X-932-77-267
PREPRINT

TM 78055

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NOVEMBER 1977

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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*Preprint from Geophysical Surveys, January 1977.

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ABSTRACT

A detailed and accurate Earth gravity field model is important to the understanding of the structure and composition of the Earth's crust and upper mantle. This paper analyzes and compares various satellite-based techniques for providing more accurate models of the gravity field.

A high-low configuration satellite-to-satellite tracking mission is recommended for the determination of both the long wavelength and short wavelength portions of the field. Satellite altimetry and satellite gradiometry missions are recommended for determination of the short wavelength portion of the field.

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INTRODUCTION

Since the beginning of the space age, satellite tracking information has been utilized to improve knowledge of the Earth's gravity field. This method arose because errors in the low degree and order spherical harmonic coefficients of the Earth's potential leave a discernable trace in the residuals of satellite tracking data. Thus, the choice of the spherical harmonic representation was a natural consequence of the use of this particular data type. The conventional method for estimating a field from measured orbit perturbations has been to express the gravity field in terms of a standard spherical harmonic expansion and to adjust the coefficients of the expansion to best fit the data according to a least squares criterion. The first coefficient to be accurately recovered from satellite data was the second harmonic, which describes the Earth's flattening at the poles relative to the Equator. Soon afterward, the third zonal harmonic was also accurately determined. As the number of satellites increased and as tracking improved it was possible to independently estimate a large number of coefficients of the gravity field. At present, estimates of spherical harmonic coefficients complete to degree and order 12 are available.

There are several reasons why a significant improvement in gravity field models is desirable. An obvious one is that with a better model every satellite mission could be performed more economically since, for a given orbit determination accuracy, less tracking data acquisition and processing would be required. Another reason is that a detailed and accurate gravity field model is useful for providing information about Earth and Ocean Physics. For instance, a global map of 2° or 3° mean gravity anomalies in conjunction with correlated magnetic anomalies will reveal much about the structure and composition of the Earth's crust and upper mantle. This information is of practical importance in suggesting mechanisms which cause motions in the outer crust and also in isolating concentrations of exploitable mineral resources. Oceanographers are interested in improved gravity field models since better models would permit a more accurate determination of the marine geoid which may be defined as that equipotential surface to which the ocean surface would cohere if no dynamic effects intervened. An accurate knowledge of this surface will permit a separation of static from dynamic influences on mean sea level, which in turn will provide knowledge of ocean circulation, storm surges and other dynamic effects on sea surface typography.

It is generally agreed that substantial improvements in present models require the utilization of new types of satellite measurement systems, new gravity field representations, and possibly new estimation techniques. The proposed measurement systems divide into the following categories.

(1) Satellite to Satellite Tracking, which is essentially a measurement of a range or a range rate between two satellites rather than a measurement between a satellite and a ground station. Proposed satellite to satellite

tracking systems can also be divided into two distinguishable categories: hi-lo configurations, and lo-lo configurations. The hi-lo configuration implies the tracking of a low altitude satellite by a high altitude relay satellite. The lo-lo configuration implies tracking between two satellites in identical low altitude orbits separated by a few hundred kilometres.

(2) Spacecraft Borne Radar Altimetry. A spacecraft borne altimeter measures the distance between the satellite and its subearth point on the Earth's surface. Altimeters have flown successfully on both the SKYLAB and GEOS-C satellites.

(3) Spacecraft Borne Gravity Gradiometry. A gravity gradiometer measures gradients of the Earth's gravity field. Gradiometers have been flown on airplanes but have not yet been placed on board satellites. We will analyze the advantages and disadvantages of each of these measuring systems. We will also provide recommendations for future spacecraft missions for estimating the gravity field.

Any attempt to estimate a detailed and global gravity field creates a difficult data reduction problem. This fact has motivated research on new estimation techniques and some of this research has been controversial. We will attempt to clarify some of the issues involved and we will express our opinion concerning proper data reduction procedures.

The difficulties encountered in mapping the long wavelength features of the gravity field (say greater than 1000 km) are different in character from those encountered in determining shorter wavelength features. Hence, in this paper the recovery of long wavelength and short wavelength features are treated as separate estimation problems in separate sections.

LONG WAVELENGTH GRAVITY FIELD ESTIMATION

A prominent feature of NASA's Applications Program is the use of satellites as platforms from which highly accurate instruments globally monitor natural phenomena. The accuracy of these instruments has led to demands for very accurate orbit determinations. As an example, the altimeter on board the GEOS-3 spacecraft has an altitude resolution of 1 to 2 metres. Comparable orbit altitude determination will be difficult to obtain (Argentiero and Garza-Robles, 1975). Another example is the effort to monitor tectonic plate motions by LASER tracking of satellites. Agreen and Smith (1973) have shown that the major difficulty is the lack of adequate orbit determination accuracy. Other types of missions have similar problems. For instance, Earth resources satellites are equipped with sophisticated imaging equipment which cannot be fully exploited without a very accurate orbit determination. The major impediment to achieving high orbit determination accuracies is the uncertainty in our present estimate of the long wavelength portion of the gravity field. A significant improvement of this estimate is necessary if NASA's applications program is to achieve its goals.

Present estimates of the long wavelength portion of the gravity field are based primarily on satellite perturbation data obtained from ground based tracking stations. The usual procedure for obtaining a gravity field from the data is to parameterize the field by means of low degree and order spherical harmonic coefficients and to adjust the coefficients according to a least squares method. It is doubtful if this mathematical procedure can be improved and we agree with Kaula (1970) who states: "Because of the characteristics of close satellite orbit dynamics and orbit determination from ground tracking, spherical harmonics will continue to be the most suitable representation of the main part of the gravity field indefinitely."

