General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.

- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.

- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.

- This document is paginated as submitted by the original source.

- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

Produced by the NASA Center for Aerospace Information (CASI)

CR151600

FINAL REPORT TO
NASA JOHNSON SPACE CENTER
CONTRACT NAS9-15171

ANALYTICAL AND
COMPUTATIONAL
MATHEMATICS, INC.

FINAL REPORT TO NASA JOHNSON SPACE CENTER

CONTRACT NAS9-15171

ANALYTICAL AND COMPUTATIONAL MATHEMATICS, INC.

1275 SPACE PARK DRIVE, SUITE 114

HOUSTON, TEXAS 77058

DECEMBER 1977

CONTENTS

1.0   INTRODUCTION

The work done by ACM under Contract NAS9-15171 covered a wide variety of tasks.   The categories of work included:

- Development of new mathematical and algorithmical techniques for solution of problems in satellite dynamics.
- Development of new solutions to satellite orbit motion.
- Dynamical analysis of Shuttle on-orbit operations.
- Development and analysis of computer software routines for use in Shuttle mission planning.
- Formulation of mathematical models of atmospheric density.

Due to the variety of the tasks and the several modifications to the contract, a separate report has been written for each task.   These reports are listed in the references. The purpose of this Final Report is to describe the objectives and scope of each task, and to give a summary of the significant results of each task.   Therefore, this report will serve as an overview and summary of the work done under Contract NAS9-15171.   In case more detailed information is required, please refer to the individual reports.   Copies of each report are available from the Technical Monitors or the JSC Technical Library.

2.0 ANALYTICAL SOLUTION OF ARTIFICIAL
SATELLITE MOTION WITH DRAG

Orbit computation methods can usually be classified
as:

Numerical - The calculations are carried out in a step-
by-step manner. High precision is possible but
computer runtime can be excessive.

Analytical - The calculations are carried out in one
step, regardless of the prediction interval. There-
fore, these methods have extremely fast computation
times.

The fast execution times of analytical satellites theories
make them very attractive as mission analysis and planning
tools. But difficult problems existed in the analytical
methods:

1) The solutions were expressed by extremely complex
formulas which required much more computer storage than the
numerical methods.

2) The accuracy of the analytical methods was restricted
by the simplicity of the model or the solution technique.

3) Unlike numerical methods, the analytical theories
were generally difficult to extend. For instance, numerical
methods may use a geopotential model of arbitrary order or
degree, where as the analytical methods were limited to a fixed
order and degree. The numerical methods can easily include
new or more accurate density models but this was impossible in
analytical methods.

4) Analytical theories suffered because the different
perturbations had never been unified under one non-singular
formulation.

These problems were the motivation behind developing a
new satellite theory. The resulting theory provides a con-
cise and efficient algorithm which has been implemented into

a working, modular computer software tool. The development
of the theory and corresponding software are described in the
following sections.

2.1 Perturbations due to Atmospheric Drag

The objective of this task was to arrive at a *fully ana-*
lytical theory for the motion of an artificial satellite per-
turbed by the J_2 term of the zonal geopotential expansion
and by a drag force which is tangential to the orbit and pro-
portional to the square of the velocity magnitude. The task
was executed in two stages:

1) Development of the drag differential equations in a
regular set of two body elements and in a form suitable for
solution by quadrature (reference 1).

2) Development of a realistic density model suitable
for application in the theory developed in stage 1 (reference 7).

The second stage is discussed in more detail in Section 4.2
of this report and so this section will restrict its discussion
to the first stage.

The drag theory was built on top of the previously devel-
oped J_2 theory in the PSϕ elements. Since the density ef-
fects only the magnitude, not the direction, of the drag force,
the transformation of the drag forces may be carried through
without specifying the density function. A canonical treatment
of this transformation of the drag effects results in a tract-
able set of differential equations. To allow the analytical
solution of these equations, an expansion is made in powers
of the eccentricity. To avoid errors, these expansions were
created in an automatic manner by a minicomputer program spe-
cially designed for this task. The result is a theory of rela-
tively concise form, programmed in FORTRAN and executable in
the interactive mode on the UNIVAC 1110.

