AIRCRAFT ATTITUDE MEASUREMENT USING A VECTOR MAGNETOMETER

Research Report

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AIRCRAFT ATTITUDE

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VECTOR MAGNETOMETER

Preface

The following report is based on work performed as a part of NASA Grant 05-017-031 by the University of Santa Clara, Department of Electrical Engineering and Computer Science.

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CHAPTER I

A VECTOR AUTOPILOT SYSTEM

1-1 INTRODUCTION

An essential requirement of an aircraft attitude control system is that deviation of the body axes relative to a reference axes frame must be sensed. In addition, to overcome the ever-present possibility of errors or failure of the sensors, various configurations of redundant sensors are usually employed to assist in detection and correction of errors. To this end, there has been a continuing effort to improve existing sensors, to develop new sensor configurations, and to develop new sensor devices.

This chapter discusses the role of a vector magnetometer[1] as a new instrument for aircraft attitude determination. Although magnetometers have played a role in the attitude measurement of missiles and satellites [Ref. 1-1], there is an apparent lack of application in aircraft systems. By providing independent measures of attitude, the solid state vector magetometer sensor system can not only assist in improving accuracy and reliability of existing systems but can also reduce component count with obvious benefits in weight and cost. Additionally, since a large number of aircraft heading reference systems depend on measurement of the Earth's magnetic field, it can be shown that by substituting a three-axis magnetometer for the remote sensing unit; both heading and attitude measurement functions can be derived using common elements, thereby further reducing the component count.

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[1]Aviation use to date has been essentially scalar magnetometry.

To investigate the feasibility of the above system, this chapter will proceed by developing a technique to determine attitude given magnetic field components. Sample calculations are then made using the Earth's magnetic field data acquired during actual flight conditions. Results of these calculations are compared graphically with measured attitude data acquired simultaneously with the magnetic data. The role and possible implementation of various reference angles are discussed along with other pertinent considerations. Finally, it is concluded that the Earth's magnetic field as measured by modern vector magnetometers can play a significant role in attitude control systems.

1-2 ATTITUDE DETERMINATION

Coordinate systems are usually defined by orthogonal right-handed sets of three unit vectors. An example of such a set is illustrated in Fig. 1-1 where the orientation of the body fixed frame used in this paper is delineated. Angular rotations are conventionally defined as rotations in the plane normal to a unit vector with the positive sense of rotation defined by the right-hand rule [Ref. 1-2].

To derive relationships of attitude variations as a function of magnetic vector component variation, we can proceed by considering matrix representations of an orthogonal transformation. If Hx, Hy, and Hz are the magnetic components measured at a desired airframe attitude and Hx', Hy', and Hz' are the components measured after any rotation of the body, vector $H' = [Hx'\ Hy'\ Hz']^T$ can be related to vector $H = [Hx\ Hy\ Hz]^T$ by an orthogonal linear transformation $H' = AH$. Here A must satisfy the orthogonality condition $AA^T = I$, where A^T is the transpose of A; additionally, the determinant of A must be unity [Ref. 1-3, 1-4].

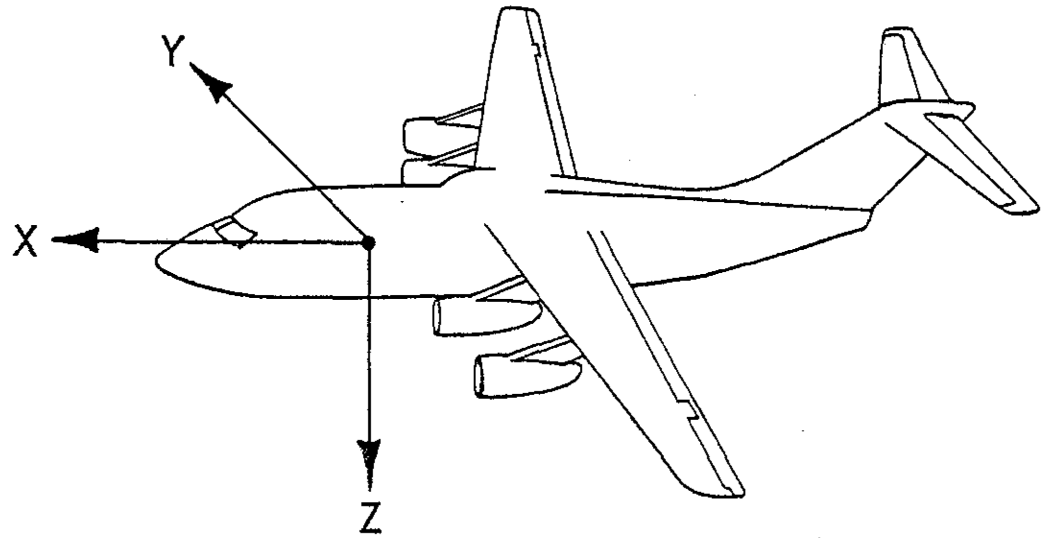

Fig. 1-1 AXIS ORIENTATION

Rotations about the z axis in Fig. 1-1 result in yaw deviations (ψ) and in new components (H'), as shown by

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} \tag{1-1}$$

Similarly, independent rotations about the y axis and the x axis result in pitch (θ) and roll (ϕ) dependent variations in the measured H components, as shown by

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} \qquad (1\text{-}2)$$

$$\begin{bmatrix} Hz' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} \qquad (1\text{-}3)$$

The effect of a combined rotation can be expressed by using the product of the transformation matrices. In addition, if the rotations are small, the total rotation experienced by applying sequential rotations is independent of the order in which the rotations are performed [Ref. 1-3,1-4].

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} \qquad (1\text{-}4a)$$

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} \cos\psi\cos\theta & \sin\psi\cos\phi + \sin\theta\cos\psi\sin\theta \\ -\sin\psi\cos\theta & \cos\psi\cos\phi - \sin\phi\sin\psi\sin\theta \\ \sin\theta & -\cos\theta\sin\phi \end{bmatrix}$$

$$\begin{matrix} \sin\phi\sin\psi - \sin\theta\cos\psi\cos\phi \\ \cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi \\ \cos\phi\cos\theta \end{matrix} \Bigg] \begin{bmatrix} Hz \\ Hy \\ Hz \end{bmatrix} \qquad (1\text{-}4b)$$

Assume that the angular variations θ, ψ, and ϕ are small enough so that the small angle approximations

$$\sin\theta \approx \theta,\ \sin\psi \approx \psi,\ \sin\phi \approx \phi,$$
$$\cos\theta \approx \cos\psi \approx \cos\phi \approx 1$$

can be made. Then, if the products of small angles (in radians) can be assumed to be much smaller than the angles alone, the expression reduces to

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} 1 & \psi & -\theta \\ -\psi & 1 & \phi \\ \theta & -\phi & 1 \end{bmatrix} \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} \tag{1-5}$$

Further modifications in the form of the matrices result in

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} -Hz & Hy & 0 \\ 0 & -Hx & Hz \\ Hx & 0 & -Hy \end{bmatrix} \begin{bmatrix} \theta \\ \psi \\ \phi \end{bmatrix} + \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} \tag{1-6}$$

By subtracting, we arrive at an expression for the difference in H components as functions of angular deviation.

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} - \begin{bmatrix} Hz \\ Hy \\ Hz \end{bmatrix} = \begin{bmatrix} \Delta Hx \\ \Delta Hy \\ \Delta Hz \end{bmatrix} = \begin{bmatrix} -Hz & Hy & 0 \\ 0 & -Hx & Hz \\ Hx & 0 & -Hy \end{bmatrix} \begin{bmatrix} \theta \\ \psi \\ \phi \end{bmatrix} \tag{1-7}$$

It is significant to note at this point that the transformation matrix is singular implying that solutions for θ, ψ, and ϕ are not independently available.

1-3 ATTITUDE DETERMINATION EMPLOYING MAGNETIC FIELD COMPONENTS

A given orthogonal set of three unit vectors can be displaced in Euclidean space by rotating the system through any angle δ about a directed rotation axis. It is also customary to represent this rotation vectorially as a directed line segment whose length is proportional to the rotation angle. This rotation is analogous to the rotation experienced by the body fixed frame of Fig. 1-1 as the aircraft experiences combined pitch, yaw, and roll variation. During flight the body fixed set rotates about this rotation axis assuming new (possibly erroneous) attitudes in space. The task of the attitude sensing system is to provide measures of compounded pitch, yaw, and roll that would result in the same attitude assuming that the rotations occurred sequentially about the x, y and z axes rather than the actual rotation axis.

It was shown in the previous section that a compounded rotation of an orthogonal set can be described by a product of respective transformation matrices. Additionally it was noted that for small angular rotations the order of multiplication is unimportant. Using the relationships of (1-7), expressions for the angular deviations in terms of measured magnetic vector components can be derived.

$$\Delta Hx = -Hz\theta + Hy\psi \tag{1-8a}$$

yields

$$\theta = (Hy\psi - \Delta Hx)/Hz \tag{1-8b}$$

$$\psi = (\Delta Hx + Hz\theta)/Hy \tag{1-8c}$$

Similarly,

$$\Delta Hy = -Hx\psi + Hz\phi \qquad (1\text{-}9a)$$

yields

$$\psi = (Hz\phi - \Delta Hy)/Hx \qquad (1\text{-}9b)$$

$$\theta = (\Delta Hy + Hx\psi)/Hz \qquad (1\text{-}9c)$$

and

$$\Delta Hz = Hx\theta - Hy\phi \qquad (1\text{-}10a)$$

yields

$$\theta = (\Delta Hz + Hy\phi)/Hx \qquad (1\text{-}10b)$$

$$\phi = (Hx\theta - \Delta Hz)/Hy \qquad (1\text{-}10c)$$

Assuming that Hx, Hy and Hz are nominal vector components as measured in a reference attitude and that Hx', Hy' and Hz' are new field components at the new attitude, then $\Delta Hx = Hx' - Hx$, $\Delta Hy = Hy' - Hy$, $\Delta Hz = Hz' - Hz$ are expressions of the incremental changes in field components. Additionally, before using (1-8), (1-9) or (1-10) to solve for attitude variations (pitch, yaw, or roll), one additional angle from an auxiliary sensor[2] must be supplied. Using one additional angle of rotation (about any one axis) the remaining two rotations can then be calculated.

To illustrate this point, flight data acquired during the flight of a NASA flown Convair 900 instrumented with a three-axis magnetometer and a Litton inertial navigation system were used to calculate roll, pitch, and yaw.

[2]It was noted following (1-7) that a unique solution for attitude variation is not possible using magnetic field data alone.

Attitude variation about each of the three axes was calculated using measured magnetic field components supported by one angle from the inertial system. The results of these calculations are plotted in Figs. 1-2 through 1-4.

It is significant to note that the rotations shown occurred simultaneously (i.e., time base is the same for all three figures). The flight was at an altitude of approximately 5000 ft at an airspeed of approximately 250 nmi/h.

Although the data used to plot the attitudes shown in Figs. 1-2 through 1-4 were not acquired specifically for this purpose, the correlations in measured and calculated attitude clearly show that; within the limits of instrument accuracy, signals proportional to attitude variation can be derived using flight data.

1-4 A POSSIBLE SYSTEM CONFIGURATION

Since the intent of this chapter is to introduce the notion that magnetometer technology has advanced to the point where three-axis magnetometers can be incorporated in aircraft attitude sensing systems on a cost effective basis, the system discussion will be limited in scope to describing a possible combined heading and attitude measurement method.

Heading references fall into three classes; 1) those that depend on the Earth's magnetic field, 2) those that depend on the use of low-drift gyroscope to retain a preset azimuth, and 3) those (gyrocompasses) that depend on sensing the Earth's rotation [Ref.1-5]. By far the greatest number of aircraft heading systems depend on the Earth's magnetic field, although many of these include gyroscopes to improve the performance characteristics.

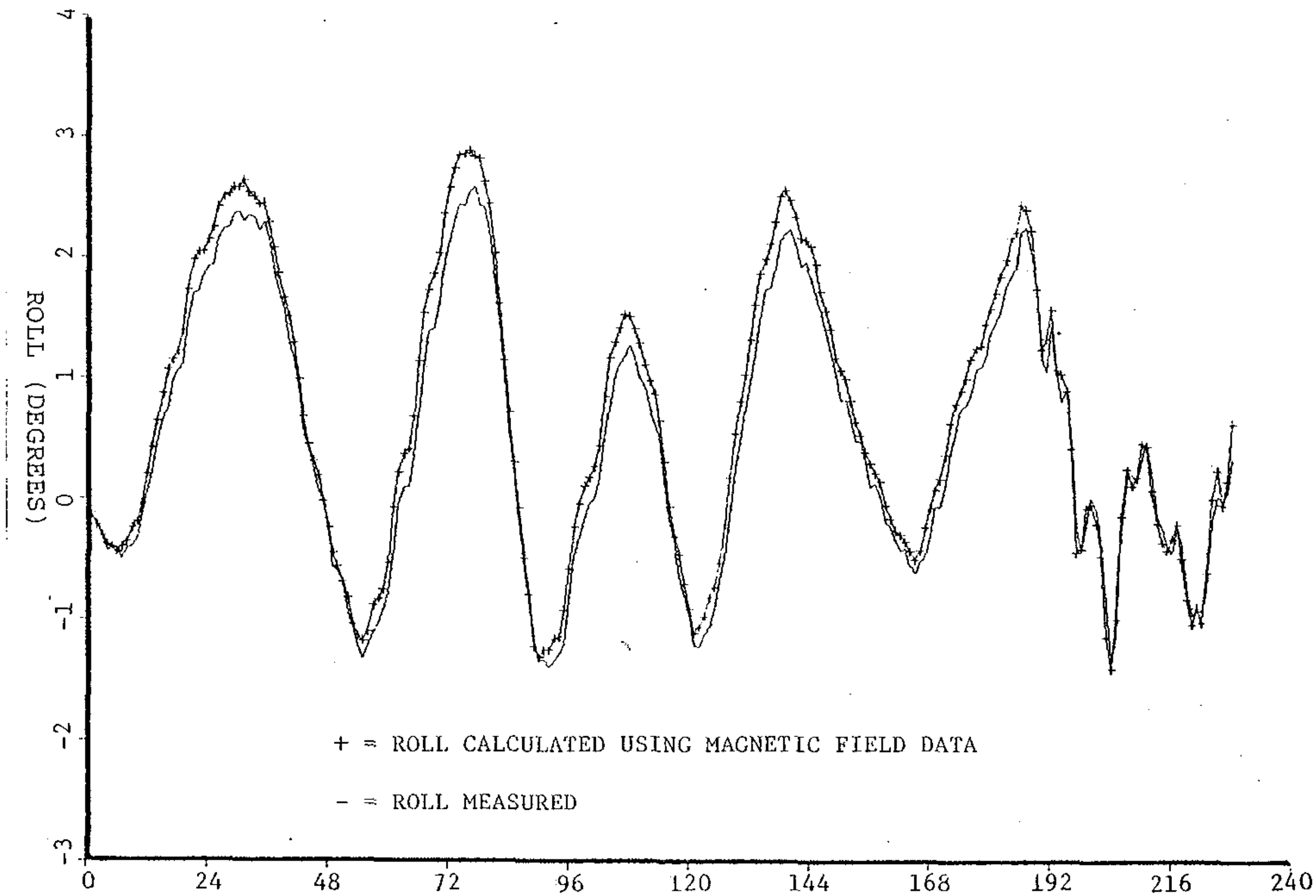

FIGURE 1-2 ROLL AXIS

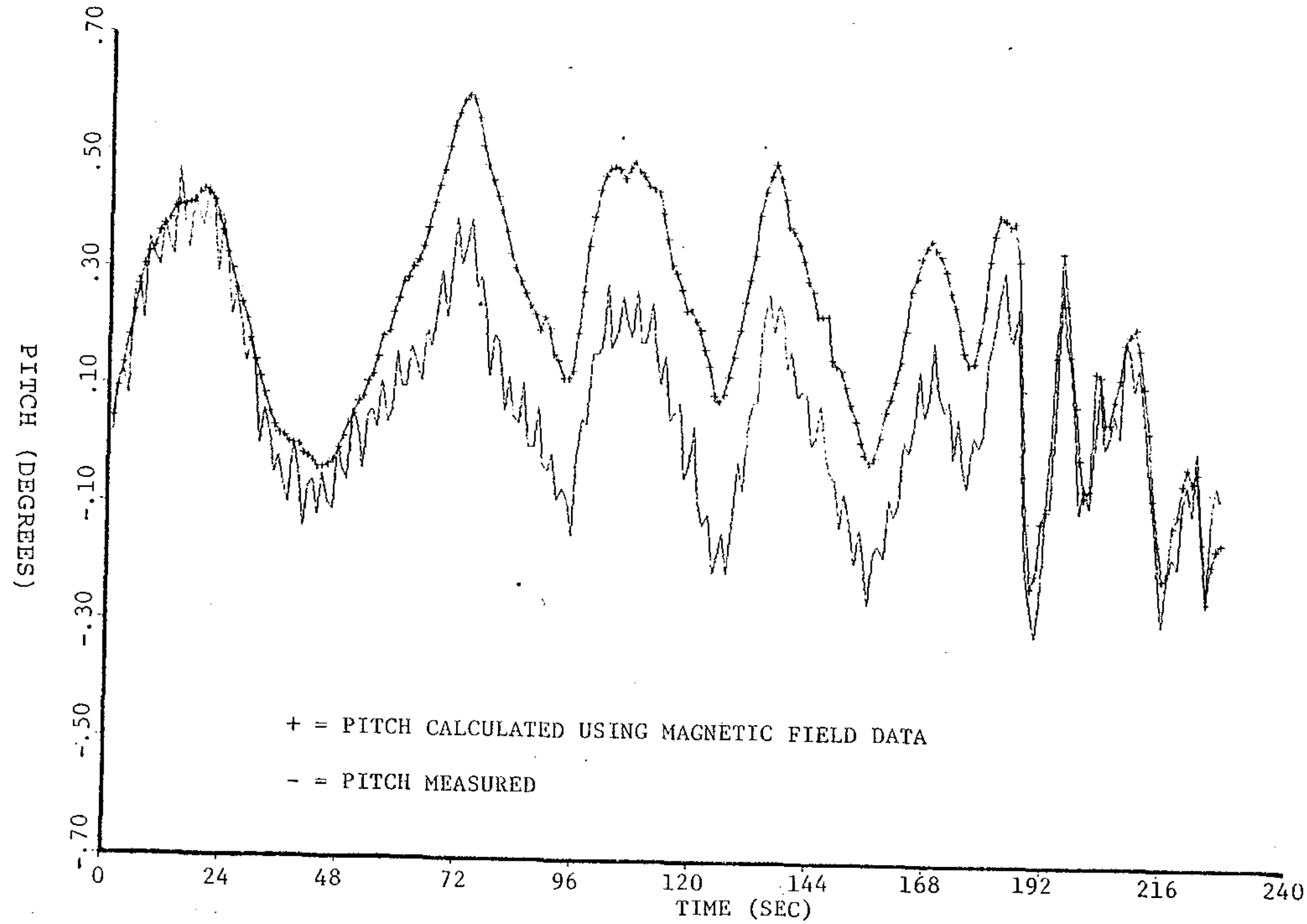

FIGURE 1-3 PITCH AXIS

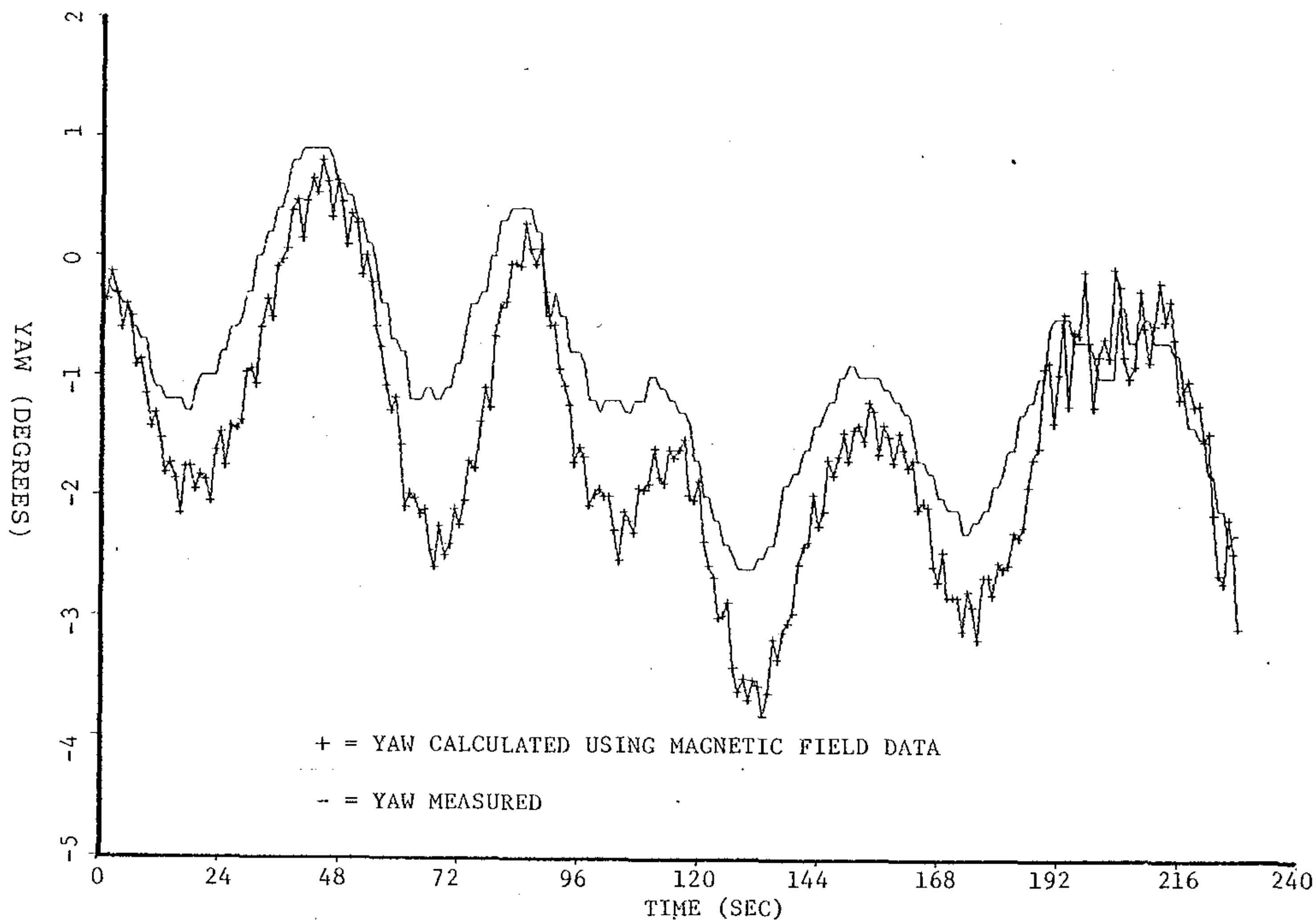

FIGURE 1-4 YAW AXIS

A popular system combination (with no gyro) is to combine a pendulous remote magnetic sensor and a synchro receiver in a null seeking circuit. The philosophy being to attempt to measure only the horizontal component of the Earth's magnetic field and to swing the receiver into alignment with it. Under acceleration, departures of the sensor unit from the horizontal result in angular heading errors ε [Ref. 1-5].

$$\varepsilon = (aH/g)\ \tan\gamma\ \sin\theta$$

where aH is the horizontal acceleration, g is the acceleration due to gravity, θ is the angle between the acceleration vector and magnetic north, and γ is the magnetic field dip angle; arctan (vertical field/horizontal field).

Accuracy of this system can be improved by incorporating a strapped-down solid state magnetic sensing unit (free of acceleration errors) that measures and displays the angle of the Earth's horizontal magnetic component relative to the aircraft. This system can be implemented as follows:

1) Determine the direction of the magnetic vector F relative to the sensors (and the airframe), by measuring the x, y and z components (Figs. 1-1 and 1-5). The direction cosines $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are the cosines of the angles α, β, γ between the magnetic vector and the positive x, y and z axes. Additionally,

$$\cos\alpha = x/(x^2 + y^2 + z^2)^{\frac{1}{2}}$$

$$\cos\beta = y/(x^2 + y^2 + z^2)^{\frac{1}{2}}$$

$$\cos\gamma = z/(x^2 + y^2 + z^2)^{\frac{1}{2}}$$

2) Using either a vertical reference[3] or knowledge of aircraft attitude, we can effectively rotate the body axes such that the x-y plane is horizontal (see Chapter II).

3) Simple application of direction cosines will yield the direction of magnetic north in the aircraft's x-y plane.

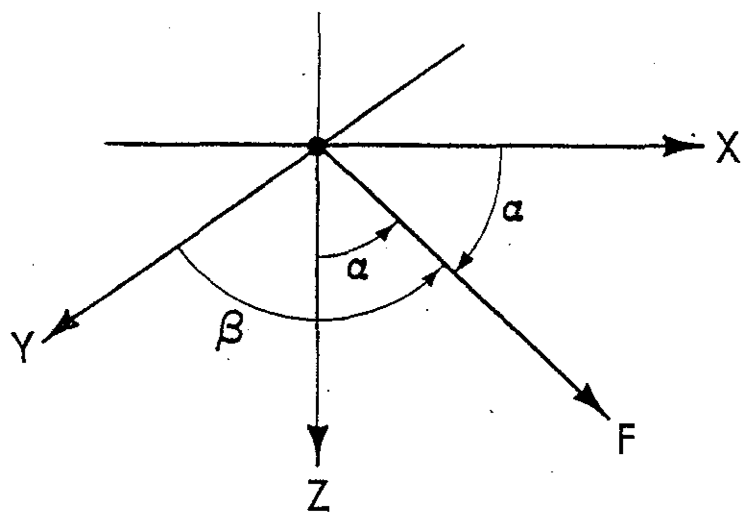

Fig. 1-5 FIELD VECTORS AND DIRECTION COSINES

Although the preceding discussion implies that heading can be determined by using a strapped-down magnetometer, there remains the problem of attitude determination. Another widely used system for obtaining a heading reference

[3]Not necessarily derived inertially [Ref. 1-11].

is to combine the relatively excellent short term stability of a directional gyroscope with the long term stability of magnetic field measurements. By slaving the directional gyroscope to the magnetic field [Ref. 1-5, sec. 10.4.7], gyroscopes with relatively large free drift error can be used to provide an excellent heading reference.

Replacement of the pendulous remote sensing unit of this type of system with a strapped-down vector magnetometer would result in both heading and attitude information on a continuous basis. This combination would operate as follows:

1) The system is initialized by determining a reference attitude (perhaps by using a primary inertial attitude system).

2) The angular position of the horizontal magnetic field component is computed as above and used to slave the directional gyroscope.

3) The directional gyroscope, with relatively good short term stability (devices with free drift of less than 0.5 deg/h have been designed), is used to determine yaw (ψ) errors.

4) For small angle deviations, (1-8), and (1-9), and (1-10) can be employed to recalculate aircraft attitude. The process loops back to step 2) closing the loop on a combined attitude and heading reference system.

The sampling frequency required to maintain an acceptable level of error is of course determined by the aircraft performance expected (angular rates) and by the gyro error (drift rate plus errors due to additional sources such as gyroscope tilt from vertical). The overall system is such that heading can be determined as before with errors due to sensor departures from horizontal substituted for long term accumulation of attitude uncertainty (this can be corrected by looping to step 1) at a frequency dependent on error rates). Additionally one gains measurements of attitude with minimal computation and replacement of a mechanical remote sensing unit with a solid state strapped-down magnetometer sensor.

1-5 OTHER CONSIDERATIONS

The characteristics of the Earth's magnetic field and its variations have long been established [Ref. 1-6-1-10]. Since the field is to be used as a reference in the attitude measurement scheme, there is a need here to discuss its adverse characteristics. Although the field does experience variation, most of the variation is either in amplitude (ionospheric contributions) or has time constants that make the variation negligible (secular variation).

In traversing local anomalies, there will, however, be deflections in the ambient field due to the additive effect of local dipoles or monopoles. The effect of local terrain caused anomalies can be visualized by picturing the main field vector oriented in space with a second modulating vector rotating at its tip. Maximum angular error would occur when this modulating vector has maximum magnitude and is positioned at right angles to the main vector.

To illustrate the effect of local anomalies one can calculate the level of anomaly required to cause an error. Since the Earth's main field is typically in the order of 0.50 G it is readily apparent that a local anomaly of approximately 0.01 G at right angles to the local field is required to cause an error of 1 deg. Furthermore, the local anomaly would have to be aligned with one of the aircraft body axes to result in one degree of attitude error in any one axis. Fortunately, anomalies with components of this magnitude positioned at right angles to the main field are extremely rare. In addition, the anomalies are localized over ore bodies or other geophysical irregularities, have magnitudes that diminish as the cube of altitude, and tend to average to zero over relatively short distances. In summary, the probability of encountering an anomaly that would cause as much as a 1 degree error is relatively small. The error, if introduced, will be short lived and, unlike drift error, will average to zero.

Fundamental to a magnetic field referenced system is the ability to measure orthogonal components of the field vector. Precision and accuracy of measurement of the components is of course specified by the desired control specifications.

Since the Earth's magnetic field varies in magnitude on a global basis between 0.3 G and 0.6 G (30,000 gamma to 60,000 gamma), it is apparent that full scale measurements of 0.6 G can be expected. Sensors mounted at right angles to the field will monitor no measureable field and thus define the lower limit of measurement to be zero. For the continental United States the declination varies between 60 and 80 deg, resulting in a range in horizontal component of 0.15 to 0.25 G with vertical component in the range of 0.4 to 0.55 G. Heading variations (yaw) result in changes of the horizontally sensed field components and would specify the maximum precision required. In addition, flight at 45 deg ± (n x 90 deg) (where n is any whole number) with respect to magnetic north results in minimum sensitivity of the x and y axes measurements. In this case sensor inputs would range between 0.106 and 0.177 G with minimum field at the north. Assuming the preceding ambient measurements, variations in component magnitude of approximately 0.0180 to 0.0305 G/deg for small angle variations can be expected.

A brief survey of commercial magnetometer manufacturers reveals that triaxial magnetometers that measure from zero to 0.6 G with linearities of 0.5 percent, noise less than ±1 mG and sensitivies of at least 2.5 V per 600 mG are currently available. In addition, these devices have a bandwidth of direct current to at least 500 Hz and are rated to have less than 1 deg error in orthogonality.

From a precision standpoint, it is apparent that variations in yaw for this worst case situation can be sensed to better than 0.1 deg with currently available magnetometer technology. The sensor technology required to implement an attitude sensing system of reasonable specifications is available (more detailed analysis is presented in Chapter III).

Although the preceding calculations indicate that for small angular variations attitude can be calculated using measured magnetic data, there is a need to consider the effects of larger finite rotations. In this case the small angle assumptions would not be valid and an Euler transformation would have to be made. Measurement of three axes of field components could be used to develop the direction cosines required to determine the orientation of the axis of rotation, the angular rotation about it, and the three angular rotations of pitch, roll, and yaw.

For the special case where the axis of rotation aligns with the magnetic vector, there would of course be no measured component changes.[4] By measuring the attitude of a second vector (not in alignment with the magnetic vector), we could resolve the ambiguous situation cited above and provide additional redundancy.

The optimum auxiliary vector would be one that could be sensed without using inertial devices. The Earth's electric field can be considered. The main reason for considering this field as a means of providing an auxiliary angular reference is that the resultant system has the potential of being completely solid state. The electric field vector can be used to determine attitude variation in a manner analogous to the magnetic vector system. Inherent limitations of each single vector system can be obviated if the vectors are not coincident.

Although Hill [Ref. 1-11] reported success in controlling pitch and roll using the electrostatic field alone, comments by Markson [Ref. 1-12] indicate that the electrostatic field is not always a reliable vertical reference. Employment of the electrostatic field for this attitude measurement system is limited to augmenting the magnetic field measurements by eliminating ambiguity of motion around the magnetic vector. The requirement of vertical electrostatic field is thus removed and replaced by a requirement that the field direction is relatively stable.

By using two independently derived vectors we have sufficient data to obviate the ambiguity just cited and we have the potential of providing redundancy as well.

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[4] An example of this would be yaw rotation while flying straight and level over the magnetic poles or roll rotation while flying towards a pole at the magnetic equator.

1-6 CONCLUSION

This chapter has identified a novel method of measuring aircraft attitude using relatively inexpensive, well developed instrumentation. It has recognized that magnetic field sensing systems have been used to some extent in attitude sensing and control of space vehicles; it has also suggested, however, that with appropriate support, magnetometers can find increased application in aircraft attitude measurement systems.

This claim is corroborated by actual flight test data. Magnetometers have evolved to a point where three axis measurements of the Earth's magnetic field can be made with sufficient precision and accuracy to enable measurement of small angle attitude variations.

This chapter has also discussed a possible system configuration combining heading determination and attitude measurement functions. By replacing the conventional remote sensing unit with a three-axis magnetometer, it has been suggested that both functions can be obtained with the hardware required previously for heading measurement alone.

As with any system, there are limitations imposed. The main limitation for a vector magnetometer system seems to be the inability to sense rotations around the magnetic vector itself. This problem is not unlike the ambiguity experienced by magnetic heading systems at high latitudes. By judiciously incorporating auxiliary instruments, not only can the ambiguities be removed but a degree of redundancy can be added while still maintaining a cost and weight advantage over comparable systems.

CHAPTER II

AN ATTITUDE INDEPENDENT REMOTE MAGNETIC INDICATOR

2-1 INTRODUCTION

Preliminary investigation [Ref.2-1] revealed that aircraft attitude can be calculated using measurements of earth's magnetic field vector and a single auxiliary rotation angle. An algorithm to compute the two remaining aircraft rotational angles was developed. Using flight data, it was demonstrated that an excellent correlation in computed versus actual aircraft attitude could be achieved. In addition to providing measurements of the magnetic field for redundant attitude computations (to improve accuracy and reliability of existing autopilot systems), it was noted that the vector magnetometer could substitute for the remote magnetic sensing unit. In this manner both heading and attitude measurements could be derived using common elements with obvious benefits in weight and cost.

This chapter discusses the mechanization of a microprocessor based computer system that uses a three axis magnetometer plus gyro data to compute heading. The magnetometer is a three axis solid state device that can be mounted in a strapped down configuration resulting in an attitude independent remote magnetic indicator. Gyro measurements of pitch and roll angle plus three axis magnetic measurements are used by the algorithm to compute aircraft heading. The system can function independently to compute heading or by simply increasing the stored program could implement the attitude computing algorithm of [Ref. 2-1] as well.

The chapter proceeds by developing an algorithm to compute aircraft heading using the strapped down magnetometer and two gyro measured angles. Practical aspects of designing the system including both hardware and software are then presented. In addition, the limitations in instrument accuracy and operation as determined by sensor errors, signal processing errors, arithmetic precision and computation speed are discussed. Considerable computational capability inherent in the system enables minimization of systematic errors. It is demonstrated that inexpensive sensors can be employed with offset and orthogonality errors compensated by microprocessor programming. Finally, it is concluded that a microprocessor based computer with a solid state magnetometer can play a significant role in aircraft instrumentation.

2-2 AN ALGORITHM TO COMPUTE AIRCRAFT HEADING

Coordinate frames are usually defined by orthogonal right-hand sets of three unit vectors. An example of such a set is illustrated in Fig. 1-1 where the orientation of the body fixed frame used in this chapter is delineated. The reference coordinate frame referred to in this chapter is oriented with axes x and y in the horizontal plane and axis z vertical (z down is positive). Pitch attitude angle (Θ) of an aircraft is defined [Ref. 2-2] as the angle between some preferred longitudinal axis and the horizontal reference. In this chapter, pitch angle is the angle between the x axis of the aircraft and the x-y plane of the reference axis set. Since angular rotations are conventionally defined as rotations in the plane normal to a unit vector with the positive sense of rotation defined by the right-hand rule [Ref. 1-2], we will define positive pitch angle (Θ) as the "nose up" or positive rotation about the y axis when the y axis is horizontal. The roll and yaw angles (Φ and Ψ) will then simply be rotations about the x and y axes respectively.

By aligning the three magnetometer axes with the respective x, y and z axes of the aircraft, we can measure magnetic field components of the aircraft at any attitude. For the trivial case where pitch (Θ) and roll (Φ) are both zero degrees, Hx and Hy are the horizontal field components and we can compute yaw from the horizontal vectors as follows:

$$\Psi_1 = \cos^{-1}(Hx/(Hx^2 + Hy^2)^{\frac{1}{2}} \qquad (2\text{-}1a)$$

or

$$\Psi_1 = \sin^{-1}(Hy/(Hx^2 + Hy^2)^{\frac{1}{2}} \qquad (2\text{-}1b)$$

We select either (2-1a) or (2-1b) based on the relative magnitudes of Hx and Hy. By minimizing the numerator of the argument we guarantee that the inverse trigonometric operation results in an angle between zero and forty-five degrees with maximum sensitivity ensured. Heading is then computed using the signs of Hx and Hy to select the appropriate equation from Table 2-1.

Hy \ Hx	NEGATIVE	POSITIVE
Negative	$\Psi = 180 - \Psi_1$	$\Psi = \Psi_1$
Positive	$\Psi = \Psi_1 + 180$	$\Psi = 360 - \Psi_1$

Table 2-1. Formulae to Compute Heading

For most cases, the pitch and roll angles are not zero and inverse rotations are required to determine the actual horizontal field components Hx and Hy. Since any aircraft attitude can be represented as a sequence of rotations about each axis beginning at some reference attitude, we can determine the reference Hx and Hy field components by performing an inverse roll followed by an inverse pitch computation[1].

The inverse roll computation can be developed by considering vector components of an arbitrary vector $\overline{H}$ in Fig. 2-1. The first set (x_2, y_2, z_2) represents the vector components measured in a reference orientation. The second set has common origin and aligns with common x axis component. It is rotated (rolled) about the x axis resulting in new y and z values. We can describe vector $\overline{H}$ in both coordinate frames as

$$\overline{H} = x_2 \cdot \hat{i}_2 + y_2 \cdot \hat{j}_2 + z_2 \cdot \hat{k}_2 \tag{2-2}$$

and

$$\overline{H} = x_3 \cdot \hat{i}_3 + y_3 \cdot \hat{j}_3 + z_3 \cdot \hat{k}_3 \tag{2-3}$$

Since the vector $\overline{H}$ is unique, we note that equations (2-2) and (2-3) are equal. Furthermore if we form dot products we solve for the horizontal components x_2, y_2, and z_2 in terms of the rotated values and the roll angle (Φ).