It is surprising how much uncertainty remains in satellite derived estimates of the long wavelength gravity field. Lerch, et al. (1974) calibrated the Goddard Earth Model 5 spherical harmonic expansion of the gravity field against actual observations of 15° by 15° mean gravity anomalies, and nominal standard deviation values were scaled to be consistent with the residuals. The resultant standard deviations are displayed as percentages of Kaula's "rule of thumb," $10^{-5}/L^2$ where L is the degree of the normalized spherical harmonic coefficient. This is an empirical formula used to approximate the power spectral density function of the gravity field in Figure 1. The coefficients to degree 12 are seen to be uncertain to within 5% to 60% of their nominal values. Agreen and Smith (1973) estimate that gravity field estimates must be improved by a factor of 7 for effective satellite monitoring of tectonic plate motions. Results generated by Koch et al. (1973) suggest that gravity field models must be improved by a factor between 7 and 8 if altitude resolution of applications satellites comparable with altimeter accuracy is to be achieved. Bryant (1975) asserts that a factor of 5 improvement in present gravity field models is necessary to obtain the orbit determination accuracy desired for the Earth resources satellites. We take as a reasonable goal for a satellite mission designed for long wavelength gravity field estimation, a factor of 10 improvement in present gravity field models.

To see what is required for the design of such a mission it is necessary to understand why although large amounts of satellite perturbation data are available, gravity field models still exhibit the significant errors shown in Figure 1. In theory the geopotential field is represented by an infinite series of spherical harmonic coefficients. Numerical procedures for estimating the gravity field from satellite perturbation data involves the recovery of coefficients below a certain degree and order. Higher degree and order coefficients are assumed to be zero. Since these coefficients are not in fact zero, a certain aliasing effect occurs. This effect can be defined as the distortion in the estimate of a parameter set which occurs when other parameters in the models used in the estimation are misrepresented. The aliasing phenomenon can be demonstrated in terms of a simple numerical example taken from Koch (1975). A natural phenomenon is modeled correctly by the quadratic $Y = x^2 + x + 1$. An investigator, however, assumes a linear model, $Y = a x + b$. This assumed model neglects the second degree term of the correct model, in effect equating it to zero. Next he performs a standard least squares fit to the three data points $Y(0) = 1$, $Y(5) = 31$, $Y(10) = 111$ using the linear model. The least squares procedure yields as a solution $Y = 11x - 7.33$. The estimated coefficients are $a = 11$, $b = -7.33$ whereas the correct values are $a = 1$, $b - 1$. Thus, neglecting the second degree term in the correct model has seriously degraded the quality of the parameter estimates. The degradation is a simple example of the aliasing phenomenon. In a similar fashion, the neglect of uncertainties in higher degree and order geopotential coefficients aliases the estimates of lower degree and order coefficients. It can be shown that a necessary and sufficient condition for the elimination of this aliasing effect is the possession of a dense and globally distributed data set. Since locations of tracking stations are limited by geographical and political considerations, it is not possible to obtain a global distribution of low altitude satellite perturbation data by conventional methods. Hence estimates of long wavelength gravity fields continue to be plagued by severe aliasing. This is essentially an observability problem and no amount of additional data collected from the same well-covered areas will substantially improve the situation.

A logical solution to the problem is suggested by the possibility of tracking a low altitude high inclination satellite by means of a high relay satellite. Numerical studies indicate that with a low satellite at a 300 km altitude there is sufficient sensitivity in the satellite to satellite tracking data to improve present estimates of geopotential coefficients to degree and order 22. We have computed the perturbations of satellite to satellite range rate sum data between a geosynchronous satellite and a satellite in a polar, circular, 300 km orbit, caused by geopotential coefficient perturbations. The 276 cosine terms of the spherical harmonic expansion of the field to degree and order 22 were perturbed by current estimates of term uncertainties as obtained from Figure 1. These individual perturbations were propagated into variations of the range rate sum data over a 24-hour period. Figure 2 is a histogram of the mean absolute values of the range rate sum variations over the data arc. The graph shows that for over 96% of the geopotential coefficients, the data perturbations caused by the difference between nominal and actual values have an average amplitude greater than the present estimate of satellite to satellite tracking accuracy of 1 mm/s.

Figure 2 demonstrates that errors in geopotential coefficients can be sensed in the data. But this does not imply that the coefficients can be decoupled and independently estimated from information supplied by the range rate sum data. To determine the recoverability of the coefficients in this sense generally requires a covariance analysis in which the individual standard deviations and the statistical correlations of the estimates are computed.

Argentiero et al. (1974) used a covariance analysis to study the possibility of estimating geopotential coefficients from satellite to satellite tracking of GEOS-3 with ATS-6 as a relay satellite. The results show that if the GEOS-3 state and ATS-6 state are simultaneously estimated with coefficients of a geopotential field to degree and order 8, then the resultant coefficient estimates are improved by one to two orders of magnitude over present estimates. The authors do not account for the aliasing effect due to uncertainties in higher degree and order coefficients. Thus, the results are no doubt optimistic. Also, the correlation coefficients between estimates of the GEOS-3 state and ATS-6 state were quite high. This suggests that there may be difficulties with such experiments in obtaining a convergence of the least squares iteration procedure. Experience to date with the ATS-6/GEOS-3 range rate sum data implies that this is the case. An ideal solution to this problem could be provided if an accurate a priori fix on the relay satellite epoch state were extracted from a combination of ranging and trilateration data. An accurate estimate of the state of a geosynchronous satellite (say to the 15 or 20 metre level) is difficult to obtain since there is very little motion between the satellite and ground based tracking stations. Schmid and Lynn (1975) report promising results with reduction of the trilateration data type and research is continuing on this subject.

An alternative configuration for a satellite to satellite tracking experiment, first suggested by Siry (1971), is provided by the dual GRAVSAT/GEOPAUSE mission. The GRAVSAT and GEOPAUSE satellites are to be coplaner in orbits perpendicular to both the Earth's equator and the ecliptic plane. The high or GEOPAUSE satellite is placed in a circular orbit at about 3.6 Earth radii above the Earth's surface. The low or GRAVSAT satellite is in a circular orbit about 300 km above the Earth's surface. Tracking between the GRAVSAT and GEOPAUSE is relayed from the GEOPAUSE to ground based tracking stations.

