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SHAPE OPTIMIZATION OF PRESSURE GRADIENT MICROPHONES

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ABSTRACT

Recently developed finite element computer programs were utilized to investigate the influence of the shape of a body on its scattering field with the aim of determining the optimal shape for a pressure gradient microphone (PGM). Circular cylinders of various aspect ratios were evaluated to choose the length to diameter ratio best suited for a dual element PGM application. Alterations of the basic cylindrical shape by rounding the edges and recessing at the centerline were also studied. It was found that for a ±1 dB deviation from a linear pressure gradient response, a circular cylinder of aspect ratio near 0.5 was most suitable, yielding a useful upper frequency corresponding to ka = 1.8. The maximum increase in this upper frequency limit obtained through a number of shape alterations was only about 20 percent. An initial experimental evaluation of a single element cylindrical PGM of aspect ratio 0.18 utilizing a piezoresistive type sensor was also performed and is compared to the analytical results.

INTRODUCTION

The consideration of the effects of microphone body scattering is of prime importance in the design of a pressure gradient microphone (PGM) for maximum frequency response. A previous study using oblate spheroids of various aspect ratios (ref. 1) showed the shape of the body determines the frequency at which scattering effects become significant.

Until recently solution of radiation and scattering by rigid bodies were restricted to simple shapes that fit a coordinate surface of a system in which the wave equation was separable and the appropriate separable functions were calculable (e.g., oblate spheroids). Within the past decade finite element approaches to the Helmholtz integral solution have been programmed on a digital computer which now enable bodies of more arbitrary shapes to be handled successfully. This study takes advantage of recently developed finite element programs (refs. 2 and 3) to investigate the influence of a body's shape on its scattering field, with the aim of determining the optimal shape for a pressure gradient microphone.

Since microphones are generally cylindrical in shape, this report first looks at cylinders of various aspect ratios to determine the length to diameter ratio best suited for a dual element PGM application. Once this aspect ratio is chosen, the effect of altering the basic circular cylindrical shape on the scattering characteristics of the body is determined.

An initial experimental evaluation of the response of a low aspect ratio PGM has also been made, and the results are compared to those computed analytically.

STATEMENT OF PROBLEM

The fact that an instrument used to measure any physical quantity can alter that which one is attempting to measure is well known to researchers in all the physical sciences. For point measurement of acoustic pressure the pertubations caused by the microphone are insignificant as long as the wavelength of the sound is much larger than the dimensions of the probe. As the sound frequency increases, however, scattering by the probe is enhanced and the measured pressure signal is no longer indicative of the field that exists when the microphone is removed.

Additional complications arise in the determination of the pressure gradient. Consider an axisymmetric body with centerline thickness Δz and maximum radius a placed normally to an incident plane acoustic wave of wavenumber k as shown in figure 1a. (Note that pressures are normalized by the amplitude of the incident pressure and that the assumed sinusoidal time variation is suppressed.) The relationship between the pressure difference across the centerline of the body divided by the centerline thickness, and the pressure gradient in the absence of the body, results from two phenomena in addition to the high frequency scattering. The first exists independent of the presence of the body and results from the approximation of a gradient by a finite difference. This free field relationship is (fig. 1b)

$$\left(\frac{\partial p_i}{\partial z}\right)_{z=0} = \left[\frac{k\Delta z/2}{\sin(k\Delta z/2)}\right]\frac{\Delta p_i}{\Delta z} \tag{1}$$

The introduction of the body into the field leads to the second phenomena. Even at low frequency, scattering leads to a phase distortion which depends on the body shape and causes the pressure difference across the body, Δp, to differ from Δp_i. Defining a body shape calibration factor K as the ratio of these two differences in the limit of zero frequency,

$$K = \lim_{ka \to 0} \frac{\Delta p_i}{\Delta p} \tag{2}$$

results in the low frequency relationship

$$\left(\frac{\partial p_i}{\partial z}\right)_{z=0} = K\left[\frac{k\Delta z/2}{\sin(k\Delta z/2)}\right]\frac{\Delta p}{\Delta z} \qquad (3)$$

A more detailed account of this development can be found in reference 1.

Deviations from this essentially linear low frequency relationship between the true gradient and the measured difference occur as the frequency increases. These deviations, caused by high frequency body scattering, are investigated through a scattering parameter σ, defined as

$$\sigma = 20 \log \left| \frac{K\left[\frac{k\Delta z/2}{\sin(k\Delta z/2)}\right]\frac{\Delta p}{\Delta z}}{\left(\frac{\partial p_i}{\partial z}\right)_{z=0}} \right| \qquad (4)$$

The extent which a given body shape will be usable as a PGM is governed by the variation of the scattering parameter with frequency. Choosing an acceptable error of say ±12 percent in the measured pressure gradient yields an acceptable variation in σ of ±1 dB. This criteria will be used to determine the highest frequency at which a given body will be acceptable for use as a PGM, i.e., yield an essentially linear response, and hence the optimal shape for the probe.

This report considers only axially symmetric bodies and plane waves incident along the axis of symmetry. Hence, the frequency parameter ka, where a is half the maximum body diameter, is an excellent parameter to use to compare bodies of different shape. This frequency parameter is equivalent to the ratio of the body circumference normal to the wave to its wavelength.

DETERMINATION OF SURFACE PRESSURES

In order to calculate the shape calibration factor K and the scattering parameter σ, the surface pressures on the axis of symmetry of the body (p_T and p_B in figure 1a) must be determined. At any point $\vec{\zeta}$ on the body, the pressure is the sum of the incident and scattered components,

$$p(\vec{\zeta}) = p_i(\vec{\zeta}) + p_s(\vec{\zeta}) \tag{5}$$

Specifying the body surface S to be rigid yields the boundary condition of zero normal velocity, or

$$v_{n_s}(\vec{\zeta}) = -v_{n_i}(\vec{\zeta}) \tag{6}$$

Since both the incident and scattered field satisfy the Helmholtz equation, the integral solution for the scattered surface pressure is (ref. 4)

$$\frac{1}{2}\, p_s(\vec{\zeta}) = \int_S p_s(\vec{\xi})\, \frac{\partial g(\vec{\xi}, \vec{\zeta})}{\partial n}\, dS(\vec{\xi}) + i\rho ck \int_S v_{n_s}(\vec{\xi}) g(\vec{\xi}, \vec{\zeta}) dS(\vec{\xi}) \tag{7}$$

where $g(\vec{\xi},\vec{\zeta}) = e^{-ik|\vec{\xi} - \vec{\zeta}|}/4\pi|\vec{\xi} - \vec{\zeta}|$ is the free space Green's function and ρc is the characteristic impedance of air. Since the incident field is specified and the scattered normal surface velocity is given by equation (6), the results for the scattered pressure contained in equation (7) will enable the surface pressures to be calculated via equation (5).

To solve the integral equation (7) the surface is divided into elements small enough such that the pressure does not vary too greatly over each element (generally, elements are thus restricted to dimensions on the order of an eighth wavelength or smaller). The solution then involves a matricial equation for the pressure on each element in terms of the given normal velocities of the elements.

Two finite element computer programs were used to obtain the results of this report. The first (SHIP) was developed at the Naval Research Laboratory (ref. 2) and is limited to circular cylinders of arbitrary aspect ratio. The other program (CHIEF) was produced at the Naval Undersea Research and Development Center (ref. 3) and extensively tested at NRL (ref. 5). CHIEF has the capability of handling arbitrary shaped axisymmetric bodies and yields excellent results as long as the length and diameter of the body are the same order of magnitude. (SHIP is not limited in this regard since the cylindrical geometry allows the Green's function integrations of equation (7) to be evaluated analytically - a procedure not feasible with the arbitrary geometry of CHIEF.)