From (2-2) we obtain

$$\overline{H} \cdot \hat{i}_2 = x_2(\hat{i}_2 \cdot \hat{i}_2) + y_2 (\hat{j}_2 \cdot \hat{i}_2) + z_2(\hat{k}_2 \cdot \hat{i}_2) \tag{2-4a}$$

$$\overline{H} \cdot \hat{i}_2 = x_2 \tag{2-4b}$$

and from (2-3) we obtain

$$\overline{H} \cdot \hat{i}_2 = x_3(\hat{i}_3 \cdot \hat{i}_2) + y_3 (\hat{j}_3 \cdot \hat{i}_2) + z_3(\hat{k}_3 \cdot \hat{i}_2) \tag{2-5a}$$

$$\overline{H} \cdot \hat{i}_2 = x_3 \tag{2-5b}$$

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[1]Since pitch is defined as the angle between the x axis and the horizontal plane we can assume that at any heading, aircraft attitude results due to a pitch followed by a roll.

then

$$x_2 = x_3 \tag{2-6}$$

Similarly,

$$\bar{H} \cdot \hat{j}_2 = y_2 = x_3(\hat{i}_3 \cdot \hat{j}_2) + y_3(\hat{j}_3 \cdot \hat{j}_2) + z_3(\hat{k}_3 \cdot \hat{j}_2) \tag{2-7a}$$

$$y_2 = y_3 \cos \Phi - z_3 \sin \Phi \tag{2-7b}$$

and

$$\bar{H} \cdot \hat{k}_2 = z_2 = x_3(\hat{i}_3 \cdot \hat{k}_2) + y_3(\hat{j}_3 \cdot \hat{k}_2) + z_3(\hat{k}_3 \cdot \hat{k}_2) \tag{2-8a}$$

$$z_2 = y_3 \sin \Phi + z_3 \cos \Phi \tag{2-8b}$$

These expressions can be summarized as

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \Phi & -\sin \Phi \\ 0 & \sin \Phi & \cos \Phi \end{bmatrix} \cdot \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} \tag{2-9}$$

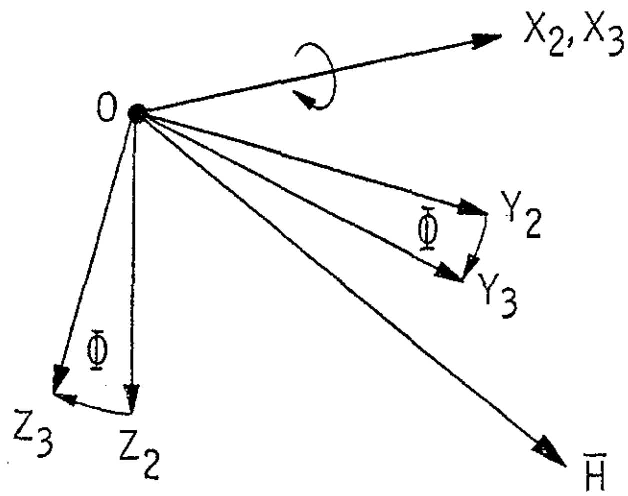

Fig. 2-1 AXES ROTATED IN ROLL

Similarly, considering an axis set rotated in pitch as shown in Fig. 2-2, we can express the reference set x_1, y_1, z_1 in terms of the rotated set x_2, y_2, z_2 as follows

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} \tag{2-10}$$

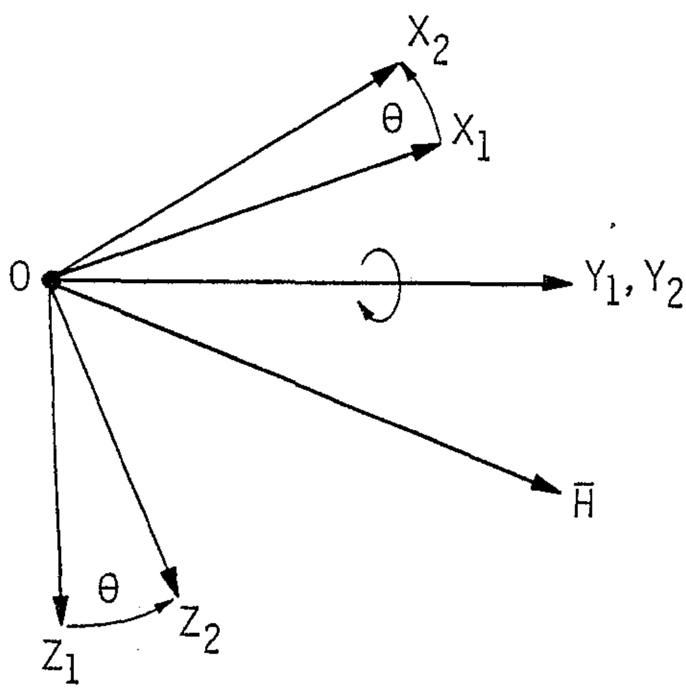

Fig. 2-2 AXES ROTATED IN PITCH

Finally, if we assume that the axis set subscripted with 3 represents components of Earth's magnetic vector measured at an arbitrary aircraft attitude, we can derive the magnetic components (Hxh, Hyh, Hzh) in the horizontal plane for a given heading

$$\begin{bmatrix} Hxh \\ Hyh \\ Hzh \end{bmatrix} = \begin{bmatrix} \cos\Theta & 0 & \sin\Theta \\ 0 & 1 & 0 \\ -\sin\Theta & 0 & \cos\Theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Phi & -\sin\Phi \\ 0 & \sin\Phi & \cos\Phi \end{bmatrix} \bullet \begin{bmatrix} Hx_3 \\ Hy_3 \\ Hz_3 \end{bmatrix} \tag{2-11a}$$

$$\begin{bmatrix} Hxh \\ Hyh \\ Hzh \end{bmatrix} = \begin{bmatrix} \cos\Theta & (\sin\Theta\sin\Phi) & \sin\Theta\cos\Phi \\ 0 & \cos\Phi & -\sin\Phi \\ -\sin\Theta & (\cos\Theta\sin\Phi) & \cos\Phi\cos\Theta \end{bmatrix} \bullet \begin{bmatrix} Hx_3 \\ Hy_3 \\ Hz_3 \end{bmatrix} \tag{2-11b}$$

The algorithm to be implemented with the microprocessor would therefore require operations as outlined in Fig. 2-3. Details of programming method, modifications to the above equations to facilitate programming and computation speed versus accuracy tradeoffs are discussed in following sections.

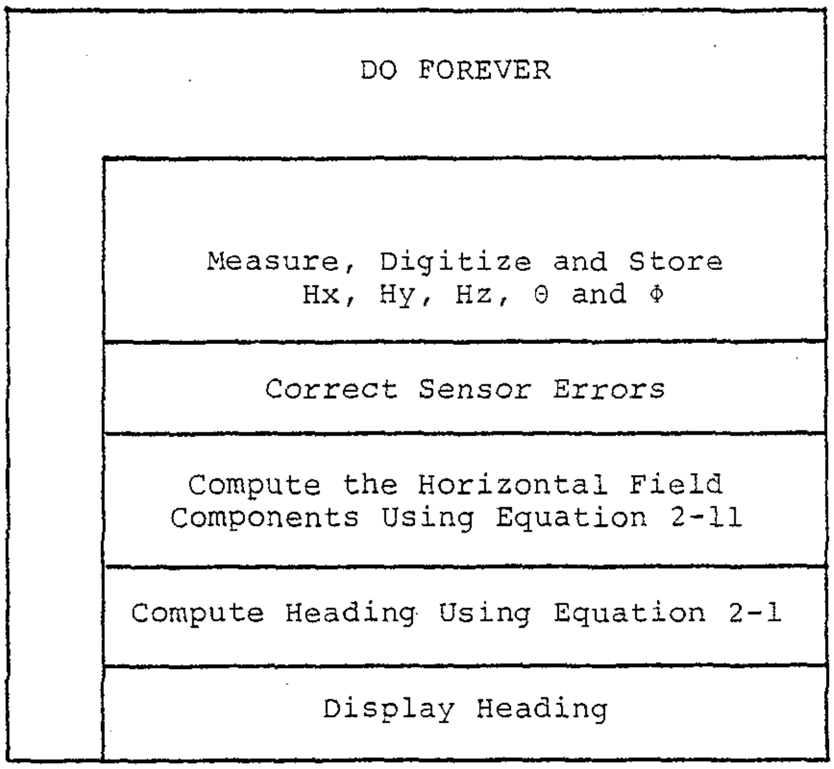

Fig. 2-3 LOGICAL OPERATIONS REQUIRED TO COMPUTE HEADING

2-3 MECHANIZATION OF THE HEADING ALGORITHM

A. General Considerations

To evaluate the performance of an integrated system experimentally, an instrument was designed to implement the algorithm developed above. Several approaches were considered to implement the heading instrument for experimentation:

1) A minicomputer implementation incorporating an HP-2100 minicomputer supported by peripheral interface and analog circuitry. Programming of the HP-2100 would have enabled the computer to control multiplexing and processing of sensor data as suggested by Parish and Lee [Ref. 2-3].

2) A hybrid system composed of a remote data acquisition system to collect data from sensors for subsequent processing by a computer (possibly an HP-2100).

3) A digital/analog electronic implementation incorporating the design of a special purpose computer to perform the required functions of a heading instrument.

The first two approaches were abandoned since it was desirable to perform the experiments at various locations remote from a computer facility and to have data available immediately without having to rely on off-line computations at a later date. The design task then evolved to the design of a special purpose computer system to implement the algorithm, provide a means for evaluating the performance of the proposed algorithm and to allow modifications to the system if required.

B. Design Criteria

Having decided on the general approach to implementing the algorithm it became necessary to consider the performance criteria desired of the instrument.

1) Accuracy
As a design goal, an absolute accuracy of $\pm 1.0°$ in heading uncertainty was selected for the laboratory implementation. This accuracy is compatible with commercially available heading systems.

2) Computation Speed
The bandwidth of the system is determined mainly by the computation speed of the computer[2]. As a design goal, complete heading updates once per second was established.

3) Flexibility
A desirable feature of the laboratory evaluation instrument was considered to be flexibility. Revisions or additions to the algorithm as predicted by experimental data should be incorporated with minimal redesign of the instrument.

2-4 CONCLUSIONS

An instrument designed to implement the heading algorithm developed above uses a three axis magnetometer to measure magnetic field data in the vicinity of an aircraft. Since the magnetometer proposed is a solid state three axis fluxgate device and is permanently mounted in a strapped down configuration, the implementation results in an attitude independent

[2] The response times of the various sensors and analog circuitry are orders of magnitude greater than the desired one second sample interval.

remote magnetic indicator[3].

Several factors will contribute to system inaccuracy. Although the major error sources can be evaluated mathematically (Chapter IV), there is a need to evaluate the implementation experimentally. Systematic errors that arise can be reduced by instrument computation. This capability (inherent with a computer based system) enables incorporation of less expensive sensors in the heading instrument with less concern with factors such as temperature regulation, sensor orthogonality and sensor offset[4].

Since the algorithm can be implemented using a microprocessor as the major computer element, the resulting instrument will have inherent computation capability, be small in size and consume relatively little power. These factors make the instrument an ideal device for aircraft application where the need for redundant distributed processing capability is invaluable.

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[3]Current remote magnetic indicators are pendulous and rely on gravity to enable measurements of the horizontal magnetic vector (not attitude independent).

[4]Assuming that the sensors have repeatable or measurable characteristics, algorithms can be developed to correct previously measured erroneous data.

CHAPTER III

DESIGN OF A MICROPROCESSOR BASED HEADING INSTRUMENT

3-1 INTRODUCTION

Progress in device and component technologies during the 1970's has led to an assortment of sophisticated integrated circuits (IC) devices [Ref. 3-1] which enable the design of instruments with a high degree of sophistication and accuracy. Of these devices, the microprocessor has to date been the most exploited component in industrial control and instrumentation applications [Ref. 3-2 through 3-7]. There have been many papers presented addressing the general application and feasibility of applying microcomputers to particular design tasks [Ref. 3-8 through 3-21].

Although much of the literature to date on microprocessors has addressed the design of commercial products (usually the final result of a carefully orchestrated effort beginning with a market survey), the design of a laboratory instrument for algorithm evaluation differs in design philosophy. In particular, the laboratory instrument is designed to evaluate a proposed algorithm under laboratory conditions. The traditional benchmark evaluations and attempts to match the microprocessor to the application is not only difficult but unnecessary. If the processor is much more powerful than necessary, the "overkill" is little noticed; but if an insufficiently endowed microprocessor is selected, the effects can be devastating. Not only will the program be difficult to write and voracious of memory, it would be difficult to change to a more powerful microprocessor part way through the project. With these considerations in mind, a general purpose, flexible microprocessor with powerful architecture and instruction set the Signetics 2650 microprocessor [Ref. 3-22] was selected.

3-2 HARDWARE DESIGN CONSIDERATIONS

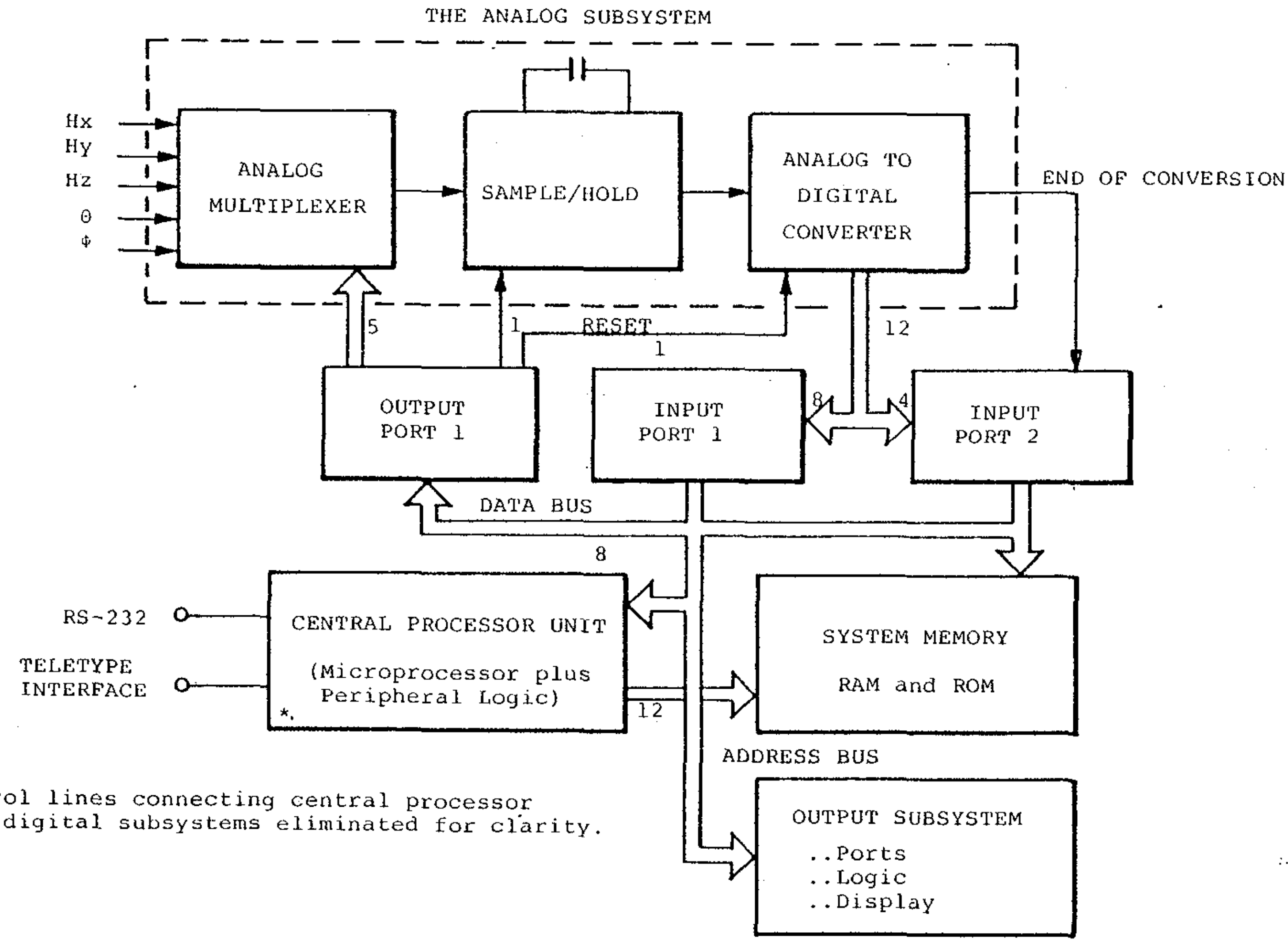
The design of a microprocessor based system begins by considering the total system level block diagram to be implemented (Fig. 3-1). Inputs from five sensors including x, y and z axis magnetic data plus pitch and roll angles (Hx, Hy, Hz, Θ and Φ) are to be multiplexed, sequentially sampled and converted to a digital representation prior to processing (executing the algorithm developed above). The main subsystem of Fig. 3-1, the central processing unit (CPU), operates under control of instructions stored in the system memory and interfaces with the input and output subsystems via data ports.

At this early stage in the design, it is significant to note that the block diagram of Fig. 3-1 differs slightly from that of a classical discrete hardware solution. The input subsystem (composed of analog multiplexer, sample and hold, and analog to digital converter) differs from a conventional data acquisition in that it is devoid of a control section. The microprocessor will control the data acquisition sampling and conversion in addition to performing the arithmetic function associated with the algorithm.

Having established a tentative block diagram of the instrument, the design continues by addressing relevant characteristics and limitations of each subsystem. These characteristics will then in term be considered in configuring the final system and program to be executed.

1) The Analog Subsystem

Composed of the analog multiplexer, sample/hold and analog to digital converter, the analog subsystem of Fig. 3-1 affects both system accuracy and throughput rate. The



*Control lines connecting central processor to digital subsystems eliminated for clarity.

Fig. 3-1. SYSTEM BLOCK DIAGRAM

well-known Shannon theorem [Ref. 3-23, 3-24] on sampling theory defines one of the basic limits on throughput rate stating that the minimum frequency for sampling must be double the highest significant frequency of the signal, including the noise on the signal. This minimum frequency is necessary, the theorem states, if the sampled signal is to contain all of the information needed for undistorted reconstruction. At a lower sampling frequency ailiasing can occur[1]. The minimum sampling rate for data to be used in this heading instrument (based on the design goal of Chapter II) then results in a system bandwidth of 30 hertz. The analog signals from each sensor are low pass filtered to reduce frequency content above 60 hertz. A survey of commercially available multiplexers, sample and hold modules and analog to digital convert modules (ADC) [Ref. 3-25 to 3-28] reveals that subsystems with throughput characteristics exceeding the requirements of a system sampled at one second intervals are readily available (pertinent specifications are discussed in more detail in Chapter IV). The limiting parameter determining total system speed performance will then be the execution time of the algorithm (a programming consideration). A further system consideration is the ability to adjust analog system offset and gain. These adjustments are made using variable resistors (trim pots) connected to appropriate leads on the sample and hold and analog to digital converter modules.

2) The Central Processing Unit (CPU)

The central processing unit (Fig. 3-2) is composed of the microprocessor (Signetics 2650) supported by peripheral logic elements (Fig. 3-2). Design of this subsystem involved medium

[1]That is, the sampled data derived from a sine wave of frequency f sampled at a frequency less than 2f can be fitted with sine waves of a frequency other than f.

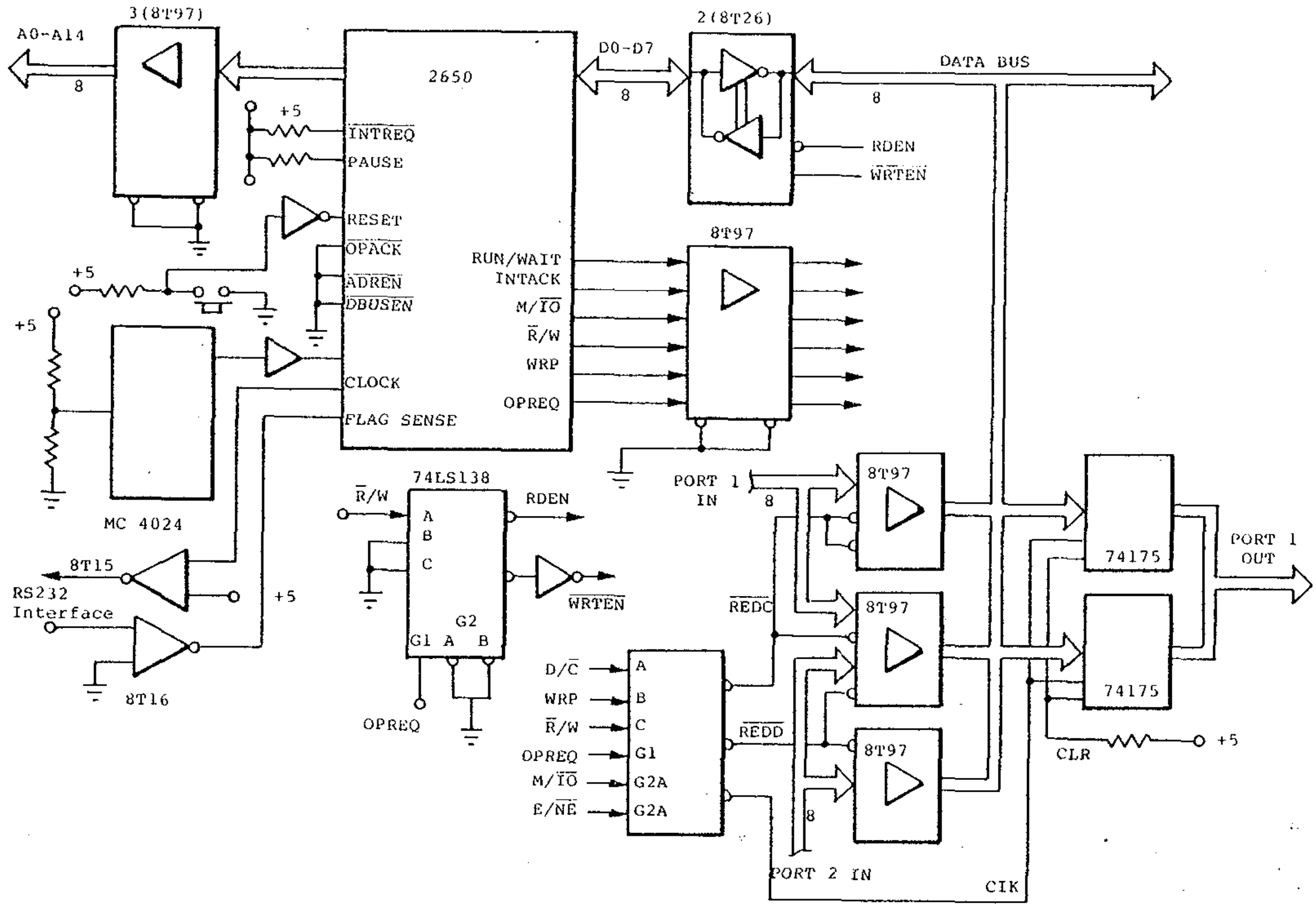


Fig. 3-2. THE CPU SUBSYSTEM

and small scale integrated circuits using well-known [Ref. 3-29 through 3-31] design techniques. To facilitate system development several features were included in the design of the CPU subsystem (features that would not necessarily be required in a production instrument). These include:

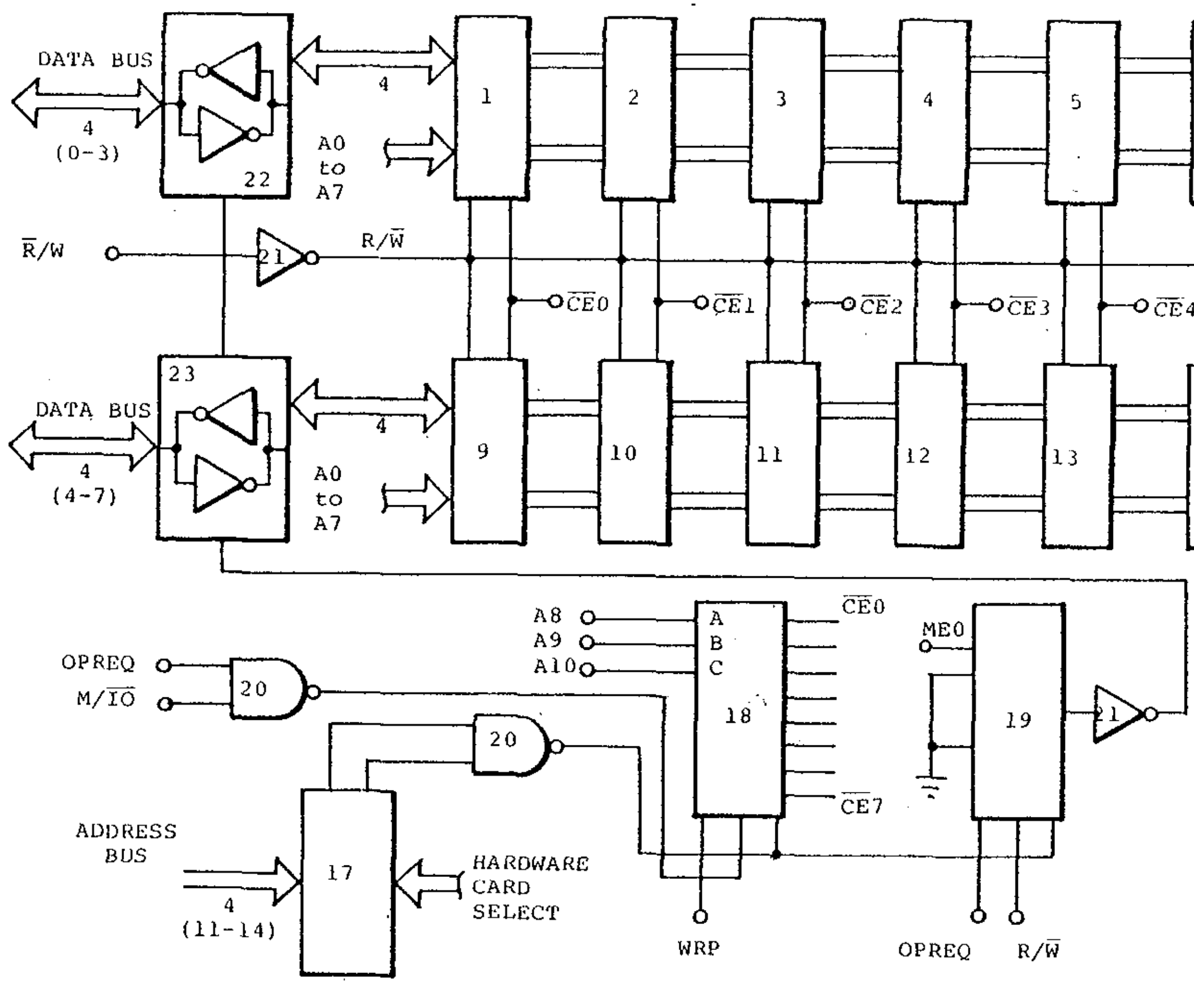
a) System reset, single step and normal run mode operation controlled by switches and logic elements.

b) An RS-232 teletype interface is included to enable manual intervention and development capability during program development. The program was developed by loading and executing instructions into the random access memory (RAM) under control of the PIPBUG[2] program.

3) The Memory Subsystem

The memory subsystem (Fig. 3-3) was organized onto cards each with two thousand byte capability. In this manner system memory could easily be expanded (or reduced) in increments of 2K bytes. The memory chips selected were organized as 256 four bit words and feature pin for pin compatibility with commercially available random access (RAM) and programmable read only memory (PROM) chips. Program segments could then be developed in RAM and finally "burned" into PROM chips for a permanent, nonvolatile operation. In this manner the system development begins with 1K bytes of memory devoted to the resident PIPBUG program (in ROM chips) with the remainder of memory allocated as RAM for both program and scratch pad usage.

[2]Signetics tradename for the 2650 resident loader and monitor program.



NUMBER	PART
1-16	2112 or 82S129
17	DM8200
18, 19	74LS138
20	7400
21	7404
22, 23	8T26

Fig. 3-3. MEMORY SUBSYSTEM (1 Card)

As the program is developed, additional memory is added in increments of 2K bytes per card or 256 bytes on the card. Modifications to the program can be easily made using the PIPBUG program and teletype.

4) The Output Subsystem

For laboratory development the output subsystem of Fig. 3-4 was designed to provide seven segment visual output of the aircraft heading with three significant digits displayed. To expediate the design cycle and to enhance system throughput rate, the outputs were designed as ports with latches and decoder driver functions provided by hardware. In other applications a hardware/software tradeoff could be made with the data decoding and driving implemented using table lookup and multiplexing controlled by the CPU.

3-3 SOFTWARE DESIGN CONSIDERATIONS

The general purpose processor selected to implement the CPU was designed to implement programmed logic and to perform conventional computer operations. This heading instrument takes advantage of both areas. Since the instrument is actually a special purpose computer under control of a stored program, the functional specialization resides in the program rather than the hardware logic. Modifications can be made relatively easily, satisfying the flexibility design goal of Chapter II.

Having decided on the tentative hardware structure described in Section 3-2 above, the program development leading to the final listing in Appendix B proceeded as follows:

1) Structured flow charts were developed depicting the total system operation as an ordered sequence of operations. Each

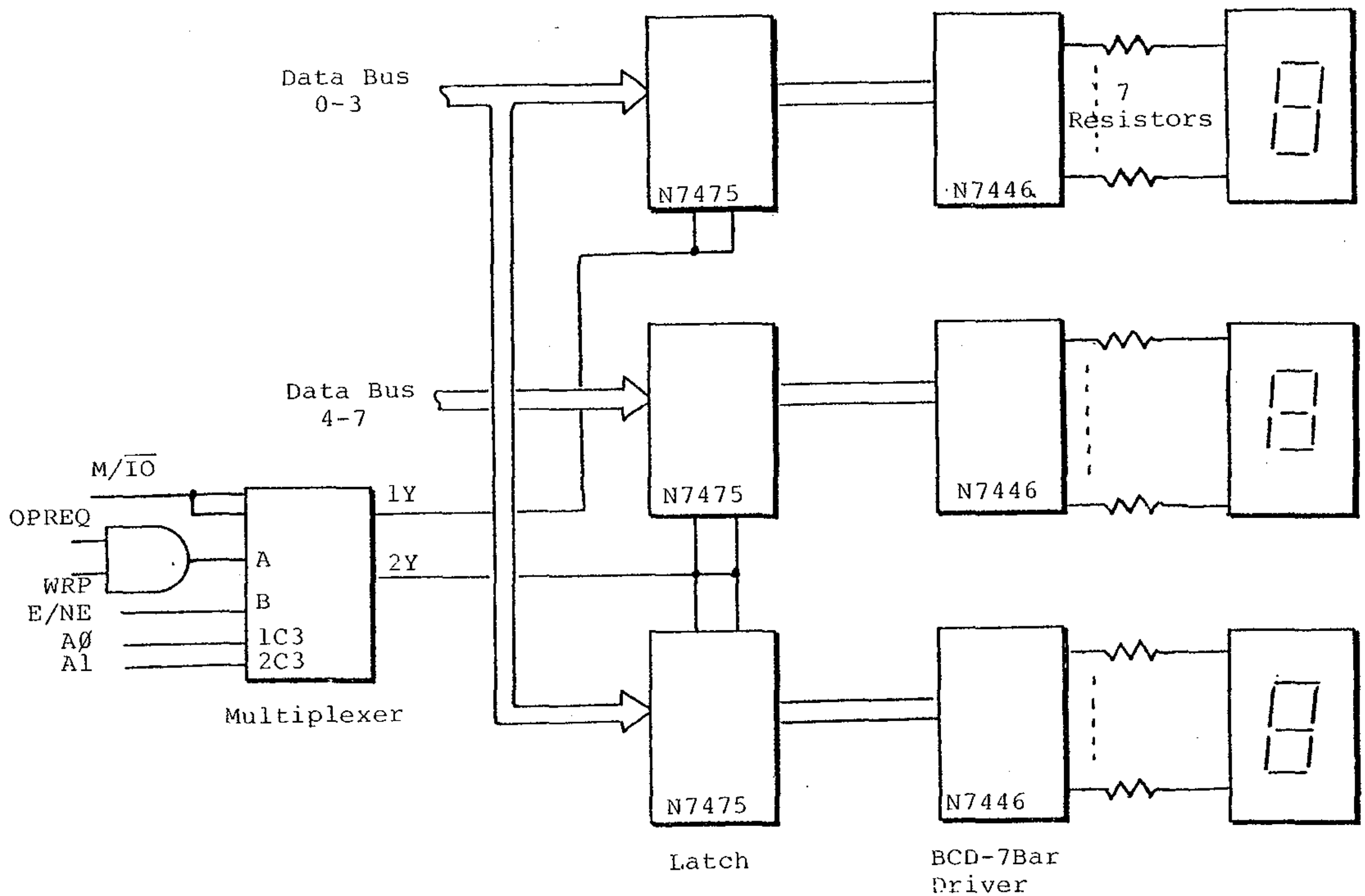


Fig. 3-4 THE OUTPUT SUBSYSTEM

operation is identified as a separate subroutine which in turn can have "nested" subroutines of its own (Fig. 3-5).

2) System accuracy requirements were next investigated (discussed in detail in Chapter IV) to ascertain the precision requirements[3] of the various subroutines.

3) The respective subroutines outlined in 1) above were developed and implemented using a cross assembler program [Ref. 3-32]. Each subroutine was then loaded into the development hardware and "debugged" prior to total program integration. The above program development depicts a top down strategy of program development [Ref. 3-33] and leads to an expedient system development with subroutines being individually developed to yield a modular program construction.

3-4 DESIGN OF SUBROUTINES

The total program consists of an overall system program composed of nested subroutines. The discussion in this section is limited in scope to the design of the more complex subroutines required to implement the solid state remote magnetic heading algorithm.

1) Subroutine "SAMP" (Fig. 3-6a)

The first portion of this subroutine is dedicated to the control function of selecting an analog channel via the multiplexer, sampling and holding the data, resetting and reading data from the analog to digital converter (ADC). Prior to or during the programming of this section, data fields in

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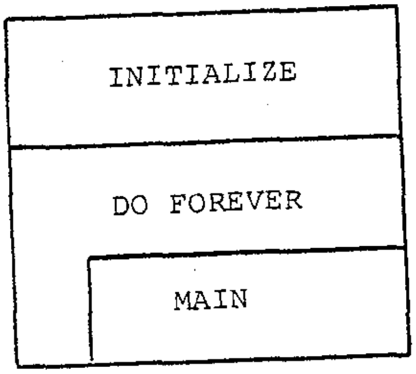
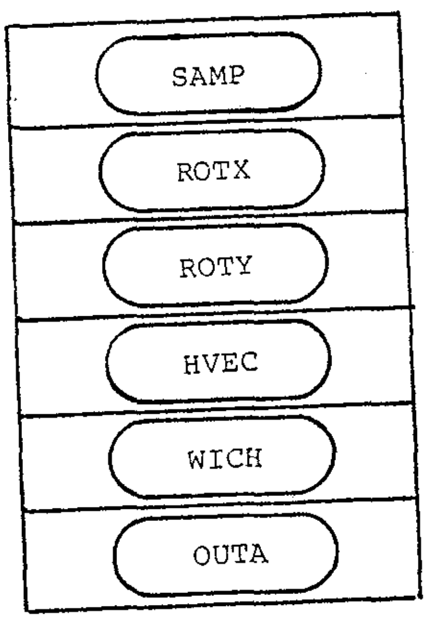
[3] This step is vital to determine whether the operations outlined in 1) above are to be carried out in a single or multiprecision manner.



Power on reset of all registers and subsystems

Compute the aircraft heading

Fig. 3-5a. SYSTEM PROGRAM



Sample all analog channels

Compute the horizontal Hx field

Compute the horizontal Hy field

Compute the horizontal field vector

Compute heading

Output the data

Fig. 3-5b. SUBROUTINE "MAIN"

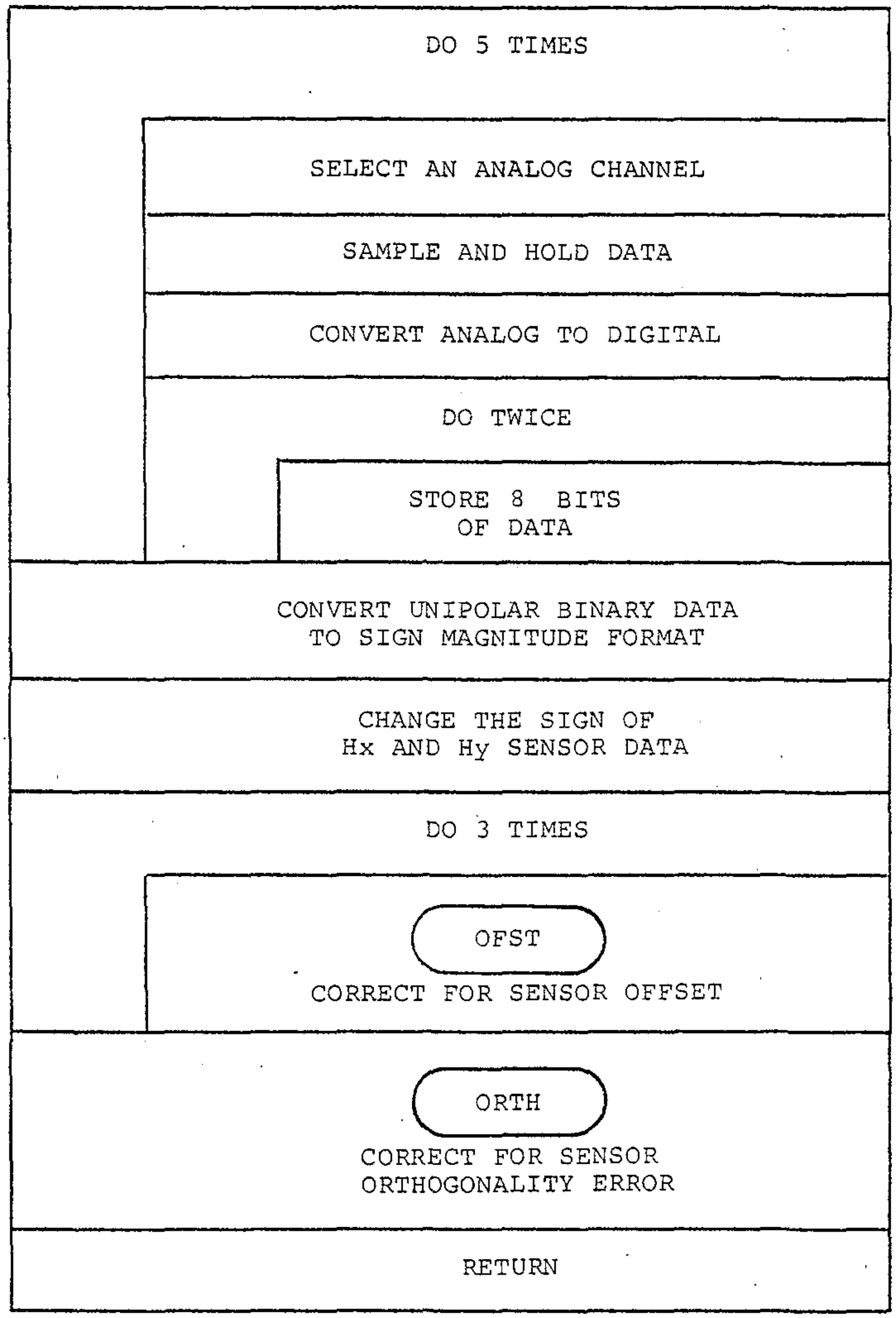


Fig. 3-6a. SUBROUTINE "SAMP"

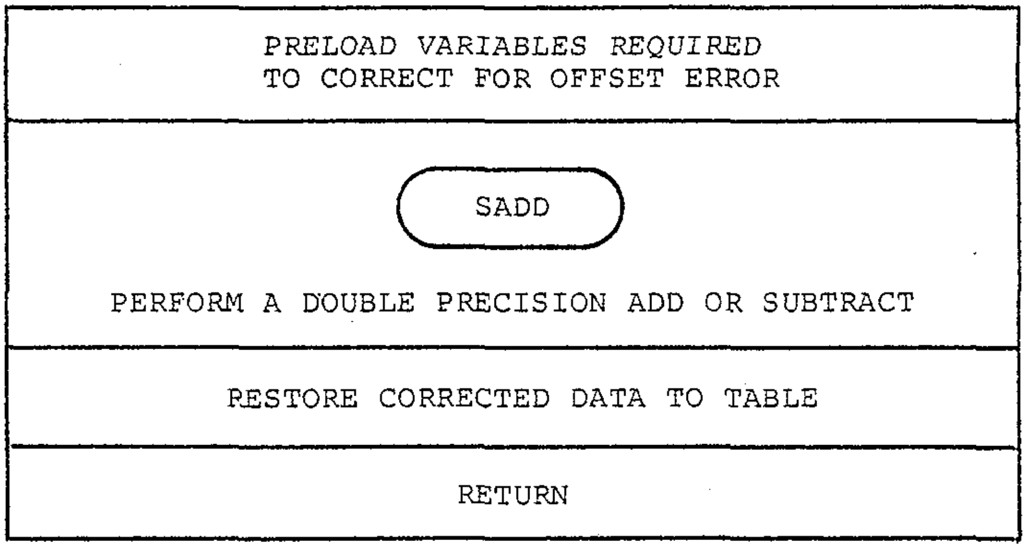


Fig. 3-6b. SUBROUTINE "OFST"

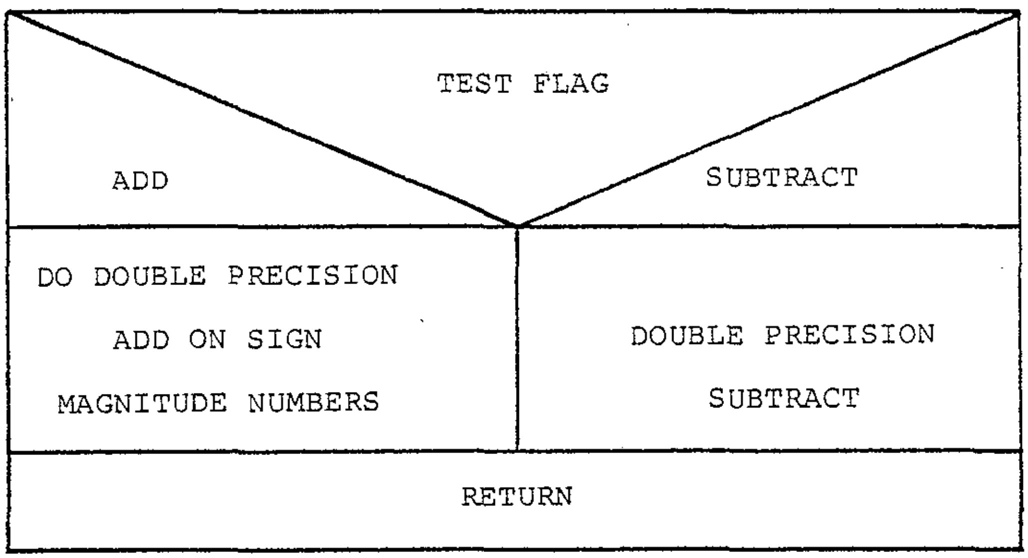


Fig. 3-6c. SUBROUTINE "SADD"

input ports 1 and 2 and output port 1 of Fig. 3-1 are allocated. Control information is then passed to the peripheral module by writing control words to output port 1. Analog to digital converter status and the 12 bit data field are sampled by reading input ports 1 and 2.