Six properly chosen tracking stations, three in the Northern Hemisphere and three in the Southern Hemisphere, are adequate to maintain constant communication with the GEOPAUSE satellite.

Koch and Argentiero (1974) used covariance analysis techniques to study the potential of the GRAVSAT/GEOPAUSE mission for determining gravity field coefficients to degree and order 8. The results show that the data obtained from the mission can yield a two orders of magnitude improvement of geopotential coefficients. The estimates were relatively independent with most of the 3200 correlation coefficients between geopotential coefficient estimates of absolute value less than 0.01. Also, there should be little difficulty in obtaining a good a priori fix on the GEOPAUSE epoch state. Hence, it should be possible to obtain convergence of the least squares iteration procedure when both satellites are estimated from the data. The results of this study should be considered as somewhat optimistic since again the effect of aliasing from higher degree and order coefficients was not taken into account.

Even with the excellent data distributions obtainable from hi-lo configuration satellite to satellite tracking experiments, geopotential aliasing is still a serious problem. Koch (1975) obtained quantitative measures of the geopotential aliasing effect for the GRAVSAT/GEOPAUSE satellite configuration. He determined that uncertainties in unadjusted coefficients of degree 12 significantly alias adjusted coefficients of degree as low as 8. Koch suggests that for a good determination of the field to degree and order 8, a field of degree and order 12 should be estimated from the data and estimates of terms of degree 9 through 12 discarded due to aliasing. These conclusions are compatible with the results of an earlier simulation of geopotential aliasing performed by Anderle et al. (1969).

In summary, a global data distribution is necessary for a significant improvement in present estimates of the long wavelength gravity field. The only feasible way to achieve such a distribution is by the satellite to satellite tracking of a low altitude, high inclination satellite using a high relay satellite. Two such configurations have been investigated: the use of a geosynchronous relay satellite, and the use of a high altitude polar satellite (GEOPAUSE) as a relay satellite. Numerical studies suggest that both configurations are capable of providing a data set from which an order of magnitude improvement in estimates of geopotential coefficients can be obtained. Other studies show that even with the excellent data distribution obtainable from these missions, data reduction procedures must be carefully designed to eliminate the effect of geopotential aliasing.

SHORT WAVELENGTH GRAVITY FIELD RECOVERY

The Data Reduction Problem

A global knowledge of gravity field fine structure is fundamental to the understanding of solid Earth and Ocean dynamics. A major goal of NASA's applications program is a global gravity field mapping sufficiently detailed to show features as small as 3°. This is equivalent to estimating spherical harmonic coefficients of the gravity field to degree and order 60.

Any effort to obtain such a global and detailed gravity field creates a severe data reduction problem. The essence of the problem is that a large number of parameters must be estimated. For instance, the spherical harmonic coefficients of the gravity field to degree and order 60 number over 3700. It is not possible to simultaneously estimate such large parameter sets. In practice, it is necessary to adjust small subsets of parameters at one time while constraining the rest to a priori values. But unless the data set and the gravity field parameterization bear a certain mathematical relationship to each other, the net effect is that uncertainties of the unadjusted terms will badly corrupt the estimates of the adjusted terms. This is the aliasing effect discussed at length in the previous section. The required mathematical relationship has been described as an orthogonality of a parameterization in a data type. The property is rigorously defined by Argentiero et al. (1976), but it may be loosely described as a relationship between a data set and a parameterization which permits a decomposition of the large dimensional estimation problem into estimation problems of much smaller dimensionality and without fear of serious aliasing. Because of these data reduction considerations, any satellite mission designed to provide a global mapping of gravity field fine structure must generate a data set which has an orthogonality relation with at least one parameterization of the gravity field.

Research on approximate representations of the gravity field have focused on the development of localized parameterizations. Such representations have the property that if a given parameter of the representation is perturbed, then the representation is perturbed only in a given localized area. Spherical harmonic coefficients do not have this property since a perturbation of a spherical harmonic coefficient disturbs the representation of the gravity field everywhere outside the reference sphere. Some of the gravity field representations which have the property are those which utilize mean gravity anomalies, surface density blocks, sample functions, and mass concentrations. Localized gravity field representations are generally recommended for gravity fine structure recovery because in some local satellite data types these parameterizations exhibit a high degree of orthogonality.

The difficulties in optimally combining diverse geodetic data types to estimate geopotential fine structure has lead to the development of an estimation procedure called *least squares collocation*. This estimator differs formally from the conventional least squares estimator which has seen general use since first introduced by Gauss in 1809. Several authors have claimed that the least squares collocation method is a more general and more powerful parameter estimation procedure than the classical least squares method. Moritz (1974a, 1974b) asserts that least squares collocation is the only parameter estimation method which permits the simultaneous and optimal processing of heterogeneous data types. Rapp (1974) states that the use of conventional least squares techniques in estimating mean gravity anomalies can lead to false or misleading results. This criticism is repeated by Uotila (1975).

Both Tapley (1975) and Rummel (1976) mention that least squares collocation appears in the literature in two formally different versions. Tapley has shown that both collocation models can be cast into a form in which standard least squares reduction procedures are applicable and that the solution so obtained is identical to the collocation solution. Argentiero and Lowrey (1977) have shown that the collocation algorithms are derivable from the well-known regression equations. From this vantage point they provide a second proof that the collocation algorithms are equivalent to standard least squares reduction procedures.

Although the two estimation procedures are mathematically equivalent, their computational properties differ. The conventional least squares procedure requires the inversion of a square matrix whose dimension is the size of the estimated parameter set. However, the least squares collocation algorithm requires the inversion of a square matrix whose dimension is the size of the data set. This is an undesirable feature since it severely limits the size of a data set which can be used in an estimation. Hence, from the vantage point of computational convenience the conventional least squares algorithm is the preferred method for estimating gravity field fine structure from geodetic data.