RESULTS AND DISCUSSION

Pressure gradient microphones can be constructed using either one or two pressure sensor elements. As will be shown, the two element design has a smaller scattering cross section but requires the use of exceptionally well phase matched pressure sensors. Thus, the optimization of each type is of equal importance; the use of either depending upon the particular application.

Dual Element PGM

The dual element PGM was modeled first using circular cylinders of various aspect ratios. Figure 2 gives the computed scattering parameter σ vs ka for cylinder length to diameter ratios (aspect ratios) ranging from 0.01 to 1.00. One can see that σ reaches a maximum value at a frequency inversely related to the aspect ratio, and then falls off rapidly at higher frequency. Since the maximum frequency response is desired, it is apparent that the cylinder which has a maximum σ corresponding to the chosen acceptable deviation from linearity is best suited for PGM application. For a 1 dB criterion an aspect ratio of about 0.5 yields the best response, giving a maximum frequency for acceptability corresponding to ka near 1.8.

The first shape deviations from the dual element circular cylinder were obtained by rounding its edges. These included elliptic, quarter-round, and half-round edged bodies of 0.5 overall aspect ratio as shown in longitudinal cross section in figure 3. The scattering parameter for these round edged bodies are compared to the circular cylinder results in the same figure. The three round edges give almost identical results and are seen to offer no more than a slight improvement over the circular cylinder.

A somewhat better frequency response is obtained through more severe shape deviations from the circular cylinders. Results for an oblate spheroid (fig. 4) demonstrate an increase in the maximum frequency for acceptable response of about 10 percent better than the circular cylinder. Additional improvement was obtained with the dual conical shape of figure 5. These results are presented for a length to diameter ratio of 0.6 rather than 0.5 because for this dual conical body an aspect ratio of 0.5 gave an increase in the maximum value of the scattering parameter above the chosen 1 dB criterion. Figure 5 shows the upper limit on ka to be about 2.2, or close to 20 percent higher than that for the circular cylinder.

Single Element PGM

Perhaps the simpliest version of a single element pressure gradient microphone is the circular cylinder with a centerline recess to accommodate the single pressure sensor. Both parabolic and straight recesses with an overall aspect ratio of 0.5 and centerline height to a diameter ratio of 0.25 were investigated. Results are shown in figure 6 along with those of a nonrecessed circular cylinder of aspect ratio 0.25. The similarity in the results for these three shapes suggests that for circular cylinders recessed at the centerline, the ratio of the centerline height to diameter forms an equivalent aspect ratio that most governs the variation in the scattering parameter. This implies that an optimal signal element PGM cannot be obtained by simply recessing the optimized dual element PGM to accommodate the single pressure sensor.

A different trend is obtained with the toroidal body of figure 7. The centerline height to total diameter ratio is again 0.25, and the overall aspect ratio is 0.5. The scattering parameter is seen to vary in a manner quite similar to that for nonrecessed bodies of the same overall aspect ratio. Although this result at first appears promising, application to a single element PGM would require the flat central portion to have a much smaller thickness to diameter ratio. To accomplish this with a toroidal model requires an increase in the overall diameter of the body, resulting in a smaller overall aspect ratio and a corresponding increase in the maximum value of the scattering parameter.

Experimental Results

A working model of a single element PGM (fig. 8) was tested in an anechoic room using a free field method where both the free space amplitude and phase of an incident wave could be determined in the absence of the model. The longitudinal cross section of the outside shape of the model is as shown at the top of figure 9. However, most of the top and bottom surfaces of the PGM consisted of a protective grid which if removed would result in the effective hard surfaced geometry also shown in figure 9. Computations of the scattering parameter for both these shapes are shown in the same figure along with the experimental data. These experimental data points represent the measured pressure gradient response of the microphone, appropriately corrected for the finite difference factor and normalized to 0 dB at low frequency. It is seen that the computations for the shape incorporating the internal details of the microphone give much better agreement with the experimental data than those for the shape simulating only the external geometry. The correct frequency at maximum response is predicted, although the measurements at this frequency give a 1 to 2 dB higher peak. This difference could possibly be due to probe stem interference or the fact that the actual sensing element was implanted into an etched out area on the internal surface, resulting in a somewhat different geometry than was modelled. Nevertheless, figure 9 does indicate that the computational method used herein recovers the main features of the scattering process and hence justifies its use in the optimization of a PGM.

Surface Pressure Variation

The variation in the calculated magnitude of the surface pressures (normalized to the incident pressure amplitude) at $ka = 2$ are depicted in figure 10 for a number of the bodies investigated. The abscissa in these plots is the ratio of the surface distance from the top centerline to the total surface distance between the centerline points; hence, the leftmost data point is closest to the top centerline and the rightmost point is near the bottom centerline. The vertical lines in the figures correspond to those positions where the analytical representation of the surface changes (e.g., edges or changes in slope). As might be expected, the shapes with the smallest variation in surface pressure are generally those with smallest magnitude scattering parameters near this frequency. Also, sharp changes in the slope of the pressure distributions occur at discontinuities in the slope of the body.

Shape Calibration Factor

Even at low frequency the presence of a body in an acoustic field causes scattering that results in a phase variation over the surface of the body. The phase difference across the centerline of the body is larger than that existing in free space, resulting in a shape calibration factor (defined by equation 2) less than unity. Since this factor cannot be determined without a detailed knowledge of the surface pressures, an attempt has been made to correlate the body's shape factor with its geometry for the wide variety of axisymmetric shapes investigated.

Referring back to figure 1, a wave communication distance is defined as the shortest distance from the plane of the incident wave front at T to the bottom centerline point B. In the free field this communication distance is simply Δz, whereas in the presence of the body it is the distance Δn shown in figure 11. The geometric factor chosen to correlate the shape factor is the ratio of the wave communication distance in free space to that in the presence of the body.

The result of this correlation is shown in figure 11. Besides the bodies discussed previously, shape factors were calculated for circular cylinders of aspect ratios as high as 20. Also shown are results for a variety of oblate spheroids as reported in reference 1. Although the results are slightly shape dependent at the higher aspect ratios, the shape calibration factor is seen to correlate quite closely with the wave communication distance ratio for the all axisymmetric bodies investigated.

General Comments

Two additional points in regard to PGM design should be noted. First, since the maximum acceptable frequency is inversely proportional to the diameter of the body (i.e., $f_{max} \sim (ka)_{max}/D$) it is obvious that a small diameter body is desired for the highest frequency response. But the sensitivity of the sensors are proportional to their size, so the best PGM could have its active elements almost as large as its diameter. Hence, the pressure recorded by an element would actually be an average over its surface, not the value at the centerline as used in the computations of this report. However, the resulting difference in the scattering parameter is small, as shown in reference 1. For the oblate spheroid tested in this reference, the differences amounted to a 0.4 dB decrease in the maximum value of the scattering parameter and a small decrease in the maximum acceptable frequency. The second point concerns the finite difference factor. This factor can be significant; for example it is 1.5 dB at $ka = 2$ for a body of aspect ratio equal to 0.5. Although this is easily taken into account when computing the scattering parameter, it may not be easy to account for it in some PGM applications. In such cases, the measured PGM response and its peak frequency will be lower than that of the computed scattering parameter. Thus, if either of these points are considered, the optimal aspect ratio of a dual element circular cylinder PGM would actually be smaller than 0.5.

CONCLUDING REMARKS

Computations of the scattering parameter for circular cylinders of various aspect ratios show that for an acceptable deviation from linearity of ±1 dB, an aspect ratio of about 0.5 is best suited for dual-element pressure gradient microphone applications. This aspect ratio should be somewhat smaller if the diameter of the microphone is not large compared to the size of the sensing elements or if the finite difference factor is not taken into account. The maximum frequency at which this PGM can be used within the ±1 dB criterion corresponds to a ka of about 1.8. Changing the shape of the edges of the PGM can increase this frequency limit by only about 20 percent.

