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# Minimal-Resource Computer Program for Automatic Generation of Ocean Wave Ray or Crest Diagrams in Shoaling Waters 

Lamont R. Poole, Stuart R. LeCroy, and W. Douglas Morris

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Lamont R. Poole<br>Langley Research Center, Hampton, Virginia<br>Stuart R. LeCroy<br>Vought Corporation, Hampton, Virginia<br>W. Douglas Morris<br>Langley Research Center, Hampton, Virginia

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## SUMMARY

An improved computer program for studying the linear ocean wave refraction process has been developed. The program features random-access modular bathymetry data storage to minimize computer resource requirements. The user can select from three bottom topography approximation techniques which provide varying degrees of bathymetry data smoothing. This allows the user to assess the sensitivity of computed results to the degree of smoothing chosen. Waveray patterns can be generated and plotted automatically with either a specified uniform initial (deepwater) ray spacing or a constrained final (nearshore) ray spacing. As an option, wave-crest patterns can be constructed and plotted by using a cubic polynomial to approximate individual crest segments between adjacent rays.

## INTRODUCTION

The prime objective of the NASA Oceanology Program is development and demonstration of the feasibility of satellite systems such as SEASAT-A for the acquisition of global data on ocean conditions. The broad-scale data provided by such systems can be used to initialize analytical models which would then provide short-term, higher resolution forecasts of ocean conditions for ship routing and offshore activities. One such analytical tool is the wave refraction model, which can predict changes in the speed and direction of waves as they travel from deep water across the continental shelf toward the coastline. Such changing patterns affect the distribution of wave energy (wave height) over the area through which the waves are traveling and along the shoreline.

Early models, such as that developed in reference 1, used graphical techniques for constructing refraction diagrams. A wave refraction diagram is a map showing either a group of wave crests at a given time or the successive positions of a particular wave crest as it moves shoreward. The first models required the construction of each wave crest as it advanced toward shore, but later models required only the construction of wave rays which are locally perpendicular to each crest. The wave-ray technique eliminated the time-consuming process of constructing each crest, at the expense of reducing the visual familiarity of the end product. This graphical ray technique was later incorporated into a computer program by Griswold in 1963 (ref. 2), but the program's usefulness was limited because a spatial grid of wave speeds was required in calculating wave-ray paths. The refraction model developed by Wilson (ref. 3) used a grid of water depths which eliminated many of the problems associated with the wave-speed grid; however, the model could not provide wave height. The model developed by Dobson (ref. 4) succeeded in coupling wave-ray construction using a grid of water depths with a finite-difference solution to the wave intensity equation developed by Munk and Arthur (ref. 5) to obtain wave heights. The Dobson model formed the basis for the model used at the Langley Research Center (LaRC) in a recent broad-scale analytical study of wave refraction in the Virginian Sea region of the mid-Atlantic continental shelf (ref. 6).

Several modifications have been incorporated into the LaRC refraction model since the Virginian Sea study. Bathymetry data are stored and used in modular form by using random-access techniques to pass data modules as required from a peripheral mass-storage file to the central memory of the computer (ref. 7). This technique allows very large geographical regions to be studied with a minimum of computer resources but does not restrict the applicability of the model to smaller areas. Ocean bottom topography can be approximated by using either the quadratic least squares, the cubic least squares, or the constrained bicubic interpolation approaches (ref. 8). With these three approximation options, the user can select different degrees of input data smoothing and assess the sensitivity of computed results to the degree of smoothing selected. In the normal mode, the spatial position of the initial point on each wave ray in a particular set is computed automatically to achieve a uniform deepwater ray density. In the previous version of the model, the spatial coordinates of each initial point were input on individual computer cards. By selecting the uniform deepwater ray density, the user can readily identify areas of relatively high or low energy on the resultant refraction diagram as areas of convergent or divergent ray groups. As an alternative, the user can select a control option which provides for automatic computation of deepwater ray spacing to satisfy specified constraints on nearshore ray density. Selection of this option could provide increased directional information in the nearshore zone while sacrificing the easy identification of high or low energy regions. As a third option, wave crests can be constructed by using a cubic polynomial to approximate the crest segment between corresponding points on adjacent rays. Selection of this crest output option also results in selection of the aforementioned controlled nearshore ray density option.

This paper presents a description of the modified LaRC wave refraction model as developed for use on the Control Data Cyber 173 or 175 computer under Network Operating System (NOS) 1.2. A central memory field length of 660008 is required, along with the peripheral mass-storage file used for input of bathymetry data. A description, flow diagram, and source code listing are provided for the main program and each subroutine. Program input and output parameters are listed and described. Sample graphic outputs are provided. A list of system subroutines used, a detailed description of the wave-crest approximation technique and the accompanying subroutine for controlling nearshore ray density, and a description of a utility program which can be used for modularizing a regional bathymetry array are included in the appendixes.

## SYMBOLS

$a_{1}, a_{2}, a_{3}$ coefficients of cubic polynomial used to approximate wave-crest segment
b
ray separation distance, meters
c wave speed, meters/second
d water depth, meters
g acceleration due to gravity, meters/second ${ }^{2}$

H
wave height, meters
refraction coefficient
shoaling coefficient
wavelength, meters
coefficient in equation governing ray separation, defined by equation (10), seconds ${ }^{-1}$
coefficient in equation governing ray separation, defined by equation (11), seconds ${ }^{-2}$
spatial coordinate along wave ray
time, seconds
alongshore coordinate, nautical miles
computational boundaries along x-axis
offshore coordinate, nautical miles
computational boundaries along y-axis
ray angle with respect to $x$-axis, degrees
ray separation factor
ray curvature, radians/meter
radian wave frequency, radians/second
angle between adjoining first-order wave-crest segments, degrees
Subscripts:
i
$j \quad$ index of computational point along ray o initial

A prime denotes differentiation with respect to water depth $d$.

## PROBLEM DESCRIPTION

The problem considered in the present paper is that of computing the refraction characteristics of ocean waves propagating from deep water across the continental shelf toward the coastline. The results of linear wave theory,
which assumes a progressive sinusoidal wave of constant frequency and small steepness, have been adopted; and steady-state conditions of constant tide height and no winds are assumed. The governing equations have been given in references 4 and 5 and are discussed briefly in this section. Also discussed is the method of solution, which is based primarily on the method presented in reference 4. Two additional features enhance the solution in terms of bathymetry data storage requirements and alternate numerical techniques for approximating ocean bottom topography.

The path of any wave ray (orthogonal to a wave crest) can be represented by the equations

$$
\begin{align*}
& \frac{d x}{d t}=c \cos \alpha  \tag{1}\\
& \frac{d y}{d t}=c \sin \alpha  \tag{2}\\
& \kappa=\frac{d \alpha}{d s}=\frac{1}{c} c^{\prime}\left(\frac{\partial d}{\partial x} \sin \alpha-\frac{\partial d}{\partial y} \cos \alpha\right) \tag{3}
\end{align*}
$$

where $d$ is the local water depth. Ray orientation relative to regional coordinates is depicted in the following sketch:


The local wave phase speed $c$ can be expressed according to linear wave theory by

$$
\begin{equation*}
c-c_{o} \tanh \frac{\sigma d}{c}=0 \tag{4}
\end{equation*}
$$

4
where $c_{0}$ is the initial (deepwater) phase speed and $\sigma$ is the fixed radian wave frequency. On the basis of properties of the hyperbolic tangent function, the quantity $\tanh \sigma d / c$ in equation (4) can be replaced by 1 if the value of $\sigma d / c$ is sufficiently large (deepwater range), or by the quantity $\sigma d / c$ if the value of $\sigma d / c$ is sufficiently small (shallow-water range). For the purposes of the present paper, deep water is assumed if the local water depth exceeds 0.25 times the initial wavelength (ref. 6). Shallow water is assumed if the local depth is less than 0.005 times the wavelength. All other depths lie within the intermediate range in which the full transcendental form of equation (4) must be used. By differentiating equation (4) with respect to d, it can be shown that

$$
\begin{equation*}
c^{\prime}=\frac{\sigma c\left[1-\left(\frac{c}{c_{0}}\right)^{2}\right]}{\frac{c^{2}}{c_{o}}+\sigma d\left[1-\left(\frac{c}{c_{o}}\right)^{2}\right]} \tag{5}
\end{equation*}
$$

By assuming that energy is neither transmitted across wave rays nor dissipated by bottom friction or percolation, the wave height at any point along a wave ray can be computed by

$$
\begin{equation*}
\mathrm{H}=\mathrm{H}_{\mathrm{o}} \mathrm{~K}_{\mathrm{s}} \mathrm{~K}_{\mathrm{r}} \tag{6}
\end{equation*}
$$

where $H_{O}$ is the initial wave height. The local shoaling coefficient $K_{S}$ can be expressed as (ref. 4)

$$
\begin{equation*}
K_{S}=\left[\frac{c_{o}}{c\left(1+\frac{2 \sigma d / c}{\sinh 2 \sigma d / c}\right)}\right]^{1 / 2} \tag{7}
\end{equation*}
$$

The local refraction coefficient $K_{r}$ is given by

$$
\begin{equation*}
K_{r}=\left(\frac{b_{0}}{b}\right)^{1 / 2}=(\beta)^{-1 / 2} \tag{8}
\end{equation*}
$$

The ray separation factor $\beta$ is governed by the differential equation

$$
\begin{equation*}
\frac{d^{2} \beta}{d t^{2}}+p \frac{d \beta}{d t}+q \beta=0 \tag{9}
\end{equation*}
$$

$$
\begin{equation*}
p=-2 c^{\prime}\left(\frac{\partial d}{\partial x} \cos \alpha+\frac{\partial d}{\partial y} \sin \alpha\right) \tag{10}
\end{equation*}
$$

and it can be shown that

$$
\left.\begin{array}{rl}
q= & c^{\prime}\left\{\sin ^{2} \alpha\left[\frac{\partial^{2} d}{\partial x^{2}}+U\left(\frac{\partial d}{\partial x}\right)^{2}\right]-2 \sin \alpha \cos \alpha\left(\frac{\partial^{2} d}{\partial x} \partial y\right.\right.
\end{array}+U \frac{\partial d}{\partial x} \frac{\partial d}{\partial y}\right)
$$

where

$$
\begin{equation*}
\mathrm{U}=\frac{-2 \sigma c^{2} / c_{o}}{\left\{\frac{\mathrm{c}^{2}}{c_{o}}+\sigma d\left[1-\left(\frac{c}{c_{o}}\right)^{2}\right]\right\}^{2}} \tag{12}
\end{equation*}
$$

The variables of primary influence in the solution of the governing equations are the local water depth and its spatial variation. For a large geographical area such as the continental shelf, a practical method for the input of bathymetry data to the refraction model is through a spatial grid format in which known depth values are supplied at the nodes of the grid. Such a format is used in the present model, but instead of storing the bathymetry data array for the entire region of interest in central memory, as with the model described in reference 4, overlapping subregional modules of bathymetry data are read as required into memory from a peripheral mass-storage file by using nonsequential random-access techniques. This method of data storage is described in detail in reference 7 and was shown to decrease computer storage requirements by 75 percent for a refraction model of the mid-Atlantic continental shelf.

In conjunction with the grid-type format for bathymetry data, a technique is required for determining depth values at intermediate points between nodes and for approximating spatial derivatives of the depth as required in equations (3), (10), and (11). The present model offers the selection of three such bottom topography approximation techniques, which are discussed in reference 8. These include the quadratic least squares technique, which was used in the model described in reference 4 , the cubic least squares technique, and the constrained bicubic interpolation technique. Both least squares techniques offer some smoothing of the input bathymetry data but do not guarantee continuity in the computed depth or its derivatives. The constrained bicubic interpolation technique guarantees continuity in the depth values and first-order
and mixed second-order partial derivatives; it has the possible disadvantage of using actual input data without smoothing.

Apart from the techniques for bathymetry data storage and bottom topography approximation, the basic solution technique used in the present model is described in detail in reference 4. The path of each wave ray is constructed in equal time steps by an iterative solution of equations (1) to (3). At each step the wave height is calculated by applying Fox's finite-difference solution (see ref. 4 for details) to equation (9) to determine the next ray separation factor and then using this result in equation (6).

## PROGRAM DESCRIPTION

The LaRC wave refraction computer program is coded in FORTRAN Extended (FTN) Version 4 for use on the Control Data Cyber 173 or 175 computer. It consists of a main program WAVE and subroutines RAYINIT, RAYCON, REFRAC, CURVE, DEPTH, HEIGHT, WRITER, PLOTR, RAYOPT, and MATRIX, whose names are descriptive of the functions they perform. The functions performed by routines WAVE, RAYCON, REFRAC, CURVE, and HEIGHT are similar to those of their counterpart routines in Dobson's model (ref. 4). Subroutine DEPTH is an expanded version of its counterpart in Dobson's model, with additional logic for alternate topography approximation techniques and for determination of the required bathymetry data module. Subroutine WRITER stores computed parameters in arrays and prints these arrays after a complete ray path has been constructed. Subroutine RAYINIT computes the spatial coordinates of the initial point on each ray in a particular set, based on the initial ray direction and a specified distance between adjacent rays and wave crests. Subroutine RAYOPT provides the user, if desired, with control over the final nearshore spacing between adjacent rays. This control option is also exercised if the user desires a plot of wave crests, which are constructed by segments in subroutine PLOTR. This subroutine also contains the logic for plotting both wave crests and wave rays. Finally subroutine MATRIX performs matrix multiplications which are required in determining coefficients in the bottom topography approximating polynomials. In addition, the program uses a number of system subroutines to satisfy various data handling and plotting requirements; these subroutines are listed in appendix A .

## Main Program WAVE

The main program WAVE is used for input and output of parameters pertinent to the ray set being computed, including setting default values and checking for input errors. The random-access disk file named TAPE1, which is used for input of bathymetry data modules, is initialized by using the system subroutine OPENMS. Initialization of parameters for each ray in the set is also performed in WAVE. If one of the plotting options is chosen, subroutine PLOTR is called to perform plot maintenance such as to open and close the plot file and to control frame advances and positioning of frame origins. The flow diagram and listing of program WAVE follow.