Several satellite missions have been proposed as capable of providing a data set from which geopotential fine structure can be recovered. These missions divide into four types: satellite altimetry missions, satellite gradiometry missions, satellite-to-satellite tracking missions using a hi-lo satellite configuration, and satellite-to-satellite tracking missions using a lo-lo satellite configuration. Each of these mission types is discussed in a separate section.

SATELLITE ALTIMETRY

If no dynamic effects such as tides intervened then the mean sea level would be an equipotential surface known as the marine geoid. After suitable corrections, the output of a satellite borne altimeter over an ocean area may be viewed as a direct observation of the height of the ocean geoid at the subsatellite point. With regard to using altimeter data for estimating gravity fields, two limitations are apparent. First, the altimeter output has significance for the gravity field only over the ocean. But, since most of the Earth is covered by oceans, this is not a fatal limitation. The second limitation is that errors in the altitude estimate of the satellite project directly onto errors in the altimeter data. Satellite borne altimeters are assumed to be accurate to within one metre. Comparable altitude resolution of the satellite is difficult to obtain. It may be possible to exploit the spectral properties of altitude errors to remove their effects on the altimeter data. This possibility has not been thoroughly investigated.

Independent studies by Gopalapilli (1974) and Argentiero et al. (1976) show that the mean gravity anomaly parameterization of the gravity field displays a high degree of orthogonality in altimeter data. This implies that local blocks of altimetry data can be used to estimate local blocks of gravity anomalies without serious aliasing. Hence, with altimeter data the data reduction problem described in a previous section can be reduced to manageable proportions.

The results generated by Gopalapilli and by Argentiero et al. appear to be quite compatible. Both studies conclude that local blocks of gravity anomalies can be estimated in local blocks of altimeter data, provided that validly estimated gravity anomalies are separated from unadjusted anomalies by at least 10°. Gravity anomalies as small as 2° can be recovered with reasonable accuracy and 5° anomalies can be recovered with an accuracy of $10\mu\ m/s^2$ (1 mgal). These results are predicated on the assumptions that altimeter data is accurate to 1 m and that various biasing effects, including those due to orbit determination error, can be removed from the data.

Rapp (1974) and Smith (1974) used the least squares collocation to simulate the recovery of mean gravity anomalies from altimeter data. When differences in assumptions concerning altimeter data densities are accounted for, the results generated by Rapp and by Smith are compatible with the results obtained by Gopalapilli and by Argentiero et al.

The numerical simulations performed to date strongly indicate that satellite altimetry is a very useful data type for recovering geopotential fine structure. Limited experience with actual satellite borne altimeter data is also encouraging. Argentiero et al. display a graph of Skylab altimeter residuals obtained during June 1973. The graph clearly shows the ability of altimeter data to reveal short wavelength features of the marine geoid. At present, work proceeds on the processing of GEOS-C altimeter data to estimate mean gravity anomalies over ocean areas and preliminary results show promise. However, the aliasing effects of satellite altitude error continues to be a limiting factor in the usefulness of altimeter data. Further research into the possibility of eliminating this error either by more accurate orbit determinations or by sophisticated filtering techniques is required.

SATELLITE GRADIOMETRY

We are concerned here with a rotating type gradiometer which appears to be the most likely one to be used on a spacecraft mission. Two such instruments are under independent development by the Hughes Research Laboratory (1971) and the Bell Aerospace Company (1971). The instruments are electro-mechanical analogues of each other and hence their outputs relate to the gravity field in a mathematically identical fashion. Figure 3 is a simplified representation of a rotating gradiometer. Accelerometers A_1, A_2, A_3, and A_4 rotate in the place of the figure at angular velocity W. The outputs of the accelerometers are combined as shown on the figure to form a continuous signal. The measurement of the instrument is taken to be the amplitude of the signal which is a function of second derivatives of the scalar potential field in the sensing plane of the instrument. If such an instrument were mounted on a polar, low altitude satellite, it would provide a global distribution of in situ observations of the gravity field. An equivalent way of expressing this fact is to state that each observation in this globally distributed set would relate to the gravity field only in terms of where the satellite was at the time of the observation rather than where the satellite had been. Localized representations of the gravity field tend to have good orthogonality properties in such a data type.

The output of a gradiometer is conventionally measured in Eotvos units (1E unit = 10^{-9} gal/cm; 1 gal = 10^{-2} m/s^2). Bench tests of rotating gradiometers indicate an accuracy of about 1E. Theoretically, in a zero gravity environment such as what is obtained on an orbiting satellite much better accuracies should be available.

Reed (1973) uses the mean gravity anomaly representation of the gravity field to show the feasibility of estimating gravity field fine structure from gradiometer data. The orthogonality properties of gravity anomalies in gradiometer data were assumed rather than demonstrated. The effects of orbit and attitude errors were also ignored. Reed's simulations imply that with the data from a rotating gradiometer with a 1E accuracy and onboard a satellite at a 300 km altitude, 5° mean gravity anomalies can be recovered with an accuracy of about 10μ m/s^2.

Argentiero and Garza-Robles (1976, a) employ the techniques of covariance analysis to study the same problem that was attacked by Reed. The effects of orbit and altitude determination errors are again neglected but the aliasing effect of unadjusted anomalies are included in this study. The gradiometer accuracy is assumed to be 0.1E. The results are that when the satellite altitude is 250 km, 5° mean gravity anomalies can be recovered with an accuracy of 10μ m/s^2. This corresponds to a recovery of the marine geoid accurate to within 0.4 m.

Argentiero and Garza-Robles (1976, b) investigated the orbit and attitude determination requirements of a gradiometer mission. They conclude that the satellite orbit must be determined to within 50 m radially and 300 m horizontally. The attitude determination requirements are a 5° resolution for satellite spin vector azimuth and 0.2° resolution for satellite spin vector elevation.

