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AN RC ACTIVE FILTER DESIGN HANDBOOK

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AN RC ACTIVE FILTER DESIGN HANDBOOK

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Preface

Electronic devices that permit signals of certain frequencies to pass while suppressing others are called filters, and this handbook describes the design of such devices.

The design of filters from fundamental principles is an extremely involved process that requires an intimate knowledge of circuit theory and associated mathematics; often, computer assistance is essential. Most scientists and engineers have neither the time nor the inclination to become filter designers, because the filter they need is simply a tool to achieve some end in their own field of endeavor. In recent years there has been a trend towards simplification and standardization of filter design and the purpose of this handbook is to assist engineers and scientists by presenting a set of standardized designs and procedures that can be applied to resistor-capacitor active filters or RC active filters. (RC active filters and their advantages and disadvantages are discussed in chapter 1.) Throughout the handbook, emphasis is placed on simplified procedures that can be used by the reader who has a minimum of knowledge about circuit design and little acquaintance with filter theory. This approach is stressed by the manner in which the handbook is organized.

The handbook has three main parts: The first part (chapter 2) is a review of some information that is essential for work with filters; it is not intended to be a comprehensive review of circuit or filter theory, but it includes certain topics (usually considered elementary) that must be understood thoroughly to avoid confusion when using the rest of the handbook. Readers who find the first part too elementary may wish to proceed directly to the second part.

The second part (chapters 3 through 6) includes design information for specific types of filter circuitry and describes simple procedures for obtaining the component values for a filter that will have a desired set of characteristics. All of the circuits have been built and tested, and pertinent information relating to their actual performance is given in this part. The mathematical knowledge required to obtain working designs from this part of the handbook involves only arithmetic and elementary algebra.

The third part (appendix) is a review of certain topics in filter theory and is intended to provide some basic understanding of how filters are designed. The review of theory is placed at the end of the handbook to emphasize the fact that a minimum of specialized knowledge is required to understand and use the design information in the second part.

Many thanks are due Ricky Cox for building and testing many of the circuits in this handbook and to Carol Carothers for the difficult job of typing much of the manuscript.

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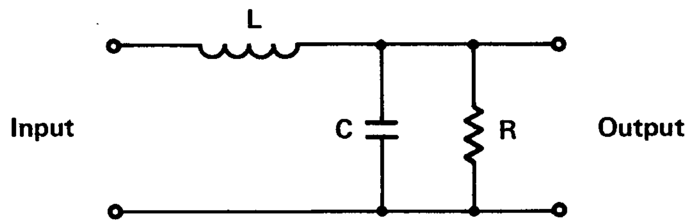
Introduction

RC ACTIVE FILTERS

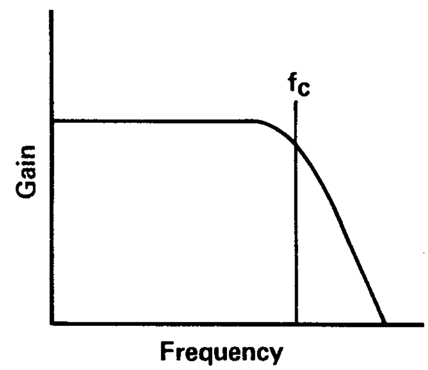
There are many ways in which filter circuits can be classified, for example, filters that use inductors and those that do not. Figure 1(a) shows a filter circuit consisting of an inductor L, a capacitor C, and a resistor R, but filters of this type may be formed without a resistor. The circuit is called an LC filter and is intended to pass a band of frequencies extending from dc to some cutoff frequency f_C. The amplitude response of such a circuit is given in figure 1(b).

There are instances in which it is desirable to use a filter that does not require inductors; filter circuits without inductors present many advantages. The circuit for a low-pass filter that has an amplitude response identical to the LC filter in figure 1(a) is shown in figure 1(c), and is seen to consist only of resistors, capacitors, and an active element in the form of an amplifier. The circuit is a characteristic example of an RC active filter. An active filter of this type includes more parts than a passive LC circuit and, in addition, requires a power supply; accordingly, the RC active circuit must have definite advantages if it is to take the place of an LC filter, but the most significant advantages accrue from the absence of an inductor. For low-frequency circuits, especially below one hertz, inductors are large and expensive. Large inductors also are not ideal because they have too much series resistance and stray capacitance. Inductors of over a few microhenries cannot be integrated either monolithically or in hybrid form; as a result, most LC filters cannot be miniaturized or mass-produced by modern microelectronic techniques. LC filters are troublesome even in discrete form because it is often difficult to obtain nonstandard values of inductances. Moreover, fabrication of inductors that have special values, or time-consuming selection of inductors that must be connected in parallel or in series, is much more inconvenient that the bridging of a number of capacitors to produce a given capacitance value. Also it is easier to select special value capacitors than inductors.

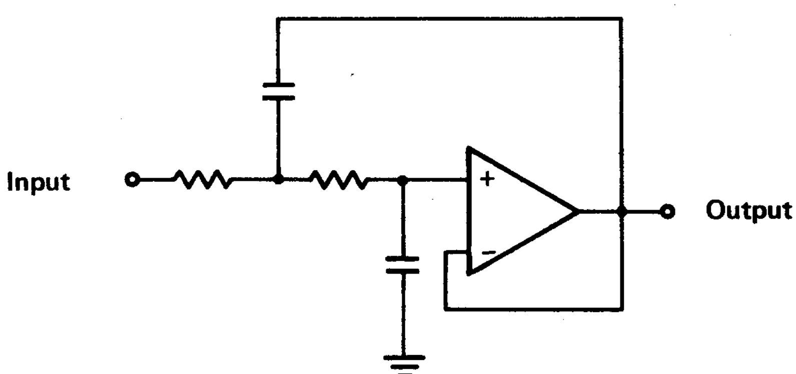
In contrast to passive LC filters, RC active filters do not need to be matched to a source or load impedance, but a low-impedance source often must be used. Also, RC active filters can provide an impedance transformation; that is, they can have a high input impedance and a low output impedance, which means that network stages are isolated and can be tuned independently without interaction.



1(a) An LC low-pass filter



1(b) Amplitude response of LC filter (gain vs. frequency).



1(c) An RC active low-pass filter.

FIGURE 1. – Two low-pass filters and their responses.

Many of the disadvantages of RC active filters stem from the use of active elements, usually operational amplifiers. Amplifier outputs ordinarily have offsets that range from a few microvolts to a few millivolts and have temperature coefficients of typically 1 to 100 microvolts per degree Celsius ($\mu V/^{\circ}C$). Also, amplifiers usually have input bias currents that may flow through input circuit resistors and produce output voltage offsets; input bias currents also are a function of temperature. Moreover, the limited frequency response of operational amplifiers defines the high-frequency responses of RC active filters; ordinarily, the maximum bandwidth is usually about 100 kilohertz (kHz), but a 1-megahertz (MHz) response can be achieved with simple designs that have only a few stages and use fast operational amplifiers and low resistance-element values. In practice, the limiting factor is the slewing rate of the operational amplifier (see the section on Slewing Rate under Operational Amplifiers in this chapter). A high slewing rate is necessary to prevent distortion of the output waveform at high frequencies when the output voltage is a few volts or more.

The advantages and disadvantages of an RC active filter, then, can be summarized as follows:

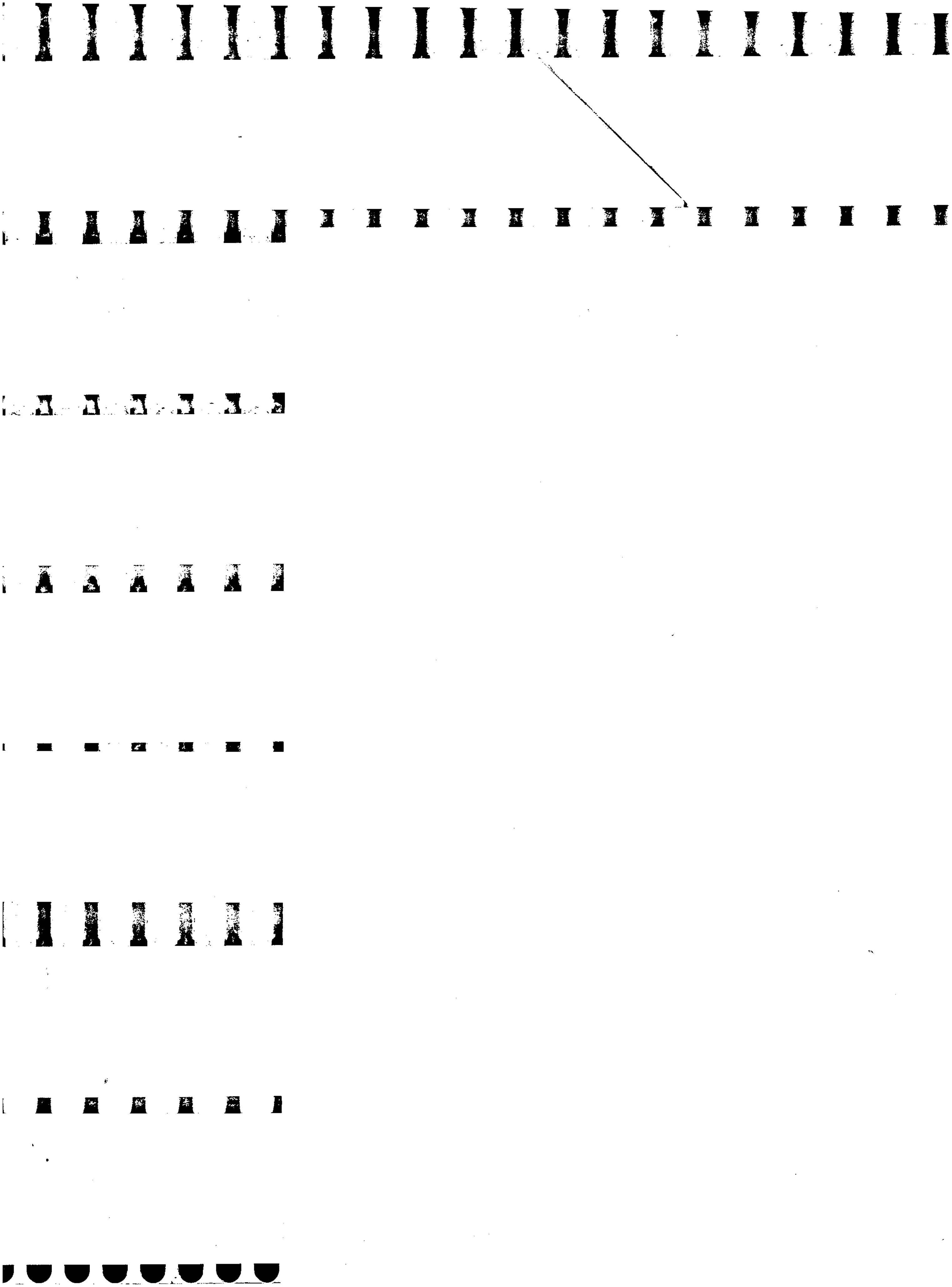
Advantages–

- Require no inductors
- Can be made light and small
- Can be mass-produced by integrated-circuit technology
- Are practical at frequencies as low as a fraction of a hertz
- Do not require impedance matching
- Can provide high-to-low impedance transformation
- Can be designed to provide gain or zero insertion loss
- Have flexible power requirements; often, when the filter is part of a circuit the same power supply can be used.

Disadvantages–

- Usually require more parts than passive LC filters
- Require a power supply
- Require active elements that limit the bandwidth and output swing and often cause dc offsets in the output.

RC active filters are now in widespread use because their advantages far exceed their disadvantages in many applications. Many types of active filters are available as off-the-shelf components from a number of commercial sources, and one is advised to purchase filters whenever possible. However, there are many instances when it is necessary to design and build an RC active filter, and it is hoped this handbook will prove useful on such occasions.



Review of Fundamentals

TYPES OF FILTERS

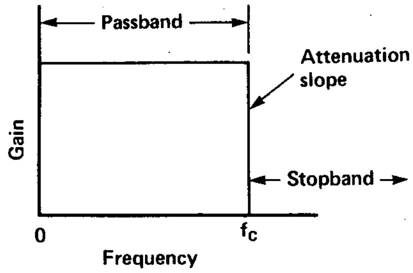
The design of any filter involves an attempt to achieve some unattainable ideal. For example, an ideal low-pass filter would have the amplitude response shown in figure 2(a), and would pass all frequencies with uniform gain in the pass region from dc to some cutoff frequency f_c; above f_c, in the stopband, all frequencies would be infinitely attenuated and the attenuation rate (slope of the curve at f_c) would be infinite and would appear as a vertical straight line at f_c.

Practical amplitude responses are not ideal; nevertheless, they are useful in many applications. Practical filters approach the ideal response in different ways; for example, some emphasize flatness in the passband, but do not have a particularly steep attenuation slope; others have steep attenuation slopes, but do not have flat gain in the passband. Certain classes of filters are designed to have linear phase response (important in minimizing overshoot or ringing in pulse circuits) at the expense of flatness of gain and steepness of attenuation. In general, a practical low-pass filter will approach the ideal by emphasizing one or more of the basic filter characteristics of passband flatness, attenuation slope, or phase linearity. Other factors, such as infinite attenuation of a particular frequency or the degree of attenuation in some given region of the stopband, are often important.

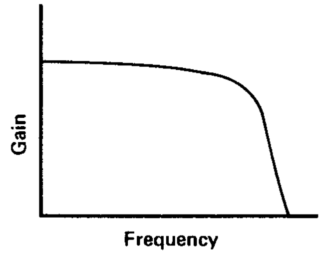
The various approximations to the ideal filter response are identified by such names as Butterworth, Chebyshev, Bessel, and others. The response curves of the common types are shown in figures 2(b) through 2(h).

Probably the most commonly used filter is the Butterworth, which has the characteristic amplitude response shown in figure 2(b). Butterworth filters have a maximally-flat response in the passband; that is, passband flatness is the ideal filter characteristic emphasized, but it is achieved at the expense of phase linearity [see figure 2(h)] and of steepness of attenuation slope. However, the attenuation slope of the Butterworth filter is quite good and, for applications where phase linearity is not important, the Butterworth response is an excellent general purpose approximation to the ideal filter.

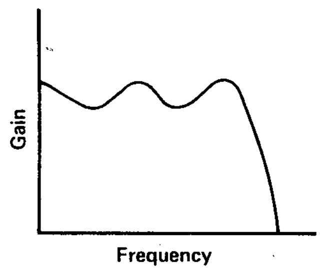
If steepness of attenuation slope, especially in the region of cutoff, is more important than passband flatness or phase linearity, the Chebyshev response, shown in figure 2(c) is often applicable; however, there is ripple in the passband. Chebyshev filters can be designed to provide different amounts of ripple, but the amplitude of the ripple in the passband remains constant for any given amount.



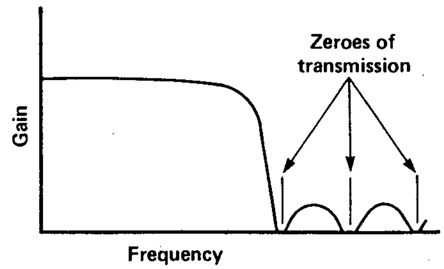
2(a) Ideal.



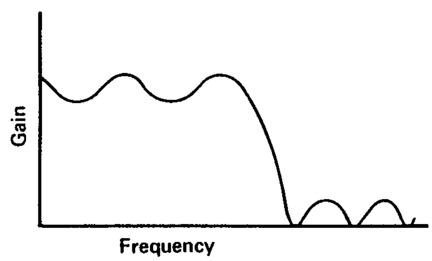
2(b) Butterworth.



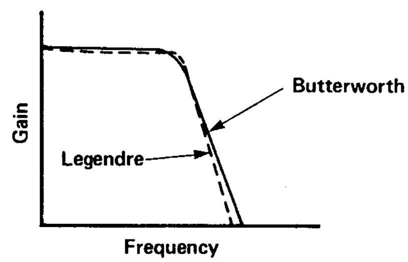
2(c) Chebyshev.



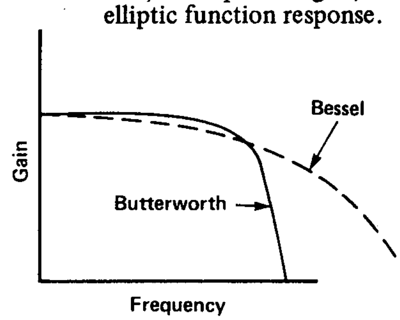
2(d) Inverse Chebyshev.



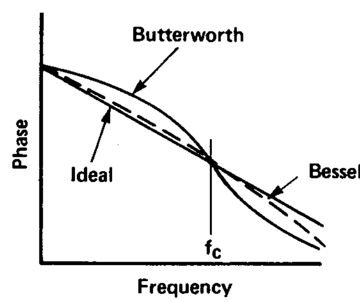
2(e) Complete Chebyshev, also called a Cauer, an elliptic integral, or an elliptic function response.



2(f) Legendre or Optimum.



2(g) Bessel.



2(h) Bessel and Butterworth phase shift.

FIGURE 2. – Types of low-pass filter response.

Both the Butterworth and the Chebyshev low-pass filters achieve infinite attenuation only at infinite frequency; that is, all the zeroes of transmission occur at infinite frequency, but at any other frequency some signals will get through, even in the stopband. If infinite attenuation at particular frequencies in the stopband is required, the Inverse Chebyshev response shown in figure 2(d) may be used; there is no ripple in the passband, but ripple does exist in the stopband and attenuation is infinite at certain frequencies.

A third filter in the Chebyshev family is the Complete Chebyshev; its response is shown in figure 2(e). The Complete Chebyshev is also called a Cauer, Elliptic-Integral, Elliptic-Function, or Zolatarev (but rarely Darlington, even though S. Darlington did much of the original work). The Chebyshev [figure 2(c)] and Inverse Chebyshev [figure 2(d)] filters are special cases of the more general class of Complete Chebyshevs. The Complete Chebyshev filter has ripple in the passband and stopband as well as infinite attenuation at certain finite frequencies.

As has been emphasized, a Butterworth filter has a maximally-flat passband response and the Chebyshev family of filters provides a good attenuation slope. On some occasions the ripples of a Chebyshev filter are not tolerable and the attenuation slope of a Butterworth filter is inadequate. Designing a Chebyshev filter that will have a very small or zero ripple does not help because Chebyshev and Butterworth filters are of the same family, and a Chebyshev filter with zero ripple is a Butterworth filter. A solution in this instance would be to use a Legendre or Optimum filter. The amplitude response for such a filter is given in figure 2(f) and, for purposes of comparison, the response of a Butterworth filter also is shown. Notice that the Legendre response is not as flat as that of the maximally-flat Butterworth response in the passband, but that the attenuation slope of the Legendre response is steeper. A key property of a Legendre filter is monotonicity; that is, for any value of gain there is a unique frequency. This is in contrast to the Chebyshev family in which a particular value of gain will occur at several frequencies because of ripple. Butterworth filters also are monotonic, but Legendre filters have the steepest possible cutoff for a filter which is monotonic.

Thus far filters have been discussed mainly in terms of their amplitude responses, which are plots of gain (or attenuation) versus frequency. However, these plots do not describe the complete transmission properties of a filter; for example, the phase characteristics of a network is one of the most important parameters of response for a filter designed for pulse work. When a rectangular pulse is passed through a Butterworth, Chebyshev, or Legendre filter, overshoot or ringing will appear on the pulse at the output. If this is undesirable, one of the members of the Gaussian family of filters can be used, the most common of which is usually called a Bessel filter since Bessel polynomials occur in the denominator of the transfer functions. Bessel filters are sometimes called Thomson filters after the originator of the design method.

If ringing or overshoot must be avoided when pulses are filtered, the phase shift between the intput and output of a filter must be a linear function of frequency; stated differently, the rate of change of phase with respect to frequency, or the group delay, must be constant. The net effect of a constant group delay in a filter is that all frequencies are delayed by the same amount; thus there is no dispersion of signals of different frequencies. Accordingly, since a pulse contains signals of different frequencies, its shape will be retained when filtered by a circuit that has a linear-phase response or constant group delay. Just as the Butterworth filter is the best approximation to the ideal of perfect flatness in a passband, so the Bessel filter is the best approximation to the ideal of perfect flatness or constancy of group delay in the passband because it has a maximally-flat group-delay response; however, this only applies to low-pass filters because high-pass and bandpass Bessel filters do not have the linear-phase property.

Figure 2(g) compares the amplitude response of a Bessel filter with a Butterworth. Note that the Bessel is a poorer approximation to the ideal both in flatness in the passband and in steepness of attenuation. Figure 2(h) compares the phase for an ideal filter, a Butterworth, and a Bessel. For an ideal filter, the group delay is constant at all frequencies and the phase shift is linear with frequency. The Butterworth filter group delay is not constant, and the plot of the phase angle versus frequency is nonlinear. In the passband region, the Bessel filter shows a reasonably linear phase-angle vs. frequency curve that is a fairly good approximation to constant group delay.

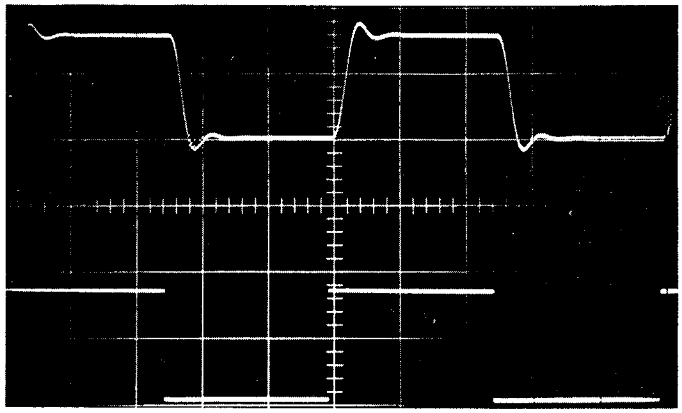
The responses of various filters to a square-wave input are illustrated by the oscillograms reproduced in figure 3. These oscillograms are obtained from actual circuits. The ringing in the Butterworth and Chebyshev filters that is the result of their nonlinear phase characteristics is evident, and the absence of ringing in the Bessel filter shows how well this type of filter approximates the desired linear-phase response; clearly, the response of the Chebyshev filter is inferior to the other two.

Transitional filters exist that have compromise characteristics that trade off the best properties of two types of filters; perhaps the most common is the Butterworth-Thomson filter that has characteristics lying between the maximally-flat amplitude of the Butterworth and the maximally-flat group delay of the Thomson filter (Bessel).

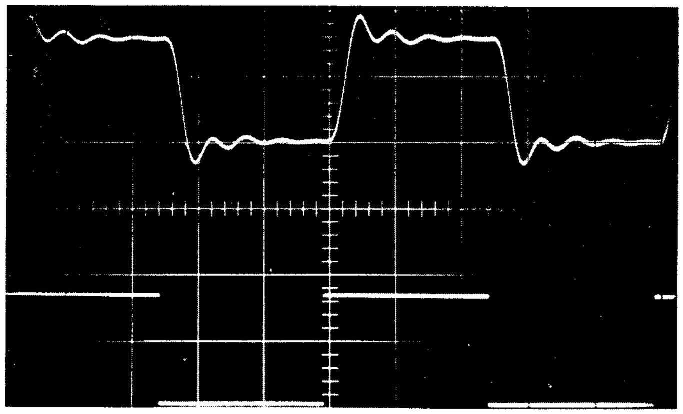
A summary of the main features of the types of filters discussed so far is given in table I. The majority of filtering requirements can be met with three types of filters for which design information is given in subsequent chapters.

DECIBELS

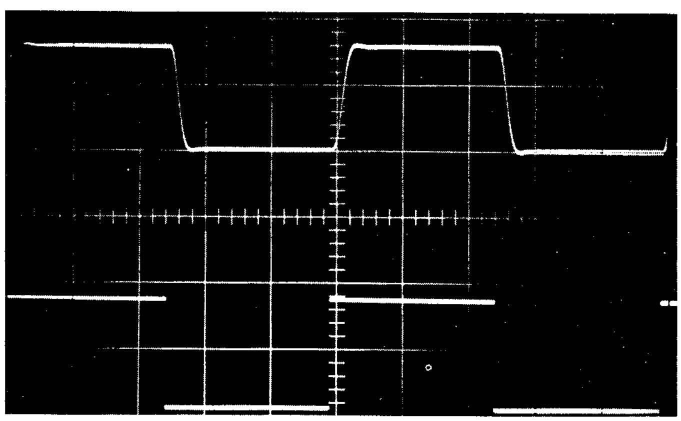
Decibel is a logarithmic expression used in filter applications as a unit of gain or loss. Characteristic of logarithmic operations, processes involving



3(a) Butterworth.



3(b) Chebyshev.



3(c) Bessel.

FIGURE 3. – Responses of various four-pole, low-pass filters to a square wave input.

TABLE I. – *Comparison of Filter Types*

Name of filter type	Main distinguishing characteristic	Remarks
Butterworth	Has maximally-flat amplitude response	The most popular general purpose filter
Chebyshev	Amplitude response has equal amplitude ripples in the pass-band	Attenuation slope is steeper than Butter-worth near cutoff
Inverse Chebyshev	Amplitude response has equal amplitude ripples in the stop-band	No passband ripple. Has zeros of trans-mission in the stop-band
Complete Chebyshev (also called Cauer, elliptic-function, elliptic-integral, or Zolatarev)	Amplitude response has equi-ripple in both pass- and stop-bands	Has zeros of trans-mission in the stop-band
Legendre	Has no passband ripple, but has steeper attenuation slope than the Butterworth	Is not maximally flat
Bessel (also called Thomson)	Phase characteristic is nearly linear in the pass region, giving maximally-flat group delay	Good for pulse circuits because ringing and overshoot are minimized. Has poor attenuation slope

multiplication or division are reduced to additions or subtractions, that in many but not all cases is advantageous because the calculation is simpler. Decibels have come to be used in a manner that is not strictly correct according to the original definition; however, the massive use of the incorrect form on manufacturers' data sheets, in books, and throughout the electronics industry in general has virtually forced a redefinition of the term decibel.

The *original* definition of decibels is embodied in figure 4. The box represents an amplifier, filter, or other device that may be passive or active. Input power P_1 across input impedance Z_1 results in an input voltage V_1 with a phase angle ϕ_1. Correspondingly, at the output there appears power P_2 in Z_2 along with V_2 and phase angle ϕ_2. The ratio of output power to input power of the circuit is:

$$\text{Power Ratio} = \frac{P_2}{P_1} = \frac{V_2^2/(Z_2 \cos\phi_2)}{V_1^2/(Z_1 \cos\phi_1)} \tag{1}$$

The power ratio computed by equation (1) has no units. Decibels are now defined as

$$\text{dB} = 10 \log_{10} \frac{P_2}{P_1} \tag{2}$$

The power gain or loss in dB is therefore found by taking the logarithm to the base 10 of the expression in equation (1) and multiplying it by 10, which gives

$$\text{dB} = 10 \log_{10} \frac{V_2^2}{V_1^2} + 10 \log_{10} \frac{Z_1}{Z_2} + 10 \log_{10} \frac{\cos\phi_1}{\cos\phi_2} \tag{3}$$

If the impedances are resistive, the last term in equation (3) can be omitted and if the impedances are equal, the last two terms can be dropped. The remaining term can be manipulated to become

$$\text{dB} = 20 \log_{10} \frac{V_2}{V_1} \tag{4}$$

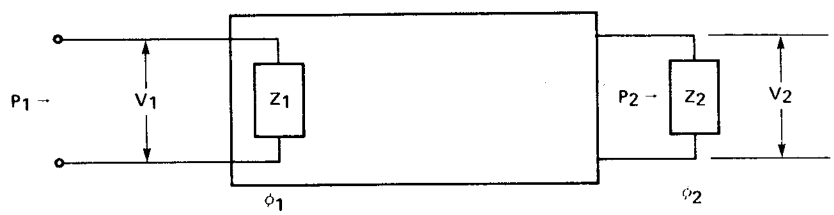


FIGURE 4. – Diagram illustrating the relationships in the definition of a decibel.

Note that equation (4) is an expression for the *voltage* gain or loss of the circuit expressed in dB. Strictly speaking, it should be used only when the input and output impedances are equal; for example, in a 600-ohm telephone system; however, the expression for voltage gain in equation (4) currently is used even when the impedances are not equal. For simplicity, and because it is used extensively in reference literature, equation (4) is used to define voltage gain in this handbook.

EXAMPLE 1: An amplifier has a voltage gain of 2; express this in dB.

SOLUTION: From equation (4),

$$dB = 20 \log_{10} 2/1 = (20)(0.3010) = 6.020$$

Accordingly, an amplifier with a gain of 2 is usually said to have a gain of 6 dB.

EXAMPLE 2: A circuit attenuates an input voltage by a factor of 2; express its gain in dB.

SOLUTION: From equation (4),

$$dB = 20 \log_{10} 1/2 = -20 \log_{10} 2/1 = -6.020$$

Note that for a gain or loss of 2, the numerical answer is the same, namely 6 dB, but the gain is indicated by a plus sign while a loss or attenuation is indicated by a minus sign.

EXAMPLE 3: An amplifier has a 3-position gain switch to provide gains of 10, 1.0, and 0.1; express these gains in dB.

SOLUTION:

Gain = 10	$dB = 20 \log_{10} 10 = 20$ dB
Gain = 1	$dB = 20 \log_{10} 1 = 0$ dB
Gain = 0.1	$dB = 20 \log_{10} 0.1 = -20$ dB.

EXAMPLE 4: Express voltage gain of 0.01, 0.1, 1, 10, 100, 1000, 10,000, 100,000, 1,000,000 in dB.

SOLUTION: From equation (4),

Gain	dB
0.01	–40
0.1	–20
1	0
10	20
100	40
1,000	60
10,000	80
100,000	100
1,000,000	120

The value of using decibels in *all* cases to describe gain is highly questionable. For example, it is probably better to say that an amplifier has a gain of 100 than to state that its gain is 40 dB. Moreover, the use of dB to simplify cascaded-stage gain calculations can result in complication rather than simplification. This is illustrated by the following example:

EXAMPLE 5: Three amplifiers with gains of 3, 4, and 5 are connected in series. Express the overall gain as a number and in dB.

SOLUTION:

$$\text{Gain} = 3 \times 4 \times 5 = 60$$
$$\text{Gain} = 9.5424 + 12.0412 + 13.9794 = 35.563 \text{ dB}$$

In the above example, it is obvious that computations have not been simplified by adding dB's instead of multiplying gains, in fact, it is clear that expressing performance in decibels does not enhance comprehension of what the amplifier chain is doing in this case. Nevertheless, decibels will be used extensively in this handbook because they are widely used in filter design and because their use provides convenient graphs of filter performance.

GRAPHICAL PLOTS OF FILTER RESPONSES

The performance of filters can be depicted in a number of ways; for example, by plotting frequency versus amplitude, phase angle, or group delay, and by the pole-zero plots that are discussed in the appendix. The most common graphical representation is a plot of amplitude versus frequency (for examples, see figure 5). A peculiarity of the scales used for attenuation and frequency is worth noting: Although the units of attenuation are dB, a logarithmic quantity, the values are spaced linearly along the vertical axis; on the other hand, frequency is not converted to a logarithmic equivalent before plotting, but the graph paper is logarithmically divided along its horizontal axis, and thus frequency is plotted logarithmically. Accordingly, a logarithmic quantity is plotted linearly along the vertical axis and a linear quantity is plotted logarithmically along the horizontal axis. Amplitude-versus-frequency curves nearly always are plotted this way.

In the design of filters, special emphasis is placed on the attenuation slope, which is a measure of the steepness of the attenuation curve in the transition between passband and stopband. The attenuation slope is usually expressed in dB per octave or dB per decade. An octave, which is a term originating in music, represents a factor of two in frequency; accordingly, 10 kHz is one octave above 5 kHz, and 2.5 kHz is an octave below 5 kHz. A decade represents a factor of 10 in similar manner. The relationship between dB-per-octave and dB-per-decade is systematic; for example:

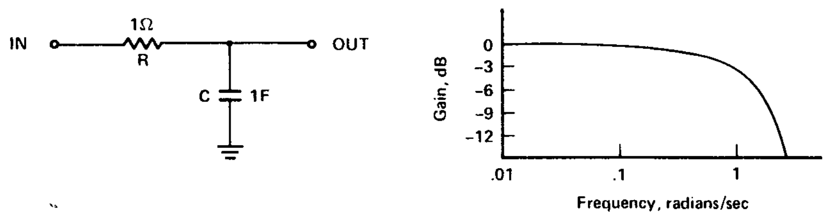
6 dB per octave = 20 dB per decade
12 dB per octave = 40 dB per decade
18 dB per octave = 60 dB per decade
24 dB per octave = 80 dB per decade
30 dB per octave = 100 dB per decade

The relationship is simple because a decade on logarithmic graph paper is 3.3219 times as much distance as an octave so that, for a straight-line attenuation curve, 6 dB per octave (more accurately 6.0206 dB per octave) is equal to (6.0206)(3.3219) = 20 dB per decade.

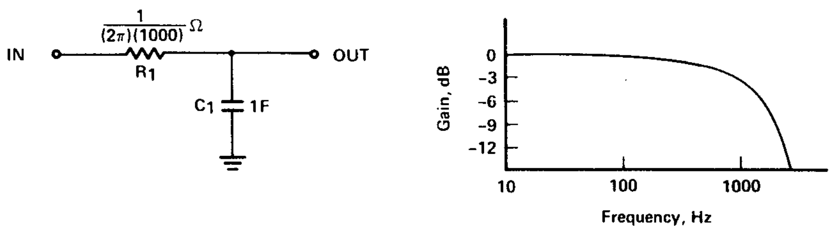
FREQUENCY AND IMPEDANCE SCALING

For purposes of standardization and simplification, a filter is initially designed to provide some convenient cutoff frequency such as $f = 1$ Hz [or, more usually, $\omega = 1$ radian per second (rad/sec)] and to incorporate convenient impedance levels such as one ohm or one farad, even though the final result might yield impractical component values. The filter is then redesigned to the desired frequency and to practical impedance levels by frequency and impedance scaling.

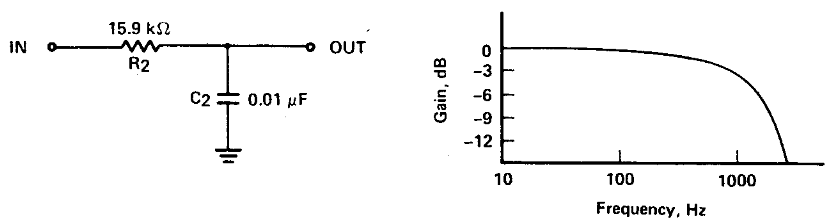
The simple low-pass filter shown in figure 5(a) has a cutoff frequency of $\omega_c = 1/(RC) = 1$ rad/sec. It is called a *normalized filter* because its cutoff frequency is 1 rad/sec and because its component values are unity (ohm-farad). The *normalized amplitude response* is shown in the figure.



5(a) Normalized filter with normalized response curve.



5(b) Frequency-scaled filter with response curve.



5(c) Impedance-scaled filter with response curve.

FIGURE 5. – Frequency and impedance scaling.

Suppose a filter is required that can be constructed from practical components and that has the same amplitude response curve shape as in figure 5(a), but with a cutoff frequency of 1000 Hz. Since ω_c is proportional to the reciprocal of the RC product, the frequency can be increased by reducing R, C, or both. The multiplying factor is the ratio of the normalized frequency $f = \omega/2\pi = 1/2\pi$ Hz, to the desired frequency of 1000 Hz. Note that frequencies should be either in radians per second or, as chosen in this case, cycles per second (Hz). Further, let it be decided to reduce R and leave C at one farad. The new resistance will be

$$R_1 = \frac{\text{(Normalized frequency) (R)}}{\text{(Desired frequency)}} = \frac{1}{(2\pi)(1000)} \text{ ohm}$$

Figure 5(b) shows the design of the filter at this stage. Three observations can be made at this point: First, the filter has been adjusted to the correct cutoff frequency of 1000 Hz. Second, the shape of the amplitude response curve has not been changed by frequency scaling, so, theoretically, the design is complete. Third, the design involves completely impractical component values of $R_1 = 0.000159$ ohm and $C_1 = 1$ farad. Fortunately, the frequency is proportional to the reciprocal of the *product* of R_1 and C_1; thus if one is increased and the other decreased by the same amount, the frequency will remain unchanged. Consequently, a more practical component selection can be obtained by converting the initial C_1 value of 1 farad to 0.01 microfarad by dividing by 10^8. To keep the cutoff frequency at 1000 Hz, R_1 must be multiplied by 10^8 to give 15,900 ohms. The final circuit and its response are shown in figure 5(c).

Note that the amplitude responses indicated in figures 5(b) and 5(c) are identical in shape and cutoff frequency, which illustrates that impedance scaling does not change the response curve in any way.

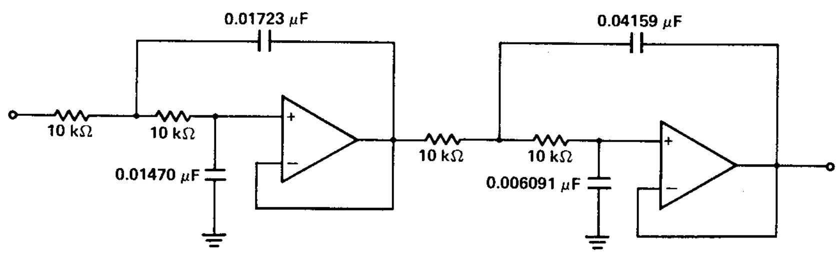
The procedures used to scale frequency and impedance are general and can be applied to any RC active filter. The rules are summarized in table II.

EXAMPLE 6: The circuit shown in figure 6(a) is for a 4-pole, low-pass filter with Butterworth response having a 3-dB cutoff frequency of 1000 Hz. Redesign for a cutoff of 10 Hz.

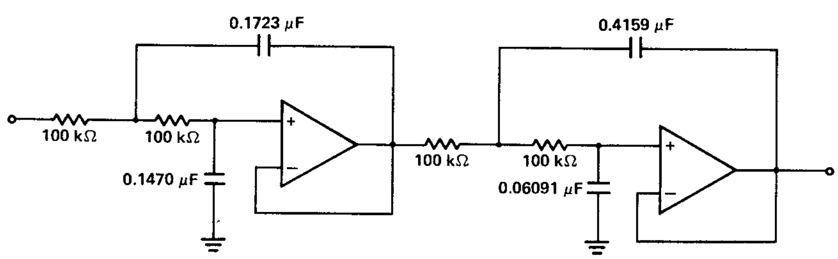
SOLUTION: As indicated in table II, the frequency is scaled from 1000 Hz to 10 Hz by multiplying all resistors by 1000/10 =100; all resistors become one megohm. The decision whether to impedance-scale or not depends on practical considerations that are covered in a succeeding section on operational amplifiers. For the present, however, assume that it is desirable to minimize offset as much as possible between the input and output of the filter. Low offset can be achieved in several ways; for example, the resistors can be left at one megohm and FET operational amplifiers can be used that have

TABLE II. – *Procedures for Scaling the Frequency and Impedance of an Active Filter*

Frequency scaling	Given a filter that has a cutoff frequency of f_c, change to a new cutoff frequency f_n as follows: Either multiply all resistor values by f_c/f_n or multiply all capacitor values by f_c/f_n
Impedance scaling	Change to more practical impedance levels as follows: Multiply all resistors by K and divide all capacitors by K, where K is any suitable constant that will bring the impedances to the desired levels. Note that K can be greater or less than unity, so that R's can be increased and C's decreased or vice versa as desired. Note that the frequency is not altered by impedance scaling.



6(a) Four-pole low-pass Butterworth filter with a 1000-Hz cutoff.



6(b) Four-pole, low-pass Butterworth filter with frequency and impedance scaled to 10 Hz.

FIGURE 6. – Example 6 Butterworth filter circuits.

sufficiently low input bias currents to produce low dc offset (in this instance, there is no need to scale impedances). Another method for reduction of dc offset applies when ordinary operational amplifiers (not FET) are used, when it is necessary to balance out their relatively large bias currents by adding two 2-megohm resistors, one between the output of each amplifier and its inverting input. It might be necessary to bypass the 2-megohm resistors to prevent oscillation or to maintain amplifier open-loop gain at higher frequencies.

A third way of reducing the dc offset from input to output would be to use smaller resistors and larger capacitors. For this purpose, choose K = 0.1 and multiply the 1-megohm resistor values by 0.1 and divide all capacitor values by 0.1 as in table II (page 18); this gives a circuit with the values shown in figure 6(b).

EXAMPLE 7: The circuit given in figure 7(a) is for a bandpass filter having a center frequency of 1000 Hz and a 3-dB bandwidth of 33.33 Hz as shown in figure 7(b). Redesign by scaling for a 200-Hz center-frequency, and find the new bandwidth.

SOLUTION: As noted in table II (page 18), the frequency is scaled by multiplying all resistors by 1000/200 = 5. Thus, R_1 = 238.8 kΩ, R_2 = 132.7 Ω, and R_3 = 477.5 kΩ; impedance scaling is not necessary since these values are reasonable, as are each of the capacitor values (0.1 microfarad). The redesigned circuit is shown in figure 7(c). Since the frequency scaling applies to all frequencies, the new bandwidth is proportional and is 33.33/5 = 6.667 Hz, providing the response curve shown in figure 7(d).

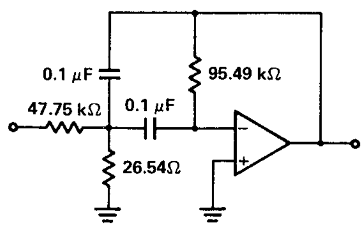
BANDWIDTH AND Q

Although Q may be defined in several ways, a general definition that applies to any system is

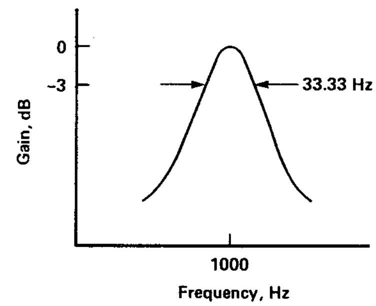
$$Q = \frac{2\pi \text{ (Peak energy storage)}}{\text{(Energy dissipated per cycle)}} \tag{5}$$

This is the fundamental definition of Q, and all other definitions are derived from it. Equation (5) applies to any type of resonant system including series-tuned and parallel-tuned circuits comprised of inductors and capacitors, transmission lines, microwave cavities, acoustic organ pipes, mechanical pendulums, and RC active circuits.

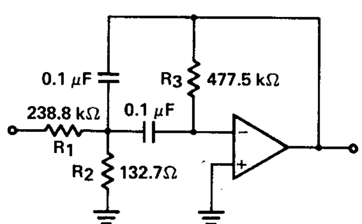
For an inductor or capacitor, Q turns out to be the ratio of the reactance to the resistance. For an inductor, $Q = \omega L/R$ and for a capacitor, $Q = 1/\omega CR$, where R in both instances is an equivalent series resistance. Applied to inductors and capacitors, Q is a measure of the quality of the component; in



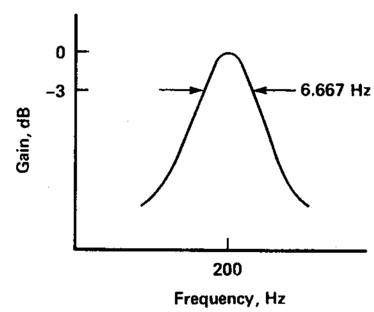
7(a) A 1000-Hz bandpass filter with a bandwidth of 33.33 Hz.



7(b) The response curve for the 1000-Hz filter.



7(c) A 200-Hz bandpass filter with a bandwidth of 6.667 Hz.



7(d) The response curve for the 200-Hz filter.

FIGURE 7. – Example 7 bandpass filter circuits.

fact, Q is an abbreviation for "quality factor." The higher the Q, the more nearly does an inductor or capacitor approach the ideal component.