Since a single element PGM measures the pressure difference across a distance small in comparison to its diameter (i.e., across the thickness of the element), its use appears to be limited by the rising portion of the frequency response curve. Hence, to maintain the chosen criterion for linearity, it can only be used to an upper frequency of about a third that of the dual element PGM of the same diameter.

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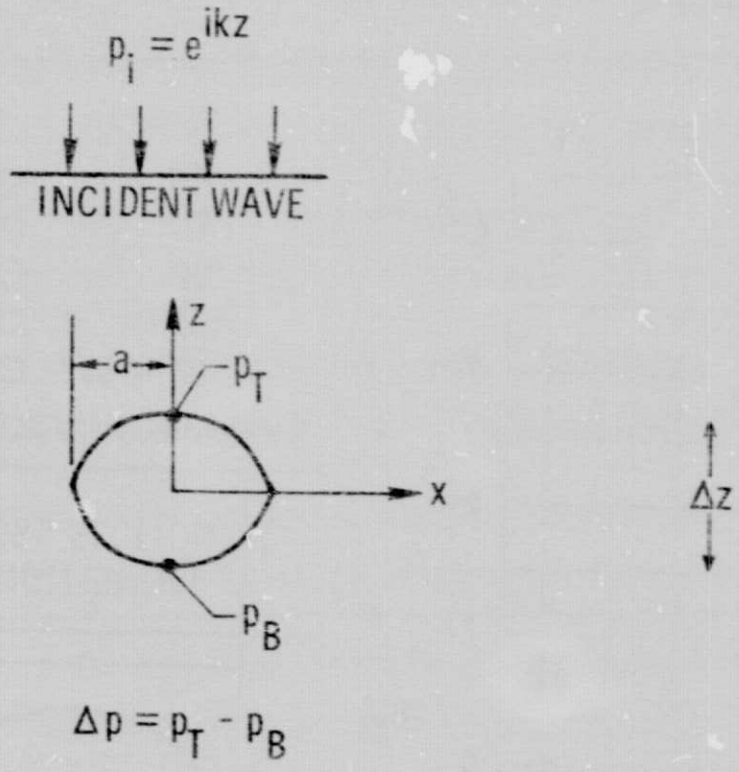

Figure 1a.- Plane wave incident normal to an axisymmetric body.

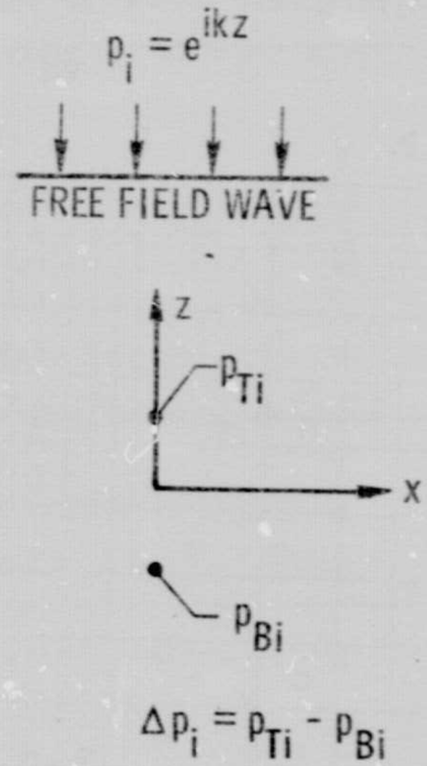

Figure 1b.- Free field plane wave.

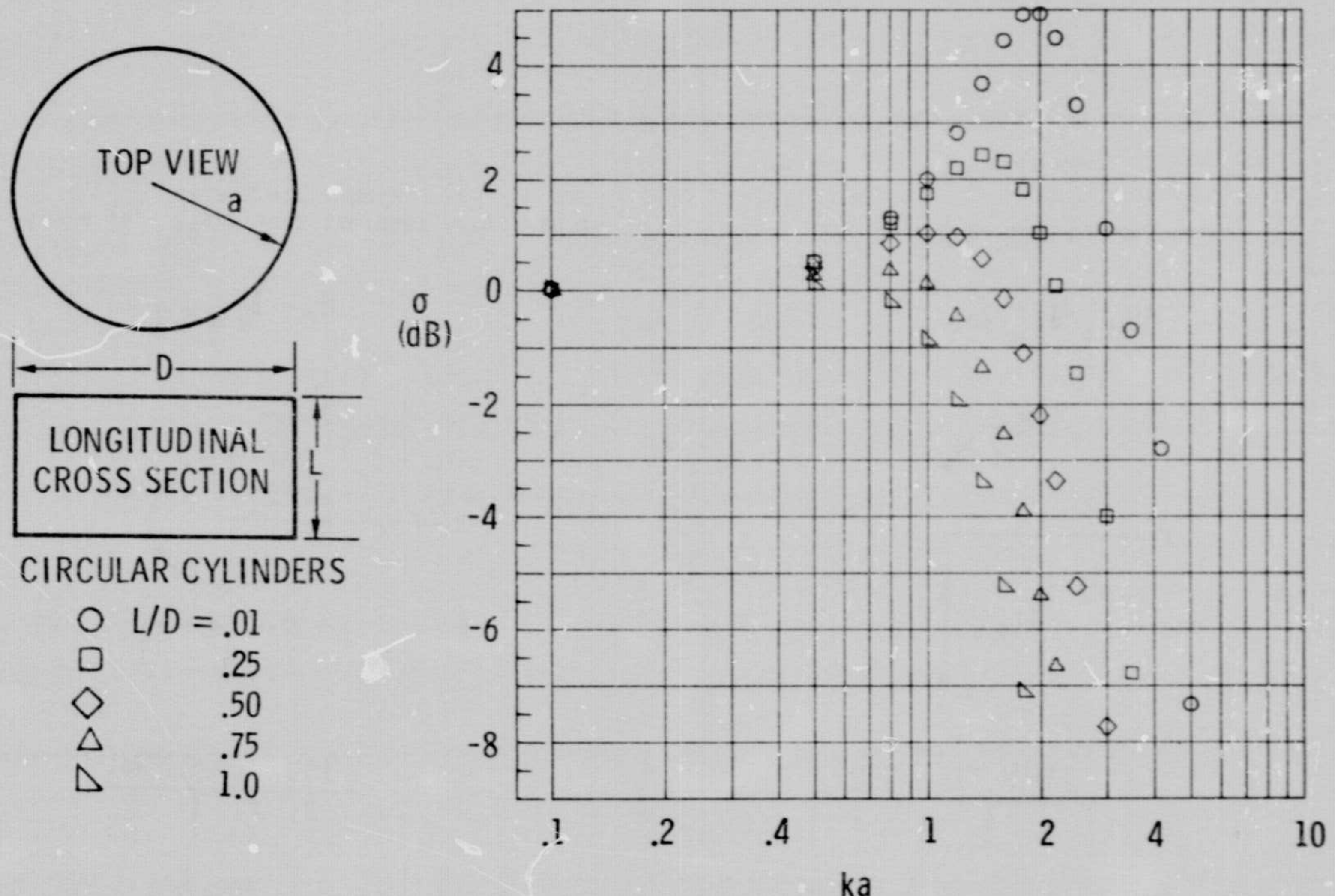

Figure 2.- Scattering parameter for circular cylinders of various aspect ratios.

