

(NASA-CR-155736) HIGH RESOLUTION 10 MU SPECTROMETRY AT DIFFERENT PLANETARY LATITUDES. A PRACTICAL HADAMARD TRANSFORM SPECTROMETER FOR ASTRONOMICAL APPLICATION Final Report, 1 Sep. 1973 - (Cornell Univ., G3/91. 05334	N78-17976 Unclas
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CORNELL UNIVERSITY

Center for Radiophysics and Space Research

ITHACA, N. Y.



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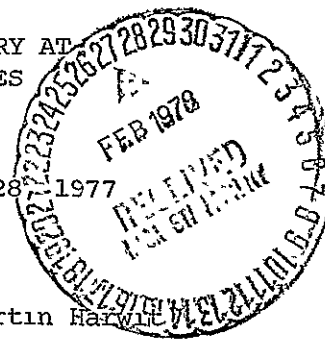
FINAL REPORT

for

HIGH RESOLUTION 10μ SPECTROMETRY AT
DIFFERENT PLANETARY LATITUDES

NASA Grant NGR 33-010-210

September 1, 1973 through April 28, 1977



Principal Investigator: Prof. Martin Harvey



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HIGH RESOLUTION 10μ SPECTROMETRY AT
DIFFERENT PLANETARY LATITUDES

NASA Grant NGR 33-101-210

September 1, 1973 through April 28, 1977

Principal Investigator: Prof. Martin Harwit
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1. Introduction

Jupiter's bands are a clear indication of atmospheric differences at different latitudes on the planet. The aim of the present study was to attempt infrared observations at different latitudes in order to obtain spectra in the 10μ region. These spectra might then help one to understand differences in chemical composition or physical structure of the well known optical features.

In order to obtain such spectra of a rotating planet, it seemed well to attempt simultaneous observations at different latitudes. We planned to use a Hadamard transform spectrometer with 15 entrance slits in order to obtain 15 simultaneous spectra, at a resolution of 0.01μ . The spectral band covered contained 255 spectral elements.

2. Results

The results obtained during the course of the study are covered in ten publications which include seven journal articles, two general review papers, and a Ph.D. thesis. These publications are listed in section 3 below. The thesis which is the best single summary of the work done under the present contract appears as the Appendix to this report.

The publications fall into two main groups: those that deal with technical aspects of the instrumentation we had to develop in order to make observations; and those that deal with observational work.

The technical difficulties encountered turned out to be rather more severe than had been initially anticipated. Many of the problems we had to solve are of a very general nature that applies to other varieties of multiplex instrumentation, as well as to other classes of Hadamard transform spectrometers. As a result, all the technically innovative work was of sufficiently general interest to warrant publication.

The astronomical observations that were undertaken are described in the thesis of Ming-Hing Tai, the graduate student who worked on the project (see Appendix). The results cited there represent the first observations undertaken with our instrument. The instrument did in fact perform with the spectral resolving power and the number of spatial and spectral elements we had anticipated in our design. Because of the above mentioned technical difficulties, we suffered considerable delays in initiating observations. Ultimately we therefore had to content ourselves with far less observing time than we had originally hoped to obtain, and the observational side of the program had to be carried out at far more modest levels than anticipated because important technical requirements had drained a disproportionately large fraction of our available funds.

Despite these setbacks, however, we feel that the results obtained--while not precisely in line with what we had set out to do--are interesting, led to a large number of publications, and constitute a worthwhile contribution to observational planetary science.

3. Publications

1. Two asymmetric Hadamard transform spectrometers, Martin Harwit, Perry G. Phillips, Leon W. King, and Daniel A. Briotta, Jr. *Applied Optics*, 13, 2669 (1974).
2. Infrared astronomy and the shuttle, Martin Harwit, *Astronautics and Aeronautics* 12, 23 October (1974) 1.
3. Masks for Hadamard transform optics, and weighing designs, Neil J. A. Sloane and Martin Harwit, *Applied Optics* 15, 207 (1976).
4. A practical multi-spectrum Hadamard transform spectrometer, Ming Hing Tai, D. A. Briotta, Jr., N. Kamath and Martin Harwit, *Applied Optics* 14, 2533 (1975).
5. Errors in Hadamard spectroscopy or imaging caused by imperfect masks, Ming Hing Tai, Martin Harwit and Neil J. A. Sloane, *Applied Optics* 14, 2678 (1975).
6. Fourier and Hadamard transform spectrometers: a limited comparison, Ming Hing Tai and Martin Harwit, *Applied Optics* 15, 2664 (1976).
7. "Hadamard Transform Analytical Systems", Martin Harwit, to be published in "Transformations in Analytical Chemistry," P. Griffiths, Ed., Plenum Press (1978).
8. Fourier and Hadamard transform spectrometers: a limited comparison II. M. Harwit and Ming Hing Tai, *Applied Optics* 16, 3070 (1977).
9. Distortion in Hadamard Transform Optics, N.J.A. Sloane, Martin Harwit and Ming-Hing Tai, accepted for publication, *Applied Optics* (1978).

10. A practical Hadamard transform spectrometer for astronomical application, Ming-Hing Tai, Cornell University Ph.D. thesis and Center for Radiophysics and Space Research Report CRSR 655 (1977).

4. Personnel Participating on the Grant

Prof. M. Harwit, principal investigator

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Mr. Gary Melnick, graduate student

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Mr. Gerald Stasavage, research technician (part time)

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Mr. Leon W. King, graduate student (part time)

Mr. Narayana Kameth, programmer (part time)

5. Financial Status

All funds provided under this contract have been spent by this time.

CENTER FOR RADIOPHYSICS AND SPACE RESEARCH
CORNELL UNIVERSITY
ITHACA, NEW YORK

January, 1977

A PRACTICAL HADAMARD TRANSFORM SPECTROMETER
FOR
ASTRONOMICAL APPLICATION

by
Ming-Hing Tai

DEDICATION

To my parents

献给 爸爸、妈妈

ACKNOWLEDGEMENTS

This project is by no means one man's product. Many people have devoted their talents to the completion of this project. I am just the one to make a summary of it.

My special thanks go to Prof. Martin Harwit, my thesis advisor. Martin set himself up as an admirable example: he always moved the heaviest equipment himself; tackled the most difficult jobs in the experiment so that he himself was responsible for any misfortunes; was earnest in discussions about problems with his student; and was modest in sharing the credit for the results. Above all, he truly expressed concern about his student, as a respectable advisor and as a warm friend.

My second thanks go to Dr. Daniel A. Briotta. Dan helped to automate the whole process and taught me the mini computer technique. Dan has shown me how to run the computer as though he were showing me how the lines in his palm run.

My thanks also go to the infrared astronomy group at Cornell. Prof. J. Houck has served on my committee and has always been available for discussion of astronomical and experimental problems. I especially thank him for his help in finishing up my degree while Martin was on sabbatical leave. Dr. Phillips built the spectrometer. The late L. King modified it. G. Stasavage built some electronic parts and H. Kondracki built the dewer. G. Melick helped with the obser-

vation, data processing and electronic parts, G. Gull with the field trip, and Drs. W. Forrest, Diward, and D. Shacck were involved in many valuable discussions. Thanks also go to Westy Dain for not only being always willing to help on chilly night's observations and tedious laboratory work, but also for proof reading my thesis.

I thank Prof. E.E. Salpeter for serving on my committee, Prof. A. Siever and Allen Chin for helping the He₃ cooled dewer, and Prof. P. Gierasch for always being available for discussing astronomical problems. I also thank G. Vincent for building many mechanical parts, B. Boettcher for figures and Eckelmaun for photographs.

The author thanks the staffs of the Kitt Peak National Observatory and McDonald Observatory for generous allotments of observing time and for assistance during the observing runs.

Finally, my thanks go to Divan Sun, Adeline Wu, and W.W. Chiu, for typing the thesis for me, and to N.H. Cheung, K.K. Mon, K.M. Lo and many of my Chinese friends for their continual help during these years.

Portions of this work were supported by the NGR 33-010-210 and NSG 1263.

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忆 秦 娥

娄 山 关

一九三五年二月

西风烈，
长空雁叫霜晨月。
霜晨月，
马蹄声碎，
喇叭声咽。

雄关漫道真如铁，
而今迈步从头越。
从头越，
苍山如海，
残阳如血。

毛泽东主席

CHAPTER I

HISTORICAL INTRODUCTION TO HADAMARD TRANSFORM SPECTROMETRY (HTS)

The idea of modulating or encoding the optical output of a spectrometer goes back to the original work of Golay (1949) and Fellgett (1951). Its purpose is to allow many different wavelengths of radiation to fall on a detector simultaneously, and thereby to increase the signal-to-noise ratio (SNR) of the resulting spectrum. This improvement comes about because each element of the spectrum is effectively viewed a larger fraction of the total available observing time. One idea is to encode or modulate each spectral wavelength exiting the spectrometer output with an audio frequency that contains the optical wavelength information. The use of a conventional wave analyzer then allows recovery of the original optical spectrum. There are many variations of this technique.

In 1968, Ibbett et al and Decker et al independently suggested the use of sequentially stepped multiplex spectrometers. In both systems radiation enters a dispersive instrument through a single slit and is analyzed at a number of exit slits. Decker et al pointed out that two constraints should be imposed on the encoding scheme:

- (1) To obtain the optimum signal to noise ratio, each spectral element should be viewed during exactly half the step positions.
- (2) To impose the smallest dynamic range requirements on the

detector amplifier system, each step position should pass light from exactly half the spectral elements. They also worked out a scheme that satisfied the two constraints for masks having elements $m=4n+2$, where n is an arbitrary integer or zero. Ibbett et al introduced the Hadamard pattern for the mask. As discussed below, this is a pattern based on a set of binary orthogonal matrices first studied by the French mathematician Jacques Hadamard. Ibbett et al also described the application of their scheme to a real time computer aided measurement.

In 1969, Sloane et al worked out a number of binary cyclic coding schemes for multiplex spectrometry and evaluated the performance of each scheme in terms of a linear, least mean square, unbiased estimate. These schemes include a Hadamard matrix H, and various modified Hadamard matrices, which these authors refer to as G matrix and S matrix (Fig. 1-1).

A Hadamard matrix H of order N is an $N \times N$ matrix H_N of +1's and -1's which satisfies:

$$H_N H_N^T = N I_N$$

where I_N is an $N \times N$ unit matrix

A modified Hadamard matrix G of order M is a partitioned matrix from the H matrix:

$$\underline{H} = \begin{bmatrix} 1 & \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & & \underline{G} & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ 1 & & & & & \end{bmatrix} \quad (M = N-1)$$

$$\underline{H} = \begin{bmatrix} + & + & + & + & + & + & + & + \\ + & + & + & - & - & - & + & - \\ + & + & - & - & - & + & - & + \\ + & - & - & - & + & - & + & + \\ + & - & - & + & - & + & + & - \\ + & - & + & - & + & + & - & - \\ + & + & - & + & + & - & - & - \\ + & - & + & + & - & - & - & + \end{bmatrix}$$

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OF POOR QUALITY

$$\underline{G} = \begin{bmatrix} + & + & - & - & - & + & - \\ + & - & - & - & + & - & + \\ - & - & - & + & - & + & + \\ - & - & + & - & + & + & - \\ - & + & - & + & + & - & - \\ + & - & + & + & - & - & - \\ - & + & + & - & - & - & + \end{bmatrix}$$

$$\underline{S} = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Figure 1-1. An 8 x 8 Hadamard matrix and two 7 x 7 cyclic matrices that can be derived from it.

where the first row and first column of \underline{H} are all +1's. A feature of the \underline{G} matrix is that it can be written in cyclic form-- a factor which we will show to be of considerable practical importance.

A modified Hadamard matrix \underline{S} is a matrix obtained from \underline{G} by replacing +1's by 0's and -1's by +1's.

The properties of \underline{H} , \underline{G} , and \underline{S} will be discussed in section 2-6.

When we talk of encoding by means of a Hadamard matrix we have the following in mind. A mask is used to modulate - open or close - a series of entrance and exit slits in a spectrometer. If a certain slit location is open, we can designate it by a +1; if it is closed we can designate it by a 0; if it can be used to subtract from the signal incident on the detector, we designate it by -1. The sequence of +1's and -1's characterizing a mask in a given modulating position corresponds to a row of a matrix. The whole set of mask positions corresponds to the set of rows of the matrix. If the sequence of mask patterns corresponds to the rows of a Hadamard matrix we say we are encoding with a Hadamard pattern.

Sloane et al also introduced the idea of using a cyclic matrix for coding masks. This greatly decreases the experimental cost and facilitates operation, since any N slits of a single mask-- $2N-1$ slits long can be used to provide one of the required mask patterns.

The first single entrance Hadamard spectrometer (HTS) was built by Decker and Harwit (1969). The spectrometer had

a single entrance slit and 19 exit slits. The exit mask was stepped manually. The authors used this spectrometer to take the spectrum of the mercury vapor 1.7μ band to demonstrate the Hadamard transformed spectrum's fidelity and freedom from systematic errors.

With 19 exit slits, the HTS had a theoretical signal-to-noise advantage of 2.18 over the conventional spectrometer, which is rather hard to verify experimentally, Decker (1971) therefore proceeded to build a 255-slit HTS. In this spectrometer, the radiation, after being decoded by the exit mask, exits along the same path it comes in. This reverse pass de-disperses the beam and allows it to be brought to a focus at the entrance plane. Thus the dimensions of the focused image are roughly the same as the dimensions of the entrance aperture, and the detector size can be minimized. This is important since sufficiently large detectors sometimes do not exist, and if available tend to be noisy. Decker experimentally verified the theoretically predicted multiplex advantage of an HTS.

DeGraauw and Veltman (1970) were the first to use an HTS for astronomical work during the 1970 solar eclipse. Houck et al (1973) subsequently used an HTS to obtain near infrared spectra of Mars from airplane altitudes.

Besides putting an encoding mask at the exit plane, one can also put another encoding mask at the entrance plane of a spectrometer. In that way the radiation is modulated at both the entrance and exit apertures. Harwit et al (1970) worked out this scheme of doubly multiplexed dispersive spec-

trometry. The double multiplexing scheme allows one to increase the total amount of radiation that can be transmitted through a spectrometer. Furthermore, by a proper reduction of the data, one can also obtain a one dimensional picture of the source at the entrance plane. For a spectrometer of m entrance slits and n exit slits, one needs $m \times n$ data points to recover m spatial spectra, with each spatial spectrum containing n spectral elements. For a homogeneous source one does not need the spatial information, so $(n + m - 1)$ data points will be enough to recover the spectra. Harwit et al (1974a) describe two schemes for recovering the spectrum with $(n + m - 1)$ data points.

In 1975 Tai et al (1975a) finished the construction of a doubly coded Hadamard transform spectrometer. The spectrometer has 15 entrance slits and 255 exit slits, which can simultaneously obtain 15 spatial spectra, each having 255 spectral elements. Tai et al (1975b) went on to give an analysis of the errors in Hadamard spectrometry caused by imperfect masks.

Besides coding the radiation at both the entrance and the exit aperture, one can go one step further and use a two dimensional mask at the entrance aperture (Harwit, 1971).

This yields a two dimensional picture at the entrance aperture, where each spatial point at the entrance has its own spectrum. To put it a different way, one obtains a two dimensional picture of the source at the entrance aperture for each color of the spectral elements.

Harwit (1973) experimentally verified the operation of imaging spectrometry, and Swift et al (1976) constructed the first Hadamard imaging spectrometer.

There are other discussions of Hadamard transform spectrometry in the literature, mostly of theoretical aspects. Nelson and Fredman (1970) give a more complete theoretical treatment of Hadamard matrix encoding. They also rediscovered a theorem due initially to Hotelling (1944) showing that the Hadamard matrix is the best design for a singly coding mask. Sloane and Harwit (1976) show the connection between Hadamard spectrometry and the mathematics of weighing designs in statistics.

There have been various comparisons of Hadamard transform spectrometry with other spectrometry. Larson et al (1974) makes a theoretical comparison of singly multiplexed Hadamard transform spectrometers and scanning spectrometers. They present a general mathematical framework for the comparison of relative performance and also verify their prediction by computer simulation of various characteristic spectra. Their results show that where the noise level is constant and independent of the incident photon flux, the determined multiplex advantage is $\sqrt{N/2}$, as predicted by Fellgett (1951). This is usually the case in a low energy region, such as the infrared. For a noise level that is signal-dependent, such as in the UV energy region, the detector is characterized by an output with statistics approaching a Poisson distribution and variance therefore proportional to the input signal. In that case the

HTS technique will be advantageous only for spectra that are characterized by a few well-defined and intense peaks on a very low intensity background. For spectra with high background, for dense spectra, or for spectra having very weak spectral features, the HTS will have no advantage over the conventional single slit (SS) technique.

Hirschfeld and Wyntjes (1973) compare Fourier transform and Hadamard transform spectrometry. They also describe various limitations of Hadamard transform spectrometry. This paper was followed by an exchange of notes between Decker (1973) and Hirschfeld and Wyntjes (1973) in the journal Applied Optics in which some of these limitations are disputed. These papers concern themselves with a number of practical matters on which opinions can vary. Here we mention these controversial papers mainly for completeness. Their contents will be discussed further below.

Wyatt and Esplin (1974) analyzed the effect of band width on noise equivalent power (NEP) for multiplex spectrometry with cryogenically cooled, cooled-background extrinsic long wavelength infrared detectors. They find that the NEP is directly proportional to band width, so multiplex schemes that require increased band width are not of real advantage. They further conclude that doubly encoded systems that are based on $m + n - 1$ measurements would have a real throughput advantage

Various other aspects of Hadamard matrices and Hadamard transform spectrometry which have not been mentioned above are covered in articles by: Baumert, Pratt et al (1969), Hirschy

et al (1971), Allen et al (1972,1973), Kowalski et al (1973), Planky et al (1974), Oliver et al (1974).

In this thesis Chapter II will describe the mathematical properties of Hadamard matrices and their application to spectroscopy. Chapter III describes the Hadamard transform spectrometer, and gives results on laboratory performance. Chapter IV gives a comparison of Hadamard transform and Fourier transform encoding in spectrometry. The output of an HTS is fed into a mini computer. The computer performs a real time inverse Hadamard transform to recover the spectrum. Chapter V describes the algorithm and programming of inverse Hadamard transform. Chapter VI discusses observational results and their interpretation.

CHAPTER II

HADAMARD MATRICES

(A) Weighing Designs

In order to understand the mathematical advantage of Hadamard transform encoding, let us look at the following examples (Sloane et al., 1976).

Suppose four objects are to be weighed, using a spring balance which makes an error e each time it is used. Assume that e is a random variable with mean zero and variance σ^2 .

First suppose the objects are weighed separately. If the unknown weights are $\psi_1, \psi_2, \psi_3, \psi_4$, the measurements are $\eta_1, \eta_2, \eta_3, \eta_4$, and the errors made by the balance are e_1, e_2, e_3, e_4 , then the four weighings give four equations:

$$\begin{array}{ll} \eta_1 = \psi_1 + e_1 & \eta_2 = \psi_2 + e_2 \\ \eta_3 = \psi_3 + e_3 & \eta_4 = \psi_4 + e_4 \end{array}$$

The best estimate of the unknown weights are the measurements themselves:

$$\begin{array}{l} \hat{\psi}_1 = \eta_1 = \psi_1 + e_1 \\ \hat{\psi}_2 = \eta_2 = \psi_2 + e_2 \end{array}$$

These are unbiased estimates:

$$E\hat{\psi}_1 = \psi_1$$

$$E\hat{\psi}_2 = \psi_2 \quad (E \text{ denotes expected value})$$

with variance or mean square error

$$E(\hat{\psi}_1 - \psi_1)^2 = E\sigma_1^2 = \sigma^2$$

On the other hand, suppose the balance is a chemical balance with two pans, and the four weighings are made as follows:

$$\begin{aligned} \eta_1 &= \psi_1 + \psi_2 + \psi_3 + \psi_4 + e_1 \\ \eta_2 &= \psi_1 - \psi_2 - \psi_3 - \psi_4 + e_2 \\ \eta_3 &= \psi_1 + \psi_2 - \psi_3 - \psi_4 + e_3 \\ \eta_4 &= \psi_1 - \psi_2 - \psi_3 + \psi_4 + e_4 \end{aligned} \quad (2-1)$$

This means that in the first weighing all four objects are placed in the left hand pan, and in the other weighings two objects are in the left pan and two in the right. (Note that the e are independent of the weights on the balance. This point is crucial). It is easy to solve for $\psi_1, \psi_2, \psi_3, \psi_4$, as long as the coefficient matrix for ψ is not singular. Thus the best estimate for ψ_1 is

$$\begin{aligned}\hat{\psi}_1 &= \frac{1}{4}(\eta_1 + \eta_2 + \eta_3 + \eta_4) \\ &= \psi_1 + \frac{1}{4}(e_1 + e_2 + e_3 + e_4)\end{aligned}$$

The variance of Ce , here C is a constant, is C^2 times the variance of e , and the variance of a sum of independent random variables is the sum of the individual variances.

Therefore the variance of $\hat{\psi}_1$ (and also of $\hat{\psi}_2, \hat{\psi}_3, \hat{\psi}_4$) is

$$\frac{4\sigma^2}{16} = \frac{\sigma^2}{4}$$

Weighing the objects together has reduced the mean square error by a factor of 4. In effect the signal to noise ratio (SNR), which is given by the root mean square (rms) error is reduced by a factor of 2.

Finally, suppose the balance is a spring balance with only one pan, so only coefficients 0 and 1 can be used. A good method of weighing the four objects is:

$$\begin{aligned}\eta_1 &= \psi_2 + \psi_3 + \psi_4 + e_1 \\ \eta_2 &= \psi_1 + \psi_2 + e_2 \\ \eta_3 &= \psi_1 + \psi_3 + e_3 \\ \eta_4 &= \psi_1 + \psi_4 + e_4\end{aligned}\quad (2-2)$$

In this case the variances of $\psi_1, \psi_2, \psi_3, \psi_4$, are $\frac{4\sigma^2}{9}, \frac{7\sigma^2}{9}, \frac{7\sigma^2}{9}, \frac{7\sigma^2}{9}$ respectively, a smaller improvement than in the previous case.

The theory of weighing designs is of immediate interest to multiplex optics, since the simultaneous measurement of the intensities of different bundles of rays is completely ana-

logous to the simultaneous weighing of different groups of weights. In measuring the intensity of radiation passed through slits in a mask, we are effectively 'weighing' that radiation.

(B) General Mathematical Formulation

One can put the problem into a more general form. Let ψ_i be the i^{th} unknown, η_j be the j^{th} measurement, e_j be the error associated with the j^{th} measurement. Let w_{ji} be the weighing coefficient of the j^{th} measurement with the i^{th} unknown. Then

$$\eta_j = \sum_{i=1}^n w_{ji} \psi_i + e_j \quad \begin{matrix} i=1 \dots n \\ j=1 \dots n \end{matrix} \quad (2-3)$$

In matrix notation:

$$\underline{\eta} = \underline{W} \underline{\psi} + \underline{e} \quad (2-4)$$

With the notation $\langle \rangle$ for ensemble averages, the error e_i has the following properties:

- (1) $\langle e_i \rangle = 0$
- (2) e_i is independent of $\underline{\psi}$
- (3) $\langle e_i e_j \rangle = 0$ if errors are assumed to be uncorrelated.
 $= \sigma^2$ if $i=j$

The problem now is the following: (i) For a particular coding matrix W , what should be the decoding matrix A ? i.e. What is A such that $\hat{\underline{\psi}} = \underline{A} \underline{\eta}$ where $\hat{\underline{\psi}}$ is an unbiased estimate of $\underline{\psi}$.

(ii) What is the best choice of \underline{W} (or \underline{A}) that will minimize the error of measurement i.e. What is \underline{W} such that

$$\epsilon = \left\langle \sum_{j=1}^n (\hat{\psi}_j - \psi_j)^2 \right\rangle$$

is a minimum.

In the absence of noise, i.e. $n=0$, it is clear from (2-4) that

$$\underline{\psi} = \underline{W}^{-1} \underline{n}$$

and therefore

$$\underline{A} = \underline{W}^{-1}$$

and

$$\hat{\underline{\psi}} = \underline{A} \underline{n} = \underline{\psi}$$

In the presence of noise,

$$\begin{aligned} \hat{\underline{\psi}} &= \underline{A} \cdot \underline{n} \\ &= \underline{A} \underline{W} \underline{\psi} + \underline{A} \underline{n} \end{aligned}$$

and with the assumed properties of the noise $\langle \underline{\psi} \rangle = \underline{\psi}$, one obtains

$$\begin{aligned} \langle \hat{\underline{\psi}} \rangle &= \underline{A} \underline{W} \langle \underline{\psi} \rangle + \underline{A} \langle \underline{e} \rangle \\ &= \underline{A} \underline{W} \underline{\psi} \end{aligned}$$

Assuming no prior knowledge of the unknowns, one may use the unbiased condition $\langle \hat{\underline{\psi}} \rangle = \underline{\psi}$. This again implies

$$\underline{A} = \underline{W}^{-1}$$

So with the assumed properties of noise and unbiased condition, the decoding matrix is just the inverse of the coding matrix.

One still has to find a coding matrix which will minimize

the uncertainty, ϵ .

The second question can be solved in the following way:
Let $\hat{\eta}_i$ be the i^{th} measurement in the absence of noise, then
for each measurement

$$\begin{aligned}\eta_i &= W_{i1}\psi_1 + W_{i2}\psi_2 + \dots + W_{in}\psi_n + e_i \\ &= \hat{\eta}_i + e_i \\ \psi_j &= A_{j1}\eta_1 + A_{j2}\eta_2 + \dots + A_{jn}\eta_n \\ &= A_{j1}(\hat{\eta}_1 + e_1) + A_{j2}(\hat{\eta}_2 + e_2) + \dots \\ &\quad + A_{jn}(\hat{\eta}_n + e_n) \\ &= (A_{j1}\eta_1 + \dots + A_{jn}\eta_n) + (A_{j2}e_2 + \dots + A_{jn}e_n) \\ &= \psi_j + \text{noise}\end{aligned}$$

The mean square of the noise term corresponding to the j^{th} unknown is therefore

$$\begin{aligned}\epsilon_j &= (A_{j1}^2 + \dots + A_{jn}^2) \sigma^2 \\ &= \Delta_j^2 \sigma^2\end{aligned}\tag{2-7}$$

where

$$\Delta_j = (A_{j1}^2 + \dots + A_{jn}^2)^{\frac{1}{2}}\tag{2-8}$$

and Δ_j represents the improvement in the SNR for the weighing design, compared to the SNR for individual weighings.

Hence, the problem of maximizing the signal to noise ratio becomes the problem of minimizing ϵ_j ; or Δ_j (Nelson and Fredman (1970)).

Sloane et al (1969) independently developed an expression for ϵ/σ^2 where

$$\begin{aligned}\epsilon/\sigma^2 &= \text{Trace } \underline{W}^{-1}(\underline{W}^{-1})^T \\ &= \text{Trace } \underline{A} \underline{A}^T\end{aligned}\quad (2-9)$$

Equation (2-8) and (2-9) amounts to the same thing because it can be seen very easily that

$$\text{Trace } \underline{A} \underline{A}^T = \sum_{j=1}^n \Delta_j^2$$

The question of minimizing ϵ had been answered by Hotelling (Hotelling, 1944) and rediscovered by Nelson and Fredman.

Hotelling has shown that for any choice of mask W with $|W_j| \leq 1$, the ϵ_i are bounded by $\epsilon_i \geq \frac{\sigma^2}{N}$, and that it is possible to have $\epsilon_i = \frac{\sigma^2}{N}$ for $i=1, \dots, N$ if and only if a Hadamard Matrix H_N of the order N exists (by taking $W=H_N$). This leads to the discussion of the Hadamard Matrix.

(C) Hadamard Matrix

A Hadamard matrix of order N is an $N \times N$ matrix H_N of +1's and -1's which satisfies:

$$\underline{H}_N \underline{H}_N^T = \underline{H} \underline{I}_N \quad (2-10)$$

where \underline{I}_N is an $N \times N$ unit matrix.

A Hadamard matrix has following properties: (Golomb(1964))

- (1) Its row vectors (or equivalently, its column vectors) are mutually orthogonal.
- (2) The Hadamard properties will not be disturbed by:
 - a. Interchanging rows,
 - b. Interchanging columns,
 - c. Changing the sign of every element in a row, or

d. Changing the sign of every element in column.

These properties enable the first row and column of every Hadamard matrix to be normalized to contain only +1's. If \underline{G} represents the remaining $M \times M$ matrix ($M = N-1$), then \underline{H} can be partitioned into

$$\underline{H} = \begin{bmatrix} 1 & 1 & 1 & 1 & \dots & \dots & 1 \\ 1 & & & & & & \\ 1 & & \underline{G} & & & & \\ \cdot & & & & & & \\ \cdot & & & & & & \\ \cdot & & & & & & \\ 1 & & & & & & \end{bmatrix} \quad (M = N-1)$$

It is conjectured that Hadamard matrices exist for all multiples of four. Further, if one of the following conditions is also satisfied,

- (1) $N = P + 1$ P prime
- (2) $N = P(P + 2) + 1$ P and $P + 2$ prime
- (3) $n = 2^m$ m an integer

then \underline{G} can be made cyclic. That is, the $(j + 1)^{\text{th}}$ row can be generated by shifting the j^{th} row one position to the left. For example, when $N=8$, we have matrices of the form shown in Fig. 1-1. Note that \underline{H} and \underline{G} are symmetric.

Another choice for \underline{W} is the matrix \underline{S} obtained from \underline{G} by replacing +1's by 0's and -1's by +1's.

The properties of the \underline{H} , \underline{G} and \underline{S} are discussed by Sloane et al. If rows i and j are any two rows of \underline{H} , \underline{G} or \underline{S} , it can be shown that their dot product is:

$$\text{In } H_N : \text{ row } i \cdot \text{ row } j = \begin{matrix} 0 & i \neq j \\ N & i = j \end{matrix}$$

$$\text{In } G_M : \text{ row } i \cdot \text{ row } j = \begin{matrix} -1 & i \neq j \\ M & i = j \end{matrix}$$

$$\text{In } S_M : \text{ row } i \cdot \text{ row } j = \begin{matrix} M/4 & i \neq j \\ M/2 & i = j \end{matrix}$$

The inverse of each matrix is:

$$H_N^{-1} = \frac{1}{N} H_N ; G_M^{-1} = \frac{1}{M+1} (G_M - J_M) ; S_M^{-1} = \frac{2}{M+1} (2S_M - J_M)$$

where J is a $M \times M$ matrix consisting entirely of -1's and $N = M+1$.

Table 2-1 gives the value of Δ for different matrices.

The matrix I represents the weighing scheme weighing each object separately. This corresponds to a conventional single slit spectrometer or to a wedge filter monochromator.

Table 2-1

MATRIX A	ELEMENTS OF A	Δ^{-1}	(FOR LARGE N)
I	1, -0	1	1
H	1, -1	\sqrt{N}	\sqrt{N}
G	1, -1	$\sqrt{N} (2 - \frac{2}{N})^{-\frac{1}{2}}$	$\sqrt{\frac{N}{2}}$
S	1, 0	$\sqrt{N} (2 - \frac{2}{N})^{-1}$	$\frac{\sqrt{N}}{2}$
* F	cosine squared functions	$\frac{N}{4} \frac{2}{N+1}$	$\sqrt{\frac{N}{8}}$

* F is the Fourier Transform case which will be discussed in Chapter IV.

If the number of measurements N is a multiple of 4, and the matrix coefficients are ± 1 , the best weighing scheme is the Hadamard matrix H . ϵ will be reduced by a factor $\frac{1}{\sqrt{N}}$ compared to weighing the unknown separately. This is the maximum advantage a weighing scheme can obtain with weighing coefficient $|W_{ij}| \leq 1$. If N is not a multiple of 4, or if the weighing coefficients are 0's and 1's, it is not possible to simultaneously minimize $\epsilon_1 \dots \epsilon_n$ and some other criterion must be used (Sloane et al., 1976). Also the errors are uniformly larger than for the H -matrix, as shown for the G and S matrices in Table 2-1 above.

It is interesting to see that a spectrometer using the Fourier Transform, such as a Michelson interferometer, has a multiplex advantage a factor of $\sqrt{8}$ lower than the H -matrix and a factor of $\sqrt{2}$ lower than the S -matrix encoding instrument.

Following are some computer simulations for S -matrix transformations with various inputs (See Fig. 2-1(a) to (c)).

INPUT	OUTPUT
1. Constant	Constant
2. Hadamard code: representing single line emission	Single line
3. Single line: 1 at the 1 st element and 0 for the rest. This represents an unknown impulse coming in during the	Hadamard code. Note, unlike the monochromater, the error propagates to other

INPUT	OUTPUT
observation.	elements.
4. Sine wave.	sine wave with different phase.
5. Square wave. The input is not a perfect wave because we have an odd number of elements. The input values have amplitudes 0 or 1 for each element.	Not a perfect square wave.
6. Straight line at a slope 1/255. This may represent a shift in baseline.	Refer to figure (2-1C).

(D) Optical Realization of Hadamard Encoding

We have made use of (modified Hadamard) S-matrices in two optical instruments: One is a Hadamard transform spectrometer having an encoding mask at the exit aperture. The other is a doubly encoded HTS which has encoding masks at both the entrance and exit apertures.

S codes can be used for both the entrance and exit masks for the HTS, with +1 standing for an open slot through which radiation is transmitted, and with 0 standing for a closed slot where radiation is blocked.

The cyclic property of the S-matrix is very desirable, for then only a single mask $2N-1$ slots wide need be constructed. Successive encoding positions are generated by stepping the mask one slot width along its length. This avoids the construction of N masks with N^2 slots.

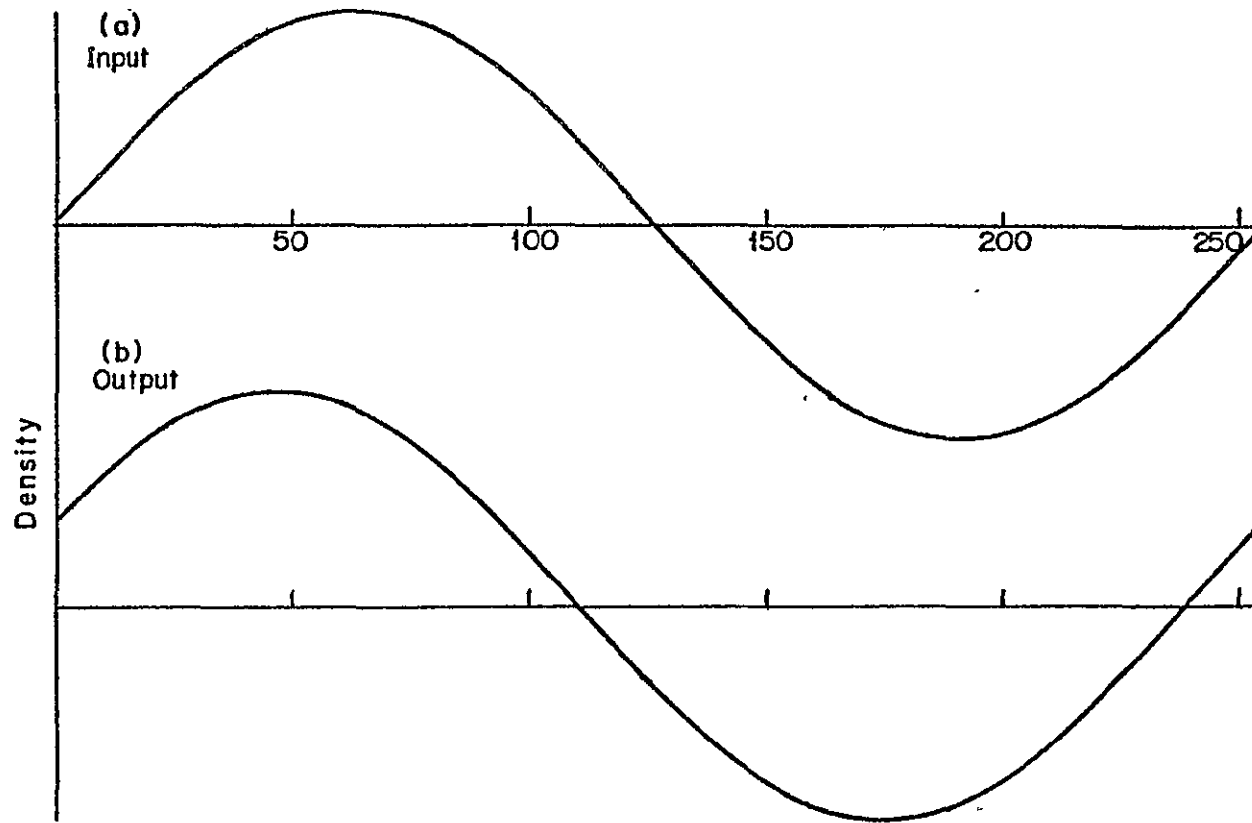


Figure 2-1a. Hadamard transformation of a Sine wave input.

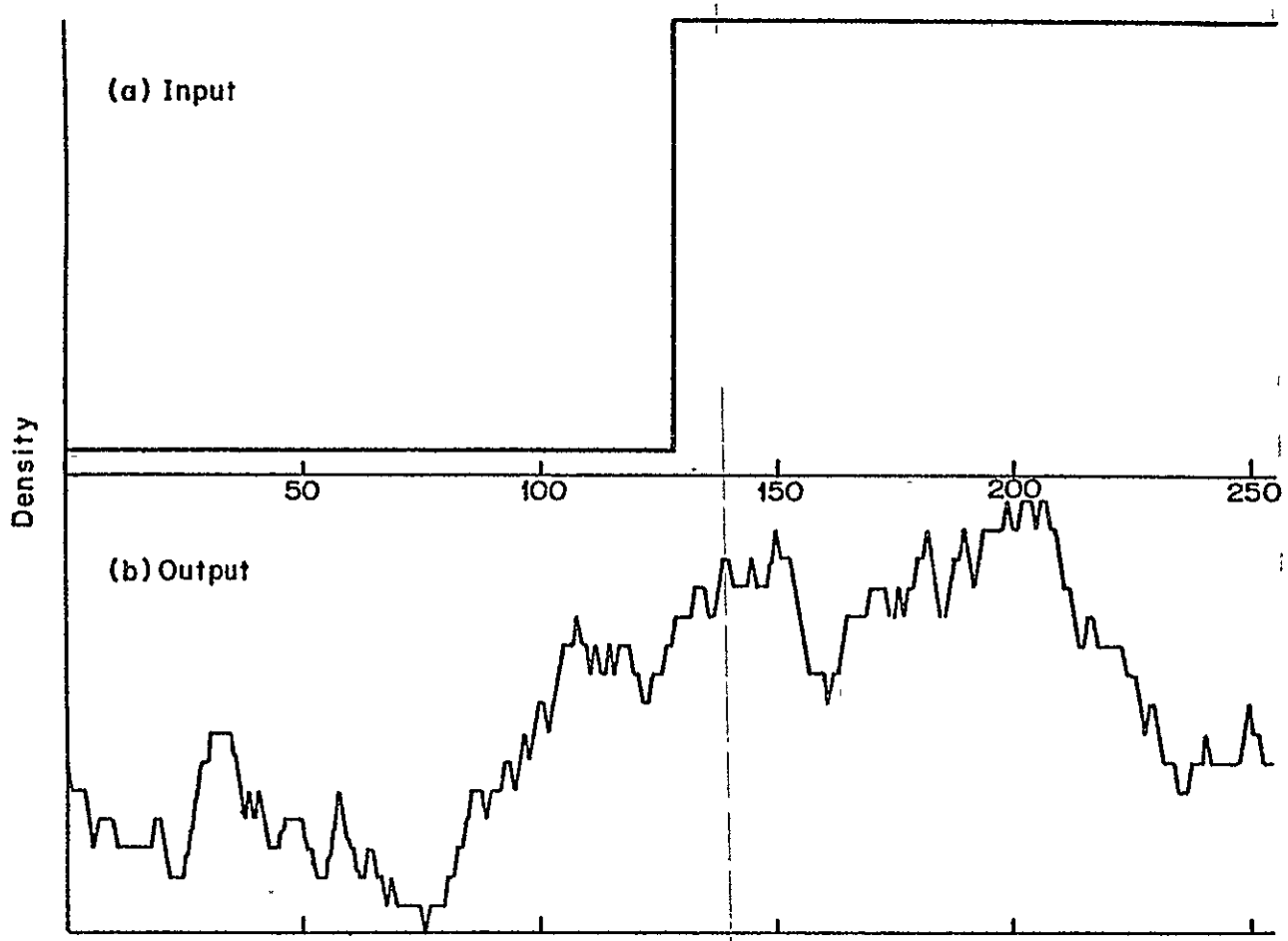


Figure 2-1b. Hadamard transformation of a square wave input

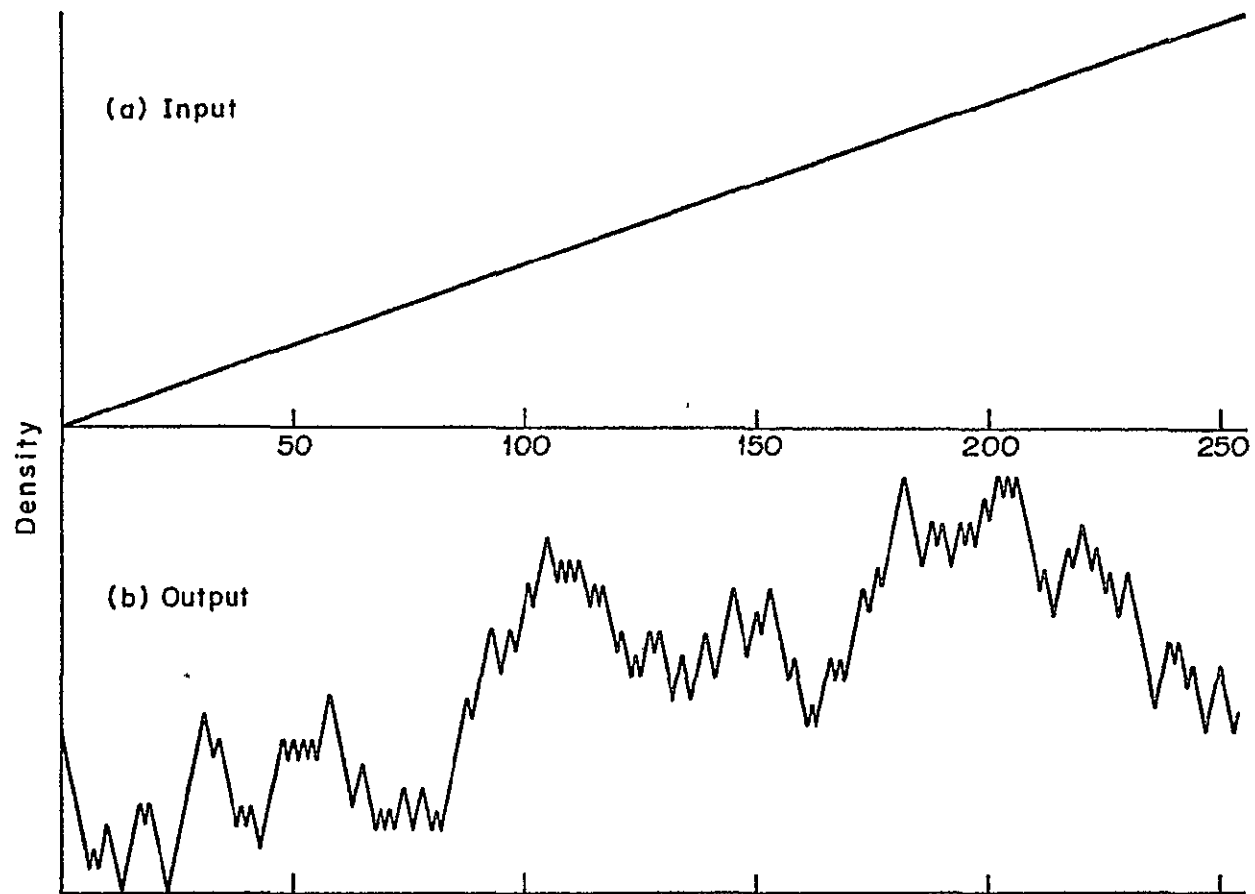


Figure 2-1c. Hadamard transformation of a straight line input.

Although H and G matrices have better coding efficiency than the S matrix, they introduce technical difficulties when used for spectrometer coding schemes. To utilize H and G matrices, one must measure the reflected as well as the transmitted radiation. In this case +1's represent reflecting slots and -1's represent transmitting slots. Therefore a minimum of two detectors must then be used, one in a subtracting mode, the other in the normal mode. The use of two detectors, however, increases the noise. Furthermore, the H matrix, with all elements +1 in the first row, makes the dynamic range of the detector system change by about a factor of two. Also its lack of the cyclic property does not allow one to generate the rest of the masks by the simple stepping technique mentioned above.

1. Hadamard Transform Spectrometer (HTS)

For the singly encoded HTS, we use the following optical arrangement (figure 2-2). Radiation passing through the single entrance slot is rendered parallel and directed to the dispersing element. The dispersed radiation is then de-collimated and focused upon the multi-slot mask at the exit aperture. The spectral elements transmitted by the mask pass through suitable post-optics and are collected onto a detector. One then makes N (in our case N=255) measurements by sequentially stepping the mask N times. The inversion procedure $\hat{\psi} = \underline{S}^{-1} \underline{n}$ recovers the spectrum. Figure (2-3) gives a 255 cyclic S-matrix code for the exit mask.