In a series-tuned circuit it is possible for the voltage across the inductor to be considerably greater than the voltage applied to the circuit. In fact, both the inductor and capacitor voltages will be nearly Q times the applied voltage, where Q is the quality factor of the overall circuit. Similarly, in a parallel-tuned circuit the circulating current will be nearly Q times the current entering the circuit.

For RC active filter applications, Q is defined in a way that allows it to be used for tuned circuits as a measure of the selectivity or sharpness of tuning. The response curve for a tuned circuit is shown in figure 8, and the quality factor, Q, may be obtained as follows:

$$Q = \frac{f_o}{f_1 - f_2} = \frac{\text{center frequency}}{\text{3 dB-bandwidth}} \tag{6}$$

where f_o is the center frequency of the tuned circuit, f_1 is the upper 3-dB frequency, and f_2 is the lower 3-dB frequency. Notice that since Q is a ratio of two frequencies, it is a dimensionless quantity, so that $Q = \omega_o/(\omega_1 - \omega_2)$ is also valid.

As explained below, f_1 and f_2 are often referred to as the half-power points: Let the power in a circuit having resistance R be P. If the voltage across the circuit is V,

$$P = \frac{V^2}{R} \tag{7}$$

If the power is halved, then

$$\frac{P}{2} = \frac{V^2}{2R} = \left(\frac{V}{\sqrt{2}}\right)^2 \cdot \frac{1}{R} \tag{8}$$

Thus the power is halved when the voltage is divided by $\sqrt{2}$. Expressing this in dB,

$$20 \log_{10} \frac{1}{\sqrt{2}} = 20 \log_{10} 0.7071 = -3 \text{ dB} \tag{9}$$

Accordingly, the half-power points occur at the frequency where the voltage is 3 dB down from the peak. It can also be shown that f_o is the geometric mean of f_1 and f_2, that is,

$$f_o^2 = f_1 f_2 \tag{10}$$

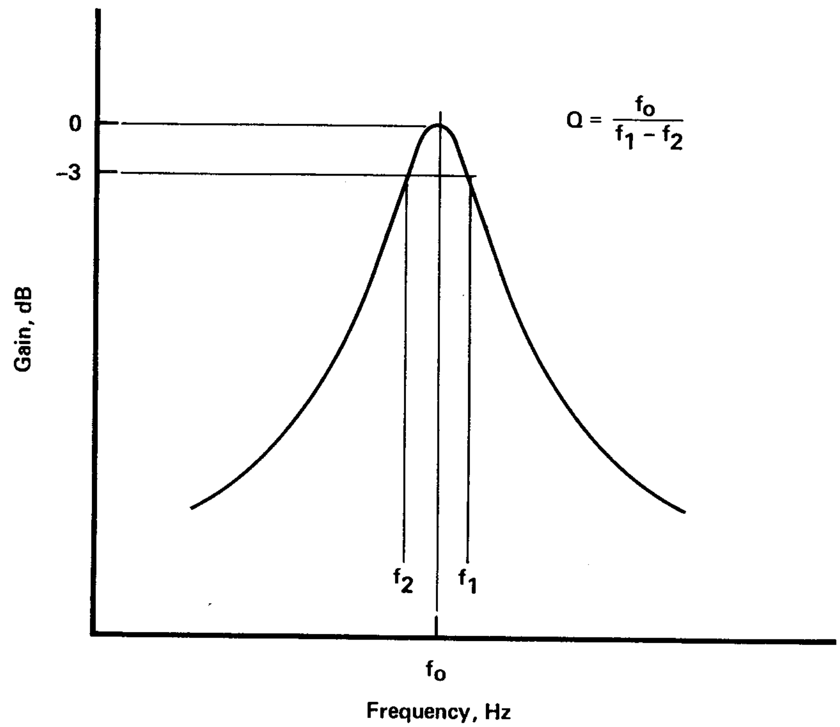


FIGURE 8. – The Q of a resonant circuit shows a sharp rise in gain over a narrow band centered at the resonant frequency f_o.

EXAMPLE 8: The circuit in figure 7(a) is for a bandpass filter having a center frequency of 1000 Hz and a 3-dB bandwidth of 33.33 Hz as shown in figure 7(b). Find the circuit Q and the 3-dB frequencies.

SOLUTION: From equation (6),

$$Q = \frac{\text{Center frequency}}{\text{3-dB bandwidth}} = \frac{1000}{33.33} = 30$$

From equation (10),

$$f_1 f_2 = (1000)^2 = 10^6 \tag{11}$$

Also,

$$f_1 - f_2 = 33.33 \text{ Hz} \tag{12}$$

Combining equations (11) and (12), and solving the resulting quadratic equation, there is obtained

$$f_1 = 1016.80 \text{ Hz}$$
$$f_2 = 983.47 \text{ Hz}$$

EXAMPLE 9: The circuit in figure 7(c) is for a bandpass filter having a center frequency of 200 Hz and bandwidth of 6.667 Hz. Find the circuit Q.

SOLUTION: This simple problem can be solved in two ways. One way is to use equation (7),

$$Q = \frac{200}{6.667} = 30$$

The other way is to observe that circuit Q is not changed by frequency-scaling or impedance-scaling. Since the circuit in figure 7(c) was scaled from a circuit having a Q of 30 (see example 8), the new circuit also must have a Q of 30.

OPERATIONAL AMPLIFIERS

Introduction

Operational amplifiers play a key role in RC active filters inasmuch as they are the *active* elements. An operational amplifier is a high-gain amplifier that has a frequency response down to dc. In linear applications, operational amplifiers are invariably used with feedback, usually negative; however, RC active circuits may also involve a mixture of positive and negative feedback. Negative feedback in RC active circuits is usually used to achieve some precise gain requirement as required by design calculations. Positive feedback is often used to modify the frequency response curve to some desired shape. Only the characteristics and limitations of operational amplifiers as applied to RC active filters will be reviewed because the theory and application of operational amplifiers are thoroughly discussed in books, articles, and manufacturers' applications notes.

The term "operational amplifier" was coined in the late 1940's to denote a type of amplifier used at that time in analog simulators as well as analog computers for the solution of differential and integral equations. It is interesting to note that in recent years developments have come full circle, because work directed towards producing highly stable active filters has resulted in a class of circuits called "infinite-gain, state-variable circuits," one of which is shown in figure 63; these circuits are composed of analog integrators, adders, and subtractors and are a type of analog computer that can be analyzed readily by standard analog-computer techniques. Initial design usually assumes an ideal operational amplifier; fortunately, it is not necessary (and it would be impossible) to take into account all the ways in which deviations of actual amplifiers from the ideal will affect filter performance. The experienced designer will consider the effect of various relevant amplifier imperfections and will account for them in the initial steps of design.

The ideal operational amplifier is represented by the symbol shown in figure 9. The amplifier has a differential input and usually, but not invariably, a single-ended output. Signals fed to the inverting, or negative, input appear 180° out of phase at the output, while signals fed to the non-inverting, or positive, input remain in phase at the output. The ideal operational amplifier has the following characteristics:

- Infinite gain
- Infinite input impedance between the + and - inputs
- Infinite common-mode impedance between each of the inputs and ground
- Infinite bandwidth
- Infinite output-current drive capability
- Infinite common-mode rejection so that only differential signals are amplified
- Zero output impedance
- Zero input current to either terminal
- Zero voltage input results in a zero voltage output
- Characteristics listed above have zero temperature coefficients.

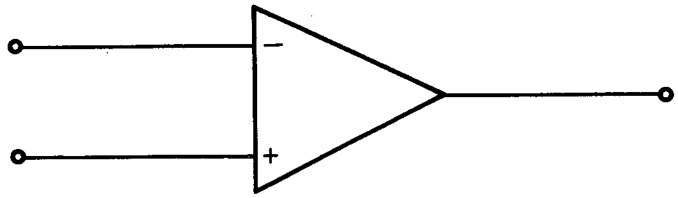


FIGURE 9. – Symbol for an ideal operational amplifier.

A practical amplifier has none of the above properties, but how closely it approximates the ideal is a measure of the quality of the amplifier. Since some feature is usually emphasized at the expense of others, an operational amplifier must be selected for the particular application in mind; for example, low input-bias current might be obtained at the expense of bandwidth, should offset be more important than high-frequency response.

The analysis of operational amplifier circuits is greatly simplified when ideal performance is assumed because the following additional assumptions can be made:

1. No matter what the output voltage is, the voltage between the + and - signal inputs is zero; this is brought about by the infinite gain of an ideal amplifier.
2. Because the ideal amplifier has infinite input impedance and requires zero current bias, any current entering the nodes at the + and - inputs must leave by some path other than the amplifier inputs.

Some basic circuits can be readily analyzed with the aid of these two assumptions.

Figure 10 is the circuit for a voltage follower, a circuit widely used in low-pass and high-pass filters. Input voltage e_1 results in output voltage e_2; note that e_2 appears not only at the output but also at the inverting or - input. From assumption number 1, there is no potential difference between the input terminals; thus e_2 must equal e_1 and the circuit gain must be $e_2/e_1 = 1$.

Figure 11 shows the circuit for an inverting amplifier. This contrasts with the voltage follower of figure 10 which has no inversion. Since the plus input is grounded and is at zero volts, the minus input must also be at zero volts (a virtual ground) as indicated by assumption number 1. According to assumption number 2, the current in the input resistor must equal the current in the feedback resistor, whereupon

$$\frac{e_1 - 0}{R} = \frac{0 - e_2}{R} \tag{13}$$

and

$$\frac{e_2}{e_1} = -1 \tag{14}$$

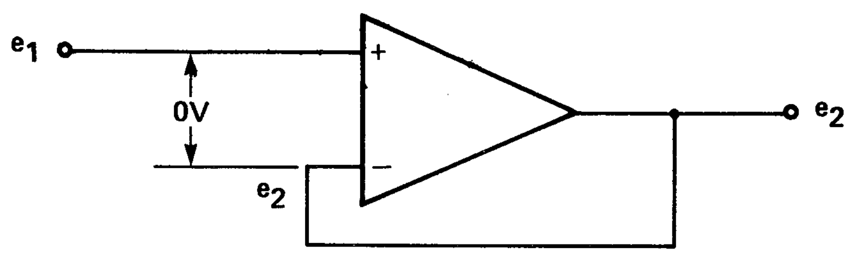


FIGURE 10. – An operational amplifier connected to function as a voltage follower.

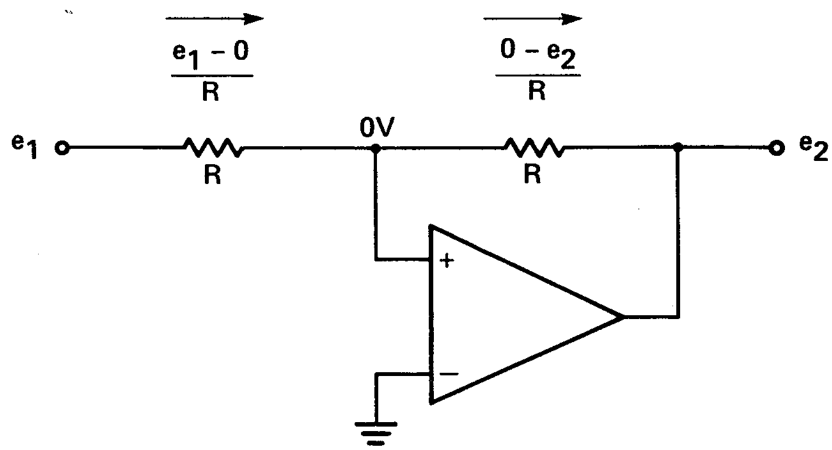


FIGURE 11. – The unity-gain inverting amplifier has equal values of resistances in the input and feedback loops.

As with the voltage follower, the gain is again unity, but the minus sign indicates a signal inversion, of a 180° phase shift between input and output.

Figure 12 shows an integrator; a circuit capable of integrating a time-variable input e_1 with respect to time. Circuit performance is easily analyzed using operational calculus in which $1/p$ represents integration and the capacitor has an impedance of $1/pC$. Again, the inverting input is at a virtual ground and currents in the resistor and capacitor are equal; hence,

$$\frac{e_1 - 0}{R} = \frac{0 - e_2}{1/pC} \tag{15}$$

and

$$e_2 = -\frac{1}{RC} \cdot \frac{1}{p} e_1 \tag{16}$$

$$= -\frac{1}{RC} \int e_1 \, dt \tag{17}$$

Figure 13 is the circuit for a 2-pole, low-pass filter. The operational amplifier is used as a voltage follower; accordingly,

$$e_4 = e_3 \tag{18}$$

Also,

$$i_1 = i_2 + i_3 \tag{19}$$

from which,

$$\frac{e_1 - e_2}{R} = \frac{e_2 - e_4}{1/pC_1} + \frac{e_2 - e_3}{R} \tag{20}$$

From assumption number 2,

$$i_3 = i_4 \tag{21}$$

so that

$$\frac{e_2 - e_3}{R} = \frac{e_3}{1/pC_2} \tag{22}$$

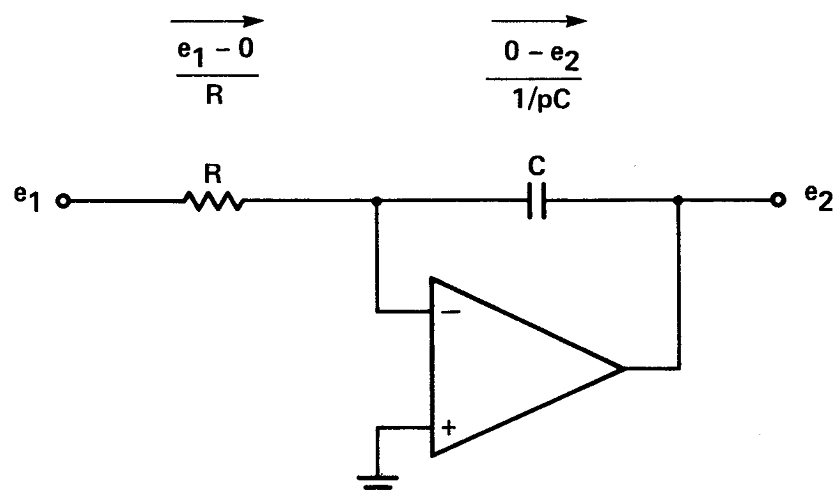


FIGURE 12. – The operational amplifier with an input resistor and a capacitor in the feedback loop performs as an integrator.

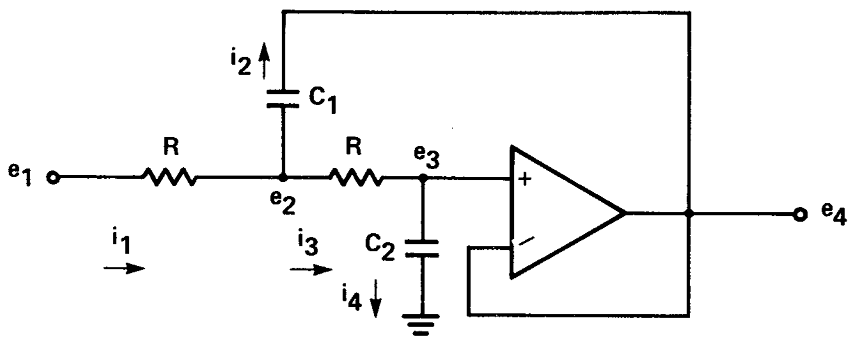


FIGURE 13. – The diagram of an elementary RC active low-pass filter.

Equations (18), (20), and (22) can be solved to give

$$\frac{e_4}{e_1} = \frac{\dfrac{1}{R^2C_1C_2}}{p^2 + p\dfrac{2}{RC_1} + \dfrac{1}{R^2C_1C_2}} \tag{23}$$

Equations of the form of equation (23) can be used to design filters, as explained in the appendix. As an example, in order for a 2-pole, low-pass filter to have a Butterworth or maximally-flat amplitude response, it must have the transfer equation

$$\frac{e_{out}}{e_{in}} = \frac{1}{p^2 + \sqrt{2}\,p + 1} \tag{24}$$

Also, the 3-dB angular frequency (ω_c rad/sec) for the low-pass filter described by equation (23) is the square root of the last term in the denominator of equation (23), or

$$\omega_c = \frac{1}{R\sqrt{C_1C_2}} \text{ rad/sec} \tag{25}$$

If a 2-pole Butterworth filter with a 3-dB angular frequency of $\omega_c = 1$ rad/sec is desired, the following choices can be made: $R = 1\Omega$, $C_1 = \sqrt{2}$ F, $C_2 = 1/\sqrt{2}$ F, for if these values are substituted into equation (23), there is obtained the Butterworth equation (24), and $\omega_c = 1$ rad/sec from equation (25). For further information on the design of this particular circuit, see the appendix.

Illustrations so far have demonstrated how simple filter circuits can be analyzed when an ideal operational amplifier is assumed to be the active element. A generalized discussion of the effects of all the nonideal operational amplifier characteristics on an arbitrary filter is outside the scope of this handbook and, indeed, may not be possible. References 4 and 18 give very complete information on operational amplifiers and provide further information on RC active filters. This handbook will, however, concentrate on certain critical characteristics that affect commonly used filters. These are:

- Finite open-loop gain and bandwidth
- Slewing rate

- Finite input-resistance
- Nonzero output-resistance
- Input offset and drift

Effect of Finite Open-Loop Gain

The amplitude-response and phase-response curves for a typical operational amplifier are given in figure 14. The low-frequency gain of the amplifier is 100 dB, or 100,000, and the low-frequency phase shift is essentially zero. At some frequency (often below 10 Hz as in this case) the gain begins to fall off, usually at 6 dB per octave. A slope of 6 dB per octave is used because it provides the basis for stability when feedback is applied properly; this comes about because the maximum phase shift is -90° for all gains from 100 to 0 dB (10^5 to 1). Negative feedback means that there is an additional -180° of phase shift outside the amplifier, and so the maximum phase shift for all usable gains is -(180 + 90) = -270°. Since it takes 360° of phase shift (input and output in phase) for an instability in the form of oscillations to occur, a 6-dB-per-octave-slope operational amplifier will be stable if negative feedback is applied properly.

A very important indicator of operational amplifier performance can be extracted from figure 14 by inspection, namely, the *gain margin* for a particular application. First note that the gain without feedback is called the *open-loop* gain, while the gain achieved by the application of feedback is called the *closed-loop* gain. The gain margin is the difference in gain between the open-loop and closed-loop gains. The open-loop and closed-loop gains and gain margin for an amplifier with a closed-loop gain of 20 dB (x 10) are shown at some frequency f in figure 14. The significance of gain margin is as follows: The gain of an operational amplifier circuit depends solely on the external feedback elements and not on its internal transistors and resistors as long as there is adequate gain margin. For example, for the 20 dB gain closed-loop plot shown in figure 14, the gain margin varies with frequency as follows:

1 Hz, gain margin = 80 dB
10 Hz, gain margin = 80 dB
100 Hz, gain margin = 60 dB
1 kHz, gain margin = 40 dB
10 kHz, gain margin = 20 dB
100 kHz, gain margin = 0 dB

For most applications of operational amplifiers, including active filters, a gain margin of 40 dB is sufficient, but even 20 dB is often adequate, depending on accuracy requirements. If 40 dB of gain margin is needed with a

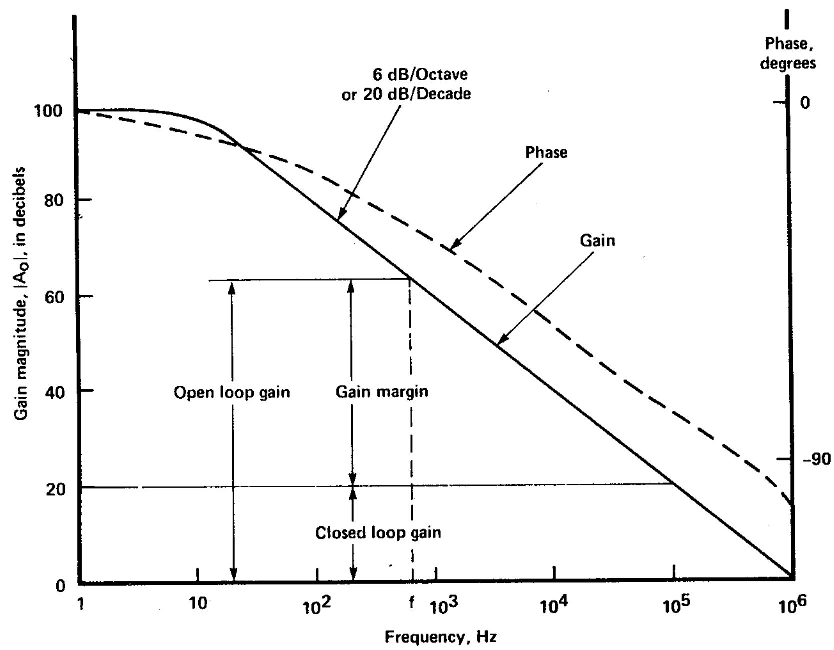


FIGURE 14. – Amplitude response of typical operational amplifier.

closed-loop gain of 20 dB, an amplifier having the response shown in figure 14 can be used only up to 1 kHz, while if 20 dB is satisfactory, then the frequency can be extended to 10 kHz. This discussion illustrates a fundamental limitation of RC active circuits; that is, the limited frequency response of the operational amplifiers used for the active elements.

The following example illustrates the effect of finite bandwidth on the voltage follower, an operational amplifier configuration used extensively in active filters.

EXAMPLE 10: Find the magnitude of the gain at 100 kHz of a voltage follower that uses the operational amplifier for which the frequency response is plotted in figure 14.

SOLUTION: Referring to figure 15, the follower has an input voltage e_1, an output voltage e_2, and an open-loop gain of A_o (it is nonideal in this respect). The output voltage is equal to the voltage difference between the input terminals of the amplifier times A_o, or

$$e_2 = A_o (e_1 - e_2) \tag{26}$$

Rearranging equation (26) gives the closed-loop gain A_c,

$$A_c = \frac{e_2}{e_1} = \frac{1}{1 + 1/A_o} \tag{27}$$

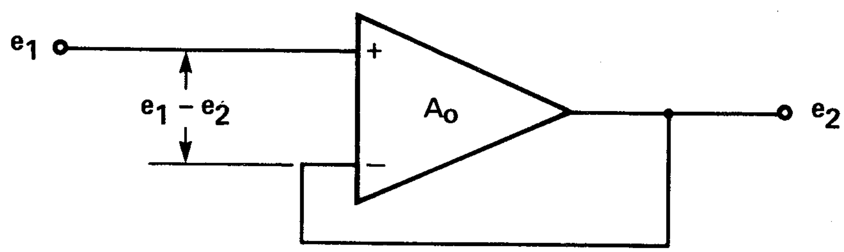


FIGURE 15. – An operational amplifier used as a voltage follower.

If A_o is infinity, as with an ideal operational amplifier, then the closed-loop gain $A_c = 1$. In this example, however, $|A_o| = 20$ dB or 10 at 100 kHz. It is not correct to substitute $|A_o| = 10$ into equation (27) because an *incorrect* answer of 0.9091 will be obtained, which is more than nine percent lower than the ideal gain of one.

In the region where the gain is falling off at 6 dB per octave, A_o is a vector quantity and must be substituted as such into equation (27). The curve in figure 14 is a plot of the *magnitude* of A_o or $|A_o|$; accordingly,

$$A_o = |A_o|(\cos\theta + j\sin\theta). \qquad (28)$$

For this example, assume θ is nearly -90°, also, $|A_o| = 10$, so that

$$A_o = -j\,10 \qquad (29)$$

Substituting A_o from equation (29) into equation (27) gives

$$A_c = \frac{1}{1 + 1/(-j\,10)} = \frac{1}{1.01}\,[1 - 0.1j] \qquad (30)$$

The magnitude of A_c is then

$$|A_c| = 0.99504 \qquad (31)$$

which is only about one-half percent below the ideal value of one.

There are factors other than gain that can affect the high-frequency performance of an operational amplifier; notably, phase shift and input capacitance. Another important factor is the slewing rate.

Slewing Rate

Slewing rate is the maximum rate at which the output can change and it is usually expressed as volts per microsecond. Often it is specified for the condition where the amplifier in the voltage-follower configuration is delivering the rated output voltage across the rated load. If inverting and noninverting slewing rates are different, the lower of the two should be specified. For some 741-type amplifiers, the slewing rate is about 0.7 V/μsec. The slewing rate is an indicator of the maximum frequency at which the rated output can be varied without significant distortion. The output frequency F and the slewing rate S are related by

$$F = \frac{S}{2\pi E} \qquad (32)$$

where E is the rated peak-output voltage.

EXAMPLE 11: A 741-type operational amplifier used as a voltage follower has a slewing rate of 0.7 V/μsec and a rated peak-to-peak output of

26 V into 2 kΩ. What is the maximum frequency at which the rated output can be delivered without significant distortion?

SOLUTION: Given

$$S = 0.7 \text{ V}/\mu\text{sec}$$

$$E = 26/2 = 13\text{-V peak}$$

then,

$$F = \frac{(0.7)}{(2\pi)(13)} = 8.6 \text{ kHz}$$

Since the numbers used are approximate, F would probably be rounded off to 10 kHz on a data sheet.

Slewing rate affects the impedance-scaling procedure as discussed previously under Frequency and Impedance Scaling. The amplifier in an active filter must supply current not only to the load, but also to the feedback elements connected to the output. Since the feedback elements are effectively part of the load, there are limitations on how low the impedance values of the RC elements can be, as illustrated in the following two examples.

EXAMPLE 12: The inverting amplifier circuit shown in figure 16(a) is intended to supply 20 V peak-to-peak at 10 kHz without distortion to a 3-kΩ load. The slewing rate S is 1 V/μsec and the rated output of the amplifier is 25 V peak-to-peak into 2 kΩ. What is the minimum value for R?

SOLUTION: Given

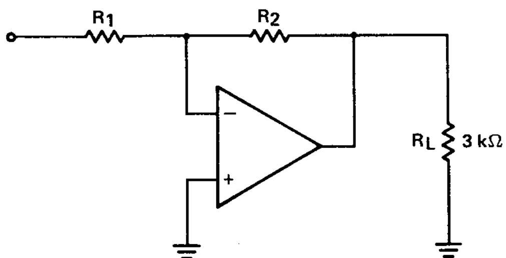
$$S = 1 \text{ V}/\mu\text{sec}$$

$$E = 25/2 = 12.5\text{-V peak}$$

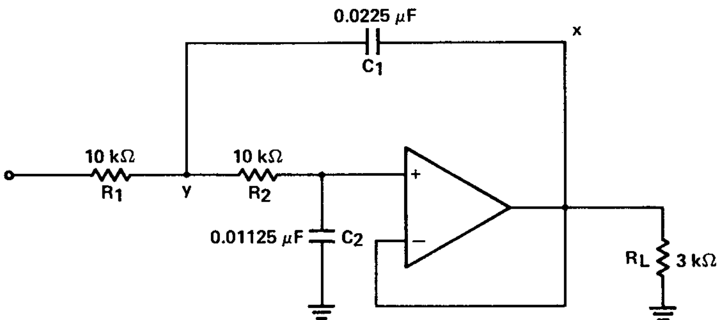
and substituting in equation (32)

$$F = \frac{(1)}{(2\pi)(12.5)} = 12.7 \text{ kHz}$$

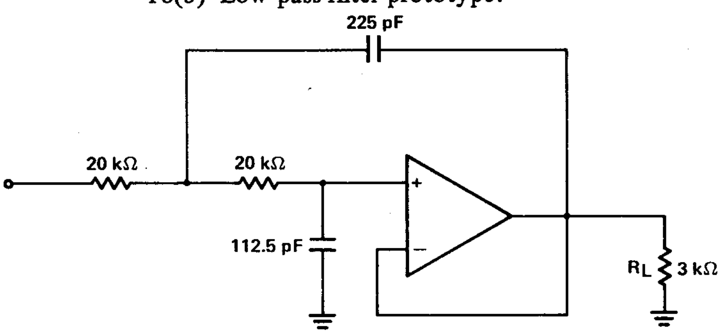
The amplifier can therefore deliver 25 V peak-to-peak at 12.7 kHz into 2 kΩ. All these numbers are typical, so it would be wise to account for the use of an atypical amplifier by derating to 20 V peak-to-peak at 10 kHz;



16(a) Inverting amplifier.



16(b) Low-pass filter prototype.



16(c) Frequency- and impedance-scaled low-pass filter.

FIGURE 16. – Circuits for showing effects of slewing rate on selection of passive element values.

that is, to the conditions set in the problem. The minimum load is therefore 2 kΩ. The load on the amplifier is the 3-kΩ external load in parallel with R_2, for which the left-hand side is effectively grounded. Since R_2 in parallel with 3 kΩ must equal 2 kΩ,

$$R_2 = \frac{\text{Product}}{\text{Difference}} = \frac{(3)(2)}{(3-2)} = 6\ \text{k}\Omega$$

A value of $R_1 = R_2$ of equal or greater than 6 kΩ would be suitable.

EXAMPLE 13: The circuit in figure 16(b) is for a low-pass Butterworth filter with a 3-dB cutoff frequency of 1 kHz. Redesign for a 50-kHz cutoff if the load is to be 3 kΩ and the output is to be 20 V peak-to-peak. The amplifier can deliver 10-mA peak.

SOLUTION: This problem involves frequency-scaling and impedance-scaling as previously described and as in table II (page 18). From table II, f_c/f_n is 1/50. First try leaving the capacitors as they are and scaling the resistors, giving $R_1 = R_2 = 0.2$ kΩ. The filter is now redesigned for 50 kHz, but it is necessary to determine whether the operational amplifier can supply the total load. An exact analysis for all conditions is fairly involved, but a workable design can be realized by solving a "worst case" situation.

The load on the filter consists of the 3-kΩ resistor R_L, and C_1; the amplifier must be able to supply current to both of these. Since the output is 20 V peak-to-peak, the maximum current into R_L is 10 V/3000Ω = 3.3 mA. It is not easy to determine the maximum current into C_1, but a worst case would be when point X on figure 16 is at +10 V and point Y at -10V, giving 20 V across C_1. (Point X at -10V and point Y at +10 V also gives 20 V across C_1.) Beyond 50 kHz, the filter output is rolling off, and a 10-V output would not be obtained from a 10-V input beyond a cutoff frequency. However, since the impedance of C_1 decreases as frequency increases, the loading by C_1 of the amplifier will get worse at higher frequencies. Therefore, consider the loading by C_1 at 100 kHz with 20 V across it. The current through C_1 is

$$i_c = \frac{\text{Voltage across } C_1}{\text{Impedance of } C_1} = \frac{V_c}{(1)/(2\pi f C_1)}$$

$$= \frac{20}{1/(2\pi)(10^5)(0.0225)(10^{-6})} = 283\ \text{mA}$$

This current is not necessarily in phase with the 3.33 mA into R_L, and an exact analysis would involve vector addition of the 283-mA and the 3.33-mA currents; however, it is obvious that the amplifier, with its 10-mA peak output current capability cannot supply its load and that capacitor C_1 is the problem.

If C_1 is scaled by a factor of 100 to increase its impedance, i_c will be reduced to 2.83 mA and, ignoring vector addition to give a worst case, the maximum current the amplifier could be called upon to supply would be 2.83 + 3.33 = 6.16 mA, which is satisfactorily below the 10-mA rating of the amplifier. From table II (page 18), it is evident that if all capacitor values are divided by 100, R_1 and R_2 (=0.2 kΩ) must also be multiplied by 100 to keep the same cutoff frequency. The final design is shown in figure 16(c).

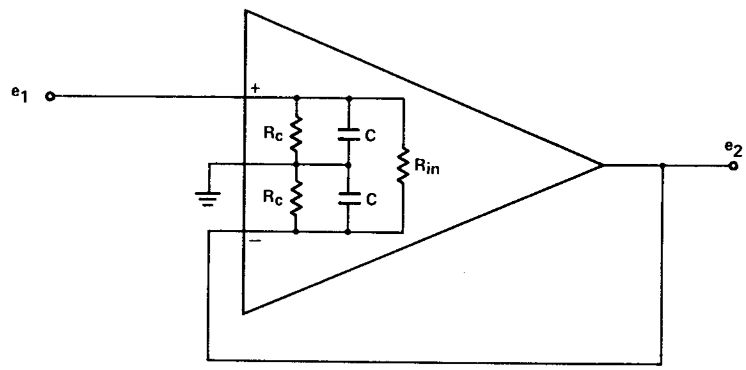
Effect of Finite Input Impedance

The effect an operational amplifier with a finite input impedance has on an active filter depends on whether the amplifier is used in an inverting or noninverting mode. The voltage follower is an example of the noninverting mode and is very commonly used in active filter circuitry; consider the circuit in figure 17(a). Between the input terminals of the amplifier there is a resistor R_{in}, and from each terminal to ground there are common-mode resistors, R_c. From each input terminal to ground there are capacitors C; for purposes of simplification, the capacitors and the resistors are assumed to be fixed in value.

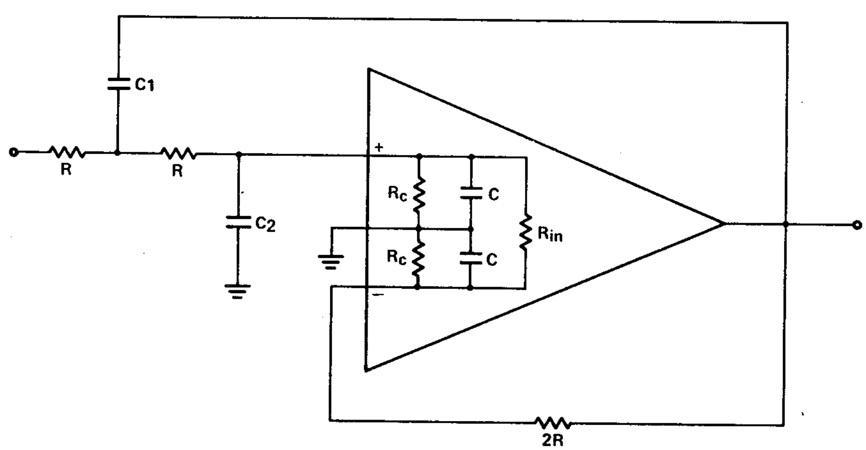
Because R_c and C are grounded, the feedback has no effect on their effective values and they modify filter design simply by paralleling any component from the input to ground. The effective value of R_{in} is determined by multiplying R_{in} by the circuit-loop gain, which is approximately the ratio of the open-loop gain to the closed-loop gain. For a follower, the closed-loop gain is nearly unity; consequently, at dc and low frequencies, the effective value of R_{in} becomes very large, but at high frequencies the effective value is reduced as the open-loop gain falls off.

EXAMPLE 14: A voltage follower has a gain of 0.999 at dc with R_{in} = 1 MΩ and R_c = 100 MΩ. What is the input resistance at dc?

SOLUTION: If e_1 in figure 17(a) is +1 V, then e_2 will be +0.999 V and the voltage across R_{in} will be 0.001 V. The current through R_{in} is (0.001 V)/(1 MΩ) = 0.001 μA. The current in R_{in} results from application of an input voltage of 1 V, so the *effective* input resistance of R_{in} is (1 V)/(0.001 μA) = 1000 MΩ. The effective input resistance appears in parallel with the R_c connected to the + input. (The other R_c is isolated



17(a) On voltage follower.



17(b) On low-pass filter.

FIGURE 17. – The effect of finite input impedance.

by the high effective R_{in}.) The net input resistance is therefore 100 MΩ in parallel with 1000 MΩ or 90.9 MΩ.

The above example illustrates what is often the case with follower circuits, namely that the input resistance is almost entirely determined by the common-mode resistance, at least at frequencies where the follower gain approaches unity.

In low-pass filters, such as in figure 16(b) and (c), the external resistors in the RC network form an attenuator with the effective input resistance and with any input capacitors. Correct circuit design involves impedance-scaling and operational amplifier selection to ensure that the attenuation is adequately small, usually less than one percent.

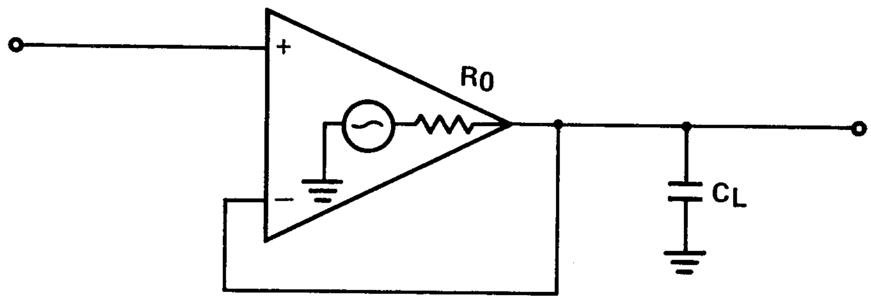
In high-pass filters, a resistor, 2R, is sometimes connected as shown in figure 17(b) in order to reduce the effects of amplifier input current, as explained in the Input Offset and Drift section that follows. It is not uncommon for such a circuit to oscillate, especially is 2R is large (a few megohms) as in low-frequency, FET operational amplifier circuits. The extra phase shift in the negative feedback loop produced by 2R and C in figure 17(b) induces oscillation, but such oscillations usually can be eliminated by paralleling 2R with a capacitor or by reducing R (and hence 2R) by impedance-scaling.

It can be shown that the main effect of the various input resistances and capacitances on the inverting configuration is to reduce the loop gain. Unless the amplifier is being used near the upper limits of its frequency range, the effect is usually small enough to be neglected.

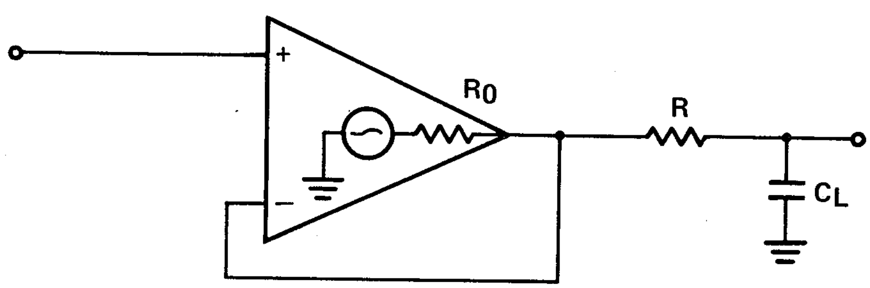
Effect of Nonzero Output Impedance

Most operational amplifiers have fairly low open-loop output resistances, 1000 ohms being typical. When negative feedback is applied, the open-loop output resistance is reduced by approximately the ratio of the open-to-closed-loop gains. At lower frequencies, where the open-loop gain is very large, the effective output impedance is often a small fraction of an ohm, thereby making the amplifier approach the true voltage source often assumed in filter design.

The main problem resulting from nonzero output resistance occurs at high frequencies when the amplifier is driving a capacitive load such as a coaxial cable (see figure 18(a). As the frequency increases, the open-loop gain drops and the effective output resistance increases. The output resistance and any load capacitances form a lag or low-pass network that introduces additional phase shift into the feedback loop. The additional phase shift adds to other phase shifts in the feedback loop and may cause oscillations. One



18(a) Without correction.



18(b) With correction.

FIGURE 18. – Effect of output resistance with capacitive loading.

solution to this problem is to reduce the load capacitance; another is to isolate the load capacitance with a resistor, R, in figure 18(b). Usually a value of 100 to 1000 Ω is sufficient, and if the following circuit has an input resistance of 1 MΩ or more, loading errors are often insignificant.

Input Offset and Drift

An ideal operational amplifier would have zero-voltage output for a zero-voltage input, but with a practical amplifier there will always be an output even when there is no input because there are two sources of offset at the amplifier input, one a voltage offset and the other a current offset. The offsets are independent of each other and their relative effect on the output depends on the particular circuit in which the amplifier is being used. Low-pass active filters are the ones most commonly affected by offsets, and it is the operational amplifier input current that usually has the greatest effect. Currents in the inverting and noninverting inputs are called bias currents and the difference between the two bias currents is called the current offset. The input currents flow through various external resistors, depending on the actual circuit used; in conjunction with the input voltage offset, input currents combine to produce a net output voltage offset.

Some low-pass filters consist of several operational amplifiers connected in series, but separated by resistors as in figure 33. For low-frequency filters, the resistors may have such large values that currents flowing through them produce large offsets that may be disadvantageous in some applications. Corrective steps include rescaling to use smaller resistor values, the use of amplifiers with low bias-currents and offset-currents and the use of cancellation techniques to reduce offset effects.

It is usually possible to cancel the effects of offsets, but the extra components needed are often undesirable ones such as potentiometers that must be set individually. It is better, where technically and economically feasible, to use an operational amplifier that has offsets small enough to be neglected. Even when offsets are cancelled, a problem remains because cancellation is only perfect at one temperature. The variation of offsets over a period of time because of aging or changes in temperature is called drift. It is usual to refer offsets and drift to the amplifier input, since the resulting values are independent of gain and circuit configuration. More importantly, the values can be compared with input signal levels as an indicator of performance.

The effects of voltage and current offsets can be studied with the help of the equivalent circuit of figure 19.

The effects of offset are readily discerned when there is no signal at either input, so R_1 and R_2 are grounded. The input bias currents are I_1 and I_2 and the input offset voltage is V_0. The offset current, I_0, is the dif-

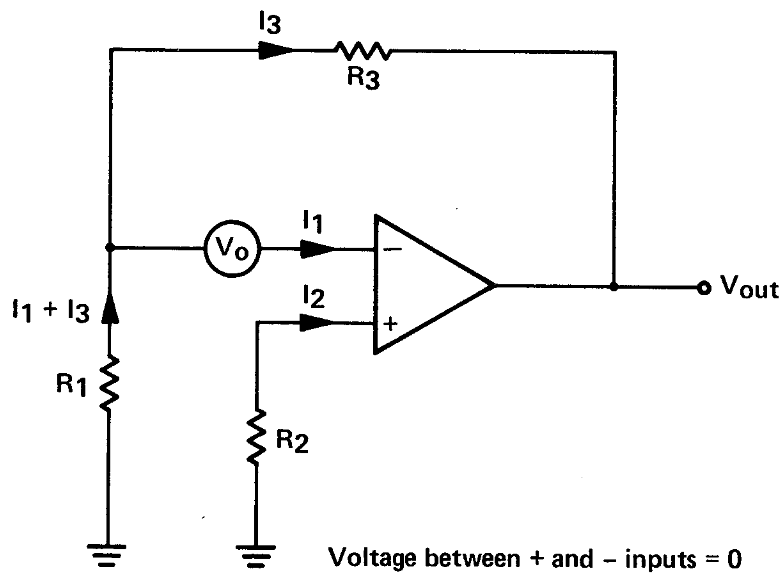


FIGURE 19. – Operational amplifier circuit showing equivalent voltage- and current-offset generators.

ference between I_1 and I_2, or $I_2 - I_1$. The output voltage, V_{out}, is the result of the sum of the various offset effects.

Only two equations need to be solved to obtain a useful expression for offset effects; since V_{out} equals the sum of the voltage drops across R_1 and R_3, then

$$V_{out} = (I_1 + I_3) R_1 + I_3 R_3 \tag{33}$$

Also, the potential drop around the loop that contains R_1, V_o, and R_2 is zero; thus

$$(I_1 + I_3) R_1 - V_o - I_2 R_2 = 0 \tag{34}$$

Equations (33) and (34) can be solved to give

$$V_{out} = V_o \left[1 + \frac{R_3}{R_1}\right] + I_2 R_2 \left[1 + \frac{R_3}{R_1}\right] - I_1 R_3 \tag{35}$$

Since the coefficients of I_1 and I_2 in equation (35) are opposite in sign, V_{out} can be minimized by making their coefficients equal. From

$$R_3 = R_2 \left[1 + \frac{R_3}{R_1}\right] \tag{36}$$

there is obtained

$$R_2 = \frac{R_1 R_3}{R_1 + R_3} \tag{37}$$

Equation (37) reveals that when R_2 is made equal to the equivalent parallel resistance of R_1 and R_3, V_{out} is minimum. Substituting equation (36) into (35) and recalling that $I_2 - I_1 = I_o$ gives

$$V_{out} = V_o \left[1 + \frac{R_2}{R_1}\right] + I_o R_3 \tag{38}$$

Equation (38) permits computation of the output offset when the inputs are shorted to ground, given the offset voltage and current (V_o and I_o) and the circuit resistances, if the effective resistances on the + and – inputs are equal. The equation gives a worst-case result, but it is also possible for the voltage and current effects to subtract and provide a lower output offset.