Sensor outputs were biased at +2.5 Volts with transfer characteristics as depicted in Fig. 3-7a [Ref. 3-34]. The ADC selected for this laboratory instrument had a binary output data format related to analog input as shown in Fig. 3.7b [Ref. 3-35]. The second function of the sampling subroutine "SAMP" was to convert data from a unipolar binary format to a sign magnitude format. Since the total transfer function from sensor input to ADC output (Fig. 3-7a and b) indicates an offset of 2.5 Volts or 1/2 the ADC output range, the sign magnitude format can be generated as shown in Fig. 3-8.

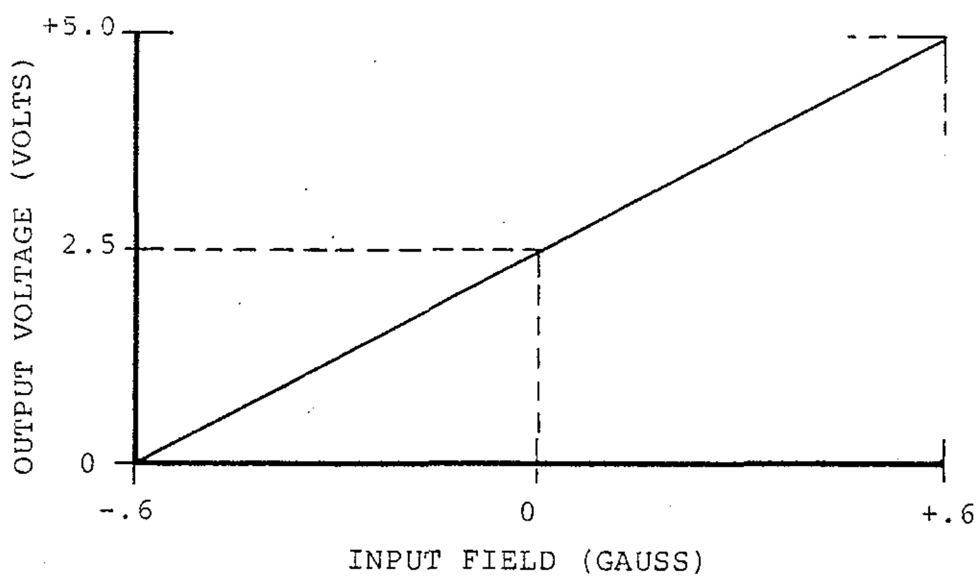


Fig. 3-7a. SENSOR TRANSFER CHARACTERISTIC

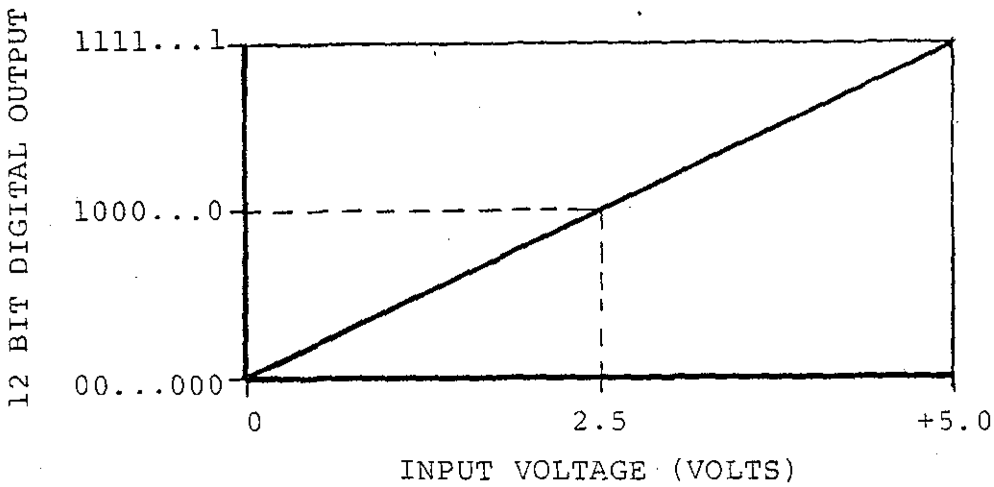


Fig. 3-7b. VDC TRANSFER CHARACTERISTICS

SIGN BIT ?

1	0
$x = x_0 + \sum_{i=1}^{11} x_i \cdot 2^i$ $(x = x_0 + x^*)$ Where: $x_0 = 0$ x^* = The 11 least significant bits of the binary number	$x = x_0$ + 2's complement of the 11 least significant bits x_0 = 1 (indicating negative quantity)

Fig. 3-8 CONVERSION OF DATA

The third function of the "SAMP" subroutine was to reverse the sign of the Hx and Hy data (to correct a test fixture problem) and to correct for sensor offsets. Although analog subsystem offsets are corrected by adjusting either the sample and hold module or the ADC, the independent sensors themselves have offsets[4]. Offset errors for the laboratory instrument were compensated by determining the offset correction term for each sensor (method described in detail in Chapter V) and then either adding or subtracting the term to the respective data during the sample subroutine. By characterizing the sensor errors[5], actual datum could be improved further during this step.

The final function of the "SAMP" subroutine was to correct for sensor orthogonality error (subroutine "ORTH"). Although the sensors were physically aligned and specified to have orthogonality characteristic [Ref. 3-34] less than ± 1 degree relative to the base coordinates, this nonorthogonality contributes appreciably to total system error (see error analysis in Chapter IV). The physical misalignment of the sensors was determined experimentally (Chapter V) and determined to be mainly a misalignment of sensor x in the x-y plane as illustrated in Fig. 3-9.

The actual data measured with the x axis sensor is then related to the true Hx and Hy values as

$$Hx^1 = Hx \cos \varepsilon - Hy \sin \varepsilon \ .$$

[4]With zero stimulus applied the sensors have a finite nonzero output. This error in the fluxgate magnetometer is a function of temperature, voltage and magnetic remanence in the sensor magnetics [Ref. 3-36].

[5]Sensor characteristics relating the temperature and power supply coefficients of offset error and nonlinearity can be derived empirically.

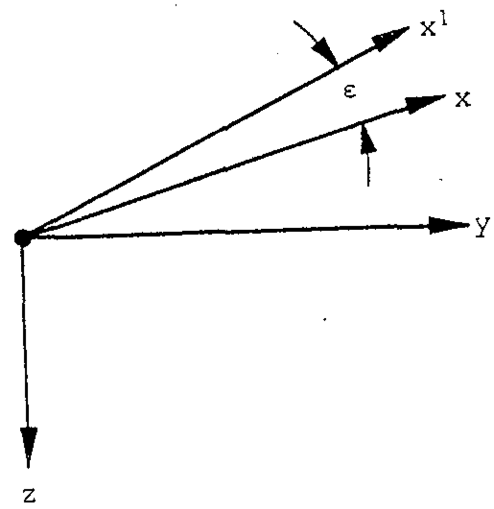


Fig. 3-9 X AXIS NONORTHOGONALITY

Using small angle approximations, we can solve for the desired true value of Hx

$$Hx^1 \simeq Hx - Hy \sin \varepsilon \qquad (3\text{-}1a)$$

$$Hx = Hx^1 + Hy \sin \varepsilon \qquad (3\text{-}1b)$$

By measuring ε (Chapter V) and storing the angle as a constant, the x axis data was then restored using equation 3-1b above in subroutine "ORTH".

2) Subroutines ROTX and ROTY

These subroutines compute arithmetic values for Hxh and Hyh of equation 2-11b using sign magnitude quantities and table lookup to determine solutions for the transcendental functions. Subroutines "SADD" and "SMPY" are nested and used to perform double precision add and multiply as required.

3) Subroutine HVEC

Following computation of the horizontal X and Y axis magnetic vector, the subroutine "MAIN" calls subroutine "HVEC" to compute the square of the horizontal vector. Vectors Hx and Hy are squared by calling subroutine "SQU" then added, yielding $H(HORIZONTAL)^2$.

4) Subroutine WICH

To compute heading, equation 2-1 (or a similar form) must be solved using the horizontal magnetic field vector and either the x or y axis horizontal field component. Although the square root operation implied in equation 2-1 could be implemented using a numerical technique [Ref. 3-37, 3-38], the computation time is decreased by using a table lookup method. Subroutine "WICH" (Fig. 3-10) compares the absolute magnitude of the two horizontal field vectors Hx and Hy to determine the relative heading of the aircraft[6] with respect to the north-south and east-west axes (Fig. 3-11).

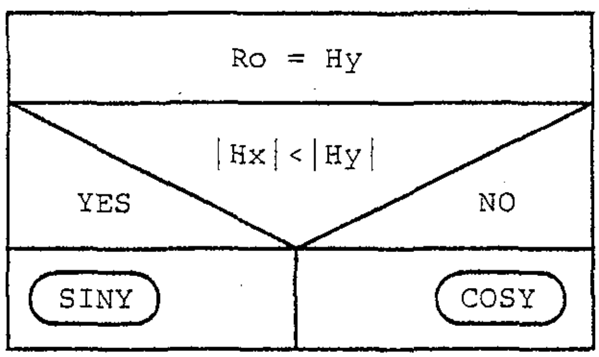


Fig. 3-10. SUBROUTINE "WICH"

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[6] If $|Hx|<|Hy|$, then an equation similar in form to 2-1a must be used.

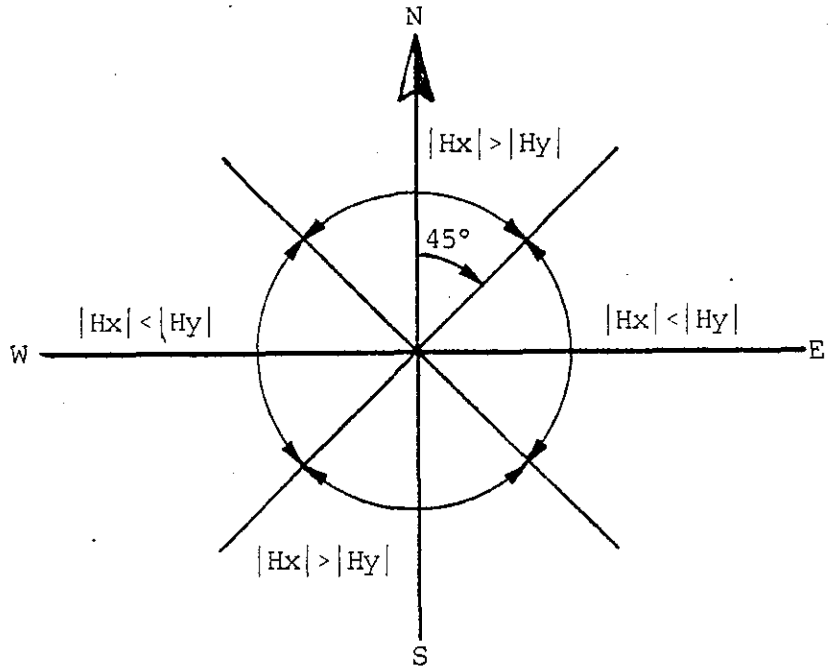

Fig. 3-11. MAGNITUDES OF Hx AND Hy RELATED AIRCRAFT HEADING

5) Subroutines COSY and SINY (Fig. 3-12, 3-13)

Depending on the relative absolute magnitudes of Hx and Hy, either "COSY" or "SINY" is called to compute aircraft heading. These subroutines invoke subroutine "DIVI" to form the quotient of the axis vector squared and the horizontal field vector squared (a double precision operation). Subroutine "ANGL" is then called to perform an associative table lookup operation using successive approximation and interpolation to complete the inverse cos squared operation. The double precision binary quantity is then converted to three digit binary coded decimal format (BCD) prior to computation of aircraft heading (subroutine "HDG").

The subroutine "SINY" of Fig. 3-12 includes a subtraction of the computed angle from 90 degrees following conversion to BCD format. This operation ensures that the angle passed to

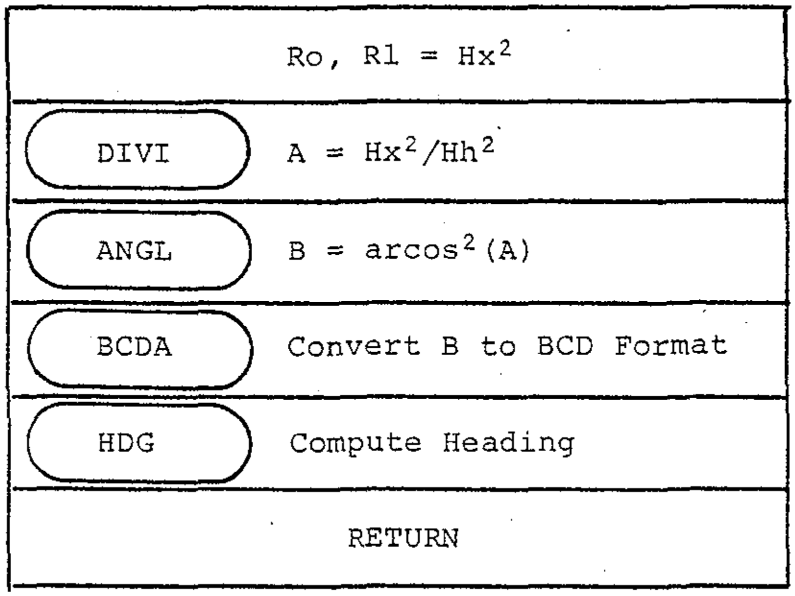

Fig. 3-12. SUBROUTINE "COSY"

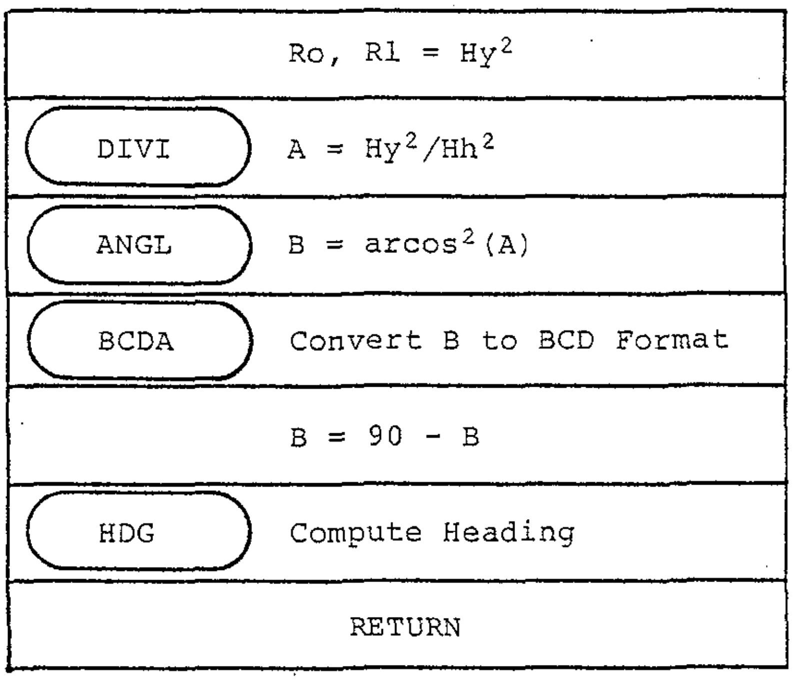

Fig. 3-13. SUBROUTINE "SINY"

the calling subroutine upon exiting either "SINY" or "COSY" is an aircraft heading angle relating sensor x to the north-south axis.

6) Subroutine HDG (Fig. 3-14)

The function of this subroutine is to compute aircraft heading having established the angle between the x axis sensor and the north-south geodetic axis. Determination of the actual heading is accomplished by comparing the signs of both the x and y axis horizontal vectors prior to computing heading (Fig. 3-15). It should be noted that all of the preceding computations leading to horizontal vector data were on sign magnitude quantities preserving the correct horizontal vector polarities[7].

3-5 CONCLUSIONS

This chapter has outlined the practical aspects of designing an instrument to evaluate both the heading algorithms and solid state magnetic indicator proposed in previous chapters. The chapter outlined a design approach that can be used to implement a microprocessor based instrument. In particular, the need to consider the total system hardware requirements while simultaneously considering the programming requirements was identified. Design proceeded by outlining a system block diagram (Fig. 3-1) with major subsystems considered. The instrument required a special purpose computer with an analog subsystem to sample and digitize five sensor signals. Timing and control of the analog subsystem plus digital processing of data was controlled by a microprocessor based central processing unit (CPU). Memory for permanent storage of

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[7]It is possible at certain attitudes to require sign reversals when computing horizontal vectors.

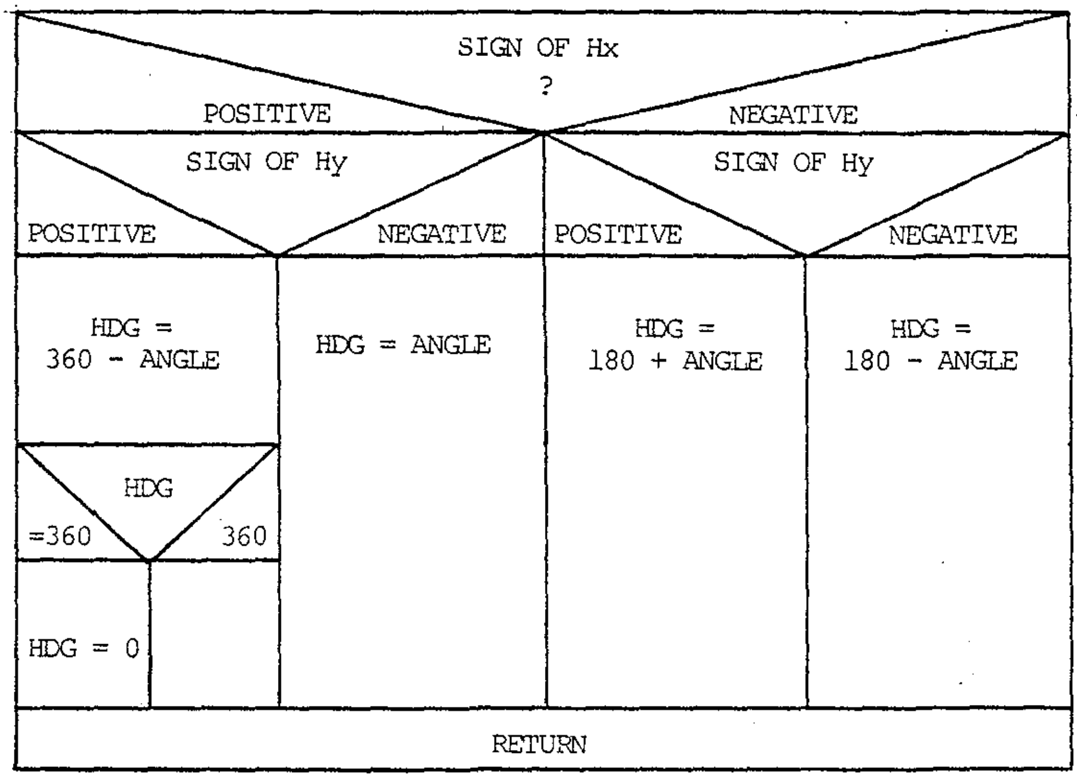

Fig. 3-14. SUBROUTINE "HDG"

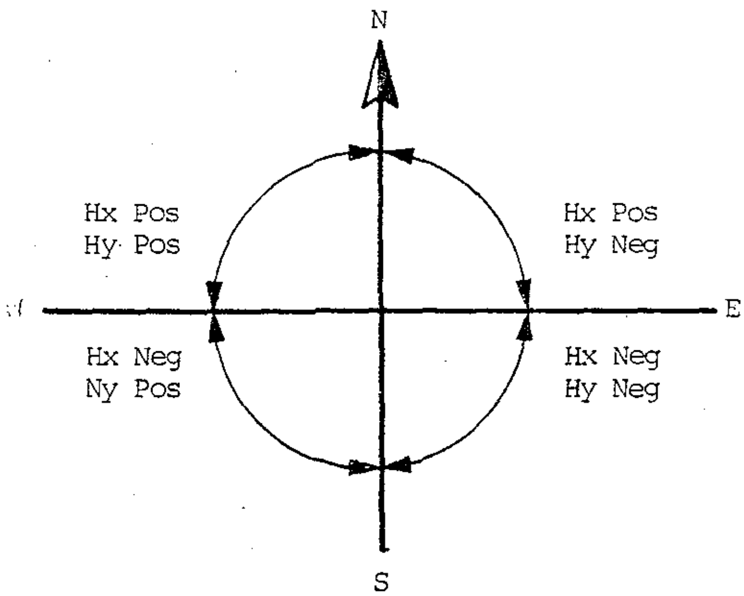

Fig. 3-15. POLARITIES OF HORIZONTAL VECTORS RELATED TO AIRCRAFT HEADING

instructions and temporary storage of data was implemented using memory chips organized on cards with 2048 byte capacity. The particular memory chips selected feature pin compatibility[8] with both read only and volatile random access versions. System inputs consisted of sensor signals from a three axis solid state fluxgate magnetometer plus two analog signals simulating gyroscope outputs. System outputs consist of visual seven segment readout displaying computed heading. In addition, an RS-232 teletype interface was provided to facilitate system development and experimentation.

By identifying the total system in block diagram form at the very beginning, the role and requirements of each subsystem as well as the supporting software were identified. The design then evolved on a modular basis with each subsystem and its supporting program developed in parallel. In this manner pin assignments for input/output ports and critical timing requirements that involved both hardware and software consideration were handled efficiently. By outlining the program requirements in flow chart form (analagous to the block diagram of the hardware subsystem), subroutines were identified facilitating a modular program development. Where possible, subroutines were shared in a nested manner avoiding replication of programming and waste of memory.

Details of error analysis and calculation of overall system throughput rate were deferred to Chapter IV. It was pointed out however, that errors induced by imprecision of data plus truncation and roundoff during processing of the algorithm were to be considered early in the design phase. These data were required to select the sensors and the analog to digital

---

[8]Memory integrated circuit (IC) devices of both types can be used in the same mechanical sockets with actual chip type being used transparent to the remainder of the system.

converter as well as to design the supportive software for the analog subsystem. In addition, the data precision requirements were necessary prior to programming the algorithm[9].

By incorporating a microprocessor as the main CPU element, considerable sophistication in both control and computing performance was achieved. The overall system was designed relatively quickly, provided a convenient laboratory instrument for evaluation of the proposed algorithms and featured inherent flexibility.

[9]Some of the subroutines required double precision manipulations to maintain overall system accuracy.

CHAPTER IV

HEADING INSTRUMENT ERROR ANALYSIS

4-1 INTRODUCTION

The heading instrument designed to evaluate the heading and solid state remote magnetic indicator algorithms is prone to error from many sources. These errors will accumulate and degrade the accuracy of aircraft heading or yaw angle computations. This chapter addresses the various error sources to determine their relative magnitudes and effects on the overall computation.

Prior to beginning the hardware design of the microprocessor based instrument many of these potential error sources were considered. Their effects were considered in establishing parameters such as word lengths, A/D converter precision, computation speeds, sampling rates, magnetometer sensor accuracies, system noise tolerance, etc. As the design of the microprocessor based system evolved, the error analysis refined. Ultimately, important limitations in instrument design and operation were identified by combined error analysis and empirical data. By carefully analyzing the source and extent of the limiting parameters (such as sensor offset and non-orthogonality), the magnitude of errors unique to this laboratory sensor array were identified. Specialized software was then added (with empiracally derived constants) to correct for the otherwise limiting sensor irregularities improving the total system performance.

In this manner, it is apparent that error analysis is an integral part of instrument design. Not only are important parameters identified early in the design cycle (prior to system block diagram development), but shortcomings in conventional sensors can be improved by judicial application of error correcting algorithms. In this case, data constants were determined after the final instrument became operational. The sensor peculiarities were analyzed empirically using the instrument itself.

The chapter begins by first identifying and carefully analyzing potential error sources in the sensors. This analysis is followed by a similar consideration of errors originating in the analog subsystem. Processing errors that originate due to the finite word length and precision of the microprocessor along with the effects of simplifications made to the algorithms are finally analyzed. The chapter then concludes with a summary of measurement errors, a sample error analysis, a comparison of predicted to measured error and a summary.

4-2 SENSOR ERRORS

The heading computation algorithm employing the remote magnetic indicator (Chapter II) is prone to error proportional to both fluxgate magnetometer sensor and gyroscope measurement errors. Errors inherent in the fluxgate magnetometer are summarized on the data sheet [Ref. 3-34]. Since the experimentation employed simulated gyroscope sensors with voltage levels accurately represented, the analysis of sensor errors will assume ideal gyroscope sensors to predict experimental data.

A) Sensor Offset Error

Magnetometer sensors exhibit error caused by both electronic and magnetic phenomena. Errors in the Develco sensors were outlined by Workentine [Ref. 4-1]. These offset errors are induced in the Develco sensors by both electronic offset voltages and currents in the respective sensor electronics and by residual magnetic fields in the magnetic mass of the sensor assemblies. Although the physical and electronic design attempts to reduce offset error, a finite non-zero output can exist when a zero input is applied.

Offset error for each sensor in the Develco model 9200C three axis magnetometer assembly is specified [Ref. 3-34] as "Zero Field Bias +2.5 Volts ±1.0%". This offset translates into a worst case maximum error voltage of

$$EOFFSET = \pm(2.5V \times 0.01) = \pm 25mV$$

Since the offset error is sensor dependent, correction cannot be made at a single physical point (as for analog subsystem offsets described in Section 4-3). Corrections can however be made to the measured data by simply adding or subtracting a constant equal to the offset magnitude following each data measurement[1].

Offset values for each sensor used in the experiment were obtained by rotating the sensor into alignment with earth's magnetic field vector to measure both positive and negative maximum values. The difference in magnetic measurement (assuming negligible analog subsystem error) is related to system offset error composed of sensor electronic and sensor plus test fixture induced magnetic offset error. The actual offset error can be calculated using these two measurements

[1]Offset corrections were made in the sample subroutine "SAMP" illustrated in Fig. 3-6a.

$$|E_{max}| = E_f + E_o$$
$$|E_{min}| = E_f - E_o$$
$$|E_{max}| - |E_{min}| = (E_f + E_o) - (E_f - E_o) = 2 E_o$$
$$E_o = (1/2) \quad (|E_{max}| - |E_{min}|)$$

where

Emax = The maximum positive voltage recorded when the sensor aligns with earth's field vector.

Emin = The maximum negative voltage recorded when the sensor aligns 180° with earth's field vector.

Ef = The magnitude of earth's magnetic vector represented in volts.

Eo = The sensor offset voltage due to both electronic and magnetic phenomena

Data recorded during x, y and z axis offset measurements as described above are recorded in Table 4-1. Since the offset error is a function of sensor magnetic permeability, the actual offset value will vary with time depending on induced magnetic fields[2].

Final offset correction values were determined by rotating two sensors in the horizontal plane around the third vertical axis and measuring offsets in two sensors at a time. Recorded data for each sensor was previously corrected for orthogonality error by the sample subroutine "SAMP" (discussion

[2]For example, magnetized screwdrivers or other tools used near the sensor will alter the residual magnetic field.

SENSOR AXIS	DATA RECORDED (HEXADECIMAL) |Emax|	|Emin|	OFFSET (HEXADECIMAL PLUS SIGN)
X	628	E78	+40
Y	640	E5D	-15
Z	637	E68	-25

Table 4-1 OFFSET DATA DERIVED BY MEASURING EARTH'S FIELD

of this correction follows in Section 4-2B). Data recorded in this manner appears in Tables 4-2 and 4-3. Final correction terms for correcting sensor offset error were calculated using these data. Offset terms to be added or subtracted from respective data channels are tabulated in Table 4-4.

By correcting system offset errors in this manner, the effective error contribution can be reduced appreciably (see final data discussion Chapter V). For a flight instrument, sensor offset characteristics as a function of temperature variation and supply voltage can be derived empirically and appropriate offset corrections made by computing the value of the correction term variable. Magnetically induced offsets can be reduced by degaussing the sensor assembly periodically.

B) Axis Alignment Errors

The error specification of [Ref. 3-34] indicates that the maximum axis alignment error is ± 1 degree relative to base referenced coordinates. This error results in sensor directional uncertainty as illustrated in Fig. 4-1. Each sensor is located within a right circular cone with axis along the true sensor axis and vertex at the common sensor origin. Although this alignment uncertainty contributes no error in determining the total magnetic vector

$$\overline{H} = (\overline{H}x^2 + \overline{H}y^2 + \overline{H}z^2)^{\frac{1}{2}},$$

there is considerable uncertainty in attempting to resolve the true magnetic field component along any axis of the reference coordinate system. This alignment uncertainty of magnetic sensors limits system performance of conventional field direction measuring apparatus [Ref. 4-1].

Protractor Heading Measurement (Degrees)	Data Measured Hx	Data Measured Hy (Units)	Protractor Heading Measurement (Degrees)	Data Measured Hx	Data Measured Hy (Units)	Error Due To Offset X	Error Due To Offset Y (Units)
0	8	-759	180	83	720	75	-39
345	199	-743	165	-112	704	87	-39
330	383	-677	150	-292	638	91	-39
315	540	-569	135	-448	527	92	-42
300	666	-421	120	-573	381	93	-40
285	749	-249	105	-658	206	91	-43
270	783	- 62	90	-696	14	87	-48
255	770	132	75	-681	-179	89	-47
240	706	314	60	-616	-362	90	-48
225	597	473	45	-504	-521	93	-48
210	449	601	30	-359	-645	90	-44
195	273	687					
180	(DATA UNAVAILABLE DUE TO TEXT FIXTURE LIMITATION)						
165							
			TOTAL OFFSETS			978	477
			AVERAGE OFFSETS			88.9	43.4

Table 4-2 X AND Y AXIS ERROR MEASURED BY ROTATING X, Y AROUND Z IN THE HORIZONTAL PLANE

Protractor Heading Measurement (Degrees)	Hz Data Measured (Units)	Protractor Heading Measurement (Degrees)	Hz Data Measured (Units)	Offset Error (Units)
0.5	0	180.5	-56	-56
315.5	-527	135.5	469	-58
270.5	-763	90.5	707	-56
225.5	-571	45.5	513	-58
		TOTAL OFFSET		-228
		AVERAGE OFFSET		-57

Table 4-3 Z AXIS OFFSET ERROR MEASURED BY ROTATING THE Z AXIS AROUND THE VERTICAL X AXIS

Sensor Axis	Total Average Offset (Units)	Required Correction	Amount of Correction Decimal	Binary	Hex
X	88.9	Subtraction	45	00101101	02D0
Y	43.4	Addition	22	00010110	0160
Z	47.0	Addition	29	00011101	01D0

Table 4-4 OFFSET CORRECTION VALUES

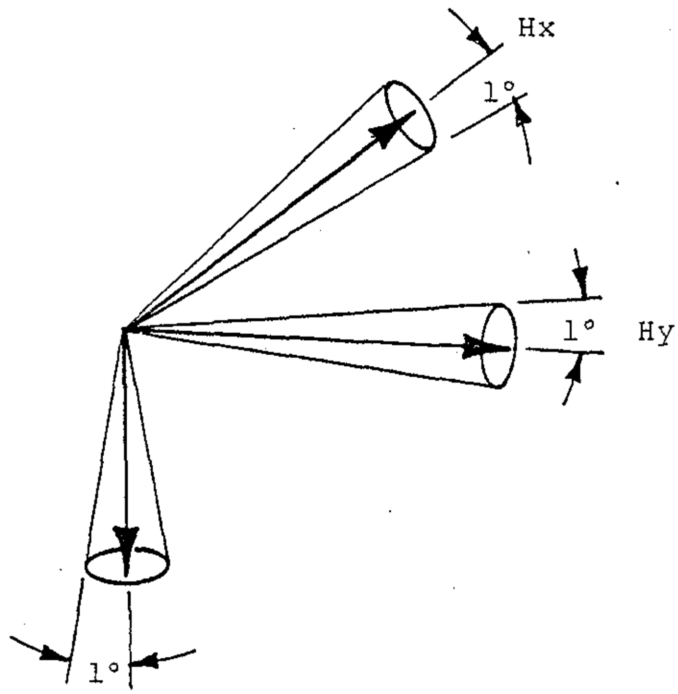

Fig. 4-1 SENSOR ALIGNMENT UNCERTAINTY

Although this error source can be reduced by physically aligning the sensors more accurately during assembly, cost of the sensors increases. Ultimately, directionality of the magnetic sensors becomes a function of the physical sensor itself and more accurate sensors are required as pointed out by Gise [Ref. 4-2]. A heading system that tolerates sensor misalignment is therefore a very desirable alternative to requiring precise alignment or more elaborate sensors.

During assembly of the Develco fluxgate magnetometer sensor array, sensor misalignment is determined by using earth's magnetic field and a precision mechanical rotation assembly. A sensor (assume the X axis) is aligned with earth's magnetic vector by positioning the sensor to maximize electrical output[3]. One of the other sensors (assume the y axis) is aligned with the rotation axis of the precision calibration assembly (Fig. 4-2) and perpendicular to the first by rotating the sensor array around the second sensor axis (y axis in this case) and adjusting its relative position until a null output is achieved at all rotation angles. Mechanical orthogonality of the sensors is then limited only by the mechanical imprecision of the calibration device (orthogonality within ±0.01 degrees can be easily achieved in the calibration tool) and by the directional characteristics of the physical sensors.

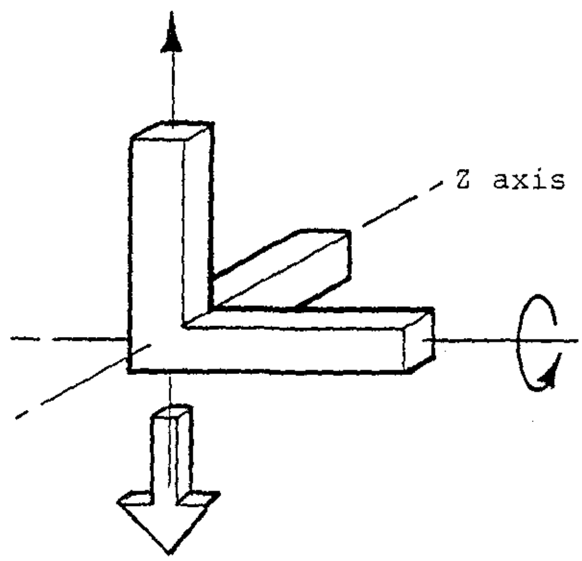

Fig. 4-2 MECHANICAL ORIENTATION OF THE MAGNETOMETER SENSORS DURING CALIBRATION

In addition to functioning as an alignment apparatus, the calibration device described above provides a convenient means

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[3]By maximizing or nulling a measurement, the mechanical positioning is a function of only the field and the resolution of the voltage measuring device obviating errors due to physical position measurement.

to characterize sensor assemblies after final assembly adjustments are made. Any misalignment of the second sensor relative to the first results in a coning of the second sensor around the rotation axis[4] with a sinusoidal output voltage that is a function of total earth's magnetic field and axis alignment error. The peak to peak voltage resulting from sensor coning is recorded during the final alignment test and made available to sensor purchasers. Coning voltages developed for the sensor assembly used with this experiment were obtained from Develco [Ref. 4-3] and are recorded in Table 4-5. Sensor misalignment for each axis can be derived using additional data provided by Develco along with additional empirical data derived by experimentation.

The total ambient magnetic field at the Develco laboratory is measured using the three sensors (applying equation 4-1) and is supplied as digital data. In our case, the total field measured was 1573 units or

$$\frac{1573 \text{ units}}{2048 \text{ units Full Scale (F.S.)}} \times 60{,}000 \text{ gamma F.S.} = 46{,}084 \text{ gamma } (\gamma)$$

$$\text{Sensitivity of the sensor} = \frac{2.5 \text{ Volts F.S.}}{60{,}000\ \gamma\text{F.S.}} = 42\ \mu\text{Volts}/\gamma$$

Considering the X axis sensor, coning resulted in a signal of 38 mV peak to peak (or 19mV peak). Misalignment of the X axis sensor from the Y-Z plane can then be calculated as

[4]Assume that the first axis is initially adjusted for maximum output to align it with earth's field and the rotation axis is perpendicular to the field.

SENSOR ASSEMBLY NO. S/N 1043-013

Rotation Axis	Coning Voltage (Peak-Peak mV)	Orthogonality Error (Degrees)
X	38	0.57
Y	8	≈0
Z	51	0.76

Table 4-5 MAGNETOMETER ORTHOGONALITY MEASUREMENTS

Peak Signal = 19 mV or 456 gamma angular misalignment

$$\varepsilon_x = \sin^{-1} \frac{456}{46{,}084}$$

$$\varepsilon_x = 0.57 \text{ degrees}$$

Similarly, the Y and Z axis have misalignment errors of $\varepsilon y \simeq 0$ and $\varepsilon z = 0.76$ degrees with respect to the X-Z and X-Y planes respectively (sensor orthogonality errors are tabulated in Table 4-5).

Having established that sensor orthogonality errors exist, the remaining task is to identify the direction that the sensor axis points relative to the other two sensor axes. Since the Y axis has relatively little orthogonality error, it will be assumed to be perpendicular to the X-Z plane. In addition, since the Hz data enters into the algorithm in a second order manner relative to the Hx and Hy measured data, correction and characterization of the Hx sensor was considered to be of primary concern. Orientation of the X axis sensor relative to the Y and Z axes was determined empirically.

Angular position of the X axis sensor can be described using the error angles εxy and εxz as delineated in Fig. 4-3. Characterization of sensor orthogonality error in terms of these two angles would enable algorithmic corrections of measured data.

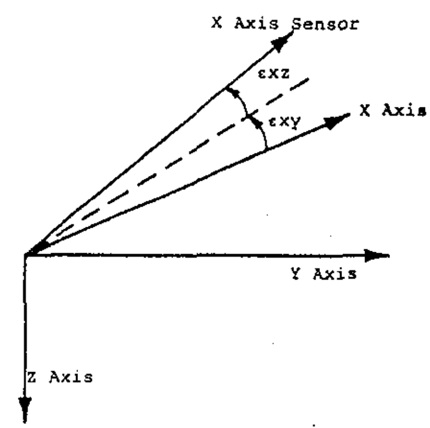

Fig. 4-3 X AXIS SENSOR ORIENTATION

1) Empirical Determination of εxz

The angle εxz (angle between the x axis sensor and the z axis of the geodetic coordinate system) was determined in several steps using the test apparatus described in Chapter V.

i) The x and y sensors were oriented in the horizontal plane with the z axis sensor vertical downward.

ii) The x and y sensors were rotated around the z axis with magnetic data measurements (corrected for sensor offset error as described in Section 4-2A) recorded in Table 4-6 for incremental rotation angles.

iii) The total horizontal field at each angular position was calculated

$$Hht = (Hx^2 + Hy^2)^{\frac{1}{2}}.$$

iv) Average horizontal field Hav was computed by averaging the results of iii) above.

v) The horizontal field deviation Hd was computed for each angular position; tabulated in Table 4-6 and plotted on Fig. 4-4.

$$Hd = (Hav - Hht)$$

The horizontal field deviation or error (as shown on Fig. 4-4) was now examined. An angular error εxz should cause the horizontal field error curve to peak at 90 and 180 degrees. Since this obviously was not the case, it was concluded that major error in x axis orthogonality was due to the component εxy.

Physical* Heading (Degrees)	Displayed** Heading (Degrees)	Measured Data (Units) Hx	Hy	Total Computed Horizontal Field (Hht) (Units)	Hd (Hav-Hht) (Units)
355	90	12	-727	727	-3
335	70	258	-682	729	-1
315	50	477	-555	732	2
295	30	637	-362	733	3
275	10	720	-127	731	1
255	350	718	127	729	-1
235	330	628	363	725	-5
215	310	464	569	734	4
195	290	242	687	728	-2
175	270	-10	731	731	1
155	250	-258	686	733	3
135	230	-476	557	733	3
115	210	-635	362	731	1
95	190	-721	123	731	1
75	170	-718	-133	730	0
55	150	-628	-371	729	-1
35	130	-460	-565	729	-1

Total Hht = 12415

Average (Hav) = 730

*Measured using a protractor on the test apparatus.
**Computed and displayed digitally by the instrument.