* A PROGRAM TO CONSTRUCT REFRACTION DIAGRAMS AND COMPUTE WAVE
* 
* 

***** THIS PROGRAA REQUIRES A GRIO DF BATHYAETRY DATA TO BE INPUT TO
象

## a progran to construct refraction diagrams and compute wave

 heights for waves moving into shoaling water．＊＊＊＊＊namelist input parameters＊＊＊＊＊
＊

```
BASIC DATA = NPUTI **
```

    DCON - MULTIPLIER TO CONVERT DEPTH UNITS TO FEET FOR
                        INTERNAL COMPUTATIONS
                (DEFAuLT - 2.1
    Deltas - minimum step leng th along ray in shallow water in *
        GRID UNITS
        (DEFAULT - 0.01)
    GRID - NUMBER OF FEET PER GRID DIVISION *
                (Input reouired) *
    GRINC - StEp LENGTH ALONG RAy in deEp water in grid units *
                (DEFAULT - 1.)
    ioutunt - output units - nautigal miles for spatial
        COORDINATES \(X, Y\) y for all other parameters,
                                    1 = U.S. customary units
                                    2 = SI UNITS (DEFAULT)
    JPRTFRQ(1) - PRINT FREQUENCY FOR DEPTH/WL > 0.5
                (DEFAULT = 20)
    JPRTFRQ(2) - PRINT FREQUENCY FOR 0.25 < DEPTH/WL < 0.5
        (DEFAULT = 10)
    JPRTFRO(3) = PRINT FREOUENCY FOR DEPTH/WL \(<0.25\)
                        (OEFAULT -5)
    KPLOT = PLOT OPTION
                                    0 - no plot (default)
                                    1. PLOT WAVE RAYS WITH UNIFORM INITIAL SPACING
                                    2 = plot wave rays with controlled final spacing
                3 - plot wave crests
    

```
    DISTMAX - MAXIMUM PERMISSIBLE FINAL SEPARATION DISTANCE *
    BETWEEN ADJACENT RAYS IN GRID UNITS *
                (DEFAULT * 999.)
    OISTMIN MINIMUM PERMISSIBLE FINAL SEPARATION DISTANCE
        BETWEEN ADJACENT RAYS IN GRID UNITS
        CDEFAULT = 25.1
    KCREST : POINT BETHEEN ADJACENT RAYS AT WHICH TO BEGIN
        PLOTTING THE CURVED CREST SEGMENT WHEN ANGLE
        BETWEEN ADJOINING FIRST-ORDER CREST SEGMENTS
        IS LESS THAN RAYANG. MAXIMUM - 7
        (DEFAULT = 1)
    RAYANG MINIMUM ACCEPTABLE ANGLE BETWEEN ADJOINING
        FIRST-ORDER CREST SEGMENTS BELOW WHICH CREST
        PLOTTING WILL BE MODIFIED (DEGREES)
                (DEFAULT = 120.)
    RAYMAX MAXIMUM PERMISSIBLE INITIAL RAY SPACING IN GRID
        UNITS
                (DEFAULT=25.)
    RAYMIN : MINIMUM PERMISSIBLE INITIAL RAY SPACING IN GRID
        UNITS
        (DEFAULT 0.01)
******** END OF NAMELIST INPUT #########
```



```
    TITL IDENTIFYING TITLE FOR EACH SET *
                                (INPUT CARD FORMAT = &AIO) *
##### DATA INPUT ORDER ##### #
* (***)
* NPUTI/SET LETITL,NPUTZ/SET 2*TITL,NPUTZ/ETC
```



```
*
        COMMON BI,B2,CO,CXY,CPRIME,DCON,DELTAS,DEP(42,32),DISTMAX,DISTMIN,
        +DRC,OTGR,DXY,GRID,GRINC,HO,IBLK(101), IGO,IOUTUNT,IREF,ISTOP,JGO,
        +JPRTFRQ(3),KCREST,KPLOT,LIMNPT,MI,MJ,NCREST,NFRST,NFRSTI,NLAST,
        +NLASTI,NORAY,NOROW,NPT,NSURFCE,NTIC,NTICOFF,NWRITE,PDX,POY,PD2X,
        +PD2Y,PD2XY,PLTLNGX,PLTLNGY,PLTSCAL,RAYANG,RAYMAX,RAYMIN,RAYSPC,
        +RCCO,RK,SK,SIG,TIOE,TITL(8),V,WL,WLO,XO,XM,XPLOT(500),XPLOTL(500),
        +XPLOT2(500), XTEST,Y0,YM,YPLOT(500),YPLOT1(500),YPLOTZ(500),YTEST
*
            NAMELIST/NPUTI/DCJN,DELTAS,GRID,GRINC,IOUTUNT,JPRTFRO,KPLOT,
            +LIMNPT,MI,MJ,NOMOD,NOROW,NOSETS,PLTLNGX,PLTLNGY,PLTSCAL,TIDE
            +/NPUT2/A,OISTMAX,DISTMIN,HO,KCREST,NCREST,NSURFCE,RAYANG,RAYMAX,
            +RAYMIN,RAYSPC,T
*
##** SET INPUT DEFAULT VALUES *##*
```

```
        DCON = 1.0
        DELTAS = 0.01
        GRIO = 0.0
        GRINC - 1.0
        IOUTUNT = 2
        IREPEAT - 0
        JPRTFRO(1) = 20
        JPRTFRQ(2) = 10
        JPRTFRQ(3) = 5
        KPLOT = O
        LIMNPT = 400
        MI = O
        MJ=O
        NOMOD = O
        NOROW = 0
        NOSETS - 1
        PLTLNGX = 10.
        PLTLNGY . 10.
        PLTSCAL=999.
        TIDE = 0.
        A = 999.
        DISTMAX - 999.
        DISTMIN = 25.
        HO = 999.
        KCREST - 1
        NCREST = 5
        NSURFCE - 1
        RAYANG - 120.
        RAYMAX=25.
        RAYMIN = 0.01
        RAYSPC = 5.
        T = 999.
* READ baSIC DATA **
        READ (5,NPUT1)
*
** CHECK FDR INPUT ERROR **
        IF (EOF(5)) 10,20
    10 PRINT 190
        GO TO 180
    20 IF (GRID.GT. 0.01 GO TO 30
        PRINT 200
        GO TO }18
    30 IF (MI .GT. O .AND. MJ .GT. O .AND. NOMOD .GT. O .AND. NORDW .GT.
        +O) GO TO 40
        PRINT 210
```

```
    40 IF (KPLOT .EQ. O .OR. PLTSCAL .LT. 999.0) GO TO 50
        PRINT 220
        GOTO 180
*
** print basic input parameters #*
    50 PRINT 230, DCON,OELTAS,GRID,GRINC,IOLTUNT,JPRTFRQ,KPLOT,
        +LIMNPT,MI,MJ,NOMOD,NOROW,NOSETS,PLTLNGX,PLTLNGY,PLTSCAL,TIDE
*
** OPEN MASS STORAGE DISK fILE (TAPEI) CONTAINING MODULAR BATHYMETRY DATA **
*
        NOMOD - NOMOD+1
        CALL OPENMS (1,IBLK,NOMOD,O)
*
** OPEN PLOT FILE IF PLOT OPTION USED **
*
    IF (KPLOT .GT. O) CALL PLOTR (1)
** GENERATE CURRENT RAY SET **
* DO 170 NOSETO1,NOSETS
        DO 170 NOSET=1,NOSETS
*
** READ TITLE INFORMATION **
*
        READ 250, TITL
*
** CHECK FOR TITLE CARD **
IF (EOF(5)) 60,70
    60 PRINT 260
        GOTO }18
    70 TITCI = 2H $
        ENCODE (10,270,TITC2) TITL(1)
        IF (TITC1 *NE. TITC2) GO TO 80
        PRINT 260
        GOTO 180
    80 CONTINUE
*
** READ INPUT PARAMETERS for CURRENT SET **
        READ (5,NPUTZ)
*
** CHECK FJR INPUT ERROR **
        IF (EOF(5)) 90,100
    90 PRINT 280
        GOTO 180
    100 IF (A .LT.9999.0.AND. HO .LT. 999.0.ANO. T .LT. 999.0) GO TO 110
        PRINT 290
```

```
        GOTO 180
*
** INITIALIZE PARAMETERS FIXED FOR CURRENT SET **
    110 SIG e 6.28318531/T
        CO = 5.1204062#7
        WLO = CO*T
        DRC = 0.25#WLO
        DTGR=GRINC/CO
        AST =A
        DXY = 999.99
        ISTOP = 0
        NORAY = 1
        UNIT = OTGR*GRID
*
** PLOT TITLE AND AXES INFORMATION IF PLOT OPTION USED **
    IF (KPLOT .GT. O) CALL PLOTR (2)
#
** PRINT SPECIAL INPUT PARAMETERS IF PLOTTING WAVE CREST OR RAYS WITH **
** CONTROLLED FINAL SPACING
        IF &KPLOT .GE. 2) PRINT 240, DISTMAX,DISTMIN,KCREST,RAYANG,
        +RAYMAX,RAYMIN
*
** FIND RAY STARTING POINT **
    120 CALL RAYINIT (X,Y,AST)
        IF (ISTOP .EQ. 1) GO TO 160
*
** INITIALIZE RAY PARAMETERS **
        A = AST
        CXY - CO
        NPT = l
        NTICOFF NCREST*(NTIC-1)
        NWRITE = 1
        WL - WLO
        B1=B2=RK=SK=1.0
        PDX=PDY=0.
        CALL DEPTH (X,Y)
*
** Calculate ray path **
    CALL RAYCON (X,Y,A)
*
** SAVE INDEX OF FIRST ANO LAST RAY POINT WITH RESPECT TO **
** REFERENCE CORNER
```

```
        NFRST - NTICDFF+1
        NLAST NTICOFF+NPT
*
* COMPUTE INITIAL RAY SPACING TO MEET FINAL CONSTRAINTS IF KPLOT *
* EQUALS 2 OR 3
*
        IF (KPLOT .GE. 2) CALL RAYOPT (IREPEAT)
        IF (IREPEAT •GT. O) GD TO 120
*
** PRINT CONSTANT RAY PARAMETERS #*
* PRINT 300, TITL,T,UNIT,RAYSPC,NCREST,NSURFCE,NORAY,NOSET
*
** PRINT DUTPUT HEADER INFORMATION **
PRINT 310
*
** PRINT OUTPUT UNITS **
* IF (IDUTUNT-2) 130,140
    130 PRINT 320
        GO TO 150
    140 PRINT 330
    150 CONTINUE
*
** PRINT COMPUTED PARAMETERS FOR CURRENT RAY **
*
        CALL URITER (X,Y,A,H)
*
** PLOT WAVE RAYS IF PLJT OPTION USED **
        IF (KPLOT .GT. O) CALL PLOTR (3)
        NORAY = NORAY+1
        GO TO 120
    160 PRINT 340, NOSET
    170 CONTINUE
        PRINT 350, NOSETS
    180 CONTINUE
*
** END PLOTTING IF PLOT OPTION USED **
    IF (KPLOT .GT. O) CALL PLOTR (4)
    STOP
190 FORMAT (IHO,#INPUT ERROR - NO DATA INPUT UNDER NPUTI#)
200 FORMAT (1HO,*INPUT ERROR = NO DATA FOR GRID*)
210 FORMAT (1HO, *INPUT ERROR = NO DATA FOR MI OR MJ OR NOMOD OR NOROW*
    +)
220 FORMAT (1HO,*INPUT ERROR - NO DATA FOR PLTSCAL*)
```

```
230 FORMAT (1H1,12X,*BASIC PROGRAM PARAMETERS*//
    +2X,*DEPTH UNIT MULT................DCON =*,FB.2I
    +2X,*MIN STEP SIZE.................DELTAS -*,F6.21
    +2x,*GRID SIZE......................GRID =#,F8.2I
    +2X, &DEEP WATER STEP SIZE...........GRINC **,F6.2I
    +2X,#OUTPUT UNIT OPTION.............IOUTUNT **,121
    +2X,*PRINT FREQ 1 (D/L>0.5).........JPRTFRQ(1) =*,13/
    +2X,*PRINT FREO 2 (0.25<D/L<0.5)....JPRTFRO(2) =*,13/
    +2X,*PRINT FREO 3 (D/L<0.25)........JPRTFRQ(3) **,13/
    +2x,*PLOT OPTION.....................KPLOT E*,12/
    +2X,#MAX NO OF RAY POINTS...........LIMNPT **,I4/
    +2X,*GRID LIMITS ABSCISSA.........MI **,I4/
    +2X,* ORDINATE.........MJ E*,141
    +2x,*NO OF DEPTH MJDULES...........NOMOD **,I31
    +2x,*NO OF ROW MODULES IN NOMOD....NUROW **,I3/
    +2x,*ND OF SETS OF RAYS.............NOSETS E*,I3/
    +2X,*PLOT X AXES LENGTH..............PLTLNGX &*,F6.21
    +2X,*PLOT Y AXES LENGTH.............PLTLNGY **,F6.21
    +2X,*PLOT SCALE FACTOR...............PLTSCAL E*,F6.2/
    +2X,*TIDE HEIGHT...................TIDE &*,F6.2I
240 FORMAT (IHO,1X, #MAX FINAL RAY SPACING..........DISTMAX
    +2X,*MIN FINAL RAY SPACING..........DISTMIN =*,FG.1/
    +2X,*INITIAL PT FOR CREST CURVE.....KCREST E*,I4/
    +2X,*MIN ANGLE BETWEEN SEGMENTS....RAYANG -&,FG.1/
    +2X,*MAX INITIAL RAY SPACING.........RAYMAX **,F6.1/
    +2X,*MIN INITIAL RAY SPACING.......RAYMIN =*,F7.2I
250 FDRMAT (8A1O)
260 FORHAT (1HO,#INPUT ERROR - NO TITLE CARD#)
270 FORMAT (AZ)
280 FDRMAT (1HO,#INPUT ERROR = NO DATA INPUT UNDER NPUT2*)
290 FORMAT (1HO,*INPUT ERROR - NO DATA FOR A OR HO DR T*)
300 FORMAT 1IH1,8A10/1X,*PERICD =*,F6.2,* SEC...*;*TIME STEP m*,F7.2,
    ** SEC...*&#RAY SPACING =*,F5.1,*&..CREST SPACING E#,I3,
    +*...BOTTOM APPROX CODE E*I2/IX,*RAY NO &*,I3,**.&SET NO E*,I3)
310 FORMAT (1HO,* POINT X Y ANGLE DEPTH LENGT
    +H SPEED REFRACTION SHOALING HEIGHT*)
320 FORMAT (11X,*(N.MI.) (N.MI.) (OEG) (FT) (FT) (FT/SEC
```



```
    + COEF COEF (M)*/)
340 FORMAT (IHO,*SET NO*,I3,* ENDED. RAYS LIE OUTSIDE BOUNDARY*)
350 FORMAT (1HO,*ALL SETS COMPLETED. NUMBER OF SETS =#,I3)
        END
```


## Subroutine RAYINIT

Subroutine RAYINIT computes the coordinates of the initial point on each ray in a particular set. It is assumed that the coastline and isobaths for the geographical region to be considered are aligned in a crude sense with the x-axis. Rays are to be initiated, depending on their initial direction, at a sequence of deepwater points along the seaward computational boundaries of the region ( $x_{0}, y_{m}, x_{m}$ ), which are set arbitrarily a distance of 4 grid units inward from the limits of the bathymetry grid. (See fig. 1.)