Theoretically, the mean square error in the spectrum

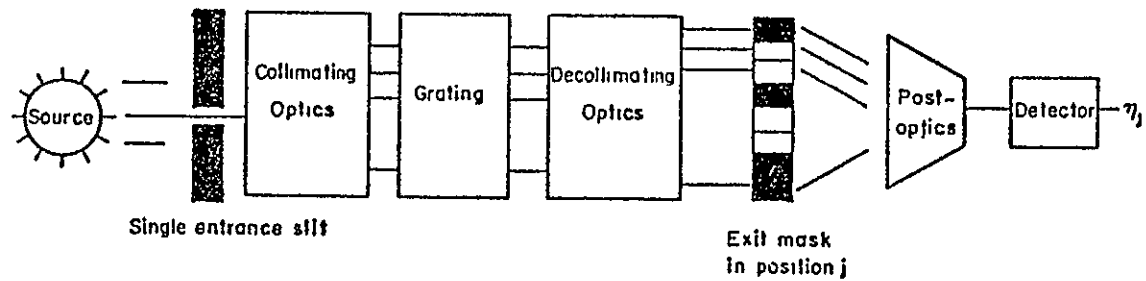
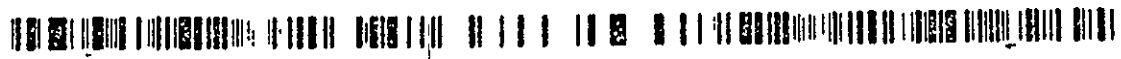


Figure 2-2. Schematic representation of HTS using the S code.



00000	00101	10001	11101	00001	11111	11001	00001
01001	11110	10101	01110	00001	10001	01011	00110
01011	11110	11110	01101	11011	10010	10100	10100
01001	01101	00011	00111	00111	10001	10110	00010
00101	11010	11110	11011	11100	00110	10011	01011
01101	01000	00100	11101	10010	01001	10000	00111
01001	00011	10001					

Figure 2-3. The 509 exit slit mask and the 255-element S code.

given by a monochromator is σ^2 . For an HTS using the S code, this error is $\frac{1}{N}(2-\frac{2}{N})^2\sigma^2$ (Sloane et al, 1969). Hence the rms gain in S/N for the HTS is $G = \frac{N\sigma^2}{(2-\frac{2}{N})^2\sigma^2} = \frac{\sqrt{N}}{2-\frac{2}{N}}$. For $N=255$, $G \sim 8.0$ (figure 2-4 gives Decker's results. The experimental gain was measured as 8.0 ± 0.3). Note that the scale in figure (2-4) are in arbitrary units, and the zero point appears to be shifted between parts (a) and (b).

2. Doubly Encoded Hadamard Transform Spectrometer (DHTS)

Figure (2-5) is a schematic representation of an optical system which has a number of entrance as well as exit slits. Instead of passing radiation through only one entrance slit, a mask M slits wide is placed at the entrance aperture. Radiation passed into the spectrometer through different combinations of open and closed slits. The dispersed radiation at the exit plane is analyzed in the same fashion as in the HTS. Encoding is accomplished by sequentially stepping one of the masks through its N different positions for each position of the other mask.

In a DHTS the entrance aperture is modulated by a $P \times P$ S matrix.

Let $\epsilon = \epsilon_{ir}$ be the $P \times P$ matrix whose rows represent P different entrance masks. $\epsilon_{ir} = 1$ for open slots and 0 for closed slots ($1 \leq i \leq P$, $1 \leq r \leq P$). Similarly let $x = x_{ij}$ represent the exit mask. When the entrance mask is in position i and the exit mask is in position j , the detector measurement η_{ij} is

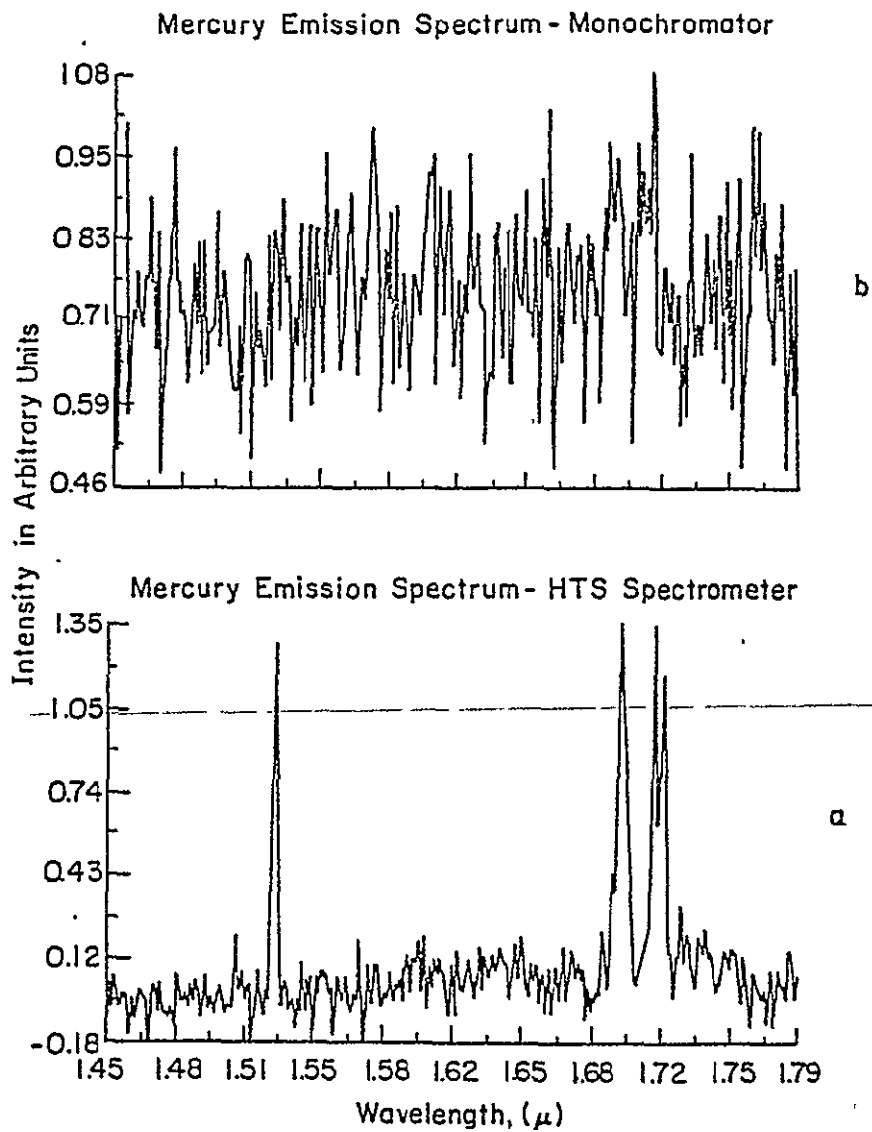


Figure 2-4. Comparison spectrum of the mercury emission lines in the 1.4-1.8 μ m region: (a, bottom) as obtained in the Hadamard-transform mode; (b, top) as obtained under identical conditions using the same optical system as a conventional monochromator. This figure is taken from Decker, 1971b.

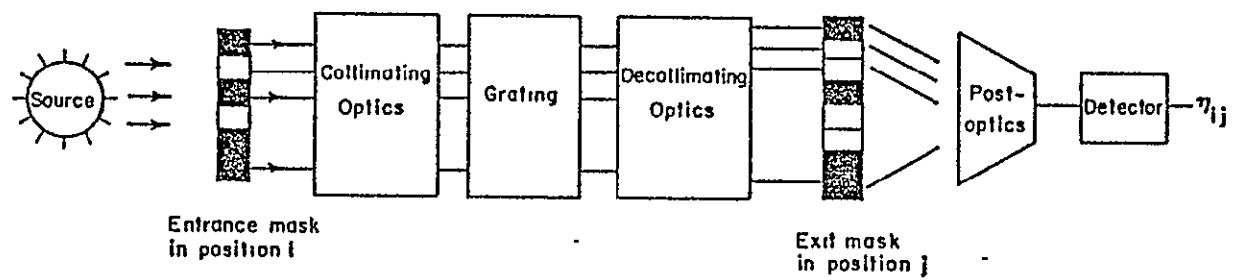


Figure 2-5. Schematic representation of DHTS.

$$n_{ij} = \sum_{r=1}^P \sum_{s=1}^N \epsilon_{ir} \psi_{rs} \chi_{sj} + v_{ij} \quad (2-11)$$

where ψ_{rs} is the spectral element produced by radiation passing through the r^{th} entrance slot and the S^{th} exit slot, v_{ij} is the noise in the $(i,j)^{\text{th}}$ measurement; it has the following properties:

$$\langle v_{ij} \rangle = 0 \quad \langle v_{ij}, v_{kl} \rangle = \sigma^2 \delta_{ik} \delta_{jl}$$

If the instrument has no optical magnification, the spectrum of radiation that passes solely through the r^{th} entrance slot to first order is shifted by r spectral channels from the spectrum passing solely through the first entrance slot.

Hence, only $P+N-1$ distinct spectral elements exist:

$$\psi_{-(P-1)}, \dots, \psi_{-1}, \psi_0, \psi_1, \dots, \psi_{N-1}$$

where

$$\psi_{rs} \equiv \psi_{r-s} \equiv \psi_t \quad (t=r-s)$$

In matrix notation one may write

$$\underline{n} = \underline{\epsilon} \underline{\psi} \underline{\chi}^T + \underline{v} \quad (2-12)$$

Employing the same analysis as one does for the HTS, i.e. using the unbiased condition $\langle \hat{\underline{\psi}} \rangle = \underline{\psi}$ and the properties of \underline{v} ,

one obtains

$$\hat{\underline{\psi}} = \underline{\epsilon}^{-1} \underline{\Pi} (X^T)^{-1}$$

where

$$\hat{\underline{\psi}} = \begin{array}{ccccccc} & \hat{\psi}_0 & \cdots & \cdots & \cdots & & \hat{\psi}_{-N+1} \\ \hat{\psi}_1 & \hat{\psi}_1 & \cdots & \cdots & \cdots & & \hat{\psi}_{-N} \\ & \vdots & & & & & \vdots \\ \hat{\psi}_{P-1} & \hat{\psi}_{P-2} & \cdots & \cdots & \cdots & & \hat{\psi}_{-N-P+2} \end{array}$$

Each row i of $\hat{\underline{\psi}}$ represents a spectrum at the exit mask for radiation that enters the instrument through the i^{th} entrance position. Hence the j^{th} diagonal gives a one-dimensional spatial picture across the entrance aperture for the spectral element j .

One may obtain an average spectrum $\hat{\underline{\psi}}_i$ by averaging all the elements in each diagonal.

$$\begin{aligned} \hat{\psi}_i &= \frac{1}{N-|t|} \sum_{r=1}^P \psi_{r,r-t} & t \geq 0 \\ &= \frac{1}{N-|t|} \sum_{r=1}^P \psi_{r,r-t} & t < 0 \end{aligned}$$

Harwit et al (1970) showed that if both the entrance and exit masks are S matrices and we define

$$\sigma_t^2 = \langle (\underline{\psi}_t - \hat{\underline{\psi}}_t)^2 \rangle$$

Then

$$\begin{aligned}\sigma_t^2 &= \frac{16}{(N+1)} \frac{N^2-1}{N-|t|} \sigma^2 \\ &\approx \frac{16\sigma^2}{(N-|t|)N^2} \quad \text{for } N \text{ large} \quad (2-13)\end{aligned}$$

where

$$t = -(N-1), \dots, (N-1)$$

If the total mean square error for the unknown is

$$\epsilon = \sum_{t=-\frac{N-1}{2}}^{\frac{N-1}{2}} \sigma_t^2$$

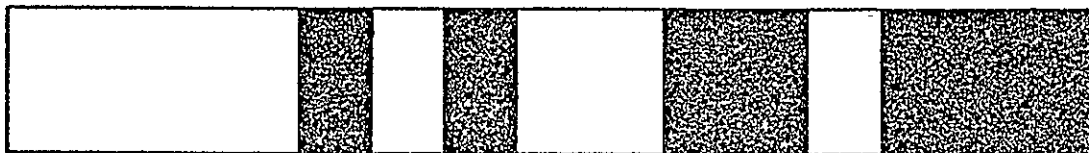
where one sums only the central element, then for the S-code

$$\epsilon = \sigma^2 \left[\frac{22.18}{N} + O\left(\frac{1}{N^2}\right) \right] \quad N \text{ large}$$

where $\sigma^2 = \text{constant} \frac{N}{T}$.

There are two points that should be made about the DHTS.

(1) It has not been shown that Hadamard codes are the best codes for such an instrument. In fact some evidence suggests that Hadamard codes are not precisely optimum for this "two-ended" operation (Harwit et al, 1974b). (2) For $P \times N$ data points the spectrum yields only $P+N-1$ spectral elements, plus a one-dimensional image. It is also possible to reconstruct the $(P+N-1)$ elements with $(P+N-1)$ data points only (Harwit et al, 1974b). Fig. (2-6) gives a 15 element cyclic S-matrix code for the entrance mask.



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1 1 1 1 0 1 0 1 1 0 0 1 0 0 0

Figure 2-6. The 29 entrance slit mask and the 15-element S code.

Table 2-2 compares three different grating spectrometers. The first column represents the conventional single entrance and exit slot instrument. N measurements are made in time T , with a mean square error σ^2 in each. The second column is for a singly multiplexed instrument with an exit mask S , and is taken from Sloane et al (1969). The last column is for the doubly multiplexed system, using Equation (2-13) for σ_t^2 , and has been multiplied by a factor of N to allow for having to make N^2 measurements in time T .

Table 2-2

	NO MASK	SHT	DHT
Δ^{-1}	1	$\frac{\sqrt{N}}{2}$	$\frac{N}{\sqrt{22.2}}$

(E) Errors in Hadamard-Spectroscopy

During the manufacture of the masks, whether by deposition of metal or by removal of metal through an etching process, it is possible to obtain a systematic error that leaves each of the opaque portions of the mask either too wide or too narrow by a fixed amount. This will cause a systematic variation in signal passing through the slit. For example if the open slit is too narrow by a fixed amount ϵ , the light passing through an open slit position will be

I_0 when the open slit is bounded by two open slits.

$I_0(1-\epsilon)$ when the open slit is bounded by one open slit and one closed slit.

$I_0(1-2\epsilon)$ when the open slit is bounded by two closed slits.

A similar analysis holds for open slits that are too wide, except that the minus sign in these expressions is replaced by a plus sign.

The spectrum of a single (spectral line resulting from such imperfect masks) is remarkably simple (Tai et al., 1975 b). Independent of the particular S-matrix mask to be used, there are always precisely four false blips present in the final spectrum. The amplitude of these blips is always the same for a fixed narrowing or widening of the transmitting slits. Two of the blips always surround the main spectral line, and a pair of adjacent blips always are some distance removed from that line. For the 255 element S-matrix, these two are located 24 and 25 elements away to the left of the parent line. The amplitude of the displaced blips is positive when the transparent slits are too wide and is negative when the slits are too narrow. In contrast, the two blips surrounding the parent line always are positive. Figure (2-7) shows the negative features accompanying the 1.7 μm mercury vapor doublet and the computer simulation of a pure spectral line input and its distorted spectrum. For the general case the reader may refer to Tai et al (1975 b).

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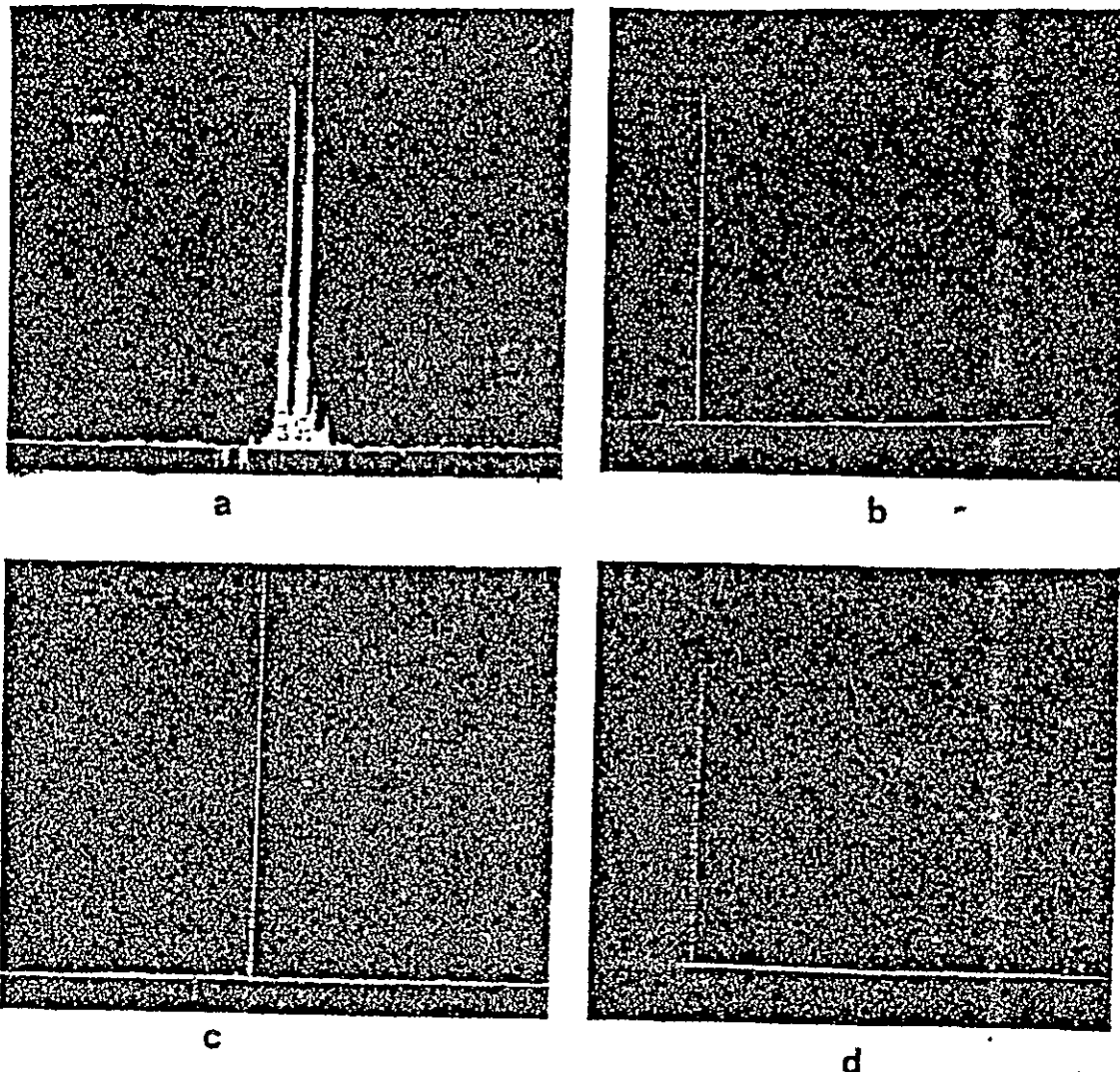


Figure 2-7. (a) Spectrum of the $1.7\mu\text{m}$ mercury vapor doublet showing negative peaks to the left at the emission peaks; (b) Shows the response we would obtain to a single spectral line with a perfect mask; (c) Shows the response for a single line with the radiation simulated as passing through a mask with slits too narrow because each opaque mask element protrudes into the adjacent transparent slot by a tenth of a slot width; (d) Shows the effect of simulating slits that are systematically too wide. Note that the main spectral line has been placed in different positions for the synthetic runs (b), (c), and (d).

CHAPTER III

INSTRUMENTATION

(A) Multislit Spectrometer

A conventional spectrometer has four essential elements, an entrance slit, a dispersive device such as a grating or prism, a set of imaging optics, and an exit slit.

Such a spectrometer has two important parameters. The first is the "resolution" R , which is a measure of how well the spectrometer can separate two neighboring lines. The second parameter is the "throughput" E . This is a measure of the light gathering capability of the system. The two parameters, R and E , are lumped together into what is called "luminosity" L , defined as (Vanasse, 1974).

$$L = E \cdot R$$

For a conventional grating spectrometer having a grating of area A_g given by WH , where W and H are the width and height respectively of the grating, the throughput is determined by the product of A_g with the solid angle Ω subtended by the slit at the collimating mirror (or lens). The solid angle is given by

$$\Omega = \frac{w \cdot l}{F^2}$$

where w and l are the width and height respectively of the slit

and F is the focal length of the collimating mirror

$$E \sim W \cdot H \cdot \frac{W \cdot h}{F^2} \quad (3-1)$$

The resolution of this instrument is

$$\begin{aligned} R &= \frac{\lambda}{\Delta\lambda} \\ &= nN \\ &= n \cdot \frac{W}{d} \end{aligned} \quad (3-2)$$

where λ is the wavelength, $\Delta\lambda$ the closest wavelength that can be separated, n the order, N the total number of lines on the grating, W the width of the grating and d the spacing between rulings.

$$\text{Since } d \sin \alpha = n\lambda \quad (3-3)$$

substitute (3-2) into (3-3)

$$\frac{W}{R} \sin \alpha = n\lambda \quad (3-4)$$

if the slit width is limited by diffraction, which is the minimum slit width, then

$$W \sim \frac{F\lambda}{W \cos \alpha} \quad (3-5)$$

substituting (3-5) into (3-4) one gets

$$R \sim \frac{F}{W} \tan \alpha \quad (3-6)$$

Comparing equation (3-1) and (3-6) one sees immediately that, for a fixed grating area $W \cdot H$, and fixed optical system, E is proportional to the slit width w and R is inversely proportional to the slit width w . This means that an increase in luminosity of the system by increasing the slit width is made at the sacrifice of resolution, and vice versa.

From (3-1) and (3-6) one obtains

$$\begin{aligned} L &\sim E \cdot R \\ &\sim W \cdot H \cdot \frac{1}{F} \end{aligned}$$

Another feature of a conventional spectrometer is that it transmits only one narrow spectral range of light to the detector, and all other spectral elements are wasted. As a result, the instrument is inefficient.

Within the past two decades there has been much research done in an effort to design new spectrometric systems with a view to maximizing the luminosity L , and to observe a number of spectral elements simultaneously. This can provide a multiplex advantage, or a wide aperture advantage. Two quite distinct modulation techniques have been employed in the past. The first depends on the wave nature of radiation, and makes use of interferometry. The Fabry-Perot interferometer, Michelson interferometer, and Mach-Zehnder interferometer (Jacquinot, 1954, 1960; Vanasse and Sakai, 1967) are instru-

ments of this type. The other technique employs dispersing spectrometers in which entrance and exit slits are replaced by opaque or transmitting masks. Golay's multislit spectrometer, Girard's Grill spectrometer and Hadamard spectrometers (Harwit et al, 1974a) are representative of these instruments.

A spectrometer, whether interferometric or mask-multiplexed, yields a multiplex advantage mainly for detector noise or amplifier noise limited applications. In these cases it can be shown that for N spectral elements, one can achieve of the order $N^{1/2}$ improvement in the overall spectral signal-to-noise ratio, S/N , over a conventional spectrometer (Chapter II).

For photon noise limited applications, the multiplexing advantage is cancelled by the N -fold increase in the photon noise attributed to the N -fold increase in the energy falling onto the detector. Nevertheless, for photon noise limitations, the throughput advantage can still be realized. The large throughput will become a disadvantage when the noise is background noise which increases faster than the noise due to the source (Harwit et al, 1974a).

In this chapter we will describe the experimental study of a Hadamard transform spectrometer (HTS) and calibration in the laboratory. Figure (3-1) is the flow chart of the whole process, starting with radiation from the telescope and ending with the output of the computer. Each component will be described.

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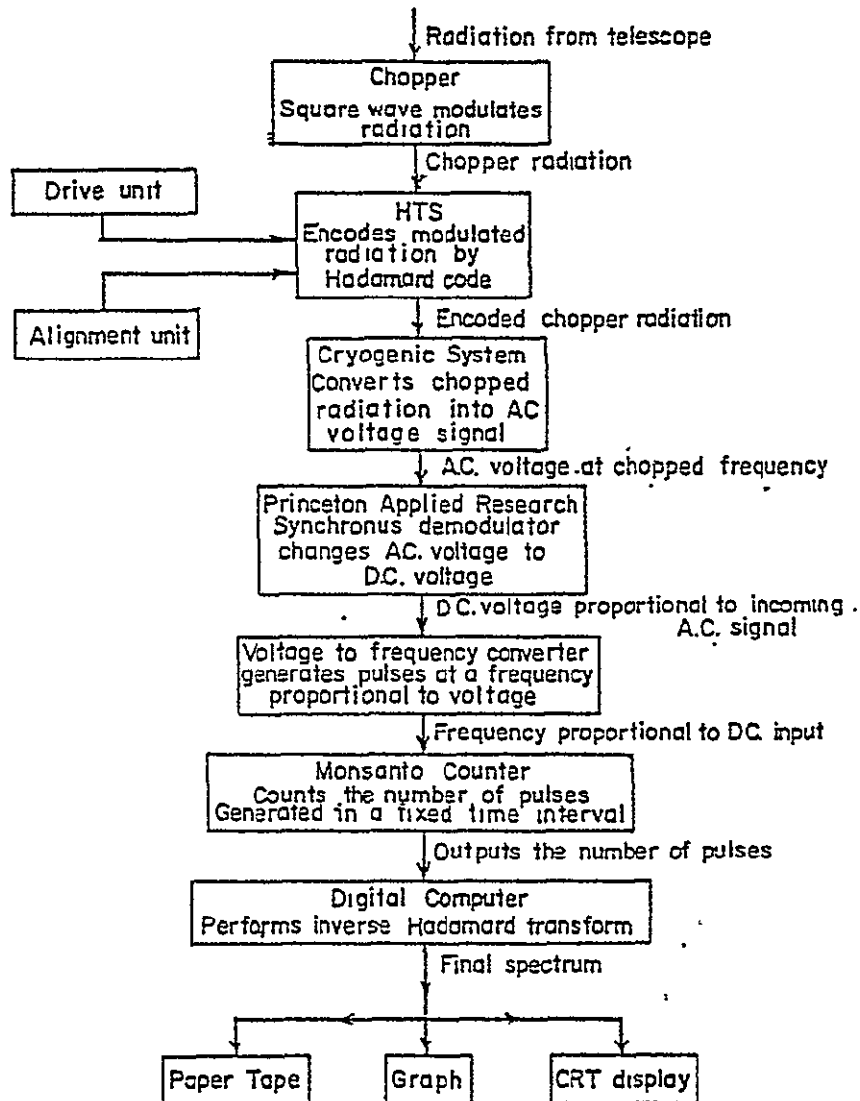


Figure 3-1. The flow chart of the data taking process, starting with radiation from the telescope and ending with the output of the computer.

(B) The Optics

1. Spectrometer

Fig. (3-2) shows the basic spatial design of the spectrometer. It works in the Ebert-Fastie mode. Radiation passing through the entrance aperture S falls upon the spherical mirror M which, in turn, renders it parallel and directs it to the grating G. The dispersed radiation is collimated by the other half of the spheroid and focused upon the exit aperture S'. A 255-slot encoding mask is located at this position. The exit focal plane is positioned in such a way that it bisects a 90° corner reflection. The corner reflector returns the radiation through the spectrometer again and displaces the beam from the center of the principal plane to one side. This reverse process dedisperses the beam and allows it to be brought to a focus at the entrance plane. The dimensions of the focused ~~image are roughly the same as the dimensions of the entrance~~ aperture (Decker 1971). These procedures allow one to use a smaller detector.

A diagonal mirror at the entrance directs the dedispersed radiation to the liquid helium cooled post optics.

The entrance mask can be a single entrance slit with any width between zero to 1.5 mm for the 1x255 program, or it can be a fifteen S-matrix code with each slit having width 0.1 mm for the 15x255 element program. In normal use the height of of entrance slit is 3.5 mm. It can be increased up to 10 mm.

M₁ is a spherical mirror with a 49.5 cm focal length. On its back it is held in place with three teflon-tipped

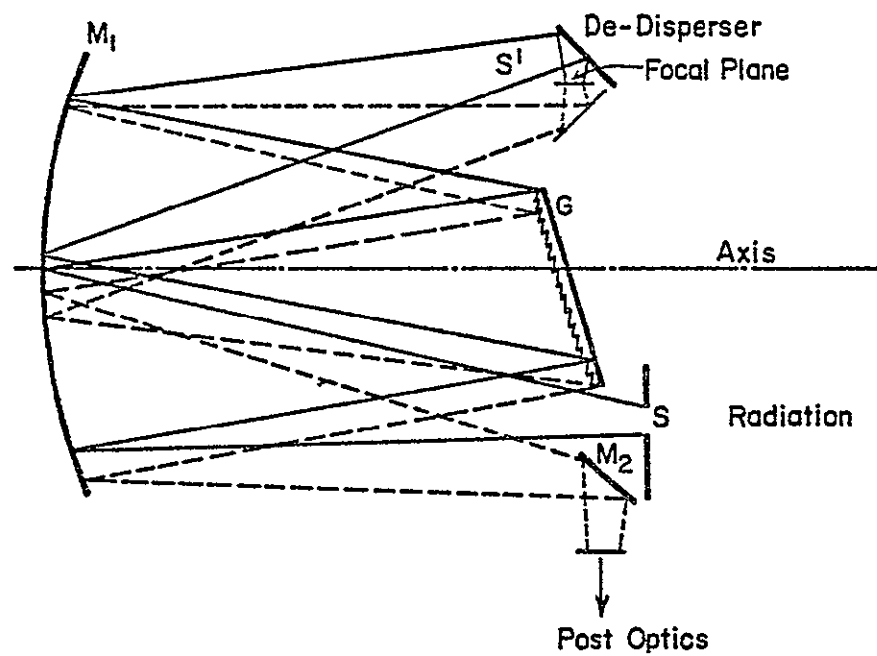


Figure 3-2. Optical path through the spectrometer. The dedisperser and exit mask are shown rotated by 90° .

screws. Three teflon-tipped springs bear on the front edges of the mirror. This design allows one to make slight adjustments in the position of M when aligning the instrument. The central part of the mirror is blocked off to reduce stray radiation.

G is a 75 mm x 75 mm grating with 20 lines/mm and blaze angle $5^{\circ}11'$. The corresponding blaze wavelength at first order is 9.03μ . Figure (3-3) is the calculated grating efficiency as a function of slit position, at two wavelengths. At 8μ the energy imaged within one slit width is 80% of the total. For 14μ the energy within one slit is 52%. The grating is mounted on a yoke which allows it to be adjusted in three mutually perpendicular directions. It is located at 0.82 focal length from the primary mirror M.

All mirrors inside the spectrometer are silver coated with a protective coating of SiO_2 . ~~The reflectivity of silver coating at 10μ is better than 97%.~~

For a multiplexing spectrometer which has N entrance and N exit slits, one wishes to image entrance slits S_1, S_2, \dots, S_n onto exit slits S'_1, S'_2, \dots, S'_n such that S_1 is imaged onto S'_1, \dots, S_n onto S'_n at a particular wavelength λ . Let δ be the angle subtended by S. Where δ and δ' are measured from the center of M. The grating equation for imaging S onto S' is

$$\sin \alpha + \sin \beta = \frac{m\lambda}{a} \quad (3-7).$$

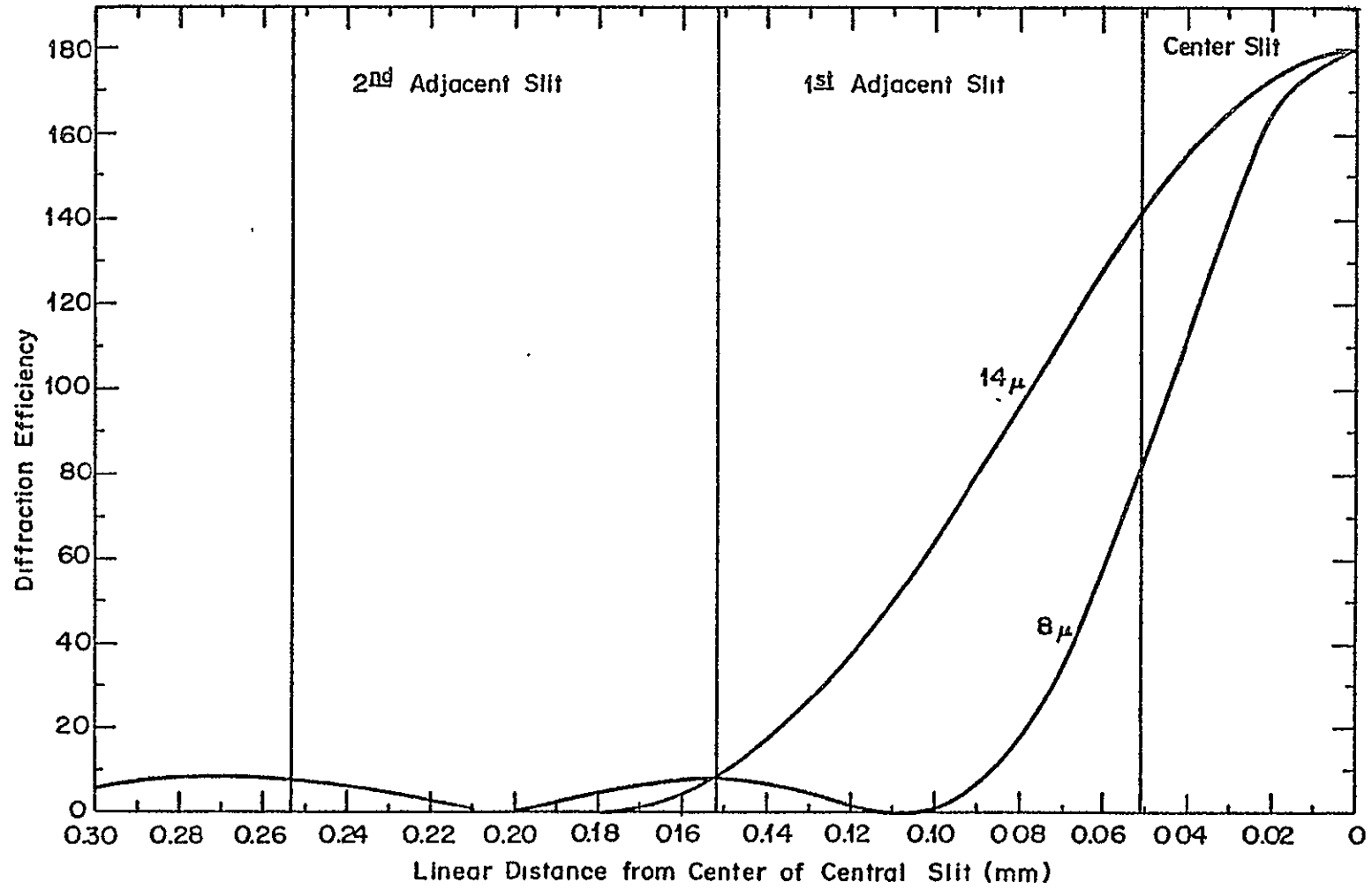


Figure 3-3. Grating diffraction efficiency as a function of slit position at 8 and 14 μm.

Differentiating (3-7) with respect to α gives

$$\frac{d\beta}{d\alpha} = - \frac{\cos \alpha}{\cos \beta} \quad (3-8)$$

The minus sign indicates that α and β change in opposite directions. $d\alpha$ is the width of the entrance slit and $d\beta$ is the width of exit slit, so $\delta \neq \delta'$ unless $\alpha = \beta$.

In our spectrometer, the exit slit width is 0.1024 mm, 2.4% larger than the entrance slit width. This is the effect of anamorphic dispersion- a magnification produced by the grating (3-8), when the lower limit on the slit width is set by diffraction. The total number of spectral elements that can be observed simultaneously is limited by the optical aberrations of any particular optical system, which set a limit on the total useful width over which the spectrum can be displayed.

2. Post Optics

The post optics consist of a liquid helium cooled Arsenic-doped silicon (As:Si) detector, with appropriate optics for focusing the radiation onto the detector (Fig. 3-4). Radiation enters the evacuated dewar through a barium fluoride window, passing through a filter with pre-selected band-width. The filtered radiation then passes through a cooled barium-fluoride filter and is focused onto the detector inside the housing by a gold coated mirror. A light baffle is partitioned in front of the detector housing.

Below is a brief discussion of each of the cryogenic components

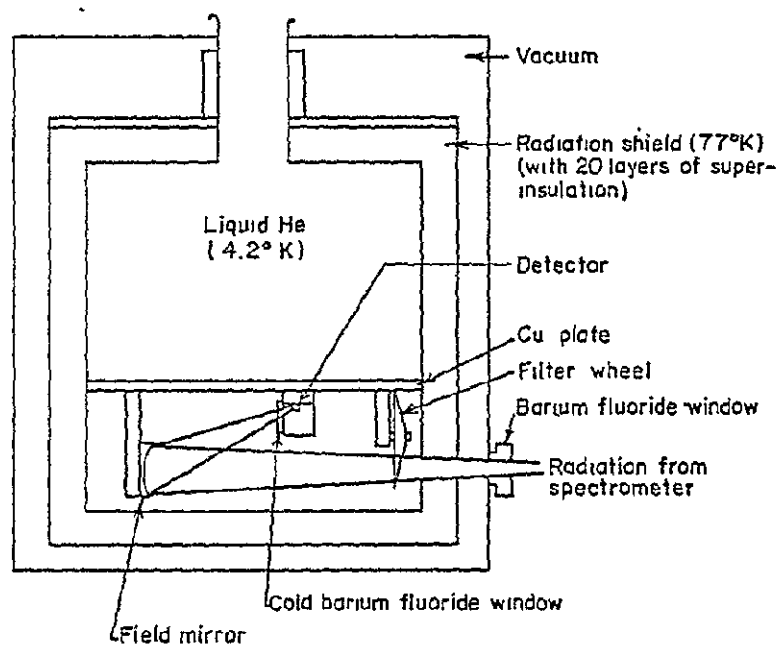


Figure 3-4. Liquid helium cooled post optics.

(a) Barium Fluoride Window

The barium fluoride window is 2 mm thick and 0.75" in diameter. - It is used to cut off wavelengths longer than 14μ . It has a transmission efficiency of around 90% out to 11μ before it starts to cut off. At 14.29μ (700 cm^{-1}), its transmission efficiency is 50%.

(b) Filters

A liquid helium cooled filter wheel with 8 filter positions is housed inside the dewar. Table 3-1 gives a brief description of each filter.

Table 3-1

<u>Filter position</u>	<u>Range</u>	<u>Band-width</u>	<u>Peak transmission efficiency</u>
1	no filter		
2	closed (blocked-by aluminum foil)		
3	$8\mu - 9.9\mu$	1.9μ	71%
4	$8.7\mu - 11.1\mu$	2.4μ	83%
5	$11\mu - 12.4\mu$	1.4μ	82%
6	$11.1\mu - 13.8\mu$	2.7μ	86%
7	$8.4\mu - 15\mu$	6.0μ	85%
8	glass	shorter than 2μ	

Filter positions 1 and 2 are for testing purposes.

The filter wheel is held in place by a spring-loaded screw.

The wheel was originally connected to the outside world through a stainless steel rod and could be changed to different positions by turning the rod. It was found that the stainless steel rod conducts too much heat from the outside into the helium containing can. When the stainless steel is replaced by a G-10 Glass Epoxy Lamitex rod, the holding time for the liquid helium of the dewar increases from 9 hours to 15 hours.

(c) Field Mirror

A gold coated mirror with focal length 7 mm and $f/0.41$ is used as a field mirror to focus the radiation onto the detector. The reflection efficiency for gold mirrors at 10μ is over 99%. The mirror has 3 degrees of freedom of adjustment, one translational and two rotational adjustments.

(d) Liquid Helium Cooled Barium Fluoride Filter

This barium fluoride window is also used to cut off the radiation longer than 14μ . Although the filter barium fluoride window cuts off radiation longer than 14μ from the outside world, it will emit radiation of its own because it is at room temperature. Since all the interference filters have a long wavelength leak between 20μ to 26μ , and since the detector will cut off radiation longer than 24μ only, there is still radiation from 20μ to 24μ that gets into the detector as background radiation. The insertion of the liquid helium cooled barium fluoride filter eliminates this peak. It was found to cut down the background radiation by a factor of four.

(e) Detector

An arsenic doped silicon detector with dimension 1.2 mm x

3.2 mm is positioned inside a housing, which has a baffle at its entrance.

Arsenic doped silicon is a N-type extrinsic semiconductor. When the detector absorbs radiation, free carriers are provided for the conduction band, thus changing the resistance of the detector. The following discussion follows the work of Putley. For further details, one can refer to Putley (1964) and Kittel (1966).

Let σ be the conductivity of the detector

e be the electric charge of the carrier

τ be the life time of free carriers

N be the density of free carriers, and

μ be the mobility of free carriers

Then,

$$\sigma_D = NeP\mu \quad (3-9)$$

and

$$\begin{aligned} \Delta\sigma_D &= \Delta J e \tau \mu \eta \\ &= \frac{\Delta P}{h\nu} e \tau \mu \eta \end{aligned} \quad (3-10)$$

where

$\Delta\sigma_D$ is the change in conductivity of the detector

ΔJ is the number of photons incident in unit time

ΔP is the radiation power incident

ν is the frequency of the incoming photons, and

η is the quantum efficiency, i.e. number of electrons freed per photon.

Since

$$R_D = \frac{l}{\sigma_D A} \quad (3-11)$$

where R_D is the resistance of the detector
 l is the length of the detector, and
 A is the area of the detector

Then

$$\begin{aligned} \Delta R_D &= - \frac{l}{A} \frac{\Delta \sigma_D}{\sigma_D^2} \\ &= - \frac{l}{A} \frac{\Delta P}{h\nu} \frac{\eta}{N^2 e \mu \tau} \end{aligned} \quad (3-12)$$

Let

$$I_D = \frac{V_B}{R_D} \quad (3-13)$$

where I_D is the detector current, and
 V_B is the bias voltage across the detector

Then

$$\begin{aligned} \Delta I_D &= - \frac{V_B}{R_D^2} \Delta R_D \\ &= \frac{V_B}{R_D} \frac{l}{A} \frac{\Delta P}{h\nu} \frac{\eta}{N^2 e \mu \tau} \end{aligned} \quad (3-14)$$

Let

$$\begin{aligned} V_o &= I_D R_L \\ &= \frac{R_L}{R_D} V_B \end{aligned} \quad (3-15)$$

where V_o is the voltage across the load resistor

R_L is the load resistance

Then

$$\begin{aligned}\Delta V_o &= \Delta I_D R_L \\ &= R_L V_B \frac{\Delta P}{h\nu} \frac{A}{\lambda} \text{ neut} \quad (3-16)\end{aligned}$$

From equation (3-16) one can calculate ΔP from ΔV_o .

(f) Procedures for Alignment of the Optics

i) Place all components in their respective positions and line them up visually. Be sure there is no mechanical binding in the mask and in the driving mechanism.

ii) Using a laser, put the spot from the entrance slot on the middle of the grating. It is suggested that only one entrance slot be used.

iii) Adjust the grating tilt until the line of dispersed dots exits at the proper position at exit. As the grating is rotated, this line of spots should remain level, not displaced normal to itself.

iv) Put one half of the dedispersing mirror combination in place. Use a T square to line it up roughly. At this point, use a mercury emission lamp with proper f-number to simulate the beam coming from the telescope. A number of colored image of the single entrance slot will be seen at the exit.

v) Adjust the spherical mirror for coarse adjustment and the position of the dedispersing mirror as a fine adjustment to bring the image to a focus on the exit plane.

vi) Adjust the grating position in its yoke until the

color image from the mercury lamp is parallel to the exit slit length.

vii) Adjust the angle of the dedisperser so that the color image reflected by it is perpendicular to the exit mask.

viii) Put in the other half of the dedisperser and line it up at an angle so the radiation falls back upon the grating. Adjust with fine adjustment screws so the grating is fully illuminated. The image of the grating will appear on itself when viewed from the diagonal, 45° , mirror which diverts the radiation to the dewar.

ix) Place the dewar on the spectrometer using $\frac{1}{4}$ " spacers to represent the thickness of the dewar bottom cover. Adjust the 45° mirror so that the dedispersed image (with color) falls on the center of the filter on the filter wheel.

x) Turn the filter wheel to position one (no filter), so that radiation can fall on the field mirror. Adjust the field mirror until the dedispersed radiation impinges upon the detector. Be sure that all the light falls onto the detector. Be sure that the mirrors accept all the radiation.

xi) Insert the housing. Be sure that the incoming radiation is clear of the housing.

xii) Put in the liquid helium shield, and the radiation shield. Put on the nose. Use GE varnish and aluminum foil to reduce openings in the baffles so that they will transmit only the bright white fringes.

xiii) When the spectrometer is on the telescope, maximize the signal by tilting and rotating the dewar.

(C) The Electronics

The data-taking process is controlled electronically to ensure a smooth process. The operator only needs to turn the switch on. The spectrometer will then automatically take data in, process it, and stop at the end of the transform indicated by the operator. All the operator has to do in the whole observation is to keep the astronomical object in the beam. Figure 3-5 shows the block diagram of electronic and computer set up. The following are brief descriptions of the electronic parts incorporated in the system.

1. Alignment Sensor

The alignment sensor (figure 3-6) is used to synchronize a Monsanto electronic counter, and the computer with the spectrometer. The circuit is shown in figure (3-6). The exit mask is continuously moving. When the exit mask is at its starting position, i.e. the first 255 slots are at the exit aperture, a light pulse goes through an alignment slot on the exit mask and is detected by a photo-cell on the other side of the exit mask. Two transistors amplify the output light curve and two IC chips change the light curve into an alignment pulse. This alignment pulse, through the drive unit, readies the counter for counting, readies the computer to accept data, and to turn on an indicator light showing that the system is taking data. One can adjust the starting position of the exit mask by adjusting the intensity of the light. After 255 data points have been obtained, the exit mask is at its other end, and another alignment pulse turns off the counter and the indicator light.