EXAMPLE 15: The circuit diagram of a 24 dB-per-octave, low-pass filter is given in figure 20. The amplifiers have offset voltages of 10 mV and offset currents of 10 nA. What is the output voltage when the input is shorted to ground?

SOLUTION: Since the problem is concerned with dc offsets, the capacitors function as open circuits and have no effect on dc performance; also, equation (38) may be used directly, noting that there are 200-kΩ input resistances at both inputs. The two stages are identical, and a worst-case condition can be obtained when all current and voltage offsets are additive. Comparing one stage of figure 20 with figure 19, it is evident that $R_1 = \infty$, $R_2 = 200\ k\Omega$, and $R_3 = 200\ k\Omega$. Hence

$$V_{out} = (10\ mV)\ (1 + \frac{200\ k\Omega}{\infty}) + (10\ nA)\ (200\ k\Omega)$$

$$= 10\ mV + 2\ mV = 12\ mV$$

The output voltage may be plus or minus depending on the polarity of the offsets. For two stages, the output would be 24 mV at most.

EXAMPLE 16: The circuit diagram of a 12 dB-per-octave, high-pass filter is given in figure 21. What is the output voltage with the input shorted to ground if the amplifier has $V_o = 10$ mV, $I_o = 10$ nA and the bias current I_2 indicated in figure 19 is 100 nA?

SOLUTION: The resistances at the plus and minus inputs of the amplifier are not equal, so equation (38) cannot be used, but equation (35) may be used because it applies to unequal resistances. Comparing figures 21 and 19, it is seen that

R_1 = (- input to ground) = ∞
R_2 = (+ input to ground) = 200 kΩ
R_3 = (feedback resistance) = 0

(The 100-kΩ resistor has no effect on the dc performance because it is isolated by two capacitors.) Substituting in equation (35),

$$V_{out} = (10\ mV)\ (1 + \frac{0}{\infty}) + (100\ nA)\ (200\ k\Omega)\ (1 + \frac{0}{\infty}) - (I_1)\ (0)$$

$$= (10\ mV) + (20\ mV) = 30\ mV$$

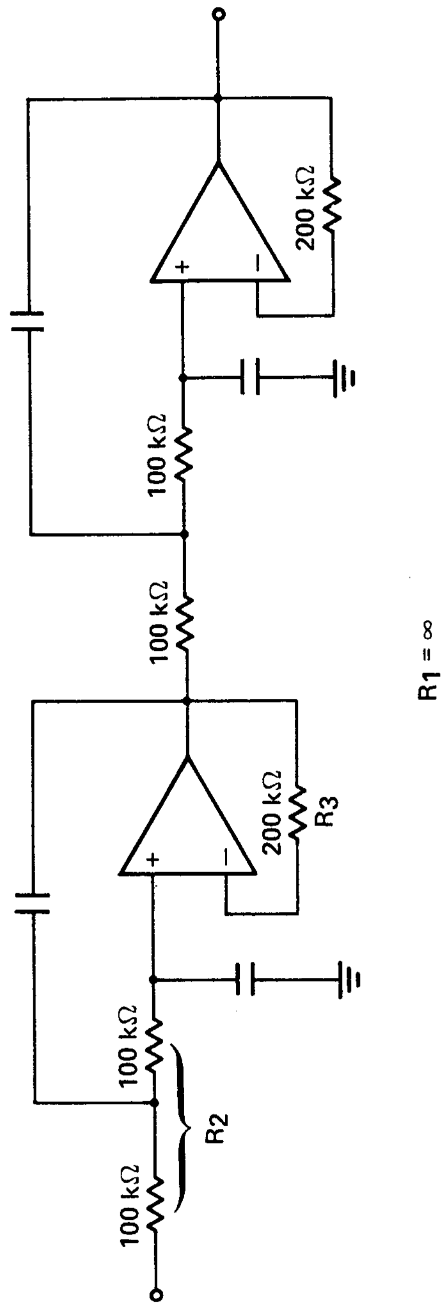


FIGURE 20. – Offset in a low-pass filter.

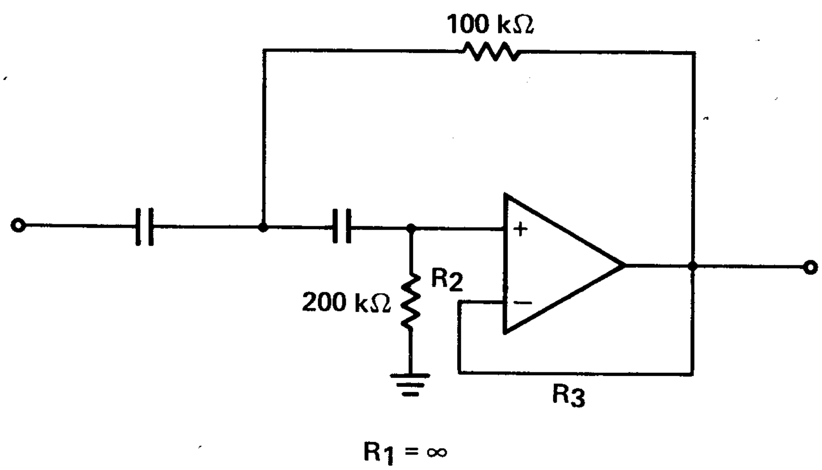


FIGURE 21. – Offset in a high-pass filter.

Drift, which represents changes in offset because of aging and temperature variations, can be studied by methods similar to those used to study offsets. Use is made of figure 19, equation (35), and equation (38), but it is necessary to make substitutions; for example, offset changes caused by variation of temperature. Following are some applicable symbols formed by using lower-case letters for temperature or time *coefficients* and upper-case letters for *dc* offsets and biases:

v_o = temperature coefficient of V_o
i_o = temperature coefficient of I_o
i_1 = temperature coefficient of I_1
i_2 = temperature coefficient of I_2

EXAMPLE 17: Figure 22 is the circuit diagram for a bandpass filter. The value for the temperature coefficient of the offset voltage, v_0, is 10 μV/°C; I_1 is 100 nA at 25°C and doubles for every 8°C increase in temperature. What is the maximum output change from 25° to 50°C?

SOLUTION: Since the resistances at the plus and minus inputs are unequal, equation (35) must be used. First calculate V_0 and I_1 changes for the range 25° to 50°C:

$$\Delta t = 50 - 25 = 25°\ C$$
$$v_o = 10\ \mu V/°C$$
$$\Delta V_o = \Delta t v_o = 250\ \mu V$$

Given the value of offset current at 25°C and that it doubles every 8°C, at any other temperature the value is,

$$I_T = \left[2^{\frac{T-25}{8}}\right] I_{25} \tag{39}$$

For T = 50°C

$$I_{50} = \left[2^{\frac{25}{8}}\right] (100\ nA) = 872\ nA$$

The change in I_1, ΔI_1, is therefore 872 - 100 - 772 nA, and this value can now be substituted into equation (35). Comparing figures 19 and 22, the resistance R_1 from the negative terminal to ground is infinite, R_2 is zero, and R_3 is 100 kΩ. It is also necessary to replace V_0 and I_1 by ΔV_0 and ΔI_1. Note that the I_1 term in equation (35) is negative because I_1 always has an opposite sign to I_2. However, all the signs in equation (35) are arbitrary;

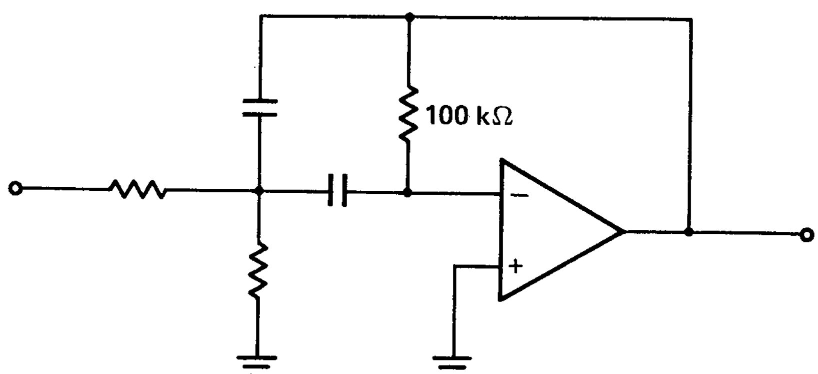


FIGURE 22. – Bandpass amplifier with temperature drift.

thus V_o can add or subtract from I_1 or I_2. For a worst case the signs are adjusted to add the effects of ΔV_o and ΔI_1,

$$\Delta V_{out} = (250\,\mu V)\ (1 + \frac{100\,k\Omega}{\infty}) + (772\,nA)\ (100\,k\Omega)$$

$$= 250\,\mu V + 77{,}200\,\mu V = 77.45\,mV$$

PASSIVE COMPONENTS

Introduction

The most important requirement of the resistors and capacitors used in filter circuits is stability. Initial accuracy is of secondary importance because any type of component can be selected or trimmed to a desired value. Once a resistor or capacitor of a given value is inserted into a filter, it is important that the value be stable; the required degree of stability depends on the application. If the cutoff frequency, center frequency, bandwidth or filter shape must be held very closely, then stable components with accurate values must be used. The requirements for stability and accuracy are more stringent for high-Q than for low-Q circuits. Sometimes advantage can be taken of the fact that resistor and capacitor values appear as RC products in the formulae for filters; thus when resistors that have a temperature coefficient (TC) of +X ppm/°C are used with capacitors that have coefficients of -X ppm/°C, a degree of compensation can be obtained.

Resistors

As usual, selection of the type of resistor to be used in the construction of a filter is a compromise between performance and cost. The best components and the most expensive should be used only where necessary. For low-Q circuits and room-temperature applications, carbon composition resistors that have a TC ranging from about 250 to 500 ppm/°C may be used. Note that the value of a component with a TC of 500 ppm/°C will change by one percent over a 20-degree range.

For most applications, the metal-film resistor is the best compromise between cost and performance. Various TC's are available, and 100 ppm/°C is typical, but resistors with coefficients of 10 ppm/°C are available. For the most demanding applications, wirewound resistors (of the noninductive type for high-frequency applications) are available with TC's of a few ppm/°C.

Resistors may also be purchased in integrated circuit form in which the temperature stability of film resistors is superior to diffused or pinch resistors. Thin-film resistors are generally superior to thick-film types for long-term stability, but high values of resistance are difficult to manufacture. The TC's of thick-film and thin-film resistors are comparable.

Capacitors

When selecting a capacitor for an RC filter, the designer must choose not only on the basis of cost and performance, as with resistors, but must also consider size. The best capacitors are not only the most expensive, but also tend to be physically large, for example, large-value polystyrene capacitors. In general, capacitors vary more with temperature, time, applied voltage, frequency, and mounting pressure than do resistors. Also, just as a resistor will have stray capacitance associated with it, so will a capacitor have losses in the form of equivalent resistances. Since filters are often designed on the basis that ideal capacitors are used, the fact that practical capacitors have losses and are therefore nonideal introduces inaccuracies to a greater or lesser degree in the final filter. There are a number of conventional ways to describe the losses in a capacitor, the three most common being the quality factor or figure of merit Q, the dissipation factor D, and the power factor PF. These are defined with the aid of the vector diagram in figure 23 as follows:

$$Q = \frac{\text{Capacitive reactance}}{\text{Effective series resistance}} = \frac{1}{\omega CR} = \tan\theta \tag{40}$$

$$D = \frac{1}{Q} = \omega CR = \frac{1}{\tan\theta} \tag{41}$$

$$PF = \cos\theta \tag{42}$$

where θ is the angle by which the current and voltage in a capacitor are out of phase. For a lossless capacitor, θ would be 90°. For most capacitors, θ is very close to 90° and it can be assumed that

$$D = \frac{1}{Q} = PF = \omega CR \tag{43}$$

because for θ close to 90°, $\cos\theta \approx 1/\tan\theta$.

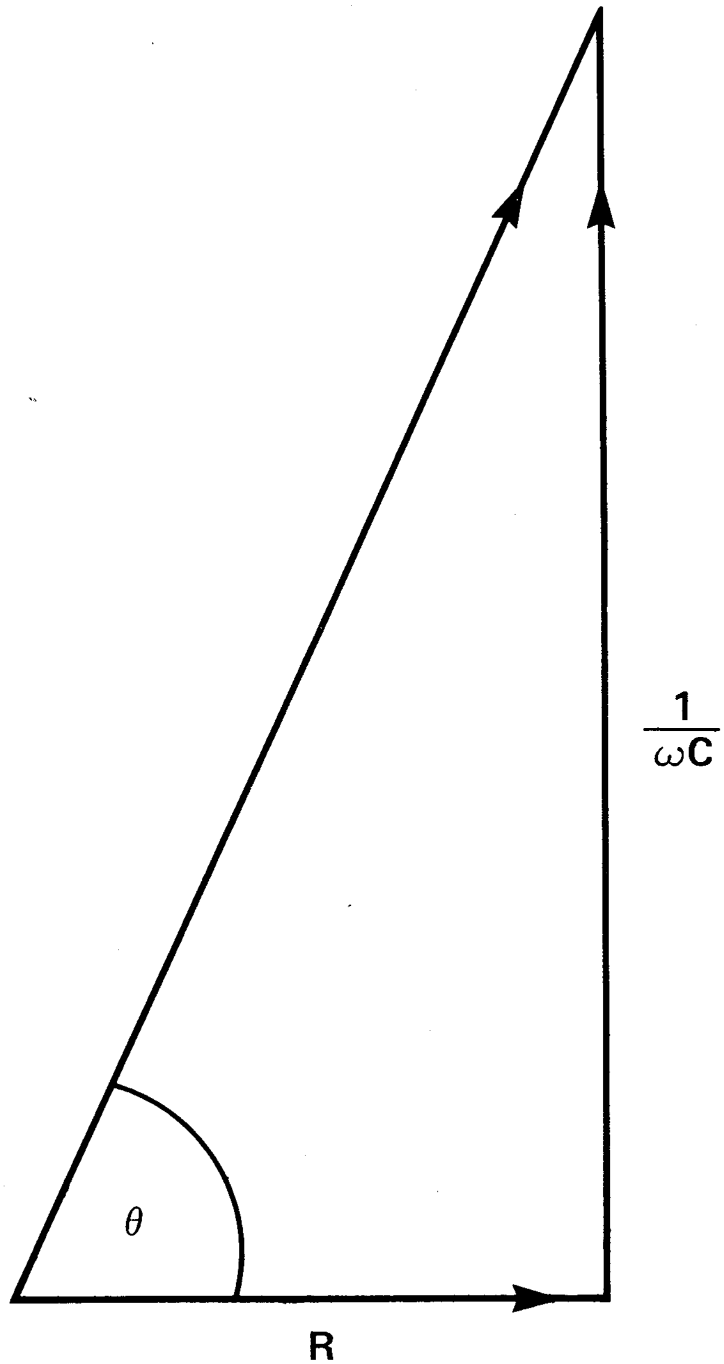


FIGURE 23. – Designation of the capacitor phase angle θ.

The performance of a capacitor depends primarily on the dielectric material used. A material with a high dielectric constant permits construction of a capacitor with a small volume because of the following relationship:

$$\text{Dielectric constant of a material} = \frac{\text{Capacitance of a capacitor with the material as dielectric}}{\text{Capacitance of the same capacitor with vacuum as dielectric}} \qquad (44)$$

Dielectrics have ac losses, dc leakage paths and, because of dielectric absorption, can retain charges that are not removed when capacitor leads are shorted. The dielectric constant, and hence capacitance, is affected by temperature, time, and applied voltage. The effect of applied voltage on capacitance can be minimized by operating a capacitor well below its maximum voltage rating. Table III lists common dielectric materials.

Selection of Odd-Value Components

An unfortumate fact about most filter designs, including those for RC active circuits, is that they usually specify nonstandard component values. If a filter is to be produced in large quantities, it is worth the cost and time to obtain special values. However, if one or only a few filters are required, the designer is confronted with the problem of generating nonstandard-value components from combinations of available components. The resistors or the capacitors in a circuit often can be scaled to be standard values; an example of this kind of scaling is shown in figure 24. Both circuits are 1000-Hz, low-pass Butterworth filters with identical responses. The circuits have been impedance-scaled (see table II, page 18) so that the circuit in figure 24(a) uses standard-value resistors and nonstandard capacitors; the opposite situation is shown in the circuit in figure 24(b). Two questions can be asked: Which of the two circuits is better and, having selected a circuit, how can one obtain the necessary odd-value components?

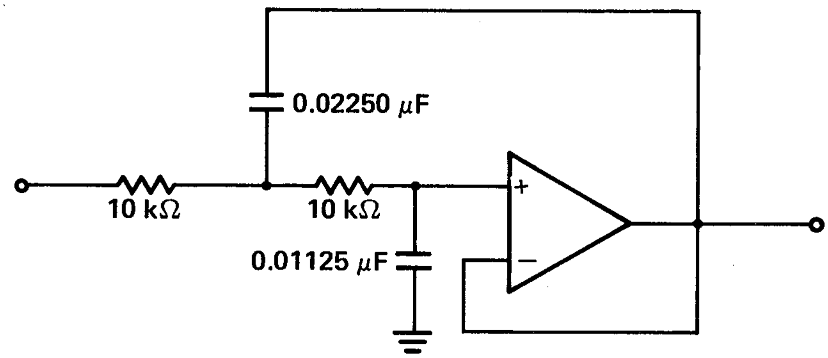
At first sight it might appear that the circuit in figure 24(b) would be the better choice since it uses standard-value capacitors of 0.01 and 0.02 μF, and it is easy to obtain the required resistors by using one-percent precision components. For example, an 11.25-kΩ resistor accurate to one percent can be obtained by combining an 11.15-kΩ one-percent resistor with a 100-Ω 10-percent resistor. Note that the 100-Ω resistor need not be a one-percent component since 10 percent of 100 Ω is 10 Ω and this discrepancy will contribute less than a 0.09-percent error to the final value of 11.25 kΩ. However, it is also assumed that one-percent capacitors are available; if they are, the filter can be built with the following components:

0.01- and 0.02 - μF capacitors (one percent)
11.15-kΩ (1-percent) and 100-Ω (10-percent) resistors

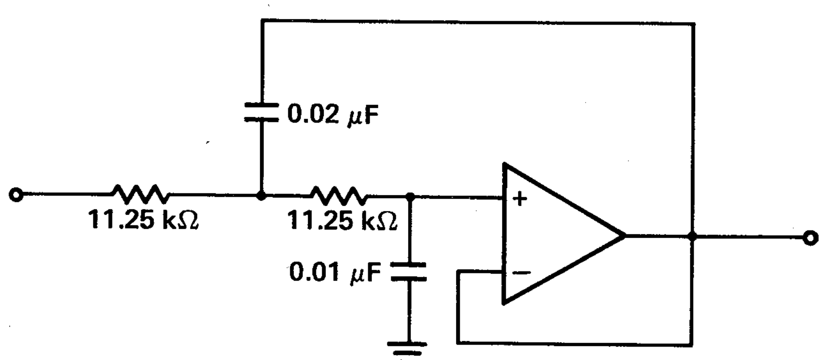
TABLE III. – *Typical Values for the Temperature Coefficient (TC) of the Dielectrics Used in Capacitors*

Dielectric	ppm/°C	Remarks
Polystyrene	± 10 to ± 100	Long-term stability, but use is limited to about +85°C.
Polytetrafluoroethylene (PFTE; Teflon)	– 100	Useful to +125°C.
Polycarbonate	+ 100	TC is small at room temperature.
Mylar	150*	Very compact types are metallized. Good for general purpose use.
Paper	± 500	Unstable with time, but low cost.
Silver-mica	± 100	Used for stable, low values.
Glass	+ 150	Used for stable, low values.
Ceramic	**	Large values have poor TC and stability with time.
Tantalum oxide	± 1000***	Only used as a last resort in RC active filters. Polarized types can be used with up to 0.5 V of reverse-bias.

* TC is very nonlinear.
** Different types vary from a few ppm to 10,000 ppm.
*** Varies widely.



24(a) Design of a 1000-Hz low-pass Butterworth filter with standard resistance values.



24(b) A 1000-Hz low-pass, Butterworth filter with standard capacitance values.

FIGURE 24. – Examples of scaling resistance and capacitance values to standard.

Usually, one-percent capacitors are not available and it is necessary to use 5-percent or 10-percent components and to select appropriate units, but if 5-percent or 10-percent tolerance capacitors are used, there is always a possibility that all in a given lot will be found to be greater than 0.01 or 0.02 μF. The only way to obtain a 0.01 - μF one-percent capacitor from a 0.01 - μF 5-percent capacitor, which turns out to be, say, 0.0105 μF, is to connect a 0.21 - μF capacitor in series with it. To trim a capacitor value by connecting a much larger value capacitor in series is feasible in this case but, at lower frequencies, trimming a 1 - μF with a 100-μF capacitor would most likely be impractical.

With luck, a 0.01 - μF capacitor might be found to be less than 0.01 μF; in this instance, trimming may be effected by connecting smaller capacitors in parallel. For example, if the 0.01 - μF 5-percent capacitor was found to be 0.0095 μF, a 0.0005 – μF (500 – pF) capacitor could be connected across it as a trimmer, and because one percent of a 0.01 - μF capacitor is 100 pF, a 500-pF 20-percent unit would thus give 0.01 μF within one percent. A 5-percent 500pF capacitor would allow for errors in bridge measurement.

Regardless of the methods employed for trimming, if the circuit of figure 24(b) is used (because it uses readily available 0.01 - μF and 0.02 - μF capacitor values), it will be necessary to select capacitors when one-percent-accuracy types are not available. Also, two components must be used for each of the 11.25-kΩ resistors. Probably it is better to use the circuit of figure 24(a), because one-percent precision resistors can be used for the 10,000-ohm components and it won't be necessary to make up special resistor values. In any event, it still is necessary to select capacitors as for the case of the circuit of figure 24(b). Suppose that capacitors of 5-percent accuracy are available and that they have nominal values of 0.022 μF and 0.01 μF, but on measurement they are found to be 0.0215 μF and 0.01 μF, respectively. The 0.0215 - μF capacitor is already within 4.5 percent of the desired value. A 0.001 - μF capacitor with a 20-percent tolerance or better in parallel with the 0.0215 - μF capacitor will provide a capacitor that is within one percent of 0.0225 μF, the desired value. The 0.0105 - μF capacitor is too small by 0.00075 μF (750 - pF), or about 6.7 percent; however, by placing a 750-pF 10-percent capacitor in parallel with it, the desired value of 0.0105 μF will be obtained to better than one percent.

SELECTION OF NUMBER OF POLES

The pole-zero approach to filter design is discussed in the appendix, but at this point it is pertinent to note that, in general, the more poles a given type of filter has, the steeper will be the attenuation slope in the transition from passband to stopband. Because an ideal filter has an infinitely steep attenuation slope, the more poles a filter has, the closer it will approach the ideal. Unfortunately, more poles are obtained at the expense of greater complexity and more parts. Compare, for example, the circuits and attenuation slopes of the Butterworth filters shown in figures 29 and 33. The filter in figure 29 is a two-pole filter of low complexity with a 12-dB-per-octave cutoff. The filter in figure 33 is an eight-pole filter of increased complexity, but it provides a 48-dB-per-octave cutoff.

For the low-pass and high-pass filters described in chapters 3 and 4, and for many types of bandpass filters, one pole essentially corresponds to 6 dB per octave, which is why the 2-pole filter has a 12-dB-per-octave cutoff slope and the 8-pole has a 48-dB-per-octave cutoff slope. Often the number of poles is equal to the number of RC sections, as is true of the low-pass and high-pass filter sections discussed in chapters 3 and 4. Sometimes a circuit may have an unequal number of resistors and capacitors; for example, the one shown in figure 22 is a bandpass circuit with three resistors and two capacitors, but the input resistor and the resistor connected to ground in this circuit are used as an input attenuator to make the filter gain practical. Theoretically, these resistors could be replaced by a single equivalent resistor; the filter characteristic would be the same, but with a higher scale factor or gain. Moreover, the circuit now would have two resistors and two capacitors, as is common with 2-pole filters.

As usual, it is the job of the designer to compromise between performance and complexity in selecting an appropriate filter design. More information and some numerical examples on the selection of the number of poles are given in the first sections of chapters 3 and 4.

Low-Pass Filters

SELECTION OF FILTER TYPE AND NUMBER OF POLES

This chapter describes the steps involved in the design of three types of low-pass filters: Butterworth, Chebyshev, and Bessel. First of all, it is necessary to select the appropriate filter for the application at hand; the three types of filters are discussed at the beginning of chapter 2 and summarized in table I. The Butterworth is a general-purpose filter that provides good attenuation characteristics and maximum possible flatness in the pass region. The Chebyshev filter has a steeper attenuation slope in the region of cutoff, but this is achieved at the expense of ripple in the passband. The Bessel filter has a poor attenuation slope and also poor flatness in the passband. It is widely used for filtering pulses because its linear-phase characteristic minimizes the overshoot that can be a problem with low-pass Butterworth or Chebyshev filters (see figure 3).

After selecting one of the three basic types of filters, the next step is to decide on the number of poles that are to be used. For economy and simplicity, it is prudent to select a filter with the least possible number of poles. Elimination of unwanted signals and the reduction of noise are the two most common factors that govern the number of poles in the filter.

EXAMPLE 1: A unity-gain filter is required to pass all frequencies up to 100 Hz with maximum flatness. Signals of 4500 Hz and higher must be attenuated by at least 50 dB. How many poles are required in the filter?

SOLUTION: Figure 25 shows the idealized amplitude response curves for a Butterworth filter (maximally-flat amplitude response) with 2, 3, 4, 6, and 8 poles, the number of poles for which designs are provided in this handbook. A 4-pole filter will provide the necessary 50-dB attenuation for frequencies of 4500 Hz and above.

EXAMPLE 2: A unity-gain filter is required to pass all frequencies up to 200 Hz with maximum flatness. Signals of 4000 Hz or more must be attenuated by 50 dB or more. How many poles are required?

SOLUTION: Figure 25 applies to filters that have a cutoff at 1000 Hz. Because frequency scaling does not affect the *shape* of the response curves, the data in the figure may be applied to the problem of a 200-Hz cutoff filter by dividing *all* numbers on the frequency scale by 5; the cutoffs are

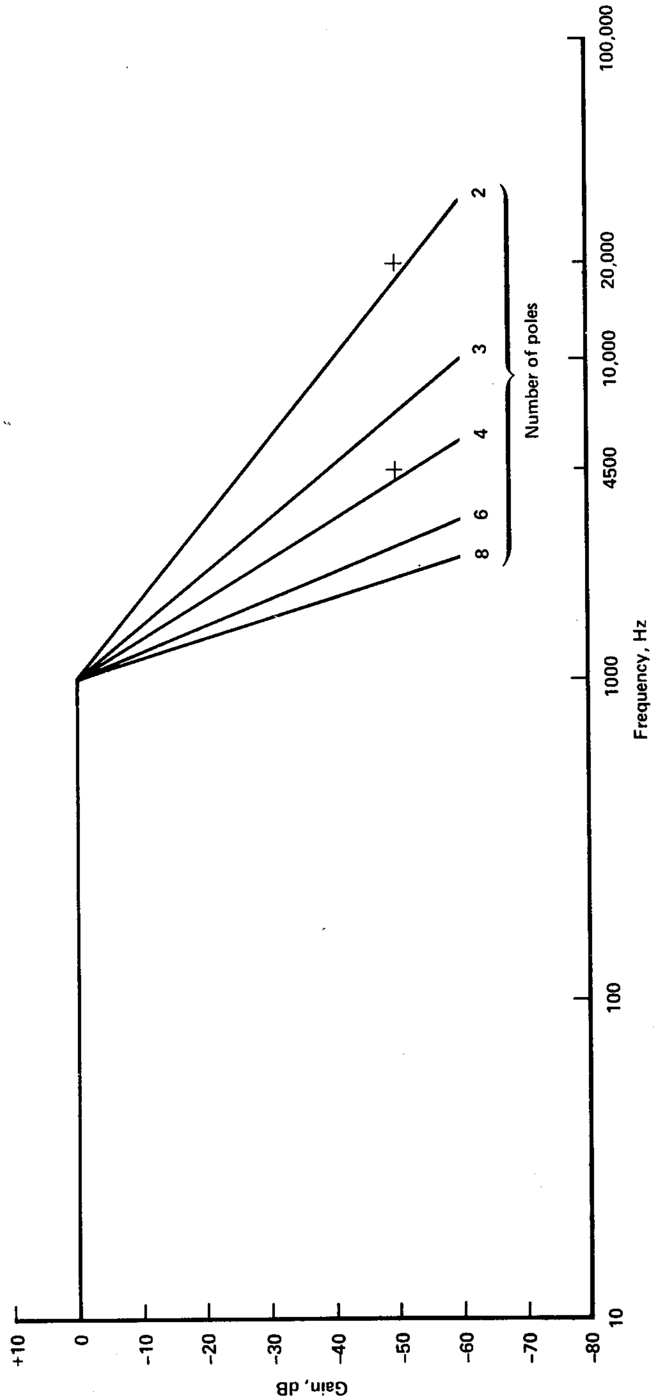


FIGURE 25. – Idealized amplitude response curves for low-pass Butterworth filters

thus converted to 200 Hz. Accordingly, the number of poles required to give 50 dB of attenuation at 4000 Hz is two (20,000 Hz divided by the scaling factor is 4000 Hz).

The data given in figures 34 to 38 and 39 to 43, respectively, may be used to find out how much attenuation Chebyshev or Bessel low-pass filters with different numbers of poles will provide. Also, figures 29 to 33 provide more accurate cutoff and attenuation information for the Butterworth filters than is shown in figure 25; however, the data in figure 25 are accurate enough for most purposes.

The problem of determining by how much a given filter will reduce noise is more involved than the selection of number of poles. It has been found convenient to study the noise performance of frequency-dependent circuits, of which filters are but one example, by means of an equivalent noise bandwidth. The equivalent noise bandwidth curve is rectangular, as indicated in figure 26, where there is displayed a low-pass filter response curve with a cutoff of f_c and the equivalent noise bandwidth curve with an infinitely steep cutoff at f_n. Once f_n is known, the usual expressions relating noise to bandwidth may be used; for example, it is known that thermal noise is reduced by the square root of the ratio by which the bandwidth is reduced.

EXAMPLE 3: A circuit has an equivalent noise bandwidth of 10 kHz and a thermal noise level of 100 μV. What will be the noise level e_n if the bandwidth is reduced to 1 kHz?

SOLUTION:

$$
\begin{aligned}
e_n &= [(1\ \text{kHz})/(10\ \text{kHz})]^{1/2} \cdot (100\ \mu\text{V}) \\
&= (0.316)(100\ \mu\text{V}) \\
&= 31.6\ \mu\text{V}
\end{aligned}
$$

Unfortunately, the noise in most instances is not simple thermal noise and it does not have the constant spectral density (that is, constant value of μV per root-hertz) required for the application of the simple square root rule illustrated in example 3. In general, noise usually is greatest at low frequencies ($1/f$ noise) and at high frequencies (f noise). The determination of the exact equivalent noise bandwidth for multipole filters with

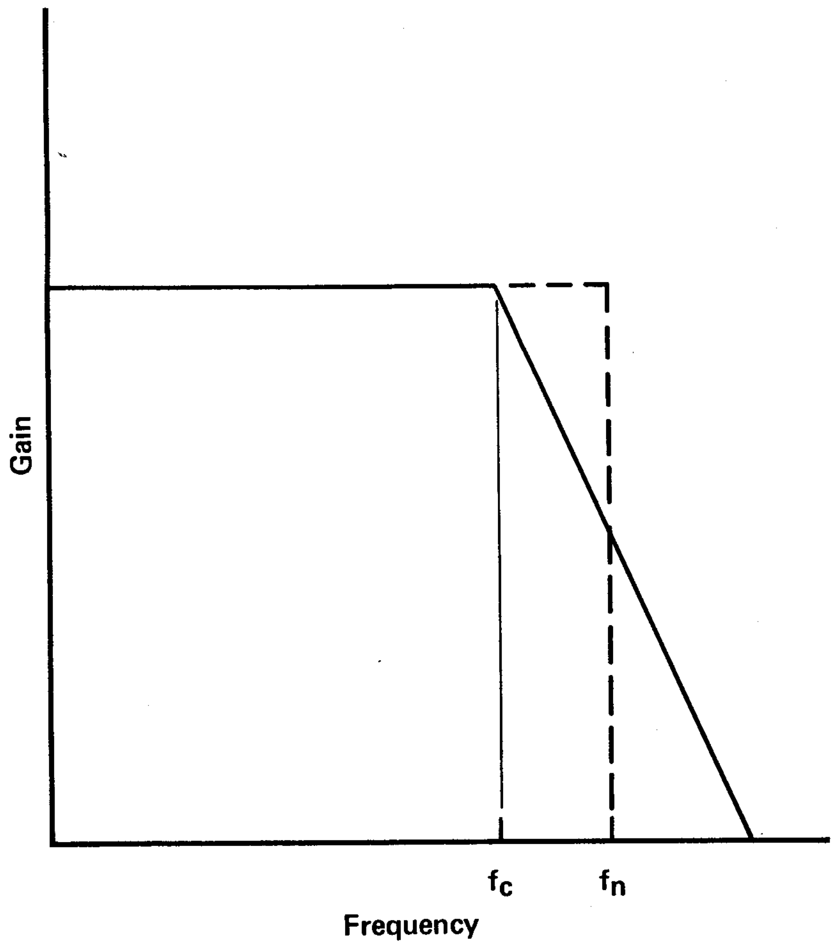


FIGURE 26. – Equivalent noise bandwidth.

the various Butterworth, Chebyshev, and Bessel responses is nontrivial. Reduced to its essentials, the analysis would proceed as follows:

Let $H(j\omega)$ = the transfer function of the filter

N_d = the spectral noise density applied to the filter

$\overline{e_n^2}$ = the squared RMS voltage, resulting from noise of density N_d being applied to the network having a transfer function $H(j\omega)$

Then

$$\overline{e_n^2} = \int_0^{\infty} (N_d)|H(j\omega)|^2 df \tag{45}$$

If the spectral noise density is assumed to be constant, as with thermal noise (but not with $1/f$ or f noise), equation (45) may be simplified by placing N_d on the left side of the integration sign. To obtain an effective noise bandwidth, the RMS voltage computed with the aid of equation (45) is equated to the value obtained from an ideal rectangular response. The simplest and most interesting result is obtained when equation (45) is applied to the case of a single-pole network, that is, one having a 6-dB-per-octave slope. Referring to figure 26, in this instance it is found that

$$f_n = \frac{\pi f_c}{2} = 1.571 f_c \tag{46}$$

The result is interesting because it represents a worst case, one that can be used to advantage because it simplifies the problem of finding out by how much a filter will reduce noise. A filter with two or more poles will remove more noise than the single-pole filter of performance represented by equation (46). Moreover, a single-pole filter is not only a worst case, it is also one of two endpoints. A single-pole filter will have more noise than a multipole filter, but the multipole filter will have more noise than the other endpoint, an ideal rectangular filter with a cutoff f_c (referring to figure 26); this state of affairs is best clarified by the following example.

EXAMPLE 4: Noise with a constant spectral density of 2 mV/Hz$^{1/2}$ is applied to a 6-pole Butterworth, 1000-Hz cutoff, low-pass filter. Find limits between which the noise must lie.

SOLUTION: First observe that the noise out of the filter will be less than for a single-pole filter with a cutoff of 1000 Hz, but greater than for an ideal rectangular filter with a 1000-Hz cutoff.

For a single-pole 1000-Hz filter, the equivalent noise bandwidth is obtained from equation (46) and figure 26:

$$f_n = (1.571)(1000) = 1571 \text{ Hz}$$

The noise output from the filter is the noise density times the square root of the effective noise bandwidth or

$$e_n = (1571)^{1/2}(2) = 79.27 \text{ mV}$$

For an ideal rectangular filter, with f_c from figure 26, the equivalent noise bandwidth is also f_c, and in this instance the noise output would be

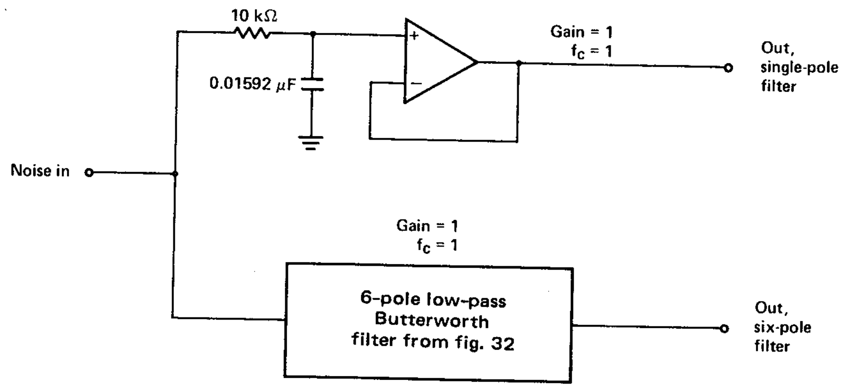
$$e_n = (1000)^{1/2}(2) = 63.25 \text{ mV}$$

For the 6-pole Butterworth filter, the noise will lie between 63.25 mV and 79.27 mV. In fact, the noise from a Butterworth filter with any number of poles will lie between these values. Two conclusions can be drawn from this analysis: First, multiple-pole, low-pass filters are little better than single-pole filters in reducing noise. Second, whenever noise voltage can be shown to lie between two closely-spaced values, there is little value in performing the extensive calculations implied by equation (45) to find the exact noise voltage for filters with various numbers of poles.

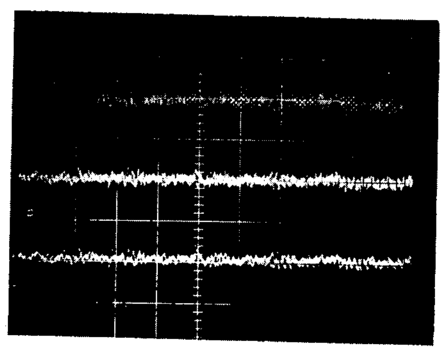
To compare the effectiveness of single-pole and multipole filters in reducing noise, 1-pole and 6-pole Butterworth low-pass filters were each fed with the same amount of noise by the arrangement shown in figure 27(a). As can be seen in the oscillogram reproduced in figure 27(b), the two filters show little difference in noise levels.

All the Butterworth and Bessel low-pass filters in figures 29 to 33 and 39 to 43, respectively, are 3 dB down at 1000 Hz. If the filters are scaled to another frequency by the techniques summarized in table II (page 18), the new cutoff frequency will also mark a 3-dB point.

With Chebyshev filters, the situation is slightly more complicated, but it is easily resolved. Plots of the amplitude-responses of 3-pole and 4-pole 1-dB-ripple Chebyshev filters are presented in figures 28(a) and 28(b), respectively. These figures are not drawn to scale in order to emphasize certain important features of the plots. The plots are values obtained from the classical formula for the amplitude response of the Chebyshev filters, which is



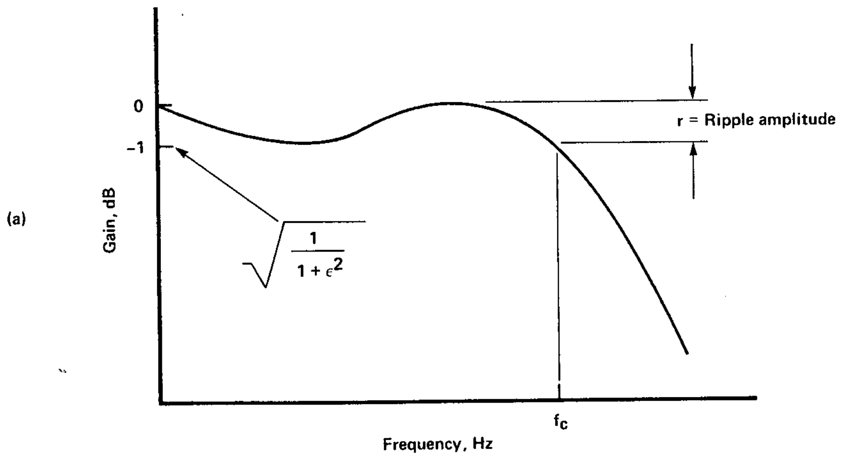
27(a) Test circuit.



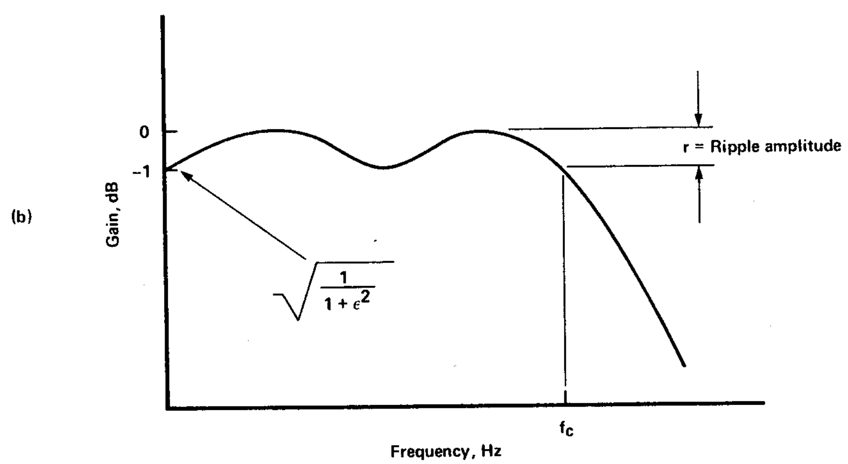
Horizontal, 10 ms/cm
Vertical, 0.1V/cm

27(b) Upper trace, input noise; middle trace, noise from one-pole filter; lower trace, noise from six-pole filter.

FIGURE 27. – Comparison of the performance of one-pole and six-pole filters with noise as the input signal.



28(a) Three-pole, low-pass.



$$\epsilon = \sqrt{10^{\frac{r}{10}} - 1}\ \text{, r in dB}$$

28(b) Four-pole, low-pass.

FIGURE 28. – Chebyshev amplitude responses.

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 C_n^2(\omega)}} \qquad (47)$$

where ϵ is a constant that controls the degree of ripple height, and $C_n(\omega)$ is the Chebyshev polynomial, which is a function of the frequency and the number of poles. For a 3-pole Chebyshev, $C_n(\omega) = 4\omega^3 - 3\omega$, and for a 4-pole, $C_n(\omega) = 8\omega^4 - 8\omega^2 + 1$.

The key features of interest at this time are the gains at dc and at the cutoff frequency f_c; at cutoff, both filters are down by an amount equal to the ripple or -1 dB, which is to be contrasted with the fact that the cutoff of many other filters is specified as occurring at -3 dB. Note that the 3-pole filter has a gain of 1 (0 dB) at dc, but that the 4-pole filter has a gain of -1 dB. In general, all 1-dB Chebyshev filters with an even number of poles will be down 1 dB at dc and at cutoff, while all those with an odd number of poles will be down 0 dB at dc and 1 dB at cutoff.

The Chebyshev filters described in this handbook differ in two ways from filters having the responses shown in figures 28(a) and 28(b): (1) All low-pass filters in this handbook, including Chebyshevs, are designed to have a gain of unity (0 dB) at dc, whether they have an even or odd number of poles; (2) All cutoffs are specified as occuring at the -3 dB point; that is, -3 dB from the maximum gain. The maximum gain for filters with an odd number of poles occurs at dc and at certain other frequencies; for filters with an even number of poles, the maximum gain occurs at frequencies other than dc.

Plots for 1 - dB-ripple Chebyshev low-pass filters are shown in figures 34 through 38; these filters are still true Chebyshevs, but the gains have been adjusted to be unity (0 dB) at 0 Hz. It is pertinent to reiterate that frequency-scaling or impedance-scaling will not affect the gains at dc or at the new cutoff.