Table 4-6 MEASUREMENT OF HORIZONTAL FIELD

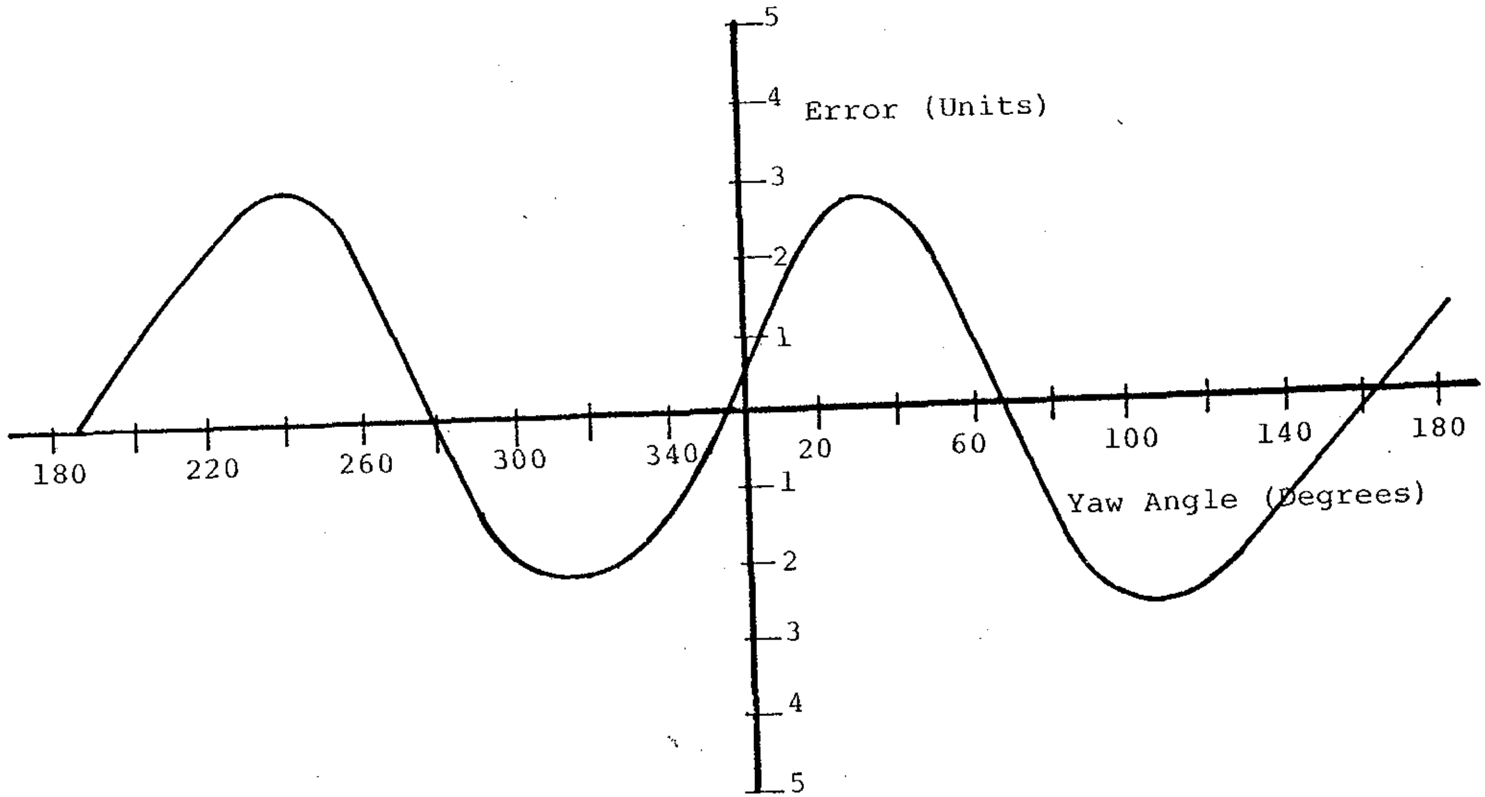

Fig. 4-4 DEVIATIONS OF THE HORIZONTAL FIELD MEASUREMENT FROM THE MEAN

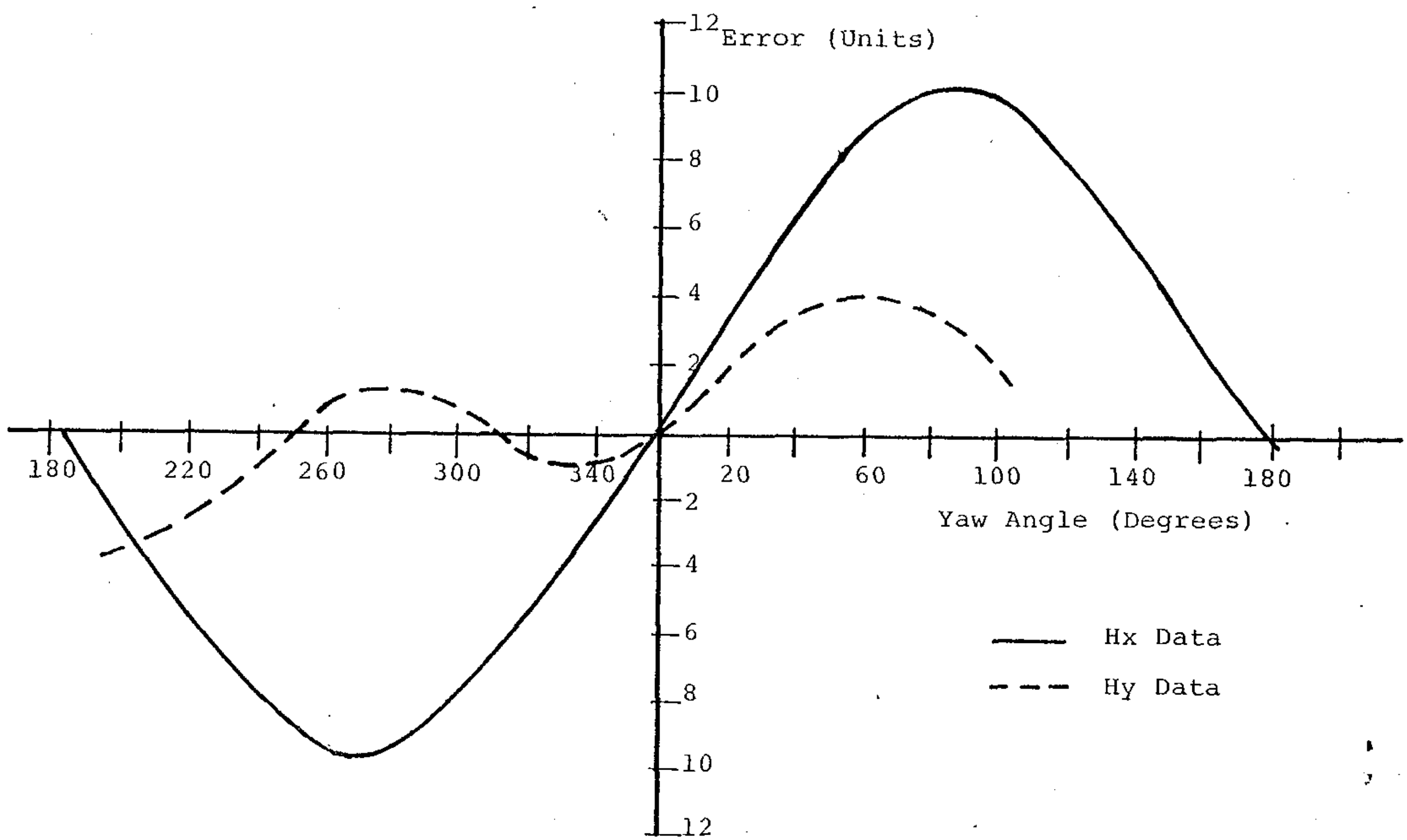

Fig. 4-5 DEVIATION OF Hx AND Hy DATA FROM THE COMPUTED FIELD COMPONENTS AS A FUNCTION OF YAW

2) Empirical Determination of εxy

The angle εxy representing x axis sensor misalignment relative to axis y was measured as follows:

Steps i) and ii) above were repeated with the exception that the calculated values for Hx and Hy (Hxc and Hyc respectively) were recorded with measured Hx and Hy data (Hxm and Hym respectively) in Table 4-7. The calculated values were obtained by assuming that the angle εxz as determined above was negligible and that the y axis sensor was perpendicular to the x-z plane. With these assumptions, we note that at the heading of zero degrees (extrapolated between display of 10 and 350 degrees of Table 4-7 and Fig. 4-5), there is no error in yaw due to either Hx or Hy. By physically rotating the sensors in fixed intervals from yaw = 0 degrees and noting that the horizontal field Hh = 730 units, we can then compute expected Hx and Hy data at respective yaw orientations.

Physical orientation of the x axis sensor is easily determined by considering orientation at the maximum error excursions. These observations are illustrated in Fig. 4-6. We note that the only possible orientation of the x axis sensor satisfying the data in Fig. 4-5 is that of Fig. 4-6.

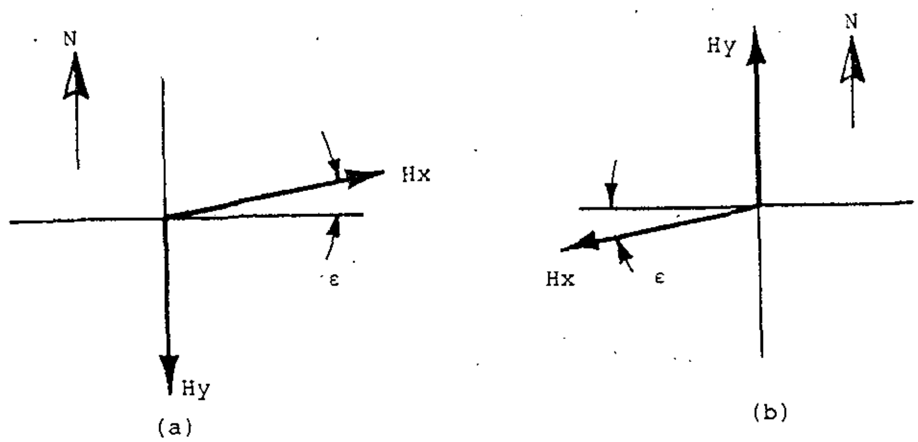

Fig. 4-6 (a) SENSORS ORIENTED AT YAW = +90 degrees
(b) SENSORS ORIENTED AT YAW = +270 degrees

Physical* Heading (Degrees)	Displayed** Heading (Degrees)	Measured Data (Units)		Computed Data (Units		Deviation (Units)	
		Hxm	Hym	Hxc	Hyc	(Hxm-Hxc)	(Hym-Hyc)
355	90	12	-727	0	-730	+12	+ 3
335	70	258	-682	250	-686	+ 8	+ 4
315	50	477	-555	469	-559	+ 8	+ 4
295	30	637	-362	632	-365	+ 5	+ 3
275	10	720	-127	719	-127	+ 1	0
255	350	718	127	719	+127	- 1	0
235	330	628	363	632	365	- 4	- 2
215	310	464	569	469	559	- 5	10
195	290	242	687	250	686	- 8	1
175	270	-10	731	0	730	-10	1
155	250	-258	686	-250	686	- 8	0
135	230	-476	557	-469	559	- 7	- 2
115	210	-635	362	-632	365	- 3	- 3
95	190	-721	123	-719	+127	- 2	- 4
75	170	-718	-133	-719	-127	1	- 6
55	150	-628	-371	-632	-365	4	- 6
35	130	-460	-565	-489	-559	9	- 6

* Measured using a protractor on the test apparatus.
** Computed and displayed digitally by the instrument.

Table 4-7 MEASURED AND COMPUTED Hx AND Hy DATA IN THE HORIZONTAL PLANE

Magnitude of the angle ϵxy can be computed as follows using data from Fig. 4-5

Max. delta from Fig. 4-5 = 10 units
Average horizontal field = 730 units

$$\epsilon xy \text{ max} = \text{Sin}^{-1} \frac{10}{730}$$

$$= 0.79 \text{ degrees}$$

We note that the angle of 0.79 degrees is approximately the same as determined by Develco during manufacture of the sensors (Table 4-5). The added error is due to test set inaccuracy.

C) Fluxgate Sensor Noise Induced Error

The analog output from the fluxgate sensors can exhibit an error due to signal uncertainty resulting from noise. Although the data sheet [Ref. 3-34] indicates that 5mV peak to peak of ripple can exist on the output, the frequency content centers in the 550 kHz range (driver frequency of the fluxgate magnetometer) and no appreciable ripple[5] exists below 60 Hz (especially when the sensor output is filtered prior to data sampling). The noise specification of less than 1 gamma peak to peak in the 1 Hz bandwidth region is also negligible. In summary, no appreciable error due to noise on the magnetometer signal lines is evident.

D) Magnetometer Gain Error

The magnetometer is specified to have gain (sensitivity) of 2.5 Volts/600 milligauss, $\pm 1\%$ which translates into a maximum signal uncertainty of

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[5]Verbally confirmed by Workentine of Develco [Ref. 4-1].

$$\pm(2.5V \times 0.01) = \pm 25 \text{ mV}.$$

This represents a sensor transfer function of 4.16 Volts/gauss or 0.24 gauss per volt. The uncertainty then can be expressed as

$$\pm(0.24 \text{ gauss} \times 0.01) = \pm 2.4 \text{ milligauss}$$
$$= \pm(2.4 \times 10^2) \text{ gamma}$$

Since this error is not corrected in the laboratory instrument it will be considered in total in the final error analysis. It is worth noting however, that should the magnetometer gain uncertainty be characterized, gain corrections for each sensor could be made during computation by the computer. In addition the error term is proportional to actual signal level applied.

E) Magnetometer Linearity Error

D.C. linearity of the magnetometer is specified to be $\pm 0.5\%$ of signal level. This uncertainty at full scale can be expressed as $\pm(2.5 \text{ Volts} \times 0.005) = \pm 12.5\text{mV}$. Alternately, linearity error can cause a signal uncertainty of ± 1.2 milligauss or $\pm(1.2 \times 10^2)$ gamma. Linearity error is also not corrected during computation and is considered in the final error analysis. By simply characterizing and correcting the linearity characteristics of each sensor, considerable improvement in system accuracy could be achieved.

4-3 ANALOG SUBSYSTEM ERROR ANALYSIS

The analog subsystem of the instrument is outlined in block diagram form in Fig. 4-7. This subsystem accepts analog signals from magnetometer and gyroscope transducers, performs a time division multiplexing between the signals and digitizes

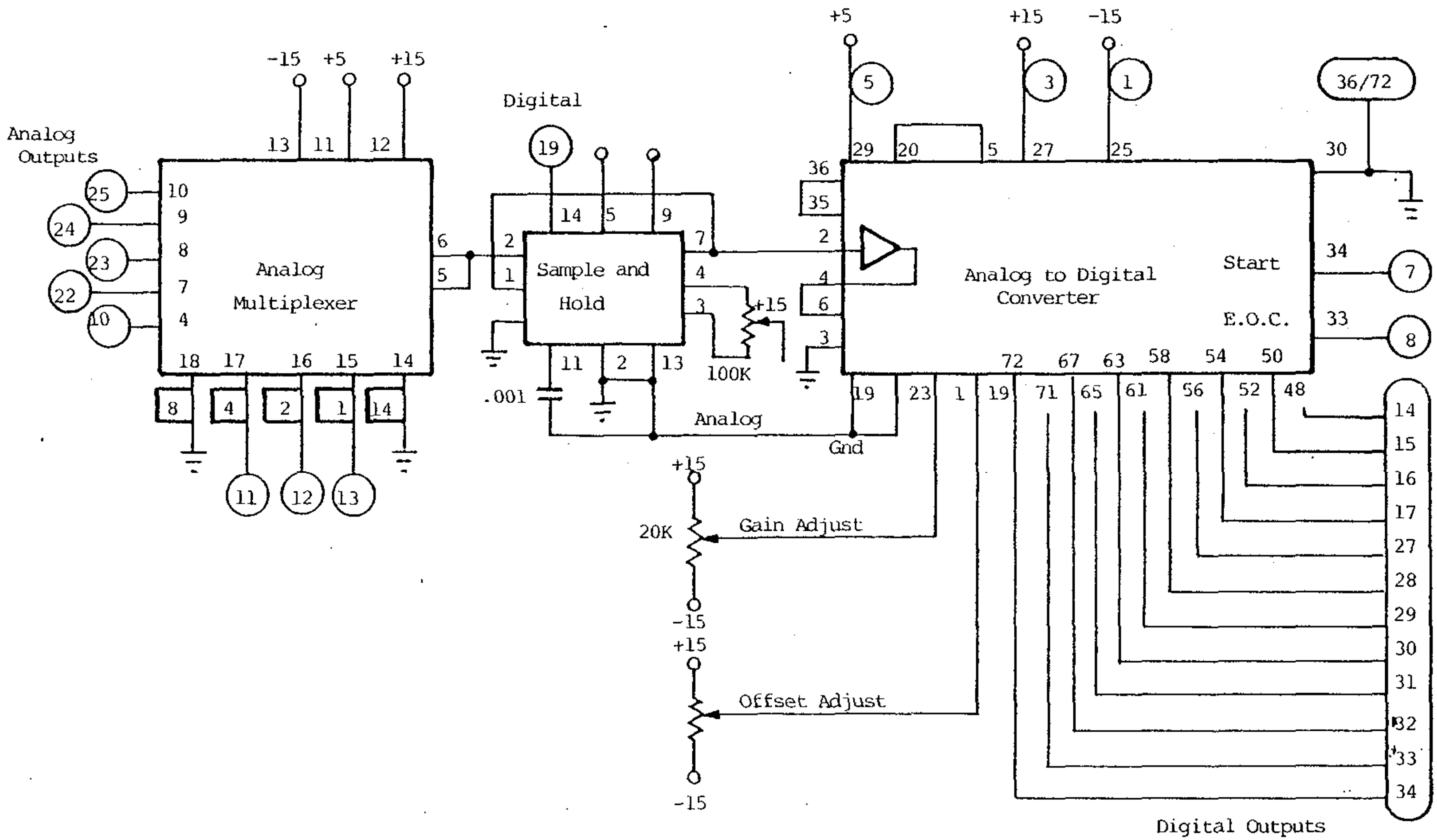


Fig. 4-7 THE ANALOG SUBSYSTEM

the respective signals prior to subsequent processing by the computer. During this data acquisition and conversion process, errors are introduced into each of the signals. This section addresses the potential error sources and computes the respective error contributions to be expected during operation of the instrument.

Although the multiplexer and sample and hold blocks of Fig. 4-7 could be eliminated (eliminating possible error sources) by digitizing each signal with a unique analog to digital converter, it can be shown that such a system would be expensive and difficult to implement. The analog to digital converter (A/D) quantizes an analog signal in a finite amount of time. Speed of conversion is predicted in a finite amount of time by both the resolution of the converter and the frequency of the signal to be converted. Time required to perform a conversion is generally called the "aperature time".

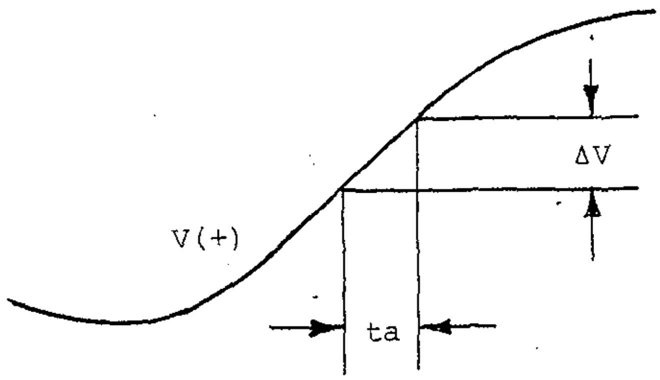


Fig. 4-8 APERATURE TIME AND AMPLITUDE UNCERTAINTY

As illustrated in Fig. 4-8, aperature time and amplitude uncertainty are related by the time rate of change of the analog signal. For the particular case of a sinusoidal signal to be converted, the maximum rate of change occurs at the zero crossing of the waveform and the amplitude change is:

$$\Delta V = \frac{d}{dt} (V \text{ Sin } wt)_{t = 0} \times ta \qquad (4\text{-}1a)$$

$$\Delta V = V \; w \; ta \qquad (4\text{-}1b)$$

$$\text{giving } \frac{\Delta V}{V} = w \; ta = 2 \; \pi f \; ta. \qquad (4\text{-}2)$$

From this result we can determine the aperature time required to digitize a 30 Hz signal to 12 bits resolution (a resolution of 1 part in 4096 or 0.0244%).

$$ta = \frac{V}{V} \times \frac{1}{2 \; f} = \frac{.000244}{6.28 \times 30} = 1.3 \times 10^{-6}$$

This result indicates that to remain within 1 bit of resolution (0.0244%) we require an aperature time of 1.3 microseconds to process analog signals varying at a rate of 30 Hertz. It can be seen that the system would require fast A/D converters plus extremely fast computational capability to accommodate this configuration of sensors and analog subsystem. By using multiplexing and sample and hold circuitry we can however reduce the number of A/D converters required to one and alleviate the aperature and processing requirements imposed above.

The operation of sampling to be used by the instrument is illustrated in Fig. 4-9 which shows an analog signal and a train of sampling pulses. The pulses are provided by the central processing unit. A switch connects the analog signal for a very short period of time to the hold circuitry charging a capacitor and storing the sampled voltage until the next sample is required. This type of sampler is called sample and hold.

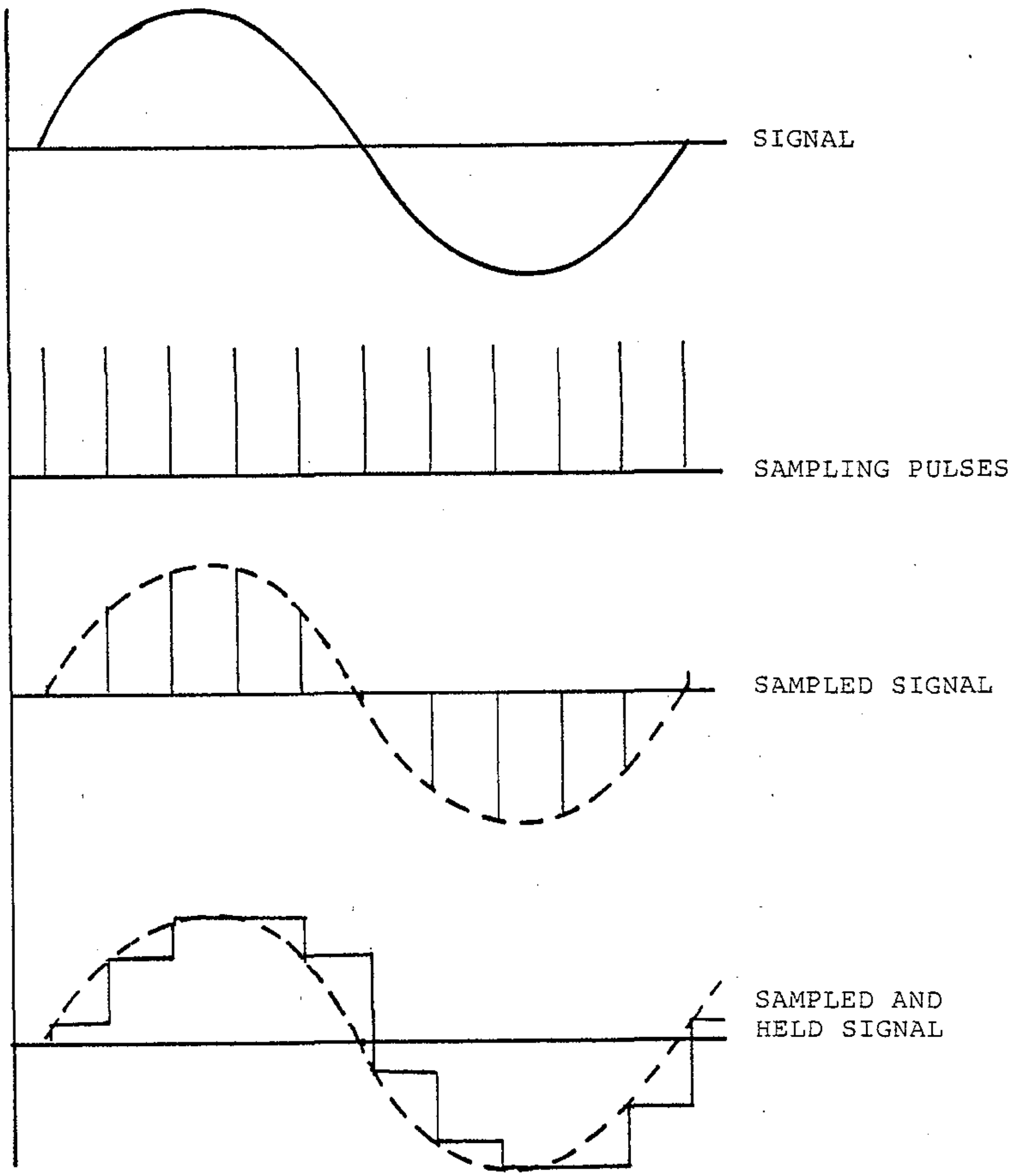


Fig. 4-9 SIGNAL SAMPLING PROCESS

A) Sampling Rate Errors

The process of uniformly sampling a function of continuous time can yield a significant source of error if the sampling period T is selected too large [Ref. 4-4, 4-5]. This error can be illustrated by considering an analog signal xa(t) that has the Fourier representation [Ref. 4-6]

$$xa(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Xa(j\Omega)e^{j\Omega t}d\Omega \qquad (4\text{-}3a)$$

$$Xa(j\Omega) = \int_{-\infty}^{\infty} xa(t)e^{-j\Omega t}dt \qquad (4\text{-}3b)$$

The sequence x(n) with values x(n) = xa(nT) is said to be derived from xa(t) by periodic sampling and T is the sampling period. The reciprocal of T is called the sampling frequency or sampling rate. In order to determine the sense in which x(n) represents the original signal xa(t), it is convenient to relate $Xa(j\Omega)$, the continuous-time Fourier transform of xa(t), to $X(e^{j\Omega})$, the discrete-time Fourier transform of the sequence x(n). From (4-3a) we note that

$$x(n) = xa(nt) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Xa(j\Omega)e^{j\Omega nt}d\Omega \qquad (4\text{-}4)$$

From the discrete-time Fourier transform we also obtain the representation [Ref. 4-4]

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega \qquad (4\text{-}5)$$

To relate the equations (4-4) and (4-5) we can express (4-4) as a sum of integrals over intervals of length $2\pi/T$, as in

$$x(n) = \frac{1}{2\pi}\sum_{r=-\infty}^{\infty}\int_{(2r-1)\pi/T}^{(2r+1)\pi/T} Xa(j\Omega)e^{j\Omega nT}d\Omega \qquad (4\text{-}6)$$

ıch term in the sum can be reduced to an integral over the ınge $-\pi/T$ to $+\pi/T$ by a change of variables to obtain

$$x(n) = \frac{1}{2\pi} \sum_{r=-\infty}^{\infty} \int_{-\pi/T}^{\pi/T} Xa\left[j(\Omega+\frac{2\pi r}{T})\right] e^{j(\Omega+\frac{2\pi r}{T})nT} d\Omega \tag{4-7a}$$

$$x(n) = \frac{1}{2\pi} \sum_{r=-\infty}^{\infty} \int_{-\pi/T}^{\pi/T} Xa(j\Omega+j\frac{2\pi r}{T}) e^{j\Omega nT} e^{j2\pi rn} d\Omega \tag{4-7b}$$

: we now change the order of integration and summation and ›te that $e^{j2\pi rn} = 1$ for all integer values of r and n, we obıin

$$x(n) = \frac{1}{2\pi} \int_{-\pi/T}^{\pi/T} \left[\sum_{r=-\infty}^{\infty} Xa(j\Omega+j\frac{2\pi r}{T})\right] e^{j\Omega nT} d\Omega \tag{4-8}$$

' substituting $\Omega = \omega/T$ we get

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[\frac{1}{T} \sum_{r=-\infty}^{\infty} Xa(\frac{j\omega}{T}+j\frac{2\pi r}{T})\right] e^{j\omega n} d\omega \tag{4-9}$$

ıich is identical in form to equation (4-5). We can therefore ıke the identification (equating like terms of (4-5) and (4-9)

$$X(e^{j\omega}) = \frac{1}{T} \sum_{r=-\infty}^{\infty} Xa(\frac{j\omega}{T}+j\frac{2\pi r}{T}) \tag{4-10}$$

; can also express (4-10) in terms of the analog frequency varıble Ω (where $\Omega = \omega/T$) as

$$X(e^{j\omega T}) = \frac{1}{T} \sum_{r=-\infty}^{\infty} Xa(j\Omega+j\frac{2\pi r}{T}) \tag{4-11}$$

The last two equations clearly reveal the relationship between the continuous-time Fourier transform and the Fourier transform of a sequence derived by sampling. For example, if Xa $(j\Omega)$ is as depicted in Fig. 4-10a then $X(e^{j\omega})$ will be as shown in Fig. 4-10b when the sampling period T is too long and as shown in Fig. 4-10C if T is short enough.

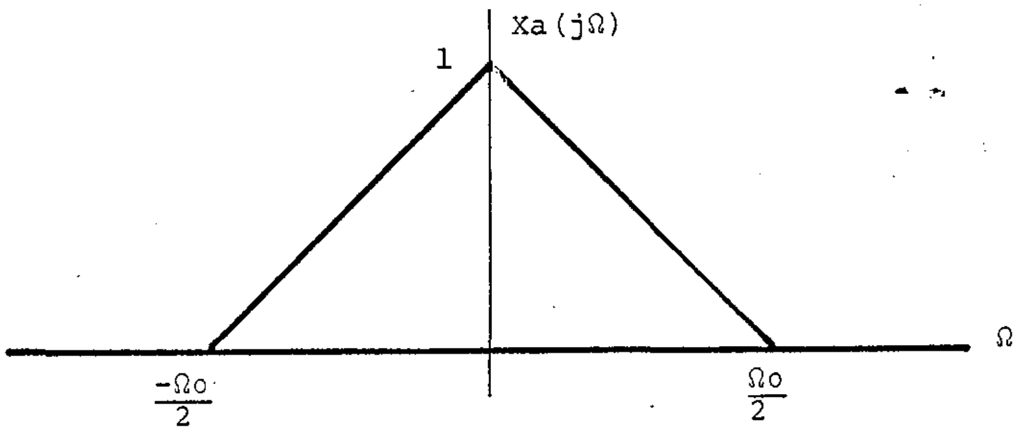
From Fig. 4-10c it is obvious that if $\frac{\Omega oT}{2} < \pi$, i.e., we sample at a rate at least twice the highest frequency of $Xa(j\Omega)$, then $X(e^{j\omega})$ is identical to $Xa(\omega/T)$ in the interval $-\pi \leq \omega \leq \pi$ and can be recovered from the samples xa(nT) by an appropriate interpolation formula.

For the remote magnetic indicator instrument designed in previous chapters, the analog signals are filtered with a low pass section reducing frequency content above 30 Hz. The sampling rate must therefore exceed 60 Hz (T<16.67 m.s.) to enable accurate dynamic operation of the system.

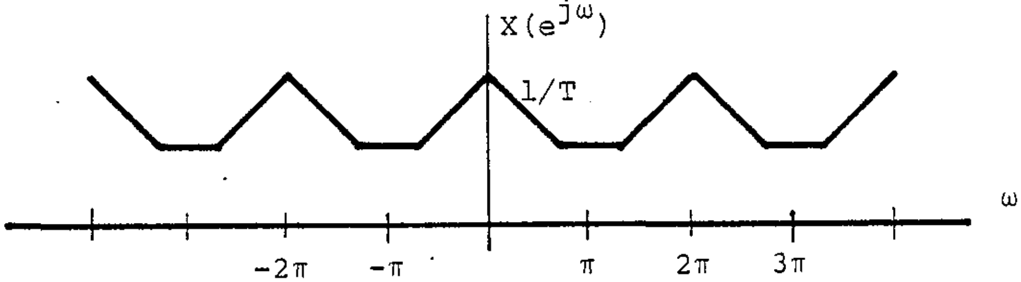
Laboratory measurements of sampling rates on the functional microprocessor based instrument revealed that the analog subsystem operated at a sampling rate of 62.5 Hz (16 m.s.) indicating that the algorithm execution rate supported a system bandwidth of 31.25 Hz. If frequency content of the analog signals is less than 31.25 Hz there is no error due to sampling.

B) Analog Multiplexer Induced Error

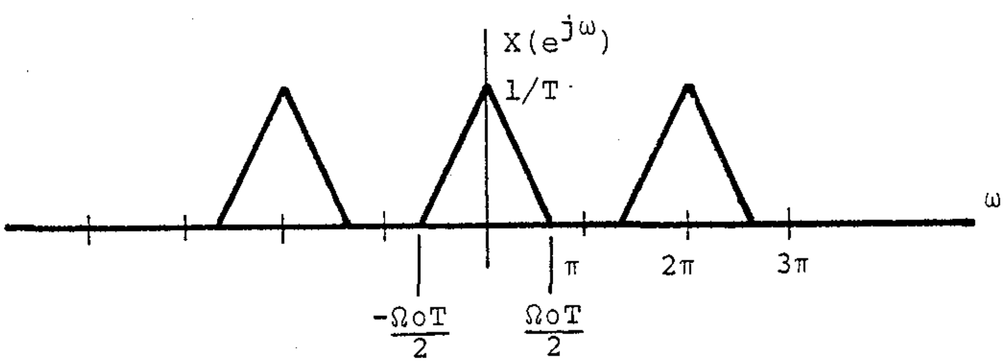
The analog multiplexer of Fig. 4-7 selectively connects one analog transducer output at a time to the input of the sample and hold subsystem. The Datel Systems, Inc., multiplexer [Ref. 4-7] selected for the remote magnetic indicator experiment features eight MOS-FET switches with associated driver circuits,



(a) Fourier transform as a continuous-time signal



(b) Fourier transform of the discrete-time signal obtained by periodic sampling (T is too large)



(c) Same as (b) except T is short enough

Fig. 4-10 FOURIER TRANSFORMS OF CONTINUOUS AND DISCRETE-TIME SIGNALS

FET pull-up to reduce propogation delays and all of the necessary decoding logic to enable random channel addressing with a four bit parallel binary input.

Several important parameters are used to characterize analog multiplexers and can contribute error.

1) Transfer Accuracy

Transfer accuracy is a function of the source impedance, switch resistance, load impedance (if the multiplexer is not buffered) and the signal frequency. It expresses the input to output error as a percentage of the input. In our case the system configuration predicates a maximum error due to transfer accuracy of ($\pm$0.01%) yielding an error term of

$$\pm 0.0001 \times 2.5 \text{ Volts} = \pm 25 \text{ mV}$$

2) Settling Time

This parameter defines the time elapsed from the application of a full scale step input to the time when the output has entered and remained within a specified error band around its final value. In our case the selected multiplexer has a maximum settling time of 1 microsecond to $\pm$0.01% full scale (F.S.) Since the control system selecting channels is implemented using a microprocessor, the minimum time between analog subsystem commands will always be greater than 3.0 microsecond[6]. The multiplexer will therefore always have settled to the final value before the sample and hold circuit (following this subsystem) can be activated with no error due to the settling time parameter.

[6]One machine cycle time for the 2650 microprocessor with 1 mHz clock frequency.

3) Throughput Rate

The highest rate at which the multiplexer can switch from channel to channel at its specified accuracy is in this case 500 kHz. Since this rate is more than four orders of magnitude greater than the operational rate of the subsystem there is no error due to throughput rate limitations.

4) Input Leakage Current

The amount of signal coupled to the output as a percentage of input signal applied to all OFF channels together can be calculated by considering the maximum leakage current specified from OFF channels to the ON channel. In our case the maximum error signal can be calculated

$$\text{Error} = \left[\, 4 \text{ (8 na x 2000 ohms source imped.)}^2 \,\right]^{\frac{1}{2}}$$

$$\text{Error} = 32 \text{ microvolts}$$

Note that in this case the voltage levels are statistically independent allowing an R.S.S. of error sources to calculate total error [Ref. 4-8, 4-9].

C) Sample and Hold Circuit Induced Errors

The sample and hold subsystem consists of a switch and capacitor arrangement as shown in Fig. 4-11. The Datel Systems, Inc., model SHM-IC-1 integrated circuit sample and hold device [Ref. 4-10] features a self-contained high gain differential input amplifier, a digitally controlled electronic switch and a high input impedance buffer amplifier. The external components used with the sample and hold circuit in the solid state remote magnetic indicator instrument consisted of the 0.001μf holding capacitor and a 100K offset trimpot. By connecting the output back to the negative input of the input amplifier (Fig. 4-11), the sample and hold subsystem operated in a unity gain, noninverting mode. When the switch is closed, the unit is in the sampling or tracking mode (Digital Control = 0 Volts), and will follow a changing input signal.

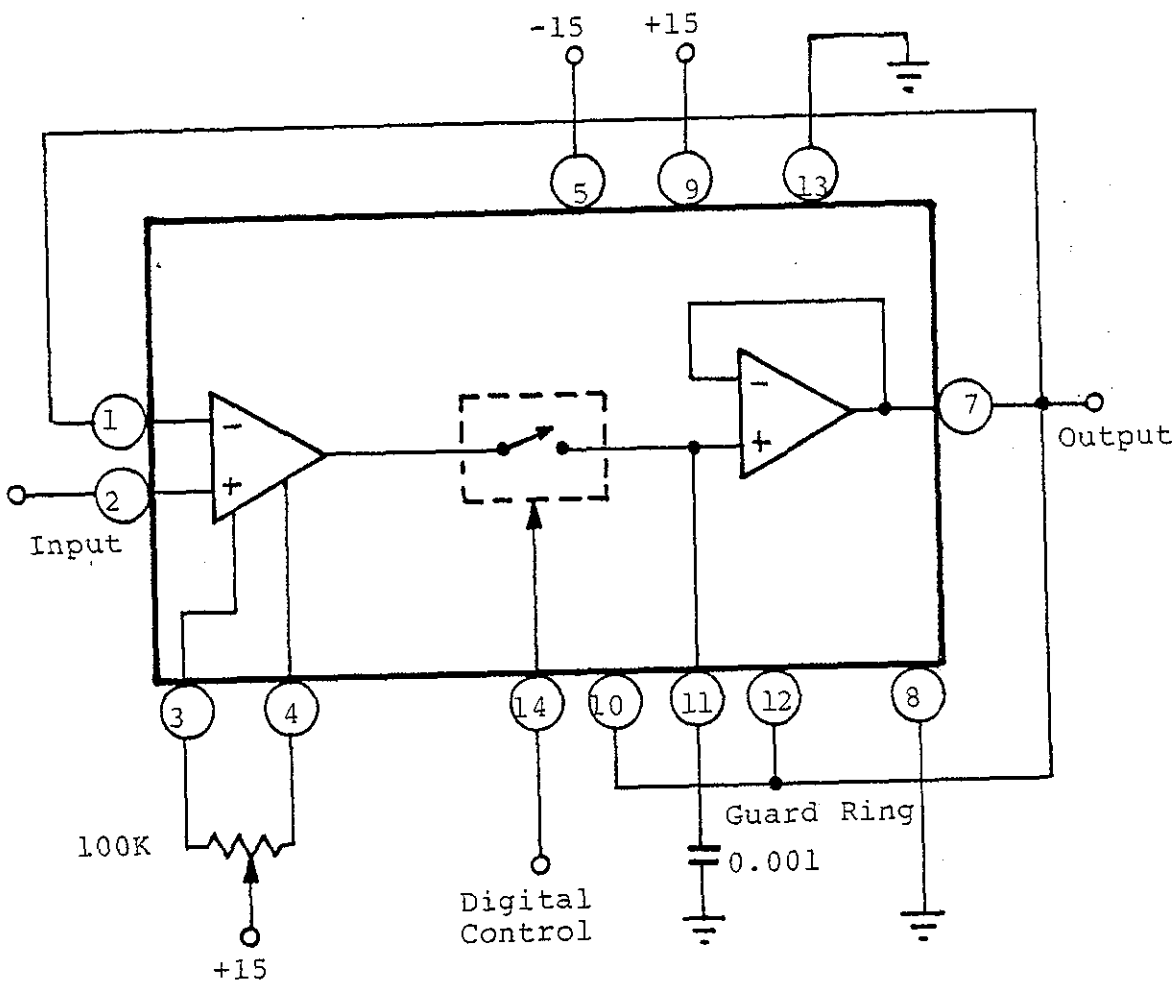


Figure 4-11 SAMPLE AND HOLD SUBSYSTEM

When the switch opens the unit is in the hold mode and retains a voltage on the capacitor for some period of time depending on capacitor and switch leakage. Sample and hold devices are characterized by a number of important parameters that must be considered in the design of a data acquisition subsystem.

1) Acquisition Time

The time lapse between the time that the sample command is given to the point where the output enters and remains within a specified error band around the input value is specified to be less than 4 microseconds time to transit from 0 to 0.1% of 10 Volts with C = 0.001 μf [Ref. 4-10]. This implies that the control signals emanating from the central processor should allow at least 4 μs acquisition time prior to entering the hold mode. We note that the sample and hold subroutine (Appendix B) executes the instruction

IORI, R3 H'80' READY TO HOLD,

a two machine cycle instruction prior to sending the hold control signal. This instruction delays control signal transmission by (2 x 3 μs) = 6 μs allowing the sample and hold circuit ample time to settle with no appreciable error due to the acquisition time parameter.