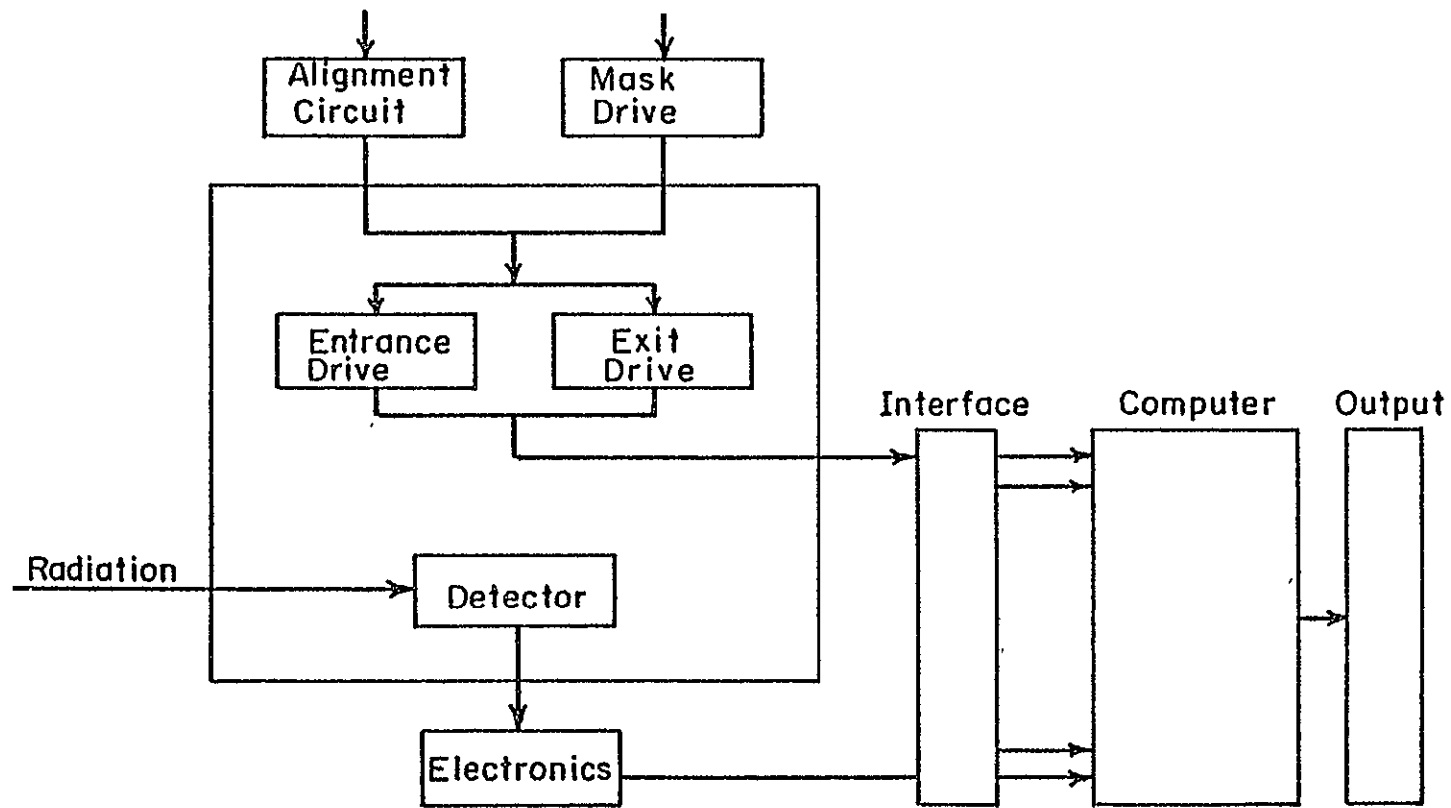


Figure 3-5. HTS drive unit block diagram.

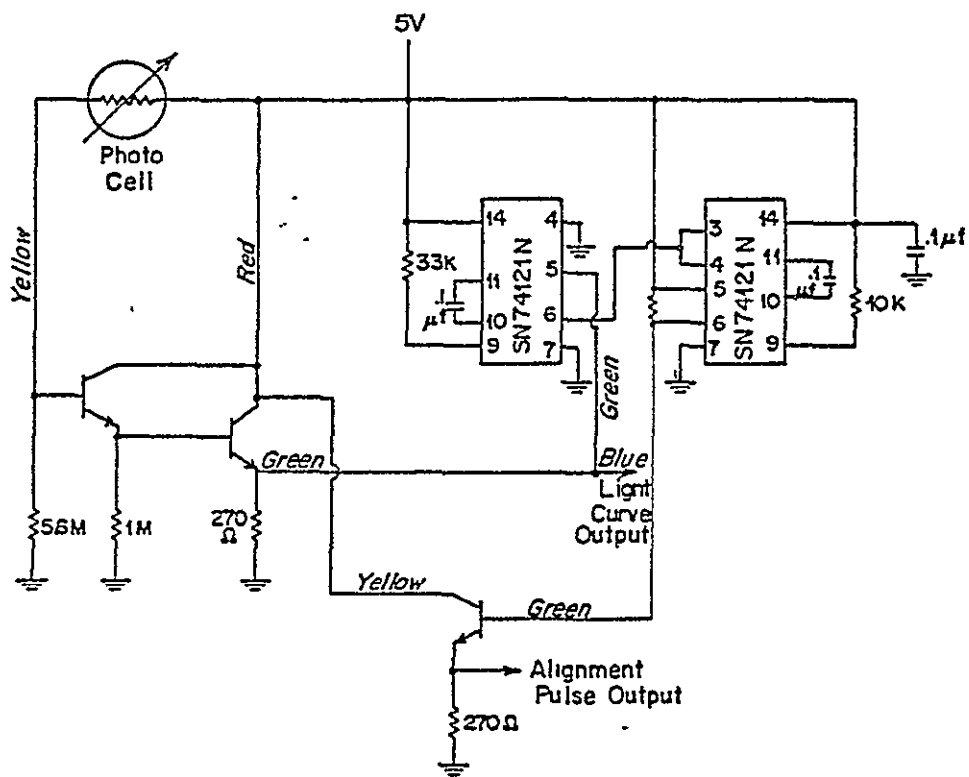


Figure 3-6. HTS alignment circuit.

The computer after taking 255 readings checks that the indicator light is off. This step ensures the synchronization of computer and spectrometer.

2. Drive Unit

A drive unit (Fig. 3-7) is used to drive the entrance and exit masks of the spectrometer. It can be set to be adjusted by pulse streams at 100 Hz, 200 Hz, or 400 Hz. The unit performs the following functions:

(a) It receives an alignment pulse from the sensor circuit(18) and generates a pulse to reset the counter for counting(17). The pulse will also ready the computer for taking the data.

(b) It drives the exit mask in a continuous mode at a displacement rate of one slot for every 41 pulses(18).

(c) After each set of 41 pulses the unit instructs the the mini-computer to read the integrated signal off the counter and then reset the counter for the next data integration.

(d) After 255 reset pulse the unit advances the entrances mask by one slot by sending the entrance mask advance motor 40 pulses (11).

3. Preamplifier

The circuit (Fig. 3-8) shows a transimpedance amplifier implemented with a Burr Brown operational amplifier. The circuit has the advantage of high speed, low susceptibility to microphonics, and detector operation at constant voltage with a high resistance load resistor.

Neglecting the voltage noise and current noise in the

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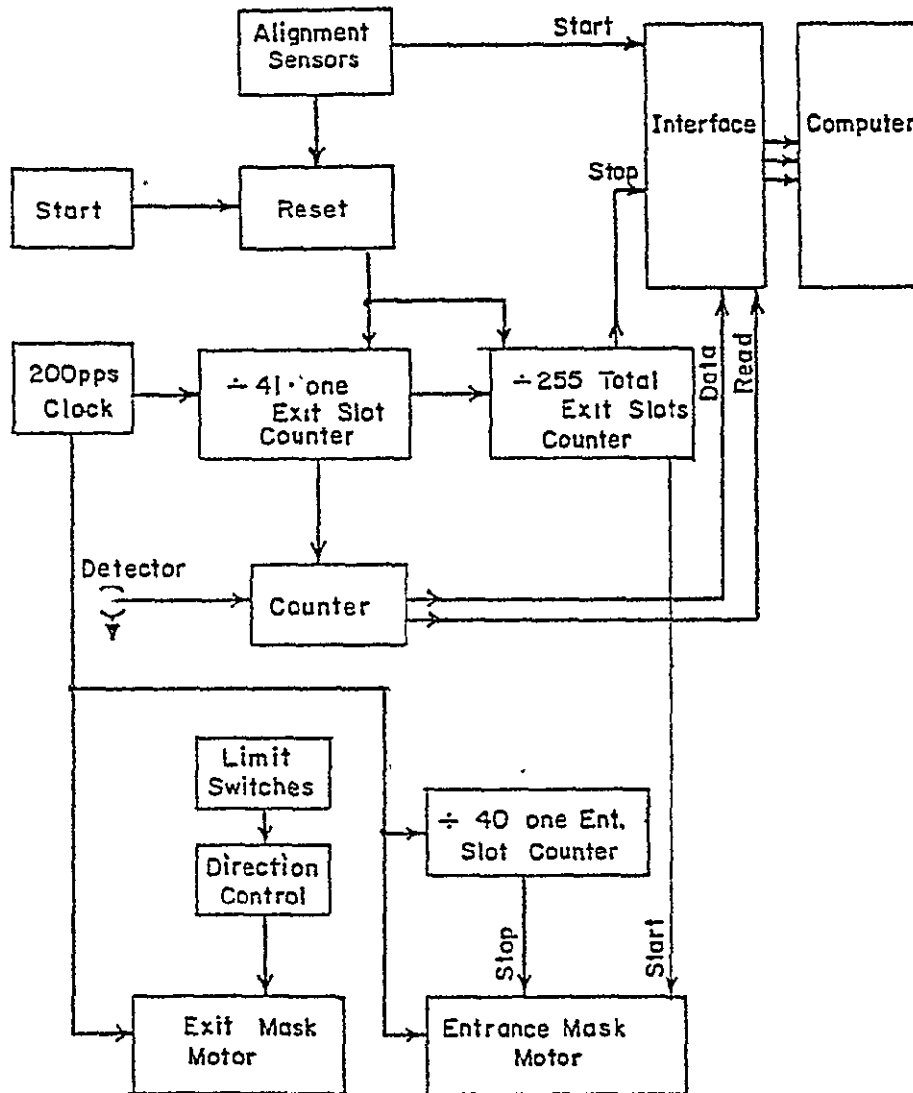


Figure 3-7. Logic diagram for mask motion and data processing.

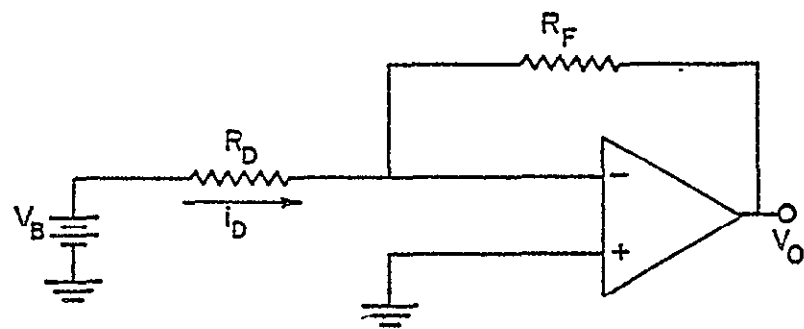


Figure 3-8. Basic detector bias and preamplifier circuit.

first order approximation, the current through the detector, I_D , also goes through the load resistor R_F because the input impedance of the operational amplifier can be taken to be very large, so

$$\begin{aligned} V_O &= I_D R_F \\ &= V_B \frac{R_F}{R_D} \end{aligned}$$

where V_O is the output voltage,

V_B is the constant bias voltage, and

R is the load resistance

(D) Laboratory Calibration

1. Calibration of Post Optics

Liquid nitrogen is used to calibrate the efficiency and sensitivity of the post optics. Liquid nitrogen is in a dewar with black paper along the wall to simulate a blackbody. Through a chopping device, the post optics will alternately see radiation from the liquid nitrogen and from the room. The difference between these two radiations gives the A.C. signal. D.C. measurements are obtained by putting liquid nitrogen directly in front of the dewar window. The following are the results of the calibration:

Dewar profile:	$f_H = 7.6$
	$f_V = 10.3$
Wavelength region:	$8.7\mu - 11.1\mu$

Band-width:	2.4 μ
Bias voltage:	15 μ
Load resistor at LH ₂ temperature:	1.2 M Ω
A.C. power:	9x10 ⁻¹⁰ watt.
A.C. signal:	272.7 mv
A.C. noise:	4.4 μ v
(NEP) A.C. detector:	5.2x10 ⁻¹³ H _Z ^{1/2}
(NEP) A.C. system:	1.25x10 ⁻¹² H _Z ^{1/2}
A.C. responsivity:	1.2 amp/watt.
D.C. power:	3.05x10 ⁻⁶ watt.
D.C. signal:	11.32V
D.C. noise:	4 μ v
(NEP) D.C. system:	1.47x10 ⁻¹² H _Z ^{1/2}
D.C. responsivity:	2.58 amp/watt.
Background noise:	NEP _{Blip} = $\sqrt{2 P_{BG} h\nu}$ = 3.5x10 ⁻¹³

where P_{BG} is the D.C. power

ν is the frequency that is assumed to be 10H_Z.

The system is a factor of 3.57 away from background limited.

2. Test of the Spectrometer

A mercury vapor lamp with emission at 1.7 μ is used with the spectrometer to test the computer program. A PbS detector and an appropriate blocking filter isolates the 1.7 μ doublet of mercury. The slit width and length for each mask is 0.15 mm and 3.5 mm. Fig. (3-9) is the mercury vapor spectrum at 1.7 μ for the 1 x 255 mode. Fig. (3-10) is the mercury vapor spectrum

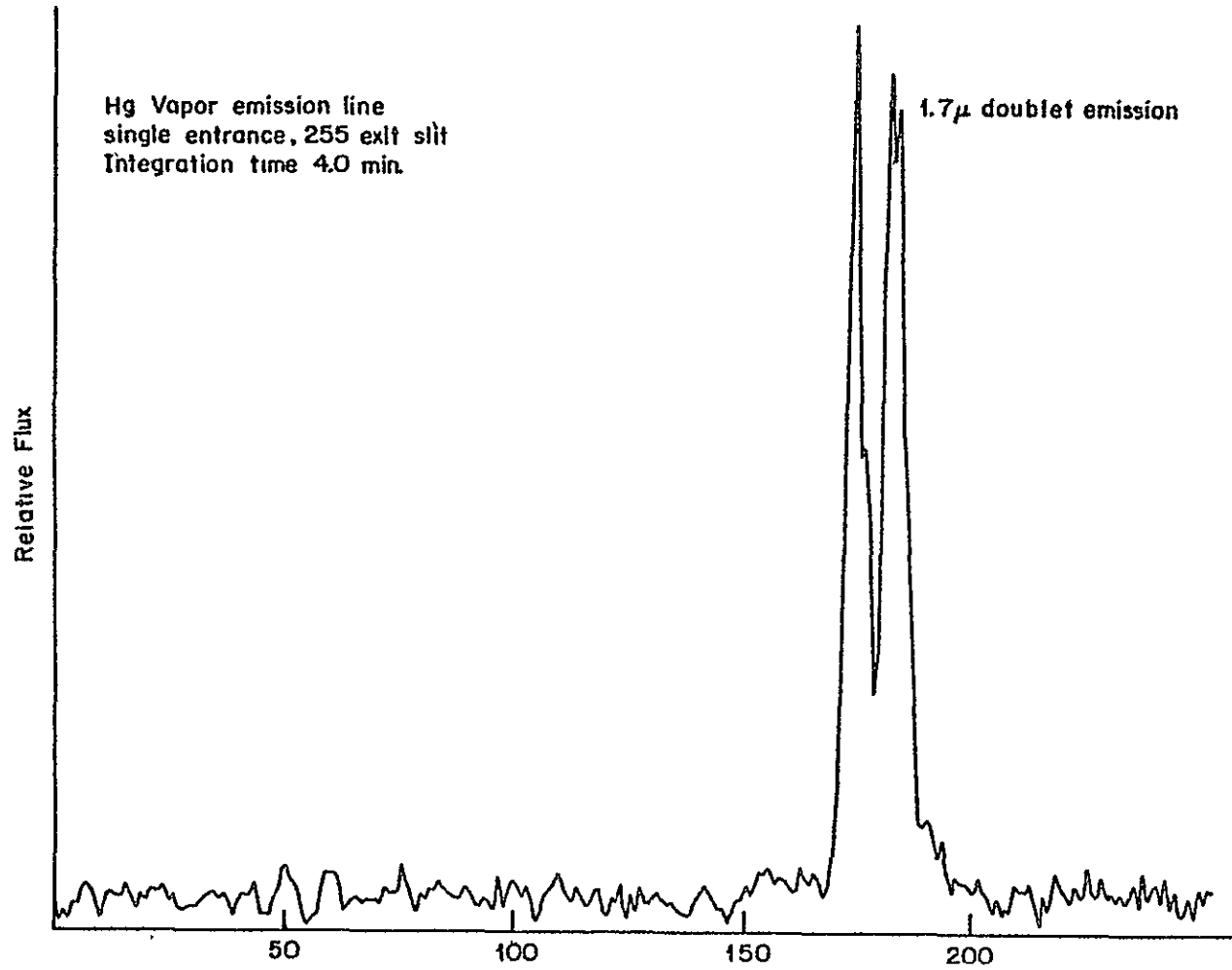


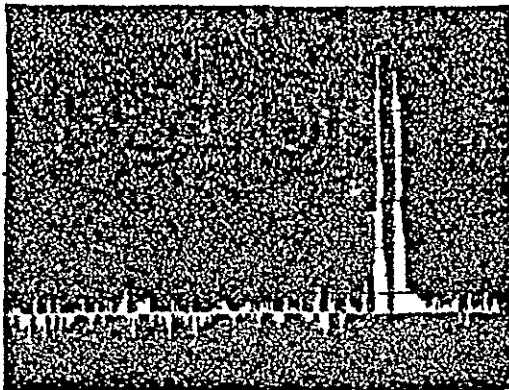
Figure 3-9. Spectrum of the mercury vapor 1.7 μ m line using the 1x255 mode.

for 15 x 255 mode. Figure (3-10a) shows the eighth of a series of fifteen individual spectra. Figure (3-10b) shows an average of all fifteen spectra, and (3-10c) shows all 15 spectra, each spectrum being displaced vertically from the next. The diagonal pattern near the right-hand edge represents a displacement of the peak between successive spectra. This represents the actual shift in spectral range between adjacent spatial elements.

Once it was clear that the instrument with the computer program worked properly in the 1.7μ region, the spectrometer was tested with the cryogenically cooled, arsenic-doped silicon detector. The transmission spectrum of polystyrene with a soldering iron as the source was obtained. The shape of the filter profile and the transmission spectrum of polystyrene showed that the instrument worked in the 10μ region.

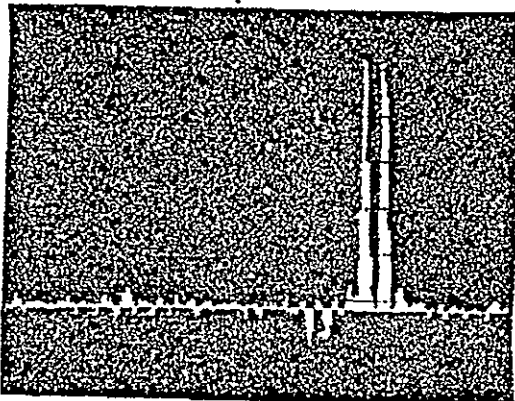
The wavelength calibration of the spectrometer is obtained by comparing the polystyrene transmission spectrum obtained by the spectrometer and the spectrum obtained by a Perkin-Elmer monochromator. The wavelength calibration is then checked against the moon spectra at 9.5μ where the atmosphere has strong absorption features.

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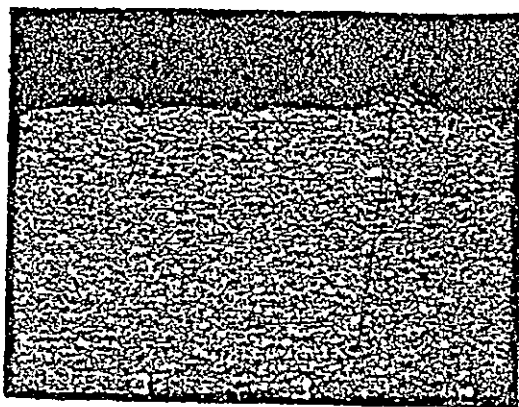
(a)

Figure 3-10.
Calibration spectra obtained for the mercury vapor emission lines at $1.69\mu\text{m}$ and $1.71\mu\text{m}$: (a) The eighth of a series of fifteen individual spectra obtained;



(b)

(b) An average of all fifteen spectra;



(c)

(c) A display of the fifteen spectra, each spectrum being displaced vertically from the next. The diagonal pattern near the right-hand edge represents a displacement of the peak between successive spectra. This represents the actual shift in spectral range between adjacent spatial elements.

CHAPTER IV

COMPARISONS BETWEEN FOURIER TRANSFORM AND HADAMARD TRANSFORM SPECTROSCOPY

Since the Michelson interferometer spectrometer (MIS) and Hadamard transform spectrometer (HTS) both have the multiplex advantage and the advantage of large through-put, there are a number of comparisons between them in the literature. In this chapter the comparisons are carried out in four different aspects: mathematically, computationally, optically and mechanically.

(A) Mathematically

Fourier transforms and Hadamard transforms can be viewed as using two different weighing schemes. Appendix A gives the mathematical analysis of the coding error for Fourier transform.

Let ξ be the path difference in a two beam interferometer. $P(\nu)$ is the power at wave number ν (i.e. the spectral density function). ν here is taken to be the inverse of wavelength, then $S(\xi)$, the power received for path difference ξ , is:

$$S(\xi) = \int_0^{\infty} P(\nu) \cos^2(2\pi\xi\nu) d\nu \quad \text{A-1}$$

$$= 1/2P_0 + 1/2 \int_0^{\infty} P(\nu) \cos(4\pi\xi\nu) d\nu \quad \text{A-2}$$

In the Fourier transform each spectral element can be viewed as having a phase modulation by a cosine squared term. Its argument depends on the stepped distance and the particular

wavelength. In the Hadamard transform each spectral element is modulated by a step function of 1 and 0 for a S-matrix coding.

One can ask oneself whether these modulating or encoding schemes are equally efficient?

In table 2-1 we stated that for large N the multiplex advantage, or the efficiency, of the H-matrix coding of elements 1 and -1 is \sqrt{N} , the S matrix coding of element 1 and 0 is $\frac{\sqrt{N}}{2}$, and for a single detector MIS the Fourier coding is $\sqrt{N/8}$. This last figure is based on calculations shown in Appendix A. The other two values were described by Sloane et al (1968). The H-matrix is an orthornormal matrix. The element -1 means "subtract" the radiation, while the +1 element means "add" the radiation. No radiation is wasted and thus the coding scheme has the highest efficiency. For S-matrix coding half the slits are open, 1, letting light pass through; and half the slits are 0, blocking the light. Half the radiation is therefore wasted each time and one can intuitively see why the efficiency of the S-matrix is only half that of the H-matrix.

Although the S-matrix is less efficient, it has the important advantage that it is cyclic; that is, the $(i+1)^{th}$ column of the S-matrix is obtained by shifting the i^{th} column cyclically one place downwards. Instead of constructing a mask of N^2 slits for N spectral elements, one constructs only one mask with $2N-1$ slits. Such a mask has two advantages. First, the cost of mask construction is reduced by $\sqrt{N}/2$ and the design of the advanced mechanism is considerably simplified,

since the total weight of the mask also decreases as $\sqrt{N}/2$. Secondly, it can be self-supporting and therefore permits the construction of a spectrometer which requires no transmission materials. In operation the mask is stepped one slit width - along the length of the mask- for each successive encoding position.

In the Fourier case, equation (A-2) of Appendix A

$$S(\xi) = 1/2P_0 + 1/2 \int_0^{\infty} P(v) \cos(4\pi\xi v) dv \quad (A-2)$$

shows that half the power goes into $1/2P_0$, the first term on the right hand side, which is not modulated at all. This reduces the efficiency by a factor of two as in the S-matrix case. The other half of the power is modulated by a cosine term. Cosine modulation gives a factor $1/\sqrt{2}$ because cosines functions do not form an orthonormal set themselves. The total Fourier modulation efficiency is therefore $1/2(\sqrt{N}/2)$. Mathematically, the S-matrix is a factor of $\sqrt{2}$ better in SNR than the Fourier transform. The true Hadamard code, which cannot be realized experimentally, as yet, is a factor of $\sqrt{8}$ better than the Fourier code.

(B) Computer Requirements

Both MIS and HTS require a digital computer to decode the data. However, in the HTS case this reduces to nothing more than a series of additions and subtractions. Hence, as much as an order of magnitude in computer time can be gained over

the Fourier decoding procedure required by MIS (Decker, 1971). In addition, HTS do not have the large zero path-length spike which is characteristic of MIS, and hence can be operated with a substantially lower dynamic range.

(C) Optics

HTS attempts to "liberate" the grating instrument from its inferior position and offers a possibility to convert a conventional scanning spectrometer into a multiplex instrument at a moderate cost. However, it is also the grating and optics that limit the capabilities of HTS.

1. Resolution

The resolution of a grating instrument is

$$\begin{aligned} R &= \frac{\lambda}{\Delta\lambda} \\ &= mN \end{aligned} \quad (4-1)$$

$$= m \frac{W}{d} \quad (4-2)$$

where m is the order of diffraction. N is the total number of rulings in the grating. W is the width of the grating. d is the separation of lines.

The MIS introduces variable path differences between two interfering beams. The resolution is determined by the maximum permissible path difference in the interferogram.

$$\Delta v = \frac{1}{2x} \quad (4-3)$$

$$= \frac{1}{4\xi} \quad (4-4)$$

where x is the maximum path difference between interfering beams.

ξ is the displacement of one of the two mirrors from the white light fringe position, and

$\Delta\nu$ is the increment in wavenumber.

Since

$$\begin{aligned} R &= \frac{\lambda}{\Delta\lambda} \\ &= \frac{1}{\lambda\Delta\nu} && \text{(Mertz a, P.5)} \\ &= \frac{2x}{\lambda} && \text{(4-5)} \end{aligned}$$

A MIS can have a resolution as high as $\sim 10^6$.

2. Slit Width

In the HTS, the minimum usable slit width is determined by the diffraction pattern

$$w \sim \frac{\lambda}{W} F \quad (4-6)$$

where F is the focal length of the imaging mirror.

It has been argued that a boxcar profile is a poor match to a sine diffraction pattern (Hirschfeld, et al, 1973, Mertz, 1976 b). In the presence of diffraction, the transmission at each point of the mask will be a complex function of the spectral distribution, the mask position, and the relative width of the nearby transparent and opaque slits.

One way to correct this is to make the slits, wider than the diffraction limit, allowing the mask's transmission to

approach the geometric optical limit. This way, however, not only the resolution of the instrument is reduced, but the total number N of spectral elements that can be observed simultaneously, also decreases.

There is another way to look at the same problem. The grating is an operator that changes the frequency domain into the spatial domain, so that intensity as a function of frequency, after passage by the grating, becomes a function of position. With diffraction effects included, the intensity has a new functional dependence on distance. The Hadamard mask code and the subsequent decoding process just translate this spatial distribution function back into its spectral domain. Therefore, an intensity pattern which is complicated by the diffraction pattern, after the Hadamard coding and decoding, should still show up the same intensity pattern. Since one knows the diffraction pattern for a given optical system, the diffraction effect on the spectral intensity distribution can be computed and corrected, so that the sine^2 diffraction effect would not make the slit width any larger than the diffraction limit. One point, however, should still be noted. The correction that would have to be applied is wavelength dependent because diffraction is wavelength dependent.

A similar problem exists in MIS (Stewart P.296) because the moveable mirror must be stopped when its maximum displacement is reached.

The sidelobes in the interferometric case is of the form

$$S(\omega') = \frac{\sin(\omega - \omega')T}{\omega - \omega'} \quad \omega \approx \omega' \quad (4-7)$$

(Stewart P.295)

which has considerably stronger side lobes than the diffraction pattern. Here, T is the time for the mirror to travel from one end to the other, and ω is the central frequency. Various schemes of apodization have been introduced to compensate for the side lobes in the interferometric case.

3. Multiplex Number

The total number of elements N that can be observed simultaneously yields the multiplex advantages. In HTS, the total aperture size is limited by aberrations largely due to off axis radiations. The aperture width is (Hirschfeld, 1973)

$$w = F \cdot S_w \quad (4-8)$$

where F is the instrument focal length and $S_w/2$ is a factor of the order of $0.05 \sim 0.1$, that describes how far off axis one can go before the aberration pushes the individual slit width up to the point where no further gain in N is possible. Since the minimum slit width, determined by diffraction effects, is $\sim 1.22\lambda f$, the total number N is

$$N = \frac{FS_w}{1.22\lambda f} \quad (4-9)$$

and is of the order of 10^3 .

For our HTS at Cornell, we have

$$F = 49.5$$

$S_w \sim 0.08$ (assume \sim medium value)

$\lambda = 10.0 \mu\text{m}$

$f = 7.5$

$N \sim 250$

Actually, however, Mertz and Flamand (1976) suggested that S_w values $\gg 0.1$ may be realized in practice.

The MIS has a very large effective value of N . This is its main gain. The total wave number range observed in MIS is

$$\begin{aligned} \nu_{\max} - \nu_{\min} &= \frac{1}{4\Delta} \\ &= \frac{1}{4\Delta\sigma} \end{aligned}$$

where Δ is the step size of the mirror, σ is the resolution in wave number. N for the MIS can be 10^6 .

4. Spectral Range

The MIS also has a broad free spectral range. Its range is limited by the beam splitter efficiency which usually varies approximately as the cosine of the wavelength. HTS free spectral range is usually about one grating order. Its free spectral range can be increased by using order sorting, but this increases the technical difficulty. Although the HTS has a smaller spectral free range, it can be set to recover only those spectral bands of particular interest throughout any spectral regions (Decker, 1971). This is impractical with a MIS.

5. Throughput

The throughput is defined in section 1-A as the product of aperture A and angular acceptance Ω

$$E = A\Omega$$

For a MIS it can be shown that $\Omega R = 2\pi$ and

$$E_{\text{MIS}} = A_m \cdot \frac{2\pi}{R}$$

where R is the resolution of the instrument. A_m is the area of the interferometer mirror. Typically, $A \sim 1 \text{ cm}^2$, and for $R \sim 10^3$, $E_{\text{MIS}} \sim 6 \times 10^{-3}$.

For the HTS, from equation (3-1) one obtains

$$E_{\text{HTS}} = A_g \left(\frac{w \cdot h}{F^2} \right)$$

where h and w are slit height and width and F is the focal length. For a double multiplexed spectrometer, the throughput of MIS and DHTS are about the same, of the order 10^{-2} cm^2 . For the same throughput, the HTS may have a worse system transmission because the HTS requires a dedispersing process.

DHTS has the additional advantage that one can construct a one dimensional picture of the source.

(D) Mechanical Requirements

The HTS mask can be made self-supporting hence no beam-splitters or transmission optics, are required. Furthermore, in the MIS, construction tolerances usually involve dimensions

and motions that have to be maintained to within fractions of wavelengths. For a HTS, the corresponding tolerances are fractions of a slit width, and these tolerances are normally two orders of magnitude more relaxed, so that this instrument will be more suitable for rugged applications and less costly.

It is clear from the above comparison that the MIS has the advantages of highest resolution, very large multiplex number and free spectral range. The HTS has the mechanical advantages and computational advantages for large N. The HTS can have on the order of 10^3 spectral elements and a resolution sufficient to resolve the rotational lines of many molecules. For most IR astronomical observations this will be sufficient. Furthermore, its potential for modifying the existing scanning spectrometer at a moderate cost make this a very worthwhile field for further study.

CHAPTER V

PROGRAMMING FOR HADAMARD TRANSFORM SPECTRAL DATA REDUCTION

An 8K Computer Automation minicomputer model L.S.I. or model Alpha-16 can be used to interface with the output of a Monsanto scalar counter which digitized the output of the detector used with the Hadamard transform spectrometer. The computer processes each data point as soon as it receives it and when the data gathering run is completed, the computed spectrum is also ready within a fraction of a second. The final spectrum can be displayed on a cathode ray tube for quick visualization, or printed on paper by a teletype machine for more detailed analysis. Also, it can be stored on paper tape for future use.

There are two inverse transformation programs: 1 x 255 for the single entrance slit and 255 exit slit Hadamard transform spectrometer, HTS, and 15 x 255 for the 15 entrance slit and 255 exit slit instrument, DHTS.

(A) HTS Program

The inverse HTS program processes the raw data obtained by the combination of a single entrance slit and 255 exit slit. This program is in double precision format. Two areas in the memory are reserved by the program to store the final spectrum. The final spectrum can be stored in either the plus beam or the

minus beam area. The plus and minus beams are arbitrarily named.

Data can come in at a rate of 10 data points per sec., 5 data points per sec., or 2.5 data points per sec., depending on how the clock driving the spectrometer is set. Since a complete pass has 255 points, each pass takes 25.5 sec., 51 sec., or 102 sec., depending on the data rate. Each pass yields one spectrum. One can take as many passes as one wants until one is satisfied with the SNR of the spectrum.

The whole program (Appendix B) is linked by the following subprograms: COMMAND, TRANSFORM, INPUT/OUTPUT, READ, CLEAR, PUNCH, GRAPH, DISPLAY, MATHEMATICAL PACKAGE and MASK. The function of each subprogram is described briefly in the following sections.

1. COMMAND: This subprogram performs two functions.

(a) It commands the computer to do one of the following functions: TRANSFORMATION, CLEAR, READ, PUNCH, GRAPH or DISPLAY.

(b) If two distinct spectra are stored in two different beam areas, the COMMAND program can take the difference and ratio of the two spectra.

2. INVERSE TRANSFORM: This is the most important subprogram in the whole program. It will take data points from the counter, transform them and enter them into either the plus or minus beam areas, or it will read the data points from the paper tape into one beam area, transform them and enter them into the other beam area. This program is called the inverse transform, since it inverts the transformation performed by the coding mask, and

yields a spectrum. The program also performs the Hadamard transform, i.e. transforms the spectrum back to raw data, from either input.

The algorithm is based on the following idea: Let ψ_j be the j^{th} spectral element and w_{ij} be the weight of the j^{th} element of the i^{th} mask. w_{ij} equals 1 for transmitted radiation and 0 for blocked radiation. Each measurement then has a value

$$\eta_i = \sum_{j=1}^{255} S_{ij} \psi_j + v_i$$

where v_i is the random detector noise satisfying the properties mentioned in Section II-B. \underline{S} is the 255 x 255 matrix. η_i is the i^{th} data point entered into the computer. The computer's job is to decode η_i to reconstruct the original spectral values ψ_j . Therefore

$$\hat{\psi}_j = \sum_{i=1}^{255} S_{ji}^{-1} \eta_i$$

where $\hat{\psi}_j$ is the unbiased estimate of ψ_j and \underline{S}^{-1} is the inverse of the S matrix.

According to the relation

$$\underline{S}^{-1} = \frac{2}{N}(2\underline{S} - \underline{J})$$

one obtains \underline{S}^{-1} by keeping all +1's in the S-matrix and replacing all 0's by -1. The matrix obtained in this way is the

inverse matrix of \underline{S} except for a constant factor $\frac{2}{255}$, which only gives a different normalization.

To reconstruct the spectral values ψ_j one needs to add or subtract each measured value to n_i different bins, according to whether \underline{S}^{-1} is plus or minus. Fig. (5-1) is a flow chart for the 1×255 transform program.

By a Hadamard transform we mean a program that transforms the spectrum back to its raw data*. This procedure is useful because by inspecting the raw data display which usually appears quite smooth, any bad data point can be easily identified, and for example, replaced by the average of its two adjacent data points. This procedure will improve the final spectrum.

The Hadamard transform turns out to be extremely easy. All one has to do is to change one statement in the inverse transform program. When S_{ij}^{-1} is -1, instead of negating the data, one just sets it to zero.

Data points are taken both with the exit mask moving in a forward and in a reverse direction. The inverse transform program takes care that when the exit mask moves in the forward direction, the spectrum is transformed into the plus beam area. When the exit mask moves in the reverse direction, the final spectrum is stored in the minus area. Not adding the

* The notation here may be a bit confusing. We use the \underline{S}^{-1} to transform the raw data into an intensity spectrum. The inverse transform uses \underline{S} to transform the intensity spectrum back into raw data.

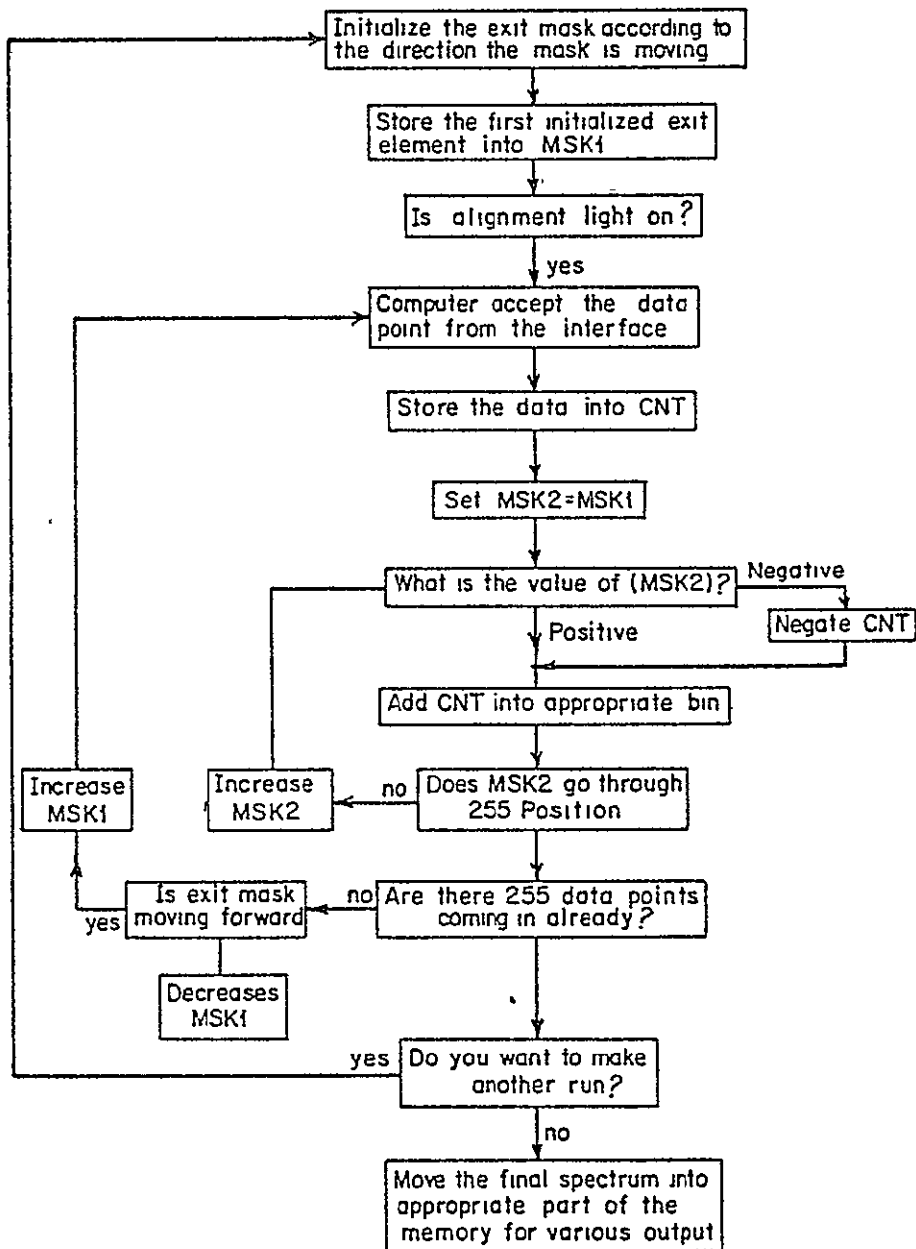


Figure 5-1. The flow chart of the 1x255 inverse transform program.

forward and backward spectrum eliminates a degradation of the final spectrum due to any asymmetry between the data taking for different directions of motion of the mask.

3. INPUT/OUTPUT: This program links up the computer, the teletype and high speed reader. It consists of the following functions: Keyboard Input, Paper Tape Input, Output to Teletype, Output Text from Buffer, Output Floating Point Number, Wait for Execute Signal, Command Error Exit, Carriage Return-Line Feed.

4. READ, CLEAR, PUNCH: This program reads the spectrum from the paper tape into either plus or minus beam areas for further manipulation, or punches the spectrum out from the beam area; it also can clear the beam area. The speed of teletype for reading is 100 words per sec. Hence it takes about 12 min. to read the spectrum. Punching has the same rate.

5. GRAPH: This program plots the graph on teletype paper, with its numerical value in floating point format. This procedure offers one the chance to inspect the spectrum in detail if it is needed. It takes about fifteen to twenty minutes to finish a spectrum, depending on the complexity of the spectrum.

6. CRT: A cathode ray display is interfaced with the output of the computer. It takes about 2 sec. to display the spectrum, with a factor of 5 higher resolution than the graph printed by the teletype on paper. One can display the spectrum at the end of any pass to see how good it is.

7. MATHEMATICAL PACKAGE: This package is supplied by the Computer Automation library tape, with a little modification

0-2

from our own on its double precision part.

8. MASK: This part contains the S^{-1} matrix, the 255 elements exhibited in the first mask position plus an additional 254 elements representing the further cycling of this mask.

(B) DHTS

The doubly encoded Hadamard transform program processes the data obtained by the various combinations of fifteen entrance slots and 255 exit slots. It is a single precision program. It can accept data at a rate of 5 data points per sec. and 2.5 data points per sec. only, because it takes a longer time to process each data point. The whole transform takes about 14 minutes. The final spectrum consists of 15 separate spectra, representing a one-dimensional color picture across the spectrometer entrance aperture. Each separate spectrum contains 255 spectral elements. The program can co-add all fifteen separate spectra yielding a sum spectrum with improved SNR.

The program consists of the following subprograms:

COMMAND, TRANSFORM, DATA, INPUT/OUTPUT, READ, CLEAR, PUNCH, GRAPH, DISPLAY, ENTRANCE MASK, EXIT MASK. (Appendix C).

Since most of the subprograms are performing the same function as their counterpart in the 1 x 255 case, except written in single precision format, their description will not be repeated here. Only the TRANSFORM, DATA, and DISPLAY programs will be discussed because they are different from those in the 1 x 255 scheme.

1. INVERSE TRANSFORM: The program can accept data either from the counter or from paper tape. In order to eliminate any asymmetry due to the different directions of motion of the exit mask, the computer will accept data only when the mask is moving in a given direction, either forward or backward, depending on the operator. If one wants to save time, one can still choose the mode in which the computer will accept data in both directions of mask motion. Hence, the program provides six modes for operation: accepting data from the counter, in the forward, backward, or both directions, and accepting data stored on the paper tape, in the forward, backward and both directions. Figure (5-2) shows the flow chart for 15 x 255 program.

Let both the entrance and the exit masks be linear arrays encoded by Reed-Muller codes. Then the matrix of spatial-spectral elements ψ is related to the matrix of measurements η by

$$\underline{s} \underline{\psi} \underline{S} = \underline{\eta}$$

To obtain the spatial-spectral information about the viewed scene we solve this equation by premultiplying the data matrix by \underline{s}^{-1} and postmultiplying by \underline{S}^{-1}

$$\underline{\psi} = \underline{s}^{-1} \underline{\eta} \underline{S}^{-1}$$

Now consider the element η_{11} . It is multiplied only by elements of the first column of \underline{s}^{-1} ; and in turn it multiplies only the elements of the first row of \underline{S}^{-1} . To a given spectral-

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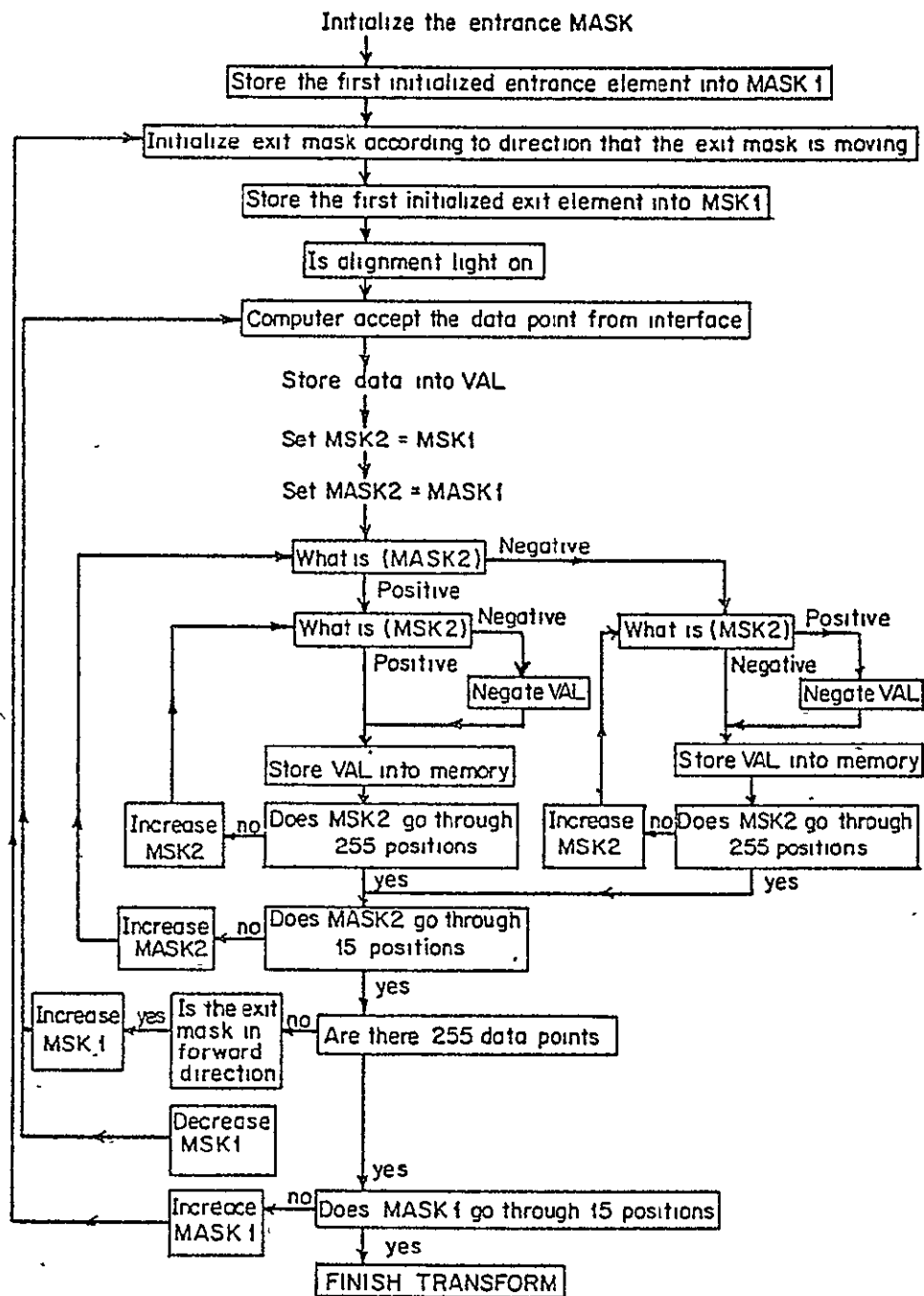


Figure 5-2. The flow chart of the 15x255 inverse transform program.

spatial element ψ_{ij} , it therefore contributes an amount $s_{i1}^{-1} \eta_{11} S_{1j}$. But the elements s_{i1}^{-1} and s_{ij}^{-1} all have values, either +1 or -1, and the result is that each element ψ_{ij} of the matrix ψ receives a contribution $+\eta_{11}$, or $-\eta_{11}$ from the reading, η_{11} .