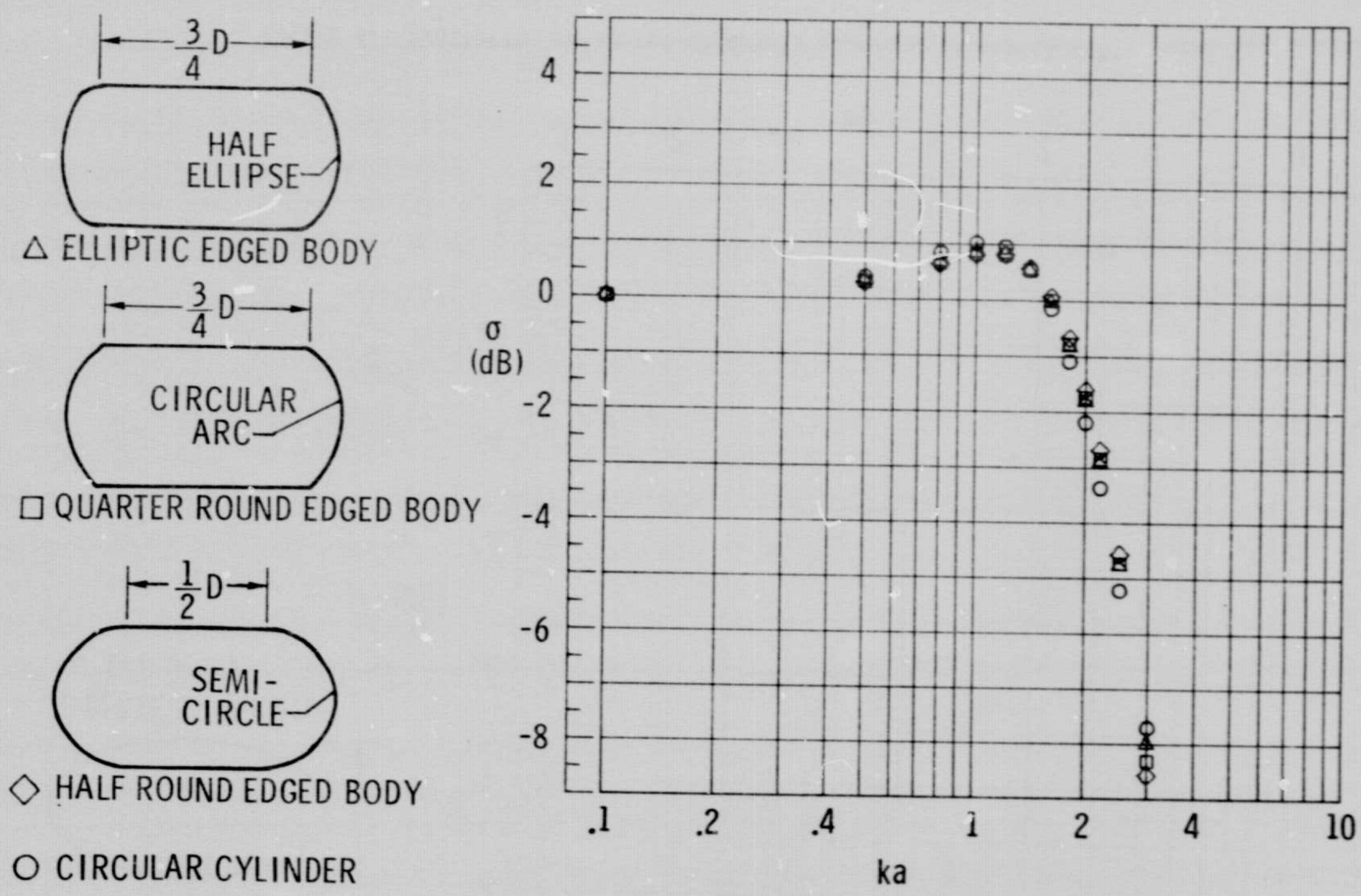

Figure 3.- Comparison of scattering parameter of circular cylinder and rounded edged bodies of aspect ratio 0.5.

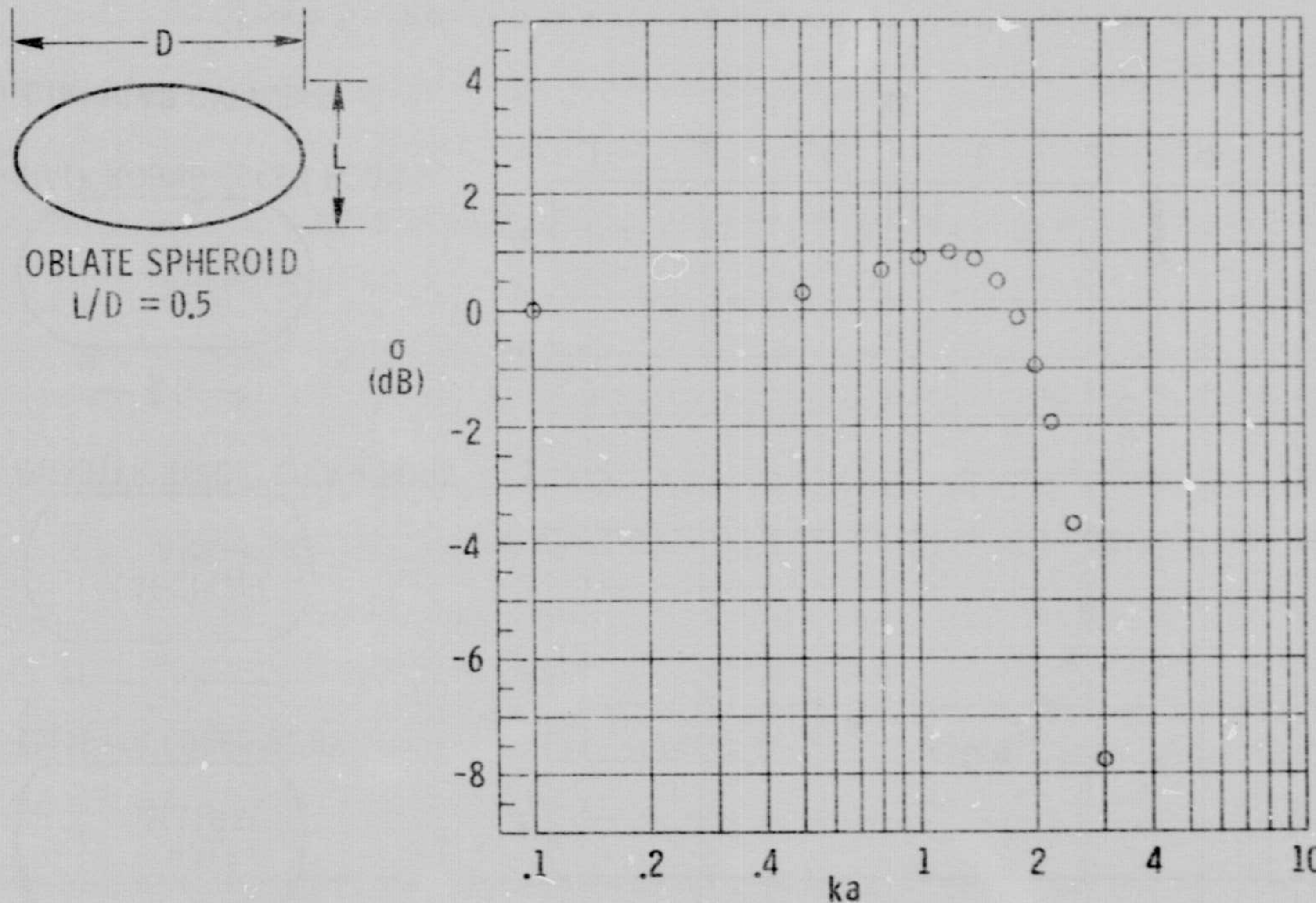

Figure 4.- Scattering parameter of oblate spheroid of aspect ratio 0.5.