Numerical studies were performed comparing the analytical

solution to a precision numerical solution. In the first step, both analytical and numerical solutions assumed the density to be a constant. The results showed the analytical constant density solution to be accurate to within 1% of the numerical solution for Shuttle type orbits. In the second step, the analytical theory with the realistic density model was compared to numerical solutions obtained by using extremely accurate and complex drag models. Even in these rigorous tests, the analytical solution showed errors of no more than 2% .

2.2 Perturbations due to an Arbitrary Number of Zonal Terms

It was the purpose of this task to derive a "complete" first order solution to the motion of a satellite about a rotationally symmetric earth. No restriction was made on the order of the geopotential expansion, and the solution was developed entirely in non singular elements. Recursive expressions are used so that a concise formulation is maintained.

The method of solution was to apply the Von Zeipel algorithm to a recursive formulation of the zonal geopotential. This is done by defining the zonal potential in the canonical and regular PSφ elements. Considerable simplications can be realized with this choice of elements. By including the true anomaly as an canonical element, the hamiltonian for a finite number of zonal harmonics becomes a finite fourier expansion in the *canonical* elements. Also, the second order integration of the mean motion is not necessary because the new formulation uses a mean motion related to the total energy. The detailed development of this theory is given in reference 2.

The result is a simple formulation which has been implemented in FORTRAN and executable on the UNIVAC 1110.

Numerical studies were conducted to determine the accuracy of the solution. In the studies, the analytical solution was compared to a precision numerical solution. Both numerical

and analytical solutions modeled the 18th order geopotential
expansion. For Shuttle-type orbits, the differences between
analytical and numerical solutions were on the order of a few
meters after 500 revolutions.

2.3 An Analytical Satellite Orbit Prediction Program (ASOP)

The analytical formulations that are discussed in Sections
2.1, 2.2 and 4.2 of this report as well as certain formulations
that were developed under a previous contract, are being con-
verted into an operational computer program. This program rep-
resents a new state-of-the-art for orbit prediction computer
software. ASOP is able to compute near-earth orbits to within
an accuracy of a few meters. Recursive equations are used
instead of complicated formulas so that the execution time of
ASOP is on the order of a few milliseconds. At present these
calculations only include the perturbations due to J_2. How-
ever, the perturbations due to atmospheric drag and all the
Earth's geopotential terms are currently being incorporated
into the existing ASOP program.

The current version of ASOP is now available to the
general user of the UNIVAC 1110-EXEC 8 system at NASA/JSC
Houston in two forms:

(1) A stand-alone program that can be used interactively
to obtain immediate results for a specific problem.

(2) A user-subroutine package that can be incorporated
into other software systems.

Both versions were designed to be small and execute quickly.

ASOP has been extensively documented (reference 3). Any
additions to the program will be documented as revisions to
reference 3.

3.0 AN ANALYTICAL STATE TRANSITION MATRIX FOR ORBITS
PERTURBED BY AN OBLATE SPHEROID

Often in orbit determination or navigation algorithms
one must predict how small deviations from some nominal orbit
will in time cause the actual orbital path to deviate from the
nominal path. If the initial deviation is small, then a
linear approximation may be used to determine the deviation
at any given time. The linear approximation, a truncated
Taylor series expansion, requires the first partial deriva-
tives of the coordinates defining the position and velocity
at a given time, with respect to the coordinates defining the
position and velocity at the initial time. This matrix of
partial derivatives is commonly called the state transition
matrix - Φ .

Φ may be determined by numerical integration. But when
one considers the fact that the transition matrix is usually
used in some iterative manner, the computational costs re-
quired by the numerical integration of the matrix differential
equation becomes prohibitive.

The alternative, of course, is to determine the transition
matrix by some analytical technique. Several analytical meth-
ods have determined expressions for the transition matrix under
the assumption that the satellite moves along a two-body orbit,
all perturbations being neglected. But this assumption may
result in non-negligible errors if the perturbations are large.

For artificial satellites orbiting near the earth, the
J_2 oblateness potential contributes a strong perturbation
which may not be neglected for accurate satellite orbit pre-
diction. Significant advantages are offered by a J_2 satel-
lite theory developed from a new set of canonical elements
proposed by Scheifele. The elements are in an extended phase

space in that eight (instead of six) variables describe the state. They also have as their independent variable, a quantity related to the true anomaly instead of time. The set is similar to the Poincare elements and are therefore named the Poincare-Similar (PS) elements. The PS elements are non-singular for bounded orbits.