This procedure is generally valid. Any reading $\eta_{k\ell}$ will make additive contributions that can only have values $+\eta_{k\ell}$ or $-\eta_{k\ell}$ to each element ψ_{ij} of the ψ matrix.

For real time decoding we therefore need the following:

(a) A memory that consists of bins containing the contributions to the elements ψ_{ij} accumulated up to any given time t in the cycle of measurements. For a device that can resolve m spatial and n spectral elements, this memory requires mn bins and of the order of mn memory words.

(b) For each acquired reading $\eta_{k\ell}$ we perform a series of additions of values either $+\eta_{k\ell}$ or $-\eta_{k\ell}$, one to each of the ψ_{ij} memory bins. But before that can be done, we need to decide on the assignment of + or - needed for a given bin.

This is done in the following way.

We store the sequence of + and - signs in one column of \underline{s}^{-1} and in one row of \underline{S}^{-1} and in one cycled permutation of each of these vectors. Let us designate these signs by their positions in these two vectors, as \underline{s}_i^{-1} and \underline{S}_j^{-1} , respectively, $i=1, \dots, m$; $j=1, \dots, n$. (Since each of these sequences is cyclic it can, respectively, be brought into its k th and ℓ th cycling position after a measurement $\eta_{k\ell}$). The elements of the two vectors then are multiplied in all possible combinations to

give a matrix having mn elements.

$$\Sigma_{ij} = s_i^{-1} S_j^{-1} \quad i=1 \dots m; j=1 \dots n$$

Each element Σ_{ij} is either + or - depending only on whether the signs s_i^{-1} and S_j^{-1} are similar or dissimilar for a particular combination of i and j values.

The additions $+n_{kl}$ or $-n_{kl}$ to the bins ψ_{ij} are made as successive elements, Σ_{ij} are computed, so that the elements Σ_{ij} need never be stored. Figure (5-3) shows the relation of Σ_{ij} to a superarray containing the set of all elements that are constructed at various stages of the computation.

When only a restricted number of spectral elements are of interest, we need to compute elements ψ_{ij} representing only selected j values. This might be useful, for example, if only certain atmospheric CO_2 absorption lines needed to be studied, and the spectral elements between were of lesser interest. In that case only $\Sigma_{k+i-1, l+j-1}$ elements corresponding to given j values need to be used, and the computing time decreases as p/n , where n is the total number of available spectral elements, and p is the actual number of interest.

One starts with the inverse of the codes, s^{-1} and S^{-1} , for the entrance and exit masks stored in the computer. One stores 509 elements of the exit mask, i.e., the 255 elements exhibited in the first mask position plus an additional 25^4 elements representing the further cycling of this mask. Similarly one stores twenty-nine elements for the entrance mask,

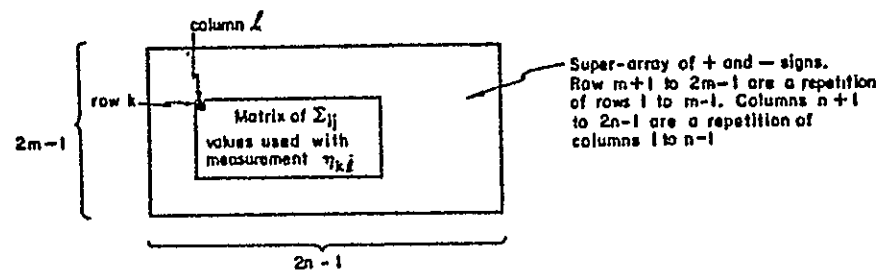


Figure 5-3. Matrices generated by the computer during the reduction of the spectral data.

representing the first fifteen elements used, plus the further cycling of fourteen elements.

For each reading $\eta_{k\ell}$ one essentially makes use of the matrix (figure 5-3) making use of elements k to $k+14$ of the stored entrance code and elements ℓ to $\ell+254$ of the stored exit code. This matrix consists of + and - signs. When s_{ik}^{-1} and $S_{\ell j}^{-1}$ have the same sign, both being + or both being -, the matrix position ij is assigned a + sign, and the reading $\eta_{k\ell}$ is added to the accumulatively stored value of ψ_{ij} . If the elements s_{ik}^{-1} and $S_{\ell j}^{-1}$ have dissimilar signs, a - sign is assigned to ij and the reading $\eta_{k\ell}$ is subtracted from the stored ψ_{ij} values. This whole process takes ~ 100 msec, and is carried out while the succeeding intensity measurement is being made.

In practice, we start with the first spatial element, $i=1$ and add or subtract the contributions to all the ψ_{ij} values, successively going from $j=1$ to $j=255$. We then repeat this procedure for i values going from 2 to 15. This whole procedure is carried out while the exit mask is moving from position ℓ to $\ell+1$ depending on whether the exit mask is moving forward or back. The entire process is then repeated for the next reading $\eta_{k,\ell+1}$. When the exit mask reaches its 255th position, ℓ remains unchanged, but the entrance mask moves from the position k to $k+1$.

For odd values of k , the exit mask moves in the direction of increasing ℓ values, and for even values of k , it moves toward decreasing values. In short, the exit mask moves back and forth as readings are taken. After the entrance mask has

moved through all its fifteen positions, and the total 3825 readings have been taken and added onto or subtracted from the ψ_{ij} elements, the run is completed.

2. DATA: Instead of processing a data point immediately as it comes in, this program stores the data in the memory, so one can display the raw data points first, correct them if there are any obvious bad points, and then transform them. This program serves the same function as the inverse transform in the 1×255 system.

3. DISPLAY: The display program allows one to display the information in a number of different ways:

(a) One can call for the spectrum corresponding to any one of the entrance slit positions and display it individually.

(b) One can display the sum of the different spectra. In order to do this, one has to take into account that the spectrum for a given entrance slit is displaced by one spectral position from adjacent entrance slit position. In other words, the wavelength for element ψ_{ij}^{-1} corresponds to the wavelength for element $\psi_{l+1,j+1}$, because of the slightly displaced light paths through the spectrometer.

(c) Finally one can display all fifteen of these spectra simultaneously, with the zero baseline of each spectrum vertically displaced from the next one. While this format is somewhat crowded, it does allow a quick comparison of the individual spectra.

(C) Correction Program:

This program is shown in Appendix D written in BASIC language. It corrects the error introduced by the imperfect mask. Instead of correcting the mask which in practice is not possible, the program corrects the final spectrum. That is much easier.

As seen in section II-D, for any spectral line I_0 , the distorted spectrum shows a line $I'_0 = I_0(1-\epsilon)$, two positive blips with amplitude $(1/2)(\epsilon I'_0/1-\epsilon)$ adjacent to the line on both sides, and two negative blips with same amplitude, i.e. $(\epsilon/2)(I'_0/1-\epsilon)$ at 24, 25 elements to the left. The correction program takes the intensity of every element, I'_0 , multiplies it by $\epsilon/2$, adds it to the elements 24 and 25 positions to the left, and subtracts it from the two adjacent elements, one on each side of the line. The final spectrum is complete except for a different normalization factor. This is a linearized correction procedure valid only for small values of ϵ , $\epsilon \ll 1$.

CHAPTER VI

ASTRONOMICAL OBSERVATION

(A) Correction Procedure

The correction of the spectra for telluric absorption and for instrumental response is a critical procedure. The correction is carried out by comparing the source spectrum (either stellar or planetary) with a lunar or solar spectrum which is taken on the same day at an airmass as close as possible to the star. The following procedures are used in the data reduction:

(1) Correct the raw star spectrum and lunar spectrum (or sun) for negative dip due to the imperfect mask as described in section II-D.

(2) Correct for different entrance slit width if necessary because different slit widths will give different resolution.

(3) Most of the astronomical infrared sources to be observed are weak sources, hence a positive offset is always added to the signal to prevent the signal becoming negative. (The electronics are confused by negative signals.) The correction procedure shown previously also take out this offset. One can take out the offset from the raw spectrum if one knows how large the offset is. Another way to correct this is by subtracting a constant intensity from the stellar spectrum until the ratio of the stellar spectrum over the Moon spectrum,

around any large telluric absorption region, such as the ozone band, is optimally corrected. By optimal correction we mean that the atmospheric feature appears as neither a positive, nor a negative band. The Moon is a strong infrared signal and does not require an offset, so the result can be used as a calibration for base line.

(4) Align the stellar spectrum and the lunar spectrum by the telluric absorption feature. The final stellar spectrum is obtained by taking the ratio of the stellar spectrum and the lunar spectrum and multiplying it by the black body temperature of the Moon. The lunar temperature is obtained by noting the phase angle of the lunar east limb where it has usually been observed, and extrapolating the temperature from the value given by Linsky (1973). This procedure assumes the lunar infrared emissivity at 8-14 μ m as unity, which is not true. Murcray et al's (1970) results for the lunar emissivity of 8-14 μ m region are shown in figure (6-1). The observation was done from a balloon. The strong feature centered at 9.6 μ m is a result of telluric ozone absorption. Murcray's result has not been used in our analysis because in the region to be discussed, 8.5 μ m-14 μ m, the Moon's emissivity is about constant except for the ozone absorption.

(B) Observation of α -Orionis

The observations of α -orionis were carried out with the 50" infrared telescope at Kitt Peak National Observatory, Arizona, in May 1974. The beam size is 18 sec. x 47 sec. The Kitt Peak 50" telescope bolometer was used with the Hadamard spectrometer. The dewar has a band pass of 8 - 14 μ m. The spec-

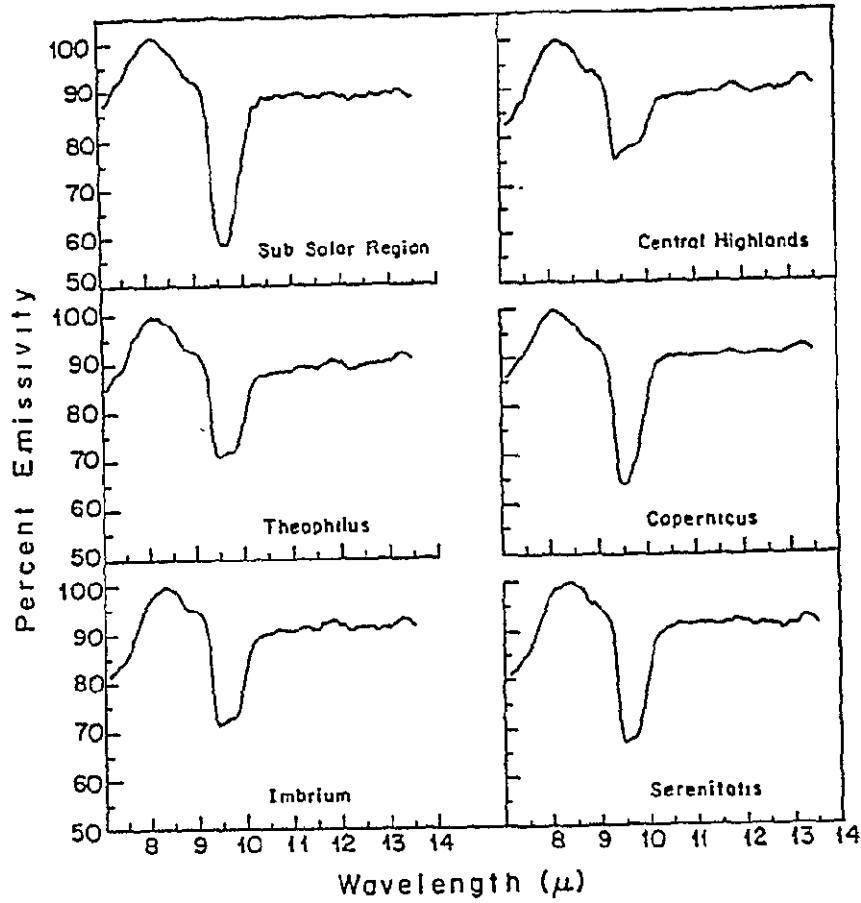


Figure 6-1. The spectral emissivity of various regions as calculated from the flight data (Murcray *et al.*, 1970).

trometer operated in the 8-11 μ m region with a resolution $\lambda/\Delta\lambda$ around 500. The chopping frequency was 10 cycles per second.

Three runs were taken. Each run consisted of 10 scans of the sources. Two lunar spectra were taken on the same day for correction purposes. For the lunar temperature we used 383°K. Figure (6-2) shows the raw spectra of α -orionis and the Moon. The α -orionis spectrum is the sum of two independent runs.

Figure (6-3) shows the ratio spectrum of α -orionis corrected for lunar temperature. Except for the region immediately around the ozone band all the telluric absorption features are gone. The region between 9.35 μ m to 9.7 μ m is unreliable because the ozone band has a very low transmission.

α -orionis is a late type super-giant with temperature around 3000°K. A stellar continuum corresponding to a 3000°K blackbody is also shown in figure(6-3), normalized to arbitrary units. The broad emission feature around 10 μ m, which is due to silicate emission, is clearly shown in the spectrum. This 10 μ m 'silicate' emission feature of α -orionis has been previously discussed by others.

Woolf and Ney (1969) renormalized Gillètt et al's spectra (1968) of α -orionis and interpreted the 10 μ m emission as coming from circumstellar dust which absorbs starlight and reradiates at infrared wavelengths. Since the emission is far more sharply peaked than a black body, the wavelength dependence of the emission probably closely mimics the wavelength dependence of the opacity of the material. In the same paper, Woolf and Ney

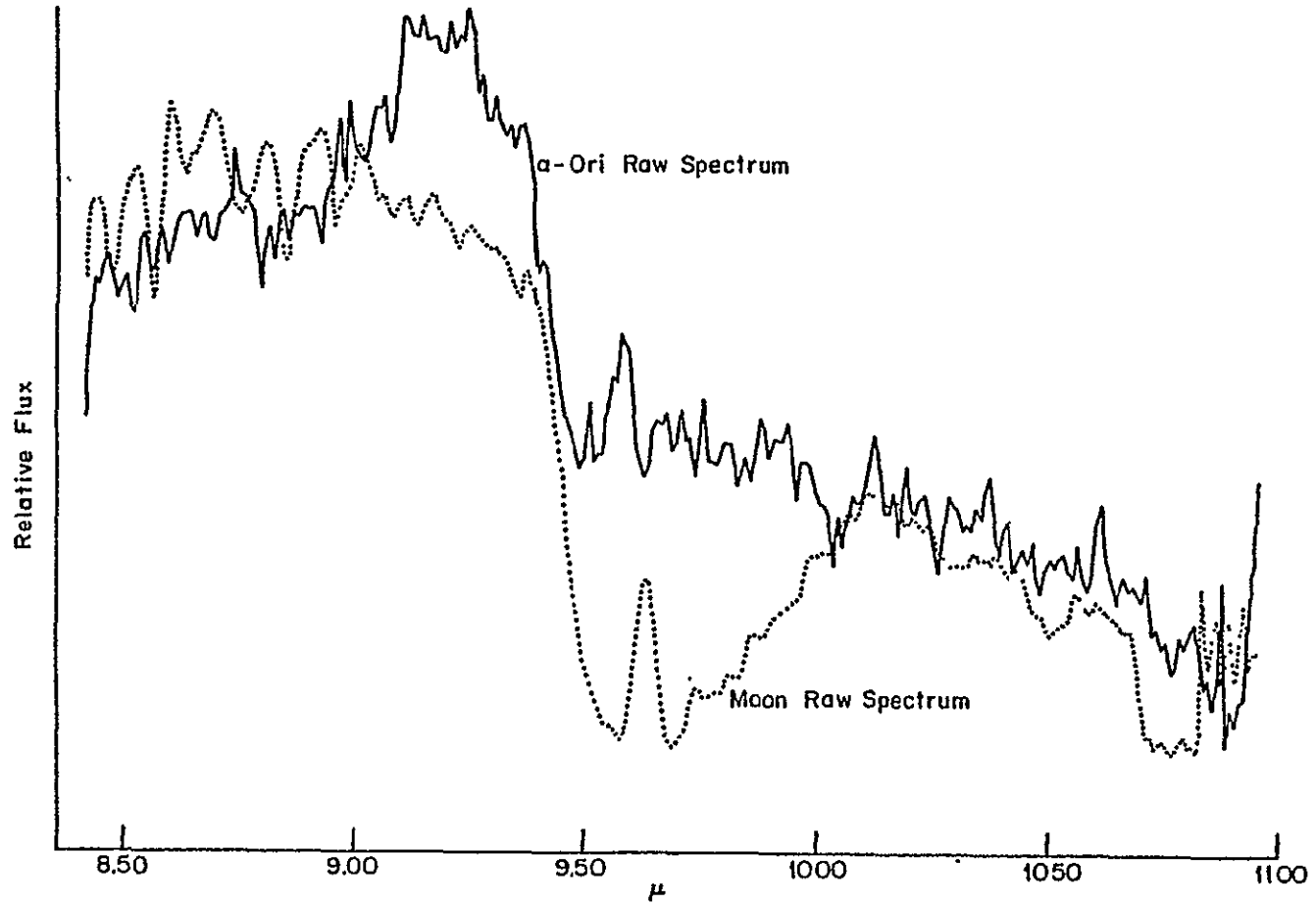


Figure 6-2. The raw spectra of α -orionis and the Moon.

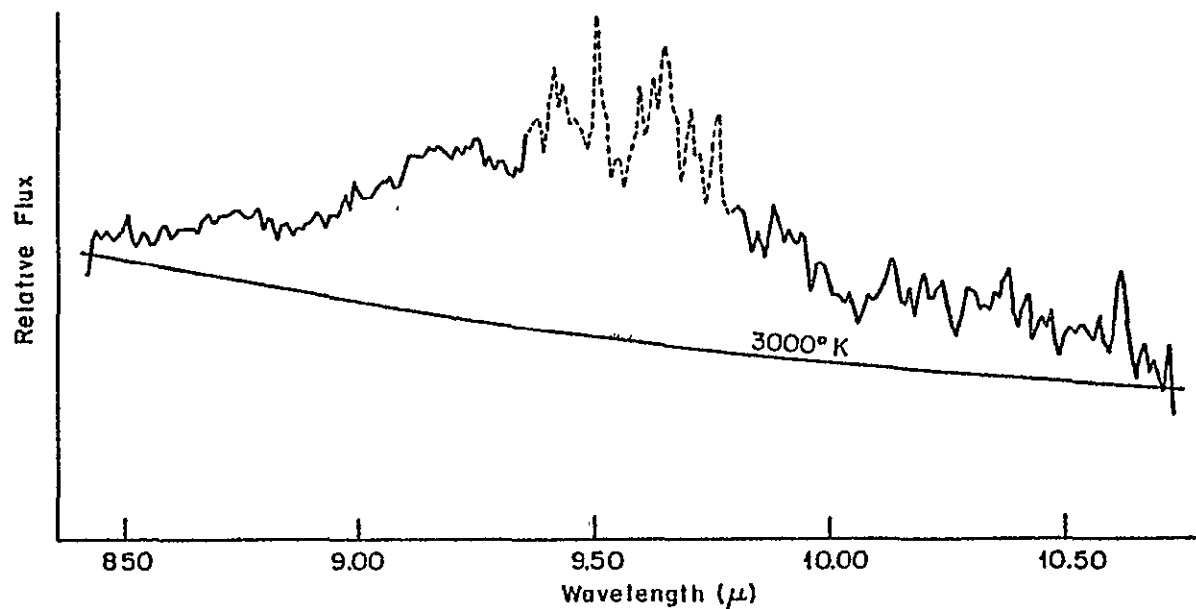


Figure 6-3. The ratio spectrum of α -orionis to the Moon corrected for lunar temperature.

also proposed that it is silicate grains which one is observing in the circumstellar dust cloud. This suggestion has been strengthened by high resolution spectra of Gamown et al (1972) with resolution $\lambda/\Delta\lambda=250$ from 850 cm^{-1} to 1100 cm^{-1} . The emissivity of silicate grains should have a second peak near $20\mu\text{m}$. This second emission feature was observed by Low and Swamy (1970) in narrow-band photometry of α -orionis. Another supporting piece of evidence for the silicate model of the α -orionis dust cloud comes from observations of silicon monoxide (SiO). Silicon monoxide is expected to be among the most abundant molecules present in the atmospheres of cool stars of normal composition. It is also a reagent in the condensation mechanism thought to produce circumstellar silicate grains (Mass et al, 1970). The presence of silicon monoxide in α -orionis was confirmed by Cuduback et al (1971). They observed SiO absorption features around $4\mu\text{m}$.

The silicates are expected to form from the material ejected by cool stars. Gilman (1969) calculated what solids would condense from gas of stellar composition as it moved away from a star and cooled. This is critically dependent on the ratio of oxygen to carbon in the gas. These elements first combine to form carbon monoxide; the subsequent development depends on which of the two is left over when all the other has been used up in this way. If carbon predominates, graphite will be the principal condensate, or under certain circumstances silicon carbide. If oxygen wins, the grains which should form are the silicates of calcium, magnesium, aluminium and iron;

combinations of these are responsible for the 10 and 20 μ m spectral features. Aluminium and calcium silicate may be rare because of the low cosmic abundance of aluminium and calcium. Woolf and Ney (1969) expected magnesium silicate, $MgSiO_3$, with some iron silicate, $FeSiO_3$, to be most abundant. For recent work on dust grains one can refer to Salpeter's paper (1974 a) on the theory of nucleation and dust grains in carbon-rich stellar atmosphere and his paper (1974 b) on formation and flow of dust grains in cool stellar atmospheres.

Laboratory spectra exist for silicate absorption features (Day, 1974). Any fine feature of the astronomically observed silicate emission may be washed out by different particle sizes and shapes, uncertainty in temperature and mixture of composition. Gammon et al (1972) examined the excess in XY Seg and O Cet and concluded that the type of silicates involved are basic rather than acidic. Day (1974) synthesized the amorphous magnesium silicate and obtained an absorption band quite similar to the material causing the interstellar 10 μ m absorption feature. He suggested that the existence of disordered structures seems a more reasonable expectation than crystalline terrestrial-type minerals. For the moment, the nature of the silicates is certainly at an unsettled stage.

Penman (1976) had measured the middle infrared reflectivities of five silicate minerals, and used Kramers-Konig analysis to obtain the optical constants of the samples. He then used the optical constants in Mie computations of the absorption properties of very small mineral grains. The final absorption

cross-section spectra compared to the observed $10\mu\text{m}$ silicate of the source W3/IRS5. All the calculated spectra have sharper features than the astronomical features due to the application of Mie theory. However, the hydrated silicate, Chloritite (hydrated Mg/Fe/Al silicate) and serpenlenite (hydrated Mg/Fe silicate) fits the observed astronomical positions correctly. They fall almost exactly at the center of the astronomical features.

To the author's knowledge only two spectra of α -orionis in 8- $14\mu\text{m}$ exist in the literature. Gillett et al (1968) (figure 6-4a) obtained results in the wavelength region from 2.8 to $14\mu\text{m}$, with a resolution $\lambda/\Delta\lambda=50$. Treffers and Cohen (1973) (figure 6-4b) obtained a spectrum from 8- $14\mu\text{m}$, and in the $20\mu\text{m}$ region with resolution 1000. Gillett's and Treffers and Cohen's spectra are shown in figure (6-4). Compared with our result, all show the $10\mu\text{m}$ emission feature if Gillett's black body curve is lowered instead of being drawn tangent to the observed data at $10\mu\text{m}$. Our spectrum shows a rather rapid dip at wavelengths beyond $10\mu\text{m}$ while Treffers and Cohen show a slower nearly constant decline. Further high resolution observations should clear this matter up. Our own instrument now operates at a resolution similar to that of Treffers and Cohen, and if used on a telescope as large as the 120 inch Lick Observatory reflector that they used, sufficiently high signal to noise ratios should be obtained.

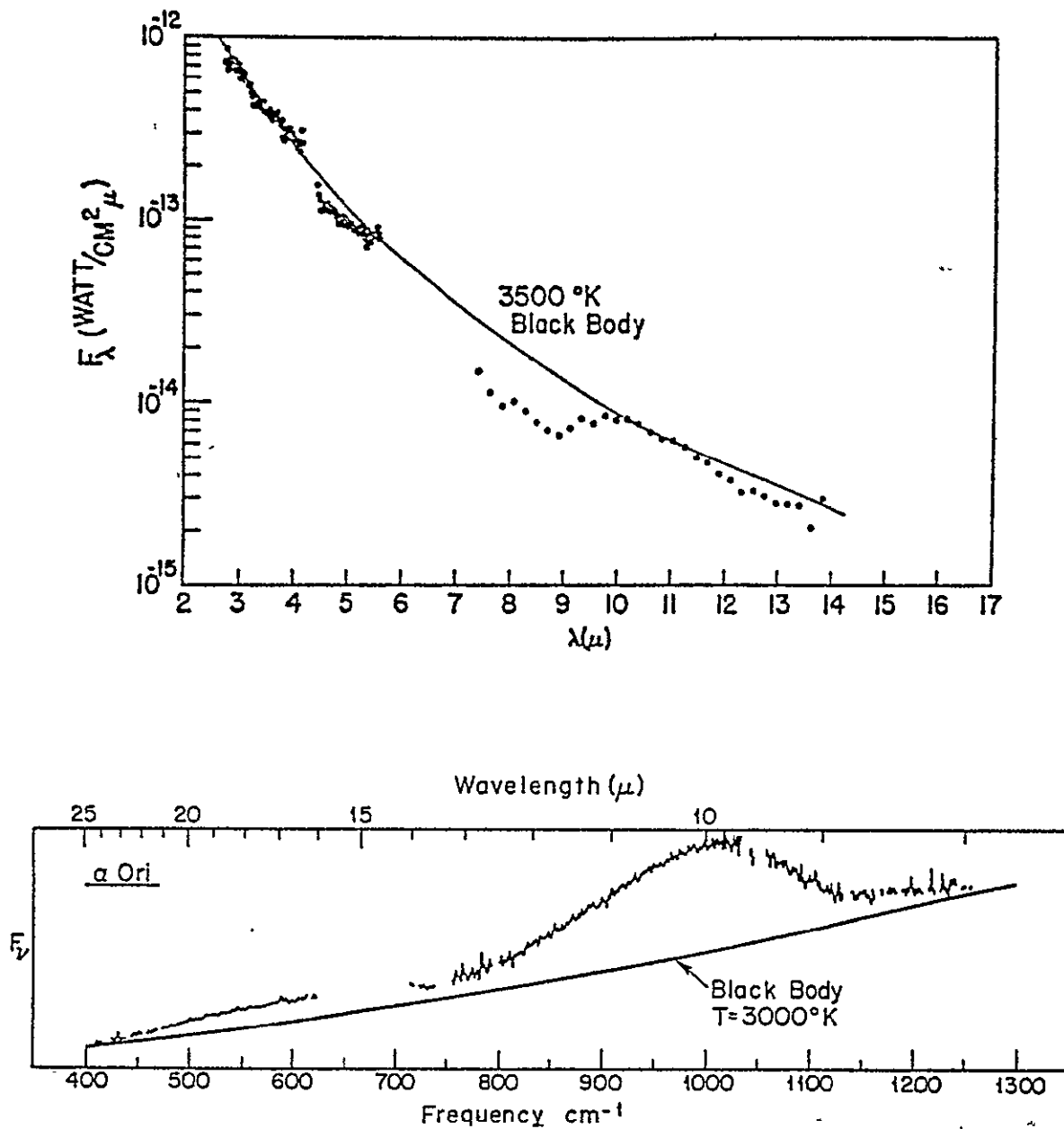


Figure 6-4. (a) Low resolution spectrum taken by Gillett et al (1969). Different symbols represent spectra taken on different nights. (b) High resolution spectrum taken by Treffers and Cohen (1973).

(C) Observations of Jupiter

The observations of Jupiter were also carried out with the 50" infrared telescope at Kitt Peak. The beam size is $18'' \times 47''$. The Kitt Peak bolometer assigned to the 50" telescope with band pass 8-14 μ m was used with the Hadamard spectrometer. For our Jupiter observations, the spectrometer operated in the 10.8-13.4 μ m region with an effective resolution $\lambda/\Delta\lambda$ around 250. The chopping frequency was 10 cycles per second.

Two runs were taken. Each run consisted of twelve scans of the whole Jovian disk. Two lunar spectra were taken on the same day for correction purposes. For the lunar temperature we used a value of 383°K. Figure (6-5) shows the raw spectrum of Jupiter and the Moon. Both the Jovian and lunar spectra are the sums of two independent runs. Figure (6-6) shows the ratio spectrum of Jupiter corrected for lunar temperature. The arrows labeled by H₂O show the position of telluric water vapor features. Most of the telluric features have been cancelled properly.

Jupiter is covered by clouds which in the visible part of the spectrum are seen from earth. Current models show three distinct cloud layers (figure 6-7, Ingersoll). The lowest layer is water ice, with maximum density at about 270°K. The middle cloud is solid ammonium hydrosulfide (NH₄SH) at about 200°K. Lewis and Prinn (1970) suggested that ultraviolet radiation from 2200 to 2700 Å is not absorbed by H₂, He, CH₄ and absorbed a little by NH₃. Therefore, the radiation in this

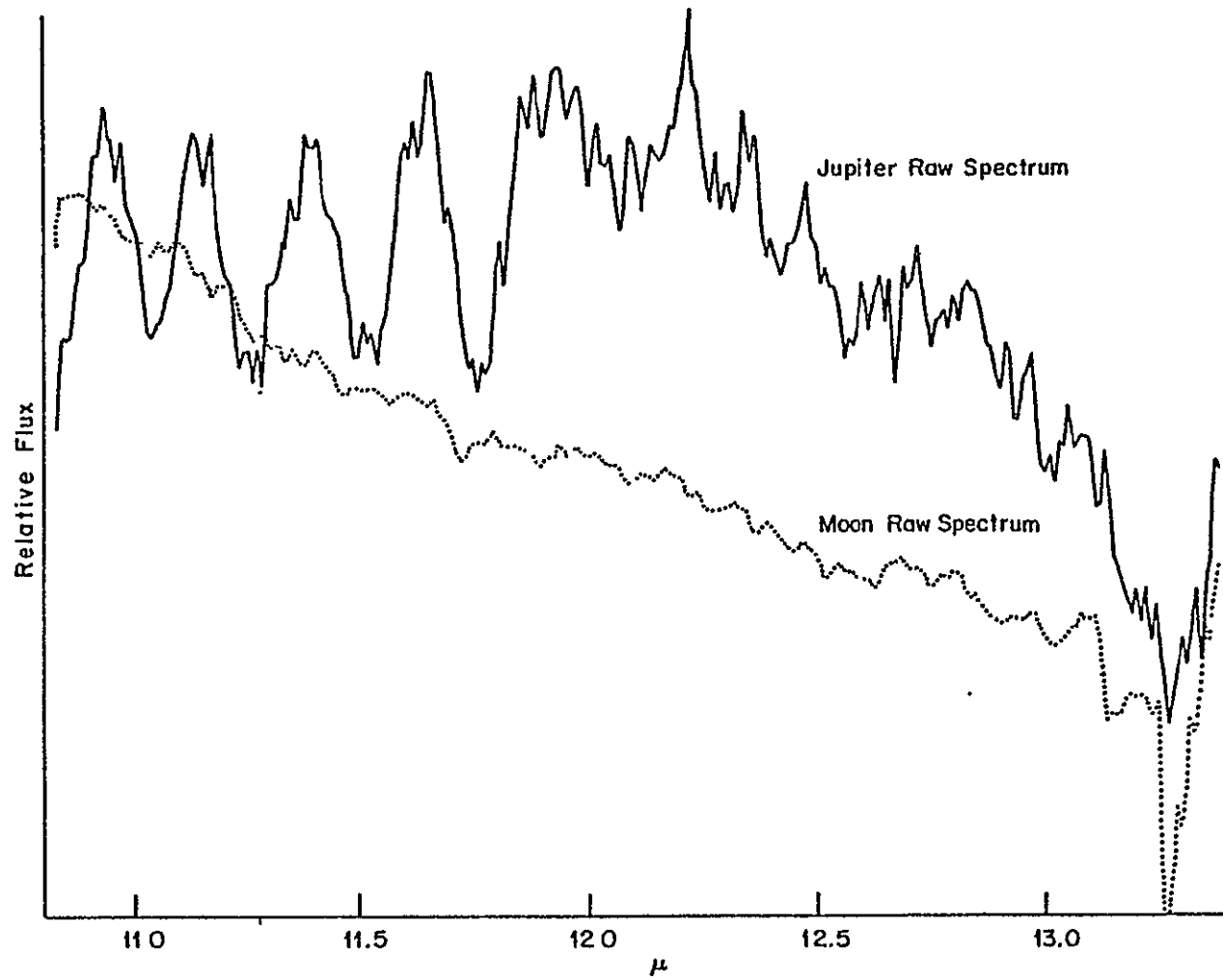


Figure 6-5. The raw spectrum of Jupiter and the Moon.

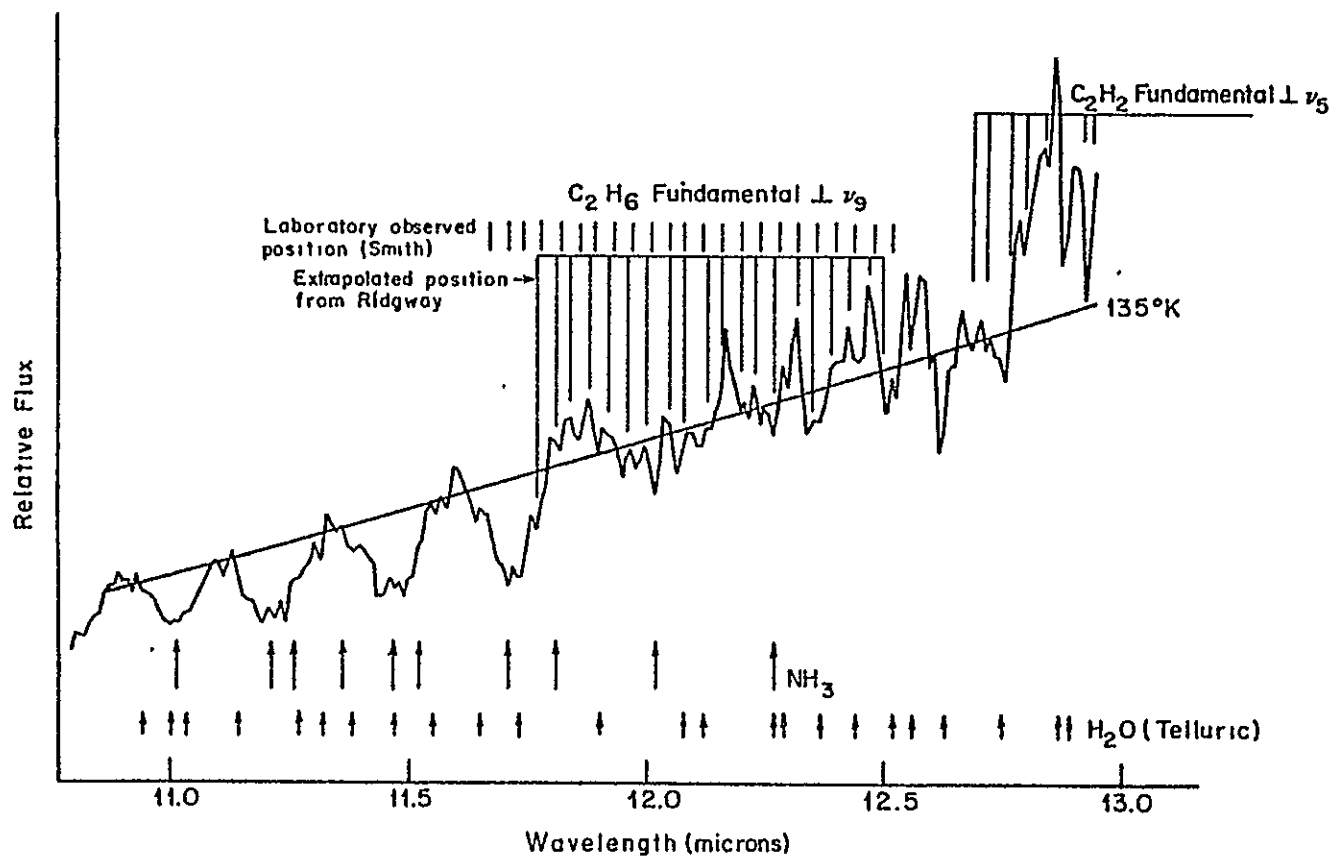


Figure 6-6. The ratio spectrum of Jupiter to the Moon corrected for lunar temperature.

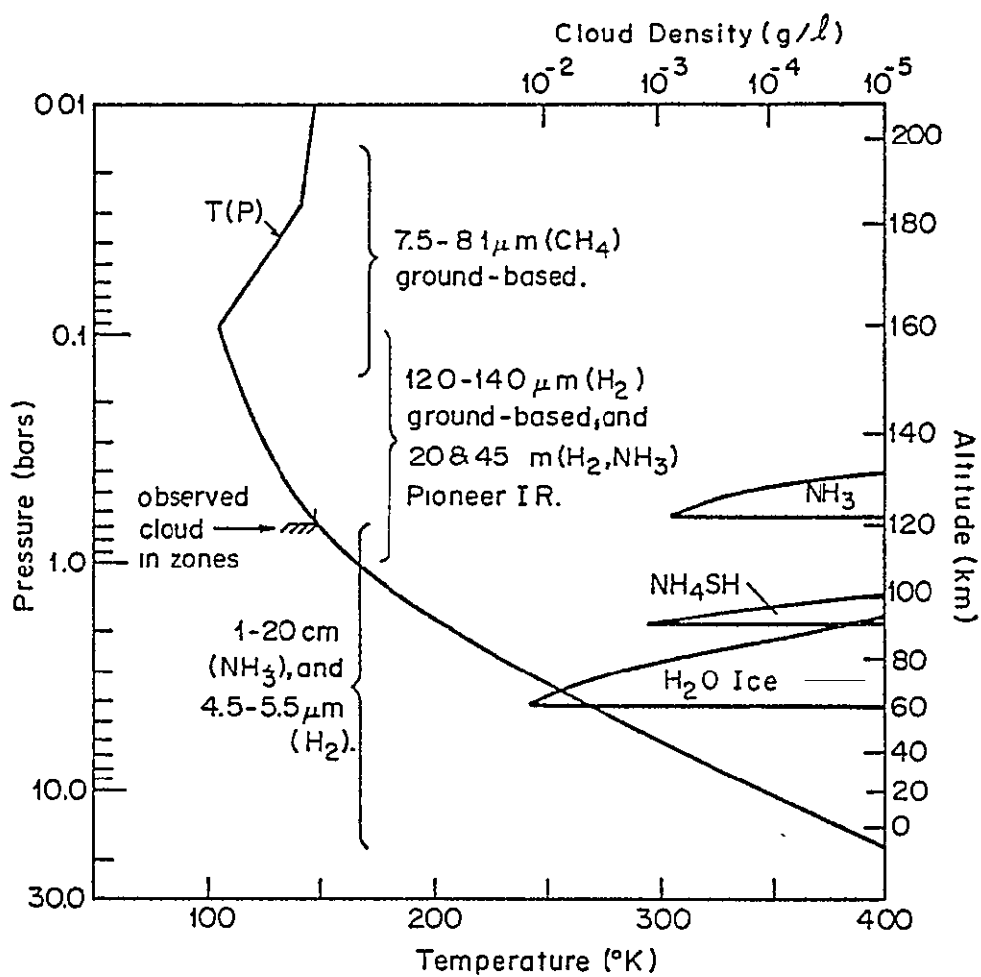


Figure 6-7. The atmospheric profile of Jupiter for a solar-composition model.

region may reach the ammonium hydrogen cloud and photolyze hydrogen sulfide there into hydrogen polysulfides (H_2S_x), elemental sulfur, and ammonium polysulfides $[(NH_4)_2S_x]$. All of these species are yellow, orange, or brown and may explain the color of zones. However, Sagan and Salpeter (1976) suggested that even under the most optimistic assumption that every H_2S photo dissociation event leads to polymerics, the implied optical depth falls short by two orders of magnitude from matching the observed values. Moreover, pure polymeric sulfur fits the observed optical properties of the Jovian red chromophores only poorly (Rages and Sagan, 1977). The upper cloud is solid ammonia at around $150^\circ K$. Solid ammonia is whitish and probably forms the white zones on the Jovian disk. The color of the Great Red Spot may be due to high altitude ultraviolet photolysis of phosphine (PH_3) into P_2H_4 and amorphous red phosphorus. The total depth of the cloud system is about 70 Km, and the pressure range probably runs from about 0.5 bar at the top to 4.5 bar at the cloud base. It is the cloud tops and above where the $10\mu m$ infrared radiation originates. Our spectrum measures a color temperature of $135^\circ K$ which is consistent with Ingersoll's picture. The radiation should come from the cloud tops because the Jovian atmosphere at 10.5 to $13\mu m$ has appreciable opacity, as discussed in the next paragraph.

The main constituents of the Jovian atmosphere are hydrogen molecules, helium molecules, methane and ammonia, with minor constituents hydrogen sulfide, water, ethane and acetylene. Table 6-1 shows their observed abundance ratio by number.

TABLE 6-1

Solar composition atmosphere (fraction by number)

Species	(1)	(2)
H ₂	0.886	0.870
He	0.112	0.128
H ₂ O	1.05 x 10 ⁻³	8.80 x 10 ⁻⁴
CH ₄	6.30 x 10 ⁻⁴	6.17 x 10 ⁻⁴
NH ₃	1.52 x 10 ⁻⁴	1.49 x 10 ⁻⁴
H ₂ S	2.90 x 10 ⁻⁵	2.56 x 10 ⁻⁵

(1) Weidenschilling and Lewis (1973).

(2) Podolak and Cameron (1974); Cameron (1973).

The table is taken from Ingrosell.

The abundances of hydrogen, methane, ammonia, and helium seem consistent judged from solar atomic abundances. For detailed information one can refer to McElroy (1973).

The opacity due to hydrogen molecules is caused by pressure induced dipole absorption. The hydrogen molecule has no permanent dipole moment, and consequently, no permanent dipole spectrum. Gaseous H_2 , however, has a weak pressure - induced dipole spectrum which absorbs significantly over the long path lengths and low pressure of the Jovian atmosphere. The induced dipole moment results from two distinct physical processes (Kranendonk and Kiss, 1959). The first takes place when the permanent quadrupole moment of one molecule induces a dipole moment in another molecule by virtue of the neighbor's polarizability. This is a long range interaction. The second physical process takes place when the overlap forces of the two adjacent molecules cause an asymmetrical distortion of their electronic charge clouds. The net induced dipole moment is modulated by the relative translational and rotational motion of the colliding pair and this modulation produces the absorption of infrared radiation. The translational spectrum is predominant at long wavelengths with its peak at $100\mu m$ at $100^\circ K$ (Trafton and Munch, 1969). In our wavelength region ($10.5 - 13\mu m$) its contribution to the opacity is negligible. The rotational hydrogen collisional spectrum, however, has its peak at $17\mu m$ and contributes a continuous opacity in our wavelength region (Th. Encrenaz, 1972).

The helium molecule also has no permanent dipole moment.

Its opacity comes from the collision with the hydrogen molecules and resembles the H_2-H_2 collision process. The collision is less important due to the smaller abundances of helium.