The initial ray in the set is found by forming a counterclockwise sequence of trial rays beginning at a reference corner of the region. For initial ray directions $\alpha_{0}>-90^{\circ}$, the reference corner lies at ( $x_{0}, y_{m}$ ), whereas for $\alpha_{0} \leqq-90^{\circ}$, it lies at $\left(x_{m}, y_{m}\right)$. For $\alpha_{0}=-180^{\circ}$, the initial ray is begun at the reference corner ( $x_{m}, y_{m}$ ), and no sequence of trial rays is formed. For all other initial directions, the counterclockwise sequence of trial rays (illustrated in fig. 1 for $\alpha_{0}>-90^{\circ}$ ) is formed, with each ray separated by a distance of RAYSPC grid units orthogonal to the initial ray direction. Steps are made inward along each trial ray in increments of DTIC = NCREST*GRINC until the test point along the ray lies within the appropriate seaward computational boundary. If that point lies in deep water, the counterclockwise sequence is continued. If the first test point lying within the computational boundary is in intermediate water, the counterclockwise sequence is terminated and the previous ray in the sequence is chosen as the initial ray in the set. The sequence is also terminated if the entire trial ray lies outside the computational boundaries or if the first intraboundary point along the trial ray lies on land. In these instances, the previous ray in the sequence is also chosen as the initial ray in the set.

The coordinates of the initial point on each ray in the set are found by reversing the procedure and forming a clockwise sequence of rays beginning with the initial ray (illustrated in fig. 2 for $\alpha_{0}>-90^{\circ}$ ). The initial point along each ray is selected as that trial point advanced along the ray by an increment of DTIC and lying both within and closest to the appropriate seaward boundary. The clockwise sequence (and the ray set) is terminated when a trial ray is formed along which no appropriate initial point can be found.

The flow diagram and listing of subroutine RAYINIT follow.


this subroutine computes the coordinates of the initial point *
FOR EACH RAY IN A CLOCKWISE SEQUENCE ALONG THE SEAWARD *
BOUNDARIES OF THE REGION. EACH RAY IS SEPARATED BY RAYSPC GRID *
UNITS FROM THE PREVIQUS GNE. COMPUTATIONS BEGIN AT A PDINT
ALONG THE RAY WHICH LIES IN DEEP WATER AND IS ADVANCED BY AN *
INTEGER HULTIPLE OF THE DEEP-WATER STEP LENGTH BETVEEN CREST *
TIC MARKS(DTIC) FROM THE CREST WHICH WOULD PASS THROUGH THE *
REFERENCE CORNER.

COMMON BI, B2,CO,CXY,CPRIME,DCON,DELTAS,DEP $442,321, D I S T M A X, D I S T M I N$, +DRC, DTGR,DXY,GRID, GRINC, HO,IBLK(IOI), IGO, IOUTUNT, IREF, ISTOP, JGO, + JPRTFRQ(3), KCREST, KPLOT,LIMNPT, HI, MJ, NCREST, NFRST, NFRST1,NLAST, +NLASTI, NORAY, NOROW,NPT,NSURFCE,NTIC,NTICOFF,NWRITE,PDX,PDY,PD2X, +PD2Y, PD 2XY, PLTLNGX, PLTLNGY, PLTSCAL,RAYANG,RAYMAX,RAYMIN,RAYSPC, +RCCO,RK,SK,SIG,TIDE,TITL(8),V,WL,WLO,XO,XM,XPLOT(500),XPLOT1(500), +XPLOT2(500), XTEST, YO, YM, YPLOT (500), YPLOT1(500), YPLOT2(500), YTEST
*
IF (NORAY .EQ. 1) GO TO 10 DXRAY=-RAYSPC*SARAD DYRAY=RAYSPC*CARAD GO TO 110
$+$
*
*

* COMPUTE FIXED DEEP-WATER SEPARATION DISTANCES FOR RAYS AND CREST
* TIC-MARKS. INITIALIZE COMPUTATIONAL BOUNDARIES.
$10 \quad \times 0=4$.
YO=4.
$X M=M I-4$
$Y M=M J-4$
ARADEAST/57.2957795
SARAD=SIN(ARAD)
CARAD=COS (ARAD)
DXRAY=-RAYSPC*SARAD
DYRAYERAYSPC\#CARAD
DTIC=NCREST*GRINC
DXTIC=DTIC\#CARAD
DYTIC=DTIC*SARAD
* 
* CHOOSE REFERENCE CORNER - (XO,YM) IF RAY ANGLE>-90 DEG.

```
        IF (AST.LT.-89.99) GO TO 20
        XC=XO
        YC=YM
        GO TO 30
    20 XC=XM
    YC=YM
#
#
IF RAY ANGLE IS -180 DEG, FIRST RAY BEGINS AT REFERENCE CDRNERE
    30 IF (AST.GT.-179.99) GO TO 40
        X=XTEST:XC
        Y=YTEST=YC
        NTICENCORN=1
        RETURN
* ANGLE >-180 DEG. START SEARCH FOR INITIAL RAY AT REFERENCE CORNER.
    40 NTIC=1
        NCORN=O
        XTEST*XC
        YTEST=YC
*
* ADVANCE COUNTERCLOCKWISE DNE RAY AND MOVE TO CORRESPONDING TEST PQINT.
    50 XTESTEXTEST-DXRAY
        YTESTEYTEST-OYRAY
        NCORN=NCORN+1
#
* BASED ON ANGLE, TEST TO SEE IF CURRENT TEST POINT IS INSIDE REGION.
    60 IF (AST.LT.-89.99) GO TO 70
        IF (XTEST.LT.XO) GO TO 90
        IF (YTEST.LT.YO) GOTO 10O
        GO TO 80
    70 IF (YTEST.GT.YM+0.01) GO TO 90
    IF (XTEST.LT.XO) GO TO 100
* INTERIOR TEST POINT FDUND. CHECK LDCAL DEPTH.
80 CALL DEPTH (XTEST,YTEST)
#
* IF DEEP WATER, ADVANCE CDUNTERCLOCKWISE DNE RAY AND MOVE TO
* CORRESPONDING TEST POINT. IF INTERMEDIATE WATER, THE PREVIOUS
* RAY IS THE INITIAL RAY - GO TO STATEMENT IIO TO MOVE TO INITIAL RAY.
    IF (DXY.LE.DRC) GO TO 11G
```

```
    GO TO 50
*
* CURRENT TEST POINT OUTSIDE REGION. ADVANCE ONE TEST POINT
* AlONG CURRENT RAY.
    90 XTEST=XTEST+DXTIC
        YTEST=YTEST+DYTIC
        NTIC=NTIC+I
        60 TO }6
* All TEST POINTS ON TRIAL RAY OUTSIDE REGION. SET PREVIOUS RAY
* TO THE INITIAL RAY.
    100 CONTINUE
    ADVANCE CLOCKWISE ONE RAY AND MOVE TO CDRRESPONDING TEST POINT.
    110 XINITOXTEST+DXRAY
        YINIT=YTEST+DYRAY
* If THE RAY PASSING THROUGH THE REFERENCE CORNER HAS BEEN COMPUTED,
* CRITERIA FOR ENDING RAY SET MUST BE TESTED. IF NOT, MAKE SURE THE
* PRESENT TEST POINT IS THE fIRST INTERIOR TEST POINT.
*
        IF (NORAY.GT.NCORN) GO TO 160
        IF (XINIT.GT.XM+.O1.JR.YINIT.GT.YM+.O1) GO TO 160
    120 XTEMP=XINIT-DXTIC
    YTEMP =YINIT-DYTIC
        IF (AST.LT.-89.99) GO TO 130
        IF(XTEMP.LT.XO-0.01.OR.YTEMP.GT.YM+0.01) GO TO 150
        GO TO 140
    130 IF (XTEMP.GT.XM+.O1.OR.YTEMP.GT.YM+.O1) GO TO 150
* NEW INITIAL TEST POINT. RESET COORDINATES aND REDUCE VALUE OF tIC-mark
* CQUNTER BY 1. GD BACK TO STATEMENT 120 AND TRY AGAIN.
    140 XINIT=XTEMP
    YINIT=YTEMP
    NTIC=NTIC-1
    GO TO 120
* CORRECT INITIAL TEST POINT FOUND.
    150 X=XTEST=XINIT
        Y=YTEST-YINIT
        RETURN
*
* RAY PASSING through reference corner has already beEN computed.
* TEST VARIOUS CRITERIA FOR ENDING RAY SET.
```

```
*
    160 IF (AST.LT.-.01) GO TO 170
*
* zero degree case. final ray already computed.
*
    ISTOP = I
    RETURN
    170 IF (AST.LT.-90.01) GO TO 210
*
* ray angle > -90 deg. ChECK location of present teSt point.
    180 IF (YINIT.LE.YM) GO TO 190
*
* Y-coordinate dF CURRENT TEST POINT GREATER THAN YM. advance one
* TEST POINT AND RECHECK lUCATION.
*
    XINIT=XINIT+OXTIC
            YINIT=YINIT+OYTIC
            NTICENTIC+1
            GO TO 180
    190 IF (XINIT.LT.XM) GO TO 200
*
* X-COORDINATE OF CURRENT TEST POINT GREATER THAN XH. NO FURTHER RAYS
* POSSIBLE.
    ISTOP=1
    RETURN
    200 XeXTEST=XINIT
            Y=YTEST=YINIT
            RETURN
* RAY ANGLE < -90 DEG. CHECK lOCATION OF CURRENT TEST POINT.
    210 1F (XINIT.LE.XH+0.OO1) GO TO 220
*
* X-COORDINATE OF CURRENT TEST POINT GREATER THAN XM. ADVANCE ONE TEST POINT
AND RECHECK LOCATION.
    XINIT=XINIT+DXTIC
            YINIT=YINIT+DYTIC
            NTICENTIC+1
            GO TO 210
    220 IF(YINIT.LE.YO) GS TO 230
* INTERIOR TEST POINT FOUND. ChECK lOCAL DEPTH.
    CALL DEPTH (XINIT,YINIT)
    IF (DXY.LE.DRC) GO TO 230
*
* new rar can be computed
    X=XTEST=XINIT
    YEYTEST=YINIT
    RETURN
*
* INItIAL test point liES Either dutSide region or in intermediate mater
* NO FURTHER RAYS POSSIBLE.
230 I STOP=1
    RETURN
    END
```

Subroutine RAYCON controls the computation of each point along the wave ray. Points which lie in deep water are computed directly in RAYCON, while the routine calls subroutine REFRAC to compute ray points lying in shoaling water. After the ray point is computed, subroutine HEIGHT is called to compute the wave height and subroutine WRITER is called to store computed parameters for printing and plotting. Computation of the ray path is terminated if the ray goes outside the computational boundaries, if the shore is reached, or if the maximum number of ray computational points is exceeded. The program flag NWRITE designates the type of termination condition which is met. The different phases of the ray path are indicated by the program parameter IGO. When the ray is in deep water, $I G O=1$. When the ray reaches shoaling water, IGO is set to 2 in subroutine REFRAC. When a termination condition has been reached, IGO is set to 3 in subroutine WRITER. The flow diagram and listing of subroutine RAYCON follow.


SUBROUTINE RAYCON $(X, Y, A)$

```
*
```



```
*
* THIS SUBROUTINE ASSUMES DEEP GATER AND CALCULATES THE NEXT
* POINT ON THE RAY. IF THE WAVE HAS ENTERED SHOALING WATER, THE
* PREVIDUSLY CALCULATED POSITION OF THE NEXT POINT IS DISCARDED.
* SUBROUTINE CURVE IS CALLED TD CALCULATE THE INITIAL RAY
* CURVATURE, WHICH IS USED IN SUBROUTINE REFRAC TO CALCULATE THE
* next point on the ray.
*
*
    COMMDN B1,B2,CD,CXY,CPRIME,DCON,DELTAS,DEP(42,32),DISTMAX,DISTMIN,
    +ORC,DTGR,DXY,GRID,GRINC,HO,IBLK(IOI),IGO,IOUTUNT,IREF,ISTOP,JGO,
    +JPRTFRQ(3),KCREST,KPLOT,LIMNPT, MI,MJ,NCREST,NFRST,NFRST1,NLAST,
    +NLASTI,NORAY,NOROW,NPT,NSURFCE,NTIC,NTICOFF,NWRITE,PDX,POY,PD2X,
    +PDZY,PD2XY,PLTLNGX, PLTLNGY, PLTSCAL,RAYANG,RAYMAX,RAYHIN,RAYSPC,
    +RCCO,RK,SK,SIG,TIOE,TITL(B),V,WL,WLO,XO,XM,XPLOT(500),XPLOT1(500),
    +XPLOT2(500), XTEST,YO,YM,YPLOT(500),YPLOT1(500),YPLOT2(500),YTEST
*
** SET RAY INITIAL CONDITIONS **
*
    ANG = A/57.2957795
    COSA = COS(ANG)
    SINA = SIN(ANG)
    H= HO
    IGO = I
    IF (NPT .EQ. 1) GJ TO 70
    10 PX = X
    PY = Y
*
## COMPUTE NEXT POINT ON RAY FOR DEEP WATER #*
*
        DS - GRINC
        x = x+DS*CosA
        Y= Y+DS*SINA
*
** TEST TO SEE IF RAY POINT OUTSIDE COMPUTATICNAL BOUNDARIES **
*
        IF (X .GE. XM .OR. X .LE. XO) GO TO 20
        IF (Y .LT. YM .AND. Y .GT. YO) GO TO }3
* DUTSIDE COMPUTATIONAL BOUNDARIES. SET FLAG AND EXIT **
*
    20 NWRITE = 4
        GO TO }7
*
```

```
** COMPUTE DEPTH **
* 30 CALL DEPTH (X,Y)
** tEST to seE if shore has beEN reached
*
IF IDXY.GT. O.1 GO TO 40
*
** Shore reached. SET flag and exit **
*
            NWRITE - 3
            GOTO 70
    40 NPT = NPT + 1
** TEST TO SEE IF mAX NO OF RAY POINTS REACHED **
*
    IF (NPT .LT. LIMNPT) GO TO 50
* max MO OF pOINTS pEACHED SET FLAG
** max no of pointS reached. SET flag **
*
    NWRITE = 5
*
** TEST TO SEE IF IN SHIALING WATER **
* 50 IF (IGO .GE. 2) GJ TO 60
            IF (DXY .GE. DRC) GO TO 70
** IN SHOALING WATER. START REFRACTION CALCULATIONS **
X
*
** COMPUTE INITIAL RAY CURVATURE **
    CALL CURVE (X,Y,ANG,FK)
*
** COMPUTE NEXT PUINT ON RAY FOR SHOALING WATER **
* 60 CALL REFRAC (X,Y,ANG,FK)
** COMPUTE WAVE HEIGHT **
*
    CALL HEIGHT (ANG,H)
*
** SAVE PRINT AND PLDT DATA **
* 70 A = 57.2957795*ANG
    CALL WRITER (X,Y,A,H)
*
** REPEAT FOR NEXT POINT ON RAY **
    IF (IGD-2) 10,40,80
    80 RETURN
        END
```