2) Hold Mode Voltage Droop

The maximum change in output voltage as a function of time is specified to be 50 mv/sec maximum using a 0.001 μf polystyrene capacitor. Since the maximum total accumulated time to completion of the analog to digital conversion can be calculated as

5 Instructions (11 machine cycles)	= 33 μs
1 Analog to Digital Conversion	= 20 μs
3 Instructions if Conversion not synchronized with instructions (7 machine cycles)	= 21 μs
	74 μs

we can then compute droop error to be 50 mv/sec x (74×10^{-6}) sec = 3.73 mv.

3) Aperature Delay

The maximum time lapse between the time of hold signal receipt to opening of the switch is specified to be 50 nsec, an insignificant length of time in the instrument. There is therefore no error due to aperature delay.

4) Offset Error

Although the maximum offset error is specified to be 20 mv maximum [Ref. 4-10], the error was eliminated using the 100K trimpot offset adjustment. There was no appreciable offset error contribution due to the sample and hold circuit.

5) Gain Error

The gain error of a sample and hold circuit is apparent during the sample mode when the transfer function of the total amplifier deviates from the ideal unity slope condition (Fig. 4-12). In the noninverting unity gain mode, the specified gain error is ±0.05% maximum yielding a signal error of

$$(\pm 0.0005) \times 5.0\ V = 250\ mV$$

This error can, however, be eliminated with the gain

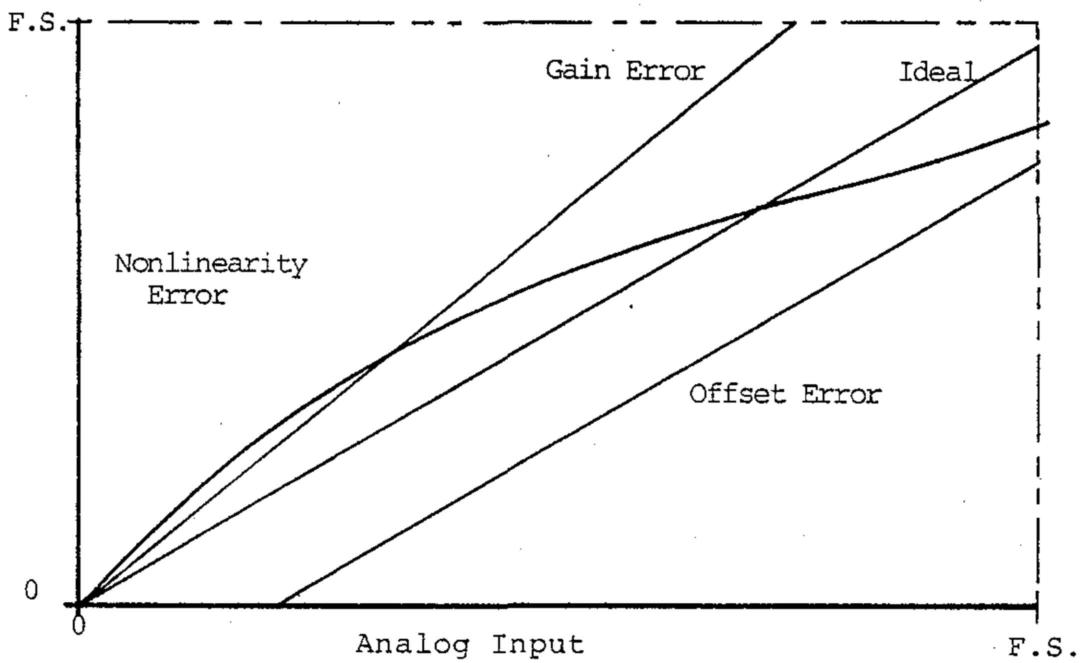


Fig. 4-12 GAIN, OFFSET AND LINEARITY ERRORS

adjustment available at the analog to digital converter. There will therefore be no appreciable net gain error due to the analog subsystem.

6) Nonlinearity Error

Nonlinearity error is apparent in the sample and hold circuit if the transfer function departs from a linear curve (Fig. 4-12). In the noninverting unity gain mode with a 0.001 μf holding capacitor the maximum nonlinearity is 0.01% resulting in a worst case signal uncertainty of (.0001) x 2.5 Volts = 25 mV.

7) Hold Mode Feedthrough

This error appears due to input signal appearing at the output when the unit is in the hold mode. Although the feedthrough varies with signal frequency and the expected signal frequencies are substantially lower than the upper frequency limits of the sample and hold device (30 Hz max. versus several kiloHertz), we consider the worst case feedthrough of 0.01% [Ref. 4-10] or 25 mV.

D) Analog to Digital Converter Induced Errors

The A/D Converter selected for the solid state magnetic indicator instrument (Datel ADC-MA12B1B) [Ref. 4-11] uses the successive approximation technique to achieve excellent linearity and speed. Important parameters that potentially contribute errors are addressed below.

1) Resolution Error

The smallest analog change that can be distinguished by the A/D converter is

$$\text{Least Significant Bit (LSB)} = \frac{\text{Full Scale}}{2^n}$$

$$\text{LSB} = \frac{5}{2^{12}} = 1.22 \text{ mV}$$

this uncertainty manifests itself as an error in computing by limiting the precision of any calculation.

2) Linearity Error

The maximum deviation from a straight line drawn between the end points of the converter transfer function are specified in [Ref. 4-11] to be $\pm$ 1/2 LSB (in our case $\pm$1.22 mV of analog signal).

3) Accuracy Error

The input to output error of the A/D converter is specified in [Ref. 4-11] to be ±0.012% F.S. ±1/2 LSB or

$$\pm(0.00012) \times 5.00V \pm 1.22\ mV = \pm 1.82\ mV \text{ Worst Case}$$

In reality, the two error terms are unrelated and the

$$\text{Rss Error} = \pm\ [(.00012 \times 5V)^2 + (1.22mV)^2]^{\frac{1}{2}}$$

$$\text{RSS Error} = \pm 1.36\ mV$$

4) Offset Error and Gain Error

Both the offset error and gain error were adjusted to zero using the trimming potentiometers (Fig. 4-7) and the calibration procedure outlined in Ref. 4-11. A reference signal of plus 1/2 LSB (1.22 mV) was applied to the converter and the offset trimming potentiometer adjusted until the output flickered equally between logic "0" and logic "1". The gain was then adjusted by setting the converter input to full scale minus 1-1/2 LSB (4.99817 Volts) and the gain trimming potentiometer was adjusted until the output flickered between logic "111...110" and logic "111...111". The above steps were repeated until no appreciable error in gain or offset was evident.

4-4 PROCESSING ERRORS

Errors in processing data accrue due to several sources including imprecision and truncation. Since the microprocessor selected for the instrument is inherently an eight bit device, single precision calculations are conducted with eight bits and double precision calculations are conducted with a total of sixteen bits. This section addresses the effects of computational precision and truncation in the various subroutines and relates these to overall computational accuracy. The various subroutines are analyzed in chronological order as they appear in the main program.

A) Subroutine "SAMP"

The sample subroutine (delineated in Fig. 3-6a) selects and digitizes analog signals by controlling respective analog subsystem modules. During the first portion of this subroutine, A/D converter data bits are stored in two consecutive bytes[7] in the computer memory. The A/D conversion precision of 12 bits is thereby preserved.

The second, third and fourth operations of the sample subroutine convert the unipolar binary format of the data to sign magnitude format, adds offset quantities and merely changes the signs of the Hx and Hy data. The operations are conducted in a double precision manner and precision of the data remains unaltered.

Correction of x axis orthogonality error is the final operation of the sample subroutine. Equation (3-1) is implemented at this point using a table lookup (for the sin function), multiplication and addition. The final result can be expressed as

$$Hx = Hx' + Hy \text{ Sin } \varepsilon$$

[7]A byte is accepted terminology for an eight bit data quantity.

where the respective quantities have the following forms

$$Hx^1 = x_1 + \sum_{i=1}^{11} a_i 2^i$$

$$Hy = x_2 + \sum_{j=1}^{11} a_j 2^j$$

$$\text{Sin } \varepsilon = \sum_{k=1}^{8} a_k 2^{-k}$$

and

x_1, x_2 are sign bits

$a_{i,j,k}$ equal 0 or 1 depending on whether the respective term is to exist or not

We can analyze the effects of imprecision and truncation by noting that the sin function has eight significant binary bits resulting in a resolution of 1/256 or 90°/256 = 0.352°.

The relative error in sin ε is computed by Dahlquist [Ref. 4-12] as follows

let $\tilde{a}$ = the approximate value of sin ε
a = the exact value of sin ε

then the relative error in $\tilde{a}$ is

$$(\tilde{a} - a)/a \text{ if } a \neq 0$$

Since data in the sin table has been truncated, maximum relative error can be as large as $\pm(1/2^{12})$ or $\pm 0.02\%$.

From the definition of relative error we obtain the following relationships between exact, estimate and estimated relative error

$$\tilde{a} = a + ar = a(1 + r)$$

If a_1, and a_2 have relative errors of $\pm 0.39\%$ and $+0.02\%$, respectively, then

$$\tilde{a}_1\tilde{a}_2 = a_1(1 \pm 0.0039)\ a\ \ (1 \pm 0.00024)$$
$$= a_1a_2(1 \pm 0.0039)(1 \pm 0.00024)$$

Thus, the relative error in $\tilde{a}_1\tilde{a}_2$ is

$$(1 \pm 0.0039)(1 \pm 0.00024) - 1 =$$
$$\pm(0.0039) \pm (0.0039)(0.00024) \pm (0.00024)$$
$$\cong \pm(0.0041)$$

Since the maximum value of Sin ε to be encountered occurs when the orthogonality error (ε) is 1 degree, sin ε = 0.017 maximum. The maximum value for Hy can be 0.6 gauss or 2048 units. Maximum error due to imprecision in the product HySinε is then

$$\text{Er Max} = (2048 \times 0.017)(1 + .0041) - (2048 \times 0.017)$$
$$= 0.1427 \text{ units}$$

Since only the integer portion is retained in the final product, insignificant error can be attributed to imprecision of the sin ε term in this case. Orthogonality error will be adequately corrected.

B) Subroutines ROTX and ROTY

These subroutines were developed in Chapter III and implement the equation of 2-11 required to compute horizontal x and y magnetic field components. Equations to be implemented by the respective subroutines are

$$Hx = Hxm \cos(\text{pitch}) + Hym \sin(\text{pitch}) \sin(\text{roll}) + Hzm \sin(\text{pitch}) \cos(\text{roll}) \quad (4\text{-}12)$$

and

$$Hy = Hym \cos(\text{roll}) - Hzm \sin(\text{roll}) \quad (4\text{-}13)$$

where Hxm, Hym and Hzm are measured field components made available from the magnetometer via the analog subsystem.

Since the transcendental functions are implemented using table lookup and are limited in precision to 8 bits, imprecision in these variables will dominate in generating error. In particular, the sin/cos terms will have relative error in the order of $\pm 1/256$ or $\pm 0.39\%$ while the measured field data has relative uncertainty of only $\pm 1/4096$ or $\pm 0.02\%$. Multiplications will result in addition of the bounds for the relative error as illustrated in section 4-4A above.

The transcendental terms above are limited in magnitude to 1.0 maximum while the field measurements can be 0.60 gauss max. In this case the individual product terms of (4-12) and (4-13) can have maximum errors of

$$Er = (2048)(1 + 0.0041) - (2048) = 8.4 \text{ units}$$

Errors in Hy and Hx (4-12 and 4-13) will be maximum when roll and pitch are at 45 degrees and the fields are equal. In this case the error in Hy will be

$$EHy = [(0.707)(2048)(1 + 0.0041) - (0.707)(2048)] -$$
$$[(0.707)(2048)(1 - 0.0041) - (0.707)(2048)]$$

$$EHy = 4.94 - 5.94 = 11.87 \text{ units}$$

Similarly, maximum error in Hx can be calculated as

$$EHx \cong [(0.707)(2048)(1.0041) - (0.707)(2048)] \times 3$$
$$EHx \cong 17.8 \text{ units maximum}$$

It should be noted that these error terms are worst case and peak at multiples of 45 degrees in yaw.

C) Subroutines COSY and SINY

These two subroutines compute the angle between the x axis sensor (when projected onto the horizontal plane) and the north-south horizontal vector of earth's magnetic field. The first two opeations of these subroutines perform double precision multiplication and division. Since the data variables involved are 12 bits in length and the computations performed preserving 16 bits, no error is introduced.

The "ANGL" subroutine called by the above two subroutines computes the desired (x axis to horizontal vector) angle by completing an associative table look up procedure. The task required is to match a given data quantity either (Hx^2/Hh^2 or Hy^2/Hh^2) with the contents of a memory cell. The address of this cell is then the required angle.

Since the table is limited in precision to 16 bits there are obviously cases where an interpolation is required to ascertain the true address[9]. The function stored in tabular form is $\cos^2\Theta$ where Θ varies from 45 to 90 degrees. Maximum error will therefore be induced while attempting to locate solutions (angles near 90 degrees if inadequate precision is provided. Error in this region due to resolution of tabular data can be examined by noting the entries in Table 4-8

Θ	$\text{Cos}^2\Theta$	Most Significant Binary Bit (2^{-x})
90	0	-
89	0.000305	12
88	0.001218	9
87	0.00274	8

Table 4-8 $\text{Cos}^2\Theta$ AND MOST SIGNIFICANT BINARY DIGITS

provided to indicate the relative magnitudes of $\text{Cos}^2\Theta$ in the region of Θ = 90 degrees. We observe that the most significant binary digit affected at 89 degrees is binary decimal digit 12 implying that the resolution of Hx^2/Hh^2 or Hy^2/Hh^2 (the argument of $\text{Cos}^2\Theta$) must be accurate to at least $1/2^{12}$ or 0.024%.

Considering the horizontal field of earth's magnetic vector as observed in laboratory experimentation at this latitude, we note that Hh is 730 units. At a heading of 89 degrees, Hx = 730 Cos 89 = 12.7 units. The argument would therefore be

$$\text{ARG} = Hx^2/Hh^2 = \frac{(12.7)^2}{(730)^2} = 0.000305$$

[9]The procedure determines the relative address by linear interpolation, then selects the closest address as the required angle for the solution.

Since, the squaring and division operations are conducted in double precision, precision is preserved and the algorithm should be able to resolve heading to at least one degree over all portions of the compass.

D) Errors Due to the Remaining Subroutines

Since all of the remaining subroutines work with data that has been rounded to a precision representing 1 degree or better and the computations involve addition or subtraction in double precision binary or binary coded decimal (BCD) format, we note that there will be no further appreciable error due to truncation or rounding.

4-5 MEASUREMENT ERROR SUMMARY

Errors due to sensors and measurement of their respective outputs were discussed in sections 4-2 and 4-3 above. Before proceeding with the analysis of errors, the total signal inaccuracy due to contribution from the many sources above will be summarized in Table 4-10. Total instrument error can then be computed by considering the propagation and enhancement of these errors during the computation process.

Since the errors in Table 4-9 are stochastically independent, we can compute error for any given signal level by finding the RSS of respective error sources. In this manner, the instrument error can be evaluated by considering all input signals with errors superimposed to produce an erroneous computation of heading.

PARAMETER	ERROR	COMMENT
1. Magnetometer		
Offset	≃0	Corrected by software
Orthogonality	≃0	Corrected by software
Noise	Negligible	
Gain	±0.01%	Proport.to signal level
Linearity	±0.01%	
2. Analog Subsystem		
Sampling	Negligible	Sampling rate & filtering adequate
Multiplexer		
Transfer Accuracy	±0.01%	Proport.to signal level
Settling Time	≃0	
Rate	≃0	
Input Leakage	≃0	
Sample and Hold		
Acquisition	≃0	
Hold	4mV	
Aperature Delay	≃0	
Offset	≃0	Corrected by software
Gain	≃0	Corrected by software
Nonlinearity	±0.01%	Proport.to signal level
Feedthrough	±0.01%	Proport.to signal level and frequency
A/D Converter		
Resolution	±1.2mV	
Accuracy	±1.4mV	
Offset	≃0	
Gain	≃0	

Table 4-9 SENSOR AND ANALOG SUBSYSTEM ERROR SUMMARY

4-6 SAMPLE ERROR ANALYSIS

Orthogonality correction using the algorithmic method can be verified by computing expected error prior to correction and comparing measured system output with the error predicition. Assuming that the angle between the x and y sensors exceeds 90 degrees as in Fig. 4-13, we can proceed to compute error by noting the following relationships

$$Hx = Hh\ Cos\ (+\psi)$$
$$Hy = Hh\ Sin\ (\psi)$$
$$Hx_1 = Hx\ Cos\ \varepsilon - Hy\ Sin\ \varepsilon$$
$$\text{True Yaw} = \psi T = Cos^{-1}\left(\frac{Hx}{(Hx^2+Hy^2)^{\frac{1}{2}}}\right)$$

Computed yaw

$$\psi m = Cos^{-1}\left[\frac{HxCos\varepsilon - HySin\varepsilon}{((HxCos\varepsilon - HySin\varepsilon)^2 + Hy^2)^{\frac{1}{2}}}\right]$$

$$= Cos^{-1}\left[\frac{HhCos\psi Cos\varepsilon - HhSin\psi Sin\varepsilon}{[(HhCos\psi Cos\varepsilon - HhSin\psi Sin\varepsilon)^2 + H^2hSin^2\psi]^{\frac{1}{2}}}\right]$$

Using small angle approximations with $\varepsilon \cong 0.79°$

$$Cos\varepsilon \cong 1 \text{ and } Sin\varepsilon \approx 0.014$$

then

$$\psi m = Cos^{-1}\left[\frac{Hh\ Cos\psi - 0.014\ Sin\psi}{[Hh^2\ (Cos\psi - 0.014\ Sin\psi)^2 + Hh^2Sin^2\psi]^{\frac{1}{2}}}\right]$$

Computed error

$$\text{Error} = \psi m - \psi$$

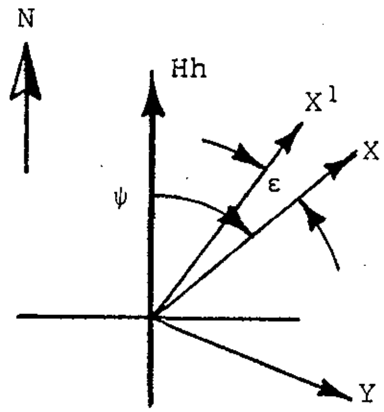


Fig. 4-13 ANGLE (X - Y)>90°

We can now evaluate computed yaw angle (ψm) given a particular yaw (ψ) and the horizontal field vector (Hh). Heading error for horizontal field vector of 730 units at various yaw angles with pitch and roll angles of zero degrees is tabulated in Table 4-10 and plotted along with actual measured yaw error (data taken during experimentation of Chapter V) in Fig. 4-14.

Heading (Degrees)	Computed Error (Degrees)	Heading (Degrees)	Computed Error (Degrees)
90	0.8	290	0.7
70	0.7	270	0.8
50	0.5	250	0.7
30	0.2	230	0.5
10	0.0	210	0.2
350	0.0	190	0.0
330	0.2	170	0.0
310	0.5	150	0.2
		130	0.5

Table 4-10 COMPUTED HEADING ERROR WITH Hh = 730 UNITS

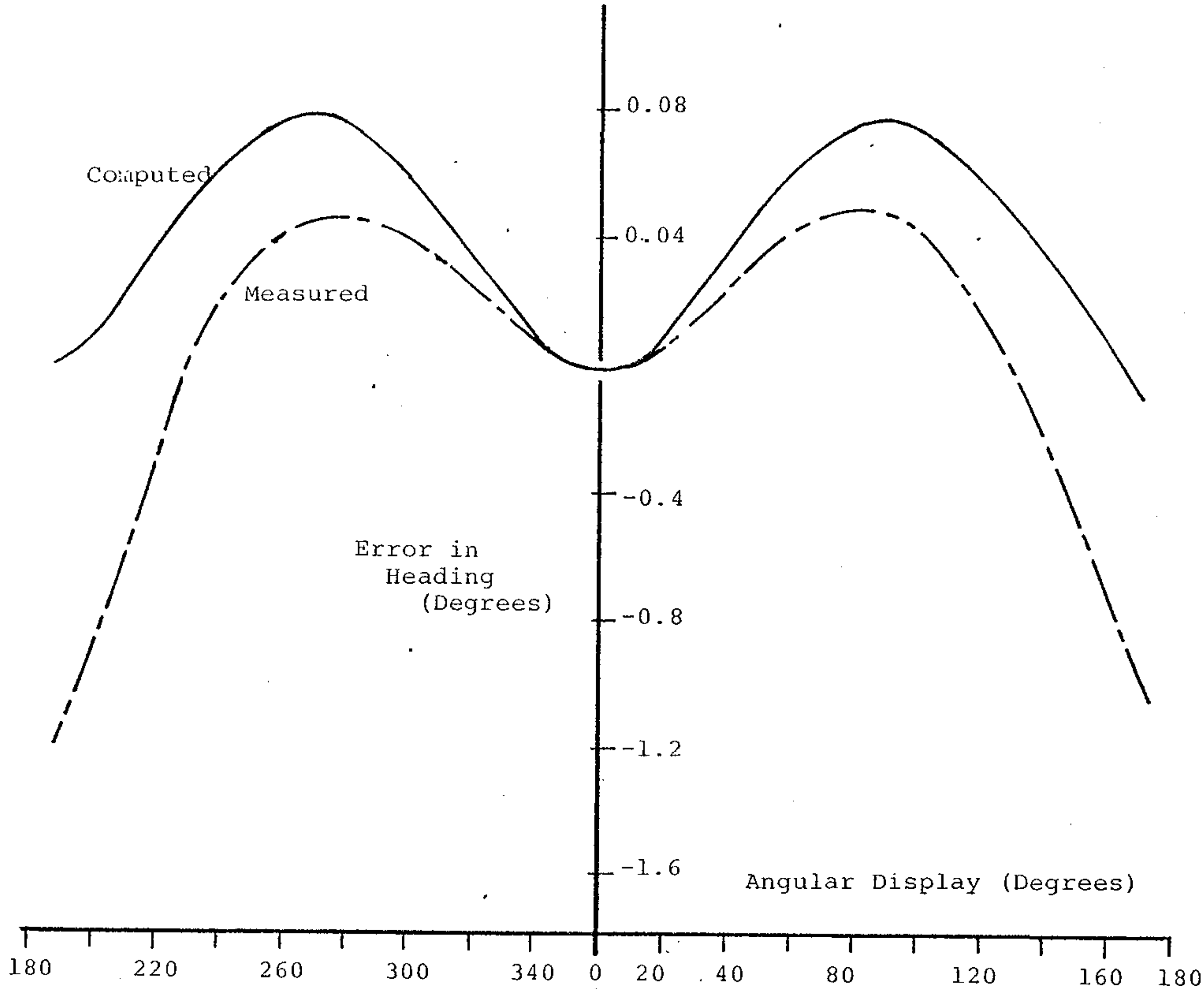


Fig. 4-14 COMPARISON OF COMPUTED AND MEASURED ORTHOGONALITY INDUCED ERROR

4-7 CONCLUSIONS

The preceeding error analysis has identified potential error sources along with relative magnitudes of error to be expected. Magnetometer sensor and analog subsystem errors were identified and analyzed individually. During this analysis it became apparent that errors due to sensor offset and nonorthogonality dominated and would severely limit total instrument performance. The relative magnitudes of these errors and their mode of contribution would have degraded system capacity.

By carefully characterizing the offset and orthogonality error it was determined that these systemmatic errors could be reduced by appropriate programming. A need to identify the extent of each error unique to the laboratory instrument imposed a need to evaluate the instrument empirically. Using earth's magnetic field and the laboratory test fixture (described in Chapter V) to provide control inputs each of the parameters was identified and measured. An algorithm with the empirically determined correction coefficients was included in the final system to reduce the error and to improve final system performance. The remaining potential error sources were tabulated and relative magnitudes noted.

Processing errors due to register precision and truncation were analyzed by considering pertinent subroutines individually. It was noted that the relative error bounds add when multiplying variables with relative error. In addition, it was noted that error accrued during processing is proportional to sensor signal levels involved. The final uncertainty is then proportional to actual aircraft attitude with error increasing as displacement from level flight occurs. Computational error is also noted to increase at particular headings causing the error function to peak at specific yaw angles.

The sample error analysis clearly shows that a correlation between sensor nonorthogonality induced error and measured (uncorrected) data exists. By predicting and computing an error function prior to experimentally verifying the result we gain confidence that the sensor characteristics derived empirically in previous sections are correct.

CHAPTER V

LABORATORY EVALUATION OF THE ATTITUDE INDEPENDENT REMOTE MAGNETIC INDICATOR AND HEADING INSTRUMENT

5-1 INTRODUCTION

This chapter addresses laboratory evaluation of the microprocessor based computer designed to implement the heading measurement instrument. An integral part of this instrument was the three axis fluxgate magnetometer used to implement the attitude independent remote magnetic indicator of Chapter II. The laboratory evaluation was designed to investigate empirically the effects of physical parameters that would otherwise be impossible to assess.

Although phenomena such as noise, magnetic field gradient, sensor orthogonality errors and offset errors can be predicated, combined effects on the proposed instrument and remote magnetic indicator are best evaluated in the laboratory. In addition, it was noted that errors due to sensor offset and nonorthogonality could be corrected by software included with the sample subroutine. Determination of the effectiveness of this correction technique necessitated laboratory measurements of the errors (to determine correction constants) and comparison of data prior to and following corrections.

The chapter begins by discussing laboratory test apparatus designed to evaluate the instrument. Actual data measured and recorded during experimentation is then presented in both tabular and graphic form to facilitate comparison and evaluation. Finally, the laboratory data is discussed and it is concluded that the remote magnetic indicator used with the heading measurement instrument results in a viable alternative to conventional heading measurement systems. The microprocessor based computer implentation of the instrument has added unique sensor measurement correction ability that enhances performance of otherwise marginal sensors. In this manner limitations in systems performance that now exist due to sensor inadequacy can be minimized without incurring the burden of using more expensive sensors.

5-2 TEST APPARATUS

A) Electronic Subsystem

The microprocessor based computer (illustrated in photos 5-1 and 5-2) was constructed on printed circuit boards consisting of a central processing card, two memory cards (2K bytes capacity each) and an output board. A separate analog subsystem card contained the multiplexer, sample and hold, analog to digital converter and trimming potentiometers. The circuit cards were all organized with edge connectors and mounted vertically into a hand wired backplane assembly as shown in photos 5-1 and 5-2.

The card in the left foreground of photo 5-1 served as the output display with three seven-segment displays displaying significant figures of system heading. A small printed circuit in the right foreground of photo 5-1 contained potentiometers used to generate analog signals proportional to roll and pitch signals (simulating gyroscope outputs). Cards shown vertically mounted in photo 5-2 can be identified from right to left as the analog subsystem, two memory cards and the central processing card. The large integrated circuit shown on the CPU card is the Signetics 2640 microprocessor.

B) Sensor Assembly

To evaluate the effects of combined aircraft pitch, roll

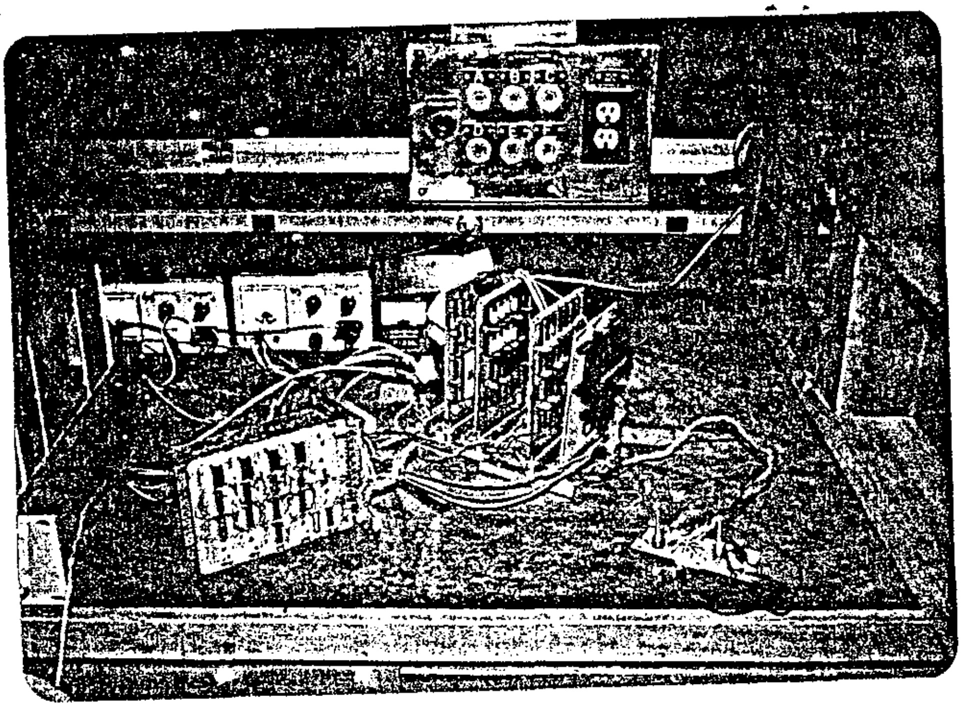


Photo 5-1 MICROPROCESSOR BASED HEADING COMPUTER

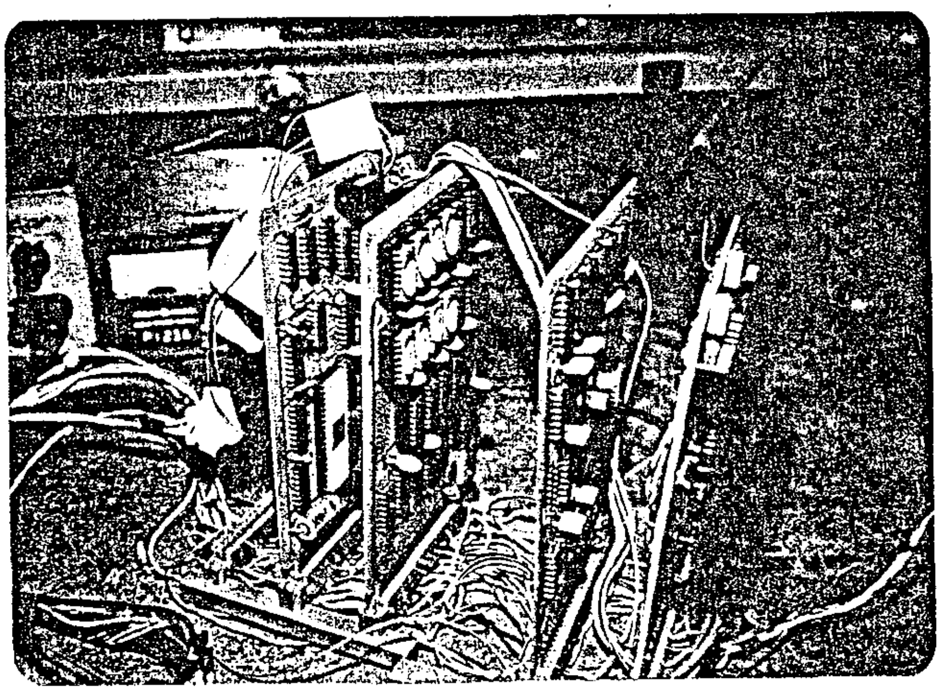


Photo 5-2 CENTRAL PROCESSOR, MEMORY AND ANALOG SUBSYSTEM

and yaw a three axis gimbal apparatus was required. In addition, since angular measurements were required, a means of measuring angular rotation in each of the three exes was provided. The gimbal apparatus as illustrated in photos 5-3 and 5-4 was fitted with large protractors centered on the rotation axes. Pointers were provided to enable angular rotation measurements on the respective protractor scales. Since the angular precision on each protractor scale resolved angular position to 0.5 degrees, angular measurements to a resolution of at least 0.5 degrees were possible. Angular position was measured by estimating the decimal place of each measurement with accuracy to ±0.5 degrees ensured.

Since the three axis magnetometer (housed in the rectangular block of photos 5-3 and 5-4) measured ambient magnetic fields the test apparatus was constructed of nonferrous material. This ensured that local fields due to residual magnetic fields in the test apparatus would be minimized. In addition, since the material had low permeability, there would be little deformation of the local field causing error due to changing field gradient.

The sensor package shown in photos 5-3 and 5-4 was physically mounted such that the sensors were centered as close to the center of the gimbal as possible. This precaution ensured that measurement error due to sensor translation was minimized[1]. During instrument evaluation, the entire gimbal assembly and sensor were leveled and mounted in a Helmholtz coil assembly as illustrated in photo 5-5. Although the coils were not activated during experimentation, the rotations in heading were

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[1]Since the local magnetic field has a nonzero gradient, field measurements include a component due to translation of the sensor axes. This component of measurement produces unacceptable error in a system designed to measure field components that change due to rotation.

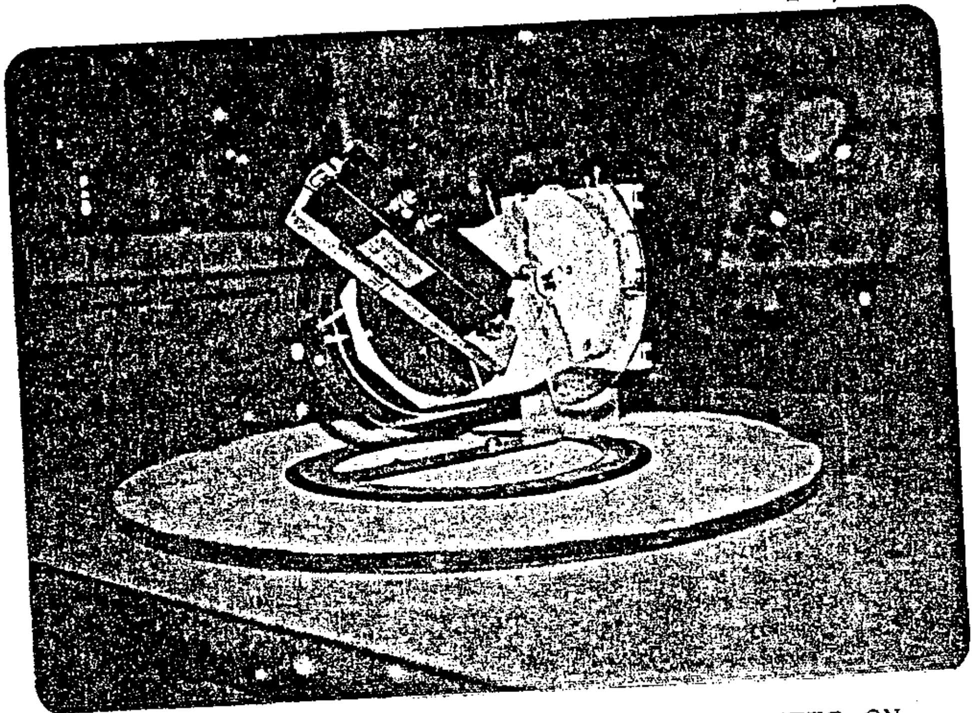

Photo 5-3 MAGNETOMETER SENSOR MOUNTED ON GIMBALLED TEST FIXTURE

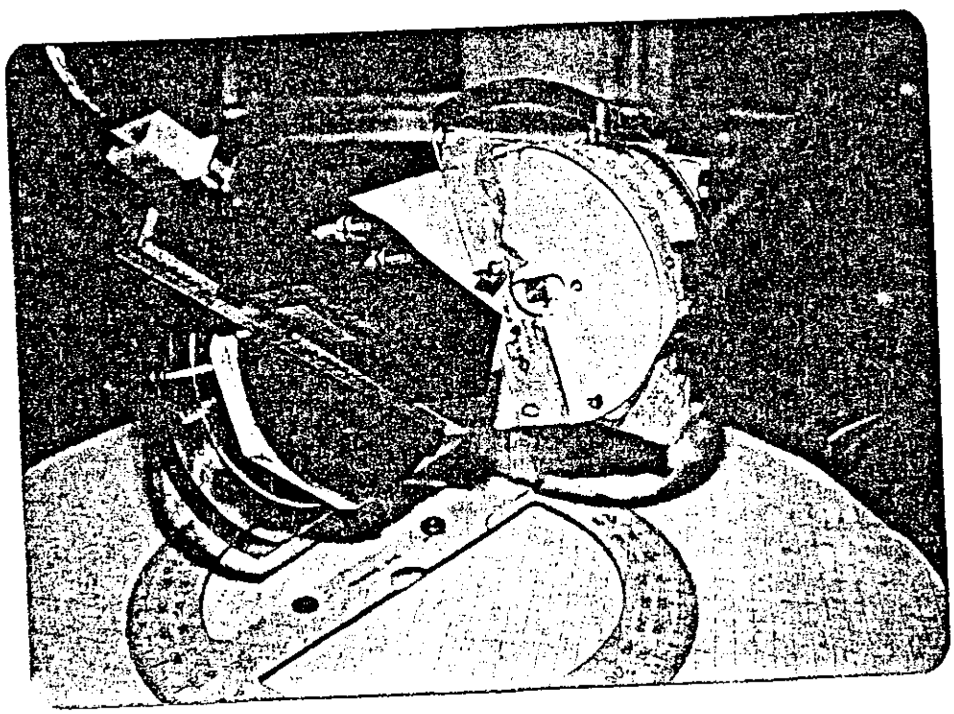

Photo 5-4 SENSOR AND GIMBAL ASSEMBLY WITH PROTRACTORS

carefully controlled since the gimbal assembly was an integral part of the Helmholtz coil fixture with the vertical rotation axis serving as the system yaw axis.

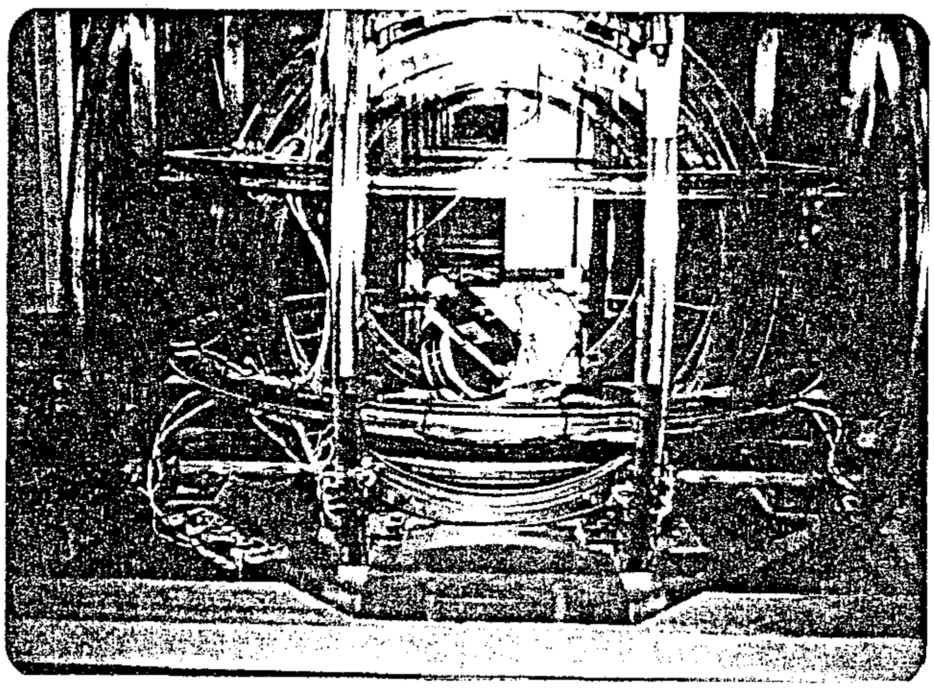

Photo 5-5 TEST FIXTURE MOUNTED IN HELMHOLTZ COIL ASSEMBLY

5-3 HEADING MEASUREMENTS WITH NO OFFSET CORRECTION

By maintaining heading of the text fixture constant (no rotation about the vertical axis) and varying both pitch and roll angle, the instrument display was observed to vary. This variation gave a direct measure of instrument error since a constant heading was maintained and a constant display was to be expected.