Ammonia is an important source of opacity at $10\mu m$ under Jovian atmospheric conditions. The $10\mu m$ band of ammonia arises from transitions through the ν_2 mode. In the ν_2 mode of vibration, the nitrogen atom oscillates vertically relative to the plane of the hydrogen atoms (figure 6-8a). The nitrogen atom is able to penetrate through the potential barrier to the other side of the hydrogen plane. This inverted position leads to inversion splitting of the levels of the ammonia molecules. The splitting generates both symmetric and antisymmetric energy levels with a given vibrational quantum number. The $10\mu m$ band of ammonia arises from transitions from the ground vibrational state to the first excited state in the ν_2 mode (figure 6-8b). Another transition, from the first excited symmetric vibrational state to the second excited asymmetric state is also in $10\mu m$ range, but the contribution due to this "hot band" is small for the low temperature in the Jovian atmosphere.

The ammonia is clearly seen in absorption in the spectrum. The centers of the bands are shown by the arrows labeled NH_3 . The ammonia absorption has been observed by different groups.

Gillett et al (1969)(figure 6-9) observed Jupiter from $2.8-14\mu m$ with low resolution $\lambda/\Delta\lambda=50$. Briefly, their results show the following: The spectrum has a depression at $3.3\mu m$ caused by CH_4 . Solar heating of the upper atmosphere via this band results in warming of the upper atmospheric layers. This

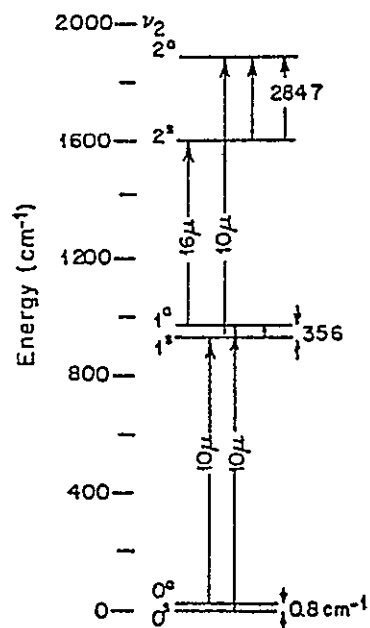
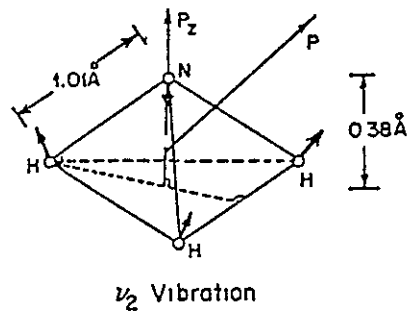


Figure 6-8. (a) Schematic representation of the NH_3 molecule. The components of angular momentum and the motion in the ν_2 vibrational mode are also shown. (b) Energy levels of the ν_2 vibrational mode of ammonia. Superscripts a and s refer to the antisymmetric and symmetric levels which arise due to inversion splitting.

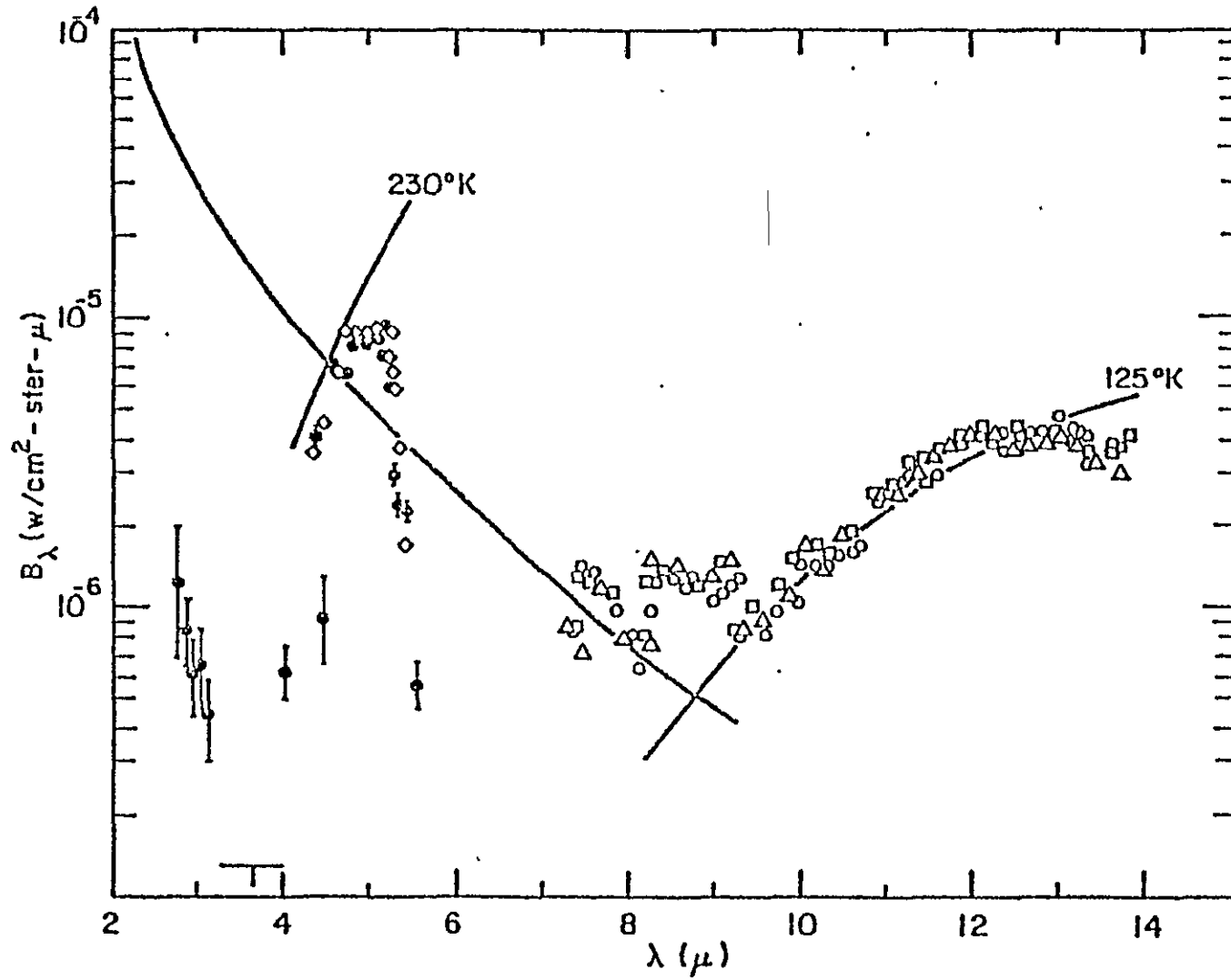


Figure 6-9. Low resolution spectrum taken by Gillett et al (1969). Different symbols represent spectrum taken on different nights. This figure is taken from their paper.

proceeds until energy is radiated at the same rate via the $7.7\mu\text{m}$ band of CH_4 . Ammonia absorption around $10\mu\text{m}$ was also detected. Judging from the CH_4 emission, these authors were the first to suggest a temperature inversion on Jupiter caused by solar heating of the $3.3\mu\text{m}$ band of CH_4 . They also showed that the NH_3 band at $10\mu\text{m}$ is saturated, and calculated that the H_2 abundance at $12.5\mu\text{m}$, assuming a temperature of 125°K , is 12 km-atm. with a pressure $P_{\text{H}_2} \sim 1/4\text{atm}$.

Aitken and Jones (1972) obtained a Jovian spectrum from $8 - 13\mu\text{m}$ at a resolution $\lambda/\Delta\lambda \sim 143$ (figure 6-10). The ammonia absorption band is again seen. They estimated that the ammonia abundance in the band is about 2.7 cm-atm. and a lapse rate $\Gamma = \frac{\partial T}{\partial h}$ at $13\mu\text{m}$ given by $H = -30\text{K}$, where H is the scale height $\sim 20\text{km}$.

The most recent published infrared spectrum is by Lacy et al (1975) who used the Lick Observatory 120" telescope. High resolution spectra were obtained at 890 cm^{-1} ($11.24\mu\text{m}$) with $\lambda/\Delta\lambda = 1780$. Medium resolution data were observed from 1000 cm^{-1} to 850 cm^{-1} ($10 \sim 12.75\mu\text{m}$) with $\lambda/\Delta\lambda$ from 250 to 333 (figure 6-11 a,b). The authors also calculated synthetic spectra, assuming that NH_3 and H_2 are the only sources of opacity. Their conclusions from comparison between observed and computed spectra follow: All of the prominent lines in their observed spectrum are saturation NH_3 bands broadened to a width many times the pressure - broadened line width. The observed 135°K continuum is primarily formed by the wings of the NH_3 line. The H_2 opacity may be important if NH_3 is unsaturated

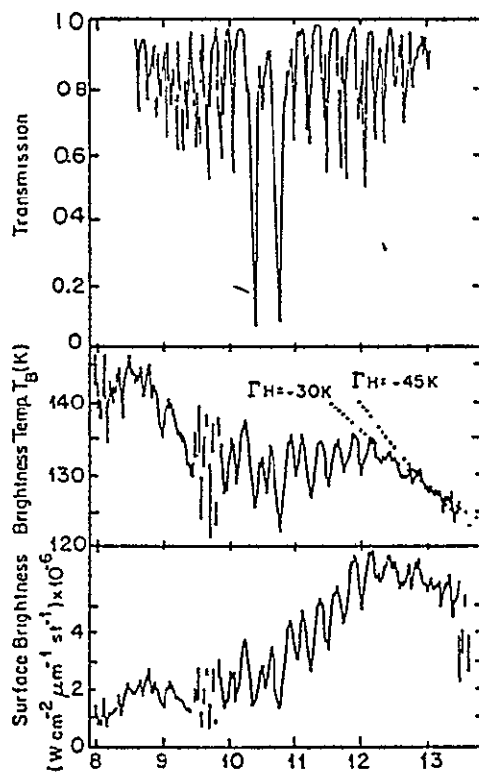


Figure 6-10. (a) Room temperature absorption spectrum of ammonia, $p=0.06$ atmos, $w=0.6$ cm atmos. (b) Brightness temperature; (c) Surface brightness of the central region of Jupiter from 8 to 13.5 μm . (Taken from Aitken and Jones).

near 135°K . A pressure of 0.125 atm. at 135°K is required to form the continuum. The minimum temperature in their synthetic model is $118 \pm 5^{\circ}\text{K}$ while the observed minimum temperature is 123°K , about 5°K larger than the derived temperature due to incomplete resolution of the features. The lapse rate at 135°K is $7.5 \pm 2.5^{\circ}\text{K}/\text{SH}$. Gillett et al (1969) estimated the lapse rate at the NH_3 saturation level is $4^{\circ}\text{K}/\text{SH}$. A discrepancy occurs in the comparison of the medium resolution data between 870 and 890 cm^{-1} . In this region the Jovian spectrum seems to be depressed by about 2°K relative to the calculated curve. The authors suggested that it may be due to an as yet unidentified minor constituent of the Jovian atmosphere.

Our spectrum has about the same resolution as the medium spectra of Lacy et al's and so the two spectra can be compared. The line positions match well. The vertical matching shows a drift towards longer wavelength. Figure (6-11a) is obtained by matching points A and B. The short wavelength side matches but the end of the long wavelength side is about 1.5 times higher. Also our spectrum seems to match the theoretical curve better at 870 to 890 cm^{-1} . Figure (6-11b) is obtained by matching A' and B'. Then the long wavelength side matches better than before but the short wavelength side is different. Also now our spectrum matches better with Lacy et al's observed result at 870 - 890 cm^{-1} . A conclusion about the discrepancy at 870 cm^{-1} between Lacy's observed and calculated spectra can not be reached at present until there is a better way for matching our and Lacy's et al's spectrum. Also it is not clear

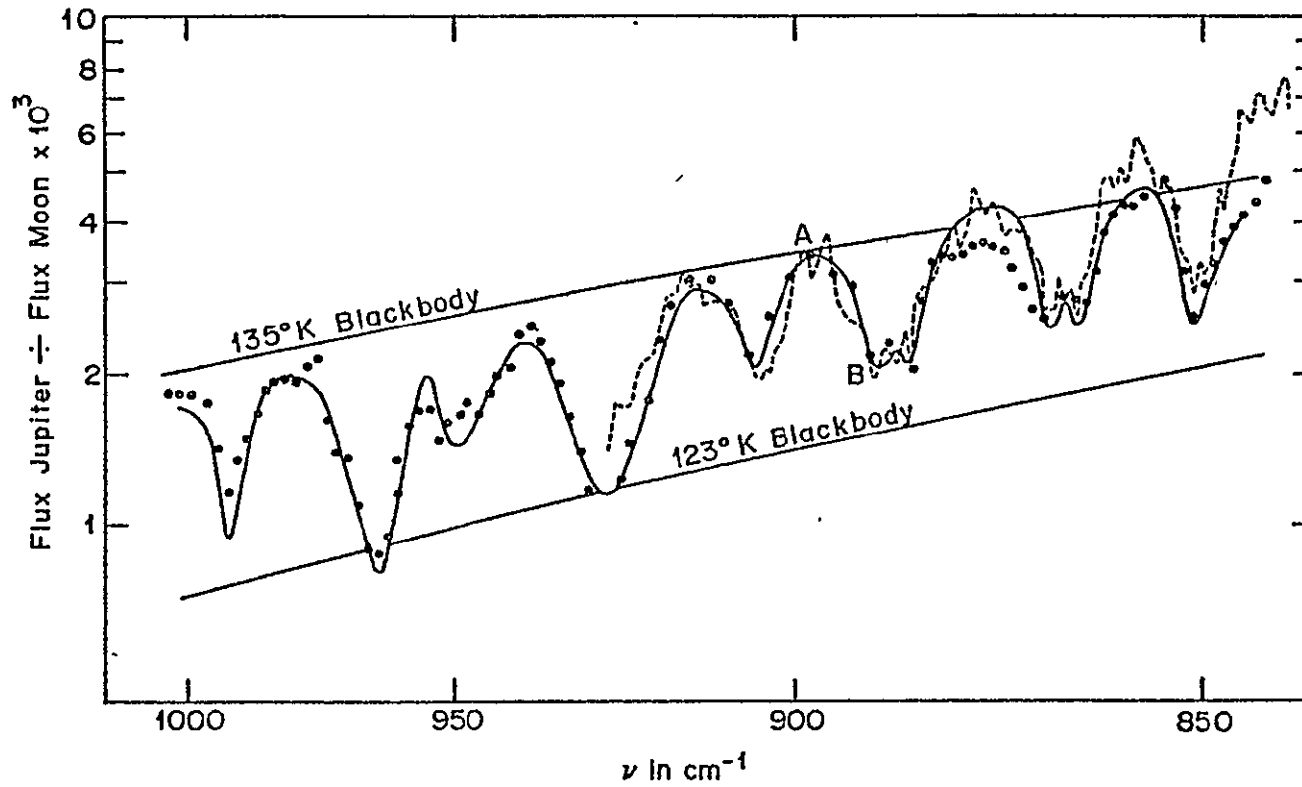


Figure 6-11a. Spectrum of the N and S polar regions of Jupiter at $3\text{-}4\text{ cm}^{-1}$ resolution divided by the spectrum of the Moon. Data points are shown as solid circles, and the solid line represents the best fitting synthetic spectrum calculated from the model calculated by Lacy *et al* (1975) (The graph is taken from Lacy *et al*). The dotted curve is our observed spectrum by matching Lacy's spectrum at points A and B.

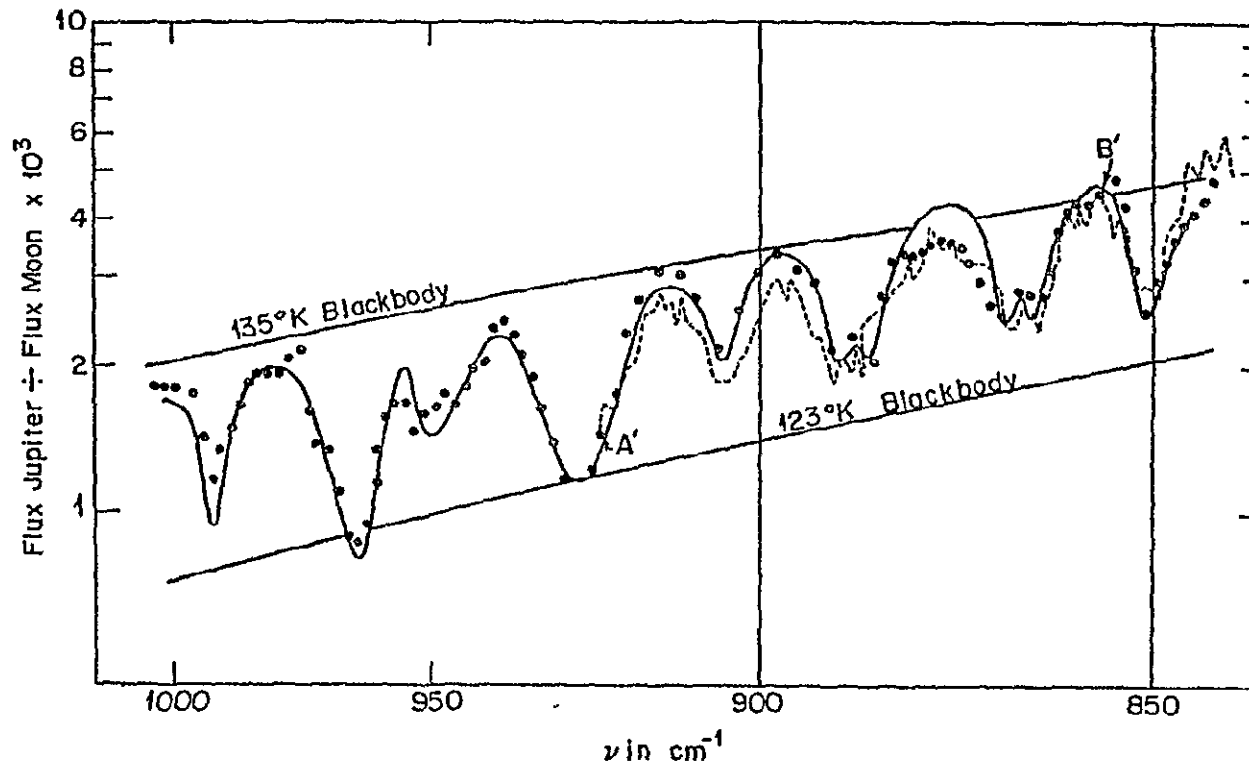


Figure 6-11b. Same as 6-11a except matching our spectrum with Lacy's at A' and B'.

whether the difference in matching of our spectrum and Lacy et al's between long wavelength and short wavelength is real or not. There could be several reasons for the difference. It could be due to the inaccuracy of the end of the spectra, because the short wavelength side is the end of our spectrum and the long wavelength side is the end of Lacy's spectrum; or it may just be due to the matching technique. More effort is needed to clarify this point.

The absorption due to NH_3 is much less important beyond 12μ , so one may be able to use the H_2 opacity to estimate the lapse rate in that region. The lapse rate can be estimated by the equation

$$T_B(\lambda_1) - T_B(\lambda_2) = \frac{\Gamma H}{2} \ln \frac{a(\lambda_1)}{a(\lambda_2)}$$

where $T_B(\lambda_1)$ is the brightness temperature at λ_1

$T_B(\lambda_2)$ is the brightness temperature at λ_2

Γ is the lapse rate

$a(\lambda_1)$ is the absorption coefficient at λ_1

$a(\lambda_2)$ is the absorption coefficient at λ_2

H is the scale height

If one chooses $\lambda_1=11.95$, $\lambda_2=12.34$ with measured brightness temperature $T_1=133.94$, $T_2=133.64$, $a(\lambda_1)$, $a(\lambda_2)$ are taken from Calpa and Ketebaar (1957), one obtains a result $\Gamma H=-43.2^\circ\text{K}$ for an H_2 opacity dominated atmosphere. Gillett et al (1969) calculated the adiabatic lapse rate $(\Gamma H)_{ad}=-42^\circ\text{K}$ for an H_2 atmosphere, while Aitken and Jones (1972) measured a value of $\Gamma H=-30^\circ\text{K}$ from their spectrum. Our value seems closer to the

value calculated by Gillett rather than to Aitken's. It is emphasized here that the estimate is based on the assumption that the H_2 opacity is the dominant opacity at wavelengths 11.95μ and 12.34μ , which may not be true.

Methane has a strong emission band at 7.7μ but does not have any band structure in our region. Methane has two important contributions to the overall thermal structure of the Jovian atmosphere. The first one is that methane has an absorption band at $3.3\mu m$ which absorbs solar energy and reradiates at $7.7\mu m$ producing an inversion temperature layer with a maximum temperature of $150^\circ K$ at an altitude $160 - 200$ km. Secondly, methane is photo dissociated by ultraviolet light in the upper atmosphere. This results in products such as ethane (C_2H_6), acetylene (C_2H_2), and ethylene (C_2H_4). Strobel (1973) estimated that column densities of C_2H_6 , C_2H_2 , C_2H_4 above the cloud top are approximately 10^{21} , 3×10^{16} , $3 \times 10^{15} cm^{-2}$ respectively. C_2H_6 and C_2H_2 were first observed by Ridgway (1973) using the 60" Kitt Peak solar telescope in the $750 - 875 cm^{-1}$ ($11.42 - 13.33\mu m$) range with resolution $\lambda/\Delta\lambda=770$, (figure 6-12a). The lines are shown in strong emission at the $140^\circ K$ temperature. The apparent lines are superpositions of many lines, each group corresponding to a sub-band. Ridgway calculated that the mixing ratios are $N(C_2H_6)/N(H_2)=4 \times 10^{-3}$ and $N(C_2H_2)/N(H_2)=8 \times 10^{-5}$. The ratio $N(C_2H_6)/N(C_2H_2)=50$ where Strobel predicts about 200. Combes et al (1974)'s (figure 6-12b) observation confirms the presence of very strong emission lines of C_2H_2 and C_2H_6 . The abundance of

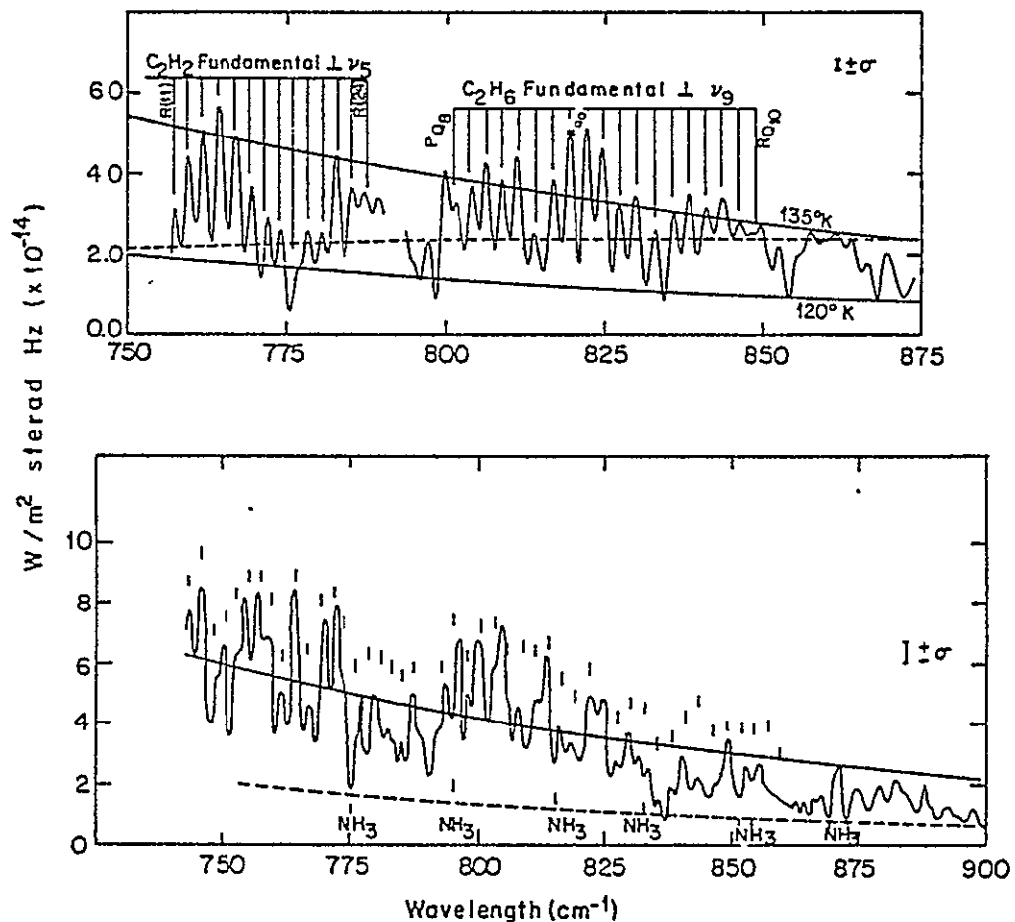


Figure 6-12. (a) Thermal emission spectrum of Jupiter corrected for absorption in the earth's atmosphere observed by Ridgway (1973). The dashed line is the predicted form of the H_2 continuum. (b) The ratio of the Jovian spectrum to the atmospheric absorption spectrum observed by Combes *et al* (1974). The solid and dashed lines are the blackbody curves at 135°K and 120°K respectively.

ethane estimated from Ridgway's spectra depends strongly on the distribution of gas temperature, which is not well-determined. If the temperature in the mesospheric inversion layer turns out to have a maximum value of 150°K, the observations indicate \sim (Ridgway 1974) $2 \times 10^{-2} \text{ gm cm}^{-1}$ of ethane in this high temperature region. Sagan and Salpeter (1976) estimate the column density of ethane molecules to be $3 \times 10^{-3} \text{ gm cm}^{-2}$ by assuming that ethane is produced by photolysis of methane by solar ultraviolet photon and destroyed mainly by eddy diffusion into the troposphere, followed by pyrolysis in deeper, hotter layers. This would be in very serious conflict with the observations, especially since only a small fraction of the theoretical column density refer to the hotter inversion region.

The ethane emission band is also shown in our spectrum (figure 6-6). In the figure the indicated emission line position was extrapolated from Ridgway's spectrum, and the laboratory observed position by Smith (1949) are also shown for comparison. Our positions agree with Ridgway's reasonable well, while Smith's seem displaced from ours by 0.01 μm , possibly due to a uncertainty in position calibration. Only a portion of the C_2H_2 spectrum can be seen in our spectral coverage. The abundance of C_2H_6 is not estimated here because the absolute amplitude of our spectrum is not well calibrated.

Our spectrum contains both ammonia and ethane features while other observers have not shown both. This will be useful because one can compute the synthetic spectra including both. Ammonia will provide us with information about the top

of the cloud layer while ethane provides us with the information about the inversion layer. A synthetic spectrum including both ethane and acetylene would be interesting because the inclusion of these new gases would affect the models, especially around the inversion. Acetylene would appear around $13\mu\text{m}$. The absorption of solar radiation by CH_4 at $3.3\mu\text{m}$ used to be thought to be radiated solely by CH_4 at $7.7\mu\text{m}$. The $7.7\mu\text{m}$ emission intensity is a critical test of a temperature inversion model and the emission intensity calculated by Wallace et al (1974) is within 25% of the value observed by Gillett et al (1969). If ethane and acetylene do contribute to emission in the thermal infrared, there must be some additional source of solar absorption in order to produce the observed inversion temperature. Additional absorption at this altitude, perhaps due to particles, is suggested by the low ultraviolet albedo of Jupiter in the wavelength region 2100 to 3600 \AA (Wallace, Caldwell and Savage, 1972).

Terrile and Westphal (1976) had imaged Jupiter at high spatial resolution at 8 - $14\mu\text{m}$. All images reveal a belt and zone structure similar to visible photographs. In the 8 - $14\mu\text{m}$ broad-band data, belts appear to be about 2°K hotter than the zone. The lowest belt-zone contrast is found in the hydrogen opacity dominated region at $12.5\mu\text{m}$, while images at $9.5\mu\text{m}$ have the greatest contrast. This is consistent with the dynamic picture that zones are rising columns of air and belts are sinking columns of air. Ammonia gas being carried upward in zones will freeze out and form a thick cloud on top of the zones,

giving a low infrared temperature to the zones, and the crystallized NH_3 particle will be carried down to the deep atmospheres in the belt where they will get sublimated. One can look deeper into clouds in the belt because of the lack of the ammonia cloud on top of it and therefore see a higher infrared temperature. Although there are a number of high resolution spectra of good spectra for methane. The observation of methane will be interesting not only because it provides us with information about the inversion layer, it is also useful to find out the temperature profile. If the temperature profile of the Jovian atmosphere is known, one can use it to find the ammonia abundance profile. Ammonia itself is not a very good tool for probing the temperature profile because it has a low vapor pressure and its variation is very sensitive to temperature changes. Observations of methane at $3.3\mu\text{m}$ and $7.7\mu\text{m}$, should be able to accomplish this.

The Hadamard transform spectrometer described in this thesis would be able to make these observations, with small modifications that would permit observations to be made at these wavelength. In addition, observations should be undertaken at 8.0 to 9.5 microns where neither ammonia nor methane have strong absorption features. At these wavelengths one would be observing the clouds. In this 8.0 to 9.5 micron region our instrument should be able to image the bands and zones of Jupiter, to probe for spectral differences and cloud features. Such observations should increase our understanding of Jupiter's cloud structure.

(D) Observations of Mercury

The observations of Mercury were made with the newly built Cornell 25" telescope at Mount Pleasant, Ithaca, New York.

The telescope has a focal ratio $f/13.5$. The spectrometer's acceptance beam size is $\widehat{7.8''} \times \widehat{78''}$. The dewar described in section III-B was used with the spectrometer. The spectrometer operated in the $10.5\sim 13\mu\text{m}$ region with a resolution $\lambda/\Delta\lambda$ around 300. The chopping frequency was 10 cycles per second.

The observational procedure was carried out a little differently from the observations of α -orionis and Jupiter. First, Sun spectra were used for correction spectra rather than lunar spectra. There are no known molecular lines in this region. Its temperature at $11.10\mu\text{m}$ is 5030°K (Sailly & Goody). The observations were made on August 3, 1976. At that time the Sun was about an hour away from Mercury, and was observed through roughly the same air mass as Mercury. Secondly, Mercury is so faint in broad day light that we were unable to see it in visible light. The way to find Mercury was the following: We pointed the telescope in the correct region and scanned for the infrared signal. The signal is so strong that one can see it go off scale on the synchronous demodulator. The pointing accuracy of the telescope is 6 sec. of time in right ascension and 25 sec. of arc in declination. Thirdly, since we could not see Mercury visually for tracking, we adapted a different method for tracking Mercury. Since our computer programming is set up in such a way that the data taken when the mask is moving forward and moving backward are stored in different

areas, we only took data when the mask was moving forward. When the mask was moving backward we maximized the signal to assure correct pointing and waited for the next forward pass. Any noise introduced when moving the telescope for maximizing the signal would have gone into the backward-pass data bins and those data points were thrown away anyway. Since each pass takes 51 sec. only, Mercury remained at essentially the same position during the forward data taking pass.

Two runs of Mercury and two runs of the Sun were taken. Each run consisted of ten scans of the sources. Figure (6-13) shows the raw spectra of Mercury and the Sun. Since Ithaca has a lot of moisture in the air during the summer time, the correction for atmospheric features is more difficult than at Kitt Peak and is done in a different way. A constant was added to the Mercury spectra such that atmospheric features in the Mercury/Sun ratio spectrum were minimized. This step ensures that the atmospheric features are largely corrected. The Mercury spectrum which has a constant added to it was then multiplied by another constant to make its amplitude as close to that of the solar spectrum as possible. The solar spectrum was then subtracted from the modified Mercury spectrum. The multiplication of the Mercury spectrum by a constant assured that atmospheric features in the two spectra had similar amplitudes before the subtraction step. This difference spectrum was then added to a "perfect" solar spectrum which is calculated according to the blackbody function appropriate to the solar temperature. What one gets from these procedures is:

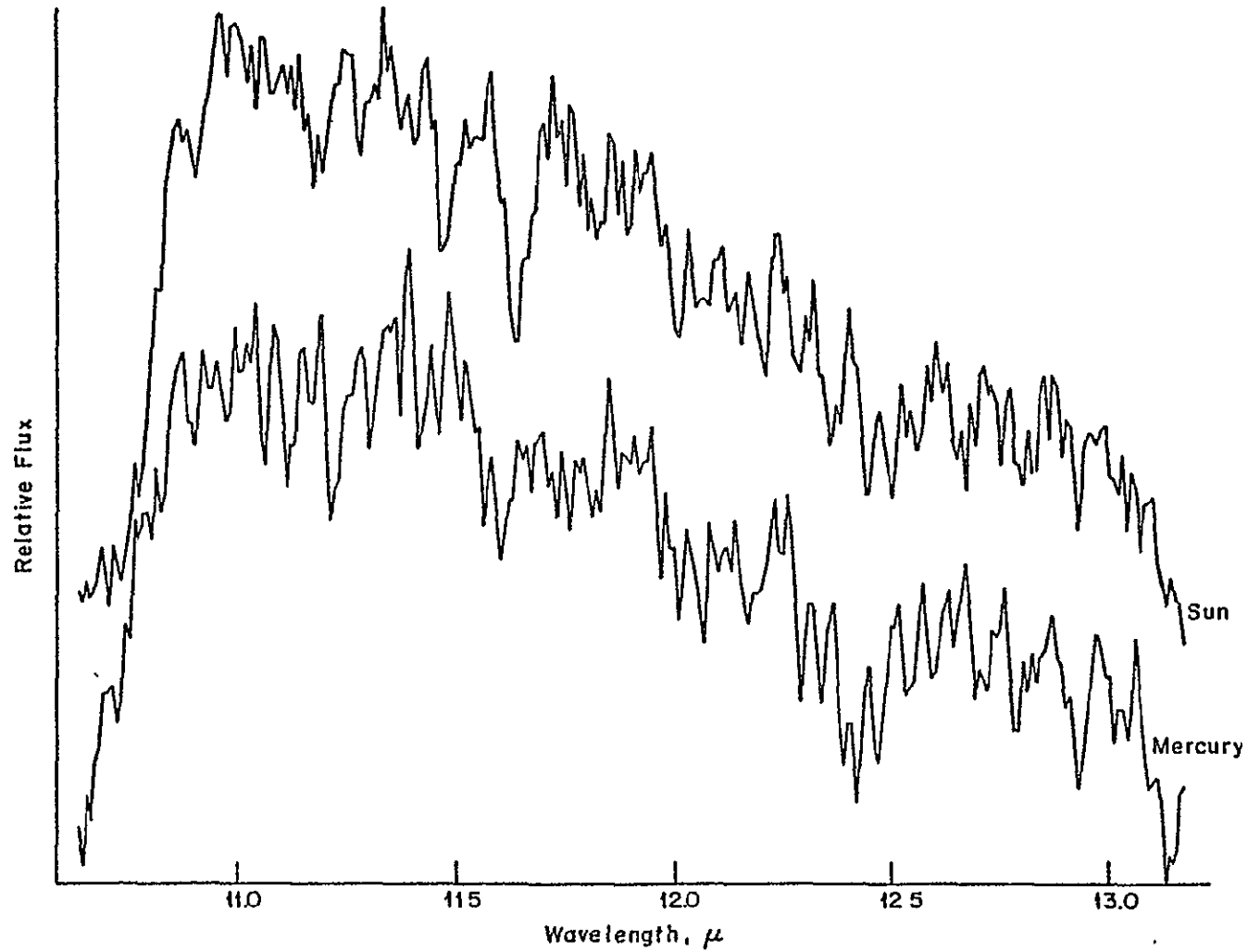


Figure 6-13. The raw spectra of Mercury and the Sun.

Final Mercury Spectrum

$$\begin{aligned}
 &= \text{observed corrected Mercury spectrum} - \text{observed solar} \\
 &\quad \text{spectrum} + \text{perfect solar spectrum} \\
 &= (\text{"perfect" Mercury spectrum} + \text{noise in the Mercury spec-} \\
 &\quad \text{trum}) - (\text{"perfect" solar spectrum} + \text{noise in the solar} \\
 &\quad \text{spectrum}) + \text{"perfect" solar spectrum} \\
 &= \text{"perfect" Mercury spectrum} + (\text{noise in the Mercury spec-} \\
 &\quad \text{trum} - \text{noise in the solar spectrum})
 \end{aligned}$$

Any systematic noise such as emission and absorption due to the sky or to the telescope will be subtracted away. The advantage of this method is that the final spectrum is obtained through subtraction rather than by division. Division, in the low signal portion of the spectrum, produces deceptive high noise spikes in the ratio spectrum. The method we have used tends to eliminate these.

Since this will be the first high resolution Mercury spectrum obtained, we will calculate Mercury's disk integrated infrared temperature and compare this temperature with the observed one. Morrison and Sagan (1967) had calculated the infrared brightness temperature of the center-of-disk as a function of phase angle and heliocentric longitude, but there is no disk integrated infrared temperature available in the literature. In the following we will discuss the factors that may affect the brightness temperature, and then present a method of calculating the disk integrated infrared brightness and compare it with our observation. A good review of thermophysics of Mercury is given by Morrison (1970).

In 1965, Pettingill and Dyce (1965) used radar to discover that Mercury has a rotation period of 59 days, two thirds of the orbital period, instead of an 88 day synchronous rotation around the Sun. This implies that Mercury has a solar day, on the planet, about 176 terrestrial days long, equal to two orbital revolutions in three rotations. Mercury's non-synchronous rotation leads to time-dependent thermal emission of the planet due to the diurnal variation of the insolation. This diurnal variation would not happen for a synchronously rotating Mercury. The diurnal variation changes the brightness temperature as a function of both phase angle and heliocentric longitude. It also allows a measurement of the thermal properties of Mercury's surface.

Because of the high eccentricity of Mercury's orbit, ($e=0.2$), the diurnal cycle of insolation is markedly different from longitude to longitude, and can differ by a factor of 2.5. The eccentricity enters in two ways. First, the variation in distance from the Sun produces a solar constant that varies by more than a factor of 2 from perihelion to aphelion. Second, the changing orbital angular velocity causes the apparent speed of the Sun across the sky to vary; near perihelion the angular velocity of revolution actually slightly exceeds the angular velocity of rotation, and the apparent planetocentric solar motion is retrograde (figure 6-14, Soter and Ulrichs, 1967). The two effects of the eccentricity reinforce one another, with the larger flux coming at a time when the angular rate of the Sun across the sky is largest. The two longitudes (180° apart)

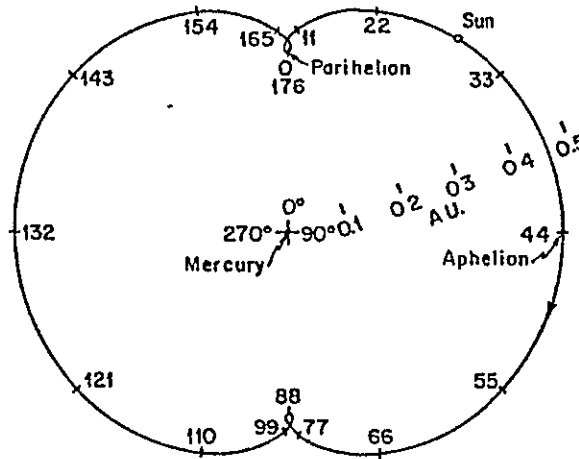


Figure 6-14. Diurnal path of the Sun about Mercury, drawn to scale. The relative positions of the Sun are marked at 11 day intervals with the planet held as a fixed reference. Planeto-graphic longitude are indicated for Mercury. (Taken from Soter and Ulrichs, 1967).

that see the Sun overhead at perihelion receive more than two and a half as much energy per period as the longitude 90° away, where the Sun is always small and rapidly moving while near the zenith.

Besides the insolation geometry, a possible atmosphere on Mercury will also affect the thermal emission. CO_2 is a major product for a possible secondary atmosphere, furthermore, since CO_2 could have been photodissociated and reduced to CO by preferential loss of oxygen, Fink et al (1973) had set up a search for a possible Mercury atmosphere of CO_2 and CO. They set up an upper limit 1.0×10^{-4} mb surface pressure for CO_2 and 2.0×10^{-5} mb for CO. Mariner 10's results also suggest that Mercury has no atmosphere although it may have a thin layer of He and other inert gas trapped by Mercury's magnetic field.

The optical observations of Mercury show that the integral spectral reflectivity of Mercury is quite similar to that for the integral moon (McCord and Adams, 1972). The Bond albedo of Mercury (de Vancouleurs, 1964) is 0.058.

In our model calculation we assume the following things. This model has been described by Murdock (1974):

1. The emission from the dark side at the phase angle we observed is negligible. On the day we made our observations, August 3, 1976, the illuminated portion amounted to 0.973 of the total disk and the dark portion was 0.017. The dark side temperature is about 110°K , which at $10\mu\text{m}$ has a flux $6.7181 \times 10^{-8} \text{ watt-cm}^{-2}\text{-}\mu^{-1}\text{-sr}^{-1}$. The flux for 500°K at $10\mu\text{m}$ is $7.1007 \times 10^{-3} \text{ watt-cm}^{-2}\text{-}\mu^{-1}\text{-sr}^{-1}$, so the contribution from the dark side

is negligible.

2. At infrared wavelengths, the radiation that reaches the observer originates very near the surface, so the infrared temperature is assumed equal to the insolation temperature. Soter and Ultichs' (1967) results show that the day time temperature is independent of the thermal properties of the surface material and determined largely by the insolation temperature.

3. The infrared emissivity we assumed was 0.9, which is the lunar value. We choose this value since Mercury's surface may be similar to the lunar surface.

In figure (6-15) we choose two coordinate systems on Mercury surface. The unprimed system is the "solar system" with the z-axis pointing towards the Sun. A is the subsolar point. The hemisphere above the plane BCD facing the Sun is the illuminated part. The primed system is the "earth system" with the z'-axis pointing towards the earth. A' is the subearth point. The hemisphere above the plane B'C'D' facing the earth is the portion that is being seen from earth. The two systems are different by an angle α with the x-axis as the common axis.

If the surface is in equilibrium with sunlight and cannot conduct heat away, the subsolar point temperature is:

$$T_o = \frac{S(1-A)}{\sigma \epsilon R^2} \quad 1/4 \quad (6-1)$$

where S is solar constant at earth, equal to $1.360 \times 10^6 \text{ erg cm}^{-2} \text{ s}^{-1}$

A is the Bond albedo for Mercury, assumed to be 0.058.

σ is the stefan-Boltzmann constant equal to $5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1}$

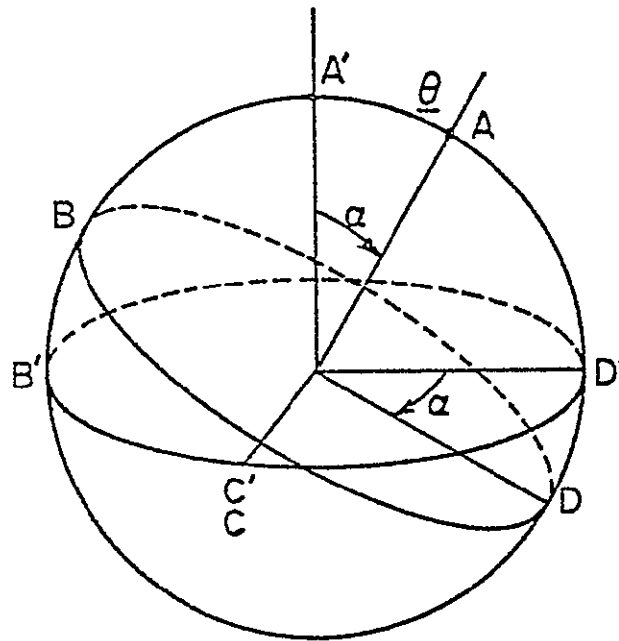


Figure 6-15.

Two coordinate systems on the surface of Mercury. The unprimed system is the "solar system" with the Z-axis pointing towards the Sun. A is the subsolar point. The primed system is the "earth system" with the Z'-axis pointing towards the earth. A' is the sub-earth point.

ϵ is the infrared emissivity at $10\mu\text{m}$ from the surface, assumed to be 0.9

R is the distance between Mercury and the Sun in astronomical units

Mercury's subsolar point temperature varies with distance from the Sun as $R^{-1/2}$ and therefore is a function of heliocentric longitude due to the eccentricity we discuss above.

In the unprimed system the temperature distribution on the surface will be concentric isothermal bands around the subsolar point. The temperature of a band at colatitude θ is given by

$$T(\theta) = T_0 \cos^{1/4} \theta \quad (6-2)$$

The total intensity at any wavelength region will be composed of the contributions at that wavelength from many isothermal regions each with its own apparent area.

Since one is interested in the flux coming to the earth one will have,

$$dF'_\lambda = I_\lambda(\theta', \phi') dA' \quad (6-3)$$

$$= I_\lambda(\theta', \phi') r^2 \sin\theta' d\theta' d\phi' \quad (6-4)$$

$$= \epsilon_\lambda(\theta', \phi') \cos\theta' r^2 \sin\theta' d\theta' d\phi' \quad (6-5)$$

where dF'_λ is the flux at wavelength, λ coming from the area dA'

I_λ is the intensity at (θ', ϕ')

dA' is the differential area on Mercury

r is the radius of Mercury

$\Xi_{\lambda}(\theta', \phi')$ = $I_{\lambda}(\theta', \phi')/\cos\theta'$ is the component of flux that radiates toward the earth.