It is instructive to compare the potential of a gradiometer mission for gravity field fine structure recovery with the potential of a satellite altimeter mission for the same task. Numerical simulations show that a rotating gradiometer with a 0.1E accuracy and on board a satellite in a 250 km altitude orbit provides a data set from which mean gravity anomalies can be estimated with an accuracy equivalent to what is obtainable from altimeter data with a 1 m accuracy. Also the orthogonality properties of localized parameterizations appear to be good in both gradiometer data and altimeter data. Thus, there should be no serious computational difficulties in estimating detailed gravity fields from either data type. But gradiometer data is useful for estimating the gravity field all over the Earth. Altimetry only has significance for the gravity field over ocean areas. Also, the orbit determination requirements for a gradiometer mission appear to be less severe than those implied by an altimeter mission. Hence, theoretical arguments favor a gradiometer mission over an altimeter mission. Practical considerations favor altimetry. The spacecraft technology required in support of an altimetry mission has been demonstrated on both the SKYLAB and the GEOS-3 satellite missions. A gradiometer has never been flown on a satellite. A space qualified rotating gradiometer is inherently a more sensitive and complex instrument than an altimeter. Considerable resources would be required to develop such an instrument with the required accuracy of 0.1E. In our opinion, the theoretical advantages of the rotating gradiometer warrant such a development.

A LO-LO CONFIGURATION SATELLITE-TO-SATELLITE TRACKING EXPERIMENT

The lo-lo configuration satellite-to-satellite tracking experiment would employ two satellites in the same circular orbit, with one following the other at a distance of a few hundred km. The hypothesis which motivates this configuration is that range rate data between the two satellites is sensitive to local anomalies but not to distant anomalies. This implies that it should be possible to estimate local blocks of gravity anomalies in local blocks of range or range rate data.

Schwarz (1970) performed the first simulation of this mission configuration. In this study the satellites are separated by 200 km. The reported results are that at a 700 km altitude 5° gravity anomalies can be recovered with an accuracy between $10\mu\,m/s^2$ (1 mgal) and $30\mu\,m/s^2$ (3 mgal). At a 200 km altitude, 2° gravity anomalies can be recovered with the same accuracy. This study has several limitations, the most serious of which is that it assumes rather than demonstrates that local blocks of gravity anomalies can be estimated in local blocks of range or range rate data generated by the experiment. If this is not the case then the data reduction problem is unsolvable and the experiment is not feasible.

Lowrey (1975) determined the signal strength of a given size gravity anomaly in range rate data generated by a lo-lo configuration satellite-to-satellite tracking experiment. This study concludes that at a satellite altitude of 300 km, gravitational features separated by less than 5° cannot be estimated from the information supplied by the range rate data.

Estes and Lancaster (1976, a) used the techniques of covariance analysis to investigate the feasibility of a lo-lo configuration system. In this study aliasing effects were properly considered as an error source. The results are that even with extremely accurate orbit determination support, recovery of gravity anomalies is unimpressive. Estes and Lancaster conclude that, "the estimation of gravitational fine structure by recovery of local sets of density blocks utilizing SST relative range rate data with the low-low configuration is not a feasible approach, principally due to the very stringent orbital accuracies required with many short data arcs."

On the basis of these results we believe that a lo-lo configuration satellite-to-satellite tracking experiment should be rejected as a means of determining geopotential fine structure.

A HI-LO CONFIGURATION SATELLITE-TO-SATELLITE TRACKING EXPERIMENT

In an earlier section we discussed the possibility of using a hi-lo configuration satellite-to-satellite tracking experiment to determine long wavelength features of the gravity field. Vonbun (1972) and others have also suggested this configuration for estimating the short wavelength features of the gravity field.

Sjogren (1975) has simulated the summed doppler data of the ATS-6/GEOS-3 satellite-to-satellite tracking experiment. He shows that by differentiating spline functions fitted to Doppler residuals one can reconstruct the anomalous acceleration profile of the satellite due to short wavelength gravity field features. He does not discuss how such acceleration profiles can be used to uniquely and accurately reconstruct the short wavelength gravity field.

The Apollo-Soyuz satellite-to-satellite tracking experiment, performed in July 1975, was the first demonstration of the capabilities of this data type to detect short wavelength features of the gravity field. The Apollo space capsule was in an approximately circular, 250 km altitude orbit. The ATS-6 relay satellite is in a geosynchronous orbit. The Apollo was tracked from the ATS-6 and the resulting range rate data was relayed to a ground tracking station at Madrid, Spain. Vonbun et al. (1975) show that the Indian Ocean anomaly was readily visible in the range rate signal from Apollo as tracked by ATS-6 during four different orbital passes. A sample signal from Revolution 8 of the Apollo spacecraft is shown in Figure 4. There is a rapid dip in the Doppler signal as it passes over the center of the anomalous low in the Indian Ocean, amounting to nearly 3 cm/s and extending over a 2 to 4 minute duration.

Hajela (1974) has investigated the feasibility of uniquely reconstructing the short wavelength gravity field from satellite-to-satellite tracking data resulting from a hi-lo configuration. In a comprehensive set of simulations he examines the possibilities of recovering 10°, 5°, and 2.5° mean gravity anomalies from range rate sum observations when the low satellite altitude is 250 km and when the low satellite altitude is 900 km. The results are relatively optimistic. However the aliasing effect of unadjusted anomalies on the estimate of adjusted anomalies is not properly accounted for in this study. The effect of uncertainty in the relay satellite epoch state is also ignored. In fact, this is likely to be a major error source. Because of these limitations, Hajela's results are difficult to interpret with regard to the overall feasibility of the mission.

Estes and Lancaster (1976, b) have also simulated the recovery of mean gravity anomalies from data generated by a hi-lo satellite-to-satellite tracking experiment. In this study the aliasing effect of unadjusted anomalies as well as uncertainties in epoch states of high and low satellites are properly considered as error sources. The high satellite is assumed to be geosynchronous and the low satellite is assumed to be in a circular, 250 km drag free orbit. The results indicate that with short arc data reduction techniques and with proper estimation procedures to reduce aliasing, 5° mean gravity anomalies can be recovered with an accuracy of $15\mu\,m/s^2$ (1.5 mgal). The high and low satellite orbit determination requirements for such accuracies are rigid but within the present state of the art.