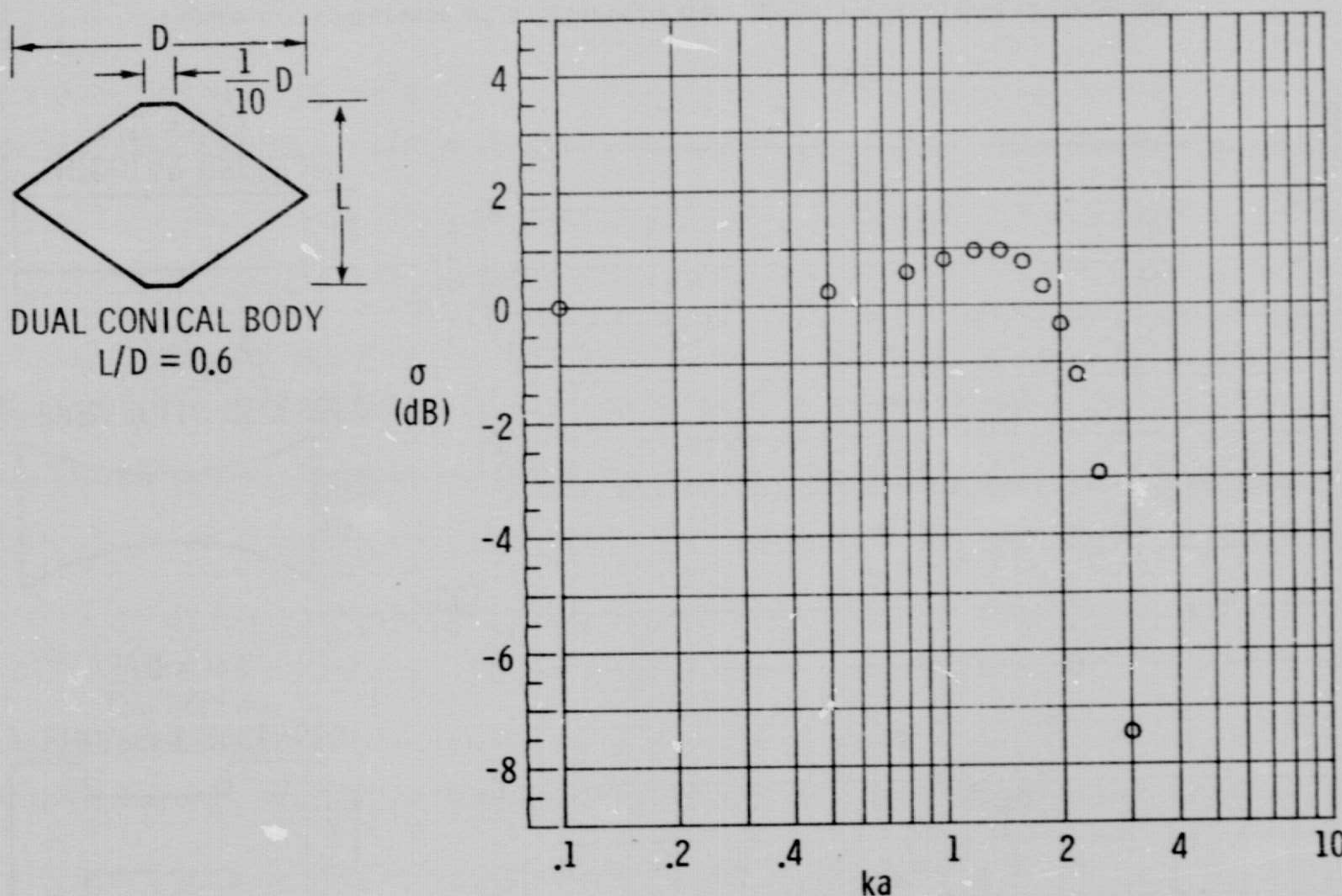

Figure 5.- Scattering parameter of dual conical body of aspect ratio 0.6.

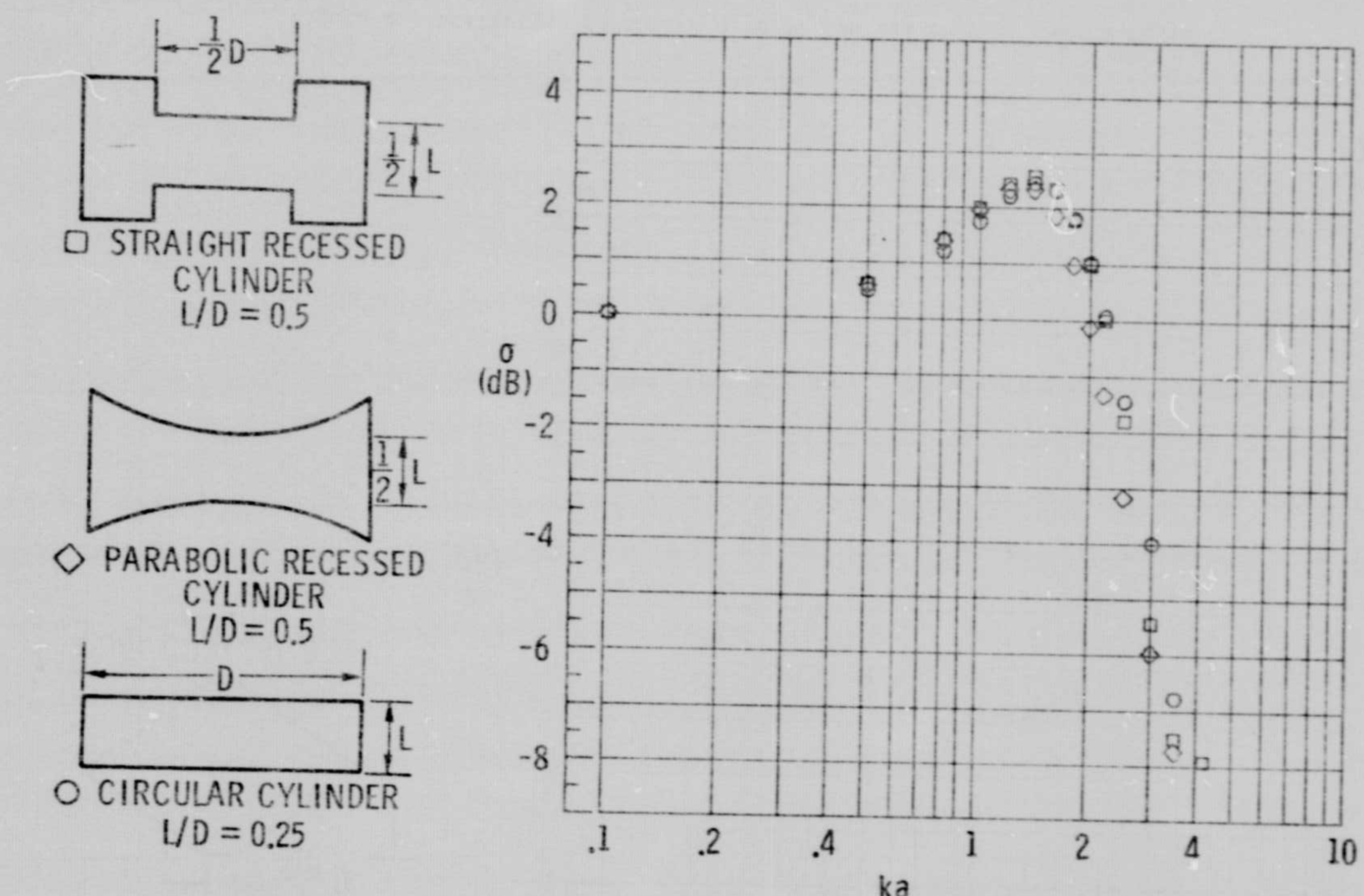

Figure 6.- Comparison of scattering parameter of recessed and nonrecessed cylinders of same centerline aspect ratio.