In subroutine REFRAC, equations (1) to (3) are solved iteratively to find the next point on the ray. If the curvature is oscillating between two values, the curvature is taken as the average of these two values. If the iteration process is converging very slowly or not at all, the ray is stopped. Subroutine CURVE is called to compute the curvature. Several ray termination conditions are checked in this subroutine. The ray is terminated if it goes outside the computational boundaries or if it reaches the shore. In addition, the ray is terminated if the wave speed has been reduced so that the incremental distance between successive ray points is less than the minimum step length allowed DELTAS which is an input parameter. The ray path phase flag IGO is set to 2 in this routine. The flow diagram and listing of subroutine REFRAC follow.


```
    SUBROUTINE REFRAC (X,Y,A,FK)
*
```



```
*
* THIS SUBROUTINE SOLVES THE REFRACTION EQUATIONS ITERATIVELY TO
* FIND THE NEXT POINT ON THE RAY. IF THE ITERATIVE PROCESS IS
OSCILLATING BETWEEN THO VALUES; THEY ARE AVERAGED. IF THE *
*
PROCEDURE IS CONVERGING VERY SLOWLY, OR NOT AT ALL,THE RAY IS
STCPPED.
#####&######################################################################
*
    COMMON B1,B2,CD,CXY,CPRIME,DCON,DELTAS,DEP(42,32I,DISTMAX,DISTMIN,
    4DRC, DTGR,DXY,GRID,GRINC,HO,IBLK(IOI),IGO,IOUTUNT,IREF,ISTOP,JGO,
    +JPRTFRO(3),KCREST,KPLOT,LIMNPT,MI,MJ,NCREST,NFRST,NFRST1,NLAST,
    +NLASTI, NORAY,NOROW,NPT,NSURFCE,NTIC,NTICOFF,NWRITE,PDX,POY,PD2X,
    +PO2Y,PD2XY,PLTLNGX,PLTLNGY,PLTSCAL,RAYANG,RAYHAX,RAYMIN,RAYSPC,
    +RCCO,RK,SK,SIG,TIDE,TITL(B),V,WL,WLO,XO,XA,XPLOT(500),XPLOTI(500),
    +XPLDTZ(500),XTEST,YO,YM,YPLOT(500),YPLOT1(500),YPLCT2(500),YTEST
*
NCUR - 1
IF (1G0-2) 10,20,110
*
** SET INITIAL CONDITIONS **
*
    10 FKM - FK
    IGO=2
    20 DS = CXY#DTGR
*
** TEST TO SEE IF STEP SIZE TOO SMALL **
*
    IF (DS .GE. DELTAS) GO TO }3
*
** STEP SIZE TOD SMALL. SET FLAG AND EXIT **
*
    NWRITE = 6
    GO TO 110
*
** COMPUTE TOLERANCE DN CONVERGENCE FOR CURVATURE **
*
    30 RESMAX = 0.00005/DS
*
** ITERATE FOR CURVATURE SOLUTION AND NEXT POINT ON RAY **
*
    40 DO 80 I=1,20
        DELA F FKH*DS
        AA = A+DELA
        AM - A+0.5*DELA
```

```
    XX=X+DS*CDS(AM)
    YY = Y+DS*SIN(AM)
*
** TEST TO SEE IF RAY PJINT OUTSIDE COMPUTATIONAL BOUNDARIES **
*
    IF (XX .GE. XH .OR. XX .LE. XO) GO TO }5
    IF (YY &LT. YM .AND. YY .GT. YO) GD TO 60
*
** OUTSIDE COMPUTATIONAL BOUNDARIES. SET FLAG AND EXIT **
    50 NWRITE = 4
    GO TO 100
*
** COMPUTE RAY CURVATURE **
60 CALL CURVE (XX,YY,AA,FKK)
*
** teSt tO SEE IF SHORE HAS bEEN REACHED **
*
    IF (DXY.GT. O.) GO TO 70
** SHORE REACHED. SET FLAG AND EXIT **
*
        NWRITE - }
        GO TO 100
    70 IF (NCUR .EQ. 2) GO TO 100
        FKM - 0.5*(FK+FKK)
        IF (I .EQ. 1) GO TO }8
*
** TEST TO SEE IF SOLUTION HAS CONVERGED **
        IF (ABS(FKP-FKM) .LE. RESMAX) GO TO 100
        IF (I .EO. 18) FK18 = FKM
    80 FKP = FKM
*
** test to SEE If SOLUTION has CONVERGED **
*
    IF (ABS(FKM-FK18) .LE. RESMAX) GO TO 90
** OIO NOT CONVERGE. SET FLAG AND EXIT **
** OIO NOT CONVERGE. SET FLAG AND EXIT **
    NWRITE=2
    GOTO 100
*
** Curvature average **
*
    90 FKM=0.5*(FKM+FK18)
        NCUR - 2
        GO TO 40
    100 X = XX
        Y = YY
        A - AA
        FK = FKK
    110 RETURN
        ENO
```

Subroutine CURVE initially calls subroutine DEPTH to determine whether the ray is in intermediate water or shallow water. It then sets the flag JGO to either 1 for intermediate water or 2 for shallow water. The local wave speed $c$ and the rate of change of wave speed with depth $c^{\prime}$ are then computed according to the appropriate equations. By using these values, the ray curvature is computed. Several parameters required in subroutine HEIGHT for computation of the wave separation factor $\beta$ are computed in this routine. The flow diagram and listing of subroutine CURVE follow.


```
*
```



```
* THIS SUBROUTINE CHECKS THE LOCAL DEPTH, CALCULATES THE WAVE
SPEED FOR THE APPROPRIATE REGIME, COMPUTES THE RATE OF CHANGE *
OF SPEED MITH RESPECT TO DEPTH, AND CALCULATES THE CURVATURE *
AT THE SPECIFIED RAY POINT.
```



```
*
            COMMON B1,B2,CO,CXY,CPRIME,DCON,DELTAS,DEP (42,32),DISTMAX,DISTMIN,
        +DRC,DTGR,DXY,GRID,GRINC,HO,IBLK(IOII,IGD,IOUTUNT,IREF,ISTOP,JGO,
        +JPRTFRQ(3),KCREST,KPLOT,LIMNPT,MI,MJ,NCREST,NFRST,NFRSTI,NLAST,
        +NLASTI,NORAY,NOROW,NPT,NSURFCE,NTIC,NTICOFF,NWRITE,PDX,PDY,PDZX,
        +PO2Y,PD2XY,PLTLNGX,PLTLNGY,PLTSCAL,RAYANG,RAYMAX,RAYMIN,RAYSPC,
        +RCCO,RK,SK,SIG,TIDE,TITL(8),V,WL,WLO,XO,XM,XPLOT(500),XPLOTI(500),
        +XPLOT2(500), XTEST,Y0,YM,YPLOT(500),YPLOTI(500),YPLOT2(500),YTEST
*
    IF (IGO.EQ. 1) GO TO 20
*
** COMPUTE DEPTH **
*
    CALL DEPTH (X,Y)
*
* TEST TO SEE IF IN INTERMEDIATE WATER **
#
    IF (200.0*OXY .GT. WL) GO TO 20
*
** IN SHALLOW WATER **
*
* TEST TO SEE IF SHORE HAS BEEN REACHEO **
*
        IF (OXY. GT. O.) GO TO 10
*
** SHORE REACHED. EXIT SUBROUTINE **
*
GO TO 60
    10 JGO=2
*
** COMPUTE WAVE SPEED FOR SHALLOW WATER **
*
        ARG = 32.1725*DXY
        CXY = SQRT(ARG)
*
** COMPUTE CPRIME FOR SHALLOW hATER **
*
    CPRIME = 16.08625/CXY
```

```
    GOTO 50
*
** IN Intermediate water **
    20 CI = CXY
        JGO=1
** ITERATE TO SOLVE FOR INTERMEDIATE WATER WAVE SPEED
*
        DO 30 I=1,50
        ARG = (DXY*SIG)/CI
        CXY = CO*TANH(ARG)
        RESID = CXY - CI
        IF (ABS(RESID) .LT. 0.0001) GO TO 40
    30 CI = 0.5*(CXY+CI)
*
** compute intermediate water cprime **
* 40 RCCO = CXY/CO
        SCMC = (1.-RCCO*RCCO)*SIG
        V = SCHC*DXY+RCCO*CXY
        CPRIME - CXY&SCMCIV
*
** COMPUTE RAY CURVATURE **
    50 FK - (SIN(A)*PDX-COS(A)*PDY)*CPRIME/CXY
    60 RETURN
        END
```

Subroutine DEPTH first determines the bathymetry data module in which the current ray point resides. If the point lies outside the boundaries of the module currently residing in central memory, the required module is read into memory by using the system subroutine READMS. Subroutine DEPTH selects the subset of data points in the module that are to be used to compute the bottom topography approximation surface. It then, on option, computes coefficients to a least squares quadratic, a least squares cubic, or a constrained bicubic polynomial which is used to approximate the local topography. The depth at the current ray point is then computed by evaluating the approximating polynomial at that point. Partial derivatives required in subroutine HEIGHT for computing the wave separation factor $B$ are also computed by evaluating analytical partial derivatives of the approximating polynomial. The input parameters TIDE and DCON are used to simulate a constant tide effect and to accommodate various units for the bathymetry data values. The flow diagram and listing of subroutine DEPTH follow.


THIS SUBRDUTINE CALLS INTO CORE THE BATHYMETRY DATA MDDULE IN WHICH THE CURRENT POINT RESIDES．IT THEN FITS A LEAST SOUARES CAN THEN BE EVALUATED BY USING THE POLYNOMIAL APPROXIMATION．

COMMIN B1，B2，CO，CXY，GPRIME，DCON，DELTAS，DEP\｛42，321，OISTMAX，DISTMIN， ＋ORC，DTGR，DXY，GRID，GRINC，HO，IBLK（IOI），IGO，IOUTUNT，IREF，ISTOP，JGO， ＋JPRTFRQ（3），KCREST，KPLOT，LIMNPT，MI，MJ，NCREST，NFRST，NFRST1，NLAST， ＋NLAST1，NQRAY，NORDW，NPT，NSURFCE，NTIC，NTICOFF，NWRITE，PDX，PDY，PD2X， ＋PDZY，PD $2 X Y$ ，PLTLNGX，PLTLNGY，PLTSCAL，RAYANG，RAYMAX，RAYMIN，RAYSPC， ＋RCCO，RK，SK，SIG，TIDE，TITL（8），V，WL，WLO，XO，XM，XPLDT（500），XPLOT1（500）， ＋XPLOTZ（500），XTEST，YO，YM，YPLOT（500），YPLOT1（500），YPLOT2（500），YTEST

## DIMENSION $C(16), D(16), S \times 1(6,12), S \times 2(10,12), 5 \times 3(16,16)$

INPUT CONSTANT MATRIX FOR QUADRATIC LEAST SQUARES，SXI，BY COLUMNS

```
    DATA (SX1(I),I=1,72)/. 30861244,2*.05322967,-.125,.05263158,
+-.125,.23684211,.19677034,010586124,-.125,=.05263158,*-125,
+. 21770335,2*.14413876,-.125,.05263158,-.125,. 23684211,.10586124,
+.19677034,-.125,-.05263158,-.125,-.08492823,.090311,.03349282,
+.125,-. 15789474,0.,-.05143541,-.06758373,-.03349282,.125,.15789474
+,0.,0-.05143541,-.03349282,-.06758373,0.,.15789474,.125,-.08492823,
+.03349282,.090311,0.,-.15789474,.125,.00598086,-.18241627,
+.12440191,.125,-.15789474,0.,.13038278,-.34031101,-.12440191,.125,
+. 15789474,0.,013038278,-.12440191,-.34031101,0.,.15789474,.125,
+.00598086,.12440191,-.18241627,0.,-.15789474,.125/
```

INPUT CONSTANT MATRIX FOR CUBIC LEAST SOUARES，SXZ，BY COLUMNS

```
DATA (SX2(I),I=1,120)/. 3125,2#-.625,-.125,.05263158,-.125,4*.25,
+. 3125,.625,-.625,-. 125,-.05263158,-. 125,-. 25,.25,-. 25,.25,.3125,
+2申.625,-.125,.05263158,-.125,4*-. 25,.3125,-.625,.625,-. 125,
+=.05263158,-.125,.25,-.25,.25,-.25,-.03125,-.02083333,.0625,.125,
+-. 1578947,0.,.08333333,-. 25,2#0.,-.03125,-.02083333,-.0625,.125,
+.1578947,0.,.08333333,.25,2#0.,-.03125,-.0625,-.02083333,0.,
+.1578947,.125,2*0.,.25,.08333333,-.03125,.0625,-.02083333,0.,
+-.1578947,.125,2#0,,-. 25,.08333333,-.03125,.02083333,-.0625,
+.125,-.1578947,0.,-.08333333,.25,2*0.,-.03125,.02083333,.0625,
+. 125,.1578947,0.,-.08333333,-. 25, 2*0.,-.03125,.0625,.02083333,
+0.,.1578447,.125,2#0,,-.25,-.08333333,-.03125,-.0625,.02083333,
+0.,-.1578947,.125,2*0.,.25,-.08333333/
```

