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APPROXIMATE METHOD FOR CALCULATING FREE VIBRATIONS OF A LARGE-WIND-TURBINE TOWER STRUCTURE

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NOTICE

APPROXIMATE METHOD FOR CALCULATING FREE VIBRATIONS OF A LARGE-WIND-TURBINE TOWER STRUCTURE

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SUMMARY

An approximate method is presented to calculate the fundamental bending and torsion natural frequencies of a typical tower structure for a wind turbine. The tower structure is modeled as a system of six masses and six springs. This simplified lumped-mass model leads to a set of ordinary differential equations involving matrices in terms of mass, mass moments of inertia, structural stiffness, or structural flexibility. Solution of the differential equations leads to the common eigenvalue or characteristic value problem.

Natural frequencies of the second and higher modes are often considerably greater than the fundamental natural frequencies for large frame structures. As a result, Dunkerley's equation can be used as a solution to the eigenvalue problem and to determine the fundamental natural frequencies of the tower structure.

The fundamental natural frequencies in bending and torsion were calculated for the MOD-0 wind turbine tower structure by the approximate method. The frequencies were calculated for the tower with and without the nacelle and rotor blades. The approximate fundamental natural frequencies for the tower agree within 18 percent with test data and NASTRAN predictions. Successful design of wind turbine tower structures depends on the proper placement of the fundamental natural frequencies and modes. Thus, an approximate method for

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predicting tower frequencies should be used to aid in the preliminary design and sizing of tower members to meet frequency placement requirements.

Once the tower design is finalized and the fundamental natural frequency placements determined with a detailed structural model, an approximate method can again be useful. It can be used to find gross errors in the detailed model, to differentiate between important modes and those caused by local member vibration, and to identify coupled mode shapes. Reasonable agreement between results obtained from the detailed model and those obtained from an approximate method allows the detailed model to be used with greater confidence in evaluating the dynamic response of the coupled wind turbine generating system. This analysis is a key step in the program goal of predicting the dynamic response of wind turbines.

INTRODUCTION

Recent shortages in the supply of energy, coupled with increasing fuel costs, have forced our Nation to research all forms of energy. The power available from the wind is now being examined with renewed interest.

The Federal wind energy program is directed by the Energy Research and Development Administration (ERDA). The program includes research and development on a variety of applications and concepts for wind energy systems. Agreement was reached that, under the overall program management of ERDA, the NASA Lewis Research Center would provide project management for a portion of the overall program.

As part of this program, Lewis has designed and constructed a wind turbine large enough to assess the technology requirements and the associated operational problems of a large wind turbine. The 100-kilowatt wind turbine has been constructed at the NASA Lewis Plum Brook Station near Sandusky, Ohio. The wind turbine consists of an open-truss steel tower, 93 feet high; a nacelle that houses the alternator, the gearbox, and the low-speed drive shaft; and two aluminum rotor blades, each 62.5 feet long. The wind turbine is designed to produce 100 kilowatts of electric power in an 18-mph wind at a rotor speed of 40 rpm.

For proper design of a wind turbine, it is necessary to perform analyses to determine the natural frequencies and associated mode shapes of the primary structures of a wind turbine, such as the tower. This report provides an approximate method for calculating the first cantilever-bending natural frequency and the first torsional natural frequency of a typical tower structure for a large wind turbine.

A typical wind turbine tower structure can also be modeled and analyzed with the aid of computer programs such as STRUDL II or NASTRAN. However, as the tower model becomes more elaborate, the time required to prepare the program for running becomes lengthy. The purpose of this analysis is to provide an approximate modeling method that can be used in the preliminary design stage to quickly calculate the fundamental natural frequencies of a typical wind turbine tower.

This report describes the mathematical model selected to represent a typical wind turbine tower structure. The simplified model can be used to compute tower natural frequencies with the aid of a pocket-type calculator. The model can also be programmed with NASTRAN or STRUDL II to eliminate some of the need for repetitious calculations. The equations of motion for the tower structure are developed and solved in closed form. The equations for the free flexural vibrations of the tower are also presented. Finally, in an appendix, the natural frequencies are determined for the ERDA-NASA MOD-0 wind turbine tower structure by using the approximate method. The frequency values calculated by the approximate method are compared with values determined by testing. In addition, these fundamental frequencies are compared with values obtained by analyzing a relatively much larger structural dynamic model of the tower with the aid of NASTRAN.

SYMBOLS

A	cross-sectional area
A_D	cross-sectional area of diagonal brace member
a	flexibility coefficient in bending
b	flexibility coefficient in torsion
d	horizontal distance; derivative
E	modulus of elasticity
F	force
f	frequency, $\omega/2\pi$
G	shear modulus
h	vertical distance
I	area moment of inertia
i,j,k,n	whole numbers
J	polar mass moment of inertia
K	stiffness coefficient
l	length of diagonal brace member
M	mass
P	axial load
q	generalized coordinate
R	radial distance
R_{EQ_i}	equivalent torsional rigidity of tubular tower legs and bracing (ref. 3)
S	h/n
T	kinetic energy
t	time
V	potential energy
w	width
x,y,z	Cartesian coordinates
δ	linear displacement
θ	cylindrical coordinate
φ	projected angle on y,z plane
ω	natural frequency
$(\dot{\ })$	first derivative with respect to time

$(\ddot{\ })$	second derivative with respect to time
1,2,3,4	tower sections

Superscript:

D	diagonal brace member
0	center of tower
1,2,3,4,5,6	tower bays

ANALYSIS

Mathematical Model Description

The wind turbine tower structure is composed of four main columns with diagonal and horizontal bracing, as shown in figure 1. Because the tower height is much greater than its average width, the tower is assumed to behave like a cantilever beam.

The mathematical model for the tower treats the mass properties and the elastic properties separately. The tower mass is accounted for by lumping portions of the total mass at node points. A typical node point is located at the intersection of a horizontal member, a diagonal brace, and the vertical leg, as shown in figure 1. The four nodal masses at each tower level are added together, resulting in a single value for each of the six bays or levels of the tower. The center of each lumped mass is located on the vertical centerline of the tower and at the same elevation of each of the horizontal structural members.

The elastic properties for the tower structure are calculated at various levels along the tower height. The elastic properties are determined by calculating the area moment of inertia of the structure at each tower level selected and by using the material mechanical properties.

The tower structure is thereby simplified to a system model composed of six lumped masses and six elastic beams. Figure 2 shows the tower structural model that is used to calculate the natural frequency of the tower in bending. The model used to calculate the natural frequency of the tower in torsion is shown in figure 3.

Equations of Motion

The equations of Lagrange (ref. 1) are used to formulate the equations of motion for the tower structural model. First, the kinetic energy of the multidegree system is expressed as

$$T = \sum_{i=1}^{n} \left[\frac{1}{2} M_i (\dot{y}_i)^2 + \frac{1}{2} M_i (\dot{z}_i)^2 + \frac{1}{2} J_i (\dot{\theta}_i)^2 \right] \tag{1}$$

Next, the potential energy of the multidegree system, neglecting the effects due to gravity, is written as

$$V = \sum_{i=1}^{n} \left[\frac{1}{2} K_{x_i} (y_i)^2 + \frac{1}{2} K_{z_i} (z_i)^2 + \frac{1}{2} K_{\theta_i} (\theta_i)^2 \right] \tag{2}$$

From the Lagrange equation,

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = 0 \tag{3}$$

where q_i represents the generalized coordinates y_i, z_i, and θ_i. Equations (1) to (3) yield

$$\left.\begin{aligned} M_i \ddot{y}_i + K_{y_i} y_i &= 0 \\ M_i \ddot{z}_i + K_{z_i} z_i &= 0 \\ J_{\theta_i} \ddot{\theta}_i + K_{\theta_i} \theta_i &= 0 \end{aligned}\right\} \tag{4}$$

These equations are uncoupled linear ordinary differential equations of equilbrium in y_i, z_i, and θ_i. Therefore, free vibrations for the idealized structure in the coordinates y, z, and θ are independent of each other. This is true when the mass center of the tower structure coincides with the center of twist of the various tower levels.