Data variations were recorded in Tables 5-1 and 5-2 and plotted on Figures 5-1 and 5-2. With only ± 10 degree variation in pitch combined with ± 30 degree variation in roll we note that the heading display varies 14 degrees. Obviously,

Roll Angle (Degrees)	Θ = 0°	Θ = 10°	Θ = -10°
-30	000	353	4
-20	000	354	2
-10	359	357	0
0	359	359	358
10	359	000	355
20	358	002	354
30	358	004	353

Table 5-1 HEADING COMPUTED AT A FIXED YAW ANGLE WITH NO OFFSET CORRECTION

Roll Angle (Degrees)	Θ = 0°	Θ = 10°	Θ = -10°
-30	46	52	38
-20	45	49	41
-10	45	47	43
0	45	45	43
10	45	43	46
20	44	42	47
30	44	41	49

Table 5-2 HEADING COMPUTED AT A FIXED YAW ANGLE WITH NO OFFSET CORRECTION

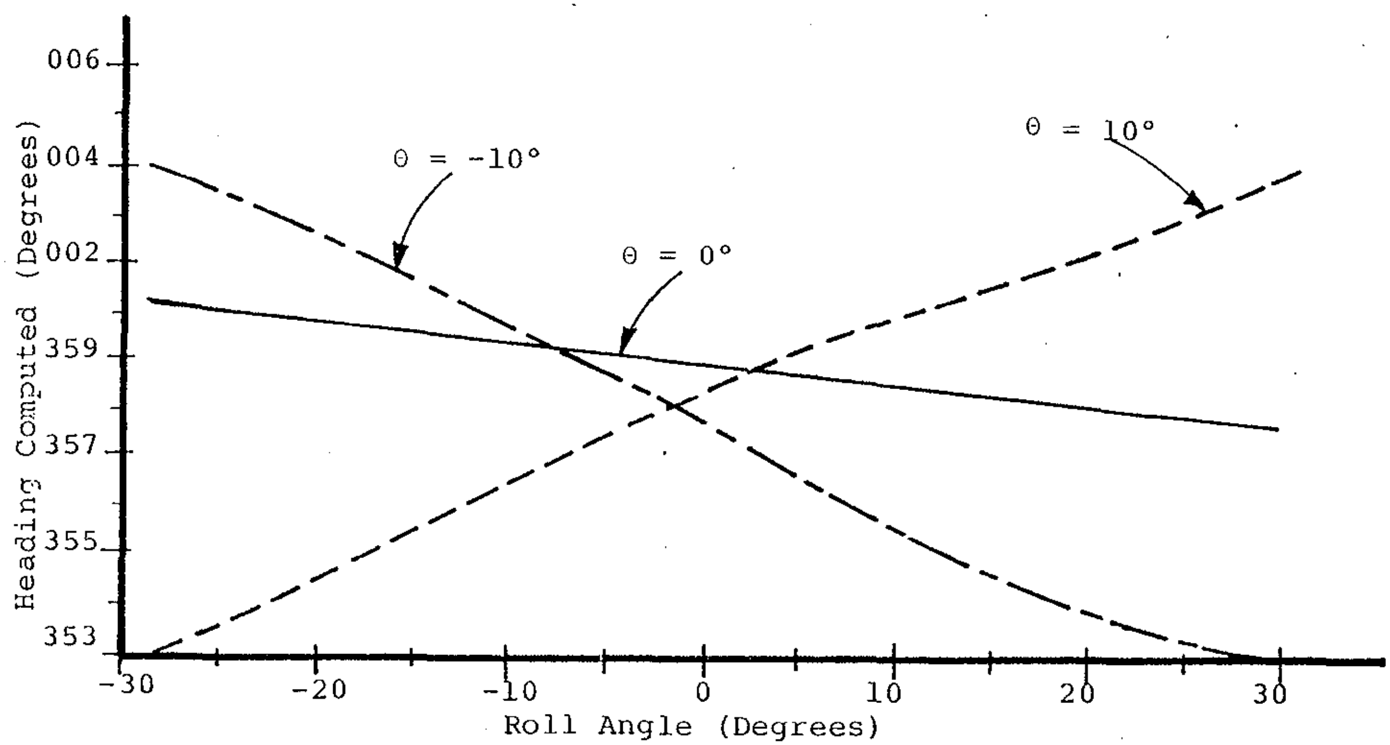

Fig. 5-1 HEADING COMPUTED AT A FIXED YAW ANGLE WITH VARYING Θ AND Φ (NO OFFSET CORRECTION)

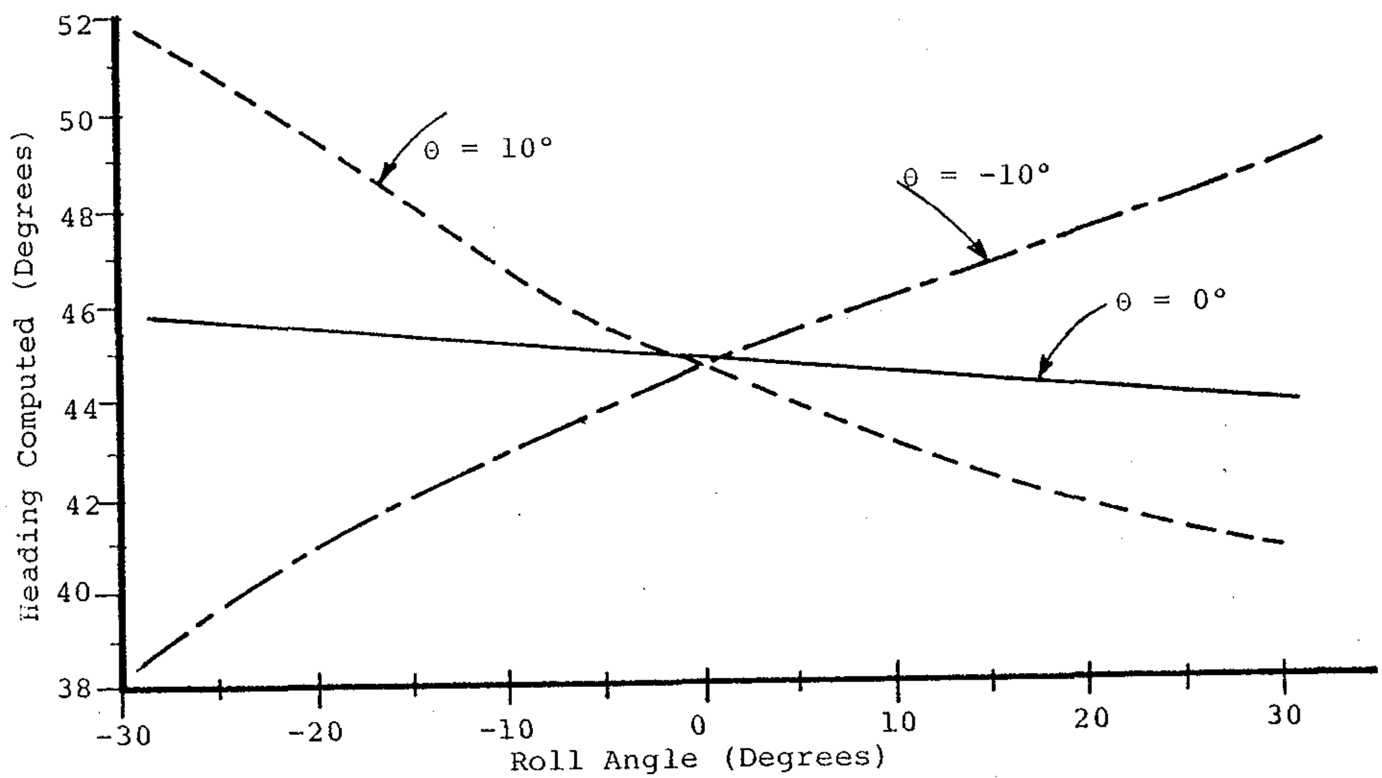

Fig. 5-2 HEADING COMPUTED AT A FIXED YAW ANGLE WITH VARYING Θ AND Φ (NO OFFSET CORRECTION)

instrument operation indicated excessive error requiring more elaborate sensors or correction of a sensor inadequacy.

5-4 HEADING MEASUREMENTS TO INVESTIGATE ORTHOGONALITY ERROR

System performance was evaluated by initially aligning the sensors with zero pitch and roll angle. Sensor Z was positioned vertically with positive direction downwards. By observing the Z axis output[2] as the test fixture was rotated about the vertical axis, adjustments were made in pitch and roll angle to minimize coning of the Z axis. Angular measurements on the respective roll and pitch axis protractors were then made to establish the initial reference attitude angles.

Heading measurement accuracy was evaluated by rotating the test fixture in the horizontal plane until the display flickered between (XX9) and (XX9+1). The rotation was then continued a very small amount until a steady display (multiple of 10 degrees) was observed[3]. Measurements ranging from 0 to 350 degrees were made by recording angular position required to produce specific heading data displays. Sets of data were recorded at various combinations of pitch and roll then tabulated in Tables 5-3 through 5-8. Relative error was computed by determining angular position expected at each display value and then computing the difference in angular positions. Errors at the roll extremes of ±44 degrees are plotted for pitch angles of plus and minus 20 degrees on Fig. 5-3 through 5-6 inclusive.

---

[2]A special subroutine was used to display Z axis data directly in BCD format on the seven bar output display.

[3]This measurement technique ensured that all heading measurements were made identically. In addition, error due to system imprecision was reduced.

Data in Tables 5-5, 5-6 and Fig. 5-3, 5-4 were recorded with no sensor orthogonality error correction implemented. Data in Tables 5-7, 5-8 and Fig. 5-5, 5-6 was recorded with the sensor orthogonality correction implemented. Comparison of these data indicate that considerable improvement in accuracy is achieved by correcting sensor orthogonality error.

5-5 CONCLUSIONS

Laboratory evaluation of the heading measurement instrument has shown that the algorithms developed in previous chapters are viable. Operation of the device in a laboratory environment has enabled empirical evaluation of the system under adverse combinations of noise, field gradient and sensor plus instrument error sources.

Test apparatus described in section 5-2 served to enable controlled simulation of roll, pitch and yaw rotations. The apparatus was nonmagnetic in nature and contributed insignificant error due to field pertebation. Mounting of protractors and pointers on the test apparatus made angular measurements possible to a precision of at least ± 0.5 degrees.

Effects of sensor offsets were evaluated in section 5-3 by recording system heading computations when only roll and pitch varied. Since the variations in Figures 5-1 and 5-2 prior to offset correction exceed the maximum excursions of Figures 5-3 and 5-4 by at least a factor of two (angular excursions in first set also less than in the record) and we note that offset errors were corrected prior to recording data in the second set of data, we conclude that offset in magnetometers can be a

PITCH ANGLE 0 DEGREES

ROLL ANGLE 0 DEGREES

Heading Displayed (Degrees)	Angular Position (Degrees)	Relative Error (Degrees)
10	275.3	0.3
30	295.2	0.2
50	315.4	0.4
70	335.5	0.5
90	355.5	0.5
130	34.8	-0.2
150	54.3	-0.7
170	74.0	-1.0
190	94.0	-1.0
210	114.4	-0.6
230	135.0	0.0
250	155.5	0.5
270	174.7	-0.3
290	195.5	0.5
310	215.2	0.2
330	235.2	0.2
350	254.7	-0.3

Table 5-3 REFERENCE DATA MEASUREMENTS OF HEADING TAKEN WITH NO ORTHOGONALITY CORRECTION

PITCH ANGLE 0 DEGREES
ROLL ANGLE 0 DEGREES

Heading Displayed (Degrees)	Angular Position (Degrees)	Relative Error (Degrees)
10	276.6	-0.4
30	296.6	-0.4
40	306.7	-0.3
50	316.7	-0.3
60	327.2	+0.2
70	337.0	0.0
90	355.9	-1.1
130	37.0	0.0
160	67.2	0.2
190	96.8	-0.2
220	126.9	-0.1
250	157.0	0.0
280	186.9	-0.1
310	216.6	-0.4
340	246.9	-0.1
350	256.4	-0.6

Table 5-4 HEADING MEASUREMENTS WITH OFFSET AND ORTHOGONALITY CORRECTIONS MADE

Heading Displayed (Degrees)	Roll = 44°		Roll = 20°		Roll = -20°		Roll = -44°	
	Angular Position	Error	Angular Position	Error	Angular Position	Error	Angular Position	Error
10	276.3	1.3	276.5	1.5	275.0	0.0	274.0	-1.0
30	296.3	1.3					293.6	-1.4
40			306.1	1.1	304.0	-1.0		
50							313.8	-1.2
60	325.8	0.8						
70			335.6	1.6	334.3	-0.7	333.8	-1.2
90	355.0	0.0	354.0	-1.0	353.1	-1.9	353.0	-2.0
130	34.3	0.7	34.3	-0.7	34.6	-0.4	34.4	-0.6
160	64.5	-0.5	64.0	-1.0	65.0	0.0	65.0	0.0
190	94.3	-0.7	94.6	-0.4	95.0	0.0	95.5	0.5
220	125.2	0.2	125.3	0.3	126.0	1.0	126.5	1.5
250	156.6	1.6	156.2	1.2	156.7	1.7	157.0	2.0
280	186.6	1.6	186.8	1.8	186.5	1.5		
310	216.8	1.8	216.8	1.8	215.9	0.9	215.9	0.9
340	246.0	1.0	246.8	1.8	245.0	0.0		
350							224.9	-0.1

TABLE 5-5 HEADING MEASUREMENTS AT PITCH = 20° WITH NO ORTHOGONALITY CORRECTION

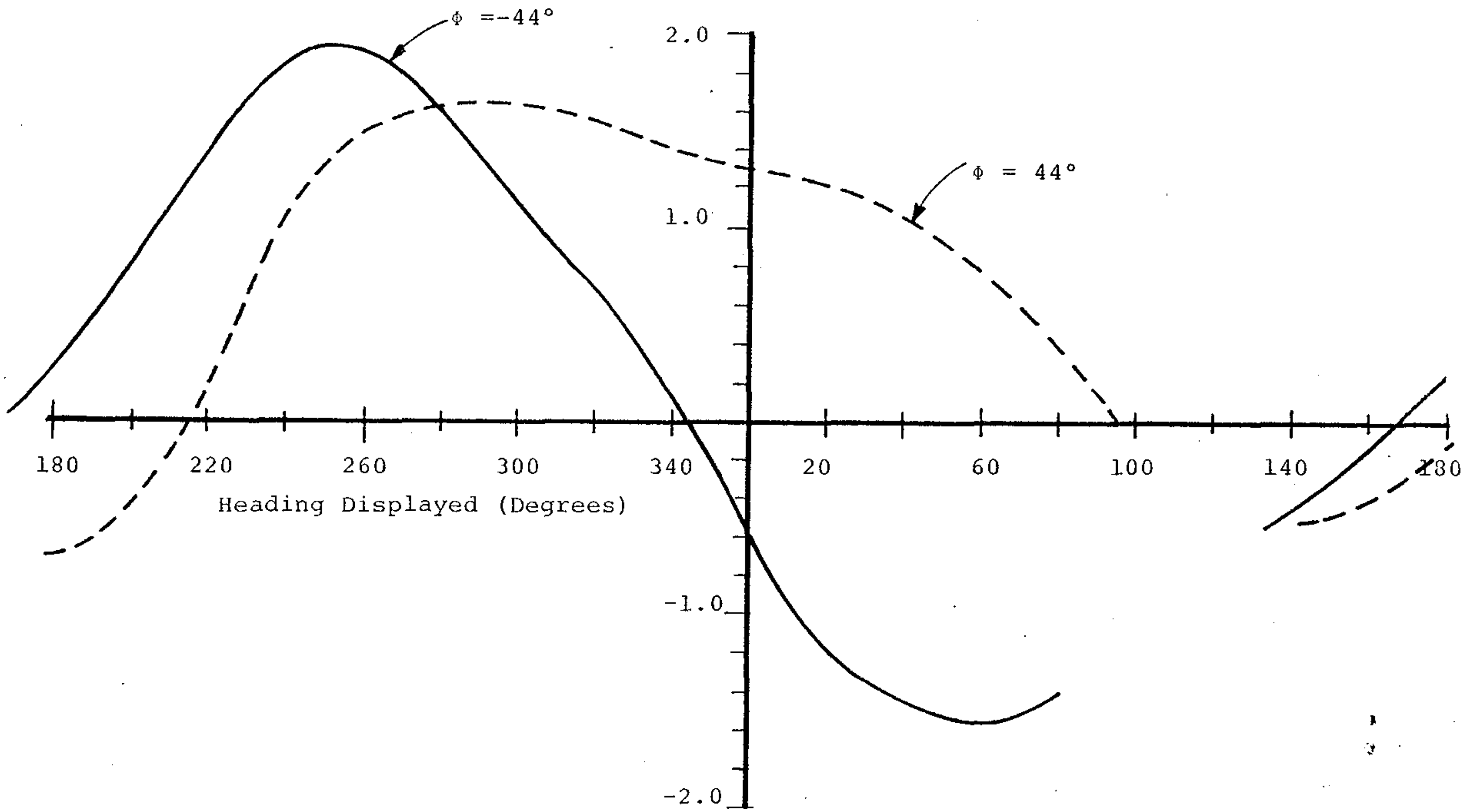

Fig. 5-3 HEADING MEASUREMENT ERROR AT PITCH = 20°
(NO ORTHOGONALITY CORRECTION)

Heading Displayed (Degrees)	Roll = 44° Angular Position	Roll = 44° Error	Roll = 20° Angular Position	Roll = 20° Error	Roll = -20° Angular Position	Roll = -20° Error	Roll = -44° Angular Position	Roll = -44° Error
10	276.1	1.1	275.1	0.1	274.2	-0.8	273.7	-1.3
30	296.2	1.2	295.9	0.9			293.6	-1.4
50	316.4	1.4	316.1	1.1			314.1	-0.9
70	336.5	1.5	336.4	1.4	335.5	0.5	334.6	-0.4
90	355.5	0.5	355.3	0.3	355.0	0.0	354.2	-0.8
130	36.0	1.0	36.4	1.4	135.7	0.7	135.7	0.7
150	55.3	0.3	55.9	0.9			55.8	0.8
170	74.5	-0.5	75.6	0.6			75.5	0.5
190	94.9	-0.1	95.3	0.3	95.5	0.5	95.5	0.5
210	115.0	0.0	115.4	0.4			115.6	0.6
220					125.5	0.5		
230	135.0	0.0	135.6	0.6			136.0	1.0
250	155.6	0.6	155.4	0.4	155.5	0.5	155.7	0.7
270	174.8	-0.2	174.8	-0.2			175.0	0.0
290	195.6	0.6	195.4	0.4			195.3	0.3
310	215.5	0.5	215.3	0.3	214.6	-0.4	214.5	-0.5
330	235.7	0.7	235.1	0.1			234.2	-0.8
350	255.6	0.6	255.2	0.2			253.8	-1.2

Table 5-6 HEADING MEASUREMENTS AT PITCH = -20° WITH NO ORTHOGONALITY CORRECTION

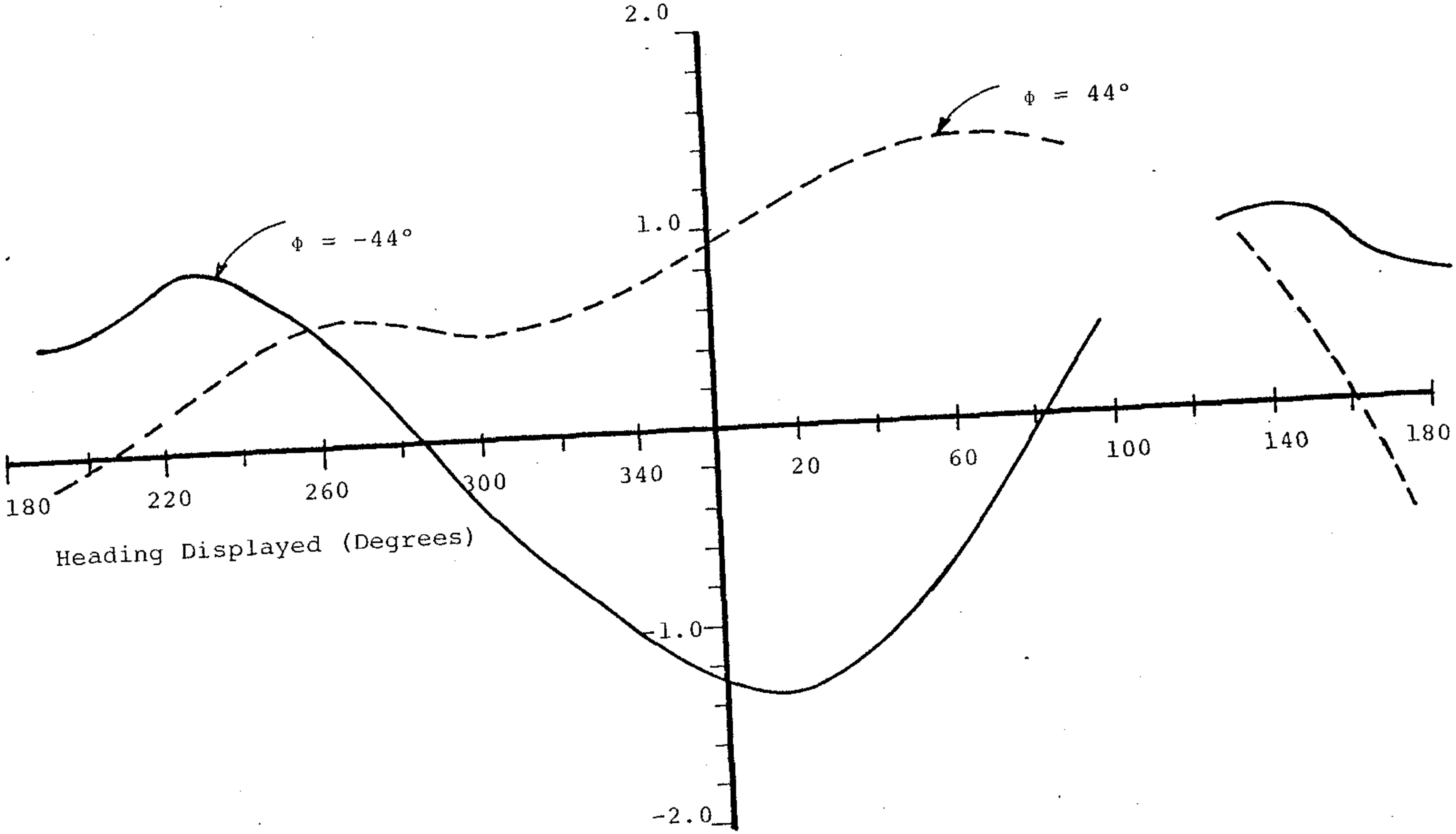

Fig. 5-4 HEADING MEASUREMENT ERROR AT PITCH = -20°
(NO ORTHOGONALITY CORRECTION)

Heading Displayed (Degrees)	Roll = 0° Angular Position	Roll = 0° Error	Roll = 44° Angular Position	Roll = 44° Error	Roll = -44° Angular Position	Roll = -44° Error
20	287.3	0.3	287.5	0.3	286.3	-0.7
40	307.2	0.2	307.5	0.3	306.0	-1.0
60	327.0	0.0	327.9	0.7	325.9	-1.1
80	346.3	-0.7	347.5	0.3	345.6	-1.4
90	355.9	-1.1	357.0	0.2	356.2	-1.8
140	46.9	-0.1	47.0	-0.2	46.5	-0.5
160	67.0	0.0	67.3	0.1	67.1	0.1
180	86.6	-0.4	86.5	-0.7	87.0	0.0
200	107.0	0.0	107.3	0.1	108.1	1.1
220	127.6	0.6	127.3	0.1	128.3	1.2
240	147.2	0.2	147.4	0.2	148.5	1.5
260	167.0	0.0	167.1	-0.1	168.5	1.5
280	187.3	0.3	187.1	-0.1	188.5	1.5
300	207.2	0.2	207.1	-0.1	208.0	1.0
320	227.3	0.3	227.0	-0.2	227.5	0.5
340	247.4	0.4	247.3	+0.1	247.4	0.4
0	266.4	-0.6	267.0	-0.2	266.0	-1.0

Table 5-7 HEADING MEASUREMENTS AT PITCH = 20° WITH OFFSET AND ORTHOGONALITY CORRECTION MADE

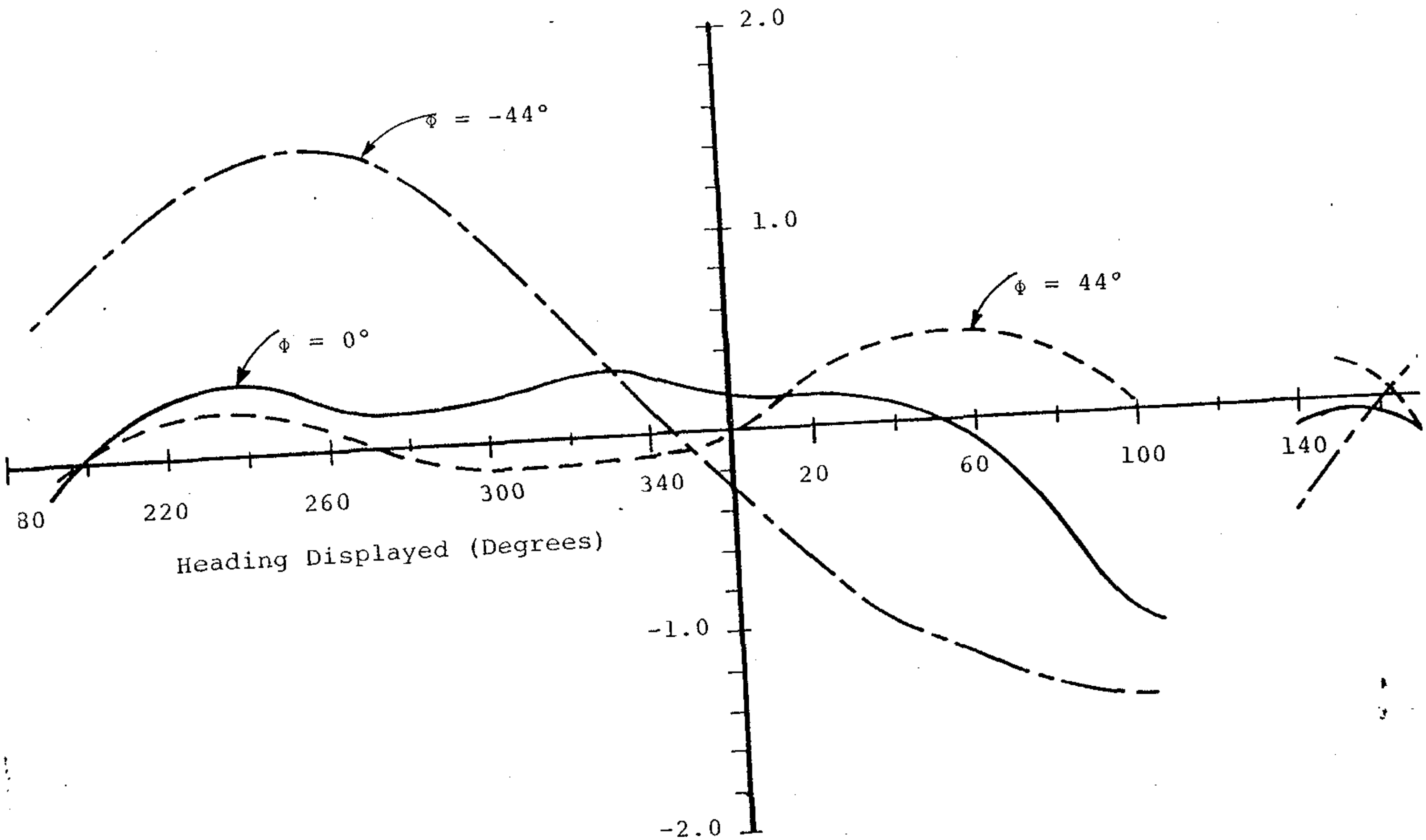

Fig. 5-5 HEADING MEASUREMENT ERROR AT PITCH - 20° (OFFSET AND ORTHOGONALITY ERROR CORRECTED)

Heading Displayed (Degrees)	Roll = 44°		Roll = -44°	
	Angular Position	Error	Angular Position	Error
20	287.0	-0.4	286.5	-0.3
40	307.0	-0.4	306.7	-0.1
60	327.0	-0.4	326.8	0.0
80	347.0	-0.4	346.8	0.0
90	356.7	-0.7	355.6	-0.2
140	47.8	0.4	48.0	1.2
160	67.7	0.3	68.0	1.2
180	87.0	-0.4	87.8	1.0
200	107.6	0.2	108.0	1.2
220	127.7	0.3	128.0	1.2
240	147.8	0.4	147.4	0.6
260	167.6	0.2	167.2	0.4
280	187.6	0.2	187.0	0.2
300	207.1	-0.3	206.5	-0.3
320	227.1	-0.3	226.6	-0.2
340	247.1	-0.3	246.9	0.1
0	266.3	-0.9	266.0	-0.8

TABLE 5-8 HEADING MEASUREMENTS AT PITCH = -20° WITH OFFSET AND ORTHOGONALITY ERROR CORRECTED

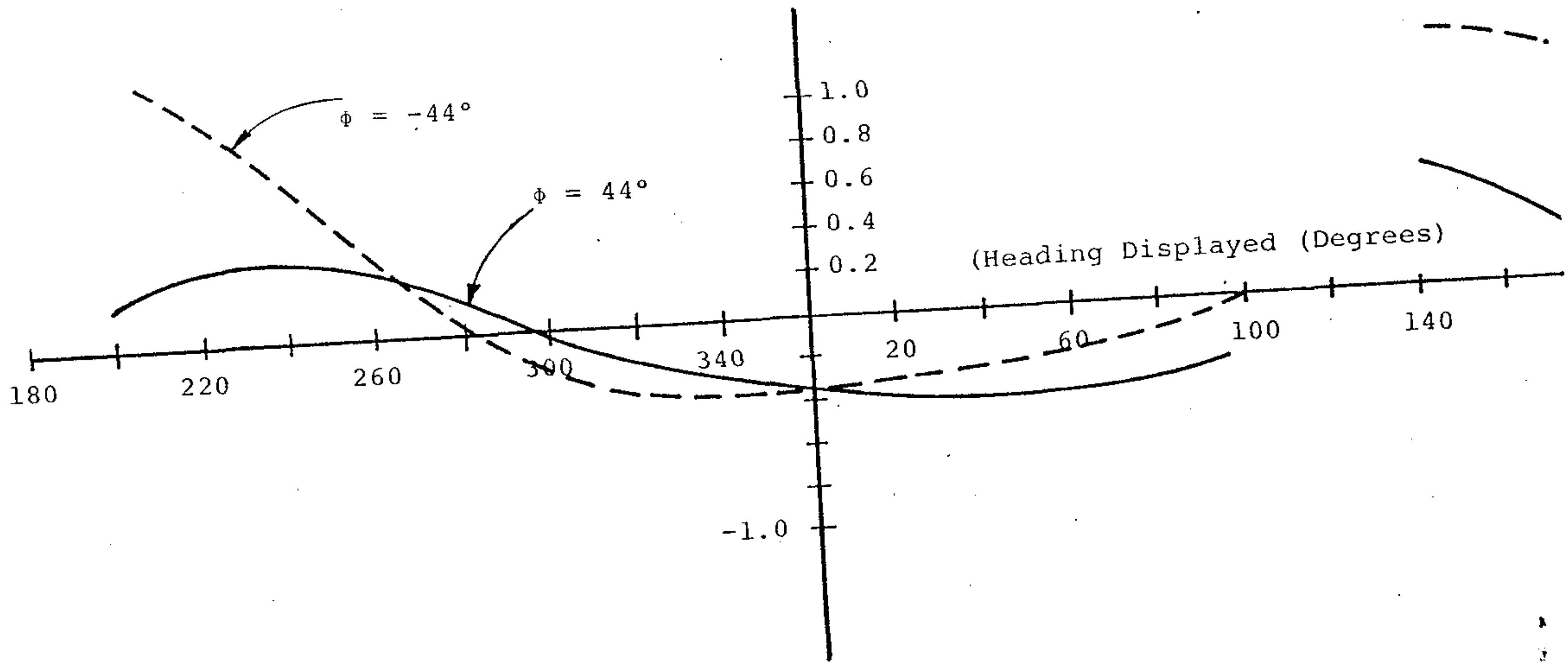

Fig. 5-6 HEADING MEASUREMENT ERROR AT PITCH = -20° (OFFSET AND ORTHOGONALITY CORRECTED)

major error source[4]. Additionally, we note that the correction of offset error in sensors has been successful. Experimental results have verified that not only can offset errors be determinded (Chapter IV), but a suitable algorithm can be implemented in the computer to improve system operation. It is postulated that offset error correction can be extended to include correction of varying offset values (functions of temperature and supply voltage) by monitoring error causing variables (example temperature) and computing correction constants prior to offset correction as above.

Errors induced by sensor nonorthogonality were predicted in Chapter IVsection 4-2 and verified by plotting expected error along with measured error in Fig. 4-14. The curves of Fig. 4-14 were plotted for heading rotations with no pitch or roll angle. To evaluate system performance and the effect of orthogonality error with combined angular rotations, measurements of heading error were plotted in Fig. 5-3 thorugh 5-6 inclusive.

Comparison of these data indicate that maximum excursions of error as a function of heading are significantly less when orthogonality corrections are made. It is also postulated that data could be improved further by similarly correcting orthogonality error in te Z axis sensor[5].

In summary, the experimental evaluation has provided insight into the operation of an attitude independent remote magnetic indicator and heading computer in the "real world"

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[4]This corroborates the observations predicted during error analysis in Chapter IV.

[5]We note that the error excursions are functions of pitch and roll and that Z axis data is used in the rotation algorithm.

environment complete with all contributing error sources. The error analysis evolved during development of the system has proven adequate in that an operational system was developed. Major error sources were measurable as predicted and the means of reducing their effects were successfully implemented. Correction of sensor offset and orthogonality error required an empirical evaluation of the respective sensor. These evaluations were performed, the errors characterized, correction coefficients determined, and correction algoritms implemented.

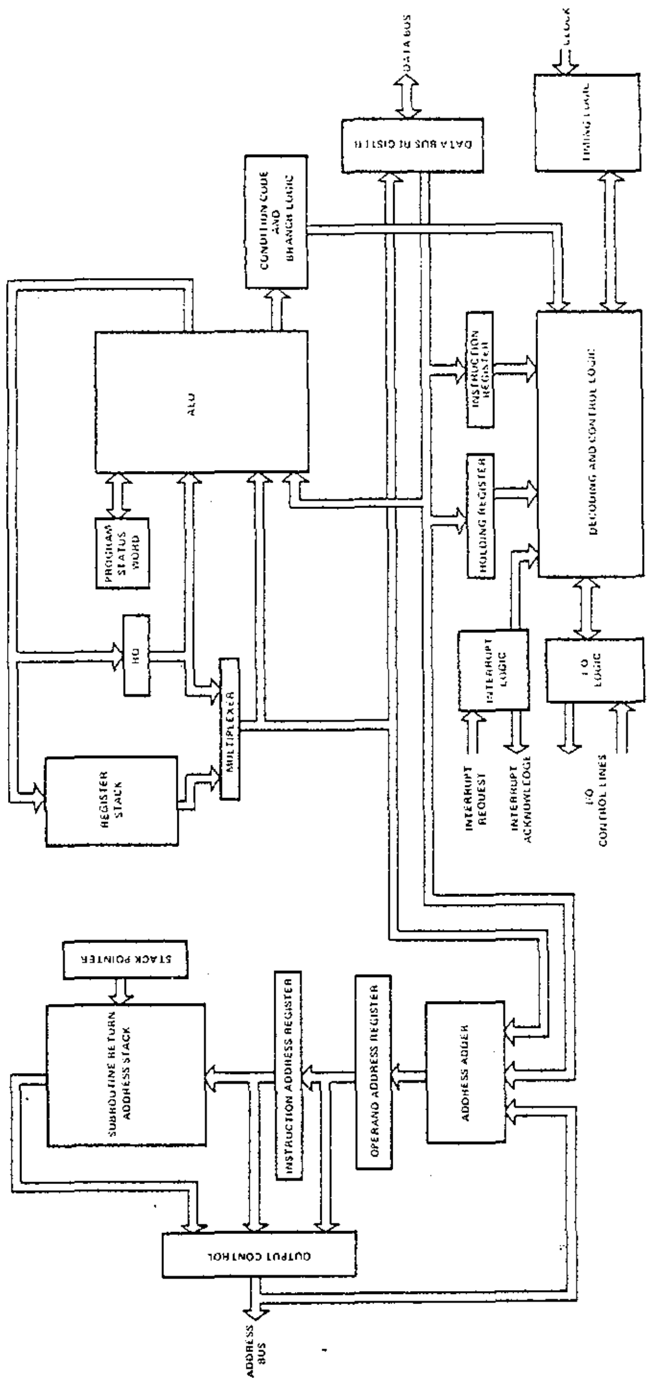
Successful implementation of these corrections was evidenced by significant reductions in system error. The correction methods presented can be extended in future with the net result that less demand is required of physical sensors if the sensor parameters can be established empirically prior to completion of instrument design. Utilization of a microprocessor in the instrument has added the computational flexibility required to facilitate accommodation of sensors with varying error magnitudes.

APPENDIX A

This appendix lists the instruction set of the Signetics 2650 microprocessor chip used to implement the heading instrument.