Because of the simplicity and accuracy of the canonical analytical solution based on PS elements, it becomes a logical choice on which to base a new analytical state transition matrix. Therefore the intention of this task was to derive a completely analytical singularity free form of the state transition matrix for orbits perturbed by an oblate spheroid.

The result is a concise, accurate and algorithmically simple state transition matrix and its inverse which includes the short period and secular effects of the zonal harmonic. The algorithm has been implemented in a working program executable in interactive mode on the UNIVAC 1110. From numerical studies it was found that the improvement over the conic matrix ranges from 2 to 4 digits better accuracy. Complete documentation on the work of this task is given in reference 4.

4.0   DEVELOPMENT AND APPLICATION OF
ATMOSPHERIC DENSITY MODELS

For accurate orbit computations for satellites perturbed
by atmospheric drag, a rather complex density model must be
employed. The Jacchia density model includes several variations
of the atmosphere character which are related directly or indi-
rectly to the density. In the most recent Jacchia model, the
exospheric temperature T_∞ , is modeled by variations due to:

1.   Averaged and daily variations in the solar flux $(F_{10.7})$.

2.   Averaged and 3 hourly variations in the geomagnetic
index (a_p) .

3.   Variations of the angle between satellite and atmos-
pheric bulge (diurnal effect) .

The exospheric temperature is then related to the density by
the solution of the diffuse equilibrium equation for the dif-
ferent constituents of the atmosphere and the desired altitude.
Other variations are then computed directly as changes in the
density; they include:

1.   Changes due to semi annual effect.

2.   Changes due to seasonal-latitudinal effect.

The causes for these variations are not exactly known but may
be modeled by fitting emperical data.

Although the model is quite accurate, it requires a great
deal of computation time and storage. The inefficiency of the
model can not be reconciled by simply neglecting some of the
effects and other models such as the 1962 standard are grossly
inaccurate for altitudes above 90km . In addition, the com-
plexity of the Jacchia model renders it useless for analytical
satellite theory applications.

The goal of this task was therefore to develop new density
models which are efficient for applications in numerical methods
or which may be applied in satellite theories. The manner in

which these goals were met are described in detail in refer-
ences 5, 6 and 7. A brief overview is given in the following
sections.

4.1 Application to Numerical Orbit Computation Methods

In an effort to avert the computational costs, two density
models have been independently developed to simulate the Jacchia
model without the costs of an inefficient algorithm. The first
model, referred to as Babb-Mueller (B-M), has been developed
in Mueller (reference 5) and numerical studies showed it to
be an efficient yet accurate model. The B-M model assumes the
density can be modeled by rather simple expressions but the
parameters of the model are unknown. These expressions can
be easily inverted i.e. given the density at a few points the
parameters can be determined. Thus the B-M model can be "cal-
ibrated" to fit the Jacchia model, by evaluating the Jacchia
model at a few specific points. The model parameters found
by calibration are implicit functions of the slowly varying
characters of the atmosphere and can be considered constants
over a limited period of about a month.

The second model, named Jacchia-Lineberry (J-L), is of
a similar nature as the B-M model. It also assumes rather
simple expressions for the density whose parameters have been
determined by a least square fit with the Jacchia model. Un-
like the B-M model, the J-L model's parameters are indeed con-
stants and thus no maintenance is required to update the para-
meters. But the J-L model does have an overhead in that a
considerable number of parameters are required for the com-
plete model. In any case, both models show large computer cost
savings over the complex Jacchia model.

In this task the algorithm for the B-M model was documented
in reference 6. Also in that reference, the complete develop-
ment of the J-L model was given. Several versions of the J-L

model were developed and implemented in a computer package on
the UNIVAC 1110. A user's guide is given in reference 6, includ-
ing software documentation. In addition, numerical studies showed
the J-L model to be very accurate for altitudes below 500km .
Run times showed the J-L model to be 3 to 9 times faster
than the Jacchia model depending on which version was used.

4.2 Application to Analytical Satellite Theories

In the past, the computation of the drag perturbations
by analytical methods has not been feasible because of their
inability to model the "real world" density.