The system of linear differential equations (4) can be written in the matrix form

$$[M] \quad \{\ddot{y}\} + [K_y] \quad \{y\} = 0 \tag{5}$$

$$[M] \quad \{\ddot{z}\} + [K_z] \quad \{z\} = 0 \tag{6}$$

$$[J_\theta] \quad \{\ddot{\theta}\} + [K_\theta] \quad \{\theta\} = 0 \tag{7}$$

The mathematical model for the tower structure is symmetric for bending in the y and z directions. As a result, the frequency solutions in the y and z directions will be identical. Since equations (5) and (6) yield identical results, only equation (6) is used in the following analysis.

Free Flexural Vibrations in z-Direction

In equation (6), the term $[K_z]$ is the stiffness coefficient matrix for the structure in the z-coordinate direction. Often it is useful to generate flexibility coefficients for the structure. Equation (6) may be rewritten, using the flexibility coefficient matrix $[a_z]$ rather than the stiffness coefficient matrix $[K_z]$, as

$$[a_z] \quad [M] \quad \{\ddot{z}\} + \{z\} = 0 \tag{8}$$

where

$$[a_z] = [K_z]^{-1}$$

and

$$[a_z] \quad [K_z] = [I]$$

the identify matrix.

Equation (8) forms a set of linear second-order differential equations whose solution is given by (ref. 1)

$$\{a\} = -w^2 \{z\} \tag{9}$$

From equations (8) and (9),

$$w^2[a_z] \quad [M] \quad \{z\} = \{z\} \tag{10}$$

or, expanding,

$$w^2[a_z] \quad [M]\begin{Bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \end{Bmatrix} = \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \end{Bmatrix} \tag{11}$$

Dividing equation (10) by w^2 yields

$$\left([a_z] \quad [M] - \frac{1}{w^2}[I]\right) \quad \{z\} = \{0\} \tag{12}$$

Equation (12) forms a set of homogeneous algebraic equations in z. ($1/w^2$ is unknown.) The solution of these equations is usually called the eigenvalue or characteristics value problem. A nontrivial solution of equation (12) can exist only if the determinant

$$\left| [a_z] \quad [M] - \frac{1}{w^2} [I] \right| = 0 \tag{13}$$

$$\begin{vmatrix} \left(a_{z_{11}} M_1 - \frac{1}{w^2}\right) & a_{z_{12}} M_2 & \cdots\cdots & a_{z_{1n}} M_n \\ a_{z_{21}} M_1 & \left(a_{z_{22}} M_2 - \frac{1}{w^2}\right) & \cdots\cdots & a_{z_{2n}} M_n \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ a_{z_{n1}} M_1 & \cdots\cdots\cdots\cdots\cdots & & a_{z_{nn}} M_n - \frac{1}{w^2} \end{vmatrix} = 0 \tag{14}$$

where the mass matrix is a diagonal matrix,

$$[M] = \begin{bmatrix} M_1 & & & & 0 \\ & M_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & 0 & & \cdot & \\ & & & & M_n \end{bmatrix}$$

and the flexibility matrix in the z-coordinate direction is

$$[a_z] = \begin{bmatrix} a_{z_{11}} & \cdots & \cdots & a_{z_{1n}} \\ a_{z_{21}} & a_{z_{22}} & \cdots & a_{z_{2n}} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ a_{z_{nn}} & \cdots & \cdots & a_{z_{nn}} \end{bmatrix}$$

Assume the roots of the frequency equation (14) are $1/w_{z_1}$, $1/w_{z_2}$, $1/w_{z_3}$, . . . , $1/w_{z_n}$. Expanding equation (14) with the assumed roots leads to the following equation from reference 2:

$$\frac{1}{w_{z_1}^2} + \frac{1}{w_{z_2}^2} + \frac{1}{w_{z_3}^2} + \ . \ . \ . \ + \frac{1}{w_{z_n}^2}$$

$$= a_{z_{11}} M_1 + a_{z_{22}} M_2 + a_{z_{33}} M_3 + \ . \ . \ . \ + a_{z_{nn}} M_n \tag{15}$$

Therefore, natural frequencies of the second and higher modes are often considerably greater than the fundamental frequency. If this holds, all terms on the left side of equation (15), except the first, may be omitted for the approximate determination of the fundamental frequency. It can be written as

$$\frac{1}{w_{z_1}^2} \cong a_{z_{11}} M_1 + a_{z_{22}} M_2 + a_{z_{33}} M_3 + \ . \ . \ . \ + a_{z_{nn}} M_n \tag{16}$$

Equation (16) is known as Dunkerley's equation (ref. 2) and allows the fundamental frequency to be determined with reasonable accuracy by using longhand calculations.

Evaluation of Flexibility Coefficients

By using the area moment principle suggested in reference 3, the flexibility coefficients can be written as

$$a_{z_{ji}} = a_{z_{ij}} = \frac{S_i}{E} \sum_{k=1}^{k=n} \frac{M_i^k x_j^k}{I_i^k} \tag{17}$$

For the case of six degrees of freedom (i = 1, 2, . . . , 6 and j = 1, 2, . . . , 6),

$$S_i = \frac{h_i}{n}$$

where h_i is the vertical distance between the tower node points and n is an arbitrary number of sections selected between node points. At each selected section, between node points, the area moment of inertia I_i^k is calculated for the tower about the y- or z-axis.

The bending moment M_i^k is calculated between two particular tower node-point levels (i = 1, 2, . . . , 6) and at a particular tower section (k = 1, 2, . . . , n). The magnitude of the bending moment M_i^k is determined by multiplying a unit horizontal force applied at a node point by the vertical distance x_j^k. The quantity x_j^k is the distance between a section of the tower (k = 1, 2, . . . , n) where the bending moment intensity M_i^k occurs and the unit force applied at a node point (j = 1, 2, . . . , 6). The quantity E is Young's modulus of elasticity for the particular tower structural material.

In appendix A of this report the first flexural frequency for the MOD-0 wind turbine tower structure is calculated by using the approximate method as described herein. Each bay of the tower having a height h_i was divided into n = 4 sections. At each of 24 sections (n = 4 sections times j = 6 bays) of the tower, the area moment of inertia was calculated. The bending moment M_i^j due to a unit horizontal force sequentially applied to each of the six node points, was computed at each section. Then equation (17) was used to calculate the flexibility coefficients $a_{z_{ij}}$, where $i = j$ for each of the six tower bays. Finally, equation (16) was used to calculate the first natural bending frequency for the MOD-0 wind turbine tower structure with and without the nacelle and rotor blades.