A hi-lo configuration satellite-to-satellite tracking experiment appears to be a feasible approach to the problem of estimating geopotential fine structure. The Apollo-Soyuz satellite-to-satellite tracking experiment conducted by Vonbun et al. has demonstrated the basic technology required for such a mission. In addition, numerical simulations show that with proper data handling procedures the data set generated by such a mission will permit a good determination of global geopotential fine structure.

SUMMARY

This paper has presented a comparison of procedures for employing satellite technology to determine the Earth's gravity field. The problems involved in estimating long wavelength gravity field features are different in character from those involved in estimating short wavelength features. Hence, the recovery of long wavelength and short wavelength components of the gravity field are treated as separate estimation problems in separate sections.

Satellite perturbations represent an excellent data type for determining the long wavelength components of the gravity field. The recovery of spherical harmonic coefficients of the Earth's gravity field from satellite perturbation data has been standard practice and it continues to be the wisest procedure for using satellites to determine the long wavelength features of the gravity field. Present estimates suffer from severe aliasing because of a non-global distribution of data. The best satellite configuration for solving this problem is that of a low altitude, polar satellite tracked by a high altitude relay satellite. Separate studies have proposed a geosynchronous orbit for the relay satellite, and a high altitude polar orbit (the GEOPAUSE concept) for the relay satellite. Numerical simulations show that both configurations are capable of providing a global data set from which an order of magnitude improvement of estimates of the long wavelength gravity field can be extracted.

The major difficulty in employing parameter estimation techniques to recover the short wavelength gravity field is that a very large number of parameters must be estimated. For this reason it is desirable for a satellite mission to provide a global data set which permits the independent estimation of smaller subsets of the parameters which represent the field. With proper corrections, the output of a satellite borne altimeter over an ocean area can be considered as an observation of geoid height. This is an in situ data type and studies show that it is possible to accurately estimate local blocks of gravity anomalies in local blocks of altimeter data. Also, the satellite technology necessary to support a satellite borne altimeter mission has been successfully demonstrated on both the SKYLAB and the GEOS-3 satellite missions. A major limitation of the use of a satellite borne altimeter for geodetic purposes is that its output relates to the gravity field only over ocean areas. Another difficulty is that requirements for altitude resolution of the spacecraft are on the order of one metre.

A spacecraft borne rotating gradiometer mission is also capable of providing a global distribution of in situ gravity field observations. Simulations show that a rotating gradiometer functioning with sufficient accuracy and on board a satellite in a 250 km altitude orbit will provide a gravity field estimate equivalent in resolution and accuracy to that obtainable by satellite altimetry. But unlike altimetry, the output of a rotating gradiometer has geodetic significance over all parts of the Earth. Also, the orbit determination requirements for a satellite gradiometer mission are less severe than the requirements for a satellite altimeter mission. Because of the inherent advantages of this instrument we recommend that a space qualified rotating gradiometer be developed for a low altitude applications satellite.

Satellite-to-satellite tracking missions in both hi-lo and lo-lo configurations have been proposed for mapping gravity field fine structure. Numerical simulations prove that the hi-lo configuration is more suitable for the task. The Apollo-Soyuz satellite-to-satellite tracking experiment has demonstrated that the data generated from this mission configuration can detect high frequency features of the Earth's gravity field. Hence a hi-lo configuration mission is recommended as a means for mapping gravity field fine structure.

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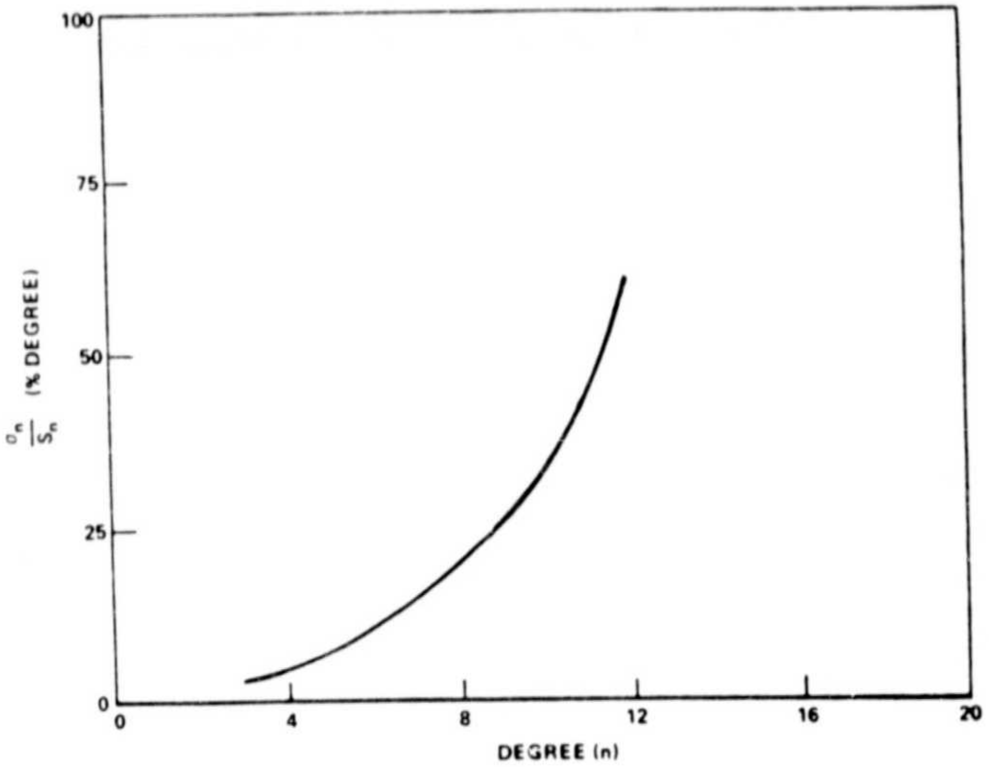