SELECTION OF VALUES

Values for the components of low-pass filters that meet the requirements of a particular application are readily obtained by modifying an existing design with the frequency-scaling and impedance-scaling methods summarized in table II (page 18).

Design performance characteristics are given in figures 29 through 43 for 1000-Hz Butterworth, Chebyshev, and Bessel filters with 2, 3, 4, 6, and 8 poles. If the application under consideration can be satisfied by a filter with a 1000-Hz cutoff, the required circuit is given, and no further design is necessary.

EXAMPLE 5: Design a 3-pole, low-pass Butterworth filter that has a 3-dB frequency of 1000 Hz.

SOLUTION: Since the required cutoff frequency is 1000 Hz, the design in figure 30 can be used directly.

EXAMPLE 6: Design a 3-pole, low-pass Butterworth filter that has a 3-dB frequency of 1000 Hz. The filter is to be used to condition signals from a circuit that has an output resistance of 15,000 ohms.

SOLUTION: Since the cutoff frequency is 1000 Hz, the design in figure 30 can be used directly; however, because the signal source acts as a 15,000-ohm series resistance, it is necessary to use a buffer amplifier to provide the low-impedance drive required by all the low-pass filters described in this section of the handbook. If a buffer is not used, the 15,000-ohm source resistance will sum with the 10,000-ohm input resistors of the filter and modify the response to the extent that the filter is no longer a Butterworth and therefore will not have the required 1000-Hz 3-dB frequency. A voltage follower, such as the one shown in figure 10, would make a suitable input buffer amplifier.

EXAMPLE 7: Design a 4-pole, low-pass Bessel filter with a 3-dB frequency of 200 Hz.

SOLUTION: The circuit given in figure 41 is a 4-pole, low-pass Bessel filter with a 3-dB frequency of 1000 Hz; accordingly, the circuit must be frequency-scaled to 200 Hz and the proper multiplying factor for all resistor values *or* all capacitor values is 1000/200 = 5 (see table II, page 18). All resistors in the modified circuit of figure 41 should be 50,000 ohms, but in practice 49. 9-kΩ one-percent resistors would be used; of course, the capacitor values remain unchanged.

In the new design, the series resistance seen by each amplifier has been increased from 20 KΩ to 100 kΩ; moreover, the output offset voltage will be increased because of unavoidable amplifier bias-current, as explained in chapter 2 under Input Offset and Drift. For example, a 741-type operational amplifier has a bias current that is typically 200 nA; therefore, each amplifier would be offset by (200 nA)(100 kΩ) = 20 mV and, since two amplifiers are involved, the total offset caused by the bias current of the operational amplifier would be typically (2)(20 mV) = 40 mV. Some 741-type amplifiers have voltage offsets as much as 1 mV; if the voltage and current offsets happen to add, which would be a worst case, the total offset could be as high as 42 mV. This usually is an acceptable offset; if not, the 741 amplifiers can be replaced with FET amplifiers that ordinarily have much lower bias currents and comparable voltage offsets. Another possibility would be to maintain the resistor values given in figure 41 and to increase all capacitor values by a factor of 5 (table II, page 18).

EXAMPLE 8: Design a 6-pole Chebyshev low-pass filter with a cutoff of 10 Hz.

SOLUTION: Figure 37 shows the circuit for a 6-pole Chebyshev low-pass filter that has a cutoff of 1000 Hz. (Refer to the first part of this chapter for a discussion of cutoff as it applies to Chebyshev filters.)

The circuit in the figure can be frequency-scaled (table II, page 18), and either the resistor values or the capacitor values can be scaled. Suppose it is decided to increase the resistor values by the multiplying factor (1000 Hz)/(10 Hz) = 100; all resistors for the new circuit will have the value of one megohm. If 741-type operational amplifiers with 200-nA bias currents are used, the offset per section will be (200nA)(2MΩ) = 0.4 V. Since there are three sections in the filter, the total offset attributable to bias currents could be as much as (3)(0.4 V) = 1.2 V. If the offset is unacceptably high, it can be reduced by using an FET or super-beta amplifier.

When low-pass Chebyshev filters with greater or less ripple than the 1-dB value used thus far are required, it is necessary to adopt the capacitor values given in table IV (for 0.25-dB and 3-dB of ripple). For example, to convert the 1-dB ripple of the filter shown in figure 34 to a filter of 0.25-dB ripple, C_1 and C_2 should be changed to 0.02831 and 0.01081 μF, respectively. The resistor values remain unchanged.

TABLE IV. – *Capacitance Values for 0.25-dB and 3-dB Ripple Chebyshev Low-Pass Filters with 1000-Hz Cutoffs*

(R = 10,000 ohms; capacitance values in microfarads)

Ripple	Number of Poles	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
0.25 dB	2	0.02831	0.01081						
	3	0.1361	0.03212	0.001765					
	4	0.03535	0.02046	0.08536	0.003318				
	6	0.04846	0.02986	0.06620	0.006839	0.1809	0.001484		
	8	0.06259	0.03937	0.07382	0.009646	0.1105	0.003214	0.3146	0.0008331
3.0 dB	2	0.04939	0.007253						
	3	0.6912	0.05777	0.0004032					
	4	0.07741	0.01670	0.1869	0.001501				
	6	0.1116	0.02557	0.1524	0.003186	0.4163	0.0006373		
	8	0.1467	0.03433	0.1731	0.004561	0.2590	0.001390	0.7376	0.0003525

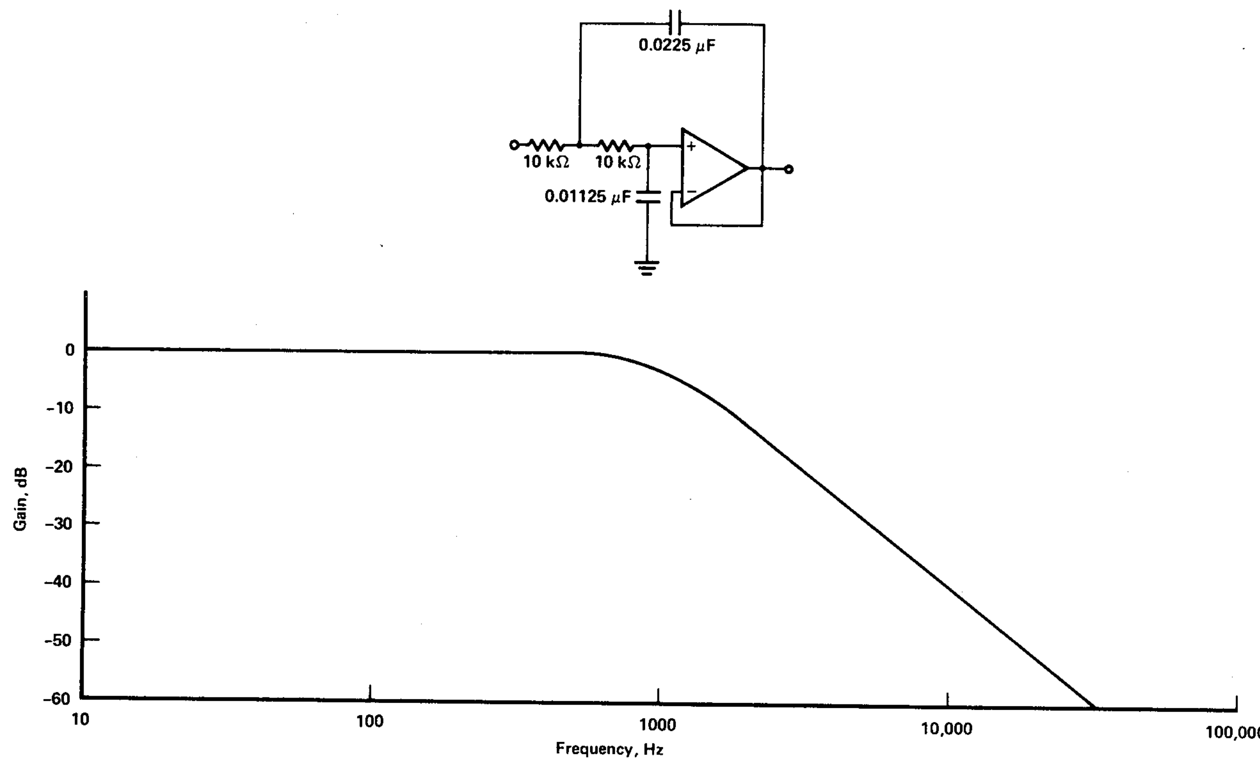


FIGURE 29. – Butterworth 2-pole low-pass filter, f_c = 1000 Hz.

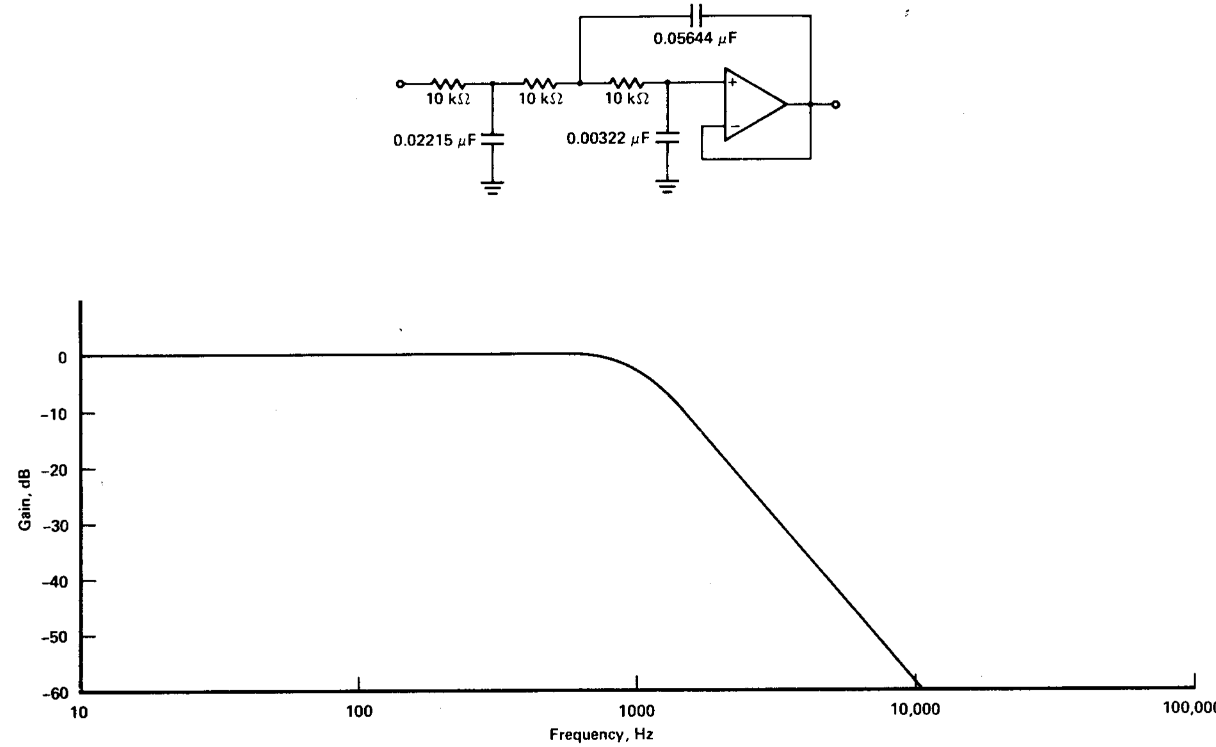


FIGURE 30. – Butterworth 3-pole low-pass filter, f_c = 1000 Hz.

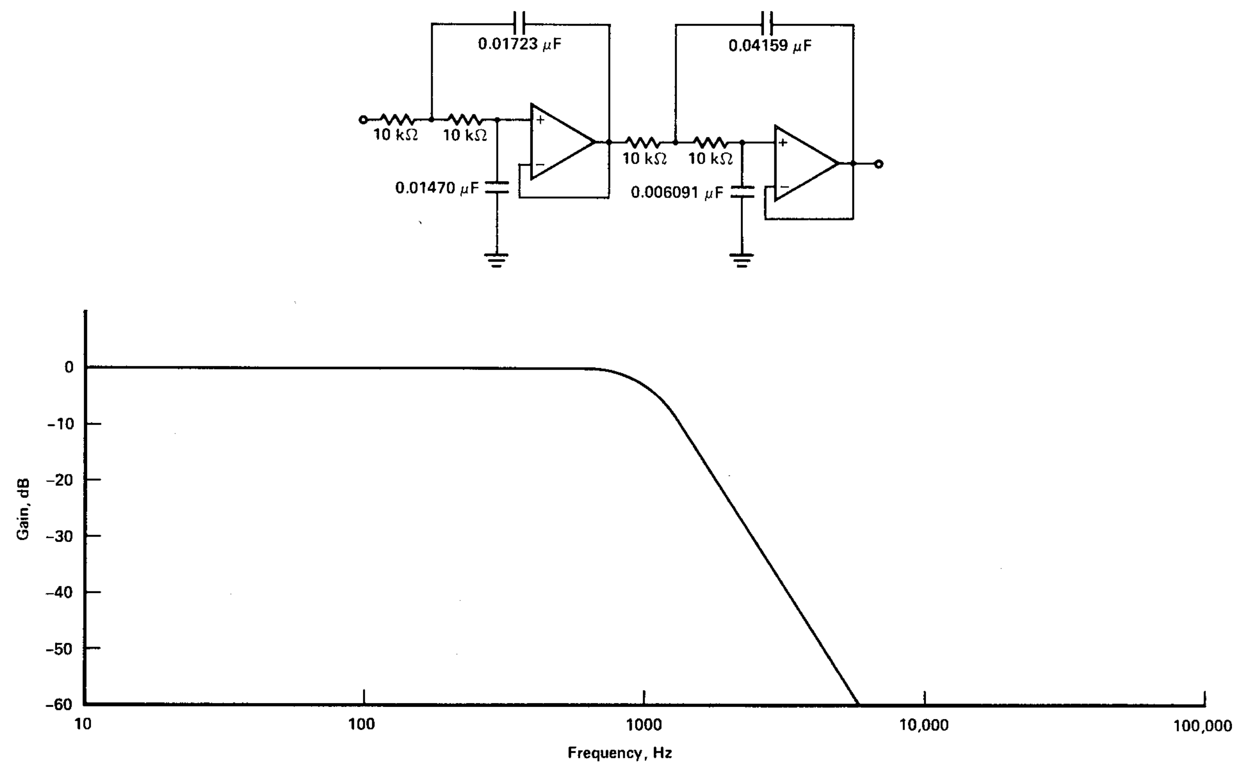


FIGURE 31. – Butterworth 4-pole low-pass filter, f_c = 1000 Hz.

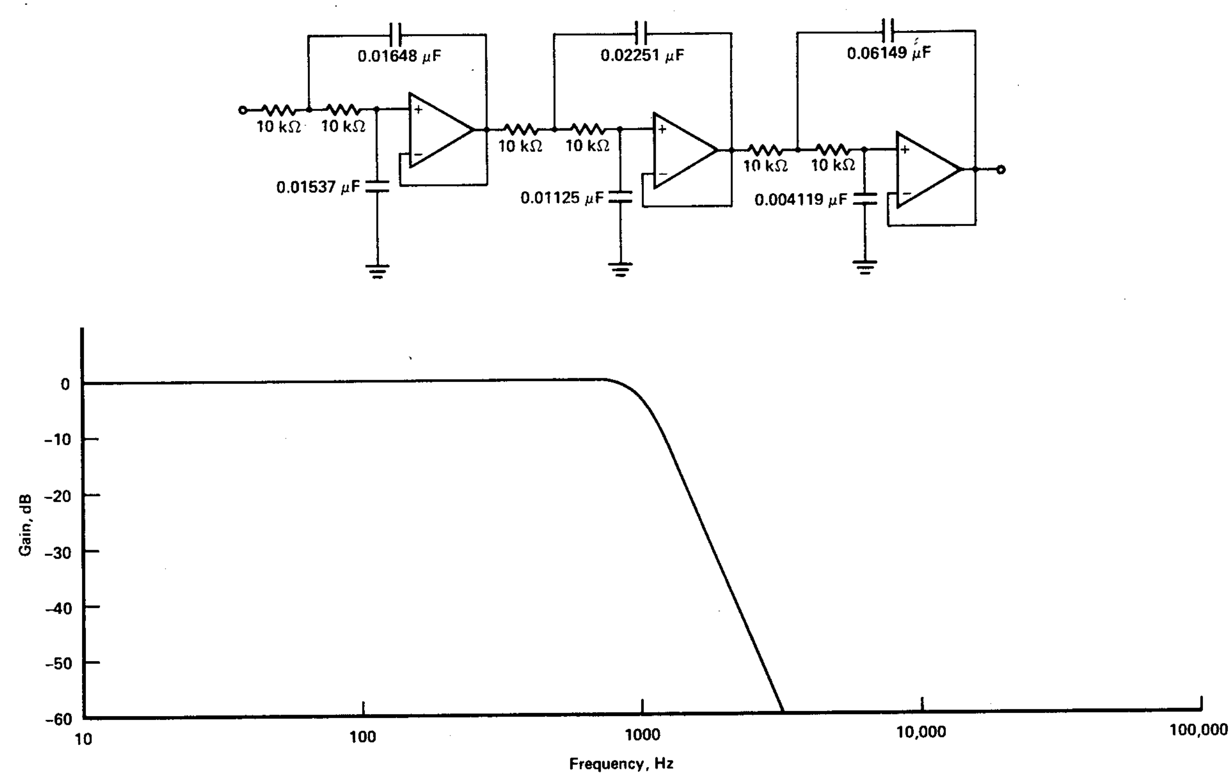


FIGURE 32. – Butterworth 6-pole low-pass filter, f_c = 1000 Hz.

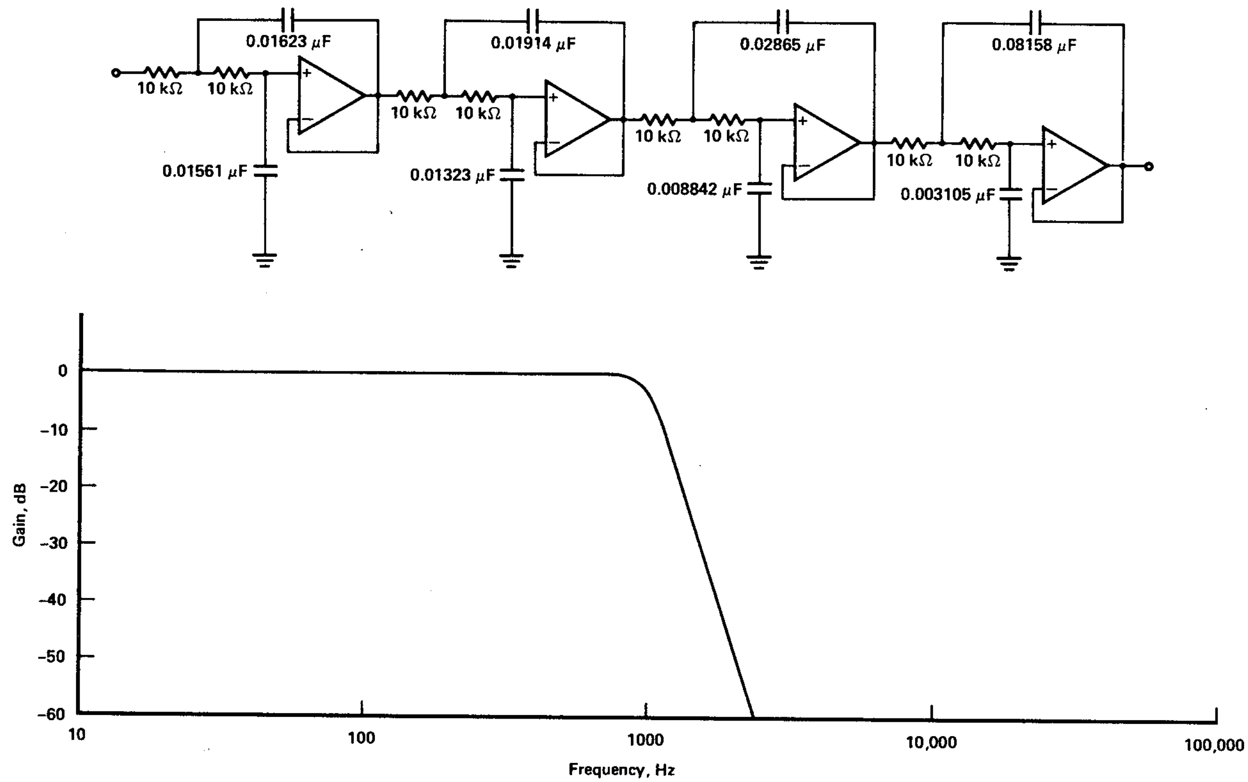


FIGURE 33. – Butterworth 8-pole low-pass filter, f_c = 1000 Hz.

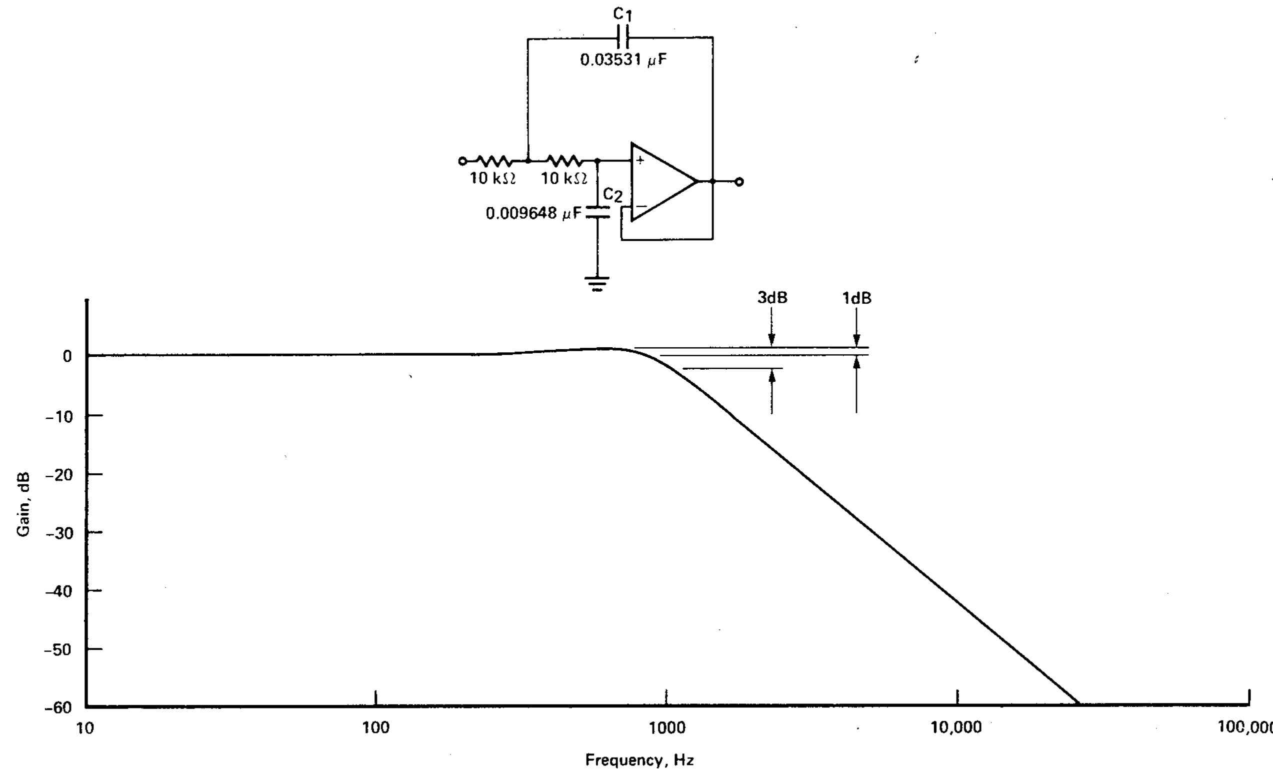


FIGURE 34. – Chebyshev 2-pole low-pass filter, f_c = 1000 Hz, 1-dB ripple.

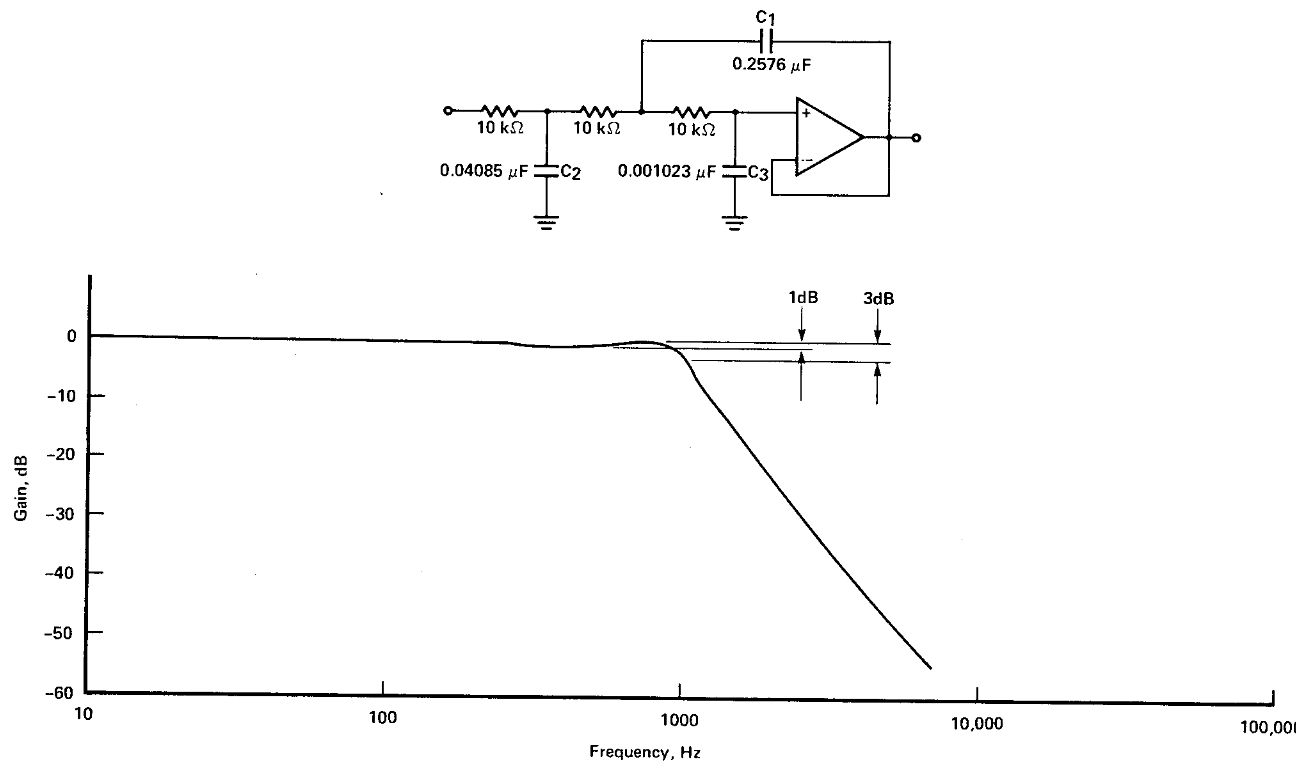


FIGURE 35. – Chebyshev 3-pole low-pass filter, f_c = 1000 Hz, 1-dB ripple.

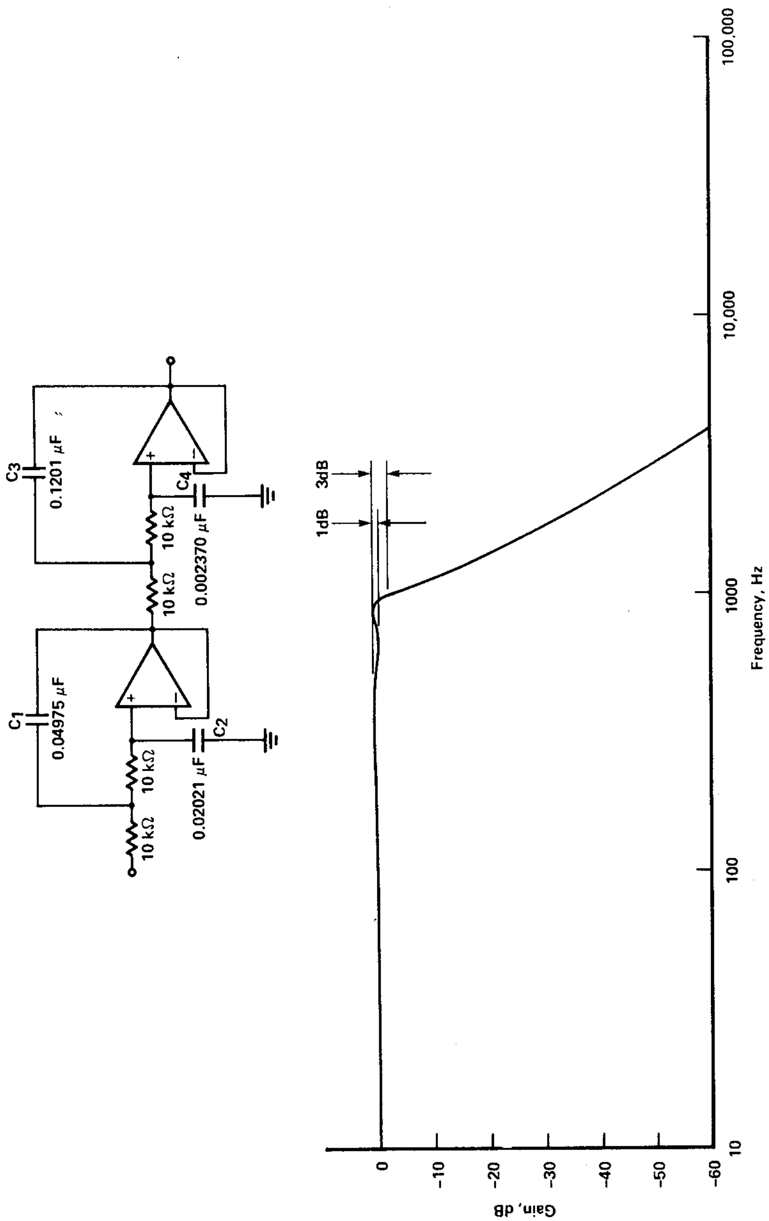


FIGURE 36. – Chebyshev 4-pole low-pass filter, f_c = 1000 Hz, 1-dB ripple.

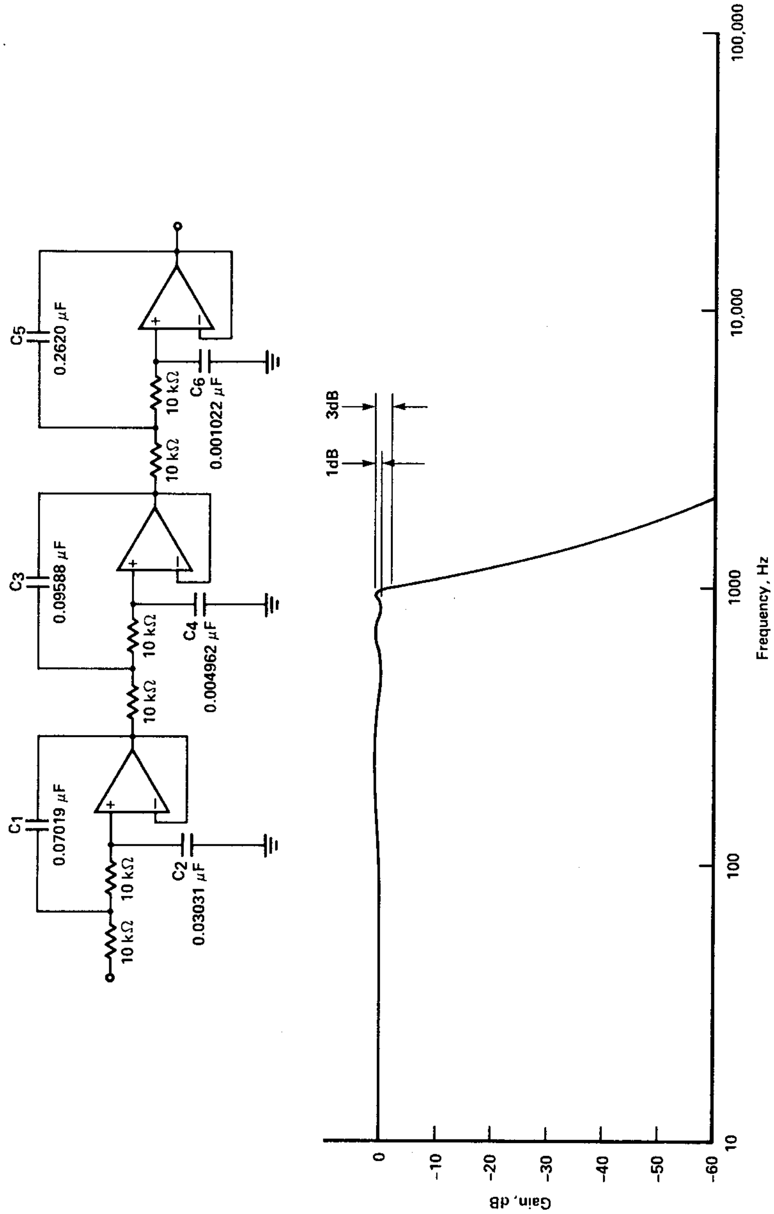


FIGURE 37. – Chebyshev 6-pole low-pass filter, f_c = 1000 Hz, 1-dB ripple.

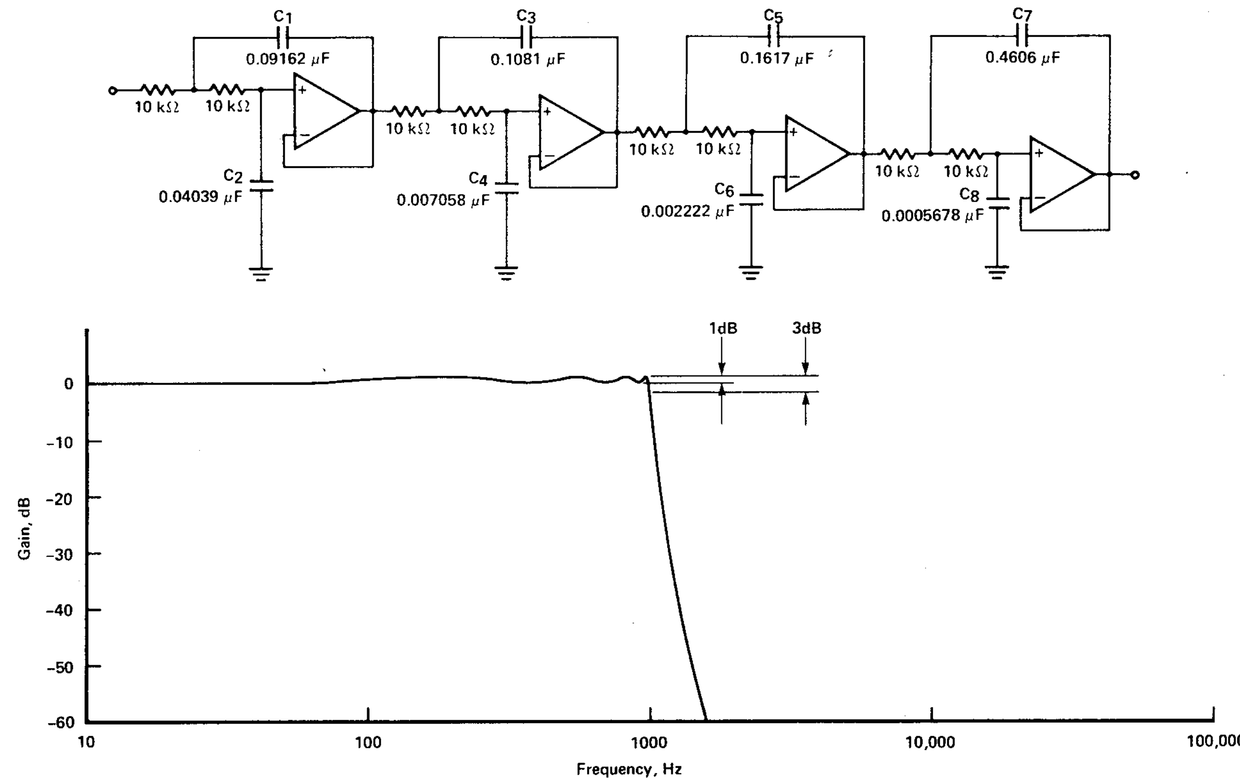


FIGURE 38. – Chebyshev 8-pole low-pass filter, f_c = 1000 Hz, 1-dB ripple.

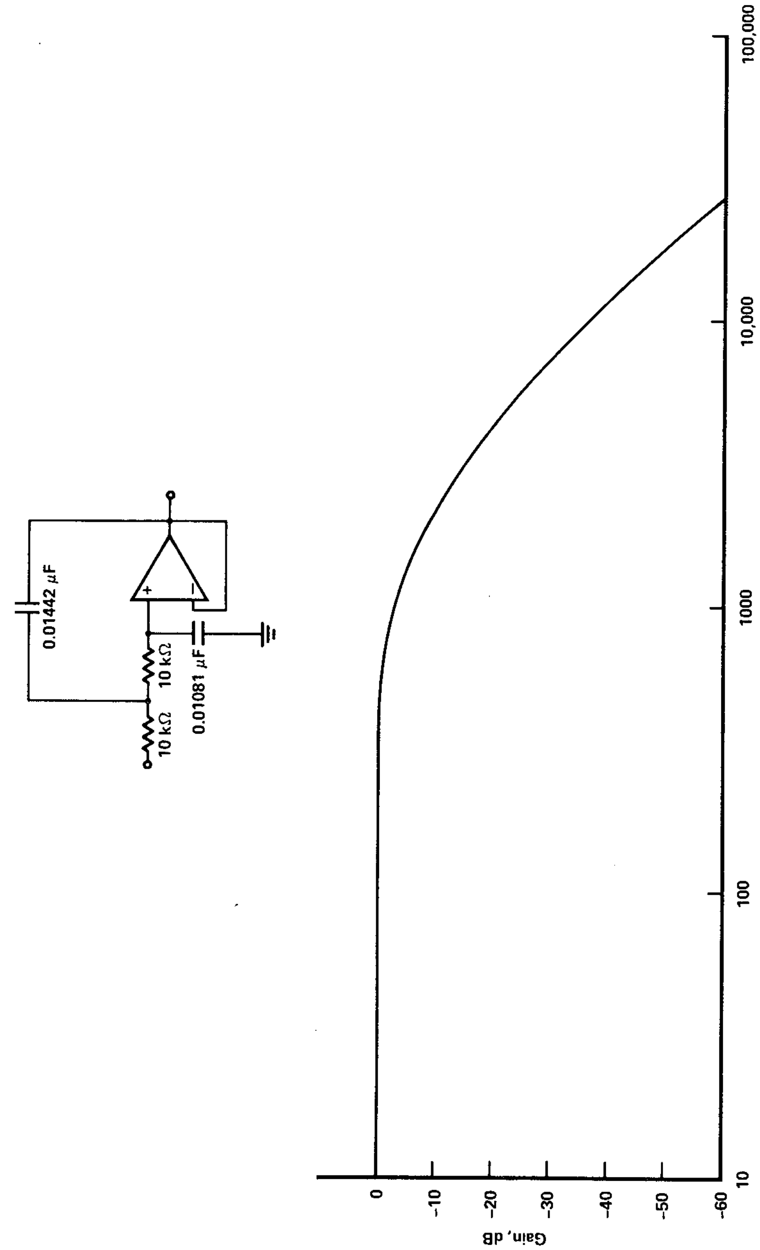


FIGURE 39. – Bessel 2-pole low-pass filter, f_c = 1000 Hz.

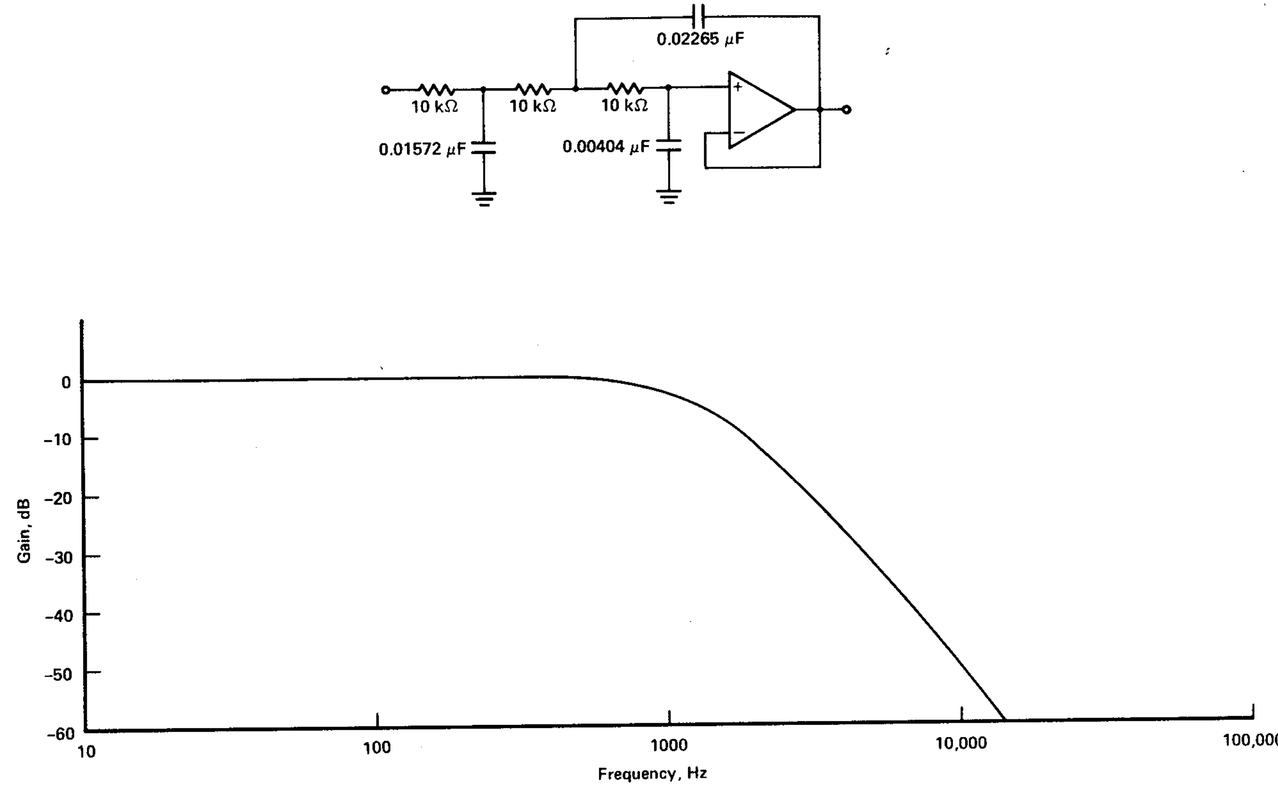
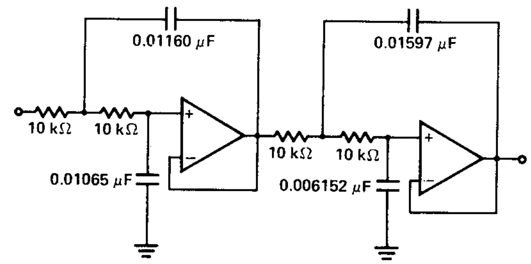


FIGURE 40. – Bessel 3-pole low-pass filter, f_c = 1000 Hz.