Free Torsional Vibrations

In equation (7) the term $[K_\theta]$ is the torsional stiffness coefficient matrix for the tower structure. Often it is useful to generate torsional flexibility coefficients for the structure. Equations (7) may be rewritten, by using the torsional flexibility coefficient matrix $[b_\theta]$ rather than the stiffness matrix $[K_\theta]$, as

$$[b_\theta] \quad [J] \quad \{\ddot{\theta}\} + \{\theta\} = 0 \tag{18}$$

where

$$[b_\theta] \quad [K_\theta] = [I] \tag{19}$$

is the identify matrix.

The solution of equation (18) is similar to that of equation (8). Therefore, the fundamental torsional frequency can be written, by using Dunkerley's equation, as

$$\frac{1}{\omega_{\theta_1}^2} \cong b_{\theta_{11}} J_1 + b_{\theta_{22}} J_2 + \ldots \; b_{\theta_{nm}} J_n \tag{20}$$

Evaluation of Torsional Flexibility Coefficient

The torsional spring constant for the tower structure can be written as

$$K_\theta = \frac{8(A_D)_i E}{h_i} w_i^2 \cos^2\varphi_i \sin\varphi_i + 4\left(\frac{12\ EI_i}{h_i^3} R_i^2\right) + 4\left(\frac{GR_{EQ_i}}{h_i}\right) \tag{21}$$

where

A_D cross-sectional area of a diagonal brace member

E modulus of elasticity

h_i vertical tower height between node points

w_i half of width of a square horizontal frame

$-\varphi_i$ angle of diagonal brace member projected on z-y plane

I_i area moment of inertia of a tubular leg taken about centroid of tube

R_i radial distance from tower x-axis to center of a tubular leg

G shear modulus

R_{EQ_i} equivalent torsional rigidity of tubular tower legs and bracing (ref. 3)

The total torsional spring constant K_θ is composed of three terms in equation (21). The first term accounts for the change in length of the diagonal crossmember bracing as located on the sides of the tower. A torsional moment vector applied along the y-axis of the tower results in tension and compression loads in these crossmembers. (This term is developed in appendix B.) The second term in equation (21) accounts for beamwise bending of each of the four main tubular legs of the tower. Again, a torsional moment vector applied along the x-axis of the tower tends to rotate the square frame structure within a horizontal y-z plane. Rotation of each frame causes bending in the tubular tower legs.

Rotation of each frame structure in the y-z plane also causes torsion in each tubular tower leg and in the diagonal cross bracing. The third term in equation (21) accounts for the torsional stiffness of these members. For a typical wind turbine tower structure, such as the MOD-0, the magnitudes of the last two terms on the right in equation (21) were found to be small as compared with the first term. As a result, the last two terms were neglected when the torsional stiffnesses for the MOD-0 tower structure were calculated in appendix C.

The first torsional frequency for the MOD-0 wind turbine tower structure was also calculated in appendix C, by the approximate method described herein. Equation (21) was used to compute the torsional stiffness coefficient at each of the six tower bays. The polar mass moment of inertia J was calculated at six tower levels. Finally, equation (20) was used to calculate the first torsional frequency of the tower structure with and without the nacelle and rotor blades, also in appendix C.

SUMMARY OF RESULTS

Calculated and experimental MOD-0 tower frequencies are compared in tables I and II. Table I presents a summary of MOD-0 tower natural frequencies without the nacelle and rotor blades. The test data were taken from reference 4. The values calculated by using NASTRAN were taken from references 4 and 5. The approximate values in table I were calculated by using the approximate method described in this report. The detailed calculations are presented in appendixes A and C. The 5.04-hertz first bending frequency calculated by the approximate method is 7 percent higher than the 4.7-hertz value determined by testing. The 8.59-hertz first torisonal frequency calculated by the approximate method is 18 percent lower than the 10.5-hertz value determined by testing.

Table II summarizes MOD-0 tower natural frequencies with the nacelle and rotor blades mounted on top of the tower. The rotor blades were oriented in the horizontal position, parallel to the ground. The test data were taken from reference 4. The values predicted by using NASTRAN were taken from references 4 and 5. The approximate values in table II were calculated by using the approximate method described in this report. The detailed calculations are contained in appendixes A and C. The 2.45-hertz first bending frequency calculated by the approximate method is 15 percent higher than the 2.1-hertz value determined by testing. The 3.78-hertz first torsional frequency calculated by the approximate method is 16 percent lower than the 4.4-hertz value determined by testing.

As a result of this analysis, an error was found in references 4 and 5. The first torsional tower frequency with the nacelle and rotor blades (horizontal) was reported as 9.8-hertz in reference 4 and 9.56-hertz in reference 5. This approximate analysis predicts the first torsional natural frequency as 3.78-hertz. Reference 4 reports that a 4.4-hertz mode "appears" to be torsion, with first-mode bending present in the north-south direction. This approximate analysis verifies that the 4.4-hertz mode is, in fact, the first torsional mode and that the 9.8-hertz mode is clearly not the first torsional mode.

For structural models that have a large number of degrees of freedom, it is very difficult to determine certain fundamental modes and frequencies. This difficulty was experienced during the work reported in reference 5, since the first torsional tower frequency was established at 9.56-hertz. If an approximate frequency analysis, as presented herein, was first conducted, the fundamental modes and frequencies as predicted by NASTRAN could have been more easily and accurately interpreted.

CONCLUSIONS

The results of this analysis show that there are important reasons for using an approximate method to evaluate the fundamental natural frequencies of a typical wind turbine tower structure. The reasons are summarized as follows:

1. The simplified model can be used during preliminary tower design to easily determine tower fundamental frequency placement.

2. The simplified model can be used to check frequency predictions resulting from the analysis of a more-detailed structural model of the tower. A detailed structural dynamic model is needed to assist in finalizing the tower design.

These checks include

a. Distinguishing between overall tower system frequencies and individual tower member frequencies
b. Identifying coupled modes
c. Checking for potential gross frequency prediction errors

APPENDIX A

APPROXIMATE CALCULATION OF FIRST FLEXURAL NATURAL FREQUENCY FOR MOD-0 TOWER

Procedure for Computing Flexibility Coefficient a_{11} for Bay 1 of Tower

The procedure for computing the flexibility coefficient a_{11} for tower bay 1 is as follows: Rewriting equation (17) from the ANALYSIS gives

$$a_{z_{ji}} = \frac{S_i}{E} \sum_{k=1}^{4} \frac{M_i^k x_j^k}{I_i^k}$$

Specializing this equation for tower bay 1, shown in figure 4, yields

$$a_{11} = \frac{S_1}{E}\left(\frac{M_1^1 x_1^1}{I_1^1} + \frac{M_1^2 x_1^2}{I_1^2} + \frac{M_1^3 x_1^3}{I_1^3} + \frac{M_1^4 x_1^4}{I_1^4}\right)$$

First, computing the values for the bending moments at each level shown in figure 4 gives the bending moment due to the horizontal 1-pound force about level 1 as

$$M_1^1 = 1.0 \text{ lb} \times 31.5 \text{ in.} = 31.5 \text{ in-lb/lb}$$

then

$$M_1^2 = 94.5 \text{ in-lb/lb}$$

$$M_1^3 = 157.5 \text{ in-lb/lb}$$

$$M_1^4 = 220.5 \text{ in-lb/lb}$$

The distances from the point where deflection due to unit load are desired to the locations of the bending moment intensity are

$$x_1^1 = 31.5 \text{ in.}$$

$$x_1^2 = 94.5 \text{ in.}$$

$$x_1^3 = 157.5 \text{ in.}$$

$$x_1^4 = 220.5 \text{ in.}$$

Each of the four vertical legs of the tower (in bays 1 and 2) is fabricated from 8.0-inch-diameter extra-heavy pipe. The cross-sectional area of the pipe is 12.76 square inches; the area moment of inertia of the pipe is 105.7 inches.^{4}

The transfer formula for computing the area moment of inertia of the pipe about the tower center is

$$I = I_0 + Ad^2$$

where

$$I_0 = 105.7 \text{ in}^4$$

and

$$A = 12.76 \text{ in}^2$$

and d is the horizontal distance from the center of the tower to the center of the tubular tower leg.