Figure 1. Present Uncertainty of Low Order Geopotential Coefficients

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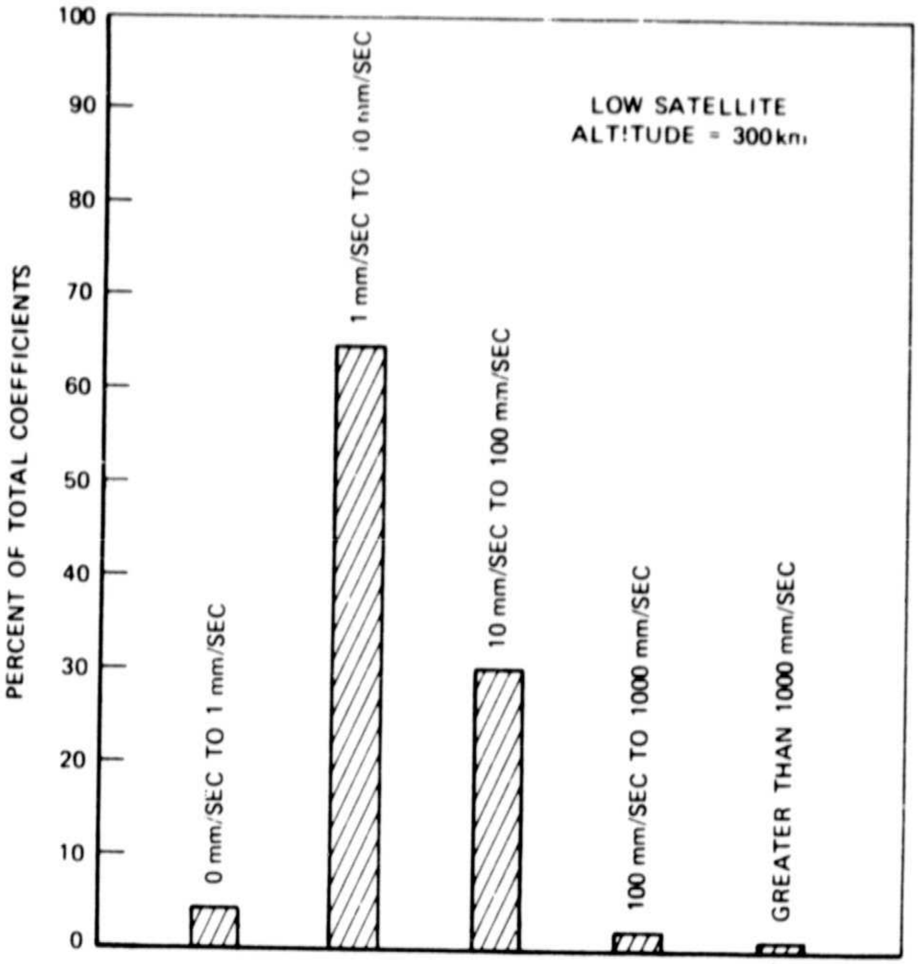

Figure 2. Signal Histogram for Geopotential Terms to Degree and Order 22 in Satellite to Satellite Tracking

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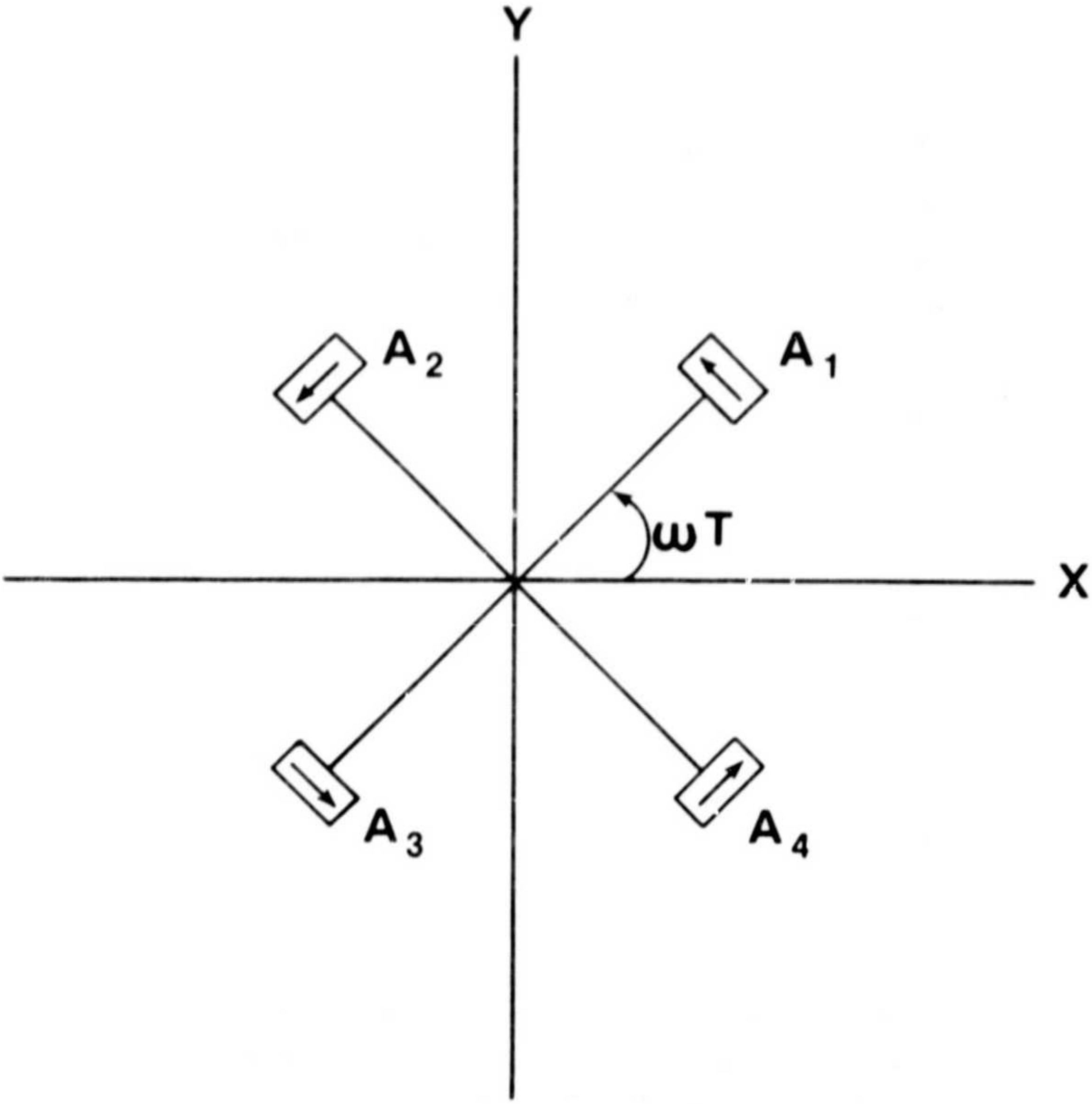