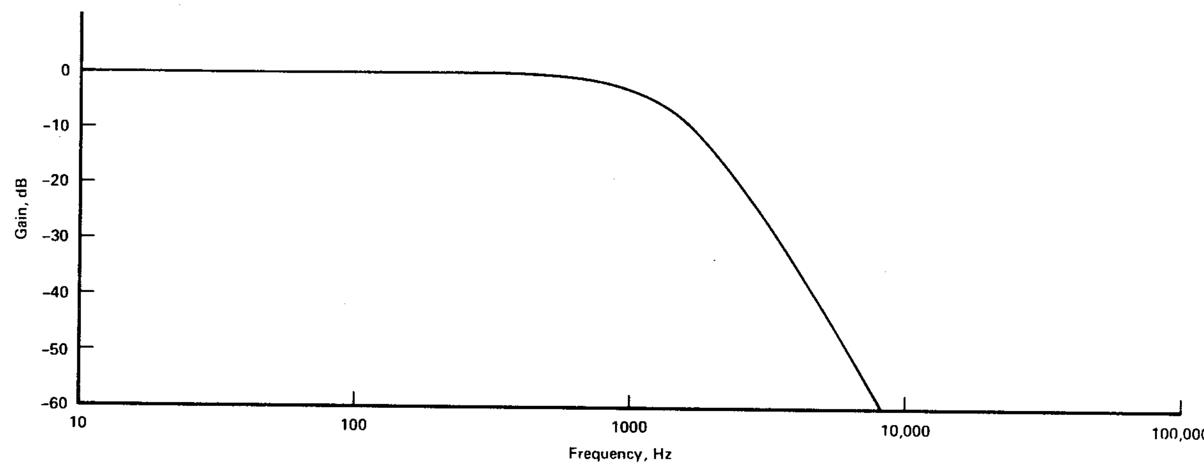


FIGURE 41. – Bessel 4-pole low-pass filter, f_c = 1000 Hz.

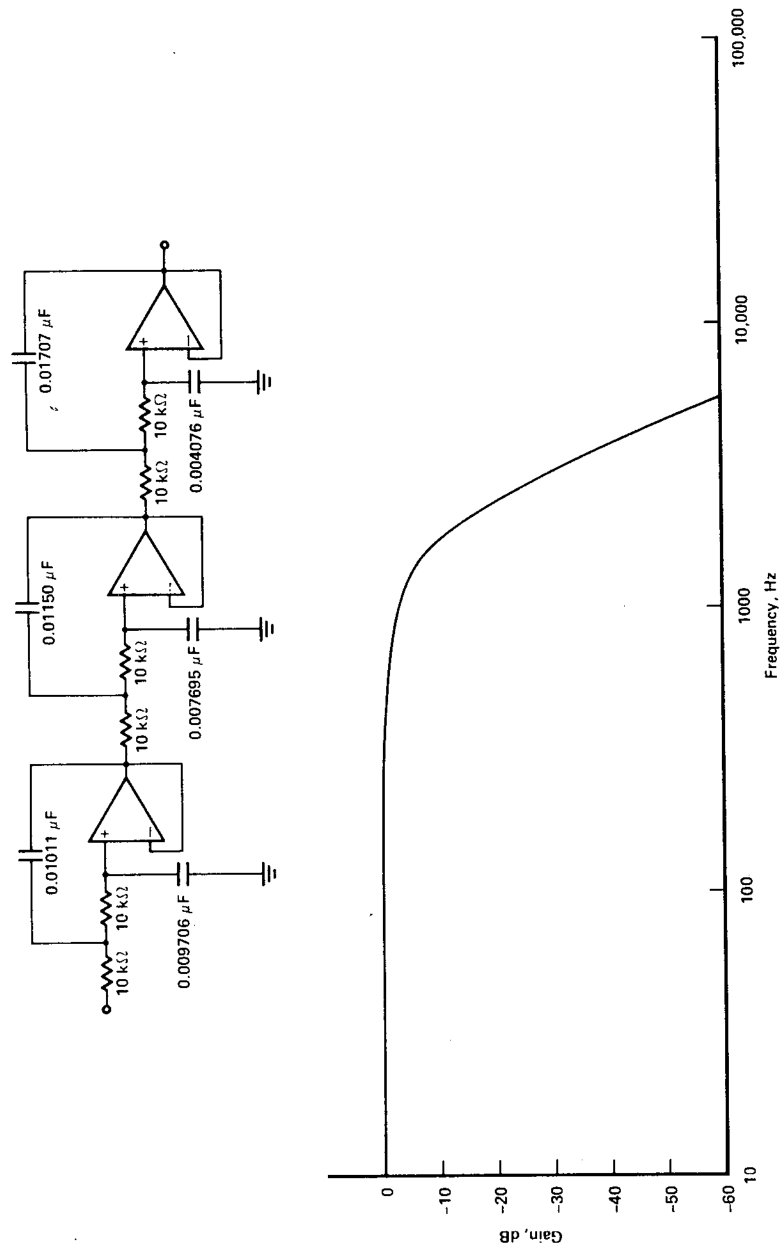


FIGURE 42. – Bessel 6-pole low-pass filter, f_c = 1000 Hz.

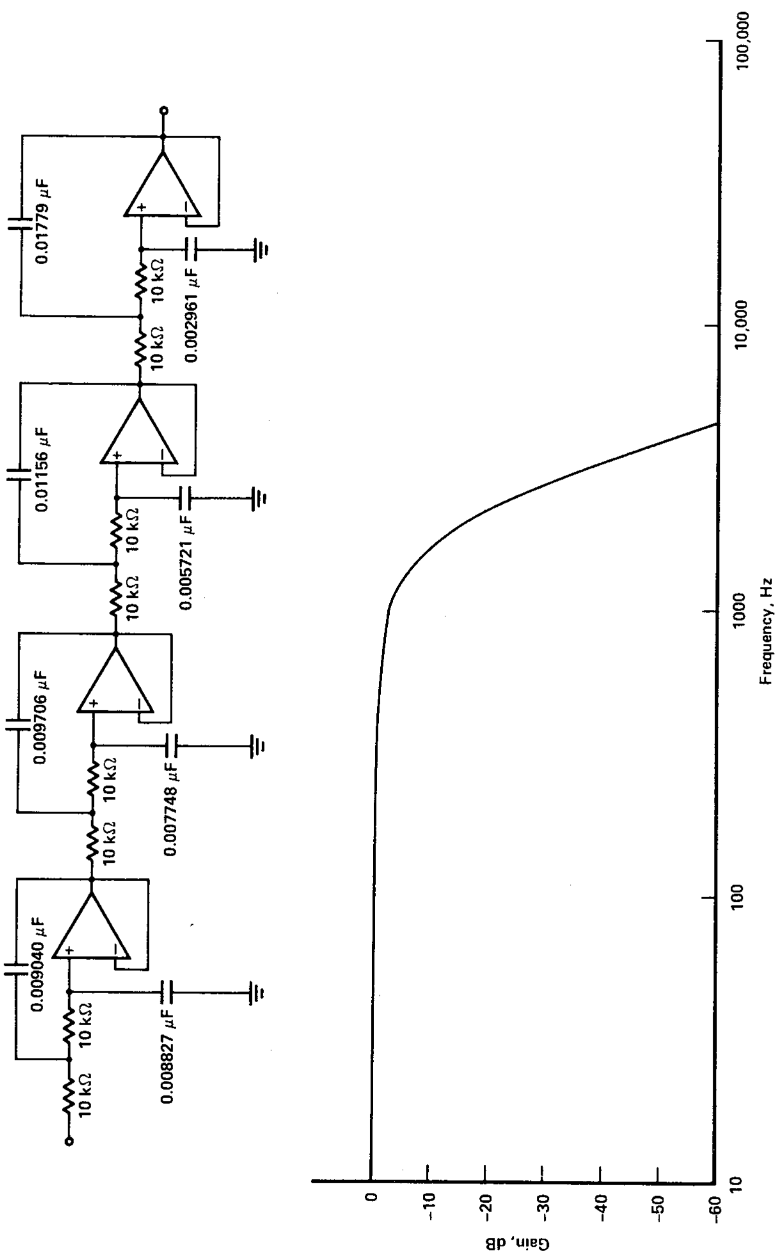


FIGURE 43. – Bessel 8-pole low-pass filter, f_c = 1000 Hz.

High-Pass Filters

SELECTION OF FILTER TYPE AND NUMBER OF POLES

This chapter describes the steps involved in the design of Butterworth, Chebyshev, and Bessel high-pass filters; the first step is to select the type of filter that is most appropriate for the application in mind, and for this purpose it is well to review the general characteristics of each type as presented in the first part of chapter 2 and summarized in table I. For example, it has been explained that the Butterworth is an excellent general purpose filter with good attenuation characteristics and the maximum possible flatness in the passband. In contrast, the Chebyshev has a steeper attenuation-slope in the region of cutoff, but this is achieved at the expense of having ripple in the passband. Bessel filters have poor attenuation-slope and poor flatness in the pass region; also, they are not very useful as high-pass filters because their linear phase properties are lost when low-frequency circuits are scaled to higher frequencies.

Having selected one of the three basic types of filters, the next step is to decide on the number of poles required. For reasons of economy and simplicity, one usually selects a filter with the least number of poles that will do the job. Usually, elimination of unwanted signals and reduction of noise are the pivotal factors in the selection of the number of poles.

EXAMPLE 1: A unity-gain filter is required to attenuate all frequencies up to 1000 Hz; maximum flatness in the passband is required and signals of 60 Hz and less must be attenuated by at least 60 dB. How many poles are required in the filter?

SOLUTION: The idealized amplitude-response curves for Butterworth (maximally-flat amplitude response) filters with 2, 3, 4, 6, and 8 poles are shown in figure 44. Inspection of this figure reveals that a 3-pole filter will provide the necessary 60-dB attenuation for frequencies of 60 Hz and below.

EXAMPLE 2: A unity-gain filter is required to attenuate all frequencies below 2000 Hz; maximum flatness in the passband is required and signals of 1000 Hz and below must be attenuated by at least 45 dB. How many poles are required?

SOLUTION: Data given in figure 44 are for filters with 1000-Hz cut-offs, but it is to be recalled that frequency-scaling does not affect the shape of response curves. Consequently, the data in the figure can be applied to the

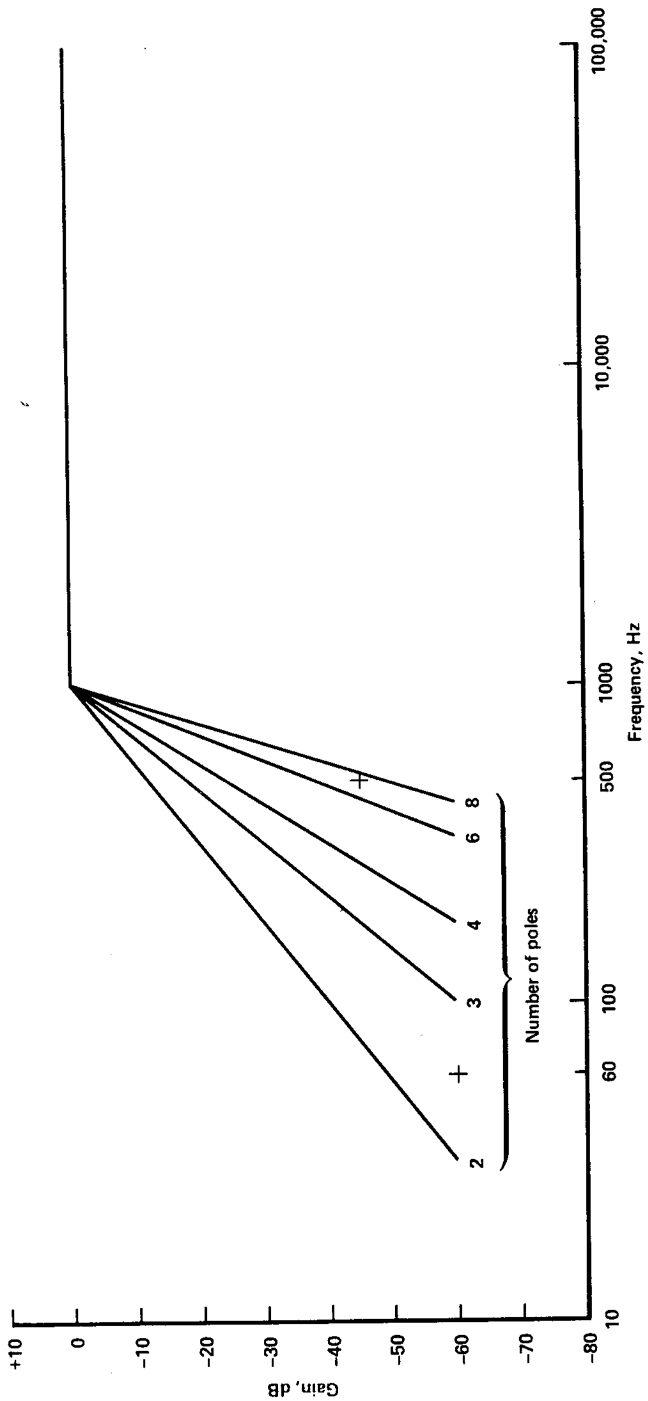


FIGURE 44. – Idealized amplitude-response curves for high-pass Butterworth filters.

problem of a 2000-Hz filter by multiplying *all* values on the frequency scale by 2 (to provide a cutoff of 2000 Hz). Continuing, the number of poles required to give 45 dB of attenuation at 1000 Hz is 8, because 500 Hz multiplied by 2 is 1000 Hz.

The attenuation provided by Chebyshev filters with different numbers of poles is summarized in figures 50 through 54 and for Bessel filters in figures 55 through 59. Determination of the extent to which a high-pass filter will reduce noise involves the same considerations that were described in chapter 3 for low-pass filters.

EXAMPLE 3: A circuit has an equivalent noise-bandwidth of 100 kHz and a thermal noise level of 200 $\mu V/Hz^{1/2}$. What will be the noise level, e_n, if the bandwidth is reduced to 3 kHz?

SOLUTION: The noise level is reduced by the square root of the ratio of the two bandwidths:

$$e_n = [(3\text{ kHz})/(100\text{ kHz})]^{1/2}\,(200\,\mu V)$$

$$= 34.6\,\mu V$$

The gains of all the Butterworth and Bessel high-pass filters in figures 45 through 49 and figures 55 through 59 are 3 dB down at 1000 Hz; thus, when frequency-scaled as described in table II, page 18, the new cutoff frequency will also occur at a 3-dB point. With Chebyshev filters, the situation is slightly more complicated and has been discussed in chapter 3. It should be recalled that all high-pass filters described in this handbook, including Chebyshevs, are designed to have a gain of unity or 0 dB at high frequencies within the passband and that all cutoffs are specified as being at the -3 dB point (that is, -3 dB from the maximum gain occurring in the passband indicated in the plots for Chebyshevs with a 1-dB ripple, figures 50 through 54).

SELECTION OF VALUES

Values for high-pass filters are readily obtained by modifying an existing design by frequency-scaling and impedance-scaling (table II, page 18) to meet the requirements for a particular application.

Designs for high-pass Butterworth, Chebyshev, and Bessel filters that have 2, 3, 4, 6, and 8 poles and a 1000-Hz cutoff are given in figures 45 through 59. If an application can use a filter with a cutoff of 1000 Hz, the circuit can be taken from the appropriate figure; no design changes are necessary.

EXAMPLE 4: Design a 3-pole, high-pass Butterworth filter with a 3-dB frequency of 1000 Hz.

SOLUTION: Since the cutoff frequency is 1000 Hz, the design in figure 46 can be used directly with no modification.

EXAMPLE 5: Design a 3-pole, high-pass Butterworth filter with a 3-dB frequency of 1000 Hz. The filter is to be used to filter signals from a circuit that includes a 0.02-μF series-output capacitor.

SOLUTION: Since the cutoff frequency is 1000 Hz, the design of figure 46 can be used; however, because there is a 0.02-μF capacitor in the output of the signal source, a buffer amplifier is necessary to provide the low-impedance drive required by all the high-pass filters described in this section of the handbook. If a buffer is not used, the 0.02-μF capacitor will modify the filter RC network to the extent that it no longer is a Butterworth and thus will not have a 1000-Hz, 3-dB cutoff frequency. A voltage follower such as that in figure 10 would make a suitable buffer, but a resistor must be connected from the + input to ground to provide a continuous dc path. The resistor, R, and the 0.02-μF source capacitor C, will of themselves form a high-pass filter. The effect of this filter on the overall circuit performance can be rendered negligible by selecting a value for R of such magnitude that at 1000 Hz the RC network will have negligible attenuation. Assume that 0.1-dB attenuation is negligible; using equation (4) in chapter 2, it is found that -0.1 dB represents a gain of 0.98855; that is, with a 1-volt input to the 0.02-μF capacitor, the voltage across the resistor should be $\geqslant$0.98855 V at 1000 Hz. By vectorial summation of voltages it can be determined that the voltage across the capacitor will be 0.15087 V. Also, the reactance of a 0.02-μF capacitor at 1000 Hz is 7.9577 kΩ. Finally, because the same current flows through both C and R,

$$R = \frac{\text{(Voltage across R) (Reactance of C)}}{\text{(Voltage across C)}}$$

$$= \frac{(0.9885)\,(7.9577)}{(0.15087)} = 52.14 \text{ k}\Omega$$

Accordingly, a resistor with value greater than 52.14 kΩ, say 56 or 68 kΩ, would be suitable.

EXAMPLE 6: Design a 4-pole, high-pass Bessel filter with a 3-dB frequency of 2000 Hz.

SOLUTION: The circuit shown in figure 57 is for a 4-pole, high-pass Bessel filter that has a 3-dB frequency of 1000 Hz. The solution is, therefore, to frequency-scale this circuit to 2000 Hz. From table II, all resistor values or all capacitor values can be multiplied by 1000/2000 or 0.5. It is probably most convenient to change all capacitors from 0.01 to 0.005 μF.

EXAMPLE 7: Design a 6-pole Chebyshev high-pass filter with a cutoff at 1 Hz.

SOLUTION: Figure 53 shows the circuit for a 6-pole Chebyshev low-pass filter with cutoff at 1000 Hz (refer to the first part of chapters 3 and 4 for a discussion of cutoff as it applies to Chebyshev filters). This circuit can be frequency-scaled (table II, page 18) and either the resistors or the capacitors can be changed; for example, increase all the capacitors to 10 μF by using a multiplying factor of 1000. The circuit is now frequency-scaled to 1 Hz, but 10 μF is an inconveniently large value; since 1 μF would be better, all capacitors should be further multiplied by 0.1 and all resistors by 10, as described in table II (page 18). The filter is still frequency-scaled to 1 Hz, but has been impedance-scaled to more convenient values. Note that the resistor on the last operational amplifier in figure 53 is 247.7 kΩ, but this has been impedance-scaled to 2,477 kΩ. Since this resistor serves as the bias resistor for the operational amplifier, an FET or super-beta type of amplifier should be used to minimize offsets resulting from input bias currents.

When high-pass Chebyshev filters with greater or less ripple than the 1-dB value used thus far are required, it is necessary to adopt the capacitor values given in table V (for filters with 0.25-dB and 3-dB of ripple). For example, to convert the 1-dB ripple of the filter shown in figure 50 to a filter of 0.25-dB ripple, R_1 and R_2 should be changed to 8.946 kΩ and 23.44 kΩ, respectively. The capacitor values remain unchanged.

TABLE V. – *Resistance Values for 0.25-dB and 3-dB Ripple Chebyshev High-Pass Filters with 1000-Hz Cutoffs*

(C = 0.01 microfarad; resistance values in thousands of ohms)

Ripple	Number of Poles	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8
0.25 dB	2	8.946	23.44						
	3	1.861	7.885	143.5					
	4	7.164	12.38	2.968	76.35				
	6	5.227	8.484	3.826	37.04	1.400	170.7		
	8	4.047	6.434	3.431	26.26	2.293	78.82	0.8051	304.0
3.0 dB	2	5.129	34.92						
	3	0.3665	4.385	628.2					
	4	3.272	15.17	1.355	168.7				
	6	2.271	9.906	1.662	79.51	0.6084	397.5		
	8	1.726	7.378	1.464	55.53	0.9780	182.2	0.3434	718.5

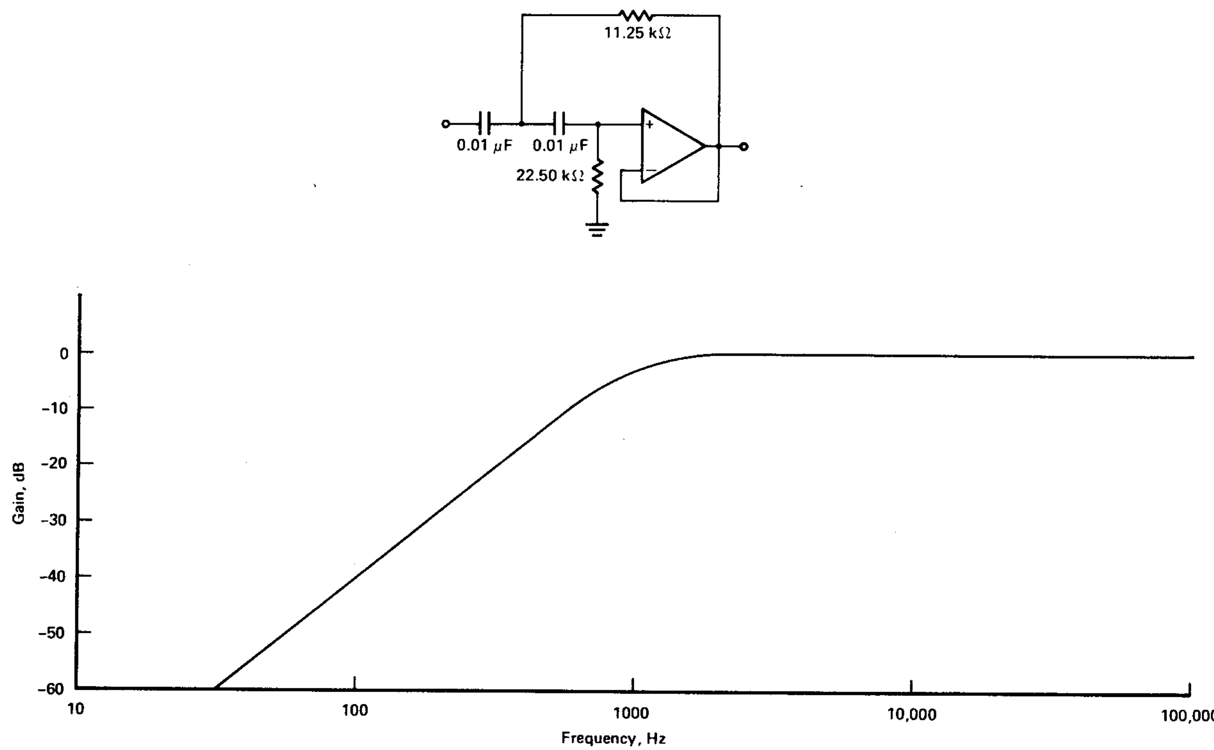


FIGURE 45. – Butterworth 2-pole high-pass filter, f_c = 1000 Hz.

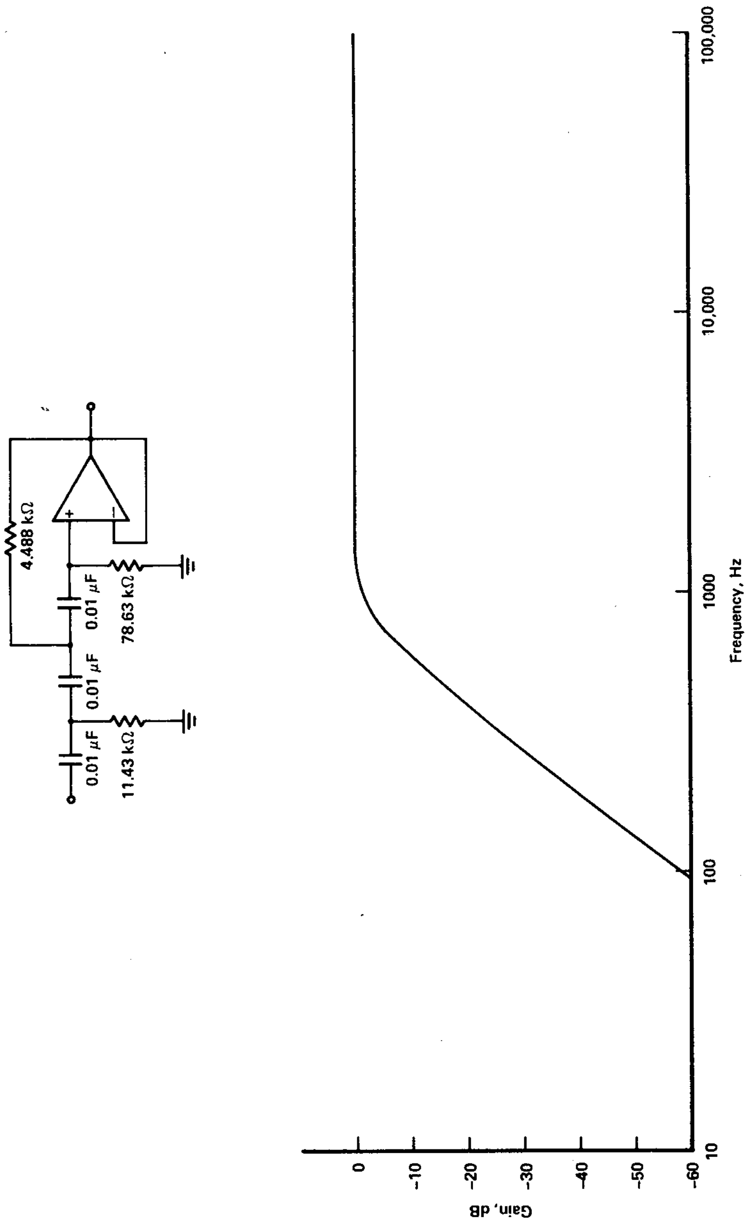


FIGURE 46. – Butterworth 3-pole high-pass filter, f_c = 1000 Hz.

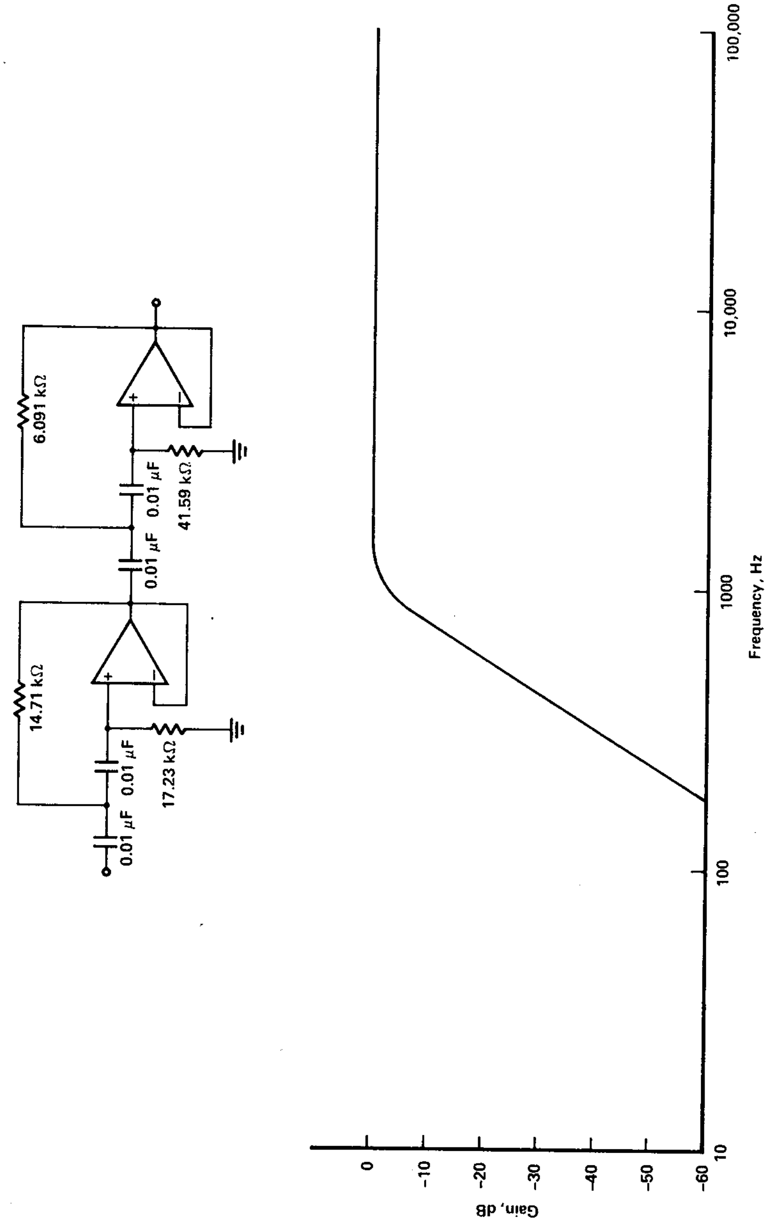


FIGURE 47. – Butterworth 4-pole high-pass filter, f_c = 1000 Hz.

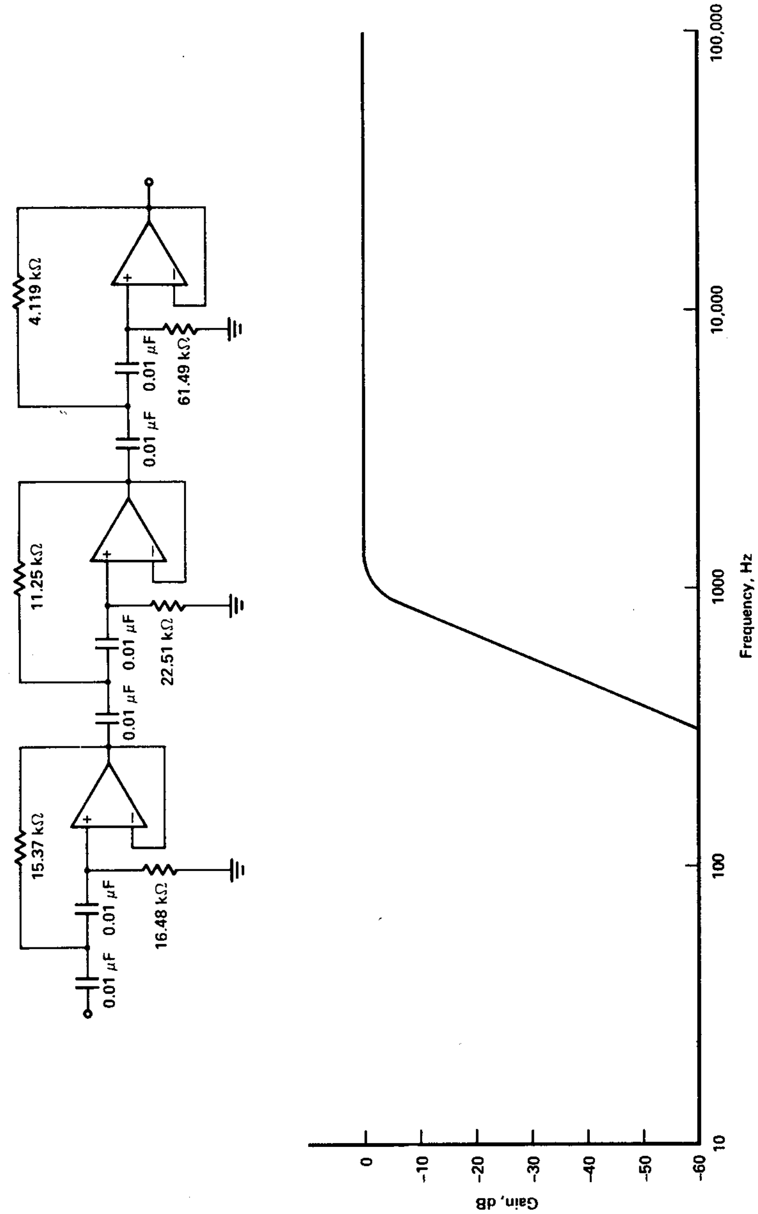


FIGURE 48. — Butterworth 6-pole high-pass filter, f_c = 1000 Hz.

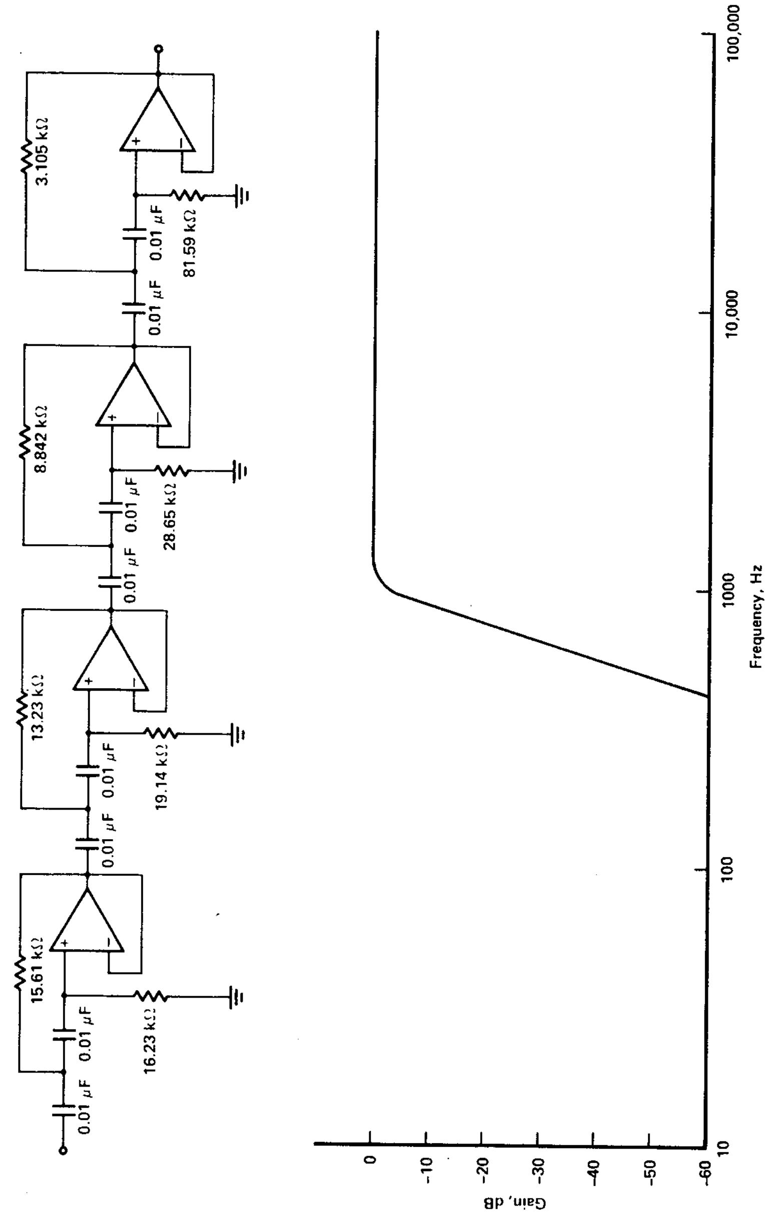


FIGURE 49. – Butterworth 8-pole high-pass filter, f_c = 1000 Hz.

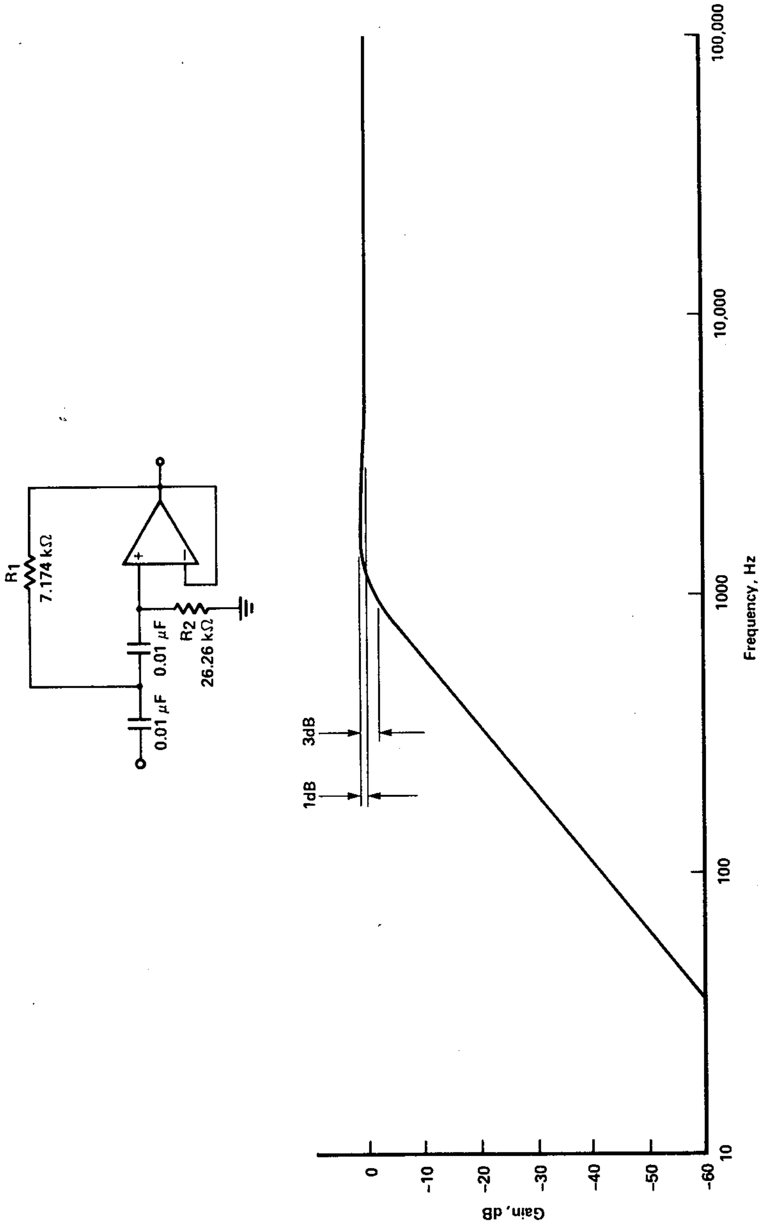


FIGURE 50. – Chebyshev 2-pole high-pass filter, f_c = 1000 Hz, 1-dB ripple.

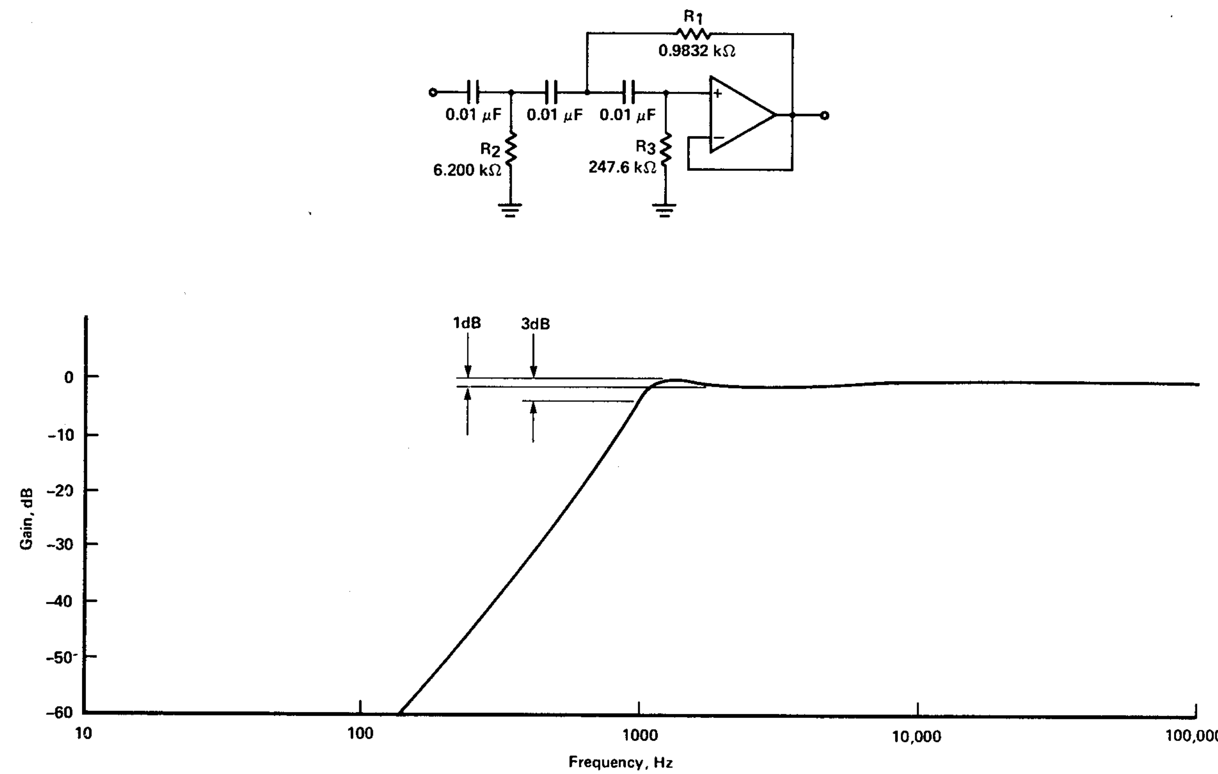


FIGURE 51. – Chebyshev 3-pole high-pass filter, $f_c = 1000$ Hz, 1-dB ripple.

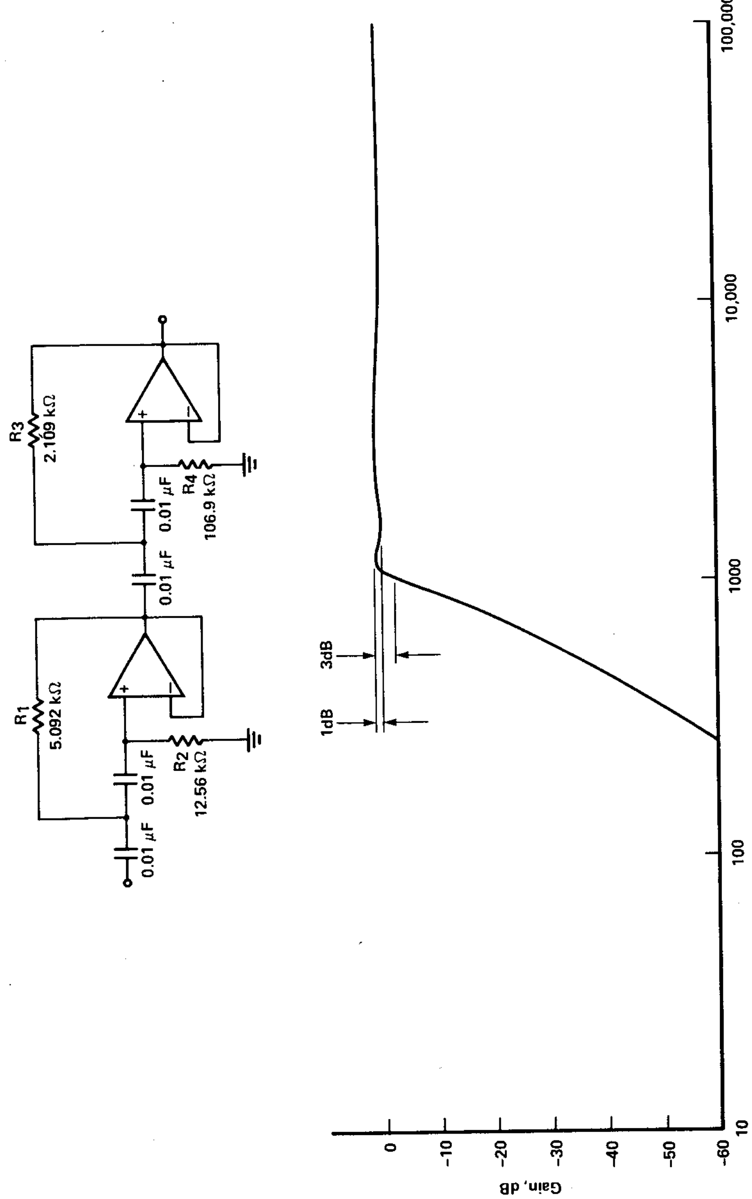


FIGURE 52. – Chebyshev 4-pole high-pass filter, f_c = 1000 Hz, 1-dB ripple.

