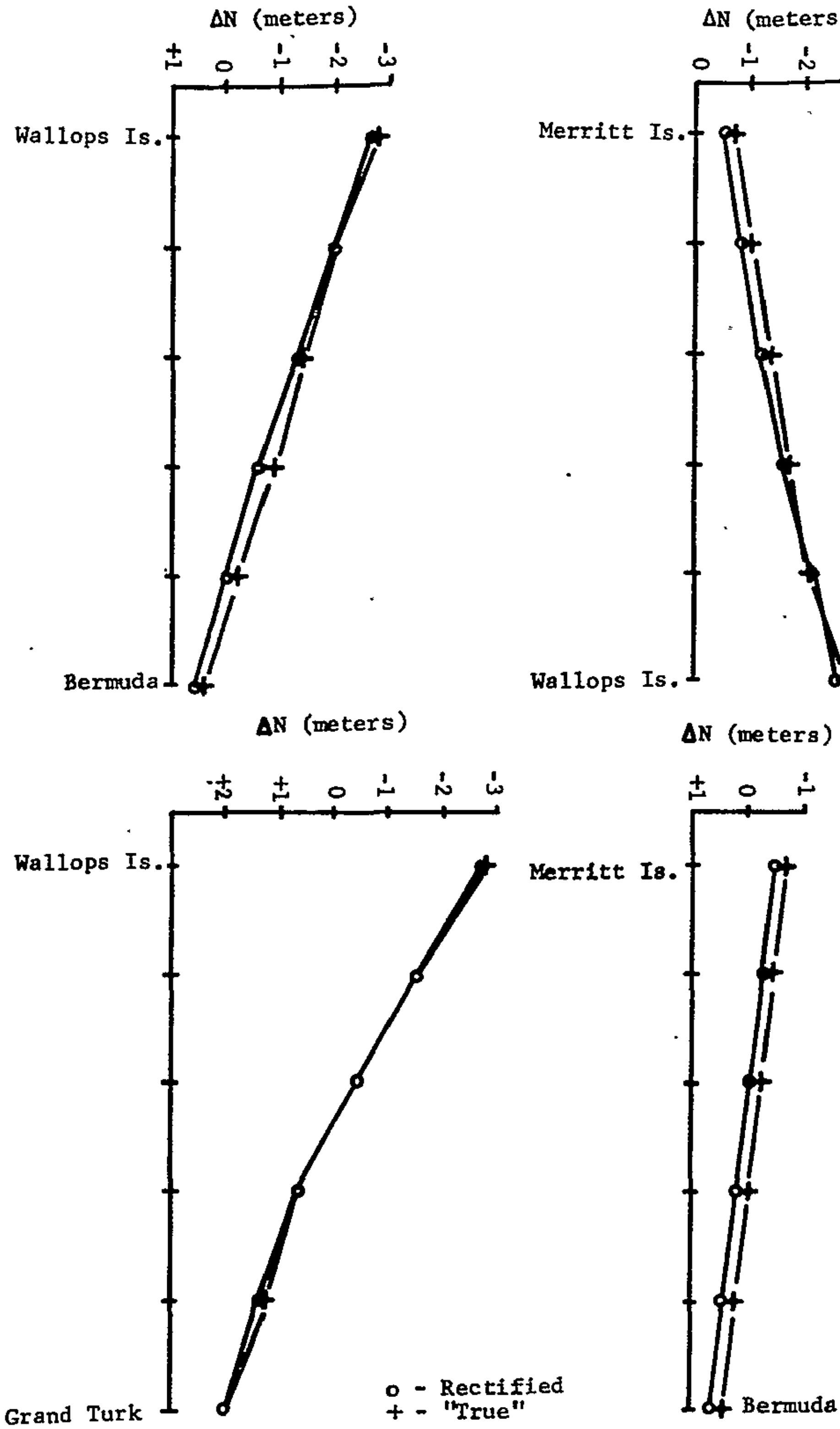


FIGURE 21. COMPARISON OF THE RECTIFIED AND "TRUE" DIFFERENCE GEOIDS FOR GSFC/C-BAND DATA

TABLE 4. DETAILED GRAVIMETRIC GEOID, N (meters)

LONGITUDES (degrees)

LATITUDES (degrees)	274	275	276	277	278	279	280	281	282	283	284	285
45	-36.4	-35.9	-36.3	-36.9	-37.0	-37.0	-36.3	-35.6	-34.8	-33.0	-31.3	-30.2
44	-35.3	-35.0	-35.4	-35.8	-36.4	-37.0	-36.9	-36.6	-35.7	-33.7	-31.5	-30.3
43	-34.5	-34.5	-34.8	-35.3	-36.0	-36.7	-36.5	-36.0	-35.3	-33.9	-32.0	-30.7
42	-34.3	-34.3	-34.8	-35.3	-35.5	-35.7	-35.1	-34.5	-34.0	-33.3	-32.3	-31.4
41	-34.3	-34.1	-34.3	-34.7	-34.6	-34.6	-34.3	-34.0	-33.6	-33.1	-32.9	-32.9
40	-34.5	-34.2	-33.7	-33.7	-34.0	-34.5	-34.2	-34.2	-34.3	-33.9	-34.0	-34.5
39	-34.1	-34.0	-33.3	-33.2	-33.7	-33.9	-33.5	-34.2	-34.9	-35.0	-35.5	-35.9
38	-33.4	-33.4	-33.0	-33.3	-33.6	-33.6	-33.7	-34.4	-35.1	-36.2	-37.5	-37.9
37	-32.2	-32.0	-31.8	-33.0	-34.1	-34.7	-34.8	-35.2	-36.3	-38.2	-39.7	-40.0
36	-31.2	-31.6	-31.2	-32.2	-34.1	-35.2	-34.8	-35.9	-37.7	-39.5	-40.8	-41.5
35	-30.7	-31.2	-30.9	-31.6	-33.1	-34.1	-34.7	-36.9	-39.3	-40.9	-42.2	-43.9
34	-30.5	-30.7	-30.6	-31.4	-32.9	-34.4	-36.0	-38.5	-40.8	-42.6	-45.0	-47.5
33	-30.2	-29.9	-29.4	-30.3	-32.1	-34.0	-36.1	-38.8	-41.1	-43.5	-46.3	-48.8
32	-29.2	-28.8	-28.5	-29.6	-31.3	-33.4	-36.0	-39.1	-41.8	-44.7	-47.1	-48.5
31	-29.7	-29.8	-29.5	-30.5	-32.0	-33.8	-36.6	-40.3	-43.4	-46.4	-48.6	-48.9
30	-30.4	-30.3	-29.9	-30.8	-32.3	-33.8	-36.8	-40.9	-44.0	-47.6	-50.4	-50.2
29	-30.6	-30.3	-29.9	-30.9	-32.4	-33.8	-36.7	-40.6	-43.7	-48.1	-51.6	-51.5
28	-31.3	-30.3	-29.0	-29.9	-31.5	-32.9	-35.7	-39.5	-42.1	-46.4	-50.0	-49.9
27	-31.3	-30.7	-29.0	-29.8	-31.3	-32.5	-35.3	-38.7	-41.6	-45.9	-49.2	-49.3
26	-29.7	-30.0	-28.6	-29.2	-30.8	-32.1	-34.8	-37.4	-40.4	-45.2	-48.2	-48.5
25	-28.3	-29.3	-28.3	-28.5	-30.8	-32.5	-34.6	-36.3	-38.8	-43.0	-46.1	-47.6
24	-27.0	-29.2	-29.0	-28.9	-31.2	-32.7	-33.8	-35.0	-36.5	-38.7	-41.4	-44.0
23	-24.4	-27.3	-27.4	-27.3	-28.8	-30.1	-31.5	-33.6	-35.8	-39.0	-41.8	-43.4
22	-22.6	-25.7	-26.1	-26.8	-28.4	-29.4	-30.6	-32.3	-35.3	-41.0	-44.9	-46.8
21	-21.5	-24.2	-25.4	-27.1	-28.5	-29.4	-30.9	-32.0	-33.6	-36.9	-39.8	-43.8
20	-18.8	-21.1	-21.9	-23.1	-23.9	-25.4	-28.9	-31.4	-33.5	-35.7	-37.4	-40.8
19	-15.7	-17.7	-17.9	-18.6	-19.8	-22.0	-25.9	-29.1	-31.7	-34.8	-38.2	-40.2