Then, from figure 4, accounting for four tubular legs, the area moments of inertia for bay 1, sections 4 to 1, are

$$I_1^4 \cong 4\left[105.7 + 12.76(134)^2\right] = 9.1758\times10^5 \text{ in}^4$$

$$I_1^3 \cong 4\left[105.7 + 12.76(147.2)^2\right] = 1.1060\times10^6 \text{ in}^4$$

$$I_1^2 \cong 4\left[105.7 + 12.76(160.3)^2\right] = 1.3121\times10^6 \text{ in}^4$$

$$I_1^1 \cong 4\left[105.7 + 12.76(173.4)^2\right] = 1.5358\times10^6 \text{ in}^4$$

Again referring to figure 4, $S_1 = 64$ in.

Finally, substituting the values for S_1, M_1, x, and I into the equation for the flexibility coefficient yields

$$a_{11} = \frac{63}{E}\left[\frac{(31.5)(31.5)}{9.1758\times10^5} + \frac{(94.5)(94.5)}{1.106\times10^6} + \frac{(157.5)(157.5)}{1.3121\times10^6} + \frac{(220.5)(220.5)}{1.5358\times10^6}\right]$$

$$= \frac{3.7623}{E} \text{ in/lb}$$

where E is the material modulus of elasticity.

Procedure for Computing Flexibility Coefficient a_{22} for Bays 1 and 2 of Tower

The procedure for computing the flexibility coefficient a_{22} for tower bays 1 and 2 is as follows:

$$a_{22} = \frac{S_1}{E} \sum_{k=1}^{4} \frac{M_1^k x_1^k}{I_1^k} + \frac{S_2}{E} \sum_{k=1}^{4} \frac{M_2^k x_2^k}{I_2^k}$$

where

$$M_2^k = x_2^k = 25.5 + (k - 1)S_2 \qquad k = 1, \ldots, 4$$

$$M_1^k = x_1^k = 204 + \frac{1}{2} S_1 + (k - 1)S_1 \qquad k = 1, \ldots, 4$$

The values for I, and S are given in table III. Subsituting these values into the preceding equation yields

$$a_{22} = \frac{26.638}{E} \text{ in/lb}$$

Procedure for Computing Flexibility Coefficient a_{33}

for Bays 1 to 3 of Tower

The procedure for computing the flexibility coefficient a_{33} for tower bays 1 to 3 is as follows:

$$a_{33} = \frac{S_1}{E} \sum_{k=1}^{4} \frac{M_1^k x_1^k}{I_1^k} + \frac{S_2}{E} \sum_{k=1}^{4} \frac{M_2^k x_2^k}{I_2^k} + \frac{S_3}{E} \sum_{k=1}^{4} \frac{M_3^k x_3^k}{I_2^k}$$

where

$$M_3^k = x_3^k = 24 + (k - 1)S_3 \qquad k = 1, \ . \ . \ . \ , 4$$

$$M_2^k = x_2^k = 192 + \frac{S_2}{E} + (k - 1)S_2 \qquad k = 1, \ . \ . \ . \ , 4$$

$$M_1^k = x_2^k = 396 + \frac{S_1}{E} + (k - 1)S_1 \qquad k = 1, \ . \ . \ . \ , 4$$

The values for I and S are given in table III. Substituting these values into the preceding equation yields

$$a_{33} = \frac{97.357}{E} \text{ in/lb}$$

Procedure for Computing Flexibility Coefficient a_{44} for Bays 1 to 4 of Tower

The procedure for calculating the flexibility coefficient a_{44} for tower bays 1 to 4 is as follows:

$$a_{44} = \frac{S_1}{E} \sum_{k=1}^{4} \frac{M_1^k x_1^k}{I_1^k} + \frac{S_2}{E} \sum_{k=1}^{4} \frac{M_2^k x_2^k}{I_2^k} + \frac{S_3}{E} \sum_{k=1}^{4} \frac{M_3^k x_3^k}{I_3^k} + \frac{S_4}{E} \sum_{k=1}^{4} \frac{M_4^k x_4^k}{I_4^k}$$

where

$$M_4^k = x_4^k = 21 + (k - 1)S_4 \qquad k = 1, \ . \ . \ . \ , 4$$

$$M_3^k = x_3^k = 168 + \frac{S_3}{2} + (k - 1)S_3 \qquad k = 1, \ . \ . \ . \ , 4$$

$$M_2^k = x_2^k = 360 + \frac{S_2}{2} + (k - 1)S_2 \qquad k = 1, \ldots, 4$$

$$M_1^k = x_1^k = 564 + \frac{S_1}{2} + (k - 1)S_1 \qquad k = 1, \ldots, 4$$

The values for I and S are given in tables III and IV. Substituting these values into the preceding equation yields

$$a_{44} = \frac{252.02}{E} \text{ in/lb}$$

Procedure for Computing Flexibility Coefficient a_{55}

for Bays 1 to 5 of Tower

The flexibility coefficient a_{55} for tower bays 1 to 5 is calculated as follows:

$$a_{55} = \frac{S_1}{E} \sum_{k=1}^{4} \frac{M_1^k x_1^k}{I_1^k} + \frac{S_2}{E} \sum_{k=1}^{4} \frac{M_2^k x_2^k}{I_2^k} + \frac{S_3}{E} \sum_{k=1}^{4} \frac{M_3^k x_3^k}{I_3^k}$$
$$+ \frac{S_4}{E} \sum_{k=1}^{4} \frac{M_4^k x_4^k}{I_4^k} + \frac{S_5}{E} \sum_{k=1}^{4} \frac{M_5^k x_5^k}{I_5^k}$$

where

$$M_5^k = x_5^k = 19.5 + (k - 1)S_5 \qquad k = 1, \ldots, 4$$

$$M_4^k = x_4^k = 156 + \frac{S_4}{2} + (k - 1)S_4 \qquad k = 1, \ldots, 4$$

$$M_3^k = x_3^k = 324 + \frac{S_3}{2} + (k - 1)S_3 \qquad k = 1, \ldots, 4$$

$$M_2^k = x_2^k = 516 + \frac{S_2}{2} + (k - 1)S_2 \qquad k = 1, \ldots, 4$$

$$M_1^k = x_1^k = 720 + \frac{S_1}{2} + (k - 1)S_1 \qquad k = 1, \ldots, 4$$

The values for I and S are given in tables III and IV. Substituting these values into the preceding equation yields

$$a_{55} = \frac{708.2}{E} \text{ in/lb}$$

Procedure for Computing Flexibility Coefficient a_{66}

for Bays 1 to 6 of Tower

The flexibility coefficient a_{66} for tower bays 1 to 6 is calculated as follows:

$$a_{66} = \frac{S_1}{E}\sum_{k=1}^{4}\frac{M_1^k x_1^k}{I_1^k} + \frac{S_2}{E}\sum_{k=1}^{4}\frac{M_2^k x_2^k}{I_2^k} + \frac{S_3}{E}\sum_{k=1}^{4}\frac{M_3^k x_3^k}{I_3^k}$$

$$+ \frac{S_4}{E}\sum_{k=1}^{4}\frac{M_4^k x_4^k}{I_4^k} + \frac{S_5}{E}\sum_{k=1}^{4}\frac{M_5^k x_5^k}{I_5^k} + \frac{S_6}{E}\sum_{k=1}^{4}\frac{M_6^k x_6^k}{I_6^k}$$

where

$$M_6^k = x_6^k = 18 + (k - 1)S_6 \qquad k = 1, \ldots, 4$$

$$M_5^k = x_5^k = 144 + \frac{S_5}{2} + (k - 1)S_5 \qquad k = 1, \ldots, 4$$

$$M_4^k = x_4^k = 300 + \frac{S_4}{2} + (k - 1)S_4 \qquad k = 1, \ldots, 4$$

$$M_3^k = x_3^k = 468 + \frac{S_3}{2} + (k - 1)S_3 \qquad k = 1, \ldots, 4$$

$$M_2^k = x_2^k = 660 + \frac{S_2}{2} + (k - 1)S_2 \qquad k = 1, \ldots, 4$$

$$M_1^k = x_1^k = 864 + \frac{S_1}{2} + (k - 1)S_1 \qquad k = 1, \ldots, 4$$

The values for I and S are given in tables III and IV. Substituting these values into the preceding equation yields

$$a_{66} = \frac{986.2}{E} \text{ in/lb}$$

The tower structure weight is estimated at 44 000 pounds. The total tower height is 93 feet. Multiplying the ratio of bay height to total tower height times the total mass of the tower structure yields the estimated mass of each tower bay. Then the mass for bay 1 is

$$M_1 = \frac{44\,000}{(32.2)(12)} \, \frac{21}{93} = 25.71 \text{ lb-sec}^2/\text{in}$$

where the height of bay 1 is 21 feet, as shown in figure 4. For the remaining tower bays the masses are

$$M_2 = 113.87\left(\frac{17}{93}\right) = 20.82\ \text{lb-sec}^2/\text{in}$$

$$M_3 = 113.87\left(\frac{16}{93}\right) = 19.59\ \text{lb-sec}^2/\text{in}$$

$$M_4 = 113.87\left(\frac{14}{93}\right) = 17.14\ \text{lb-sec}^2/\text{in}$$

$$M_5 = 113.87\left(\frac{13}{93}\right) = 15.92\ \text{lb-sec}^2/\text{in}$$

$$M_6 = 113.87\left(\frac{12}{93}\right) = 14.70\ \text{lb-sec}^2/\text{in}$$

Rewriting Dunkerley's equation gives

$$\frac{1}{(\omega_1)^2} \cong a_{11}M_1 + a_{22}M_2 + a_{33}M_3 + a_{44}M_4 + a_{55}M_5 + a_{66}M_6$$

Substituting the values for the masses and the flexibility coefficients then yields

$$\frac{1}{(\omega_1)^2} \cong \frac{1}{E}\Big[(3.76)(25.71) + (26.64)(20.82) + (97.36)(19.59)$$

$$+ (252.)(17.14) + (535.7)(15.92) + (986.2)(14.7)\Big]$$

$$(\omega_1)^2 = \frac{E}{29\,904}$$

where E is 30×10^6 psi. The frequency is therefore estimated as

$$f_1 = \frac{1}{2\pi}\,\omega_1 \cong \frac{1}{2\pi}\left(\frac{30\times10^6}{29\,904}\right)^{1/2} = 5.04\ \text{Hz}$$

The tower first bending frequency is reduced by the addition of the stairway, the nacelle, and the rotor blades. These components of the wind turbine are shown in reference 4, with their associated weights. The tower frequency with these components added is easily estimated. The stair weight (12 000 lbm) is divided into six equal weights and lumped at each node point. The nacelle and rotor blade weight (34 000 lbm) is lumped at node 6. Then the new masses are

$$M_1 = 25.7 + \frac{12\ 000}{(32.2)(12)}\left(\frac{1}{6}\right) = 25.7 + 5.2 = 30.9 \text{ lb-sec}^2/\text{in}$$

$$M_2 = 20.8 + 5.2 = 26.0 \text{ lb-sec}^2/\text{in}$$

$$M_3 = 19.6 + 5.2 = 24.8 \text{ lb-sec}^2/\text{in}$$

$$M_4 = 17.1 + 5.2 = 22.3 \text{ lb-sec}^2/\text{in}$$

$$M_5 = 15.9 + 5.2 = 21.1 \text{ lb-sec}^2/\text{in}$$

Finally,

$$M_6 = 14.7 + 5.2 + \frac{34\ 000}{(12)(32.2)} = 19.9 + 88 = 107.9 \text{ lb-sec}^2/\text{in}$$

Substituting the new values for the masses and the same values for the flexibility coefficients into Dunkerley's equation yields

$$\frac{1}{(\omega_1)^2} \cong \frac{1}{E}\Big[(3.76)(30.9) + (26.64)(26.) + (97.36)(24.8) + (252)(22.3) + (535.7)(21.1) + (986.2)(107.9)\Big]$$

$$(\omega_1)^2 \cong \frac{E}{126\ 600}$$

The frequency is therefore

$$f_1 = \frac{\omega_1}{2\pi} \cong \frac{1}{2\pi}\left(\frac{30\times10^6}{126\ 600}\right)^{1/2} \cong 2.45\ \text{Hz}$$

APPENDIX B

DEVELOPMENT OF TORSIONAL SPRING CONSTANT

FOR MOD-0 TOWER

The derive the torsional spring constant K_θ for a typical bay of the tower structure, four horizontal forces F are applied to the node points of the tower structure as shown in figure 5. This set of forces cause a rotation θ in the horizontal plane. Rotation is allowed by extension of the diagonal members a_1b_2, a_2b_1, a_2b_3, a_3b_2, etc. Extension of a single diagonal member may be expressed as

$$\delta_D = \frac{P_D l_D}{A_D E_D}$$

where

P_D axial load in diagonal member

l_D length of diagonal member

A_D cross-sectional area of diagonal member

E_D material modulus of elasticity for diagonal member

From figure 5, $\delta_1 = R\theta$, $\delta_2 = R\theta/\sqrt{2}$, and $w = R/\sqrt{2}$. Then $\delta_2 = w\theta$.

As shown in figure 6 the horizontal force component of the axial load P_D in the diagonal member is expressed as $F = P_D \cos \varphi$ or

$$F = \frac{\delta_D A_D E_D}{l_D} \cos \varphi$$

Again from figure 6, $\delta_D = \delta_2 \cos \varphi = w \cos \varphi$. Then

$$F = \frac{A_D E_D}{l_D} w\theta \cos^2\varphi$$

From figure 5, $l_D = h/\sin\varphi$. Then

$$F = \frac{A_D E_D}{h} w\theta \cos^2\varphi \sin\varphi$$

For unit values of F and θ,

$$F = \frac{A_D E_D}{h} w \cos^2\varphi \sin\varphi$$

Next, accounting for the existence of two diagonals on one side, one carrying a load in tension and the other carrying an equal but opposite load in compression,

$$F = \frac{2A_D E_D}{h} w \cos^2\varphi \sin\varphi$$

Finally, the unit value of the torsional moment M is written as $M = 4(Fw)$ or

$$M = \frac{8A_D E_D}{h} w^2 \cos^2\varphi \sin\varphi$$

The unit value of M is also the torsional spring constant K_θ.