In developing a density model for the analytical theory,
one is severely restricted by the fact that the model must be
in the form of a fourier series in the true longitude. As is
the case in most analytical theories, the perturbation must
be written in a fourier series to facilitate the solution of
the differential equations of motion. Usually, the solution
is obtained by quadrature.

Several density models have been developed to predict
accurately the density at any point in space and time. Exam-
ples are the Jacchia model and the USSR model. But both models
are extremely complicated and too unwieldly for analytical
satellite theories. In the analytical theory of Brouwer and
Hori, the density was assumed to be an exponential function
of the position radius of the satellite. The exponential must
then be expanded in a Poisson series so the quadrature can be
performed. This model has several difficulties. It first has
a problem with convergence, which Brouwer points out. Secondly,
it is simply a poor model for describing the dynamic atmosphere.
The density is extremely effected by such factors as the level
of solar activity and whether it is summer or winter, day or
night. Thus the model in Brouwer, Hori theory is simply in-
adequate.

Recently an extremely simple density model (referred to here as B-M) has been developed to match the Jacchia model to a high degree of accuracy (reference 5). The variations in the density due to changes in the height and changes in the relationship of the vehicle and sun position (diurnal effect) are included explicitly. Long period variations such as the changes in the average solar activity and semi-annual variations are included implicitly in the coefficients of the model. The value of the coefficients are determined by a procedure called "calibration". The simple formulation allows the model to be inverted, i.e. given the density at different points in space (as determined from Jacchia) one can compute the coefficients of the B-M model. Since the coefficients are implicit functions of long period effects, they can be considered constants over a limited period of about a month.

Even this extremely simple model cannot be applied in the analytical drag theory because it cannot be written in the form of a fourier series. However the technique of the B-M model does give important insight and direction to follow.

In all the models discussed, the representations of the density are considered to be global. In other words, given any position in the atmosphere one can determine its density. The approach taken in this task was to develop a model which expresses the density along a particular orbit. The coefficients in the model are calibrated with the Jacchia model in a manner similar to the B-M model. But in this case, the coefficients in the model are not only implicit functions of the long period variations in the atmosphere but also the orbital elements which describe the orbit. The result is a density model which can be written in a fourier series and easily implemented into the drag theory. Since the orbital elements are perturbed by J_2 and drag, the coefficients in the model must be updated periodically or corrected in some manner to

reflect the changes. Since the perturbations are small, the updating would be infrequent. Details of the development can be found in reference 7.

The new density model has several distinct advantages. It has a rather subtle yet extremely important advantage in that the model is a power expansion in ζ_1 ; i.e., the basic expansion used in Scheifele's development of the differential equations.

Another advantage of the new density model is its ability to simulate *any* density model. In this task the Jacchia model was chosen for the simulations, but any other model could have been used. If more accurate density models are developed, they may also be chosen for simulation and thus the analytical drag theory can reflect the accuracy of the newly developed models.

Finally, it is concluded that the analytical satellite theory need not be limited by a simplified drag model. With the approaches developed in this report, it is feasible to make the accuracy of the analytical theory competitive with *precision* numerical methods, while retaining a concise formulation and quickness of execution.

5.0 ORBITAL MOTION OF THE SOLAR POWER SATELLITE

It has been proposed to put a series of large satellites into geosynchronous orbit for the purpose of collecting solar energy and redirecting it toward the earth via microwave radiation. Preliminary studies are being carried out at JSC on the feasibility of these Solar Power Satellites (SPS).

The large area of the collecting surface (approximately 144 square kilometers) means that solar radiation pressure will cause significant perturbations on the SPS orbit. In fact solar pressure will be as important as gravitational perturbations. The purpose of this task was to determine the effects of solar radiation pressure on the SPS orbit. It was shown that the eccentricity of the orbit can get rather large (.08) even when it is initially zero. This is the primary difference between the SPS orbit and other geosynchronous satellite orbits.

The SPS configuration being considered here is described in a study report by the Johnson Space Center. However, the results in this study are applicable to any geosynchronous satellite that resembles a flat surface that continually faces the sun. The main purpose of this study was to investigate the orbital evolution of the SPS over its expected thirty year lifetime. An additional goal was to describe the motion with analytical formulas. These formulas could then be used as a basis for developing an orbit control theory that will minimize station keeping costs.