TABLE 4. DETAILED GRAVIMATIC GEOID, N (meters)
(Continued)

LONGITUDES (degrees)

LATITUDES (degrees)	286	287	288	289	290	291	292	293	294	295	296	297	298
45	-29.6	-29.0	-28.2	-27.4	-26.5	-26.0	-25.2	-23.9	-22.7	-21.6	-20.6	-19.1	-17.2
44	-29.5	-28.7	-28.3	-27.8	-27.2	-26.5	-25.3	-23.9	-23.3	-22.8	-21.7	-19.7	-17.2
43	-30.3	-29.3	-28.8	-28.4	-27.8	-27.1	-26.0	-24.8	-24.3	-24.1	-23.3	-21.3	-18.8
42	-31.2	-30.5	-30.2	-29.8	-29.2	-28.7	-27.6	-26.3	-26.1	-26.5	-26.1	-24.8	-23.2
41	-32.6	-32.3	-32.5	-32.0	-31.4	-31.1	-29.7	-28.4	-28.9	-29.6	-29.0	-28.0	-26.8
40	-34.6	-34.6	-35.3	-35.2	-34.6	-34.2	-33.1	-31.7	-31.8	-32.2	-31.3	-30.0	-28.6
39	-36.6	-37.3	-38.3	-38.1	-37.3	-36.9	-36.4	-35.2	-34.3	-33.5	-32.2	-30.7	-29.6
38	-38.8	-40.2	-40.8	-40.3	-39.6	-39.2	-38.7	-37.7	-36.3	-35.0	-33.4	-31.8	-30.7
37	-41.1	-42.4	-42.8	-42.4	-41.8	-41.4	-40.7	-39.4	-37.8	-36.3	-35.1	-33.6	-32.0
36	-43.1	-44.5	-44.8	-44.1	-43.4	-43.0	-42.2	-40.7	-38.8	-37.2	-36.4	-35.2	-33.7
35	-46.0	-47.0	-47.2	-46.4	-45.8	-45.0	-44.1	-42.6	-40.5	-38.7	-37.7	-36.7	-35.4
34	-49.1	-49.6	-49.6	-48.8	-48.0	-46.7	-45.6	-44.2	-42.0	-39.6	-38.1	-37.0	-35.8
33	-50.2	-50.8	-50.6	-49.6	-48.9	-47.6	-46.3	-44.9	-42.4	-39.2	-37.6	-37.2	-36.2
32	-50.2	-51.5	-51.5	-50.8	-50.3	-49.0	-47.4	-45.6	-42.9	-39.7	-38.2	-37.6	-36.7
31	-50.3	-52.1	-52.5	-52.0	-51.6	-50.4	-48.6	-46.4	-43.9	-41.7	-40.1	-38.7	-37.3
30	-50.7	-52.0	-52.6	-52.6	-52.6	-51.8	-50.2	-47.8	-45.5	-44.0	-42.8	-41.4	-39.9
29	-51.5	-52.3	-53.2	-53.7	-54.2	-53.3	-51.9	-50.0	-47.8	-46.3	-45.6	-44.6	-43.5
28	-49.8	-50.9	-52.1	-53.0	-53.5	-52.5	-51.4	-50.4	-48.4	-46.9	-46.4	-45.8	-45.2
27	-49.5	-51.1	-52.3	-53.0	-53.7	-52.9	-52.1	-51.2	-49.5	-48.2	-47.7	-47.4	-47.1
26	-48.6	-50.1	-51.2	-52.3	-53.5	-53.3	-52.8	-51.9	-50.2	-49.3	-48.9	-48.9	-48.7
25	-47.7	-48.7	-49.7	-50.8	-52.3	-53.0	-53.0	-52.0	-50.3	-49.6	-49.4	-49.6	-49.4
24	-44.3	-45.3	-46.6	-47.8	-49.5	-51.5	-53.0	-52.7	-51.2	-50.4	-50.4	-50.7	-50.5
23	-44.2	-45.2	-47.2	-48.9	-50.4	-52.4	-53.7	-53.8	-53.1	-52.7	-52.6	-52.5	-51.9
22	-48.4	-50.3	-51.3	-52.6	-55.1	-55.9	-55.5	-55.2	-55.0	-55.0	-54.9	-54.4	-53.6
21	-48.9	-53.0	-54.2	-55.4	-57.9	-58.6	-58.6	-59.3	-59.9	-59.7	-58.5	-57.6	-56.6
20	-44.7	-47.3	-49.0	-51.3	-54.1	-58.4	-62.5	-65.3	-65.8	-64.4	-62.9	-62.9	-61.9
19	-40.8	-41.2	-42.0	-45.1	-49.6	-55.2	-59.4	-60.3	-59.1	-58.3	-59.2	-62.4	-65.2

TABLE 5. DIFFERENCE GEOID (WFC/C-BAND), ΔN (meters)

LONGITUDES (degrees)

NORTH LATITUDES (degrees)	274	275	276	277	278	279	280	281	282	283	284	285
45	-10.3	-10.2	-10.0	-9.9	-9.7	-9.6	-9.4	-9.3	-9.1	-9.0	-8.9	-8.7
44	-9.8	-9.6	-9.5	-9.3	-9.2	-9.0	-8.9	-8.7	-8.6	-8.4	-8.3	-8.1
43	-9.2	-9.1	-8.9	-8.8	-8.6	-8.5	-8.3	-8.2	-8.0	-7.9	-7.7	-7.6
42	-8.7	-8.5	-8.4	-8.2	-8.1	-7.9	-7.8	-7.6	-7.5	-7.3	-7.2	-7.0
41	-8.2	-8.0	-7.9	-7.7	-7.6	-7.4	-7.2	-7.1	-6.9	-6.8	-6.6	-6.4
40	-7.7	-7.5	-7.4	-7.2	-7.0	-6.9	-6.7	-6.6	-6.4	-6.2	-6.1	-5.9
39	-7.2	-7.0	-6.9	-6.7	-6.5	-6.4	-6.2	-6.0	-5.9	-5.7	-5.5	-5.4
38	-6.7	-6.5	-6.4	-6.2	-6.0	-5.9	-5.7	-5.5	-5.4	-5.2	-5.0	-4.9
37	-6.2	-6.1	-5.9	-5.7	-5.5	-5.4	-5.2	-5.0	-4.9	-4.7	-4.5	-4.4
36	-5.8	-5.6	-5.4	-5.2	-5.1	-4.9	-4.7	-4.6	-4.4	-4.2	-4.0	-3.9
35	-5.3	-5.1	-5.0	-4.8	-4.6	-4.4	-4.3	-4.1	-3.9	-3.7	-3.6	-3.4
34	-4.9	-4.7	-4.5	-4.3	-4.2	-4.0	-3.8	-3.6	-3.5	-3.3	-3.1	-2.9
33	-4.5	-4.3	-4.1	-3.9	-3.7	-3.6	-3.4	-3.2	-3.0	-2.8	-2.7	-2.5
32	-4.0	-3.9	-3.7	-3.5	-3.3	-3.1	-2.9	-2.8	-2.6	-2.4	-2.2	-2.0
31	-3.6	-3.5	-3.3	-3.1	-2.9	-2.7	-2.5	-2.3	-2.2	-2.0	-1.8	-1.6
30	-3.3	-3.1	-2.9	-2.7	-2.5	-2.3	-2.1	-1.9	-1.8	-1.6	-1.4	-1.2
29	-2.9	-2.7	-2.5	-2.3	-2.1	-1.9	-1.8	-1.6	-1.4	-1.2	-1.0	-.8
28	-2.5	-2.3	-2.1	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-.8	-.6	-.4
27	-2.2	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-.8	-.6	-.4	-.2	-.1
26	-1.9	-1.7	-1.5	-1.3	-1.1	-.9	-.7	-.5	-.3	-.1	.1	.3
25	-1.6	-1.4	-1.2	-1.0	-.8	-.6	-.4	-.2	0.0	.2	.4	.6
24	-1.3	-1.1	-.9	-.7	-.5	-.3	-.1	.1	.3	.6	.8	1.0
23	-1.0	-.8	-.6	-.4	-.2	0.0	.2	.4	.6	.8	1.1	1.3
22	-.7	-.5	-.3	-.1	.1	.3	.5	.7	.9	1.1	1.3	1.5
21	-.5	-.3	-0.0	.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.8
20	-.2	-0.0	.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.9	2.1
19	-0.0	.2	.4	.6	.8	1.0	1.2	1.5	1.7	1.9	2.1	2.3