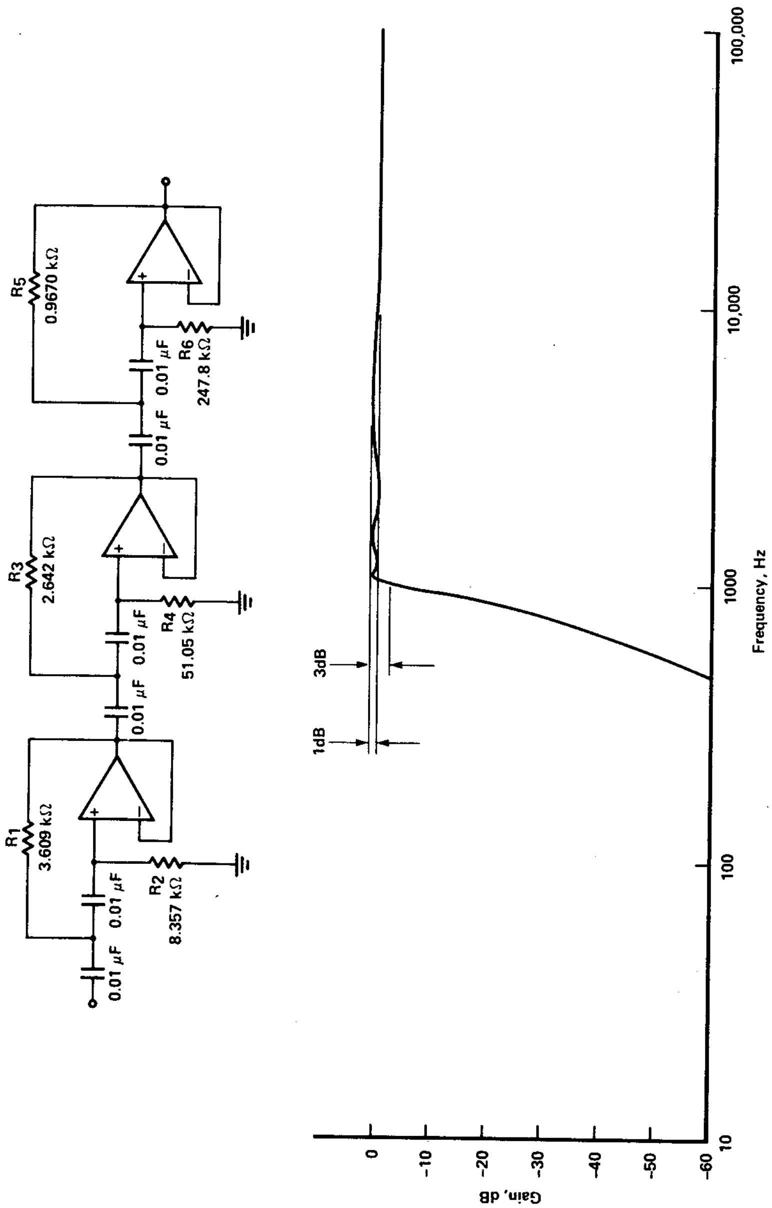


FIGURE 53. – Chebyshev 6-pole high-pass filter, f_c = 1000 Hz, 1-dB ripple.

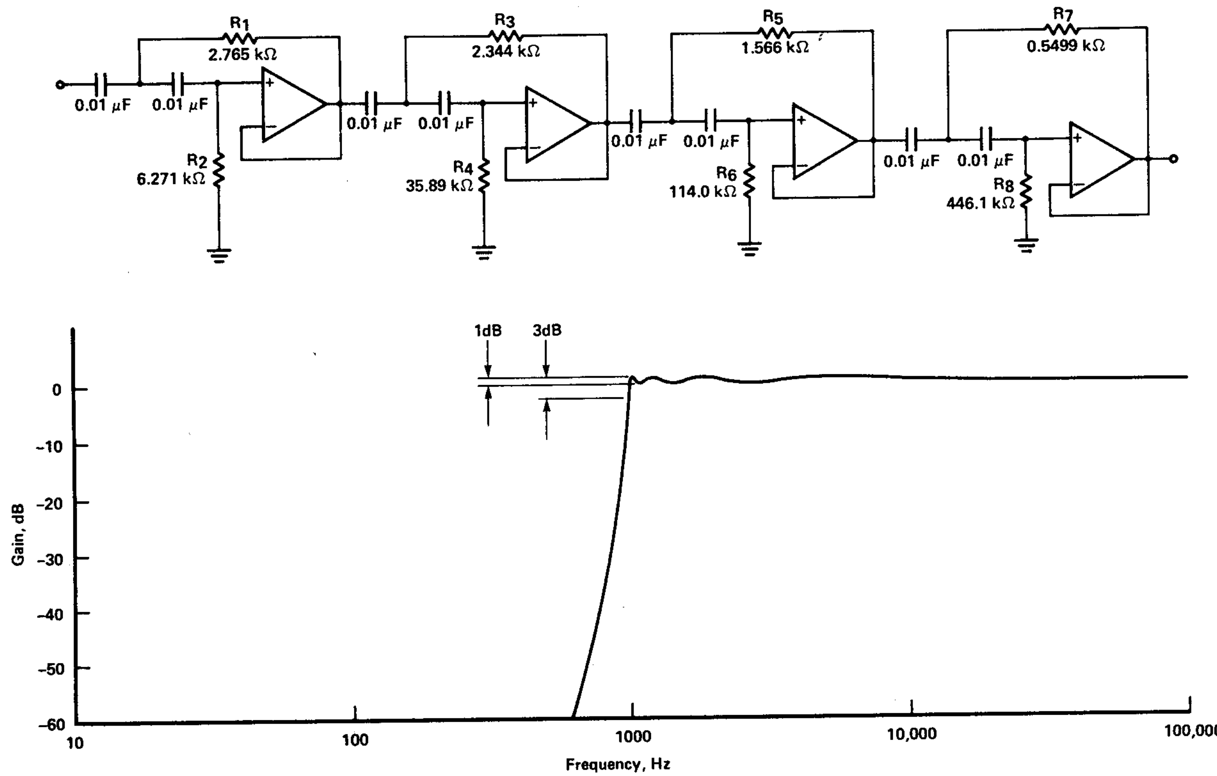


FIGURE 54. – Chebyshev 8-pole high-pass filter, f_c = 1000 Hz, 1-dB ripple.

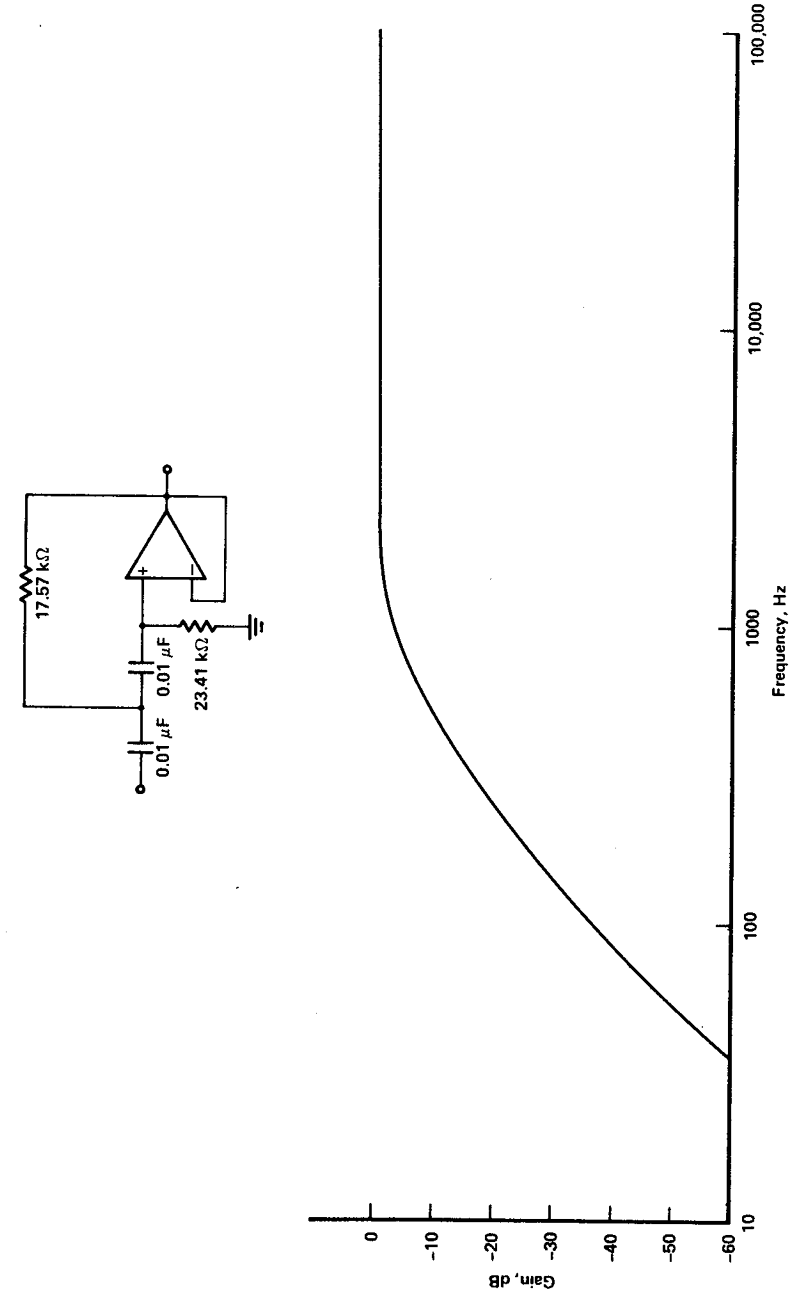


FIGURE 55. – Bessel 2-pole high-pass filter, f_c = 1000 Hz.

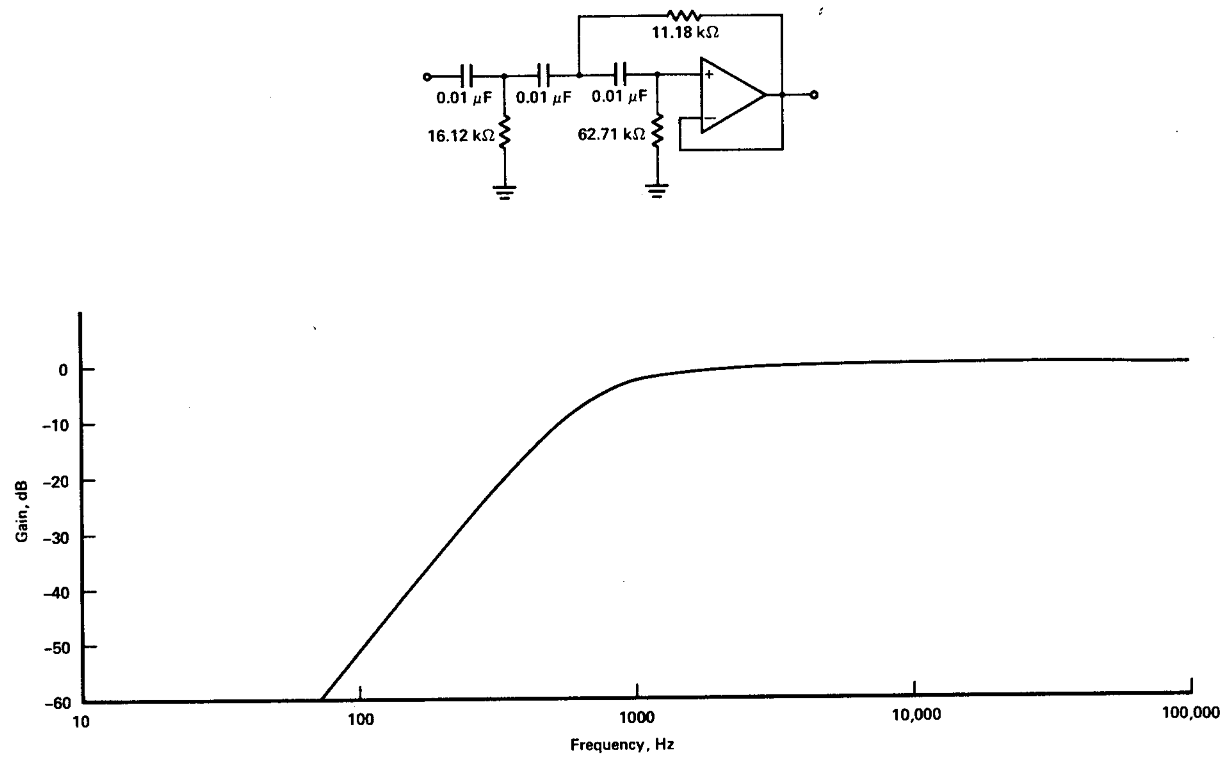


FIGURE 56. – Bessel 3-pole high-pass filter, f_c = 1000 Hz.

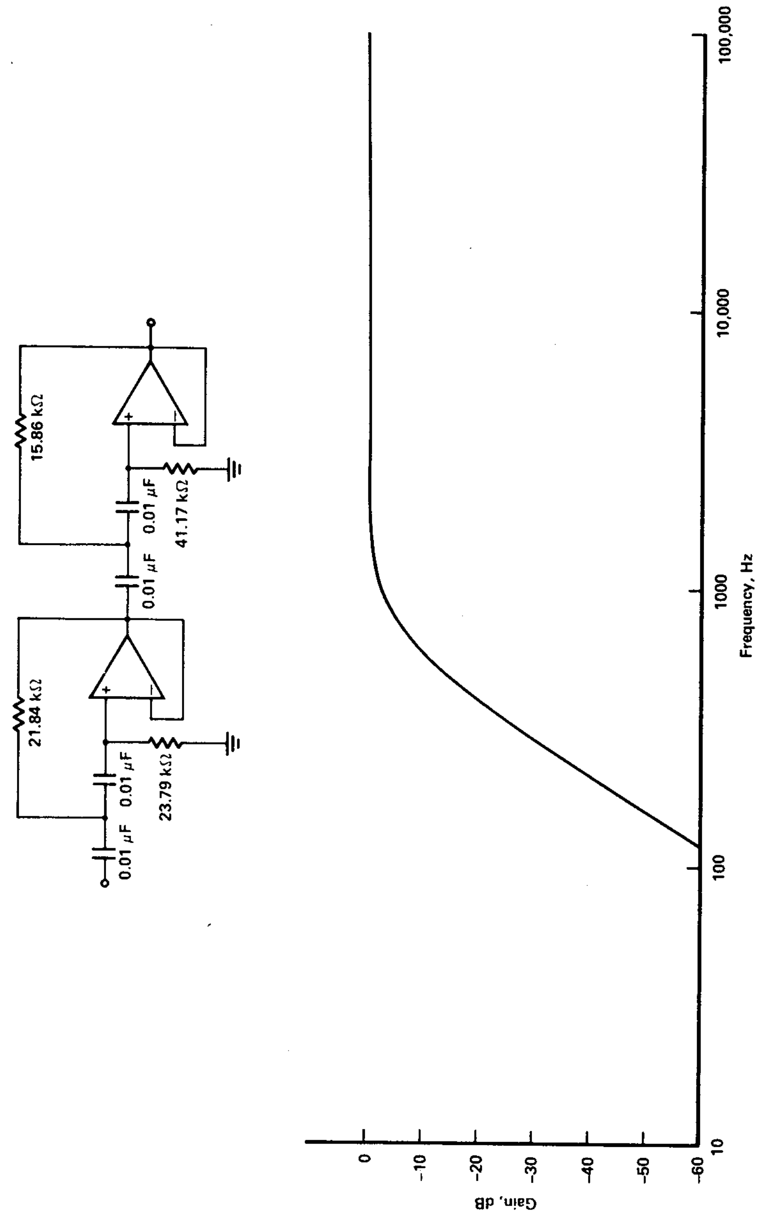


FIGURE 57. – Bessel 4-pole high-pass filter, f_c = 1000 Hz.

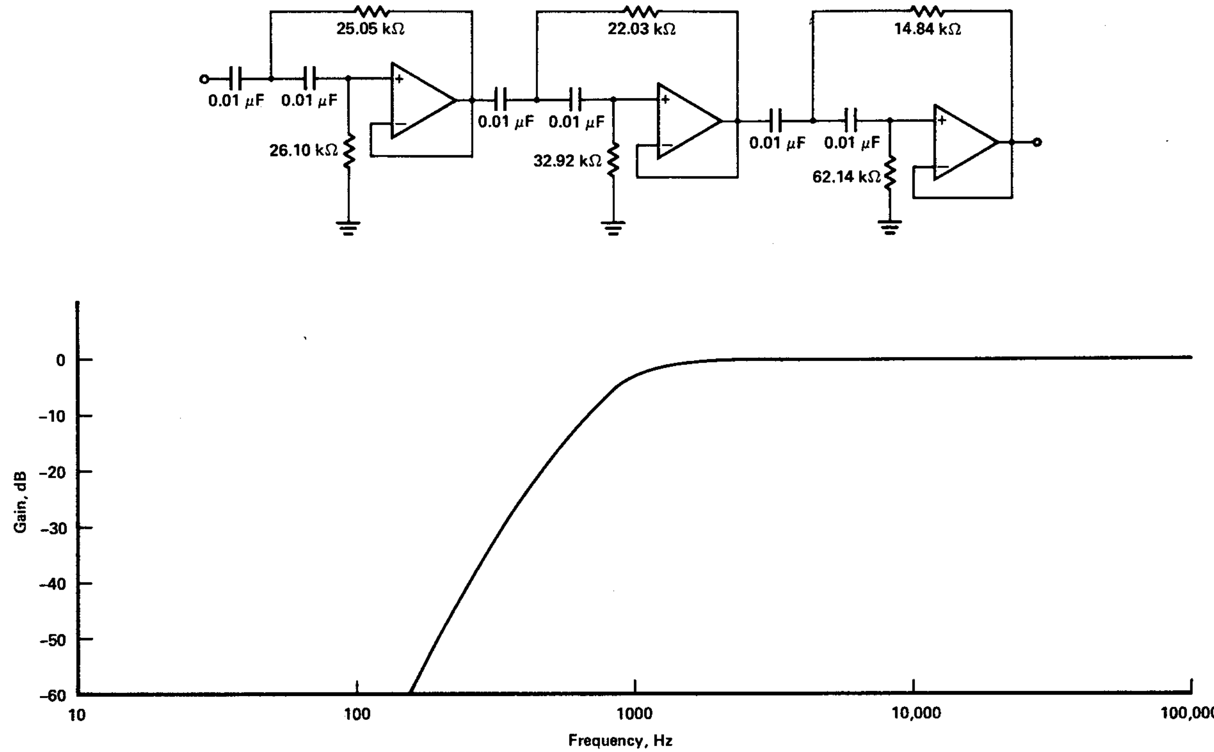


FIGURE 58. – Bessel 6-pole high-pass filter, f_c = 1000 Hz.

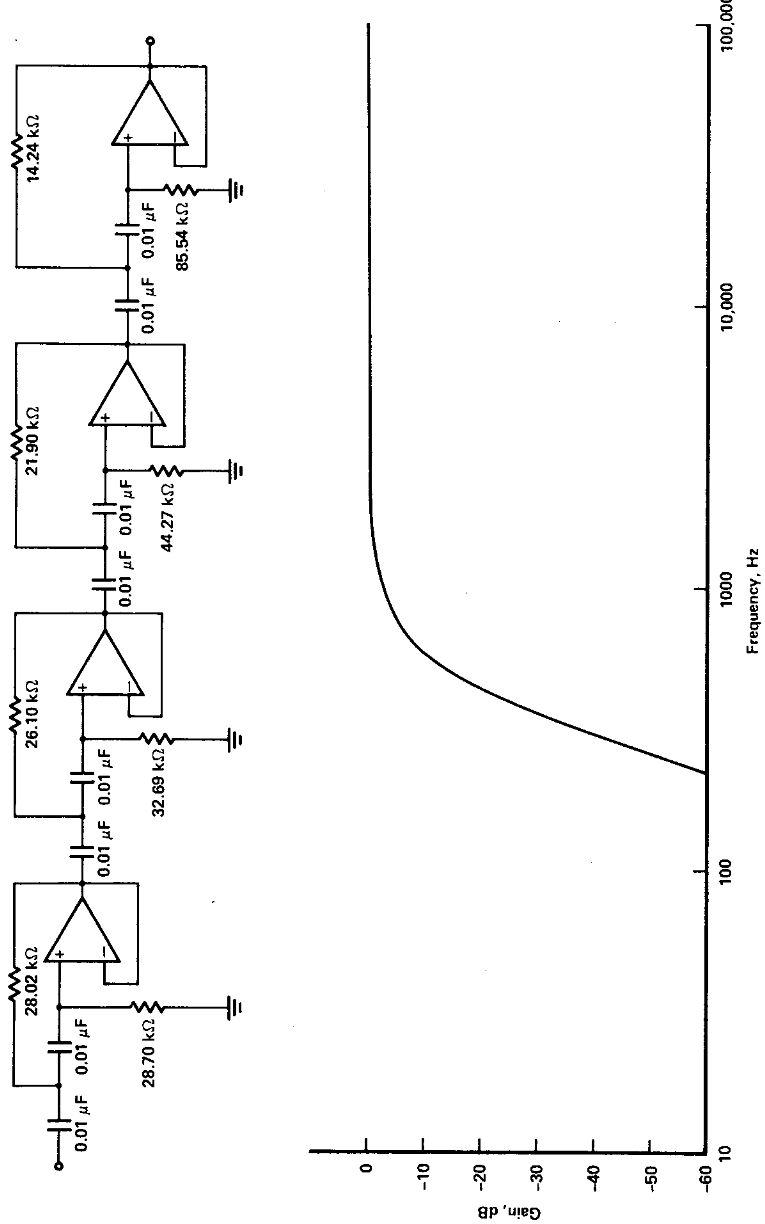


FIGURE 59. – Bessel 8-pole high-pass filter, f_c = 1000 Hz.

Bandpass Filters

TYPES OF BANDPASS FILTERS

Four types of bandpass filters are discussed in this chapter:

1. Relatively wideband filters made by connecting in series the low-pass and high-pass filters described in chapters 3 and 4.
2. Relatively narrowband filters built from one or more operational amplifier circuits; commercially available "pole-pairs" also may be used.
3. Filters designed by the method of low-pass–to–bandpass transformation.
4. Digital bandpass filters.

DESIGN BY COMBINING LOW-PASS AND HIGH-PASS FILTERS

If the design specifies lower and upper cutoff frequencies that are not too close, the circuits given in chapters 3 and 4 may be used as the basis for design of a bandpass filter; figure 60 shows how the combination of a low-pass filter with the response indicated in figure 60(a) and a high-pass filter with the response shown in figure 60(b) yields the response curve of figure 60(c). A bandpass circuit is shown in figure 60(d).

The procedure for combining circuits is straightforward, but it must be realized that the *high-pass* filter determines the *lower* cutoff and the *low-pass* filter determines the *higher* cutoff, as is indicated in figure 60. A specific example will clarify the design procedure.

EXAMPLE 1: Design a bandpass filter with a lower cutoff of 100 Hz and a higher cutoff of 1000 Hz by cascading high-pass and low-pass filters. Attenuation-slopes should be 18 dB per octave, and the high-pass and low-pass filters are to be of the Butterworth type.

SOLUTION: Designs for 18-dB-per-octave Butterworth low-pass and high-pass filters are given in figures 30 and 46, respectively. Figure 30 gives the design for a cutoff of 1000 Hz; this circuit does not need to be modified. On the other hand, it will be necessary to frequency-scale the circuit in figure 46 from 1000 to 100 Hz (table II). Suppose it is elected to increase all capacitors by the multiplying factor of 10, thereby establishing that all capacitors in the high-pass section will be 0.1 μF; the final circuit diagram is given in figure 60(d). In theory, it does not matter whether the low-pass or the high-pass section is placed first, but in practice the high-pass section

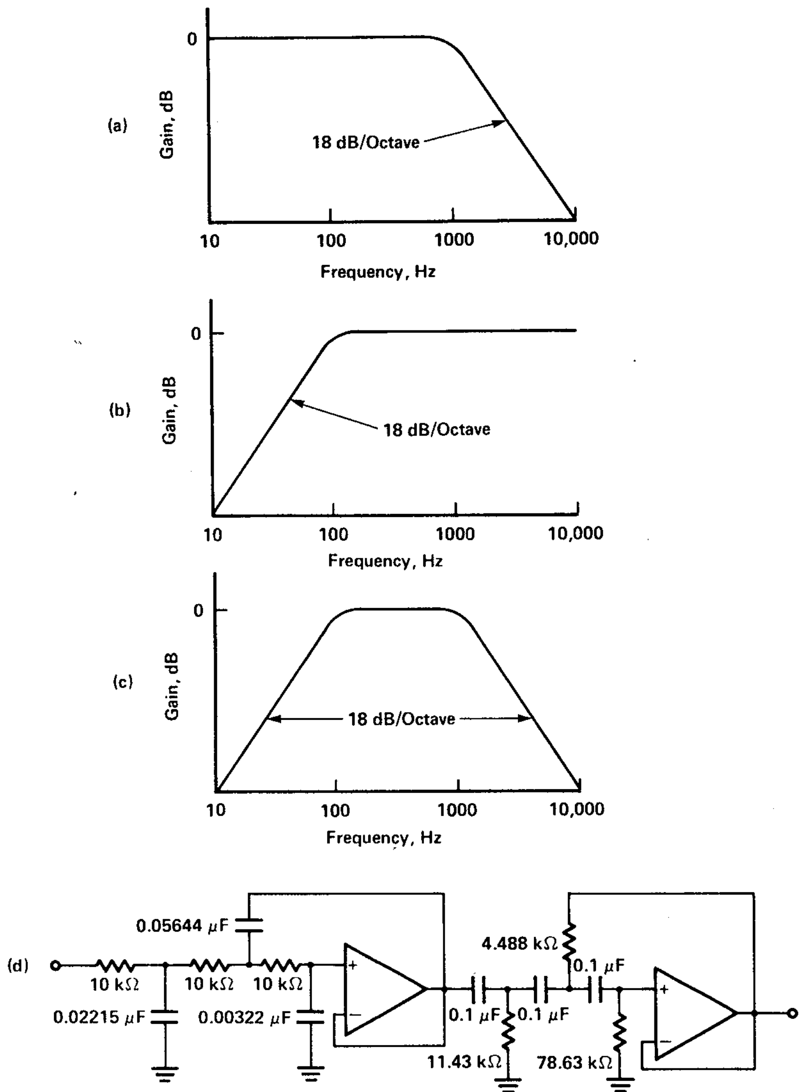


FIGURE 60. – Bandpass design by combining low-pass and high-pass filters.

is usually placed last, for in this position output offset voltages can only be caused by flow of bias currents through the 78.63-kΩ resistor. In contrast, if the low-pass filter were placed last, output offset would arise from current flow in the combination of the 78.63-kΩ plus the three 10-kΩ resistors in the low-pass section.

It can be seen by inspection of the low-pass and high-pass filter responses presented in chapters 3 and 4 that as the lower and upper cutoffs approach each other, the bandpass region has more and more insertion loss. Although attenuation losses can always be made up in an amplifier section, the filter response will have an undesirable shape. (A method for designing bandpass filters with a desired shape—Butterworth, Bessel, or Chebyshev—is given in the third section of this chapter.) How much attenuation and departure from an ideal filter shape can be tolerated depends on the particular application. It is possible to estimate attenuation and departure by studying the curves given for the standard filters in chapter 3 and 4 or, in the case of Butterworth filters, by calculations similar to the following:

For a low-pass, unity-gain Butterworth filter of n poles and with a 3-dB cutoff at f_c, the magnitude of the gain at any frequency, f, is given by

$$|A_L| = \sqrt{\frac{1}{1 + (\frac{f}{f_c})^{2n}}} \tag{48}$$

For a high-pass, unity-gain Butterworth filter of n poles and a 3-dB cutoff at f_c, the magnitude of the gain at any frequency f is

$$|A_H| = \sqrt{\frac{(\frac{f}{f_c})^{2n}}{1 + (\frac{f}{f_c})^{2n}}} \tag{49}$$

For a bandpass Butterworth filter, the gain at any frequency can be obtained by multiplying the gain of the low-pass section by the gain of the high-pass section. Equations (48) and (49) are therefore useful in checking the pass region and cutoff gains of bandpass Butterworth filters assembled by connecting low-pass and high-pass filters in series.

EXAMPLE 2: A Butterworth bandpass filter is to be built by cascading a low-pass filter that has a 1000-Hz cutoff and a high-pass filter that has a 100-

Hz cutoff; both filters are 3-pole and have unity gain. Find the bandpass filter gain at the center frequency as well as at 100 and 1000 Hz.

SOLUTION: Since both high-pass and low-pass filters have the same number of poles (three), geometric symmetry (as will be explained later) can be used to find the center frequency f_0. Looking ahead in this section, and invoking equation (52),

$$f_o = [(100)(1000)]^{1/2} = 316.23 \text{ Hz}$$

but at 316.23 Hz, equations (48) and (49) provide

$$|A_L| = 0.9995 \text{ or } -0.00434 \text{ dB}$$

$$|A_H| = 0.9995 \text{ or } -0.00434 \text{ dB}.$$

Hence, at 316.23 Hz, the overall circuit gain is

$$|A| = |A_L|\,|A_H| = 0.9990 \text{ or } -0.00868 \text{ dB}$$

Now, at 100 Hz, equations (48) and (49) show that

$$|A_L| = 0.9999995 \text{ or } -0.00000434 \text{ dB}$$

$$|A_H| = 0.7071 \text{ or } -3.010 \text{ dB},$$

whereupon, at 100 Hz the overall circuit gain is found to be

$$|A| = |A_L|\,|A_H| = 0.7071 \text{ or } -3.010 \text{ dB}.$$

Proceeding as before, at 1000 Hz, equations (48) and (49) give the following,

$$|A_L| = 0.7071 \text{ or } -3.010 \text{ dB}$$

$$|A_H| = 0.9999995 \text{ or } -0.00000434 \text{ dB}$$

for which it is found that at 1000 Hz the overall circuit gain is

$$|A| = |A_L|\,|A_H| = 0.7071 \text{ or } -3.010 \text{ dB}.$$

Summarizing, the results indicate that the filter has 3-dB points at 100 and 1000 Hz and essentially no loss at mid-frequency.

NARROWBAND FILTERS

Narrowband, peaked-response filters with the general type of response shown in figure 61 can be constructed in a variety of ways, but they usually are identified by supplying numerical values for the following characteristics:

f_o = center frequency

$$Q = \text{selectivity} = f_o/(\text{BW}) \tag{50}$$

$$\text{BW} = \text{3-dB bandwidth} = (f_2 - f_1) \tag{51}$$

A_o = gain at frequency f_o

$$f_o^2 = f_1 f_2 \tag{52}$$

The last expression, in which the square of the center frequency is equated to the product of the two 3-dB cutoff frequencies, is a particular case of the geometric mean symmetry associated with the filters under discussion, and because of this symmetry, the product of *any* two frequencies at which the same attenuation occurs is the square of the center frequency.

EXAMPLE 3: A bandpass filter has a response of the type indicated in figure 61; the center frequency is 1000 Hz and the bandwidth is 100 Hz. What is the circuit Q and what are the two 3-dB cutoff frequencies?

SOLUTION: Given that the center frequency f_o is 1000 Hz and the bandwidth (equation (51)) is 100 Hz, it is necessary to identify the two frequencies f_2 and f_1 as dictated by equation (52). By solving the quadratic expression and ignoring negative frequencies, there are obtained

$$f_1 = 951.25 \text{ Hz}$$

$$f_2 = 1051.25 \text{ Hz}$$

As a check, note that $f_2 - f_1 = 100$ Hz and $f_1 f_2 = 10^6 = f_o^2$. Continuing, equation (50) provides the selectivity,

$$Q = \frac{1000}{100} = 10$$

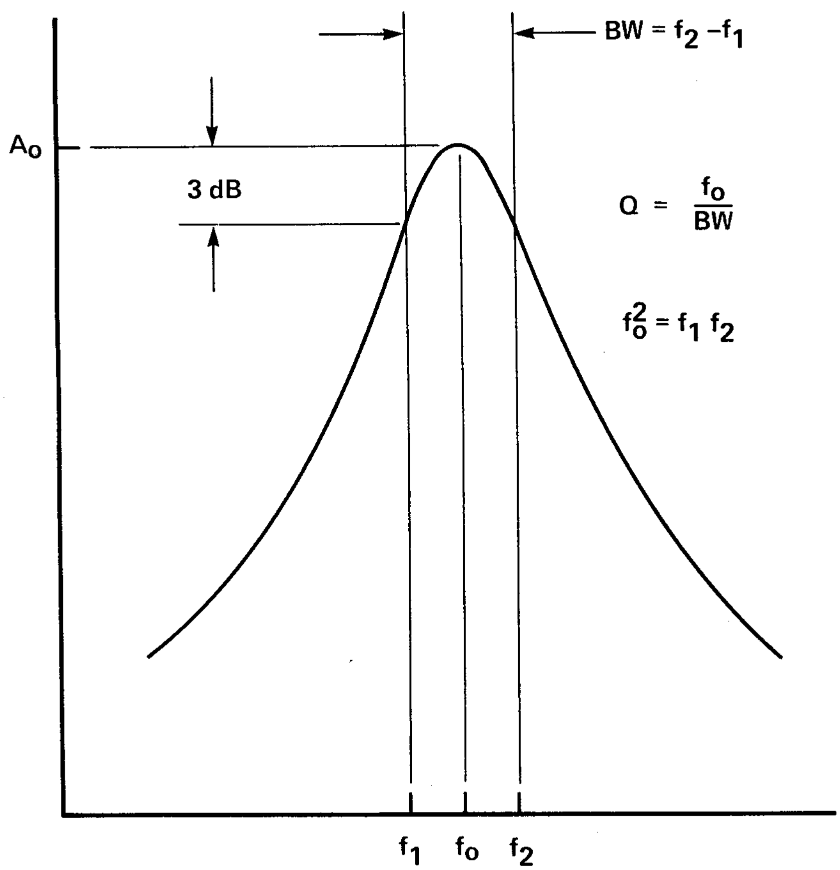


FIGURE 61. – Narrowband bandpass-filter response.

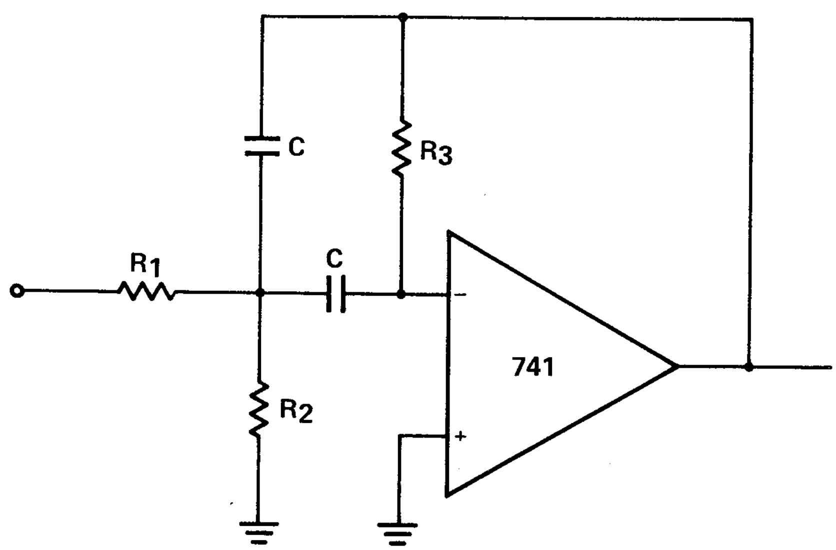


FIGURE 62. – Single operational amplifier bandpass filter.

Single Operational Amplifier Circuits

The circuit diagram of a very useful filter that requires only one operational amplifier is given in figure 62; the response of the filter is shown in figure 61.

The circuit of figure 62 is very useful for filters that have Q-values (selectivities) of up to 30 at frequencies as high as 10 kHz with a center frequency gain of unity. The exact values for Q and gain depend on the degree of match of the components and on the operational amplifier used. The circuit can be operated at higher frequencies with lower Q's, but significantly higher Q's are not usually practical, even at lower frequencies.

The design equations for the circuit given in figure 62 are as follows: Let f_o = center frequency, A_o = gain at center frequency, and B = bandwidth; then

$$A_o = -\frac{R_3}{2R_1} \tag{53}$$

$$B = \frac{1}{\pi R_3 C} \tag{54}$$

$$R_2 = \frac{R_1}{R_1 R_3 (2\pi f_o C)^2 - 1} \tag{55}$$

$$Q = \frac{f_o}{B} \tag{56}$$

The negative sign in equation (53) indicates that the input and output are 180° out of phase at the center frequency.

An important feature of the circuit can be deduced from equation (55), because R_2 appears in this equation, but not in equations (53) and (54); it is possible to use R_2 to tune the circuit *without* affecting center-frequency gain or the bandwidth. Since a major problem with all RC active filter circuits is the selection of components to achieve proper tuning, the ability to "final-tune" with a single resistor is a great convenience. Note that changing R_2 varies f_o without changing B. From equation (56) it follows that varying R_2 *will* affect the circuit Q; however, since R_2 is usually used to fine-tune the frequency, the effect is small.

EXAMPLE 4: Find values for the circuit in figure 62 that will give a Q of 30 with a gain of -1 at a center frequency of 1000 Hz.

SOLUTION: Equation (56) indicates that B must be 33.33 Hz; it is possible to choose C to be any convenient value (impedance-scale as per table II should the resistor values be impractical). Suppose C is chosen to be 0.1 μF; then, equation (54) provides R_3 = 95.49 kΩ, equation (53) shows R_1 = 47.75 kΩ, and equation (55) indicates R_2 = 26.54 Ω. Actually, the design is complete at this stage because the capacitors and the three resistors are all reasonable in value; however, R_1 and R_3 are close to standard one-percent resistor values of 49.9 kΩ and 100 kΩ, and it would be more convenient to use these values than to trim or select resistors to fit the computed values. However, if the standard values are used, the filter will be detuned. Fortunately, the standard values can be used if the filter is retuned by calculating a new value for R_2. The bandwidth and Q will be changed, but the effect will be small. An additional bonus in tuning with R_2 is that C does not have to be selected to within one percent of 0.1 μF; a pair of 0.1-μF nominal value capacitors *matched* to one percent are used and the filter is retuned with R_2. Thus with R_1 = 49.9 kΩ, R_3 = 100 kΩ, and C = 0.1 μF,

equation (53) gives A_o = −1
equation (54) gives B = 31.83 Hz
equation (56) gives Q = 31.41
equation (55) gives R_2 = 25.34 Ω

Using R_1 = 49.9 kΩ and R_3 = 100 kΩ, the bandwidth is 31.83 Hz instead of the required 30 Hz, a difference that usually is inconsequential. To simplify tuning, R_2 could be made from a fixed resistor of 15 ohms in series with a 20-ohm potentiometer used as a variable resistor.

Multiple Operational Amplifier Circuits

The circuit of figure 62 is not generally suitable for applications that require Q-values of 50 or more; for Q-values of 50 to 500, a different circuit must be used. Figure 63 shows the circuit diagram of one version of what has been called an analog computer filter, a state-variable filter, a dual integrator filter, a biquad, and a ring-of-three. The circuit has two advantages over the single operational amplifier circuit shown in figure 62; first, it is capable of providing stable Q-values of about 500, and second, its performance is much more insensitive to component values.

The design equations for the circuit in figure 63 are as follows: Let f_o = center frequency, A_o = gain at center frequency, and B = bandwidth. Then,

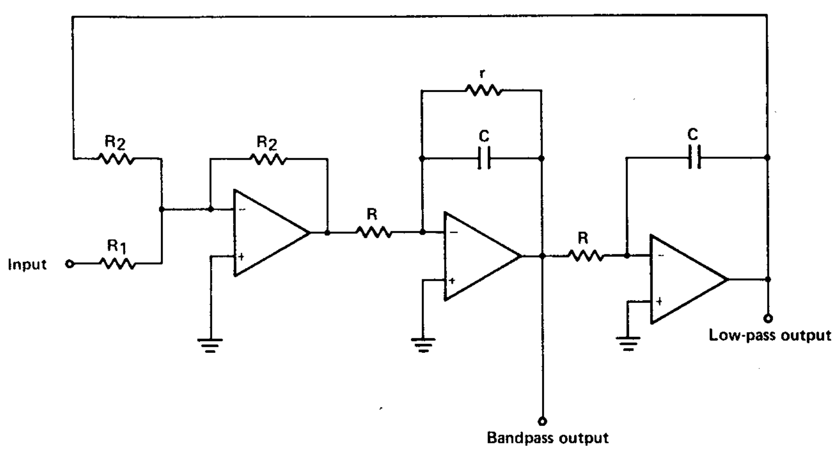


FIGURE 63. – State-variable bandpass filter.

$$A_o = \frac{R_2}{R_1} \cdot \frac{r}{R} \tag{57}$$

$$f_o = \frac{1}{2\pi CR} \tag{58}$$

$$Q = \frac{f_o}{B} = \frac{r}{R} \tag{59}$$

Note that R_1 appears only in the gain equation (57); accordingly, the gain can be adjusted independently of other parameters by varying R_1. Also, the Q of the circuit can be adjusted with a single resistor, r, without detuning the filter, because r does not appear in the center frequency expression, equation (58); however, if r is changed to vary Q, then R_1 must be changed in proportion to keep A_o constant, as indicated in equation (57).

EXAMPLE 5: Find values for the circuit of figure 63 that will give a Q of 100 with a gain of +1 at 1000 Hz. The input resistance should be one megohm.

SOLUTION: As usual, it is possible to choose any value for C as a starting point; later, if the calculations result in impractical values for other components, more appropriate values usually can be obtained by impedance-scaling (see table II, page 18). Let C = 0.1 μF; then

equation (58) furnishes R = 1.592 kΩ,
equation (59) provides r = 159.2 kΩ, and
equation (57) shows that R_1/R_2 = 100.

From elementary operational amplifier theory, the input resistance of the filter is equal to the value of R_1; because R_1 is one megohm, equation (57) shows that R_2 = 10,000 ohms.

A variation of the state-variable circuit (Kerwin and Shaffer, reference 15) is shown in figure 64. For this circuit,

$$f_o = \frac{1}{2\pi RC} \quad \text{(vary R)} \tag{60}$$

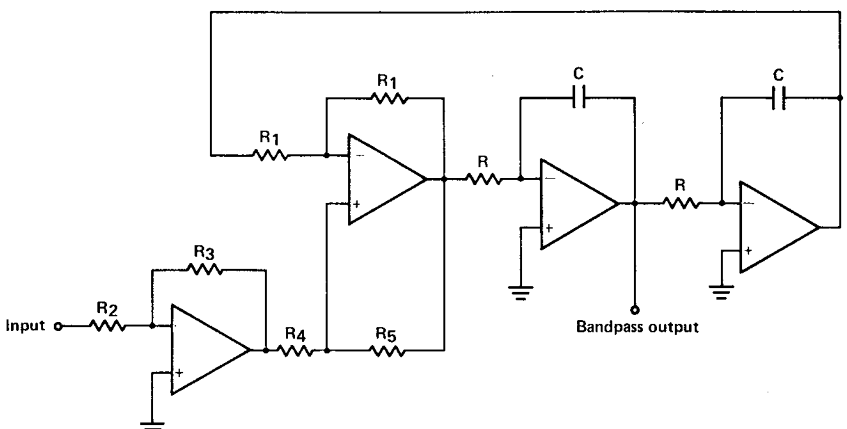


FIGURE 64. – State-variable bandpass filter with independent tuning and selectivity controls (ref. 15).

$$A_o = \frac{R_3R_5}{R_2R_4} \quad (\text{vary } R_3) \tag{61}$$

$$Q = \frac{R_4 + R_5}{2R_4} \quad (\text{vary } \frac{R_5}{R_2}) \tag{62}$$

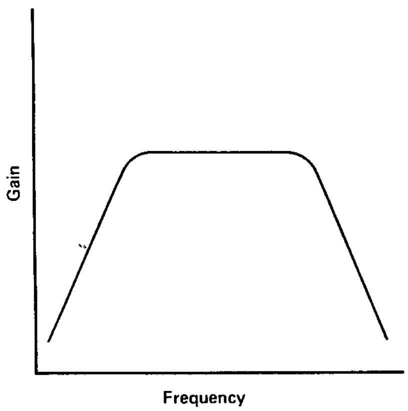
The advantage of the circuit shown in figure 64 is the ease with which f_o, A_o, and Q can be varied independently of each other; varying R changes f_o, but not A_o or Q and varying R_3 changes A_o, but not f_o or Q. Ganging R_5 and R_2 and varying the ratio R_5/R_2 changes Q but not f_o or A_o.