The perturbing forces acting on the satellite were studied in detail. To a first approximation, three types of forces can be considered separately since they have different effects on the orbit.

(1) Longitude dependent tesseral terms in the earth's geopotential cause a slow drift of the satellite's mean longitude.

(2) Sun and moon gravity cause a rotation of the orbital plane.

(3) Solar radiation pressure will cause an increase in the orbital eccentricity.

The following is a summary of the main results of the study. The detailed reports are given in references 8 and 9.

(1) An analytical solution was developed, giving the motion of orbital eccentricity and line of apsides as a function of time. This solution has a concise formulation and is accurate to within a few percent.

(2) It was shown that the eccentricity of the sun's orbit about the earth causes a linear increase in the eccentricity of the SPS orbit.

(3) The eccentricity and inclination of the SPS orbit can be controlled by choosing certain nonzero values of eccentricity and inclination.

(4) Graphical data on the orbital elements was put into appropriate form for use in mission planning or feasibility studies.

(5) Explicit equations were developed for the daily motion in longitude and latitude. It was shown that, for small eccentricities and inclinations, the ground track will be an ellipse centered on the equator and mean longitude.

6.0   A REFINED NUMERICAL METHOD FOR CALCULATIONS
OF SATELLITE ORBITS

Solutions to ordinary differential equations are usually obtained through the application of either analytical or numerical solution methods. Each approach has its own set of advantages and disadvantages. A particular method is usually chosen according to how it may satisfy foreseen applications.

Analytical methods require extensive development of mathematical formulas. Once they have been obtained, the formulas can be used to study certain global properties of the solution. Also, the analytical formulas can be used in computing machines to provide extremely rapid numerical calculations.

There is relatively little apriori mathematical development of numerical methods. They allow extremely accurate numerical calculations of the solution and are usually not difficult to program on the computer. However, the solution must be developed step-by-step, which can take large amounts of computer time. Also, very little qualitative or global information on the solution is available.

For some applications, it may be advantageous to develop new specialized methods that combine features of both the analytical and the numerical methods. This is the suggested approach for those applications that require repetitive solutions of the differential equations, but for which no analytical solutions are available. Therefore, the advantage in speed of an analytical solution might be partially realized by using a limited application of an analytical method. Numerical step-by-step calculations still need to be done, but usually in a more efficient manner.

This task concerns a new approach for the solution of artificial satellite trajectory problems. The basic idea is to apply an analytical solution method (the Method of Averages)

to an appropriate formulation of the orbital mechanics equations of motion (the KS-element differential equations). The result is a set of transformed equations of motion that are more amenable to numerical solution.

An important problem that is found when attempting the numerical solution of the satellite differential equations, is that the maximum size of the numerical integration steps is limited by the high frequency (short period) tesseral terms contained in the geopotential. This problem appears even with the KS formulation. Even though their amplitudes are small and may be considered negligible for many applications, the short period terms cause the following practical problems:

(i) Since the tesseral terms depend on time, the orbital frequency based on the total energy is no longer constant. If small steps are not taken to "track" the high frequency oscillations, large down range errors can result.

(ii) If the tesseral terms were simply neglected in the numerical integration, larger steps could be taken but unacceptable intrack errors would result.

(iii) Evaluation of all tesseral terms in the numerical integration force model consumes a major part of the computation time.

An approach to solving the three above problems is developed in this task. Basically, the idea is to carry out a numerical integration without the tesseral terms. In order to avoid the second of the above problems, the KS elements are initialized with a mean frequency (total energy). The mean values are obtained via a specialized application of the Method of Averages. Since the force model contains only zonal geopotential terms plus, possibly, atmospheric drag and external bodies, computer run time and stability problems associated with the tesseral terms are avoided.

A Modified Method of Averages was developed and applied

to the KS differential equations. This theory was then used to develop an Averaging Initialization Algorithm. Tests show that this algorithm can decrease the computation time required for predicting near earth orbits. For prediction intervals of one or more days, the computation time will be reduced by a factor of three to four. The method is particularly suited to iterative applications where accurate in-orbit satellite position information is required.