APPENDIX C

APPROXIMATE CALCULATION OF FIRST TORSIONAL NATURAL FREQUENCY FOR MOD-0 TOWER

From the equation for K_θ developed in appendix B and the structural properties listed in table V, the values for K_{θ_1}, K_{θ_2}, . . . , K_{θ_6} are calculated. Then the torsional flexibility coefficients are

$$b_{11} = \frac{1}{K_{\theta_1}} = \frac{1}{13.5\times10^9} = 7.407\times10^{-11} \text{ rad/in-lb}$$

$$b_{22} = \frac{1}{K_{\theta_1}} + \frac{1}{K_{\theta_2}} = 22.96\times10^{-11} \text{ rad/in-lb}$$

$$b_{33} = \frac{1}{K_{\theta_1}} + \frac{1}{K_{\theta_2}} + \frac{1}{K_{\theta_3}} = 32.39\times10^{-11} \text{ rad/in-lb}$$

$$b_{44} = \frac{1}{K_{\theta_1}} + \frac{1}{K_{\theta_2}} + \frac{1}{K_{\theta_3}} + \frac{1}{K_{\theta_4}} = 45.24\times10^{-11} \text{ rad/in-lb}$$

$$b_{55} = \frac{1}{K_{\theta_1}} + \frac{1}{K_{\theta_2}} + \frac{1}{K_{\theta_3}} + \frac{1}{K_{\theta_4}} + \frac{1}{K_{\theta_5}} = 65.48\times10^{-11} \text{ rad/in-lb}$$

and

$$b_{66} = \frac{1}{K_{\theta_1}} + \frac{1}{K_{\theta_2}} + \frac{1}{K_{\theta_3}} + \frac{1}{K_{\theta_4}} + \frac{1}{K_{\theta_5}} + \frac{1}{K_{\theta_6}} = 97.02\times10^{-11} \text{ rad/in-lb}$$

Calculate Polar Mass Moments of Inertia

Next the lumped polar mass moments of inertia $J_1, J_2, \ldots, J_6$ must be calculated for each bay of the tower. The lumped masses $M_1, M_2, \ldots, M_6$ for each bay of the tower had been previously calculated. The distances $d_1, d_2, \ldots, d_6$ from the center of the tower to a node point are shown in figure 7.

Each mass is assumed to be lumped at a node point on the tower. Then

$$J_1 = 2M_1(d_1)^2 = 2(25.7)(127.5)^2 = 8.356\times10^5 \text{ lb-in-sec}^2$$

$$J_2 = 2M_2(d_2)^2 = 2(20.8)(85)^2 = 3.006\times10^5 \text{ lb-in-sec}^2$$

$$J_3 = 2M_3(d_3)^2 = 2(19.6)(69.5)^2 = 1.893\times10^5 \text{ lb-in-sec}^2$$

$$J_4 = 2M_4(d_4)^2 = 2(17.1)(59.)^2 = 1.190\times10^5 \text{ lb-in-sec}^2$$

$$J_5 = 2M_5(d_5)^2 = 2(15.9)(49.25)^2 = 7.713\times10^4 \text{ lb-in-sec}^2$$

$$J_6 = 2M_6(d_6)^2 = 2(14.7)(40.25)^2 = 4.763\times10^4 \text{ lb-in-sec}^2$$

Calculate Polar Mass Moment of Inertia for Nacelle Mounted on Tower with Rotor Blades Oriented Vertically and Horizontally

The polar mass moment of inertia with the blades mounted vertically (fig. 8) is

$$J_V = \sum Md^2 = \frac{30\ 000}{32.2 \times 12}(75)^2 + \frac{4000}{32.2 \times 12}(165)^2$$

$$= 436\ 724 + 281\ 832 = 718\ 556 \text{ lb-in-sec}^2$$

With the blades mounted horizontally (fig. 8),

$$J_H = \sum Md^2 = \frac{30\ 000}{32.2 \times 12}(75)^2 + \frac{4000}{32.2 \times 12}(165)^2 + \frac{4000}{32.2 \times 12}(270)^2$$

$$= 1\ 437\ 214 \text{ in-lb-sec}^2$$

Rewriting Dunkerley's equation gives

$$\frac{1}{(\omega_1)^2} \cong b_{11}J_1 + b_{22}J_2 + b_{33}J_3 + b_{44}J_4 + b_{55}J_5 + b_{66}J_6$$

Substituting the values for the torsional flexibility coefficients and the polar mass moments of inertia then yields

$$\frac{1}{(\omega_1)^2} \cong 7.407\times10^{-11}(8.356\times10^5) + 22.96\times10^{-11}(3.006\times10^5)$$

$$+ 32.39\times10^{-11}(1.893\times10^5) + 45.24\times10^{-11}(1.19\times10^5)$$

$$+ 65.48\times10^{-11}(7.713\times10^4) + 97.02\times10^{-11}(4.763\times10^4)$$

$$= 3.4278\times10^{-4}$$

$$(\omega_1)^2 \cong 0.29175\times10^4$$

$$\omega_1 \cong 54.0 \text{ rad/sec}$$

The first torsional frequency is therefore

$$f_1 = \frac{1}{2\pi}\,\omega_1 \cong \frac{54.0}{2\pi} = 8.59 \text{ Hz}$$

Calculate First Torsional Frequency of Tower with

Nacelle and Blades Mounted Vertically

Modifying the last term of Dunkerley's equation to account for the polar mass moment of inertia of the nacelle and blades in the vertical position gives

$$b_{66}(J_6 + J_V) = 97.02\times10^{-11}\ (4.763\times10^4 + 7.185\times10^5)$$

Then

$$\frac{1}{(\omega_1)^2} = 3.4278\times10^{-4} + b_{66}J_V = 3.4278\times10^{-4} + 97.02\times10^{-11}\ (7.185\times10^5)$$

$$= 10.399\times10^{-4}$$

$$\omega_1^2 = 9.616\times10^2$$

$$\omega = 31.0 \text{ rad/sec}$$

The first torsional frequency is therefore

$$f = \frac{1}{2\pi}(\omega) = \frac{31}{2\pi} = 4.93 \text{ Hz}$$

Calculate First Torsional Frequency of Tower with Nacelle and Blades Mounted Horizontally

Modifying the last term of Dunkerley's equation to account for the polar mass moment of inertia of the nacelle and blades in the horizontal position gives

$$b_{66}(J_6 + J_H) = 97.02\times10^{-11}\ (4.763\times10^4 + 1.4732\times10^6)$$

Then

$$\frac{1}{(\omega_1)^2} = 3.4278\times10^{-4} + b_{66}J_H = 3.4278\times10^{-4} + 97.02\times10^{-11}\ (1.4732\times10^6)$$

$$= 17.7208\times10^{-4}$$

$$(\omega_1)^2 = 5.6431\times10^2$$

$$\omega_1 = 23.75 \text{ rad/sec}$$

The first torsional frequency is therefore

$$f = \frac{\omega_1}{2\pi} = \frac{23.75}{2\pi} = 3.78 \text{ Hz}$$

REFERENCES

1. Hurty, Walter C. and Rubinstein, Moshe F.: Dynamics of Structures. Prentice-Hall, Inc., 1964.

2. Thomson, William Tyrrell: Mechanical Vibrations. Second ed. Prentice-Hall, Inc., 1953.

3. Blodgett, Omer W.: Design of Welded Structures. James F. Lincoln Arc Welding Foundation, 1966.

4. Linscott, Bradford S.; Shapton, William R.; and Brown, David: Tower and Rotor Blade Vibration Test Results for a 100-Kilowatt Wind Turbine. NASA TM X-3426, 1976.