TABLE 5. DIFFERENCE GEOID (WFC/C-BAND), ΔN (meters)
(Continued)

LONGITUDES (degrees)

NORTH LATITUDES (degrees)	286	287	288	289	290	291	292	293	294	295	296	297	298
45	-8.6	-8.4	-8.3	-8.1	-8.0	-7.8	-7.7	-7.5	-7.4	-7.2	-7.1	-6.9	-6.8
44	-8.0	-7.8	-7.7	-7.5	-7.4	-7.2	-7.1	-6.9	-6.8	-6.6	-6.5	-6.3	-6.2
43	-7.4	-7.3	-7.1	-6.9	-6.8	-6.6	-6.5	-6.3	-6.2	-6.0	-5.9	-5.7	-5.6
42	-6.8	-6.7	-6.5	-6.4	-6.2	-6.1	-5.9	-5.7	-5.6	-5.4	-5.3	-5.1	-5.0
41	-6.3	-6.1	-6.0	-5.8	-5.7	-5.5	-5.3	-5.2	-5.0	-4.9	-4.7	-4.5	-4.4
40	-5.7	-5.6	-5.4	-5.3	-5.1	-4.9	-4.8	-4.6	-4.4	-4.3	-4.1	-4.0	-3.8
39	-5.2	-5.1	-4.9	-4.7	-4.6	-4.4	-4.2	-4.1	-3.9	-3.7	-3.6	-3.4	-3.2
38	-4.7	-4.5	-4.4	-4.2	-4.0	-3.9	-3.7	-3.5	-3.4	-3.2	-3.0	-2.8	-2.7
37	-4.2	-4.0	-3.8	-3.7	-3.5	-3.3	-3.2	-3.0	-2.8	-2.6	-2.5	-2.3	-2.1
36	-3.7	-3.5	-3.3	-3.2	-3.0	-2.8	-2.7	-2.5	-2.3	-2.1	-2.0	-1.8	-1.6
35	-3.2	-3.0	-2.9	-2.7	-2.5	-2.3	-2.2	-2.0	-1.8	-1.6	-1.4	-1.3	-1.1
34	-2.7	-2.6	-2.4	-2.2	-2.0	-1.8	-1.7	-1.5	-1.3	-1.1	-.9	-.8	-.6
33	-2.3	-2.1	-1.9	-1.7	-1.6	-1.4	-1.2	-1.0	-.8	-.6	-.5	-.3	-.1
32	-1.8	-1.7	-1.5	-1.3	-1.1	-.9	-.7	-.6	-.4	-.2	0.0	.2	.4
31	-1.4	-1.2	-1.0	-.9	-.7	-.5	-.3	-.1	.1	.3	.5	.6	.8
30	-1.0	-.8	-.6	-.4	-.2	-.1	.1	.3	.5	.7	.9	1.1	1.3
29	-.6	-.4	-.2	-0.0	.2	.4	.5	.7	.9	1.1	1.3	1.5	1.7
28	-.2	-0.0	.2	.4	.6	.7	.9	1.1	1.3	1.5	1.7	1.9	2.1
27	.1	.3	.5	.7	.9	1.1	1.3	1.5	1.7	1.9	2.1	2.3	2.5
26	.5	.7	.9	1.1	1.3	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9
25	.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.1	3.3
24	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6
23	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.7	3.9
22	1.7	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2
21	2.0	2.2	2.4	2.6	2.9	3.1	3.3	3.5	3.7	3.9	4.1	4.3	4.5
20	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.7	4.0	4.2	4.4	4.6	4.8
19	2.5	2.7	2.9	3.1	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.9	5.1

TABLE 6. DIFFERENCE GEOID (OSU-275), ΔN (meters)

LONGITUDES (degrees)

LATITUDES (degrees)	274	275	276	277	278	279	280	281	282	283	284	285
45	-3.5	-3.8	-4.0	-4.3	-4.6	-4.9	-5.2	-5.5	-5.8	-6.1	-6.3	-6.6
44	-4.3	-4.6	-4.9	-5.2	-5.5	-5.7	-6.0	-6.3	-6.6	-6.8	-7.1	-7.4
43	-5.1	-5.3	-5.6	-5.9	-6.2	-6.5	-6.7	-7.0	-7.3	-7.6	-7.8	-8.1
42	-5.8	-6.0	-6.3	-6.6	-6.9	-7.1	-7.4	-7.7	-7.9	-8.2	-8.4	-8.7
41	-6.4	-6.6	-6.9	-7.2	-7.4	-7.7	-8.0	-8.2	-8.5	-8.7	-9.0	-9.2
40	-6.9	-7.2	-7.4	-7.7	-8.0	-8.2	-8.5	-8.7	-9.0	-9.2	-9.5	-9.7
39	-7.4	-7.7	-7.9	-8.2	-8.4	-8.7	-8.9	-9.2	-9.4	-9.6	-9.9	-10.1
38	-7.8	-8.1	-8.3	-8.6	-8.8	-9.0	-9.3	-9.5	-9.8	-10.0	-10.2	-10.4
37	-8.1	-8.4	-8.6	-8.9	-9.1	-9.4	-9.6	-9.8	-10.0	-10.3	-10.5	-10.7
36	-8.4	-8.7	-8.9	-9.1	-9.4	-9.6	-9.8	-10.1	-10.3	-10.5	-10.7	-10.9
35	-8.7	-8.9	-9.1	-9.3	-9.6	-9.8	-10.0	-10.2	-10.4	-10.6	-10.9	-11.1
34	-8.8	-9.0	-9.3	-9.5	-9.7	-9.9	-10.1	-10.3	-10.5	-10.7	-10.9	-11.1
33	-8.9	-9.2	-9.4	-9.6	-9.8	-10.0	-10.2	-10.4	-10.6	-10.8	-11.0	-11.2
32	-9.0	-9.2	-9.4	-9.6	-9.8	-10.0	-10.2	-10.4	-10.6	-10.8	-11.0	-11.1
31	-9.0	-9.2	-9.4	-9.6	-9.8	-10.0	-10.2	-10.4	-10.5	-10.7	-10.9	-11.1
30	-9.0	-9.2	-9.3	-9.5	-9.7	-9.9	-10.1	-10.3	-10.4	-10.6	-10.8	-10.9
29	-8.9	-9.1	-9.2	-9.4	-9.6	-9.8	-10.0	-10.1	-10.3	-10.5	-10.6	-10.8
28	-8.7	-8.9	-9.1	-9.3	-9.4	-9.6	-9.8	-9.9	-10.1	-10.3	-10.4	-10.6
27	-8.6	-8.7	-8.9	-9.1	-9.2	-9.4	-9.6	-9.7	-9.9	-10.0	-10.2	-10.3
26	-8.4	-8.5	-8.7	-8.8	-9.0	-9.2	-9.3	-9.5	-9.6	-9.7	-9.9	-10.0
25	-8.1	-8.3	-8.4	-8.6	-8.7	-8.9	-9.0	-9.2	-9.3	-9.4	-9.6	-9.7
24	-7.8	-8.0	-8.1	-8.3	-8.4	-8.6	-8.7	-8.8	-8.9	-9.1	-9.2	-9.3
23	-7.5	-7.6	-7.8	-7.9	-8.1	-8.2	-8.3	-8.4	-8.6	-8.7	-8.8	-8.9
22	-7.1	-7.3	-7.4	-7.5	-7.7	-7.8	-7.9	-8.0	-8.2	-8.3	-8.4	-8.5
21	-6.7	-6.9	-7.0	-7.1	-7.3	-7.4	-7.5	-7.6	-7.7	-7.8	-7.9	-8.0
20	-6.3	-6.5	-6.6	-6.7	-6.8	-6.9	-7.1	-7.2	-7.3	-7.4	-7.5	-7.5
19	-5.9	-6.0	-6.1	-6.3	-6.4	-6.5	-6.6	-6.7	-6.8	-6.9	-7.0	-7.0