Documentation of theoretical developments and numerical experiments is given in reference 10. The computer subroutine package for the initialization algorithm has been implemented in the KSFAST program and is documented in reference 13.

7.0 A COMPARATIVE STUDY OF THE UNIFIED SYSTEM FOR
ORBIT COMPUTATION AND THE FLIGHT DESIGN SYSTEM

The high flight rate anticipated for the Shuttle Transportation System (STS) era beginning in the 1980's is causing a review of the mission design and analysis computer software structures. The rather individual case-by-case planning procedures and software products which were used for earlier missions are no longer adequate when considering the projected budgets for the STS operations. Standardized planning tools are absolutely necessary in the environment of shuttle flight design since many tasks are repetitive. Also, profit must be taken from the experience gained with earlier missions and from the advances made in the areas of flight design techniques and computer hardware and software standards.

In this study, two recent software systems were described and compared. One is the Flight Design System (FDS), currently being developed at MPAD of NASA/JSC[+]. The other is the Unified System for Orbit Computation (USOC), currently operational at MAD of ESA/ESOC[++]. The complete report on the results of this study is given in reference 11.

The FDS software consists of a set of functional and utility processors, components of the data base structure and the executive logic. It is designed as an efficient production tool for flight design and analysis tasks, to be accomplished for the support of operational shuttle missions and orbital flight tests. It also includes an interface with a system for automated generation of flight planning documents.

[+] Mission Planning and Analysis Division of NASA/Johnson Space Center, Houston, Texas.

[++] Mission Analysis Division of the European Space Agency/ European Space Operations Centre, Darmstadt, Germany.

USOC has been used for the planning, analysis and design of spacecraft missions, both earth orbital and interplanetary. It makes use of an automated approach to the *generation* of problem specific application programs from a library of functional modules. A module selector program interfaces with the user and selects the necessary modules for a particular application. The USOC design expressly facilitates quick modifications by the analyst.

The FDS is a *production* oriented software product, mainly designed to satisfy flight design needs of the class of shuttle missions which are considered as standard missions or deviations of such. It is not intended to satisfy the total analysis needs of MPAD . One of the goals of this study was to evaluate the approach of USOC for the planning and design of *unique* shuttle flight phases.

The two software systems were compared on the basis of the following considerations:
- Scope of applications
- Software system structures
- Hardware/software requirements to host computer system
- Software flexibility
- System transportability
- Unique features of each system

Although there exists some overlapping of the scope of applications of USOC and FDS , the structures of these two systems are very different, according to their different basic requirements. Their requirements are in fact, incompatible. While the FDS is designed to meet the requirements of a standardized production tool, USOC is designed for rapid generation of particular application programs. The main emphasis of USOC is put on the adaptability to new types of missions.

A software system having a USOC-like structure, adapted

to the specific needs of MPAD , would be appropriate to
support planning tasks in the area of unique STS missions.
There appears to be a need for such an additional system
within MPAD .

8.0 ANALYTICAL FORMULATION OF SELECTED ACTIVITIES
OF THE REMOTE MANIPULATOR SYSTEM

The Remote Manipulator System (RMS) will be used by NASA for Shuttle on-orbit payload (P/L) deployment and retrieval. Large simulation software is being used by several organizations to support the development of the RMS . However, for certain analyses such as evaluation of optimum RMS maneuvers, interactions with attitude and orbit control of the shuttle, etc., much simpler and faster mathematical models are needed. In addition, analytical closed form solutions will allow an estimation of operational limits of the system. These limits are not easily detectable with the big simulation programs that require considerable computing time. Finally, the transparency and redundancy of a simplified mathematical model are further advantages to be expected.

The purpose of this task was to develop new methods of analysis for the Shuttle Orbiter based RMS . The methods are based on simplified kinematical models. The motions of the simplified mathematical model are described either by explicit analytical formulas or by concise numerical algorithms.

The purposes of the new methods are to:
- Provide insight into the coupled motion of the Orbiter - RMS - Payload system.
- Provide fast answers to typical problems in the design and analysis of Shuttle missions requiring the RMS.
- Provide qualitative and quantitative results on the motion to be used as a check on numerical simulation software.
- Isolate and analyze those RMS operations that are most critical from the point of view of mission success or RMS design limits.