5. Chamis, C. C. and Sullivan, T. L.: Free Vibrations of the ERDA-NASA 100-kW Wind Turbine. NASA TM X-71879, 1976.

TABLE I. - SUMMARY OF MOD-0 TOWER NATURAL FREQUENCIES

WITHOUT NACELLE AND ROTOR BLADES

Tower mode	Approximate values	Test data[a]	NASTRAN predictions[a]
	Tower frequency, Hz		
First bending in north-south direction	5.04	4.7	4.76
First torsional	8.59	10.5	10.1

[a]From reference 4.

TABLE II. - SUMMARY OF MOD-0 TOWER NATURAL FREQUENCIES

WITH NACELLE AND ROTOR BLADES ON TOWER IN

HORIZONTAL POSITION

Tower mode	Approximate values	Test data[a]	NASTRAN predictions[a]
	Tower frequency, Hz		
First bending in north-south direction	2.45	2.1	2.15
First torsional	3.78	4.4[b]	(c)

[a]From reference 4.
[b]Erronenously reported as 9.8 Hz in reference 4.
[c]Erronenously reported as 9.56 Hz in references 4 and 5.

TABLE III. - MOD-0 TOWER STRUCTURE SECTION

PROPERTIES - BAYS 1 TO 3

Bay	Section	Area moment of inertia at center of tower, I_0, in^4	Cross-sectional area of center of tower, A_0, in^2	Horizontal distance, d, in.	Area moment of inertia, I, in^4	S
1	4	105.7	12.76	134.0	9.1758×10^5	63
	3	↓	↓	147.2	1.1060×10^6	↓
	2			160.3	1.3121	
	1			173.4	1.5358	
2	4	105.7	12.76	122.2	7.6244×10^5	51
	3	↓	↓	111.6	6.3568	↓
	2			100.9	5.2044	
	1			90.3	4.1672	
3	4	72.5	8.40	83.062	2.3208×10^5	48
	3	↓	↓	79.187	2.1096	↓
	2			75.312	1.9084	
	1			71.437	1.7174	

TABLE IV. - MOD-0 TOWER STRUCTURE SECTION

PROPERTIES - BAYS 4 TO 6

Bay	Section	Area moment of inertia at center of tower, I_0, in^4	Cross-sectional area of center of tower, A_0, in^2	Horizontal distance, d, in.	Area moment of inertia, I, in^4	S
4	4	72.5	8.40	68.187	1.5649×10^5	42
	3	↓	↓	65.562	1.4470	↓
	2			62.937	1.3337	
	1			60.312	1.2250	
5	4	72.5	8.40	57.781	1.1246×10^5	39
	3	↓	↓	55.349	1.0321	↓
	2			52.906	9.4327×10^4	
	1			50.469	8.5862	
6	4	72.5	8.40	48.125	7.8099×10^4	36
	3	↓	↓	45.875	7.0993	↓
	2			43.625	6.4228	
	1			41.375	5.7803	

TABLE V. - DIMENSIONS AND TORSIONAL SPRING CONSTANTS FOR EACH TOWER BAY

[Modulus of elasticity, E, 30×10^6 lb/in^2.]

Bay	Cross-sectional area at center of tower, A_D, in^2	Width, w, in.	Vertical distance, h, in.	Projected angle on y,z plane, φ, deg	Spring constants,[a] K_θ, in-lb/deg
1	3.17	127.5	252	54.5	13.50×10^9
2	3.17	85.	204	58.0	6.43
3	5.72	69.5	192	51.2	10.60
4	5.34	59.0	168	52.6	7.78
5	4.96	49.2	156	55.2	4.94
6	4.96	40.2	144	58.1	3.17

[a] $K_{\theta_i} \cong \frac{8A_D E}{h_i} w_i^2 \cos^2\varphi_i \sin\varphi_i$, where $i = 1, 2, \ldots, 6$.

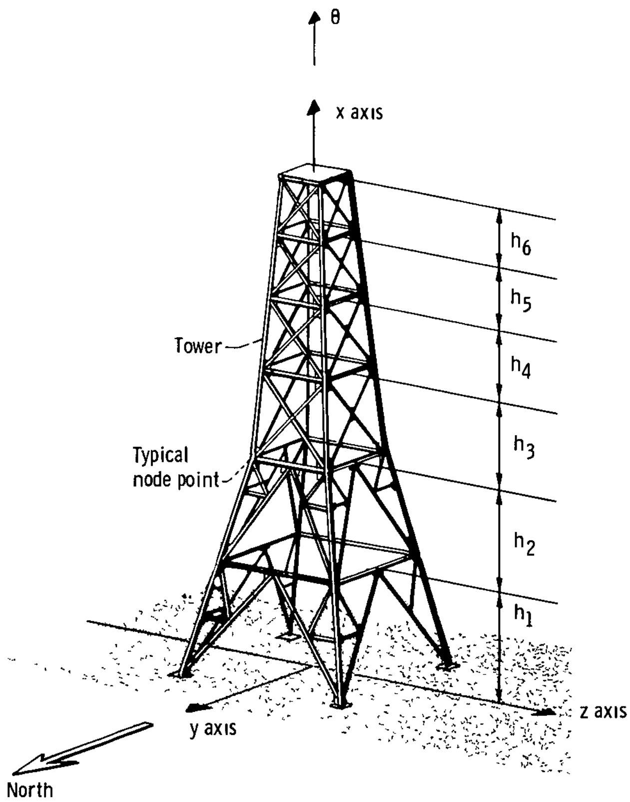


Figure 1. - Typical tower structure.

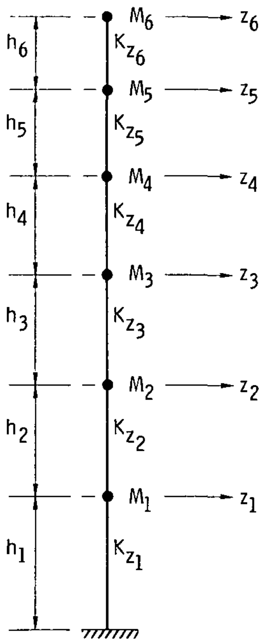


Figure 2. - Model of tower structure for flexural vibration - six degrees of freedom.