TABLE 6. DIFFERENCE GEOID (OSU-275), ΔN (meters)
(Continued)

LONGITUDES (degrees)

LATITUDES (degrees)	286	287	288	289	290	291	292	293	294	295	296	297	298
45	-6.9	-7.2	-7.4	-7.7	-8.0	-8.2	-8.5	-8.7	-9.0	-9.2	-9.5	-9.7	-10.0
44	-7.7	-7.9	-8.2	-8.4	-8.7	-9.0	-9.2	-9.5	-9.7	-10.0	-10.2	-10.4	-10.7
43	-8.3	-8.6	-8.9	-9.1	-9.4	-9.6	-9.9	-10.1	-10.3	-10.6	-10.8	-11.0	-11.3
42	-9.0	-9.2	-9.5	-9.7	-9.9	-10.2	-10.4	-10.7	-10.9	-11.1	-11.4	-11.6	-11.8
41	-9.5	-9.7	-10.0	-10.2	-10.5	-10.7	-10.9	-11.1	-11.4	-11.6	-11.8	-12.0	-12.2
40	-10.0	-10.2	-10.4	-10.7	-10.9	-11.1	-11.3	-11.6	-11.8	-12.0	-12.2	-12.4	-12.6
39	-10.3	-10.6	-10.8	-11.0	-11.2	-11.5	-11.7	-11.9	-12.1	-12.3	-12.5	-12.7	-12.9
38	-10.7	-10.9	-11.1	-11.3	-11.5	-11.7	-12.0	-12.2	-12.4	-12.6	-12.8	-13.0	-13.1
37	-10.9	-11.1	-11.4	-11.6	-11.8	-12.0	-12.2	-12.4	-12.6	-12.7	-12.9	-13.1	-13.3
36	-11.1	-11.3	-11.5	-11.7	-11.9	-12.1	-12.3	-12.5	-12.7	-12.9	-13.0	-13.2	-13.4
35	-11.3	-11.5	-11.6	-11.8	-12.0	-12.2	-12.4	-12.6	-12.8	-12.9	-13.1	-13.3	-13.4
34	-11.3	-11.5	-11.7	-11.9	-12.1	-12.2	-12.4	-12.6	-12.8	-12.9	-13.1	-13.2	-13.4
33	-11.4	-11.5	-11.7	-11.9	-12.1	-12.2	-12.4	-12.6	-12.7	-12.9	-13.0	-13.2	-13.3
32	-11.3	-11.5	-11.7	-11.8	-12.0	-12.2	-12.3	-12.5	-12.6	-12.8	-12.9	-13.1	-13.2
31	-11.2	-11.4	-11.6	-11.7	-11.9	-12.0	-12.2	-12.3	-12.5	-12.6	-12.7	-12.9	-13.0
30	-11.1	-11.3	-11.4	-11.6	-11.7	-11.9	-12.0	-12.1	-12.3	-12.4	-12.5	-12.7	-12.8
29	-10.9	-11.1	-11.2	-11.4	-11.5	-11.6	-11.8	-11.9	-12.0	-12.1	-12.3	-12.4	-12.5
28	-10.7	-10.8	-11.0	-11.1	-11.2	-11.4	-11.5	-11.6	-11.7	-11.9	-12.0	-12.1	-12.2
27	-10.4	-10.6	-10.7	-10.8	-11.0	-11.1	-11.2	-11.3	-11.4	-11.5	-11.6	-11.7	-11.8
26	-10.1	-10.3	-10.4	-10.5	-10.6	-10.7	-10.8	-10.9	-11.0	-11.1	-11.2	-11.3	-11.4
25	-9.8	-9.9	-10.0	-10.1	-10.3	-10.4	-10.5	-10.6	-10.6	-10.7	-10.8	-10.9	-11.0
24	-9.4	-9.5	-9.6	-9.7	-9.8	-9.9	-10.0	-10.1	-10.2	-10.3	-10.4	-10.4	-10.5
23	-9.0	-9.1	-9.2	-9.3	-9.4	-9.5	-9.6	-9.7	-9.7	-9.8	-9.9	-10.0	-10.0
22	-8.6	-8.7	-8.8	-8.9	-8.9	-9.0	-9.1	-9.2	-9.3	-9.3	-9.4	-9.4	-9.5
21	-8.1	-8.2	-8.3	-8.4	-8.5	-8.5	-8.6	-8.7	-8.7	-8.8	-8.8	-8.9	-8.9
20	-7.6	-7.7	-7.8	-7.9	-7.9	-8.0	-8.1	-8.1	-8.2	-8.2	-8.3	-8.3	-8.4
19	-7.1	-7.2	-7.3	-7.3	-7.4	-7.5	-7.5	-7.6	-7.6	-7.7	-7.7	-7.7	-7.8

TABLE 7. DIFFERENCE GEOID (GSFC/C-BAND), ΔN (meters)

LONGITUDES (degrees)

LATITUDES (degrees)	274	275	276	277	278	279	280	281	282	283	284	285
45	-7.3	-7.2	-7.1	-6.9	-6.8	-6.7	-6.5	-6.4	-6.3	-6.1	-6.0	-5.9
44	-6.8	-6.7	-6.6	-6.4	-6.3	-6.2	-6.0	-5.9	-5.8	-5.6	-5.5	-5.4
43	-6.4	-6.2	-6.1	-6.0	-5.8	-5.7	-5.6	-5.4	-5.3	-5.2	-5.0	-4.9
42	-5.9	-5.8	-5.6	-5.5	-5.4	-5.2	-5.1	-5.0	-4.8	-4.7	-4.6	-4.4
41	-5.5	-5.3	-5.2	-5.1	-4.9	-4.8	-4.7	-4.5	-4.4	-4.2	-4.1	-4.0
40	-5.1	-4.9	-4.8	-4.7	-4.5	-4.4	-4.2	-4.1	-4.0	-3.8	-3.7	-3.5
39	-4.7	-4.5	-4.4	-4.2	-4.1	-4.0	-3.8	-3.7	-3.6	-3.4	-3.3	-3.1
38	-4.3	-4.1	-4.0	-3.9	-3.7	-3.6	-3.4	-3.3	-3.2	-3.0	-2.9	-2.7
37	-3.9	-3.8	-3.6	-3.5	-3.4	-3.2	-3.1	-2.9	-2.8	-2.6	-2.5	-2.4
36	-3.6	-3.4	-3.3	-3.1	-3.0	-2.9	-2.7	-2.6	-2.4	-2.3	-2.1	-2.0
35	-3.2	-3.1	-2.9	-2.8	-2.7	-2.5	-2.4	-2.2	-2.1	-1.9	-1.8	-1.7
34	-2.9	-2.8	-2.6	-2.5	-2.3	-2.2	-2.0	-1.9	-1.8	-1.6	-1.5	-1.3
33	-2.6	-2.5	-2.3	-2.2	-2.0	-1.9	-1.7	-1.6	-1.5	-1.3	-1.2	-1.0
32	-2.3	-2.2	-2.0	-1.9	-1.7	-1.6	-1.4	-1.3	-1.2	-1.0	-.9	-.7
31	-2.0	-1.9	-1.8	-1.6	-1.5	-1.3	-1.2	-1.0	-.9	-.7	-.6	-.5
30	-1.8	-1.6	-1.5	-1.3	-1.2	-1.1	-.9	-.8	-.6	-.5	-.3	-.2
29	-1.5	-1.4	-1.2	-1.1	-1.0	-.8	-.7	-.5	-.4	-.2	-.1	.1
28	-1.3	-1.2	-1.0	-.9	-.7	-.6	-.4	-.3	-.1	0.0	.1	.3
27	-1.1	-1.0	-.8	-.7	-.5	-.4	-.2	-.1	.1	.2	.4	.5
26	-.9	-.8	-.6	-.5	-.3	-.2	-0.0	.1	.3	.4	.6	.7
25	-.7	-.6	-.4	-.3	-.1	0.0	.2	.3	.5	.6	.8	.9
24	-.6	-.4	-.3	-.1	0.0	.2	.3	.5	.6	.8	.9	1.1
23	-.4	-.3	-.1	0.0	.2	.3	.5	.6	.8	.9	1.1	1.2
22	-.3	-.1	0.0	.2	.3	.5	.6	.8	.9	1.1	1.2	1.4
21	-.1	0.0	.2	.3	.5	.6	.8	.9	1.1	1.2	1.3	1.5
20	-0.0	.1	.3	.4	.6	.7	.9	1.0	1.2	1.3	1.5	1.6
19	.1	.2	.4	.5	.7	.8	1.0	1.1	1.3	1.4	1.6	1.7