- Investigate further extensions of the mathe-
  matical model in order to take account of
  arm flexibility, drive motor characteristics
  and RMS control laws.

The equations of motion for a general treatment of the
coupled motion of Orbiter and P/L were derived. It was shown
that P/L capture and P/L deployment are governed by the
same equations. Also, all phases of coupled motion are de-
scribed by the same general equation of motion.

In order to demonstrate the applicability and efficiency
of the derived equations, some example problems are given.
The corresponding equations were simplified to the extent of
these special cases. Explicit analytical solutions are pre-
sented for the most interesting RMS phases of operation.
These are the following:

- P/L capture,
- P/L stowage into cargo bay and its removal from
  the cargo bay,
- P/L deployment.

The analytical formulations were used to study problems
in mission planning and on-orbit operations. Recommendations
are given on maximum capture velocity of payloads, optimum
capture philosophy, payload stowage and deployment techniques,
and hardware modifications. A detailed report on the study
is given in reference 12.

9.0 MAINTENANCE AND MODIFICATION
OF THE KSFAST PROGRAM

Since the initial release of the KSFAST program, a few
minor problems have been detected, recommendations for improve-
ments have been made and new theories for the KS elements
have been developed and proven. Therefore, the KSFAST pro-
gram has been modified to incorporate some of these improve-
ments and to correct deficiencies.

There have been three basic areas of modification:
a) Changing some of the numerical constants to the
more generally accepted values.
b) Changing the input/output units from Earth radii
and minutes to Kilometers, seconds and days.
c) Incorporation of the mean element theory for
approximating some of the Earth's geopotential
terms.

The first two modifications were done to make the KSFAST
program more acceptable by the using public. The third in-
volved a considerable programming effort to implement, but
resulted in significant savings in program execution time.
(For a complete description of the mean element theory, see
reference 10).

A revised version of the KSFAST program documentation
(reference 13) has been published, including all recent modi-
fications.

REFERENCES

1. Scheifele, G.; Mueller, A. and Starke, S.: A Singularity
    Free Analytical Solution of Artificial Satellite Motion
    With Drag, ACM Technical Report TR-103, 1977.

2. Mueller, A.: Recursive Analytical Solution Describing
    Artificial Satellite Motion Perturbed By An Arbitrary
    Number of Zonal Terms, paper presented at the AAS/AIAA
    Astrodynamics Conference, Jackson, Wyoming, September,1977.

3. Starke, S.: An Analytical Satellite Orbit Prediction Pro-
    gram (ASOP), NASA Johnson Space Center Report JSC-13094,
    1977.

4. Mueller, A.: An Analytical State Transition Matrix For
    Orbits Perturbed By An Oblate Spheroid, ACM Technical
    Report TR-104, 1977.

5. Mueller, A.: Atmospheric Density Models, ACM Technical
    Report TR-106, 1977.

6. Lineberry, E. and Mueller, A.: Efficient Atmospheric
    Density Models For Numerical Computations, NASA Johnson
    Space Center Report, (in preparation).

7. Mueller, A.: An Atmospheric Density Model For Application
    In Analytical Satellite Theories, ACM Technical Report
    TR-107, 1977.

8. Graf, O.: Orbital Motion of the Solar Power Satellite,
    ACM Technical Report TR-105, 1977.

9. Graf, O.: Effects of Solar Radiation Pressure On The
    Orbit Of The Solar Power Satellite, paper presented at
    the AAS/AIAA Astrodynamics Conference, Jackson, Wyoming,
    September, 1977.

10. Graf, O.; Mueller, A. and Starke, S.: The Method Of
    Averages Applied To The KS Differential Equations,
    ACM Technical Report TR-109, 1977.

11. Maag, W.: A Comparative Study Of The Unified System For
    Orbit Computation And The Flight Design System, ACM
    Technical Report TR-108, 1977.

12. Zimmerman, K.: Analytical Formulation Of Selected Activ-
    ities Of The Remote Manipulator System, ACM Technical
    Report TR-110, 1977.

13. Starke, S.: A Satellite Orbit Prediction Program (KSFAST),
    Revision 1, NASA Johnson Space Center Report, 1977.