State-variable type circuits are available commercially from a number of companies and are often identified as universal active filters because they have three separate outputs, low-pass, high-pass, and bandpass. The circuit of figure 63 needs to be modified to provide all three outputs because the output of the input summer is not a true high-pass section even though bandpass and low-pass outputs are available from the integrator sections as shown. Additional discussion about the modification to give a true high-pass section will be found in chapter 6, where the high-pass and low-pass sections of such a filter are summed to give a notch.

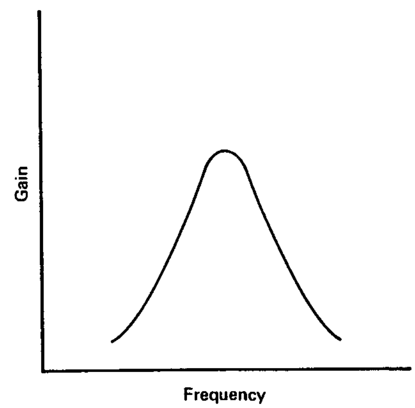
Low-Pass-to-Bandpass Transformation

So far, two basic approaches to the design of bandpass filters have been discussed. The first involved the combination of low-pass and high-pass filters to give a response such as that in figure 65(a); the second involved several methods of designing circuits to give the response in figure 65(b). Neither of these approaches is suitable for the design of filters that have the response shown in figure 65(c). The response is too narrowband to be made from combinations of practical low-pass and high-pass circuits, but it has a pass region that cannot be obtained with a simple peaking circuit.

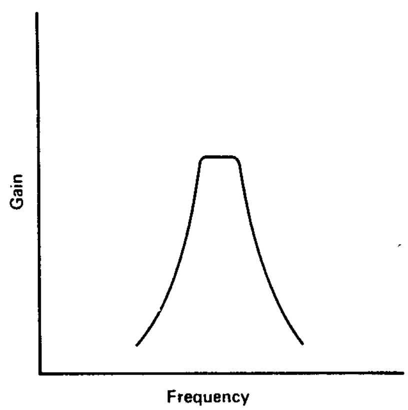
The design of filters that have the type of response shown in figure 65(c) is more involved than the circuits discussed so far, but the procedure is still straightforward, though more lengthy. In essence, the method consists of connecting in series a number of filters having the type of response shown in figure 65(b) and determining the response each individual filter must have. For example, three filters having the responses shown in figure 65(d) could be cascaded to produce the response shown in figure 65(c). (This technique,



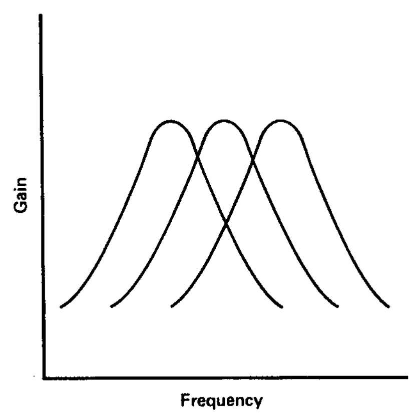
65(a) Wideband.



65(b) Narrowband peaked.



65(c) Narrowband with flat passband.



65(d) The responses of three filters which could be cascaded to provide the response shown in (c).

FIGURE 65. – Types of bandpass filter response.

as well as a related, but different technique, is described in detail in reference 3.) Note that the design of an individual filter section with a response of the type shown in figure 65(b) involves finding its Q, center-frequency f_0, and center-frequency gain A_0; these quantities specify the section completely.

The easiest way to find the design parameters of the sections to be cascaded is to use a low-pass-to-bandpass transformation; a summary of the steps involved is indicated in figure 66. Figure 66(a) shows a desired bandpass response; knowing the design requirements of this bandpass response, it is possible to find an equivalent, or prototype low-pass response, as shown in figure 66(b). The response is *equivalent* because when a low-pass-to-bandpass transformation is performed on the equations for the low-pass filter, the result is equations for the desired bandpass filter. The equations permit determination of the Q, f_0, and A_0 of each of the n required sections indicated in figure 66(c) and 66(d).

The design procedure is best illustrated by means of numerical examples. An overview of the procedure, given below, is followed by two examples.

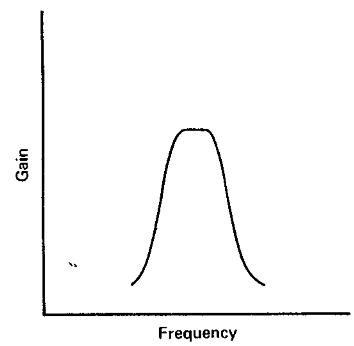
1. Define the desired bandpass filter requirements in terms of *shape factors* (shape factors and how they are found are described in design examples in this chapter).
2. Find the low-pass filter that has the same shape factor as the bandpass filter being designed.
3. Perform a low-pass-to-bandpass transformation to find Q, f_0, and A_0 for the individual sections of which the final bandpass filter design is comprised.
4. Design the individual bandpass sections.

EXAMPLE 6: Find the shape factor, S, for the bandpass Chebyshev filter shown in figure 67, and find its equivalent low-pass prototype.

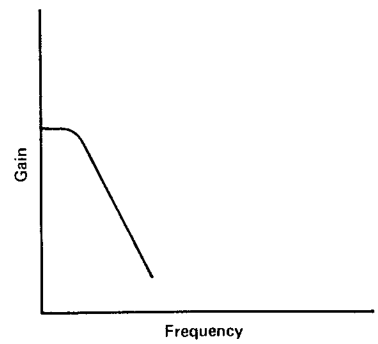
SOLUTION: *Shape factor* is the ratio of two bandwidths at two specified attenuations. In this instance, the two attenuations are 50 dB and 3 dB, and the corresponding bandwidths are 1350 Hz and 450 Hz, respectively. Therefore S is 1350/450 or 3.

Since the bandpass filter is a Chebyshev with a 1-dB ripple, the low-pass prototype must also be a Chebyshev with 1-dB ripple. Referring to figure 36, which shows the response of a 4-pole, low-pass Chebyshev filter, it is evident that attenuations of 50 dB and 3 dB occur at approximately 3000 Hz and 1000 Hz, respectively. The shape factor for this low-pass filter is therefore 3000/1000 = 3 also, and the filter is suitable for transformation to the bandpass version of figure 67.

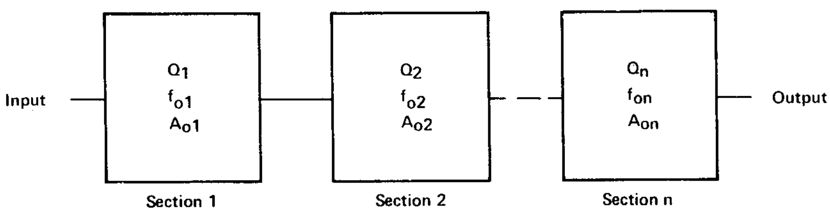
Two important points need be stressed: First, the two specified attenuations must be the same for the low-pass prototype as for the bandpass filter, although the actual attenuations are arbitrary; for example, 10 dB and 40 dB



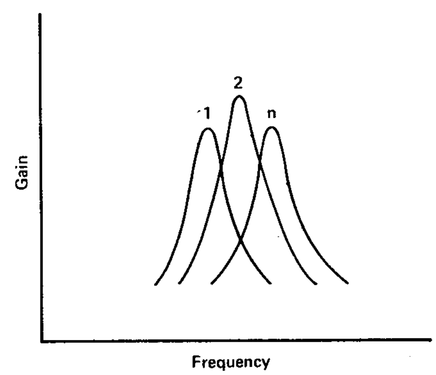
66(a) Required bandpass response.



66(b) Low-pass equivalent.



66(c) Bandpass filter sections showing design parameters of each section.



66(d) Responses of individual sections.

FIGURE 66. – Low-pass–to–bandpass transformation.

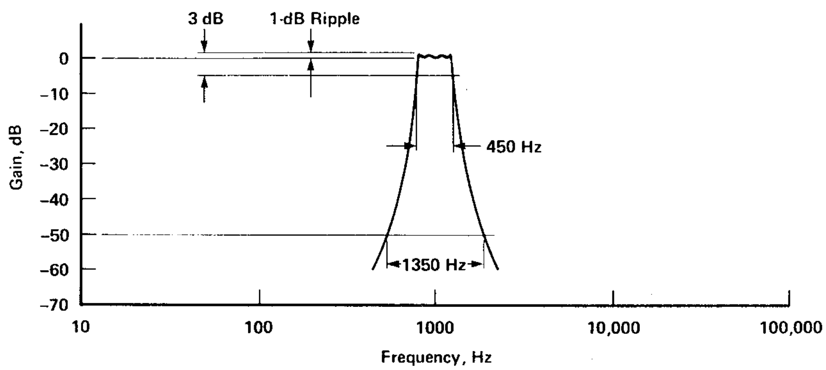


FIGURE 67. – Shape factor for a Chebyshev bandpass filter.

could have been used in place of 3 dB and 50 dB. Second, note that shape factor is the ratio of frequencies and, therefore, is dimensionless; accordingly, one could use a low-pass prototype that has a cutoff at 1 radian/sec instead of 1000 Hz (or any frequency). A 4-pole, low-pass Chebyshev filter will always have the same shape factor regardless of its cutoff. The important fact to be garnered from the comparison of bandpass and low-pass shape factors is that only a 4-pole, low-pass prototype can be transformed into the bandpass filter with the response given in figure 67.

EXAMPLE 7: Find the shape factor S for the bandpass Butterworth filter shown in figure 68 and find the equivalent low-pass prototype.

SOLUTION: At 50 dB and at 3 dB, the bandwidths are 1360 Hz and 200 Hz, respectively. Accordingly, the shape factor S is 1360/200 or 6.8. Now, since the bandpass filter is a Butterworth, the prototype must also be a Butterworth. Referring to figure 30 (the response of a 3-pole, Butterworth low-pass filter), it is found that attenuations of 50 dB and 3 dB occur at 7000 Hz and 1000 Hz, respectively. The shape factor for this low-pass filter is 7 and may be considered suitably close to 6.8 for most practical purposes. The 3-pole prototype is, therefore, suitable for transformation to the bandpass filter of figure 68.

EXAMPLE 8: How many sections of the type shown in figure 66(c), each with its own Q, f_0, and A_0, would be required to build the Chebyshev and Butterworth bandpass filters described in examples 6 and 7?

SOLUTION: The number of sections is equal to the number of poles in the low-pass prototype. From shape factor considerations, the Chebyshev low-pass prototype was found to have 4 poles in example 6; therefore, 4 sections are required. Since the Butterworth low-pass prototype was found to require 3 poles in example 7, 3 sections are required to be included in the bandpass filter.

EXAMPLE 9: Design a bandpass, 1-dB ripple Chebyshev filter that has the response shown in figure 67.

SOLUTION: From examples 6 and 8 in this chapter it has been found that the response shown in figure 67 requires a low-pass, 1-dB ripple Chebyshev prototype with 4 poles. When the prototype has an *even* number of poles, as in this case, the resulting 4 bandpass sections can be grouped in pairs having equal Q's. Therefore, the final design will have 4 sections, all

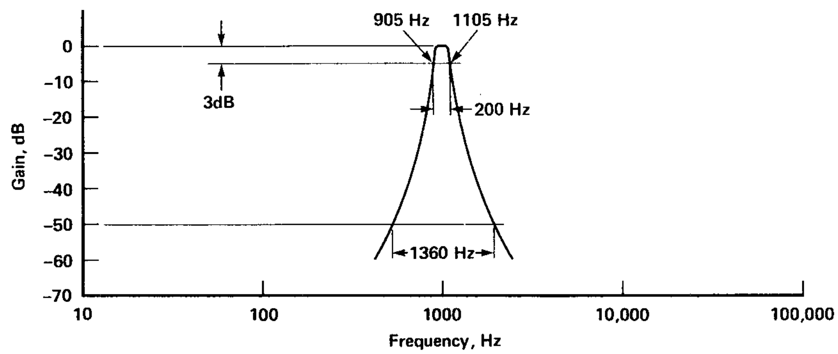


FIGURE 68. – Shape factor for a Butterworth bandpass filter.

with different center frequencies and two sections will have one value for Q while the others will have another value for Q.

First find the two values for Q with the help of constants listed in table VI(a). Data in this table show that there are 4 constants for a 4-pole 1-dB-ripple Chebyshev low-pass prototype. These constants are: $a_1 = 0.25198$; $b_1 = 0.63983$; $a_2 = 0.88970$; and $b_2 = 0.26503$. The values of a_1 and b_1 are used to calculate the Q of the first two sections, and the values of a_2 and b_2 are used to calculate the Q of the last two sections. Following are the formulas used to calculate Q:

Let Q_o = Q of bandpass filter being designed.

$$\text{Then } Q_o = \frac{\text{Center frequency of filter being designed}}{\text{3-dB bandwidth of filter being designed}} \tag{63}$$

$$\text{Calculate } X = \left(\frac{2Q_o}{b}\right)^2 \tag{64}$$

$$\text{Calculate } Y = X + \frac{a}{b^2} \tag{65}$$

$$\text{Calculate } Q = \left[\frac{Y + \sqrt{Y^2 - X}}{2}\right]^{1/2} \tag{66}$$

The center frequency is found from the two 3-dB frequencies using equation (52):

$$f_o^2 = f_1 f_2 = (800)(1250) = 10^6$$

$$f_o = 1000 \text{ Hz}$$

From equation (63), $Q_o = \frac{1000}{450} = 2.22222$

From equation (64), $X = 48.2508$, using b_1

From equation (65), $Y = 48.8664$, using a_1 and b_1

From equation (66), $Q = 6.97268$ for the first two sections.

TABLE VI. – *Low-Pass-to-Bandpass Transformation Constants**

(a) Low-pass prototype with 2, 4, 6, or 8 poles

Filter type	Poles	a_1	b_1	a_2	b_2	a_3	b_3	a_4	b_4
Butterworth	2	1.0000	1.4142						
	4	1.0000	1.8478	1.0000	0.76537				
	6	1.0000	1.9319	1.0000	1.4142	1.0000	0.51764		
	8	1.0000	1.9616	1.0000	1.6629	1.0000	1.1111	1.0000	0.39018
Chebyshev 0.25-dB ripple	2	0.82713	1.1242						
	4	0.35013	0.90030	0.89445	0.37292				
	6	0.17506	0.65680	0.55946	0.48081	0.94387	0.17599		
	8	0.10280	0.50860	0.35573	0.43117	0.71341	0.28810	0.96634	0.10117
Chebyshev 1-dB ripple	2	0.74363	0.90154						
	4	0.25198	0.63983	0.88970	0.26503				
	6	0.11906	0.45350	0.53246	0.33198	0.94587	0.12151		
	8	0.068447	0.34742	0.33206	0.29453	0.70486	0.19680	0.96848	0.069107
Chebyshev 3-dB ripple	2	0.70712	0.64452						
	4	0.19592	0.41128	0.90282	0.17032				
	6	0.088793	0.28533	0.52175	0.20888	0.95470	0.076454		
	8	0.050290	0.21695	0.32087	0.18392	0.70352	0.12289	0.97410	0.043155

(b) Low-pass prototype with 3 poles

Filter type	a	b
Butterworth	1.0000	1.0000
Chebyshev, 0.25-dB ripple	0.85278	0.61236
Chebyshev, 1-dB ripple	0.82938	0.45135
Chebyshev, 3-dB ripple	0.83873	0.29854

*Constants for Bessel filters are not given because these filters lose their linear phase property in the low-pass–to–bandpass transformation process and therefore are not generally useful as bandpass filters.

All answers are expressed in 6 significant figures because equation (68) [to be presented later] involves subtraction of large numbers that are relatively equal. Even though in practice it is sufficient to know Q to only 2 or 3 significant figures, it is necessary to retain the value for Q of 6.97268 for use in equation (68). A value of Q accurate to 3 significant figures is eventually sufficient, so Q = 6.97 is applied to the circuit of figure 69 in the first two filter sections.

By a similar set of calculations, equations (64), (65), and (66) give Q = 17.1361 when Q_o is again set equal to 2.22222, but a_2 = 0.88970 and b_2 = 0.26503. The abridged value of Q = 17.1 is used in figure 69 for the blocks representing the last two sections.

Having found the Q requirements of all 4 sections, it is now possible to proceed to the calculations of the center frequency of each section. The following formulas are used, with the center frequency f_o = 1000 and the Q of the bandpass filter being designed Q_o = 2.22222. Obtaining b as before from table VI(a):

$$\text{Calculate } Z = \frac{bQ}{Q_o} \tag{67}$$

$$f_{o1} = \frac{f_o}{2}\left[Z + \sqrt{Z^2 - 4}\right] \tag{68}$$

$$f_{o2} = \frac{f_o^2}{f_{o1}} \tag{69}$$

From equation (67), Z = 2.0076 since b_1 = 0.63983

From equation (68), f_{o1} = 1091.1 Hz

From equation (69), f_{o2} = 916.5 Hz

These center frequencies may be inserted into the first two sections of figure 69. By a similar process, but using b_2 = 0.26503 and with f_o = 1000 Hz and Q_o = 2.22222, equations (67), (68), and (69) give f_{o3} = 1232.1 Hz and f_{o4} = 811.6 Hz. These frequencies may be inserted into the last two sections of figure 69.

Thus far, there have been found all the Q's and center frequencies of the four individual bandpass sections that will give the bandpass Chebyshev response of figure 67 when connected in series. The final step is to set the gain

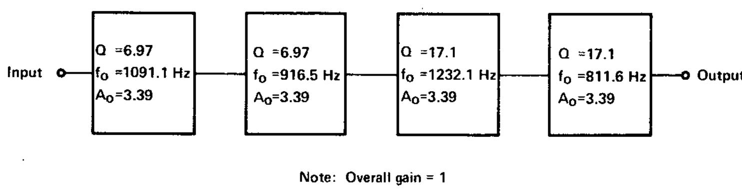


FIGURE 69. – Sections for 1-dB ripple Chebyshev bandpass filter.

of each section so that the overall gain of the final filter is unity or 0 dB at the center frequency of 1000 Hz.

To find the required gain for each section, first assume that the gain of each of the 4 sections *at its own center frequency* is unity; then find the gain of each section at the *center frequency of the final filter*, which is 1000 Hz. Multiplying the four 1000-Hz gains together will give the overall gain of the final filter at its center frequency of 1000 Hz, provided the gains of the individual sections are all unity at their own center frequencies. The gains of the individual sections can be adjusted to bring the overall gain at 1000 Hz to unity as desired. The following will clarify the procedure.

So far there has been computed for the center frequencies and Q's of the individual sections:

$$f_{o1} = 1091.1 \text{ Hz}, Q_1 = 6.97$$

$$f_{o2} = 916.5 \text{ Hz}, Q_2 = 6.97$$

$$f_{o3} = 1232.1 \text{ Hz}, Q_3 = 17.1$$

$$f_{o4} = 811.6 \text{ Hz}, Q_4 = 17.1$$

Considering the first section, for a center frequency gain of 1, the gain at any other frequency f can be computed by the following equation, provided the circuit Q of the section is known:

$$A_1 = \frac{\dfrac{f_{o1}f}{Q_1}}{\sqrt{(f_{o1}^2 - f^2)^2 + \left(\dfrac{f_{o1}f}{Q_1}\right)^2}} \tag{70}$$

For this section, $f_{o1} = 1091.1$ Hz and Q = 6.97; using equation (70) it is found that the gain of the section at $f = 1000$ Hz is 0.635. The gains of all 4 sections at 1000 Hz can be calculated by substituting the appropriate f_o and Q into equation (70) with f always 1000 Hz. Accordingly,

1st section gain at 1000 Hz = 0.635
2nd section gain at 1000 Hz = 0.635
3rd section gain at 1000 Hz = 0.137
4th section gain at 1000 Hz = 0.137
Overall gain at 1000 Hz = (0.635)(0.635)(0.137)(0.137) = 0.00757

In order to raise the overall gain to 1, it is necessary to multiply by a factor of 1/(0.00757) or 132.1. Multiplying the gain at one frequency multiplies the gain by the same amount at all frequencies; accordingly each section should be designed to have a gain of 3.39 at its own center frequency because (3.39)(3.39)(3.39)(3.39) = 132.1. The block diagram design can now be completed by inserting the gain values into figure 69.

It remains to design the individual filter sections now that their Q, f_0, and A_0 values are known. Design procedures have already been given for single operational amplifier realizations and for state-variable realizations in the third section of chapter 5. In general, a state-variable realization is to be preferred because the resulting filter has higher stability. Also, state-variable circuits in the form of universal active filters or pole pairs are commercially available and can be set to any f_0, Q, and A_0 within specifications by adding a few external components. When setting up a filter it is important to be certain that each section has its correct f_0, Q, and A_0 before it is connected to the others.

Butterworth and Chebyshev bandpass filters may be designed by the method of low-pass–to–bandpass transformation using the constants listed in table VI. Filters with 2-, 4-, 6-, and 8-pole, low-pass prototypes are designed using the constants listed in table VI(a). The method and formulas are the same as for the filter in example 9. Constants for Bessel filters are not given because in the low-pass–to–bandpass transformation process the filters lose their linear phase property and thus are not generally useful.

The procedure can also be used for 3-pole filters but a simple modification is required, as is illustrated by the following example.

EXAMPLE 10: Design a bandpass Butterworth filter that provides the response shown in figure 68.

SOLUTION: The solution to example 7 in this chapter reveals that the response shown in figure 68 requires a low-pass Butterworth prototype with three poles. With a 3-pole prototype, the three bandpass sections are grouped into a pair of sections of equal Q's and one odd section of a different Q. Accordingly, the final design will include three bandpass sections, each with different center frequencies and the Q's indicated above.

First find the value of Q for the two sections that have equal Q's; the method was described in example 9 and it initially requires finding the proper constants in table VI. There are two: a = 1, b = 1; both a and b are used to calculate the Q of the two sections having equal Q's, but only b is used for calculation of the Q of the odd section.

The center frequency from the two 3-dB frequencies is found by use of equation (52):

$$f_o^2 = f_1 f_2 = (905)(1105) = 10^6$$

$$f_o = 1000$$

Now, equations (63), (64), (65), and (66) are used to obtain the Q's of the equal-Q sections:

From equation (63), $Q_o = \frac{1000}{200} = 5$

From equation (64), X = 100.000

From equation (65), Y = 101.000

From equation (66), Q = 10.0375.

Note that Q is expressed as a value of six significant figures because equation (68) (used later in this example) involves the subtraction of large, nearly equal, numbers. Even though in practice it is necessary to know Q to only two or three significant figures, it is important to retain the value 10.0375 for use in equation (68). Eventually, however, the value of Q will need to be accurate only to about three significant figures; in fact, a Q of 10 can be inserted into the first two filter sections of figure 70(a).

Computation of the Q of the odd section is straightforward; if Q_o is the Q of the filter being designed ($Q_o = 5$ in this case), then the Q of the odd section is found from

$$Q = \frac{Q_o}{b} \tag{71}$$

where b is the constant obtained from table VI. In this instance, b = 1 and it follows that

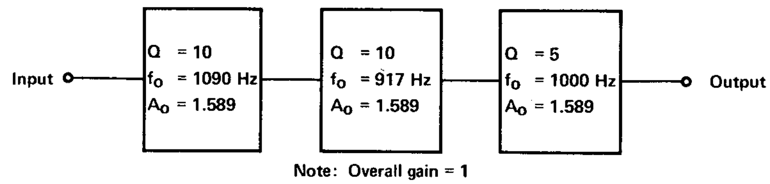
$$Q = \frac{5}{1} = 5$$

This value for Q is inserted into the block for the third section in figure 70(a).

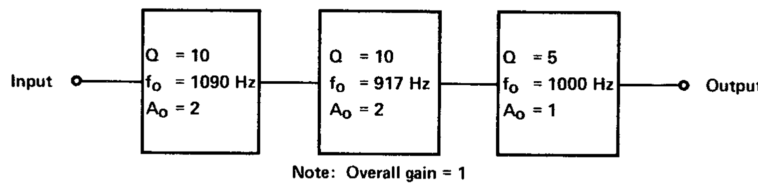
Now that the Q requirements of all three sections have been found, the next step is the calculation of the center frequency of each section. The same formulas as were used in example 9 in this chapter are used for the two sections that have the same Q, namely equations (67), (68), and (69).

Let f_o and Q_o be the center frequency and Q of the bandpass filter being designed; for this example:

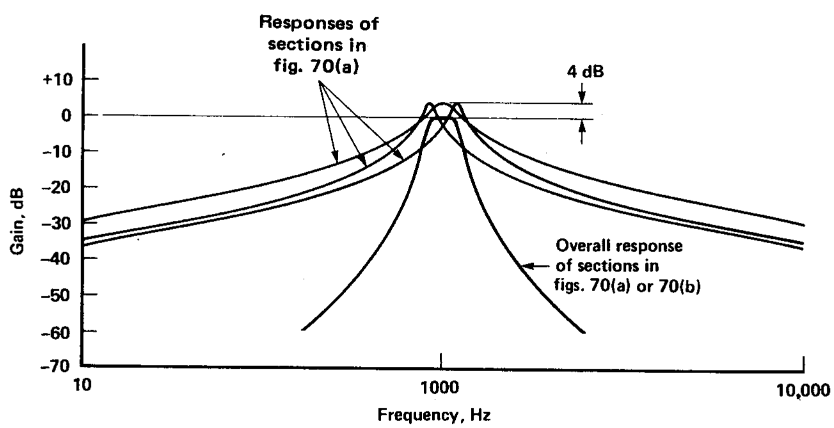
$$f_o = 1000 \qquad Q_o = 5$$



70(a) Butterworth bandpass filter sections.



70(b) Sections with same response as (a) but with unequal section-gains.



70(c) Individual and final filter-response curves.

FIGURE 70. – Sections for Butterworth bandpass filter.

Obtaining b = 1 as before from table VI(b) and using $Q_o = 5$ and Q = 10.0375, there can be obtained:

From equation (67), Z = 2.00750

From equation (68), $f_{o1} = 1090.43$ Hz

From equation (69), $f_{o2} = 917.066$ Hz

These center frequencies may be inserted into the first two sections in figure 70(a).

For the center frequency of the odd section, note that f_o is the center frequency of the filter being designed (1000 Hz in this case), and the center frequency f of the odd section is given by equation (72):

$$f = f_o \tag{72}$$

Clearly, in this case f = 1000 Hz, and this value may be inserted into the third section in figure 70(a).

Up to the present, there have been calculated the Q's and center frequencies of all three individual bandpass sections that will give the bandpass Butterworth response shown in figure 68 when connected in series. The last step is to find the gain of each section so that the overall gain of the final combination of filters is unity or 0 dB at the center frequency of 1000 Hz.

The procedure used to find the gain of each section is the same as the one used for the Chebyshev case in example 9; first, assume that the gain of each of the three sections at its *own center frequency* is unity and then find the gain of each section at the *center frequency of the final filter* (1000 Hz). Provided the gains of the individual sections are all unity at their own center frequencies, multiply the three gains of each filter at 1000 Hz to give the overall gain of the final filter at its center frequency of 1000 Hz. As noted before, the gains of the individual sections then can be adjusted to bring the overall gain at 1000 Hz to unity. Continuing with the design example will clarify the procedure.

Thus far in the example, the center frequencies and Q's of the individual sections are known:

f_{o1} = 1090.43 Hz, Q_1 = 10.0375

f_{o2} = 917.066 Hz, Q_2 = 10.0375

f_{o3} = 1000 Hz, Q_3 = 5

For a center frequency gain of 1, the gain at any other frequency, f, can be obtained from equation (70) when the circuit Q of the section is known. In fact, equation (70) provides the following:

$$\begin{aligned} \text{1st section gain at 1000 Hz} &= 0.4993 \\ \text{2nd section gain at 1000 Hz} &= 0.4993 \\ \text{3rd section gain at 1000 Hz} &= 1 \\ \text{Overall gain at 1000 Hz} &= (0.4993)(0.4993)(1) = 0.2493 \end{aligned}$$

In order to raise the overall gain to 1, it is necessary to multiply by a factor of 1/(0.2493) or 4.011; for example, the gain of each section can be increased by 1.5888 because (1.5888)(1.5888)(1.5888) = 4.011. Moreover, since multiplying the gain of a filter at one frequency multiplies the gain at all frequencies by the same amount, each section should be designed to have a gain of 1.5888 at its center frequency. Now the block diagram design can be completed by inserting the gain values into figure 70(a). However, filters with unequal gains could be used because the same overall response can be achieved without having equal gains. For example, if the gain multiplying factor is rounded off from 4.011 to 4, gains of 2, 2, and 1 for the three filter sections could also be used; the resulting filter block diagram for this case is shown in figure 70(b).

The responses of the three filter sections for the case where the gains are all equal to 1.5888 (+4 dB) are shown in figure 70(c) along with the final filter that has a center frequency gain of 1 (0 dB).

DIGITAL BANDPASS FILTERS

The digital bandpass filters to be discussed in this section are digital in the sense that they use digital integrated circuits and digital switching techniques. However, the signals processed by the filters are analog, not digital; consequently, the word digital as used here refers to the filtering technique and not to the signals being filtered. Digital filters are also called commutating filters.

Before discussing specific circuits, it is of interest to note some of the main advantages of the type of digital bandpass filter covered in this handbook, but it is also necessary to have a clear understanding of one of the disadvantages of high-Q nondigital circuits. The nature of this disadvantage is best illustrated by the following example.

EXAMPLE 11: A bandpass filter with a response of the type shown in figure 65(b) has a Q of 250, a center frequency of 1000 Hz, and a center-frequency gain of 1. If the frequency of the input signal drifts lower than 1000 Hz, the output will drop, but even when the input remains at 1000

Hz, the output may drop if the center frequency of the filter changes because of component drift. How much will the output be reduced if the frequency is reduced by 0.1 percent, 0.2 percent, and one percent?

SOLUTION: The percentage frequency changes are converted to frequency changes:

0.1% drop in 1000 Hz means the signal is reduced to 999 Hz
0.2% drop in 1000 Hz means the signal is reduced to 998 Hz
1.0% drop in 1000 Hz means the signal is reduced to 990 Hz

The gain of a bandpass filter designed for one frequency but used at any other frequency is given by equation (70); for the filter under consideration at this time, $f_{o1} = 1000$ Hz and $Q_1 = 250$. Solving for frequencies of $f = 999$, 998, and 990 Hz, there are obtained

at 999 Hz, gain = 0.894 or −1 dB
at 998 Hz, gain = 0.707 or −3 dB
at 990 Hz, gain = 0.195 or −14 dB

The values indicate that the output is very susceptible to changes either in the frequency of the signal source or in the center frequency of the filter. It takes only a one percent change in frequency to produce a change of -14 dB in filter gain. The reason for this sensitivity is the high Q of the filter (250); reducing Q will help, but the effectiveness of the filtering also is reduced. One solution to this problem is a digital filter.

A digital filter can be used advantageously in many instances because it can be made to track an input frequency so that the detuning effects revealed in example 11 will not occur. Unfortunately, a digital filter requires a clock-frequency input, but there are many instances where a clock input is available; for example, in phase-sensitive detection systems that use lock-in amplifiers, a reference signal of the same frequency as the signal is always available and usually a multiple of this signal also can be made available. As will be made evident further on, a clock running at a multiple of the signal frequency is usually necessary with digital filters. The clock can be crystal-controlled for absolute accuracy, nontracking circuits; that is, circuits that must operate without a reference signal.

The type of digital filter that will be discussed in the following paragraphs is shown in outline form in figure 71. The filter consists of N low-pass sections made from a single resistor and N capacitors; only one filter section is in the circuit at any instant, because the sections are alternately switched into the circuit in sequential fashion by rotation of switch S (ordinarily a solid-state switch). The switching rate is N times the center frequency of the

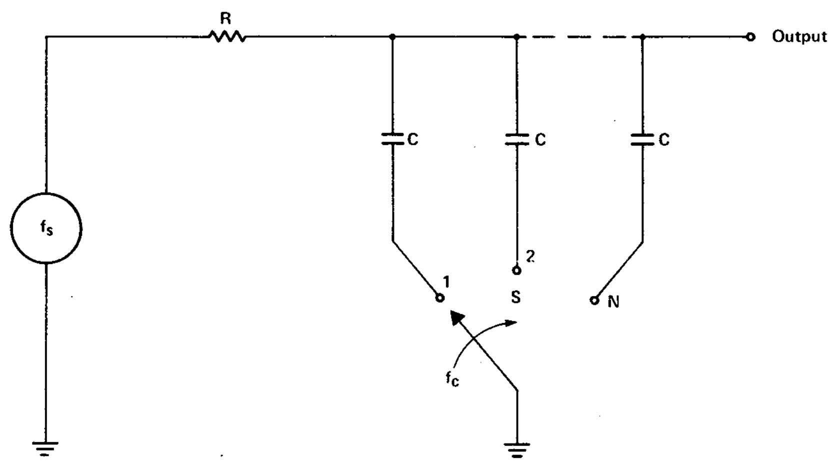


FIGURE 71. – Principle of digital filter.

bandpass filter being designed and the bandwidth of the filter is 2/N times the bandwidth of the low-pass section. The following terms are used in the design of a filter such as shown in figure 71:

f_s = center frequency of bandpass filter being designed
N = number of low-pass sections = number of capacitors
$f_c = Nf_s$ = clock frequency required to drive switch S
B = bandwidth of bandpass filter = $1/(\pi NRC)$

EXAMPLE 12: A digital bandpass filter is to be used to process a 1000-Hz signal; the low-pass sections of the filter consist of a 100-kΩ resistor and eight 0.1 - μF capacitors. Find the clock frequency required to drive the rotating switch; find the Q and bandwidth of the bandpass filter.

SOLUTION: The statement of the problem establishes that f_s = 1000 Hz and N = 8. Accordingly,

the clock frequency $f_c = Nf_s$ = 8000 Hz, and

$$\text{the bandwidth } B = \frac{1}{\pi NRC} = \frac{1}{(\pi)(8)(10^5)(10^{-7})} = 3.98 \text{ Hz.}$$

Rounding off the bandwidth B to a value of 4, the selectivity of the filter is:

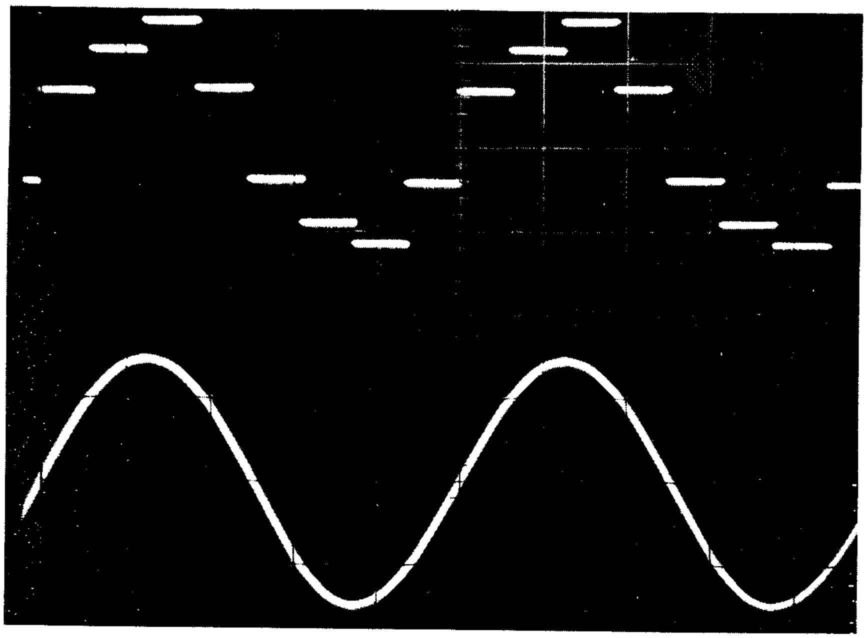
$$Q = \frac{f_s}{B} = \frac{1000}{4} = 250$$

The output of the circuit shown in figure 71 has the stepped appearance indicated in figure 72, although the waveform can be filtered further to produce the smoothed sine-wave output that is also shown in figure 72.

Before considering an example of the design of a practical digital filter, it is necessary to be aware that a filter of the 8-section type shown in figure 71 yields a multiple bandpass response of the type shown in figure 73, and in certain applications this form of response may create problems. Note that the circuit has its maximum response at dc and that the response at 1000 Hz is slightly less; in fact, there are bandpass responses at all harmonics of 1000 Hz up to 8000 Hz, the latter being the clock frequency at which there is zero response. In general, the first harmonic frequency at which no response occurs is always the clock frequency, but other zero-responses occur at multiples of the clock frequency. In the case of an 8-section digital filter, zero responses occur at 8000 Hz, 16,000 Hz, 24,000 Hz, and so on.

A peculiarity of digital filters is that the output frequency is not necessarily the same as the input frequency; this is, in contrast to the analog-

Without bandpass filtering.



72(a)

72(b)

With bandpass filtering.

FIGURE 72. – Output of digital filter.

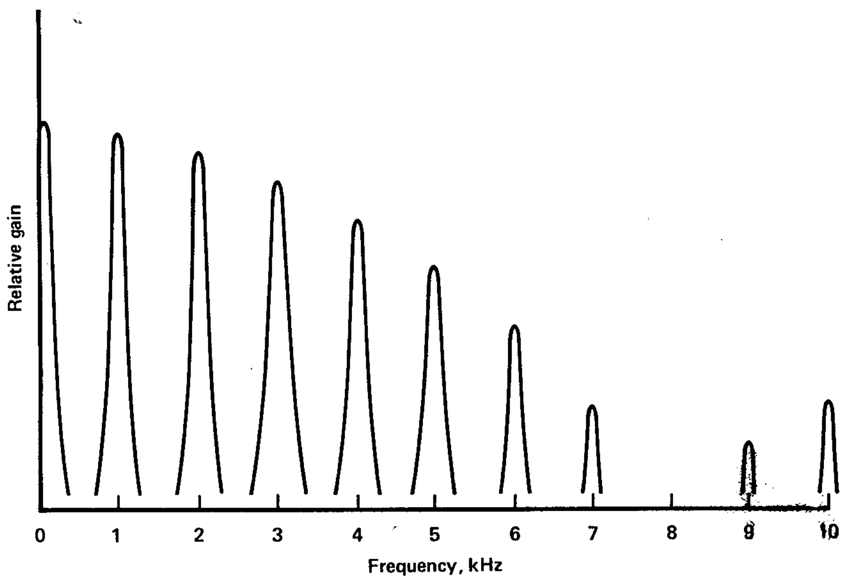


FIGURE 73. – Multiple bandpass response of an 8-section digital filter.

type filters that attenuate a signal but do not change its frequency. As an illustration, the following list shows the output frequencies obtained for various sine-wave inputs to the 8-section digital filter.

Input frequency Hz	Fundamental frequency of output, Hz	Input frequency Hz	Fundamental frequency of output, Hz
1000	1000	8,000	0
2000	2000	9,000	1000
4000	4000	10,000	2000
6000	2000	12,000	4000
7000	1000		

Data in the list show, for example, that if the filter of figure 71 is supplied with 1000-Hz, 7000-Hz, or 9000-Hz inputs, the output will be a wave form with a 1000-Hz fundamental frequency in each case. This observation is very significant for it shows that not only must an *output* filter be used to remove the steps shown in figure 72, but an *input* filter also must be used to remove the harmonics in the input signal that can generate unwanted and confusing output signals. If the input filter is omitted, unwanted frequencies will appear in the output. If the output filter is omitted, the output is a stepped wave form, not a sine wave.

The complete digital filter shown in block-diagram form in figure 74 is based on the design given in example 12; it has a Q of 250 and a center frequency of 1000 Hz. The input and output filters also have center frequencies of 1000 Hz, but have lower Q's of 30, so that slight frequency changes in the input signal do not produce large changes in output amplitude. Recall that the high-Q digital section can track input frequency changes, but the low-Q analog sections cannot.

It is reasonable to ask why a digital filter is used if it requires two additional analog filters for proper performance. The answer is that the filter in figure 74 is a practical, easily realizable device that provides a high Q with low-Q analog filters. For example, it is possible to obtain extremely high-Q values with the circuit in figure 74 and the circuit can be made to perform well at much higher frequencies than a corresponding device made exclusively of analog filters.

A practical realization of the block diagram of figure 74 is shown in figure 75. The CD4011 is used as an 8000-Hz clock to drive the rotating switch. In phase-sensitive, lock-in amplifier applications, the clock signal would come from the generator providing the phase detector reference. In these instances, there would be an 8000-Hz clock to drive the digital filter and the clock frequency would be counted down to 1000 Hz to provide the phase detector

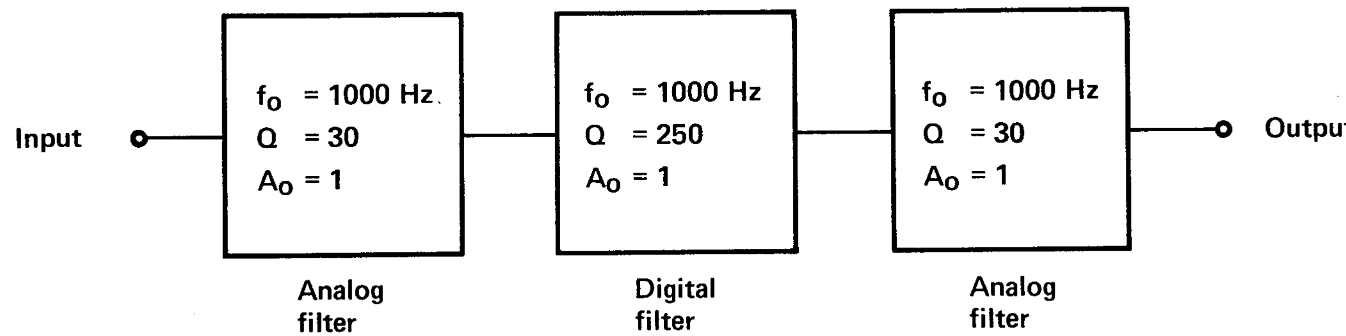


FIGURE 74. – Digital filter with analog-input and analog-output filters.