TABLE 7. DIFFERENCE GEOID (GSFC/C-BAND), ΔN (meters)
(Continued)

LONGITUDES (degrees)

LATITUDES (degrees)	286	287	288	289	290	291	292	293	294	295	296	297	298
45	-5.7	-5.6	-5.5	-5.3	-5.2	-5.1	-4.9	-4.8	-4.7	-4.6	-4.4	-4.3	-4.2
44	-5.2	-5.1	-5.0	-4.8	-4.7	-4.6	-4.4	-4.3	-4.2	-4.0	-3.9	-3.8	-3.7
43	-4.7	-4.6	-4.5	-4.3	-4.2	-4.1	-4.0	-3.8	-3.7	-3.6	-3.4	-3.3	-3.2
42	-4.3	-4.2	-4.0	-3.9	-3.7	-3.6	-3.5	-3.3	-3.2	-3.1	-3.0	-2.8	-2.7
41	-3.8	-3.7	-3.6	-3.4	-3.3	-3.2	-3.0	-2.9	-2.8	-2.6	-2.5	-2.4	-2.2
40	-3.4	-3.3	-3.1	-3.0	-2.9	-2.7	-2.6	-2.5	-2.3	-2.2	-2.1	-1.9	-1.8
39	-3.0	-2.9	-2.7	-2.6	-2.5	-2.3	-2.2	-2.0	-1.9	-1.8	-1.6	-1.5	-1.4
38	-2.6	-2.5	-2.3	-2.2	-2.1	-1.9	-1.8	-1.6	-1.5	-1.4	-1.2	-1.1	-1.0
37	-2.2	-2.1	-2.0	-1.8	-1.7	-1.5	-1.4	-1.3	-1.1	-1.0	-.9	-.7	-.6
36	-1.9	-1.7	-1.6	-1.4	-1.3	-1.2	-1.0	-.9	-.8	-.6	-.5	-.3	-.2
35	-1.5	-1.4	-1.2	-1.1	-1.0	-.8	-.7	-.5	-.4	-.3	-.1	0.0	.1
34	-1.2	-1.1	-.9	-.8	-.6	-.5	-.4	-.2	-.1	.1	.2	.3	.5
33	-.9	-.7	-.6	-.5	-.3	-.2	-0.0	.1	.2	.4	.5	.7	.8
32	-.6	-.4	-.3	-.2	-0.0	.1	.3	.4	.5	.7	.8	1.0	1.1
31	-.3	-.2	-0.0	.1	.3	.4	.5	.7	.8	1.0	1.1	1.2	1.4
30	-0.0	.1	.2	.4	.5	.7	.8	1.0	1.1	1.2	1.4	1.5	1.7
29	.2	.3	.5	.6	.8	.9	1.1	1.2	1.3	1.5	1.6	1.8	1.9
28	.4	.6	.7	.9	1.0	1.2	1.3	1.4	1.6	1.7	1.9	2.0	2.1
27	.7	.8	.9	1.1	1.2	1.4	1.5	1.7	1.8	1.9	2.1	2.2	2.4
26	.9	1.0	1.1	1.3	1.4	1.6	1.7	1.9	2.0	2.1	2.3	2.4	2.6
25	1.0	1.2	1.3	1.5	1.6	1.8	1.9	2.0	2.2	2.3	2.5	2.6	2.8
24	1.2	1.4	1.5	1.6	1.8	1.9	2.1	2.2	2.4	2.5	2.6	2.8	2.9
23	1.4	1.5	1.7	1.8	1.9	2.1	2.2	2.4	2.5	2.7	2.8	2.9	3.1
22	1.5	1.7	1.8	1.9	2.1	2.2	2.4	2.5	2.7	2.8	3.0	3.1	3.2
21	1.6	1.8	1.9	2.1	2.2	2.4	2.5	2.7	2.8	2.9	3.1	3.2	3.4
20	1.8	1.9	2.0	2.2	2.3	2.5	2.6	2.8	2.9	3.1	3.2	3.3	3.5
19	1.8	2.0	2.1	2.3	2.4	2.6	2.7	2.9	3.0	3.2	3.3	3.4	3.6

TABLE 8. RECTIFIED GEOID FOR WFC/C-BAND DATA (N,meters)

LATITUDES (degrees) \ LONGITUDES (degrees)	274	275	276	277	278	279	280	281	282	283	284	285
45	-46.7	-46.1	-46.3	-46.8	-46.7	-46.6	-45.7	-44.9	-43.9	-42.0	-40.2	-38.9
44	-45.1	-44.6	-44.9	-45.1	-45.6	-46.0	-45.8	-45.3	-44.3	-42.1	-39.8	-38.4
43	-43.7	-43.6	-43.7	-44.1	-44.6	-45.2	-44.8	-44.2	-43.3	-41.8	-39.7	-38.3
42	-43.0	-42.8	-43.2	-43.5	-43.6	-43.6	-42.9	-42.1	-41.5	-40.6	-39.5	-38.4
41	-42.5	-42.1	-42.2	-42.4	-42.2	-42.0	-41.5	-41.1	-40.5	-39.9	-39.5	-39.3
40	-42.2	-41.7	-41.1	-40.9	-41.0	-41.4	-40.9	-40.8	-40.7	-40.1	-40.1	-40.4
39	-41.3	-41.0	-40.2	-39.9	-40.2	-40.3	-39.7	-40.2	-40.8	-40.7	-41.0	-41.3
38	-40.1	-39.9	-39.4	-39.5	-39.6	-39.5	-39.4	-39.9	-40.5	-41.4	-42.5	-42.8
37	-38.4	-38.1	-37.7	-38.7	-39.6	-40.1	-40.0	-40.2	-41.2	-42.9	-44.2	-44.4
36	-37.0	-37.2	-36.6	-37.4	-39.2	-40.1	-39.5	-40.5	-42.1	-43.7	-44.8	-45.4
35	-36.0	-36.3	-35.9	-36.4	-37.7	-38.5	-39.0	-41.0	-43.2	-44.6	-45.8	-47.3
34	-35.4	-35.4	-35.1	-35.7	-37.1	-38.4	-39.8	-42.1	-44.3	-45.9	-48.1	-50.4
33	-34.7	-34.2	-33.5	-34.2	-35.8	-37.6	-39.5	-42.0	-44.1	-46.3	-49.0	-51.3
32	-33.2	-32.7	-32.2	-33.1	-34.6	-36.5	-38.9	-41.9	-44.4	-47.1	-49.3	-50.5
31	-33.3	-33.3	-32.8	-33.6	-34.9	-36.5	-39.1	-42.6	-45.6	-48.4	-50.4	-50.5
30	-33.7	-33.4	-32.8	-33.5	-34.8	-36.1	-38.9	-42.8	-45.8	-49.2	-51.8	-51.4
29	-33.5	-33.0	-32.4	-33.2	-34.5	-35.7	-38.5	-42.2	-45.1	-49.3	-52.6	-52.3
28	-33.8	-32.6	-31.1	-31.9	-33.3	-34.5	-37.1	-40.7	-43.1	-47.2	-50.6	-50.3
27	-33.5	-32.7	-30.8	-31.4	-32.7	-33.7	-36.3	-39.5	-42.2	-46.3	-49.4	-49.4
26	-31.6	-31.7	-30.1	-30.5	-31.9	-33.0	-35.5	-37.9	-40.7	-45.3	-48.1	-48.2
25	-29.9	-30.7	-29.5	-29.5	-31.6	-33.1	-35.0	-36.5	-38.8	-42.8	-45.7	-47.0
24	-28.3	-30.3	-29.9	-29.6	-31.7	-33.0	-33.9	-34.9	-36.2	-38.1	-40.6	-43.0
23	-25.4	-28.1	-28.0	-27.7	-29.0	-30.1	-31.3	-33.2	-35.2	-38.2	-40.7	-42.1
22	-23.3	-26.2	-26.4	-26.9	-28.3	-29.1	-30.1	-31.6	-34.4	-39.9	-43.6	-45.3
21	-22.0	-24.5	-25.4	-26.9	-28.1	-28.8	-30.1	-31.0	-32.4	-35.5	-38.2	-42.0
20	-19.0	-21.1	-21.7	-22.7	-23.3	-24.6	-27.9	-30.2	-32.1	-34.1	-35.5	-38.7
19	-15.7	-17.5	-17.5	-18.0	-19.0	-21.0	-24.7	-27.6	-30.0	-32.9	-36.1	-37.9