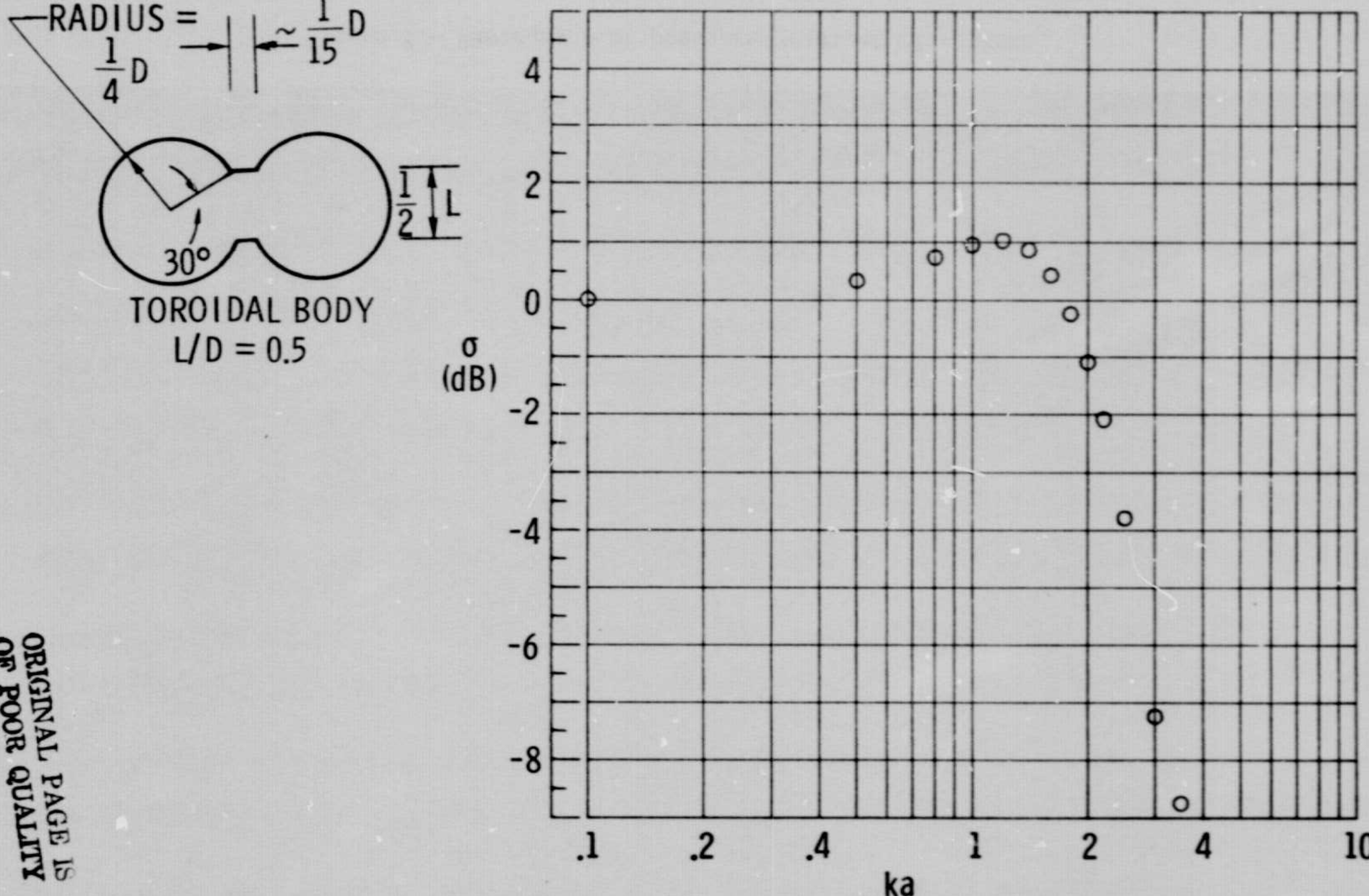

Figure 7.- Scattering parameter of toroidal body of aspect ratio 0.5

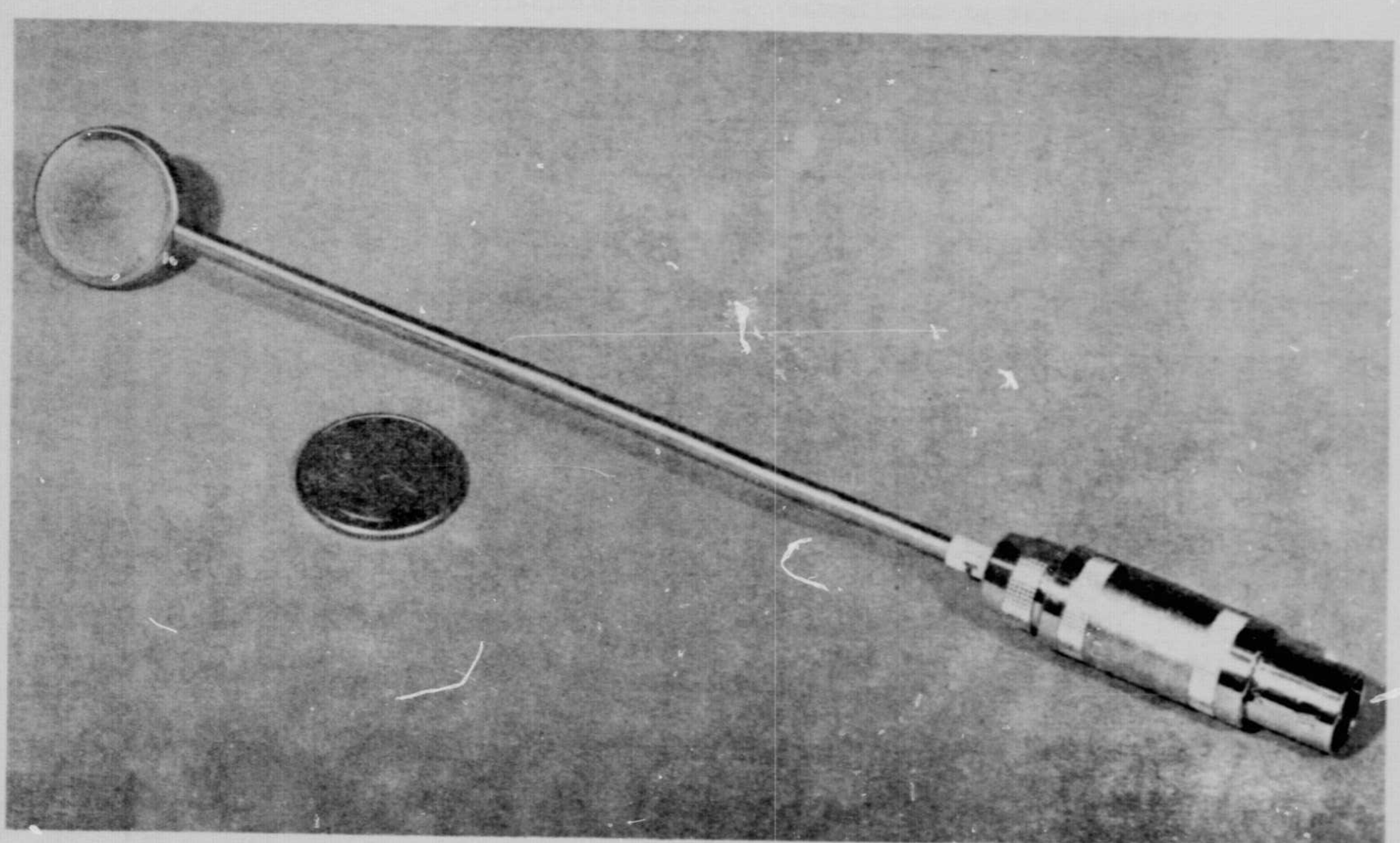

Figure 8.- Photograph of pressure gradient microphone.