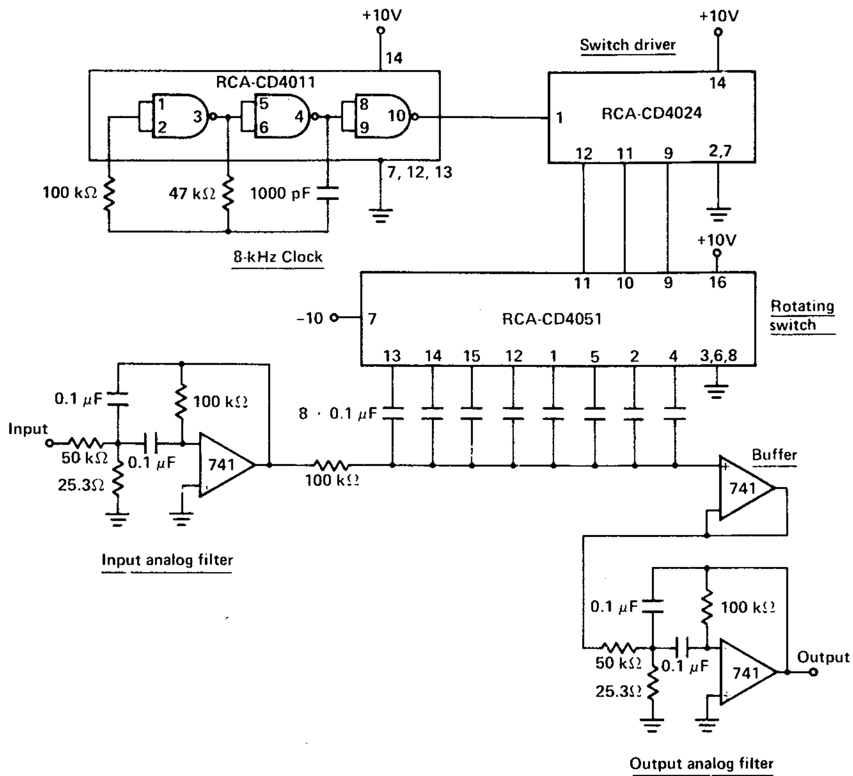


FIGURE 75. – Practical digital circuit with f_o = 1000 Hz and Q = 250, with low-Q, analog-input and analog-output filters.

reference. The CD4024 decodes the clock signal to provide appropriate drive signals for the solid-state, rotating-switch CD4051. The frequency response of this circuit is shown in figure 76; note the suppression of harmonics of 1000-Hz

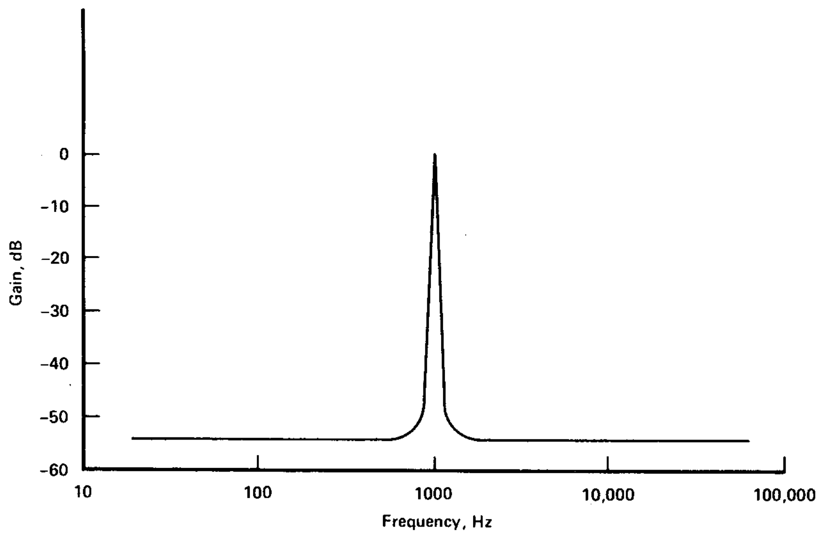


FIGURE 76. – Response of circuit in figure 75.

Band-Reject Filters

TYPES OF BAND-REJECT FILTERS

Band-reject filters can be designed by several techniques, of which three will be discussed here:

1. Combinations of the low-Q, low-pass and high-pass filters discussed in chapters 3 and 4. The low-pass and high-pass sections should be summed; that is, connected in parallel, not in series. This is in contrast to bandpass filters that are assembled from low-pass and high-pass sections connected in series.
2. Combinations that use available 3-terminal, null- or notch-networks with active feedback to one of the three terminals.
3. Combinations of high-Q, low-pass and high-pass filter outputs from the state-variable active filters that are based on the dual integrator circuit discussed in the third section "Multiple Operational Amplifier Circuits" of chapter 5.

Very sharp notch filters with extremely narrow reject regions must be used with caution. For example, such filters are very frequently used for the elimination of 60-Hz interference and sometimes the filters are required to pass dc to perhaps 58 Hz and to pass from 62 Hz to some high frequency while rejecting 60 Hz. In most instances, however, it is far better to get rid of 60-Hz interference by some means other than a notch filter. In fact, before resorting to notch filters, all attempts should be made to ensure correct shielding and proper grounding along with the use of opto-isolators, differential amplifiers with high common-mode rejection, and other related techniques.

Notch filters are extremely sensitive to component mismatch; for example, significant reduction of 60 Hz with a high-Q notch having a theoretical depth of, say, 50 dB, cannot be realized with components matched to within one percent – at least a 0.1-percent match is necessary; moreover, high stability components must be used to maintain the notch depth.

COMBINING THE OUTPUTS OF LOW-Q, LOW-PASS AND HIGH-PASS FILTERS

The design of band-reject filters comprised of combinations of low-Q, low-pass and high-pass filters is straightforward; however, the low-pass and high-pass sections must be connected in parallel paths to sum their outputs, in contrast to the series connection required for bandpass filters.

EXAMPLE 1: Figure 77(a) shows a 1-kHz low-pass filter and a 10-kHz high-pass filter. What would be the output if the two filters were connected in series? What would be the output if the inputs of the two filters were to be connected together and their outputs fed to a summer?

SOLUTION: The series connection of the two filters is shown in block diagram form in figure 77(b); clearly, only signals having frequencies between dc and about 1 kHz will pass through the first filter. However, the second filter rejects all signals below about 10 kHz; consequently, the output is zero for any input signal frequency.

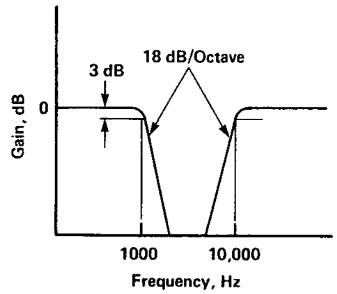
The other connection described above is shown in figure 77(c). Input A of the summing amplifier receives all signals from dc to 1 kHz. Input B of the summing amplifier receives all signals above 10 kHz. Since the summing amplifier does not discriminate against any frequency, its output contains signals from both dc to 1 kHz and 10 kHz and above. Its output response is therefore the same as that of figure 77(a).

EXAMPLE 2: Design a circuit that will provide the response shown in figure 77(a). Butterworth response is required in the pass regions.

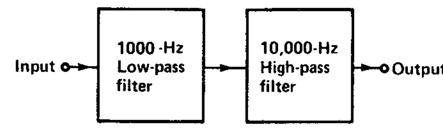
SOLUTION: Since both low-pass and high-pass sections require 18-dB/octave attenuation-slopes, both should be 3-pole filters. The design of the 1-kHz low-pass filter can be obtained directly from figure 30. The 10-kHz section can be obtained from figure 46 by frequency-scaling (see table II, page 18). As a result of the scaling procedure, all capacitors are changed in figure 46 from 0.01 to 0.001 μF. The outputs of the filters are summed; the final circuit is shown in figure 77(d).

TWIN-T WITH FEEDBACK

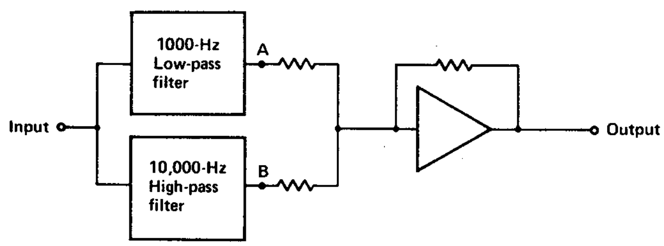
It is impractical to achieve a response such as that indicated by the solid line in figure 78(a) with the filter combinations described in the preceding section. One way to obtain this type of response is to apply feedback to any one of a number of three terminal networks, as indicated in figure 78(b). Of the many networks available, the twin-T is perhaps the best known; this is an arrangement of resistors and capacitors that produces the type of response shown by the dashed line in figure 78(a). In fact, if the response represented by the dashed line is adequate, a twin-T or other null network combined with a voltage follower used as a buffer may be employed without the need to feedback to the third terminal. In a circuit of the type shown in figure 78(b), the feedback from the amplifier output to C would be dis-



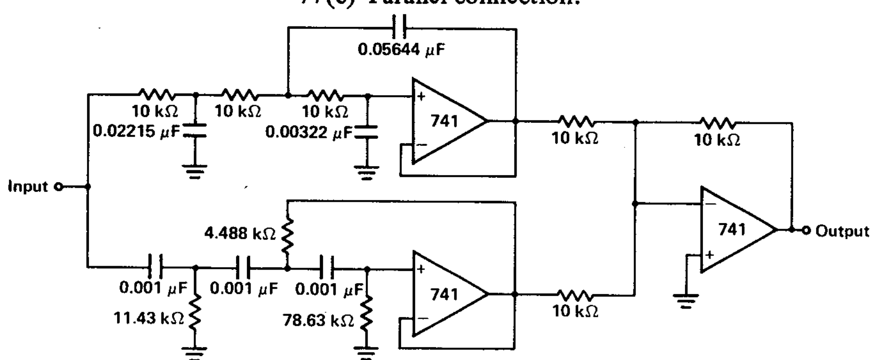
77(a) Low-pass and high-pass responses.



77(b) Series connection.

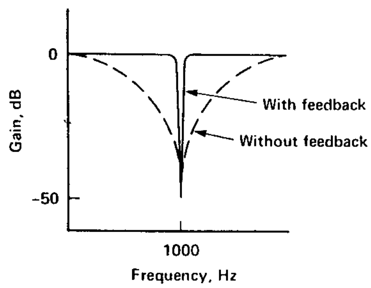


77(c) Parallel connection.

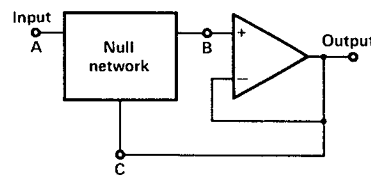


77(d) Practical circuit.

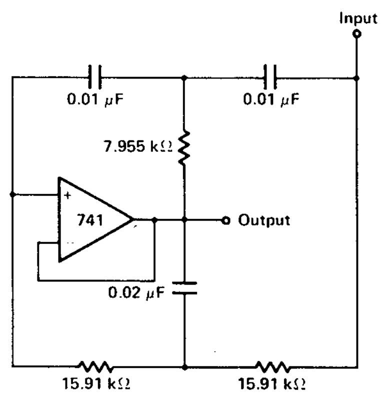
FIGURE 77. – Band rejection by summing low-Q, low-pass and high-pass sections.



78(a) Responses.



78(b) Basic circuit.



78(c) Practical circuit.

FIGURE 78. – Null network with feedback.

connected and terminal C would be grounded. In circuits connected this way, component-matching requirements are not as severe as when feedback is used.

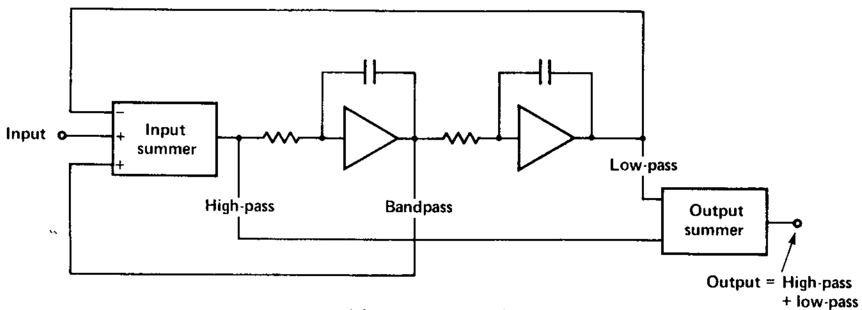
Figure 78(c) shows a circuit diagram for a filter designed to provide a response with a sharp notch such as represented by the solid-line response in figure 78(a). The values are for a 1-kHz notch, but the notch can be shifted to any frequency by frequency-scaling and impedance-scaling (table II, page 18). By using components selected to 0.1 percent or better, a null of -50 dB was achieved in an actual circuit.

COMBINING THE LOW-PASS AND HIGH-PASS OUTPUTS OF STATE-VARIABLE FILTERS

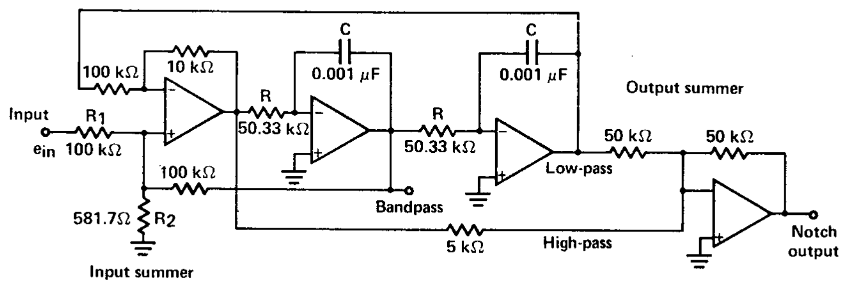
State-variable filters can be designed to have high-pass, bandpass, and low-pass outputs. If the low-pass output is derived from the high-pass by pure dual integration as shown in figure 79, the low-pass and high-pass outputs will be exactly 180° out of phase, because each integrator introduces a phase-lag of 90°; if the amplitudes of the low-pass and high-pass outputs are equal at one particular frequency, summing these outputs will result in zero voltage output (equal voltages of opposite polarity). However, even when the outputs are not equal in amplitude, they can be summed in a manner that will produce a null.

The state-variable circuit used to produce a notch in the response of a filter is slightly different from the one first introduced in figure 63. Referring to this figure, note the presence of resistor r that prevents the first integrator from having 90° of phase shift at all frequencies, thereby complicating the design equations. The circuit, as modified in figure 79(a), omits the resistor r of figure 63 that was used to set the circuit Q so that the two integrators can provide the required phase-shift of exactly 180°. Also, for control of Q, feedback is applied from the output of the first integrator, which is the bandpass output, to the input summer and an output summer is included to sum the high-pass and low-pass outputs so that a null is available at one frequency. A circuit of this type has a confusingly large number of different values for gain, since gain depends on the frequency as well as the particular output under consideration. Gains are listed below and on figures 79(c), 79(d) and 79(e) along with other important circuit parameters.

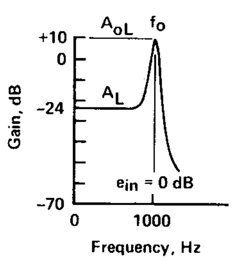
f_o = notch frequency = resonant frequency of the low-pass, high-pass and bandpass outputs
A_o = gain to bandpass output at f_o
A_{oL} = gain to low-pass output at f_o
A_{oH} = gain to high-pass output at f_o
A_L = gain to low-pass output at low frequencies



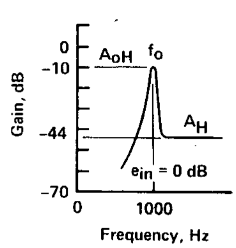
79(a) Basic circuit.



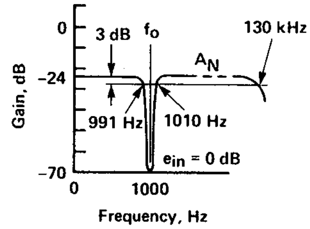
79(b) Practical circuit.



79(c) Low-pass response.



79(d) High-pass response.



79(e) Notch response.

FIGURE 79. – State-variable notch filter.

A_H = gain to high-pass output at high frequencies
A_N = notch output-gain in the bandpass region
Q_o = Q of low-pass, high-pass and bandpass outputs
R_P = the resistance of all the resistors on the noninverting input of the input summing amplifier in parallel; that is,

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{10^5} \tag{73}$$

The design equations for this circuit are as follows:

$$f_o = \frac{1}{2\pi\sqrt{10}\,CR} \tag{74}$$

$$Q_o = \frac{10^6}{(\sqrt{10})(11)(R_P)} \tag{75}$$

$$A_o = \frac{10^5}{R_1} \tag{76}$$

$$A_{oL} = \sqrt{10}A_o \tag{77}$$

$$A_{oH} = A_o/\sqrt{10} \tag{78}$$

$$A_L = A_{oL}/Q \tag{79}$$

$$A_H = A_{oH}/Q \tag{80}$$

$$A_N = A_L \tag{81}$$

EXAMPLE 4: Design a circuit based on figure 79(b) that has a notch at 1 kHz, a circuit Q of 50, and a gain to the bandpass output of unity; C = 1000 pF. Find A_{oL}, A_{oH}, A_L, A_H, and A_N.

SOLUTION:

From equation (74), $R = 50.33\ k\Omega$

From equation (76), $R_1 = 100\ k\Omega$

From equation (75), $R_P = 575.0\ \Omega$

From equation (73), $R_2 = 581.7\ \Omega$

From equation (77), $A_{oL} = \sqrt{10} = +10$ dB

From equation (78), $A_{oH} = 1/\sqrt{10} = -10$ dB

From equation (79), $A_L = \sqrt{10}/50 = -24$ dB

From equation (80), $A_H = 1/(\sqrt{10})(50) = -44$ dB

From equation (81), $A_N = -24$ dB

Refer to figures 79(c), 79(d) and 79(e) for the meaning of the gains calculated above. Theoretically, the notch should be infinitely deep, but in a practical circuit that inevitably had component mismatches, a depth of only -70 dB was achieved; the 3-dB width of the notch was 19 Hz as shown in figure 79(e). The dynamic range of this circuit is an important factor in its use; with a 1-V RMS input, the low-pass output is 10 dB higher at f_0 for a circuit Q of 50, or 8.9 V peak-to-peak. With higher Q's, the dynamic range is correspondingly larger.

Since the high-pass output gain is 0.1 times the low-pass gain, the output summer uses 5-kΩ and 50-kΩ summing resistors to equalize the gains and produce the notch; the feedback resistor in the output summer gives an overall gain of -24 dB outside the notch region. The final circuit is shown in figure 79(b) and the response in figure 79(e).

Circuits of the type shown in figure 79(b) are available commercially in the form of universal active filters. Resistors (R) are added externally to tune the filter, and extra capacitors can be added in parallel with the two capacitors (C) if necessary, Often R_2 is an internal resistor of a value that gives a Q of 1, but it can be paralleled with other resistors to raise the Q. The output summer must be provided by the user, although an undedicated operational amplifier is sometimes provided on the integrated-circuit unit for this purpose.

Notch filters can also be built using digital techniques. In this approach, there is first designed a digital bandpass filter of the type shown in figures 74 and 75 that has a peak response at the notch frequency and then the output of the digital filter and the signal input are made to be equal in amplitude, but 180° out of phase. Summing of the two signals provides a cancellation, thereby producing a notch. If the 180° of phase-shift and the equality of amplitude can be maintained with a high degree of accuracy, a very sharp, deep notch can be obtained. The notch depth provided by this type of filter can be very stable and much less dependent on component drift than straight analog filters. A filter produced by this technique is described in reference 20.

Topics in Filter Design

INTRODUCTION

A knowledge of the topics covered and referenced in this appendix is not necessary for the understanding and application of the design procedures for any of the filters discussed in previous chapters; however, the design procedures and data given previously can be applied more effectively when the user has some idea of the derivation of the various formulas and of how filters are basically designed. The discussion below is intended to serve as an introduction to the material in the references.

TRANSFER FUNCTIONS

The design of filters involves a detailed consideration of input/output relationships because a filter may be required to pass or attenuate input signals so that the output amplitude-versus-frequency curve has some desired shape. The mathematical expression relating to the output to the input is called the transfer function.

It is possible to study quite generalized expressions for transfer functions, but in order to understand certain essential points, attention will be focused on three specific transfer functions: those for low-pass, high-pass, and band-pass filters of the second order. The term "second order" refers to the fact that the transfer equations involve terms no higher than p^2. Second-order transfer functions describe 2-pole filters; third-order functions involve p^3 and describe 3-pole filters, and so on. For steady-state, sinusoidal, non-transient signals, p is an operator which can be expressed as $p = j\omega = j2\pi f$. Second-order transfer functions are a useful starting point in the study of filters because they are foundations upon which more complex filters can be built. For example, all the low-pass and high-pass filters discussed in chapters 3 and 4 are based on second-order transfer functions. The band-pass filters in chapter 5, including those using low-pass-to-bandpass transformations, also are based on 2-pole designs.

Typical second-order transfer functions for low-pass, high-pass, and bandpass filters are:

$$\text{Low-pass:}\quad \frac{\text{Output}}{\text{Input}} = \frac{H}{p^2 + bp + a} \tag{A-1}$$

$$\text{Bandpass:}\quad \frac{\text{Output}}{\text{Input}} = \frac{Hp}{p^2 + bp + a} \tag{A-2}$$

$$\text{High-pass:}\quad \frac{\text{Output}}{\text{Input}} = \frac{Hp^2}{p^2 + bp + a} \tag{A-3}$$

Note that all three transfer functions have the same denominator and that the numerators have increasing powers of p. That the equations do represent low-pass, high-pass, and bandpass filters can be seen by noting that p is proportional to frequency and by studying the behavior of the equations at such points as $\omega = 0$ and $\omega = \infty$. For example, putting $\omega = 0$ into equation (A-1) gives output/input = H/a, while $\omega = \infty$ gives output/input = 0, which indicates that equation (A-1) describes a filter that passes dc with a gain of H/a and attenuates infinite and high frequencies; in other words, it describes a low-pass filter.

The numerator and the denominator of equation (A-2) can be divided by p and then p can be replaced with $j\omega$ to give

$$\frac{\text{Output}}{\text{Input}} = \frac{H}{j\omega + b + \frac{a}{j\omega}} = \frac{H}{j\omega + b - \frac{ja}{\omega}} \tag{A-4}$$

At $\omega = 0$ and $\omega = \infty$, equation (A-4) reduces to zero; when $\omega = a/\omega$ or $\omega^2 = a$, the output/input is equal to H/b, which indicates that equation (A-4) describes a filter that attenuates low and high frequencies and passes midband frequencies; in other words, equation (A-4) describes a bandpass filter.

The numerator and denominator of equation (A-3) can be divided by p^2 and p can be replaced with $j\omega$ to give

$$\frac{\text{Output}}{\text{Input}} = \frac{H}{1 + \frac{b}{j\omega} - \frac{a}{\omega^2}} \tag{A-5}$$

Equation (A-5) reduces to 0 at $\omega = 0$ and to H at $\omega = \infty$, indicating that the equation describes a filter that passes high frequencies and attenuates low frequencies; in other words, the equation describes a high-pass filter.

Although equation (A-1) describes a low-pass filter, there are many different types of low-pass filters; for example, chapter 3 contains design information for Butterworth, Chebyshev, and Bessel low-pass filters. The *type* of filter, that is, the shape of its response curve, is determined by the constants a and b in equation (A-1); the constant H is a multiplier that affects the filter gain. Constants a and b in equations (A-2) and (A-3) also determine the type of filter and H is again a multiplying factor affecting the gain.

There are available tables listing constants such as a and b for various types of filters. (H is a scaling factor chosen by the designer to give a specific gain.) Typically, a and b are given as coefficients in polynomial functions for various types of filters. Butterworth polynomials for low-pass filters up to 4 poles are as follows:

Number of Poles	Unfactored form	Factored form
1	$\dfrac{1}{p + 1}$	--
2	$\dfrac{1}{p^2 + \sqrt{2}p + 1}$	--
3	$\dfrac{1}{p^3 + 2p^2 + 2p + 1}$	$\dfrac{1}{(p + 1)(p^2 + p + 1)}$
4	$\dfrac{1}{p^4 + 2.61p^3 + 3.41p^2 + 2.61p + 1}$	$\dfrac{1}{(p^2 + 0.765p + 1)(p^2 + 1.85p + 1)}$

For example, in equation (A-1), $H = 1, b = \sqrt{2}$, and $a = 1$ would give a Butterworth response. The factored form is given for the 3-pole and 4-pole cases to illustrate that higher order transfer functions can be reduced to products of second order functions or less. This means, for example, that a 4-pole filter can be made from two second order filters, as can be seen by comparing figures 29 and 31.

The following simple example illustrates how a filter may be designed with the aid of transfer function polynomials.

EXAMPLE A-1: Design a two-pole, low-pass Butterworth filter with a passband gain of unity and a cutoff of 1 kHz. Use the circuit of figure A-1.

SOLUTION: First obtain an expression for the output/input transfer function, or e_3/e_1. Note that e_3 appears in two places in figure A-1 because of voltage-follower action. Two equations can be obtained by summing currents; the currents are obtained by dividing the voltage across the particular component by its impedance. From $i_1 = i_2 + i_3$, it follows that

$$\frac{e_1 - e_2}{R_1} = \frac{e_2 - e_3}{1/pC_1} + \frac{e_2 - e_3}{R_2} \tag{A-6}$$

From $i_3 = i_4$, it is evident that

$$\frac{e_2 - e_3}{R_2} = \frac{e_3}{1/pC_2} \tag{A-7}$$

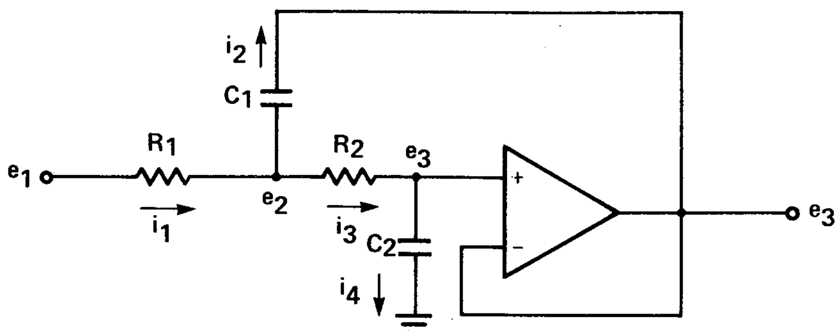
Equations (A-6) and (A-7) can be solved by eliminating e_2 to give

$$\frac{e_3}{e_1} = \frac{1/R_1R_2C_1C_2}{p^2 + p\,\dfrac{(R_1 + R_2)}{R_1R_2C_1} + \dfrac{1}{R_1R_2C_1C_2}} \tag{A-8}$$

For equation (A-8) to represent a Butterworth filter, it must have the same form as the fundamental Butterworth equation; as was indicated earlier, the form is

$$\frac{\text{Output}}{\text{Input}} = \frac{1}{p^2 + \sqrt{2}p + 1} \tag{A-9}$$

By equating coefficients between equations (A-8) and (A-9), values for R_1, R_2, C_1, and C_2 can be derived to make the circuit in figure A-1 a Butterworth filter. Examination of the coefficients reveals that any two components out of the group consisting of R_1, R_2, C_1, and C_2 can be chosen arbitrarily, and then the equations can be solved for the other two. Suppose the selected coefficients are: $R_1 = R_2 = 1\Omega$. Then, by equating coefficients there are obtained $C_1 = \sqrt{2}F$ and $C_2 = 1/\sqrt{2}F$. These values provide the Butterworth filter, but it is necessary to determine its cutoff frequency and



For ω = 1 radian/sec, $R_1 = R_2 = 1\Omega$
$C_1 = \sqrt{2}F$
$C_2 = 1/\sqrt{2}F$

For f = 1000 Hz, $R_1 = R_2 = 10\ k\Omega$
$C_1 = 0.0225\ \mu F$
$C_2 = 0.01125\ \mu F$

FIGURE A-1. – Butterworth 2-pole low-pass filter.

to complete the design for a cutoff of 1 kHz. An expression for the gain magnitude can be obtained by substituting p = $j\omega$ in equation (A-9):

$$\frac{\text{Output}}{\text{Input}} = \text{Gain} = G = \frac{1}{(1 - \omega^2) + j\omega\sqrt{2}} \tag{A-10}$$

Multiplying the numerator and denominator of equation (A-10) by $(1 - \omega^2) - j\omega\sqrt{2}$ gives

$$G = \frac{1}{(\omega^4 + 1)}\left[(1 - \omega^2) - j\omega\sqrt{2}\right] \tag{A-11}$$

Taking the square root of the sum of the squares of the real and imaginary quantities inside the square brackets in equation (A-11) gives

$$|G| = \frac{(\omega^4 + 1)^{1/2}}{(\omega^4 + 1)} \tag{A-12}$$

Thus the magnitude of the gain is

$$|G| = \frac{1}{(\omega^4 + 1)^{1/2}} \tag{A-13}$$

At $\omega = 0$, equation (A-13) shows the gain $|G|$ is equal to 1, and the gain at dc is unity. In order to find the 3-dB frequency, the gain $|G|$ is set equal to -3 dB, or 0.7071 (which is $1/\sqrt{2}$). Then, equation (A-13) indicates $\omega = 1$ radian/sec. Accordingly, the 3-dB frequency of the Butterworth filter represented by equation (A-9) is $\omega = 1$ radian/sec or $f = 1/2\pi$ Hz. Because the required filter is to have a cutoff at 1 kHz, it is necessary to frequency-scale the circuit at hand (see table II in chapter 2, page 18). The appropriate multiplying factor is $1/2000\pi$; thus C_1 and C_2 are converted to values that provide a 1-kHz cutoff. In summary, the circuit values are

$$R_1 = R_2 = 1\,\Omega$$

$$C_1 = \frac{\sqrt{2}}{2000\pi}\ \mathrm{F}$$

$$C_2 = \frac{1}{(\sqrt{2})(2000\pi)}\ \mathrm{F}$$

However, these values are impractical and so it is necessary to obtain more realizable values. Suppose it is decided to multiply all resistor values by 10,000 and to divide all capacitor values by 10,000 (see table II, chapter 2). The new values are:

$$R_1 = R_2 = 10\mathrm{k}\Omega$$
$$C_1 = 0.0225\ \mu\mathrm{F}$$
$$C_2 = 0.01125\ \mu\mathrm{F}$$

The resulting circuit in figure A-1 is the same as that in figure 29 (chapter 3) and the design is complete.

Polynomial expressions are used to define other types of filters, such as Bessel and Chebyshev. The design of these filters uses the same principles as were used in example A-1, although more sophisticated analysis techniques are useful for multiple-section filters.

Q AND SECOND-ORDER BANDPASS TRANSFER FUNCTIONS

The second-order bandpass transfer function has been given in equation (A-2) as

$$T(p) = \frac{Hp}{p^2 + bp + a} \tag{A-14}$$

The term H is an arbitrary multiplying factor, and equation (A-14) may be written in the form:

$$T(p) = \frac{Hp}{p^2 + \frac{\omega_o p}{Q} + \omega_o^2} \tag{A-15}$$

so that

$$\omega_o = \sqrt{a} = \text{resonant frequency, radians per second} \quad \text{(A-16)}$$

$$Q = \frac{\sqrt{a}}{b} = \frac{\text{resonant frequency}}{\text{bandwidth}} \quad \text{(A-17)}$$

$$BW = b = \text{bandwidth, radians per second} \quad \text{(A-18)}$$

$$A_o = \frac{HQ}{\omega_o} = \text{gain at the center frequency} \quad \text{(A-19)}$$

It can be shown that equation (A-15) represents a curve of the shape shown in figure A-2, when T(p) is plotted against frequency; the maximum value occurs at ω_o and is HQ/ω_o, and therefore peaking is increased when Q is increased. The coefficient b in equation (A-14) is the bandwidth at the 3-dB points and, by comparing coefficients in equations (A-14) and (A-15) it is evident that $b = \omega_o/Q$.

EXAMPLE A-2: The filter in figure 62 (chapter 5) and example 4 (chapter 5) was designed using design equations (53) through (56) (chapter 5) to have f_o = 1 kHz, Q = 30, and A_o = -1. Write a transfer function for this filter in the form of equation (A-14) without using equation (A-15).

SOLUTION: The filter is reproduced in figure A-3. By current-summing techniques similar to those used in example A-1 the transfer function is found to be

$$\frac{e_2}{e_1} = \frac{-\frac{1}{R_1C}\cdot p}{p^2 + \frac{2}{R_3C}\cdot p + \frac{1}{R_3C^2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \quad \text{(A-20)}$$

Putting in values for the various resistors and capacitors, equation (A-20) reduces to

$$\frac{e_2}{e_1} = \frac{-209.4p}{p^2 + 209.4p + (39.48)(10^6)} \quad \text{(A-21)}$$

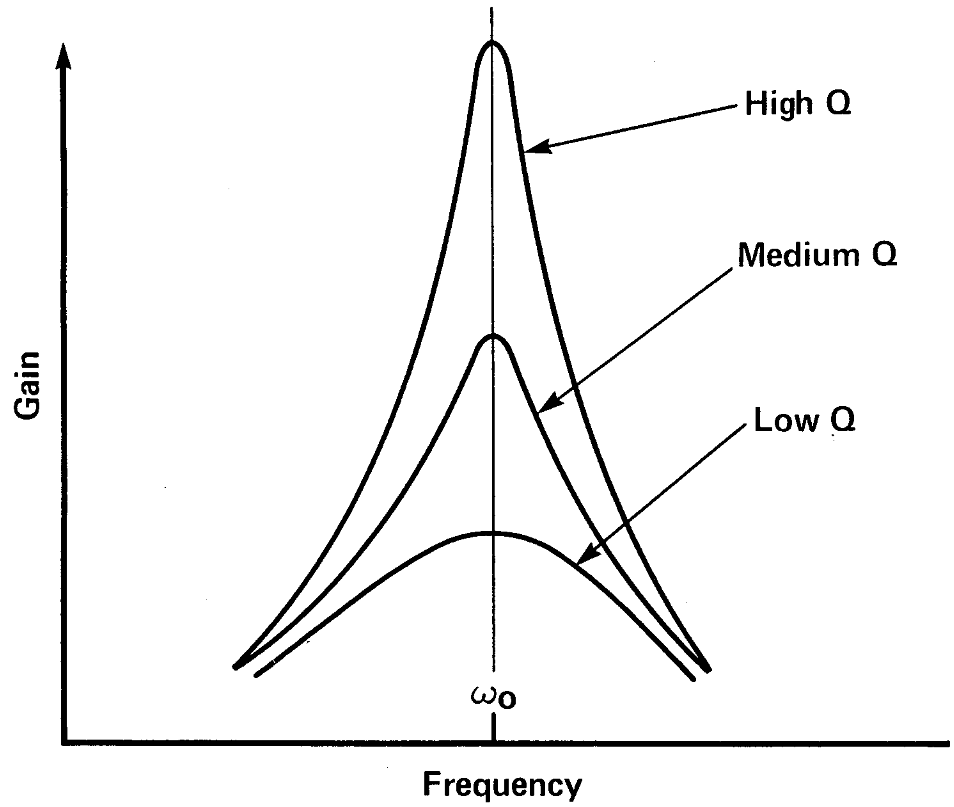
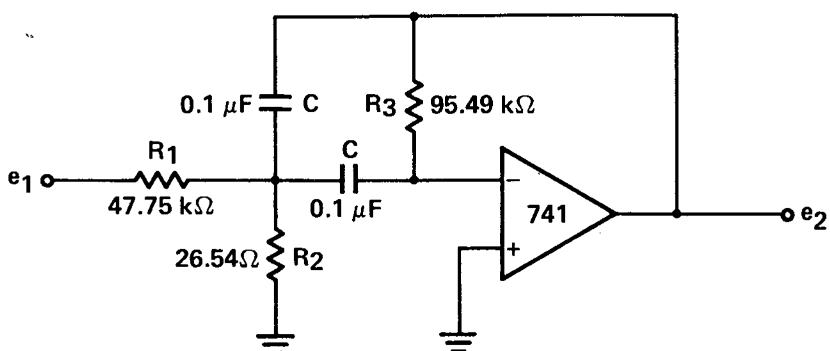


FIGURE A-2. – Effect of varying Q.



$Q = 30$
$f_o = 1000$ Hz
$A_o = -1$
$B = 33.3$ Hz

FIGURE A-3. – Bandpass filter with $f_o = 1000$ Hz, $Q = 30$, and $A_o = -1$.

so that in equation (A-14), $H = -209.4$, $a = (39.48)(10^6)$ and $b = 209.4$ As a check:

Equation (A-16) gives $\omega_o = (6.283)(10^3)$ radians per second

and $f_o = \omega_o/2\pi = 1$ kHz

Equation (A-17) gives $Q = 30$

and H = coefficient of p in numerator = -209.4

Equation (A-19) gives $A_o = -1$

POLES AND ZEROS

It has been shown that the performance of filters can be described by quadratic expressions such as equations (A-1), (A-2), and (A-3). If appropriate numbers are inserted into equation (A-1), the transfer function for a Butterworth low-pass filter is obtained:

$$T(p) = \frac{1}{p^2 + \sqrt{2}p + 1} \qquad \text{(A-22)}$$

The expression can be written in factored form:

$$T(p) = \frac{1}{(p - p_1)(p - p_2)} \qquad \text{(A-23)}$$

where

$$p_1 = -\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} \qquad \text{(A-24)}$$

and

$$p_2 = -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} \qquad \text{(A-25)}$$

The terms p_1 and p_2 are the roots of the denominator of equation (A-22) and are called the *poles*. When roots of the numerator exist, they are called the *zeros*. Equation (A-22) has two poles and is therefore the transfer function of a two-pole filter. The terms p_1 and p_2 can be plotted on a pole-zero diagram as in figure A-4. Poles are represented by crosses; zeros, when they exist, are represented by small circles.

Pole-zero plots are useful for two main reasons. First, the location of the poles and zeros on the pole-zero diagram completely characterizes a circuit, except for a scale factor. Secondly, having described a desired circuit in terms of its poles and zeros, it is possible to design or synthesize a circuit with poles and zeros at the specified points; in other words, a desired circuit can be realized from pole-zero considerations.

A great deal of information is to be gleaned from pole-zero plots; see, for example, references 2, 5, and 9.

SENSITIVITY

When applied to active filters, sensitivity is a measure of the change in some particular performance characteristic caused by a change in the value of one or more components in the filter. For example, if a one-percent change in a resistor value produced a 10-percent change in the Q of a filter, the sensitivity of Q to that resistor would probably be considered high. On the other hand, if a one-percent change in the resistor R produced a 0.1 percent change in Q, the sensitivity would probably be considered low. One way of expressing sensitivity in mathematical form would be

$$S_R^Q = \frac{dQ/Q}{dR/R} \tag{A-26}$$

Strictly speaking, equation (A-26) applies only to infinitesimally small changes, but, in practice, it can be applied to changes of the order of 10 percent. For a one-percent change in R and a 10-percent change in Q, S_R^Q would be (10/100)/(1/100) = 10. For a one-percent change in R and a 0.1-percent change in Q it would be (0.1/100)/(1/100) = 0.1

The expression given in equation (A-26) is an example of *classical sensitivity*. Also in common use is *root sensitivity* in which the effect considered is the change in *position* of the poles and zeros of the filter function. Sensitivity may be defined in a number of other ways, and a variety of other symbols may be used, but it is usual to use a superscript to denote the performance characteristic and a subscript to denote the element producing the change as was done in equation (A-26).

The subject of sensitivity applied to active filters is covered extensively in references 1, 2, 8, 9, 13, and 14.

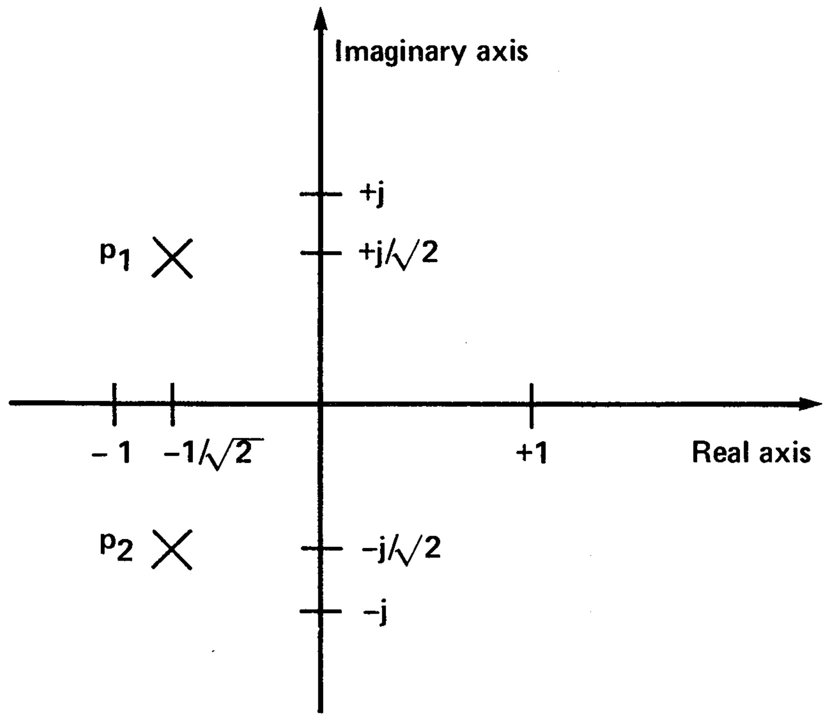
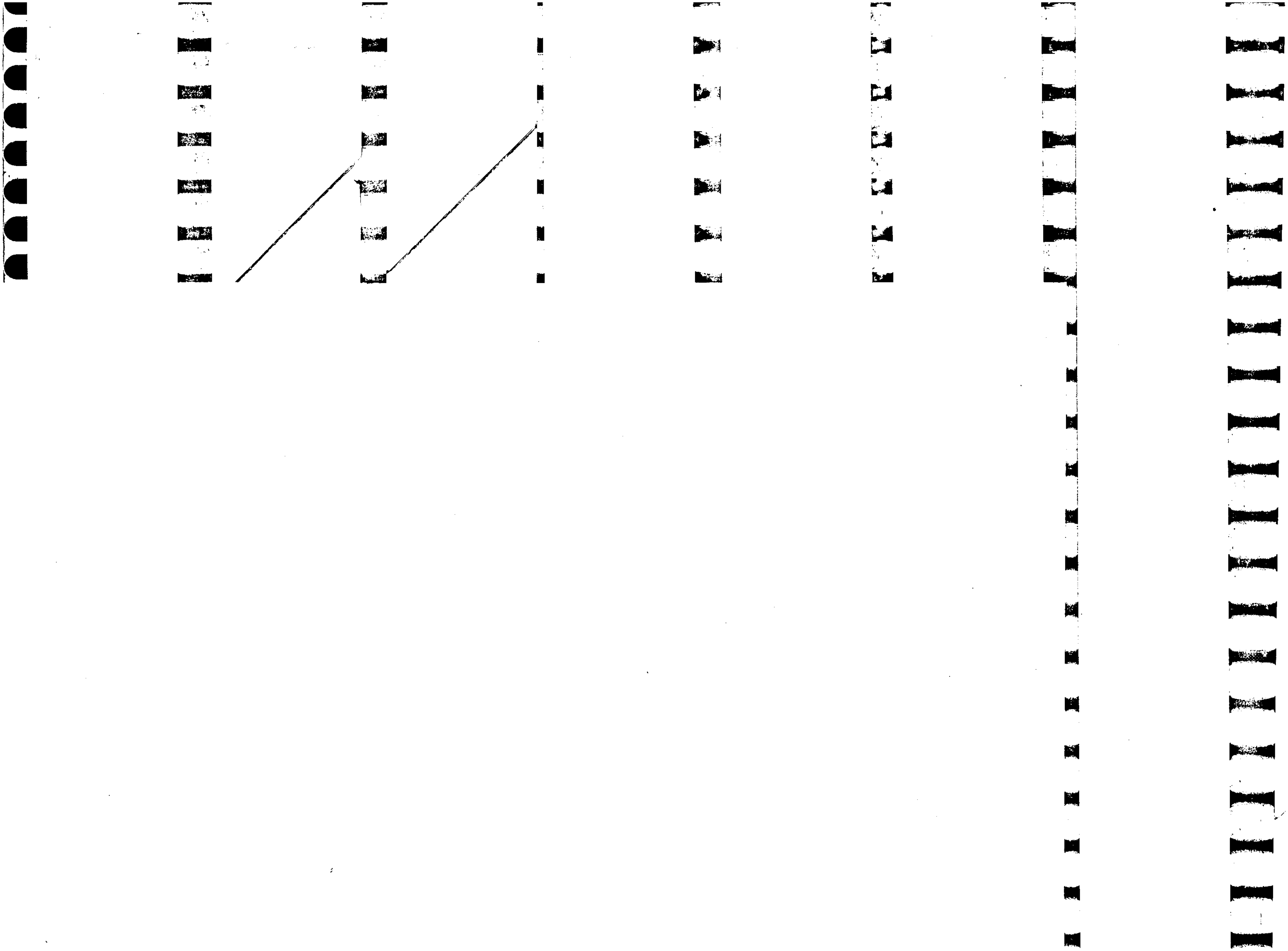


FIGURE A-4. – Pole-zero diagram for 2-pole, low-pass Butterworth filter.



References

There is a very large amount of literature on the subject of RC active circuits. For example, references 2, 8, 9, and 16 contain bibliographies with over 1000 journal articles, reports, and books on the subject. The references listed below include work done at or under grant to NASA/Ames Research Center and also include other useful articles and books that are specifically mentioned in this handbook.

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