TABLE 8. RECTIFIED GEOID FOR WFC/C-BAND DATA (N,meters)
(Continued)

LONGITUDES (degrees)

LATITUDES (degrees)	286	287	288	289	290	291	292	293	294	295	296	297	298
45	-38.2	-37.4	-36.5	-35.5	-34.5	-33.8	-32.9	-31.4	-30.1	-28.8	-27.7	-26.0	-24.0
44	-37.5	-36.5	-36.0	-35.3	-34.6	-33.7	-32.4	-30.8	-30.1	-29.4	-28.2	-26.0	-23.4
43	-37.7	-36.6	-35.9	-35.3	-34.6	-33.7	-32.5	-31.1	-30.5	-30.1	-29.2	-27.0	-24.4
42	-38.0	-37.2	-36.7	-36.2	-35.4	-34.8	-33.5	-32.0	-31.7	-31.9	-31.4	-29.9	-28.2
41	-38.9	-38.4	-38.5	-37.8	-37.1	-36.6	-35.0	-33.6	-33.9	-34.5	-33.7	-32.5	-31.2
40	-40.3	-40.2	-40.7	-40.5	-39.7	-39.1	-37.9	-36.3	-36.2	-36.5	-35.4	-34.0	-32.4
39	-41.8	-42.4	-43.2	-42.8	-41.9	-41.3	-40.6	-39.3	-38.2	-37.2	-35.8	-34.1	-32.8
38	-43.5	-44.7	-45.2	-44.5	-43.6	-43.1	-42.4	-41.2	-39.7	-38.2	-36.4	-34.6	-33.4
37	-45.3	-46.4	-46.6	-46.1	-45.3	-44.7	-43.9	-42.4	-40.6	-38.9	-37.6	-35.9	-34.1
36	-46.8	-48.0	-48.1	-47.3	-46.4	-45.8	-44.9	-43.2	-41.1	-39.3	-38.4	-37.0	-35.3
35	-49.2	-50.0	-50.1	-49.1	-48.3	-47.3	-46.3	-44.6	-42.3	-40.3	-39.1	-38.0	-36.5
34	-51.8	-52.2	-52.0	-51.0	-50.0	-48.5	-47.3	-45.7	-43.3	-40.7	-39.0	-37.8	-36.4
33	-52.5	-52.9	-52.5	-51.3	-50.5	-49.0	-47.5	-45.9	-43.2	-39.8	-38.1	-37.5	-36.3
32	-52.0	-53.2	-53.0	-52.1	-51.4	-49.9	-48.1	-46.2	-43.3	-39.9	-38.2	-37.4	-36.3
31	-51.7	-53.3	-53.5	-52.9	-52.3	-50.9	-48.9	-46.5	-43.8	-41.4	-39.6	-38.1	-36.5
30	-51.7	-52.8	-53.2	-53.0	-52.8	-51.9	-50.1	-47.5	-45.0	-43.3	-41.9	-40.3	-38.6
29	-52.1	-52.7	-53.4	-53.7	-54.0	-52.9	-51.4	-49.3	-46.9	-45.2	-44.3	-43.1	-41.8
28	-50.0	-50.9	-51.9	-52.6	-52.9	-51.8	-50.5	-49.3	-47.1	-45.4	-44.7	-43.9	-43.1
27	-49.4	-50.8	-51.8	-52.3	-52.8	-51.8	-50.8	-49.7	-47.8	-46.3	-45.6	-45.1	-44.6
26	-48.1	-49.4	-50.3	-51.2	-52.2	-51.8	-51.1	-50.0	-48.1	-47.0	-46.4	-46.2	-45.8
25	-46.9	-47.7	-48.5	-49.4	-50.7	-51.2	-51.0	-49.8	-47.9	-47.0	-46.6	-46.5	-46.1
24	-43.1	-43.9	-45.0	-46.0	-47.5	-49.3	-50.6	-50.1	-48.4	-47.4	-47.2	-47.3	-46.9
23	-42.7	-43.5	-45.3	-46.8	-48.1	-49.9	-51.0	-50.9	-50.0	-49.4	-49.1	-48.8	-48.0
22	-46.7	-48.3	-49.1	-50.2	-52.5	-53.1	-52.5	-52.0	-51.6	-51.4	-51.1	-50.4	-49.4
21	-46.9	-50.8	-51.8	-52.8	-55.0	-55.5	-55.3	-55.8	-56.2	-55.8	-54.4	-53.3	-52.1
20	-42.4	-44.8	-46.3	-48.4	-51.0	-55.1	-59.0	-61.6	-61.8	-60.2	-58.5	-58.3	-57.1
19	-38.3	-38.5	-39.1	-42.0	-46.2	-51.6	-55.6	-56.3	-54.9	-53.9	-54.6	-57.5	-60.1

TABLE 9. RECTIFIED GEOID FOR OSU-275 Data (N,meters)

LONGITUDES (degrees)