$$\begin{aligned} \text{SIGNAL} &= (A_1 + A_3) - (A_2 + A_4) \\ &= 2\,(\nabla_{xx} - \nabla_{yy}) \sin 2\omega T - 4\,\nabla_{xy} \cos 2\omega T \\ \text{AMP} &= 2\,[(\nabla_{xx} - \nabla_{yy})^2 + 4\,\nabla_{xy}^2]^{1/2} \end{aligned}$$

Figure 3. Output Signal of Rotating Gradiometer

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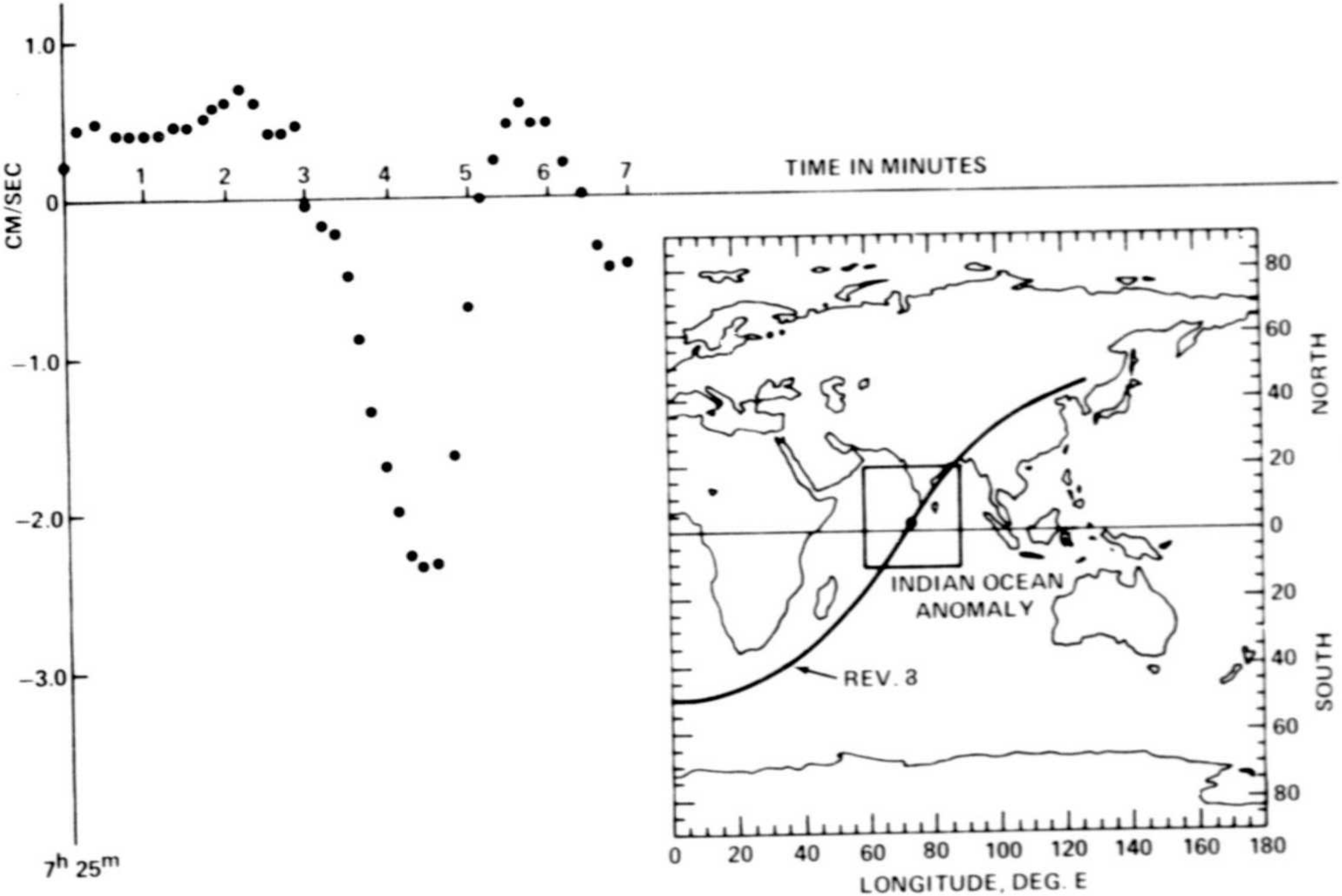

Figure 4. Doppler Signal From ATS-6/Apollo Over The Indian Ocean Anomaly

CAPTIONS

Figure 1. Present uncertainty of low order geopotential coefficients.

Figure 2. Signal histogram for geopotential terms to degree and order 22 in satellite-to-satellite tracking.

Figure 3. Output signal of rotating gradiometer

Figure 4. Doppler signal from ATS-6/Apollo over the Indian Ocean Anomaly.