Instruction	Operands	Description	Length (bytes)
LOAD/STORE INSTRUCTIONS			
LODZ	r	Load Register Zero	1
LODI,r	v	Load Immediate	2
LODR,r	(*)a	Load Relative	2
LODA,r	(*)a(,X)	Load Absolute	3
STRZ	r	Store Register Zero	1
STRR,r	(*)a	Store Relative	2
STRA,r	(*)a(,X)	Store Absolute	3
ARITHMETIC INSTRUCTIONS			
ADDZ	r	Add to Register Zero	1
ADDI,r	v	Add Immediate	2
ADDR,r	(*)a	Add Relative	2
ADDA,r	(*)a(,X)	Add Absolute	3
SUBZ	r	Subtract from Register Zero	1
SUBI,r	v	Subtract Immediate	2
SUBR,r	(*)a	Subtract Relative	2
SUBA,r	(*)a(,X)	Subtract Absolute	3
LOGICAL INSTRUCTIONS			
ANDZ	r	And to Register Zero	1
ANDI,r	v	And Immediate	2
ANDR,r	(*)a	And Relative	2
ANDA,r	(*)a(,X)	And Absolute	3
IORZ	r	Inclusive or to Register Zero	1
IORI,r	v	Inclusive or Immediate	2
IORR,r	(*)a	Inclusive or Relative	2
IORA,r	(*)a(,X)	Inclusive or Absolute	3
EORZ	r	Exclusive or to Register Zero	1
EORI,r	v	Exclusive or Immediate	2
EORR,r	(*)a	Exclusive or Relative	2
EORA,r	(*)a(,X)	Exclusive or Absolute	3
COMPARISON INSTRUCTIONS			
COMZ	r	Compare to Register Zero	1
COMI,r	v	Compare Immediate	2
COMR,r	(*)a	Compare Relative	2
COMA,r	(*)a(,X)	Compare Absolute	3

Instruction	Operands	Description	Length (bytes)
ROTATE INSTRUCTIONS			
RRR,r		Rotate Register Right	1
RRL,r		Rotate Register Left	1
BRANCH INSTRUCTIONS			
BCTR,v	(*)a	Branch on Condition True Relative	2
BCFR,v	(*)a	Branch on Condition False Relative	2
BCTA,v	(*)a	Branch on Condition True Absolute	3
BCFA,v	(*)a	Branch on Condition False Absolute	3
BRNR,r	(*)a	Branch on Register Non-Zero Relative	2
BRNA,r	(*)a	Branch on Register Non-Zero Absolute	3
BIRR,r	(*)a	Branch on Incrementing Register Relative	2
BIRA,r	(*)a	Branch on Incrementing Register Absolute	3
BDRR,r	(*)a	Branch on Decrementing Register Relative	2
BDRA,r	(*)a	Branch on Decrementing Register Absolute	3
BXA	(*)a(,x)	Branch Indexed Absolute, Unconditional	3
ZBRR	(*)a	Zero Branch Relative, Unconditional	2
SUBROUTINE BRANCH/RETURN INSTRUCTIONS			
BSTR,v	(*)a	Branch to Subroutine on Condition True, Relative	2
BSFR,v	(*)a	Branch to Subroutine on Condition False, Relative	2
BSTA,v	(*)a	Branch to Subroutine on Condition True, Absolute	3
BSFA,v	(*)a	Branch to Subroutine on Condition False, Absolute	3
BSNR,r	(*)a	Branch to Subroutine on Non-Zero Register, Relative	2
BSNA,r	(*)a	Branch to Subroutine on Non-Zero Register, Absolute	3
BSXA	(*)a(,x)	Branch to Subroutine, Indexed, Unconditional	3
RETC,v		Return From Subroutine, Conditional	1
RETE,v		Return From Subroutine and Enable Interrupt, Conditional	1
ZBSR	(*),a	Zero Branch to Subroutine Relative, Unconditional	2
PROGRAM STATUS INSTRUCTIONS			
LPSU		Load Program Status, Upper	1
LPSL		Load Program Status, Lower	1
SPSU		Store Program Status, Upper	1
SPSL		Store Program Status, Lower	1
CPSU	v	Clear Program Status, Upper, Selective	2
CPSL	v	Clear Program Status, Lower, Selective	2
PPSU	v	Preset Program Status, Upper, Selective	2
PPSL	v	Preset Program Status, Lower, Selective	2
TPSU	v	Test Program Status, Upper, Selective	2
TPSL	v	Test Program Status Lower, Selective	2
INPUT/OUTPUT INSTRUCTIONS			
WRTD,r		Write Data	1
REDD,r		Read Data	1
WRTC,r		Write Control	1
REDC,r		Read Control	1
WRTE,r	v	Write Extended	2
REDE,r	v	Read Extended	2
MISCELLANEOUS INSTRUCTIONS			
HALT		Halt, Enter Wait State	1
DAR,r		Decimal Adjust Register	1
TMI,r	v	Test Under Mask Immediate	2
NOP		No Operation	1



BLOCK DIAGRAM

APPENDIX B

This appendix contains alisting of the assembly language program used to implement the remote magnetic indicator heading algorithm. The program was assembled on the A2650 cross assembler program operational on the HP 2100 computer at the University of Santa Clara.

```
PIP ASSEMBLER VERSION SCU LEVEL 1 HEADING INSTRUMENT ASSEMLY PROGRAM  1976 PAGE    2

LINE  ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE

                                        ****************************************************
        0000                            R0      EQU      0
        0001                            R1      EQU      1
        0002                            R2      EQU      2
        0003                            R3      EQU      3
        0000                            ON      EQU      0
        0000                            Z       EQU      0
        0001                            P       EQU      1
        0002                            N       EQU      2
        0000                            EQ      EQU      0
        0001                            GT      EQU      1
        0001                            C       EQU      1
        0002                            LT      EQU      2
        0003                            UN      EQU      3
        0002                            COM     EQU      H'02'
        0010                            RS      EQU      H'10'
        0008                            WC      EQU      H'08'
        000F                            LOWR    EQU      H'0F'
        00F0                            UPPR    EQU      H'F0'
        0040                            FLAG    EQU      H'40'
        0020                            II      EQU      H'20'
        00FF                            NSAM    EQU      H'FF'
        0001                            MSB     EQU      1           OUTPUT
        0002                            LSB     EQU      2           OUTPUT
        0004                            THTA    EQU      4
        0008                            PHI     EQU      8
        0010                            SBIT    EQU      16
                                        ****************************************************
                                        ****    DATA STORAGE IN RAM      ****
                                                ORG      H'450'
        0450                            DATA    RES      10          RAW DATA
        045A                            DATM    RES      10          CORRECTED DATA
        0464                            LUPS    RES      1
        0465                            TEMP    RES      2
        0467                            SIND    RES      2
        0469                            SINR    RES      2
        046B                            HH2     RES      2           HH SQUARED DATA
        046D                            SIGN    RES      1
        046E                            HX2     RES      2           HX SQUARED DATA
        0470                            HY2     RES      2
        0472                            HZ2     RES      2
        0474                            QUAD    RES      1
        0475                            COSR    RES      2
        0477                            RSLT    RES      2
        0479                            OPR1    RES      2
        047B                            OPR2    RES      2
        047D                            STEM    RES      1
        047E                            SRES    RES      1
        047F                            DATX    RES      2
        0481                            DATY    RES      2
        0483                            DATZ    RES      2
        0485                            COST    RES      1
```

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PIP ASSEMBLER VERSION SCU LEVEL 1 HEADING INSTRUMENT ASSEMLY PROGRAM  1976 PAGE    3

LINE  ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE

 55         0446                        SIGX   RES      1
 56         0447                        SIGY   RES      1
 57         0448                        SIGZ   RES      1
 58         0449                        ABUF   RES      1
 59         044A                        BBUF   RES      1
 60         044B                        CBUF   RES      1
 61         044C                        DBUF   RES      1
 62         044D                        TBUF   RES      1
 63         044E                        OSET   RES      6
 64         0454                        SOST   RES      6
 65                                     ***************************************************
 66                                     ****   CONSTANTS IN ROM        ****
 67                                            ORG     H'500'
 68   0500  0500  00 01 03 04           SIN    DATA    H'00,01,03,04,06,07,09,0A,0C,0E,0F'
                  06 07 09 0A
                  0C 0E 0F
 69   050B        11 12 14 15                  DATA    H'11,12,14,15,17,19,1A,1C,1D,1F,20'
                  17 19 1A 1C
                  1D 1F 20
 70   0516        22 24 25 27                  DATA    H'22,24,25,27,28,2A,2B,2D,2E,30,31'
                  28 2A 2B 2D
                  2E 30 31
 71   0521        33 35 36 38                  DATA    H'33,35,36,38,39,3B,3C,3E,3F,41,42'
                  39 3B 3C 3E
                  3F 41 42
 72   052C        44 45 47 48                  DATA    H'44,45,47,48,4A,4B,4D,4E,50,51,53'
                  4A 4B 4D 4E
                  50 51 53
 73   0537        54 56 57 59                  DATA    H'54,56,57,59,5A,5C,5D'    61 DEG.
                  5A 5C 5D
 74   053E        5F 60 61 63                  DATA    H'5F,60,61,63,64,66,67,69,6A,6C,6D'
                  64 66 67 69
                  6A 6C 6D
 75   0549        6E 70 71 73                  DATA    H'6F,70,71,73,74,75,77,78,7A,7B,7C'
                  74 75 77 78
                  7A 7B 7C
 76   0554        7E 7F 80 82                  DATA    H'7E,7F,80,82,83,84,86,87,88,8A,8B,8C'
                  83 84 86 87
                  88 8A 8B 8C
 77   0560        8E 8F 90 92                  DATA    H'8E,8F,90,92,93,94,95,97,98,99,9B'
                  93 94 95 97
                  98 99 9B
 78   056B        9C 9D 9E 9F                  DATA    H'9C,9D,9E,9F,A1,A2,A3,A4,A6,A7,A8'
                  A1 A2 A3 A4
                  A6 A7 A8
 79   0576        A9 AA AB AD                  DATA    H'A9,AA,AB,AD,AE,AF,B0,B1,B2,B3,B5'
                  AE AF B0 B1
                  B2 B3 B5
 80   0581        B6 B7 B8 B9                  DATA    H'B6,B7,B8,B9,BA,BB,BC,BD,BF,C0,C1,C2'
                  BA BB BC BD
                  BF C0 C1 C2
 81   058D        C3 C4 C5 C6                  DATA    H'C3,C4,C5,C6,C7,C8,C9,CA,CB,CC,CD'
                  C7 C8 C9 CA
                  CB CC CD
```

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FIP ASSEMBLER VERSION SCU LEVEL 1 HEADING INSTRUMENT ASSEMLY PROGRAM  1976 PAGE    4

LINE  ADDR  LABL   B1 B2 B3 B4 ERROR SOURCE

 82   0598         CE CF 00 01               DATA     H,CE,CF,00,D1,02,03,03,04,05,06,07'
                   D2 D3 D3 D4
                   D5 D6 D7
 83   05A3         D8 D9 D9 DA               DATA     H,D8,D9,D9,DA,D8,DC,DD,DD,DE,DF,E0'
                   D8 DC DD DD
                   DE DF E0
 84   05AE         E1 E1 E2 E3               DATA     H,E1,E1,E2,E3,E3,E4,E5,E6,E6,E7,E8'
                   E3 E4 E5 E6
                   E6 E7 E8
 85   05B9  05B9   FF FF FF FF         COS   DATA     H,FF,FF,FF,FF,FF,FF,FF,FF,FF,FF'
                   FF FF FF FF
                   FF FF
 86   05C3         FF FF FF FF               DATA     H,FF,FF,FF,FF,FF,FE,FE,FE,FE,FE'
                   FF FF FE FF
                   FE FE
 87   05CD         FE FD FD FD               DATA     H,FE,FD,FD,FD,FD,FC,FC,FC,FC,FB'
                   FD FC FC FC
                   FC FB
 88   05D7         FB FB FB FA               DATA     H,FB,FB,FB,FA,FA,FA,F9,F9,F9,F8'
                   FA FA F9 F9
                   F9 F8
 89   05E1         F8 F7 F7 F7               DATA     H,F8,F7,F7,F7,F6,F6,F5,F5,F4,F4'
                   F6 F6 F5 F5
                   F4 F4
 90   05EB         F4 F3 F3 F2               DATA     H,F4,F3,F3,F2,F2,F1,F1,F0,EF,EF'
                   F2 F1 F1 F0
                   EF EF
 91   05F5         EE EE ED ED               DATA     H,EE,EE,ED,ED,EC,EB,EB,EA,EA,E9,E8,E8'
                   EC EB EB EA
                   EA E9 E8 E8
 92   0601         E7 E6 E6 E5               DATA     H,E7,E6,E6,E5,E4,E3,E3,E2,E1,E1'
                   E4 E3 E3 E2
                   E1 E1
 93   060B         E0 DF DE DD               DATA     H,E0,DF,DE,DD,DD,DC,DB,DA,D9,D9'
                   DD DC DB DA
                   D9 D9
 94   0615         D8 D7 D6 D5               DATA     H,D8,D7,D6,D5,D4,D3,D3,D2,D1,D0'
                   D4 D3 D3 D2
                   D1 D0
 95   061F         CF CE CD CC               DATA     H,CF,CE,CD,CC,CB,CA,C9,C8,C7,C6'
                   CB CA C9 C8
                   C7 C6
 96   0629         C5 C4 C3 C2               DATA     H,C5,C4,C3,C2,C1,C0,BF,BE,BD,BC'
                   C1 C0 BF BE
                   BD BC
 97   0633         BB BA B9 B8               DATA     H,BB,BA,B9,B8,B7,B6,B5,B3,B2,B1'
                   B7 B6 B5 B3
                   B2 B1
 98   063D         B0 AF AE AD               DATA     H,B0,AF,AE,AD,AB,AA,A9,A8,A7,A6,A4'
                   AB AA A9 A8
                   A7 A6 A4
 99   0648         A3 A2 A1 9F               DATA     H,A3,A2,A1,9F,9E,9D,9C,9B,99,98,97,95'
                   9E 9D 9C 9B
                   99 98 97 95
```

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PIP ASSEMBLER VERSION SCU LEVEL 1 HEADING INSTRUMENT ASSEMLY PROGRAM  1976 PAGE    5

LINE  ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE

100   0654        94 93 92 90              DATA    H'94,93,92,90,8F,8E,8C,8B,8A,88'
                  8F 8E 8C 8B
                  8A 88
101   065E        87 86 84 83              DATA    H'87,86,84,83,82,80,7F,7E,7C,7B'
                  82 80 7F 7E
                  7C 7B
102   0668        7A 78 77 75              DATA    H'7A,78,77,75,74,73,71,70,6E,6D,6C'
                  74 73 71 70
                  6E 6D 6C
103   0673        6A 69 67 66              DATA    H'6A,69,67,66,64,63,61,60,5F,5D,5C'
                  64 63 61 60
                  5F 5D 5C
104   067E        5A 59 57 56              DATA    H'5A,59,57,56,54,53,51,50,4E,4D'
                  54 53 51 50
                  4E 4D
105   0688        4B 4A 48 47              DATA    H'4B,4A,48,47,45,44,42,41,3F,3E,3C'
                  45 44 42 41
                  3F 3E 3C
106   0693        3B 39 38 36              DATA    H'3B,39,38,36,35,33,31,30,2E,2D'
                  35 33 31 30
                  2E 2D
107   069D        2B 2A 28 27              DATA    H'2B,2A,28,27,25,24,22,20,1F,1D'
                  25 24 22 20
                  1F 1D
108   06A7        1C 1A 19 17              DATA    H'1C,1A,19,17,15,14,12,11,0F,0E'
                  15 14 12 11
                  0F 0E
109   06B1        0C 0A 09 07              DATA    H'0C,0A,09,07,06,04,03,01,00'
                  06 04 03 01
                  00
110   06BA  06BA  7F 7B 77 72        COSM  DATA    H'7F,7B,77,72,6E,69,65,61,5C,58,54'
                  6E 69 65 61
                  5C 58 54
111   06C5        50 4B 47 43              DATA    H'50,4B,47,43,3F,3C,38,34,31,2D,2A'
                  3F 3C 38 34
                  31 2D 2A
112   06D0        27 23 20 1D              DATA    H'27,23,20,1D,1B,18,15,13,11,0E,0C'
                  1B 18 15 13
                  11 0E 0C
113   06DB        0B 09 07 06              DATA    H'0B,09,07,06,04,03,02,01,01,00,00,00,00'
                  04 03 02 01
                  01 00 00 00
                  00
114   06E8  06E8  FF 88 12 9E        COSL  DATA    H'FF,88,12,9E,2F,C5,63,08,B7,72,38,0C'
                  2F C5 63 08
                  B7 72 38 0C
115   06F4        F0 E3 E8 FF              DATA    H'F0,E3,E8,FF,2B,6C,C3,32,B9,59,15,EC'
                  2B 6C C3 32
                  B9 59 15 EC
116   0700        F0 F2 22 72              DATA    H'E0,F2,22,72,E2,73,26,FB,F4,10,52,B8'
                  E2 73 26 FB
                  F4 1  52 B8
```

```
LINE  ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE

117   070C        43 F5 CD CC              DATA     H'43,F5,CD,CC,F1,3E,B3,4F,13,00'
                  F1 3E B3 4F
                  13 00
118   0716  0716  01 02 04 08        BCD   DATA     H'01,02,04,08,16,32,64,00'
                  16 32 64 00
119   071E  071E  03                 SINE  DATA     H'03'
120                                  *************************************************
121                                        ORG      H'750'
122                                  *************************************************
123                                  *  INIALIZATION OF THE INSTRUMENT               *
124                                  *  PARAMETERS AFFECTED INCLUDE:                 *
125                                  *   (1) PR. STATUS LOWER/UPPER                  *
126                                  *   (2) CONTENTS OF REGS. C&D ON CPU BOARD      *
127                                  *************************************************
128                                  ****  INITIALIZE                ****
129   0750        76 20                    PPSU     II           INHIBIT INTERRUPTS
130   0752        75 FF                    CPSL     H'FF'        CLEAR ALL
131   0754        77 02                    PPSL     COM          LOGICAL COMPARISONS
132   0756        04 FF                    LODI,R0  NSAM
133   0758        F0                       WRTD,R0               NO SAMPLE/HLD OR MUX
134   0759        74 FF                    CPSL     H'FF'        CLEAR ALL
135                                  **
136                                  ****  START MAIN PROGRAM        ****
137   075B  075B  3F 07 80           MAIN  BSTA,UN  SAMP         SAMPLE ALL DATA CHANNELS
138   075E        3F 08 E6                 BSTA,UN  ROTX         CORRECT HX DATA
139   0761        3F 0A 71                 BSTA,UN  ROTY         CORRECT HY DATA
140   0764        3F 0A 09                 BSTA,UN  HVEC         COMPUTE HORIZ. VECTOR
141   0767        3F 07 70                 BSTA,UN  WICH         CALC. HEADING & O/P
142   076A        3F 08 97                 BSTA,UN  OUTA         O/P ROLL &PITCH
143   076D        1F 07 5B                 BCTA,UN  MAIN         LOOP FOREVER
144                                  *************************************************
145                                  *
146                                  *
147   0770  0770  77 02              WICH  PPSL     COM
148   0772        0C 04 70                 LODA,R0  HY2
149   0775        EC 04 6E                 COMA,R0  HX2
150   0778        1D 08 14                 BCTA,GT  COSY
151   077B        1E 08 2B                 BCTA,LT  SINY
152   077E        0C 04 71                 LODA,R0  HY2+1        LS BITS
153   0781        EC 04 6F                 COMA,R0  HX2+1
154   0784        1D 08 14                 BCTA,GT  COSY
155   0787        1F 08 2B                 BCTA,UN  SINY
156                                  *************************************************
```

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PIP ASSEMBLER VERSION SCU LEVEL 1 HEADING INSTRUMENT ASSEMLY PROGRAM  1976 PAGE   7

LINE  ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE
                                             ORG        H'780'
                                  ***********************************************
                                  * A SUBROUTINE TO SAMPLE DATA FROM ALL SENSORS  *
                                  *  REGISTER "D" CONTROLS SAMPLE/HOLD & MUX      *
                                  *   BITS 0,1,&2 SELECT HX,HY,HZ,THETA,PHI       *
                                  * DATA IS STORED IN DOUBLE PRECISION FORM       *
                                  * BEGINNING AT "DATA" WITH HX(MSB) FOLLOWED BY  *
                                  *     HX(LSB),HY,HZ,PITCH,ROLL.                 *
                                  *   (1) CONVERTS A/D DATA TO SIGN MAGNITUDE     *
                                  *   (2) CHANGES SIGN OF HY & HZ(ORIENTATION)    *
                                  *   (3) CORRECTS FOR OFFSET ERROR               *
                                  *  EXIT WITH CORRECTED DATA IN TABLE "DATA"     *
                                  ***********************************************
                                  ****   BEGIN SUBROUTINE          ****
      0780  0780  05 FF           SAMP   LODI,R1   -1              DATA INDEX
      0782        06 05                  LODI,R2   5               LOOP COUNTER
      0784        07 01                  LODI,R3   1               1ST SAMPLE
      0786        75 09                  CPSL      C,WC
      0788  0788  F3                NEXT WRTD,R3                   SELECT & SAMPLE DATA
      0789        67 80                  IORI,R3   H'80'           READY TO HOLD
      078B        F3                     WRTD,R3                   HOLD DATA
      078C        04 80                  LODI,R0   H'80'
      078E        B0                     WRTC,R0                   RESET A/D
      078F        20                     EORZ      R0
      07C0        B0                     WRTC,R0                   START A/D
      07C1  07C1  70                TEST REDD,R0                   READ LSB
      07C2        F4 01                  TMI,R0    H'01'
      07C4        1A 7B                  BCTR,N    TEST            BRANCH BACK IF EOC=1
      07C6        24 FF                  EORI,R0   H'FF'
      07C8        44 F0                  ANDI,R0   UPPR            STRIP OFF 4 LS BITS
      07CA        CD 24 51               STRA,R0   DATA+1,R1,+     STORE LS 1/3
      07CD        30                     REDC,R0                   READ MS 2/3
      07CE        24 FF                  EORI,R0   H'FF'
      07D0        CD 24 4F               STRA,R0   DATA-1,R1,+     STORE MSB
      07D3        87 01                  ADDI,R3   1               POINT TO NEXT DATA
      07D5        47 0F                  ANDI,R3   LOWR            POINT TO CHAN. ONLY
      07D7        FA 5F                  BDRR,R2   NEXT            TEST LOOP COUNTER
                                  ***********************************************
                                  *   CONVERT TO SIGN MAGNITUDE
      07D9        77 08                  PPSL      WC              ENABLE CARRY
      07DB        06 FF                  LODI,R2   -1              INDEX
      07DD        07 07                  LODI,R3   7               LOOP COUNTER
      07DF  07DF  0E 24 50          CONT LODA,R0   DATA,R2,+       GET MSB OF DATA
      07E2        1A 13                  BCTR,N    RSTO            BR. IF SIGN=1
      07E4        C1                     STRZ      R1
      07E5        20                     EORZ      R0              CLEAR R0
      07E6        77 01                  PPSL      C               SET UP FOR SUBT.
      07E8        AE 24 50               SUBA,R0   DATA,R2,+       SUB. LS BITS
      07EB        CE 64 5A               STRA,R0   DATM,R2         STORE NEW DATA
      07EE        20                     EORZ      R0
      07EF        A1                     SUBZ      R1              SUB. MS BITS
      07F0        64 80                  IORI,R0   H'80'           SET SIGN=1 (NEG)
```

```
ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE

07F2        CE 64 59                  STRA,R0   DATM-1,R2   STORE MS BITS
07F5        18 08                     BCTR,UN   FINI
07F7  07F7  44 7F               RSTO  ANDI,R0   H'7F'       SIGN BIT = 0
07F9        CE 64 5A                  STRA,R0   DATM,R2     STORE NEW DATA
07FC        0E 24 50                  LODA,R0   DATA,R2,+   GET  LS DATA
07FF        CE 64 5A                  STRA,R0   DATM,R2     STORE AS NEW DATA
0802  0802  F8 58               FINI  BDRR,R3   CONT
                                *   CHANGEBSIGN OF HY &HZ (SENSOR ORIENTATION)
0804        0C 04 5C                  LODA,R0   DATM+2      CHANGE SIGN OF HY & HZ
0807        0D 04 5E                  LODA,R1   DATM+4
080A        24 80                     EORI,R0   H'80'
080C        25 80                     EORI,R1   H'80'
080E        CC 04 5C                  STRA,R0   DATM+2
0811        CD 04 5E                  STRA,R1   DATM+4
                                *   CORRECT FOR OFFSET ERROR IN HX,HY, & HZ CHANNELS
0814        07 00                     LODI,R3   0           DO HX 1ST
0816        3F 08 2A                  BSTA,UN   OFST
0819        07 02                     LODI,R3   2           HY
081B        3F 08 2A                  BSTA,UN   OFST
081E        07 04                     LODI,R3   4           HZ
0820        3F 08 2A                  BSTA,UN   OFST
0823        C0                        NOP
0824        07 07                     LODI,R3   7
                                *   CORRECT FOR ORTHOGONALITY ERROR IN HX AXIS
0826        3F 0F 88                  BSTA,UN   ORTH
0829        17                        RETC,UN
                                ***************************************************
```

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PIP ASSEMBLER VERSION SCU LEVEL 1 HEADING INSTRUMENT ASSEMLY PROGRAM  1976 PAGE    9

LINE  ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE

 238                                        ************************************************
 239                                        ************************************************
 240  082A  082A  0F 64 5A                  OFST    LODA,R0    DATM,R3      MS BYTE OF DATA
 241  082D        C1                                STRZ       R1
 242  082E        44 7F                             ANDI,R0    H'7F'        STRIP SIGN
 243  0830        45 80                             ANDI,R1    H'80'        SAVE SIGN
 244  0832        CC 04 77                          STRA,R0    RSLT
 245  0835        CD 04 7E                          STRA,R1    SAES
 246  0838        0F 64 5B                          LODA,R0    DATM+1,R3    LS BYTE OF DATA
 247  083B        CC 04 78                          STRA,R0    RSLT+1
 248  083E        0F 64 8E                          LODA,R0    OSET,R3      MS BYTE OF OFFSET
 249  0841        CC 04 65                          STRA,R0    TEMP
 250  0844        0F 64 8F                          LODA,R0    OSET+1,R3
 251  0847        CC 04 66                          STRA,R0    TEMP+1
 252  084A        0F 64 94                          LODA,R0    SOST,R3
 253  084D        CC 04 7D                          STRA,R0    STE
 254  0850        75 02                             CPSL       COM
 255  0852        77 10                             PPSL       RS           BANK=1
 256  0854        3F 09 98                          BSTA,UN    SADD
 257  0857        75 10                             CPSL       RS           BANK=0
 258  0859        0C 04 77                          LODA,R0    RSLT
 259  085C        CF 64 5A                          STRA,R0    DATM,R3
 260  085F        0C 04 78                          LODA,R0    RSLT+1
 261  0862        CF 64 5B                          STRA,R0    DATM+1,R3
 262  0865        17                                RETC,UN
 263                                        ************************************************
 264                                        *       BINARY MULT. FOR A TWO-BYTE INTEGER    *
 265                                        *       MULT. BY A SINGLE BYTE INTEGER         *
 266                                        *       MULTIPLIER IS IN OPR1+1                *
 267                                        *       MULTIPLICAND IS IN OPR2,OPR2+1         *
 268                                        *       RESULT WILL BE IN RSLT,RSLT+1          *
 269                                        *       (2 BYTES ARE DISCARDED )               *
 270                                        ************************************************
 271                                        ****    BEGIN SUBROUTINE                ****
 272  0866  0866  77 08                     SMPY    PPSL       WC           SET MODE
 273  0868        20                                EORZ       R0
 274  0869        CC 04 77                          STRA,R0    RSLT         CLEAR RESULT
 275  086C        CC 04 78                          STRA,R0    RSLT+1       CLEAR RESULT+1
 276  086F        CC 04 79                          STRA,R0    OPR1
 277  0872        07 08                             LODI,R3    8            LOAD COUNT
 278  0874  0874  0E 04 7A                  LOCO    LODA,R2    OPR1+1       GET MULTIPLIER
 279  0877  0877  52                        LOOO    RRR,R2                  ROT. RIGHT WITH CARRY
 280                                        *  THIS ROTATES MULTIPLIER BY 1 BIT TO GET LSB INTO CARRY
 281  0878        20                                EORZ       R0           CLEAR R0
 282  0879        D0                                RRL,R0                  GET CARRY INTO LSB
 283  087A        F8 00                             BDRR,R0    LOC4         BRANCH IF C=0
 284  087C  087C  05 02                     LOC1    LODI,R1    2            INDEX
 285  087E  087E  0D 64 76                  LOC2    LODA,R0    RSLT-1,R1    ADD MULTIPLICAND TO PRODUCT
 286  0881        8D 64 7A                          ADDA,R0    OPR2-1,R1
 287  0884        CD 64 76                          STRA,R0    RSLT-1,R1
 288  0887        F9 75                             BDRR,R1    LOC2         FINISH THE ADD
 289                                        *
```

```
ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE

0889  0889  05 FE           LOC4  LODI,R1   -2             ROT. THE PRODUCT
088B  088B  0D 63 79        LOC5  LODA,R0   RSLT-256+2,R1
088E        50                    RRR,R0
088F        CD 63 79              STRA,R0   RSLT-256+2,R1
0892        D9 77                 BIRR,R1   LOC5
0894        F5 61                 BDRR,R3   LOO0           FINISH THE LOOP
                            ********************************************
0896        17                    RETC,UN
                            ********************************************
                            *  A SUBROUTINE TO OUTPUT PITCH & ROLL DATA   *
                            *  CALLED BY MAIN PROG. , FETCHES CONVERTED DATA*
                            *  FROM TABLE "DATM" THEN O/P'S BCD ANGLE + SIGN*
                            ********************************************
0897  0897  0C 04 60        OUTA  LODA,R0   DATM+6         FETCH THETA (MSB)
089A        C3                    STRZ      R3
089B        44 80                 ANDI,R0   H'80'          SAVE SIGN
089D        1A 10                 BCTR,N    NTH            BR. ON NEG THETA
089F        2C 04 62              EORA,R0   DATM+8         COMPARE SIGNS
08A2        1A 06                 BCTR,N    NPHI
08A4        04 07                 LODI,R0   7
08A6        D4 10                 WRTE,R0   SRIT           BOTH POSITIVE
08A8        1B 16                 BCTR,UN   AGL
08AA  08AA  20              NPHI  EORZ      R0
08AB        D4 10                 WRTE,R0   SRIT           PHI NEG.
08AD        1B 0F                 BCTR,UN   AGL
08AF  08AF  2C 04 62        NTH   EORA,R0   DATM+8
08B2        19 06                 BCTR,P    NPH
08B4        04 03                 LODI,R0   3
08B6        D4 10                 WRTE,R0   SRIT           PITCH NEG.& ROLL POS.
08B8        1B 04                 BCTR,UN   AGL
08BA  08BA  04 08           NPH   LODI,R0   8
08BC        D4 10                 WRTE,R0   SRIT           BOTH NEGATIVE
08BE  08BE  3F 08 CC        AGL   BSTA,UN   LOD            FORM ANGLE (BCD)
08C1        D5 04                 WRTE,R1   THTA           OUTPUT PITCH
08C3        0F 04 62              LODA,R3   DATM+6         FETCH ROLL
08C6        3F 08 CC              BSTA,UN   LOD            ANGLE(BCD)
08C9        D5 08                 WRTE,R1   PHI            OUTPUT PHI
08CB        17                    RETC,UN
                            ********************************************
08CC  08CC  47 7F           LOD   ANDI,R3   H'7F'
08CE        CF 04 7C              STRA,R3   OPR2+1         MULTIPLICAND
08D1        20                    EORZ      R0
08D2        CC 04 7B              STRA,R0   OPR2           CLEAR LS BYTE
08D5        04 5A                 LODI,R0   H'5A'          FACTOR(.35156)
08D7        CC 04 7A              STRA,R0   OPR1+1         MULTIPLIER
08DA        3F 08 66              BSTA,UN   SMPY           BINARY ANGLE
08DD        0C 04 78              LODA,R0   RSLT+1         ANGLE
08E0        3F 0C 64              BSTA,UN   BCDA           ANGLE(BCD)
08E3        25 FF                 EORI,R1   H'FF'
08E5        17                    RETC,UN
                            ********************************************
```

```
LINE  AUDR  LABL  B1 B2 B3 B4 ERROR SOURCE

                                   ****************************************************
                                   *   S.R. ROTX TO CORRECT HX DATA USING ALGORITHM  *
                                   *       HX=HX**COS(PITCH)+SIN(PITCH)*(HYM*SIN(    *
                                   *              ROLL)+HZM*COS(ROLL))               *
                                   *   ENTER WITH "RAW DATA" IN TABLE DATM           *
                                   *   EXIT WITH MODIFIED HX IN "DATX"               *
                                   *   CALLS S.R. HZRI                               *
                                   *   CALLS S.R.S. "SADD" & "SMPY" TO DO             *
                                   *   SIGNED MAGNITUDE ADD/SUBT AND MULTIPLY         *
                                   ****************************************************
      08E6  08E6  77 09            ROTX  PPSL      WC+C       ARITH WITH CARRY. C=1
      08E8        0C 04 60               LODA,R0   DATM+6     THETA (PITCH)
      08EB        C1                     STRZ      R1
      08EC        C3                     STRZ      R3
      08ED        45 7F                  ANDI,R1   H'7F'      STRIP OFF SIGN
      08EF        47 80                  ANDI,R3   H'80'      SAVE SIGN
      08F1        CF 04 67               STRA,R3   SINO       SIGN THETA
      08F4        0D 65 00               LODA,R0   SIN,R1     SIN(THETA)
      08F7        CC 04 68               STRA,R0   SINO+1
      08FA        0D 65 B9               LODA,R0   COS,R1     COS(THETA)
      08FD        CC 04 65               STRA,R0   COST
      0900        CC 04 7A               STRA,R0   OPR1+1     MULTIPLIER
      0903        0C 04 5A               LODA,R0   DATM       HX MSBITS
      0906        C1                     STRZ      R1
      0907        44 7F                  ANDI,R0   H'7F'      STRIP OFF SIGN
      0909        45 80                  ANDI,R1   H'80'      SAVE SIGN
      090B        CC 04 78               STRA,R0   OPR2       MULTIPLICAND
      090E        CD 04 96               STRA,R1   SIGX
      0911        0C 04 5B               LODA,R0   DATM+1
      0914        CC 04 7C               STRA,R0   OPR2+1
      0917        3F 0B 66               BSTA,UN   SMPY       FORM HX* COS (THETA)
      091A        0C 04 77               LODA,R0   RSLT       MOVE PRODUCT
      091D        CC 04 8B               STRA,R0   CBUF
      0920        0C 04 78               LODA,R0   RSLT+1
      0923        CC 04 8C               STRA,R0   DBUF
      0926        3F 0F 40               BSTA,UN   HZRI
      0929        0C 04 68               LODA,R0   SINO+1     SIN (THETA)
      092C        CC 04 7A               STRA,R0   OPR1+1     MULTIPLIER
      092F        3F 0B 66               BSTA,UN   SMPY       HZ*SIN(THETA)
      0932        0C 04 67               LODA,R0   SINO       SIGN OF THETA
      0935        CC 04 7E               STRA,R0   SGES       FOR USE BY SADD
      0938        0D 04 96               LODA,R1   SIGX
      093B        CD 04 70               STRA,R1   STEM
      093E        0C 04 8B               LODA,R0   CBUF
      0941        0D 04 8C               LODA,R1   DBUF
      0944        CC 04 65               STRA,R0   TEMP
      0947        CD 04 66               STRA,R1   TEMP+1
      094A        75 62                  CPSL      COM        FLAG = ADD
      094C        3F 09 98               BSTA,UN   SADD       FORM NEW HX
      094F        0C 04 78               LODA,R0   RSLT+1     MOVE NEW HX
      0952        CC 04 40               STRA,R0   DATX+1
      0955        0C 04 77               LODA,R0   RSLT
```

```
ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE

0958        CC 04 7F                    STRA,R0   DATX
095B        17                          RETC,UN
                                  ****************************************************
                                  *  SUBROUTINE TO SQUARE TWO 2 BYTE VALUES          *
                                  *  ENTER WITH VALUE  IN R1(MSB) & R2(LSB)          *
                                  *  EXIT WITH 2 MSB IN RSLT & RSLT+1                *
                                  ****************************************************
                                  ****  BEGIN                    ****
095C  095C  77 08                 SQU   PPSL      WC
095E        20                          EORZ      R0
095F        CC 04 77                    STRA,R0   RSLT           CLEAR TEMP STORAGE
0962        CC 04 78                    STRA,R0   RSLT+1
0965        07 10                       LODI,R3   16             COUNTER
0967        45 7F                       ANDI,R1   H'7F'
0969        CD 04 7B                    STRA,R1   OPR2
096C        CE 04 7C                    STRA,R2   OPR2+1
096F  096F  51                    ROTA  RRR,R1                   ROTATE MULTIPLIER
0970        52                          RRR,R2
0971        20                          EORZ      R0
0972        D0                          RRL,R0                   GET LS BIT INTO R0
0973        F8 12                       BDRR,R0   NOAD           NO ADD IF C=0
0975        0C 04 78                    LODA,R0   RSLT+1
0978        8C 04 7C                    ADDA,R0   OPR2+1
097B        CC 04 78                    STRA,R0   RSLT+1
097E        0C 04 77                    LODA,R0   RSLT
0981        8C 04 7B                    ADDA,R0   OPR2
0984        1F 09 8A                    BCTA,UN   NOAD+3
0987  0987  0C 04 77              NOAD  LODA,R0   RSLT
098A        50                          RRR,R0
098B        CC 04 77                    STRA,R0   RSLT
098E        0C 04 78                    LODA,R0   RSLT+1
0991        50                          RRR,R0                   ROTATE LS 1/2 RESULT
0992        CC 04 78                    STRA,R0   RSLT+1
0995        FB 58                       BDRR,R3   ROTA           CONT. MULT. IF REQ.
                                  ****************************************************
0997        17                          RETC,UN                  END OF SUBROUTINE
```

REPRODUCIBILITY OF THE ORIGINAL PAGE IS POOR


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LINE  ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE

431                                   ************************************************
432                                   *   S.R. SADD TO PERFORM DOUBLE PRECISION       *
433                                   *   SIGN & MAGNITUDE ADD/SUBT                   *
434                                   *   ADDEND (SUBTRAHEND) IN TEMP, TEMP+1         *
435                                   *   AUGEND (MINUEND) IN RSLT, RSLT+1            *
436                                   *   STEM = SIGN OF TERM IN TEMP SRES = SIG      *
437                                   *   COM BIT IS FLAG ((0=ADD) 1= SUBT)           *
438                                   *   SIGNED TOTAL LEFT IN RSLT, RSLT+1           *
439                                   ************************************************
440  099A  099A  77 09          SADD   PPSL      WC+C          ARITH WITH CARRY! SET CARRY
441  099A        07 02                 LODI,R3   2             INDEX
442  099C        B5 02                 TPSL      COM           TEST FLAG
443  099E        1C 0A 18              BCTA,UN   SSUB          BR. IF SUBT.
444                                *   START ADD HERE
445  09A1        0F 04 70              LODA,R3   STEM          SIGN OF TEMP
446  09A4        03                    LODZ      R3            EX. OR WITH SIGN RESULT
447  09A5        2C 04 7E              EORA,R0   SRES
448  09A8        1E 09 CA              BCTA,N    DIFR          BR. IF SIGNS DIFF
449  09AB        75 01                 CPSL      C
450  09AD        0C 04 66              LODA,R0   TEMP+1        FORM SUM
451  09B0        8C 04 78              ADDA,R0   RSLT+1
452  09B3        CC 04 78              STRA,R0   RSLT+1
453  09B6        0C 04 65              LODA,R0   TEMP
454  09B9        8C 04 77              ADDA,R0   RSLT
455  09BC        9C 09 F9              BCFA,Z    RTRN
456  09BF        0D 04 78              LODA,R1   RSLT+1        GET LS BITS
457  09C2        9C 09 F9              BCFA,Z    RTRN
458  09C5        20                    EORZ      R0
459  09C6        CC 04 77              STRA,R0   RSLT          ZERO RSLT = POS
460  09C9        17                    RETC,UN
461  09CA  09CA  06 02          DIFR   LODI,R2   2             CTR/INDEX
462  09CC        03                    LODZ      R3
463  09CD        1E 0A 02              BCTA,N    XNEG          BR. IF NX = NEG
464  09D0  09D0  0E 64 64       SUP1   LODA,R0   TEMP-1,R2     FORM TEMP - RSLT
465  09D3        AE 64 76              SUBA,R0   RSLT-1,R2
466  09D6        CE 64 76              STRA,R0   RSLT-1,R2
467  09D9        FA 75                 BDRR,R2   SUP1
468  09DB        B5 01                 TPSL      C
469  09DD        1C 0A 12              BCTA,UN   IZER          BR. IF C = 1
470  09E0  09E0  77 01          TNEG   PPSL      C
471  09E2        06 02                 LODI,R2   2             INDEX/CTR
472  09E4  09E4  20             SUP2   EORZ      R0            FORM 2'S COMPLEMENT
473  09E5        AE 64 76              SUBA,R0   RSLT-1,R2     IF C = 0
474  09E8        CE 64 76              STRA,R0   RSLT-1,R2
475  09EB        FA 77                 BDRR,R2   SUP2
476  09ED        C1                    STRZ      R1
477  09EE        9C 09 FE              BCFA,Z    NRES
478  09F1        0D 04 78              LODA,R1   RSLT+1
479  09F4        9C 09 FE              BCFA,Z    NRES
480  09F7        1B 01                 BCTR,UN   NOW
481  09F9  09F9  03             RTRN   LODZ      R3
482  09FA  09FA  CC 04 77       NOW    STRA,R0   RSLT
```

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 PIP ASSEMBLER VERSION SCU LEVEL 1 HEADING INSTRUMENT ASSEMLY PROGRAM  1976 PAGE   14

LINE  ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE

483   09FD        17                          RETC,UN
484   09FE  09FE  64 80               NRES    IORI,R0    H'80'
485   0A00        18 7R                       BCTR,UN    NOW
486                                   *   START HERE IF TERM IN "TEMP" IS NEGATIVE
487   0A02  0A02  0E 64 76            XNEG    LODA,R0    RSLT-1,R2    FORM DIFFERENCE
488   0A05        AE 64 64                    SUBA,R0    TEMP-1,R2
489   0A08        CE 64 76                    STRA,R0    RSLT-1,R2
490   0A0B        FA 75                       BDRR,R2    XNEG         LOOP ONCE
491   0A0D        B5 01                       TPSL       C
492   0A0F        9C 09 E0                    BCFA,UN    TNEG
493   0A12  0A12  44 7F               TZER    ANDI,R0    H'7F'        RESULT IS POS
494   0A14        CC 04 77                    STRA,R0    RSLT
495   0A17        17                          RETC,UN
496                                   ***************************************************
497                                   *   START HERE IF SUBTRACTION IS REQUIRED
498                                   *   I.E. (RSLT,RSLT+1) - (TEMP, TEMP+1)
499   0A18  0A18  0C 04 7E            SSUB    LODA,R0    SRES         SIGN OF RESULT
500   0A1B        C3                          STRZ       R3
501   0A1C        06 02                       LODI,R2    2
502   0A1E        2C 04 7D                    EORA,R0    STEM
503   0A21        9E 0A 36                    BCFA,N     SUBR         BR, IF SIGNS SAME
504   0A24        75 01                       CPSL       C
505   0A26  0A26  0E 64 64            SLUP    LODA,R0    TEMP-1,R2
506   0A29        8E 64 76                    ADDA,R0    RSLT-1,R2
507   0A2C        CE 64 76                    STRA,R0    RSLT-1,R2
508   0A2F        FA 75                       BDRR,R2    SLUP
509   0A31        63                          IORZ       R3
510   0A32        CC 04 77                    STRA,R0    RSLT
511   0A35        17                          RETC,UN
```

```
LINE  ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE

 513                                    *    START HERE IF BOTH TERMS SAME SIGN
 514  0A36  0A36  77 01                 SUBR  PPSL      C              C=1
 515  0A38  0A38  0C 64 76              BLUP  LODA,R0   RSLT-1,R2      RSLT = TEMP - RSLT
 516  0A3B        AE 64 64                    SUBA,R0   TEMP-1,R2
 517  0A3E        CE 64 76                    STRA,R0   RSLT-1,R2
 518  0A41        FA 75                       BDRR,R2   BLUP
 519  0A43        B5 01                       TPSL      C              TEST CARRY
 520  0A45        1C 0A 5F                    BCTA,UN   FINS
 521  0A48        06 02                       LODI,R2   2              START 2'S COMPL
 522  0A4A        77 01                       PPSL      C
 523  0A4C  0A4C  20                    CLUP  EORZ      R0             LOOP TO DO
 524  0A4D        AE 64 76                    SUBA,R0   RSLT-1,R2      2'S COMPL.
 525  0A50        CE 64 76                    STRA,R0   RSLT-1,R2
 526  0A53        FA 77                       BDRR,R2   CLUP
 527  0A55        0D 04 7E                    LODA,R1   SGES           TEST SIGN
 528  0A58        16                          RETC,N                   RTN IF NEG.
 529  0A59        64 80                       IORI,R0   H'80'
 530  0A5B        CC 04 77                    STRA,R0   RSLT
 531  0A5E        17                          RETC,UN
 532  0A5F  0A5F  C1                    FINS  STRZ      R1
 533  0A60        9C 0A 6A                    BCFA,Z    FAN
 534  0A63        0D 04 78                    LODA,R1   RSLT+1
 535  0A66        9C 0A 6A                    BCFA,Z    FAN
 536  0A69        17                          RETC,UN
 537  0A6A  0A6A  63                    FAN   IORZ      R3             SIGN = STEM
 538  0A6B        CC 04 77                    STRA,R0   RSLT
 539  0A6E        77 02                       PPSL      COM
 540  0A70        17                          RETC,UN
 541                                    **************************************************
```