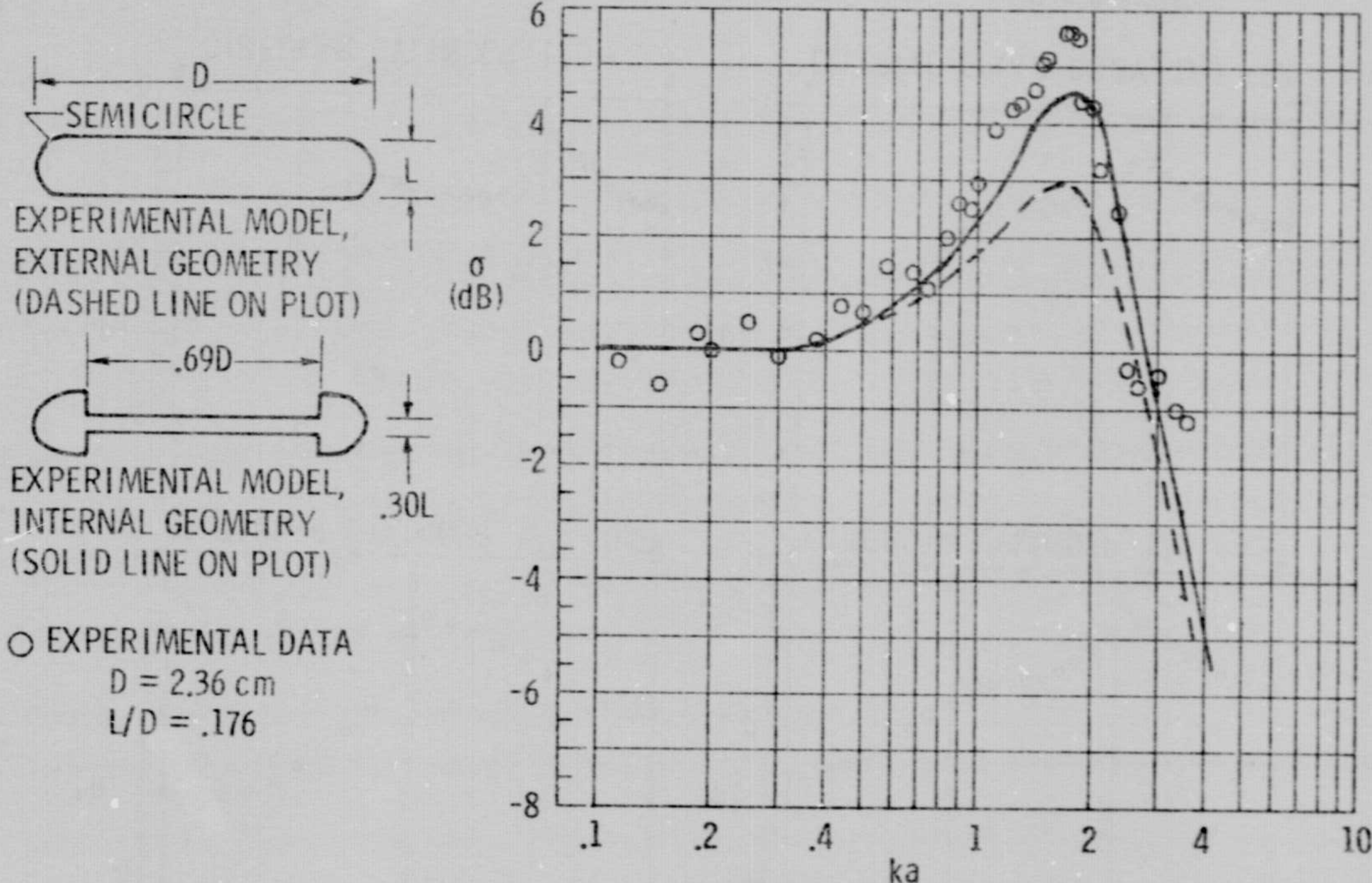

Figure 9.- Scattering parameter of experimental model - measured and computed data.

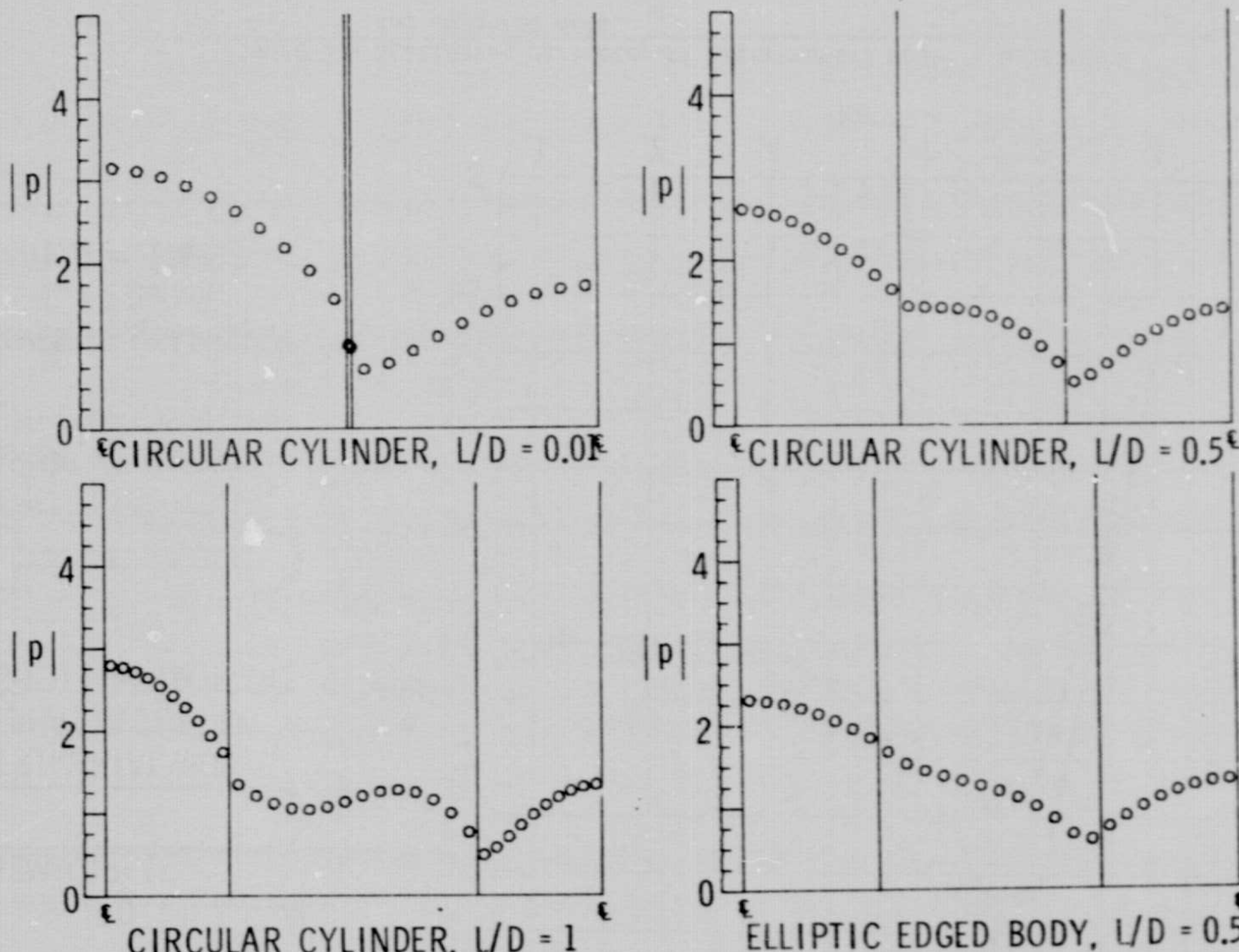

Figure 10.- Variation of surface pressure amplitude with surface distance from top centerline to bottom centerline (ka = 2).