LATITUDES (degrees)	274	275	276	277	278	279	280	281	282	283	284	285
45	-39.9	-39.7	-40.3	-41.2	-41.6	-41.9	-41.5	-41.1	-40.6	-39.1	-37.6	-36.8
44	-39.6	-39.6	-40.3	-41.0	-41.9	-42.7	-42.9	-42.9	-42.3	-40.5	-38.6	-37.7
43	-39.6	-39.8	-40.4	-41.2	-42.2	-43.2	-43.2	-43.0	-42.6	-41.5	-39.8	-38.8
42	-40.1	-40.3	-41.1	-41.9	-42.4	-42.8	-42.5	-42.2	-41.9	-41.5	-40.7	-40.1
41	-40.7	-40.7	-41.2	-41.9	-42.0	-42.3	-42.3	-42.2	-42.1	-41.8	-41.9	-42.1
40	-41.4	-41.4	-41.1	-41.4	-42.0	-42.7	-42.7	-42.9	-43.3	-43.1	-43.5	-44.2
39	-41.5	-41.7	-41.2	-41.4	-42.1	-42.6	-42.4	-43.4	-44.3	-44.6	-45.4	-46.0
38	-41.2	-41.5	-41.3	-41.9	-42.4	-42.6	-43.0	-43.9	-44.9	-46.2	-47.7	-48.3
37	-40.3	-40.4	-40.4	-41.9	-43.2	-44.1	-44.4	-45.0	-46.3	-48.5	-50.2	-50.7
36	-39.6	-40.3	-40.1	-41.3	-43.5	-44.8	-44.6	-46.0	-48.0	-50.0	-51.5	-52.4
35	-39.4	-40.1	-40.0	-40.9	-42.7	-43.9	-44.7	-47.1	-49.7	-51.5	-53.1	-55.0
34	-39.3	-39.7	-39.9	-40.9	-42.6	-44.3	-46.1	-48.8	-51.3	-53.3	-55.9	-58.6
33	-39.1	-39.1	-38.8	-39.9	-41.9	-44.0	-46.3	-49.2	-51.7	-54.3	-57.3	-60.0
32	-38.2	-38.0	-37.9	-39.2	-41.1	-43.4	-46.2	-49.5	-52.4	-55.5	-58.1	-59.6
31	-38.7	-39.0	-38.9	-40.1	-41.8	-43.8	-46.8	-50.7	-53.9	-57.1	-59.5	-60.0
30	-39.4	-39.5	-39.2	-40.3	-42.0	-43.7	-46.9	-51.2	-54.4	-58.2	-61.2	-61.1
29	-39.5	-39.4	-39.1	-40.3	-42.0	-43.6	-46.7	-50.7	-54.0	-58.6	-62.2	-62.3
28	-40.0	-39.2	-38.1	-39.2	-40.9	-42.5	-45.5	-49.4	-52.2	-56.7	-60.4	-60.5
27	-39.9	-39.4	-37.9	-38.9	-40.5	-41.9	-44.9	-48.4	-51.5	-55.9	-59.4	-59.6
26	-38.1	-38.5	-37.3	-38.0	-39.8	-41.3	-44.1	-46.9	-50.0	-54.9	-58.1	-58.5
25	-36.4	-37.6	-36.7	-37.1	-39.5	-41.4	-43.6	-45.5	-48.1	-52.4	-55.7	-57.3
24	-34.8	-37.2	-37.1	-37.2	-39.6	-41.3	-42.5	-43.8	-45.4	-47.8	-50.6	-53.3
23	-31.9	-34.9	-35.2	-35.2	-36.9	-38.3	-39.8	-42.0	-44.4	-47.7	-50.6	-52.3
22	-29.7	-33.0	-33.5	-34.3	-36.1	-37.2	-38.5	-40.3	-43.5	-49.3	-53.3	-55.3
21	-28.2	-31.1	-32.4	-34.2	-35.8	-36.8	-38.4	-39.6	-41.3	-44.7	-47.7	-51.8
20	-25.1	-27.6	-28.5	-29.8	-30.7	-32.3	-36.0	-38.6	-40.8	-43.1	-44.9	-48.3
19	-21.6	-23.7	-24.0	-24.9	-26.2	-28.5	-32.5	-35.8	-38.5	-41.7	-45.2	-47.2

TABLE 9. RECTIFIED GEOID FOR OUS-275 DATA (N,meters), (Continued)

LONGITUDES (degrees)

LATITUDES (degrees)	286	287	288	289	290	291	292	293	294	295	296	297	298
45	-36.5	-36.2	-35.6	-35.1	-34.5	-34.2	-33.7	-32.6	-31.7	-30.8	-30.1	-28.8	-27.2
44	-37.2	-36.6	-36.5	-36.2	-35.9	-35.5	-34.5	-33.4	-33.0	-32.8	-31.9	-30.1	-27.9
43	-38.6	-37.9	-37.7	-37.5	-37.2	-36.7	-35.9	-34.9	-34.6	-34.7	-34.1	-32.3	-30.1
42	-40.2	-39.7	-39.7	-39.5	-39.1	-38.9	-38.0	-37.0	-37.0	-37.6	-37.5	-36.4	-35.0
41	-42.1	-42.0	-42.5	-42.2	-41.9	-41.8	-40.6	-39.5	-40.3	-41.2	-40.8	-40.0	-39.0
40	-44.6	-44.8	-45.7	-45.9	-45.5	-45.3	-44.4	-43.3	-43.6	-44.2	-43.5	-42.4	-41.2
39	-46.9	-47.9	-49.1	-49.1	-48.5	-48.4	-48.1	-47.1	-46.4	-45.8	-44.7	-43.4	-42.5
38	-49.5	-51.1	-51.9	-51.6	-51.1	-50.9	-50.7	-49.9	-48.7	-47.6	-46.2	-44.8	-43.8
37	-52.0	-53.5	-54.2	-54.0	-53.6	-53.4	-52.9	-51.8	-50.4	-49.0	-48.0	-46.7	-45.3
36	-54.2	-55.8	-56.3	-55.8	-55.3	-55.1	-54.5	-53.2	-51.5	-50.1	-49.4	-48.4	-47.1
35	-57.3	-58.5	-58.8	-58.2	-57.8	-57.2	-56.5	-55.2	-53.3	-51.6	-50.8	-50.0	-48.8
34	-60.4	-61.1	-61.3	-60.7	-60.1	-58.9	-58.0	-56.8	-54.8	-52.5	-51.2	-50.2	-49.2
33	-61.6	-62.3	-62.3	-61.5	-61.0	-59.8	-58.7	-57.5	-55.1	-52.1	-50.6	-50.4	-49.5
32	-61.5	-63.0	-63.2	-62.6	-62.3	-61.2	-59.7	-58.1	-55.5	-52.5	-51.1	-50.7	-49.9
31	-61.5	-63.5	-64.1	-63.7	-63.5	-62.4	-60.8	-58.7	-56.4	-54.3	-52.8	-51.6	-50.3
30	-61.8	-63.3	-64.0	-64.2	-64.3	-63.7	-62.2	-59.9	-57.8	-56.4	-55.3	-54.1	-52.7
29	-62.4	-63.4	-64.4	-65.1	-65.7	-64.9	-63.7	-61.9	-59.8	-58.4	-57.9	-57.0	-56.0
28	-60.5	-61.7	-63.1	-64.1	-64.7	-63.9	-62.9	-62.0	-60.1	-58.8	-58.4	-57.9	-57.4
27	-59.9	-61.7	-63.0	-63.8	-64.7	-64.0	-63.3	-62.5	-60.9	-59.7	-59.3	-59.1	-58.9
26	-58.7	-60.4	-61.6	-62.8	-64.1	-64.0	-63.6	-62.8	-61.2	-60.4	-60.1	-60.2	-60.1
25	-57.5	-58.6	-59.7	-60.9	-62.6	-63.4	-63.5	-62.6	-60.9	-60.3	-60.2	-60.5	-60.4
24	-53.7	-54.8	-56.2	-57.5	-59.3	-61.4	-63.0	-62.8	-61.4	-60.7	-60.8	-61.1	-61.0
23	-53.2	-54.3	-56.4	-58.2	-59.8	-61.9	-63.3	-63.5	-62.8	-62.5	-62.5	-62.5	-61.9
22	-57.0	-59.0	-60.1	-61.5	-64.0	-64.9	-64.6	-64.4	-64.3	-64.3	-64.3	-63.8	-63.1
21	-57.0	-61.2	-62.5	-63.8	-66.4	-67.1	-67.2	-68.0	-68.6	-68.5	-67.3	-66.5	-65.5
20	-52.3	-55.0	-56.8	-59.2	-62.0	-66.4	-70.6	-73.4	-74.0	-72.6	-71.2	-71.2	-70.3
19	-47.9	-48.4	-49.3	-52.4	-57.0	-62.7	-66.9	-67.9	-66.7	-66.0	-66.9	-70.1	-73.0

TABLE 10. RECTIFIED GEOID FOR GSFC/C-BAND DATA (N,meters)

LONGITUDES (degrees)