$\Xi_{\lambda}(\theta', \phi')$ is a complicated function of (θ', ϕ') because the temperature distribution is a complicated function of (θ', ϕ') . However, one can convert the system to the "sun coordinate" where $\Xi_{\lambda}(\theta, \phi)$ is a simple function.

$$\text{Since } dA' = dA \quad (6-6)$$

$$\cos\theta' = \cos\theta\cos\alpha - \sin\theta\sin\phi\sin\alpha \quad (6-7)$$

equation(6-5) becomes

$$dF_{\lambda} = \Xi_{\lambda}(\theta) [\cos\theta\cos\alpha - \sin\theta\sin\phi\sin\alpha] r^2 \sin\theta d\theta d\phi$$

where

$$\Xi_{\lambda}(\theta) = \frac{C_1}{\lambda^5} \frac{1}{e^{C_2/kT(\theta)} - 1} \quad (6-8)$$

where $C_1 = 1.1909 \times 10^4 \text{ watt cm}^{-1} \mu^{-1} \text{sr}^{-1}$

$$C_2 = 1.4388 \times 10^4 \mu\text{K}$$

and $T(\theta)$ is given by (6-2)

so

$$F_{\lambda} = r^2 \int d\theta \int d\phi \Xi_{\lambda}(\theta) (\sin\theta\cos\theta\cos\alpha - \sin^2\theta\sin\phi\sin\alpha) \quad (6-9)$$

The integral (6-9) can be separated into two parts, for $\theta \leq \underline{\theta}$

$$\text{where } \underline{\theta} = \frac{\pi}{2} - \alpha \quad (6-10)$$

is the limit of the cap shared by both the Sun and the earth, i.e. the limit of integration over $d\theta$ can ϕ go from 0 to 2π . For $\theta > \underline{\theta}$, the limit of the integral over d is constrained by

the physical condition that some part on ϕ that is illuminated by the Sun can not be seen from the earth.

So (6-9) becomes

$$F_{\lambda} = r^2 \int_0^{\theta} d\theta \int_0^{2\pi} d\phi \epsilon_{\lambda}(\theta) (\sin\theta \cos\theta \cos\alpha - \sin^2\alpha \sin\phi \sin\alpha) \\ + r^2 \int_{\frac{\theta}{2}}^{\frac{\pi}{2}} d\theta \int_{\phi_1(\theta)}^{\phi_2(\theta)} d\phi \epsilon_{\lambda}(\theta) (\sin\theta \cos\theta \cos\alpha - \sin^2\theta \sin\phi \sin\alpha) \quad (6-11)$$

To find $\phi_1(\theta)$ and $\phi_2(\theta)$ one notices that ϕ is given when $\theta' = 90^\circ$.

From (6-7)

$$\cos\theta' = \cos\theta \cos\alpha - \sin\theta \sin\phi \sin\alpha \\ \theta' = \frac{\pi}{2} \Rightarrow \sin\phi = \cot\alpha \cot\theta \quad (6-12)$$

and

$$\phi = \sin^{-1}(\cot\alpha \cot\theta) \quad (6-13)$$

Since ϕ will also be symmetric about the y-axis, equation (6-11) can be rewritten as:

$$F_{\lambda} = r^2 \int_0^{\frac{\pi}{2}} d\theta \int_0^{2\pi} d\phi \epsilon_{\lambda}(\theta) (\sin\theta \cos\theta \cos\alpha - \sin^2\theta \sin\phi \sin\alpha) \\ + \int_{\frac{\pi}{2}}^{\frac{\pi}{2} - \alpha} d\theta \int_{\pi - \sin^{-1}(\cot\alpha \cot\theta)}^{2\pi + \sin^{-1}(\cot\alpha \cot\theta)} d\phi \epsilon_{\lambda}(\theta) (\sin\theta \cos\theta \cos\alpha - \sin^2\theta \sin\phi \sin\alpha) \quad (6-14)$$

After evaluating the integral one obtains the following:

$$F_{\lambda} = 2\pi r^2 \cos\alpha \left[\epsilon_{\lambda}(\theta) \frac{\sin^2\theta}{2} \right]_0^{\frac{\pi}{2} - \alpha} + \pi r^2 \cos\alpha \left[\epsilon_{\lambda}(\theta) \frac{\sin^2\theta}{2} \right]_{\frac{\pi}{2}}^{\frac{\pi}{2} - \alpha}$$

$$\begin{aligned}
& + 2r^2 \int_{\frac{\pi}{2} - \alpha}^{\frac{\pi}{2}} d\theta \sin^{-1}(\cot\alpha \cot\theta) \sin\theta \cos\theta \cos\alpha \Xi_{\lambda}(\theta) \\
& + r^2 \left[\frac{\Xi_{\lambda}(\theta) (\cos\theta / \sin^{-2}\alpha - \cos^2\theta + \sin^{-2}\alpha \sin^{-1} \frac{\cos\theta}{|\sin\alpha|})}{2} \right]_{\frac{\pi}{2}}^{\frac{\pi}{2} - \alpha}
\end{aligned}
\tag{6-15}$$

where

$$\left[\frac{\Xi_{\lambda}(\theta) \frac{\sin^2\theta}{2}}{2} \right]_{\frac{\pi}{2}}^{\frac{\pi}{2} - \alpha}$$

is the mean of $(\Xi_{\lambda}(\theta) \frac{\sin^2\theta}{2})$ in the interval of θ from 0 to $\frac{\pi}{2} - \alpha$. The same meaning applies to the third term of (6-15).

As a check of equation (6-15), if $\alpha=0$, that means when subsolar point and subearth point coincide, let $\Xi(\theta)=\text{constant}$ evaluating (6-15) gives:

$$F = \pi r^2 \Xi$$

If $\alpha = \frac{\pi}{2}$, that means the subsolar point and subearth point are 90° apart. With $\Xi(\theta)$ assumed to be constant, equation (6-15) gives

$$F = \frac{\pi r^2}{2} \Xi$$

which is as one expects since one is seeing half of Mercury.

Equation (6-15) can be readily integrated on a computer.

It is applied to our case with the following physical parameters:

Date: August 3, 1976 .

Phase Angle: 53°

Radius vector: 0.414 A.U.

Orbital longitude: 198.09°

Mercury perihelion point: 0.3075 A.U.

Subsolar temperature at perihelion point: 700°K

Subsolar temperature at $\alpha=53^\circ$: 603°K

Equation (6-15) was computed on a LSI mini computer at each wavelength, from 10.6 to 13.2 μ m. The program and the result are shown in the Appendix D. The integral was divided into twenty steps. $\epsilon(\theta)$ was evaluated from equation (6-8)

$$\epsilon_{\lambda}(\theta) = \frac{C_1}{\lambda^5} \frac{1}{e^{C_2/kT(\theta)} - 1}$$

where $T(\theta) = T_0 \cos^{1/4} \theta$

The calculated spectrum is a measure of color temperature and is used to compare with the observed spectrum. Figure (6-16) shows the final Mercury spectrum connected for solar temperature, with a number of blackbody slopes shown to match. The calculated spectrum (cross) matches the blackbody temperature 525°K, which also matches the observed spectrum. We concluded that the best fit lies in the 500°K region. Murdock (1974) measured a effective brightness temperature at 10.8 μ m at the same phase angle to be around 650°K. Our results disagree with his results.

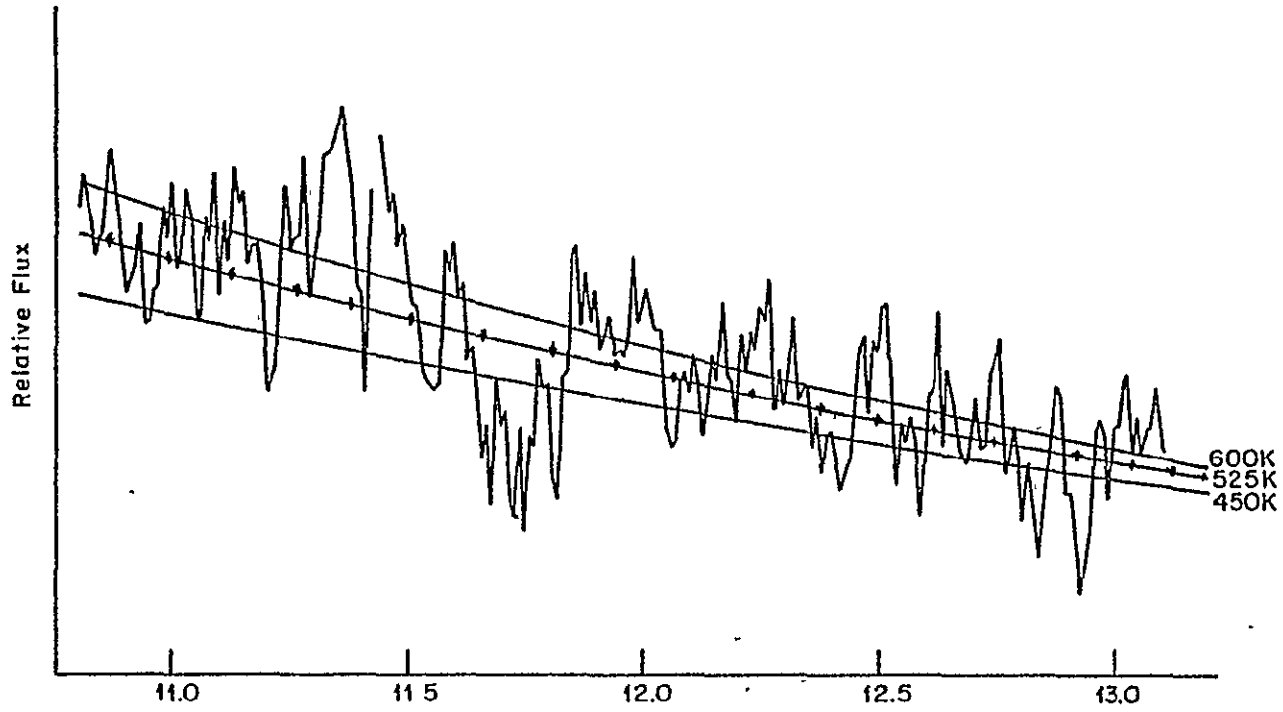


Figure 6-16. The final Mercury spectrum, corrected for solar temperature, with a number of blackbody slopes shown to match.

APPENDIX A
ESTIMATE OF CODING ERROR FOR FOURIER
TRANSFORM SPECTROMETRY

Let ξ be the path difference in a two beam interferometer. $P(\nu)$ is the power at wavelength ν , (i.e. the spectral density function) ν here is taken to be $1/\lambda$. $S(\xi)$ is the power received for path difference ξ . Then (p.96 Stewart)

$$S(\xi) = \int_0^{\infty} P(\nu) \cos^2(2\pi\xi\nu) d\nu \quad \text{A-1}$$

$$= 1/2P_0 + 1/2 \int_0^{\infty} P(\nu) \cos(4\pi\xi\nu) d\nu \quad \text{A-2}$$

The reciprocal Fourier property is (Morse and Feshbach P.454) that if

$$F(\xi) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \cos(\xi\nu) f(\nu) d\nu$$

Then $f(\nu) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \cos(\xi\nu) F(\xi) d\xi$

which implies, neglecting the constant term, that

$$P(\nu) = 16 \int_0^{\infty} \cos(4\pi\xi\nu) s(\xi) d\xi \quad \text{A-3}$$

Suppose we take measurement at $(N+1)$ equally seperated steps in the variable ξ . Let the step length be τ , then

$$\xi = n\tau$$

In general τ is chosen such that

$$\tau = \frac{1}{4(\nu_{\max} - \nu_{\min})} \quad \text{A-4}$$

which, if $\nu_{\min} \ll \nu_{\max}$, effectively implies sampling twice per cycle (Stewart, p.303).

One can now write the integral for $P(\nu)$ as

$$P(\nu) = \frac{1}{16} \sum_{n=0}^N \tau S(n\tau) \cos(4\pi\nu n\tau) \quad \text{A-5}$$

Now consider frequency $\nu = \nu_{\min} + m\delta$

$$\text{where } \delta = \frac{\nu_{\max} - \nu_{\min}}{N} \quad \text{A-6}$$

and m is an integer. $m=0, 1, \dots, N$

The $\delta P(\nu)$ is the power in the resolved spectral band width δ about frequency ν

$$\delta P(\nu) = \frac{1}{16} \tau \delta \sum_{n=0}^N S(n\tau) \cos(4\pi\nu n\tau) \quad \text{A-7}$$

$$\begin{aligned} \text{but } \tau \delta &= \frac{\delta}{4(\nu_{\max} - \nu_{\min})} \\ &= \frac{\delta}{4N\delta} \\ &= \frac{1}{4N} \end{aligned} \quad \text{A-8}$$

Therefore

$$\delta P(v) = \frac{4}{N} \sum_{n=0}^N S(n\tau) \cos(4\pi v n \tau) \quad \text{A-9}$$

Writing this out in matrix form, with $\theta_1 = 4\pi\tau v_{\min}$, $\theta_2 = 4\pi\tau(v_{\min} + \delta)$ and with $\pi(v_{\min}) \equiv \delta P(v_{\min})$ we have

$$\begin{bmatrix} \pi(v_{\min}) \\ \pi(v_{\min} + \delta) \\ \vdots \\ \pi(v_{\max} - \delta) \\ \pi(v_{\max}) \end{bmatrix} = \delta \begin{bmatrix} P(v_{\min}) \\ P(v_{\min} + \delta) \\ \vdots \\ P(v_{\max} - \delta) \\ P(v_{\max}) \end{bmatrix} - \frac{4}{N} \begin{bmatrix} 1 & \cos\theta_1 & \cdots & \cos N\theta_1 \\ 1 & \cos\theta_2 & \cdots & \cos N\theta_2 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \cos\theta_{N-1} & \cdots & \cos N\theta_{N-1} \\ 1 & \cos\theta_N & \cdots & \cos N\theta_N \end{bmatrix} \begin{bmatrix} S(0) \\ S(1) \\ \vdots \\ S(N) \end{bmatrix}$$

$$\text{or } \pi = W^{-1}S \quad \text{A-10}$$

From (2-8)

$$\begin{aligned} \Delta_j &= (A_{j1} + \cdots + A_{jN})^{1/2} \\ &= \sqrt{\frac{16}{N^2}} (1 + \cos^2\theta_j + \cdots + \cos^2N\theta_j)^{1/2} \\ &= \frac{4}{N} \left(\sum_{n=0}^N \cos^2 n\theta_j \right)^{1/2} \\ &\sim \frac{4}{N} \sqrt{\frac{N+1}{2}} \\ &\sim \sqrt{\frac{8}{N}} \quad \text{for } N \gg 1 \quad \text{A-11} \end{aligned}$$

The SNR improvement is the reciprocal of this quantity.

APPENDIX B

0001			NAM	CMND	
0002			EXTR	IER, ØTT, ØTL, CRLF	
0003			EXTR	ERR, XEQ, ØFPA	
0004			EXTR	XA, XB, FIX, FLT	
0005	0100		ABS	: 100	
0006			*PRINT	NAMÉ	
0007	0100	0110	ZAR		
0008	0101	F900	JST	ØTL	
		0000			
0009	0102	017B	DATA	NAME	
0010	0103	F201	JMP	S+2	
0011			*		
0012			*COMMAND	INTERPRETER	
0013			*		
0014	0104	0300	CMND	ENT	
0015	0105	0A00		EIN	
0016	0106	4005		CIE	ENABLE PANIC BUTTON
0017	0107	F900	JST	CRLF	PRINT PROMPT CHARACTER
		0000			
0018	0108	C687	LAP	: 87	
0019	0109	F900	JST	ØTT	
		0000			
0020	010A	C6AA	LAP	'*'	
0021	010B	F900	JST	ØTT	
		0000			
0022	010C	1200	RØV		CLEAR MATH FLAG
0023	010D	0108	ZXR		
0024	010E	F900	JST	IER	INPUT COMMAND
		0000			
0025	010F	C0D3	CAI	'S'	
0026	0110	F25C	JMP	STATUS	
0027	0111	C0D4	CAI	'T'	
0028	0112	E24E	L DX	TRANS	
0029	0113	C0D2	CAI	'R'	
0030	0114	E24D	L DX	READ	
0031	0115	C0C3	CAI	'C'	
0032	0116	E24C	L DX	CLÉAR	
0033	0117	3809	JXN	ØKF	
0034	0118	1400	SØV		SET MATH FLAG
0035	0119	C0D0	CAI	'P'	
0036	011A	E249	L DX	PUNCH	
0037	011B	C0C7	CAI	'G'	
0038	011C	E248	L DX	GRAPH	
0039	011D	C0D6	CAI	'V'	
0040	011E	E249	L DX	CRT	
0041	011F	3801	JXN	ØKF	
0042	0120	F900	BAD JST	ERR	
		0000			
0043	0121	EA48	ØKF STX	FUNC	SAVE CALL ADDRESS
0044	0122	0108	ZXR		
0045	0123	F900	JST	IER	FETCH BEAM
		0000			
0046	0124	COAB	CAI	'+'	

0047	0125	E24C	LDX	NP	
0048	0126	CUAD	CAI	'-'	
0049	0127	E24F	LDX	NM	
0050	0128	33 35	JXN	ØKB	
0051	0129	3249	JØR	BAD	IF MATH FLAG CLEAR
0052	012A	0523	XRP		RESET INDICES
0053	012B	E900	STX	XA	
		0000			
0054	012C	E900	STX	XB	
		0000			
0055	012D	EB1D	STX	*XC	
0056	012E	C5FF	LXM	255	RESET COUNT
0057	012F	EA3B	STX	CNT	
0058	0130	0108	ZXR		
0059	0131	C0C4	CAI	'D'	
0060	0132	E233	LDX	F5E	
0061	0133	C0D2	CAI	'R'	
0062	0134	E232	LDX	FDV	
0063	0135	28 55	JXZ	BAD	
0064	0136	EA35	STX	MATH	SAVE MATH POINTER
0065	0137	F900	JST	XEQ	WAIT FOR GO
		0000			
0066			*MATH LOOP		
0067	0138	FB33	MLP JST	*MATH	
0068	0139	9002	DATA	:9002, :9202, :9402	
	013A	9202			
	013B	9402			
0069	013C	D900	IMS	XA	BUMP COUNTERS
		0000			
0070	013D	D900	IMS	XB	
		0000			
0071	013E	211A	JAZ	ØKM	MATH ØK?
0072	013F	1340	SAØ		NØ!
0073	0140	C6D2	LAP	'R'	PRINT ERROR MSG.
0074	0141	3203	JØR	ØVFL	
0075	0142	F900	JST	ØTL	
		0000			
0076	0143	0134	DATA	UNDER	
0077	0144	F202	JMP	FLØW	
0078	0145	F900	ØVFL JST	ØTL	
		0000			
0079	0146	0137	DATA	ØVER	
0080	0147	C600	FLØW LAP	Ø	
0081	0148	F900	JST	ØTL	
		0000			
0082	0149	013A	DATA	MSG	
0083	014A	F900	JST	FLT	
		0000			
0084	014B		XC REF		
0085	014C	0179	DATA	XCF	
0086	014D	F900	JST	ØFPA	
		0000			
0087	014E	0179	DATA	XCF	

0088	014F	C68D		LAP	:8D	
0089	0150	F900		JST	ØTL	
		0000				
0090	0151	0197		DATA	ZERØ	
0091	0152	B707		LDA	*XC	CLEAR BAD ELEMENT
0092	0153	1050		ALA	1	
0093	0154	8A14		ADD	TP	
0094	0155	0043		TAX		
0095	0156	0110		ZAR		
0096	0157	9C00		STA	@0	
0097	0158	9C01		STA	@1	
0098	0159	DFOE	ØKM	IMS	*XC	
0099	015A	DA10		IMS	CNT	
0100	015B	F623		JMP	MLP	
0101	015C	E20C		LDX	TP	
0102	015D	F201		JMP	ØKB+1	
0103	015E	F900	ØKB	JST	XEQ	WAIT FOR GØ
		0000				
0104	015F	FBOA		JST	*FUNC	CALL FUNCTION
0105	0160	F65B		JMP	CMND+1	
0106	0161		TRANS	REF		
0107	0162		READ	REF		
0108	0163		CLEAR	REF		
0109	0164		PUNCH	REF		
0110	0165		GRAPH	REF		
0111	0166		FSB	REF		
0112	0167		FDV	REF		
0113	0168		CRT	REF		
0114	0169	1400	TP	DATA	:1400	
0115	016A	0000	FUNC	DATA	0	
0116	016B	0000	CNT	DATA	0	
0117	016C	0000	MATH	DATA	0	
0118	016D	4006	STATUS	CID		KILL PANIC SWITCH
0119	016E	0110		ZAR		
0120	016F	F900		JST	ØTL	
		0000				
0121	0170	01A1		DATA	TATUS	
0122	0171	F900		JST	ØFPA	
		0000				
0123	0172	1000	NP	DATA	:1000	
0124	0173	0110		ZAR		
0125	0174	F900		JST	ØTL	
		0000				
0126	0175	01A7		DATA	NMM	
0127	0176	F900		JST	ØFPA	
		0000				
0128	0177	1200	NM	DATA	:1200	
0129	0178	F673		JMP	CMND+1	
0130	0179	0000	XCF	RES	2,0	
0131	017B	8D3A	NAME	DATA	:8D3A, :8AA0	
		017C				
0132	017D	C8D4		TEXT	'HTS: IX255'	
		017E				
		D3BA				

	017F	A0B1			
	0180	D3B2			
	0181	B5B5			
0133	0182	8D3A		DATA	: 8D3A, 0
	0183	0000			
0134	0184	8AD5	UNDER	DATA	: 8AD5
0135	0185	CEC4		TEXT	'NDER'
	0186	C5D2			~
0136	0187	8ACF	ØVER	DATA	: 8ACF
0137	0188	D6C5		TEXT	'VER'
	0189	D2A0			~
0138	018A	C6CC	MSG	TEXT	'FLOW ØCCURRED'
	018B	CFD7			~
	018C	A0CF			
	018D	C3C3			
	018E	D5D2			
	018F	D2C5			
	0190	C4A0			
0139	0191	C1D4		TEXT	'AT ELEMENT'
	0192	A0C5			~
	0193	CCC5			
	0194	CDC5			
	0195	CEA4			
0140	0196	A000		DATA	: A000
0141	0197	ACA0	ZERØ	TEXT	', REPLACED'
	0198	D2C5			~
	0199	D0CC			
	019A	C1C3			
	019B	C5C4			
0142	019C	A0C2		TEXT	' BY ZERØ'
	019D	D9A0			~
	019E	DAC5			
	019F	D2CF			
0143	01A0	AE3D		DATA	: AE3D
0144	01A1	D4C1	TATUS	TEXT	'TATUS: N+='
	01A2	D4D5			~
	01A3	D3BA			
	01A4	A0CE			
	01A5	ABBD			
0145	01A6	A000		DATA	: A000
0146	01A7	ACA0	NMM	TEXT	', N-= '
	01A8	CEAD			~
	01A9	BDA0			
0147	01AA	0000		DATA	0
0148				END	
0000	ERRORS				

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0001          NAM  TRANS
0002          NAM  RTTR, RTOP
0003          EXTR DI SP
0004          EXTR IKB, ØTL, ØFPA, CRL F, ERR
0005          EXTR DPACC, DPFLT
0006          EXTR DPN:, DPSUB:, DPDI V:, DPSM:
0007          EXTR XB, XC, FAD
0008          EXTR DPFIX
0009 0000     REL  0
0010
0011          *
0011          * IX255 HTS PRØCESSØR
0012          *
0013 0000 0800 TRANS ENT
0014 0001 E2BC      LDX  BPNT
0015 0002 1328      LLX  1          INDIRECT BIT ØN
0016 0003 1400      SØV  .
0017 0004 11A8      RFX  1
0018 0005 EA88      STX  BEAM
0019 0006 EA88      STX  BEAM+1
0020 0007 B2A0      LDA  Z CT          CLEAR BUFFERS
0021 0008 9AA0      STA  CNT
0022 0009 E2A1      LDX  NT
0023 000A 0110      ZAR
0024 000B 9C00      CLR  STA  ø0
0025 000C 0128      IXR
0026 000D DA9B      IMS  CNT
0027 000E F603      JMP  CLR
0028 000F C6BF      LAP  '?'
0029 0010 F900      JST  ØTL
0030 0011 0137      DATA MØDE
0031 0012 F900      JST  IKB
0032 0013 COCD      CAI  'M'
0033 0014 F205      JMP  MØN
0034 0015 COD4      CAI  'T'
0035 0016 F208      JMP  TAPE
0036 0017 COC9      CAI  'I'
0037 0018 F209      JMP  INVS
0038 0019 F900      JST  ERR
0039 001A 0110      MØN  ZAR
0040 001B 0210      CAR
0041 001C 9AA0      STA  TETR
0042 001D 0110      ZAR
0043 001E F20D      JMP  NEXT
0044 001F 0110      TAPE ZAR
0045 0020 9A9C      STA  TETR
0046 0021 F202      JMP  T4
0047 0022 C601      INVS LAP  1
0048 0023 9A99      STA  TETR
0049 0024 0108      T4  ZXR
0050 0025 F900      JST  IKB
0051 0026 COAB      CAI  '+'
0052 0027 0408      CXR
0053 0028 COAD      CAI  '-'
0054 0029 0108      ZXR

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0055	002A	EAE E		STX	TREM	
0056	002B	0110		ZAR		
0057	002C	E292	NEXT	L DX	MASK	
0058	002D	0128		IXR		
0059	002E	0210		CAR		CHANGE DIRECTION
0060	002F	9A7F		STA	DIR	
0061	0030	C0FF		CAI	: FF	
0062	0031	F203		JMP	F0WD	
0063	0032	0030		TXA		
0064	0033	8A7A		ADD	H255	
0065	0034	0048		TAX		
0066	0035	EA76	F0WD	STX	MSK 1	
0067	0036	C7F5		LAM	245	
0068	0037	9A79		STA	DCT2	
0069	0038	C7FF		LAM	255	START DATA COUNT
0070	0039	9A76		STA	DCT	
0071	003A	FAB1		JST	TURNON	WAIT FOR LIGHT
0072			*INPUT LOOP			
0073	003B	FABB	ILP	JST	M0NS	INPUT
0074	003C	9A6C		STA	CNT	&SAVE
0075	003D	EA6C		STX	CNT+1	
0076	003E	C7FF		LAM	255	SPECTRUM COUNT
0077	003F	9A72		STA	SCT	
0078	0040	B26B		LDA	MSK 1	RESET MASK POINTER
0079	0041	9A6B		STA	MSK 2	
0080	0042	B271		LDA	IB	GET BUFFER POINTER
0081	0043	9A0F		STA	IBP	
0082			*TRANSFER COLUMN TO			INPUT BUFFER
0083	0044	E265	TLP	L DX	CNT+1	
0084	0045	0110		ZAR		
0085	0046	D366		IMS	*MSK 2	
0086	0047	F208		JMP	T1A	
0087	0048	B274		LDA	TETR	
0088	0049	2183		JAL	5+4	
0089	004A	0110		ZAR		
0090	004B	0108		ZXR		
0091	004C	F204		JMP	T1	
0092	004D	B25B		LDA	CNT	
0093	004E	F900		JST	DPN:	
0094	004F	F201		JMP	T1	
0095	0050	B258	T1A	LDA	CNT	
0096	0051	DA5B	T1	IMS	MSK 2	
0097	0052	F900		JST	DPACC	ADD
0098	0053	0000	IBP	DATA	0	
0099	0054	C202		AXI	2	BUMP POINTER
0100	0055	EE02		STX	IBP	
0101	0056	DA5B		IMS	SCT	DONE?
0102	0057	F613		JMP	TLP	NØ
0103	0058	E253		L DX	MSK 1	YES, MOVE COLUMN
0104	0059	0128		IXR		+ IIF FWD
0105	005A	B254		LDA	DIR	
0106	005B	C000		CAI	0	
0107	005C	C302		SXI	2	- IIF REV.
0108	005D	EA4E		STX	MSK 1	

0109	005E	DA52		IMS	DCT2	
0110	005F	F201		JMP	S+2	
0111	0060	40C7		SEL	24,7	
0112	0061	DA4E		IMS	DCT	END OF ROW?
0113	0062	F627		JMP	ILP	NO
0114			*END OF ROW	STOP	TEST	
0115	0063	B259		LDA	TETR	
0116	0064	2081		JAM	S+2	
0117	0065	F209		JMP	STOP	
0118	0066	3408		JSS	STOP	SS DOWN?
0119	0067	48C7		SSN	:C7	NO, STOP DOWN?
0120	0068	F206		JMP	STOP	YES
0121	0069	FA56		JST	ACC	NO, SAVE DATA
0122	006A	E24D		LDX	NØEM	
0123	006B	0110		ZAR		
0124	006C	F100		JMP	DISP	
0125	006D	B241	RTTR	LDA	DIR	
0126	006E	F642		JMP	NEXT	
0127	006F	E243	STOP	LDX	NI	
0128	0070	C601		LAP	1	
0129	0071	F100		JMP	DISP	
0130	0072	C6BF	RTOP	LAP	'?'	
0131	0073	F900		JST	ØTL	
0132	0074	011F		DATA	MSG	
0133	0075	F900		JST	IKB	
0134	0076	COCE		CAI	'N'	ABORT LAST PASS?
0135	0077	F203		JMP	NØ	NO
0136	0078	COD9		CAI	'Y'	YES
0137	0079	F203		JMP	YES	YES
0138	007A	F608		JMP	RTOP	WRONG ENTRY
0139	007B	FA44	NØ	JST	ACC	SAVE DATA
0140	007C	F203		JMP	Y2	
0141	007D	C6AC	YES	LAP	','	PRINT ABORT
0142	007E	F900		JST	ØTL	
0143	007F	0128		DATA	AET	
0144	0080	B234	Y2	LDA	CT	SET COUNTER
0145	0081	9A30		STA	SCT	
0146	0082	0350		ARP		RESET SUBSCRIPTS
0147	0083	9900		STA	XB	
0148	0084	9900		STA	XC	
0149	0085	E230		LDX	NPEM	
0150			*ADD TO BEAM	ARRAY		
0151	0086	EE33	ALP	STX	IBP	
0152	0087	B400		LDA	Ø0	GET DATA
0153	0088	E401		LDX	Ø1	
0154	0089	F900		JST	DPFLT	FLOAT IT
0155	008A	9A21		STA	MSK1	SAVE IT
0156	008B	EA21		STX	MSK2	
0157	008C	F900		JST	FAD	ADD TO BEAM
0158	008D	00AC		DATA	MSK1	
0159	008E	0000	BEAM	RES	2,0	
0160	0090	D900		IMS	XB	BUMP SUBSCRIPTS
0161	0091	D900		IMS	XC	
0162	0092	E63F		LDX	IBP	

0163	0093	C202		AXI	2	
0164	0094	DA1D		IMS	SCT	DONE?
0165	0095	F60F		JMP	ALP	NO
0166			*PRINT	PASS	COUNT	
0167	0096	F900		JST	CRLF	
0168	0097	C702		LAM	2	
0169	0098	9A23		STA	CEN1	
0170	0099	E21C		LDX	NPEM	
0171	009A	0110	MK	ZAR		
0172	009B	E401		LDX	01	
0173	009C	F900		JST	DPFLT	
0174	009D	9A0E		STA	MSK1	
0175	009E	EA0E		STX	MSK2	
0176	009F	F900		JST	0FPA	
0177	00A0	00AC		DATA	MSK1	
0178	00A1	0110		ZAR		
0179	00A2	F900		JST	0TL	
0180	00A3	0130		DATA	RUNS	
0181	00A4	E212		LDX	NMEM	
0182	00A5	DA16		IMS	CEN1	
0183	00A6	F60C		JMP	MK	
0184	00A7	F7A7	T2	RTN	TRANS	
0185			*DATA	STORAGE		
0186	00A8	FA00	ZCT	DATA	-1536	
0187	00A9	0000	CNT	DATA	0,0	
	00AA	0000				
0188	00AB	1400	NT	DATA	:1400	
0189	00AC	0000	MSK1	DATA	0	
0190	00AD	0000	MSK2	DATA	0	
0191	00AE	00FF	H255	DATA	255	
0192	00AF	0000	DIR	DATA	0	
0193	00E0	0000	DCT	DATA	0	
0194	00B1	0000	DCT2	DATA	0	
0195	00B2	0000	SCT	DATA	0	
0196	00B3	1800	NI	DATA	:1800	
0197	00B4	1802	IB	DATA	:1802	
0198	00B5	FE00	CT	DATA	-512	
0199	00B6	1400	NPEM	DATA	:1400	
0200	00B7	1600	NMEM	DATA	:1600	
0201	00B8	0000	N0EM	DATA	0	
0202	00B9	0000	TREM	DATA	0	
0203	00BA	9002	FLS	DATA	:9002	
0204	00BB	9202	MINS	DATA	:9202	
0205	00BC	0000	CEN1	DATA	0	
0206	00BD	0000	TETR	DATA	0	
0207	00BE	1000	BPNT	DATA	:1000	
0208	00BF		MASK	REF		
0209			*			
0210			*ADD	INPUT	ARRAY	TO
0211			*			
0212	00C0	0800	ACC	ENT		
0213	00C1	B604		LDA	TETR	
0214	00C2	2083		JAM	S+4	
0215	00C3	B60A		LDA	TREM	

0216	00C4	0210		CAR		
0217	00C5	F201		JMP	S+2	
0218	00C6	B617		LDA	DIR	
0219	00C7	C000		CAI	0	
0220	00C8	F202		JMP	S+3	
0221	00C9	E613		LDX	NPEM	
0222	00CA	F201		JMP	S+2	
0223	00CB	E614		LDX	NMEM	
0224	00CC	EE14		STX	NØEM	
0225	00CD	DC01		IMS	@1	BUMP CØUNTER
0226	00CE	C202		AXI	2	
0227	00CF	EA12		STX	TBP	SAVE TEMP PØINTER
0228	00DD	C7FF		LAM	255	DATA CØUNT
0229	00D1	9E28		STA	CNT	
0230	00D2	E61F		LDX	NI	INPUT PØINTER
0231	00D3	C202	A1	AXI	2	BUMP CØUNTER
0232	00D4	EE81		STX	IBP	+SAVE
0233	00D5	B618		LDA	TETR	
0234	00D6	2188		JAL	T5	
0235	00D7	B400		LDA	@0	
0236	00D8	E401		LDX	@1	
0237	00D9	1328		LLX	1	
0238	00DA	1B86		LLR	7	
0239	00DB	13A8		LRX	1	
0240	00DC	1356		LLA	7	
0241	00DD	10D6		ARA	7	
0242	00DE	F202		JMP	T6	
0243	00DF	B400	T5	LDA	@0	
0244	00E0	E401		LDX	@1	
0245	00E1	F900	T6	JST	DPACC	
0246	00E2	0000	TBP	DATA	0	
0247	00E3	C202		AXI	2	BUMP TEMP
0248	00E4	EE02		STX	TBP	
0249	00E5	E692		LDX	IBP	
0250	00E6	0110		ZAR		
0251	00E7	9C00		STA	@0	
0252	00E8	9C01		STA	@1	
0253	00E9	DE40		IMS	CNT	DØNE?
0254	00EA	F617		JMP	A1	NØ
0255	00EB	F72B		RTN	ACC	
0256			*			
0257			*WAIT. FØR ALIGNMENT PULSE			
0258			*			
0259	00EC	0800	TURNØN	ENT		
0260	00ED	4006		CID		DISAÆLE AUTO
0261	00EE	48C1		SSN	: C1	LIGHT ØFF?
0262	00EF	F601		JMP	S-1	NØ
0263	00F0	0E00		SEN	..	YES BYTE ØN
0264	00F1	49C1		SEN	: C1	LIGHT ØN?
0265	00F2	F601		JMP	S-1	NØ
0266	00F3	0F00		SWM	..	YES, BYTE ØFF
0267	00F4	40C4		SEL	: C4	CLEAR FLAG
0268	00F5	4005		CIE		ENABÆLE AUTO
0269	00F6	F70A		RTN	TURNØN	

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0270
0271      *
0272      *INPUT FROM MØNSANTØ
0273      *  CØNVERT BCD TØ BINARY
0274      *
0274  00F7  0800  MØNS  ENT
0275  00F8  B63B          LDA  TETR
0276  00F9  208F          JAM  T3
0277  00FA  C6FF          LAP  255
0278  00FB  8E4B          ADD  DCT
0279  00FC  1050          ALA  1
0280  00FD  E644          LDX  TREM
0281  00FE  3802          JXN  S+3
0282  00FF  8A08          ADD  DMTR
0283  0100  F201          JMP  S+2
0284  0101  8A05          ADD  DPTR
0285  0102  0048          TAX
0286  0103  B400          LDA  @0
0287  0104  E401          LDX  @1
0288  0105  F900          JST  DPFIX
0289  0106  F70F          RTN  MØNS
0290  0107  1002  DPTR  DATA : 1002
0291  0108  1202  DMTR  DATA : 1202
0292  0109  49C6  T3    SEN  : C6      FLAG
0293  010A  F601          JMP  S-1      NØ
0294  010B  5AC6          INX  : C6      YES, INPUT TØ X
0295  010C  2843          JXZ  S-3      IGNØRE ZEROES
0296  010D  C704          LAM  4        SET DIGIT CØUNT
0297  010E  9E65          STA  .CNT
0298  010F  0110          ZAR          CLEAR TALLY—
0299  0110  9E63          STA  MSK2
0300  0111  F206          JMP  M2
0301  0112  1350  M1    LLA  1        X10
0302  0113  9E66          STA  MSK2
0303  0114  1351          LLA  2
0304  0115  8E68          ADD  MSK2
0305  0116  9E69          STA  MSK2
0306  0117  0110          ZAR
0307  0118  1B03  M2    LLL  4        GET NEXT DIGIT
0308  0119  8E6C          ADD  MSK2      ADD TØ TALLY
0309  011A  DE71          IMS  CNT      LAST DIGIT?
0310  011B  F609          JMP  M1        NØ
0311  011C  0048          TAX
0312  011D  0110          ZAR
0313  011E  F727          RTN  MØNS      YES
0314          *TEXT STØRAGE
0315  011F  8D3A  MSG   DATA : 8D3A
0316  0120  C1C2          TEXT 'ABØRT LAST RUN?'
      0121  CFD2
      0122  D4A0
      0123  CCC1
      0124  D3D4
      0125  A0D2
      0126  D5CE
      0127  BFA0

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0317 0128 8D8A ABT DATA :8D8A
0318 0129 B1A0 TEXT '1 RUN ABORTED, '
      012A D2D5
      012B CEA0
      012C C1C2
      012D CFD2
      012E D4C5
      012F C4AC
0319 0130 A0D2 RUNS TEXT ' RUNS KEPT'
      0131 D5CE
      0132 D3A0
      0133 CBC5
      0134 D0D4
0320 0135 8D8A DATA :8D8A, 0
      0136 0000
0321 0137 8D8A MØDE DATA :8D8A
0322 0138 CDCF TEXT 'MØNSANTØ, TAPE ØR INVERSE?'
      0139 CED3
      013A C1CE
      013B D4CF
      013C ACD4
      013D C1D0
      013E C5A0
      013F CFD2
      0140 A0C9
      0141 CED6
      0142 C5D2
      0143 D3C5
      0144 BFA0
0323 END
0000 ERRØRS

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0001          NAM      1KB, I ER, BIPT, XEQ
0002          NAM      ØTT, ERR, CRL F
0003          NAM      ØTL, ØFPA
0004 0000      REL      0
0005          *
0006          * PANIC SWITCH
0007          *
0008 0000  FB00      J ST      *CMND
0009 0001          CMND      REF
0010          *
0011          *KEYBOARD INPUT
0012          *
0013 0002  08 00    IKB      ENT
0014 0003  4038          SEL      7, 0      AUTO-ECHØ
0015 0004  4039          SEL      7, 1      KBD MØDE
0016 0005  59 39          RDA      7, 1      READ ØN FLAG
0017 0006  403C          SEL      7, 4      RESET
0018 0007  F705          RTN      IKB
0019 0008  08 00    I ER      ENT
0020 0009  FE07          J ST      IKB
0021 000A  C0DF          CAI      :DF      BACK ARROW?
0022 000B  FFOA          J ST      *CMND      YES
0023 000C  C08 A          CAI      :8 A      LINE FEED?
0024 000D  FF0C          J ST      *CMND      YES
0025 000E  F706          RTN      I ER      NEITHER, RETURN
0026          *
0027          *PAPER TAPE INPUT
0028          * DSO = 0 FOR TTY
0029          *      1 FOR HSR
0030          *
0031 000F  08 00    BIPT      ENT
0032 0010  58 01    BIP2      ISA      READ SWITCHES
0033 0011  13D0          LRA      1      DSO UP FOR TTY
0034 0012  220A          JØS      HSR      DØWN FOR HSR
0035 0013  493B          SEN      7, 3      TTY BUSY?
0036 0014  F604          JNP      BIP2      YES
0037 0015  403A          SEL      7, 2      NØ, STEP READER
0038 0016  48 39    WT      SSN      7, 1      FLAG?
0039 0017  F203          JMP      IT      YES
0040 0018  0150          IAR
0041 0019  2149          JAZ      BIP2      NØ, BUMP CØUNT
0042 001A  F604          JMP      WT      RESTART IF TIME UP
0043 001B  58 38    IT      INA      7, 0      ELSE CHECK FLAG AGAIN
0044 001C  F70D          RTN      BIPT      INPUT FROM TTY
0045 001D  49 33    HSR      SEN      6, 3      & RETURN
0046 001E  F60E          JMP      BIP2      HSR BUSY?
0047 001F  4032          SEL      6, 2      YES
0048 0020  48 35    WH      SSN      6, 5      NØ, STEP READER
0049 0021  F203          JMP      IH      FLAG?
0050 0022  0150          IAR      YES
0051 0023  2153          JAZ      BIP2      NØ, BUMP CØUNT
0052 0024  F604          JMP      WH      RESTART IF TIME UP
0053 0025  58 35    IH      INA      6, 5      ELSE CHECK FLAG AGAIN
          INPUT FROM HSR

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0054 0026 F717      RTN      BIPT      & RETURN
0055          *
0056          *WAIT FØR EXECUTE SIGNAL
0057          *
0058 0027 0800 XEQ      ENT
0059 0028 FE20      JST      IER      INPUT
0060 0029 C08D      CAI      :8D      CARRIAGE RETURN?
0061 002A F703      RTN      XEQ      YES, RETURN
0062 002B F603      JMP      XEQ+1,    NØ, GET MØRE
0063          *
0064          *ØUTPUT TØ TTY
0065          *
0066 002C 0800 ØTT      ENT
0067 002D 403C      SEL      7, 4      RESET INTERFACE
0068 002E 6D3B      WRA      7, 3      WRITE ØN NØT BUSY
0069 002F 493B      SEN      7, 3      DØNE?
0070 0030 F601      JMP      $-1      NØ
0071 0031 F705      RTN      ØTT      YES
0072          *
0073          *CØMMAND ERRØR EXIT
0074          *
0075 0032 0800 ERR      ENT
0076 0033 C6DF      LAP      :DF      PRINT ARRØW
0077 0034 FE08      JST      ØTT
0078 0035 FF34      JST      *CMND     RESTART CØMMAND
0079          *
0080          *CARRIAGE-RETURN, LINE FEED
0081          *
0082 0036 0800 CRLF     ENT
0083 0037 C68D      LAP      :8D      CR
0084 0038 FE0C      JST      ØTT
0085 0039 C68A      LAP      :8A      LF
0086 003A FE0E      JST      ØTT
0087 003B F705      RTN      CRLF
0088          *
0089          *ØUTPUT TEXT FRØM BUFFER
0090          *
0091 003C 0800 ØTL      ENT
0092 003D 8A0F      ADD      CAI      MAKE CØMPARE INSTRUCTION
0093 003E 9A06      STA      ØT2     &SAVE IT
0094 003F 9A09      STA      ØT3
0095 0040 E704      LDX      *ØTL     GET TEXT PØINTER
0096 0041 DE05      IMS      ØTL     SET RETURN ADDRESS
0097 0042 B400 ØT1      LDA      ØØ      GET WØRD
0098 0043 11D7      RRA      8       PRINT FIRST BYTE
0099 0044 FE18      JST      ØTT
0100 0045 C000 ØT2      CAI      0       LAST ØNE
0101 0046 F70A      RTN      ØTL     YES, RETURN
0102 0047 1157      RLA      8       PRINT SECØND BYTE
0103 0048 FE1C      JST      ØTT
0104 0049 C000 ØT3      CAI      0       LAST ØNE?
0105 004A F70E      RTN      ØTL     YES RETURN
0106 004B 0128      IXR
0106          BUMP PØINTER