INPUT CONSTANT MATRIX FOR CONSTRAINED BICUBIC INTERPOLATION，SX3，BY COLUMNS








 $+-1.01 .1$

```
IF (NSURFCE .EQ. 2) GO TO 10
* Place local origin at the central cell corner closest to x ano y axes for
* CONSTRAINED BICUBIC INTERPOLATION AND QUADRATIC LEAST-SQUARES
        XP=AMOD (X,1.)
        YP=AMOD (Y,l.)
        GOTO 20
    place local origin at the center of the central cell for cubic least-squares
    10 XP=AMOD (X,1,)-.5
        YP&AMOD (Y,1.)-.5
    CalCulate regional indiCES of NODAL POINT PI aND THE MODULE ROW AND
    COLUMN NUMBERS USING GENERAL EQUATIONS.
20 I1=X+1.
        IM=I1/39+1
        J1=Y+1.
        JM=J1/29+1
        IF (NPT.EQ.1) GO TO 30
IS CURRENT CELL SAME AS THE PREVIOUS ONE. IF SO, THERE IS NO NEED TO DO
ANYTHING AND THE PROGRAM SKIPS TO CALCULATION OF THE LOCAL DEPTH, DXY.
        IF (II.NE.IIP) GO TO 30
        IF (Jl.EQ.JIP) GO TO 150
30 I1P=11
        JlP=Jl
*
* CAlculate local indices of nooal point pi uSING general equations.
        ILI=MOD (I1,39)
        JLl=MDD (J1,29)
        IF (ILI.GT.1) GOTO 40
*
* IF ILI IS leSS than 2, we muSt uSE the alternate equations for module
* ROW NUMBER AND LOCAL ROW INDEX. THUS, SET IMADD=-1 AND ILIADD=39.
        IMADD=-1
        ILIADD=39
        GO TO 50
    40 IMADO=0
        ILIADO=0
    50 IF (JLI.GT.1) GOTO 60
*
* IF JLI IS LESS THAN 2, WE MUST USE THE ALTERNATE EQUATIONS fOR MODULE
COLUMN NUMBER AND LOCAL COLUMN INDEX. THUS, SET JMADD=-1 AND JLIADD=29.
        JMADD=-1
        JLIADD=29
        GO TO 70
    60 JMADD=0
        JLIADD=0
70 IM=IM+IMADO
        JM=JM+JMADD
        I=ILI+ILIADD
        J=JLI+JLIADD
        INDEX=(JM-1)*NOROW+IM
        IF (NPT.EQ.1) GO TO 80
```

```
* If INDEX = Indexp, the Current module is the same as the previdus one
* AND THERE IS NO NEEO TO READ THE RANDOM-ACCESS FILE. SKIP TO FILLING
* THE D ARRAY.
80
    IF (INDEX.EQ.INDEXP) GO TO 90
    INDEXPEINDEX
    READ MASS STORAGE DISK FILE (TAPEl) CONTAINING BATHYMETRY DATA MODULE
    CALL READMS (1,DEP,1344,INDEX)
90 IF (NSURFCE.EO.3) GO TO 100
FILL VECTOR FOR QUADRATIC OR CUBIC LEAST SQUARES APPROXIMATION
    D(1) = DEP(I,J)
    D(2)= DEP(I+1,J)
    D(3) = DEP(1+1,J+1)
    D(4) = DEP(I,J+1)
    D(5) = DEP(1+2,J)
    D(6)= DEP(I+2,J+1)
    D(7) - DEP(I+1,J+2)
    D(8) = DEP(I,J+2)
    O(9)= DEP(I-1,J+1)
    D(10) = DEP(I-1,J)
    D(11) - DEP(I,J-1)
    D(12)= DEP(I+1,J-1)
    GO TO 110
FILL VECTOR FOR CONSTRAINED BICUBIC INTERPOLATION
100 D(1)=DEP(I,J)
    D(2)=0.5*(DEP(I+1,J)-DEP(I-1,J))
    D(3)=0.5*(DEP(I,J+1)-DEP(I,J-1))
    D(4)=0.25#(DEP(I+1,J+1)-DEP(I+1,J-1)-DEP(I-1,J+1)+DEP(I-1,J-1))
    D(5)=DEP(I+1,J)
    D(6)=0.5#(DEP(I+2,J)-DEP(I,J))
    D(7)=0.5*(DEP(I+1,J+1)-DEP(I+1,J-1))
    D(8)=0.25*(DEP(I+2,J+1)-DEP(I+2,J-1)-DEP(I;J+1)+DEP(I,J-1))
    D(9)=DEP(I+1,J+1)
    D(10)=0.5*(DEP(I+2,J+1)-DEP(I, J+1))
    D(11)=0.5#(DEP(I+1,J+2)-DEP(I+1,J))
    D(12)=0.25*(DEP(I+2,J+2)-DEP(I+2,J)-DEP(I,J+2)+DEP(I,J))
    D(13)=DEP(I,J+1)
    D(14)=0.5*(DEP(I+1,J+1)-DEP(I-1,J+1))
    D(15)=0.5*(DEP(I,J+2)-DEP(I,J))
    D(16)=0.25*(DEP(I+1,J+2)-DEP(I+1,J)-DEP(I-1,J+2)+DEP(I-1,J))
110 IF (NSURFCE-2) 120,130,140
*
* matrix mulTIPLICatION. [C] - [SXI][D]
120 CALL MATRIX (6,12,SX1,D,C)
    GO TO 150
FIND CDEFFICIENTS OF CUBIC LEAST SOUARES POLYNOMIAL BY
MATRIX MULTIPLICATION. [C] - [SX2][D]
```

```
    130 CALL MATRIX (10,12,SX2,D,C)
    GO TO 150
*
*
*
*
    140 CALL MATRIX (16,16,SX3,D,C)
*
150 DXY-C(1)+(C(3)+(C(6)+C(10)*YP)*YP)*YP+(C(2)+(C(5)+(C(9)+C(11)*YP)*
        &YP)*YP)*XP+(C(4)+(C(8)+(C(12) +C(13)*YP)*YP)*YP)*XP**2+(C(7) +
        +(C(14)+(C(15) +C(16)*YP)*YP)*YP)*XP** 
*
    COMPUTE PARTIAL DERIVATIVES OF LOCAL DEPTH WITH RESPECT TO SPATIAL
    COORDINATES FROM POLYNONIAL APPROXIMATION
        PDX=C(2)+(C(5)+(C(9)+C(11)*YP)*YP)*YP+2.*(C(4)+(C(8)+(C(12)+C(13)*
        +YP)*YP)*YP)*XP+3.*XP** 2*(C(7)+(C(14)+(C(15)+C(16)*YP)*YP)*YP)
        PDY=C(3)+(C(5)+(C(8)+C(14)#XP)*XP) #XP+2.*YP*(C(6)+(C(9)+(C(12)+
        +C(15)*XP)*XP)*XP)+3.*YP**2*(C(10)+(C(11)+(C(13)+C(16)*XP)*XP)*XP)
        PD2X*2.*(C(4)+(C(B)+(C(12)+C(13)*YP)*YP)*YP)+6.*XP*(C(7)+(C(14) +
        +(C(15)+C(16)*YP)*YP)*YP)
            PD2Y=2.*(C(6) +(C(9)+(C(12)+C(15)*XP) #XP)*XP) +6**YP*(C(10) +(C(11) +
        +(C(13)+C(16)*XP)*XP)*XP)
            P02XY=C(5)+2.#(C(7)*YP+C(8)*XP)+3.*(C(11)*YP**2+C(14)*XP**2)+
        +4.*C(12)*XP*YP+6.*XP*YP*(C(13)*YP+C(15)*XP)+9.*C(16)*XP**2*YP**2
*
    CHANGE DEPTH UNITS BY DCON
        DXY=DXY#DCON+TIDE
        PDX=PDX*DCON
        PDY=PDY*OCON
        PD2X=PD2X&DCON
        PD2Y=PD2Y&DCON
        PDZXY=POZXY*OCON
        RETURN
        END
```

Subroutine HEIGHT calculates the local shoaling coefficient (eq. (7)) and then computes the refraction coefficient (eq. (8)) by using the value of $\beta$ calculated at the previous ray point. These coefficients are then used to compute the local wave height. Fox's finite-difference solution for wave intensity is solved to give a new value of $B$. The flow diagram and listing of subroutine HEIGHT follow.


```
    SUBROUTINE HEIGHT (A,H)
由
```



```
#
THIS SUBROUTINE CALCULATES THE SHOALING COEFFICIENT, SK, AND
COMPUTES THE REFRACTION COEFFICIENT, RK, USING THE VALUE OF
BETA AT THE PREVIOUS POINT ON THE RAY. THESE ARE THEN USED TO
CALCULATE THE LOCAL UAVE HEIGHT. THEN THE FINITE-DIFFERENCE
FORM DF THE EQUATIDN OF WAVE INTENSITY IS SOLVED TO GIVE A NEW
VALUE DF BETA.
*
```



```
*
COMMON B1,B2,CO,CXY,CPRIME,DCON,DELTAS,DEP(42,32),OISTMAX,DISTMIN,
+ORC,DTGR,DXY,GRID,GRINC,HO,IBLK(IO1),IGD,IOUTUNT,IREF,ISTOP,JGO,
4JPRTFRQ(3),KCREST,KPLOT,LIHNPT,MI,MJ,NCREST,NFRST,NFRST1,NLAST,
+NLAST1,NORAY,NOROW,NPT,NSURFCE,NTIC,NTICOFF,NWRITE,POX,PDY,PD2X,
+PD2Y, PD 2XY,PLTLNGX,PLTLNGY,PLTSCAL,RAYANG, RAYMAX,RAYMIN,RAYSPC,
+RCCO,RK,SK,SIG,TIDE,TITL(8),V,WL,WLO,XO,XM,XPLOT(500),XPLOT1(500),
*XPLOTZ(500),XTEST,YO,YM,YPLOT(500),YPLOT1(500),YPLOT2(500),YTEST
#
IF (1GO -LT. 2) GO TO 30
WL WLO#RCCO
GN = 12.5663706*DXY/WL
CG = (1,0+GN/SINH(GN))#CXY
IF (CG.LT. O.) GI TO 30
SK = SORT(CO/CG)
#
** COMPUTE REFRACTION COEF USING MUNK-ARTHUR EON **
*
    RK = ABS(1./B2)
    RK = SORT(RK)
*
** COMPUTE WAVE HEIGHT **
*
    H = HO*SK*RK
*
** TEST TO SEE IF IN SHALLOW DR INTERMEDIATE WATER **
*
    IF (JGO .EQ. 2) GO TO 10
*
** INTERMEDIATE WATER VALUE FOR U **
*
    U=-2.*SIG*RCCO*CXY/(V*V)
    GO TO 20
*
** SHALLOW WATER VALUE FOR U **
*
    10 U = -0.5/DXY
*
** COMPUTE RAY SEPARATION FACTOR **
*
20 cosa - cos(A)
    SINA - SIN(A)
    P = - (COSA*PDX+SINA*PDY)*CPRIMEFDTGR*2.
        O = ((PDZX+U*PDX*PDX)*SINA*SINA-(PD2XY+U*PDX*PDY)*2.*SINA*COSA*
        +(PD2Y+U#PDY*PDY)*COSA*COSA)*CPRIME*CXY*DTGR*DTGR*2.
            B3 - ((P-2.)*B1+(4.-0)*B2)/(P+2.)
            B1=B2
            B2-B3
30 RETURN
    END
```

Subroutine WRITER is used for storing computed parameters in appropriate arrays and then for printing the same parameters after the complete ray path has been constructed. The output units for spatial coordinates ( $x, y$ ) are in nautical miles. The output units for all other computed parameters can be in either U.S. Customary Units or the International System of Units (SI), as specified by the parameter IOUTUNT. The first and last points on a ray are always printed, and intermediate points are printed according to the print frequencies specified by the input array JPRTFRQ. Position data are saved in this routine to be used in the plotting subroutine PLOTR. The flag IGO is set to 3 when the ray has reached a termination condition, and a message specifying the particular termination condition is printed. The flow diagram and listing of subroutine WRITER follow.


```
    SUBROUTINE WRITER (X,Y,A,H)
隹
```



```
*
* THIS SUBROUTINE STORES AND PRINTS COMPUTED RAY PARAMETERS AND
* THE RAY TERMINATION CONDITION. X AND Y COORDINATES ARE SAVED
* IN THIS SUBROUTINE FOR EVENTUAL PLOTTING.
申
```



```
*
    COMMON B1,B2,CO,CXY,CPRIME,DCON,DELTAS,DEP(42,32),DISTMAX,DISTMIN,
    +DRC,DTGR,DXY,GRID,GRINC,HO,IBLK(101),IGO,IOUTUNT,IREF,ISTOP,JGO,
    +JPRTFRQ(3),KCREST,KPLOT,LIMNPT,MI,HJ,NCREST,NFRST,NFRST1,NLAST,
    +NLASTI,NORAY,NOROW,NPT,NSURFCE,NTIC,NTICOFF,NWRITE,PDX,PDY,PD2X,
    +PD2Y,PD 2XY,PLTLNGX,PLTLNGY,PLTSCAL,RAYANG,RAYMAX,RAYMIN,RAYSPC,
    +RCCO,RK,SK,SIG,TIDE,TITL(8),V,WL,WLO,XO,XM,XPLOT(500),XPLOTI(500),
    +XPLOTZ(500), XTEST,YO, YM,YPLOT(500),YPLOT1(500),YPLOT2(500),YTEST
#
    DIMENSION NPT1(400),X1(400),Y1(400),A1(400),DXY1(400),WL1(400),
        +CXY1(400),RK1(400),SK1(400),H1(400)
*
    IF (IGO.EQ. 3) GO TO 30
    IF (NWRITE .GE. 2) IGO=3
*
** SET OUTPUT UNITS **
*
    IF (NPT .GT. 1) GO TO 10
    NC = O
    AMULT = 1.
    XYMULT = 1./6076.1155
    IF (IOUTUNT .EQ. 2) AMULT = 0.3048
*
** SET PRINT FREQUENCY **
*
    10 IPRINT = JPRTFRQ(3)
    IF (DXYIWL .GT. 0.25 .AND. DXYIWL .LT. 0.5) IPRINT = JPRTFRO(2)
    IF (DXY/WL .GE. O.5) IPRINT = JPRTFRO(1)
*
** SAVE X AND Y RAY CODRDINATES FOR PLOTTING WHICH ARE INDEX **
** WITH RESPECT TO REFERENCE CORNER
*
    XPLOT(NPT+NTICOFF) = X
    YPLOT(NPT+NTICOFF) = Y
*
** SAVE fIRST AND LAST POINT **
    IF (NPT .EQ. 1 .OR. NWRITE .GE. 2) GO TO 2O
```

```
** PRINT OUTPUT CONTROL **
* IF (MOD (NPT,IPRINT).NE. O) GO TO 50
*
** SAVE COMPUTED RAY PARAMETERS **
*
    20 NC=NC +1
    NPTI(NC) - NPT
    X1(NC) - XYMULT#GRID#X
    YI(NC) = XYMULT*GRID*Y
    A1(NC) - A
    DXYI(NC) - AMULT*DXY
    WLI(NC) = AMULTFWL
    CXY1(NC) E AMULT*CXY
    RKI(NC) = RK
    SKI(NC) = SK
    HI(NC) AMULT*H
    GO TO 50
*
** PRINT STORED ARRAY OF COMPUTED RAY PARAMETERS **
*
    30 DO 40 I=1,NC
    40 PRINT 60,NPT1(I),XI(I),YI(I),A1(I),DXY1(I),WLI(I),CXY1(I),
        +RKI(I),SKI(I),HI(I)
*
** PRINT RAY TERMINATION CONDITIONS **
*
    IF (NWRITE .EQ. 2) PRINT 70
    IF (NWRITE EQ. 3) PRINT 80
    IF (NWRITE .EQ. 4) PRINT 90
    IF (NWRITE .EO. 51 PRINT 100
    IF (NWRITE .EQ. 6) PRINT 110
    50 RETURN
60 FORMAT (3X,I4,2F10.2,2X,2F9.2,2F8.2,F10.3,F11.3,F9.2)
70 FORMAT (1HO,* RAY STOPPEO, NO CONVERGENCE FOR CURVATURE*)
80 FORMAT (1HO,* RAY STOPPED, REACHED SHORE#)
90 FORMAT (1HO,* RAY STOPPED, REACHED BOUNDARY*)
100 FORMAT (1HO;* RAY STOPPED, NUMBER DF POINTS EXCEEDS MAXIMUM*)
110 FORMAT (1HO,* RAY STOPPED, INCREMENTAL DISTANCE ALONG RAY LESS TH
        +AN MINIMUM*)
            END
```