LATITUDES (degrees)	274	275	276	277	278	279	280	281	282	283	284	285
45	-43.7	-43.1	-43.4	-43.8	-43.8	-43.7	-42.8	-42.0	-41.1	-39.1	-37.3	-36.1
44	-42.1	-41.7	-42.0	-42.2	-42.7	-43.2	-42.9	-42.5	-41.5	-39.3	-37.0	-35.7
43	-40.9	-40.7	-40.9	-41.3	-41.8	-42.4	-42.1	-41.4	-40.6	-39.1	-37.0	-35.6
42	-40.2	-40.1	-40.4	-40.8	-40.9	-40.9	-40.2	-39.5	-38.8	-38.0	-36.9	-35.8
41	-39.8	-39.4	-39.5	-39.8	-39.5	-39.4	-39.0	-38.5	-38.0	-37.3	-37.0	-36.9
40	-39.6	-39.1	-38.5	-38.4	-38.5	-38.9	-38.4	-38.3	-38.3	-37.7	-37.7	-38.0
39	-38.8	-38.5	-37.7	-37.4	-37.8	-37.9	-37.3	-37.9	-38.5	-38.4	-38.8	-39.0
38	-37.7	-37.5	-37.0	-37.2	-37.3	-37.2	-37.1	-37.7	-38.3	-39.2	-40.4	-40.6
37	-36.1	-35.8	-35.4	-36.5	-37.5	-37.9	-37.9	-38.1	-39.1	-40.8	-42.2	-42.4
36	-34.8	-35.0	-34.5	-35.3	-37.1	-38.1	-37.5	-38.5	-40.1	-41.8	-42.9	-43.5
35	-33.9	-34.3	-33.8	-34.4	-35.8	-36.6	-37.1	-39.1	-41.4	-42.8	-44.0	-45.6
34	-33.4	-33.5	-33.2	-33.9	-35.2	-36.6	-38.0	-40.4	-42.6	-44.2	-46.5	-48.8
33	-32.8	-32.4	-31.7	-32.5	-34.1	-35.9	-37.8	-40.4	-42.6	-44.8	-47.5	-49.8
32	-31.5	-31.0	-30.5	-31.5	-33.0	-35.0	-37.4	-40.4	-43.0	-45.7	-48.0	-49.2
31	-31.7	-31.7	-31.3	-32.1	-33.5	-35.1	-37.8	-41.3	-44.3	-47.1	-49.2	-49.4
30	-32.2	-31.9	-31.4	-32.1	-33.5	-34.9	-37.7	-41.7	-44.6	-48.1	-50.7	-50.4
29	-32.1	-31.7	-31.1	-32.0	-33.4	-34.6	-37.4	-41.1	-44.1	-48.3	-51.7	-51.4
28	-32.6	-31.5	-30.0	-30.8	-32.2	-33.5	-36.1	-39.8	-42.2	-46.4	-49.9	-49.6
27	-32.4	-31.7	-29.8	-30.5	-31.8	-32.9	-35.5	-38.8	-41.5	-45.7	-48.8	-48.8
26	-30.6	-30.8	-29.2	-29.7	-31.1	-32.3	-34.8	-37.3	-40.1	-44.8	-47.6	-47.8
25	-29.0	-29.9	-28.7	-28.8	-30.9	-32.5	-34.4	-36.0	-38.3	-42.4	-45.3	-46.7
24	-27.6	-29.6	-29.3	-29.0	-31.2	-32.5	-33.5	-34.5	-35.9	-37.9	-40.5	-42.9
23	-24.8	-27.6	-27.5	-27.3	-28.6	-29.8	-31.0	-33.0	-35.0	-38.1	-40.7	-42.2
22	-22.9	-25.8	-26.1	-26.6	-28.1	-28.9	-30.0	-31.5	-34.4	-39.9	-43.7	-45.4
21	-21.6	-24.2	-25.2	-26.8	-28.0	-28.8	-30.1	-31.1	-32.5	-35.7	-38.5	-42.3
20	-18.8	-21.0	-21.6	-22.7	-23.3	-24.7	-28.0	-30.4	-32.3	-34.4	-35.9	-39.2
19	-15.6	-17.5	-17.5	-18.1	-19.1	-21.2	-24.9	-28.0	-30.4	-33.4	-36.6	-38.5

TABLE 10. RECTIFIED GEOID FOR GSFC/C-BAND DATA (N,meters)
(Continued)

LONGITUDES (degrees)

LATITUDES (degrees)	286	287	288	289	290	291	292	293	294	295	296	297	298
45	-35.3	-34.6	-33.7	-32.7	-31.7	-31.1	-30.1	-28.7	-27.4	-26.2	-25.0	-23.4	-21.4
44	-34.7	-33.8	-33.3	-32.6	-31.9	-31.1	-29.7	-28.2	-27.5	-26.8	-25.6	-23.5	-20.9
43	-35.0	-33.9	-33.3	-32.7	-32.0	-31.2	-30.0	-28.6	-28.0	-27.7	-26.7	-24.6	-22.0
42	-35.5	-34.7	-34.2	-33.7	-32.9	-32.3	-31.1	-29.6	-29.3	-29.6	-29.1	-27.6	-25.9
41	-36.4	-36.0	-36.1	-35.4	-34.7	-34.3	-32.7	-31.3	-31.7	-32.2	-31.5	-30.4	-29.0
40	-38.0	-37.9	-38.4	-38.2	-37.5	-36.9	-35.7	-34.2	-34.1	-34.4	-33.4	-31.9	-30.4
39	-39.6	-40.2	-41.0	-40.7	-39.8	-39.2	-38.6	-37.2	-36.2	-35.3	-33.8	-32.2	-31.0
38	-41.4	-42.7	-43.1	-42.5	-41.7	-41.1	-40.5	-39.3	-37.8	-36.4	-34.6	-32.9	-31.7
37	-43.3	-44.5	-44.8	-44.2	-43.5	-42.9	-42.1	-40.7	-38.9	-37.3	-36.0	-34.3	-32.6
36	-45.0	-46.2	-46.4	-45.5	-44.7	-44.2	-43.2	-41.6	-39.6	-37.8	-36.9	-35.5	-33.9
35	-47.5	-48.4	-48.4	-47.5	-46.8	-45.8	-44.8	-43.1	-40.9	-39.0	-37.8	-36.7	-35.3
34	-50.3	-50.7	-50.5	-49.6	-48.6	-47.2	-46.0	-44.4	-42.1	-39.5	-37.9	-36.7	-35.3
33	-51.1	-51.5	-51.2	-50.1	-49.2	-47.8	-46.3	-44.8	-42.2	-38.8	-37.1	-36.5	-35.4
32	-50.8	-51.9	-51.8	-51.0	-50.3	-48.9	-47.1	-45.2	-42.4	-39.0	-37.4	-36.6	-35.6
31	-50.6	-52.3	-52.5	-51.9	-51.3	-50.0	-48.1	-45.7	-43.1	-40.7	-39.0	-37.5	-35.9
30	-50.7	-51.9	-52.4	-52.2	-52.1	-51.1	-49.4	-46.8	-44.4	-42.8	-41.4	-39.9	-38.2
29	-51.3	-52.0	-52.7	-53.1	-53.4	-52.4	-50.8	-48.8	-46.5	-44.8	-44.0	-42.8	-41.6
28	-49.4	-50.3	-51.4	-52.1	-52.5	-51.3	-50.1	-49.0	-46.8	-45.2	-44.5	-43.8	-43.1
27	-48.8	-50.3	-51.4	-51.9	-52.5	-51.5	-50.6	-49.5	-47.7	-46.3	-45.6	-45.2	-44.7
26	-47.7	-49.1	-50.1	-51.0	-52.1	-51.7	-51.1	-50.0	-48.2	-47.2	-46.6	-46.5	-46.1
25	-46.7	-47.5	-48.4	-49.3	-50.7	-51.2	-51.1	-50.0	-48.1	-47.3	-46.9	-47.0	-46.6
24	-43.1	-43.9	-45.1	-46.2	-47.7	-49.6	-50.9	-50.5	-48.8	-47.9	-47.8	-47.9	-47.6
23	-42.8	-43.7	-45.5	-47.1	-48.5	-50.3	-51.5	-51.4	-50.6	-50.0	-49.8	-49.6	-48.8
22	-46.9	-48.6	-49.5	-50.7	-53.0	-53.7	-53.1	-52.7	-52.3	-52.2	-51.9	-51.3	-50.4
21	-47.3	-51.2	-52.3	-53.3	-55.7	-56.2	-56.1	-56.6	-57.1	-56.8	-55.4	-54.4	-53.2
20	-42.9	-45.4	-47.0	-49.1	-51.8	-55.9	-59.9	-62.5	-62.9	-61.3	-59.7	-59.6	-58.4
19	-39.0	-39.2	-39.9	-42.8	-47.2	-52.6	-56.7	-57.4	-56.1	-55.1	-55.9	-59.0	-61.6

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