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0107 004C F60A      JMP      0T1      LOOP
0108 004D C000  CAI      CAI      0
0109                      *
0110                      *0UTPUT FL0ATING P0INT NUMBER
0111                      *
0112 004E 0300  0FPA  ENT
0113 004F E701      LDX      *0FPA  GET P0INTER
0114 0050 DE02      IMS      0FPA  SET RETURN ADDRESS
0115 0051 EA01      STX      0PT   SAVE P0INTER
0116 0052 FB0E      JST      *FAS  C0NVERT T0 ASCII
0117 0053 0000  0PT  DATA  0
0118 0054 0059      DATA  BUF
0119 0055 0110      ZAR
0120 0056 FE1A      JST      0TL   SET END FLAG
0121 0057 0059      DATA  BUF   PRINT NUMBER
0122 0058 F70A      RTN      0FPA
0123 0059 0000  BUF  RES      8,0
0124 0061      FAS  REF
0125                      END
0000 ERR0RS

```

0001			NAM	READ, CLEAR	
0002			NAM	PUNCH, GRAPH	
0003			EXTR	IKB, ØTT, BIPT	
0004			EXTR	ØTL, CRLF, FPLØT, ØFPA	
0005			EXTR	XA, XB, XC, FAD	
0006	0000		REL	0	
0007			*		ORIGINAL PAGE IS
0008			* READ PAPER TAPE & ADD TØ BUFFER		OF POOR QUALITY
0009			*		
0010	0000	0800	READ	ENT	
0011			*INITIALIZE VARIABLES		
0012	0001	1128	PLX	1	SET INDEX BIT
0013	0002	1400	SØV	-	
0014	0003	11A3	RRX	1	
0015	0004	EA33	STX	R5	& SAVE PØINTER
0016	0005	EA34	STX	R5+2	
0017	0006	0528	XRP		RESET SUBSCRIPTS
0018	0007	E900	STX	XA	
		0000			
0019	0008	E900	STX	XB	
		0000			
0020	0009	E900	STX	XC	
		0000			
0021	000A	B236	LDA	TP	SET INPUT BUFFER PØINTER
0022	000B	9A38	STA	MPT	
0023	000C	B235	LDA	CT	SET INPUT CØUNT
0024	000D	9A37	STA	CNT	
0025			*SKIP LEADER & LABEL		
0026	000E	C68A	LAP	:8A	LINE FEED
0027	000F	F900	JST	ØTT	
		0000			
0028	0010	F900	JST	BIPT	
		0000			
0029	0011	2141	JAZ	S- 1	SKIP LEADER
0030	0012	F900	R1 JST	ØTT	ECHØ LABEL
		0000			
0031	0013	F900	JST	BIPT	READ TAPE
		0000			
0032	0014	C092	CAI	:92	END ØF LABEL?
0033	0015	F201	JMP	R2	YES
0034	0016	F604	JMP	R1	NØ
0035	0017	F900	R2 JST	BIPT	READ TAPE
		0000			
0036	0018	CØFF	CAI	:FF	FILE MARK?
0037	0019	F201	JMP	R3	YES
0038	001A	F603	JMP	R2	NØ
0039			*READ LØØP		
0040	001B	F900	R3 JST	BIPT	LØWER BYTE
		0000			
0041	001C	1B87	LLR	8	SAVE
0042	001D	F900	JST	BIPT	UPPER BYTE
		0000			
0043	001E	1B07	LLL	8	RESTØRE

0044	001F	E224		L DK	MPT	SAVE LOW BITS
0045	0020	9C01		STA	01	
0046	0021	F900		JST	BIPT	LOWER BYTE
		0000				
0047	0022	1B87		LLR	8	
0048	0023	F900		JST	BIPT	UPPER BYTE
		0000				
0049	0024	1B07		LLL	8	
0050				*CONVERT BASIC-F.P. TO CAI-F.P.		
0051	0025	2109		JAZ	Z1	O=O=0
0052	0026	0048		TAX		SAVE SIGN
0053	0027	3081		JAP	S+2	ABS. VAL.
0054	0028	0310		NAR		
0055	0029	1B87		LLR	8	REMOVE MSB
0056	002A	1328		LLX	1	
0057	002B	8A17		ADD	D64	FIX CHARACTERISTIC
0058	002C	1B07		LLL	8	
0059	002D	13C0		LA0		RECOVER SIGN
0060	002E	11D0		RRA	1	
0061	002F	9B14	Z1	STA	*MPT	SAVE HI BITS
0062	0030	DA13		IMS	MPT	BUMP POINTER
0063	0031	DA12		IMS	MPT	TWICE!
0064	0032	DA12		IMS	CNT	MORE?
0065	0033	F618		JMP	R3	YES
0066				*ADD INPUT BUFFER TO BEAM ARRAY		
0067				*PANIC SWITCH DISABLED FOR DURATION		
0068	0034	4006		CID		DISABLE AUTO
0069	0035	B20C		LDA	CT	RESET COUNTER
0070	0036	9A0E		STA	CNT	
0071	0037	F900	R4	JST	FAD	F.P. ADD
		0000				
0072	0038	0000	R5	DATA	0, :9400, 0	
	0039	9400				
	003A	0000				
0073	003B	D900		IMS	XA	BUMP SUBSCRIPTS
		0000				
0074	003C	D900		IMS	XB	
		0000				
0075	003D	D900		IMS	XC	
		0000				
0076	003E	DA06		IMS	CNT	MORE?
0077	003F	F608		JMP	R4	YES
0078	0040	F740		RTN	READ	
0079				*STORAGE		
0080	0041	1400	TP	DATA	: 1400	
0081	0042	FF00	CT	DATA	-256	
0082	0043	0040	D64	DATA	64	
0083	0044	0000	MPT	DATA	0	
0084	0045	0000	CNT	DATA	0	
0085				*		
0086				*CLEAR BEAM ARRAY		
0087				*		
0088	0046	0800		CLEAR	ENT	

0089	0047	B605		LDA	CT	SET COUNT
0090	0048	9E03		STA	CNT	
0091	0049	0110		ZAR		CLEAR
0092	004A	9C00	C1	STA	@0	HI BITS
0093	004B	9C01		STA	@1	LO BITS
0094	004C	C202		AXI	2	BUMP POINTER
0095	004D	DE08		IMS	CNT	DONE?
0096	004E	F604		JMP	C1	NØ
0097	004F	F709		RTN	CLEAR	
0098			*			
0099			*PUNCH BEAM ARRAY - BASIC FORMAT			
0100			*			
0101	0050	0800	PUNCH	ENT		
0102	0051	FA22		JST	LEAD	LEADER
0103	0052	B610		LDA	CT	START COUNTER
0104	0053	9E0E		STA	CNT	
0105	0054	F900	P1	JST	IKB	ECHO LABEL
		0000				
0106	0055	C092		CAI	:92	CTRL/TAPE?
0107	0056	F201		JMP	P2	YES
0108	0057	F603		JMP	P1	NØ
0109	0058	C6FF	P2	LAP	:FF	PUNCH FILE MARK
0110	0059	F900		JST	ØTT	
		0000				
0111	005A	B401	P3	LDA	@1	PUNCH LO BITS
0112	005B	F900		JST	ØTT	
		0000				
0113	005C	13D7		LRA	8	
0114	005D	F900		JST	ØTT	
		0000				
0115	005E	B400		LDA	@0	GET HI BITS
0116	005F	C202		AXI	2	BUMP POINTER
0117	0060	EE1C		STX	MPT	AND SAVE
0118			*CONVERT CAI-F.P. TO BASIC-F.P.			
0119	0061	210A		JAZ	Z2	C=0=0
0120	0062	0048		TAX		SAVE SIGN
0121	0063	1350		LLA	1	CLEAR A15
0122	0064	1B87		LLR	8	SPLIT
0123	0065	9622		SUB	D64	FIX CHARACTERISTIC
0124	0066	1400		SOV		INSERT MSB
0125	0067	1150		RLA	1	
0126	0068	1B06		LLL	7	RE-FORMAT
0127	0069	1329		LLX	2	RECOVER SIGN
0128	006A	3201		JØR	S+2	CORRECT FOR IT
0129	006B	0310		NAR		
0130	006C	F900	Z2	JST	ØTT	PUNCH HI BITS
		0000				
0131	006D	1B87		LLR	8	
0132	006E	F900		JST	ØTT	
		0000				
0133	006F	E62B		LIX	MPT	RECOVER POINTER
0134	0070	DE2B		IMS	CNT	DONE?
0135	0071	F617		JMP	P3	NØ

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0136 0072 FA01          JST   LEAD   YES, PUNCH LEADER
0137 0073 F723          RTN   PUNCH
0138                *PUNCH 5" ØF LEADER
0139 0074 0800          LEAD  ENT
0140 0075 C732          LAM   50
0141 0076 9E31          STA   CNT
0142 0077 0110          ZAR
0143 0078 F900          L2    JST   ØTT
      0000
0144 0079 DE34          IMS   CNT
0145 007A F602          JMP   L2
0146 007B F707          RTN   LEAD
0147                *
0148                *PLOT DATA ARRAY
0149                *
0150 007C 0300          GRAPH ENT
0151 007D EA06          STX   T      SAVE COUNT POINTER
0152 007E C202          AXI   2
0153 007F EA08          STX   PT     SAVE DATA POINTER
0154 0080 0110          ZAR   PRINT COUNT
0155 0081 F900          JST   ØTL
      0000
0156 0082 008A          DATA CTX
0157 0083 F900          JST   ØFPA
      0000
0158 0084 0000          T     DATA 0
0159 0085 F900          JST   CRLF
      0000
0160 0086 C7FF          LAM   255    PLOT DATA
0161 0087 F900          JST   FPLØT
      0000
0162 0088 0000          PT    DATA 0
0163 0089 F70D          RTN   GRAPH
0164 008A 8A3A          CTX   DATA :8A3A
0165 008B C3CF          TEXT  'COUNT = '
      008C D5CE          -
      008D D4A0          -
      008E BDA0
0166 008F 0000          DATA 0
0167                END
0000 ERRORS

```


0050	002A	0000	RP4	DATA	O, SCALE, MAX
	002B	0053			
	002C	0051			
0051	002D	DE03		IMS	RP4
0052	002E	DE04		IMS	RP4
0053	002F	FB2D		JST	*FIX
0054	0030	0051		DATA	MAX, CNT
	0031	004F			
0055	0032	B21C		LDA	CNT
0056	0033	0048		TAX	
0057	0034	3085		JAP	P05
0058	0035	C6AD		LAP	'-'
0059	0036	FB20		JST	*0TT
0060	0037	C6B0		LAP	'0'
0061	0038	FB1E		JST	*0TT
0062	0039	F20E		JMP	0UT
0063	003A	C6A0	P05	LAP	' '
0064	003B	FB1B		JST	*0TT
0065	003C	2809		JXZ	MARK
0066	003D	C6B0		LAP	'0'
0067	003E	FB18		JST	*0TT
0068	003F	0503		NXR	
0069	0040	EA0E		STX	CNT
0070	0041	C6AA		LAP	'*'
0071	0042	F201		JMP	\$+2
0072	0043	FB13		JST	*0TT
0073	0044	DA0A		IMS	CNT
0074	0045	F602		JMP	\$-2
0075	0046	C6AA	MARK	LAP	'*'
0076	0047	FB0F		JST	*0TT
0077	0048	FB0F	0UT	JST	*CRLF
0078	0049	DA04		IMS	NR0W
0079	004A	F627		JMP	LP2
0080	004B	FB0C		JST	*CRLF
0081	004C	DE4C		IMS	FPL0T
0082	004D	F74D		RTN	FPL0T
0083	004E	0800	NR0W	HLT	
0084	004F	0800	CNT	HLT	
0085	0050	8000	B15	DATA	:8000
0086	0051	0000	MAX	RES	2,0
0087	0053	0000	SCALE	RES	2,0
0088	0055	4348	F50	DATA	:4348,0
	0056	0000			
0089	0057		0TT	REF	
0090	0058		CRLF	REF	
0091	0059		0TL	REF	
0092	005A		XA	REF	
0093	005B		FCP	REF	
0094	005C		FI1V	REF	
0095	005D		FIX	REF	
0096	005E		FMP	REF	
0097	005F		FDV	REF	
0098	0060		0FPA	REF	

```
0099 0061 D3C3 SF TEXT 'SCAL FACTOR ='
      0062 C1CC ~
      0063 85C6 ~
      0064 C1C3
      0065 D4CF
      0066 D2A0
      0067 BDA0
0100 END
```

```

0001
0002 * CRT - FLOATING POINT PLOTTER
0003 *
0004 * CALLING SEQUENCE:
0005 * JST *CRT
0006 * (RETURN)
0007 *
0008 NAM CRT
0009 0000 REL 0
0010 0000 0800 CRT ENT
0011 0001 C7FF LAM 255
0012 0002 9A5F STA NR0W
0013 0003 9A5F STA CNT
0014 0004 49C4 SEN 24, 4
0015 0005 F601 JMP $- 1
0016 0006 40C7 SEL 24, 7
0017 0007 C202 AXI 2
0018 0008 0030 TXA
0019 0009 A261 IOR B15
0020 000A 9A0E STA RP1
0021 000B 9A0E STA RP2
0022 000C 9A10 STA RP3
0023 000D 9A13 STA RP4
0024 000E 9A36 STA RP5
0025 000F 0110 ZAR
0026 0010 9A54 STA MIN
0027 0011 9A54 STA MIN+ 1
0028 0012 0350 ARP
0029 0013 9B5B STA *XA
0030 0014 F204 JMP MOVE
0031 0015 FB5A LP1 JST *FCP
0032 0016 0000 RP1 DATA 0, MAX
0017 0067
0033 0018 2183 JAL INC1
0034 0019 FB57 MOVE JST *FMV
0035 001A 0000 RP2 DATA 0, MAX
001B 0067
0036 001C FB53 INC1 JST *FCP
0037 001D 0000 RP3 DATA 0, MIN
001E 0065
0038 001F 3083 JAP INC2
0039 0020 FB50 JST *FMV
0040 0021 0000 RP4 DATA 0, MIN
0022 0065
0041 0023 DB4B INC2 IMS *XA
0042 0024 DA3E IMS CNT
0043 0025 F610 JMP LP1
0044 *
0045 0026 FB4F JST *CRLF
0046 0027 FB4E JST *CRLF
0047 0028 C6BD LAP '='
0048 0029 FB4D JST *0TL
0049 002A 0079 DATA SF

```

0050			*		
0051	002B	FB46		JST	*FSB
0052	002C	0067		DATA	MAX,MIN,MAX
	002D	0065			-
	002E	0067			
0053	002F	FB43		JST	*FDV
0054	0030	006C		DATA	F250,MAX,SCALE
	0031	0067			
	0032	0069			
0055	0033	FB44		JST	*ØFPA
0056	0034	0069		DATA	SCALE
0057	0035	FB40		JST	*CRLF
0058			*		
0059	0036	FB3D		JST	*FMP
0060	0037	0069		DATA	SCALE,MIN,MAX
	0038	0065			
	0039	0067			
0061	003A	FB3A		JST	*FIX
0062	003B	0067		DATA	MAX,MIN
	003C	0065			
0063	003D	B227		LDA	MIN
0064	003E	0310		NAR	-
0065	003F	3081		JAP	S+2
0066	0040	0110		ZAR	
0067	0041	9A22		STA	X
0068	0042	0350		ARP	
0069	0043	9B2B		STA	*XA
0070			*		
0071	0044	FB2F	LP2	JST	*FMP
0072	0045	0000	RP5	DATA	0,SCALE,MAX
	0046	0069			
	0047	0067			
0073	0048	DB26		IMS	*XA
0074	0049	FB2B		JST	*FIX
0075	004A	0067		DATA	MAX,CNT
	004B	0063			
0076	004C	E217		LDK	X
0077	004D	B215		LDA	CNT
0078	004E	49C4	DØT	SEN	24,4
0079	004F	F601		JMP	S-1
0080	0050	49C2		SEN	24,2
0081	0051	F601		JMP	S-1
0082	0052	6EC2		ØTX	24,2
0083	0053	2107		JAZ	DØNE
0084	0054	3183		JAG	PØS
0085	0055	00A3		DKR	
0086	0056	0150		IAR	
0087	0057	F609		JMP	DØT
0088	0058	0128	PØS	IXR	
0089	0059	00D0		DAR	
0090	005A	F60C		JMP	DØT
0091	005B	B208	DØNE	LDA	X
0092	005C	8A11		ADD	H256

0093	005D	9A06		STA	X
0094	005E	DA03		INS	NRØW
0095	005F	F61B		JMP	LP2
0096	0060	FB15		JST	*CRLF
0097	0061	F761		RTN	CRT
0098			*		
0099	0062	0000	NRØW	DATA	0
0100	0063	0000	CNT	DATA	0
0101	0064	0000	X	DATA	0
0102	0065	0000	MIN	RES	2,0
0103	0067	0000	MÅX	RES	2,0
0104	0069	0000	SCALE	RES	2,0
0105	006B	8000	B15	DATA	:8000
0106	006C	447A	F250	DATA	:447A,0
	006D	0000			
0107	006E	0100	H256	DATA	256
0108			*		
0109	006F		XA	REF	
0110	0070		FCP	REF	
0111	0071		FMV	REF	
0112	0072		FSB	REF	
0113	0073		FDV	REF	
0114	0074		FIP	REF	
0115	0075		FIX	REF	
0116	0076		CRLF	REF	
0117	0077		ØTL	REF	
0118	0078		ØFPA	REF	
0119			*		
0120	0079	D3C3	SF	TEXT	'SCALE FACTØR ='
	007A	C1CC			'
	007B	C5A0			
	007C	C6C1			
	007D	C3D4			
	007E	CFD2			
	007F	A0BD			
0121			*		
0122				END	
0000	ERRØRS				

ORIGINAL PAGE IS
OF POOR QUALITY

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0001          NAM      VIEW
0002          EXTR    DPN: , DPSUB: , DPDI V: , DPSI:
0003          NAM      DISP
0004          EXTR    RTR, RTOP
0005  0000  0800  VIEW  ENT
0006  0001  9A71  DISP  STA  VEVR
0007  0002  C202          AXI  2
0008  0003  EA66          STX  RP1D      POINTER
0009  0004  EA66          STX  RP2D
0010  0005  C7FF          LAM  255
0011  0006  9A6A          STA  CNT
0012  0007  9A64          STA  NR0W
0013  0008  0110          ZAR          INITIALIZE MAX AND MIN
0014  0009  9A63          LDA  MIN
0015  000A  9A63          STA  MIN+1
0016  000B  9A63          STA  MAX
0017  000C  9A63          STA  MAX+1
0018  000D  B400  STCM  LDA  e0
0019  000E  D260          CMS  MAX
0020  000F  F20A          JMP  MIN1      A1<MAX
0021  0010  F205          JNP  MAX1      A1>MAX
0022  0011  B401          LDA  e1      A1=MAX
0023  0012  D25D          CMS  MAX+1
0024  0013  F206          JMP  MIN1      A1=MAX A2<MAX+1
0025  0014  9A5B          STA  MAX+1      A1=MAX A2>MAX+1
0026  0015  F211          JMP  FICM
0027  0016  9A58  MAX1  STA  MAX
0028  0017  B401          LDA  e1
0029  0018  9A57          STA  MAX+1
0030  0019  F20D          JMP  FICM
0031  001A  B400  MIN1  LDA  e0
0032  001B  D251          CMS  MIN
0033  001C  F206          JMP  MIN2      A1<MIN
0034  001D  F209          JMP  FICM      A1>MIN
0035  001E  B401          LDA  e1      A1=MIN
0036  001F  D24E          CMS  MIN+1
0037  0020  F202          JMP  MIN2      A1=MIN A2<MIN+1
0038  0021  F205          JMP  FICM      A1=MIN A2>MIN+1
0039  0022  F204          JMP  FICM      A1=MIN A2=MIN+1
0040  0023  B400  MIN2  LDA  e0
0041  0024  9A48          STA  MIN
0042  0025  B401          LDA  e1
0043  0026  9A47          STA  MIN+1
0044  0027  E242  FICM  LDX  RP1D
0045  0028  C202          AXI  2
0046  0029  EA40          STX  RP1D
0047  002A  DA46          IMS  CNT
0048  002B  F61E          JMP  STCM
0049          *
0050  002C  B242          LDA  MAX
0051  002D  E242          LDX  MAX+1
0052  002E  F900          JST  DPSUB:
          0000

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0053	002F	006D		DATA	MIN
0054	0030	1050		ALA	1
0055	0031	1329		LLX	2
0056	0032	3201		JØR	S+2
0057	0033	0150		IAR	
0058	0034	13A3		LRX	1
0059	0035	9A39		STA	MAX
0060	0036	EA39		STX	MAX+1
0061			*		
0062	0037	B235		LDA	MIN
0063	0038	E235		LDX	MIN+1
0064	0039	F900		JST	DPDIV:
		0000			
0065	003A	006F		DATA	MAX
0066	003B	9A2D		STA	TEM
0067	003C	10D1		ARA	2
0068	003D	0310		NAR	
0069	003E	8A2A		ADD	TEM
0070	003F	10D5		ARA	6
0071			*		
0072	0040	0310		NAR	
0073	0041	3081		JAP	S+2
0074	0042	0110		ZAR	
0075	0043	9A2E		STA	X
0076			*		
0077	0044	E226		LDX	RP2D
0078	0045	B400	LPD	LDA	@0
0079	0046	E401		LDX	@1
0080	0047	F900		JST	DPDIV:
		0000			
0081	0048	006F		DATA	MAX
0082	0049	9A1F		STA	TEM
0083	004A	10D1		ARA	2
0084	004B	0310		NAR	
0085	004C	8A1C		ADD	TEM
0086	004D	10D5		ARA	6
0087			*		
0088	004E	E223		LDX	X
0089	004F	49C4	DØT	SEN	24, 4
0090	0050	F601		JMP	S-1
0091	0051	49C2		SEN	24, 2
0092	0052	F601		JMP	S-1
0093	0053	6EC2		ØTX	24, 2
0094	0054	2107		JAZ	DØNE
0095	0055	3183		JAG	PØS
0096	0056	00A8		DXR	
0097	0057	0150		IAR	
0098	0053	F609		JMP	DØT
0099	0059	0128	PØS	IXR	
0100	005A	00D0		DAR	
0101	005B	F60C		JMP	DØT
0102			*		
0103	005C	B215	DØNE	LDA	X

0104	005D	8A16		ADD	H256
0105	005E	9A13		STA	X :
0106	005F	E20B		L DX	RP2D
0107	0060	C202		AXI	2
0108	0061	EA09		STX	RP2D
0109	0062	DA09		IMS	NRØW
0110	0063	F61E		JNP	LPD
0111	0064	B20E		L DA	VEVR
0112	0065	2101		JAZ	S+2
0113	0066	F100		JMP	RTØP
		0000			
0114	0067	F100		JMP	RTTR
		0000			
0115	0068	F768		RTN	VIEW
0116	0069	0000	TEM	DATA	0
0117	006A	0000	RP1D	DATA	0
0118	006B	0000	RP2D	DATA	0
0119	006C	0000	NRØW	DATA	0
0120	006D	0000	MIN	RES	2, 0
0121	006F	0000	MAX	RES	2, 0
0122	0071	0000	CNT	DATA	0
0123	0072	0000	X	DATA	0
0124	0073	0000	VEVR	DATA	0
0125	0074	0100	H256	DATA	256
0126				END	
0000	ERRORS				

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0001			NAM	DPACC	
0002			NAM	DPFLT	
0003			NAM	DPFIX	
0004			EXTR	DPN:	
0005			EXTR	DPN	
0006	0000		REL	0	
0007			*		
0008			*DØUELE PRECISION ACCUMULATE		
0009			*		
0010	0000	0800	DPACC	ENT	
0011	0001	9A10		STA	TEMP
0012	0002	0030		TKA	
0013	0003	E703		L DX	*DPACC
0014	0004	1200		RØV	
0015	0005	8C01		ADD	@1
0016	0006	820C		AND	MASK
0017	0007	9C01		STA	@1
0018	0008	B209		LDA	TEMP
0019	0009	3204		JØR	DC1
0020	000A	1200		RØV	
0021	000B	0150		IAR	
0022	000C	3201		JØR	DC1
0023	000D	0110		ZAR	
0024	000E	8C00	DC1	ADD	@0
0025	000F	9C00		STA	@0
0026	0010	DE10		IMS	DPACC
0027	0011	F711		RTW	DPACC
0028	0012	0000	TEMP	DATA	0
0029	0013	7FFF	MASK	DATA	:7FFF
J			*		
0031			*DØUELE PRECISION TO CAI-F. P.		
0032			*		
0033	0014	0800	DPFLT	ENT	
0034	0015	3801		JXN	DPF1
0035	0016	2113		JAZ	DPF4
0036	0017	BA13	DPF1	EMA	BITS
0037	0018	0110		ZAR	
0038	0019	BA11		EMA	BITS
0039	001A	9A11		STA	SIGN
0040	001B	3081		JAP	DPF2
0041	001C	F900		JST	DPN:
		0000			
0042	001D	1328	DPF2	LLX	1
0043	001E	1B00	DPF3	LLL	1
0044	001F	DA0B		IMS	BITS
0045	0020	3242		JØR	DPF3
0046	0021	1B88		LLR	9
0047	0022	BA08		EMA	BITS
0048	0023	0310		NAR	
0049	0024	8A08		ADD	D160
0050	0025	1356		LLA	7
0051	0026	A204		IØR	BITS
0052	0027	BA04		EMA	SIGN

```

0053 0028 8205          AND    DPM
0054 0029 A202          IØR    SIGN
0055 002A F716    DPF4  RTN    DPFLT
0056 002B 0000    BITS   DATA  0
0057 002C 0000    SIGN   DATA  0
0058 002D 00A0    D160   DATA  160
0059 002E 8000    DPM    DATA  :8000
0060 002F 0800    DPFIX  ENT
0061 0030 9E04          STA    SIGN
0062 0031 821C          AND    DPP
0063 0032 13D6          LRA    7
0064 0033 9606          SUB    D160
0065 0034 9E09          STA    BITS
0066 0035 B609          LDA    SIGN
0067 0036 8218          AND    MANT
0068 0037 A218          IØR    SBIT
0069 0038 BE0D          EMA    BITS
0070 0039 2109          JAZ    DPFX2
0071 003A 3190          JAG    DPFX4
0072 003B BE10          EMA    BITS
0073 003C 1B07          LLL    8
0074 003D 1B30    DPFX1 LLR    1
0075 003E DE13          IMS    BITS
0076 003F F602          JMP    DPFX1
0077 0040 1B00          LLL    1
0078 0041 13A8          LFX    1
0079 0042 F201          JMP    $+2
0080 0043 BE18    DPFX2  EMA    BITS
0081 0044 BE18          EMA    SIGN
0082 0045 3083          JAP    DPFX3
0083 0046 BE1A          EMA    SIGN
0084 0047 F900          JST    DPN:
0085 0048 F201          JMP    $+2
0086 0049 BE1D    DPFX3  E1A    SIGN
0087 004A F71B          RTN    DPFIX
0088 004B 0118    DPFX4  ZAK
0089 004C 1400          SOV
0090 004D F71E          RTN    DPFIX
0091 004E 7FFF    DPP    DATA  :7FFF
0092 004F 007F    MANT   DATA  :7F
0093 0050 0080    SBIT   DATA  :80
0094          END
0000 ERRØRS

```

APPENDIX C

0001			NAM	CMND	
0002			EXTR	IER, ØTT, ØTL, CRLF	
0003			EXTR	ERR, XEQ	
0004	0100		ABS	: 100	
0005			*-PRINT	NAME	
0006	0100	0110	ZAR		
0007	0101	F900	JST	ØTL	
		0000			
0008	0102	012B	DATA	NAME	
0009	0103	F201	JMP	S+2	
0010			*		
0011			*COMMAND	INTERPRETER	
0012			*		
0013	0104	0800	CMND	ENT	
0014	0105	0A00		EIN	
0015	0106	4005		CIE	ENABLE PANIC BUTTON
0016	0107	F900	JST	CRLF	PRINT PROMPT CHARACTER
		0000			
0017	0108	C687	LAP	:87	
0018	0109	F900	JST	ØTT	
		0000			
0019	010A	C6AA	LAP	'*'	
0020	010B	F900	JST	ØTT	
		0000			
0021	010C	0108	ZXR		
0022	010D	F900	JST	IER	INPUT COMMAND
		0000			
0023	010E	C0D4	CAI	'T'	
0024	010F	E213	LDX	TRANS	
0025	0110	C0D2	CAI	'R'	
0026	0111	E212	LDX	READ	
0027	0112	C0C3	CAI	'C'	
0028	0113	E211	LDX	CLEAR	
0029	0114	380A	JXN	ØKF	
0030	0115	C0D0	CAI	'P'	
0031	0116	E20F	LDX	PUNCH	
0032	0117	C0C7	CAI	'G'	
0033	0118	E20F	LDX	FPLØT	
0034	0119	C0D6	CAI	'V'	
0035	011A	E20E	LDX	CRT	
0036	011B	C0C4	CAI	'D'	
0037	011C	E20A	LDX	DATUM	
0038	011D	3801	JXN	ØKF	
0039	011E	F900	BAD JST	ERR	
		0000			
0040	011F	EA0A	ØKF STX	FUNC	SAVE CALL ADDRESS
0041	0120	F900	JST	XEQ	WAIT FOR GO
		0000			
0042	0121	FB08	JST	*FUNC	CALL FUNCTION
0043	0122	F61D	JMP	CMND+1	
0044	0123		TRANS REF		
0045	0124		READ REF		
0046	0125		CLEAR REF		

0047	0126		PUNCH	REF	
0048	0127		DATUM	REF	
0049	0128		FPLØT	REF	
0050	0129		CRT	REF	
0051	012A	0000	FUNC	DATA	0
0052	012B	B1B5	NAME	TEXT	'15*255 FULL HTS'
	012C	AAB2			
	012D	B5B5			
	012E	A0C6			
	012F	D5CC			
	0130	CCA0			
	0131	C8D4			
	0132	D3A0			
0053	0133	8D3A		DATA	:8D3A,0
	0134	0000			
0054				END	
0000	ERRØRS				

Line	Code	Address	Label	Operation	Comments
0001				NAM	TRANS
0002				NAM	RTTR
0003				EXTR	DISP
0004	0000			REL	0
0005	0000	08 00	TRANS	ENT	
0006	0001	C6BF		LAP	'?'
0007	0002	F900		JST	*ØTL
		8 127			
0008	0003	013E		DATA	MØTA
0009	0004	F900		JST	*IKB
		8 128			
0010	0005	1200		RØV	
0011	0006	C0CD		CAI	'M'
0012	0007	F203		JMP	MØN
0013	0008	C0D4		CAI	'T'
0014	0009	F204		JMP	TAP
0015	000A	F900		JST	*ERR
		8 12A			
0016	000B	F900	MØN	JST	*CRLF
		8 129			
0017	000C	0108		ZXR	
0018	000D	F20F		JMP	BEG
0019	000E	F900	TAP	JST	*CRLF
		8 129			
0020	000F	C68A		LAP	:8A
0021	0010	F900		JST	*ØTT
		8 126			
0022	0011	F900		JST	*BIPT
		8 125			
0023	0012	2141		JAZ	\$- 1
0024	0013	F900	T1	JST	*ØTT
		8 126			
0025	0014	F900		JST	*BIPT
		8 125			
0026	0015	C092		CAI	:92
0027	0016	F201		JMP	T2
0028	0017	F604		JMP	T1
0029	0013	F900	T2	JST	*BIPT
		8 125			
0030	0019	C0FF		CAI	:FF
0031	001A	F201		JMP	T4
0032	001B	F603		JMP	T2
0033	001C	C401	T4	LXP	1
0034	001D	E900	BEG	STX	TETR
		012B			
0035	001E	B100		LDA	MASK 0
		0122			
0036	001F	9AF3		STA	MASK 1
0037	0020	F900		JST	*CRLF
		8 129			
0038	0021	C6BF		LAP	'?'
0039	0022	F900		JST	*ØTL
		8 127			

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0040	0023	0130		DATA	DI RE
0041	0024	F900		JST	*IKB
		8128			
0042	0025	C0C6		CAI	'F'
0043	0026	F205		JMP	FØRD
0044	0027	C0C2		CAI	'B'
0045	0028	F208		JMP	BACK
0046	0029	C0CF		CAI	'Ø'
0047	002A	F20A		JMP	BØTH
0048	002B	FBFE		JST	*ERR
0049	002C	C71E	FØRD	LAM	30
0050	002D	9AE2		STA	CNT2
0051	002E	0108		ZXR	
0052	002F	0408		CXR	
0053	0030	F207		JMP	TKF
0054	0031	C71E	BACK	LAM	30
0055	0032	9ADD		STA	CNT2
0056	0033	0108		ZXR	
0057	0034	F203		JMP	TKF
0058	0035	C70F	BØTH	LAM	15
0059	0036	9AD9		STA	CNT2
0060	0037	0358		AXP	
0061	0038	EAF5	TKF	STX	DIR1
0062	0039	E2F1		L DX	TETR
0063	003A	2802		JXZ	S+3
0064	003B	C70F		LAM	15
0065	003C	9AD3		STA	CNT2
0066	003D	B2D4		LDA	ZCT
0067	003E	9AD2		STA	CNT3
0068	003F	E2E3		L DX	IBC
0069	0040	0110		ZAR	
0070	0041	9C00	CLR	STA	@0
0071	0042	0128		IXR	
0072	0043	DÂCD		IMS	CNT3
0073	0044	F603		JMP	CLR
0074	0045	EAD1		STX	IBP
0075	0046	E2E4		L DX	TETR
0076	0047	2803		JXZ	NEXT
0077	0048	E2E5		L DX	DIR1
0078	0049	3801		JXN	NEXT
0079	004A	0210		CAR	
0080	004B	E2D3	NEXT	L DX	MASK
0081	004C	0128		IXR	
0082	004D	0210		CAR	
0083	004E	C000		CAI	0
0084	004F	C2FF		AXI	255
0085	0050	EAB8		STX	MSK1
0086	0051	9ABA		STA	DIR
0087	0052	C7F5		LAM	245
0088	0053	9ACC		STA	DCT2
0089	0054	C7FF		LAM	255
0090	0055	9AB7		STA	DCT
0091	0056	B2D4		LDA	TETR

009 2	0057	2107		JAZ	TUR1
009 3	0058	FBCC	T3	JST	*BIPT
009 4	0059	1357		LLA	8
009 5	005A	9AD4		STA	TEMT
009 6	005B	FBC9		JST	*BIPT
009 7	005C	A2D2		IØR	TEMT
009 8	005D	9AB0		STA	VAL
009 9	005E	F209		JMP	PRØC
0100	005F	FA39	TUR1	JST	TURNØN
0101	0060	FA93	ILP	JST	MØNS
0102	0061	9AAC		STA	VAL
0103	0062	B2CB		LDA	DIR1
0104	0063	C001		CAI	1
0105	0064	F203		JMP	PRØC
0106	0065	D2A6		CMS	DIR
0107	0066	F235		JMP	FINØNE
0108	0067	F234		JMP	FINØNE
0109	0068	C70F	PRØC	LAM	15
0110	0069	9AA5		STA	CNT1
0111	006A	B2A3		LDA	MASK1
0112	006B	9AA3		STA	MASK2
0113	006C	B2B6		LDA	IBC
0114	006D	9AA3		STA	IB
0115	006E	C7FF	SPA	LAM	255
0116	006F	9AA5		STA	SCT
0117	0070	B293		LDA	MSK1
0118	0071	9A93		STA	MSK2
0119	0072	B2A3		LDA	IB
0120	0073	9AA3		STA	IBP
0121	0074	0110		ZAR	
0122	0075	D39E		CMS	*MASK2
0123	0076	F201		JMP	S+2
0124	0077	F213		JMP	TLP2
0125	0078	E295	TLP1	LDX	VAL
0126	0079	0110		ZAR	
0127	007A	D38F		CMS	*MSK2
0128	007B	F201		JMP	S+2
0129	007C	0503		NXR	
0130	007D	0030		TXA	
0131	007E	3B93		ADD	*IBP
0132	007F	9B97		STA	*IBP
0133	0080	DA39		IMS	MSK2
0134	0081	DA95		IMS	IBP
0135	0082	DA92		IMS	SCT
0136	0083	F60B		JMP	TLP1
0137	0084	DA3F		IMS	MASK2
0138	0085	C6FF		LAP	255
0139	0086	3A3F		ADD	IB
0140	0087	9A3E		STA	IB
0141	0088	DA36		IMS	CNT1
0142	0089	F61B		JMP	SPA
0143	008A	F211		JMP	FINØNE
0144	008B	E232	TLP2	LDX	VAL

0145	008C	0110		ZAR	
0146	008D	D37C		CMS	*MSK2
0147	008E	0508		NXR	
0148	003F	0030		TXA	
0149	0090	8B86		ADD	*IBP
0150	0091	9B85		STA	*IBP
0151	0092	DA77		IMS	MSK2
0152	0093	DA33		IMS	IBP
0153	0094	DA80		IMS	SCT
0154	0095	r60A		JMP	TLP2
0155	0096	DA7D		IMS	MASK2
0156	0097	C6FF		LAP	255
0157	0098	8A7D		ADD	IB
0158	0099	9A7C		STA	IB
0159	009A	DA74		IMS	CNT1
0160	009B	F62D		JMP	SPA
0161	009C	E26C	FINONE	LDK	MSK1
0162	009D	0128		IXR	
0163	009E	B26D		LDA	DIR
0164	009F	C000		CAI	0
0165	00A0	C302		SXI	2
0166	00A1	EA67		STX	MSK1
0167	00A2	DA7D		IMS	DCT2
0168	00A3	F201		JMP	s+2
0169	00A4	40C7		SEL	24, 7
0170	00A5	DA67		IMS	DCT
0171	00A6	F201		JMP	TUR2
0172	00A7	F204		JMP	TUR3
0173	00A8	B282	TUR2	LDA	TETR
0174	00A9	2101		JAZ	s+2
0175	00AA	F652		JMP	T3
0176	00AB	F64B		JMP	ILP
0177	00AC	C7FF	TUR3	LAM	255
0178	00AD	9A6E		STA	CNT7
0179	00AE	E263		LDK	IBP
0180	00AF	0110		ZAR	-
0181	00B0	9C00	CLR1	STA	e0
0182	00B1	0128		IXR	
0183	00B2	DA69		IMS	CNT7
0184	00B3	F603		JMP	CLR1
0185	00B4	C7F1		LAM	241
0186	00B5	9A63		STA	CNT4
0187	00B6	C70F		LAM	15
0188	00B7	9A62		STA	CNT5
0189	00B8	9A62		STA	CNT6
0190	00B9	B269		LDA	IBC
0191	00BA	9A63		STA	IBD
0192	00BB	C70E		LAM	14
0193	00BC	9A60		STA	HM14
0194	00BD	B260	NXBIN	LDA	IBD
0195	00BE	9A60		STA	IBE
0196	00BF	B35F	NXR0W	LDA	*IBE
0197	00C0	10D3		ARA	4

0198	00C1	8B55		ADD	*IBP
0199	00C2	9B54		STA	*IBP
0200	00C3	B254		LDA	H256
0201	00C4	8A5A		ADD	IBE
0202	00C5	9A59		STA	IBE
0203	00C6	DA53		IMS	CNT5
0204	00C7	F603		JMP	NXR0W
0205	00C8	B252		LDA	CNT6
0206	00C9	9A50		STA	CNT5
0207	00CA	DA53		IMS	IBD
0208	00CB	DA4B		IMS	IBP
0209	00CC	DA4C		IMS	CNT4
0210	00CD	F610		JMP	NXBIN
0211	00CE	B24B		LDA	CNT5
0212	00CF	0150		IAR	
0213	00D0	9A4A		STA	CNT6
0214	00D1	9A48		STA	CNT5
0215	00D2	C701		LAM	1
0216	00D3	9A45		STA	CNT4
0217	00D4	DA48		IMS	HM14
0218	00D5	F618		JMP	NXBIN
0219	00D6	0110		ZAR	
0220	00D7	9A49		STA	OFFSET
0221	00D8	F100		JMP	DISP
		0000			
0222	00D9	B251	RTTR	LDA	TETR
0223	00DA	2104		JAZ	T5
0224	00DB	B252		LDA	DIR1
0225	00DC	3184		JAG	T6
0226	00DD	0210		CAR	
0227	00DE	F206		JMP	T7
0228	00DF	B24E	T5	LDA	DIR1
0229	00E0	2182		JAL	S+3
0230	00E1	B22A	T6	LDA	DIR
0231	00E2	F202		JMP	T7
0232	00E3	B223		LDA	DIR
0233	00E4	3101		JAN	S+2
0234	00E5	DA2D	T7	IMS	MASK1
0235	00E6	DA29		IMS	CNT2
0236	00E7	F69C		JMP	NEXT
0237	00E8	F7E3		RTN	TRANS
0238	00E9	0300	TURN0N	ENT	
0239	00EA	4006		CID	
0240	00EB	48C1		SSN	:C1
0241	00EC	F601		JMP	S-1
0242	00ED	0E00		SEM	
0243	00EE	49C1		SEN	:C1
0244	00EF	F601		JMP	S-1
0245	00F0	0F00		SWM	
0246	00F1	40C4		SEL	:C4
0247	00F2	4005		CIE	
0248	00F3	F70A		RTN	TURN0N
0249	00F4	0300	M0NS	ENT	

0250	00F5	49C6		SEN	: C6
0251	00F6	F601		JMP	S-1
0252	00F7	5AC6		INX	: C6
0253	00F8	2843		JXZ	S-3
0254	00F9	C704		LAM	4
0255	00FA	9A10		STA	CNT
0256	00FB	0110		ZAR	
0257	00FC	9A0D		STA	MSK2
0258	00FD	F206		JMP	M2
0259	00FE	1350	M1	LLA	1
0260	00FF	9A0A		STA	MSK2
0261	0100	1351		LLA	2
0262	0101	8A08		ADD	MSK2
0263	0102	9A07		STA	MSK2
0264	0103	0110		ZAR	
0265	0104	1B03	M2	LLL	4
0266	0105	8A04		ADD	MSK2
0267	0106	DA04		IMS	CNT
0268	0107	F609		JMP	M1
0269	0108	F714		RTN	MONS
0270	0109	0000	MSK1	DATA	0
0271	010A	0000	MSK2	DATA	0
0272	010B	0000	CNT	DATA	0
0273	010C	0000	DIR	DATA	0
0274	010D	0000	DCT	DATA	0
0275	010E	0000	VAL	DATA	0
0276	010F	0000	CNT1	DATA	0
0277	0110	0000	CNT2	DATA	0
0278	0111	0000	CNT3	DATA	0
0279	0112	F010	ZCT	DATA	-4080
0280	0113	0000	MASK1	DATA	0
0281	0114	0000	MASK2	DATA	0
0282	0115	0000	SCT	DATA	0
0283	0116	0000	IB	DATA	0
0284	0117	0000	IBP	DATA	0
0285	0118	0100	H256	DATA	256
0286	0119	0000	CNT4	DATA	0
0287	011A	0000	CNT5	DATA	0
0288	011B	0000	CNT6	DATA	0
0289	011C	0000	CNT7	DATA	0
0290	011D	FFF2	HM14	DATA	-14
0291	011E	0000	IBD	DATA	0
0292	011F	0000	IBE	DATA	0
0293	0120	0000	DCT2	DATA	0
0294	0121	0000	ØFFSET	DATA	0
0295	0122		MASK0	REF	
0296	0123		IBC	REF	
0297	0124		MASK	REF	
0298	0125		BIPT	REF	
0299	0126		ØTT	REF	
0300	0127		ØTL	REF	
0301	0128		IKB	REF	
0302	0129		CRLF	REF	

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0303	012A		ERR	REF	
0304	012B	0000	TETR	DATA	0
0305	012C	0000	TEDA	DATA	0
0306	012D	0FF0	TCT1	DATA	4080
0307	012E	0000	DIR1	DATA	0
0308	012F	0000	TENT	DATA	0
0309	0130	8D3A	DI RE	DATA	:8D3A
0310	0131	C6CF		TEXT	'FØRWARD, BACKWARD, ØR BØTH?'
	0132	D2D7			
	0133	C1D2			
	0134	C4AC			
	0135	C2C1			
	0136	C3CB			
	0137	D7C1			
	0138	D2C4			
	0139	ACCF			
	013A	D2A0			
	013B	C2CF			
	013C	D4C8			
	013D	BFA0			
0311	013E	8D3A	MØTA	DATA	:8D3A
0312	013F	C6D2		TEXT	'FROM MØNSANTØ ØR FROM TAPE?'
	0140	CFCD			
	0141	A0CD			
	0142	CFCE			
	0143	D3C1			
	0144	CED4			
	0145	CFA0			
	0146	CFD2			
	0147	A0C6			
	0148	D2CF			
	0149	CDA0			
	014A	D4C1			
	014B	DOC5			
	014C	BFA0			
0313				END	
0000	ERRØRS				

0001				NAM	DATUM, RTDA
0002				EXTR	DISD
0003	0000	0300	DATUM	ENT	
0004	0001	C6BF		LAP	'?'
0005	0002	FB63		JST	*ØTL
0006	0003	0067		DATA	DIRE
0007	0004	FB71		JST	*IKB
0008	0005	C0C6		CAI	'F'
0009	0006	F205		JMP	FØRD
0010	0007	C0C2		CAI	'B'
0011	0008	F208		JMP	BACK
0012	0009	C0CF		CAI	'Ø'
0013	000A	F20A		JMP	BØTH
0014	000B	FB6C		JST	*ERR
0015	000C	C71E	FØRD	LAM	30
0016	000D	9A73		STA	DNT2
0017	000E	0108		ZXR	
0018	000F	0408		CXR	
0019	0010	F207		JMP	PKF
0020	0011	C71E	BACK	LAM	30
0021	0012	9A6E		STA	DNT2
0022	0013	0108		ZXR	
0023	0014	F203		JMP	PKF
0024	0015	C70F	BØTH	LAM	15
0025	0016	9A6A		STA	DNT2
0026	0017	0353		AXP	
0027	0018	EA60	PKF	STX	DIR1
0028	0019	B260		LDA	ZCT1
0029	001A	9A60		STA	CT1
0030	001B	E25B		LDX	IBC
0031	001C	EA5F		STX	IB
0032	001D	0110		ZAR	
0033	001E	9C00	CLR	STA	@0
0034	001F	0128		IXR	
0035	0020	DA5A		IMS	CT1
0036	0021	F603		JMP	CLR
0037	0022	9A61		STA	RØN
0038	0023	0210	NEXT	CAR	
0039	0024	9A58		STA	DIR0
0040	0025	C7F5		LAM	245
0041	0026	9A58		STA	DDT2
0042	0027	C7FF		LAM	255
0043	0028	9A55		STA	DDT
0044	0029	FA19		JST	DURNØN
0045	002A	FA23	DLP	JST	DØNS
0046	002B	9A54		STA	DAT
0047	002C	B24C		LDA	DIR1
0048	002D	C001		CAI	1
0049	002E	F203		JMP	STØ
0050	002F	D24D		GMS	DIR0
0051	0030	F232		JMP	DLP2
0052	0031	F231		JMP	DLP2
0053	0032	B24D	STØ	LDA	DAT