Subroutine PLOTR is used for segment-by-segment construction of wave crests and for plotting the ray diagrams. The logic for constructing the crest patterns is detailed in appendix B. The plotting mode is controlled by the input parameter KPLOT. Axes, labels, and notations are the same for both the ray and crest modes of plotting. The input parameters PLTLNGX and PLTLNGY control the size of the plotting area, and the input parameter PLTSCAL scales the data to fit this area. Actual plotting is performed by using system subroutines which are described in appendix A. The flow diagram and listing of subroutine PLOTR follow.


```
    SUBROUTINE PLOTR (KOPT)
*
```



```
*
* THIS SUBROUTINE PLOTS RAY AND CREST DATA. *
```



```
*
            COMMON B1, 82,CO,CXY,CPRIME,DCON,DELTAS,DEP(42,32),DISTMAX,DISTMIN,
            +DRC,DTGR,DXY,GRID,GRINC,HD,IBLK(IOI),IGO,IDUTUNT,IREF,ISTOP, JGO,
            +JPRTFRO(3),KCREST,KPLOT,LIMNPT,MI,MJ,NCREST,NFRST,NFRST1,NLAST,
            +NLAST1,NORAY,NOROW,NPT,NSURFCE,NTIC,NTICOFF,NWRITE,PDX,PDY,PO2X,
            +PD2Y,PDZXY,PLTLNGX,PLTLNGY,PLTSCAL,RAYANG,RAYMAX,RAYMIN,RAYSPC,
            4RCCO,RK,SK,SIG,TIDE,TITL(8),V,WL,WLD,XO,XH,XPLOT(500),XPLOTI(500),
            +XPLOT2(500),XTEST,Y0,YM,YPLOT(500),YPLOT1(500),YPLOT2(500),YTEST
*
            DIMENSION XPL(400),YPL(400)
            GO TO (10,30,100,230), KOPT
*
** OPEN PLOT FILE AND SET PLOT SPEED **
    10 CALL PSEUDO
            CALL LEROY
            EPS = 0.001
            DO 20 I=1,500
    20 XPLOT(I)=YPLOT(I)=XPLOTI(I)=YPLOTI(I)=XPLOT2(I)=YPLOTZ(I)=0.0
            GO TO 240
*
** ADVANCE PLOT FRAME AND SET CONSTANTS **
*
    30 CALL NFRAME
        AH=GRID/6076.1155
        COSRAY=COS(RAYANG/57.2957795)
*
** SET ORIGIN **
        CALL CALPLT (3.,1.5,-3)
*
** PLOT TITLE INFORMATION **
*
        CALL NOTATE (-1.5,1.0,0.15,TITL,90.,60)
        IF (NSURFCE-2) 40,50,60
    40 CALL NOTATE (-1.2,3.0,0.125,23HOUADRATIC LEAST SQUARES,90.,23)
        GO TO 70
    50 CALL NOTATE (-1,2,3,0,0.125,19HCUBIC LEAST SOUARES,90.,19)
        GO TO 70
    60 CALL NOTATE (-1,2,3,0,0.125,33HCONSTRAINED BICUBIC INTERPOLATION,
```

$+90 ., 331$
*

* P PLOT AND LABEL AXES **
$70 \quad \mathrm{Y} 1=-0.069$
TMAJ=10./PLTSCAL
LMT=PLTLNGY\#PLTSCAL+10.
DO 80 I=10,LHT,10
FPN=1-10
CALL NUMBER $(-0.15, Y 1,0.12, F P N, 90,-1)$
$80 \quad Y 1=Y 1+T M A J$
X1=0.06
LMT=PLTLNGX*PLTSCAL+10.
DO 90 I=10,LMT, 10
FPN=I-10
CALL NUMBER $(\times 1,-0.4,0.12, F P N, 90 .,-1)$
$90 \times 1=X 1+$ TMAJ
$Y$ M $=$ PLTLNGY/2.0-0.26
CALL NOTATE $(-0.6, Y H, 0.15,4 H Y, N M, 90 ., 4)$
$X M=P L T L N G X / 2.0+0.08$
CALL NOTATE (XH, $-1.20,0.15,4 \mathrm{HX}, \mathrm{NH}, 90 ., 4)$


CALL CALPLT (O.,P PLTLNGY,3)
CALL CALPLT (PLTLNGX,PLTLNGY,2)
CALL CALPLT (PLTLNGX,O.,2)
GO 10240
100 IF (KPLOT .EQ. 3) GO TO 140
申
** PLOT RAY **
* 

NS $=0$
DO 110 I =NFRST, NLAST
NSONS+1
XPL(NS) =AM*XPLOT(I)/PLTSCAL
110 YPL(NS) =AM*YPLOT(I)/PLTSCAL
IF (MOD (NORAY,5) •NE. O) GO TO 120
*

* put ray number on plat **
* 

$X N O=X P(11)+0.03$
YNO = YPL(1) +0.03
CALL NUMBER (XNO,YNO,0.07,FLOAT(NORAY),90.,-1)
*
** PUT + SYMBOL AT EVERY NCREST POINT ON RAY **
*
$12000130 \quad I=1, N S$,NCREST
130 CALL PNTPLT (XPL(I), YPL(I), -22,1)
CALL DRAW (XPL, YPL,NS)

```
        GO TO 190
    140 IF (NORAY .LT. 3) GO TO 190
*
** DETERMINE FIRST AND LAST RAY POINT HITH RESPECT TO REFERENCE **
** CORNER FOR PLOTTING CREST DATA
*
        MSI - NFRST
        IF (NFRSTI -GT. NFRST) MSI=NFRSTI
        IF (NFRST2 .GT. NFRST .AND. NFRST2 .GT. NFRSTII MSIUNFRST2
        MS2 - NLAST
        IF (NLAST1 -LT. NLAST) HS2ONLASTI
        MS1=MS1+1
        MS2=MS2-1
*
** PLOT CREST DATA **
*
    DO 180 I=MSI,MSZ,NCREST
*
** Calculate vector components of ray slopes and first-order **
** CREST SEGMENTS
    DXS1=XPLOT1(I+1)-XPLOT1(I-1)
    DYSI=YPLOT1(I+1)-YPLOT1(I-1)
    IF (ABS(DYSI) ©LT. EPSS) DYSI=SIGN(EPS,DYSI)
*
*
    SL1=-DXS1/DYS1
    OXPT=XPLOT(I+1)-XPLOT(I-1)
    DYPT=YPLOT(I+1)-YPLOT(I-1)
    IF (ABS(DYPT) \bulletLT. EPS) DYPT=SIGN(EPS,DYPT)
    SL2=-DXPT/DYPT
OXPS=XPLOT(I)-XPLOTI(I)
IF (ABS(DXPS) \bulletLT. EPS) DXPS=SIGN(EPS,DXPS)
DYPS=YPLDT(I)-YPLJTI(I)
D1=SQRT(DXPS**2+DYPS**2)
IF (DI elT. EPS) DI=EPS
IF (I .GT. NLAST2) GO TO 160
DXS12=XPLOT2(I)-XPLOTI(I)
OYS12=YPLOT2(I)-YPLOT1(I)
D2=SQRT(DXS12**2+DYS12**2)
IF (D2 .LT. EPS) D2-EPS
*
** CALCULATE aNGLE bETWEEN ADJOINING FIRST-ORDER CREST SEGMENTS **
*
        THETA=(DXPS*DXS12+DYPS*DYS12)/(D1*D2)
        JS=1
        IF (THETA .GT. COSRAY) JS=KCREST
*
** calculate coefficients of cubic curve **
```

```
*
    160 A1-SL1
        A2=(3.0*DYPS-DXPS*(2.0*SLI+SL2))/DXPS**2
        A3=(-2, O*OYPS + DXPS * (SLI+SL2))/OXPS** 3
        DX=DXPS/6.0
        NS=O
申
** Calculate and draw pointS ON CUBIC CURVE BETWEEN TWO RAYS **
*
        IF (JS GE. 7) GO TO 180
        DO 170 J=JS,7
        NS=NS+1
        XPL(NS)=(XPLOT1(I)+(J-1)*DX)*AM/PLTSCAL
        D1-(J-1)*DX
        YPL(NS)=(YPLOT1(I)+01*(A1+D1*(A2+01*A3)))*AM/PLTSCAL
        IF (YPL(NS) \bulletLT. 0.0) YPL(NS)=0.0
    170 CONTINUE
        CALL DRAW (XPL,YPL,NS)
    180 CONTINUE
    190 IF (NORAY .LT. 2) GO TO 210
*
** SAVE PREVIOUS TWO RAYS *
*
        DO 200 I=NFRST1,NLASTI
        XPLOT2(I) = XPLOT1(I)
    200 YPLOT2(I) - YPLOT1(I)
    210 DO 220 I=NFRST,NLAST
        XPLOTI(I) = XPLOT(I)
    220 YPLOT1(I) YPLOT(I)
        NFRST2 = NFRST1
        NLAST2 = NLASTI
        NFRST1 NFRST
        NLAST1 NLAST
        GO TO 240
*
** ENO PLOTTING **
*
    230 CALL CALPLT(0.,0.,999)
    240 RETURN
        END
```

This subroutine determines the initial ray spacing required to satisfy specified final (nearshore) ray spacing conditions. Final ray spacing is defined as the distance between the shorewardmost synchronous points on adjacent rays. The routine would normally be used in conjunction with the wavecrest plotting option in order to facilitate the construction of curved crest segments between adjacent rays. A detailed discussion of this routine and pertinent input parameters is given in appendix C. The flow diagram and listing of subroutine RAYOPT follow.

+DRC, DTGR,OXY,GRID, GRINC,HO,IBLK(IO1),IGO,IOUTUNT,IREF,ISTOP,JGO,
+JPRTFRQ(3), KCREST, KPLOT,LIMNPT, MI, MJ, NCREST,NFRST,NFRST1,NLAST,
+NLAST1, NORAY, NOROW, NPT, NSURFCE, NTIC, NTICOFF, NWRITE, PDX, POY, PD $2 X$,
+PD2Y, PO 2XY,PLTLNGX, PLTLNGY,PLTSCAL,RAYANG,RAYMAX,RAYMIN,RAYSPC,
+RCCO,RK,SK,SIG,TIDE,TITL(8),V,WL,WLO,XO,XH,XPLDT(500),XPLOT1(500),
+XPLOT2(500), XTEST, YO, YM, YPLDT (500), YPLOTI(500), YPLOT2(500), YTEST
*
IF (NORAY .LT. 2) GO TO 80
NTEST = NLAST
IF (NLAST •GT. NLASTI) NTEST=NLASTI
*

* COMPUTE FINAL SPACING BETWEEN ADJACENT RAYS **
* DIST - SORT((XPLOT (NTEST)-XPLOT1(NTEST)) $\ddagger=2+(Y P L O T(N T E S T)-~$
+YPLOTL(NTEST))*\#2)
IF (MINDIST.GT. O) GO TO 30
IF (DIST•LT. DISTMAX) GO TO 20
* 
* 
*     * SEPARATIDN AND REPEAT. MAX NO OF PASSES EQUAL 10
* MAXDIST = 1
IF (NPASS -GT. 10 ) GO TO 80
* 

** IF INITIAL RAY SPACING LESS THAN MINIMUM, SET TO MINIMUM **
*
IF (RAYSPC •LE. RAYMIN) GO TO 10
RAYZ RAYSPC
DIST2 OIST
RAYSPC $=0.5$ *RAYSPC
GO TO 70
10 RAYSPC RAYMIN
GO TO 80
20 MINDIST - 1
NPASS=1
30 IF (NPASS .GT. 10) GO TO 80
IF (DIST •LT. DISTMIN) GO TO 40
IF (DIST.LE. DISTMAX) GO TO 80

```
    RAY2 = RAYSPC
    DIST2 = DIST
    MAXDIST = 1
    GO TO 50
** IF FINAL DISTANCE BETWEEN RAYS IS TOO SMALL, FINO INITIAL RAY
** SEPARATION BY NEWTON-RAPHSON METHOD ANO REPEAT. MAX NO
** OF PASSES EQUAL 10
    40 RAYI RAYSPC
    DIST1 = DIST
    IF (MAXDIST .GT. O) GO TO 50
    RAYSPC E 2.0*RAYSPC
    GO TO 60
    50 DISTM - 0.5*(DISTMIN+DISTMAX)
    RAYSPC = RAYI+(DISTM-DISTI)*(RAY2-RAY1)/(DIST2-DIST1)
*
** If INITIAL RAY SPACING GREATER THAN MAXIMUM, SET TO MAXIMUM **
    60 IF (RAYSPC .LT. RAYMAX) GO TO 70
        RAYSPC = RAYMAX
        GO TO 80
    70 XTEST XTESTL
        YTEST = YTESTL
        NTIC NTICL
        NPASS = NPASS$1
        IREPEAT - 1
        GO TO 90
** GOOD PASS, RESET AND UPDATE INITIAL CONOITIONS **
    80 MAXDIST - O
    IREPEAT = 0
    MINDIST - O
    XTESTL * XTEST
    YTESTL = YTEST
    NTICL = NTIC
    NPASS = 1
    90 RETURN
    END
```

This is an auxiliary routine incorporated to perform the matrix multiplication required in the determination of the coefficients for the topography approximating polynomial used in subroutine DEPTH. The subroutine multiplies the invariant matrix $S X$ which depends only on the topography approximating technique selected, by a column vector of depth values $D$ to obtain the resultant column vector of coefficients $C$ to be used with the particular polynomial. The flow diagram and listing of subroutine MATRIX follow.


SUBROUTINE MATRIX (IR,JC,SX,D,CI
*


The computer program is coded in FORTRAN Extended (FTN) Version 4 for use on the Control Data Cyber 173 or 175 computers under NOS 1.2. A central memory field length of 660008 locations is required, along with a peripheral massstorage file for input of bathymetry data as discussed subsequently. Central processing unit (CPU) time requirements for program execution vary with the size of the geographic region belng considered, the spatial density of wave rays being computed within the region, and the graphic output mode selected.

## Bathymetry Data Input

As a first step, a bathymetry data array corresponding to a square grid with a uniform spacing of GRID feet per division must be developed for the geographic region to be investigated. It is recommended that the data array be developed by using a special transverse Mercator map projection centered within the region of interest in order to minimize error in lengthy ray paths due to curvature of the Earth. Advantages of using such a projection are discussed in reference 6, and the projection itself is discussed in detail in reference 9. For input to the computer program, the regional bathymetry array must be divided into overlapped modules of dimensions 42 by 32. Modularization must be performed as a separate function and can be done using the utility program CREMOD, which is described in detail in appendix D. During program execution, the modularized bathymetry data file TAPE1 is inıtialized as a random-access file in main program WAVE by using the system subroutine OPENMS. Modules are then read from the file to central memory as required in subroutine DEPTH by using the system subroutine READMS.