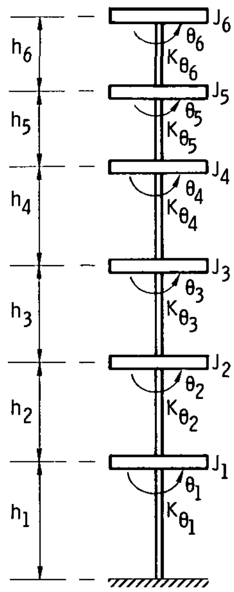
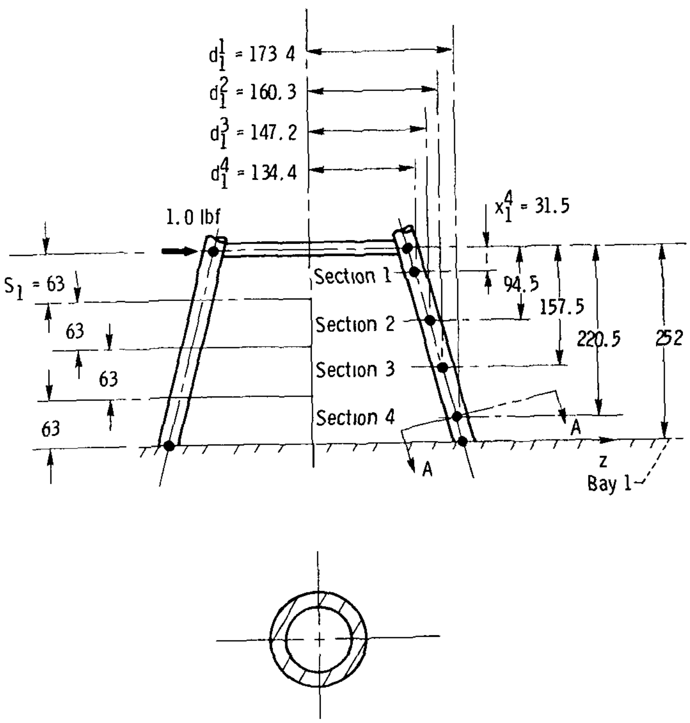


Figure 3 - Model of tower structure for torsional vibration - six degrees of freedom.



Section A-A: 8.0-inch-diameter extra-heavy pipe; cross-sectional area, 12.76 square inches, area moment of inertia, 105.7 inches^4

Figure 4. - Location of sections within tower bay 1. (Dimensions are in inches.)

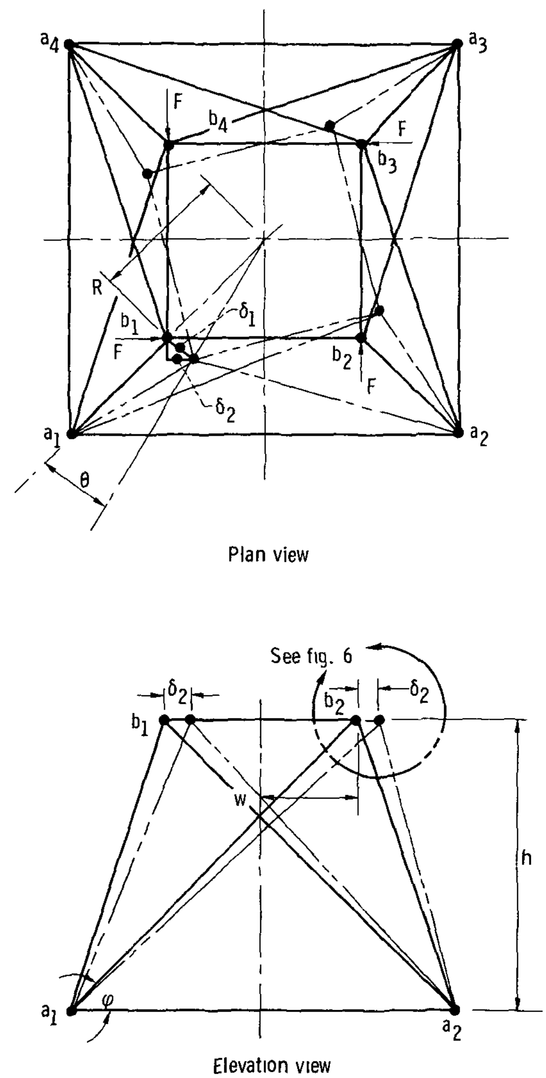


Figure 5. - Displacement of typical tower bay due to torsion.

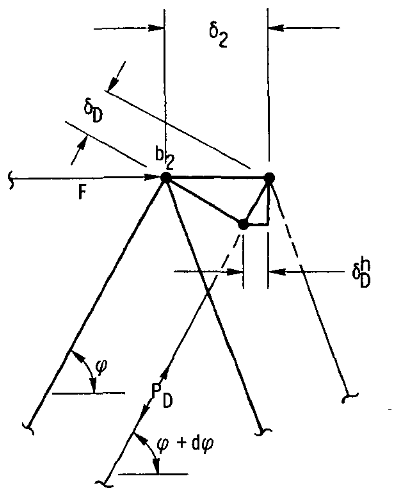


Figure 6. - Displacement of typical diagonal brace member at node point.

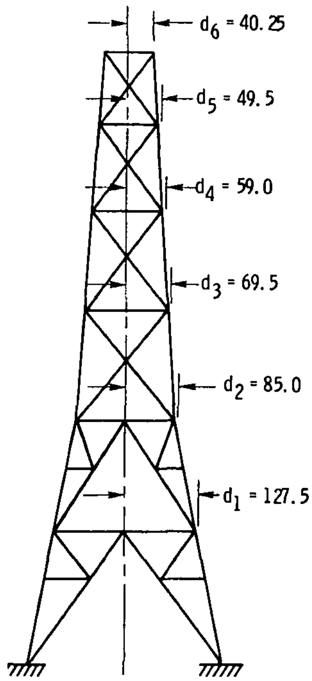


Figure 7. - Arrangement of tower structure. (Dimensions are in inches.)

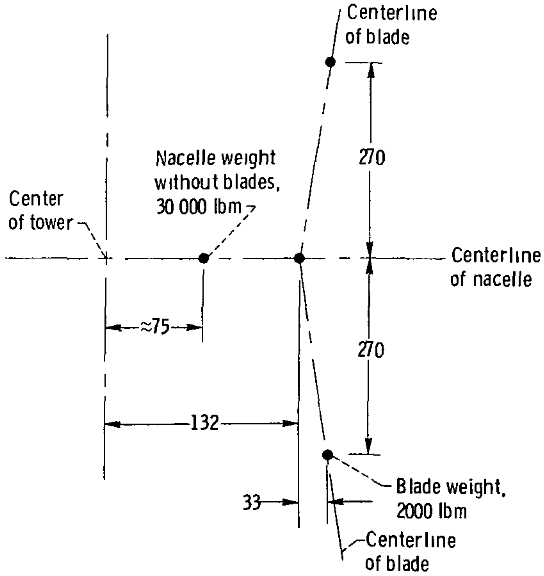


Figure 8. - Plan view of centers of gravity and locations of MOD-0 wind turbine components. (Dimensions are in inches.)

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16 Abstract

A set of ordinary differential equations are derived for a simplified structural dynamic lumped-mass model of a typical large-wind-turbine tower structure. Dunkerley's equation is used to arrive at a solution for the fundamental natural frequencies of the tower in bending and torsion. The ERDA-NASA 100-kW wind turbine tower structure is modeled, and the fundamental frequencies are determined by the simplified method described. The approximate fundamental natural frequencies for the tower agree within 18 percent with test data and predictions analyzed by means of NASTRAN.

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