0054	0033	E248		L DX	IB
0055	0034	9C00		STA	@0
0056	0035	DA46		IMS	IB
0057	0036	DA48		IMS	DDT2
0058	0037	F201		JMP	S+2
0059	0038	40C7		SEL	24, 7
0060	0039	DA44		IMS	DDT
0061	003A	F610		JMP	DLP
0062	003B	DA48		IMS	RØN
0063	003C	B247	DLP1	L DA	RØN
0064	003D	2101		JAZ	S+2
0065	003E	F100		JMP	DISD
		0000			
0066	003F	B23D	RTDA	L DA	DIRO
0067	0040	DA40		IMS	DNT2
0068	0041	F61E		JMP	NEXT
0069	0042	F742		RTN	DATUM
0070	0043	0800	DURNØN	ENT	
0071	0044	4006		CI D	
0072	0045	48C1		SSN	: C1
0073	0046	F601		JMP	S- 1
0074	0047	0E00		SBM	
0075	0048	49C1		SEN	: C1
0076	0049	F601		JMP	S- 1
0077	004A	0F00		SWM	
0078	004B	40C4		SEL	: C4
0079	004C	4005		CI E	
0080	004D	F70A		RTN	DURNØN
0081	004E	0800	DØN S	ENT	
0082	004F	49C6		SEN	: C6
0083	0050	F601		JMP	S- 1
0084	0051	5AC6		INX	: C6
0085	0052	2843		JXZ	S- 3
0086	0053	C704		LAM	4
0087	0054	9A2D		STA	CNT
0088	0055	0110		ZAR	
0089	0056	9A2C		STA	MSK2
0090	0057	F206		JMP	112
0091	0058	1350	M1	LLA	1
0092	0059	9A29		STA	MSK2
0093	005A	1351		LLA	2
0094	005B	8A27		ADD	MSK2
0095	005C	9A26		STA	MSK2
0096	005D	0110		ZAR	
0097	005E	1B03	M2	LLL	4
0098	005F	8A23		ADD	MSK2
0099	0060	DA21		IMS	CNT
0100	0061	F609		JMP	M1
0101	0062	F714		RTN	DØN S
0102	0063	DA1A	DLP2	IMS	DDT
0103	0064	F63A		JMP	DLP
0104	0065	F629		JMP	DLP1
0105	0066		ØTL	REF	

0106	0067	8 D3 A	DI RE	DATA	: 8 D3 A
0107	0068	C6CF		TEXT	' FØRWARD, BACKWARD ØR BØTH? '
	0069	D2D7			-
	006A	C1D2			
	006B	C4AC			
	006C	C2C1			
	006D	C3CB			
	006E	D7C1			
	006F	D2C4			
	0070	A0CF			
	0071	D2A0			
	0072	C2CF			
	0073	D4C8			
	0074	BFA0			
0108	0075		CRLF	REF	
0109	0076		IKB	REF	
0110	0077		IBC	REF	
0111	0078		ERR	REF	
0112	0079	0000	DIR1	DATA	0
0113	007A	F00B	ZCT1	DATA	- 408 5
0114	007B	0000	CT1	DATA	0
0115	007C	0000	IB	DATA	0
0116	007D	0000	DIR0	DATA	0
0117	007E	0000	DDT	DATA	0
0118	007F	0000	DDT2	DATA	0
0119	0080	0000	DAT	DATA	0
0120	0081	0000	DNT2	DATA	0
0121	0082	0000	CNT	DATA	0
0122	0083	0000	MSK2	DATA	0
0123	0084	0000	RØN	DATA	0
0124				END	
0000	ERRØRS				

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0001          NAM      IKB, I ER, BIPT, XEQ
0002          NAM      ØTT, ERR, CRLF
0003          NAM      ØTL, ØFPA, ØDEC
0004 0000      REL      0
0005          *
0006          * PANIC SWITCH
0007          *
0008 0000  FB00      JST      *CMND
0009 0001      CMND    REF
0010          *
0011          *KEYBOARD INPUT
0012          *
0013 0002  0800      IKB      ENT
0014 0003  4038          SEL      7, 0      AUTO-ECHO
0015 0004  4039          SEL      7, 1      KBD MODE
0016 0005  5939          RDA      7, 1      READ ON FLAG
0017 0006  403C          SEL      7, 4      RESET
0018 0007  F705          RTN      IKB
0019 0003  0800      I ER     ENT
0020 0009  FE07          JST      IKB
0021 000A  CODF          CAI      : DF      BACK ARROW?
0022 000B  FFOA          JST      *CMND    YES
0023 000C  C08A          CAI      : 8A      LINE FEED?
0024 000D  FFOC          JST      *CMND    YES
0025 000E  F706          RTN      I ER     NEITHER, RETURN
0026          *
0027          *PAPER TAPE INPUT
0028          * DSO = 0 FØR TTY
0029          * 1 FØR HSR
0030          *
0031 000F  0800      BIPT     ENT
0032 0010  5801      BIP2     ISA          READ SWITCHES
0033 0011  13D0          LRA      1          DSO UP FØR TTY
0034 0012  220A          JØS      HSR      DØWN FØR HSR
0035 0013  493B          SEN      7, 3      TTY BUSY?
0036 0014  F604          JMP      BIP2     YES
0037 0015  403A          SEL      7, 2      NØ, STEP READER
0038 0016  4839      WT      SSN      7, 1      FLAG?
0039 0017  F203          JMP      IT          YES
0040 0018  0150          IAR          NØ, BUMP CØUNT
0041 0019  2149          JAZ      BIP2     RESTART IF TIME UP
0042 001A  F604          JMP      WT          ELSE CHECK FLAG AGAIN
0043 001B  5838      IT      INA      7, 0      INPUT FRØM TTY
0044 001C  F70D          RTN      BIPT     & RETURN
0045 001D  4933      HSR     SEN      6, 3      HSR BUSY?
0046 001E  F60E          JIIP     BIP2     YES
0047 001F  4032          SEL      6, 2      NØ, STEP READER
0048 0020  4835      WH      SSN      6, 5      FLAG?
0049 0021  F203          JMP      IH          YES
0050 0022  0150          IAR          NØ, BUMP CØUNT
0051 0023  2153          JAZ      BIP2     RESTART IF TIME UP
0052 0024  F604          JMP      WH          ELSE CHECK FLAG AGAIN
0053 0025  5835      IH      INA      6, 5      INPUT FRØM HSR

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0054 0026 F717          RTN      BIPT      & RETURN
0055                      *
0056                      *WAIT FOR EXECUTE SIGNAL
0057                      *
0058 0027 0800 XEQ      ENT
0059 0028 FE20          JST      IER      INPUT
0060 0029 C03D          CAI      :8D      CARRIAGE RETURN?
0061 002A F703          RTN      XEQ      YES, RETURN
0062 002B F603          JMP      XEQ+1    NO, GET MORE
0063                      *
0064                      *OUTPUT TO TTY
0065                      *
0066 002C 0800 ØTT     ENT
0067 002D 403C          SEL      7, 4    RESET INTERFACE
0068 002E 6D3B          WRA      7, 3    WRITE ON NOT BUSY
0069 002F 493B          SEN      7, 3    DONE?
0070 0030 F601          JMP      5-1     NO
0071 0031 F705          RTN      ØTT     YES
0072                      *
0073                      *COMMAND ERROR EXIT
0074                      *
0075 0032 0800 ERR     ENT
0076 0033 C6DF          LAP      :DF      PRINT ARROW
0077 0034 FE03          JST      ØTT
0078 0035 FF34          JST      *CMD    RESTART COMMAND
0079                      *
0080                      *CARRIAGE-RETURN, LINE FEED
0081                      *
0082 0036 0800 CRLF    ENT
0083 0037 C68D          LAP      :8D      CR
0084 0033 FE0C          JST      ØTT
0085 0039 C68A          LAP      :8A      LF
0086 003A FE0E          JST      ØTT
0087 003B F705          RTN      CRLF
0088                      *
0089                      *OUTPUT TEXT FROM BUFFER
0090                      *
0091 003C 0300 ØTL     ENT
0092 003D 8A0F          ADD     CAI      MAKE COMPARE INSTRUCTION
0093 003E 9A06          STA     ØT2     &SAVE IT
0094 003F 9A09          STA     ØT3
0095 0040 E704          LDK     *ØTL    GET TEXT POINTER
0096 0041 DE05          IMS     ØTL    SET RETURN ADDRESS
0097 0042 B400 ØT1     LDA     00      GET WORD
0098 0043 11D7          RRA     8       PRINT FIRST BYTE
0099 0044 FE13          JST     ØTT
0100 0045 C000 ØT2     CAI     0       LAST ONE
0101 0046 F70A          RTN     ØTL    YES, RETURN
0102 0047 1157          RLA     8       PRINT SECOND BYTE
0103 0048 FE1C          JST     ØTT
0104 0049 C000 ØT3     CAI     0       LAST ONE?
0105 004A F70E          RTN     ØTL    YES RETURN
0106 004B 0128          IXR

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0107 004C F60A      JMP      ØT1 .      LOOP
0108 004D C000  CAI      CAI      0
0109
0110          *ØOUTPUT FLOATING POINT NUMBER
0111          *
0112 004E 0800  ØFPA  ENT
0113 004F E701          LDX      *ØFPA  GET POINTER
0114 0050 DE02          IMS      ØFPA  SET RETURN ADDRESS
0115 0051 EA01          STX      ØPT   SAVE POINTER
0116 0052 FB0E          JST      *FAS  CONVERT TO ASCII
0117 0053 0000  ØPT   DATA  0
0118 0054 0059          DATA  BUF
0119 0055 0110          ZAR
0120 0056 FE1A          JST      ØTL   PRINT NUMBER
0121 0057 0059          DATA  BUF
0122 0058 F70A          RTN      ØFPA
0123 0059 0000  BUF   RES      8,0
0124 0061          FAS      REF
0125          *
0126          * ØDEC ØUTPUT DECIMAL (+/-DDDDD)
0127          ** ØDEC CONVERTS THE BINARY VALUE IN THE
0128          *A REG AND PRINTS IT AS A SIGNED 5
0129          * DIGIT DECIMAL NUMBER ON THE TELETYPE.
0130          * AR AND OV ARE DESTROYED.
0131          *
0132          * LDA VAL AR = VALUE
0133          * SWM MUST BE IN WORD MODE
0134          * JST *ØDEC CALL ROUTINE
0135          * *** RETURN XR UNCHANGED
0136 0062 0800  ØDEC  ENT
0137 0063 EA19          STX      S      SAVE XR
0138 0064 C4AB          LXP      '+'
0139 0065 3082          JAP      $+3
0140 0066 0310          NAR
0141 0067 C202          AXI      2      MAKE VALUE +
0142 0068 9A15          STA      V      MAKE SIGN -
0143 0069 0030          TXA
0144 006A FE3E          JST      ØTT   PRINT SIGN
0145 006B B215          LDA      STRT
0146 006C 9A13          STA      PTR   INITIALIZE TEL PTR
0147 006D C705          LAM      5
0148 006E 9A10          STA      T      SET FOR 5 DIGITS
0149 006F B20E  Ø1   LDA      V
0150 0070 C4AF          LXP      :AF   ZERO TO -1
0151 0071 930E          SUB      *PTR
0152 0072 0128          IXR
0153 0073 30C2          JAP      $-2
0154 0074 8B0B          ADD      *PTR
0155 0075 9A08          STA      V
0156 0076 0030          TXA
0157 0077 FE4B          JST      ØTT   PRINT DIGIT
0158 0078 DA07          IMS      PTR
0159 0079 DA05          IMS      T

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0160 007A F60B      JMP      01
0161 007B E201      LDX     S      RESTORE XR
0162 007C F71A      RTN     0DEC    RETURN
0163 007D 0800      S       HLT     TEMP FOR XR
0164 007E 0800      V       HLT     VALUE
0165 007F 0800      T       HLT     COUNT
0166 0030 0800      PTR     HLT     POINTER
0167 0031 0032      STRT   DATA   TBL     TABLE ADDR
0168 0032 2710      TBL    DATA   10000, 1000, 100
      0033 03E3
      0034 0064
0169 0035 000A      DATA   10, 1
      0036 0001
0170                      END
0000 ERRORS

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0001			NAM	READ, PUNCH, CLEAR	
0002			EXTR	1KB, ØTT, BIPT, ØTL, CRLF, IER, ERR	
0003	0000		REL	0	
0004	0000	0800	READ	ENT	
0005	0001	F900		JST	CRLF
		0000			
0006	0002	F900		JST	IER
		0000			
0007	0003	C0B0		CAI	'0'
0008	0004	F203		JMP	REZE
0009	0005	C0C7		CAI	'G'
0010	0006	F204		JMP	RESN
0011	0007	F900		JST	ERR
		0000			
0012	0008	E21F	REZE	LDA	IBC
0013	0009	B21F		LDA	ZCT0
0014	000A	F204		JMP	RDL P
0015	000B	B21C	RESN	LDA	IBC
0016	000C	3A1E		ADD	SNO
0017	000D	0048		TAX	
0018	000E	B21B		LDA	ZCT1
0019	000F	9A1C	RDL P	STA	CNT
0020	0010	C68A		LAP	:8A
0021	0011	F900		JST	ØTT
		0000			
0022	0012	F900		JST	BIPT
		0000			
0023	0013	2141		JAZ	\$-1
0024	0014	F900	R1	JST	ØTT
		0000			
0025	0015	F900		JST	BIPT
		0000			
0026	0016	C092		CAI	:92
0027	0017	F201		JMP	R2
0028	0018	F604		JMP	R1
0029	0019	F900	R2	JST	BIPT
		0000			
0030	001A	C0FF		CAI	:FF
0031	001B	F201		JMP	R3
0032	001C	F603		JMP	R2
0033	001D	F900	R3	JST	BIPT
		0000			
0034	001E	1357		LLA	8
0035	001F	9A0E		STA	TEMP
0036	0020	F900		JST	BIPT
		0000			
0037	0021	A20C		IØR	TEMP
0038	0022	8C00		ADD	@0
0039	0023	9C00		STA	@0
0040	0024	0128		IXR	
0041	0025	DA06		IMS	CNT
0042	0026	F609		JMP	R3
0043	0027	F727		RTN	READ

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0044	0028		IBC	REF		
0045	0029	F010	ZCT0	DATA	-4030	
0046	002A	FF01	ZCT1	DATA	-255	
0047	002B	0EF1	SNO	DATA	3325	
0048	002C	0000	CNT	DATA	0	
0049	002D	0000	PNT	DATA	0	
0050	002E	0000	TEMP	DATA	0	
0051			*			
0052			* PUNCH			
0053	002F	0800	PUNCH	ENT		
0054	0030	F900		JST	CRLF	
		0000				
0055	0031	F90C		JST	IER	
		0000				
0056	0032	C0B0		CAI	'0'	
0057	0033	F203		JMP	PNZE	
0058	0034	C0C7		CAI	'G'	
0059	0035	F204		JMP	PN SN	
0060	0036	F900		JST	ERR	
		0000				
0061	0037	E60F	PNZE	L DX	IBC	
0062	0038	B60F		L DA	ZCT0	
0063	0039	F204		JMP	PNLP	
0064	003A	B612	PN SN	L DA	IBC	
0065	003B	8E10		ADD	SNO	
0066	003C	0048		TAX		
0067	003D	B613		L DA	ZCT1	
0068	003E	9E12	PNLP	STA	CNT	
0069	003F	FA0D		JST	LEAD	
0070	0040	F900	P1	JST	IKB	ECHO LABEL
		0000				
0071	0041	C092		CAI	:92	CTRL/TAPE
0072	0042	F201		JMP	P2	YES
0073	0043	F603		JMP	P1	NØ
0074	0044	C6FF	P2	LAP	:FF	PUNCH FILE MARK
0075	0045	F900		JST	ØTT	
		0000				
0076	0046	B400	P3	L DA	@0	PUNCH BITS
0077	0047	FA0D		JST	ØTW	
0078	0048	0128		IXR		
0079	0049	DE1D		IMS	CNT	
0080	004A	F604		JMP	P3	
0081	004B	FA01		JST	LEAD	
0082	004C	F71D		RTN	PUNCH	
0083			*PUNCH	'5" ØF	LEADER	
0084	004D	0800	LEAD	ENT		
0085	004E	C732		LAM	50	
0086	004F	9E22		STA	PNT	
0087	0050	0110		ZAR		
0088	0051	F900	L2	JST	ØTT	
		0000				
0089	0052	DE25		IMS	PNT	
0090	0053	F602		JMP	L2	

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009 1	0054	F707		RTN	LEAD
009 2			*		
009 3	0055	03 00	ØTW	ENT	
009 4	0056	11D7		RRA	8
009 5	0057	F9 00		JST	ØTT
		0000			
009 6	0058	1157		RLA	8
009 7	0059	F9 00		JST	ØTT
		0000			
009 8	005A	F705		RTN	ØTW
009 9			*CLEAR	PROGRAM	
0100	005B	08 00	CLEAR	ENT	
0101	005C	F9 00		JST	CRLF
		0000			
0102	005D	F9 00		JST	IER
		0000			
0103	005E	C0B0		CAI	'0'
0104	005F	F203		JMP	CLZE
0105	0060	C0C7		CAI	'G'
0106	0061	F204		JMP	CLSN
0107	0062	F9 00		JST	ERR
		0000			
0108	0063	E63B	CLZE	L DX	IBC
0109	0064	B63B		L DA	Z CTO
0110	0065	F204		JMP	CLLP
0111	0066	B63E	CLSN	L DA	IBC
0112	0067	8E3C		ADD	SNO
0113	0068	0048		TAX	
0114	0069	B63F		L DA	Z CT I
0115	006A	9E3E	CLLP	STA	CNT
0116	006B	0110		ZAR	
0117	006C	9C00	CR	STA	@0
0118	006D	0128		IXR	
0119	006E	DE42		IMS	CNT
0120	006F	F603		JMP	CR
0121	0070	F715		RTN	CLEAR
0122				END	
0000	ERRORS				

0001				NAM	FPLØT
0002	0000			REL	0
0003	0000	05 00	FPLØT	ENT	
0004	0001	C70F		LAM	15
0005	0002	9ABB		STA	NRØW
0006	0003	B2D0		LDA	IBC
0007	0004	9ABA		STA	IB
0008	0005	0108		ZXR	
0009	0006	FBCF		JST	*CRLF
0010	0007	FBD1		JST	*IER
0011	0008	COB0		CAI	'0'
0012	0009	F221		JMP	FZ RØ
0013	000A	COB1		CAI	'1'
0014	000B	F222		JMP	FØNE
0015	000C	COB2		CAI	'2'
0016	000D	F222		JMP	FTWØ
0017	000E	COB3		CAI	'3'
0018	000F	F222		JMP	FTHRE
0019	0010	COB4		CAI	'4'
0020	0011	F222		JMP	FFØUR
0021	0012	COB5		CAI	'5'
0022	0013	F222		JMP	FFIVE
0023	0014	COB6		CAI	'6'
0024	0015	F222		JMP	FSIX
0025	0016	COB7		CAI	'7'
0026	0017	F222		JMP	FSEVE
0027	0018	COB8		CAI	'8'
0028	0019	F222		JMP	FEIGH
0029	001A	COB9		CAI	'9'
0030	001B	F222		JMP	FNINE
0031	001C	COC1		CAI	'A'
0032	001D	F222		JMP	FTEN
0033	001E	COC2		CAI	'B'
0034	001F	F222		JMP	FEL E
0035	0020	COC3		CAI	'C'
0036	0021	F222		JMP	FTWVE
0037	0022	COC4		CAI	'D'
0038	0023	F222		JMP	FTHD
0039	0024	COC5		CAI	'E'
0040	0025	F222		JMP	FFRTN
0041	0026	COC6		CAI	'F'
0042	0027	F222		JMP	FFØTN
0043	0028	COC7		CAI	'G'
0044	0029	F222		JMP	FSX TN
0045	002A	FBAF		JST	*ERR
0046	002B	0110	FZ RØ	ZAR	
0047	002C	9A9C		STA	CALL
0048	002D	F22A		JMP	FØØP1
0049	002E	C601	FØNE	LAP	1
0050	002F	F21E		JMP	FSTØ
0051	0030	C602	FTWØ	LAP	2
0052	0031	F21C		JMP	FSTØ
0053	0032	C603	FTHRE	LAP	3

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0054	0033	F21A		JMP	FSTØ
0055	0034	C604	FFØUR	LAP	4
0056	0035	F218		JMP	FSTØ
0057	0036	C605	FFIVE	LAP	5
0058	0037	F216		JMP	FSTØ
0059	0038	C606	FSIX	LAP	6
0060	0039	F214		JMP	FSTØ
0061	003A	C607	FSEVE	LAP	7
0062	003B	F212		JMP	FSTØ
0063	003C	C608	FEIGH	LAP	8
0064	003D	F210		JMP	FSTØ
0065	003E	C609	FNINE	LAP	9
0066	003F	F20E		JMP	FSTØ
0067	0040	C60A	FTEN	LAP	10
0068	0041	F20C		JMP	FSTØ
0069	0042	C60B	FEL E	LAP	11
0070	0043	F20A		JMP	FSTØ
0071	0044	C60C	FTWVE	LAP	12
0072	0045	F208		JMP	FSTØ
0073	0046	C60D	FTHD	LAP	13
0074	0047	F206		JMP	FSTØ
0075	0048	C60E	FFRTN	LAP	14
0076	0049	F204		JMP	FSTØ
0077	004A	C60F	FFVTN	LAP	15
0078	004B	F202		JMP	FSTØ
0079	004C	C610	FSXTN	LAP	16
0080	004D	F200		JMP	FSTØ
0081	004E	9A7A	FSTØ	STA	CALL
0082	004F	0310		NAR	
0083	0050	9A79		STA	FRØW
0084	0051	B26D		LDA	IB
0085	0052	DA77	FRØNØ	IMS	FRØW
0086	0053	F201		JMP	S+2
0087	0054	F202		JMP	S+3
0088	0055	8A75		ADD	H255
0089	0056	F604		JMP	FRØNØ
0090	0057	9A67		STA	IB
0091	0058	B270	FØØP1	LDA	CALL
0092	0059	2101		JAZ	S+2
0093	005A	F204		JMP	S+5
0094	005B	B26C		LDA	ZCT
0095	005C	0150		IAR	
0096	005D	9A63		STA	CNT
0097	005E	F203		JMP	S+4
0098	005F	C7FF		LAM	255
0099	0060	0150		IAR	
0100	0061	9A5F		STA	CNT
0101	0062	B25C		LDA	IB
0102	0063	9A5E		STA	IPB1
0103	0064	B35D		LDA	*IPB1
0104	0065	9A5E		STA	MAX
0105	0066	DA5B	LØØP2	IMS	IPB1
0106	0067	B35A		LDA	*IPB1

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0107	0068	D25B		CMS	MAX
0108	0069	F201		JMP	S+2
0109	006A	9A59		STA	MAX
0110	006B	DA55		IMS	CNT
0111	006C	F606		JMP	LOOP2
0112	006D	0350		ARP	
0113	006E	9A56		STA	SCALE
0114	006F	9A56		STA	ROUND
0115	0070	B256		LDA	H50
0116	0071	D252		CMS	MAX
0117	0072	F202		JMP	S+3
0118	0073	F20C		JMP	LOOP
0119	0074	F20B		JMP	LOOP
0120	0075	B24E		LDA	MAX
0121	0076	0103		ZXR	
0122	0077	924F		SUB	H50
0123	0078	0128		IXR	
0124	0079	31C2		JAG	S-2
0125	007A	EA4A		STX	SCALE
0126	007B	1200		R0V	
0127	007C	11A8		RRX	1
0128	007D	3201		J0R	S+2
0129	007E	0128		IXR	
0130	007F	EA46		STX	ROUND
0131	0080	FB55	LOOP	JST	*CRLF
0132	0081	FB54		JST	*CRLF
0133	0082	C6BD		LAP	'='
0134	0083	FB54		JST	*0TL
0135	0084	00CD		DATA	SF
0136	0085	C6A0		LAP	' '
0137	0086	FB50		JST	*0TT
0138	0087	B23D		LDA	SCALE
0139	0088	FB4C		JST	*0DEC
0140	0089	C601		LAP	1
0141	008A	9A41		STA	R0N0
0142	008B	FB4A		JST	*CRLF
0143	008C	FB49		JST	*CRLF
0144	008D	B23E		LDA	R0N0
0145	008E	FB48		JST	*0TT
0146	008F	C7FF		LAM	255
0147	0090	9A2F		STA	NC0L
0148	0091	B22D		LDA	IB
0149	0092	9A30		STA	IPB2
0150	0093	FB42	LOOP1	JST	*CRLF
0151	0094	B32E		LDA	*IPB2
0152	0095	FB3F		JST	*0DEC
0153	0096	C6A0		LAP	' '
0154	0097	FB3F		JST	*0TT
0155	0098	C6B0		LAP	'0'
0156	0099	FB3D		JST	*0TT
0157	009A	B323		LDA	*IPB2
0158	009B	2192		JAL	CLOSE
0159	009C	0103		ZXR	

0160	009D	9227		SUB	SCALE
0161	009E	0128		IXR	
0162	009F	31C2		JAG	S-2
0163	00A0	8A24		ADD	SCALE
0164	00A1	00A3		DXR	
0165	00A2	D223		CMS	R0UND
0166	00A3	F202		JMP	S+3
0167	00A4	0000		N0P	
0168	00A5	0128		IXR	
0169	00A6	0030		TXA	
0170	00A7	2186		JAL	CL0SE
0171	00A3	C6AA		LAP	'*'
0172	00A9	0503		NXR	- -
0173	00AA	EA16		STX	CNT
0174	00AB	FB2B		JST	*0TT
0175	00AC	DA14		IMS	CNT
0176	00AD	F602		JMP	S-2
0177	00AE	DA14	CL0SE	IMS	IPB2
0178	00AF	0000		N0P	
0179	00B0	DA0F		IMS	NC0L
0180	00B1	F61E		JMP	L00P1
0181	00B2	FB23		JST	*CRLF
0182	00B3	B215		LDA	CALL
0183	00B4	2101		JAZ	S+2
0184	00B5	F206		JMP	FSH
0185	00B6	C6FF		LAP	255
0186	00B7	8A07		ADD	IB
0187	00B8	9A06		STA	IB
0188	00B9	DA12		IMS	R0N0
0189	00BA	DA03		IMS	NR0W
0190	00BB	F628		JMP	L00P1
0191	00BC	FE19	FSH	JST	*CRLF
0192	00BD	F7BD		RTN	FPL0T
0193	00BE	0000	NR0W	DATA	0
0194	00BF	0000	IB	DATA	0
0195	00C0	0000	NC0L	DATA	0
0196	00C1	0000	CNT	DATA	0
0197	00C2	0000	IPB1	DATA	0
0198	00C3	0000	IPB2	DATA	0
0199	00C4	0000	MAX	DATA	0
0200	00C5	0000	SCALE	DATA	0
0201	00C6	0000	R0UND	DATA	0
0202	00C7	0032	H50	DATA	50
0203	00C8	F10F	ZCT	DATA	-3825
0204	00C9	0000	CALL	DATA	0
0205	00CA	0000	FR0W	DATA	0
0206	00CB	00FF	H255	DATA	255
0207	00CC	0000	R0N0	DATA	0
0208	00CD	D3C3	SF	TEXT	'SCALE FACT0R='
	00CE	C1CC			-
	00CF	C5A0			
	00D0	C6C1			
	00D1	C3D4			

	00D2	CFD2		
	00D3	BDA0		
0209	00D4		IBC	REF
0210	00D5		ØDEC	REF
0211	00D6		CRLF	REF
0212	00D7		ØTT	REF
0213	00D8		ØTL	REF
0214	00D9		IER	REF
0215	00DA		ERR	REF
0216				END
0000	ERRØRS			

0001				NAM	CRT
0002				NAM	DISP
0003				NAM	DISD
0004				EXTR	RTDA
0005				EXTR	RTTR
0006	0000			REL	0
0007	0000	0800	CRT	ENT	
0008	0001	C70F		LAM	15
0009	0002	9900		STA	NRØW
		0112			
0010	0003	49C4		SEN	24, 4
0011	0004	F601		JMP	S- 1
0012	0005	40C7		SEL	24, 7
0013	0006	0110		ZAR	
0014	0007	9900		STA	ØFFSET
		0113			
0015	0008	C601		LAP	1
0016	0009	9900	DISP	STA	TR1
		0116			
0017	000A	3106		JAN	S+7
0018	000B	B100		LDA	AD16
		0126			
0019	000C	8900		ADD	IBC
		0133			
0020	000D	9900		STA	IB
		0114			
0021	000E	0110		ZAR	
0022	000F	9900		STA	CALL
		0117			
0023	0010	F25F		JMP	MXMI
0024	0011	B100		LDA	IBC
		0133			
0025	0012	9900		STA	IB
		0114			
0026	0013	0103		ZXR	
0027	0014	F900		JST	*CRLF
		8133			
0028	0015	F900		JST	*IER
		8134			
0029	0016	COB0		CAI	'0'
0030	0017	F221		JMP	ZERØ
0031	0018	COB1		CAI	'1'
0032	0019	F222		JMP	ØNE
0033	001A	COB2		CAI	'2'
0034	001B	F222		JMP	TWØ
0035	001C	COB3		CAI	'3'
0036	001D	F222		JMP	THREE
0037	001E	COB4		CAI	'4'
0038	001F	F222		JMP	FØUR
0039	0020	COB5		CAI	'5'
0040	0021	F222		JMP	FIVE
0041	0022	COB6		CAI	'6'
0042	0023	F222		JMP	SIX

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0043	0024	COB7		CAI	'7'
0044	0025	F222		JMP	SEVEN
0045	0026	COB3		CAI	'8'
0046	0027	F222		JMP	EIGHT
0047	0028	COB9		CAI	'9'
0048	0029	F222		JMP	NINE
0049	002A	COC1		CAI	'A'
0050	002B	F222		JMP	TEN
0051	002C	COC2		CAI	'B'
0052	002D	F222		JMP	ELE
0053	002E	COC3		CAI	'C'
0054	002F	F222		JMP	TWVE
0055	0030	COC4		CAI	'D'
0056	0031	F222		JMP	THD
0057	0032	COC5		CAI	'E'
0058	0033	F222		JMP	FRTN
0059	0034	COC6		CAI	'F'
0060	0035	F222		JMP	FVTN
0061	0036	COC7		CAI	'G'
0062	0037	F222		JMP	SXTN
0063	0038	FBFC		JST	*ERR
0064	0039	0110	ZERØ	ZAR	
0065	003A	9ADC		STA	CALL
0066	003B	F22E		JMP	LØØP1
0067	003C	C601	ØNE	LAP	1
0068	003D	F222		JMP	STØRE
0069	003E	C602	TWØ	LAP	2
0070	003F	F220		JMP	STØRE
0071	0040	C603	THREE	LAP	3
0072	0041	F21E		JMP	STØRE
0073	0042	C604	FØUR	LAP	4
0074	0043	F21C		JMP	STØRE
0075	0044	C605	FIVE	LAP	5
0076	0045	F21A		JMP	STØRE
0077	0046	C606	SIX	LAP	6
0078	0047	F218		JMP	STØRE
0079	0048	C607	SEVEN	LAP	7
0080	0049	F216		JMP	STØRE
0081	004A	C603	EI GHT	LAP	8
0082	004B	F214		JMP	STØRE
0083	004C	C609	NINE	LAP	9
0084	004D	F212		JMP	STØRE
0085	004E	C60A	TEN	LAP	10
0086	004F	F210		JMP	STØRE
0087	0050	C60B	EL E	LAP	11
0088	0051	F20E		JMP	STØRE
0089	0052	C60C	TWVE	LAP	12
0090	0053	F20C		JMP	STØRE
0091	0054	C60D	THD	LAP	13
0092	0055	F20A		JMP	STØRE
0093	0056	C60E	FRTN	LAP	14
0094	0057	F208		JMP	STØRE
0095	0058	C60F	FVTN	LAP	15

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0096 0059 F206      JMP      STØRE
0097 005A C610      SX TN   LAP      16
0098 005B F204      JMP      STØRE
0099 005C 0103      DI SD   ZXR
0100 005D EAB3          STX      TR1
0101 005E E2D4          LDX      IBC
0102 005F EAB4          STX      IB
0103 0060 9AB6      STØRE   STA      CALL
0104 0061 0310          NAR
0105 0062 9AB5          STA      RØWVA
0106 0063 B2B0          LDA      IB
0107 0064 DAB3      RØWNØ   IMS      RØWVA
0108 0065 F201          JMP      S+2
0109 0066 F202          JMP      S+3
0110 0067 8ABB          ADD      H255
0111 0068 F604          JMP      RØWNØ
0112 0069 9AAA          STA      IB
0113 006A B2AC      LØØP1   LDA      CALL
0114 006B 2101          JAZ      S+2
0115 006C F203          JMP      S+4
0116 006D B2AC          LDA      ZCT
0117 006E 9AAC          STA      CNT
0118 006F F202          JMP      S+3
0119 0070 C7FF      MXMI    LAM      255
0120 0071 9AA9          STA      CNT
0121 0072 B2A1          LDA      IB
0122 0073 9AA8          STA      IBP1
0123 0074 0110          ZAR
0124 0075 9AA3          STA      MIN
0125 0076 9AA8          STA      MAX
0126 0077 B3A4      LØØP2   LDA      *IBP1
0127 0078 D2A6          CMS      MAX
0128 0079 F202          JMP      S+3
0129 007A 9AA4          STA      MAX
0130 007B F203          JMP      S+4
0131 007C D2A1          CMS      MIN
0132 007D 9AA0          STA      MIN
0133 007E 0000          NØP
0134 007F DA9C          IMS      IBP1
0135 0080 DA9A          IMS      CNT
0136 0081 F60A          JMP      LØØP2
0137 0082 0350          ARP
0138 0083 9A9C          STA      SCALE
0139 0084 9A9C          STA      RØUND
0140 0085 B299          LDA      MAX
0141 0086 9297          SUB      MIN
0142 0087 9A97          STA      MAX
0143 0088 B28D          LDA      TR1
0144 0089 2106          JAZ      SCA
0145 008A B28C          LDA      CALL
0146 008B 2101          JAZ      S+2
0147 008C F203          JMP      S+4
0148 008D B294          LDA      H100

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0149	008E	9A98		STA	T0TD0T
0150	008F	F202		JMP	S+3
0151	0090	B293	SCA	LDA	H200
0152	0091	9A95		STA	T0TD0T
0153	0092	B294		LDA	T0TD0T
0154	0093	D28B		CMS	MAX
0155	0094	F202		JMP	S+3
0156	0095	F20C		JMP	L00P
0157	0096	F20B		JMP	L00P
0158	0097	B287		LDA	MAX
0159	0098	0108		ZXR	
0160	0099	928D		SUB	T0TD0T
0161	009A	0128		IXR	
0162	009B	31C2		JAG	S-2
0163	009C	EA83		STX	SCALE
0164	009D	1200		R0V	
0165	009E	11A3		RFX	1
0166	009F	3201		J0R	S+2
0167	00A0	0128		IXR	
0168	00A1	EA7F		STX	R0UND
0169	00A2	B273	L00P	LDA	TR1
0170	00A3	210B		JAZ	MI
0171	00A4	FB93		JST	*CRLF
0172	00A5	FB92		JST	*CRLF
0173	00A6	C6BD		LAP	'='
0174	00A7	FB8F		JST	*0TL
0175	00A8	012C		DATA	SF
0176	00A9	C6A0		LAP	' '
0177	00AA	FB8B		JST	*0TT
0178	00AB	B274		LDA	SCALE
0179	00AC	FB8C		JST	*0DEC
0180	00AD	FB8A		JST	*CRLF
0181	00AE	FB89		JST	*CRLF
0182	00AF	B26E	MI	LDA	MIN
0183	00B0	0310		NAR	
0184	00B1	0108		ZXR	
0185	00B2	926D		SUB	SCALE
0186	00B3	0128		IXR	
0187	00B4	31C2		JAG	S-2
0188	00B5	8A6A		ADD	SCALE
0189	00B6	00A8		DXR	
0190	00B7	D269		CMS	R0UND
0191	00B8	F202		JMP	S+3
0192	00B9	0000		N0P	
0193	00BA	0128		IXR	
0194	00BB	EA6C		STX	X1
0195	00BC	B26B	L00P4	LDA	X1
0196	00BD	9A6B		STA	X
0197	00BE	C7FF		LAM	255
0198	00BF	9A59		STA	NC0L
0199	00C0	B253		LDA	1B
0200	00C1	9A5B		STA	1BP2
0201	00C2	B250	L00P3	LDA	0FFSET

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0202	00C3	8A65		ADD	X
0203	00C4	9A66		STA	TØTØFF
0204	00C5	B357		LDA	*IBP2
0205	00C6	1200		RØV	
0206	00C7	1150		RLA	1
0207	00C8	2207		JØS	NEG
0208	00C9	11D0		RRA	1
0209	00CA	0108		ZXR	
0210	00CB	9254		SUB	SCALE
0211	00CC	0128		IXR	
0212	00CD	31C2		JAG	S-2
0213	00CE	8A51		ADD	SCALE
0214	00CF	F206		JMP	S+7
0215	00D0	1400	NEG	SØV	
0216	00D1	11D0		RRA	1
0217	00D2	0108		ZXR	
0218	00D3	8A4C		ADD	SCALE
0219	00D4	00A8		DXR	
0220	00D5	20C2		JAM	S-2
0221	00D6	9249		SUB	SCALE
0222	00D7	00A8		DXR	
0223	00D8	D248		CMS	RØUND
0224	00D9	F202		JMP	S+3
0225	00DA	0000		NØP	
0226	00DB	0128		IXR	
0227	00DC	0030		TXA	
0228	00DD	9A4C		STA	DØTNØ
0229	00DE	E24C		LDX	TØTØFF
0230	00DF	49C4	DØT	SEN	24, 4
0231	00E0	F601		JMP	S-1
0232	00E1	49C2		SEN	24, 2
0233	00E2	F601		JMP	S-1
0234	00E3	6EC2		ØTX	24, 2
0235	00E4	B231		LDA	TR1
0236	00E5	2108		JAZ	S+9
0237	00E6	B230		LDA	CALL
0238	00E7	2101		JAZ	S+2
0239	00E8	F205		JMP	S+6
0240	00E9	B240		LDA	DØTNØ
0241	00EA	8A40		ADD	TØTØFF
0242	00EB	0048		TAX	
0243	00EC	6EC2		ØTX	24, 2
0244	00ED	F20B		JMP	DØNE
0245	00EE	B23B		LDA	DØTNØ
0246	00EF	2109		JAZ	DØNE
0247	00F0	3184		JAG	PØS
0248	00F1	00A8		DXR	
0249	00F2	0150		IAR	
0250	00F3	9A36		STA	DØTNØ
0251	00F4	F615		JMP	DØT
0252	00F5	0128	PØS	IXR	
0253	00F6	00D0		DAR	
0254	00F7	9A32		STA	DØTNØ

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0255	00F3	F619		JMP	DØT
0256	00F9	B22F	DØNE	LDA	X
0257	00FA	8A2A		ADD	H256
0258	00FB	9A2D		STA	X
0259	00FC	DA20		IMS	IBP2
0260	00FD	DA1B		IMS	NCØL
0261	00FE	F63C		JMP	LØØP3
0262	00FF	B216		LDA	TR1
0263	0100	3104		JAN	S+5
0264	0101	B215		LDA	CALL
0265	0102	2101		JAZ	S+2
0266	0103	F100		JMP	RTDA
		0000			
0267	0104	F100		JMP	RTTR
		0000			
0268	0105	B211		LDA	CALL
0269	0106	2101		JAZ	S+2
0270	0107	F203		JMP	FIN
0271	0108	C60A		LAP	10
0272	0109	8A09		ADD	ØFFSET
0273	010A	9A03		STA	ØFFSET
0274	010B	B217		LDA	H255
0275	010C	8A07		ADD	IB
0276	010D	9A06		STA	IB
0277	010E	DA03		IMS	NRØW
0278	010F	F653		JMP	LØØP4
0279	0110	FB27	FIN	JST	*CRLF
0280	0111	F100		RTN	CRT
		8000			
0281	0112	0000	NRØW	DATA	0
0282	0113	0000	ØFFSET	DATA	0
0283	0114	0000	IB	DATA	0
0284	0115	0000	IBQ	DATA	0
0285	0116	0000	TR1	DATA	0
0286	0117	0000	CALL	DATA	0
0287	0118	0000	RØWVA	DATA	0
0288	0119	0000	NCØL	DATA	0
0289	011A	F10E	ZCT	DATA	-3826
0290	011B	0000	CNT	DATA	0
0291	011C	0000	IBP1	DATA	0
0292	011D	0000	IBP2	DATA	0
0293	011E	0000	MIN	DATA	0
0294	011F	0000	MAX	DATA	0
0295	0120	0000	SCALE	DATA	0
0296	0121	0000	RØUND	DATA	0
0297	0122	0064	H100	DATA	100
0298	0123	00FF	H255	DATA	255
0299	0124	00C3	H200	DATA	200
0300	0125	0100	H256	DATA	256
0301	0126	0EF1	AD16	DATA	3825
0302	0127	0000	TØTDØT	DATA	0
0303	0128	0000	X1	DATA	0
0304	0129	0000	X	DATA	0

0305	012A	0000	DØTNØ	DATA	0
0306	012B	0000	TØTØFF	DATA	0
0307	012C	D3C3	SF	TEXT	'SCALE FACTØR='
	012D	C1CC			~
	012E	C5AØ			-
	012F	C6C1			
	0130	C3D4			
	0131	CFD2			
	0132	BDAØ			
0308	0133		IBC	REF	
0309	0134		IER	REF	
0310	0135		ERR	REF	
0311	0136		ØTT	REF	
0312	0137		ØTL	REF	
0313	0138		CRLF	REF	
0314	0139		ØDEC	REF	
0315				END	
0000	ERRØRS				

PAGE 0001

0001			NAM	MASKO
0002	0000		REL	0
0003	0000	0001	DATA	1
0004	0001	0001	DATA	1
0005	0002	0001	DATA	1
0006	0003	0001	DATA	1
0007	0004	FFFF	DATA	-1
0008	0005	0001	DATA	1
0009	0006	FFFF	DATA	-1
0010	0007	0001	DATA	1
0011	0008	0001	DATA	1
0012	0009	FFFF	DATA	-1
0013	000A	FFFF	DATA	-1
0014	000B	0001	DATA	1
0015	000C	FFFF	DATA	-1
0016	000D	FFFF	DATA	-1
0017	000E	FFFF	DATA	-1
0018	000F	0001	DATA	1
0019	0010	0001	DATA	1
0020	0011	0001	DATA	1
0021	0012	0001	DATA	1
0022	0013	FFFF	DATA	-1
0023	0014	0001	DATA	1
0024	0015	FFFF	DATA	-1
0025	0016	0001	DATA	1
0026	0017	0001	DATA	1
0027	0018	FFFF	DATA	-1
0028	0019	FFFF	DATA	-1
0029	001A	0001	DATA	1
0030	001B	FFFF	DATA	-1
0031	001C	FFFF	DATA	-1
0032			END	
0000	ERRORS			

0001				NAM	IBC
0002	0000			REL	0
0003	0000	0000	IBC	DATA	0000
0004				END	
0000	ERRORS				

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```
1      DEF FNR(X)=INT(100*X+0.5)/100
10     DIM Ø(255),F(255),X(255)
20     PRINT "SPECTRUM CORRECTED FOR NEGATIVE DIP"
25     PRINT
26     PRINT
30     CALL (20)
40     CALL (5,Ø(0),256,2)
50     LET F(0)=Ø(0)
60     FOR N=1 TO 255
70     LET I1=N-1
75     IF I1>0 THEN 85
80     LET I1=255
85     LET I2=N+1
90     IF I2<256 THEN 100
95     LET I2=I2-255
100    LET I3=N+24
105    IF I3<256 THEN 115
110    LET I3=I3-255
115    LET I4=N+25
120    IF I4<256 THEN 130
125    LET I4=I4-255
130    LET C=(Ø(I3)+Ø(I4)-Ø(I1)-Ø(I2))/20
140    LET F(N)=Ø(N)+C
150    NEXT N
160    PRINT "A: 1 FOR DISPLAY, 2 FOR GRAPH, 3 FOR TAPE"
170    PRINT "B: 0 FOR ORIGINAL, 1 FOR FINAL"
180    PRINT
200    PRINT "A=";
210    INPUT A
220    PRINT "B=";
230    INPUT B
240    PRINT "RESOLUTION D=";
250    INPUT D
260    PRINT "INITIAL WAVE LENGTH LO=";
270    INPUT LO
280    PRINT
290    PRINT
300    IF A=3 THEN 730
310    LET Z=0
320    LET M=1E10
330    FOR N=1 TO 255
```

```
350   IF B=1 THEN 400
360   LET X(N)=Ø(N)
370   GØTØ 410
400   LET X(N)=F(N)
410   IF X(N)>=M THEN 430
420   LET M=X(N)
430   IF X(N)<=Z THEN 450
440   LET Z=X(N)
450   NEXT N
460   IF M>=0 THEN 510
470   LET SØ=255/(Z-M)
480   LET S1=48/(Z-M)
490   LET YØ=-M*S
500   GØTØ 540
510   LET SØ=255/Z
520   LET S1=48/Z
530   LET YØ=0
540   PRINT "MAX=";Z;"MIN=";M
550   IF A=2 THEN 630
560   PRINT "SCALE FACTØR=";SØ
570   FØR N=1 TØ 255
580   LET E=INT(SØ*X(N)+0.5)
590   CALL (3,N,YØ,2,E)
600   NEXT N
620   GØTØ 160
630   PRINT "SCALE FACTØR=";S1
635   PRINT
636   PRINT
640   FØR N=1 TØ 255
650   LET E=INT(S1*X(N)+0.5)
660   LET L=LØ+(N-1)*D
670   PRINT FNR(L);TAB(8);X(N)
715   NEXT N
730   IF B=0 THEN 760
740   IF B=1 THEN 780
750   GØTØ 200
760   CALL(6,Ø(Ø),256,2)
770   GØTØ 790
780   CALL(6,F(Ø),256,2)
790   CALL(6,Ø,Ø,3)
800   STØP
```

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