## Card Input

Other parameters required for program execution are input in card form by using standard Control Data NAMELIST. The program is designed to compute a number of sets (NOSETS) of refraction diagrams with a single computer run. Parameters which are fixed for all sets of diagrams to be computed are input under the NAMELIST group name NPUT1 and are listed with their default values in table I. If the user desires to use default values, only the parameters specifying the modular bathymetry grid configuration must be supplied. Parameters in effect for a single set are input under the NAMELIST group name NPUT2 and are listed with their default values in table II. If the user desires to use default values with this group, only the wave period and initial wave height and direction must be specified. In addition, a title card of up to 80 alphanumeric characters is required before each group of NPUT2 cards as identification of the conditions in effect for that set. Normal termination of program execution and closure of the plot file is effected after NOSETS of ray diagrams have been computed. However, in the event of user oversight, an end of file (EOF) is checked on card input on both NPUT1 and NPUT2 to insure proper termination.

At the beginning of the printed output, the basic program input parameters are identified and the values assigned to them are printed. Omission by the user of any mandatory input parameter causes the program to print an appropriate error message and then terminate. If KPLOT is specified as 2 or 3, the additional input parameters required for those plotting options are identified and their values printed. At the start of printed values for each ray, a header is printed which gives the values of constants valid for the current ray set along with title information for the current ray. Computed ray parameters are printed at specified increments in the ray computational point index NPT; these increments are input in the array JPRTFRQ. Output units for spatial coordinates ( $\mathrm{x}, \mathrm{y}$ ) are in nautical miles. Output units for other computed parameters can be either U.S. Customary Units or SI Units, as specified by the input parameter IOUTUNT. The computed parameters are printed in column format in the following order:

| NPT | ray computational point index |
| :--- | :--- |
| X | alongshore coordinate x , nautical miles |
| Y | offshore coordinate y , nautical miles |
| A | ray direction $\alpha$, degrees |
| DXY | water depth $d$, feet or meters |
| WL | wavelength $L$, feet or meters |
| C | wave speed $c$, feet/second or meters/second |
| RK | refraction coefficient $K_{r}$ |
| SK | wave height $H, f e e t$ or meters |

At the end of printed parameters for each ray, the condition which led to ray termination is printed.

The plotted output, depending upon the value of KPLOT, gives either ray diagrams or crest diagrams. The length of each axis is determined by the input parameters PLTLNGX and PLTLNGY. Scaling of the data and the axis scales are controlled by the parameter PLTSCAL. Major tick marks are added at 10 -nauticalmile intervals, and minor tick marks are added at 1 -nautical-mile intervals along both axes. In plotting ray diagrams, a plus symbol (+) is drawn at increments of NCREST computational points along each ray, and every fifth ray is numbered. Initial ray spacing is specified by RAYSPC in the default mode or is computed to meet constraints on final (nearshore) ray spacing if that option is selected.

## Sample Cases

In order to illustrate the input and output options of the program, three sample cases are shown. The cases represent ray and crest diagrams for the Baltimore Canyon region of the mid-Atlantic continental shelf, extending from near Wachapreague Inlet, Virginia, to near Manasquan Inlet, New Jersey. The bathymetry data array for this region was divided into a total of 72 modules of dimensions 42 by 32 , with 8 rows and 9 columns of data modules. For each sample case, the basic program parameters, the input parameters for the ray set, and the computed parameters for a representative ray within the set are shown. Also the ray or crest diagram constructed for the set in question is presented. It should be noted that the land masses and isobaths have been added to the basic diagrams for display purposes.

Case 1.- For this case, a ray diagram was generated with equal initial spacing between the computed rays. KPLOT was specified as 1, and IOUTUNT was specified as 1, resulting in output parameters other than $x$ and $y$ being given in U.S. Customary Units. This case required 10 seconds for computer processing. Figure 3 gives the ray diagram for sample case 1. A listing of the input parameters for sample case 1 is given as follows:

NPUT 1

| DCON | 1.0 | A . . . . . | -45.0 |
| :---: | :---: | :---: | :---: |
| DELTAS | 0.01 | H0 | 6.0 |
| GRID | 3038.06 | NCREST . . . | 5 |
| GRINC | 1.0 | NSURFCE . | 1 |
| IOUTUNT | 1 | RAYSPC . . | 5.0 |
| JPRTFRQ(1) | 20 | T . . . . . | 12.0 |
| JPRTFRQ(2) | 10 |  |  |
| JPRTFRQ(3) | 5 |  |  |
| KPLOT | 1 |  |  |
| LIMNPT | 400 |  |  |
| MI | 300 |  |  |
| MJ | 254 |  |  |
| NOMOD | 72 |  |  |
| NOROW | 8 |  |  |
| NOSETS | 1 |  |  |
| PLTLNGX | 12.0 |  |  |
| PLTLNGY | 10.0 |  |  |
| PLTSCAL | 12.5 |  |  |
| TIDE | 0.0 |  |  |

DELTAS . . . . . . 0.01
GRID
3038.06

GRINC . . . . . . . 1.0
IOUTUNT
1
JPRTFRQ(1) . . . . . 20
JPRTFRQ(2) . . . . . 10
JPRTFRQ(3) . . . . . 5
KPLOT . . . . . . . 1
LIMNPT . . . . . . . 400
MI . . . . . . . . . 300
MJ . . . . . . . . . 254
NOMOD . . . . . . . 72
NOROW . . . . . . . 8
NOSETS . . . . . . . 1
PLTLNGX . . . . . . 12.0
PLTLNGY . . . . . . 10.0
PLTSCAL . . . . . . 12.5
TIDE . . . . . . . . 0.0

NPUT2

A . . . . . . . . -45.0
но . . . . . . . . . . 6.0
NCREST . . . . . . . . 5
NSURFCE . . . . . . . 1
RAYSPC . . . . . . . . 5.0
T . . . . . . . . . . 12.0

## BASIC PROGRAM PARAMETERS













Case 2.- This case illustrates a ray diagram in which the final (nearshore) ray separation distance is controlled. KPLOT was set equal to 2 and the limiting conditions on initial and final ray separation distances are printed at the end of the program parameter list. The output units are in SI Units as specified by setting IOUTUNT equal to 2 . This case required 22 seconds for computer processing. Figure 4 shows the ray diagram for this sample case. The listing of the input parameters for sample case 2 is given as follows:

NPUT1 NPUT2


## BASIC PROGRAM PARAMETERS



Case 3.- The crest diagrams constructed from the rays computed in sample case 2 are illustrated by this sample case. KPLOT was set to 3 to specify this plotting option. The output units are U.S. Customary as specified by setting IOUTUNT to 1. The resulting crest diagram is shown in figure 5. This case required 22 seconds for computer processing. The listing of the input parameters for sample case 3 is given as follows:

NPUT 1
NPUT2

| DCON | 1.0 | A | -45.0 |
| :---: | :---: | :---: | :---: |
| DELTAS | 0.01 | H0 | 6.0 |
| GRID | 3038.06 | NCREST | 5 |
| GRINC | 1.0 | NSURFCE | 1 |
| IOUTUNT | 1 | RAYSPC | 1.0 |
| JPRTFRQ(1) | 20 | T | 12.0 |
| JPRTFRQ(2) | 10 | DISTMAX | 50.0 |
| JPRTFRQ(3) | 5 | DISTMIN | 30.0 |
| KPLOT | 3 | KCREST | 3 |
| LIMNPT | 400 | RAYANG | 120.0 |
| MI | 300 | RAYMAX | 15.0 |
| MJ | 254 | RAYMIN | 0.01 |
| NOMOD | 72 |  |  |
| NOROW | 8 |  |  |
| NOSETS | 1 |  |  |
| PLTLNGX | 12.0 |  |  |
| PLTLNGY | 10.0 |  |  |
| PLTSCAL | 12.5 |  |  |
| TIDE. | 0.0 |  |  |

## BASIC PROGRAM PARAMETERS

| DEPTH UNIT MULT................DCON | 1.00 |
| :---: | :---: |
| MIN STEP SIZE..................DELTAS | . 01 |
| GRID SIZE.......................GRID | = 3038.06 |
| DEEP WATER STEP SILE...........GRINC | 1.00 |
| OUTPUT UNIT OPTION..............IOUTUNT | - 1 |
| PRINT FREQ 1 (DIL>0.5)........JPRTFRQ(1) | - 20 |
| PRINT FREQ 2 (0.25<0/L<0.5)...JPRTFRQ(2) | $=10$ |
| PRINT FREQ 3 (D/L<0.25).......JJPRTFRQ(3) | - 5 |
| PLOT OPTION.....................KPLOT | - 3 |
| MAX NO OF RAY POINTS...........lIMNPT | $=400$ |
| GRID LIMITS ABSCISSA.........MI | - 300 |
| ORDINATE.........MJ | - 254 |
| NO DF DEPTH MODULES........... ${ }^{\text {NTM MOD }}$ | - 72 |
| NO DF ROW MODULES IN NOMOD.... NJROW | - 8 |
| NO OF SETS OF RAYS............NJSETS | - 1 |
| PLOT $X$ AXES LENGTH..............PLTLNGX | $=12.00$ |
| PLOT Y AXES LENGTH.............PLTLNGY | - 10.00 |
| PLOT SCALE FACTOR.............PPLTSCAL | - 12.50 |
| TIDE HEIGHT.....................tIDE | - 0.00 |
| MAX FINAL RAY SPACING.........DISTMAX | - 50.0 |
| MIN FINAL RAY SPACING..........DISTMIN | 30.0 |
| INITIAL PT FOR CREST CURVE.....KCREST | - 3 |
| MIN ANGLE BETWEEN SEGMENTS....RAYANG | - 120.0 |
| MAX INITIAL RAY SPACING....... RAYMAX | - 15.0 |
| MIN INITIAL RAY SPACING........RAYMIN | . 01 |

RAY STOPPED, REACHED BOUNDARY


DESCRIPTION OF SYSTEM SUBROUTINES
The program described in this paper uses several system subroutines to satisfy various data handling and plotting requirements. This appendix lists the names of subroutines and their functions.

AXES Draws a line, annotates the value of the variable at specified intervals with or without tick marks, and provides an axis identification label.

CALPLT Creates a reference point for plotting and ends plotting.
CLOSMS Closes the random-access mass-storage file.
DRAW Draws a line between specified points.
ENCODE Reformats data in memory.
LEROY Sets plot speed.
NFRAME Advances the plot frame and sets the plot origin.
NOTATE Draws alphanumeric information for plot annotation and labeling.
NUMBER Converts a floating point number to binary coded decimal (BCD) format and draws the resulting alphanumeric characters.

OPENMS Initializes the random-access mass-storage file.
PNTPLT Draws a given symbol centered on a given coordinate value.
PSEUDO Initializes and writes a plot vector file.
READMS Transfers data from mass storage to central memory.
WRITMS Transfers data from central memory to mass storage.

## APPENDIX B

## CONSTRUCTION OF WAVE-CREST DIAGRAMS

The interaction of ocean waves with coastal bottom topography can be examined most readily by plotting wave-crest patterns rather than ray patterns. Wave-crest patterns can be constructed by connecting the same point in time on adjacent rays by a curve which is locally orthogonal to each of the rays it intersects. The simplest smooth curve which can be drawn perpendicular to two adjacent rays is represented by a third-order equation. A third-order curve passing through point $j$ on ray $i-1$ can be written

$$
\begin{equation*}
y=y_{i-1, j}+a_{1}\left(x-x_{i-1, j}\right)+a_{2}\left(x-x_{i-1, j}\right)^{2}+a_{3}\left(x-x_{i-1, j}\right)^{3} \tag{B1}
\end{equation*}
$$

where $a_{1}, a_{2}$, and $a_{3}$ are coefficients to be determined. The coefficients can be found by constraining the curve to pass also through point $j$ on ray $i$ and constraining the slope of the cubic curve (dy/dx) to be equal to the normals at point $j$ on rays $i-1$ and $i$ (fig. 6(a)). The slopes required for the curve to be orthogonal to rays $i-1$ and $i$ at point $j$ can be approximated by

$$
\begin{align*}
& \left(\frac{d y}{d x}\right)_{i-1, j}=\frac{x_{i-1, j-1}-x_{i-1, j+1}}{y_{i-1, j+1}-y_{i-1, j-1}}  \tag{B2}\\
& \left(\frac{d y}{d x}\right)_{i, j}=\frac{x_{i, j-1}-x_{i, j+1}}{y_{i, j+1}-y_{i, j-1}} \tag{B3}
\end{align*}
$$

With these results, the three coefficients in equation (B1) are determined to be

$$
\begin{align*}
& a_{1}=\left(\frac{d y}{d x}\right)_{i-1, j}  \tag{B4}\\
& a_{2}=\frac{3\left(y_{i, j}-y_{i-1, j}\right)-\left[2\left(\frac{d y}{d x}\right)_{i-1, j}+\left(\frac{d y}{d x}\right)_{i, j}\right]\left(x_{i, j}-x_{i-1, j}\right)}{\left(x_{i, j}-x_{i-1, j}\right)^{2}} \tag{B5}
\end{align*}
$$

$$
\begin{equation*}
a_{3}=\frac{-2\left(y_{i, j}-y_{i-1, j}\right)+\left[\left(\frac{d y}{d x}\right)_{i-1, j}+\left(\frac{d y}{d x}\right)_{i, j}\right]\left(x_{i, j}-x_{i-1, j}\right)}{\left(x_{i, j}-x_{i-1, j}\right)^{3}} \tag{B6}
\end{equation*}
$$

The cubic equation is used to determine the points on the crest segment between two adjacent rays. To give the crest curve a smooth appearance, seven points are used to describe the crest segment.

When waves approach intermediate water and refraction begins to occur, adjacent rays can cross. Straightforward construction of crest segments in such situations is probably unrealistic in a physical sense and could often result in a zigzag crest pattern. To circumvent this problem, logic has been added to delete all or part of the crest segment between crossing adjacent rays i-1 and i. In order to signal the onset of this situation, the angle between adjoining first-order crest segments is computed. In figure 6(b), the angle $\theta$ between the first-order segment joining point $j$ on ray i-2 to point $j$ on ray $i-1$, and the first-order segment joining point $j$ on ray $i-1$ to point $j$ on ray $i$ is given by
$\theta=\cos ^{-1}\left[\frac{\left(x_{i}-x_{i-1}\right)\left(x_{i-2}-x_{i-1}\right)+\left(y_{i}-y_{i-1}\right)\left(y_{i-2}-y_{i-1}\right)}{\sqrt{\left(x_{i}-x_{i-1}\right)^{2}+\left(y_{i}-y_{i-1}\right)^{2}} \sqrt{\left(x_{i-2}-x_{i-1}\right)^{2}+\left(y_{i-2}-y_{i-1}\right)^{2}}}\right]$

Whenever this angle is less than a specified input parameter (RAYANG), the curved crest segment connecting point $j$ on ray $i-1$ to point $j$ on ray $i$ is modified. Since a maximum of seven points are used to describe the crest segment, the modification is accomplished by starting the segment at an intermediate point between the rays rather than at point $j$ on ray $i-1$. This results in crest patterns being disjointed when adjacent rays i-1 and $i$ cross. The program input parameter specifying the point at which to begin construction of the modified curved crest segment is KCREST and can be any number between 1 and 7 inclusive. The disjointed effect is illustrated in figure 5 with RAYANG equal to $120^{\circ}$ and KCREST equal to 3 . In this case only two-thirds of the crest curve is drawn when the angle between adjoining first-order crest segments is less than $120^{\circ}$. The logic for this procedure is incorporated in subroutine PLOTR. Crest plotting is initiated when the input parameter KPLOT is equal to 3 .