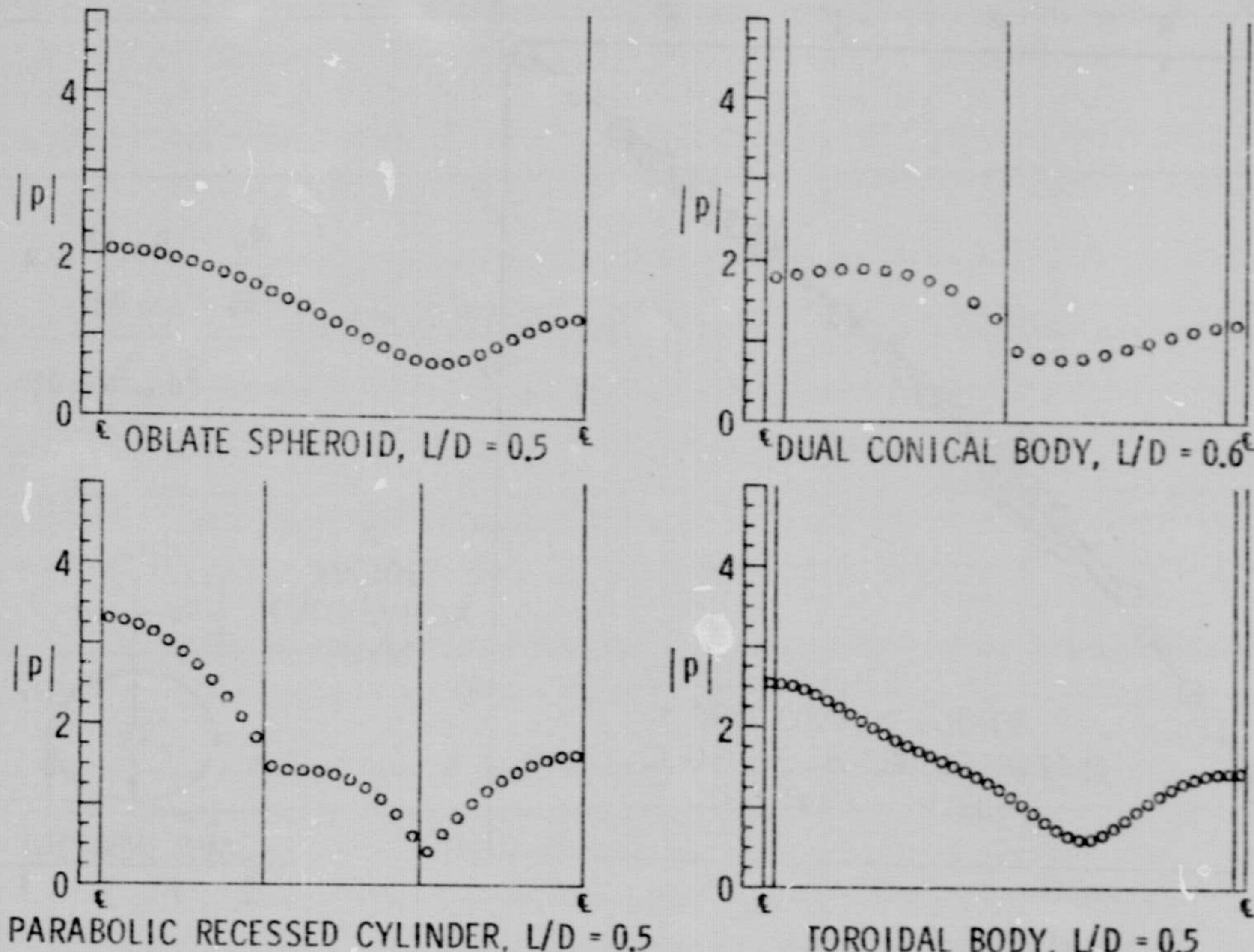

Figure 10.- Concluded.

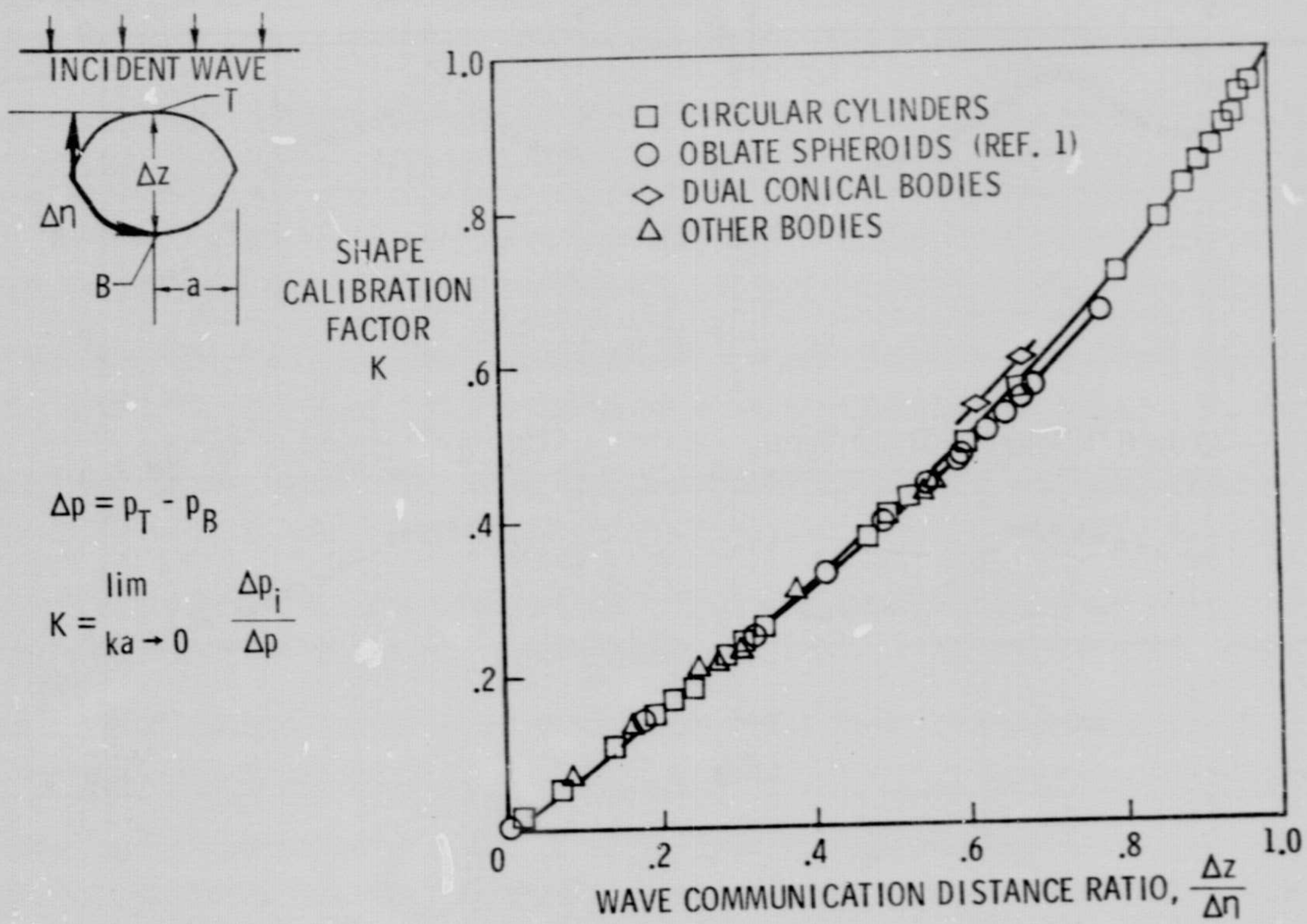

Figure 11.- Variation of shape calibration factor with wave communication distance ratio.

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16. Abstract

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