```
LINE  ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE
                                   ************************************************
                                   *   S.R. ROTY TO ADJUST HY DATA USING ALGORITHM  *
                                   *        HY=HYM*COS(ROLL)-HZM*SIN(ROLL)          *
                                   *   USES SIGN AND MAGNITUDE DATA FROM TABLE DATM *
                                   *   STORES NEW DATA INTO DATY, DATY+1            *
                                   *   CALLS S.R.S. "SADD" & "SMPY" TO PERFORM SIGNED*
                                   *   MAGNITUDE ARITHMETIC                         *
                                   ************************************************
      0A71  0A71  77 09            ROTY   PPSL      WC+C
      0A73        0C 04 62                LODA,R0   DATM+R      GET ROLL ANGLE PHI
      0A76        C1                      STRZ      R1
      0A77        45 7F                   ANDI,R1   H'7F'       STRIP OFF SIGN
      0A79        2C 04 5E                EORA,R0   DATM+4      DETERMINE SIGN OF PROD.
      0A7C        44 80                   ANDI,R0   H'80'
      0A7E  0A7E  CC 04 7D         AHED   STRA,R0   STEM
      0A81        0D 65 39                LODA,R0   COS,R1      COS (PHI)
      0A84        CC 04 75                STRA,R0   COSR
      0A87        0D 65 30                LODA,R0   SIN,R1      SIN (PHI)
      0A8A        CC 04 7A                STRA,R0   OPR1+1      MULTIPLIER
      0A8D        0C 04 5E                LODA,R0   DATM+4      GET HZ
      0A90        44 7F                   ANDI,R0   H'7F'       STRIP SIGN
      0A92        CC 04 7B                STRA,R0   OPR2        MULTIPLICAND
      0A95        0C 04 5F                LODA,R0   DATM+5
      0A98        CC 04 7C                STRA,R0   OPR2+1
      0A9B        3F 0A 66                BSTA,UN   SMPY        FORM Z2$SIN(PHI)
      0A9E        0C 04 77                LODA,R0   RSLT        MOVE PRODUCT
      0AA1        CC 04 65                STRA,R0   TEMP
      0AA4        0C 04 78                LODA,R0   RSLT+1
      0AA7        CC 04 66                STRA,R0   TEMP+1
                                   *      BEGIN FORMING 2ND TERM
      0AAA        0C 04 5C                LODA,R0   DATM+2      GET HY MS BITS
      0AAD        C1                      STRZ      R1
      0AAE        45 7F                   ANDI,R1   H'7F'       STRIP SIGN
      0AB0        CD 04 7B                STRA,R1   OPR2        MULTIPLICAND
      0AB3        44 80                   ANDI,R0   H'80'       SAVE SIGN
      0AB5        CC 04 7E                STRA,R0   SRES        SIGN OF RESULT
      0AB8        0C 04 5D                LODA,R0   DATM+3      HY LS BITS
      0ABB        CC 04 7C                STRA,R0   OPR2+1
      0ABE        0C 04 75                LODA,R0   COSR
      0AC1        CC 04 7A                STRA,R0   OPR1+1      MULTIPLIER
      0AC4        3F 0A 66                BSTA,UN   SMPY
                                   *      NOW FORM NEW HY
      0AC7        77 02                   PPSL      COM
      0AC9        3F 09 4B                BSTA,UN   SADD
      0ACC        0C 04 77                LODA,R0   RSLT        MOVE ROTATED HY
      0ACF        CC 04 41                STRA,R0   DATY
      0AD2        0C 04 78                LODA,R0   RSLT+1
      0AD5        CC 04 42                STRA,R0   DATY+1
      0AD8        17                      RETC,UN
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LINE  ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE
                                 **************************************************
                                 *  S.R. HVEC TO COMPUTE HORIZ. FIELD VECTOR       *
                                 *  HH2 = HX (HORIZ)↑2 + HY (HORIZ) ↑2             *
                                 *  GETS HX & HY FROM (DATX,DATX+1) & (DATY,DATY+1)*
                                 *  RESULT INTO HH2,HH2+1                          *
                                 **************************************************
                                 HVEC   PPSL     WC
      0AD9  0AD9  77 08                 LODA,R1  DATX         GET ROTATED HX
      0ADB        0D 04 7F              LODA,R2  DATX+1
      0ADE        0E 04 40              BSTA,UN  SQU          SQUARE HX
      0AE1        3F 04 5C              LODA,R0  RSLT         MOVE RESULT
      0AE4        0C 04 77              STRA,R0  HX2
      0AE7        CC 04 8E              LODA,R0  RSLT+1
      0AEA        0C 04 78              STRA,R0  HX2+1
      0AED        CC 04 5F              LODA,R1  DATY         GET HY (ROTATED)
      0AF0        0D 04 41              LODA,R2  DATY+1
      0AF3        0E 04 42              BSTA,UN  SQU          SQUARE HY
      0AF6        3F 04 5C              LODI,R2  2
      0AF9        06 02          LUPF   LODA,R0  RSLT-1,R2    MOVE HY2
      0AFB  0AFB  0E 64 76              STRA,R0  HY2-1,R2
      0AFE        CE 64 6F              BDRA,R2  LUPF
      0B01        FE 04 FB              CPSL     C
      0B04        75 01                 LODI,R2  2
      0B06        06 02          LUPH   LODA,R0  HX2-1,R2     SUM HX↑2+HY↑2
      0B08  0B08  0E 64 5D              ADDA,R0  RSLT-1,R2
      0B0B        8E 64 76              STRA,R0  HH2-1,R2
      0B0E        CE 64 6A              BDRR,R2  LUPH
      0B11        FA 78                 RETC,UN
      0B13        17
                                 **************************************************
                                 *  S.R. TO COMPUTE YAWL = ANGLE FROM THE HORIZ.   *
                                 *                  VECTOR COMP. HH                *
                                 *                                                 *
                                 *  ENTER WHEN MAGN(HX)< MAGN(HY)                  *
                                 *    YAWL = ARCOS(HX↑2/HH↑2)                      *
                                 *    HH2= DIVISOR       OPR1...= HX2...= DIVIDEND *
                                 *    RSLT= QUOTIENT                               *
                                 **************************************************
                                 ****   BEGIN                 INDEX
                                 COSY   LODI,R1  2            LOAD OPR1+1,OPR1
      0B14  0B14  05 02          LUPE   LODA,R0  HX2-1,R1
      0B16  0B16  0D 64 5D              STRA,R0  OPR1-1,R1
      0B19        CD 64 7B              BDRR,R1  LUPE         LOOP ONCE
      0B1C        F9 78                 BSTA,UN  DIVI         FORM HX2/HH2
      0B1E        3F 08 4A              BSTA,UN  ANGL         FIND ARCOS
      0B21        3F 08 7B              BSTA,UN  BCDA         CONVERT TO BCD
      0B24        3F 0C 66              BSTA,UN  HDG          COMPUTE HEADING
      0B27        3F 0C 7C              RETC,UN
      0B2A        17
                                 **************************************************
```

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PIP ASSEMBLER VERSION SCU LEVEL 1 HEADING INSTRUMENT ASSEMLY PROGRAM  1976 PAGE  16

ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE

                                   ****************************************************
                                   *  A SUBROUTINE TO COMPUTE HEADING WHEN             *
                                   *  ( +/-)135 < YAW <(+/-)45 DEGREES                 *
                                   ****************************************************
0B2B  0B2B  05 02                  SINY    LODI,R1    2             INDEX
0B2D  0B2D  0D 64 6F               LUP     LODA,R0    HY2-1,R1      LOAD OPR1+1,OPR1
0B30        CD 64 78                       STRA,R0    OPR1-1,R1
0B33        F9 78                          BDRR,R1    LUP
0B35        20                             EORZ       R0
0B36        3F 0A 4A                       BSTA,UN    DIVI          FORM HY2/HH2
0B39        3F 0B 9B                       BSTA,UN    ANGL          FORM ARCOS(HY2/HH2)
0B3C        3F 0C 64                       BSTA,UN    BCDA          CONVERT TO BCD
0B3F        04 90                          LODI,R0    H'90'         CALC. 90-ANGLE
0B41        77 09                          PPSL       C+WC
0B43        A1                             SUBZ       R1
0B44        94                             DAR,R0
0B45        C1                             STRZ       R1
0B46        3F 0C 7C                       BSTA,UN    HDG
0B49        17                             RETC,UN
                                   ****************************************************
```

```
LINE  ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE

662                                 **************************************************
663                                 *  A SUBROUTINE TO DIVIDE HX2/HH2 OR HY2/HH2      *
664                                 *  ENTER WITH HX2 OR HY2 IN OPR1 & OPR1+1         *
665                                 *  EXIT WITH QUOTIENT IN RSLT,RSLT+1              *
666                                 **************************************************
667   0B4A  0B4A  20                DIVI   EORZ     R0
668   0B4B        CC 04 77                 STRA,R0  RSLT        CLR QUOTIENT
669   0B4E        CC 04 78                 STRA,R0  RSLT+1
670   0B51        07 11                    LODI,R3  17          CTR.
671   0B53        0E 04 6C                 LODA,R2  HH2+1
672   0B56        0D 04 6B                 LODA,R1  HH2
673   0B59  0B59  77 0A             LOPI   PPSL     COM+C+WC    LOGICAL COMP.,C=1, WITH C
674   0B5B        E5 00                    COMI,R1  0
675   0B5D        1C 0B 93                 BCTA,Z   ZER
676   0B60        ED 04 79                 COMA,R1  OPR1
677   0B63        19 19                    BCTR,GT  GTD         BR. IF HH+2>HX+2
678   0B65        1A 05                    BCTR,LT  LTD         BR. IF HH+2<HX+2
679   0B67        EE 04 7A                 COMA,R2  OPR1+1
680   0B6A        19 12                    BCTR,GT  GTD         BR. IF HH>HX (LS BITS)
681                                 ****   HERE IF HH<HX OR HH=HX (ALL BITS)
682   0B6C  0B6C  0C 04 7A          LTD    LODA,R0  OPR1+1
683   0B6F        A2                       SUBZ     R2          FORM DIVIDEND-DIVISOR
684   0B70        CC 04 7A                 STRA,R0  OPR1+1
685   0B73        0C 04 79                 LODA,R0  OPR1
686   0B76        A1                       SUBZ     R1
687   0B77        CC 04 79                 STRA,R0  OPR1
688   0B7A        77 01                    PPSL     C
689   0B7C        1B 02                    BCTR,UN  JMP
690   0B7E  0B7E  75 01             GTD    CPSL     C
691   0B80  0B80  0C 04 78          JMP    LODA,R0  RSLT+1
692   0B83        D0                       RRL,R0               LSB=1 OR 0
693   0B84        CC 04 78                 STRA,R0  RSLT+1      STR. QUOTIENT  (C=0)
694   0B87        0C 04 77                 LODA,R0  RSLT
695   0B8A        D0                       RRL,R0               LSB=1 OR 0
696   0B8B        CC 04 77                 STRA,R0  RSLT        STR. QUOTIENT  (C=0)
697   0B8E        51                       RRR,R1               ROT. DIVISOR WITH C
698   0B8F        52                       RRR,R2
699   0B90        FB 47                    BDRR,R3  LOPI        CONT. UNTIL DONE
700                                 ****   HERE RSLT=COS(YAW1)
701   0B92        17                       RETC,UN
702   0B93  0B93  E6 00             ZER    COMI,R2  0
703   0B95        9C 0B 60                 BCFA,Z   LOPI+7
704   0B98        1F 0B 7E                 BCTA,UN  GTD
705                                 **************************************************
706                                 **************************************************
707                                 *  A SUBROUTINE TO CALCULATE THE ARCUS FUNCTION   *
708                                 *  THE ALGORITHM USES SUCCESSIVE APPROXIMATION    *
709                                 *     & TABLE LOOK-UP TO ITERATE TO THE SOLUTION*
710                                 *  BEGIN WITH ARG. IN RSLT,RSLT+1                 *
711                                 *  EXIT WITH ARCOS(ARG) IN R0                     *
712                                 **************************************************
713   0B9B  0B9B  77 0A             ANGL   PPSL     C+WC+COM
```

```
ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE
0B9D        0E 04 77                    LODA,R2   RSLT         MS BYTE OF ARG.
0BA0        05 17                       LODI,R1   23           1ST ESTIMATE
0BA2        07 17                       LODI,R3   23           TWICE 1ST INCREMENT
0BA4  0BA4  0D 66 8A              LUPA  LODA,R0   COSM,R1      COS(EST.)
0BA7        75 01                       CPSL      C            MODIFY & TEST CTR.
0BA9        53                          RRR,R3
0BAA        18 1F                       BCTR,Z    LAST
0BAC        E2                          COMZ      R2
0BAD        19 00                       BCTR,GT   ELTA         BR. IF EST. < ANGLE
0BAF        1A 12                       BCTR,LT   EGTA         BR. IF EST. > ANGLE
0BB1  0BB1  E4 01                 EQL1  COMI,R0   1
0BB3        1D 0C 43                    BCTA,GT   QUIT
0BB6        1C 0C 49                    BCTA,EQ   ONE
0BB9        1F 0C 27                    BCTA,UN   ITLS         MS BITS ARE EQU. & < 1
0BBC  0BBC  75 01                 ELTA  CPSL      C            INCR. EST.
0BBE        01                          LODZ      R1
0BBF        83                          ADDZ      R3
0BC0        C1                          STRZ      R1
0BC1        1B 61                       BCTR,UN   LUPA         CONTINUE LOOPING
0BC3  0BC3  77 01                 EGTA  PPSL      C            DECR. EST.
0BC5        01                          LODZ      R1
0BC6        A3                          SUBZ      R3
0BC7        C1                          STRZ      R1
0BC8        18 5A                       BCTR,UN   LUPA         LOOP BACK
0BCA  0BCA  07 0A                 LAST  LODI,R3   10           LAST ITERATIONS
0BCC  0BCC  0D 66 8A              LUPB  LODA,R0   COSM,R1
0BCF        E2                          COMZ      R2
0BD0        19 05                       BCTR,GT   ELT
0BD2        1A 09                       BCTR,LT   EGT
0BD4        1F 08 B1                    BCTA,UN   EQL1         TEST VS QUTY 1
0BD7  0BD7  75 01                 ELT   CPSL      C
0BD9        85 01                       ADDI,R1   1
0BDB        1B 04                       BCTR,UN   TSTR
0BDD  0BDD  77 01                 EGT   PPSL      C
0BDF        A5 01                       SUBI,R1   1
0BE1  0BE1  FB 69                 TSTR  BDRR,R3   LUPB         LOOP IF R3#0
                                  *  BEGIN INTERPOLATING USING MSBITS
0BE3        0D 66 8A                    LODA,R0   COSM,R1
0BE6        E2                          COMZ      R2
0BE7        19 1B                       BCTR,GT   IGT
                                  *  START IF COS(EST.) < ARG.
0BE9        C3                          STRZ      R3           TEMP ST. COS(EST.)
0BEA        0D 66 89                    LODA,R0   COSM-1,R1
0BED        77 01                       PPSL      C
0BEF        A3                          SUBZ      R3
0BF0        75 01                       CPSL      C
0BF2        50                          RRR,R0
0BF3        75 01                       CPSL      C
0BF5        83                          ADDZ      R3
0BF6        E2                          COMZ      R2           INTERP. VALUE VS ARG
0BF7        1E 0C 43                    BCTA,LT   QUIT
0BFA        77 01                       PPSL      C
```

```
LINE  ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE

766   0BFC        A5 01                      SUBI,R1   1
767   0BFE        1F 0C 43                   BCTA,UN   QUIT
768                                    *   START HERE IF COS(EST.) > ARG.
769   0C01  0C01  E4 02              IGT     COMI,R0   2
770   0C03        98 07                      BCFR,EQ   NEQ2        BR. IF COS(EST) # 2
771   0C05        75 01                      CPSL      C
772   0C07        85 01                      ADDI,R1   1
773   0C09        1F 0C 43                   BCTA,UN   QUIT
774   0C0C  0C0C  0D 66 E1           NEQ2    LODA,R0   COSM+1,R1
775   0C0F        C3                         STRZ      R3
776   0C10        0D 66 EA                   LODA,R0   COSM,R1
777   0C13        77 01                      PPSL      C
778   0C15        A3                         SUBZ      R3
779   0C16        75 01                      CPSL      C
780   0C18        50                         RRR,R0
781   0C19        75 01                      CPSL      C
782   0C1B        83                         ADDZ      R3
783   0C1C        E2                         COMZ      R2
784   0C1D        1E 0C 43                   BCTA,LT   QUIT
785   0C20        75 01                      CPSL      C
786   0C22        85 01                      ADDI,R1   1
787   0C24        1F 0C 43                   BCTA,UN   QUIT
788                                    *   EXAMINE LS BITS
789   0C27  0C27  75 01              ITLS    CPSL      C           BEGIN ITERATION ON LSBITS
790   0C29        05 2A                      LODI,R1   42          ESTIMATE OF ADDRESS
791   0C2B        0E 04 78                   LODA,R2   RSLT+1
792   0C2E        E6 81                      COMI,R2   H'81'       1ST ESTIMATE
793   0C30        1D 0C 43                   BCTA,GT   QUIT
794   0C33        85 01                      ADDI,R1   1
795   0C35        E6 31                      COMI,R2   H'31'        2ND EST.
796   0C37        1D 0C 43                   BCTA,GT   QUIT
797   0C3A        85 01                      ADDI,R1   1
798   0C3C        E6 09                      COMI,R2   H'09'       3D EST.
799   0C3E        1D 0C 43                   BCTA,GT   QUIT
800   0C41        85 01                      ADDI,R1   1
801   0C43  0C43  01                 QUIT    LODZ      R1
802   0C44        75 01                      CPSL      C
803   0C46        84 20                      ADDI,R0   45
804                                  ****************************************************
805   0C48        17                         RETC,UN
806   0C49  0C49  05 28              ONE     LODI,R1   40
807   0C4B        0D 66 E9                   LODA,R0   COSL+1,R1
808   0C4E        C3                         STRZ      R3
809   0C4F        0D 66 E8                   LODA,R0   COSL,R1
810   0C52        77 01                      PPSL      C
811   0C54        A3                         SUBZ      R3
812   0C55        75 01                      CPSL      C
813   0C57        50                         RRR,R0
814   0C58        75 01                      CPSL      C
815   0C5A        83                         ADDZ      R3
816   0C5B        EC 04 78                   COMA,R0   RSLT+1
817   0C5E        1A 63                      BCTR,LT   QUIT
```

```
LINE  ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE

 818  0C60        05 29                     LODI,R1    41
 819  0C62        1B 5F                     BCTR,UN    QUIT
 820                                  ***********************************************
 821                                  ***********************************************
 822                                  *  A SUBROUTINE TO CONVERT BINARY ANGLE TO BCD  *
 823                                  *  ENTR WITH R0=ANGLE(BIN) & EXIT R1= BCD EQUIV. *
 824                                  ***********************************************
 825  0C64  0C64  07 07               BCDA  LODI,R3    7             CTR.
 826  0C66        05 00                     LODI,R1    0             CLR TOTAL
 827  0C68  0C68  75 01               BRCH  CPSL       C             CLR. C
 828  0C6A        D0                        RRL,R0
 829  0C6B        C2                        STRZ       R2            MOVE RESULT
 830  0C6C        9A 0A                     BCFR,N     NINC          BR. IF MS BIT=0
 831  0C6E        01                        LODZ       R1
 832  0C6F        75 01                     CPSL       C
 833  0C71        84 66                     ADDI,R0    H'66'         BCD ADD 2 BYTES
 834  0C73        8F 67 15                  ADDA,R0    BCD-1,R3      INCR. BCD TOTAL
 835  0C76        94                        DAR,R0
 836  0C77        C1                        STRZ       R1
 837  0C78        02                        LODZ       R2            RESULT TO R0
 838  0C79  0C79  FB 6D               NINC  BDRR,R3    BRCH          GO BACK TIL DONE
 839                                  ****  LEAVE WITH BCD EQUIV. OF YAW1 IN R1
 840  0C7B        17                        RETC,UN
 841                                  ***********************************************
```

```
LINE  ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE
                              ************************************************
                              *  A SUBROUTINE TO CALCULATE HEADING & OUTPUT   *
                              *       THREE BCD DIGITS PLUS SIGN              *
                              ************************************************
      0C7C  0C7C  77 09       HDG     PPSL     WC+C
      0C7E        0C 04 7F            LODA,R0  DATX          SIGN OF HX ?
      0C81        1A 1B               BCTR,N   YAW0          BR. IF NEG
                              *  SOLVE FOR HEADING (BCD FORM)
      0C83        0C 04 81            LODA,R0  DATY          GET HY
      0C86        1A 10               BCTR,N   HYN           BR. ON HY=NEG.
                              *
                              * START HERE ON POS. HY & HX
      0C88        F5 00               COMI,R1  0
      0C8A        18 0C               BCTR,EQ  HYN
      0C8C        04 60               LODI,R0  H'60'
      0C8E        A1                  SUBZ     R1
      0C8F        94                  DAR,R0
      0C90        C3                  STRZ     R3
      0C91        06 03               LODI,R2  H'03'
      0C93        A6 00               SUBI,R2  0
      0C95        96                  DAR,R2
      0C96        1B 27               BCTR,UN  FINE
                              *
                              * ENTER HERE IF HX=POS. & HY=NEG.
      0C98  0C98  06 00       HYN     LODI,R2  H'00'
      0C9A        01                  LODZ     R1
      0C9B        C3                  STRZ     R3
      0C9C        1B 21               BCTR,UN  FINE
                              *
                              *  ENTER HERE HX=NEG & HY= +/-
      0C9E  0C9E  0C 04 81    YAW0    LODA,R0  DATY          TEST SIGN OF HY
      0CA1        1A 12               BCTR,N   NHY
```

```
LINE  ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE

876                                     *   ENTER HERE IF HX=NEG & HY=POS
877   0CA3        04 80                         LODI,R0    H'80'
878   0CA5        75 01                         CPSL       C
879   0CA7        84 66                         ADDI,R0    H'66'
880   0CA9        81                            ADDZ       R1
881   0CAA        94                            DAR,R0
882   0CAB        C3                            STRZ       R3
883   0CAC        06 01                         LODI,R2    H'01'
884   0CAE        86 66                         ADDI,R2    H'66'
885   0CB0        86 00                         ADDI,R2    0
886   0CB2        96                            DAR,R2
887   0CB3        18 0A                         BCTR,UN    FINE
888                                     *   ENTER HERE IF HX & HY = NEG
889   0CB5  0CB5  04 80                 NHY     LODI,R0    H'80'
890   0CB7  0CB7  A1                    DO      SUBZ       R1          SUB. YAW1 LSB
891   0CB8        94                            DAR,R0
892   0CB9        C3                            STRZ       R3
893   0CBA        06 01                         LODI,R2    H'01'
894   0CBC        A6 00                         SUBI,R2    0
895   0CBE        96                            DAR,R2
896                                     *
897                                     *
898                                     ****  READOUT OF DATA    ***********
899   0CBF  0CBF  26 FF                 FINE    EORI,R2    H'FF'
900   0CC1        06 01                         WRTE,R2    MSB         MSB OUT
901   0CC3        27 FF                         EORI,R3    H'FF'
902   0CC5        07 02                         WRTE,R3    LSB         LSB OUT
903   0CC7        17                            RETC,UN
904                                     ***********************************************
905                                             ORG        H'E40'
906                                     ***********************************************
907                                     *   S.R. TO CALC. THE HZ COMPONENT IN X-Z PLANE  *
908                                     *      HZ=HY*SIN(PHI) + HZ*COS(PHI)              *
909                                     *   EXIT WITH HZ IN OPR2,OPR2+1                  *
910                                     ***********************************************
911   0E40  0E40  0D 04 62              HZR1    LODA,R1    DATM+8      GET PHI
912   0E43        0E 04 62                      LODA,R2    DATM+8
913   0E46        45 80                         ANDI,R1    H'80'       SAVE SIGN
914   0E48        46 7F                         ANDI,R2    H'7F'       STRI SIGN
915   0E4A        CD 04 8D                      STRA,R1    TBUF        HOLD SIGN
916   0E4D        0E 65 00                      LODA,R0    SIN,R2      SIN(PHI)
917   0E50        CC 04 69                      STRA,R0    SINR
918   0E53        0E 65 89                      LODA,R0    COS,R2      COS(PHI)
919   0E56        CC 04 7A                      STRA,R0    OPR1+1
920   0E59        0C 04 5E                      LODA,R0    DATM+4      HZ
921   0E5C        0D 04 5F                      LODA,R1    DATM+5
922   0E5F        CC 04 7B                      STRA,R0    OPR2
923   0E62        CD 04 7C                      STRA,R1    OPR2+1
924   0E65        3F 0B 66                      BSTA,UN    SMPY        HZ*COS(PHI)
925   0E68        0C 04 77                      LODA,R0    RSLT
926   0E6B        0D 04 78                      LODA,R1    RSLT+1
927   0E6E        CC 04 89                      STRA,R0    ABUF
```

```
PIP ASSEMBLER VERSION SCU LEVEL 1 HEADING INSTRUMENT ASSEMLY PROGRAM  1976 PAGE   25

LINE  ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE
                                            STRA,R1     BBUF
928   0E71        CD 04 8A                  LODA,R0     SINR
929   0E74        0C 04 69                  STRA,R0     OPR1+1
930   0E77        CC 04 7A                  LODA,R0     DATM+2      HY
931   0E7A        0C 04 5C                  LODA,R1     DATM+3
932   0E7D        0D 04 5D                  STRZ        R2
933   0E80        C2                        ANDI,R0     H'7F'
934   0E81        44 7F                     ANDI,R2     H'80'
935   0E83        46 80                     EORA,R2     TRUF        DETERMINE SIGN
936   0E85        2E 04 8D                  STRA,R0     OPR2
937   0E88        CC 04 7B                  STRA,R1     OPR2+1
938   0E8B        CD 04 7C                  STRA,R2     SRES
939   0E8F        CE 04 7E                  BSTA,UN     SMPY        HY*SIN(PHI)
940   0E91        3F 08 66                  LODA,R0     ABUF
941   0E94        0C 04 89                  LODA,R1     BBUF
942   0E97        CD 04 8A                  STRA,R0     TEMP
943   0E9A        CC 04 85                  STRA,R1     TEMP+1
944   0E9D        CD 04 86                  CPSL        COM         ADD
945   0EA0        75 02                     EORZ        R0
946   0EA2        20                        STRA,R0     STEM        HZ*COS(PHI)=POS.
947   0EA3        CC 04 7D                  BSTA,UN     SADD
948   0EA6        3F 09 98                  LODA,R0     RSLT
949   0EA9        0C 04 77                  LODA,R1     RSLT+1
950   0EAC        0D 04 78                  STRA,R0     OPR2
951   0EAF        CC 04 7B                  STRA,R1     OPR2+1
952   0EB2        CD 04 7C                  PPSL        COM
953   0EB5        77 02                     RETC,UN
954   0EB7        17
```

```
PIP ASSEMBLER VERSION SCU LEVEL 1 HEADING INSTRUMENT ASSEMLY PROGRAM  1976 PAGE   26

LINE  ADDR  LABL  B1 B2 B3 B4 ERROR SOURCE

                                    *************************************************
                                    *  S.R. TO CORRECT NONORTHOGONALITY ERRORS IN HX *
                                    *   ENTER ONCE EACH SAMPLE CORRECTING HX SAMPLE  *
                                    *         HX=HX(MEAS) + HY(MEAS)*SIN(ERROR)      *
                                    *   LOCATE SIN(ERROR) IN SINE                    *
                                    *   USES "SMPY" TO FORM PRODUCT & "SADD" TO SUM  *
                                    *   EXIT WITH MODIFIED HX IN DATM,DATM+1         *
                                    *************************************************
      0EB8  0EB8  75 01             ORTH   CPSL      C
      0EBA        0C 04 5C                 LODA,R0   DATM+2      HY MSB
      0EBD        0D 04 5D                 LODA,R1   DATM+3      HY LSB
      0EC0        0E 07 1E                 LODA,R2   SINE        SIN(ERROR)
      0EC3        C3                       STRZ      R3
      0EC4        44 7F                    ANDI,R0   H'7F'       STRIP SIGN
      0EC6        47 80                    ANDI,R3   H'80'       SAVE  SIGN
      0EC8        CC 04 78                 STRA,R0   OPR2
      0ECB        CD 04 7C                 STRA,R1   OPR2+1
      0ECE        CE 04 7A                 STRA,R2   OPR1+1
      0ED1        CF 04 7E                 STRA,R3   SRES        SIGN OF HY*SIN(ERR)
      0ED4        3F 08 66                 BSTA,UN   SMPY        FORM PROD.
      0ED7        0C 04 5A                 LODA,R0   DATM        HX MSB
      0EDA        0D 04 5B                 LODA,R1   DATM+1      HX LSB
      0EDD        C2                       STRZ      R2
      0EDE        44 7F                    ANDI,R0   H'7F'
      0EE0        46 80                    ANDI,R2   H'80'
      0EE2        CC 04 65                 STRA,R0   TEMP
      0EE5        CD 04 66                 STRA,R1   TEMP+1
      0EE8        CE 04 70                 STRA,R2   STEM
      0EEB        75 02                    CPSL      COM
      0EED        3F 09 98                 BSTA,UN   SADD        FORM SUM
      0EF0        0C 04 77                 LODA,R0   RSLT
      0EF3        0D 04 78                 LODA,R1   RSLT+1
      0EF6        CC 04 5A                 STRA,R0   DATM        MODIFIED HX
      0EF9        CD 04 5B                 STRA,R1   DATM+1
      0EFC        77 02                    PPSL      COM
      0EFE        17                       RETC,UN
                                    *************************************************
                                           END

TOTAL ASSEMBLER ERRORS =     0
```

APPENDIX C

The transcendental functions used throughout the heading computation algorithm were implemented using a table look up procedure. To generate the respective look up tables in computer memory data was first generated using algol programs. This technique expedited modifications to tabular data and provided output data in a convenient (hexadecimal) format.

Programs that calculated Cos (Θ) and Cos^2 (Θ) to eight bit and sixteen bit resolution respectively are included.

PAGE 001

```
00000 HPAL,L,"RP"
00000 BFGIN
00001 &*************************************************!
00001 &   A PROGRAM TO GENERATE A COS*COS TABLE FOR A   *
00001 &       MICROPROCESSOR BASED SYSTEM               !
00001 &  ADDRESS, ANGLE , COS(ANGLE), AND DATA IN       !
00001 &       BOTH BINARY AND HEX FORMAT IS TABULATED   !
00001 &*************************************************!
00001 INTEGER A,B,C,D,E,F;
00010 REAL THETA,STHETA,AR,BSTH;
00020 REAL ARRAY AN[0:256,0:16];
21070 INTEGER ARRAY VAL[0:15]:="0","1","2","3","4","5",
21102            "6","7","8","9","A",
21107            "B","C","D","E","F";
21114 INTEGER ARRAY MULT[0:3]:=8,4,2,1;
21124 INTEGER ARRAY HEX[0:256,0:3];
23136 WRITE (6,#(1H1));
23175 WRITE (6,#(//,9X,"A",5X,"THETA",4X,"COS*COS",5X,
23226            "BINARY",14X,"HEX",//));
23242 WRITE (6,#(5X,"(ADDR.)",3X,"(DEG)",18X," DATA ",//));
23276 FOR A:= 45 TO 90 DO
23304       BEGIN
23304       AR:=A;
23310       THETA:= AR*PI/180;
23316       STHETA:=((COS(THETA))^2);
23330       B:=0;
23332       BSTH:=STHETA;
23336       WHILE B<17 DO
23341          BEGIN
23342          BSTH:= 2*BSTH;
23350          IF BSTH>1 THEN
23355            BEGIN
23355            AN[A,B]:=1;
23367            BSTH:=BSTH-1;
23375            END ELSE  AN[A,B]:=0;
23435          B:=B+1;
23440          END;
23441          D:=0;
23443          FOR C:=0 TO 3 DO
23451            BEGIN
23451            D:=4*C;
23455            F:=0;
23457            FOR E:=0 TO 3 DO
23465              BEGIN
23465              F:=F+MULT[E]*AN[A,D+E];
23520              HEX[A,C]:= VAL[F];
23533            END;
23537            END;
23543         THETA:= THETA*180/PI;
23553        WRITE (6,#(5X,I5,2(3X,F8.6),3X,16(1 ,5X,4A2),A,
23604            THETA,STHETA,  FOR B:= 0 TO 15 DO
23620            AN[A,B], FOR D:= 0 TO 3 DO HEX[A,D]);
23653        END;
23657 END$

PROGRAM= 023663  ERRORS=000
```

A (ADDR.)	THETA (DEG)	COS*COS	BINARY DATA	HEX
45	45.00000	.500000	0111111111111111	7 F F F
46	46.00000	.482550	0111101110001000	7 8 8 8
47	47.00000	.465122	0111011100010010	7 7 1 2
48	48.00000	.447736	0111001010011110	7 2 9 E
49	48.99999	.430413	0110111000101111	6 E 2 F
50	50.00000	.413176	0110100111000101	6 9 C 5
51	51.00000	.396044	0110010101100011	6 5 6 3
52	52.00001	.379039	0110000100001000	6 1 0 8
53	53.00000	.362181	0101110010110111	5 C 8 7
54	54.00000	.345491	0101100001110010	5 8 7 2
55	55.00000	.328990	0101010000111000	5 4 3 8
56	56.00000	.312697	0101000000001100	5 0 0 C
57	57.00000	.296632	0100101111110000	4 B F 0
58	58.00000	.280814	0100011111100011	4 7 E 3
59	58.99999	.265264	0100001111101000	4 3 E 8
60	60.00001	.250000	0011111111111111	3 F F F
61	61.00000	.235040	0011110000101011	3 C 2 B
62	62.00000	.220403	0011100001101100	3 8 6 C
63	63.00000	.206107	0011010011000011	3 4 C 3
64	64.00000	.192169	0011000100110010	3 1 3 2
65	65.00000	.178606	0010110110111001	2 D B 9
66	66.00000	.165435	0010101001011001	2 A 5 9
67	66.99998	.152671	0010011100010101	2 7 1 5
68	68.00000	.140330	0010001111101100	2 3 E C
69	69.00000	.128427	0010000011100000	2 0 E 0
70	70.00000	.116978	0001110111110010	1 D F 2
71	71.00000	.105995	0001101100100010	1 B 2 2
72	72.00000	.095491	0001100001110010	1 8 7 2
73	73.00000	.085481	0001010111100010	1 5 E 2
74	73.99998	.075976	0001001101110011	1 3 7 3
75	75.00000	.066987	0001000100100110	1 1 2 6
76	76.00000	.058526	0000111011111011	0 E F B
77	77.00002	.050603	0000110011110100	0 C F 4
78	78.00000	.043227	0000101100010000	0 B 1 0
79	79.00000	.036408	0000100101010010	0 9 5 2
80	80.00000	.030154	0000011110111000	0 7 B 8
81	81.00000	.024472	0000011001000011	0 6 4 3
82	82.00000	.019349	0000010011110101	0 4 F 5
83	83.00000	.014852	0000001111001101	0 3 C D
84	84.00002	.010926	0000001011001100	0 2 C C
85	84.99998	.007596	0000000111110001	0 1 F 1
86	86.00000	.004866	0000000100111110	0 1 3 E
87	87.00000	.002739	0000000010110011	0 0 B 3
88	88.00000	.001218	0000000001001111	0 0 4 F
89	89.00000	.000305	0000000000010011	0 0 1 3
90	90.00000	.000000	0000000000000000	0 0 0 0

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```
00000 REAL,L,"RP"
00000 BEGIN
00001 &*************************************************;
00001 &   A PROGRAM TO GENERATE A COS TABLE FOR A        *
00001 &      MICROPROCESSOR BASED SYSTEM                 ;
00001 &  ADDRESS, ANGLE , COS(ANGLE), AND DATA IN        ;
00001 &      BOTH BINARY AND HEX FORMAT IS TABULATED     ;
00001 &*************************************************;
00001 INTEGER A,B,C,D,E,F;
00010 REAL THETA,STHETA,AR,BSTH;
00020 REAL ARRAY AN[0:256,0:16];
21070 INTEGER ARRAY VAL[0:15]:="0","1","2","3","4","5",
21102           "6","7","8","9","A",
21107           "B","C","D","E","F";
21114 INTEGER ARRAY MULT[0:3]:=8,4,2,1;
21124 INTEGER ARRAY HEX[0:256,0:3];
23136 WRITE (6,#(1H1));
23175 WRITE (6,#(//,9X,"A",5X,"THETA",4X,"COS(THETA)",2X, "BINARY",6X,"HEX",
23240          /));
23243 WRITE (6,#(5X,"(ADDR.)",3X,"(DEG)",18X," DATA ",//));
23277 FOR A:= 45 TO 90 DO
23305     BEGIN
23305     AR:=A;
23311     THETA:= AR*PI/180;
23317     STHETA:= COS(THETA);
23323     B:=0;
23325     BSTH:=STHETA;
23331     WHILE B<9  DO
23334       BEGIN
23335       BSTH:= 2*BSTH;
23343       IF BSTH>1 THEN
23350         BEGIN
23350         AN[A,B]:=1;
23362         BSTH:=BSTH-1;
23370         END ELSE  AN[A,B]:=0;
23403       B:=B+1;
23406       END;
23407       D:=0;
23411       FOR C:=0 TO 3 DO
23417         BEGIN
23417         D:=4*C;
23450         F:=0;
23452         FOR E:=0 TO 3 DO
23460           BEGIN
23460           F:=F+MULT[E]*AN[A,D+E];
23513           HEX[A,C]:= VAL[F];
23526         END;
23532         END;
23536      THETA:= THETA*180/PI;
23546     WRITE (6,#(5X,I5,2(3X,F8.6),3X,8F1.0,5X,4A2),A,THETA,STHETA,
23605          FOR B:=0 TO 7 DO AN[A,B],FOR D:=0 TO 3 DO HEX[A,D]);
23646     END;
23652 END;
```

PROGRAM= 023656   ERRORS=000

A (ADDR.)	THETA (DEG)	COS(THETA)	BINARY DATA	HEX
45	45.00000	.707107	1.11.1.1	8 5
46	46.00000	.694659	1.11...1	8 1
47	47.00000	.681998	1.1.111.	A E
48	48.00000	.669131	1.1.1.11	A B
49	48.99999	.656059	1.1..111	A 7
50	50.00000	.642788	1.1..1..	A 4
51	51.00000	.629320	1.1....1	A 1
52	52.00001	.615662	1..111.1	9 D
53	53.00000	.601815	1..11.1.	9 A
54	54.00000	.587785	1..1.11.	9 6
55	55.00000	.573576	1..1..1.	9 2
56	56.00000	.559193	1...1111	8 F
57	57.00000	.544639	1...1.11	8 B
58	58.00000	.529919	1....111	8 7
59	58.99999	.515038	1.....11	8 3
60	60.00001	.500000	.1111111	7 F
61	61.00000	.484810	.11111..	7 C
62	62.00000	.469471	.1111...	7 8
63	63.00000	.453991	.111.1..	7 4
64	64.00000	.438371	.111....	7 0
65	65.00000	.422618	.11.11..	6 C
66	66.00000	.406737	.11.1...	6 8
67	66.99998	.390731	.11..1..	6 4
68	68.00000	.374607	.1.11111	5 F
69	69.00000	.358368	.1.11.11	5 B
70	70.00000	.342020	.1.1.111	5 7
71	71.00000	.325568	.1.1..11	5 3
72	72.00000	.309017	.1..1111	4 F
73	73.00000	.292372	.1..1.1.	4 A
74	73.99998	.275637	.1...11.	4 6
75	75.00000	.258819	.1....1.	4 2
76	76.00000	.241922	..1111.1	3 D
77	77.00002	.224951	..111..1	3 9
78	78.00000	.207912	..11.1.1	3 5
79	79.00000	.190809	..11....	3 0
80	80.00000	.173648	..1.11..	2 C
81	81.00000	.156435	..1.1...	2 8
82	82.00000	.139173	..1...11	2 3
83	83.00000	.121869	...11111	1 F
84	84.00002	.104528	...11.1.	1 A
85	84.99998	.087156	...1.11.	1 6
86	86.00000	.069757	...1...1	1 1
87	87.00000	.052336	....11.1	0 D
88	88.00000	.034900	....1...	0 8
89	89.00000	.017452	.....1..	0 4
90	90.00000	-.000000	....	0 0

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