## APPENDIX C

## DISCUSSION OF SUBROUTINE RAYOPT

When wave rays approach shallow water and refraction occurs, cases of extreme convergence or divergence can occur, resulting in either large areas completely devoid of rays or small areas covered by many rays. To allow the user a degree of control over nearshore ray spacing, the subroutine RAYOPT was developed. In subroutine RAYOPT, desired nearshore ray spacing is achieved by perturbation of the initial ray spacing. Upper and lower bounds specified by input parameters are imposed on both the initial ray spacing and the final ray spacing. If desired results are not obtained in 10 iterations, then the 10 th iteration is specified as the solution.

To start the RAYOPT procedure, an initially small value is input or assigned by default to the input parameter RAYSPC, which is the initial ray spacing. The initial ray in the set is found by using subroutine RAYINIT. A second ray is begun at a distance RAYSPC from the initial ray, its complete path is generated, and its final separation distance with respect to the preceding ray is computed. If this distance is greater than the input parameter DISTMAX, the current RAYSPC is halved and a new trial ray is generated. Conversely, if this distance is less than the input parameter DISTMIN, the current RAYSPC is doubled and a new trial ray is generated. This procedure continues until the final separation distance lies between the limits DISTMIN and DISTMAX. If at anytime in the iteration process RAYSPC falls below the input value RAYMIN or exceeds the input value RAYMAX, the iteration process terminates and RAYSPC is set to the respective limit. In some cases, the final ray separation distance oscillates between values of DISTMIN and DISTMAX. When this occurs, an acceptable initial ray spacing is computed by using a Newton-Raphson iteration scheme. For all cases, the final iterated value for RAYSPC is used as the first trial value in computing the next ray.

Graphical results obtained by using subroutine RAYOPT can be seen by comparing figures 3 and 4. Figure 3 presents a ray diagram computed with a uniform initial ray spacing (RAYSPC) of 5 grid units, with no control exercised over final spacing. There are large gaps between adjacent rays in the nearshore region in this diagram. In figure 4, final ray spacing is controlled with DISTMIN equal to 30 grid units and DISTMAX equal to 50 grid units. The initial ray spacing is constrained to lie between 0.01 grid units (RAYMIN) and 15 grid units (RAYMAX). As can be seen, the nearshore ray spacing is much more uniform than that in figure 3. In deeper areas in which no refraction occurs, ray spacing is increased by the subroutine until either RAYMAX or DISTMAX is exceeded. User control over the final ray spacing is particularly useful in cases in which crest curves are to be constructed between adjacent rays, as described in appendix B. Figure 7 shows the complete crest pattern constructed for the ray diagram shown in figure 3, which was generated by using a constant initial ray spacing of 5 grid units. In constructing the crest patterns of figure 7, RAYANG was specified as $120^{\circ}$ and KCREST was specified as 1. This crest pattern does not accurately depict nearshore wave fronts because of the

## APPENDIX C

sparsity of wave rays in certain nearshore areas. The crest pattern of figure 5 was constructed for the ray diagram of figure 4 , in which final ray spacing was controlled. Here, a more realistic version of the wave fronts for the nearshore zone is seen.

An experimental period may be required on the part of the user to obtain an acceptable crest pattern for a particular application. Care should be taken when selecting values for the input parameters required in generating crest patterns. The following discussion on each of the required input parameters is included as a guide in specifying these parameters for any application:

DISTMAX This input constant is the maximum permissible final ray separation distance. Too large a number causes wide nearshore ray separation, such as that seen in figure 3 , and results in unsatisfactory crest patterns (fig. 7). Too small a number causes too many rays to be generated and results in patterns similar to that shown in figure 8, where DISTMAX is equal to 30 grid units. In order for the iteration process to converge in a few cycles, this number should differ from DISTMIN by a reasonable margin.

DISTMIN This parameter is the minimum permissible final ray separation distance. This number should be large enough to insure sufficient nearshore separation between adjacent rays.

KCREST This parameter controls modification of the curved crest segment to be drawn between adjacent rays which cross. Normally, a value equal to 2 or 3 produces an acceptable diagram. Failure to draw at least part of the curved segment (i.e., setting KCREST $=7$ ) can result in large gaps in the pattern, as seen in figure 9 (compared with fig. 5).

RAYANG

RAYMAX $\quad$ This parameter is the maximum permissible initial ray spacing. RAYMAX should be set to a large value to allow an optimum search for an acceptable initial ray point. In addition, only a few rays are required to define the crest pattern in situations in which the rays remain in deep water over their entire path.

RAYMIN
This parameter is the minimum permissible initial ray spacing. In all instances, this number should be set to an arbitrarily small value in order that final ray constraints can be met.

## APPENDIX D

## UTILITY PROGRAM CREMOD

In the wave refraction computer model described in this paper, bathymetry data are stored and retrieved in the form of overlapped modules of dimensions 42 by 32. Random-access techniques are used to pass data modules as required from a peripheral mass-storage file to the central memory of the computer. The potential user, however, generally has bathymetry data available in the form of an array of dimensions MI by MJ covering the entire region of interest. This appendix describes a utility program CREMOD which can be used to modularize the regional array supplied by the user. Upon completion of execution of this program, the modularized data reside on the peripheral file TAPE1, which then can be saved as a permanent file by the user for later input to the refraction model.

It is assumed that the regional array is supplied in card deck form and that the data are arranged in columnwise order with the first column corresponding to the $x$-axis of the region. The total number of rows of data MI and the total number of columns of data MJ are required input parameters. The user must also designate the format (FORMT) with which the data cards are read. To avoid possible extraneous results, it is strongly recommended that the format be chosen so that the input record length is an integral divisor of MI. The regional array is divided into a total of NOMOD overlapped modules which are indexed by columns, as shown in figure 10, and written to the disk file TAPE1 by using system subroutine WRITMS. If the regional configuration is such that portions of certain modules extend beyond the boundaries of the MI by MJ array (as illustrated in fig. 10), data values within those portions are set equal to the values in the corresponding column of row MI or in the corresponding row of column MJ, as the case requires. Values for NOROW, the number of module rows, NOCOL, the number of module columns, and NOMOD are printed when modularization is complete.

Program CREMOD uses the system subroutines OPENMS, WRITMS, and CLOSMS for initializing, writing records to, and closing the mass-storage file TAPE1. Reference 10 provides a detailed description of each of these routines. On the Control Data Cyber 175 computer under NOS 1.2 , program CREMOD required a central memory field length of 640008 and, for a sample case in which MI and MJ were both 100 , required 4 seconds of CPU time. The user should again be reminded to save the file TAPE1 as a permanent file after execution of program CREMOD. The flow diagram and listing of program CREMOD follow.

APPENDIX D


## APPENDIX D

```
        PROGRAM CREHDD (INPUT,DUTPUT,TAPE5=INPUT,TAPEI)
        DIMENSION DEP(420,32), DMOD(42,32),IBLK(101)
        READ (5,180) MI,MJ,FORMT
    DETERMINE NO. OF MODULE ROWS ANO COLUMNS.
        NDCDL MJ/29
        NCOUT = MOD (MJ,29)
        IF (NCDUT &T. 4) GO TD 10
        NOCOL NOCOL+1
        GOTO 20
    10 NCDUT = NCDUT+29
    20 NDROW - MI/39
        NROUT - MOD (MI,39)
        IF (NROUT &LT. 4) GO TO 30
        NOROW - NOROW+1
        GO TO 40
    30 NROUT = NRDUT+39
    4O NOAUD NOROW*NDCOL
    OPEN MASS STORAGE FILE, TAPEI.
        CALL DPENMS (1,IBLK,NOMOO+1,O)
    READ 3 COHPLETE COLUMNS OF DATA.
        READ (5,FORHT) ((OEP(I,J),I=1,MI),J=1,3)
        DO 170 NC=1,NOCOL
        IF (NC .LT. NUCOL) GO TO }7
* FINAL MODULE COLUMN. ONLY (NCOUT-3) SU8-COLUMNS OF DATA LEFT.
        IF (NCOUT EEQ. 32) GO TO }7
        READ (5,FORMT) ({DEP(I,J),I=1,MI),J=4,NCOUT)
    SET VALUES IN EXTERIOR COLUANS - TO VALUES IN LAST CDLUMN.
        J1 - NCDUT+1
        DO 60 J=Jl,32
        OO 50 I=1,MI
        DEP(I,J) DEP(I,NCOUT)
    50 CONTINUE
    60 CONTINUE
        GO TO 80
    FULL HODULE COLUMN AVAILABLE. READ COLUMNS 4 THRU 32.
```

```
    70 READ (5,FDRMT) ((DEP(I,J),I=1,MI),J=4,32)
    FILL DMOD ARRAY FOR NOROW MODULES IN THIS COLUMN. WRITE EACH TO
    MASS STORAGE FILE.
    80 DO 140 NR=1,NOROW
        I2ERO = 39*(NR-1)
        NM = (NC-1)*NOROW+NR
        IF (NR .LT. NOROW) GO TO 90
* finAl module row. nrout-3 more sub-rows of data available.
        IMAX = NROUT
        GO TO 100
    90 IMAX = 42
    100 DO 130 J=1,32
        DO 110 [=1,IMAX
        DMOD(I,J) = DEP(IZERD+I,J)
    110 CONTINUE
        IF (IMAX .EQ. 42) GO TO 130
        II = IMAX+1
        OO 120 I= I1,42
        DMOD(I,J) DMOD(IMAX,J)
    120 CONTINUE
    130 CONTINUE
    CALL WRITMS (1,DMOD,1344,NM)
    140 CONTINUE
    IF (NC .EQ. NOCOL) GO TO }17
* SUB-COLUMNS 30-32 WILL BE SUB-COLUMNS 1-3 OF NEXT mODULE.
        DO 160 J=1,3
        DO 150 I=1,MI
        DEP(I,J)= DEP(I,J+29)
    150 CONTINUE
    160 CONTINUE
    170 CONTINUE
        PRINT 190, NOROW,NOCOL,NOMOD
        CALL CLOSMS (1)
        RETURN
*
    180 FORMAT (2I5,A10)
    190 FORMAT (2X,*NOROW =*,I 3/2X,*NOCOL **,I 3/2X,*NOMOD **,I3)
        ENO
```

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TABLE I.- DESCRIPTION OF ELEMENTS IN NAMELIST NPUT1

| Variable name | Description | $\begin{aligned} & \text { Default } \\ & \text { value } \end{aligned}$ |
| :---: | :---: | :---: |
| DCON | Multiplier to convert depth units to feet for internal computations | 1.0 |
| DELTAS | Minimum step length along ray in shallow water, grid units | 0.01 |
| GRID | Number of feet per grid division | None |
| GRINC | Step length along ray in deep water, grid units | 1.0 |
| IOUTUNT | Output units: nautical miles for spatial coordinates $x$ and $y$; for all other parameters; 1 = U.S. Customary Units; $2=$ SI Units | 2 |
| JPRTFRQ(1) | Print frequency of ray parameters when depth is greater than one-half the wavelength, time steps | 20 |
| JPRTFRQ (2) | Print frequency when depth is greater than onefourth the wavelength but less than one-half the wavelength, time steps | 10 |
| JPRTFRQ(3) | Print frequency when depth is less than onefourth the wavelength, time steps | 5 |
| LIMNPT | Maximum number of computation points along a ray | 400 |
| KPLOT | ```Plotting option: 0 = No plot 1 = Plot wave rays with uniform initial spacing 2 = Plot wave rays with controlled final spacing 3 = Plot wave crests``` | 0 |
| MI | Total number of rows in regional bathymetry data array | None |
| MJ | Total number of columns in regional bathymetry data array | None |
| NOMOD | Total number of 42 by 32 bathymetry data modules; maximum allowed is 100 | None |
| NOROW | Number of modules in $x$-direction | None |
| NOSETS | Number of sets of rays to be computed | 1 |
| PLTLNGX | Length of x-axis for plot, inches | 10.0 |
| PLTLNGY | Length of y -axis for plot, inches | 10.0 |
| PLTSCAL | Scale factor for $x$ - and $y$-axes; change in $x$ or $y$ value per inch of plot (input required for plotting only) | None |
| TIDE | Height of tide, feet | 0.0 |

TABLE II.- DESCRIPTION OF ELEMENTS IN NAMELIST NPUT2

| Variable name | Description | Default value |
| :---: | :---: | :---: |
| A | Initial ray direction, degrees | None |
| H0 | Deepwater wave height, feet | None |
| NCREST | Spacing of crest tick marks or crest curves; integer multiple of program computational time step | 5 |
| NSURFCE | Bottom topography approximation technique: <br> 1 = Quadratic least squares <br> $2=$ Cubic least squares <br> 3 = Constrained bicubic interpolation | 1 |
| RAYSPC | Initial spacing between wave rays in deep water, grid units | 5.0 |
| T | Wave period, seconds | None |
| DISTMAX ${ }^{\text {a }}$ | Maximum permissible final separation distance between adjacent rays, grid units | 999. |
| DISTMIN ${ }^{\text {a }}$ | Minimum permissible final separation distance between adjacent rays, grid units | 25. |
| KCREST ${ }^{\text {a }}$ | Point (between 1 and 7) between adjacent rays at which to begin plotting curved crest segment when angle between adjoining first-order segments is less than RAYANG | 1 |
| RAYANGa | Minimum acceptable angle between adjoining firstorder crest segments, below which crest plotting is modified, degrees | 120. |
| RAYMAX ${ }^{\text {a }}$ | Maximum permissible initial ray spacing, grid units | 25. |
| RAYMIN ${ }^{\text {a }}$ | Minimum permissible initial ray spacing grid units | 0.01 |

[^0]

Figure 1.- Determination of starting point for first ray for ray direction greater than $-90^{\circ}$.


Figure 2.- Determination of ray starting points for ray direction greater than $-90^{\circ}$.


Figure 3.- Ray diagram with equally spaced initial points.


Figure 4.- Ray diagram with controlled final (nearshore) spacing.


[^1]
(a) Crest curve connecting two rays.

(b) Angle between adjoining first-order crest segments.

Figure 6.- Wave-crest geometry.


Figure 7.- Wave-crest diagram with equally spaced initial ray points.


Figure 8.- Wave-crest diagram resulting from generation of too many rays due to small value of DISTMAX.


Figure 9.- Wave-crest diagram constructed with KCREST $=7$.


Figure 10.- Sequence of bathymetry data modules.


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[^0]:    a These parameters are used only for ray sets in which nearshore ray spacing is controlled (KPLOT $=2$ or 3 ).

[^1]:    Figure 5.- Wave-crest diagram using modified segments in areas of crossing rays.

